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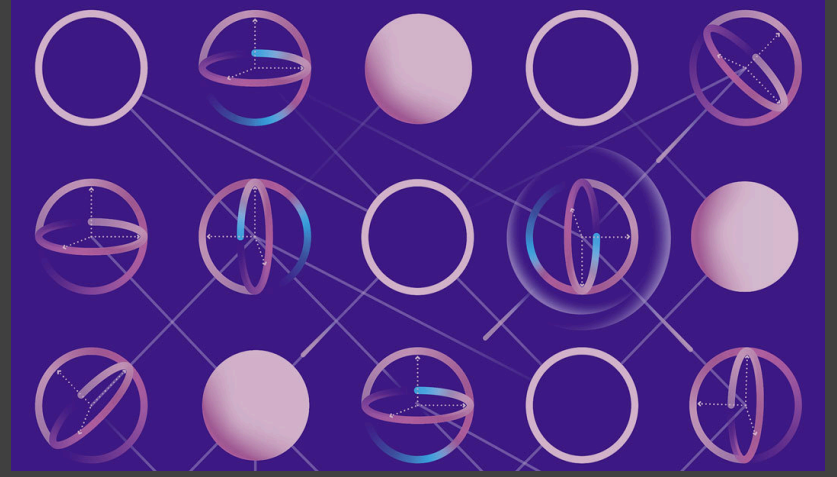
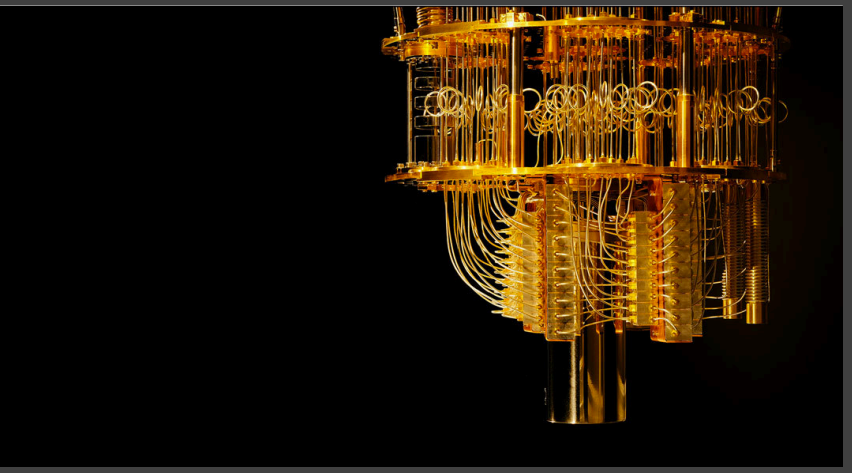
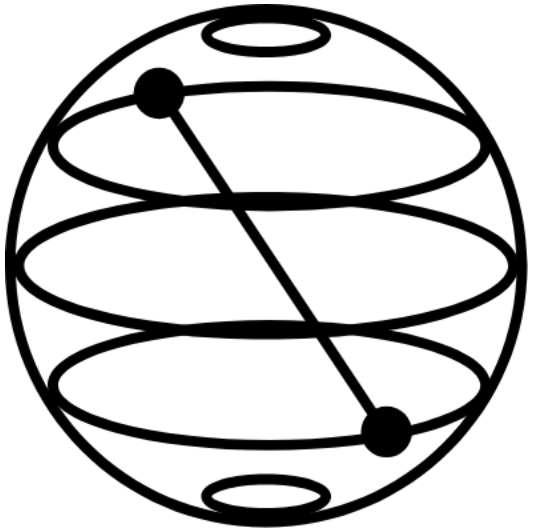


IBM





Quantum Computing principles

IBM Client Center Montpellier

JM Torres | torresjm@fr.ibm.com

27 mars – 2 avril mars 2020

Agenda:

 Turing machine and other computational models

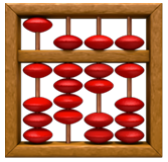
 reversible computation

 algorithmic complexity

 Definition of a quantum bit

 Controlling a qubit

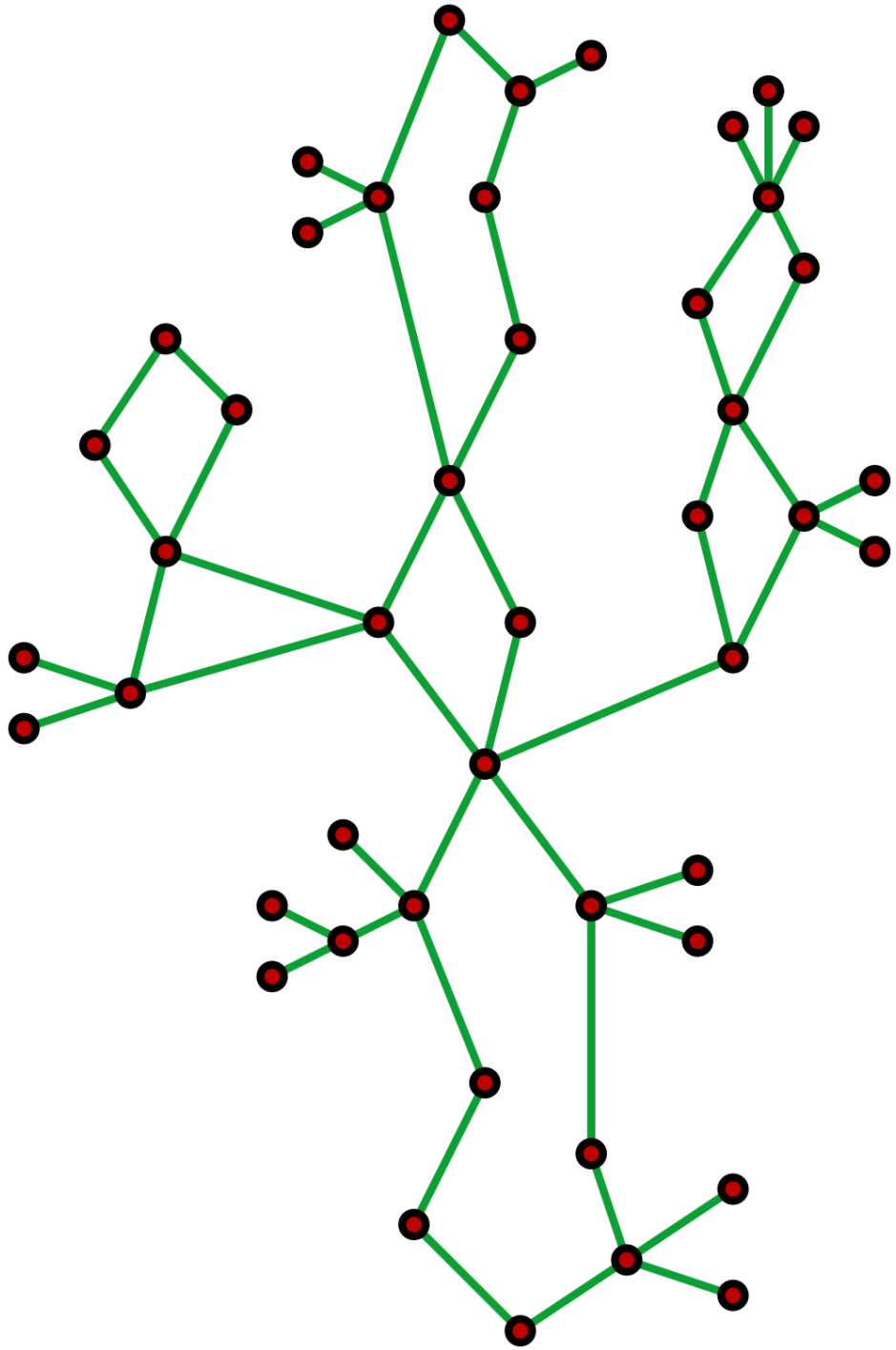
 Working with many qubits



Turing machine and other computational models

a graph

How can we build a planar graph of order n with minimal diameter (and what is that value) ?



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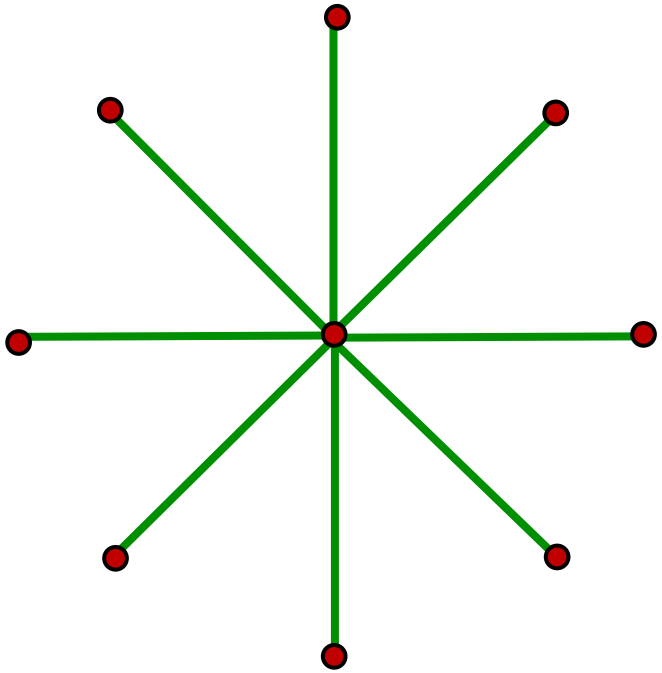
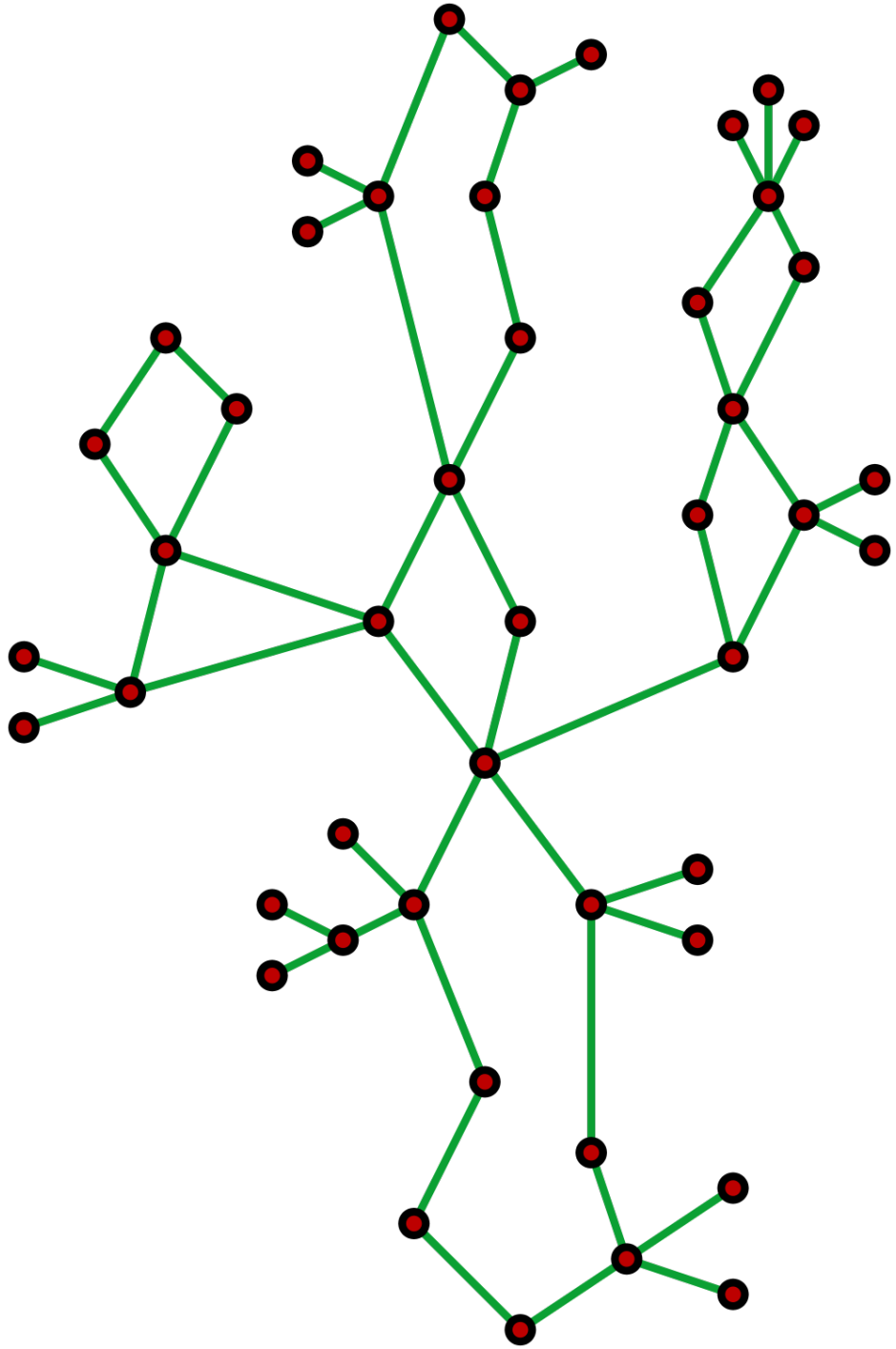
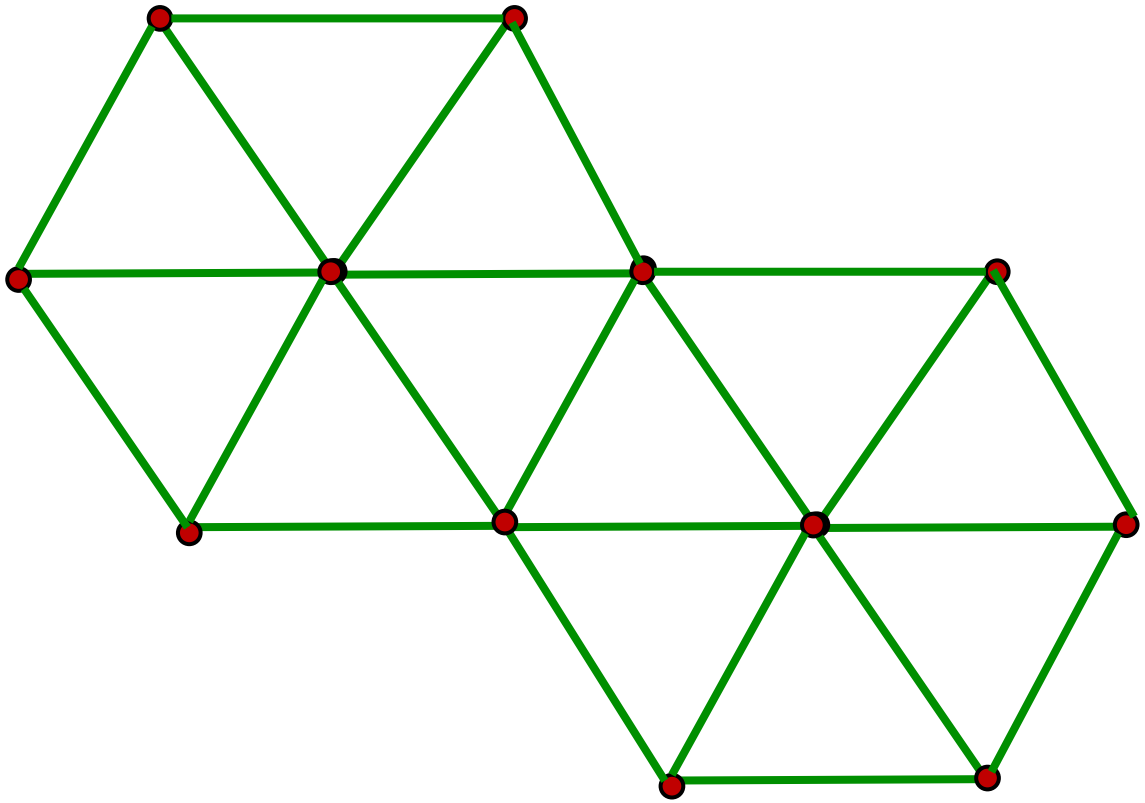


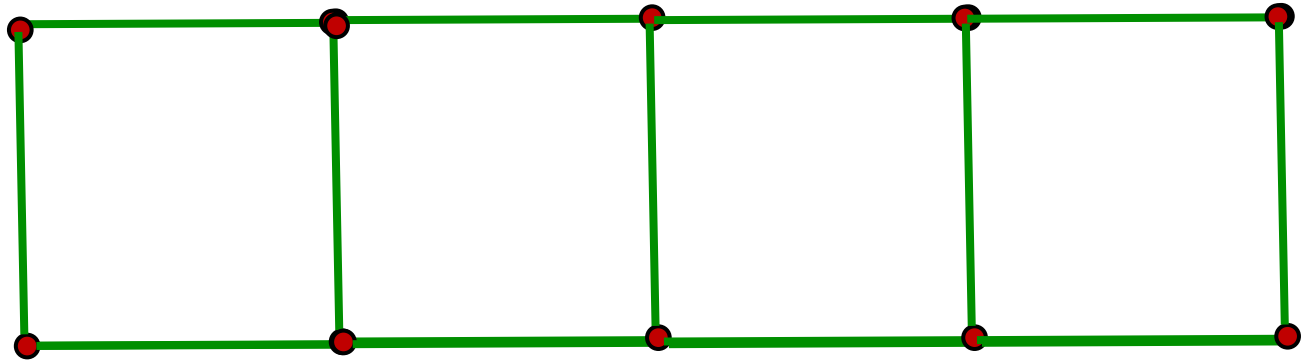
How can we build a planar graph of order *n, all vertices having max order 6,* with minimal diameter (and what is that value) ?

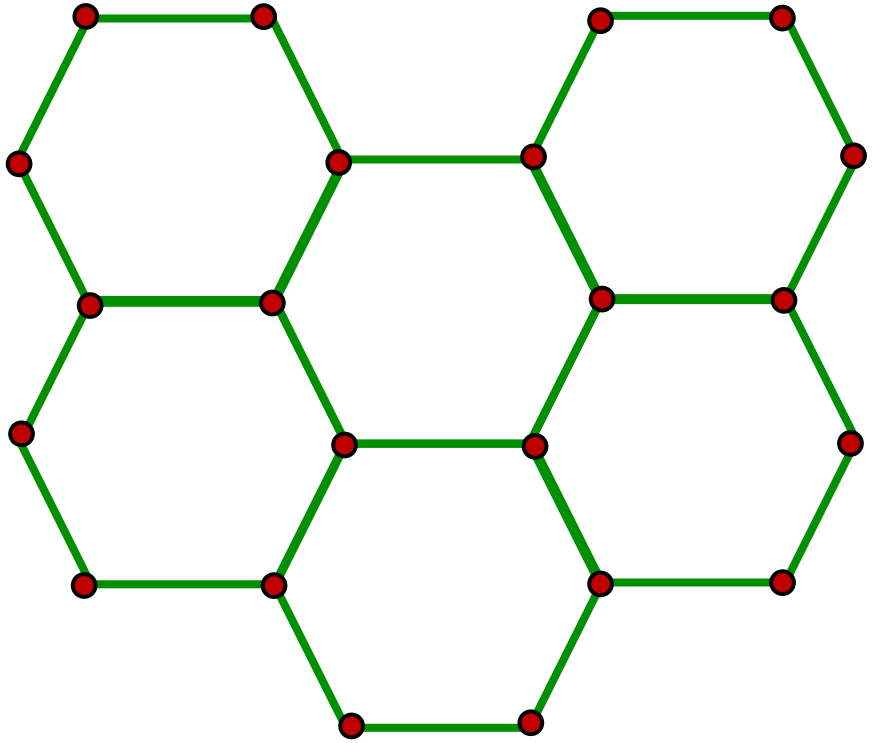


this ? …

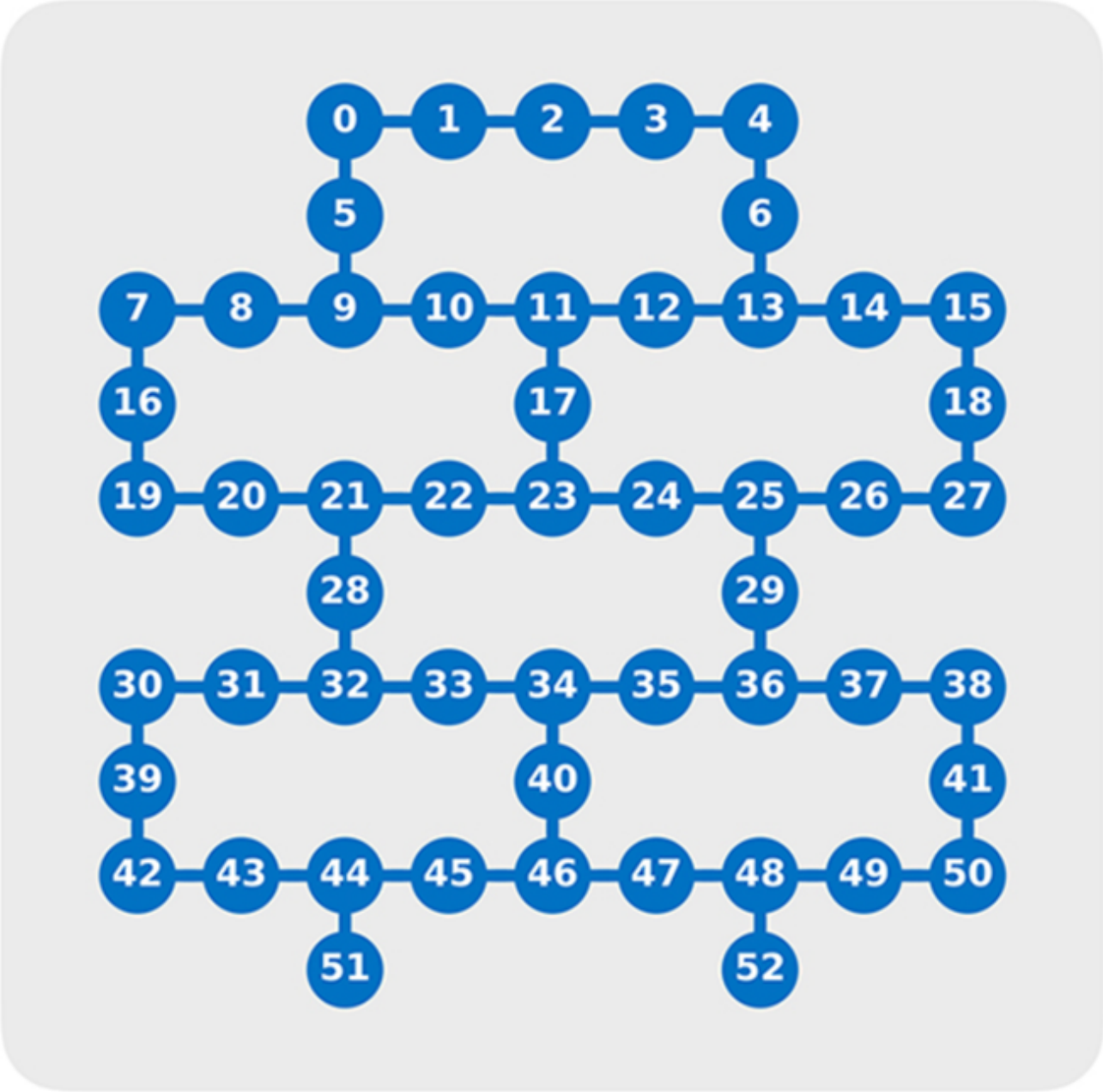


with order 3 vertices …



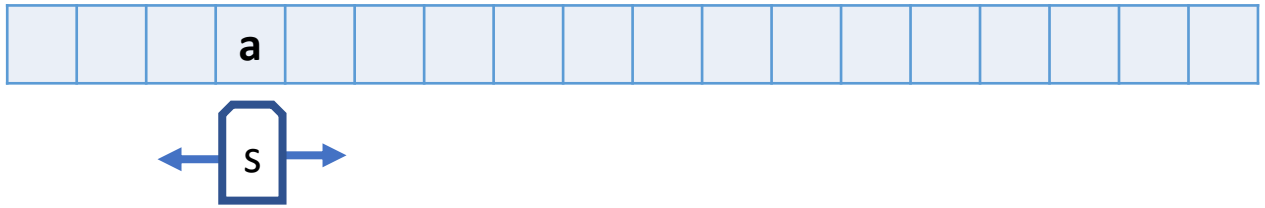


IBM 53 qubits (“Rochester”)



turing machine, circuits

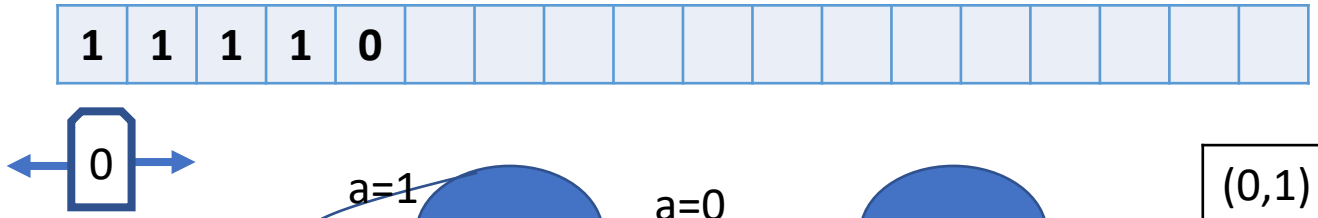
Turing Machine



- infinite tape strip
- read/write head
- a state set : $s_0,s_1,s_2,..$
- alphabet of symbols: $a_0,a_1,a_2,…$
- transition table = list of :

(current state; symbol) ➜ (new state, new symbol, move: ➡/⬅/🛑)

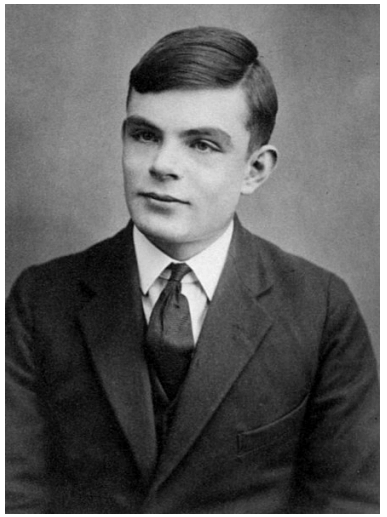
For example the following machine increments a number *n* by one unit (*n* ➜ *n+1) :*



(0,1) ➜ (0,1, ➡)
(0,0) ➜ (1,1, 🛑)

Church-Turing thesis

Church-Turing thesis : **any computation can be simulated by a Turing Machine**

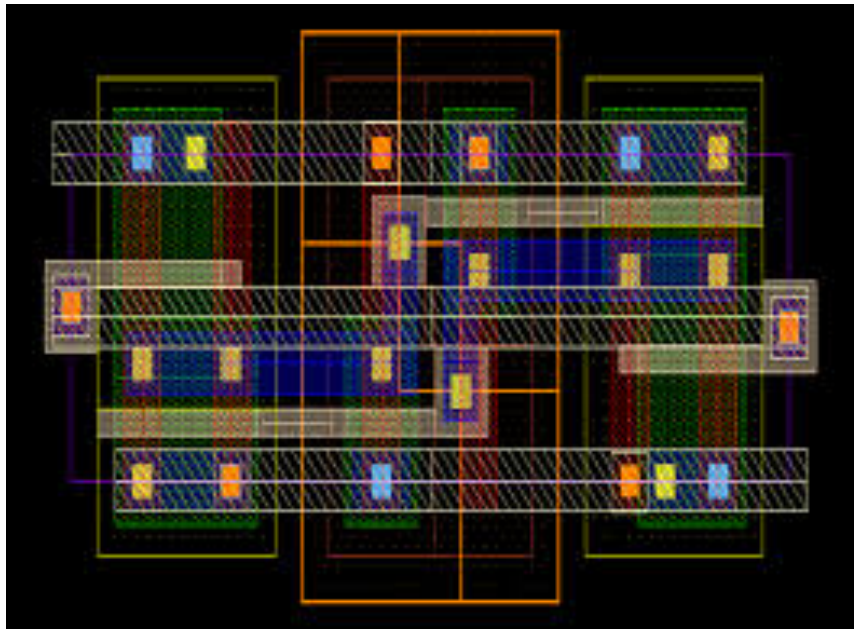


1936 : Alan Turing states there exists Universal Turing Machines (can simulate any Turing Machine)

2002 : Yurii Roghozin proves an Universal Turing Machine can be build using only 4 states, 9 symbols and 24 transitions.

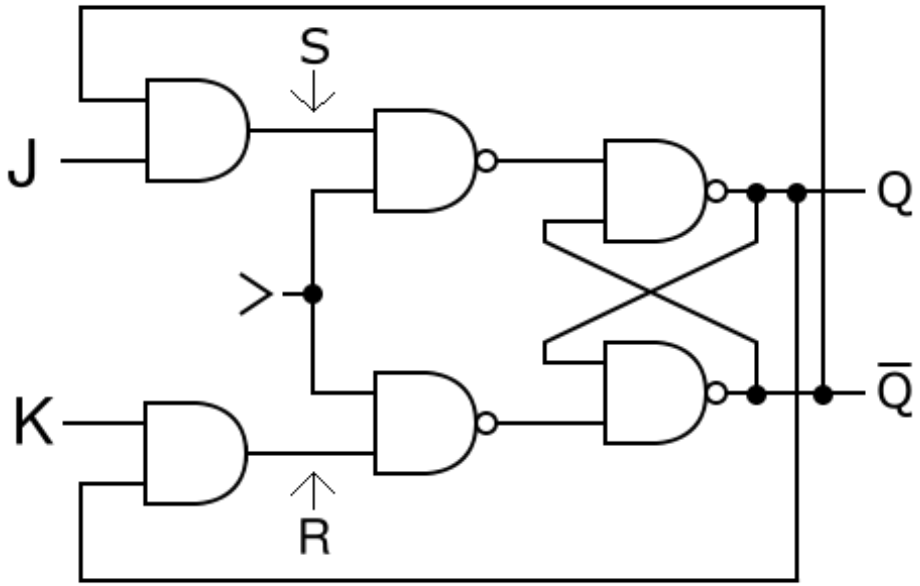
but… computation times are not practical, and the tape can be very long…

circuit model



← this is a bit

Logique Booléenne



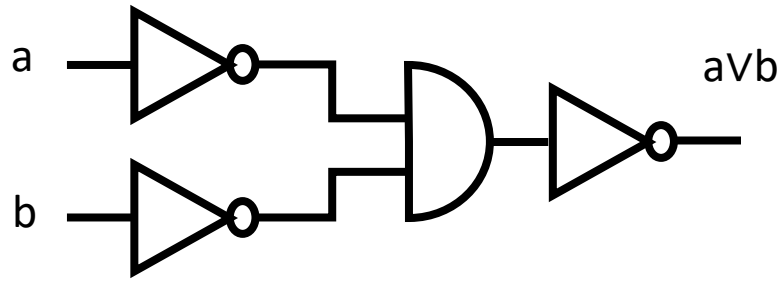
← this is a bit

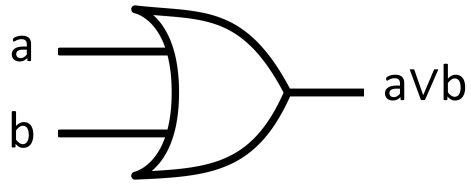
{NOT, AND} : is a universal gate set

universality

Morgan theorem : OR gate can be build from AND and NOT :

$$a \vee b = \neg(\neg a \wedge \neg b)$$





a	b	¬a	¬b	¬a∧¬b	¬(¬a∧¬b)
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

{NOT, AND} : is a universal gate set

Any logical function can be calculated with NOT and AND gates (and OR) :

universality

let f : n+1 bits → {0,1}

we can define : $f_0(X_1,X_2,\ldots,X_n) = f(\mathbf{0},X_1,X_2,\ldots,X_n)$

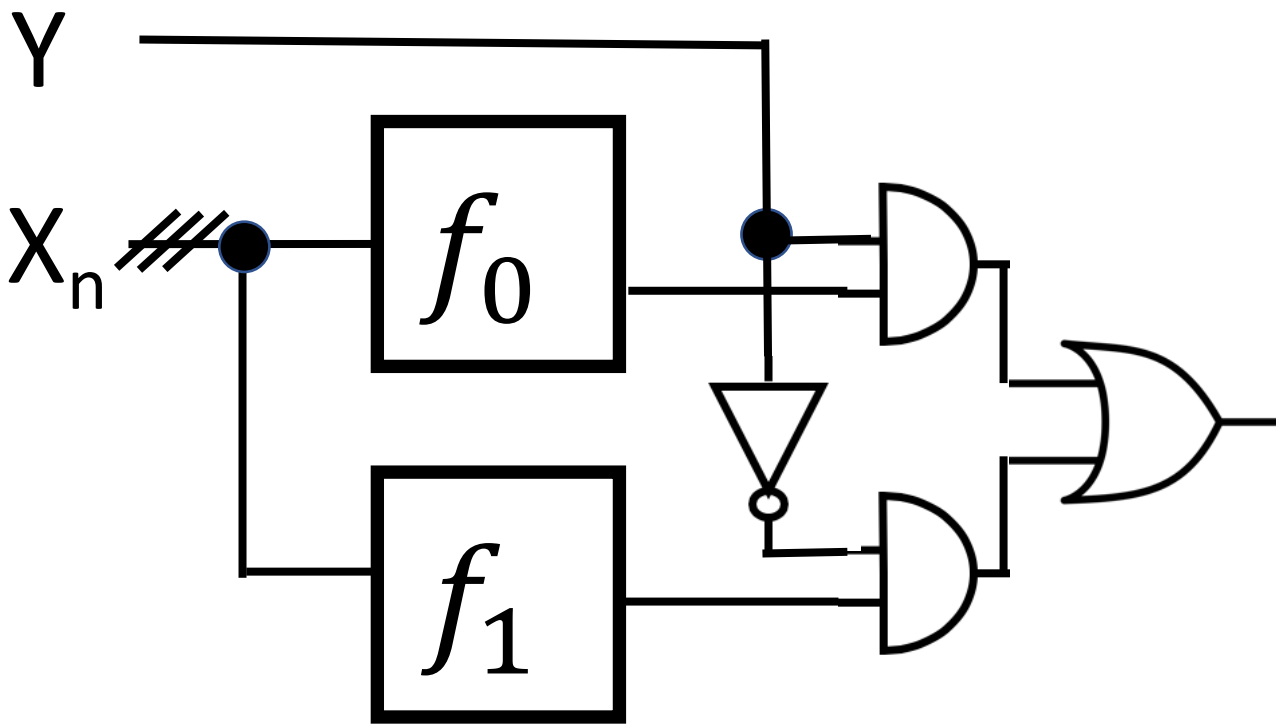
and. $f_1(X_1,X_2,\ldots,X_n) = f(\mathbf{1},X_1,X_2,\ldots,X_n)$

if f_0 & f_1 can be calculated with AND and NOT, then f can be as well :

$$f(Y,X_1,X_2,\ldots,X_n) = (\neg Y \wedge f_0) \vee (Y \wedge f_1)$$

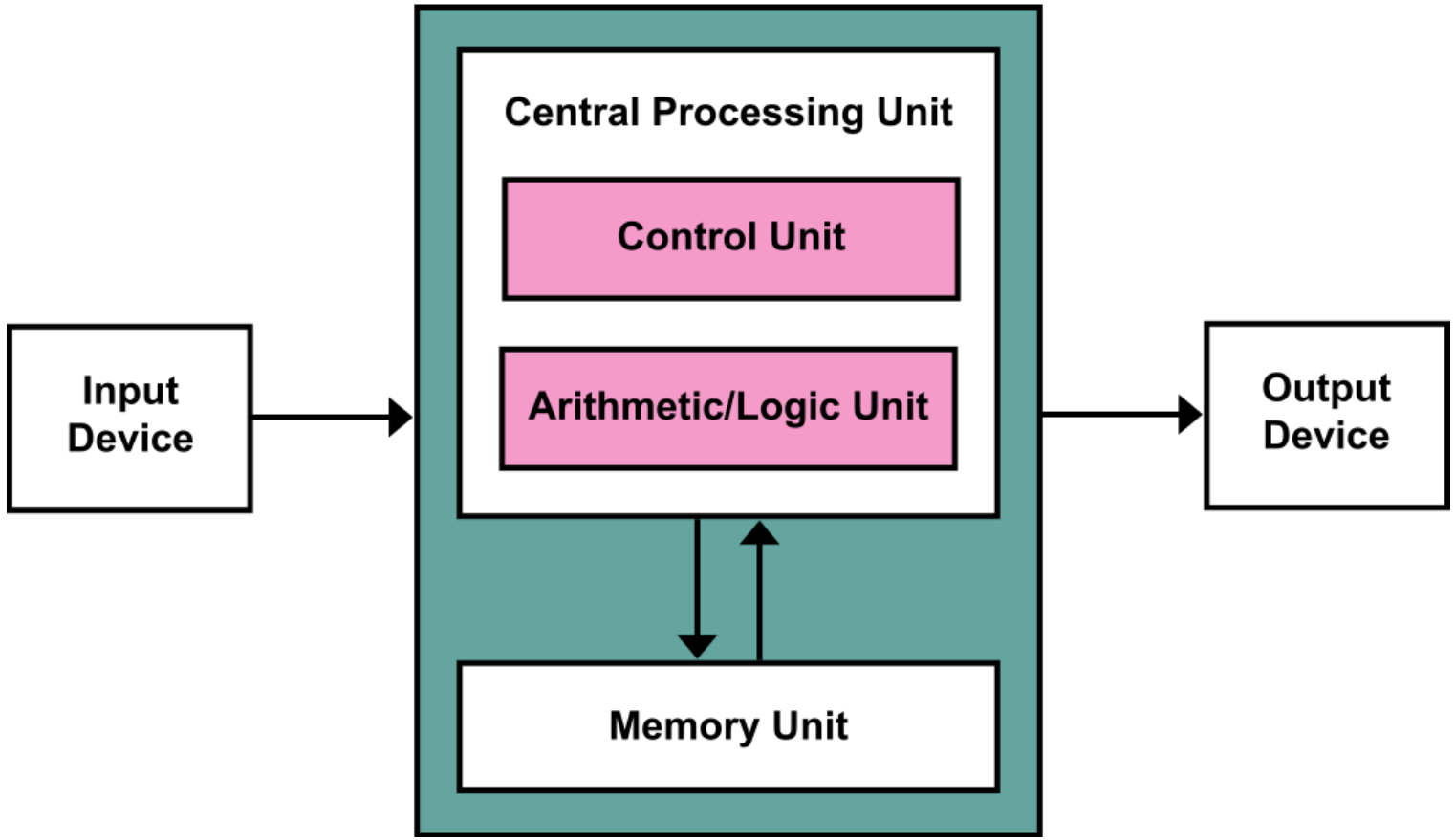
But, the gate count increases exponentially (size and power consumption as well)

2^n

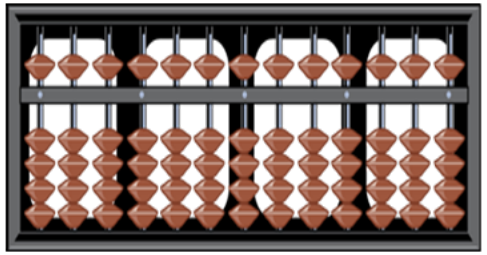


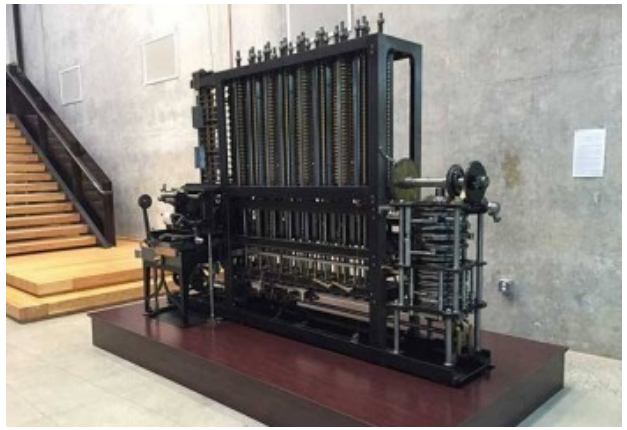
“classical” computers

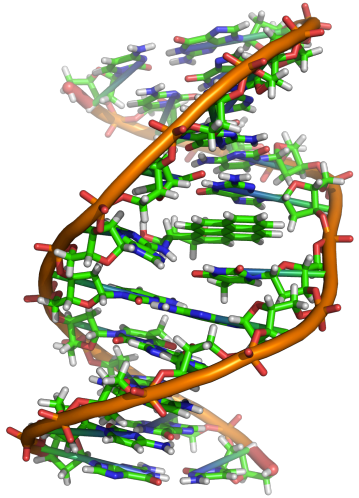
Von Neuman architecture (ALU+Memory) can work-around the circuit size problem, but the computation time issue is not solved :

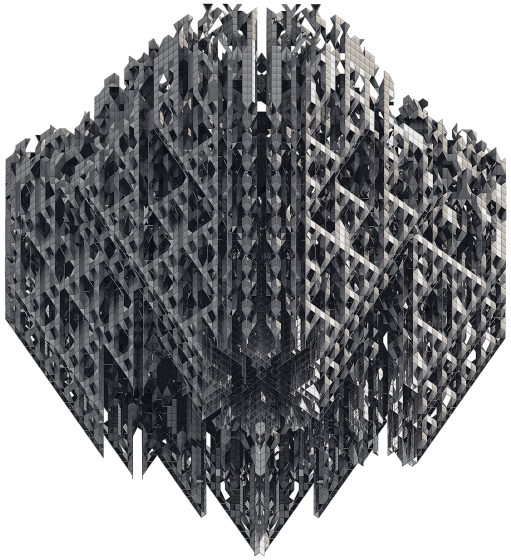


Other computational models







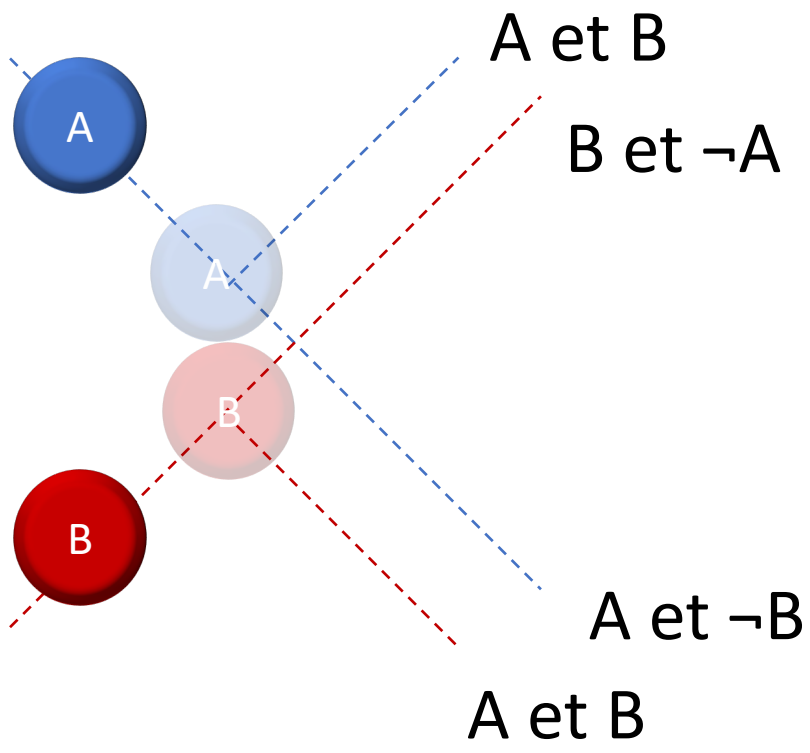


Q

reversible computation

reversible computation

Reversible mechanical computer : « billard balls Machine » (BBM)
Edward Fredkin & Tommaso Toffoli - 1982



http://jameslin.name/bball/

Reversible mechanical computer : « billard balls Machine » (BBM)
Edward Fredkin & Tommaso Toffoli - 1982

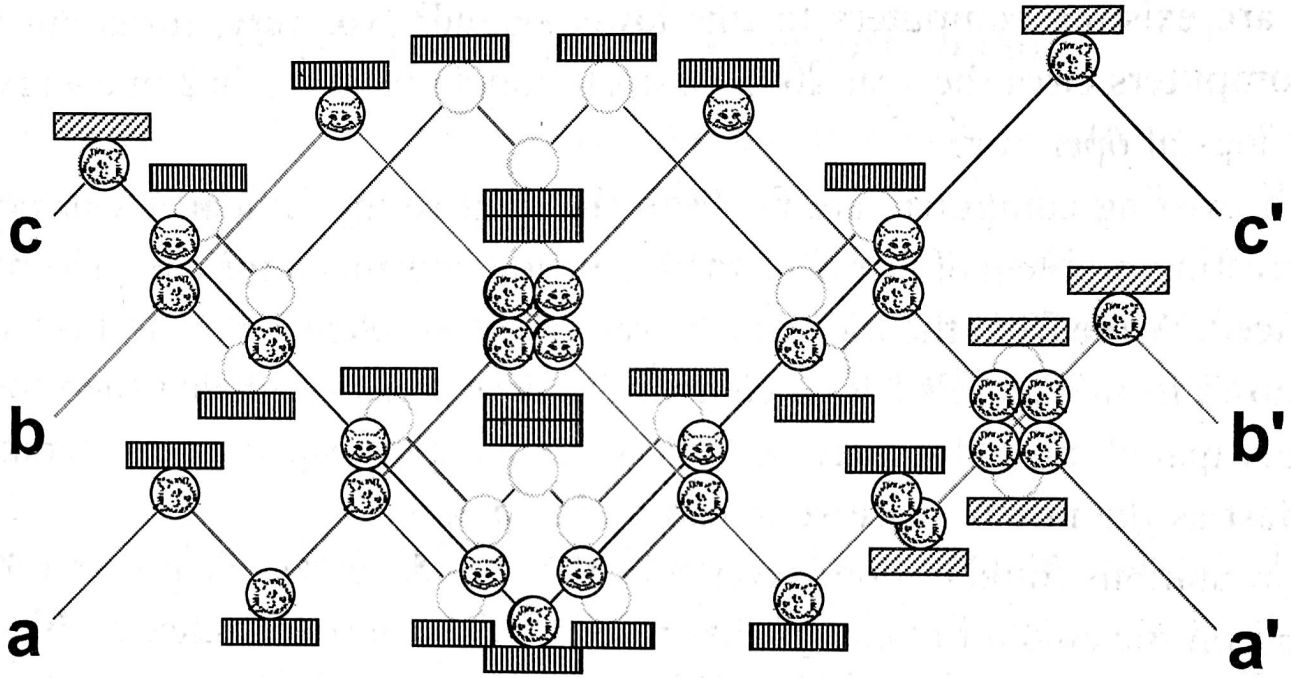


Figure 3.14. A simple billiard ball computer, with three input bits and three output bits, shown entering on the left and leaving on the right, respectively. The presence or absence of a billiard ball indicates a 1 or a 0, respectively. Empty circles illustrate potential paths due to collisions. This particular computer implements the Fredkin classical reversible logic gate, discussed in the text.

Michael Nielsen and Isaac Chuang (2000). *Quantum Computation and Quantum Information*. Cambridge: Cambridge University Press. (ISBN 0-521-63503-9).

🏭 Computation and energy:

Landauer Principle (first form) : when a bit is erased in a computer, the value of energy given to its environment is at minimum : kTLog2

kTLog2

Landauer Principle (second form) : when a computer erases a bit of information, its environment entropy raises by at least kTLog2 :

$$3.10^{-21}\ J$$

Current computers operate at 500 times this limit

Reversible computation (of which Quantum Computing) is not constrained by this limit.

Rolf Landauer. Irreversibility and heat generation in the computing process. IBM Journal of Research and Development, 5:183, 1961.

algorithmic complexity

algorithmic complexity

Information technology complexity : f(espace, temps)

For a given problem P, looking at time complexity is studying how the number of instructions changes to solve this problem as a function of a parameter *n* which represents the size of this problem.
Let C(*n*) be that number of instructions, we look for an evaluation of C(*n*) when $n \longrightarrow \infty$

One way to do this is to compare C(n+1) with to C(n)

Variation de C(n)	Complexité	Notation « grand O »
C(n+1) = C(n)	constant	$\mathcal{O}(1)$
C(n+1) = C(n) + ε (par exemple C(n+n) = C(n) + 1)	Logarithmic	$\mathcal{O}(\log(n))$
C(n+1) = C(n) + k	Linear (kn)	$\mathcal{O}(n)$
C(n+1) = C(n) + n	Polynomial	$\mathcal{O}(n^2)$
C(n+1) = C(n) + n^k	Polynomial	$\mathcal{O}(n^{k+1})$
C(n+1) = C(n)*2	Exponential	$\mathcal{O}(2^n)$
C(n+1) = C(n)*n	Exponential	$\mathcal{O}(n!)$
…		

Exemple: recherche dans une liste non ordonnée:

```
# is  « y » in the list ?
list = [u,g,r,e,x,d,t,l,o,p,q,m,h,z,a,v,n,i,k,f,b,c,s,d,w…];

for (i = 0; i < len(list); i++) {
        if (liste[i] == y) {
                found  = 1;
                print(« found at position », i);
                break;
        }
}
if (found == 0) {
        print(y, « not found in list »);
}

long = len(list);
for (i = 0; i < long; i++) {
        if (list[i] == y) {
                found  = 1;
                print(« found at position », i);
                break;
        }
}
if (found == 0) {
        print(y, « not found in list »);
}
```

Dealing cards to *n* players :

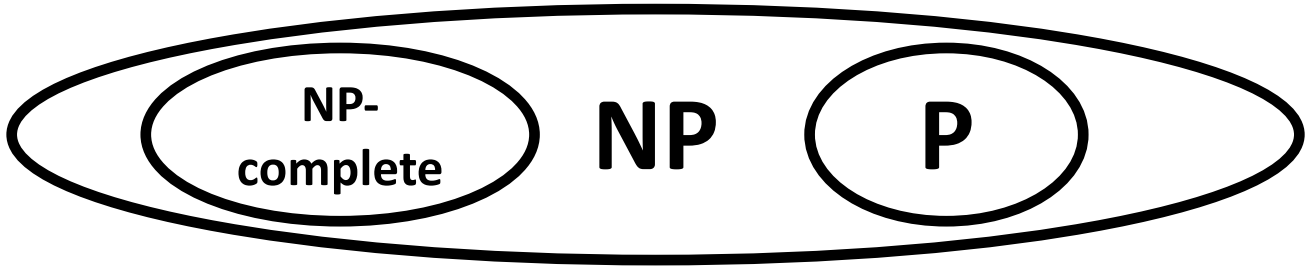
complexity examples

Distribution process	$\mathcal{O}$	example
Deal 52 cards, ∀ the players quantity.	$\mathcal{O}(1)$	Fetch 10 first lines of a data list.
Deal one card for the group of n/2 players, deal another card to the half group of the remaining players … and son on… (128 players will be dealt 8 cards)	$\mathcal{O}(\log_2(n))$	Search for one value in a sorted list.
Deal one card to each player	$\mathcal{O}(n)$	Search for one value from a non sorted list
Deal *n* cards to n^{th} player (or deal n cards to each player)	$\mathcal{O}(n^2)$	Process all pairs of a set.
Deal 1 card to first player, 2 cards to 2^{nd} player, 4 cards to 3^{rd}, 8 to 4^{th}, and so on : to each player deal twice the number of cards compared to previous player.	$\mathcal{O}(2^n)$	When we need to compute on all subsets of a set.

P vs NP

P & NP are two sets of decision problems :

- a problem *p* can be solved efficiently if there is an algorithm of polynomial order solving *p*. Then *p* is in P.
- there are problems for which we know of an exponential algorithm that can solve them, but we do not know if there is a polynomial algorithm to solve them. However we know a polynomial algorithm to verify a solution. Those are NP problems.
- It is not known if P = NP.
- We know that there is a class of NP problems that collect all the difficulty of any NP problem. These are called NP-complete.
- for any one of the NP-complete problem :
  - If we can prove no polynomial algorithm can solve it : then P ≠ NP
  - if one polynomial algorithm is found, then P = NP.



« 3SAT » statisfiability problem :

Satisfiability: For a logical expression F using *n* booleans B_i , we ask if there exists a combination of values for B_i so that F = True ?

3SAT has the following expression (for example) :

$F = (B_0+B_3+B_4)*(\neg B_1+B_5+\neg B_7)*(B_2+B_3+\neg B_4)*\ldots*(\neg B_0+B_1+B_6)$

Solving the 3SAT problem requires the evaluation of F for all possible combination of the values of the n booleans.

It is a NP-complete problem.

Note that 2SAT is in class P:
$(F = (B_0+B_3)*(\neg B_1+B2)*(B_2+B_3)*\ldots*(\neg B_0+B_4))$

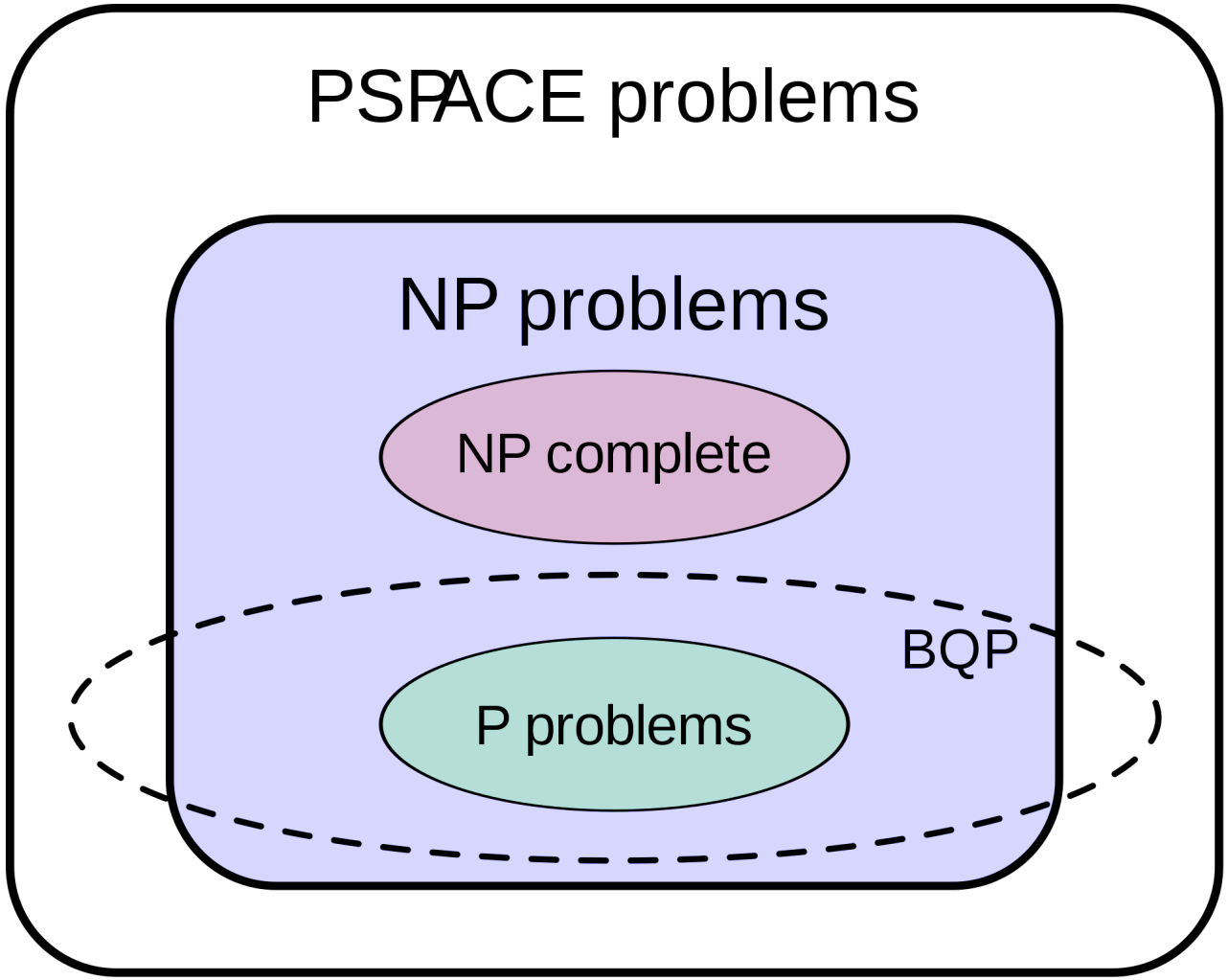
NP complete examples

CLIQUE (graph theory): a CLIQUE in a non oriented graph is a set of vertices that are all connected together. The CLIQUE size is the number of vertices it contains. For a graph G, is there a CLIQUE of size N ?
SUM of SUBSETS (arithmetic) : given a finite set E of integer, and an integer s : is there a subset of E for which the sum of its elements adds up to s ?
MAXCUT (graph theory) Given a graph, with weighted edges, a cut is a subset of vertices. The cut weight is the summation of weight for edges having one end inside the subset and the other outside. A cut is called a maxcut if it has the maximum possible weight of all cuts.
KNAP SACK(combinatory optimization) : given a set of objects for which we know wheight and value : what subset of object should I put in the KnapSack with the maximum value under a maximum weight value.

https://fr.wikipedia.org/wiki/Liste_de_probl%C3%A8mes_NP-complets

BQP is the category of algorithms that can be solved efficiently with a quantum algorithm :

Assumed relation between BQP and other complexity classes



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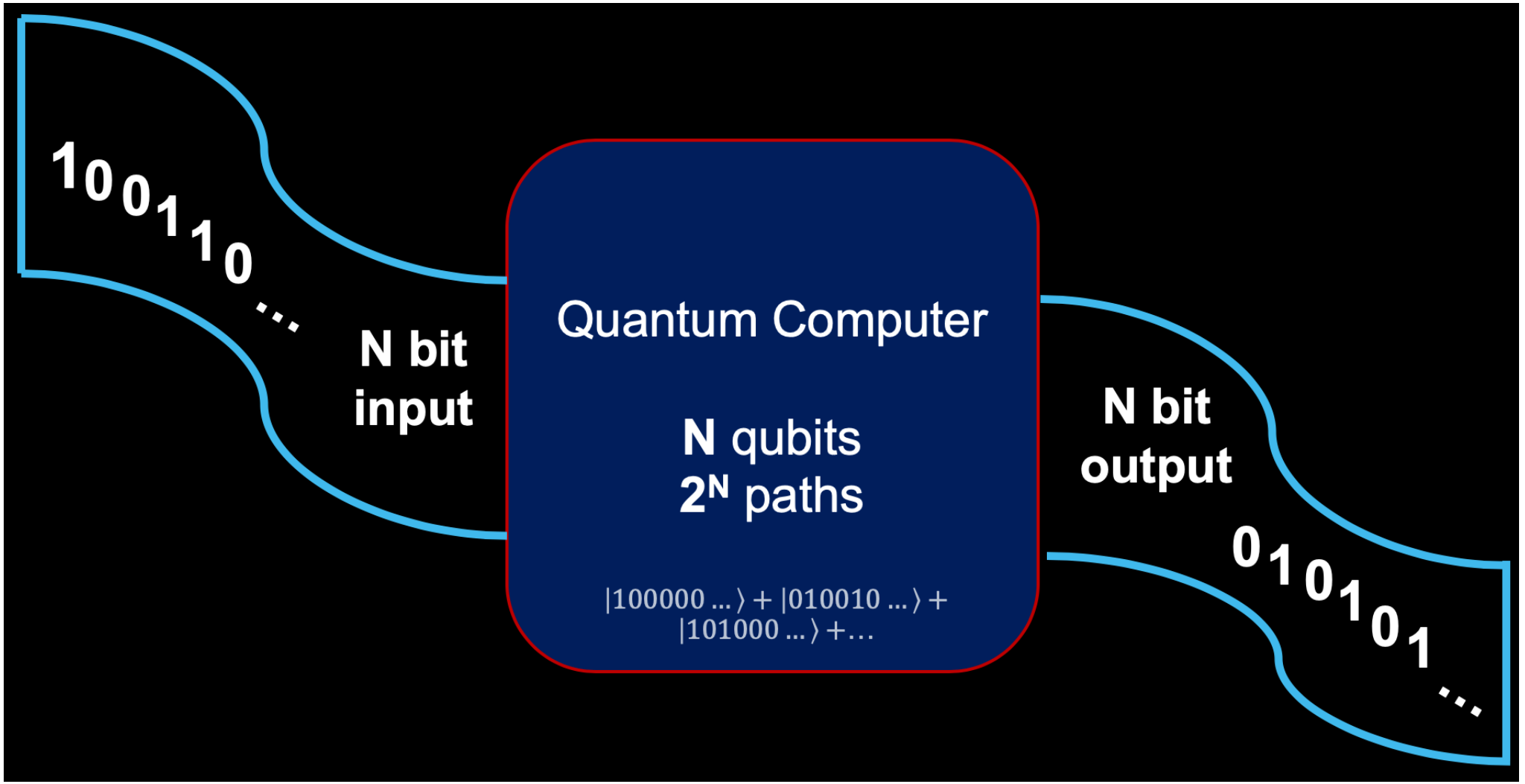
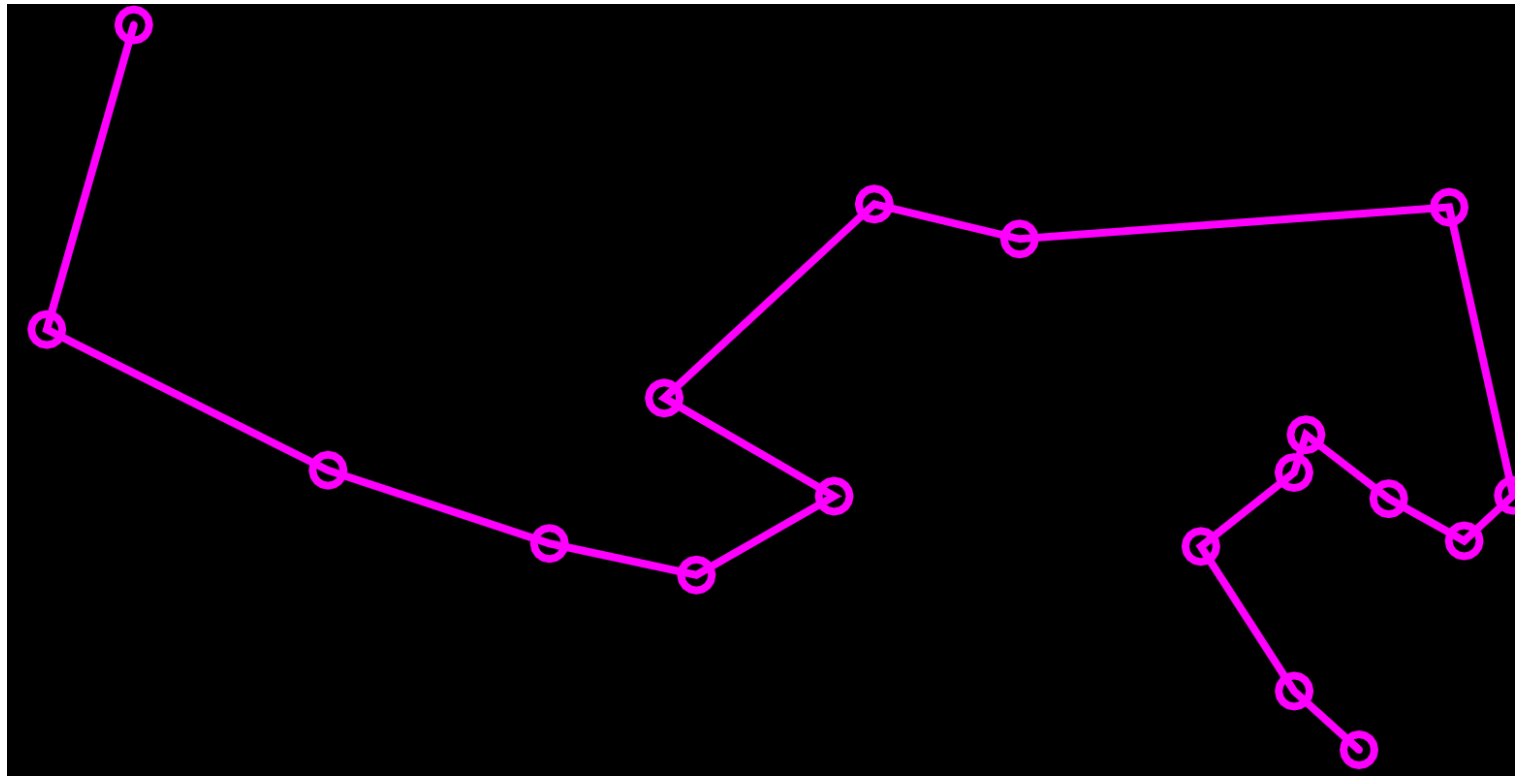

1 0 0 1 1 0 …
N bit input
Quantum Computer
N qubits
2^N paths
|100000 …⟩ + |010010 …⟩ +
|101000 …⟩ +…
N bit output
0 1 0 1 0 1 …

more …

🚗 démo : Voyageur de commerce (TSP)

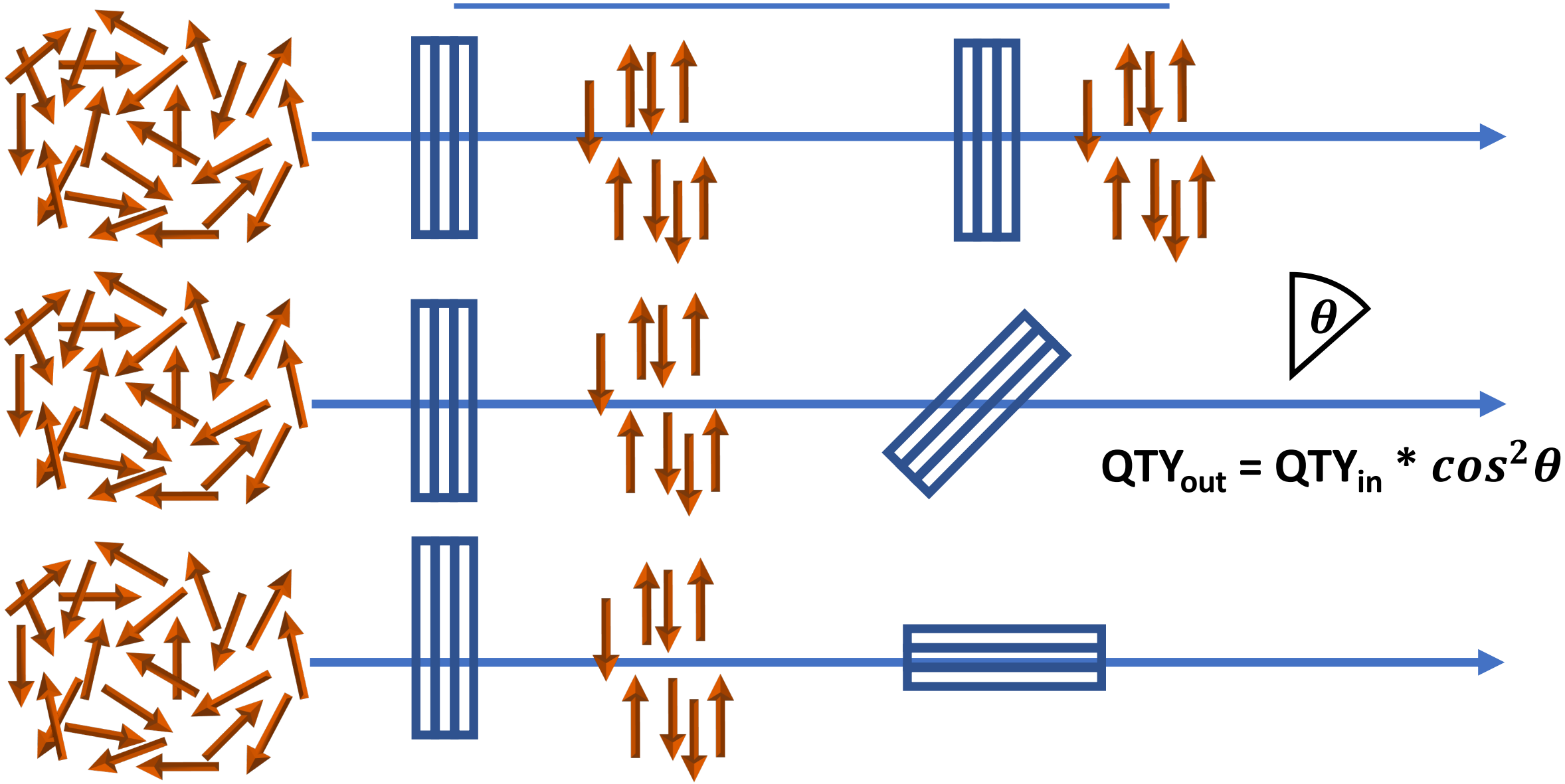
Démo “Voyageur de Commerce” (TSP)



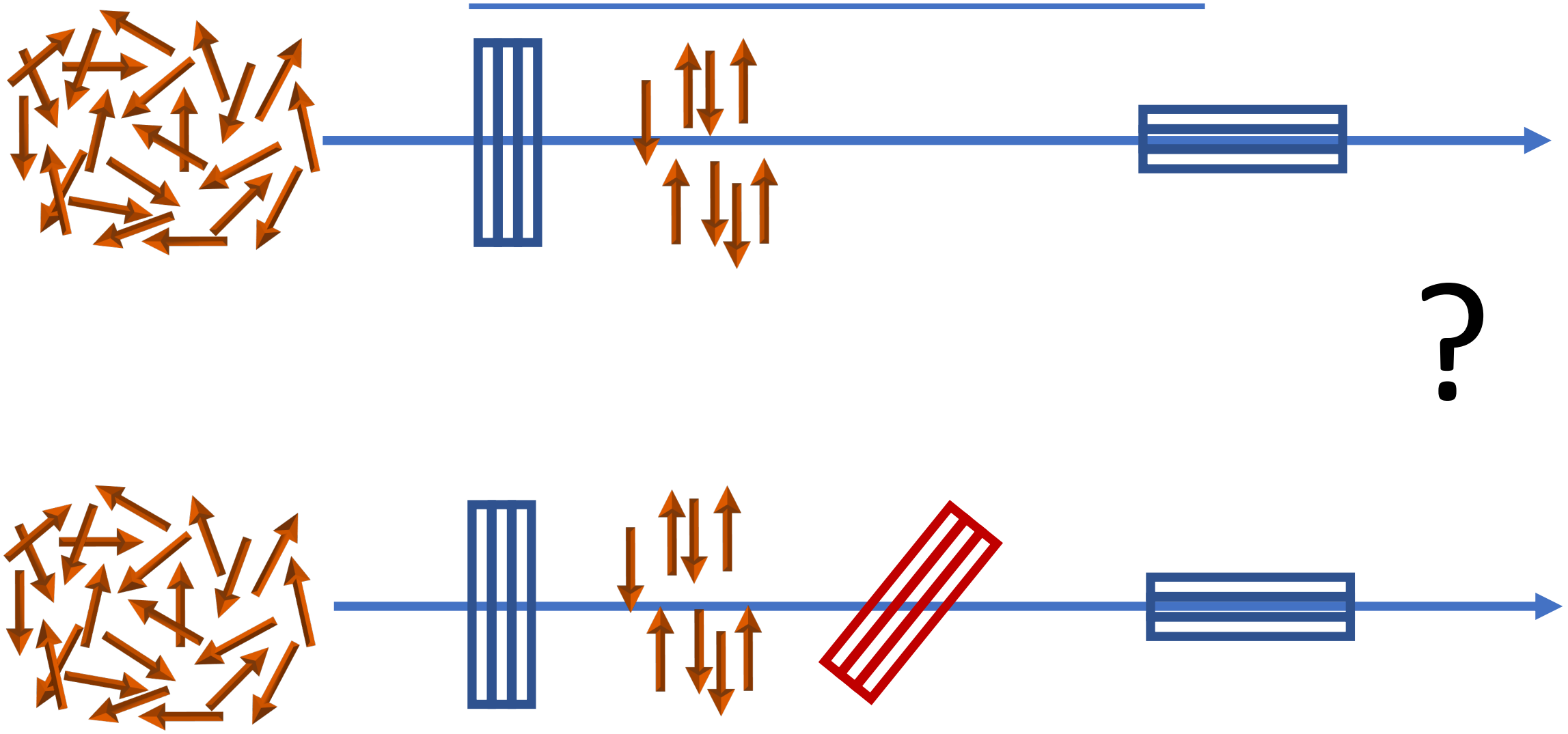
Youtube : The Coding Train. Genetic Algorithms

Definition of a quantum bit

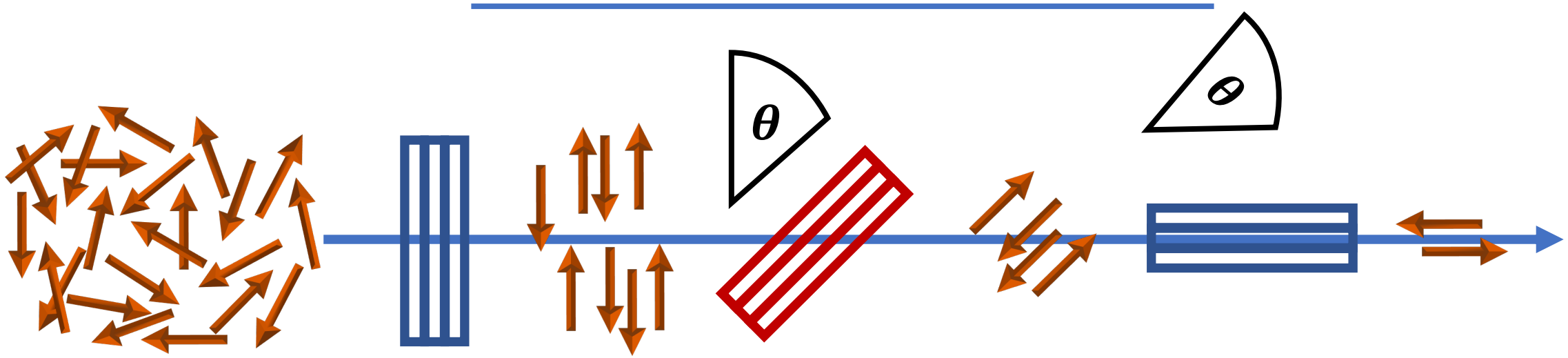
Polarized photons (A)



Polarized photons (B)

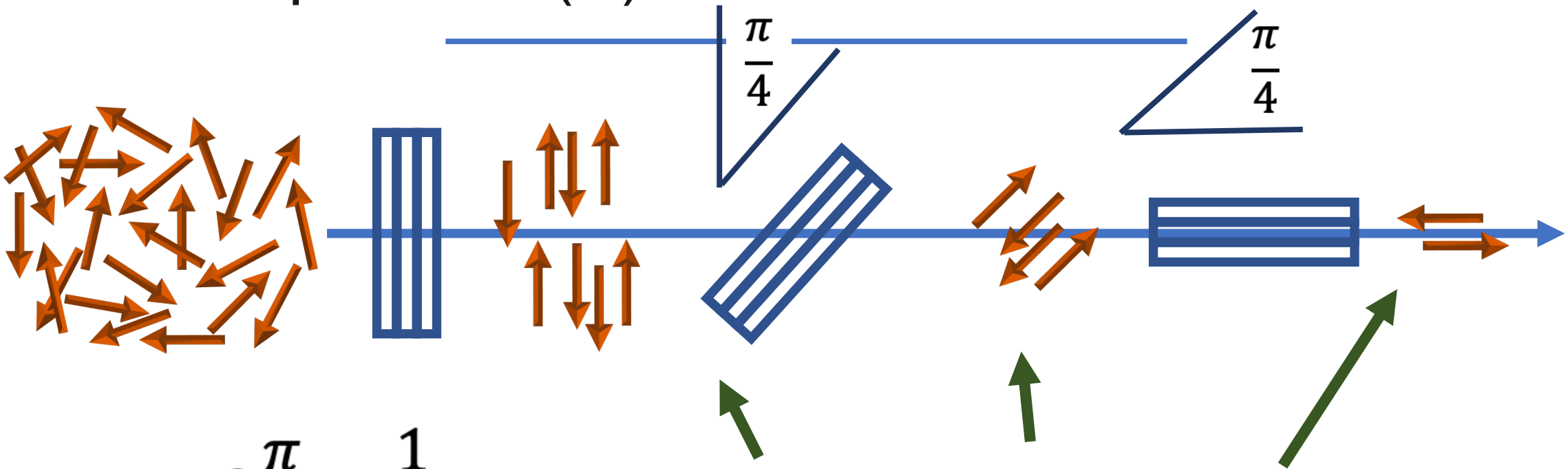


Polarized photons (B)



$$QTY_{out} = QTY_{in} * cos^2\theta$$

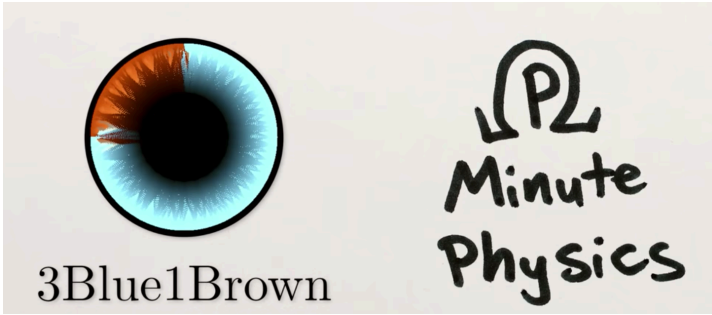
Polarized photons (B)



$$cos^2 \frac{\pi}{4} = \frac{1}{2}$$

100 → 50 → 25

More, and better: https://youtu.be/zcqZHYo7ONs



Thought experiment (1/2)

Consider the spin as a magnetic moment wich we can imagine as a rotation of the particule around itself (this is only an image)

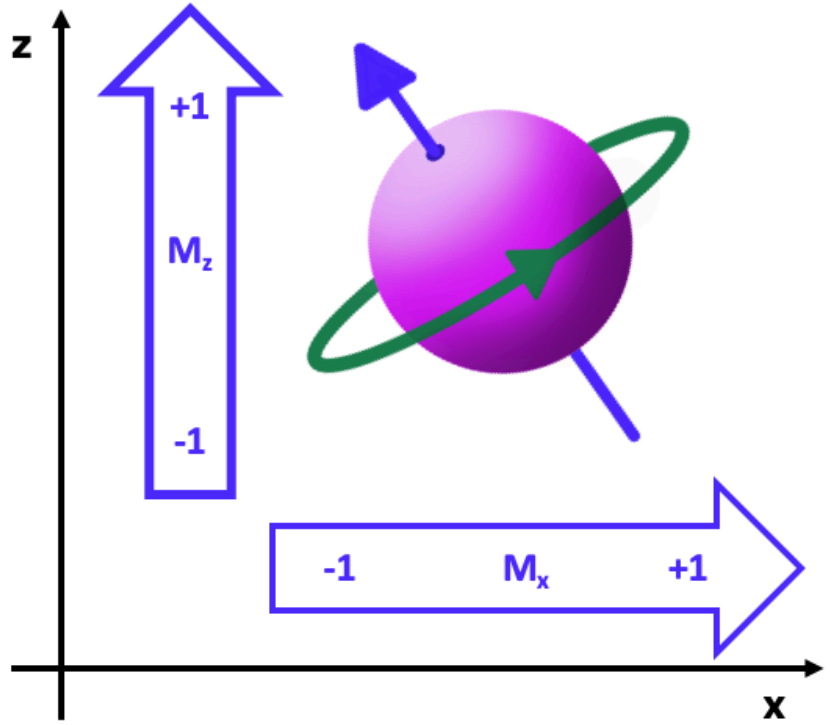
Looking at the two following statements:

- A: measuring the spin along z axis results in +1
- B: measuring the spin along x axis results in +1

I want to study (AvB == 1) ?

With classical reasonning :

1. if I get a result +1 on z : the experiment is over : AvB=1.
   - if not, I measure on x : if I get +1 : experiment is over : AvB=1
     - else AvB = 0
2. The other way around (starting with x) : same thing :
   - if I get +1 on x : experiment is over : BvA=1.
   - if not, I measure on Z, if I get +1 : experiment is over : BvA=1
     - else BvA = 0

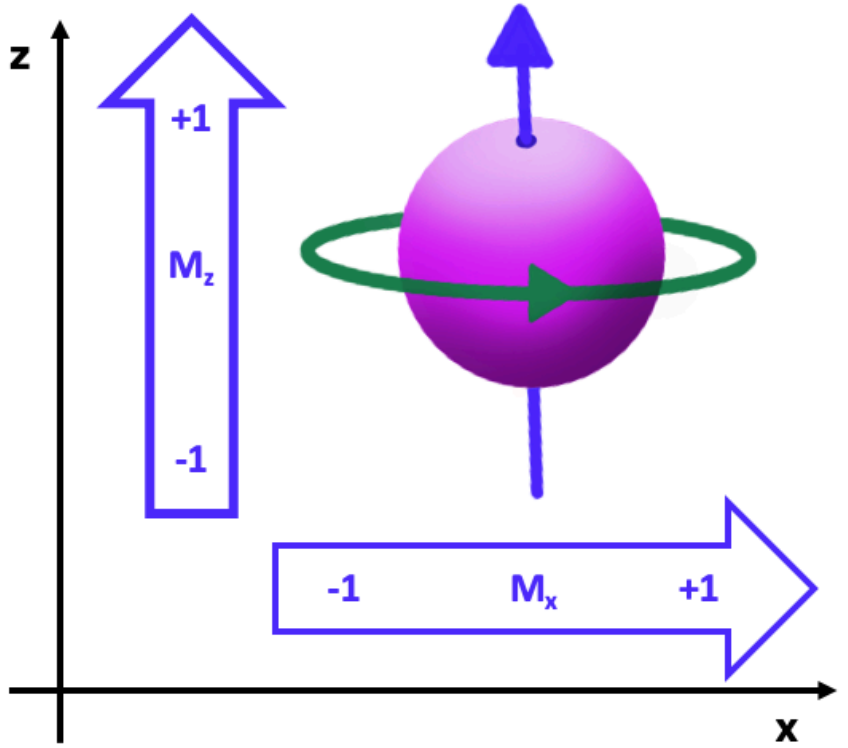


Thought experiment (1/2)

Now with quantum mechanics:

Assume we have prepared the spin on +1 over z.

1. I measure +1 on z : experience is over : AvB=1
2. Now with the same starting condition, let's evaluate (BvA==1)?
   - measuring on x : result will be +1 half of the time and -1 the other half. If I found +1 experience is over : BvA = 1 (50% chances)
     - If I found -1, now I have to measure against z axis, but now the previous measurement has moved the spin along x, so measuring on z will give -1 in 50% of cases, and +1 in 50% of cases.
   - So at the end BvA=1 in 75% of the cases.

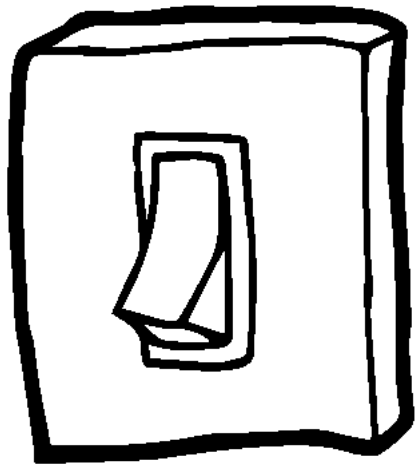


Conclusion : if an interaction has enough energy to perform a measurement of one property of the system, it also has enough energy to modify the system.

In other words : we cannot learn anything from a quantum system without changing it.

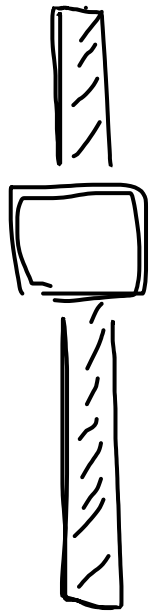
bits & qubits

0



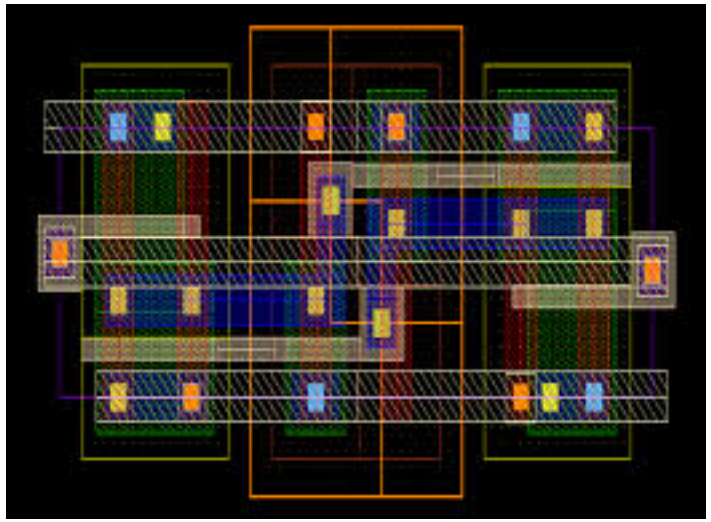
1

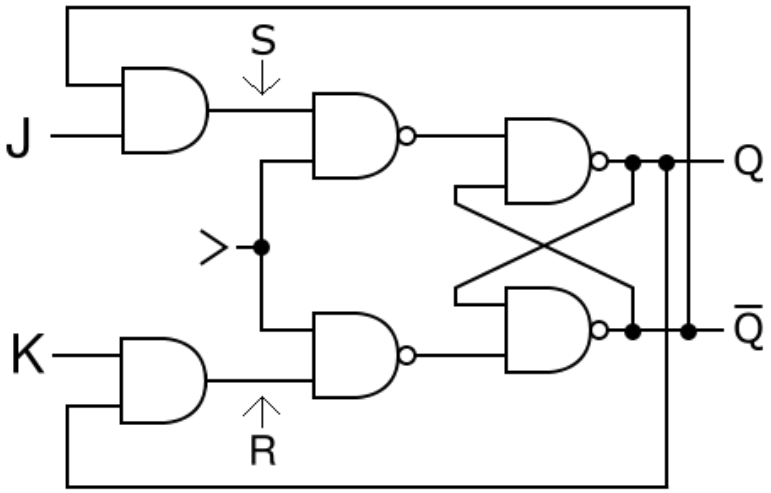
0



1

Classical bits properties





- well defined state : 0 or 1, no other value permitted,
- reading its value does not change it,
- information is local.

Classical computers use classical bits

« bits » : (BInary digiTS) : smallest piece of information :

True /False, Yes/No, Vcc/0

Usually represented with the values 0 or 1.

Single bits operations : Identity, Not, Set, Unset:

a	¬a	Iden(a)	Set(a)	Unset(a)
0	1	0	1	0
1	0	1	1	0

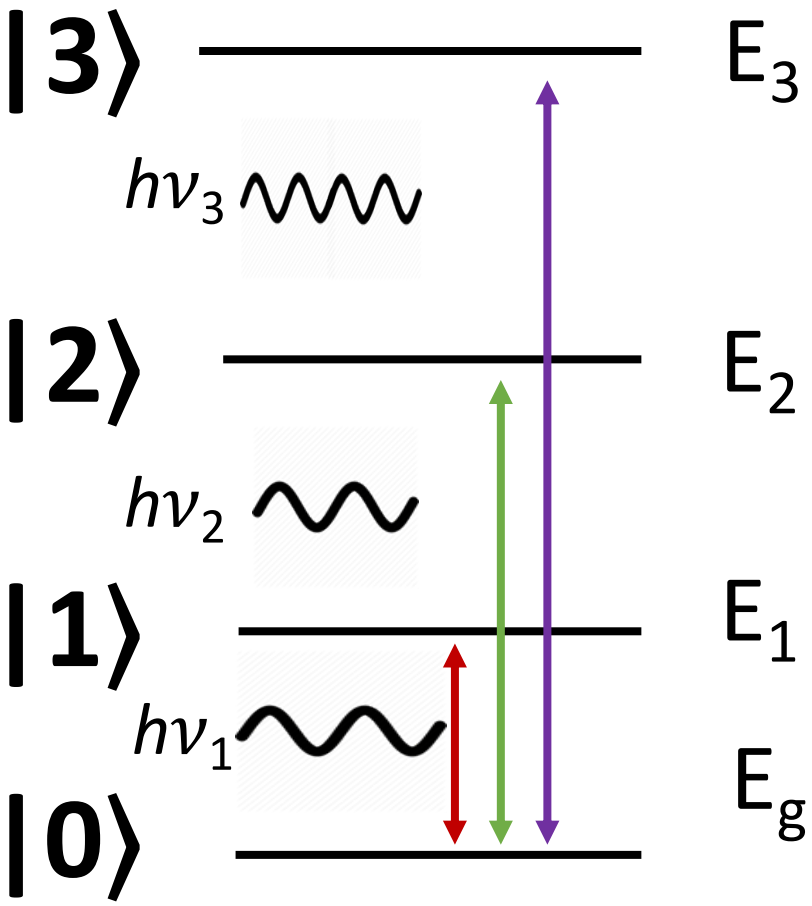
2 bits operations « Boolean Logic » :

a	b	a∧0	a∧b [AND]	a∧¬b	a	¬a∧b	b	(a∨b)∧¬(a∧b) [a⊕b] [XOR]	¬(a∨b)	¬a∧¬b	¬a∧¬b∨(a∧b)	¬b	a∨(¬a∧¬b)	¬a	a∨b	¬(a∧b) [NAND]	a∨1
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Quantum object behaviour

Quantification

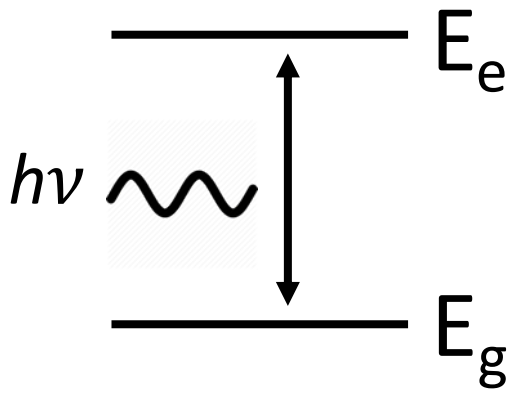
Superposition



Quantum bit : qubit

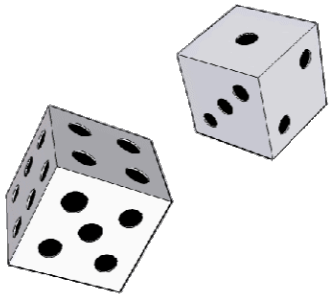
$|1\rangle$

$|0\rangle$



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Superposition

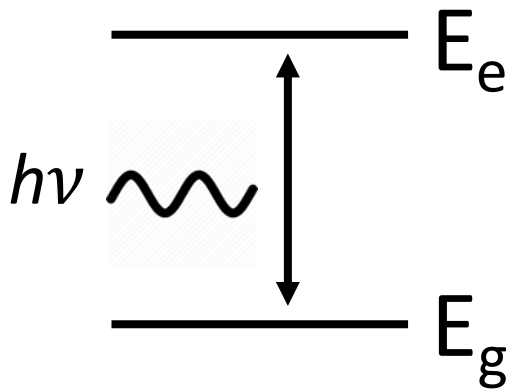


Quantum bit : qubit

Similar to a bit, a qubit is the basic unit of information in quantum computing, and can be in one of those two states :

$|1\rangle$

$|0\rangle$

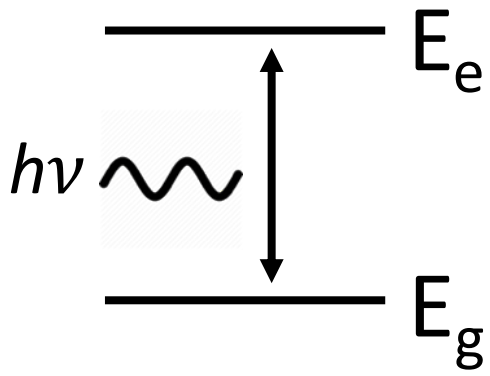


The qubit, however, can also be in a ***superposition state :*** a linear combination of the basic states $|0\rangle$ and $|1\rangle$ **:**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Quantum bit : qubit

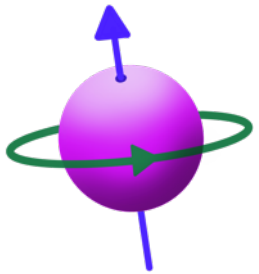
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



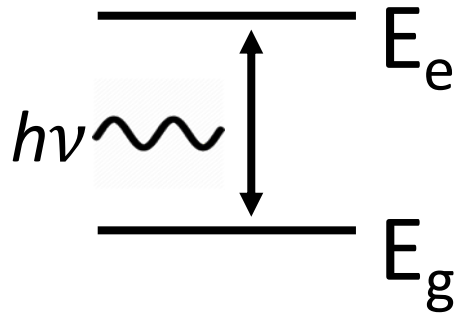
\+
- Can be controlled to a chosen state
- State can be read (sort of)

\-
- hν is typically a very small quantity of energy
- Goes back to it's ground state after a specific time

Quantum bit : 3 rules



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



1. For any possible state: the measurement can only result in

$$|0\rangle \text{ or } |1\rangle$$

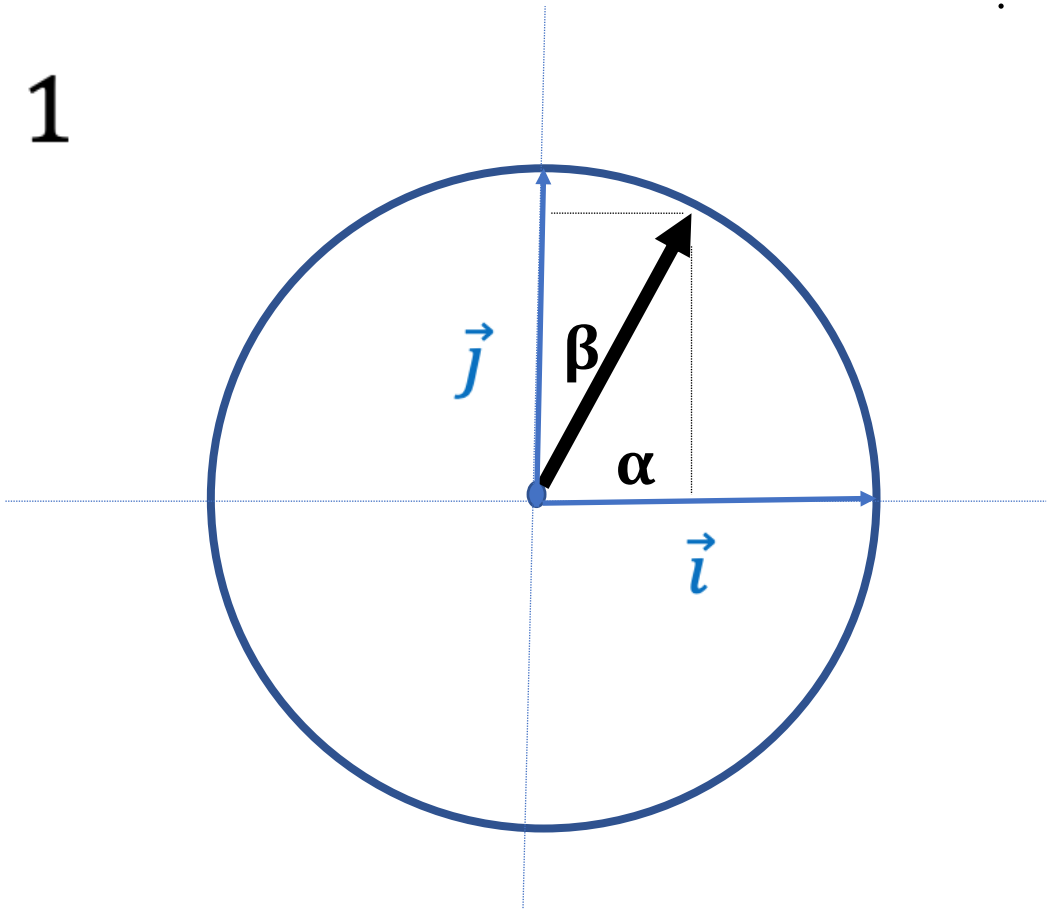
2. Probability of measuring $|0\rangle$ is α^2, probability of measuring $|1\rangle$ is β^2. *(1)*

3. When the measure is done, the superposition is lost.

(1) α and β are complex numbers the probabilities are $|\alpha|^2 = \alpha\alpha^*$ and $|\beta|^2 = \beta\beta^*$

Quantum bits properties

$$\alpha^2 + \beta^2 = 1$$

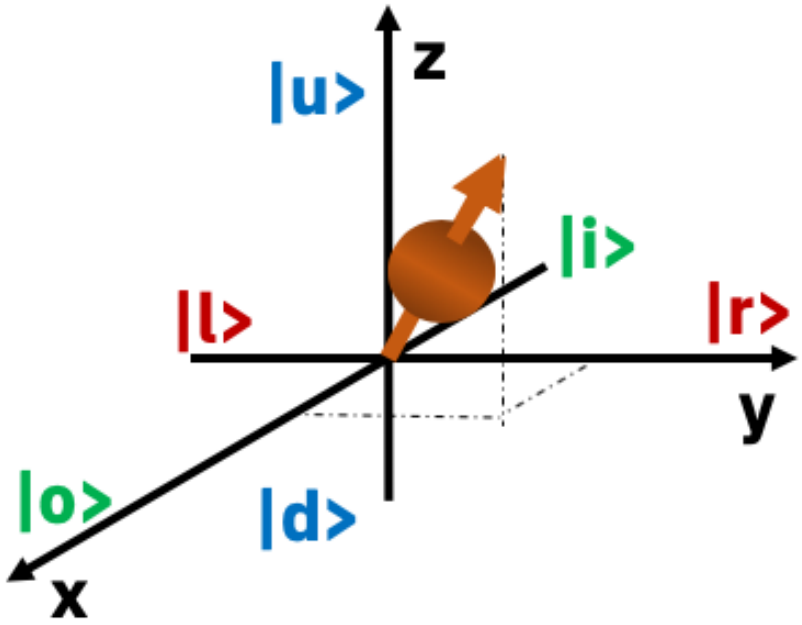


This was not enough

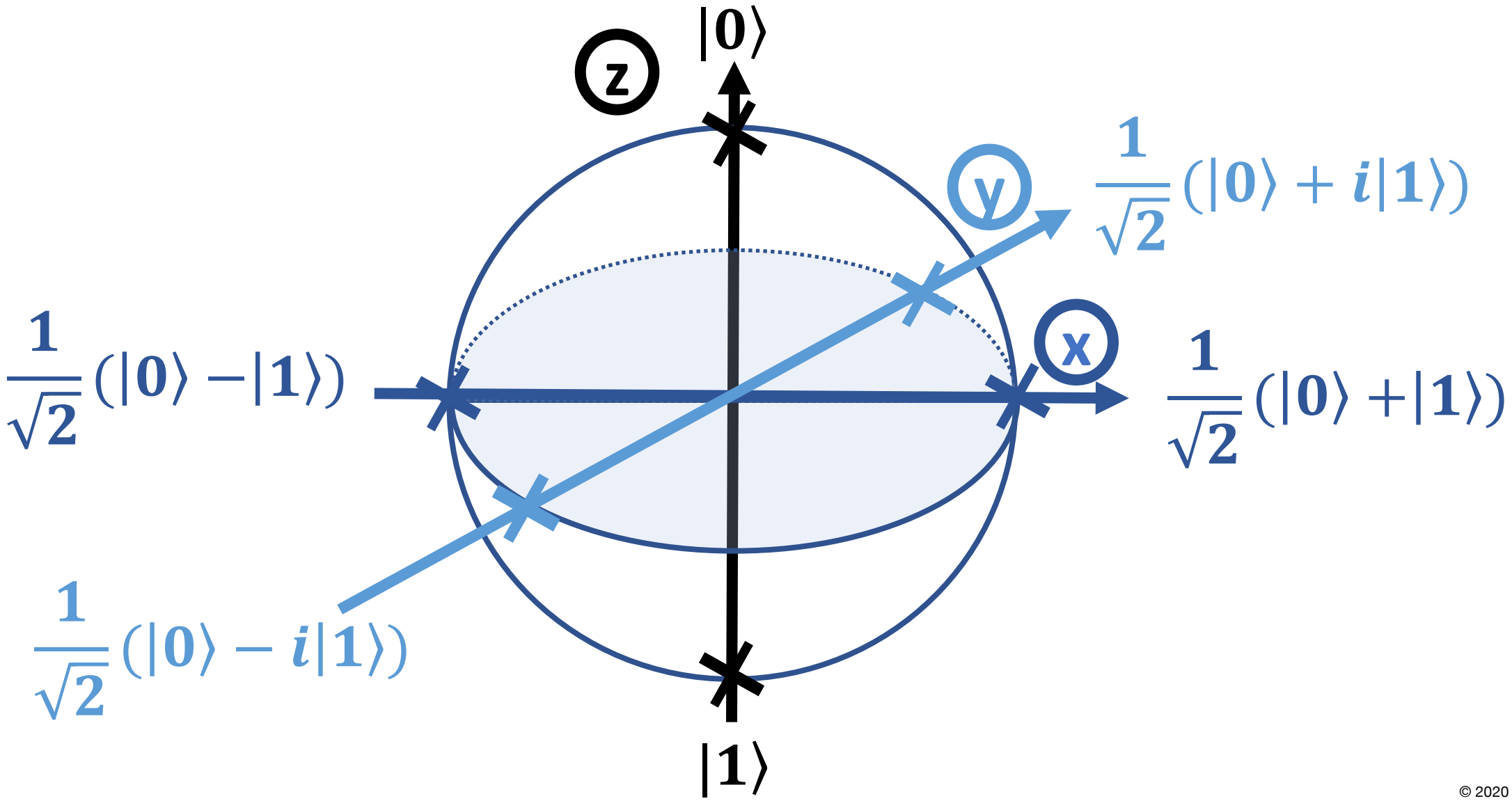
With the spin example, an image could be the rotation around itself :

with a measurement device, that can be positionned along the 3 (x,y,z), the possible states can be noted:

|up⟩, |down⟩, |left⟩, |right⟩, |in⟩, |out⟩, same in short: |u⟩, |d⟩, |l⟩, |r⟩, |i⟩, |o⟩,



The qubit states can be shown on the « Bloch Sphere »:



Quantum bit : qubit

A **qubit** is a 2 dimensionnal vector space, with a basis $|0\rangle$ and $|1\rangle$:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad (\equiv \overrightarrow{OM} = a\vec{\imath} + b\vec{\jmath})$$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \; |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} ; \; |\psi\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and :

$$\langle\psi| = (\alpha^* \quad \beta^*)$$

so :

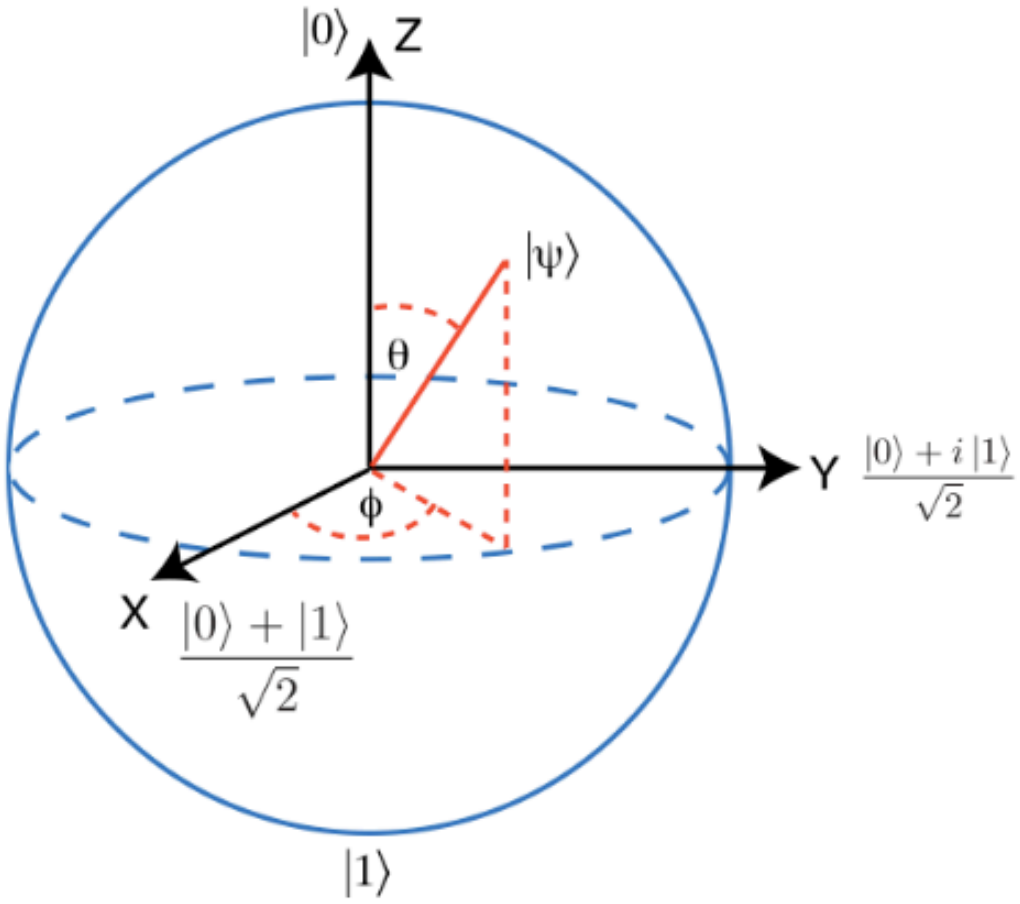
$$\langle\psi|\psi\rangle = |\psi|^2 = \alpha\alpha^* + \beta\beta^*$$

Controlling a qubit

The Bloch Sphere

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$



The Bloch sphere

Controlling a qubit (as a classical bit)

$$ID|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle, \quad ID|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$SET|0\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle, \quad SET|1\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$UNSET|0\rangle = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle, \quad UNSET|1\rangle = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$NOT|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle, \quad NOT|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

Qubit : operators

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$ = **identity**

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

(= **NOT** because $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ goes to $|1\rangle$, and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ goes to $|0\rangle$)

Some operations can't be done :

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha + \beta \end{pmatrix}$$

but many other are possible! …

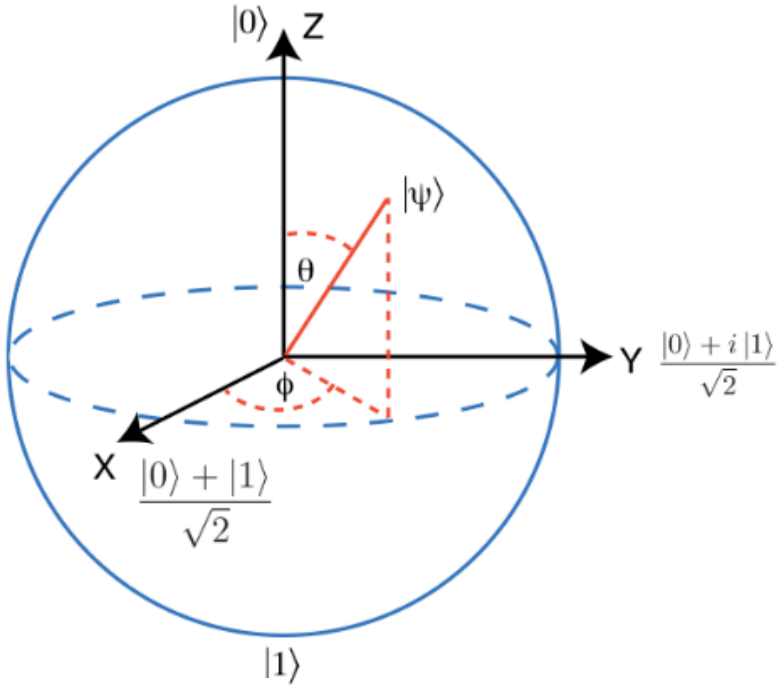
Bit Flip– X gate, NOT

- Rotation around the x-axis by π (Bit-Flip (0→1, 1→0)

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$



The Bloch sphere

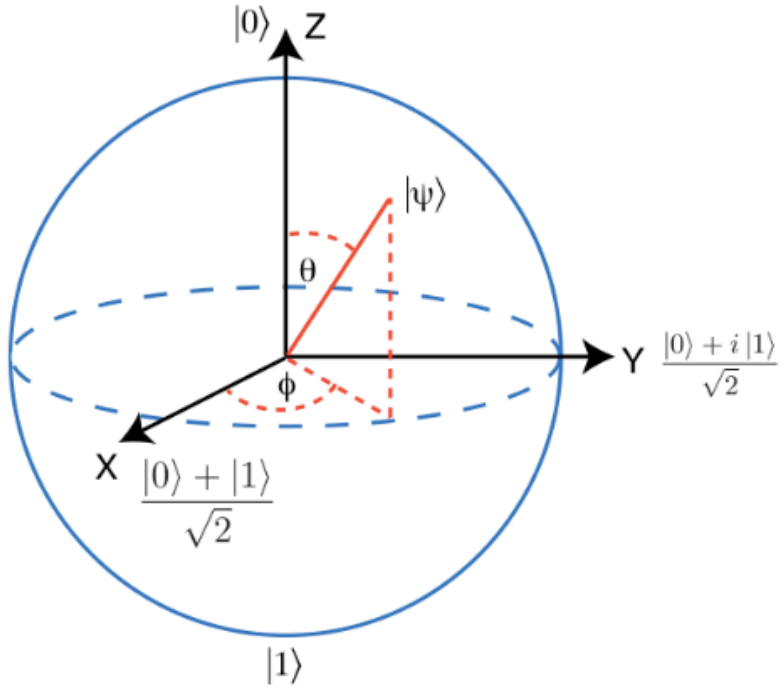
Y gate

- Rotation around the y-axis by π

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$Y|0\rangle = -i|0\rangle$

$Y|1\rangle = i|1\rangle$



The Bloch sphere

Z gate

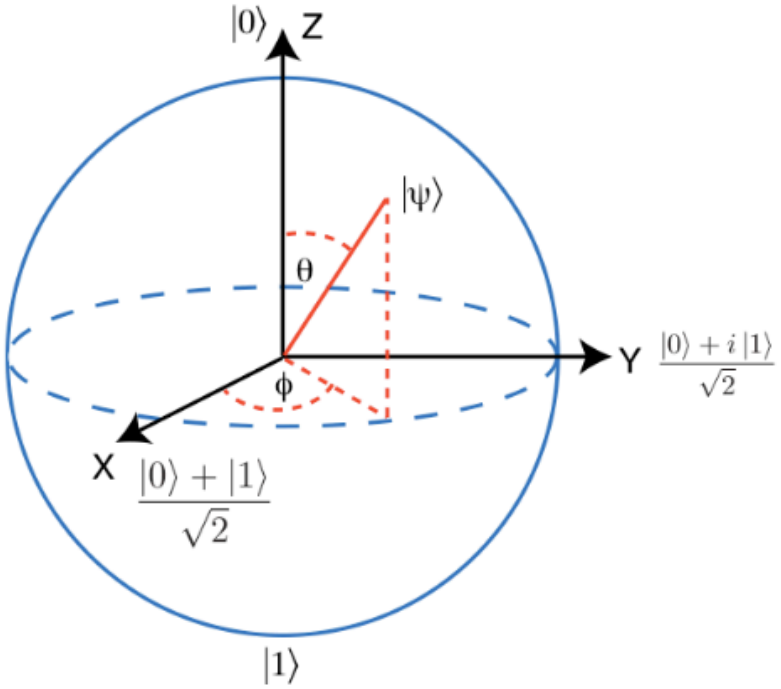
Z

- Rotation around the Z-axis by π

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$



The Bloch sphere

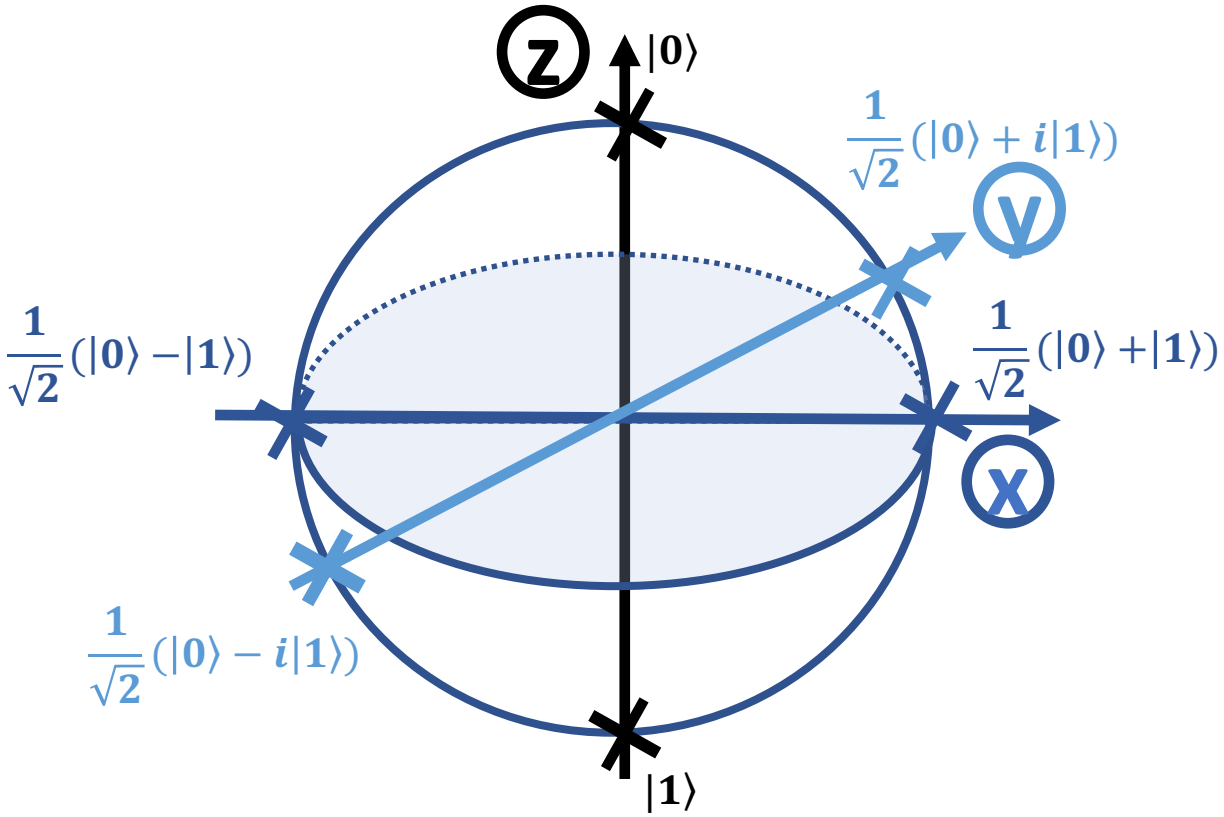
Summary

Rotation of $\boldsymbol{\pi}$ around the X, Y, Z axis

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Properties:

$$X^2 = Y^2 = Z^2 = I$$

$$XY = iZ\,; ZX = iY\,; YZ = iX$$

$$XY = -YX\,; YZ = -ZY; XZ = -ZX$$

and many more …

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$R_X(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \qquad R_Y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \qquad R_z(\theta) = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$$

$$U_3(\theta, \phi, \lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i\lambda+i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

U3 has the effect of rotating a qubit in the initial |0⟩ state to one with an arbitrary superposition and relative phase

$$U_2(\phi, \lambda) = U_3(\frac{\pi}{2}, \phi, \lambda) = \begin{pmatrix} 1 & -e^{i\lambda} \\ e^{i\phi} & e^{i\lambda+i\phi} \end{pmatrix}$$

From this gate, the Hadamard is done by H=U2(0,π)

$$U_1(\lambda) = U_3(0,0,\lambda) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}$$

The U1 gate is known as the phase gate and is essentially the same as Rz(λ).

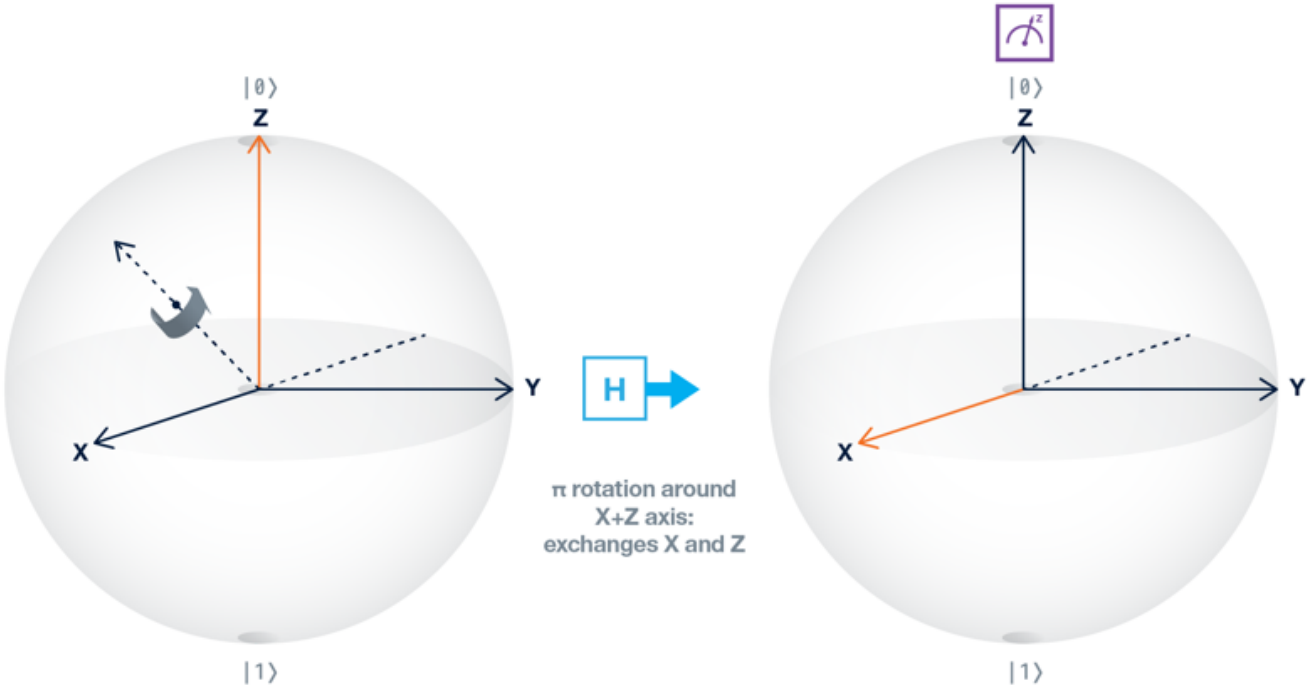
Creating Superposition – Hadamard gate

Produces equal-weighted combination of the states $|0\rangle$ and $|1\rangle$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

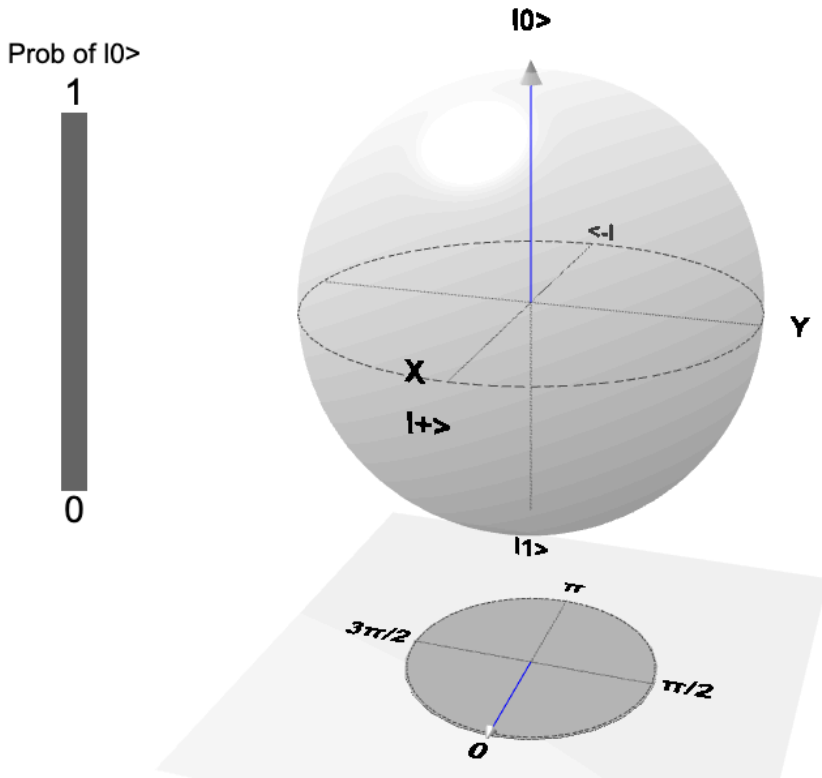


Superposition is one of the two crucial properties giving quantum computers their special ability

Manipulation de la sphère de Bloch

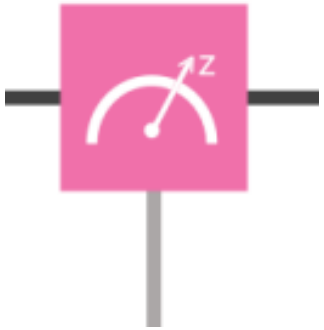
https://javafxpert.github.io/grok-bloch/

$$|\psi\rangle = \sqrt{1.00}\,|0\rangle + (\sqrt{0.00})e^{i\,0} \quad |1\rangle$$



Opération de mesure

Quantum measurement



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

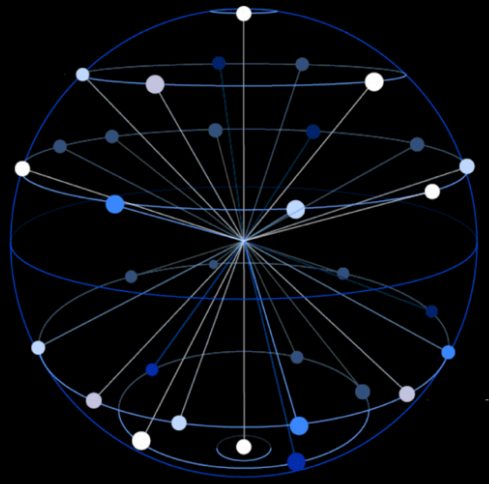
$$|\alpha|^2 + |\beta|^2 = 1$$

If a qubit in some state $|\psi\rangle$ is measured in the standard basis, the **result 0 is obtained with probability $|\alpha|^2$**, and the **result 1 is obtained with the complementary probability $|\beta|^2$**.

Working with many qubits

Multiple qubits

Universal quantum computers leverage quantum mechanical properties of superposition and entanglement to create states that scale exponentially with number of qubits, or quantum bits.

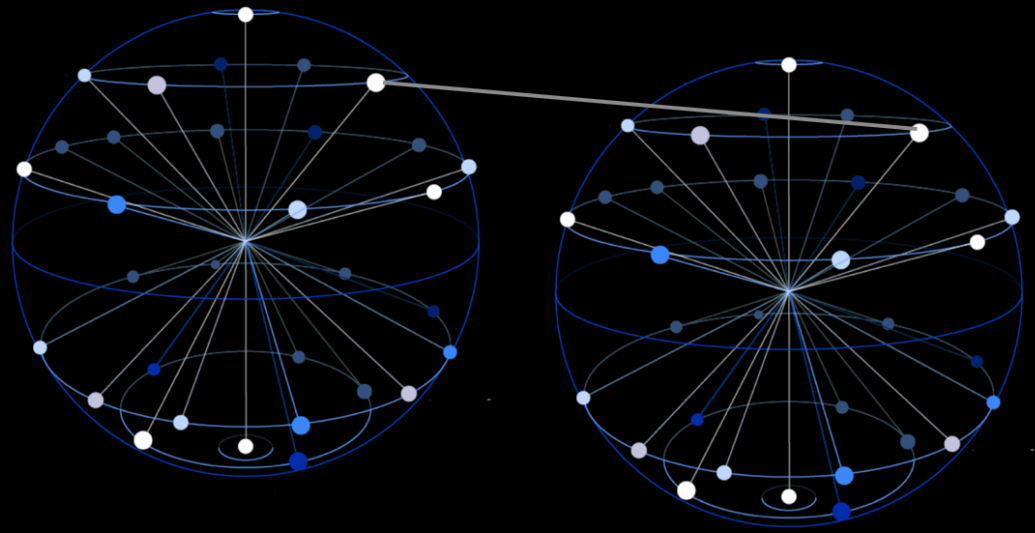


Superposition

A single quantum bit can exist in a superposition of 0 and 1:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

and N qubits allow for a superposition of all possible 2^N combinations.



Entanglement

The states of entangled qubits cannot be described independently of each other.

Two qubits system:

qubit A is defined in its states space : $|\boldsymbol{\psi}_A\rangle = \boldsymbol{\alpha}|0_A\rangle + \boldsymbol{\beta}|1_A\rangle$

qubit B is defined in its states space : $|\boldsymbol{\psi}_B\rangle = \boldsymbol{\gamma}|0_B\rangle + \boldsymbol{\delta}|1_B\rangle$

The two qubits system (A & B), is described by the product of the two single states :

$$|\boldsymbol{\Psi}_{AB}\rangle = |\boldsymbol{\psi}_A\rangle \; | \; \boldsymbol{\psi}_B\rangle = \left(\boldsymbol{\alpha}|0_A\rangle + \boldsymbol{\beta}|1_A\rangle\right) \times \left(\boldsymbol{\gamma}|0_B\rangle + \boldsymbol{\delta}|1_B\rangle\right)$$

$$|\boldsymbol{\Psi}\rangle = \boldsymbol{\alpha\gamma}|0_A\rangle|0_B\rangle + \boldsymbol{\alpha\delta}|0_A\rangle|1_B\rangle + \boldsymbol{\beta\gamma}|1_A\rangle|0_B\rangle + \boldsymbol{\beta\delta}|1_A\rangle|1_B\rangle$$

$$|\boldsymbol{\Psi}\rangle = \boldsymbol{\alpha\gamma}|00\rangle + \boldsymbol{\alpha\delta}|01\rangle + \boldsymbol{\beta\gamma}|10\rangle + \boldsymbol{\beta\delta}|11\rangle$$

Two qubits system:

$$|\boldsymbol{\Psi}\rangle = \boldsymbol{\alpha\gamma}|00\rangle + \boldsymbol{\alpha\delta}|01\rangle + \boldsymbol{\beta\gamma}|10\rangle + \boldsymbol{\beta\delta}|11\rangle$$

The two qubit system is defined in a 4 dimension state, with basis vectors:

$$|00\rangle\ ,\ |01\rangle,\ |10\rangle,\ |11\rangle$$

General state is :

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle \quad et \quad |a|^2+|b|^2+|c|^2+|d|^2 = 1$$

for 3 qubits :

$$|\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$$

(with sum of squared components equals 1)

And so on for 4 qubits…, n qubits

Two qubit states calculations (tensor product)

$$\begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix}\otimes\begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix}=\begin{pmatrix}\alpha_1\alpha_2\\ \alpha_1\beta_2\\ \beta_1\alpha_2\\ \beta_1\beta_2\end{pmatrix}$$

$$|0\rangle\otimes|0\rangle=|00\rangle=\begin{pmatrix}1\\ 0\end{pmatrix}\otimes\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 0\\ 0\end{pmatrix}$$

$$|0\rangle\otimes|1\rangle=|01\rangle=\begin{pmatrix}1\\ 0\end{pmatrix}\otimes\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}0\\ 1\\ 0\\ 0\end{pmatrix}$$

$$|1\rangle\otimes|0\rangle=|10\rangle=\begin{pmatrix}0\\ 1\end{pmatrix}\otimes\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 1\\ 0\end{pmatrix}$$

…

And so on:

$$\begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix}\otimes\begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix}\otimes\begin{pmatrix}\alpha_3\\ \beta_3\end{pmatrix}=\begin{pmatrix}\alpha_1\alpha_2\alpha_3\\ \alpha_1\alpha_2\beta_3\\ \alpha_1\beta_2\alpha_3\\ \alpha_1\beta_2\beta_3\\ \beta_1\alpha_2\alpha_3\\ \beta_1\alpha_2\beta_3\\ \beta_1\beta_2\alpha_3\\ \beta_1\beta_2\beta_3\end{pmatrix},$$

$$\text{eg: } |101\rangle=\begin{pmatrix}0\\ 1\end{pmatrix}\otimes\begin{pmatrix}1\\ 0\end{pmatrix}\otimes\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\\ 0\\ 1\\ 0\\ 0\\ 0\end{pmatrix}$$

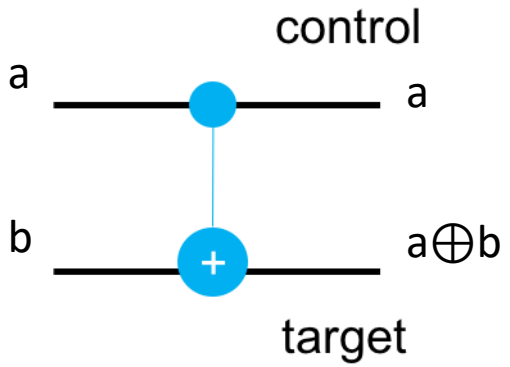
Two qubit states calculations (tensor product)

$$\begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix}\otimes\begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix}=\begin{pmatrix}\alpha_1\alpha_2\\ \alpha_1\beta_2\\ \beta_1\alpha_2\\ \beta_1\beta_2\end{pmatrix}$$

$$|0>\otimes|0>=|00>=\begin{pmatrix}1\\ 0\end{pmatrix}\otimes\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 0\\ 0\end{pmatrix}$$

$$|0>\otimes|1>=|01>=\begin{pmatrix}1\\ 0\end{pmatrix}\otimes\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}0\\ 1\\ 0\\ 0\end{pmatrix}$$

Multi–qubits Gates : CNOT gate

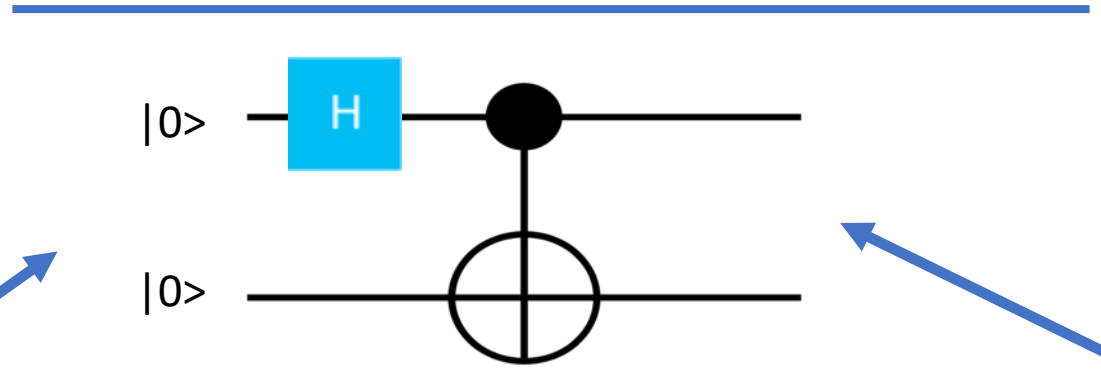


- CNOT = Controlled-NOT Gate
- Inverts a target qubit according to the state of the control qubit

CNOT		
a	b	a⊕b
0	0	0
0	1	1
1	0	1
1	1	0

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Example Bell-State

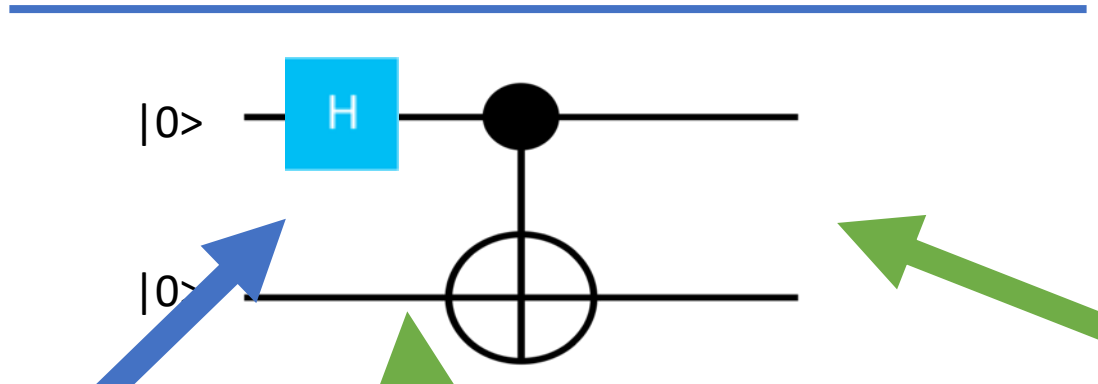


$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|00\rangle+|10\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Example Bell-State



$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

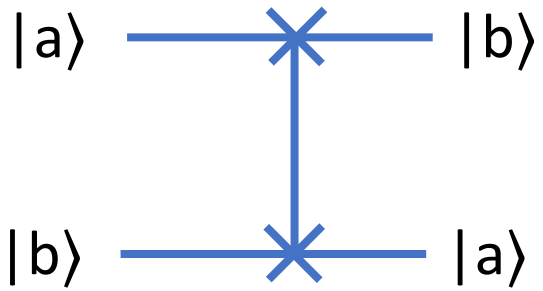
$$H = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$Cx = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|00\rangle+|10\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Multi–qubits Gates : SWAP gate

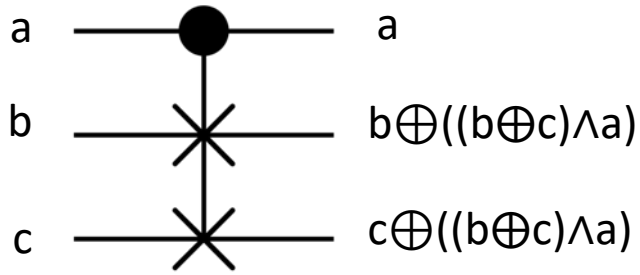


- Swaps qubit states

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SWAP			
a	b	a	b
0	0	0	0
0	1	1	0
1	0	0	1
1	1	1	1

Multi–qubits Gates : CSWAP (Fredkin gate)



- CSWAP = Controlled-Swap Gate
- Swaps two target qubits according to the state of the control qubit

CSWAP					
a	b	c	(b⊕c)∧a	b⊕((b⊕c)∧a)	c⊕((b⊕c)∧a)
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	0	1	1

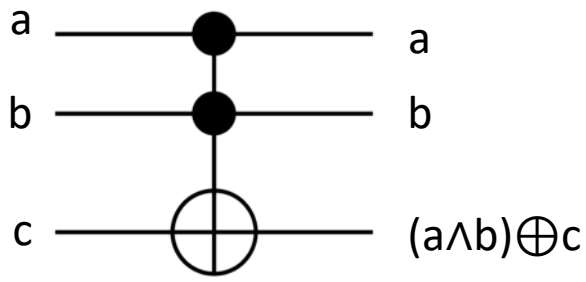
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Basis: |000>,|001>,|010>,|011>,|100>,|101>,|110>,|111>

Note:

- If c==0 : c⊕((b⊕c)∧a) = a ∧ b
- If c==1: b⊕((b⊕c)∧a) = a ∨ b
- If b==0 & c ==1 : c⊕((b⊕c)∧a) = ¬a

Multi–qubits Gates : CCNOT (Toffoli gate)



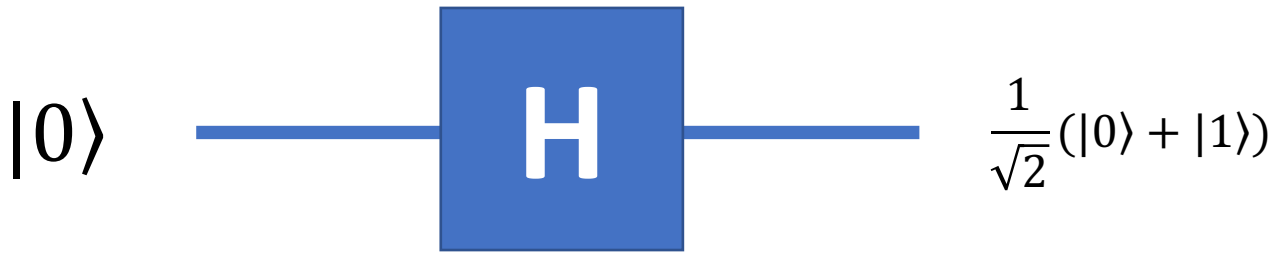
- CCNOT = Control-Control-NOT Gate
- Inverts a target qubit according to the state of the two control qubits

CCNOT			
a	b	c	(a∧b)⊕c
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

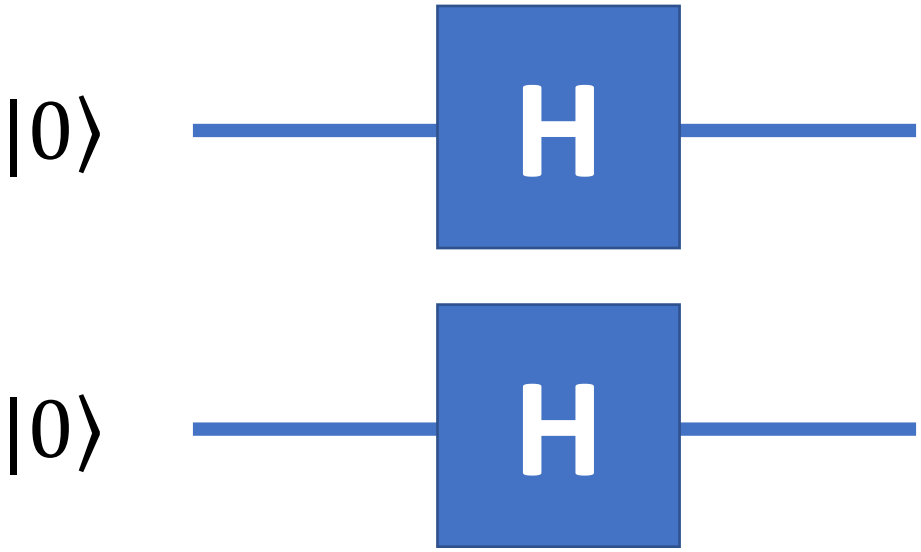
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Basis: |000>,|001>,|010>,|011>,|100>,|101>,|110>,|111>

$H^{\otimes n}$

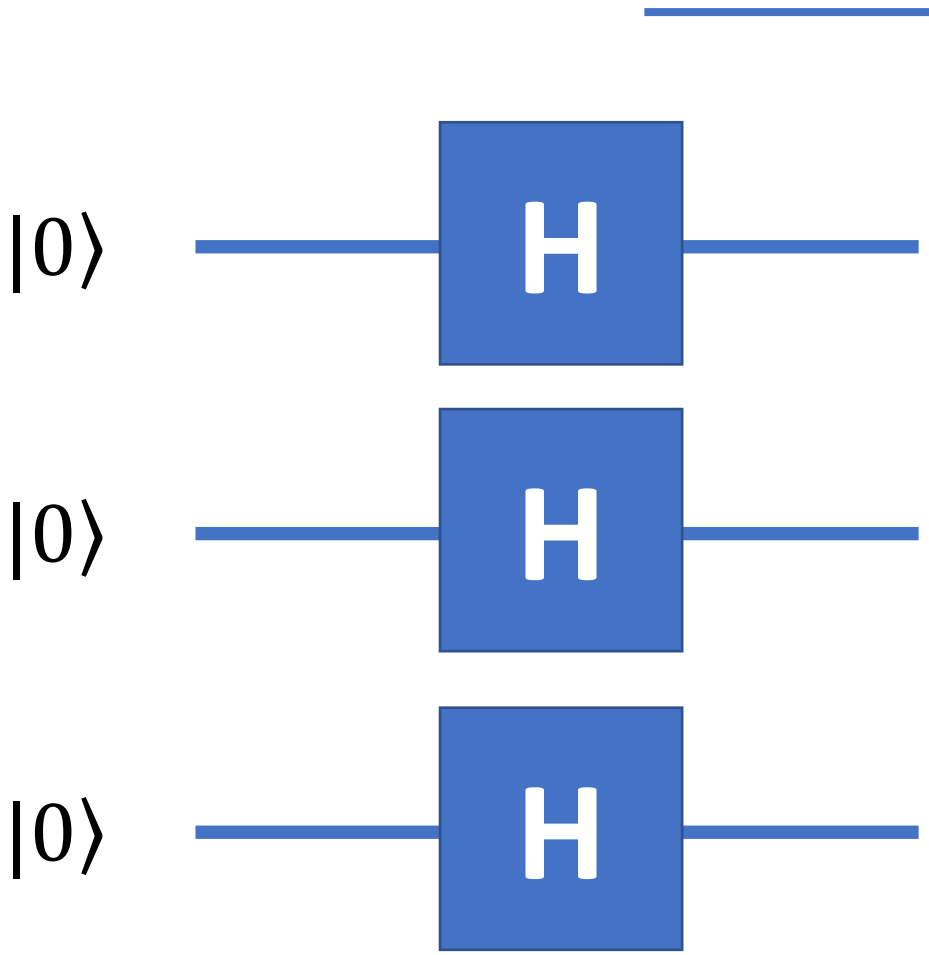


$H^{\otimes n}$



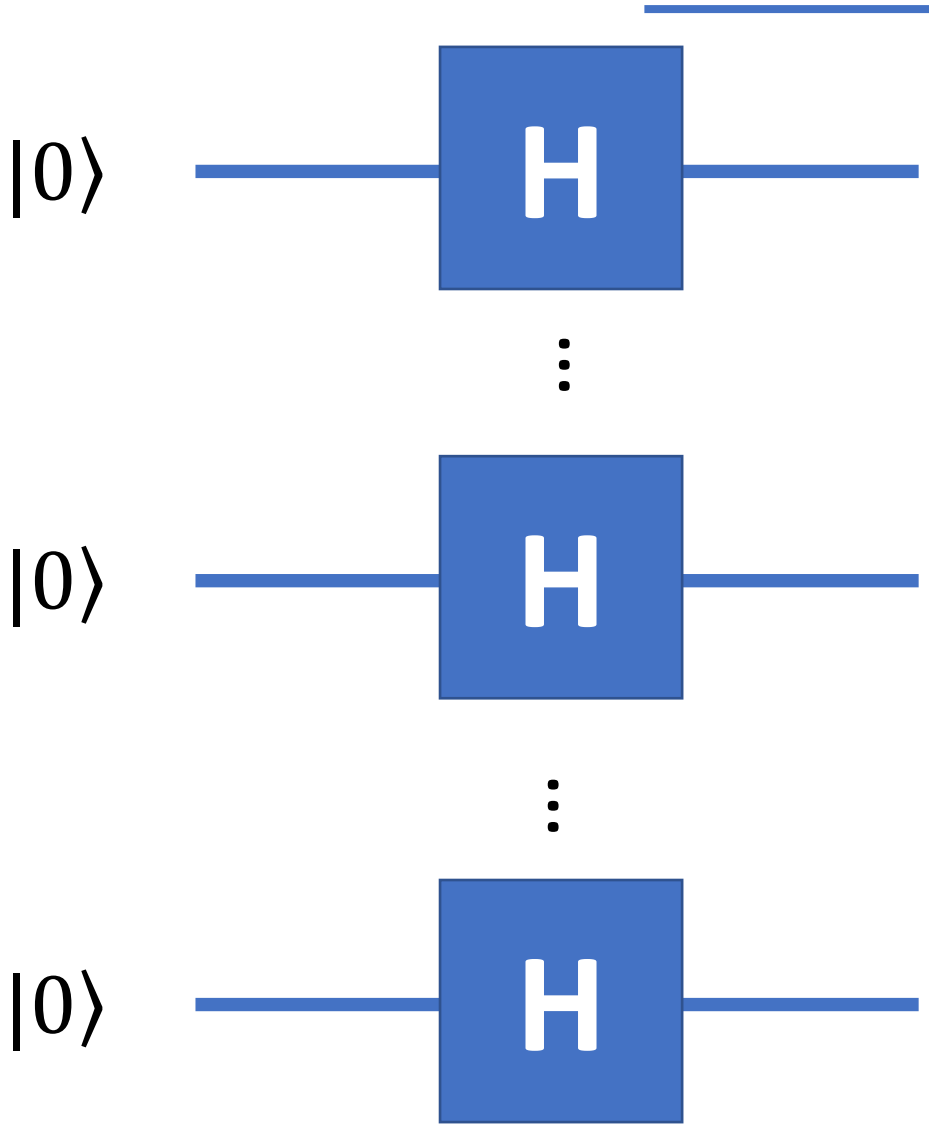
$$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$H^{\otimes n}$



$$\frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$$

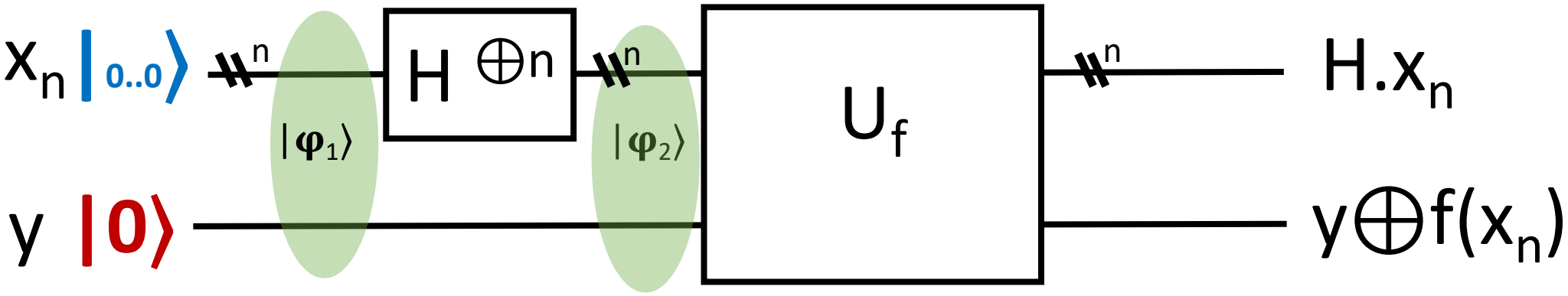
H^n



$$\frac{1}{\sqrt{2^n}}(|0 \dots 0\rangle + |0 \dots 1\rangle + \cdots$$

$$+|1 \dots 10\rangle + |1 \dots 11\rangle)$$

“Parallelism”



$$|\varphi_1\rangle = |0..0,0\rangle \qquad |\varphi_2\rangle = \frac{1}{\sqrt{2^n}}(|0..0,0\rangle + \ldots + |1\ldots1,0\rangle)$$

Output state contains all results of *f* applied to all possible values for x_n, namely : *f*(0..0), *f*(0..1), … and *f*(1..1), while U_f has been calculated only once.

Doing something usefull with that is another story… and that is all about quantum algorithms…