Understanding and Pricing Digital Options

1 Introduction

A digital option, also known as a binary option, pays off a fixed amount if the underlying asset price is above (for a call) or below (for a put) the strike price at expiration. Unlike vanilla options, the payoff of a digital option is not continuous but binary—typically \$0 or \$1.

2 The Black-Scholes Model for Digital Options

To price a digital option using the Black-Scholes model, we calculate the present value of the payoff, discounted at the risk-free rate, and multiplied by the probability that the option will expire in-the-money. The formula for a digital call option is:

$$Call\ Price = e^{-rT} N(d_2) \tag{1}$$

And for a digital put option:

$$Put\ Price = e^{-rT} N(-d_2) \tag{2}$$

where e^{-rT} is the discount factor, $N(\cdot)$ is the cumulative distribution function of the standard normal distribution, and d_2 is given by:

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \tag{3}$$

S is the current underlying asset price, K is the strike price, T is the time to maturity, r is the risk-free rate, and σ is the volatility of the asset.

3 Numerical Example

Consider a digital call option with the following parameters:

- Current price of the underlying asset, $S = 100$
- Strike price, $K = 80$
- Time to maturity, $T = 1$ year

- Annual risk-free interest rate, $r = 0.05$ (5%)
- Volatility of the underlying asset, $\sigma = 0.4$ (40%)

Using the provided Python code, the digital call option price is calculated to be \$0.65, with an intrinsic value of \$1.00 and a time value of -\$0.35.

Below are the graphs illustrating the payoff and current value of the digital option:

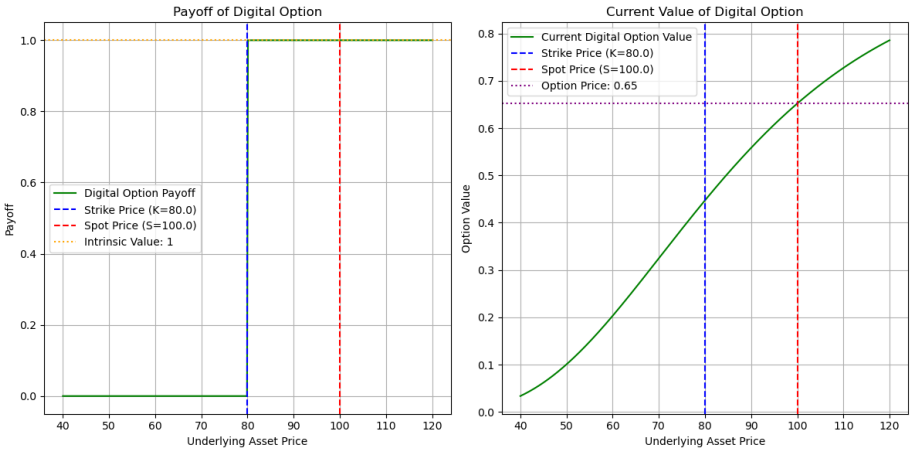

Figure 1: Digital option example

4 Conclusion

Digital options are a type of exotic option with a unique payoff structure. They are widely used in speculative trading and hedging and can be priced using the Black-Scholes model, as demonstrated by the numerical example and Python code.