ARMA Models (Part 2) and ARIMA

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Seminar on 14 Mar 2023

In this moment, I will review about Order Determination.

The table below is the behaviour of the ACF dan PACF of a casual and invertible ARMA Models to identify the model.

	MA(q)	AR(p)	ARMA(p,q) $(p > 0, q > 0)$
ACF	Cuts off after lag q	Tails off	Tails off
PACF	Tails off	Cuts off after lag p	Tails off

Behavior of the ACF and PACF of ARMA models

In [1]:

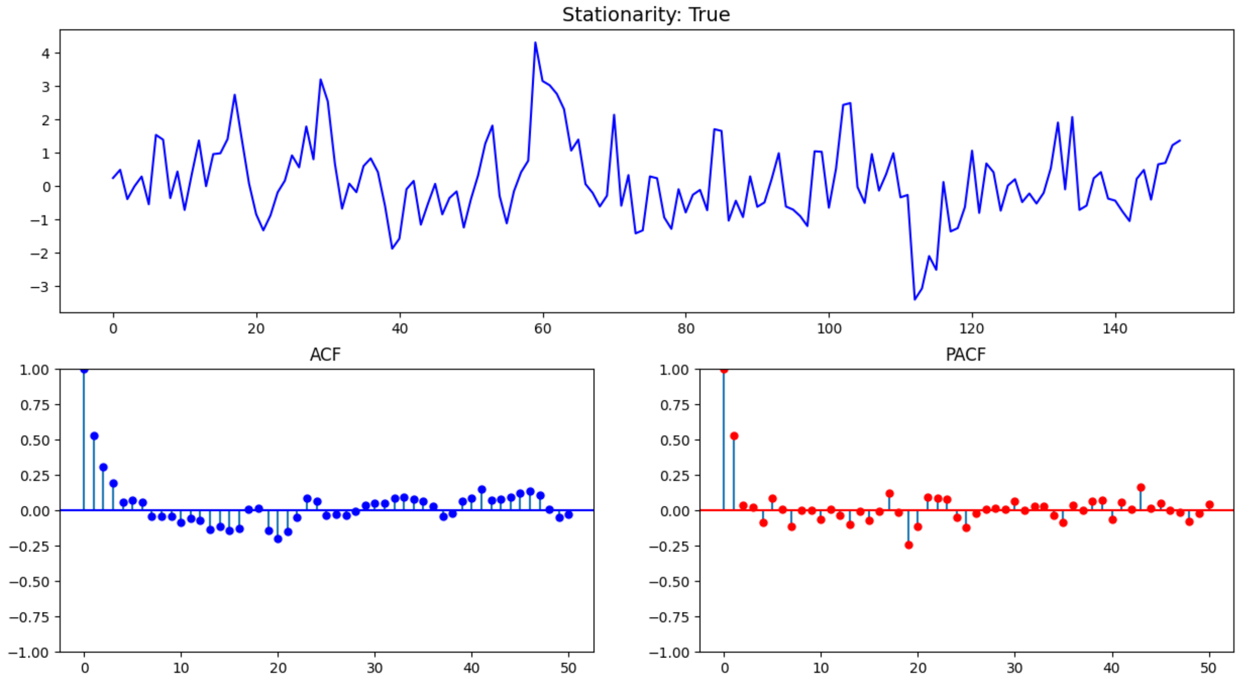
```
## initializing requirement
import pandas as pd
import matplotlib.pyplot as plt

from luwiji.time_series import illustration, demo
```

Choosing the order of Autoregressive AR(p)

In [2]:

```
demo.AR1_simulation()
```

alpha1 0.60

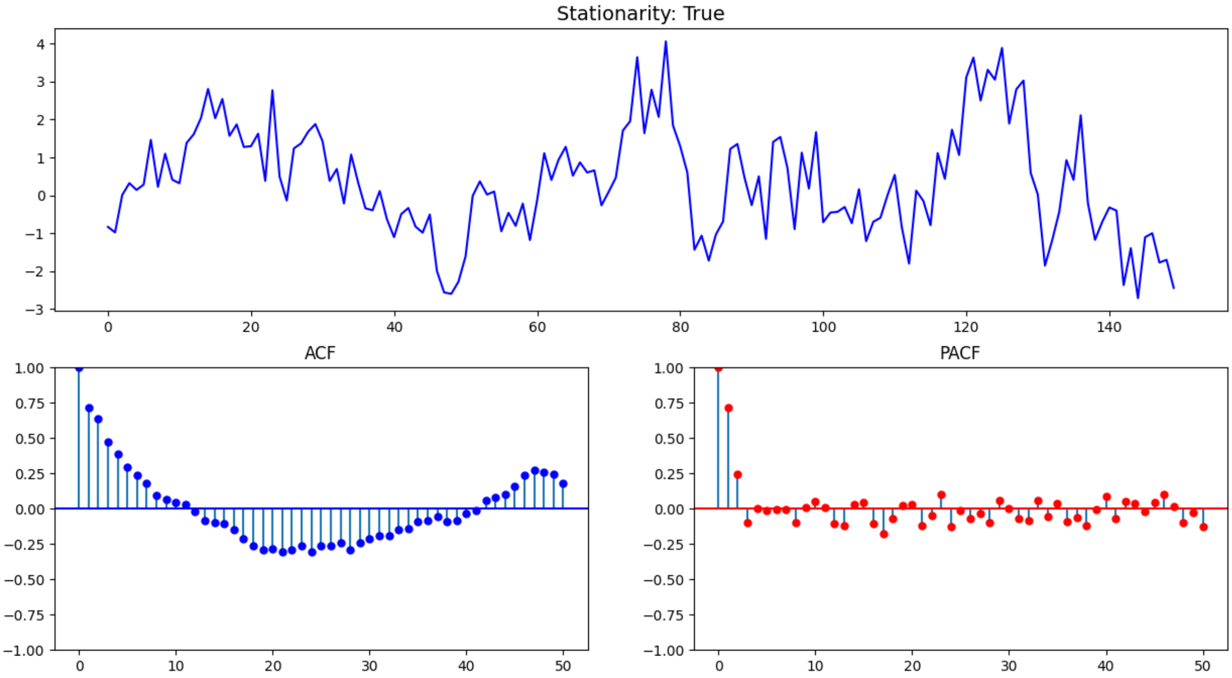


In [3]:

```
demo.AR2_simulation()
```

alpha1 0.50

alpha2 0.30

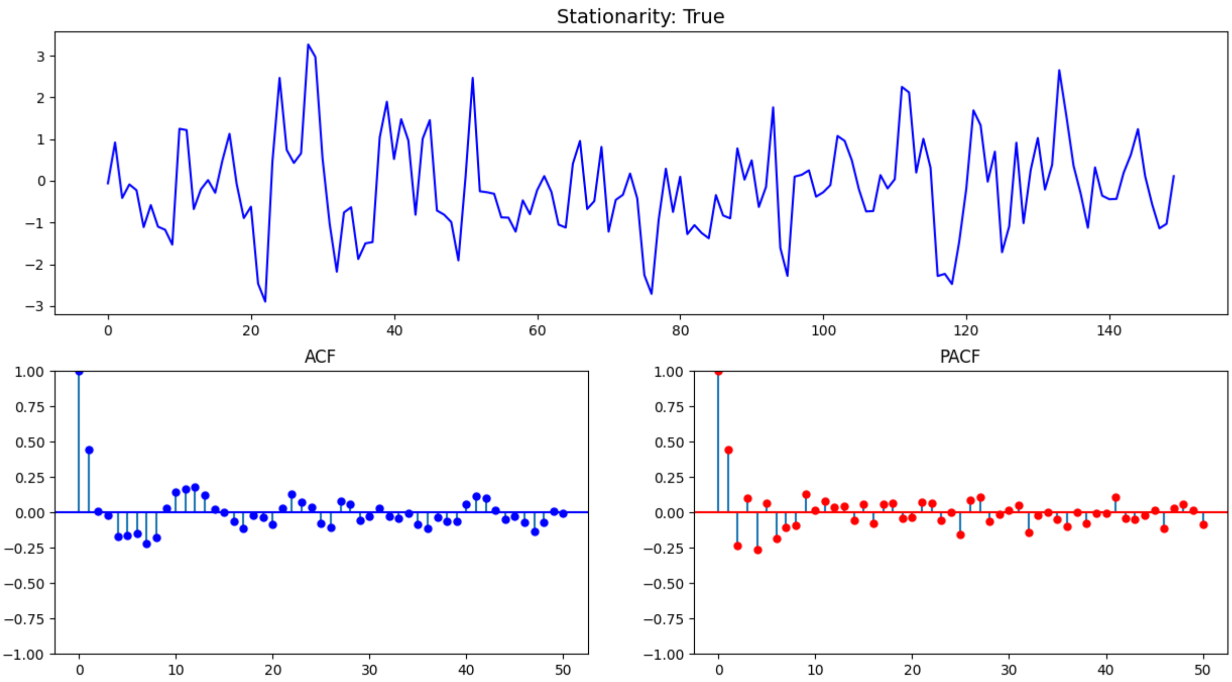


Choosing the order of Moving Average MA(q)

In [4]:

```
demo.MA1_simulation()
```

theta1 0.60

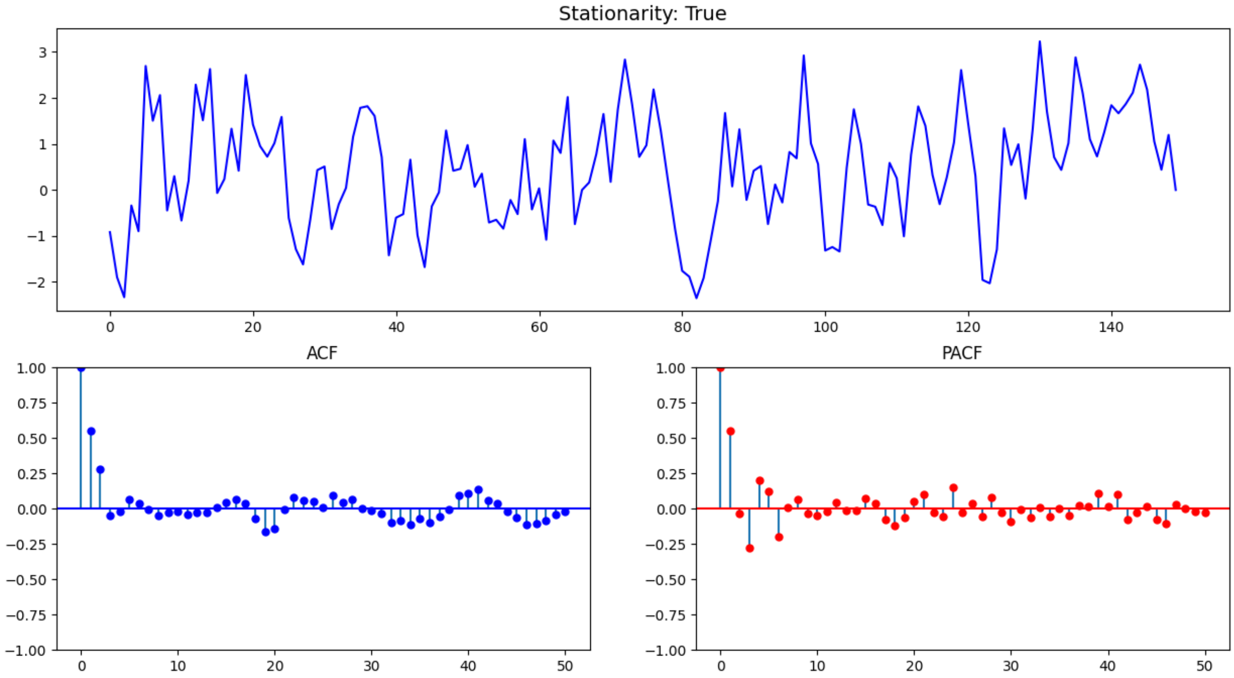


In [5]:

```
demo.MA2_simulation()
```

theta1 0.60

theta2 0.60



Choosing the order of ARMA(p,q)

In [6]:

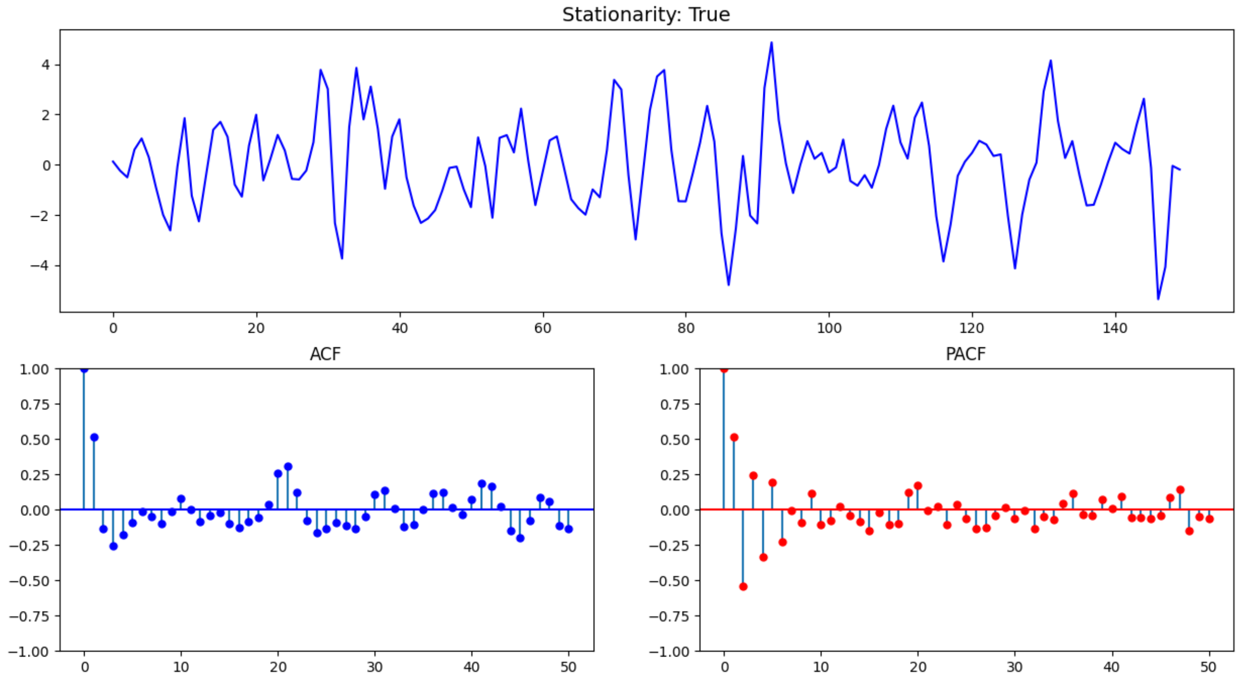
```
demo.AR2MA2_simulation()
```

alpha1 0.70

alpha2 −0.40

theta1 0.80

theta2 −0.50



It is quite difficult to determine the order using the value of ACF and PACF for the combination like above, i.e. classical way by True ACF and PACF (theoritically). Therefore, there are another way by using information criteria such as AIC, BIC, HQIC, and so forth.

Another difficulty is that if the time series data is nonstationary. In this case, we can do differencing to get stationary time series.

Nonstationary Time Series

Suppose $\{X_t\}$ satisfies ARMA(p,q) models as following:

$$X_t = \mu + \sum_{i=1}^{p} \varphi_i X_{t-i} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$$

$$\varphi(B)X_t = \mu + \theta(B)\epsilon_t$$

where:

- $\{\epsilon_t\} \sim WN(0, \sigma_\epsilon^2)$
- $E[X_s \epsilon_t] = 0$ if $s < t$
- μ is a constant term
- $\varphi_p \neq 0$
- $\theta_q \neq 0.$
- $\varphi(z) = 1 - \varphi_1 z - \varphi_2 z^2 - \cdots - \varphi_p z^p$
- $\theta(z) = 1 + \theta_1 z + \theta_2 z^2 + \cdots + \theta_q z^q$

Here we can make a new model $\{Y_t\}$ that satisfies $Y_t = \nabla^d X_t = (1 - B)^d X_t$.

If $\{Y_t\}$ is stationary, then we can build an ARMA(p,q) model for it as follows:

$\varphi(B)Y_t = \mu + \theta(B)\epsilon_t$ or $\varphi(B)(1 - B)^d X_t = \mu + \theta(B)\epsilon_t$

Definition

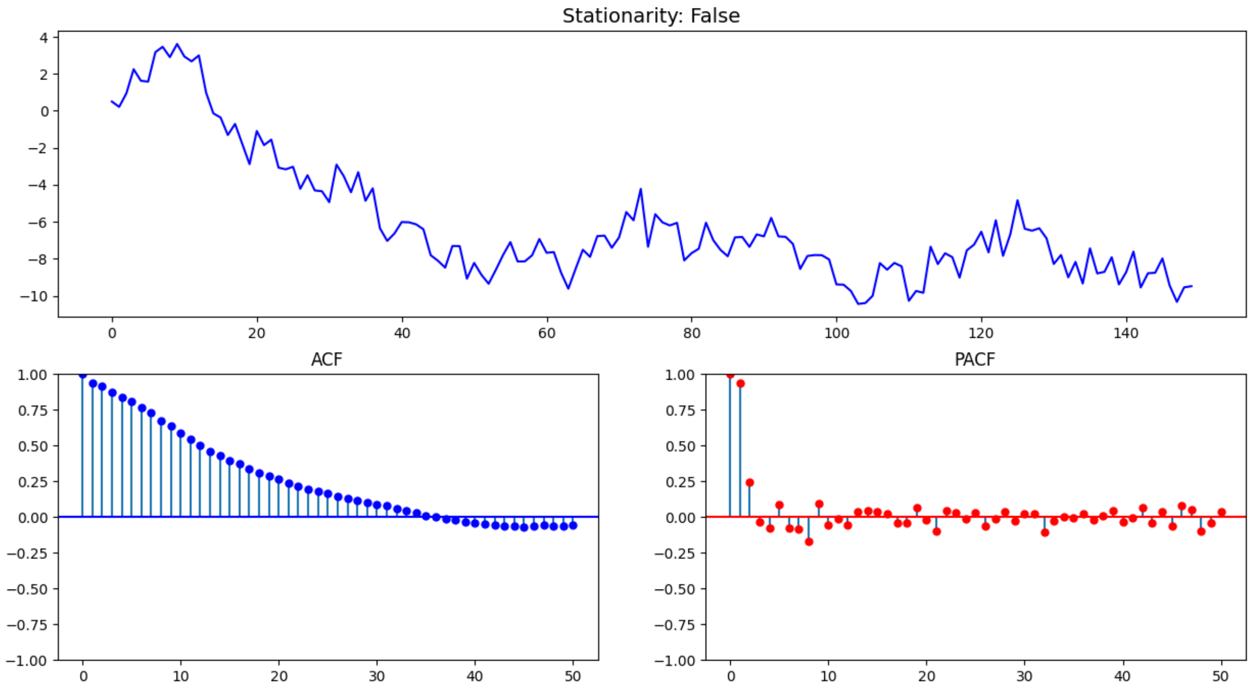
1. Equation $\varphi(B)(1-B)^d X_t = \mu + \theta(B)\epsilon_t$ with $Y_t = (1-B)^d X_t$ is stationary is called an ARIMA(p,d,q) model.
2. A time series $\{X_t\}$ that satisfies $\varphi(B)(1-B)^d X_t = \mu + \theta(B)\epsilon_t$ is said to be ARIMA(p,d,q) process.

Example 1

Below is the example of nonstationary time series.

In [7]:

```
demo.nonstationarity_simulation()
```

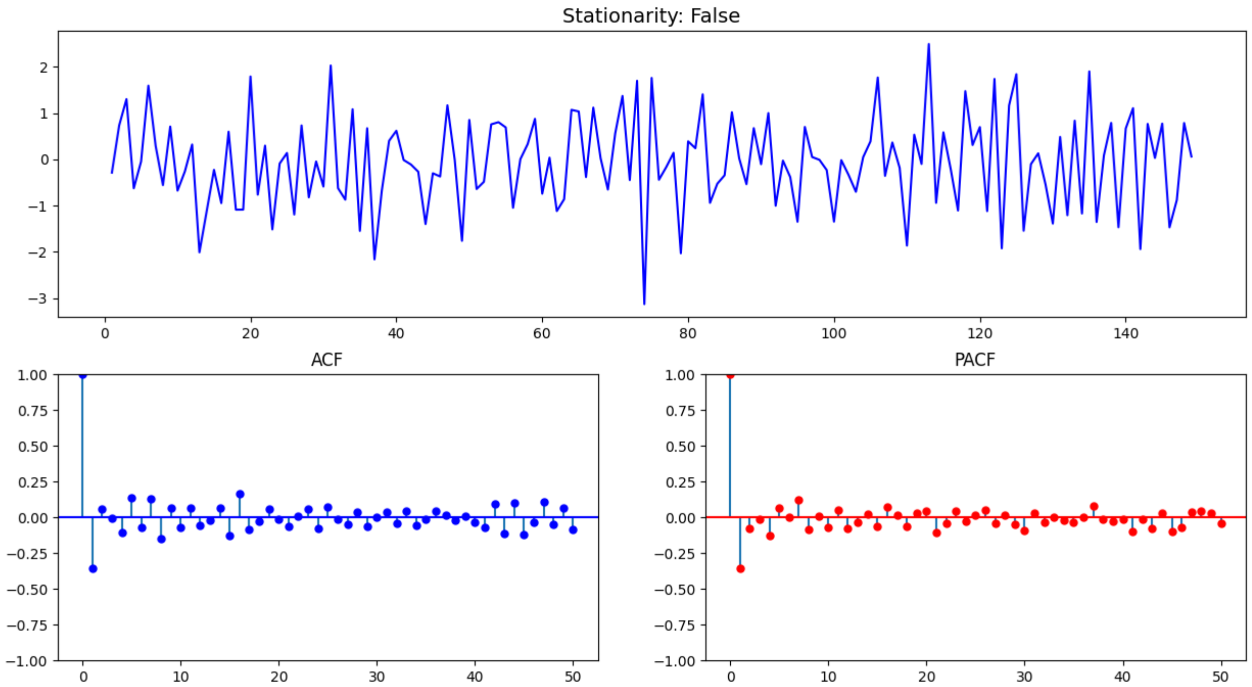


As for the PACF, maybe we can see that it cuts of after lag 1 or 2 for guessing AR(1) or AR(2), but the ACF shows its value that almost all significant and decreasing slowly. In addition, the data plot shows the trend which is the sign of nonstationarity.

Here we will try to do differencing one time.

In [8]:

```
demo.nonstationarity_simulation(diff=True)
```



In this result, we can say that the model is ARIMA(1,1,1).

Overview of AIC, BIC, HQIC

Akaike Information Criterion (AIC)

```
AIC = -2(maximized log likelihood) + 2(No. of estimated parameters)
```

Bayesian Information Criterion (BIC)

```
BIC = -2(maximized log likelihood)+log(n)(No. of estimated parameters)
```

Hannan-Quinn Information Criterion (HQIC)

```
HQIC = -2(maximized log likelihood) + log log(n)(No. of estimated parameters)
```

Above are information criterions of model that we can use to choose the model by minimizing those values. However they are not good to select the order of differencing (d) of an ARIMA (p,d,q). Then it is recommended to do differencing in advance.

Example 2

In [9]:

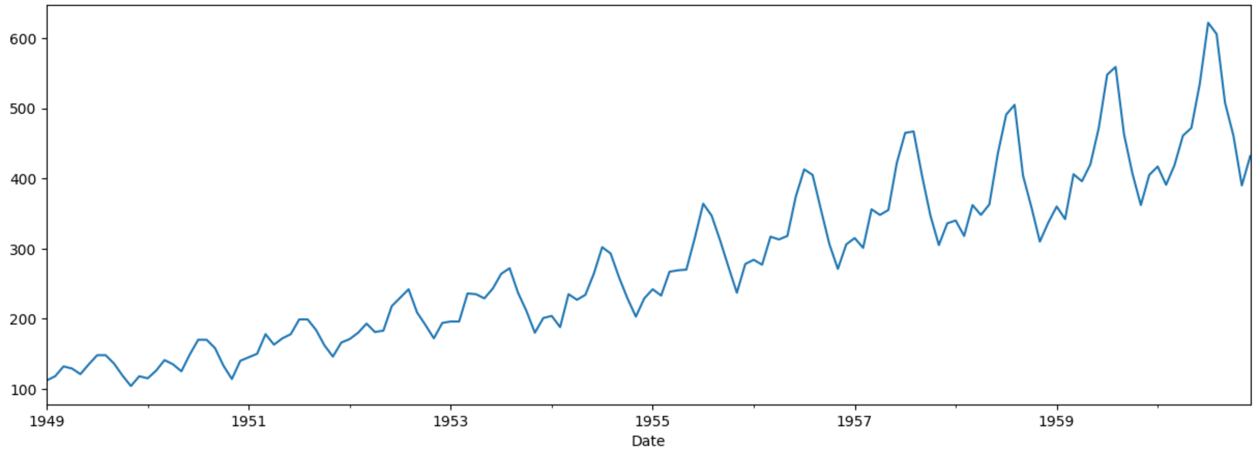
```
from pmdarima import auto_arima
```

In [10]:

```
df = pd.read_csv("data/airline.csv", index_col="Date", parse_dates=["Date"])
ts = df.passengers
X_train, X_test = ts[:-25],ts[-25:]
ts.plot(figsize=(15,5))
```

Out[10]:

```
<Axes: xlabel='Date'>
```



In [11]:

```
arima = auto_arima(X_train, seasonal=True, m=12, information_criterion="bic", trace=True, suppress_warnings=True, ran
```

```
Performing stepwise search to minimize bic
 ARIMA(2,0,2)(1,1,1)[12] intercept   : BIC=827.245, Time=1.29 sec
 ARIMA(0,0,0)(0,1,0)[12] intercept   : BIC=905.268, Time=0.02 sec
 ARIMA(1,0,0)(1,1,0)[12] intercept   : BIC=814.145, Time=0.35 sec
 ARIMA(0,0,1)(0,1,1)[12] intercept   : BIC=863.209, Time=0.13 sec
 ARIMA(0,0,0)(0,1,0)[12]             : BIC=1054.346, Time=0.02 sec
 ARIMA(1,0,0)(0,1,0)[12] intercept   : BIC=810.275, Time=0.03 sec
 ARIMA(1,0,0)(0,1,1)[12] intercept   : BIC=814.218, Time=0.19 sec
 ARIMA(1,0,0)(1,1,1)[12] intercept   : BIC=818.746, Time=0.50 sec
 ARIMA(2,0,0)(0,1,0)[12] intercept   : BIC=812.370, Time=0.06 sec
 ARIMA(1,0,1)(0,1,0)[12] intercept   : BIC=813.000, Time=0.05 sec
 ARIMA(0,0,1)(0,1,0)[12] intercept   : BIC=859.035, Time=0.10 sec
 ARIMA(2,0,1)(0,1,0)[12] intercept   : BIC=815.411, Time=0.06 sec
 ARIMA(1,0,0)(0,1,0)[12]             : BIC=814.329, Time=0.01 sec

Best model:  ARIMA(1,0,0)(0,1,0)[12] intercept
Total fit time: 2.814 seconds
```

In [12]:

```
arima.summary()
```

Out[12]:

SARIMAX Results

Dep. Variable:	y	**No. Observations:**	119
Model:	SARIMAX(1, 0, 0)x(0, 1, 0, 12)	**Log Likelihood**	-398.128
Date:	Tue, 14 Mar 2023	**AIC**	802.257
Time:	13:51:55	**BIC**	810.275
Sample:	01-01-1949	**HQIC**	805.507
	- 11-01-1958		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
intercept	5.6255	2.077	2.709	0.007	1.555	9.696
ar.L1	0.7908	0.067	11.868	0.000	0.660	0.921
sigma2	98.9310	11.756	8.415	0.000	75.889	121.973

Ljung-Box (L1) (Q):	1.87	**Jarque-Bera (JB):**	2.57
Prob(Q):	0.17	**Prob(JB):**	0.28
Heteroskedasticity (H):	1.36	**Skew:**	-0.08
Prob(H) (two-sided):	0.36	**Kurtosis:**	3.74

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [13]:

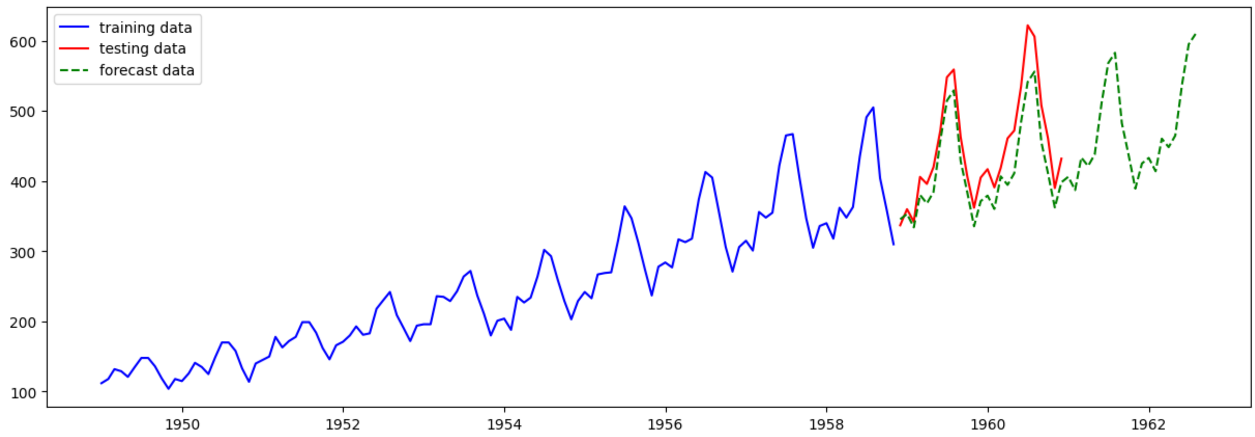
```
nf = len(X_test)+20 # predict additional 20 data in future from the test data
pred = arima.predict(nf)
```

In [14]:

```
plt.figure(figsize=(15,5))
plt.plot(X_train, "b-", label="training data")
plt.plot(X_test, "r-", label="testing data")
plt.plot(pred, "g--", label="forecast data")
plt.legend()
```

Out[14]:

<matplotlib.legend.Legend at 0x1659f7dc0>



Example 3

In [15]:

```
nao = pd.read_csv("data/nao.csv", header=0)
```

In [16]:

```
nao.head()
```

Out[16]:

	year	month	index
0	1950	1	0.92
1	1950	2	0.40
2	1950	3	-0.36
3	1950	4	0.73
4	1950	5	-0.59

In [17]:

```
tidx = pd.date_range('1950-01',periods=len(nao),freq="M")
nao.index=tidx
nao_ts = nao['index']
```

In [18]:

```
nao.head()
```

Out[18]:

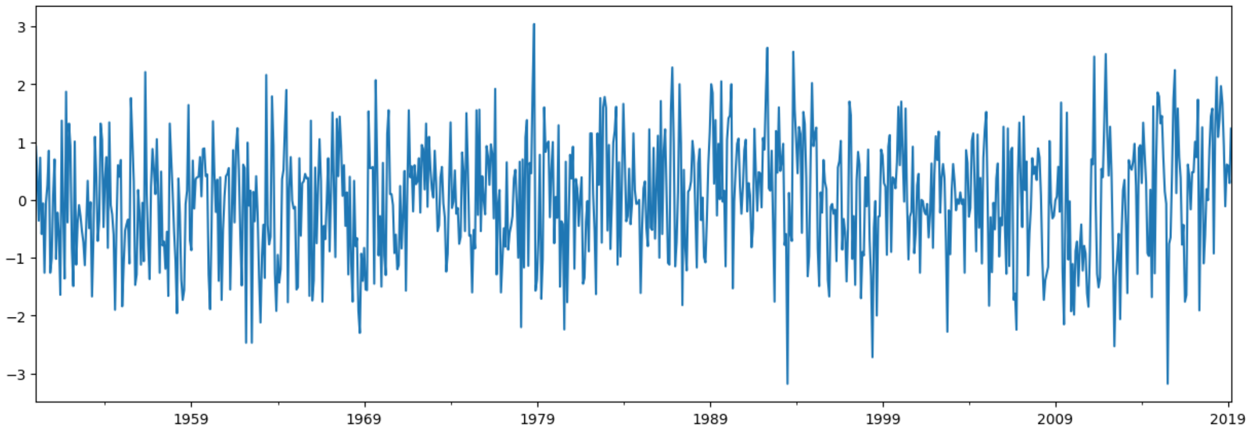
	year	month	index
1950-01-31	1950	1	0.92
1950-02-28	1950	2	0.40
1950-03-31	1950	3	-0.36
1950-04-30	1950	4	0.73
1950-05-31	1950	5	-0.59

In [19]:

```
X_train, X_test = nao_ts[:-25],nao_ts[-25:]
nao_ts.plot(figsize=(15,5))
```

Out[19]:

<Axes: >



In [20]:

```
arima = auto_arima(X_train, information_criterion="aic", trace=True, suppress_warnings=True, random=42)
```

```
Performing stepwise search to minimize aic
 ARIMA(2,0,2)(0,0,0)[0] intercept   : AIC=2282.724, Time=0.15 sec
 ARIMA(0,0,0)(0,0,0)[0] intercept   : AIC=2308.217, Time=0.01 sec
 ARIMA(1,0,0)(0,0,0)[0] intercept   : AIC=2280.177, Time=0.02 sec
 ARIMA(0,0,1)(0,0,0)[0] intercept   : AIC=2282.394, Time=0.03 sec
 ARIMA(0,0,0)(0,0,0)[0]             : AIC=2306.441, Time=0.01 sec
 ARIMA(2,0,0)(0,0,0)[0] intercept   : AIC=2281.772, Time=0.03 sec
 ARIMA(1,0,1)(0,0,0)[0] intercept   : AIC=2281.836, Time=0.05 sec
 ARIMA(2,0,1)(0,0,0)[0] intercept   : AIC=2283.749, Time=0.11 sec
 ARIMA(1,0,0)(0,0,0)[0]             : AIC=2278.319, Time=0.01 sec
 ARIMA(2,0,0)(0,0,0)[0]             : AIC=2279.905, Time=0.02 sec
 ARIMA(1,0,1)(0,0,0)[0]             : AIC=2279.970, Time=0.03 sec
 ARIMA(0,0,1)(0,0,0)[0]             : AIC=2280.553, Time=0.02 sec
 ARIMA(2,0,1)(0,0,0)[0]             : AIC=2281.883, Time=0.05 sec

Best model:  ARIMA(1,0,0)(0,0,0)[0]
Total fit time: 0.538 seconds
```

In [21]:

```
arima.summary()
```

Out[21]:

SARIMAX Results

Dep. Variable:	y	**No. Observations:**	806
Model:	SARIMAX(1, 0, 0)	**Log Likelihood**	-1137.159
Date:	Tue, 14 Mar 2023	**AIC**	2278.319
Time:	13:51:55	**BIC**	2287.703
Sample:	01-31-1950	**HQIC**	2281.923
	- 02-28-2017		
Covariance Type:	opg		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
ar.L1	0.1916	0.034	5.678	0.000	0.125	0.258
sigma2	0.9837	0.053	18.398	0.000	0.879	1.089

Ljung-Box (L1) (Q):	0.02	**Jarque-Bera (JB):**	4.65
Prob(Q):	0.90	**Prob(JB):**	0.10
Heteroskedasticity (H):	0.97	**Skew:**	-0.09
Prob(H) (two-sided):	0.79	**Kurtosis:**	2.68

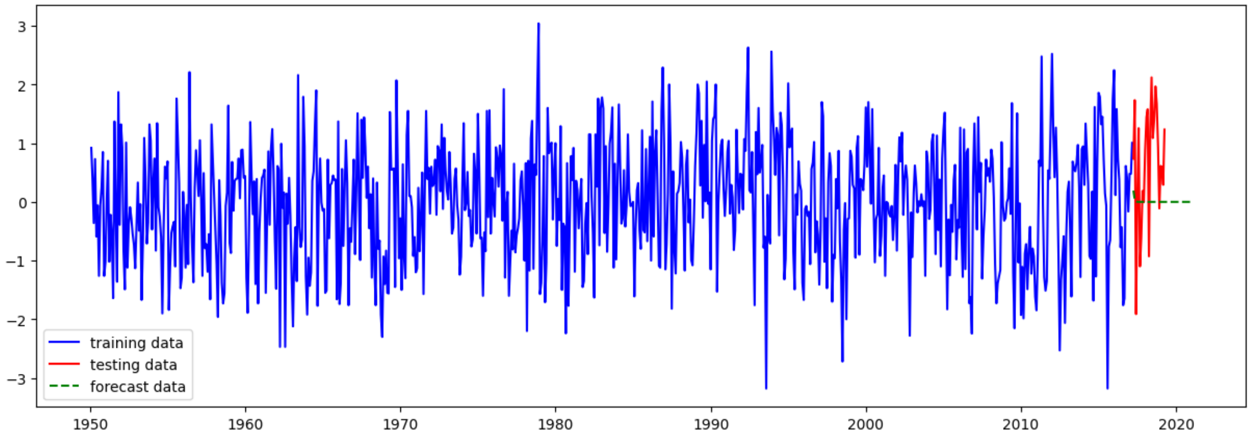
Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [22]:

```
nf = len(X_test)+20 # predict additional 20 data in future from the test data
pred = arima.predict(nf)

plt.figure(figsize=(15,5))
plt.plot(X_train, "b-", label="training data")
plt.plot(X_test, "r-", label="testing data")
plt.plot(pred, "g--", label="forecast data")
plt.legend()
```

Out[22]:

```
<matplotlib.legend.Legend at 0x165b05ed0>
```



In [ ]: