Bond Pricing and Duration

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Duration

- To deal with the ambiguity of the “maturity” of a bond making many payments, we need a measure of the average maturity of the bond’s promised cash flows.

- **Duration**: The average lifetime of a debt security’s stream of payments. It is a measure of effective maturity.

$$DUR = \sum_{t=1}^{n} t \frac{CF_t}{(1+i)^t} \Big/ P$$

where P is the bound price, and CF_t is the cash flow at time t

- $CF_t = C$ for $t = 1, 2, \ldots, n-1$ and $CF_t = C + F$.

Duration

- Define the weight w_t as the ratio of discounted cash flow over price

$$w_t = \frac{CF_t/(1+i)^t}{P}$$

- Duration is

$$DUR = \sum_{t=1}^{n} t \cdot w_t$$

Duration

- Example: Calculating duration on a $1, 000 2-year 10% coupon bond when its interest rate is 10%.

Year	Cashflow	PV of Cashflow	Weight [%]	Weighted Maturity	
1	$100	$90.91	9.091	0.09091	
2	$1,100	$909.09	90.909	1.81818	
Total				1.90909	(duration)

Duration

- Key facts about duration:
  - All else equal, when the maturity of a bond lengthens, the duration rises as well.
  - All else equal, when interest rates rise, the duration of a coupon bond fall.
  - The higher is the coupon rate on the bond, the shorter is the duration of the bond.

- Duration is additive: the duration of a portfolio of securities is the weighted-average of the durations of the individual securities, with the weights equaling the proportion of the portfolio invested in each.

Duration and Interest-Rate Risk

- How is the duration used to measure interest rate risk?

$$\%\Delta P \approx -DUR \cdot \frac{\Delta i}{1+i}$$

where $\%\Delta P$ is the rate of capital gain and i is the interest rate.

- Example: Consider the 10-year Treasury bond with duration of 9.1 years.
  - If $i \uparrow 10\%$ to 11%, then $\%\Delta P \approx -9.1 \cdot \frac{0.01}{1+0.1} = -8.27\%$

Duration and Interest-Rate Risk

- We take derivative of price P wrt interest rate i

$$P = \sum_{t=1}^{n} \frac{CF_t}{(1+i)^t} = \sum_{t=1}^{n} CF_t (1+i)^{-t}$$

$$\frac{dP}{di} = \sum_{t=1}^{n} CF_t (-t)(1+i)^{-t-1} = -(1+i)^{-1} \sum_{t=1}^{n} t \cdot CF_t (1+i)^{-t}$$

$$\frac{\Delta P}{\Delta i} \approx \frac{dP}{di} = -\frac{1}{1+i} \sum_{t=1}^{n} t \frac{CF_t}{(1+i)^t}$$

- Consider percentage change by dividing current price

$$\frac{\Delta P}{P} \approx -\frac{1}{1+i} \underbrace{\sum_{t=1}^{n} t \frac{CF_t}{(1+i)^t} \frac{1}{P}}_{\text{Duration}} \Delta i = -DUR \frac{\Delta i}{1+i}$$

Duration and Interest-Rate Risk

- The greater is the duration of a security, the greater is the percentage change in the market value of the security for a given change in interest rates.

- Therefore, the greater is the duration of a security, the greater is its interest-rate risk.

- The longer the maturity of a bond, ceteris paribus, the greater the sensitivity of the bond's price to interest rate changes.