# Lecture Note.
# COMPOUND RATE OF RETURN

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# The Compound Rate of Return

- Is the rate at which an investment would have grown if it had grown at the same rate every period and the profits were reinvested at the end of each period (typically one year).

- Let:
  - $P$ = Principal (or initial Capital)
  - $A$ = Accrued amount (principal + net profit) over the investment period
  - $t$ = time in decimal years; e.g., 6 months is calculated as $t = 0.5$ years. Five years means $t = 5$.
  - $n$ = number of compounding periods per unit of time $t$

- The compound interest formula is

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

# The Compound Rate of Return

- Thus the per period RoR is:

$$r = n \left( \left( \frac{A}{P} \right)^{\frac{1}{nt}} - 1 \right)$$

- When $n = 1,$ we get the annual RoR:

$$r = \left( \frac{A}{P} \right)^{\frac{1}{t}} - 1$$

- Multiply by 100 to convert r into a percentage

# The Continuously Compound RoR

- With continuous compounding, the number of times compounding occurs per period approaches infinity or $n \to \infty$.

- Then using our original equation

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

to solve for $A$ as $n \to \infty$ we have:

$$A = P\left(\lim_{n \to \infty}\left(1 + \frac{r}{n}\right)^{nt}\right)$$

We introduce a variable $m = \frac{n}{r}$, so that $n = mr$. Note that as $n \to \infty$ so does $m$.

# The Continuously Compound RoR

- Replacing $n$ in our equation with $mr$ and cancelling $r$ in the numerator of $r/n$ we get:

$$A = P \left( \lim_{m \to \infty} \left( 1 + \frac{1}{m} \right)^m \right)^{rt}$$

- Let us use the definition of the constant $e$:

$$e = \lim_{m \to \infty} \left( 1 + \frac{1}{m} \right)^m$$

- Replacing $\lim_{m \to \infty} \left( 1 + \frac{1}{m} \right)^m$ by $e$, we get the continuous compounding formula

$$A = P \cdot e^{rt}$$

- Thus, we have that the Continuously Compounded RoR is $r = \dfrac{\ln \left( \frac{A}{P} \right)}{t}$

- **LEVERAGE = Initial Investment / Initial Capital**

- **Increasing leverage:**
  - Increases RoR when returns are positive
  - Reduces RoR when returns are negative

- **Excess Returns:**
  - r(strategy) – r(safe)
  - r(safe) is the treasury interest rate