INNA WILLIAMS

Section 2.1

$\boxed{7}$ Number of op. per sec (RATE) $= \frac{4000^2}{0.002}$

Operation count $= \frac{2}{3}n^3$, $n = 9000$

$$Time_{9000\times9000} = \frac{\frac{2}{3}n^3 \cdot 0{,}002}{4000^2}$$

$$Time(9000) = \frac{2}{3} \cdot \frac{9000^3 \cdot 0.002}{4000^2} = \frac{2\cdot 9^3 \cdot \cancel{10^9} \cdot 2 \cdot \cancel{10^{-3}}}{3\cdot 4^2 \cdot \cancel{10^6}} =$$

$$= \frac{2\cdot 9^3 \cdot 2}{3\cdot 4^2} = \frac{243}{4} = 60.75 \approx 61 \text{ seconds}$$

Answer: Time needed to solve a general system of 9000 equations in 9000 unknowns equal $\boxed{61}$ seconds

$\boxed{6}$ $RATE = \frac{5000^2}{0.005}$

$$Time = \frac{\text{Number of Operations}}{RATE} = \frac{\left(n^2 + \frac{2}{3}n^3\right)\cdot 0.005}{5000^2}$$

$$Time(5000) = \frac{\left(5000^2 + \frac{2}{3}\cdot 5000^3\right)\cdot 0.005}{5000^2} =$$

$$= \frac{5000^2}{5000^2}\left(1 + \frac{2\cdot 5000}{3}\right)\cdot 0.005 = \left(1 + \frac{10000}{3}\right)\cdot \frac{5}{1000} =$$

$$= \frac{5}{1000} + \frac{5}{1000}\cdot\frac{10000}{3} = 0.005 + \frac{50}{3} = 0.005 + 16.6667$$

$$= 16.672 \approx 17 \text{ seconds}$$

Answer: It will take $\boxed{17}$ seconds

to do a Complete Gaussien elimination.

Scanned by CamScanner

INNA WILLIAMS

Section 2.2.

$\boxed{2a}$

$$A = \begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 0 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = U$$

$$L = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$L \times U = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{vmatrix} = A$$

<u>Answer:</u>

$$L = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \quad U = \begin{vmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix}$$

$\boxed{4a}$

$$A = \begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{vmatrix} = L \cdot U = \underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}}_{L} \times \underbrace{\begin{vmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix}}_{U} \leftarrow \text{from } 2a$$

$$\begin{vmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{vmatrix} \times \underbrace{\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}}_{x} = \underbrace{\begin{vmatrix} 0 \\ 1 \\ 3 \end{vmatrix}}_{b}$$

$LC = b \quad , U \cdot x = C$

$$LC = b \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} c_1 \\ c_2 \\ c_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 3 \end{vmatrix} \Rightarrow \begin{matrix} c_1 \cdot 1 = 0 & \Rightarrow c_1 = 0 \\ c_1 \cdot 2 + c_2 = 1 & \Rightarrow c_2 = 1 \\ c_1 + c_3 = 3 & \Rightarrow c_3 = 3 \end{matrix}$$

$$U \cdot x = c \Rightarrow \begin{vmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} \times \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 3 \end{vmatrix} \Rightarrow \begin{matrix} 3x_3 = 3 & \Rightarrow x_3 = 1 \\ x_2 = 1 & \Rightarrow x_2 = 1 \\ 3x_1 + 1 \cdot 1 + 2 \cdot 1 = 0 & \Rightarrow x_1 = -1 \end{matrix}$$

Check if $\{x_1, x_2, x_3\} = \{1, 1, -1\}$ is a solution

$$\left.\begin{matrix} 3x_1 + x_2 + 2x_3 = 3 \cdot (-1) + 1 + 2 = 0 \\ 6 \cdot x_1 + 3 \cdot x_2 + 4 \cdot x_3 = 6(-1) + 3 + 4 = 1 \\ 3 \cdot x_1 + x_2 + 5 \cdot x_3 = 3(-1) + 1 + 5 = 3 \end{matrix}\right\} \Rightarrow x_1, x_2, x_3 \text{ is a solution}$$

Answer: $\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ -1 \end{vmatrix}$

$$\boxed{\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ -1 \end{vmatrix}}$$

INNA WILLIAMS

Section 2.3

$\boxed{1|b}$

$$A = \begin{vmatrix} 1 & 5 & 1 \\ -1 & 2 & -3 \\ 1 & -7 & 0 \end{vmatrix} \qquad \|A\|_\infty - ?$$

$$\|A\|_\infty = \max \begin{Bmatrix} Sum\{|1|+|5|+|1|\} = 7 \\ Sum\{|-1|+|2|+|-3|\} = 6 \\ Sum\{|1|+|-7|+|0|\} = 8 \end{Bmatrix} = \boxed{8}$$

Answer: $\|A\|_\infty = 8$

$\boxed{2|b}$

$$A = \begin{vmatrix} 1 & 2.01 \\ 3 & 6 \end{vmatrix} \qquad \|A\|_\infty = \max \begin{Bmatrix} Sum\{|1|+|2.01|\} \\ Sum\{|3|+|6|\} \end{Bmatrix} =$$

$$= \max \begin{Bmatrix} 3.01 \\ 9 \end{Bmatrix} = \boxed{9}$$

Condition Number $= \|A\|_\infty \times \|A^{-1}\|_\infty$

$$A^{-1} = \frac{1}{6 - 3 \cdot 2.01} \cdot \begin{vmatrix} 6 & -2.01 \\ -3 & 1 \end{vmatrix} = (-33.33) \cdot \begin{vmatrix} 6 & -2.01 \\ -3 & 1 \end{vmatrix}$$

$$\|A^{-1}\|_\infty = |-33.33| \cdot \max \begin{Bmatrix} Sum\{|6|+|-2.01|\} \\ Sum\{|-3|+|1|\} \end{Bmatrix} =$$

$$= 33.33 \cdot \max(8.01, 4) = 33.33 \cdot 8.01 = 267.0$$

$Condition\_Number = \|A\|_\infty \cdot \|A^{-1}\|_\infty = 9 \cdot 267 = 24..$

Answer: Condition Number = 2403

$\|A\|_\infty = 9$

$\boxed{3|e}$ $FE = |x - x_a| = \left|\begin{matrix}1\\1\end{matrix}\right| - \left|\begin{matrix}-2\\4.0001\end{matrix}\right| = \left|\begin{matrix}-3\\-3.0001\end{matrix}\right|$

$\|x - x_a\|_\infty = \boxed{3.0001}$

$BE = b - A \cdot x_a = \left|\begin{matrix}2\\2.0001\end{matrix}\right| - \left|\begin{matrix}1 & 1\\1.0001 & 1\end{matrix}\right| \cdot \left|\begin{matrix}-2\\4.0001\end{matrix}\right| =$

$= \left|\begin{matrix}2\\2.0001\end{matrix}\right| - \left|\begin{matrix}2.0001\\1.9999\end{matrix}\right| = \left|\begin{matrix}0.0001\\0.0002\end{matrix}\right|$

$\|b - A \cdot x_a\|_\infty = \boxed{0.0002}$

Error Magnification factor $= \dfrac{3.0001 / 1}{0.0002 / 2.0001} =$

$= \boxed{30002.5}$

Answer: $FE = 3.0001$

$BE = 0.0002$

$EMF = 30002.5$

INNA WILLIAMS

Section 2.5.

$\boxed{1}\boxed{6}$ $[u_0\ v_0\ w_0] = [0, 0, 0]$

$$\begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} \times \begin{vmatrix} u \\ v \\ w \end{vmatrix} = \begin{vmatrix} 0 \\ 2 \\ 0 \end{vmatrix}$$

Jacobi equations are:

$$u_{k+1} = \frac{1}{2} \cdot v_k$$

$$v_{k+1} = \frac{1}{2}(2 + u_k + w_k)$$

$$w_{k+1} = \frac{1}{2} v_k$$

Step 1

$$u_1 = \frac{0}{2} = \boxed{0}$$

$$v_1 = \frac{0+0+2}{2} = \boxed{1}$$

$$w_1 = \frac{0}{2} = \boxed{0}$$

Step 2

$$u_2 = \boxed{\frac{1}{2}}$$

$$v_2 = \frac{0+0+2}{2} = \boxed{1}$$

$$w_2 = \boxed{\frac{1}{2}}$$

Gauss-Seidel equations are:

$$u_{k+1} = \frac{1}{2} v_k$$

$$v_{k+1} = \frac{u_{k+1} + w_k + 2}{2}$$

$$w_{k+1} = \frac{1}{2} \cdot v_{k+1}$$

Step 1

$$u_1 = \boxed{0}$$

$$v_1 = \frac{0+0+2}{2} = \boxed{1}$$

$$w_1 = \frac{0+1}{2} = \boxed{\frac{1}{2}}$$

Step 2

$$u_2 = \boxed{\frac{1}{2}}$$

$$v_2 = \frac{\frac{1}{2}+\frac{1}{2}+2}{2} = \boxed{\frac{3}{2}}$$

$$w_2 = \frac{\frac{3}{2}}{2} = \boxed{\frac{3}{4}}$$

Answer :

Jacobi :

Step 1
$$\begin{vmatrix} u_1 \\ v_1 \\ w_1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$$

Step 2
$$\begin{vmatrix} u_2 \\ v_2 \\ w_2 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{vmatrix}$$

Gauss-Seidel :

Step 1
$$\begin{vmatrix} u_1 \\ v_1 \\ w_1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ \frac{1}{2} \end{vmatrix}$$

Step 2
$$\begin{vmatrix} u_2 \\ v_2 \\ w_2 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} \\ \frac{3}{2} \\ \frac{3}{4} \end{vmatrix}$$

INNA WILLIAMS

Section 2.5

$\boxed{26}$

$$u - 8v - 2w = 1$$
$$u + v + 5w = 4$$
$$3u - v + w = -2$$

Strictly diagonal:

$$\left|\begin{array}{ccc|c} 3 & -1 & 1 & -2 \\ 1 & -8 & -2 & 1 \\ 1 & 1 & 5 & 4 \end{array}\right|$$

Jacoby:

$$u_{k+1} = \frac{1}{3}\left(-2-(-v_k+w_k)\right) = \frac{1}{3}\left(-2+v_k-w_k\right)$$
$$v_{k+1} = -\frac{1}{8}\left(1-(v_k-2w_k)\right) = -\frac{1}{8}\left(1-u_k+2w_k\right)$$
$$w_{k+1} = \frac{1}{5}\left(4-(v_k-2w_k)\right) = \frac{1}{5}\left(4-u_k-v_k\right)$$

Step 1

$$u_1 = \frac{1}{3}\cdot(-2) = \boxed{-\frac{2}{3}}$$
$$v_1 = -\frac{1}{8}(1) = \boxed{-\frac{1}{8}}$$
$$w_1 = \frac{1}{5}(4) = \boxed{\frac{4}{5}}$$

Step 2

$$u_2 = \frac{1}{3}\left(-2-\frac{1}{8}-\frac{4}{5}\right) = \frac{1}{3}\left(\frac{-80-5-32}{40}\right) = \frac{-117}{120} = \boxed{-\frac{39}{40}}$$
$$v_2 = -\frac{1}{8}\left(1+\frac{2}{3}+2\cdot\frac{4}{5}\right) = -\frac{1}{8}\left(\frac{15}{15}+\frac{10}{15}+\frac{24}{15}\right) = \boxed{-\frac{49}{120}}$$
$$w_2 = \frac{1}{5}\left(4+\frac{2}{3}+\frac{1}{8}\right) = \frac{1}{5}\left(\frac{96}{24}+\frac{16}{24}+\frac{3}{24}\right) = \frac{115}{5\cdot 24} = \boxed{\frac{23}{24}}$$

Gauss-Seidel:

$$u_{k+1} = \frac{1}{3}\left(-2+v_k-w_k\right)$$
$$v_{k+1} = -\frac{1}{8}\left(1-u_{k+1}+2w_k\right)$$
$$w_{k+1} = \frac{1}{5}\left(4-u_{k+1}-v_{k+1}\right)$$

Step 1:

$$u_1 = \boxed{-\frac{2}{3}}$$
$$v_1 = -\frac{1}{8}\left(1+\frac{2}{3}+0\right) = -\frac{1}{8}\cdot\frac{5}{3} = \boxed{-\frac{5}{24}}$$
$$w_1 = \frac{1}{5}\left(\frac{24\cdot 4}{24}+\frac{16}{24}+\frac{5}{24}\right) = \boxed{\frac{39}{40}}$$

Step 2

$$u_2 = \frac{1}{3}\left(-2-\frac{5}{24}-\frac{39}{40}\right) = -\frac{3056}{960\cdot 3} = \boxed{-\frac{191}{180}}$$
$$v_1 = -\frac{1}{8}\left(1+\frac{191}{180}+2\cdot\frac{39}{40}\right) = \boxed{-\frac{361}{720}}$$
$$w_1 = \frac{1}{5}\left(4+\frac{361}{720}+\frac{191}{180}\right) = \boxed{\frac{89}{80}}$$

Answer :

Jacobi :

Step 1 : $\begin{vmatrix} u_1 \\ v_1 \\ w_1 \end{vmatrix} = \begin{vmatrix} -\frac{2}{3} \\ -\frac{1}{8} \\ \frac{4}{5} \end{vmatrix}$ Step 2 : $\begin{vmatrix} u_2 \\ v_2 \\ w_2 \end{vmatrix} = \begin{vmatrix} -\frac{39}{40} \\ -\frac{49}{120} \\ \frac{23}{24} \end{vmatrix}$

Gauss-Seidel:

Step 1 : $\begin{vmatrix} u_1 \\ v_1 \\ w_1 \end{vmatrix} = \begin{vmatrix} -\frac{2}{3} \\ -\frac{5}{24} \\ \frac{39}{40} \end{vmatrix}$ Step2 $\begin{vmatrix} u_2 \\ v_2 \\ w_2 \end{vmatrix} = \begin{vmatrix} -\frac{191}{180} \\ -\frac{361}{720} \\ \frac{89}{80} \end{vmatrix}$

INNA WILLIAMS

Section 2.5

$\boxed{36}$ $\omega = 1.25$

$$\begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} \cdot \begin{vmatrix} u \\ v \\ w \end{vmatrix} = \begin{vmatrix} 0 \\ 2 \\ 0 \end{vmatrix}$$

$\boxed{SOR}$

$$u_{k+1} = (1-\omega)\cdot u_k + \omega \frac{v_k}{2}$$

$$v_{k+1} = (1-\omega)\cdot v_k + \omega \cdot \frac{u_{k+1} + w_k + 2}{2}$$

$$w_{k+1} = (1-\omega)\cdot w_k + \omega \cdot \frac{v_{k+1}}{2}$$

Step 1:

$$u_1 = \left(1 - \frac{3}{2}\right)\cdot 0 + \frac{3}{2}\cdot 0 = \boxed{0}$$

$$v_1 = \left(1 - \frac{3}{2}\right)\cdot 0 + \frac{3}{2}\cdot \frac{0+0+2}{2} = \boxed{\frac{3}{2}}$$

$$w_1 = \left(1 - \frac{3}{2}\right)\cdot 0 + \frac{3}{2}\cdot \frac{3}{2\cdot 2} = \boxed{\frac{9}{8}}$$

Step 2:

$$u_2 = \left(1 - \frac{3}{2}\right)\cdot 0 + 1.5\cdot \frac{3}{2\cdot 2} = \boxed{\frac{9}{8}}$$

$$v_2 = \left(1 - \frac{3}{2}\right)\cdot \frac{3}{2} + \frac{3}{2}\cdot\left(\frac{9}{8} + \frac{9}{8} + 2\right)/2 =$$

$$= -\frac{3}{4} + \frac{51}{16} = \boxed{\frac{39}{16}}$$

$$w_2 = \left(1 - \frac{3}{2}\right)\cdot \frac{9}{8} + \frac{3}{2}\cdot \frac{39}{16\cdot 2} = -\frac{9}{16} + \frac{117}{64}$$

$$= \boxed{-\frac{81}{64}}$$

Answer: Step 1:

$$\begin{vmatrix} u_1 \\ v_1 \\ w_1 \end{vmatrix} = \begin{vmatrix} 0 \\ \frac{3}{2} \\ \frac{9}{8} \end{vmatrix}$$

Step 2:

$$\begin{vmatrix} u_2 \\ v_2 \\ w_2 \end{vmatrix} = \begin{vmatrix} \frac{9}{8} \\ \frac{39}{16} \\ -\frac{81}{64} \end{vmatrix}$$