Inna Williams

###

Check writtern homework

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Section 10.1

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1

a)

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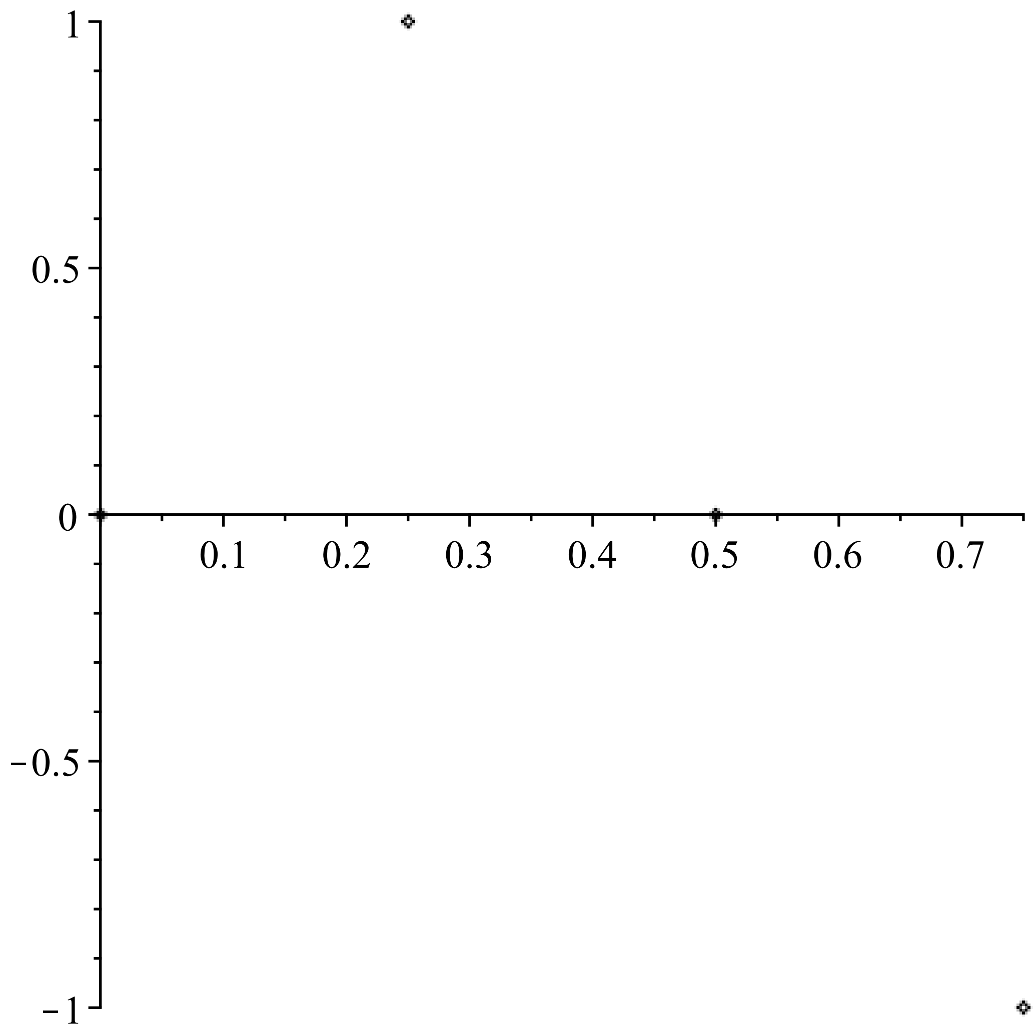
> $with(DiscreteTransforms) : with(LinearAlgebra) : with(plots) :$

> $V := Vector(4, [0, 1, 0, -1])$

$$V := \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad \textbf{(1)}$$

> $rule := i \rightarrow \frac{(i-1)}{4} : T := Vector(4, rule) :$

> $dataplot1 := pointplot(T, V) : display(dataplot1)$



> $Z := FourierTransform(V)$

$$Z := \begin{bmatrix} 0.\,I \\ -I \\ 0.\,I \\ I \end{bmatrix} \tag{2}$$

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The answer is the same as in written homework

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3

b

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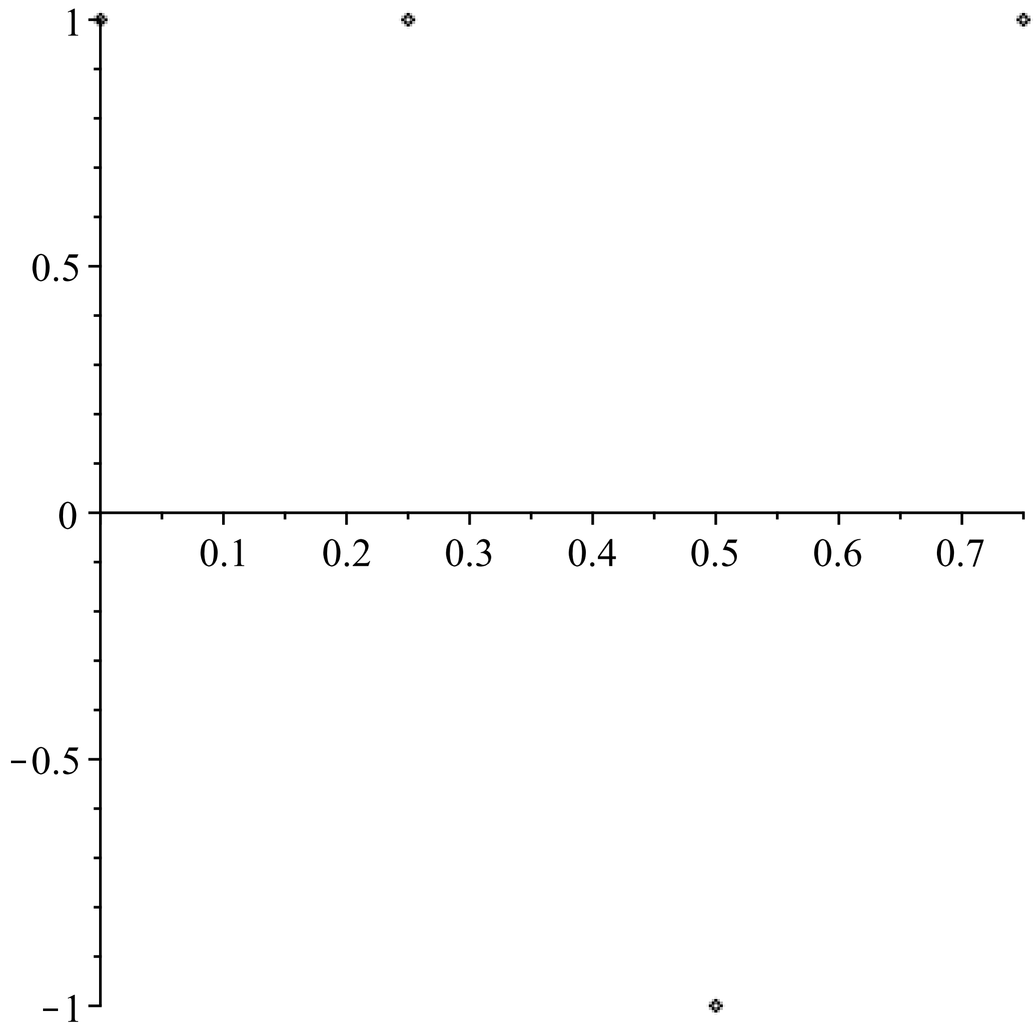
> $V := Vector(4, [1, 1, -1, 1])$

(3)

$$V := \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \tag{3}$$

> $rule := i \rightarrow \frac{(i-1)}{4} : T := Vector(4, rule) :$

> $dataplot1 := pointplot(T, V) : display(dataplot1)$



> $Z := InverseFourierTransform(V)$

$$Z := \begin{bmatrix} 1. + 0.\,I \\ 1. + 0.\,I \\ -1. + 0.\,I \\ 1. + 0.\,I \end{bmatrix} \tag{4}$$

****
The answer is the same as in written homework
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Section 10.2

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Computer Problems

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1

b

Find the order 8 trigonometric interpolating function P8(t) for the following data:

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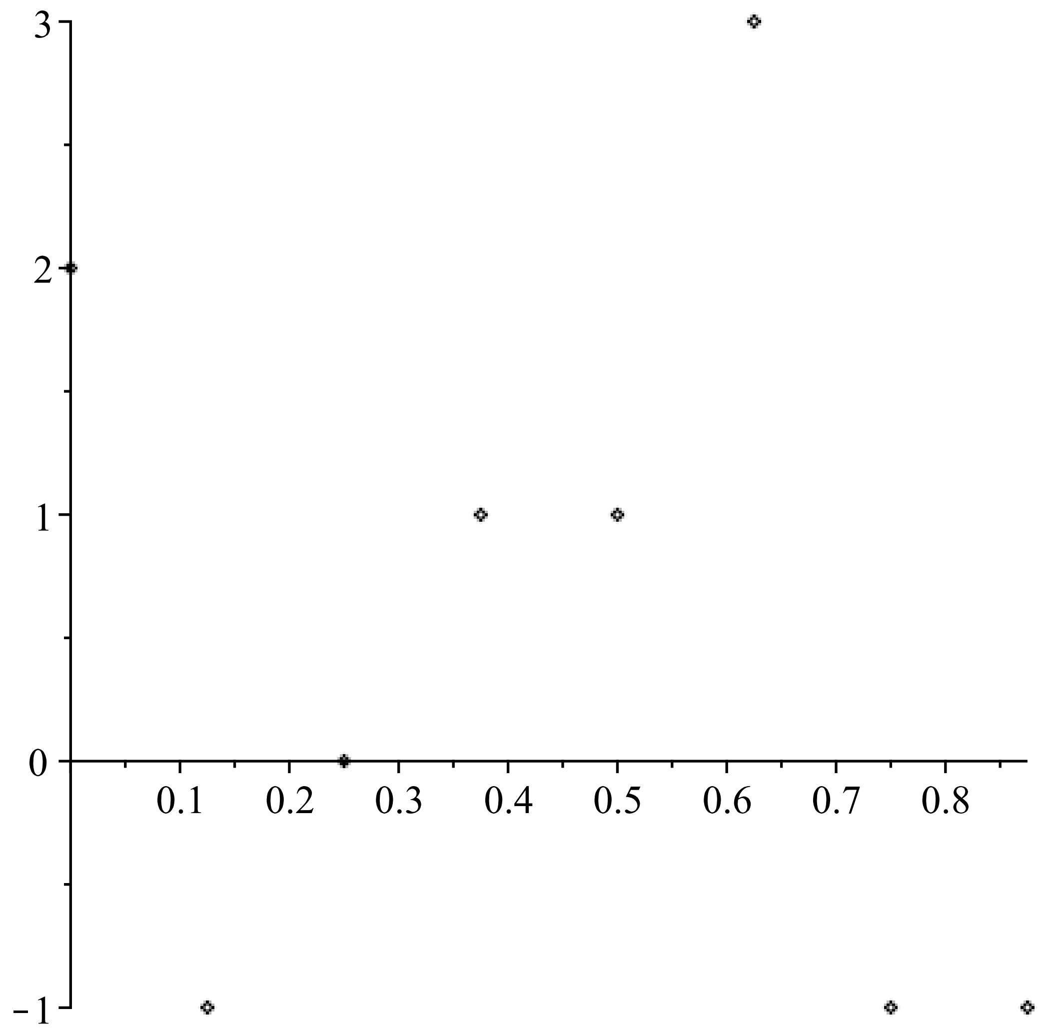
> $V := Vector(8, [2, -1, 0, 1, 1, 3, -1, -1])$

$$V := \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 1 \\ 3 \\ -1 \\ -1 \end{bmatrix} \qquad \textbf{(5)}$$

> $rule := i \rightarrow \frac{(i-1)}{8} : T := Vector(8, rule) :$

> $dataplot1 := pointplot(T, V) : display(dataplot1)$



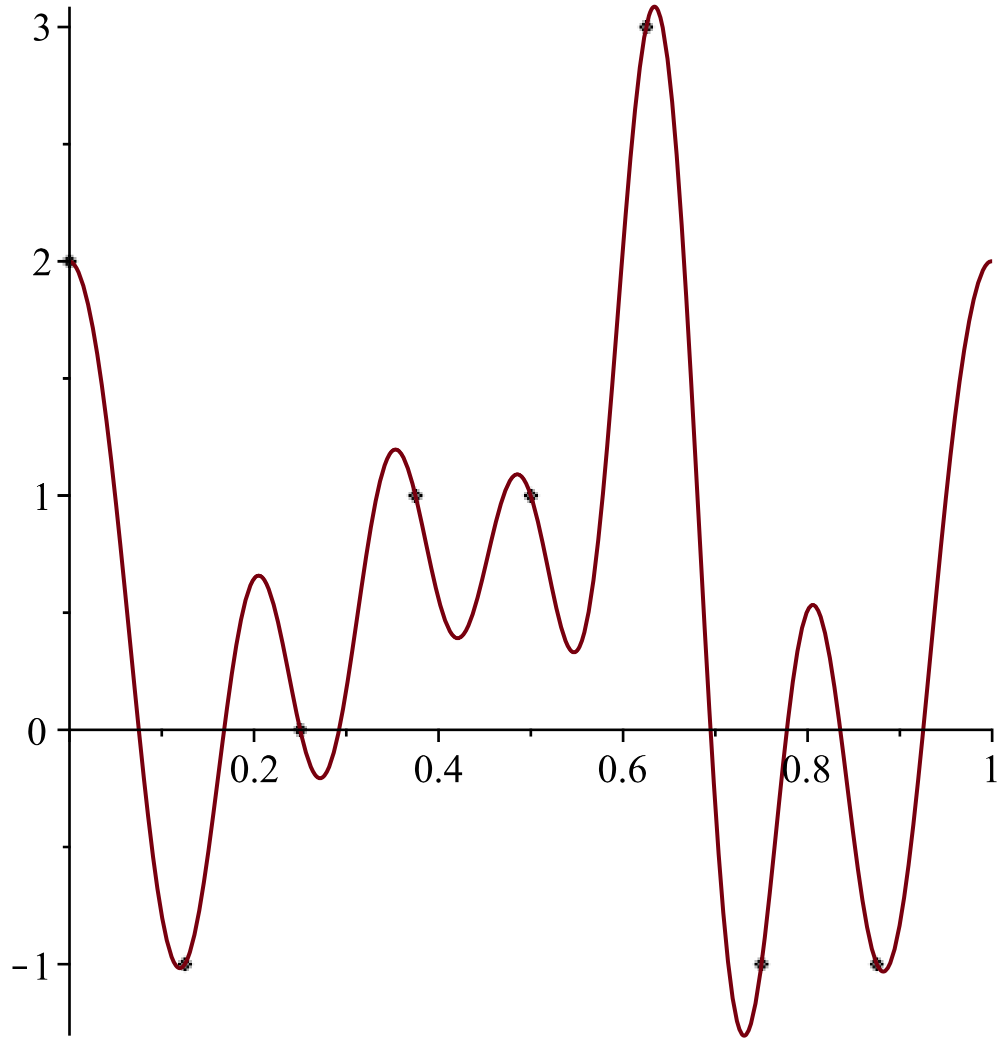
> $Z := FourierTransform(V)$

$$Z := \begin{bmatrix} 1.41421356237310 + 0.\,\mathrm{I} \\ -1.14644660940673 + 0.146446609406726\,\mathrm{I} \\ 1.41421356237310 - 0.707106781186548\,\mathrm{I} \\ 1.85355339059327 + 0.853553390593274\,\mathrm{I} \\ 0.\,\mathrm{I} \\ 1.85355339059327 - 0.853553390593274\,\mathrm{I} \\ 1.41421356237310 + 0.707106781186548\,\mathrm{I} \\ -1.14644660940673 - 0.146446609406726\,\mathrm{I} \end{bmatrix} \quad \textbf{(6)}$$

> $P := t \rightarrow evalf\left(\frac{1}{\mathrm{sqrt}(8)} \cdot add\left(\mathrm{Re}(Z[k]) \cdot \cos\left(2 \cdot (k-1) \cdot \pi \cdot t\right) - \mathrm{Im}(Z[k]) \cdot \sin\left(2 \cdot (k-1) \cdot \pi \cdot t\right), k = 1..8\right)\right):$

>

> $plot1 := plot(P, 0..1) : display(dataplot1, plot1)$



>

> $a := \mathrm{Re}(Z); b := \mathrm{Im}(Z)$

$$a := \begin{bmatrix} 1.41421356237310 \\ -1.14644660940673 \\ 1.41421356237310 \\ 1.85355339059327 \\ 0. \\ 1.85355339059327 \\ 1.41421356237310 \\ -1.14644660940673 \end{bmatrix}$$

$$b := \begin{bmatrix} 0. \\ 0.146446609406726 \\ -0.707106781186548 \\ 0.853553390593274 \\ 0. \\ -0.853553390593274 \\ 0.707106781186548 \\ -0.146446609406726 \end{bmatrix} \quad \textbf{(7)}$$

$> c := 0; d := 1; n := 8;\quad a[1] := evalf\left(\frac{a[1]}{n^{\frac{1}{2}}}\right) : a[2..8] := evalf\left(\frac{2}{n^{\frac{1}{2}}} \cdot a[2..8]\right) : b[1] := evalf\left(\frac{b[1]}{n^{\frac{1}{2}}}\right) : b[2..8] := evalf\left(\frac{2}{n^{\frac{1}{2}}} \cdot b[2..8]\right) : a; b;$

$$c := 0$$

$$d := 1$$

$$n := 8$$

$$\begin{bmatrix} 0.500000000044868 \\ -0.810660171852566 \\ 1.00000000008974 \\ 1.31066017189743 \\ 0. \\ 1.31066017189743 \\ 1.00000000008974 \\ -0.810660171852566 \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 0.103553390602566 \\ -0.500000000044868 \\ 0.603553390647434 \\ 0. \\ -0.603553390647434 \\ 0.500000000044868 \\ -0.103553390602566 \end{bmatrix} \quad \textbf{(8)}$$

$> P := (k) \rightarrow\ a[k] \cdot \cos(2 \cdot \pi \cdot (k-1) \cdot t) - b[k] \cdot \sin(2 \cdot \pi \cdot (k-1) \cdot t)$

$$P := k \mapsto a_k \cos(2\,\pi\,(k-1)\,t) - b_k \sin(2\,\pi\,(k-1)\,t) \quad \textbf{(9)}$$

$> \quad P8t := P(1) + P(2) + P(3) + P(4)$

$$P8t := 0.500000000044868 - 0.810660171852566\cos(2\pi t) - 0.103553390602566\sin(2\pi t) + 1.00000000008974\cos(4\pi t) + 0.500000000044868\sin(4\pi t) + 1.31066017189743\cos(6\pi t) - 0.603553390647434\sin(6\pi t) \tag{10}$$

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Section 10.3

##

2. nd 8 for the
following data points:
b
Find the least squares trigonometric approximating functions of orders 4, 6, and 8 for the following data points

****

> $V := Vector(8, [1, 0, -2, 1, 3, 0, -2, 1])$

$$V := \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 3 \\ 0 \\ -2 \\ 1 \end{bmatrix} \tag{11}$$

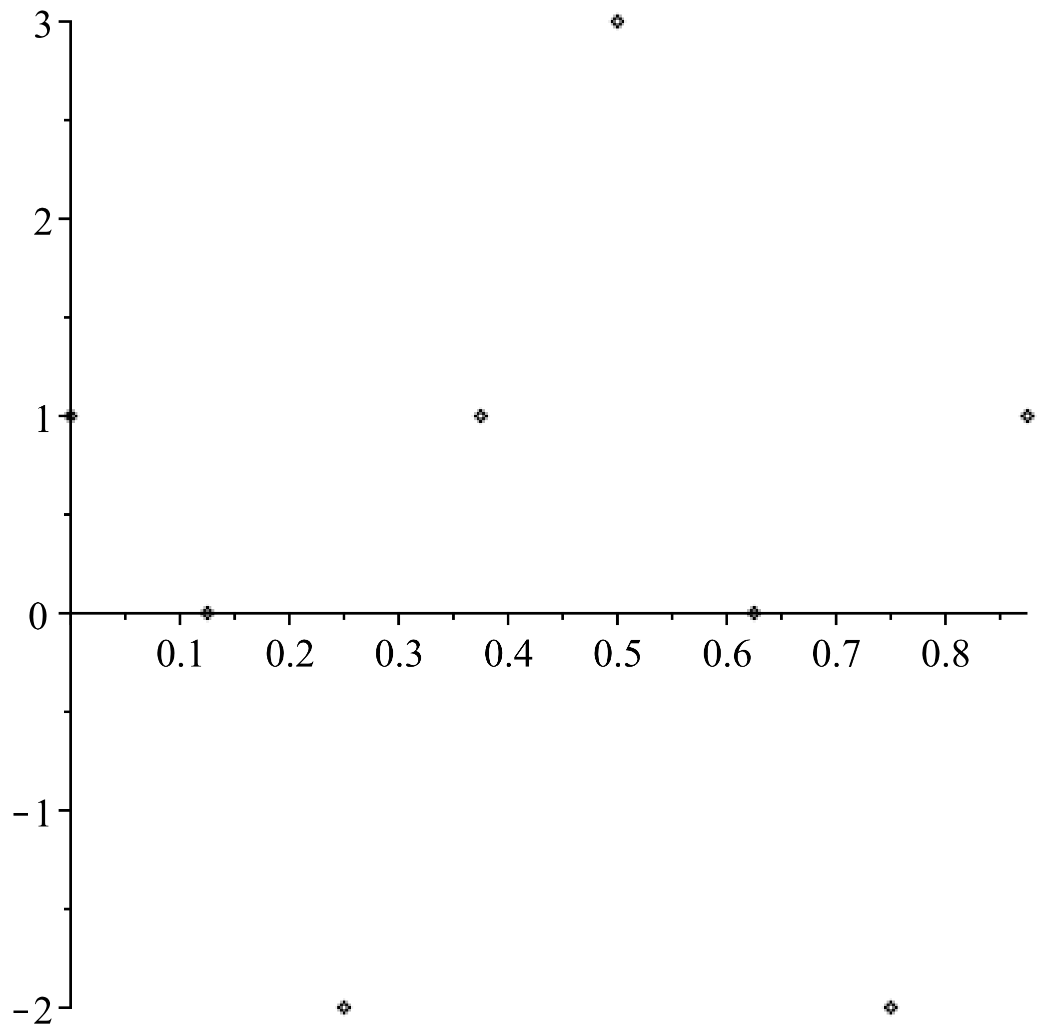
>

> $Z := FourierTransform(V)$

$$Z := \begin{bmatrix} 0.707106781186548 + 0.\,I \\ -0.707106781186548 + 0.\,I \\ 2.82842712474619 + 0.707106781186548\,I \\ -0.707106781186548 + 0.\,I \\ -0.707106781186548 + 0.\,I \\ -0.707106781186548 + 0.\,I \\ 2.82842712474619 - 0.707106781186548\,I \\ -0.707106781186548 + 0.\,I \end{bmatrix} \tag{12}$$

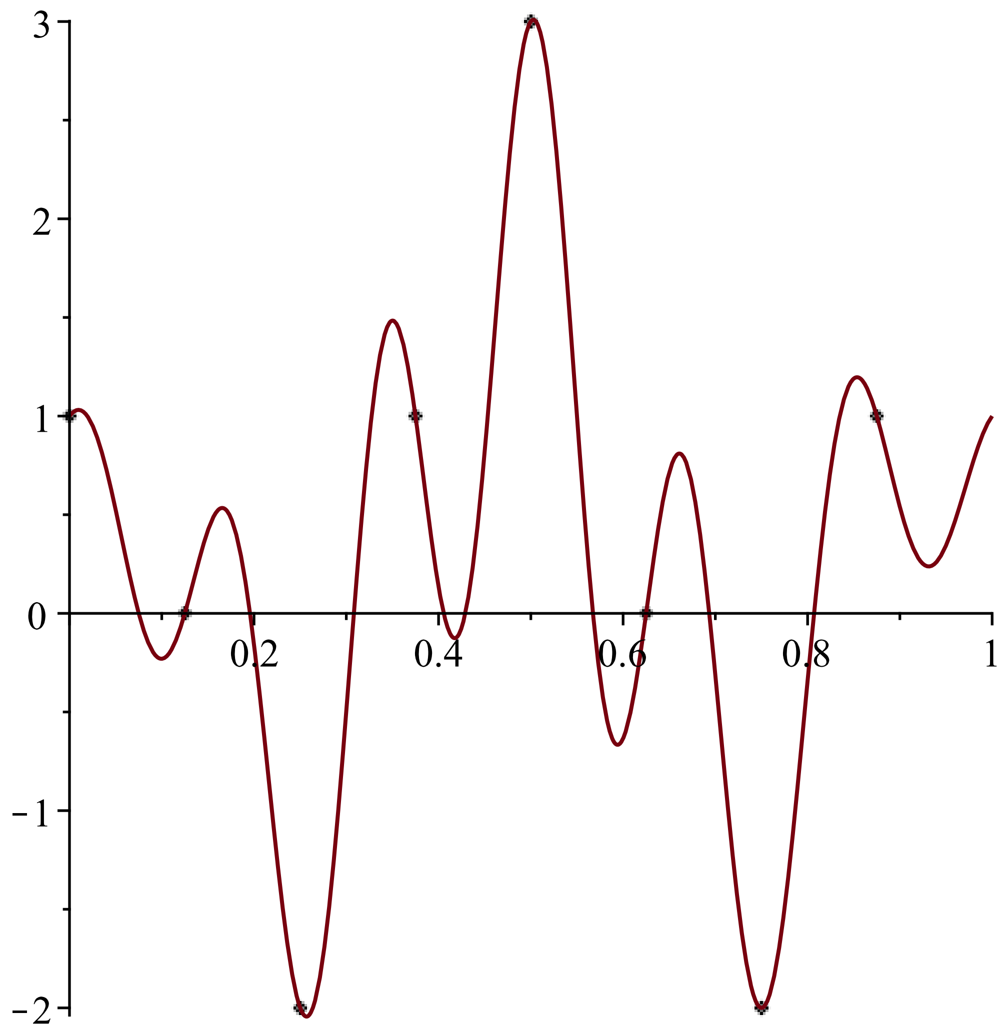
> $rule := i \rightarrow \frac{(i-1)}{8} : T := Vector(8, rule) :$

> $dataplot1 := pointplot(T, V) : display(dataplot1)$



> $P := t \to evalf\left(\frac{1}{\mathtt{sqrt(8)}} \cdot add\left(\mathrm{Re}(Z[k]) \cdot \cos\left(2 \cdot (k-1) \cdot \pi \cdot t\right) - \mathrm{Im}(Z[k]) \cdot \sin\left(2 \cdot (k-1) \cdot \pi \cdot t\right), k = 1..8\right)\right):$

> $plot1 := plot(P, 0..1) : display(dataplot1, plot1)$




```
> Pdeg4 := t → evalf( 1/sqrt(8) · add( Re(Z[k]) · cos(2·(k − 1)·π·t) − Im(Z[k]) · sin(2·(k − 1)·π·t), k = 1 .. 4 ) ) :
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> plot2 := plot(Pdeg4, 0 ..1, color = blue) : display(plot1, plot2, dataplot1)
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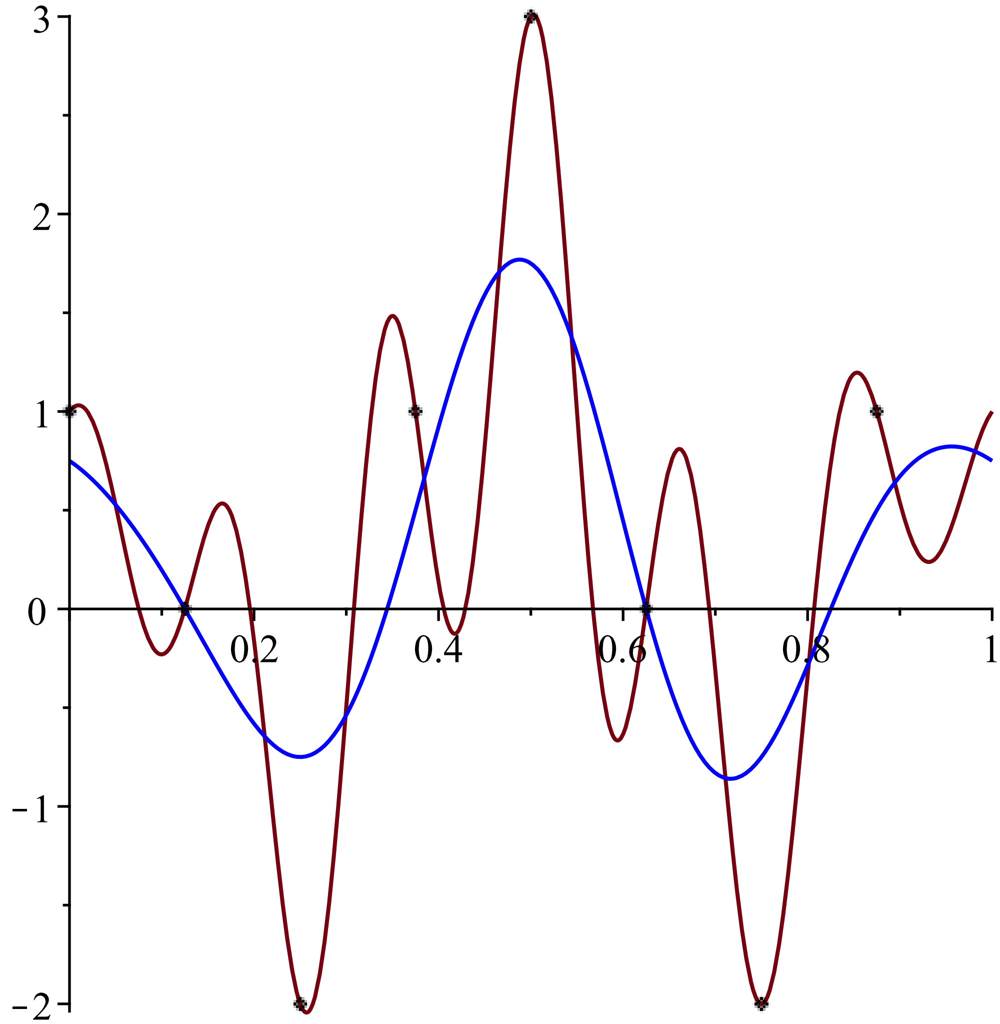
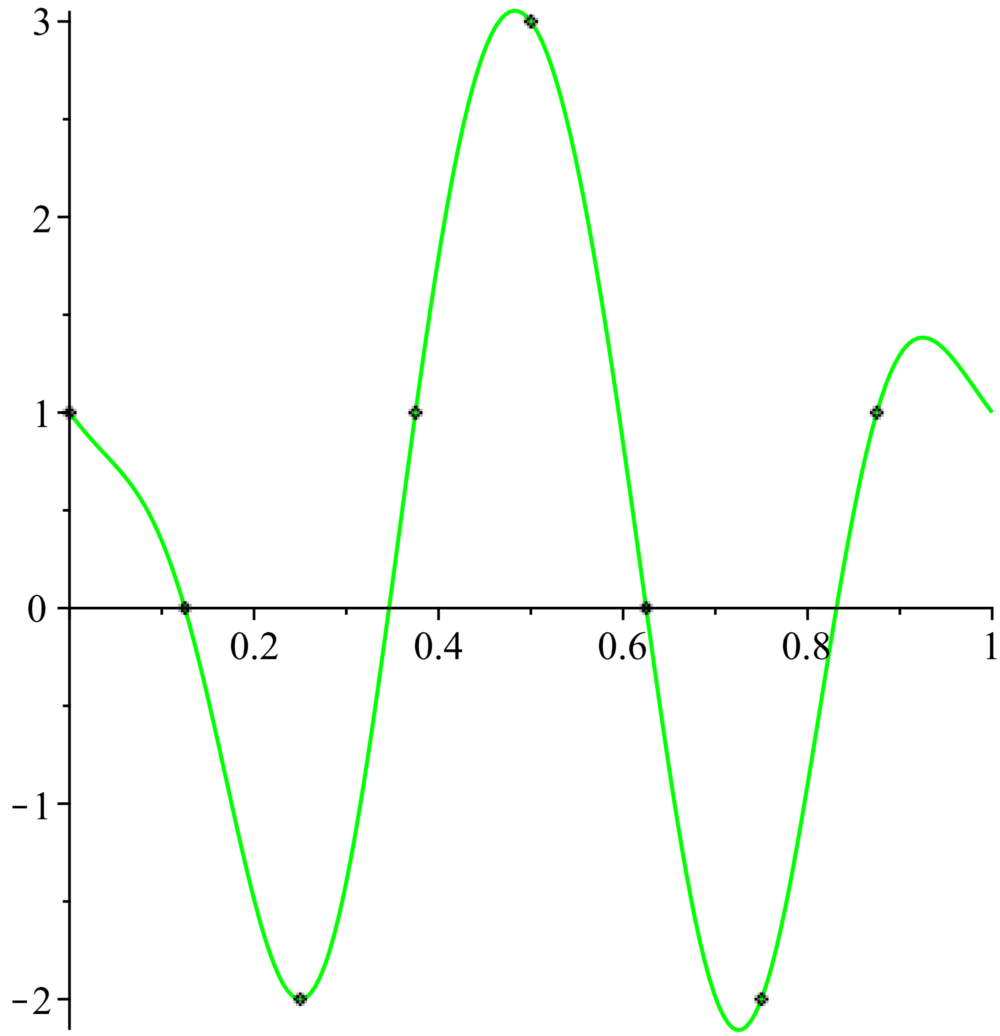


> $Peven := t \rightarrow evalf\left(\frac{1}{\mathtt{sqrt}(8)}\cdot \mathrm{Re}(Z[1]) + \frac{2}{\mathtt{sqrt}(8)}\cdot\ add\left(\mathrm{Re}(Z[k])\cdot\cos(2\cdot(k-1)\cdot\pi\cdot t) - \mathrm{Im}(Z[k])\cdot\sin(2\cdot(k-1)\cdot\pi\cdot t), k = 2..4\right) + \frac{1}{\mathtt{sqrt}(8)}\cdot \mathrm{Re}(Z[5])\cdot\cos(8\cdot\pi\cdot t)\right):$

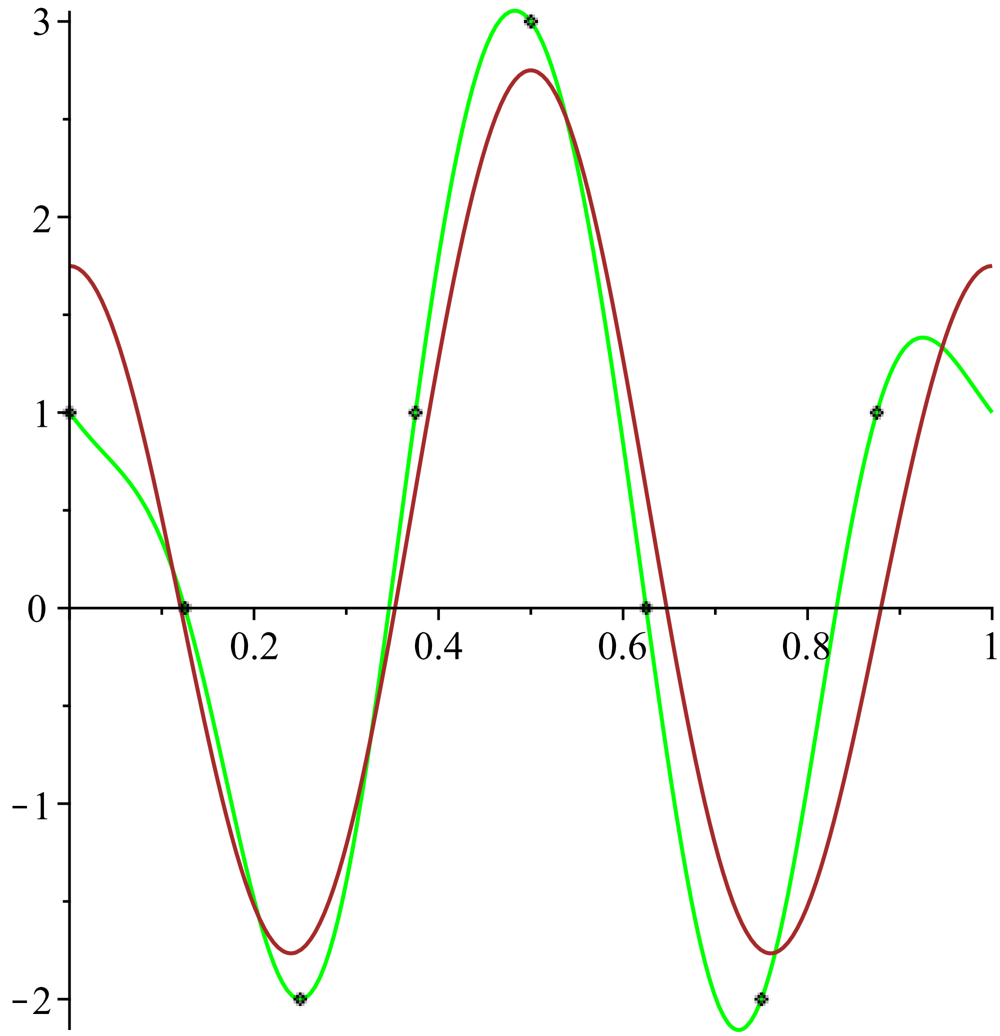
> $plot3 := plot(Peven, 0..1, color = green) : display(plot3, dataplot1)$



And we compute the reduced least-square trigonometric polynomial of order 4 (showed in brown)

> $Pevendeg4 := t \rightarrow evalf\left(\frac{1}{\mathrm{sqrt}(8)} \cdot \mathrm{Re}(Z[1]) + \frac{2}{\mathrm{sqrt}(8)} \cdot \ add\left(\mathrm{Re}(Z[k]) \cdot \cos\left(2 \cdot (k-1) \cdot \pi \cdot t\right) - \mathrm{Im}(Z[k]) \cdot \sin\left(2 \cdot (k-1) \cdot \pi \cdot t\right), k = 2..2\right) + \frac{2}{\mathrm{sqrt}(8)} \cdot \mathrm{Re}(Z[3]) \cdot \cos(4 \cdot \pi \cdot t)\right) :$

> $plot4 := plot(Pevendeg4, 0..1, color = brown) : display(plot3, plot4, dataplot1)$



3
2
1
0
−1
−2
0.2
0.4
0.6
0.8
1

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