Project
Hermit Crabs
Mathematical Modeling MAT 484
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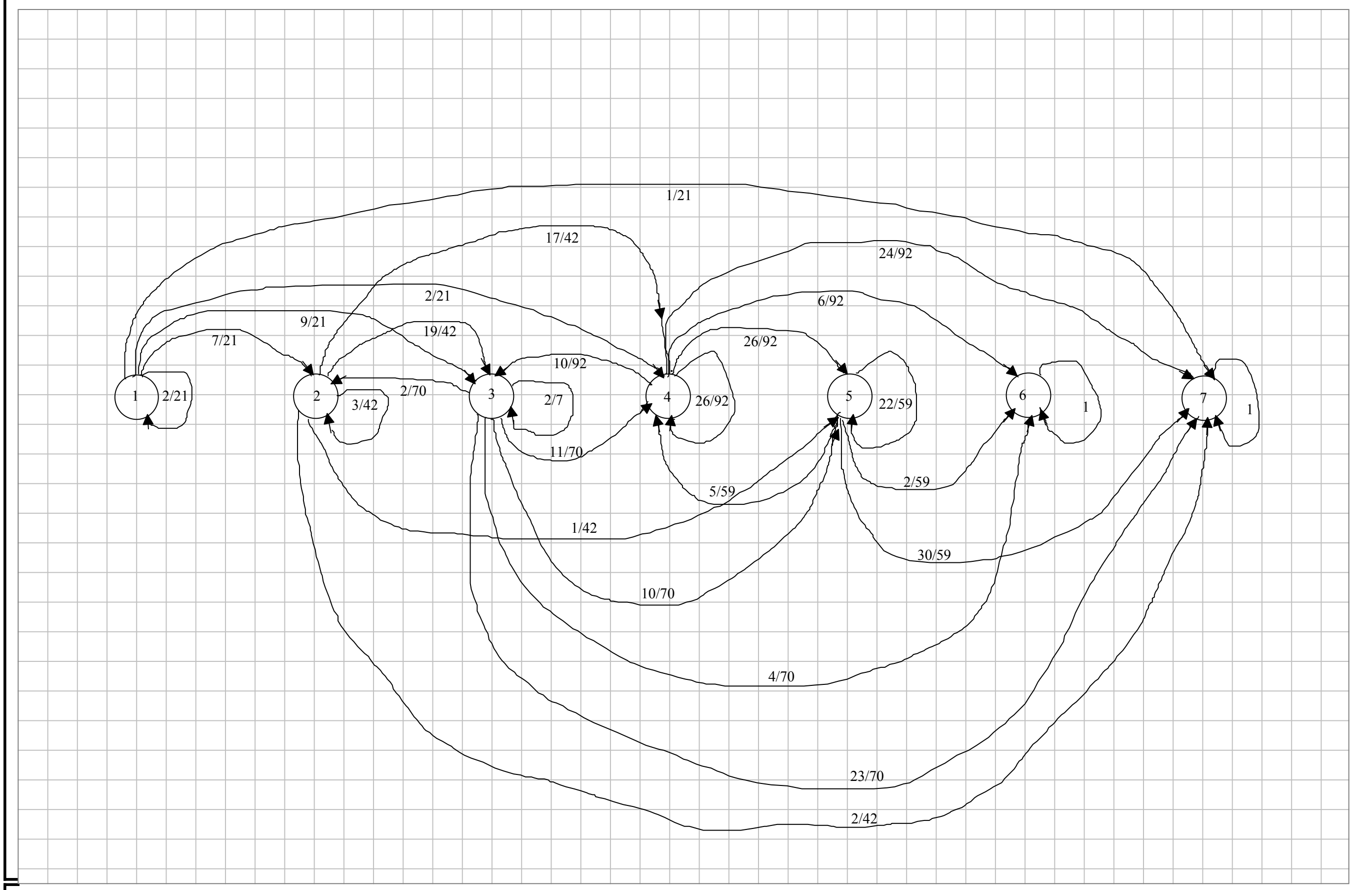
Hermit Crabs

The hermit crab (Pagarus longicarpus) does not have a hard protective shell to protect its body. It uses discarded shells to carry around as portable shelters. These empty shells are rare commodities in tide pools and are available only when their occupants die. In an experiment at Long Island Sound tide pools, an empty shell was dropped into the water to initiate a chain of vacancies. This experiment wa repeated large number of times as vacancies flowed from larger to generally smaller shells. The states in this experiment are the various shell sizes.
There are seven states in a model. States 6 and 7 are absorbing states.

Transition Matrix for Absorbing Markov Chain.

	1	2	3	4	5	6	7
1	2/21	0	0	0	0	0	0
2	7/21	3/42	2/70	0	0	0	0
3	9/21	19/42	20/70	10/92	0	0	0
4	2/21	17/42	11/70	26/92	5/59	0	0
5	0	1/42	10/70	26/92	22/59	0	0
6	0	0	4/70	6/92	2/59	1	0
7	1/21	2/42	23/70	24/92	30/59	0	1

State Diagram



> $with(LinearAlgebra)$:

> $T := evalf\left(Matrix\left(\left[\left[\frac{2}{21}, 0, 0, 0, 0, 0, 0\right], \left[\frac{7}{21}, \frac{3}{42}, \frac{2}{70}, 0, 0, 0, 0\right], \left[\frac{9}{21}, \frac{17}{42}, \frac{20}{70}, \frac{10}{92}, 0, 0, 0\right], \left[\frac{2}{21}, \frac{17}{42}, \frac{11}{70}, \frac{26}{92}, \frac{5}{59}, 0, 0\right], \left[0, \frac{1}{42}, \frac{10}{70}, \frac{26}{92}, \frac{22}{59}, 0, 0\right], \left[0, 0, \frac{4}{70}, \frac{6}{92}, \frac{2}{59}, 1, 0\right], \left[\frac{1}{21}, \frac{2}{42}, \frac{23}{70}, \frac{24}{92}, \frac{30}{59}, 0, 1\right]\right]\right)\right)$

$$:= \begin{bmatrix} 0.09524 & 0. & 0. & 0. & 0. & 0. & 0. \\ 0.33333 & 0.07143 & 0.02857 & 0. & 0. & 0. & 0. \\ 0.42857 & 0.40476 & 0.28571 & 0.10870 & 0. & 0. & 0. \\ 0.09524 & 0.40476 & 0.15714 & 0.28261 & 0.08475 & 0. & 0. \\ 0. & 0.02381 & 0.14286 & 0.28261 & 0.37288 & 0. & 0. \\ 0. & 0. & 0.057143 & 0.06522 & 0.033898 & 1. & 0. \\ 0.04762 & 0.04762 & 0.32857 & 0.26087 & 0.50847 & 0. & 1. \end{bmatrix}$$

> $A := evalf\left(Matrix\left(\left[\left[\frac{2}{21}, 0, 0, 0, 0\right], \left[\frac{7}{21}, \frac{3}{42}, \frac{2}{70}, 0, 0\right], \left[\frac{9}{21}, \frac{17}{42}, \frac{20}{70}, \frac{10}{92}, 0\right], \left[\frac{2}{21}, \frac{17}{42}, \frac{11}{70}, \frac{26}{92}, \frac{5}{59}\right], \left[0, \frac{1}{42}, \frac{10}{70}, \frac{26}{92}, \frac{22}{59}\right]\right]\right)\right)$

> $$\begin{bmatrix} 0.09524 & 0. & 0. & 0. & 0. \\ 0.33333 & 0.07143 & 0.02857 & 0. & 0. \\ 0.42857 & 0.40476 & 0.28571 & 0.10870 & 0. \\ 0.09524 & 0.40476 & 0.15714 & 0.28261 & 0.08475 \\ 0. & 0.02381 & 0.14286 & 0.28261 & 0.37288 \end{bmatrix}$$

> $B := evalf\left(Matrix\left(\left[\left[0, 0, \frac{4}{70}, \frac{6}{92}, \frac{2}{59}\right], \left[\frac{1}{21}, \frac{2}{42}, \frac{23}{70}, \frac{24}{92}, \frac{30}{59}\right]\right]\right)\right)$

> $$\begin{bmatrix} 0. & 0. & 0.05714 & 0.06522 & 0.03390 \\ 0.04762 & 0.04762 & 0.32857 & 0.26087 & 0.50847 \end{bmatrix}$$

> $Id := Matrix([[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 1]])$

$$Id := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

In order to calculate the expected length of the crab vacancy we need to calculate Fundamental Matrix.

Fundamental Matrix F = (I - A)^(-1)

> $F := MatrixInverse(Id - A)$

$$\begin{bmatrix} \mathbf{1.10526} & \mathbf{-0.0} & \mathbf{-0.0} & \mathbf{-0.0} & \mathbf{-0.0} \\ \mathbf{0.42778} & \mathbf{1.10012} & \mathbf{0.04582} & \mathbf{0.00733} & \mathbf{0.00099} \\ \mathbf{1.00811} & \mathbf{0.75376} & \mathbf{1.48902} & \mathbf{0.23829} & \mathbf{0.032201} \\ \mathbf{0.67383} & \mathbf{0.85663} & \mathbf{0.41435} & \mathbf{1.53863} & \mathbf{0.2079} \\ \mathbf{0.54955} & \mathbf{0.59951} & \mathbf{0.52766} & \mathbf{0.74794} & \mathbf{1.69567} \end{bmatrix}$$

Absorbtion Probabilities

> $Absorbtion := B \cdot F$

$$\begin{bmatrix} \mathbf{0.12018} & \mathbf{0.11926} & \mathbf{0.13000} & \mathbf{0.13932} & \mathbf{0.07288} \\ \mathbf{0.85945} & \mathbf{0.82835} & \mathbf{0.86782} & \mathbf{0.86033} & \mathbf{0.92707} \end{bmatrix}$$

Probability that starting from occupying Shell 1 will reach Shell 6 = 0.120
Probability that starting from occupying Shell 1 will reach Shell 7 = 0.859

Probability that starting from occupying Shell 2 will reach Shell 6 = 0.119
Probability that starting from occupying Shell 2 will reach Shell 7 = 0.828

Probability that starting from occupying Shell 3 will reach Shell 6 = 0.130
Probability that starting from occupying Shell 3 will reach Shell 7 = 0.868

Probability that starting from occupying Shell 4 will reach Shell 6 = 0.139
Probability that starting from occupying Shell 4 will reach Shell 7 = 0.860

Probability that starting from occupying Shell 5 will reach Shell 6 = 0.073
Probability that starting from occupying Shell 5 will reach Shell 7 = 0.927

Limiting Steady-State Matrix L = lim(T^n), n->infinity

> $LimitingSteadyStateMatrix := evalf(MatrixPower(T, 15))$

$$\begin{bmatrix} 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00004 & 0.00003 & 0.00002 & 0.0000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00010 & 0.00007 & 0.00004 & 0.00004 & 0.00000 & 0.00000 & 0.00000 \\ 0.00021 & 0.00000 & 0.00009 & 0.00008 & 0.00000 & 0.00000 & 0.00000 \\ \mathbf{0.12014} & \mathbf{0.1192353} & \mathbf{0.12998} & \mathbf{0.13930} & \mathbf{0.07287} & 1.00000 & 0.00000 \\ \mathbf{0.85911} & \mathbf{0.828118} & \mathbf{0.86768} & \mathbf{0.86020} & \mathbf{0.92700} & 0.00000 & 1.00000 \end{bmatrix}$$

We can see that when power of transition matrix approching infinity the partial matrix (cells(6,1) - cell(7,5)) aproches Absorbtion = B*F Matrix.

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Expected Length Of The Crab Vacancy Chains

Summing the columns of the Fundamental matrix F we will get the expected length of vacancy chains.

> $ExpectedLengthOfVacancyChains := Vector([add(F[i, 1], i = 1..5), add(F[i, 2], i = 1..5), add(F[i, 3], i = 1..5), add(F[i, 4], i = 1..5), add(F[i, 5], i = 1..5)])$

$$ExpectedLengthOfVacancyChains := \begin{bmatrix} 3.76452639127018 \\ 3.31000862999884 \\ 2.47683941189274 \\ 2.53219315884294 \\ 1.93678285919612 \end{bmatrix} \qquad \textbf{(2)}$$

Expected length resulting from starting occupying Shell 1 = 3.765
Expected length resulting from starting occupying Shell 2 = 3.310
Expected length resulting from starting occupying Shell 3 = 2.477
Expected length resulting from starting occupying Shell 4 = 2.532
Expected length resulting from starting occupying Shell 5 = 1.937

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