INNA WILLIAMS

Section 4.3

$\boxed{1}$ (a) $\begin{bmatrix} 4 & 0 \\ 3 & 1 \end{bmatrix}$ $\quad y_1 = A_1 = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\quad A_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\|y_1\|_2 = \sqrt{16+9} = 5 = r_{11} \quad q_1 = \frac{y_1}{\|y_1\|_2} = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5} \end{bmatrix}$$

$$y_2 = A_2 - q_1 \cdot q_1^T \cdot A_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5} \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{12}{25} \\ \frac{9}{25} \end{bmatrix} = \begin{bmatrix} -\frac{12}{25} \\ \frac{16}{25} \end{bmatrix} = y_2$$

$$\|y_2\|_2 = \sqrt{\frac{144}{625} + \frac{256}{625}} = \sqrt{\frac{16}{25}} = \frac{4}{5} = r_{22}$$

$$q_2 = \frac{y_2}{\|y_2\|_2} = \frac{5}{4} \cdot \begin{vmatrix} -\frac{12}{25} \\ \frac{16}{25} \end{vmatrix} = \begin{vmatrix} -\frac{3}{5} \\ \frac{4}{5} \end{vmatrix}$$

$$r_{12} = q_1^T \cdot A_2 = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{3}{5} \quad \Rightarrow$$

$$q = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} \qquad R = \begin{vmatrix} 5 & \frac{3}{5} \\ 0 & \frac{4}{5} \end{vmatrix}$$

Answer: $A = Q \cdot R = \begin{vmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{vmatrix} \times \begin{vmatrix} 5 & \frac{3}{5} \\ 0 & \frac{4}{5} \end{vmatrix}$

Section 4.3

2 (b) $A = \begin{vmatrix} -4 & -4 \\ -2 & 7 \\ 4 & -5 \end{vmatrix}$ $A_1 = \begin{vmatrix} -4 \\ -2 \\ 4 \end{vmatrix}$ $A_2 = \begin{vmatrix} -4 \\ 7 \\ -5 \end{vmatrix}$

$$\|y_1\|_2 = \sqrt{16+4+16} = \sqrt{36} = 6 = r_{11} \quad y_1 = A_1 = \begin{vmatrix} -4 \\ -2 \\ 4 \end{vmatrix}$$

$$q_1 = \frac{y_1}{\|y_1\|_2} = \frac{1}{6} \cdot \begin{vmatrix} -4 \\ -2 \\ 4 \end{vmatrix} = \begin{vmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{vmatrix}$$

$$y_2 = A_2 - q_1 \cdot q_1^T \cdot A_2 = \begin{vmatrix} -4 \\ 7 \\ -5 \end{vmatrix} - \begin{vmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{vmatrix} \times \begin{vmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{vmatrix} \cdot \begin{vmatrix} -4 \\ 7 \\ -5 \end{vmatrix}$$

$$y_2 = \begin{vmatrix} -4 \\ 7 \\ -5 \end{vmatrix} - \begin{vmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{vmatrix} \times [-3] = \begin{vmatrix} -4 \\ 7 \\ -5 \end{vmatrix} - \begin{vmatrix} 2 \\ 1 \\ -2 \end{vmatrix} = \begin{vmatrix} -6 \\ 6 \\ -3 \end{vmatrix}$$

$$\|y_2\|_2 = \sqrt{36+36+9} = \sqrt{81} = 9 = r_{22}$$

$$q_2 = \frac{y_2}{\|y_2\|_2} = \frac{1}{9} \begin{vmatrix} -6 \\ 6 \\ -3 \end{vmatrix} = \begin{vmatrix} -\frac{2}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{vmatrix} \qquad r_{12} = q_1^T \cdot A_2 = \begin{vmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{vmatrix} \cdot \begin{vmatrix} -4 \\ 7 \\ -5 \end{vmatrix} = -3$$

Answer:

$$Q = \begin{vmatrix} -\frac{2}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{vmatrix} \quad R = \begin{vmatrix} 6 & -3 \\ 0 & 9 \end{vmatrix} \quad A = Q \cdot R = \begin{vmatrix} -\frac{2}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{vmatrix} \times \begin{vmatrix} 6 & -3 \\ 0 & 9 \end{vmatrix}$$

Section 12.1

$\boxed{5}$ (a) $\lambda = \{3, 1, 4\}$ Power iteration will converge to the largest eigenvalue $= 4$, Rate$= \frac{\lambda_2}{\lambda_1} = \frac{3}{4}$

Answer: It will converge to $\lambda = 4$ with convergense Rate $= \frac{3}{4}$

$\boxed{9}$ (a) $A = \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$

$A - \lambda I = \begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix}$ $\det|A - \lambda I| = (1-\lambda)(3-\lambda) - 8 = 0$

$\Rightarrow \lambda^2 - 4\lambda - 5 = 0 \Rightarrow \lambda_{12} = \frac{4 \pm \sqrt{16+20}}{2} = \frac{4 \pm 6}{2}$, $\lambda_1 = 5$, $\lambda_2 = -1$

$\lambda_1 = 5$

$\begin{vmatrix} 1-5 & 2 \\ 4 & 3-5 \end{vmatrix} \cdot \begin{vmatrix} v_1 \\ v_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix} \Rightarrow \begin{vmatrix} -4 & 2 & 0 \\ 4 & -2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 4 & -2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \end{vmatrix}$

$\Rightarrow 2v_1 - v_2 = 0 \Rightarrow v_2 = 2v_1 \Rightarrow \{1, 2\}$

$\lambda = -1$

$\begin{vmatrix} 1+1 & 2 \\ 4 & 3+1 \end{vmatrix} \cdot \begin{vmatrix} v_1 \\ v_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 2 & 2 & 0 \\ 4 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} \Rightarrow$

$\Rightarrow v_1 + v_2 = 0 \Rightarrow v_2 = -v_1$ $\{-1, 1\}$

(b) $x_0 = \{1, 0\}$

Step1: $x_1 = A \cdot u_0 = \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 4 \end{vmatrix}$

$\lambda_1 = u_0^T \cdot x_1 = [1, 0] \begin{bmatrix} 1 \\ 4 \end{bmatrix} = 1$ $u_1 = \frac{x_1}{\|x_1\|_2} = \frac{1}{\sqrt{17}} \begin{vmatrix} 1 \\ 4 \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \end{vmatrix}$

Step2: $x_2 = A \cdot u_1 = \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} \times \begin{vmatrix} \frac{1}{\sqrt{17}} \\ \frac{4}{\sqrt{17}} \end{vmatrix} = \begin{vmatrix} \frac{9}{\sqrt{17}} \\ \frac{16}{\sqrt{17}} \end{vmatrix}$

$\lambda_2 = u_1^T \cdot x_2 = \begin{vmatrix} \frac{1}{\sqrt{17}} & \frac{4}{\sqrt{17}} \end{vmatrix} \begin{vmatrix} \frac{9}{\sqrt{17}} \\ \frac{16}{\sqrt{17}} \end{vmatrix} = \frac{73}{17}$

$u_2 = \frac{x_2}{\|x_2\|_2} = \frac{1}{\sqrt{\frac{81}{17} + \frac{256}{17}}} \cdot \begin{vmatrix} \frac{9}{\sqrt{17}} \\ \frac{16}{\sqrt{17}} \end{vmatrix} = \sqrt{\frac{17}{337}} \times \begin{vmatrix} \frac{9}{\sqrt{17}} \\ \frac{16}{\sqrt{17}} \end{vmatrix} = \begin{vmatrix} \frac{9}{\sqrt{337}} \\ \frac{16}{\sqrt{337}} \end{vmatrix}$

Step 3:

$x_3 = A \cdot u_2 = \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} \cdot \begin{vmatrix} \frac{9}{\sqrt{337}} \\ \frac{16}{\sqrt{337}} \end{vmatrix} = \begin{vmatrix} \frac{41}{\sqrt{337}} \\ \frac{84}{\sqrt{337}} \end{vmatrix}$

$\lambda_3 = u_2^T \cdot x_3 = \begin{vmatrix} \frac{9}{\sqrt{337}} & \frac{16}{\sqrt{337}} \end{vmatrix} \times \begin{vmatrix} \frac{41}{\sqrt{337}} \\ \frac{84}{\sqrt{337}} \end{vmatrix} = \frac{1713}{337}$

$u_3 = \frac{x_3}{\|x_3\|_2} = \frac{1}{\sqrt{\frac{41^2 + 84^2}{337}}} \times \begin{vmatrix} \frac{41}{\sqrt{337}} \\ \frac{84}{\sqrt{337}} \end{vmatrix} = \sqrt{\frac{337}{8737}} \times \begin{vmatrix} \frac{41}{\sqrt{337}} \\ \frac{84}{\sqrt{337}} \end{vmatrix} = \begin{vmatrix} \frac{41}{\sqrt{8737}} \\ \frac{84}{\sqrt{8737}} \end{vmatrix}$

Answer: $u_1 = \begin{vmatrix} \frac{1}{\sqrt{17}} & \frac{4}{\sqrt{17}} \end{vmatrix}$ $\lambda_1 = 1$

$u_2 = \begin{vmatrix} \frac{9}{\sqrt{337}} & \frac{16}{\sqrt{337}} \end{vmatrix} \approx \begin{vmatrix} 0.4903 & 0.8716 \end{vmatrix}$

$\lambda_2 = \frac{73}{17} = 4.29$

$u_3 = \begin{vmatrix} \frac{41}{\sqrt{8737}} \\ \frac{84}{\sqrt{8737}} \end{vmatrix} = \begin{vmatrix} 0.4386 & 0.8987 \end{vmatrix}$

$\lambda_3 = \frac{1713}{337} \approx 5.08$