

performed by Inna Williams

```
#####  
data Problem_5_1_5_5;  
input  X Y Z;  
  
cards;  
1 3 15  
7 13 7  
8 12 5  
3 4 14  
4 7 10  
;  
/*    a    */  
proc corr data=Problem_5_1_5_5;  
title "Pearson Correlation Coefficient Between X and Y, X and  
Z";  
var X;  
with Y Z;  
Run;  
proc sgplot data=Problem_5_1_5_5;  
scatter x=Z y=X;  
run;  
/*    b    */  
proc corr data=Problem_5_1_5_5;  
title "Pearson Correlation Coefficient Between variables X, Y  
Z";  
var X Y Z;  
run;
```

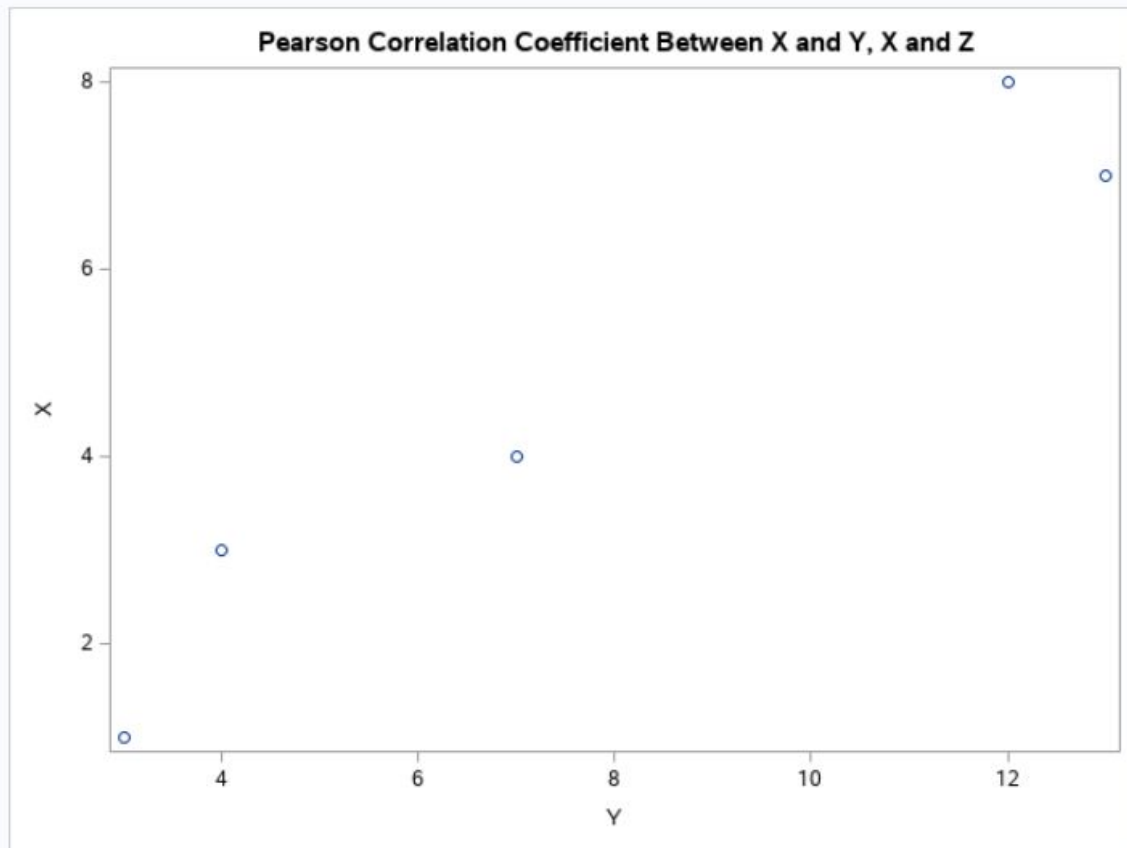
Pearson Correlation Coefficient Between X and Y, X and Z

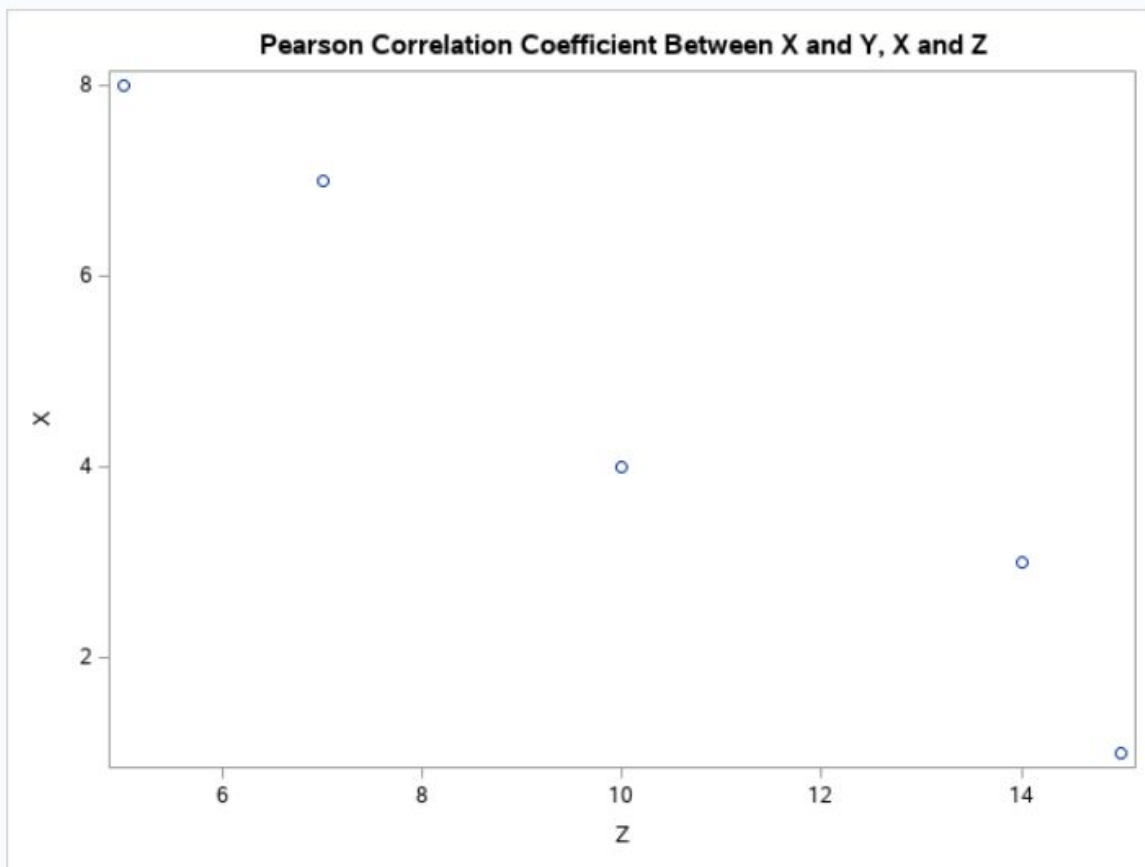
The CORR Procedure

| | |
|-------------------|-----|
| 2 With Variables: | Y Z |
| 1 Variables: | X |

| Simple Statistics | | | | | | |
|-------------------|---|----------|---------|----------|---------|----------|
| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
| Y | 5 | 7.80000 | 4.54973 | 39.00000 | 3.00000 | 13.00000 |
| Z | 5 | 10.20000 | 4.32435 | 51.00000 | 5.00000 | 15.00000 |
| X | 5 | 4.60000 | 2.88097 | 23.00000 | 1.00000 | 8.00000 |

| Pearson Correlation Coefficients, N = 5 Prob > r under H0: Rho=0 | |
|---|--------------------|
| | X |
| Y | 0.96509 0.0078 |
| Z | -0.97525 0.0047 |





Interpretation Of X Vs Y and X vs Z

X vs Y $R=0.96509$ $p=0.0078$

This means that X and Y have Very strong linear positive correlation. If Y increasing then X also increasing.

X vs Z $R=-0.97525$ $p=0.0047$

This means that X and Y have Very strong linear negative correlation. If Y increasing then X decreasing.

$H_0 \rightarrow \text{correlation} = \text{zero}$

$H_a \rightarrow \text{correlation not} = \text{zero}$

$p\text{-value} < 0.0078$, $p=0.0047$ (less than 0.05 level). This means that we reject H_0 We have an evidence that correlations between X and Y, and X and Z is not zero.

Graph shows also that X and Y and X and Z have linear relationship.

Pearson Correlation Coefficient Between variables X, Y Z

The CORR Procedure

3 Variables: X Y Z

| Simple Statistics | | | | | | |
|-------------------|---|----------|---------|----------|---------|----------|
| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
| X | 5 | 4.60000 | 2.88097 | 23.00000 | 1.00000 | 8.00000 |
| Y | 5 | 7.80000 | 4.54973 | 39.00000 | 3.00000 | 13.00000 |
| Z | 5 | 10.20000 | 4.32435 | 51.00000 | 5.00000 | 15.00000 |

| Pearson Correlation Coefficients, N = 5 Prob > r under H0: Rho=0 | | | |
|---|--------------------|--------------------|--------------------|
| | X | Y | Z |
| X | 1.00000 | 0.96509 0.0078 | -0.97525 0.0047 |
| Y | 0.96509 0.0078 | 1.00000 | -0.96317 0.0084 |
| Z | -0.97525 0.0047 | -0.96317 0.0084 | 1.00000 |

Interpretation for X, Y, Z Correlations

X vs Y $R=0.96509$ $p=0.0078$

This means that X and Y have Very strong linear positive correlation. If Y increasing then X is also increasing.

X Vs Z $R=-0.97525$ $p=0.0047$

This means that X and Y have Very strong linear negative correlation. If Z increasing then X is decreasing.

Y vs X $R=0.96509$ $p=0.0078$

This means that Y and X have Very strong linear positive correlation. If X increasing then Y is also increasing.

Y Vs Z $R=-0.96317$ $p=0.0084$

This means that Y and Z have Very strong linear negative correlation. If Z increasing then Y is decreasing.

Z vs X $R=0.97525$ $p=0.0047$

This means that Z and X have Very strong linear positive correlation. If X increasing then

Z is also increasing.

Z Vs Y R=0.96317 p=0.0084

This means that Z and Y have Very strong linear negative correlation. If Y increasing then Z is decreasing.

for each case above

Ho -> correlation =zero

Ha -> correlation not = zero

p-value < ... (less than 0.05 level).This means that

we reject Ho We have an evidence that correlations between X,Y Z are not zero.

#####

5_5 a

#####

proc reg data=Problem_5_1_5_5;

title "Regression Y on X";

model Y=X;

Run;

Regression Y on X

The REG Procedure
Model: MODEL1
Dependent Variable: Y

| | |
|-----------------------------|---|
| Number of Observations Read | 5 |
| Number of Observations Used | 5 |

| Analysis of Variance | | | | | |
|----------------------|----|----------------|-------------|---------|--------|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 77.11928 | 77.11928 | 40.73 | 0.0078 |
| Error | 3 | 5.68072 | 1.89357 | | |
| Corrected Total | 4 | 82.80000 | | | |

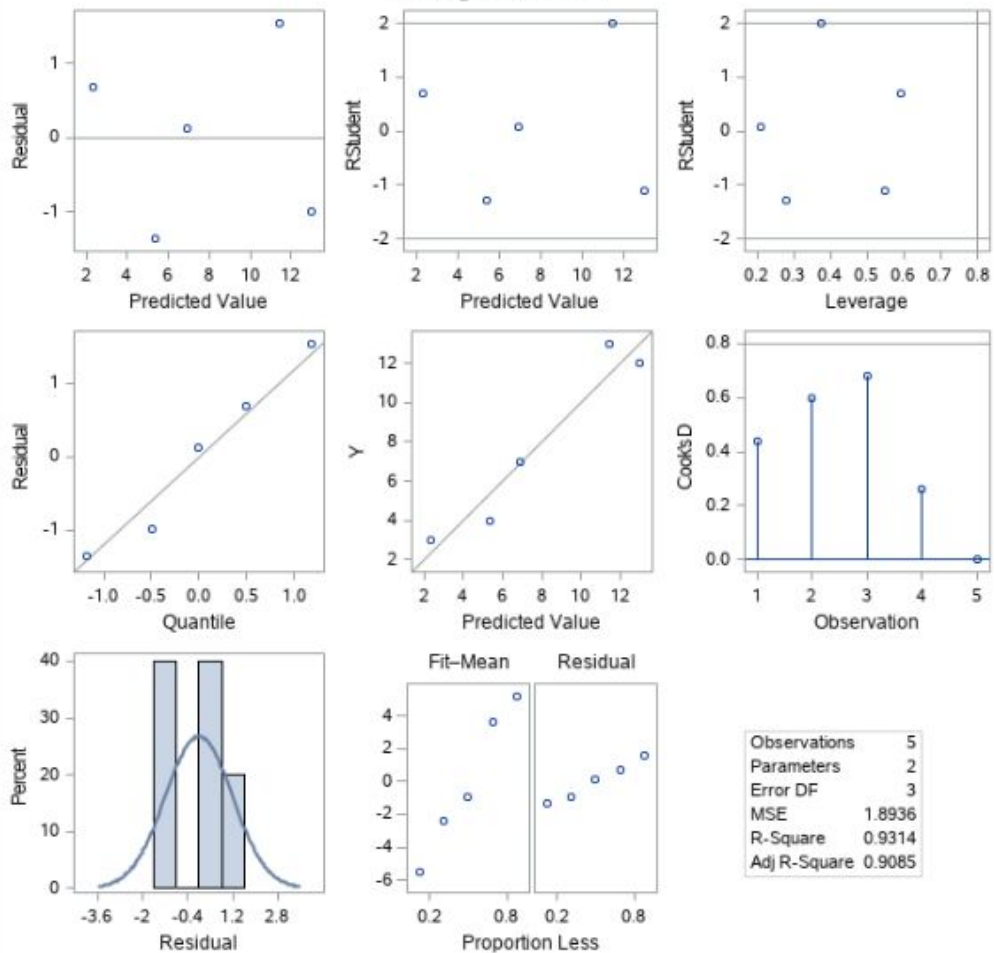
| | | | |
|----------------|----------|----------|--------|
| Root MSE | 1.37607 | R-Square | 0.9314 |
| Dependent Mean | 7.80000 | Adj R-Sq | 0.9085 |
| Coeff Var | 17.64195 | | |

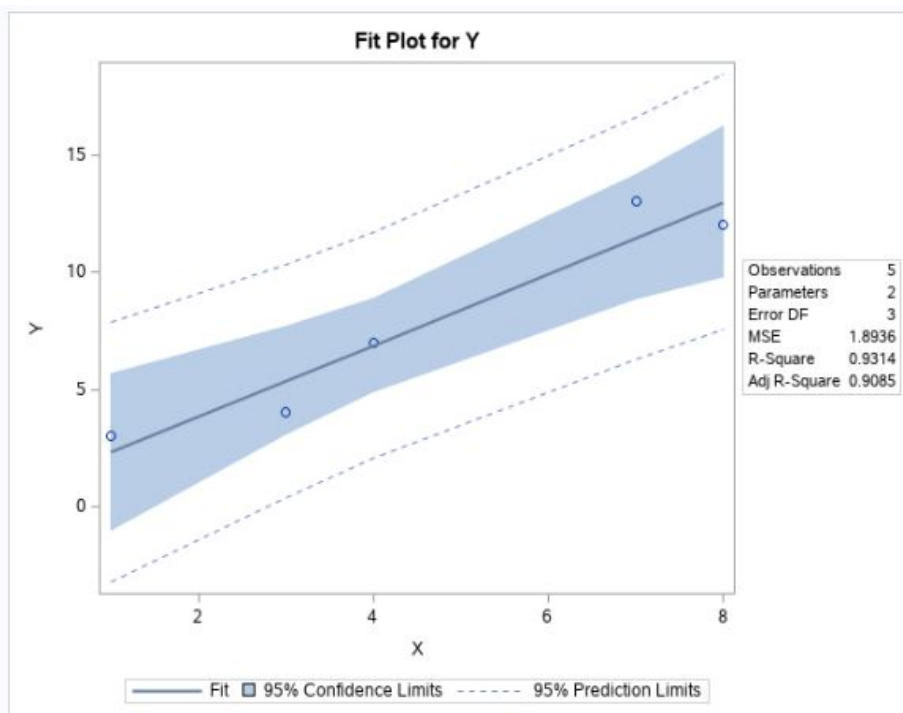
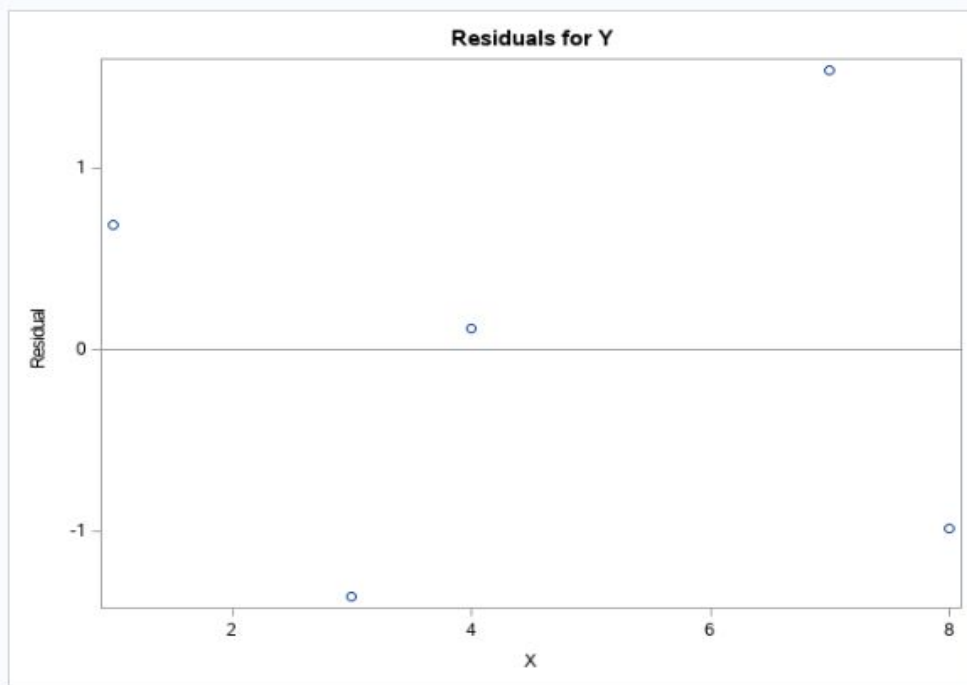
| Parameter Estimates | | | | | |
|---------------------|----|--------------------|----------------|---------|---------|
| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > t |
| Intercept | 1 | 0.78916 | 1.25920 | 0.63 | 0.5753 |
| X | 1 | 1.52410 | 0.23882 | 6.38 | 0.0078 |

Regression Y on X

The REG Procedure
Model: MODEL1
Dependent Variable: Y

Fit Diagnostics for Y





```
#####  
5_5 b)
```

```
#####  
Intercept = 0.78916 prob|t| = 0.5753  
Slope      = 1.52410 prob|t| = 0.0078
```

```
#####  
5_5 c)
```

```
#####  
Ho -> slope is zero  
Ha -> slope is not zero  
p-value < 0.0078 (less than 0.05 level).This means that  
we reject Ho We have an evidence that slope not equal zero
```

```
Ho -> intercept is zero  
Ha -> Intercept is not zero  
p-value = 0.5753 (> than 0.05 level).This means that  
we fail to reject Ho We do not have enough evidence  
to conclude that the slope is not equal zero.
```

```
#####  
5_3
```

```
#####  
data Problem_5_3;  
input AGE SBP;  
cards;  
15 116  
20 120  
25 130  
30 132  
40 150  
50 148  
;
```

```
proc corr data=Problem_5_3;  
title "Problem 5_3 SBP vs Age";  
var SBP AGE;  
Run;
```


Problem 5_3 SBP vs Age

The CORR Procedure

2 Variables: SBP AGE

| Simple Statistics | | | | | | |
|-------------------|---|-----------|----------|-----------|-----------|-----------|
| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
| SBP | 6 | 132.66667 | 14.00952 | 796.00000 | 116.00000 | 150.00000 |
| AGE | 6 | 30.00000 | 13.03840 | 180.00000 | 15.00000 | 50.00000 |

| Pearson Correlation Coefficients, N = 6 Prob > r under H0: Rho=0 | | |
|---|-------------------|-------------------|
| | SBP | AGE |
| SBP | 1.00000 | 0.95258 0.0033 |
| AGE | 0.95258 0.0033 | 1.00000 |

SBP vs Age $R=0.95258$ $p=0.0033$

This means that SBP and AGE have Very strong linear positive correlation. If AGE increasing then SBP is also increasing.

Age vs SBP $R=0.95258$ $p=0.0033$

This means that AGE and SBP have Very strong linear negative correlation. If SBP increasing then AGE is also increasing.

#####

5_8

#####

data DOSE_RESPONSE;

input DOSE SBP DBP;

Label

DOSE='Dose'

SBP='Systolic Blood Pressure'

DBP='Diastolic Blood Pressure'

;

cards;

4 180 110

1 190 108

4 178 100

8 170 100

8 180 98

```

8 168 88
16 160 80
16 172 86
16 170 86
32 140 80
32 130 72
32 128 70
;
proc reg data=DOSE_RESPONCE;
title 'Problem 5_8 Dose Response';
model SBP DBP =DOSE;
Run;

```

Problem 5_8 Dose Response

The REG Procedure
Model: MODEL1
Dependent Variable: SBP Systolic Blood Pressure

| | |
|-----------------------------|----|
| Number of Observations Read | 12 |
| Number of Observations Used | 12 |

| Analysis of Variance | | | | | |
|----------------------|----|----------------|-------------|---------|--------|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 4176.55854 | 4176.55854 | 103.61 | <.0001 |
| Error | 10 | 403.10813 | 40.31081 | | |
| Corrected Total | 11 | 4579.66667 | | | |

| | | | |
|----------------|-----------|----------|--------|
| Root MSE | 6.34908 | R-Square | 0.9120 |
| Dependent Mean | 163.83333 | Adj R-Sq | 0.9032 |
| Coeff Var | 3.87533 | | |

| Parameter Estimates | | | | | | |
|---------------------|-----------|----|--------------------|----------------|---------|---------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t |
| Intercept | Intercept | 1 | 188.82998 | 3.06430 | 61.62 | <.0001 |
| DOSE | Dose | 1 | -1.69469 | 0.16649 | -10.18 | <.0001 |

Problem 5_8 Dose Response

The REG Procedure

Model: MODEL1

Dependent Variable: DBP Diastolic Blood Pressure

| | |
|-----------------------------|----|
| Number of Observations Read | 12 |
| Number of Observations Used | 12 |

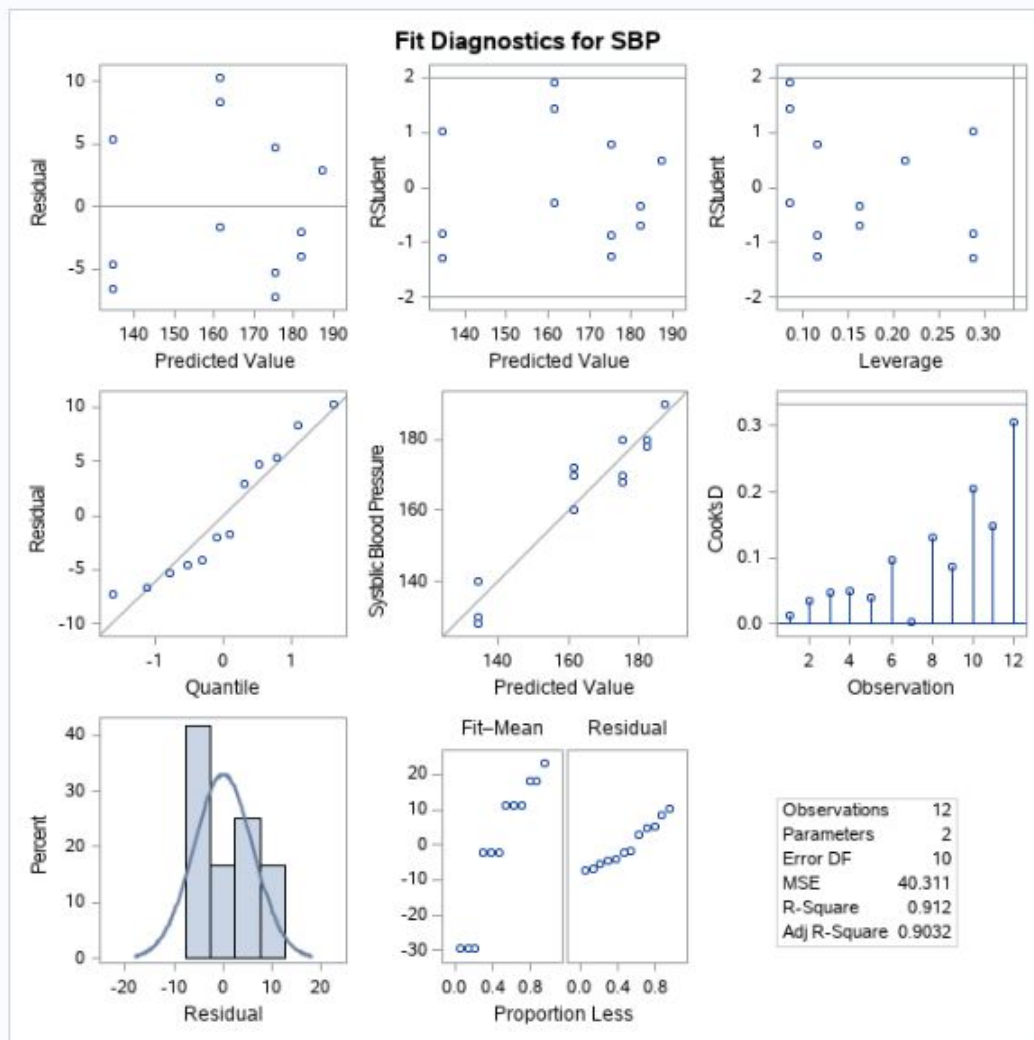
| Analysis of Variance | | | | | |
|----------------------|----|----------------|-------------|---------|--------|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 1606.54100 | 1606.54100 | 47.10 | <.0001 |
| Error | 10 | 341.12567 | 34.11257 | | |
| Corrected Total | 11 | 1947.66667 | | | |

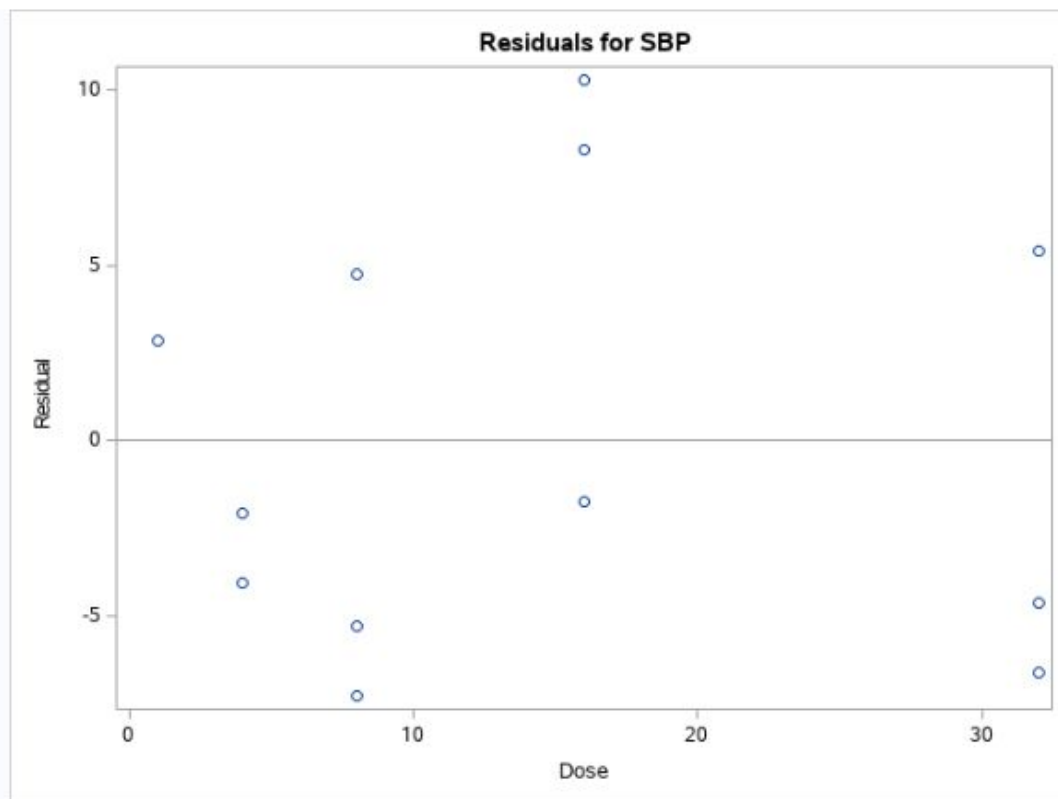
| | | | |
|----------------|----------|----------|--------|
| Root MSE | 5.84060 | R-Square | 0.8249 |
| Dependent Mean | 89.83333 | Adj R-Sq | 0.8073 |
| Coeff Var | 6.50159 | | |

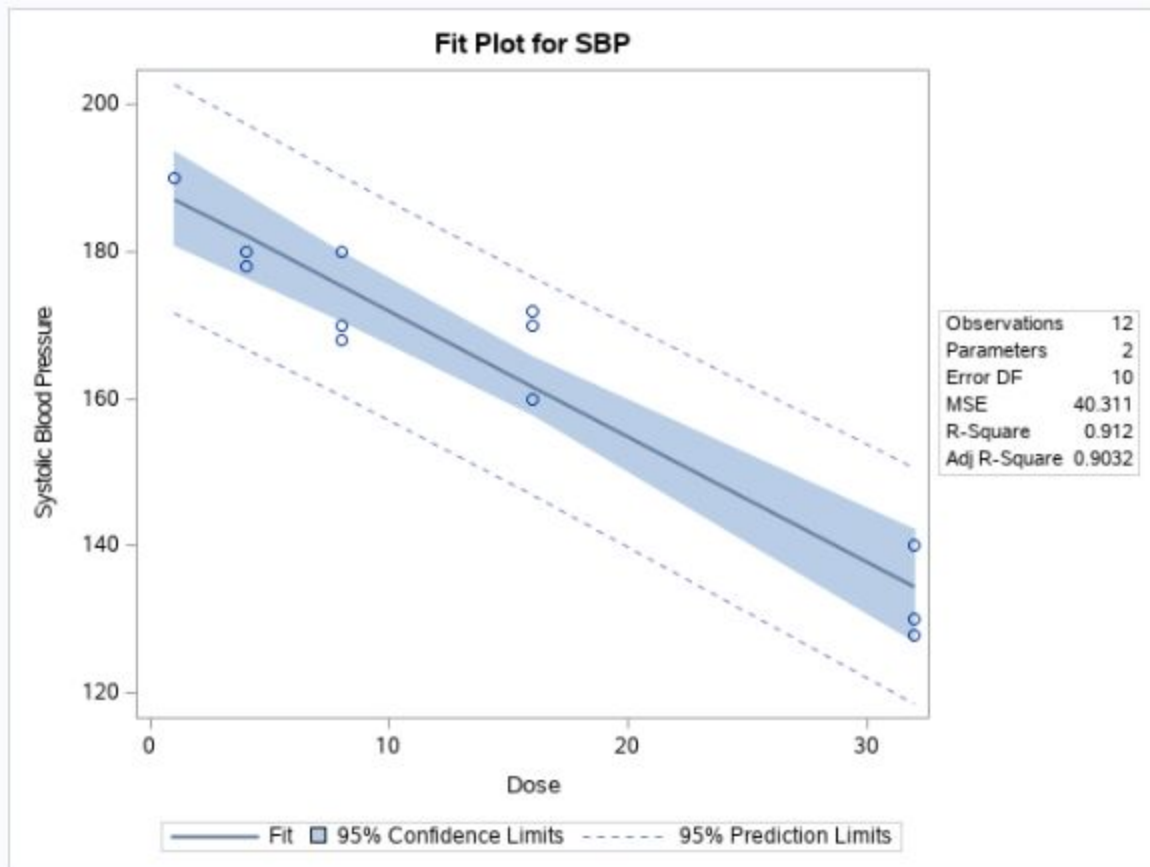
| Parameter Estimates | | | | | | |
|---------------------|-----------|----|--------------------|----------------|---------|---------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t |
| Intercept | Intercept | 1 | 105.33643 | 2.81888 | 37.37 | <.0001 |
| DOSE | Dose | 1 | -1.05106 | 0.15316 | -6.86 | <.0001 |

Problem 5_8 Dose Response

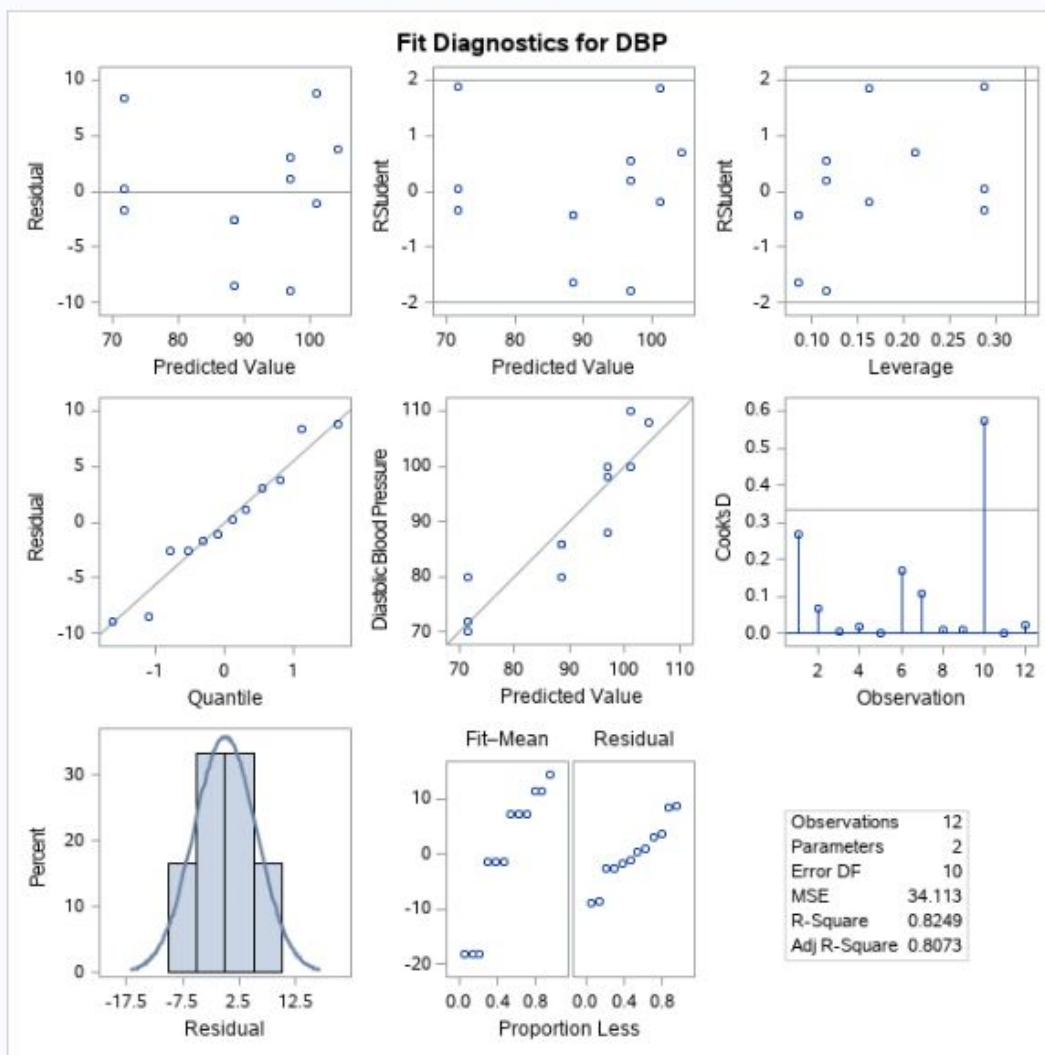
The REG Procedure
Model: MODEL1
Dependent Variable: SBP Systolic Blood Pressure

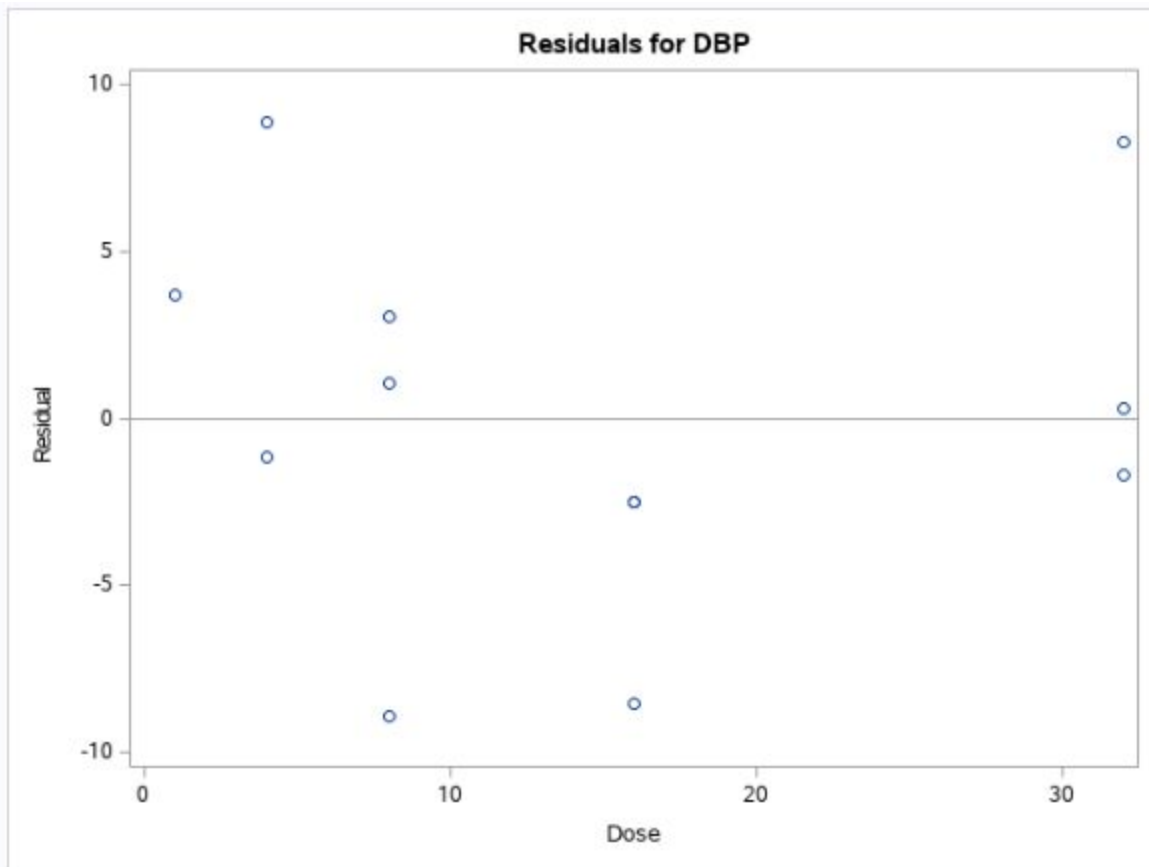


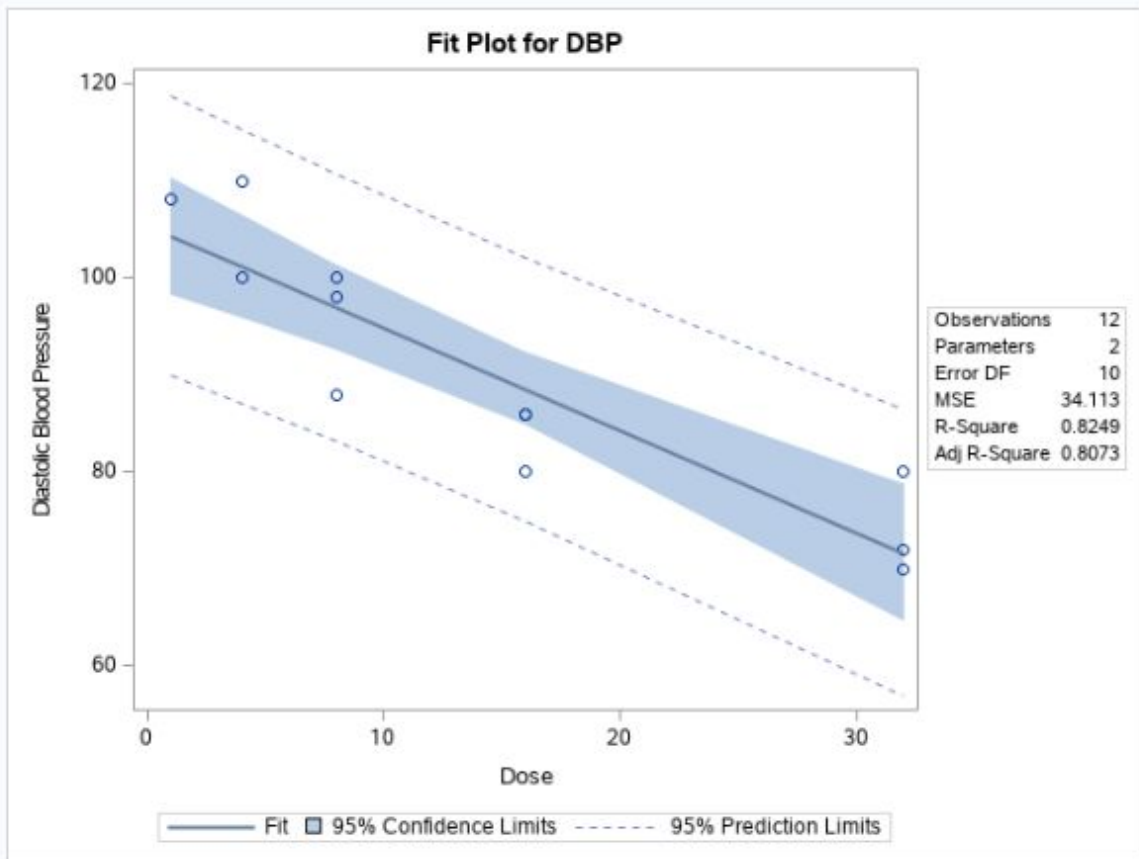




The REG Procedure
 Model: MODEL1
 Dependent Variable: DBP Diastolic Blood Pressure







SBP VS DOSE

Intercept = 188.82998 Slope = -1.69469

DBP VS DOSE

Intercept = 105.33643 Slope = -1.05106

#####

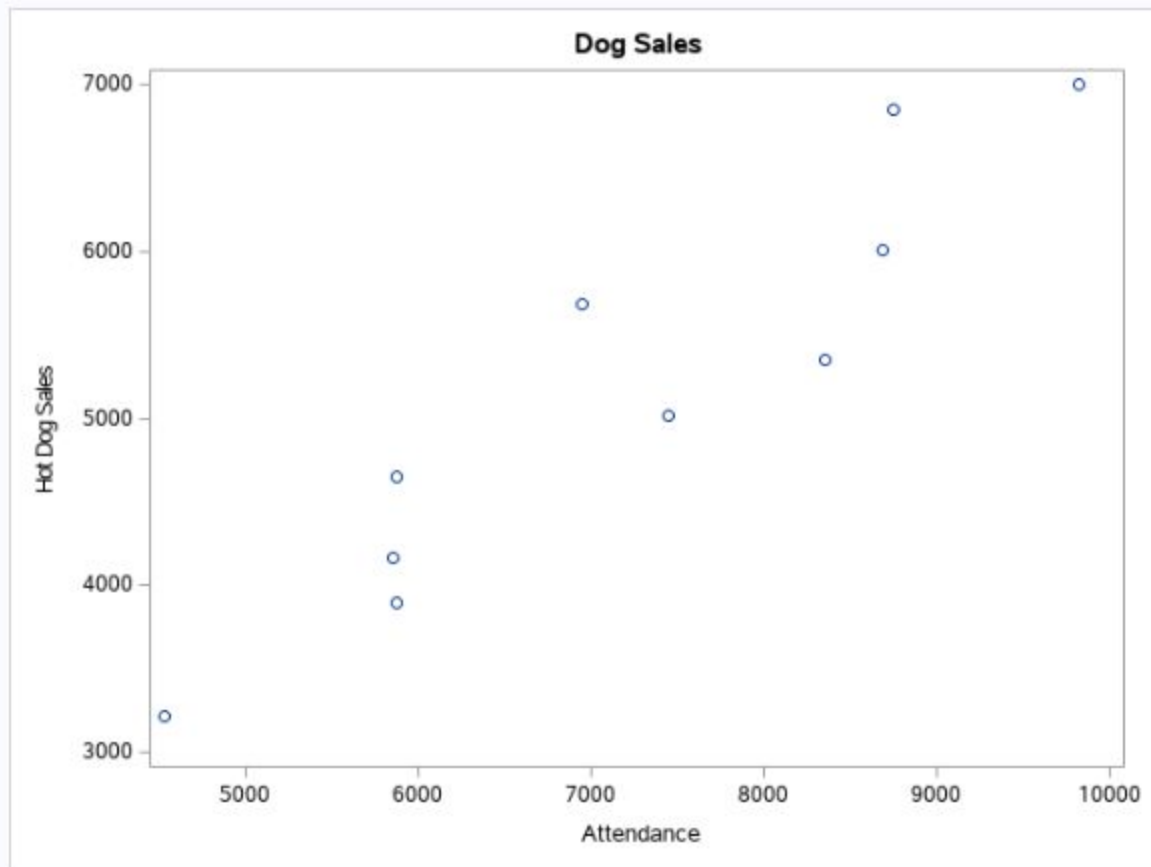
The daily attendance and the number of hot dog sales at a local ballpark are studied over a period of games. Given the following data, answer the following questions.

#####

(a) Plot the data using proc sgplot.

#####

```
data DOG_SALES;
input ATT SALES;
Label
    ATT="Attendance"
    SALES="Hot Dog Sales";
;
cards;
8747 6845
5857 4168
8360 5348
6945 5687
8688 6007
4534 3216
7450 5018
5874 4652
9821 7001
5873 3896
;
proc sgplot data=DOG_SALES;
title 'Dog Sales';
scatter x=ATT y=SALES;
Run;
```



#####

(b) Find the correlation coefficient and test its significance.

#####

```
proc corr data=DOG_SALES;  
title 'Correlation Coefficient Dog Sales';  
Var ATT SALES;  
Run;
```



Attendance vs Sale Coefficient of Correlation $R=0.93748$

with significance $p<.0001$

This means that Attendance vs Sale have Very strong linear positive correlation. If Sales is increasing then Attendance is also increasing.

Sale vs Attendance Coefficient of Correlation $R=0.93748$

with significance $p<.0001$

This means that Sale vs Attendance have Very strong linear positive correlation. If Attendance is increasing then Sales is also increasing.

testing for significance

$H_0 \rightarrow$ correlation = zero

$H_a \rightarrow$ correlation not = zero

$p\text{-value} < 0.0001$ (less than 0.05 level). This means that we reject H_0 We have an evidence that correlation is not Zero.

#####

(c)

Find the regression line to predict hot dogs sales based on attendance.

#####

```
proc reg data=DOG_SALES;
title 'Regression Coefficient Dog Sales';
model SALES=ATT / clb;
run;
```

Regression Coefficient Dog Sales

The REG Procedure
Model: MODEL1
Dependent Variable: SALES Hot Dog Sales

| | |
|-----------------------------|----|
| Number of Observations Read | 10 |
| Number of Observations Used | 10 |

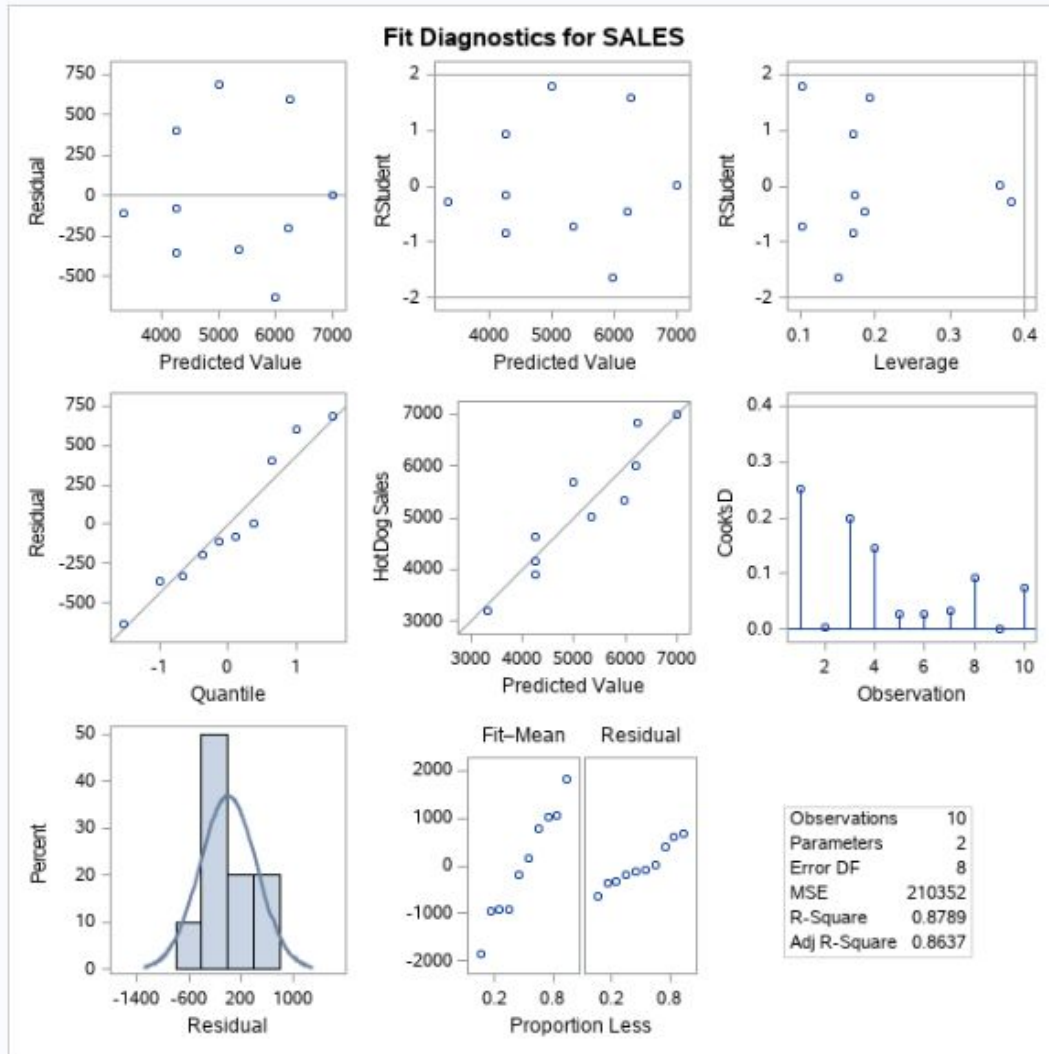
| Analysis of Variance | | | | | |
|----------------------|----|----------------|-------------|---------|--------|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 12209634 | 12209634 | 58.04 | <.0001 |
| Error | 8 | 1682814 | 210352 | | |
| Corrected Total | 9 | 13892448 | | | |

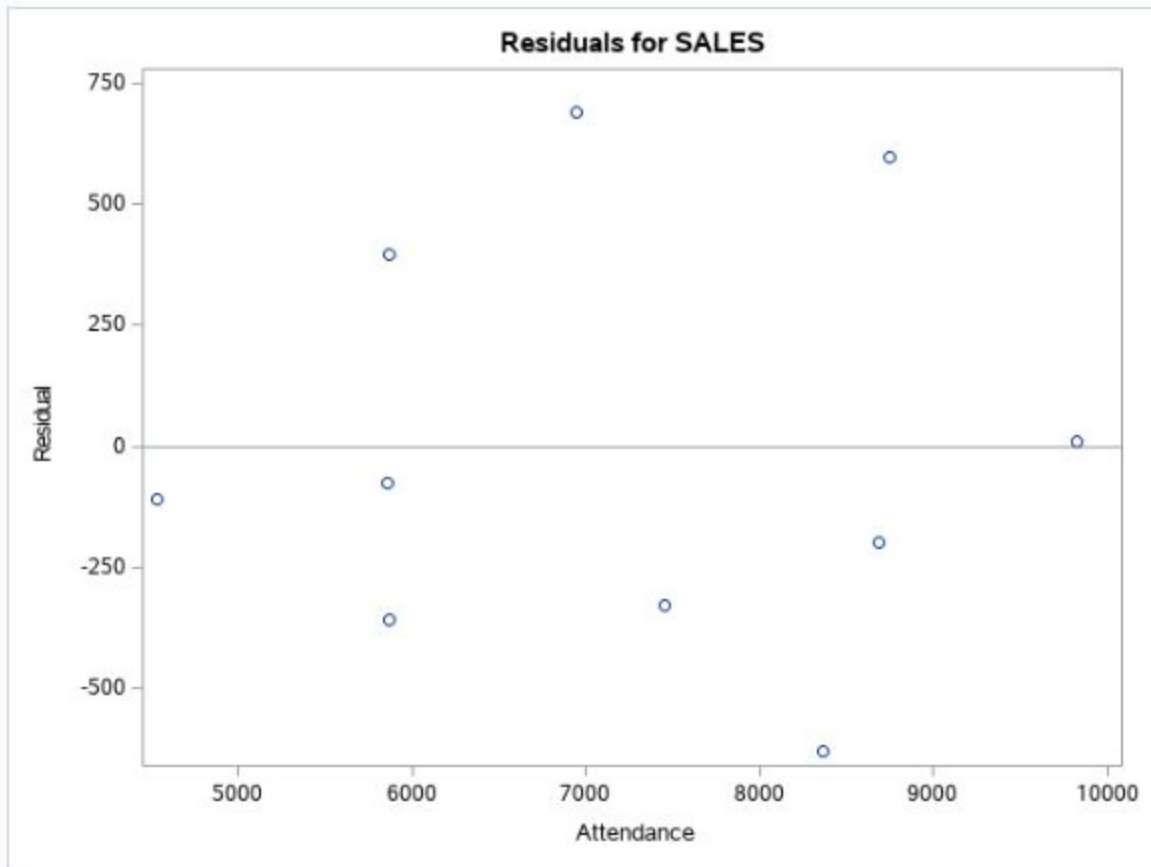
| | | | |
|----------------|------------|----------|--------|
| Root MSE | 458.64114 | R-Square | 0.8789 |
| Dependent Mean | 5183.80000 | Adj R-Sq | 0.8637 |
| Coeff Var | 8.84759 | | |

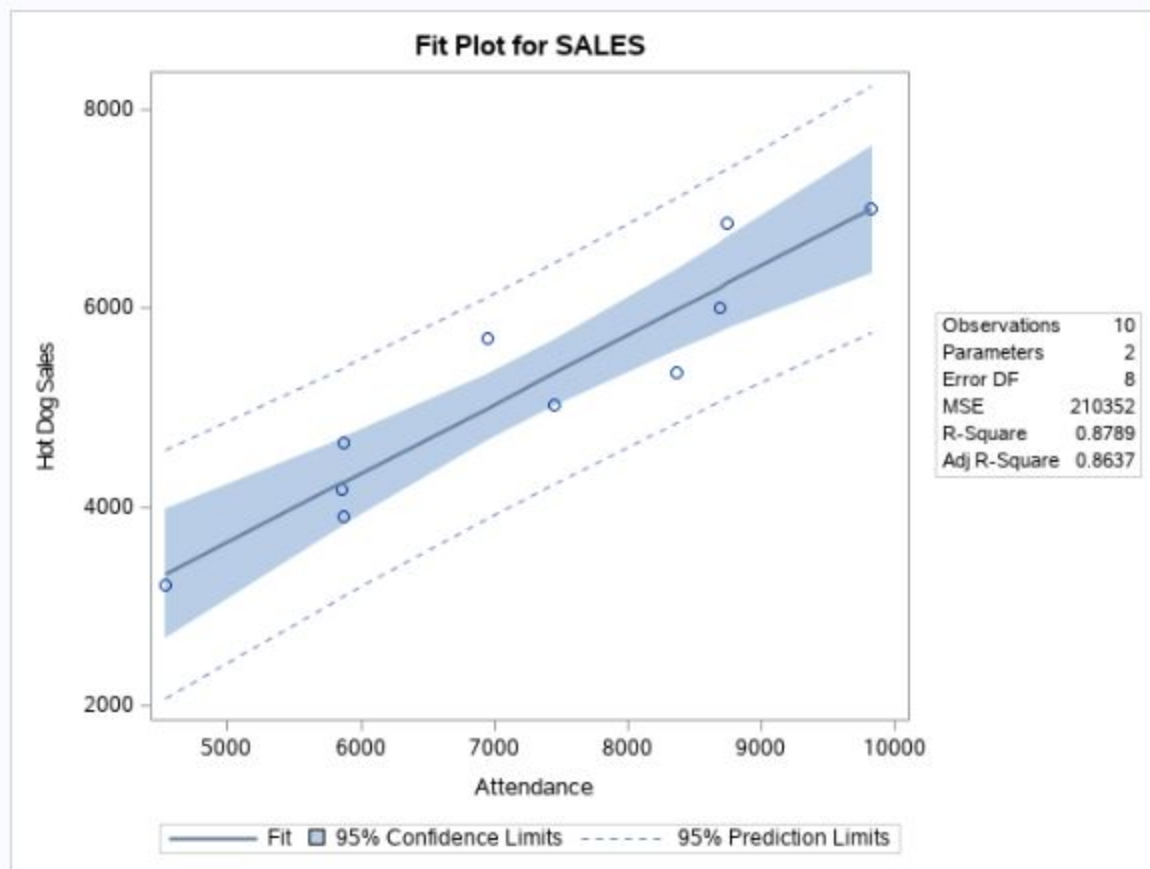
| Parameter Estimates | | | | | | | | |
|---------------------|------------|----|--------------------|----------------|---------|---------|-----------------------|------------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t | 95% Confidence Limits | |
| Intercept | Intercept | 1 | 179.41977 | 672.68015 | 0.27 | 0.7964 | -1371.78344 | 1730.62298 |
| ATT | Attendance | 1 | 0.89362 | 0.09104 | 7.62 | <.0001 | 0.48367 | 0.90356 |

Regression Coefficient Dog Sales

The REG Procedure
Model: MODEL1
Dependent Variable: SALES Hot Dog Sales







Regression Line:

$\text{hot_dog_sales} = 179.41977 + 0.69362 * \text{Attendance}$

#####

(d) What is the estimate of standard deviation (root MSE)?

#####

Root MSE = 458.64114

It is the square root of the Residual(or error)

#####

(e) Report and interpret the 95% confidence interval for the slope coefficient.

#####

95% CI for Slope = [0.48367, 0.90356]

0.48367 0.90356

with 95% confidence for each attender there is an associated increase in dog sales between [0.48367, 0.90356]

#####

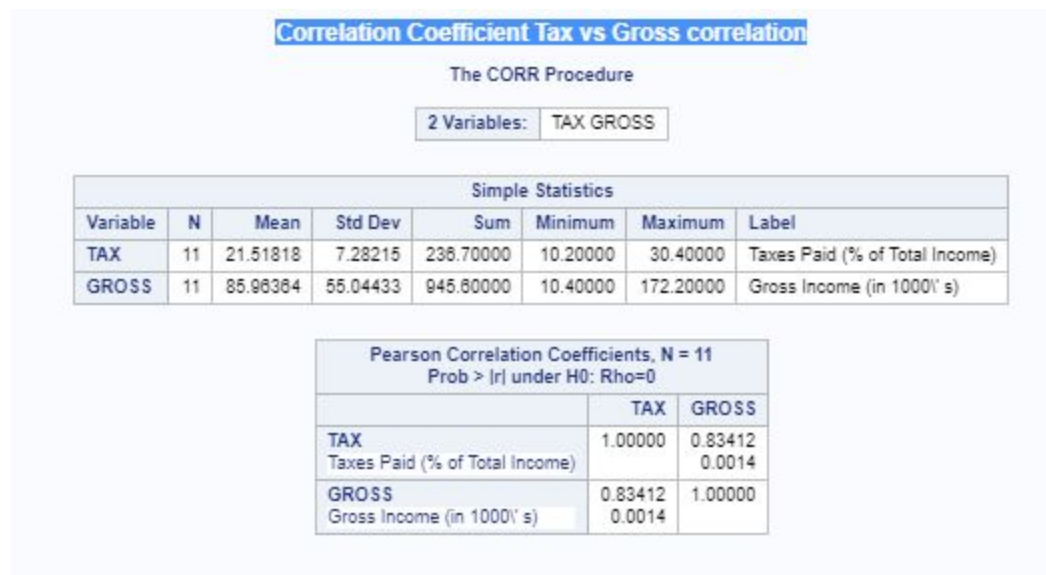
2. Although the income tax system is structured so that people with higher incomes should pay a higher percentage of their incomes in taxes, there are many loopholes and tax shelters available for individuals with higher incomes. A sample of 2017 tax returns gave the data listed in the table.

#####

```
data TAX_DATA;
input INDIVIDUAL GROSS TAX;
Label
    GROSS='Gross Income (in 1000\'\' s)'
    TAX='Taxes Paid (% of Total Income)'
;
cards;
1 44.2 16.0
2 92.0 20.1
3 17.0 11.1
4 54.0 24.3
5 10.4 10.2
6 172.2 30.4
7 63.9 27.3
8 125.9 27.9
9 83.6 16.2
10 167.7 29.8
11 114.7 23.4
;
```

```
#####
(a) Compute the sample correlation coefficient and
interpret the results.
```

```
#####
proc corr data=TAX_DATA;
title 'Correlation Coefficient Tax vs Gross correlation';
var TAX GROSS ;
Run;
```



TAX vs Gross : Correlation Coefficient = 0.83412 significance = 0.0014
This means that Correlation Coefficient > 0.8 and therefore
it is strong positive correlation. If Gross is increasing then Tax is
increasing.
Ho -> correlation between Tax and Gross Income equal zero
Ha -> correlation between Tax and Gross Income !=zero
because significance =0.0014 < 0.05 we reject Ho. We have a evidence
that correlation between Tax and Gross Income is not zero.

```
#####
(b) Compute r-squared and interpret results.
```

```
#####
proc reg data=TAX_DATA;
```

```

title 'Regression Tax vs Gross';
model TAX=GROSS /clb;
run;

```

Regression Tax vs Gross

The REG Procedure
Model: MODEL1
Dependent Variable: TAX Taxes Paid (% of Total Income)

| | |
|-----------------------------|----|
| Number of Observations Read | 11 |
| Number of Observations Used | 11 |

| Analysis of Variance | | | | | |
|----------------------|----|----------------|-------------|---------|--------|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 368.96003 | 368.96003 | 20.58 | 0.0014 |
| Error | 9 | 161.33633 | 17.92626 | | |
| Corrected Total | 10 | 530.29636 | | | |

| | | | |
|----------------|----------|----------|--------|
| Root MSE | 4.23394 | R-Square | 0.6958 |
| Dependent Mean | 21.51818 | Adj R-Sq | 0.6620 |
| Coeff Var | 19.67611 | | |

| Parameter Estimates | | | | | | | | |
|---------------------|--------------------------|----|--------------------|----------------|---------|---------|-----------------------|----------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t | 95% Confidence Limits | |
| Intercept | Intercept | 1 | 12.03199 | 2.44988 | 4.91 | 0.0008 | 6.49003 | 17.57395 |
| GROSS | Gross Income (in 1000's) | 1 | 0.11035 | 0.02432 | 4.54 | 0.0014 | 0.05533 | 0.16538 |

$R\text{-squared} = \text{Sum Of Squares Of Model} / \text{Sum Of Squares Of Total}$
 $= 368.960 / 530.296 = 0.6958$

$R\text{-squared} = 0.6958$

R-square is the proportion of the variance explained by independent variables.

R-square means that out of total variance 69.58% explained by Variance of the model. (in our case this is a Gross) or 69.58% of total tax variance can be predicted by the variable Gross income.

#####

(c) Is the correlation coefficient significant at $\alpha = 0.05$?

#####

Correlation Coefficient Tax vs Gross correlation

The CORR Procedure

2 Variables: TAX GROSS

| Simple Statistics | | | | | | | |
|-------------------|----|----------|----------|-----------|----------|-----------|--------------------------------|
| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum | Label |
| TAX | 11 | 21.51818 | 7.28215 | 236.70000 | 10.20000 | 30.40000 | Taxes Paid (% of Total Income) |
| GROSS | 11 | 85.96364 | 55.04433 | 945.60000 | 10.40000 | 172.20000 | Gross Income (in 1000' s) |

| Pearson Correlation Coefficients, N = 11 Prob > r under H0: Rho=0 | | |
|--|-------------------|-------------------|
| | TAX | GROSS |
| TAX Taxes Paid (% of Total Income) | 1.00000 | 0.83412 0.0014 |
| GROSS Gross Income (in 1000' s) | 0.83412 0.0014 | 1.00000 |

Coefficient Of Correlation = 0.83412 p=0.0014

Ho - >Correlation between Tax paid and Gross Income is zero

Ha -> correlation between Tax paid and Gross income is not zero

p-value =0.0014 < alpha=0.05 -> We reject Ho. We conclude that we have evidence that correlation between tax paid and the gross income is not zero for alpha=0.05.

#####

(d) Compute the estimated line of regression.

(e)What is the estimate of standard deviation (root MSE) ?

#####

```
proc reg data=TAX_DATA;  
title 'Regression Tax vs Gross';  
model TAX=GROSS /clb;  
run;
```

Estimated line of regression

The REG Procedure
Model: MODEL1
Dependent Variable: TAX Taxes Paid (% of Total Income)

| | |
|-----------------------------|----|
| Number of Observations Read | 11 |
| Number of Observations Used | 11 |

| Analysis of Variance | | | | | |
|----------------------|----|----------------|-------------|---------|--------|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 368.98003 | 368.98003 | 20.58 | 0.0014 |
| Error | 9 | 161.33633 | 17.92626 | | |
| Corrected Total | 10 | 530.29636 | | | |

| | | | |
|----------------|----------|----------|--------|
| Root MSE | 4.23394 | R-Square | 0.6958 |
| Dependent Mean | 21.51818 | Adj R-Sq | 0.6620 |
| Coeff Var | 19.67611 | | |

| Parameter Estimates | | | | | | | | |
|---------------------|--------------------------|----|--------------------|----------------|---------|---------|-----------------------|----------|
| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > t | 95% Confidence Limits | |
| Intercept | Intercept | 1 | 12.03199 | 2.44988 | 4.91 | 0.0008 | 6.49003 | 17.57395 |
| GROSS | Gross Income (in 1000's) | 1 | 0.11035 | 0.02432 | 4.54 | 0.0014 | 0.05533 | 0.16538 |

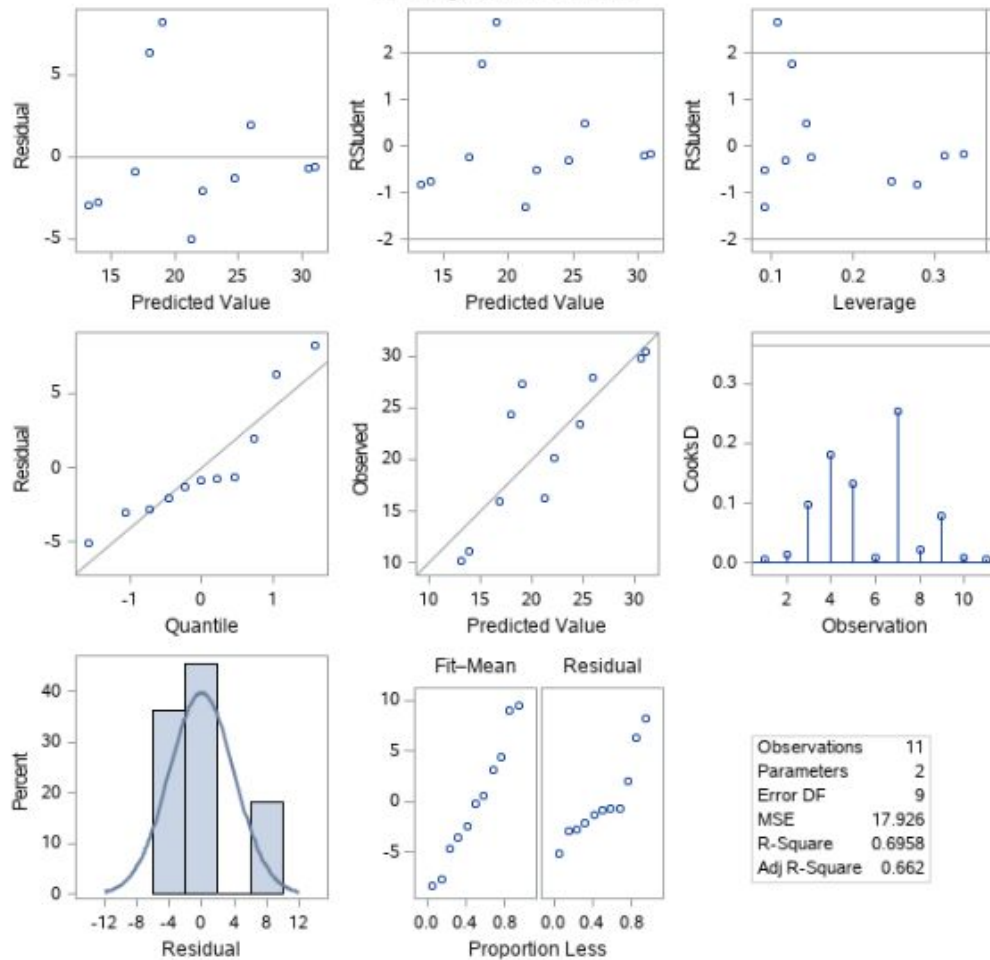
Estimated line of regression

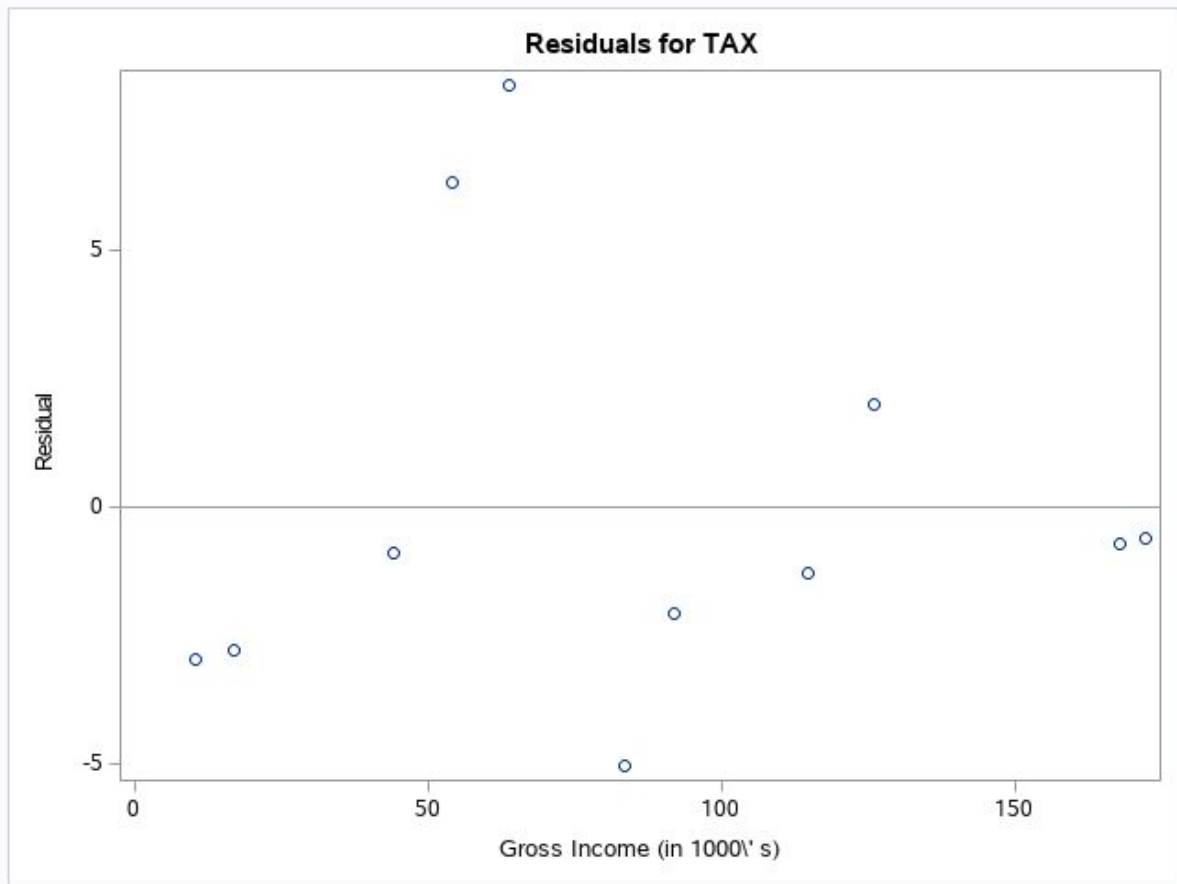
The REG Procedure

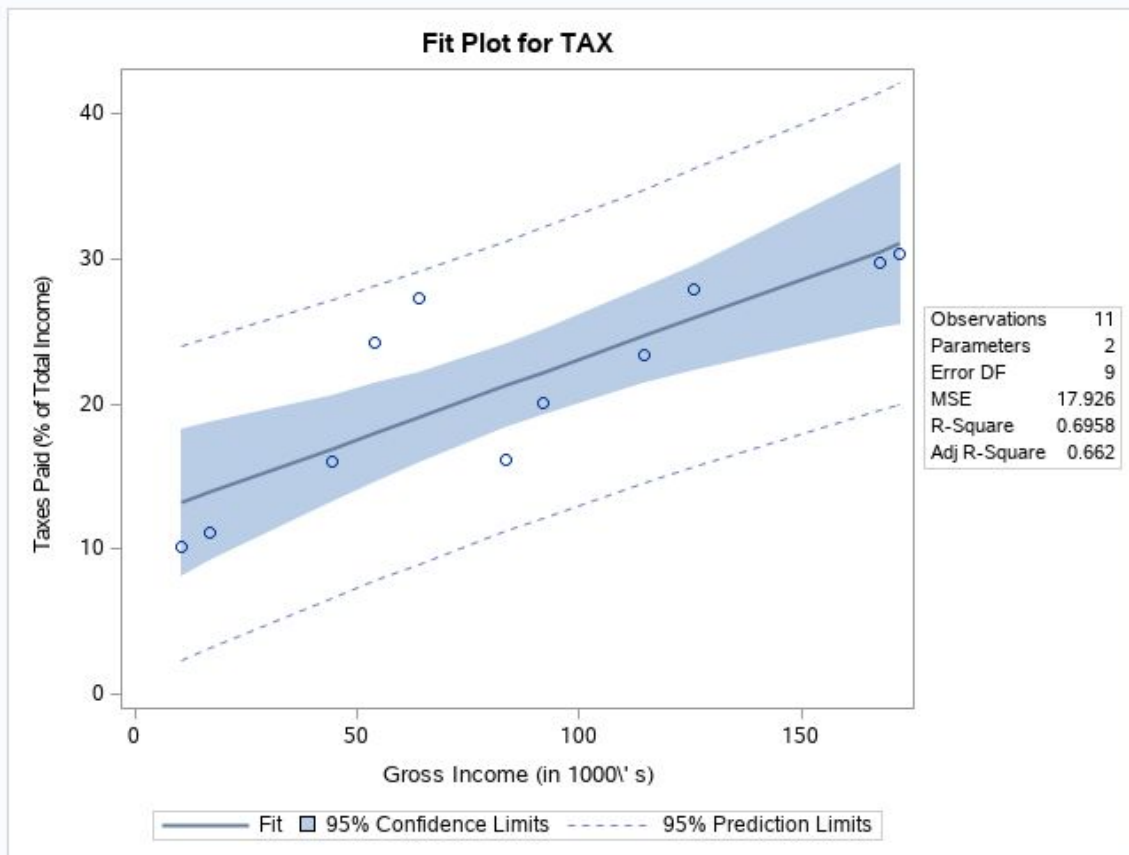
Model: MODEL1

Dependent Variable: TAX Taxes Paid (% of Total Income)

Fit Diagnostics for TAX







We calculated Intercept and slope, therefore estimated line of regression is:

$$\text{TaxPaid} = 12.03199 + 0.11035 * \text{GrossIncome}$$

The estimate of standard deviation (square root MSE):

$$\text{root MSE} = 4.23394$$

Root MSE is a square root of the mean square residual.

#####

(f) Predict the mean percentage of income paid in taxes by individuals with a gross income of \$80,000. Report

and interpret the confidence interval for this estimate.

#####

$\text{TAX}(\$80000) = 12.03199 + 0.11035 * 80 = 20.85999$

```
data TAX_DATA;
set TAX_DATA end=last;
output;
if last then do;
TAX=.;
GROSS=80;
output;
end;
run;

proc reg data=TAX_DATA;
model TAX=GROSS / clb;
output out=TAX_DATA_out (where=(TAX=.)) predicted=TAX_hat
LCLM=LCL_mean UCLM=UCL_mean;
run;

proc print data=TAX_DATA_OUT ;
title 'Report Prediction Of Tax Data';
var GROSS TAX_hat LCL_mean UCL_mean;
Run;
```

Report Prediction Of Tax Data

| Obs | GROSS | TAX_hat | LCL_mean | UCL_mean |
|-----|-------|---------|----------|----------|
| 1 | 80 | 20.8601 | 17.9537 | 23.7665 |

for 80000 of gross income
estimated tax paid = 20.8601% with
95% CI= [17.9537 , 23.7665]
the mean tax of 80000 income will be in the interval
[17.9537 , 23.7665].

