INNA WILLIAMS

Section 4.1

1 a

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$A^T A = A^T \cdot b$$

$$A^T A = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 5 & 4 \\ 4 & 6 \end{vmatrix}$$

$$A^T b = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} 3 \\ 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 5 \\ 8 \end{vmatrix}$$

Normal equation:

$$\begin{vmatrix} 5 & 4 \\ 4 & 6 \end{vmatrix} \times \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 5 \\ 8 \end{vmatrix}$$

$$\left|\begin{array}{cc|c} 5 & 4 & 5 \\ 4 & 6 & 8 \end{array}\right| = \left|\begin{array}{cc|c} 5 & 4 & 5 \\ 2 & 3 & 4 \end{array}\right| = \left|\begin{array}{cc|c} 10 & 8 & 10 \\ 10 & 15 & 20 \end{array}\right| = \left|\begin{array}{cc|c} 0 & -7 & -10 \\ 2 & 3 & 4 \end{array}\right|$$

$$= \left|\begin{array}{cc|c} 0 & -21 & -30 \\ 14 & 21 & 28 \end{array}\right| = \left|\begin{array}{cc|c} 0 & 7 & 10 \\ 14 & 0 & -2 \end{array}\right| \Rightarrow \begin{array}{l} x_1 = -\frac{2}{14} = -\frac{1}{7} \\ x_2 = \frac{10}{7} \end{array}$$

$$\bar{x} = \left[-\frac{1}{7}, \frac{10}{7}\right]$$

$$e = b - Ax = \begin{vmatrix} 3 \\ 1 \\ 1 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{vmatrix} \cdot \begin{vmatrix} -\frac{1}{7} \\ \frac{10}{7} \end{vmatrix} = \begin{vmatrix} 3 \\ 1 \\ 1 \end{vmatrix} - \begin{vmatrix} \frac{15}{7} \\ \frac{10}{7} \\ \frac{3}{7} \end{vmatrix} = \begin{vmatrix} \frac{2}{7} \\ -\frac{3}{7} \\ \frac{4}{7} \end{vmatrix}$$

$$e^2 = \begin{vmatrix} \frac{4}{49} \\ \frac{9}{49} \\ \frac{1}{49} \end{vmatrix}$$

$$Sum(e^2) = \frac{4}{49} + \frac{9}{49} + \frac{1}{49} = \frac{14}{49} = \frac{2}{7}$$

$$Norm = \sqrt{Sum(e^2)} = \sqrt{\frac{2}{7}} =$$

Answer : $\hat{x} = \left[-\frac{1}{7}, \frac{10}{7} \right]$

$$Sum(e^2) = \frac{2}{7}$$

$$Norm = \sqrt{\frac{2}{7}} = 0.5345$$

INNA WILLIAMS

Section 4.1

8 (b) $(1,2), (3,2), (4,1), (6,3)$

$y = C_1 + C_2 t$ $\quad A^T A = A^T B \rightarrow$ normal equation

$$\begin{vmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{vmatrix} \times \begin{vmatrix} C_1 \\ C_2 \end{vmatrix} = \begin{vmatrix} 2 \\ 2 \\ 1 \\ 3 \end{vmatrix} \qquad A = \begin{vmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{vmatrix} \quad b = \begin{vmatrix} 2 \\ 2 \\ 1 \\ 3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{vmatrix} = \begin{vmatrix} 4 & 14 \\ 14 & 62 \end{vmatrix} = A^T \cdot A$$

$$A^T \cdot b = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{vmatrix} \cdot \begin{vmatrix} 2 \\ 2 \\ 1 \\ 3 \end{vmatrix} = \begin{vmatrix} 8 \\ 30 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 14 \\ 14 & 62 \end{vmatrix} \times \begin{vmatrix} C_1 \\ C_2 \end{vmatrix} = \begin{vmatrix} 8 \\ 30 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 14 & 8 \\ 14 & 62 & 30 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 4 \\ 7 & 31 & 15 \end{vmatrix} = \begin{vmatrix} 14 & 49 & 28 \\ 14 & 62 & 30 \end{vmatrix} =$$

$$2 \begin{vmatrix} 14 & 49 & 28 \\ 0 & 13 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 4 \\ 0 & 13 & 2 \end{vmatrix} = \begin{vmatrix} 26 & 91 & 52 \\ 0 & 91 & 14 \end{vmatrix} = \begin{vmatrix} 26 & 0 & 38 \\ 0 & 91 & 14 \end{vmatrix} =$$

$$= \begin{vmatrix} 13 & 0 \\ 0 & 13 \end{vmatrix} \begin{vmatrix} 19 \\ 2 \end{vmatrix} \Rightarrow \quad c_1 = \frac{19}{13} \qquad c_2 = \frac{2}{13}$$

$y = \frac{19}{13} + \frac{2}{13} \cdot 2$ *

$$\boxed{y = \frac{19}{13} + \frac{2}{13} X}$$

X	y	line	e^2
1	2	1,6153846	0.1479
3	2	1.9230769	0.0059
4	1	2.0769230	1.1598
6	3	2.3846154	0.3787

$Sum(e^2) = 1,69231$

$NORM = \sqrt{sum(e^2)} = \sqrt{1,69231} = 1,3$

$RMSE = 0.6504 = \frac{NORM}{\sqrt{4}} = \frac{NORM}{2} = \frac{1.3}{2}$

Answer: Line fit: $y = \frac{19}{13} + \frac{2}{13} \cdot X$

$RMSE = 0,6504$

Section 4.1

96

$(1,2), (3,2), (4,1), (6,3)$

$y = c_1 + c_2 t + c_3 t^2$

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{vmatrix} \qquad b = \begin{vmatrix} 2 \\ 2 \\ 3 \\ 1 \end{vmatrix}$$

Sol: $A^T A = A^T b$

$$A^T \cdot A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \\ 1 & 9 & 16 & 36 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 14 & 62 \\ 14 & 62 & 308 \\ 62 & 308 & 1634 \end{vmatrix}$$

$$A^T \cdot b = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \\ 1 & 9 & 16 & 36 \end{vmatrix} \times \begin{vmatrix} 2 \\ 2 \\ 3 \\ 1 \end{vmatrix} = \begin{vmatrix} 8 \\ 30 \\ 144 \end{vmatrix}$$

Normal equation

$$\begin{vmatrix} 4 & 14 & 62 \\ 14 & 62 & 308 \\ 62 & 308 & 1634 \end{vmatrix} \times \begin{vmatrix} c_1 \\ c_2 \\ c_3 \end{vmatrix} = \begin{vmatrix} 8 \\ 30 \\ 144 \end{vmatrix}$$

$$\left|\begin{array}{ccc|c} 2 & 7 & 31 & 4 \\ 7 & 31 & 154 & 15 \\ 31 & 154 & 817 & 72 \end{array}\right| = \left|\begin{array}{ccc|c} 14 & 49 & 217 & 28 \\ 14 & 62 & 308 & 30 \\ 31 & 154 & 817 & 72 \end{array}\right| =$$

$$\begin{vmatrix} 0 & -13 & -91 & -2 \\ 7 & 31 & 154 & 15 \\ 31 & 154 & 817 & 72 \end{vmatrix} = \begin{vmatrix} 0 & -13 & -91 & -2 \\ 217 & 961 & 4774 & 465 \\ 217 & 1078 & 5719 & 504 \end{vmatrix} =$$

$$= \begin{vmatrix} 0 & -13 & -91 & -2 \\ 0 & -117 & -945 & -39 \\ 31 & 154 & 817 & 72 \end{vmatrix} = \begin{vmatrix} 0 & -1521 & -10647 & -234 \\ 0 & 1521 & 12285 & 507 \\ 31 & 154 & 817 & 72 \end{vmatrix} =$$

$$= \begin{vmatrix} 0 & 0 & 1638 & 273 \\ 0 & 117 & 945 & 39 \\ 31 & 154 & 817 & 72 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1547910 & 257985 \\ 0 & 191646 & 1547910 & 63882 \\ 31 & 154 & 817 & 72 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1638 & 273 \\ 0 & 191646 & 0 & -194103 \\ 31 & 154 & 817 & 72 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1338246 \\ 0 & 191646 & 0 \\ 50778 & 252252 & 1338246 \end{vmatrix}$$

$$\begin{vmatrix} 223044 \\ -194103 \\ 117936 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1638 & 273 \\ 0 & 191646 & 0 & -194103 \\ 50778 & 252252 & 0 & -105105 \end{vmatrix} =$$

$$\begin{vmatrix} 0 & 0 & 1638 \\ 0 & 191646 & 0 \\ 9731400588 & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 1638 & 273 \\ 0 & 4834308679 2 & & -489628699 56 \\ 9731400588 & 4834308679 2 & 0 & 201429528 30 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1638 & 273 \\ 0 & 191646 & 0 & -194103 \\ 9731400588 & 0 & 0 & 288199171 26 \end{vmatrix}$$

$$c_3 = \frac{273}{1638} = 0,1667$$

$$c_2 = -\frac{194103}{191646} = -1,0128$$

$$c_1 = \frac{288199171 26}{9731400588} = 2.9615$$

X	y	parabola	e	e^2	Sum of e^2
1	2	2.1153847	-0.1153846	0.0133136	0.6923769
3	2	1.4230770	0.5769231	0.3328402	
4	1	1.5769231	-0.5769231	0.3328402	
6	3	2.8846154	0.1153846	0.0133136	

$$NORM = \sqrt{Sum\ e^2} = \sqrt{0.6923769} = 0.8320503$$

$$RMSE = \frac{NORM}{\sqrt{4-1}} = \frac{NORM}{2} = \frac{0.8320503}{2} = 0.4160251$$

Answer: Parabola:

$$y = \frac{28819917126}{97314005888} - \frac{194103}{191646}x + \frac{273}{1638}x^2$$

or: $y = 2.9615 - 1.0128 \cdot x + 0.1667 \cdot x^2$

RMSE = 0.4160

Norm = 0.8321

RMSE for parabola = 0.4160 <
RMSE for line = 0.6504
therefore parabola is better fit
in this case.