INNA WILLIAMS

Section 3.5.

$\boxed{1}$ (b) $(1,1), (0,0), (-2,0), (-2,1)$

$b_x = 3(x_2 - x_1) = 3(0-1) = -3$

$c_x = 3(x_3 - x_2) - b_x = 3(-2-0)+3 = -3$

$d_x = x_4 - x_1 - b_x - c_x = -2-1+3+3 = 3$

$b_y = 3(y_2 - y_1) = 3(0-1) = -3$

$c_y = 3(y_3 - y_2) - b_y = 3\cdot(0-0)+3 = 3$

$d_y = y_4 - y_1 - b_y - c_y = 1-1+3-3 = 0$

Answer: $x(t) = 1 - 3t - 3t^2 + 3t^3$

$y(t) = 1 - 3t + 3t^2$

$\boxed{2}$ (b) $x(t) = 3 + 4t - t^2 + 2t^3$

$y(t) = 2 - t + t^2 + 3t^3$

$\boxed{x_1 = 3}$, $b_x = 4$, $c_x = -1$, $d_x = 2$

$3(x_2 - x_1) = b_x \Rightarrow 3(x_2 - 3) = 4$

$\Rightarrow \boxed{x_2 = 13/3}$

$3(x_3 - x_2) - b_x = c_x \Rightarrow 3(x_3 - \frac{13}{3}) - 4 = -1$

$\Rightarrow x_3 = \frac{-1+4}{3} + \frac{13}{3} = \boxed{\frac{16}{3} = x_3}$

$x_4 - x_1 - b_x - c_x = d_x \Rightarrow$

$x_4 - 3 - 4 + 1 = 2 \Rightarrow$

$\boxed{x_4 = 8}$

$\boxed{y_1 = 2}$, $b_y = -1$, $c_y = 1$, $d_y = 3$

$3(y_2 - y_1) = b_y \Rightarrow 3(y_2 - 2) = -1$, $\Rightarrow \boxed{y_2 = \frac{5}{3}}$

$3(y_3 - y_2) - b_y = c_y \Rightarrow 3(y_3 - \frac{5}{3}) + 1 = 1 \Rightarrow \boxed{y_3 = \frac{5}{3}}$

$y_4 - y_1 - b_y - c_y = d_y \Rightarrow y_4 - 2 + 1 - 1 = 3 \Rightarrow \boxed{y_4 = 5}$

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Answer:

$$\begin{cases} (x_1, y_1) \to (3, 2) \\ (x_2, y_2) \to \left(\frac{13}{3}, \frac{5}{3}\right) \\ (x_3, y_3) \to \left(\frac{16}{3}, \frac{5}{3}\right) \\ (x_4, y_4) \to (8, 5) \end{cases}$$

$$(3, 2), \left(\frac{13}{3}, \frac{5}{3}\right), \left(\frac{16}{3}, \frac{5}{3}\right), (8, 5)$$

INNA WILLIAMS

$\boxed{7}$ $(-1,0) \to (x_1, y_1)$ $(1,0) \to (x_4, y_4)$

passes through $x=0$, $y=1$ and has vertical tangents at midpoint

vertical tangents: $\frac{x_2 - x_1}{y_2 - y_1} = 0$ $\frac{x_4 - x_3}{y_4 - y_3} = 0$

$\Downarrow$ $\Downarrow$

$x_2 = x_1 = -1$ $x_4 = x_3 = 1$

the points are: $(-1,0), (-1, y_2) (1, y_3) (1,0)$

$\boxed{b_x} = 3(x_2 - x_1) = 3(-1+1) = \boxed{0}$, $\boxed{c_x} = 3(x_3 - x_2) - b_x = 3(1+1) - 0 = \boxed{6}$

$\boxed{d_x} = 1+1-6-0 = \boxed{-4}$, $\boxed{b_y} = 3(y_2 - y_1) = 3(y_2 - 0) = \boxed{3y_2}$,

$\boxed{c_y} = 3(y_3 - y_2) - b_y = 3(y_3 - y_2) - 3y_2 = \boxed{3y_3 - 6y_2}$

$\boxed{d_y} = (y_4 - y_1 - b_y - c_y) = (0-0) - 3y_2 - 3y_3 + 6y_2 = \boxed{3y_2 - 3y_3}$

Bezier curves are:

$x(t) = -1 + 0 \cdot t + 6t^2 - 4t^3 \Rightarrow$ $\boxed{x(t) = -1 + 6t^2 - 4t^3}$

$y(t) = 0 + 3y_2 \cdot t + (3y_3 - 6y_2)t^2 + (3y_2 - 3y_3)^3 \Rightarrow$

$\Rightarrow$ $\boxed{y(t) = 3y_2 t + (3y_3 - 6y_2)t^2 + (3y_2 - 3y_3)t^3}$

passes through $x=0$, $y=1$:

$x(t) = -1 + 6t^2 - 4t^3 = 0$ or $+4t^3 - 6t^2 + 1 = 0$

roots can be found trying: $\pm 1, \pm 4, \pm\sqrt{4}, \left(\pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}\right)$

try $\frac{1}{2}$: $4 \cdot \frac{1}{8} - 6 \cdot \frac{1}{4} + 1 = \frac{1}{4} - \frac{6}{4} + 1 = 0 \Rightarrow x\left(\frac{1}{2}\right) = 0$

now we have to solve $\boxed{y\left(\frac{1}{2}\right) = 1}$

$$y\left(\tfrac{1}{2}\right) = 3\cdot y_2\cdot\tfrac{1}{2} + (3y_3 - 6y_2)\cdot\tfrac{1}{4} + (3y_2 - 3y_3)\,\tfrac{1}{8} = 1$$

$$\cancel{\tfrac{3}{2} y_2} + \tfrac{3}{4} y_3 - \cancel{\tfrac{3}{2} y_2} + \tfrac{3}{8} y_2 - \tfrac{3}{8} y_3 - 1 = 0$$

$$\tfrac{3}{8} y_3 + \tfrac{3}{8} y_2 = 1 \Rightarrow \tfrac{3}{8} y_3 = 1 - \tfrac{3}{8} y_2 \Rightarrow y_3 = \tfrac{8}{3}\left(1 - \tfrac{3}{8}\right) y_2 \Rightarrow$$

$\Rightarrow \boxed{y_3 = \frac{8}{3} - y_2}$ → any linear combination of y_2 and y_3 of this form will satisfy

for any y_2 the Bezier curves would be

$$\left.\begin{aligned} x(t) &= -1 + 6t^2 - 4t^3 \\ y(t) &= 3y_2 + \left(3\left(\tfrac{8}{3} - y_2\right) - 6y_2\right)t^2 + \left(3y_2 - 3\left(\tfrac{8}{3} - y_2\right)\right)t^3 \end{aligned}\right\}$$

$$\Rightarrow \boxed{\begin{cases} x(t) = -1 + 6t^2 - 4t^3 \\ y(t) = 3y_2 + (8 - 9y_2)t^2 + (6y_2 - 8)t^3 \end{cases}} \to \text{any } y_2$$

if we set $y_2 = y_3$ then from $\left\{y_3 = \frac{8}{3} - y_2,\ y_2 = y_3\right\} \Rightarrow$

$$\Rightarrow \left\{\begin{aligned} y_3 + y_2 &= \tfrac{8}{3} \\ y_2 &= y_3 \end{aligned}\right\} \Rightarrow 2y_2 = \tfrac{8}{3},\ \boxed{y_2 = y_3 = \tfrac{4}{3}} \to \text{for symmetric Bezier curve}$$

$$\Rightarrow y(t) = 3\cdot\tfrac{4}{3} + \left(8 - 9\cdot\tfrac{4}{3}\right)t^2 + \left(6\cdot\tfrac{4}{3} - 8\right)t^3 = 4 + (-4)\cdot t^2 + 0\cdot t^3 \Rightarrow$$

$\boxed{y(t) = -4t^2 + 4}$ → for symmetric Bezier curve

Answer: $\boxed{\text{symmetric Bezier curve} = \begin{cases} x(t) = -1 + 6t^2 - 4t^3 \\ y(t) = 4 - 4t^2 \end{cases}}$

$\boxed{\text{for arbitrary } y_2 = \begin{cases} x(t) = -1 + 6t^2 - 4t^3 \\ y(t) = 3y_2 + (8 - 9y_2)t^3 + (6y_2 - 8)t^3 \end{cases}}$