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I)

Problem 9.

A behavioral psyhologyst places a rat each day in the cage with 2 doors, A and B The rat can go through door A, where it recieves electric shock, or through door B, where it receive some food. A records is made of the door through which the rat passes. At the start of the experiment, on Monday, the rat is equally likely to go through door A as through door B. After going through door A, and receiving a shock, the probability of going through the same door on the next day is 0.3. After going throgh door B, and receiving food, the probability of going through the same door on the next day is 0.6

a)

Write the transition matrix for the Markov process.

A	B	
0.3	**0.4**	**A**
0.7	**0.6**	**B**

A	B	
0.3	0.4	A
0.7	0.6	B

b) What is the probability of the rat going through door A on Thursday(the 3rd day after starting the experiment)?

> $with(LinearAlgebra):$

> $T := Matrix([[0.3, 0.4], [0.7, 0.6]])$

$$T := \begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 0.6 \end{bmatrix} \tag{1}$$

> $X0 := \langle 0.5, 0.5 \rangle$

$$X0 := \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \tag{2}$$

> $X3 := MatrixPower(T, 3).X0$

$$X3 := \begin{bmatrix} 0.363500000000000 \\ 0.636500000000000 \end{bmatrix} \tag{3}$$

Answer:

The probability that the rat going through door A On a 3rd day is 0.364

c) ***What is the steady state vector?***

> $(evalues, evectors) := Eigenvectors(T)$

$$evalues, evectors := \begin{bmatrix} -0.100000000000000 + 0.\ \mathrm{I} \\ 1. + 0.\ \mathrm{I} \end{bmatrix}, \begin{bmatrix} -0.707106781186548 + 0.\ \mathrm{I} & -0.496138938356834 + 0.\ \mathrm{I} \\ 0.707106781186547 + 0.\ \mathrm{I} & -0.868243142124459 + 0.\ \mathrm{I} \end{bmatrix} \quad (4)$$

>

> $domvector := Column(evectors, 2)$

$$domvector := \begin{bmatrix} -0.496138938356834 + 0.\ \mathrm{I} \\ -0.868243142124459 + 0.\ \mathrm{I} \end{bmatrix} \quad (5)$$

> $steady := \frac{domvector}{Norm(domvector, 1)}$

$$steady := \begin{bmatrix} -0.363636363749188 + 0.\ \mathrm{I} \\ -0.636363636561079 + 0.\ \mathrm{I} \end{bmatrix} \quad (6)$$

Answer:

Steady state vector is :

$$\boldsymbol{steady := \begin{bmatrix} 0.36 \\ 0.64 \end{bmatrix}}$$

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Problem 11

A study has determined that the occupation of a boy, as an adult, depends upon the ocupation of his father and is given by the following transition matrix where
P = professional
F = farther
L = laborer

		Father's	Ocupation	
Son's		P	F	L
Ocupation	P	0.8	0.3	0.2
	F	0.1	0.5	0.2
	L	0.1	0.2	0.6

a) What is the probability that the grandchild of the proffesional will also be a professional?

> $T := Matrix([[0.8, 0.3, 0.2], [0.1, 0.5, 0.2], [0.1, 0.2, 0.6]])$

(7)

$$T := \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \quad (7)$$

> $X0 := \langle 0.333, 0.333, 0.333 \rangle$

$$X0 := \begin{bmatrix} 0.333 \\ 0.333 \\ 0.333 \end{bmatrix} \quad (8)$$

> $X2 := MatrixPower(T, 2).X0$

$$X2 := \begin{bmatrix} 0.486180000000000 \\ 0.236430000000000 \\ 0.276390000000000 \end{bmatrix} \quad (9)$$

Answer:
The probability that the grandchild will be a professional is 0.486

b) In a long run, what proportion of the population will be farmers?

> $(evalues, evectors) := Eigenvectors(T)$

$$evalues, evectors := \begin{bmatrix} 1.00000000000000 + 0.\ \mathrm{I} \\ 0.561803398874989 + 0.\ \mathrm{I} \\ 0.338196601125011 + 0.\ \mathrm{I} \end{bmatrix}, \quad (10)$$

$[[-0.866448577718212 + 0.\ \mathrm{I}, -0.809016994374947 + 0.\ \mathrm{I}, 0.309016994374947 + 0.\ \mathrm{I}], [-0.324918216644329 + 0.\ \mathrm{I}, 0.309016994374948 + 0.\ \mathrm{I}, -0.809016994374948 + 0.\ \mathrm{I}], [-0.379071252751718 + 0.\ \mathrm{I}, 0.500000000000001 + 0.\ \mathrm{I}, 0.500000000000000 + 0.\ \mathrm{I}]]$

> $domvector := Column(evectors, 1)$

$$domvector := \begin{bmatrix} -0.866448577718212 + 0.\ \mathrm{I} \\ -0.324918216644329 + 0.\ \mathrm{I} \\ -0.379071252751718 + 0.\ \mathrm{I} \end{bmatrix} \quad (11)$$

> $steady := \dfrac{domvector}{Norm(domvector, 1)}$

$$steady := \begin{bmatrix} -0.551724137948382 + 0.\ \mathrm{I} \\ -0.206896551730643 + 0.\ \mathrm{I} \\ -0.241379310352417 + 0.\ \mathrm{I} \end{bmatrix} \quad (12)$$

check using the Xn = 1000

> $X1000 := MatrixPower(T, 1000).X0$

$$X1000 := \begin{bmatrix} 0.551172413793150 \\ 0.206689655172431 \\ 0.241137931034503 \end{bmatrix} \quad \textbf{(13)}$$

Answer:
In the long run the 20.69% will be farmers.
##
Problem 13.
A new mass transit system has just gone into operation. The transit authority has made studies that predict the percentage of commuters who will change to mass transit (M) The following transition matrix has been obtained:

		This	year
		M	A
Next	M	0.7	0.2
Year	A	0.3	0.8

Suppose that the population of the area remains constant, and that initially 30% of the commuters use mass transit and 70% use their automobiles.
X0 = [0.3, 0.7]
a) What percentage of commuters will be using the mass transit system after 1 year? after 2 years?

> $T := Matrix([[0.7, 0.2], [0.3, 0.8]])$

$$T := \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \quad \textbf{(14)}$$

> $X0 := \langle 0.3, 0.7 \rangle$

$$X0 := \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix} \quad \textbf{(15)}$$

> $X1 := MatrixPower(T, 1).X0$

$$X1 := \begin{bmatrix} 0.350000000000000 \\ 0.650000000000000 \end{bmatrix} \quad \textbf{(16)}$$

> $X2 := MatrixPower(T, 2).X0$

$$X2 := \begin{bmatrix} 0.375000000000000 \\ 0.625000000000000 \end{bmatrix} \quad \textbf{(17)}$$

Answer:
35% of commuters will be using mass transit after one year
37.5% of commuters will be using mass transit after second year

b) What percentage of the commuters will be using the mass transit system in the long run?

> $(evalues, evectors) := Eigenvectors(T)$

$$evalues, evectors := \begin{bmatrix} 0.500000000000000 + 0.\ I \\ 1. + 0.\ I \end{bmatrix}, \begin{bmatrix} -0.707106781186548 + 0.\ I & -0.554700196225229 + 0.\ I \\ 0.707106781186547 + 0.\ I & -0.832050294337844 + 0.\ I \end{bmatrix} \quad (18)$$

> $domvector := Column(evectors, 2)$

$$domvector := \begin{bmatrix} -0.554700196225229 + 0.\ I \\ -0.832050294337844 + 0.\ I \end{bmatrix} \quad (19)$$

> $steady := \dfrac{domvector}{Norm(domvector, 1)}$

$$steady := \begin{bmatrix} -0.399999999893055 + 0.\ I \\ -0.599999999839583 + 0.\ I \end{bmatrix} \quad (20)$$

Check steady with with X large

> $X100 := MatrixPower(T, 100).X0$

$$X100 := \begin{bmatrix} 0.400000000000002 \\ 0.600000000000003 \end{bmatrix} \quad (21)$$

Answer:

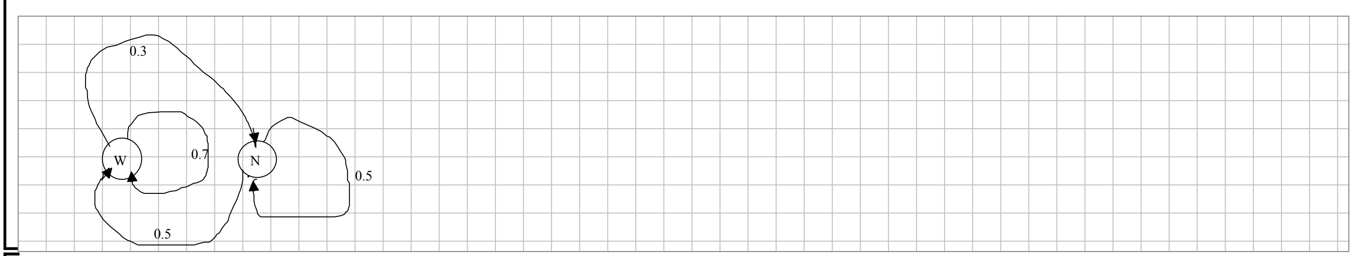
40% of commuters will be using the mass transit system in the long run

##

II) Mooney-Swift Chapter 3 handout:

Problem 6.

A copy machine is always in one of 2 states, either working or broken(not working). If it is working, there is 7-% chance that it will be working tomorrow. If it is broken there is 50% chance it will be broken tomorrow. Assume that one day is a natural time stemp. a) draw a state diagram



b) Formulate the transition Matrix

	W	N
W	0.7	0.5
N	0.3	0.5

c)Assuming the machine is working today.

$X0 := \langle 1, 0 \rangle$

$$X0 := \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{22}$$

> $T := Matrix([[0.7, 0.5], [0.3, 0.5]])$

$$T := \begin{bmatrix} 0.7 & 0.5 \\ 0.3 & 0.5 \end{bmatrix} \tag{23}$$

> $X0 := \langle 1, 0 \rangle$

$$X0 := \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{24}$$

What is the probability it will be working tomorrow?

> $X1 := MatrixPower(T, 1).X0$

$$X1 := \begin{bmatrix} 0.700000000000000 \\ 0.300000000000000 \end{bmatrix} \tag{25}$$

Answer: The probability it will be working tomorrow = 0.700

The next day?

> $X2 := MatrixPower(T, 2).X0$

$$X2 := \begin{bmatrix} 0.640000000000000 \\ 0.360000000000000 \end{bmatrix} \tag{26}$$

Answer: The probability it will be working next day = 0.640

After one week?

> $X7 := MatrixPower(T, 7).X0$

$$X7 := \begin{bmatrix} 0.625004800000000 \\ 0.374995200000000 \end{bmatrix} \tag{27}$$

Answer: The probability it will be working after one week = 0.625

After one month?

> $X30 := MatrixPower(T, 30).X0$

$$X30 := \begin{bmatrix} 0.624999999999999 \\ 0.375000000000000 \end{bmatrix} \tag{28}$$

Answer: The probability it will be working after one month = 0.625

d) What is the long term probability that the copy machine will be working on any given day

> $(evalues, evectors) := Eigenvectors(T)$

$$evalues, evectors := \begin{bmatrix} 1. + 0.\ \mathrm{I} \\ 0.200000000000000 + 0.\ \mathrm{I} \end{bmatrix}, \tag{29}$$

$$\begin{bmatrix} 0.857492925712544 + 0.\ \mathrm{I} & -0.707106781186548 + 0.\ \mathrm{I} \\ 0.514495755427527 + 0.\ \mathrm{I} & 0.707106781186548 + 0.\ \mathrm{I} \end{bmatrix}$$

> $domvector := Column(evectors, 1)$

$$domvector := \begin{bmatrix} 0.857492925712544 + 0.\ \mathrm{I} \\ 0.514495755427527 + 0.\ \mathrm{I} \end{bmatrix} \quad \mathbf{(30)}$$

> $steady := \dfrac{domvector}{Norm(domvector, 1)}$

$$steady := \begin{bmatrix} 0.625000000038019 + 0.\ \mathrm{I} \\ 0.375000000022811 + 0.\ \mathrm{I} \end{bmatrix} \quad \mathbf{(31)}$$

Answer:

The probability that the copy machine will be working on any given day = 0.625

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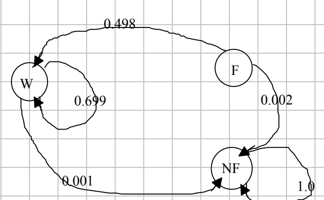
Problem 7.

A slightly more refined model of a copy machine has tree states:
working
brocken and fixable
brocken and infixable

a) Draw state diagram



b) Formulate the treansition Matrix

	W	F	NF
W	0.699	0.498	0
F	0.3	0.5	0
NF	0.001	0.002	1.0

> $T := Matrix([[0.699, 0.498, 0], [0.3, 0.5, 0], [0.001, 0.002, 1]])$

$$T := \begin{bmatrix} 0.699 & 0.498 & 0 \\ 0.3 & 0.5 & 0 \\ 0.001 & 0.002 & 1 \end{bmatrix} \quad (32)$$

c) Compute the fundamental matrix and interpret the results
How long this machine will last
How much of the time will it be working, and how much of that time will it be under repair(brocken and fixable)

Limiting steady matrix is

$$A := \begin{bmatrix} 0.699 & 0.498 \\ 0.3 & 0.5 \end{bmatrix}$$

$$A := \begin{bmatrix} 0.699 & 0.498 \\ 0.3 & 0.5 \end{bmatrix} \quad (33)$$

> $A := Matrix([[0.699, 0.498], [0.3, 0.5]])$

$$A := \begin{bmatrix} 0.699 & 0.498 \\ 0.3 & 0.5 \end{bmatrix} \quad (34)$$

> $Id := Matrix([[1, 0], [0, 1]])$

$$Id := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (35)$$

>

> $F := MatrixInverse(Id - A)$

$$F := \begin{bmatrix} 454.545454545446 & 452.727272727264 \\ 272.727272727268 & 273.636363636359 \end{bmatrix} \quad (36)$$

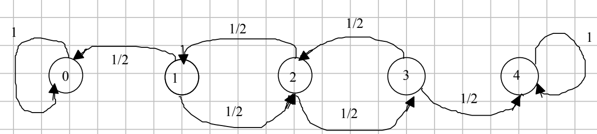
Answer:

454 days working , 272 broken but fixable

The machine will last $\{ NumberOfDays = 454 + 272 = 726\}$
726 days

III) Random (Drunkard's) walk.
A man walks along a four-block stretch of Park Avenue (see picture). If he is at corner 1, 2, or 3, then he walks to the left or right with equal probability. He continues until he reaches corner 4, which is a bar, or corner 0, which is his home. If he reaches either home or the bar, he stays there.

a)Construct the Markov chain and the associated transition matrix with states 0, 1, 2, 3, 4 (states 0 and 4 are absorbing states).



	1	2	3	0	4
1	0	0.5	0	0	0
2	0.5	0	0.5	0	0
3	0	0.5	0	0	0
0	0.5	0	0	1	0
4	0	0	0.5	0	1

b) Find the limiting steady-state matrix L (by hand) using the appropriate formulas.

$$\begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

$$\text{Id - A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \end{bmatrix} = \begin{matrix} 1 & -0.5 & 0 \\ -0.5 & 1 & -0.5 \\ 0 & -0.5 & 1 \end{matrix}$$

Det(Id - A) = 1 (1 - 0.25) + 0.5(-0.5 - 0) + 0 * (0.25 - 0) = 0.75 - 0.25 = 1/2

If determinant is not zero then matrix inverse exist.

$$\text{Transpose (Id - A)} = \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & -0.5 \\ 0 & -0.5 & 1 \end{bmatrix}$$

1 -0.5 = 3/4 -0.5 -0.5 = 0.5 -0.5 1 = 1/4
-0.5 1 0 1 0 -0.5

-0.5 0 = -0.5 1 0 = 1 1 -0.5 = -0.5
-0.5 1 0 1 0 -0.5

-0.5 0 = 0.25 1 0 = 0.5 1 -0.5 = 3/4
1 -0.5 -0.5 -0.5 -0.5 1

$$\text{Adj(Id - A)} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

$$\text{(Id - A)\^{}-1 = 1/det(Id - A) * Adj(Id - A)} = 2 * \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$$

$$\mathbf{L} = \text{B * (Id - A)\^{}-1} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \; X \; \begin{bmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

> $B := Matrix([[0.5, 0, 0], [0, 0, 0.5]])$

$$B := \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \quad (37)$$

> $Id := Matrix([[1, 0, 0], [0, 1, 0], [0, 0, 1]])$

$$Id := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (38)$$

> $A := Matrix([[0, 0.5, 0], [0.5, 0, 0.5], [0, 0.5, 0]])$

$$A := \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \end{bmatrix} \quad (39)$$

> $M := Id - A$

$$M := \begin{bmatrix} 1. & -0.500000000000000 & 0. \\ -0.500000000000000 & 1. & -0.500000000000000 \\ 0. & -0.500000000000000 & 1. \end{bmatrix} \quad (40)$$

> **F := MatrixInverse(M)**

$$F := \begin{bmatrix} 1.50000000000000 & 1.00000000000000 & 0.500000000000000 \\ 1.00000000000000 & 2.00000000000000 & 1.00000000000000 \\ 0.500000000000000 & 1.00000000000000 & 1.50000000000000 \end{bmatrix} \quad (41)$$

absorbtion probabilities:

> $Absorbtion := B \cdot F$

$$Absorbtion := \begin{bmatrix} 0.750000000000000 & 0.500000000000000 & 0.250000000000000 \\ 0.250000000000000 & 0.500000000000000 & 0.750000000000000 \end{bmatrix} \quad (42)$$

> $T := Matrix([[0, 0.5, 0, 0, 0], [0.5, 0, 0.5, 0, 0], [0, 0.5, 0, 0, 0], [0.5, 0, 0, 1, 0], [0, 0, 0.5, 0, 1]])$

$$T := \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0 & 1 \end{bmatrix} \quad (43)$$

> $L := MatrixPower(T, 10)$

(44)

$$L := \begin{bmatrix} 0.0156250000000000 & 0. & 0.0156250000000000 & 0. & 0. \\ 0. & 0.0312500000000000 & 0. & 0. & 0. \\ 0.0156250000000000 & 0. & 0.0156250000000000 & 0. & 0. \\ 0.734375000000000 & 0.484375000000000 & 0.234375000000000 & 1. & 0. \\ 0.234375000000000 & 0.484375000000000 & 0.734375000000000 & 0. & 1. \end{bmatrix} \tag{44}$$

>

c) What is the probability that starting from state 1 the drunkard will reach home?

The probability that starting from state 1 the drunkard will reach home (state 0) is 0.75

d) What is the expected number of steps (starting from state 1) that the drunkard will take before reaching one of the absorbing states?

from Fundamental matrix F :

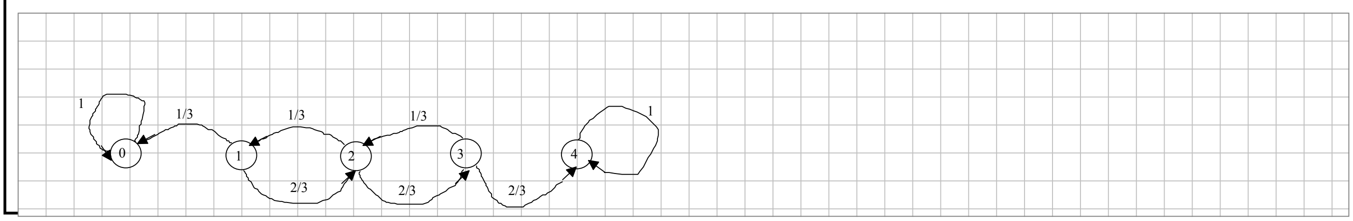
The sum of entries in column ‘j=1’ = number of steps for a system started in state 1 to be absorbed.

Number_Of_Steps = 1.5 + 1.0 + 0.5 = 3

Answer:

Number_Of_Steps = 3 *steps* if *start* in *state* 1 to *be obsorbed.*

e) repeat parts a)-d) ***a***)



	1	2	3	4	0
1	0	1/3	0	0	0
2	2/3	0	1/3	0	0
3	0	2/3	0	0	0
4	0	0	2/3	1	0
0	1/3	0	0	0	

b) Find the limiting steady-state matrix L (by hand) using the appropriate formulas.

$$A = \begin{bmatrix} 0 & \frac{1}{3} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} 0 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 0 \end{bmatrix} \qquad \mathrm{Id - A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & \frac{1}{3} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ -\frac{2}{3} & 1 & -\frac{1}{3} \\ 0 & -\frac{2}{3} & 1 \end{bmatrix}$$

Det(Id - A) = 1* (1 - 2/9) + 1/3 * (-2/3 + 0) + 0 * (0.25 - 0) = 0.25 - 0.25 = 7/9 - 2/9 = 5/9

If determinant not zero then matrix has an inverse

$$\text{Transpose (Id - A)} = \begin{bmatrix} 1 & \frac{-2}{3} & 0 \\ \frac{-1}{3} & 1 & \frac{-2}{3} \\ 0 & \frac{-1}{3} & 1 \end{bmatrix}$$

1 -2/3 = 7/9 -1/3 -2/3 = 1/3 -1/3 1 = 1/9
-1/3 1 0 1 0 -1/3

-2/3 0 = 2/3 1 0 = 1 1 -2/3 = 1/3
-1/3 1 0 1 0 -1/3

-2/3 0 = 4/9 1 0 = 2/3 1 -2/3 = 7/9
1 -2/3 -1/3 -2/3 -1/3 1

$$\text{Adj(Id - A)} = \begin{bmatrix} \frac{7}{9} & \frac{1}{3} & \frac{1}{9} \\ \frac{2}{3} & 1 & \frac{1}{3} \\ \frac{4}{9} & \frac{2}{3} & \frac{7}{9} \end{bmatrix}$$

$$\text{F = (Id - A)^-1 = 1/det(Id - A) * Adj(Id - A) = 9/5 *} \begin{bmatrix} \frac{7}{9} & \frac{1}{3} & \frac{1}{9} \\ \frac{2}{3} & 1 & \frac{1}{3} \\ \frac{4}{9} & \frac{2}{3} & \frac{7}{9} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{\frac{7}{5}} & \mathbf{\frac{3}{5}} & \mathbf{\frac{1}{5}} \\ \mathbf{\frac{6}{5}} & \mathbf{\frac{9}{5}} & \mathbf{\frac{3}{5}} \\ \mathbf{\frac{4}{5}} & \mathbf{\frac{6}{5}} & \mathbf{\frac{7}{5}} \end{bmatrix}$$

$$\boldsymbol{Absorbtion} := B \cdot F \;=\; \text{B * (Id - A)^-1} = \begin{bmatrix} 0 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 0. \end{bmatrix} \; X \; \begin{bmatrix} \frac{7}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{6}{5} & \frac{9}{5} & \frac{3}{5} \\ \frac{4}{5} & \frac{6}{5} & \frac{7}{5} \end{bmatrix} = \begin{bmatrix} \mathbf{\frac{8}{15}} & \mathbf{\frac{4}{5}} & \mathbf{\frac{14}{15}} \\ \mathbf{\frac{7}{15}} & \mathbf{\frac{1}{5}} & \mathbf{\frac{1}{15}} \end{bmatrix}$$

> $A := Matrix\left(\left[\left[0, \frac{1}{3}, 0\right], \left[\frac{2}{3}, 0, \frac{1}{3}\right], \left[0, \frac{2}{3}, 0\right]\right]\right)$

(45)

$$A := \begin{bmatrix} 0 & \frac{1}{3} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \end{bmatrix} \tag{45}$$

> $B := Matrix\left(\left[\left[0, 0, \frac{2}{3}\right], \left[\frac{1}{3}, 0, 0\right]\right]\right)$

$$B := \begin{bmatrix} 0 & 0 & \frac{2}{3} \\ \frac{1}{3} & 0 & 0 \end{bmatrix} \tag{46}$$

> $Id := Matrix([[1, 0, 0], [0, 1, 0], [0, 0, 1]])$

$$Id := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{47}$$

> $F := MatrixInverse(Id - A)$

$$F := \begin{bmatrix} \frac{7}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{6}{5} & \frac{9}{5} & \frac{3}{5} \\ \frac{4}{5} & \frac{6}{5} & \frac{7}{5} \end{bmatrix} \tag{48}$$

> $Absorbtion := evalf(B \cdot F)$

$$Absorbtion := \begin{bmatrix} 0.5333333333 & 0.8000000000 & 0.9333333333 \\ 0.4666666667 & 0.2000000000 & 0.06666666667 \end{bmatrix} \tag{49}$$

> $T := Matrix\left(\left[\left[0, \frac{1}{3}, 0, 0, 0\right], \left[\frac{2}{3}, 0, \frac{1}{3}, 0, 0\right], \left[0, \frac{2}{3}, 0, 0, 0\right], \left[0, 0, \frac{2}{3}, 1, 0\right], \left[\frac{1}{3}, 0, 0, 0, 1\right]\right]\right)$

$$T := \begin{bmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 1 \end{bmatrix} \tag{50}$$

> $X15 := evalf(MatrixPower(T, 15))$

$$X15 := \begin{bmatrix} 0. & 0.001141829130 & 0. & 0. & 0. \\ 0.002283658261 & 0. & 0.001141829130 & 0. & 0. \\ 0. & 0.002283658261 & 0. & 0. & 0. \\ 0.5315064067 & 0.7972596101 & 0.9324198700 & 1. & 0. \\ 0.4662099350 & 0.1993149025 & 0.06643830084 & 0. & 1. \end{bmatrix} \tag{51}$$

>

c) What is the probability that starting from state 1 the drunkard will reach home?

The probability that starting from state 1 the drunkard will reach home (state 0) is 0.47

>

d) What is the expected number of steps (starting from state 1) that the drunkard will take before reaching one of the absorbing states?

from Fundamental matrix F :
The sum of entries in column ‘j=1’ = number of steps for a system started in state 1 to be absorbed.

$$\text{Number\_Of\_Steps} = \frac{7}{5} + \frac{6}{5} - \frac{4}{5} = \frac{17}{5} = 3.4 \sim 4$$

Answer:

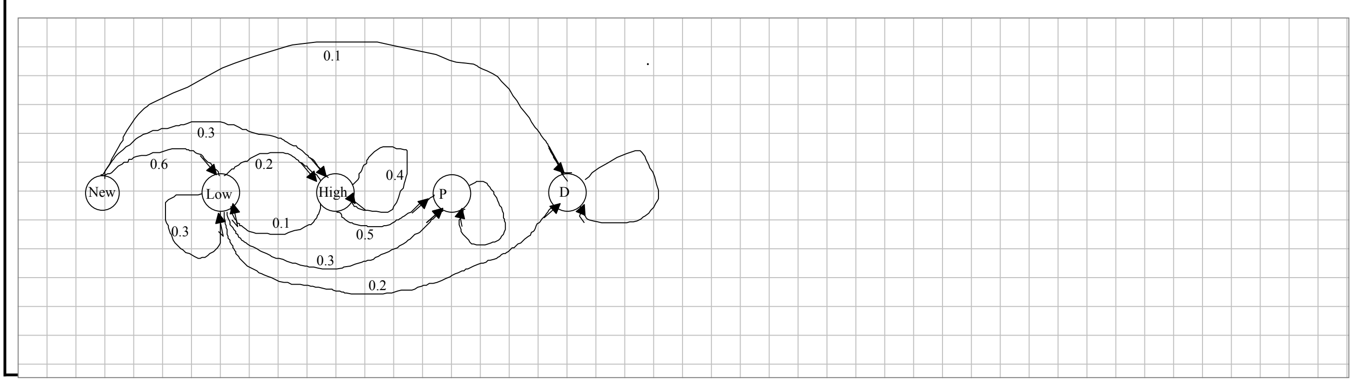
Number_Of_Steps exact(3.4) or rounded ~ 4 *steps* if *start* in *state* 1 the druckard will take before raching one of the absorbibg states

IV) Telemarketing: Absorbing Markov chains are used in marketing to model the probability that a customer who is contacted by telephone will eventually buy a product.
Consider a prospective customer who has never been called about purchasing a product. After one call, there is a 60% chance that the customer will express a low degree of interest in the product, a 30% chance of a high degree of interest, and a 10% chance the customer will be deleted from the company’s list of prospective customers.
Consider a customer who currently expresses a low degree of interest in the product. After another call, there is a 30% chance that the customer will purchase the product; a 20% chance the person will be deleted from the list, a 30% chance that the customer will still possess a low degree of interest, and a 20% chance that the customer will express a high degree of interest.
Consider a customer who currently expresses a high degree of interest in the product. After another call, there is a 50% chance that the customer will have purchased the product, a 40% chance that the customer will still have a high degree of interest, and a 10% chance that the customer will have a low degree of interest.

>

a) Construct a Markov chain with three transient states (new prospective, low interest and high interest customer), and two absorbing states (deletion, purchase) and the associated transition matrix

>



	New	Low	High	P	D
New	0	0	0	0	0
Low	0.6	0.3	0.1	0	0
High	0.3	0.2	0.4	0	0
P	0	0.3	0.5	1	0
D	0.1	0.2	0	0	1

>

b) What is the probability that a new prospective customer will eventually purchase the product?

> $T := Matrix([[0, 0, 0, 0, 0], [0.6, 0.3, 0.1, 0, 0], [0.3, 0.2, 0.4, 0, 0], [0, 0.3, 0.5, 1, 0], [0.1, 0.2, 0, 0, 1]])$

$$T := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0.3 & 0.1 & 0 & 0 \\ 0.3 & 0.2 & 0.4 & 0 & 0 \\ 0 & 0.3 & 0.5 & 1 & 0 \\ 0.1 & 0.2 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(52)}$$

```
> Id := Matrix([[1,0,0], [0,1,0], [0,0,1]])
```

$$Id := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{(53)}$$

> $A := Matrix([[0.0, 0.0, 0.0], [0.6, 0.3, 0.1,], [0.3, 0.2, 0.4]])$

$$A := \begin{bmatrix} 0. & 0. & 0. \\ 0.6 & 0.3 & 0.1 \\ 0.3 & 0.2 & 0.4 \end{bmatrix} \quad \textbf{(54)}$$

> $IdMinusA := Id - A$

(55)

$$IdMinusA := \begin{bmatrix} 1. & 0. & 0. \\ -0.600000000000000 & 0.700000000000000 & -0.100000000000000 \\ -0.300000000000000 & -0.200000000000000 & 0.600000000000000 \end{bmatrix} \quad (55)$$

```
> F := MatrixInverse(IdMinusA)
```

$$F := \begin{bmatrix} 1. & -0. & -0. \\ 0.975000000000000 & 1.50000000000000 & 0.250000000000000 \\ 0.825000000000000 & 0.500000000000000 & 1.75000000000000 \end{bmatrix} \quad (56)$$

> $B := Matrix([[0.0, 0.3, 0.5], [0.1, 0.2, 0]])$

$$B := \begin{bmatrix} 0. & 0.3 & 0.5 \\ 0.1 & 0.2 & 0 \end{bmatrix} \quad (57)$$

> $Absorbtion := B \cdot F$

$$Absorbtion := \begin{bmatrix} 0.705000000000000 & 0.700000000000000 & 0.950000000000000 \\ 0.295000000000000 & 0.300000000000000 & 0.0500000000000000 \end{bmatrix} \quad (58)$$

> $X10 := evalf(MatrixPower(T, 10))$

$X10 :=$ (59)

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. \\ 0.000586091100000000 & 0.000325589100000000 & 0.000325486700000000 & 0. & 0. \\ 0.00117172140000000 & 0.000650973400000001 & 0.000651075800000001 & 0. & 0. \\ 0.703476600900000 & 0.699153662900000 & 0.949153637300000 & 1. & 0. \\ 0.294765586600000 & 0.299869774600000 & 0.0498698002000000 & 0. & 1. \end{bmatrix}$$

>

Answer:

From Absorbtion Matrix :

The probability that a new prospective customer will eventually purchase the product = 0.705 (first row for purchase, first column for New) of the Absorbtion matrix)

c) What is the probability that a low-interest prospective customer will ever be deleted from the list?

Answer

The probability that low interest prospective will ever be deleted from the list = 0.3 (from Absorbtion matrix , last row for deleted and second column for Low)

d) On the average, how many times will a new prospective customer be called before either purchasing the product or being deleted from the list?"

Answer.

From Fundamental Matrix F

$$\textbf{(Ave\_Num\_Cals} = \sum_{i=k}^{n} col\_1\_values \quad (\textbf{for } new\ Customers\ sum\ col\,umn\ 1\ values)$$

Ave_Num_Cals = 1.0 + 0.975 + 0.825 = 2.8 rounds to ~ 3

(exact 2.8) or rounded ~ 3 times on average new perspective customer have to called before either purchasing the product or being deleted from the list.

>

>