INNA WILLIAMS

Section 12.3

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$$A = \begin{vmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{vmatrix} \quad |A - \lambda I| = \begin{vmatrix} \frac{3}{2}-\lambda & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2}-\lambda \end{vmatrix} = 0$$

$$\left(\frac{3}{2}-\lambda\right)^2 - \frac{1}{4} = 0 \Rightarrow \left(\frac{3}{2}-\lambda-\frac{1}{2}\right)\left(\frac{3}{2}-\lambda+\frac{1}{2}\right) = 0$$

$$\Rightarrow (1-\lambda)(2-\lambda) = 0 \Rightarrow \boxed{\lambda_1 = 2} \quad \boxed{\lambda_2 = 1}$$

$\lambda_1 = 2$

$$\begin{vmatrix} \frac{3}{2}-\frac{4}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2}-\frac{4}{2} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ -1 & -1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} \Rightarrow -x_1 - x_2 = 0$$

$$\Rightarrow -x_1 = x_2$$

$$v_1 = \begin{vmatrix} -1 \\ 1 \end{vmatrix} \quad \text{Normalized } v_1 = \begin{vmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{vmatrix}$$

$\lambda_2 = 1$

$$\begin{vmatrix} \frac{3}{2}-\frac{2}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2}-\frac{2}{2} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} \Rightarrow -x_1 + y_2 = 0 \Rightarrow$$

$$\Rightarrow x_1 = x_2$$

$$v_2 = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \quad \text{Normalized } v_2 = \begin{vmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{vmatrix}$$

Answer: $A = U \cdot S \cdot V^T = \begin{vmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} \times \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \times \begin{vmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix}$

Expands into ellipse with major axis of length = 4 along line $y = -x$

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2 (b) $A = \begin{vmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{vmatrix}$ Step 1: $A \cdot A^T = \begin{vmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{vmatrix} \cdot \begin{vmatrix} 6 & 8 \\ -2 & \frac{3}{2} \end{vmatrix} =$

$= \begin{vmatrix} 40 & 45 \\ 45 & \frac{265}{4} \end{vmatrix}$ $A - I\lambda = \begin{vmatrix} 40-\lambda & 45 \\ 45 & \frac{265}{4}-\lambda \end{vmatrix} = 0 \Rightarrow$

$(40-\lambda)\left(\frac{265}{4}-\lambda\right) - 45^2 = 0 \Rightarrow 4\lambda^2 - 425\lambda + 2500 = 0$

$\lambda_1 = 100$, $\lambda_2 = 25/4$

$\lambda_1 = 100$

$\left|\begin{array}{cc|c} -60 & 45 & 0 \\ 45 & -\frac{135}{4} & 0 \end{array}\right| = \left|\begin{array}{cc|c} -60 & 45 & 0 \\ 180 & -135 & 0 \end{array}\right| = \left|\begin{array}{cc|c} -180 & 135 & 0 \\ 180 & -135 & 0 \end{array}\right| = \left|\begin{array}{cc|c} 0 & 0 & 0 \\ 4 & -3 & 0 \end{array}\right| \Rightarrow$

$\Rightarrow 4X_1 - 3X_2 = 0 \Rightarrow X_1 = \frac{3}{4}X_2 \Rightarrow V_1 = \begin{vmatrix} \frac{3}{4} \\ 1 \end{vmatrix}$

$\|V_1\| = \sqrt{\frac{9}{16} + \frac{16}{16}} = \frac{5}{4} \Rightarrow V_1(\text{norm}) = \begin{vmatrix} \frac{3}{5} \\ \frac{4}{5} \end{vmatrix}$

$\lambda_2 = \frac{25}{4}$

$\left|\begin{array}{cc|c} \frac{135}{4} & 45 & 0 \\ 45 & \frac{240}{4} & 0 \end{array}\right| = \left|\begin{array}{cc|c} 135 & 180 & 0 \\ 180 & 240 & 0 \end{array}\right| = \left|\begin{array}{cc|c} 3 & 4 & 0 \\ 3 & 4 & 0 \end{array}\right| = \left|\begin{array}{cc|c} 0 & 0 & 0 \\ 3 & 4 & 0 \end{array}\right| \Rightarrow$

$\Rightarrow 3X_1 + 4X_2 = 0 \Rightarrow X_1 = -\frac{4}{3}X_2 \Rightarrow V_2 = \begin{vmatrix} -\frac{4}{3} \\ 1 \end{vmatrix}$

$\|V_2\| = \sqrt{\frac{16}{9} + \frac{9}{9}} = \frac{5}{3} \Rightarrow V_2(\text{norm}) = \begin{vmatrix} -\frac{4}{5} \\ \frac{3}{5} \end{vmatrix}$

$\Rightarrow U = \begin{vmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{vmatrix}$ $S = \begin{vmatrix} \sqrt{100} & 0 \\ 0 & \sqrt{\frac{25}{4}} \end{vmatrix} = \begin{vmatrix} 10 & 0 \\ 0 & \frac{5}{2} \end{vmatrix}$

Step 2: $A^T \cdot A = \begin{vmatrix} 6 & 8 \\ -2 & \frac{3}{2} \end{vmatrix} \cdot \begin{vmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{vmatrix} = \begin{vmatrix} 100 & 0 \\ 0 & \frac{25}{4} \end{vmatrix}$

$|A - \lambda I| = \begin{vmatrix} 100-\lambda & 0 \\ 0 & \frac{25}{4}-\lambda \end{vmatrix} = 0 \Rightarrow (100-\lambda)\left(\frac{25}{4}-\lambda\right) = 0 \Rightarrow$

$\lambda_1 = 100, \quad \lambda_2 = \frac{25}{4}$

$\lambda_1 = 100$

$\begin{vmatrix} 0 & 0 \\ 0 & \frac{25}{4} - \frac{400}{4} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & -\frac{375}{4} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} \Rightarrow$

$\Rightarrow 0 X_1 + 1 \cdot X_2 = 0 \Rightarrow X_1 = 1, \; X_2 = 0 \quad V_1 = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$

$\lambda_2 = \frac{25}{4}$

$\begin{vmatrix} \frac{400}{4} - \frac{25}{4} & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} \frac{375}{4} & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} \Rightarrow$

$\Rightarrow 1 \cdot X_1 + 0 \cdot X_2 = 0 \quad X_2 = 1, \; X_1 = 0 \Rightarrow V_2 = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

Step 3.

$A = U \cdot S \cdot V_t = \begin{vmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{vmatrix} \times \begin{vmatrix} 10 & 0 \\ 0 & \frac{5}{2} \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$U \cdot S = \begin{vmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{vmatrix} \quad U \cdot S \cdot V_t = \begin{vmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{vmatrix} = A$

$\Rightarrow$ Answer: $A = \begin{vmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{vmatrix} \times \begin{vmatrix} 10 & 0 \\ 0 & \frac{5}{2} \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

U S V^t