INNA WILLIAMS

$\boxed{1}$ $\boxed{a}$ $x = [0, 1, 0, -1]$ DFT, $n = 4$

$$\omega = e^{-\frac{2\pi i}{4}} = e^{-\frac{\pi}{2} i} = \cos\frac{\pi}{2} - i \sin\frac{\pi}{2} = 0 - i \cdot 1 = -i$$

$$\begin{vmatrix} y0 \\ y1 \\ y2 \\ y3 \end{vmatrix} = \frac{1}{\sqrt{4}} \times \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{vmatrix} \times \begin{vmatrix} 0 \\ 1 \\ 0 \\ -1 \end{vmatrix} = \frac{1}{2} \times \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{vmatrix} \times \begin{vmatrix} 0 \\ 1 \\ 0 \\ -1 \end{vmatrix} = \begin{vmatrix} 0 \\ -i \\ 0 \\ i \end{vmatrix}$$

Answer: $y = \begin{bmatrix} 0 \\ -i \\ 0 \\ i \end{bmatrix}$

$\boxed{3}$ $\boxed{b}$ $x = [1, 1, -1, 1]$ Inverse DFT

$$\omega = e^{-\frac{2\pi i}{4}} = e^{-\frac{\pi}{2} i} = \cos\frac{\pi}{2} - i \sin\frac{\pi}{2} = 0 - i \cdot 1 = -i$$

$$F_4^{-1} \Rightarrow \begin{vmatrix} 1 \\ 1 \\ -1 \\ 1 \end{vmatrix} = \frac{1}{\sqrt{4}} \times \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{vmatrix} \times \begin{vmatrix} 1 \\ 1 \\ -1 \\ 1 \end{vmatrix} = \frac{1}{2} \times \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & -i \end{vmatrix} \times \begin{vmatrix} 1 \\ 1 \\ -1 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ -1 \\ 1 \end{vmatrix}$$

Answer: $F_4^{-1}(y) = \begin{vmatrix} 1 \\ 1 \\ -1 \\ 1 \end{vmatrix}$

Section 10.2

$\boxed{1}$ $\boxed{a}$ $x = [0, 1, 0, -1]$ $t = [0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}]$

DFT for $n=4$, $c=0$, $d=1$, $\ominus$

$$P_n(t) = \frac{a_0}{\sqrt{n}} + \frac{2}{\sqrt{n}} \sum_{k=1}^{n/2-1} \left(a_k \cdot \cos \frac{2\pi k(t-c)}{d-c} - b_k \cdot \sin \frac{2\pi k(t-c)}{d-c} \right) +$$

$$+ \frac{a_{n/2}}{\sqrt{n}} \cdot \cos \frac{n\pi(t-c)}{d-c}$$

$$\begin{vmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{vmatrix} = \frac{1}{\sqrt{4}} \times \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{vmatrix} \times \begin{vmatrix} 0 \\ 1 \\ 0 \\ -1 \end{vmatrix} = \frac{1}{2} \times \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{vmatrix} \times \begin{vmatrix} 0 \\ 1 \\ 0 \\ -1 \end{vmatrix} = \begin{vmatrix} 0 \\ -i \\ 0 \\ i \end{vmatrix}$$

$a_0 = 0$ $b_0 = 0$

$a_1 = 0$ $b_1 = -1$

$a_2 = 0$ $b_2 = 0$

$a_3 = 0$ $b_3 = 1$

$$P_4(t) = 0 + \frac{2}{\sqrt{4}} \left(0 \cdot \cos 2\pi \cancel{k} t + 1 \cdot \sin 2\pi \cancel{k} t \right) + \underbrace{\frac{0}{\sqrt{4}} \cos 4\pi}_{=0} \Rightarrow$$

$$\boxed{P_4(t) = \sin 2\pi t}$$