1.
Suppose an animal lives three years. The first year it is immature and does not reproduce.
The second year it is an adolescent and reproduces at a rate of 0.8 female offspring per female individual.
The last year it is an adult and produces 3.5
Female offspring per female individual. Further suppose that 80% of the first year female survice
to become second-year female, and 90% of the second-year female survive to become third year females.
All third year females die. We are interested in modeling only the female portion of this population.

X1(n+1) = 0 * X1(n) + 0.8 * X2(n) + 3.5 * X3(n)
X2(n+1) = 0.8 * X1(n)
X3(n+1) = 0.9 * X2(n)

b)Construct Leslie Matrix

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    | 0.0    0.8  3.5  |
L = | 0.8    0.0  0.0  |
    | 0.0    0.9  0.0  |
```

c). Compute the Eigenvalues for this matrix. From this determine if the population will eventually grow or decline.

> *with*(*LinearAlgebra*);

> $L := Matrix([[0.0, 0.8, 3.5], [0.8, 0.0, 0.0], [0.0, 0.9, 0.0]])$

$$L := \begin{bmatrix} 0. & 0.8 & 3.5 \\ 0.8 & 0. & 0. \\ 0. & 0.9 & 0. \end{bmatrix} \quad \textbf{(1)}$$

> $(evalues, evectors) := Eigenvectors(L)$

$$evalues, evectors := \begin{bmatrix} 1.51697199548312 + 0.\,I \\ -0.758485997741562 + 1.04206670914584\,I \\ -0.758485997741562 - 1.04206670914584\,I \end{bmatrix},$$

$$\begin{bmatrix} -0.852490143513951 + 0.\,I & -0.797296044248156 + 0.\,I & -0.797296044248156 + -0.\,I \\ -0.449574624213126 + 0.\,I & 0.291228710186860 + 0.400112519594083\,I & 0.291228710186860 - 0.400112519594083\,I \\ -0.266726849933015 + 0.\,I & 0.106215690704993 - 0.328835908777124\,I & 0.106215690704993 + 0.328835908777124\,I \end{bmatrix}$$

>

Answer: We can see that dominant eigenvalue are $1.51697199548312 + 0.\,I > 0 \Rightarrow$ *the rate of population growth* = $\sqrt{(1.51697199548312^2 + 0.\,I^2)} \sim 1.52$ or rate of population will grow each year by 52%

d) Suppose that the population of 100 first year females are released into a steady area. Track the female population over 10 years

> $X0 := \langle 100, 0, 0 \rangle$

$$X0 := \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(2)}$$

> $X10 := MatrixPower(L, 1)\,.X0$

$$X10 := \begin{bmatrix} 1571.41172224000 \\ 1491.63356160000 \\ 889.958891520000 \end{bmatrix} \quad \textbf{(3)}$$

e) Compute the eigenvector assosiated with the dominant eigenvalue. Normalize both this

eigenvector and
the population distribution after 10 years.

> $domvector := Column(evectors, 1)$

$$domvector := \begin{bmatrix} -0.852490143513951 + 0.\,I \\ -0.449574624213126 + 0.\,I \\ -0.266726849933015 + 0.\,I \end{bmatrix} \quad (4)$$

> $steady := \dfrac{domvector}{Norm(domvector, 1)}$

$$steady := \begin{bmatrix} -0.543405595593935 + 0.\,I \\ -0.286573831138325 + 0.\,I \\ -0.170020573084048 + 0.\,I \end{bmatrix} \quad (5)$$

> $\dfrac{X10}{Norm(X10, 1)}$

$$\begin{bmatrix} 0.397523415738679 \\ 0.377341762216485 \\ 0.225134822031069 \end{bmatrix} \quad (6)$$

We can see if number of years -> ∞ then

{

$0.397523415738679 \rightarrow (\sqrt{(-0.543405595593935^2 + 0.\,I^2)}) = 0.543405595593935\}$

$0.377341762216485 \rightarrow (\sqrt{(-0.286573831138325^2 + 0.\,I^2)}) = 0.286573831138325\}$

$0.225134822031069 \rightarrow (\sqrt{(-0.170020573084048^2 + 0.\,I^2)}) = 0.170020573084048\}$

}

We can see with number of years =>∞ population distribution

does not depend on X0 : { < i, 0 , 0 > , i >= 1} and defined by $\begin{bmatrix} 0.5434055955939351 \\ 0.2865738311383245 \\ 0.17002057308404764 \end{bmatrix}$

54% for immature, 28% for adolescent, 17% for adults

2. Trace the stand of trees through time steps (50, 100, 150, 200, 250) and explain how we know that this model can not possibly be valid

> $L := Matrix([[12, 26, 6], [0.30, 0.92, 0.0], [0.0, 0.18, 0.67]])$

$$L := \begin{bmatrix} 12 & 26 & 6 \\ 0.30 & 0.92 & 0. \\ 0. & 0.18 & 0.67 \end{bmatrix} \quad (7)$$

> $X0 := \langle 1, 0, 0 \rangle$

$$X0 := \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

> $(evalues, evectors) := Eigenvectors(L)$

$$evalues, evectors := \begin{bmatrix} 12.6663361370626 + 0.\,I \\ 0.334059874481654 + 0.\,I \\ 0.589603988455727 + 0.\,I \end{bmatrix}$$

$$\begin{bmatrix} -0.999673943410001 + 0.\,I & 0.864707583271779 + 0.\,I & 0.409708449480100 + 0.\,I \\ -0.0255315512448800 + 0.\,I & -0.442728298820715 + 0.\,I & -0.372015794832196 + 0.\,I \\ -0.000383090234515860 + 0.\,I & 0.237218146134724 + 0.\,I & 0.832912501298895 + 0.\,I \end{bmatrix}$$

\>

\> $domvector := Column(evectors, 1)$

$$domvector := \begin{bmatrix} -0.999673943410001 + 0.\,I \\ -0.0255315512448800 + 0.\,I \\ -0.000383090234515860 + 0.\,I \end{bmatrix} \quad (9)$$

\> $steady := \dfrac{domvector}{Norm(domvector, 1)}$

$$steady := \begin{bmatrix} -0.974731932469559 + 0.\,I \\ -0.0248945353111606 + 0.\,I \\ -0.000373532077195206 + 0.\,I \end{bmatrix} \quad (10)$$

In a long run normilized distribution must be approching steady vector modules

\> $X50 := MatrixPower(L, 50).X0$

$$X50 := \begin{bmatrix} 1.28383385942066\ 10^{55} \\ 3.27889610285332\ 10^{53} \\ 4.91984629115363\ 10^{51} \end{bmatrix} \quad (11)$$

\> $\dfrac{X50}{Norm(X50, 1)}$

$$\begin{bmatrix} 0.974731932641829 \\ 0.0248945353155604 \\ 0.000373532077261223 \end{bmatrix} \quad (12)$$

\> $X100 := MatrixPower(L, 100).X0$

$$X100 := \begin{bmatrix} 1.74204463860363\ 10^{110} \\ 4.44916087436075\ 10^{108} \\ 6.67577957332085\ 10^{106} \end{bmatrix} \quad (13)$$

\> $\dfrac{X100}{Norm(X100, 1)}$

$$\begin{bmatrix} 0.974731932775445 \\ 0.0248945353189729 \\ 0.000373532077312426 \end{bmatrix} \quad (14)$$

\> $X150 := MatrixPower(L, 150).X0$

$$X150 := \begin{bmatrix} 2.36379458340278\ 10^{165} \\ 6.03710269096867\ 10^{163} \\ 9.05841977049204\ 10^{161} \end{bmatrix} \quad (15)$$

> $\frac{X150}{Norm(X150, 1)}$

$$\begin{bmatrix} 0.974731932292981 \\ 0.0248945353066509 \\ 0.000373532077127539 \end{bmatrix} \quad (16)$$

> $X200 := MatrixPower(L, 200) . X0$

$$X200 := \begin{bmatrix} 3.20745215633689\ 10^{220} \\ 8.19179389788594\ 10^{218} \\ 1.22914436939120\ 10^{217} \end{bmatrix} \quad (17)$$

> $\frac{X200}{Norm(X200, 1)}$

$$\begin{bmatrix} 0.974731932462880 \\ 0.0248945353109901 \\ 0.000373532077192647 \end{bmatrix} \quad (18)$$

> $X250 := MatrixPower(L, 250) . X0$

$$X250 := \begin{bmatrix} 4.35221800042393\ 10^{275} \\ 1.11155119766025\ 10^{274} \\ 1.66783602337301\ 10^{272} \end{bmatrix} \quad (19)$$

> $\frac{X250}{Norm(X250, 1)}$

$$\begin{bmatrix} 0.974731932648030 \\ 0.0248945353157188 \\ 0.000373532077263599 \end{bmatrix} \quad (20)$$

> $Test := \langle 1696, 485, 82 \rangle$

$$Test := \begin{bmatrix} 1696 \\ 485 \\ 82 \end{bmatrix} \quad (21)$$

> $evalf\left(\frac{Test}{Norm(Test, 1)}\right)$

$$\begin{bmatrix} 0.7494476359 \\ 0.2143172779 \\ 0.03623508617 \end{bmatrix} \quad (22)$$

>

>

Answer :

A current census in *a particular stand finds that* 1696 *young*, 485 *mature* and 82 *old redwoods*= >
total = 1696 + 485 + 82 = 2263

probability of young = 1696/2263 = 0.749 but model gives 0.974731932502808
0.79 does not approches 0.97
probability of mature = 485/2263 = 0.214 but model gives 0.0248945353120098
0.214 does not approch 0.0248945353120098 ,
probability of old = 82 / 2263 = 0.036 *but model gives* 0.00037
0.036 does not approch 0.00037

III. Consider a colony of beetles. Assume that the beetles live at most 4 years. Divide the female population into four age groups: 0-1, 1-2 , 2-3, 3-4, each spanning 1 year. Suppose an initial number of females is 120, and that they are spread over the four age g roups 70:25:15:10. Let the average birth rates per beetle of the fours age groups be 0, 3, 1, 1 per year, and assume that the survival probabilites of the four groups are 0.4, 0.2, 0.1, and 0 (no one survives past year 4).

	0-1	1-2	2-3	3-4
Initial Population (X0)	70	25	15	10
Average Birth rate	0	3	1	1
Probability Of Survival	0.4	0.2	0.1	0

i) Construct the 4x4 Leslie matrix.

X1(n+1) = 0 *70 X1(n) + 3 * 25 X2(n) + 1 * 15 * X3(n) + 1 * 10 *X4(n)
X2(n+1) = 0.4 * 70 X1(n)
X3(n+1) = 0.2 * 25 X2(n)
X4(n+1) = 0.1 * 15 *X3(n)

=>

$$\begin{vmatrix} X1(n) \\ X2(n) \\ X3(n) \\ X4(n) \end{vmatrix} = \begin{vmatrix} 0 & 3 & 1 & 1 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \end{vmatrix} \text{ X } \begin{vmatrix} 70 \\ 25 \\ 15 \\ 10 \end{vmatrix}$$

ii) Compute by hand the population vector after 2 years.

X2 = L^2*

$$\begin{bmatrix}70\\25\\15\\10\end{bmatrix} = \begin{bmatrix}0 & 3 & 1 & 1\\0.4 & 0 & 0 & 0\\0 & 0.2 & 0 & 0\\0 & 0 & 0.1 & 0\end{bmatrix} \cdot \begin{bmatrix}0 & 3 & 1 & 1\\0.4 & 0 & 0 & 0\\0 & 0.2 & 0 & 0\\0 & 0 & 0.1 & 0\end{bmatrix} \cdot \begin{bmatrix}70\\25\\15\\10\end{bmatrix} = \begin{bmatrix}1.2 & 0.2 & 0.1 & 0\\0 & 1.2 & 0.4 & 0.4\\0.08 & 0 & 0 & 0\\0 & 0.02 & 0 & 0\end{bmatrix} \cdot \begin{bmatrix}70\\25\\15\\10\end{bmatrix}$$

$=$

$$\begin{bmatrix}1.2\cdot70 + 0.2\cdot25 + 0.1\cdot15\\1.2\cdot25 + 0.4\cdot15 + 0.4\cdot10\\0.08\cdot70\\0.02\cdot25\end{bmatrix} = \begin{bmatrix}84 + 5 + 1.5\\30 + 6 + 4\\5.6\\0.5\end{bmatrix} = \begin{bmatrix}90.5\\40\\5.6\\0.5\end{bmatrix}$$

Answer : $\begin{bmatrix}\mathbf{90.5}\\\mathbf{40}\\\mathbf{5.6}\\\mathbf{0.5}\end{bmatrix}$

iii) Use Maple (or Excel) to estimate the population vector after 5 years, a nd compute the age-group probability distribution vector.

> $L := Matrix([[0,3,1,1],[0.4,0,0,0],[0,0.2,0,0],[0,0,0.1,0]])$

$$L := \begin{bmatrix}0 & 3 & 1 & 1\\0.4 & 0 & 0 & 0\\0 & 0.2 & 0 & 0\\0 & 0 & 0.1 & 0\end{bmatrix} \quad \textbf{(23)}$$

> $X0 := \langle 70, 25, 15, 10\rangle$

$$X0 := \begin{bmatrix}70\\25\\15\\10\end{bmatrix} \quad \textbf{(24)}$$

> $X5 := MatrixPower(L, 5)\, X0$

$$X5 := \begin{bmatrix}159.360000000000\\46.8640000000000\\10.0880000000000\\0.724000000000000\end{bmatrix} \quad \textbf{(25)}$$

> $\dfrac{X5}{Norm(X5, 1)}$

$$\begin{bmatrix} 0.734256068129280 \\ 0.215927311601472 \\ 0.0464807681682240 \\ 0.00333585211675200 \end{bmatrix} \quad (26)$$

iv) Use Maple (and eigenvector/eigenvalue analysis) to determine the long-trend growth factor/growth rate and the age-group distribution.

> $(evalues, evectors) := Eigenvectors(L)$

$$evalues, evectors := \begin{bmatrix} 1.13006925756369 + 0.\ I \\ -1.06389498642639 + 0.\ I \\ -0.0330871355686483 + 0.0745606669566616\ I \\ -0.0330871355686483 - 0.0745606669566616\ I \end{bmatrix}$$

$$\begin{bmatrix} -0.9410360775 + 0.\ I & 0.9339684780 + 0.\ I & -0.04829790470 + 0.01588946228\ I & -0.04829790470 - 0.01588946228\ I \\ -0.3330897009 + 0.\ I & -0.3511506267 + 0.\ I & 0.1672826359 + 0.1848730596\ I & 0.1672826359 - 0.1848730596\ I \\ -0.05895031630 + 0.\ I & 0.06601227211 + 0.\ I & 0.2479498470 - 0.5587460397\ I & 0.2479498470 + 0.5587460397\ I \\ -0.005216522431 + 0.\ I & -0.006204773305 + 0.\ I & -0.7493844443 + 0.\ I & -0.7493844443 + -0.\ I \end{bmatrix}$$

The Largest eigen value is = 1.13 => that growth rate factor = 1.13 or population will grow by 13% per year

> $domvector := Column(evectors, 1)$

$$domvector := \begin{bmatrix} -0.941036077546071 + 0.\ I \\ -0.333089700917922 + 0.\ I \\ -0.0589503163082285 + 0.\ I \\ -0.00521652243114011 + 0.\ I \end{bmatrix} \quad (27)$$

Age group probability distribution vector:

> $steady := \dfrac{domvector}{Norm(domvector, 1)}$

$$steady := \begin{bmatrix} -0.703161674512983 + 0.\ I \\ -0.248891532906195 + 0.\ I \\ -0.0440488989927537 + 0.\ I \\ -0.00389789375278807 + 0.\ I \end{bmatrix} \quad (28)$$

v) In the long-run, what percentage of beetles can be expected to be 2-year olds?
Answer:
from Age group probability distribution vector we can see that 2 year old will be in the 3rd row ,
so it will be 4.4048 % of 2-year olds

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