Inna Williams

Problem* 1. *Suppose you open a savings account with
\$1000 *intial deposit that pays* 0.5 *% interest each month*
You withdraw \$50 each month.

(i) Construct a model (recursive equation) for the amount of xn in your acount after n month

Answer:

$$x(n+1) = 1.005\cdot x(n) - 50\,,\ x(0) = 1000$$

(*iii*) *Use the explicit*· (*close form*) *solution* to *compute x12* and *compare with*· (*ii*)

$$x1 := 1.005\cdot x0 - 50$$
$$x2 := 1.005\cdot x1 - 50 = 10.05\cdot(1.005\ \ x0 - 50) - 50 = \left(1.005^{2}\right)\cdot x0 - 1.005\cdot 50 - 50$$
$$x3 := 1.005\cdot x2 - 50 = 1.005\cdot\left(\left(1.005^{2}\right)\cdot x0 - 1.005\cdot 50 - 50\right) - 50 = \left(1.005^{3}\right)\cdot x0 - \left(1.005^{2}\right)\cdot 1.005\cdot 50 - 50$$
$$xn := \left(1.005^{n}\right)\cdot x0 - 50\cdot\left(SUM(1.005)^{k-1}, k=1\ \textbf{to}\ n\right) = \left(1.005^{n}\right)\cdot x0 - 50\cdot\frac{\left(1 - 1.005^{n}\right)}{1 - 1.005} = \left(1.005^{n}\right)\cdot x0 + 50\cdot\frac{\left(1 - 1.005^{n}\right)}{0.005}$$

closed form :

$$xn = \left(1.005^{n}\right)\cdot x0 + 50\cdot\frac{\left(1 - 1.005^{n}\right)}{0.005}$$
$$xn = \left(1.005^{n}\right)\cdot x0 + 10000\cdot\left(1 - 1.005^{n}\right)$$
$$xn = 1.005^{n}\cdot(x0 - 10000) + 10000$$

Answer:

if $n = 12\qquad x12 = 1.005^{12}\cdot(x0 - 10000) + 10000$

X12 = 444.899692

I used Maple below to calculate the result of the formula above.

> $x12 := 1.005^{12}\cdot(1000 - 10000) + 10000$

$$x12 := 444.899692 \tag{1}$$

(*ii*) *Use Excel* or *Maple* to *track the balance* in *your account* for *one year*· (12 *month*)

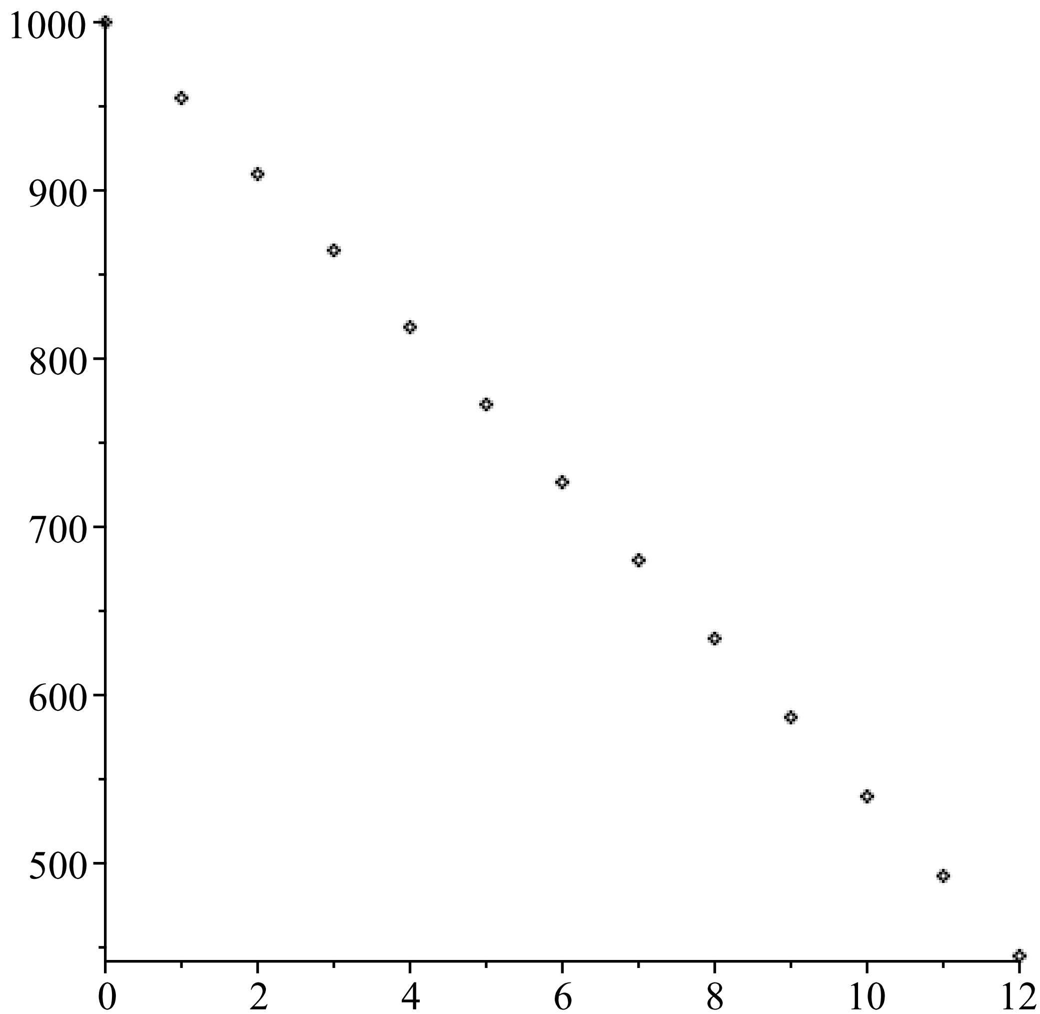
> $money := rsolve(\{x(n+1) = 1.005\cdot x(n) - 50, x(0) = 1000\}, x(n))$

$$money := -9000\left(\frac{201}{200}\right)^{n} + 10000 \tag{2}$$

> $seq(evalf(money), n = 0..12)$

1000., 955., 909.7750000, 864.3238750, 818.6454944, 772.7387218, 726.6024155, 680.2354275, 633.6366047, 586.8047877, 539.7388116, 492.4375057, 444.8996932 (3)

> $with(plots) : pointplot(\{seq([n, evalf(money)], n = 0..12)\})$



Result received in (ii) for 12 month are the same as a result was found in (iii) using formula found in (iii) are the same.

x12 = 444.89969

(V) What is the maximum amount that you could withdraw each month so that the account will never get depleted

$$xn = (1.005^n) \cdot x0 + W \cdot \frac{(1 - 1.005^n)}{0.005} > 0$$

$$W > -\frac{(1.005^n) \cdot x0}{\frac{(1 - 1.005^n)}{0.005}} \rightarrow W > -(1.005^n) \cdot x0 \cdot \frac{0.005}{(1 - 1.005^n)} \rightarrow$$

$$W > (1.005^n) \cdot x0 \cdot \frac{0.005}{(1.005^n - 1)}$$

$$\text{limit}\left(\frac{1.005^n \cdot x0 \cdot 0.005)}{1.005^n - 1}, n \rightarrow \text{infinity}\right) =$$

$$\text{limit}\left(\frac{x0 \cdot 0.005}{1 - \frac{1}{1.005^n}}, n \to \text{infinity}\right)$$

$$\text{limit}\left(\frac{1}{1.005^n}, n \to \text{infinity}\right) = 0$$

$$x0 \cdot 0.005 \cdot \ limit \cdot (\ 1\ , n \to \text{infinity}) = x0 \cdot 0.005$$

in our case x0 = 1000 -> W > 1000 * 0.005 -> W > 5

Answer:

5 is the maximum amount that can be withdrown each month so that the account will never get depleted.

If the amount more then 5 it will get depleted if n -> infinity

>

(IV) Investigate what happens with the account in a long run. If account get depleted, find the exact time when this happens.

close form found in (iii) is $xn = 1.005^n \cdot (x0 - 10000) + 10000$,

when account gets depleted xn = 0 -> $1.005^n \cdot (x0 - 10000) + 10000 = 0$

$$1.005^n = -\frac{10000}{x0 - 10000} = \frac{10000}{10000 - x0}$$

n * log(1.005) = log(10000 / (10000 - x0))
n = log(10000 / (10000 - x0)) / log(1.005)
n = log(0.9) / log(1.005)
n = log(10/9) / log(1.005) = log(1.111) / log(1.005)
Use Maple to calculate n

> $n := \log(1.111) / \log(1.005)$

$$n := 21.10468865 \qquad (4)$$

Answer:

Account gets depleted when n > 21.104 or n = 22

>

Problem 2. A patient is given an initial dose of 100mg of a prescription drug. The body filters out 60% of the drug every 24 hours and patient is administered an aditional dose of 50 mg everyday.

(i) Construct a model (recursive equation) for the amount of Xn of medicine in the body after n days.

Xn+1 = Xn - Xn * 0.6 + 50 ->

Answer:

Model : Xn+1 = Xn * 0.4 + 50 , X0 = 100

(ii) Use Excel or Maple to truck the drug amount in the body during the first week (7 days).

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> drugamount := rsolve({x(0) = 100, x(n+1) = .4*x(n)+50}, x(n))
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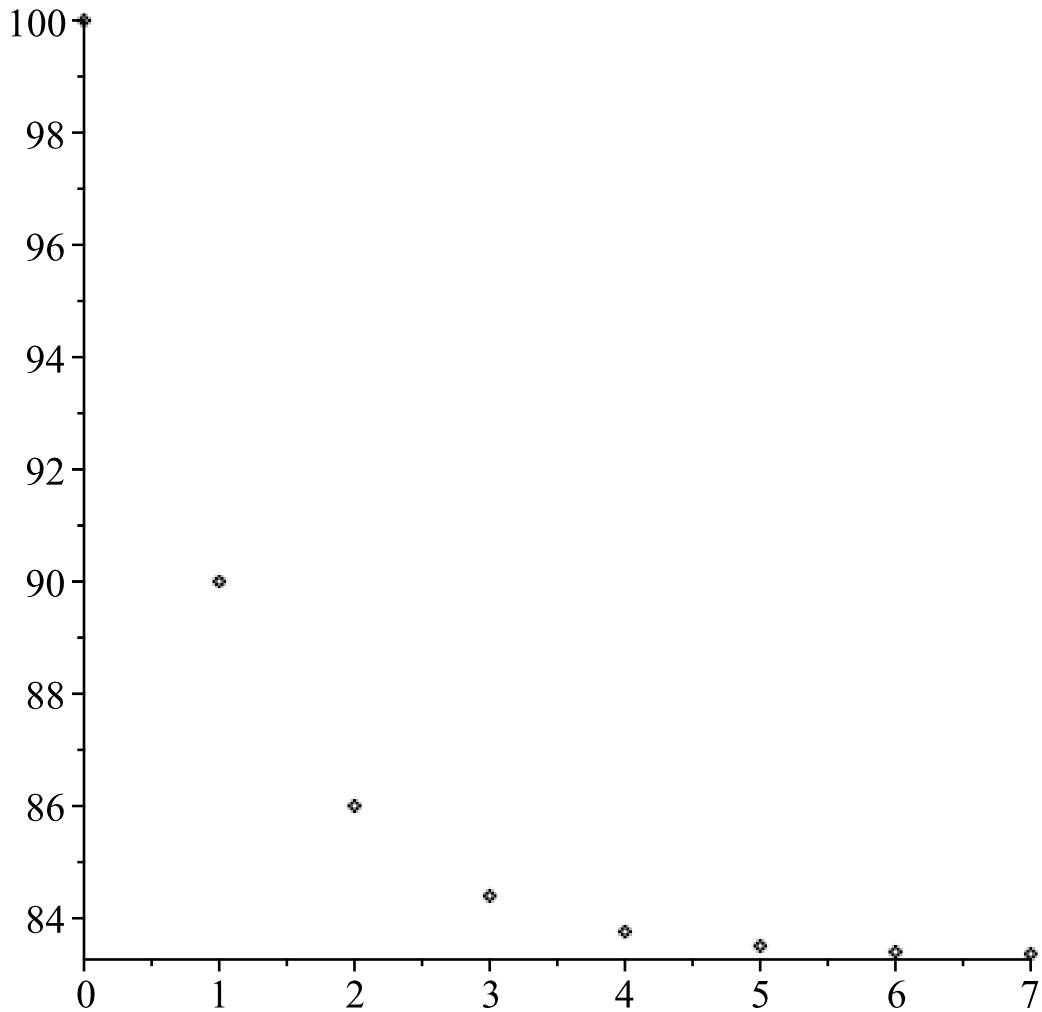
(5)

$$drugamount := \frac{50\left(\frac{2}{5}\right)^n}{3} + \frac{250}{3} \tag{5}$$

> $seq(evalf(drugamount), n = 0..7)$

$$100., 90., 86., 84.40000000, 83.76000000, 83.50400000, 83.40160000, 83.36064000 \tag{6}$$

> $with(plots) : pointplot(\{seq([n, evalf(drugamount)], n = 0..7)\})$



(iii) Investigate what happens with the amount of medicine in a body in a long run.

dynamic system Xn+1 = r * Xn + b   has an explicit solution
Xn = r ^n * X0 + b * (1 - r ^ n) / (1 - r)
in our system r = 0.4, b =50 , X0=100, using a general formula we get:

Xn = 0.4^n * X0 + 50 * (1 - 0.4^n)/(1 - 0.4)
Xn = 0.4 ^ n * X0 + 83.33 *(1 - 0.4^n)

limit(Xn, n= infinity) = limit((0.4 ^ n * X0 + 83.33 *(1 - 0.4^n)), n=infinity) =
limit((0.4 ^ n * X0),n=infinity) + 83.33 * limit(1, n=infinity) + 83.33 * limit(0.4 ^ n * X0),n= infinity)

limit((0.4 ^ n * X0),n=infinity) = 0
->
limit(Xn, n= infinity) = 83.33

if we use maple function to check

> $seq(evalf(drugamount), n = 0..20)$

100., 90., 86., 84.40000000, 83.76000000, 83.50400000, 83.40160000, 83.36064000, 83.34425600, 83.33770240, 83.33508096, 83.33403238, 83.33361295, 83.33344518, 83.33337807, 83.33335123, 83.33334049, 83.33333620, 83.33333448, 83.33333379, 83.33333352 **(7)**

Answer:

Closed form solution of the dynamic equation for amount of medicine when n -> infinity converges to 83.33

(IV) An amount of 150 mg or higher of a drug in the body is considered unsafe. Can the daily dose of 50 mg be increased without exceeding the safe value.

If yes, by how much?

let dose amout = dose
general solution for Xn is:
Xn = 0.4^n * X0 + dose * (1 - 0.4^n)/(1 - 0.4)
Xn = 0.4^n * X0 + dose * (1 - 0.4^n)/(0.6)
the following iequality will give us the solution for the amount of dose without exceeding the value 150:

0.4^n * X0 + dose * (1 - 0.4^n)/(0.6) > 150
0.4 ^n * X0 * 0.6 + dose *(1 - 0.4 ^n) > 150 * 0.6
dose > (90 - 0.4^n * X0 * 0.6) / (1 - 0.4^n)
limit((90 - 0.4^n * X0 * 0.6) / (1 - 0.4^n), n -> ifinity) =
limit(90 / (1 - 0.4^n), n -> infinity) + limit((0.4^n * X0 * 0.6) / (1 - 0.4^n), n -> ifinity)
limit(0.4^n, n-> infinity) = 0, -> limit(90 / (1 - 0.4^n), n -> infinity) = 90 (1)
limit((0.4^n * X0 * 0.6) / (1 - 0.4^n), n -> ifinity) =

X0 * 0.6 limit($\left(\frac{1}{\frac{1}{0.4^n}+1}, n \rightarrow \text{infinity}\right) = 0 \quad \cdot (2)$

from (1) and (2) limit((90 - 0.4^n * X0 * 0.6) / (1 - 0.4^n), n -> ifinity) = 90.

Answer:

Safe value of the drug in the body is < 150.
Amount can be increased to dose less then 90 (< 90) without exceeding the safe value. When the dose = 90 Amount of the drug in the body converges to 150 and 150 is not safe
There for Dose starting from 90 becomes unsafe.

Problem 3. Complete the steps highlighted in class for finding explicit (close form) solution for Fibonnaci equation Xn+1 = Xn + Xn-1 (1)that models his famous question. "How many pairs of rabbit can be produced in one year starting with one pair if it is supposed that every month each pair begets a new pair which from the second monthon becomes productive.

(a) Show that Xn = C * $\lambda^n \cdot (2)$ ***can be a solution of the recursive equasion*** **if and** ***only*** **if**

λ ***satisfies the equation*** $\lambda^2 - \lambda - 1 = 0$, ***so*** $\lambda = \frac{(1 \pm \sqrt{5})}{2}$

Prove :

$$Xn+1 = Xn + Xn\text{-}1, \qquad Xn = C * \boldsymbol{\lambda}^{n}$$

$$C * \lambda^{n+1} = C \cdot \lambda^{n} + C * \lambda^{n-1}$$

$$\lambda^{n+1} = \lambda^{n} + \lambda^{n-1}$$

$$\lambda^{n+1} - \lambda^{n} - \lambda^{n-1} = 0$$

$$\lambda^{n}\left(\lambda - 1 - \frac{1}{\lambda}\right) = 0$$

for all n = 0,1,2, ... $\lambda^{n} \neq 0 \quad \Rightarrow$

$$\lambda - 1 - \frac{1}{\lambda} = 0 \quad \Rightarrow \lambda^{2} - \lambda - 1 = 0, \ \boldsymbol{\lambda} = \frac{\mathbf{1} \pm \sqrt{\mathbf{1} - \mathbf{4} \cdot (\mathbf{1})}}{\mathbf{2}} \quad \Rightarrow \boldsymbol{\lambda} = \frac{(\mathbf{1} \pm \sqrt{\mathbf{5}})}{\mathbf{2}}$$

(b) Show by direct substitution that Xn = C1 * $\boldsymbol{\lambda 1}^{n}$ + C2 * $\boldsymbol{\lambda 2}^{n}$ *is also a solution of the recurrent relation,*

where C1, C2 are arbitrary real coefficients, $\boldsymbol{\lambda 1} = \frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}, \ \boldsymbol{\lambda 2} = \frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}$

Prove:

$$C1 * \boldsymbol{\lambda 1}^{n+1} + C2 * \boldsymbol{\lambda 2}^{n+1} = C1 * \boldsymbol{\lambda 1}^{n} + C2 * \boldsymbol{\lambda 2}^{n} + C1 * \boldsymbol{\lambda 1}^{n-1} + C2 * \boldsymbol{\lambda 2}^{n-1}$$

$$C1 * \boldsymbol{\lambda 1}^{n+1} + C2 * \boldsymbol{\lambda 2}^{n+1} - C1 * \boldsymbol{\lambda 1}^{n} - C2 * \boldsymbol{\lambda 2}^{n} - C1 * \boldsymbol{\lambda 1}^{n-1} - C2 * \boldsymbol{\lambda 2}^{n-1} = 0$$

$$C1 * (\boldsymbol{\lambda 1}^{n+1} - \boldsymbol{\lambda 1}^{n} - \boldsymbol{\lambda 1}^{n-1}) + C2 * (\boldsymbol{\lambda 1}^{n+1} - \boldsymbol{\lambda 1}^{n} - \boldsymbol{\lambda 1}^{n-1}) = 0$$

$$C1 * \boldsymbol{\lambda 1}^{n} \cdot \left(\boldsymbol{\lambda 1}^{2} - \boldsymbol{\lambda 1} - 1\right) + C2 \cdot \boldsymbol{\lambda 2}^{n} \cdot \left(\boldsymbol{\lambda 2}^{2} - \boldsymbol{\lambda 2} - \mathbf{1}\right) = 0$$

$$C1 * \boldsymbol{\lambda 1}^{n} \cdot \left(\boldsymbol{\lambda 1}^{2} - \boldsymbol{\lambda 1} - 1\right) + C2 \cdot \boldsymbol{\lambda 2}^{n} \cdot \left(\boldsymbol{\lambda 2}^{2} - \boldsymbol{\lambda 2} - \mathbf{1}\right) = 0$$

$$C1 * \boldsymbol{\lambda 1}^{n} \cdot \left(\boldsymbol{\lambda 1} - \frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}\right) \cdot \left(\boldsymbol{\lambda 1} - \frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}\right) +$$

$$C2 \cdot \boldsymbol{\lambda 2}^{n} \cdot \left(\boldsymbol{\lambda 2} - \frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}\right) \cdot \left(\boldsymbol{\lambda 2} - \frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}\right) = 0 \ ,$$

Substitute for $\boldsymbol{\lambda 1} = \frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}, \ \boldsymbol{\lambda 2} = \frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}$ *gives us* $\Rightarrow$

$$C1 * \boldsymbol{\lambda 1}^{n} \cdot \left(\frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}} - \frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}\right) \cdot \left(\frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}} - \frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}\right) +$$

$$C2 \cdot \boldsymbol{\lambda 2}^{n} \cdot \left(\frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}} - \frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}\right) \cdot \left(\frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}} - \frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}\right) = 0 \ , \Rightarrow$$

$0 = 0 \Rightarrow$ C1 * $\boldsymbol{\lambda 1}^{n}$ + C2 * $\boldsymbol{\lambda 2}^{n}$ ***is also a solution*** *of the rucurrent relation,*
where C1, C2 are arbitrary real coefficients,

(c) Use part (b) to find the particular solution corresponding to initial conditions x0 =1, x1 =1. In other words find the values of C1 and C2

$$Xn = C1 * \left(\frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n} + C2 * \left(\frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n}, \ X0 = 1, X1 = 1$$

Solution: Solve the system of 2 equations:

$$C1 * \left(\frac{(\mathbf{1} + \sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{0} + C2 * \left(\frac{(\mathbf{1} - \sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{0} = 1$$

C1 *

$$\left(\frac{(\mathbf{1}+\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{1} + C2 * \left(\frac{(\mathbf{1}-\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{1} = 1 \quad \Rightarrow$$

$$C1 \quad + \quad C2 \quad = 1$$
$$C1 \cdot (1 + \sqrt{\mathbf{5}}) + C2(1 - \sqrt{\mathbf{5}}) = 2 \quad \Rightarrow$$

$$\left|\begin{array}{cc|c} 1 & 1 & 1 \\ 1+\sqrt{\mathbf{5}} & 1-\sqrt{\mathbf{5}} & 2 \end{array}\right| \Rightarrow$$

$$\left|\begin{array}{cc|c} 1 & 1 & 1 \\ 1+\sqrt{\mathbf{5}} - (1-\sqrt{\mathbf{5}}) & 1-\sqrt{\mathbf{5}} - (1-\sqrt{\mathbf{5}}) & 2 - (1-\sqrt{\mathbf{5}}) \end{array}\right| \Rightarrow$$

$$\left|\begin{array}{cc|c} 1 & 1 & 1 \\ 2\cdot\sqrt{5} & 0 & 1+\sqrt{\mathbf{5}} \end{array}\right| \Rightarrow \left|\begin{array}{cc|c} 2\cdot\sqrt{5} & 2\cdot\sqrt{5} & 2\cdot\sqrt{5} \\ 2\cdot\sqrt{5} & 0 & 1+\sqrt{\mathbf{5}} \end{array}\right| \Rightarrow$$

$$\left|\begin{array}{cc|c} 0 & 2\cdot\sqrt{5} & 2\cdot\sqrt{5} - 1 - \sqrt{5} \\ 2\cdot\sqrt{5} & 0 & 1+\sqrt{\mathbf{5}} \end{array}\right| \Rightarrow$$

$$\left|\begin{array}{cc|c} 0 & 2\cdot\sqrt{5} & -(1-\sqrt{5}) \\ 2\cdot\sqrt{5} & 0 & 1+\sqrt{\mathbf{5}} \end{array}\right| \Rightarrow$$

$$C1 \cdot 0 + C2 \cdot 2\cdot\sqrt{5} = -(1-\sqrt{5})$$
$$C1 \cdot 2\cdot\sqrt{5} + C2 \cdot 0 = 1 + \sqrt{\mathbf{5}} \quad \Rightarrow$$
$$C2 = -\frac{(1-\sqrt{5})}{2\cdot\sqrt{5}} \quad , \quad C1 = \frac{1+\sqrt{5}}{2\cdot\sqrt{5}}$$

Verify C1 and C2 by substituting in the equasions for X0 and X1 :

$$X0 = \frac{1+\sqrt{5}}{2\cdot\sqrt{5}} - \frac{(1-\sqrt{5})}{2\cdot\sqrt{5}} = \frac{(2\cdot\sqrt{5})}{2\cdot\sqrt{5}} = 1$$

$$X1 = \frac{1+\sqrt{5}}{2\cdot\sqrt{5}} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^{1} - \frac{(1-\sqrt{5})}{2\cdot\sqrt{5}} \cdot \left(\frac{1-\sqrt{5}}{2}\right)^{1} =$$

$$\frac{1+2\sqrt{5}+5}{4\cdot\sqrt{5}} - \frac{1-2\sqrt{5}+5}{4\cdot\sqrt{5}} = \frac{1+2\sqrt{5}+5-1+2\sqrt{5}-5}{4\cdot\sqrt{5}} = \frac{4\cdot\sqrt{5}}{4\cdot\sqrt{5}} = 1$$

Answer: $\mathbf{C2 = -\frac{(1-\sqrt{5})}{2\cdot\sqrt{5}}} \quad , \quad \mathbf{C1 = \frac{1+\sqrt{5}}{2\cdot\sqrt{5}}}$

(d) Discuss the long term behavior of the solution in part c

$$\text{limit}\left(C1 * \left(\frac{(\mathbf{1}+\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n} + C2 * \left(\frac{(\mathbf{1}-\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n}, n \to \text{infinity}, \mathit{C1}, \mathit{C2\ constants}\right) =$$

$$C1 * \text{limit}\left(\left(\frac{(\mathbf{1}+\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n,}, n \to \text{infinity}\right) + C2 \cdot \mathit{limit}\left(\left(\frac{(\mathbf{1}-\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n}, n \to \text{infinity}\right) =$$

$$\text{limit}\left(\left(\frac{(\mathbf{1}+\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n,}, n \to \text{infinity}\right) = \mathit{limit}\left(1.618^{n}, n \to \mathit{infiity}\right\} = \text{infinity}$$

$$\Rightarrow$$

$$\text{limit}\left(\left(\frac{(\mathbf{1}-\sqrt{\mathbf{5}})}{\mathbf{2}}\right)^{n,}, n \to \text{infinity}\right) = \mathit{limit}\left(1.618^{n}, n \to \mathit{infiity}\right\} = 0$$

Xn -> infinity, if n -> infinity

Problem 4. Consider recursive relation Xn+1 = Xn + 2Xn-1, X0=1, X1=1

(a) How would you interpret this model in terms of Fibonacci's rabit model.

Answer:
The newly-born rabits are 3 females and one male, rabits are to able to mate at age of one month so that at the end of it second month a female can produce another 3 females and one male. Every month from the second month 3 females and one male are born.

(b) Use Excel (or) Maple to calculate X12

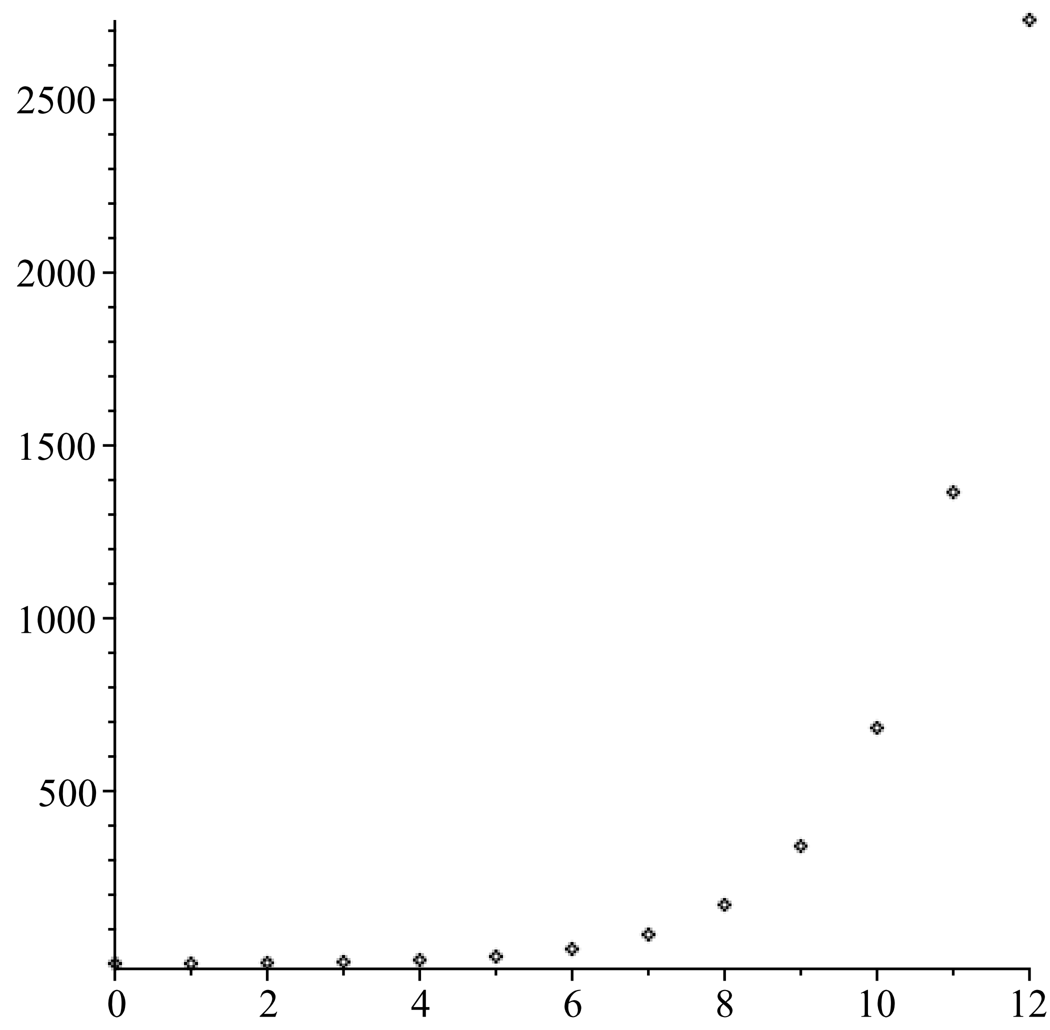
> $fib := rsolve(\{x(n+1) = x(n) + 2.0 \cdot x(n-1), x(0) = 1, x(1) = 1\}, x(n))$

$$fib := \frac{(-1)^n}{3} + \frac{2\,2^n}{3} \tag{8}$$

> $seq(evalf(fib), n = 0..12)$

$$1., 1., 3., 5., 11., 21., 43., 85., 171., 341., 683., 1365., 2731. \tag{9}$$

> $with(plots) : pointplot(\{seq([n, evalf(fib)], n = 0..12)\})$



(c) Follow the steps in problem 3 to find explicit (closed form) formula for Xn and calculate X12.
Solution:

Xn+1 = Xn + 2 * Xn-1, Xn = C * $\boldsymbol{\lambda}^n$

$C * \lambda^{n+1} = C \cdot \lambda^n + 2 * C * \lambda^{n-1}$

$C \cdot \lambda^n * (\lambda^{n+1} - \lambda^n + 2 * \lambda^{n-1}) = 0$

$C * \lambda^n \left(\lambda - 1 - \frac{2}{\lambda}\right) = 0$

$C * \lambda^n \left(\lambda^2 - \lambda - 2\right) = 0$

$\lambda = \frac{\mathbf{1 + \sqrt{(1 - 4 \cdot (1) \cdot (-2))}}}{\mathbf{2}} = \frac{\mathbf{1 \pm 3}}{\mathbf{2}}$

$\lambda 1 = 2,\ \lambda 2 = -1$

$Xn = C1 * \boldsymbol{\lambda}1^n + C2 \cdot \lambda 2^n,\ X0 = 1, X1 = 1$

find C1 and C2

$Xn = C1 * \mathbf{2}^n + C2 \cdot (-1)^n,\ X0 = 1, X1 = 1$

from the following system of equations we will find C1 and C2

$C1 * \mathbf{2}^0 + C2 \cdot (-1)^0 = 1$

$C1 * \mathbf{2}^1 + C2 \cdot (-1)^1 = 1 \quad \Rightarrow$

$C1 + C2 = 1$

$2 * C1 - C2 = 1 \quad \Rightarrow$

$\begin{matrix} 1 & 1 & | & 1 \\ 2 & -1 & | & 1 \end{matrix} \quad \Rightarrow$

$\begin{matrix} 0 & 3 & | & 1 \\ 2 & -1 & | & 1 \end{matrix} \quad \Rightarrow$

$\begin{matrix} 0 & 3 & | & 1 \\ 6 & -3 & | & 3 \end{matrix} \quad \Rightarrow$

$\begin{matrix} 0 & 3 & | & 1 \\ 6 & 0 & | & 4 \end{matrix} \quad \Rightarrow$

$3 * C2 = 1$

$6 * C1 = 4 \quad \Rightarrow$

$C1 = \frac{2}{3},\quad C2 = \frac{1}{3}$

There for $Xn = \frac{2}{3} \cdot 2^n + \frac{1}{3} \cdot (-1)^n$

$X12 = \frac{2}{3} \cdot 2^{12} + \frac{1}{3} \cdot (-1)^{12}$

$X12 = \frac{2}{3} \cdot 2^{12} + \frac{1}{3}$

$X12 = \frac{2}{3} \cdot 2^{12} + \frac{1}{3} = 2730.66 + 0.33 = 2730.99$

> $eval\left(\frac{2}{3} \cdot 2^{12} + \frac{1}{3}\right)$

$$2731 \tag{10}$$

Aswer:

closed form formula for $Xn = \frac{2}{3} \cdot 2^n + \frac{1}{3} \cdot (-1)^n$

The value for X12 = 2731 is the same as we got in (b)