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##
Section 4.3
##

```
> with(LinearAlgebra) :
```

Use the Matlab QR factorization to find the least squares solutions and 2-norm error of the following inconsistent systems

a)

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{3} \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} \mathbf{3} \\ \mathbf{5} \\ \mathbf{5} \\ \mathbf{5} \end{bmatrix}$$

Solve Ax=b
Using QR factorization we need to solve
We need to solve

$Rx = Q^T b$

```
> A := Matrix([[1, 1], [2, 1], [1, 2], [0, 3]])
```

$$A := \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \\ 0 & 3 \end{bmatrix} \quad \mathbf{(1)}$$

```
> b := <3, 5, 5, 5>
```

$$b := \begin{bmatrix} 3 \\ 5 \\ 5 \\ 5 \end{bmatrix} \quad \mathbf{(2)}$$

```
> Q, R := evalf(QRDecomposition(A))
```

$$Q, R := \begin{bmatrix} 0.4082482906 & 0.05063696836 \\ 0.8164965809 & -0.2025478735 \\ 0.4082482906 & 0.3544587785 \\ 0. & 0.9114654304 \end{bmatrix}, \begin{bmatrix} 2.449489743 & 2.041241452 \\ 0. & 3.291402944 \end{bmatrix} \quad \mathbf{(3)}$$

```
> sol := LinearSolve(Transpose(R).R, Transpose(R).Transpose(Q).b)
```

(4)

$$sol := \begin{bmatrix} 1.61538461630120 \\ 1.66153846099252 \end{bmatrix} \quad (4)$$

****

Answer:

$$[x1,x2]=\begin{bmatrix} \mathbf{1.61538461630120} \\ \mathbf{1.66153846099252} \end{bmatrix}$$

b)

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 2 \\ 3 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 10 \\ 5 \\ 10 \\ 5 \end{bmatrix}$$

(5)

> $A := Matrix([[1, 2, 2], [2,-1, 2], [3, 1, 1], [1, 1,-1]]); b := \langle 10, 5, 10, 3\rangle$

$$A := \begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 2 \\ 3 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$b := \begin{bmatrix} 10 \\ 5 \\ 10 \\ 3 \end{bmatrix} \quad (6)$$

> $Q, R := evalf(QRDecomposition(A))$

$$Q, R := \begin{bmatrix} 0.2581988897 & 0.7115947169 & 0.6289624339 \\ 0.5163977793 & -0.6294876342 & 0.3754999605 \\ 0.7745966692 & 0.08210708271 & -0.2487687237 \\ 0.2581988897 & 0.3010593033 & -0.6336561833 \end{bmatrix}, \begin{bmatrix} 3.872983346 & 1.032795559 & 2.065591118 \\ 0. & 2.435843454 & -0.05473805515 \\ 0. & 0. & 2.393812249 \end{bmatrix} \quad (7)$$

> $sol := LinearSolve(Transpose(R).R, Transpose(R).Transpose(Q).b)$

(8)

$$sol := \begin{bmatrix} 2.05882352883262 \\ 2.37254901953936 \\ 1.57843137287748 \end{bmatrix} \tag{8}$$

Answer:

$$[x1,x2,x3]=\begin{bmatrix} 2.0588 \\ 2.3725 \\ 1.5784 \end{bmatrix}$$

##

Section 12.1

##

1. Using the supplied code (or code of your own) for the Power Iteration method, find thedominant eigenvector of A, and estimate the dominant eigenvalue by calculating a Rayleigh quotient. Compare your conclusions with the corresponding part of Exercise 5.

****

```
> PowerIteration :=proc(A, v, k)
  local u, x, i, lam; x := v;
  i := 1;
  while i ≤ k do
     u := evalf( x / Norm(x, 2) );
     x := A.u;
     lam := u.x; i := i + 1;
  end do;
  u := x / Norm(x, 2) ;
  return(lam, u);
  end proc:
```

a)

$$\mathbf{A} = \begin{bmatrix} 10 & -12 & -6 \\ 5 & -5 & -4 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -12 & -6 \\ 5 & -5 & -4 \\ -1 & 0 & 3 \end{bmatrix} \tag{9}$$

```
> A := Matrix([[10,-12,-6], [5,-5,-4], [-1, 0, 3]])
```

(10)

$$A := \begin{bmatrix} 10 & -12 & -6 \\ 5 & -5 & -4 \\ -1 & 0 & 3 \end{bmatrix} \quad (10)$$

```
> PowerIteration(A, <1, 0, 0>, 10)
```

$$4.05075208624448724, \begin{bmatrix} 0.599566575529715 \\ 0.577057617702386 \\ -0.554548850472259 \end{bmatrix} \quad (11)$$

> $Eigenvectors(A)$

$$\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 2 \\ -\frac{1}{2} & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (12)$$

Answer:
Converges to 4 and the eigenvector corresponding to the largest eigenvalue = [-1, -1, 1]

###

Section 12.2

##

Construct a 4x4 symmetric matrix and use the QR algorithm (in Maple) to approximate the eigenvalues/eigenvectors for 5, 10, 20 iterations.
Then compare your answers with the results of the Eigenvectors command

****

```
> QRalgorithm := proc(A, k)
   local Q, x, i, lam, Qbar, R; Q := IdentityMatrix(RowDimension(A));
   i := 1; Qbar := Q; R := A;
   while i ≤ k do
      Q, R := evalf(QRDecomposition(R.Q));
      Qbar := Qbar.Q; i := i + 1;
   end do;
   return(Diagonal(R.Q), Qbar);
  end proc:
```

> $A := Matrix([[8, 2, 3, 4], [2, 1, 5, 6], [3, 5, 1, 7], [4, 6, 7, 8]])$

$$A := \begin{bmatrix} 8 & 2 & 3 & 4 \\ 2 & 1 & 5 & 6 \\ 3 & 5 & 1 & 7 \\ 4 & 6 & 7 & 8 \end{bmatrix} \quad (13)$$

> $qr\_values\_5 := QRalgorithm(A, 5)$

(14)

$$qr\_values\_5 := \begin{bmatrix} 18.8954143041867 \\ 5.22501646610950 \\ -3.98857181502880 \\ -2.13185895468923 \end{bmatrix}, \tag{14}$$

$$\begin{bmatrix} 0.444965802977347 & -0.892739670099887 & -0.0692489742030934 & 0.0150298103458064 \\ 0.397312655073697 & 0.162925039732380 & 0.599229193422089 & 0.675664457281970 \\ 0.446107251933993 & 0.288069465452248 & -0.770845581662161 & 0.351854219735893 \\ 0.667185437529684 & 0.305756922886893 & 0.204758496950925 & -0.647649792640825 \end{bmatrix}$$

> $qr\_values\_10 := QRalgorithm(A, 10)$

$$qr\_values\_10 := \begin{bmatrix} 18.8956145426746 \\ 5.38266582345502 \\ -4.14641647050334 \\ -2.13186389504802 \end{bmatrix}, \tag{15}$$

$$\begin{bmatrix} 0.441530050629705 & -0.893156761553116 & -0.0832396854692259 & -0.0198335186592888 \\ 0.398284434587157 & 0.263870426050538 & -0.556776753919728 & -0.679515675502075 \\ 0.446764576530559 & 0.152847099582243 & 0.811670865748564 & -0.343845289148790 \\ 0.668447557175489 & 0.330577013102434 & -0.155760092248327 & 0.647792787319703 \end{bmatrix}$$

> $qr\_values\_20 := QRalgorithm(A, 20)$

$$qr\_values\_20 := \begin{bmatrix} 18.8956145433958 \\ 5.39543569264719 \\ -4.15918653773431 \\ -2.13186369773052 \end{bmatrix}, \tag{16}$$

$$\begin{bmatrix} 0.441523502797114 & -0.895471632017345 & -0.0528695223367321 & -0.0198082395578903 \\ 0.398286195300690 & 0.244784996138131 & -0.565192103350201 & -0.679710451696946 \\ 0.446765934914478 & 0.180289463317795 & 0.806126549987538 & -0.343592628107525 \\ 0.668449925193015 & 0.325125470001690 & -0.167100822539675 & 0.647723275248343 \end{bmatrix}$$

> $values := evalf(Eigenvectors(A)):$

```
> egenvalues:=Re(values[1])
```

$$egenvalues := \begin{bmatrix} 18.89561456 \\ 5.395506304 \\ -2.131863717 \\ -4.159257137 \end{bmatrix} \tag{17}$$

> $egenvectors := \frac{\text{abs}(Column(\ values[2], 1))}{Norm(\text{abs}(Column(\ values[2], 1)), 1)}, \frac{\text{abs}(Column(\ values[2], 2))}{Norm(\text{abs}(Column(\ values[2], 2)), 1)}, \frac{\text{abs}(Column(\ values[2], 3))}{Norm(\text{abs}(Column(\ values[2], 3)), 1)}$

$$egenvectors := \begin{bmatrix} 0.225840271334371 \\ 0.203724291456343 \\ 0.228521785969956 \\ 0.341913651200000 \end{bmatrix}, \begin{bmatrix} 0.544110968328722 \\ 0.147780213716408 \\ 0.110862112156112 \\ 0.197246706000000 \end{bmatrix}, \begin{bmatrix} 0.0117150775008967 \\ 0.401997180998273 \\ 0.203208719102045 \\ 0.383079022300000 \end{bmatrix} \quad (18)$$

> $egenvectors\_qr\_5 := \frac{Column(\ qr\_values\_5[2], 1)}{Norm(Column(\ qr\_values\_5[2], 1), 1)}, \frac{Column(\ qr\_values\_5[2], 2)}{Norm(Column(\ qr\_values\_5[2], 2), 1)}, \frac{Column(\ qr\_values\_5[2], 3)}{Norm(Column(\ qr\_values\_5[2], 3), 1)}$

$$egenvectors\_qr\_5 := \begin{bmatrix} 0.227537516825073 \\ 0.203169624123336 \\ 0.228121207660279 \\ 0.341171651173120 \end{bmatrix}, \begin{bmatrix} -0.541221271888520 \\ 0.0987729123951446 \\ 0.174641418664489 \\ 0.185364397075633 \end{bmatrix}, \begin{bmatrix} -0.0421201398994090 \\ 0.364476409206086 \\ -0.468860717636466 \\ 0.124542733468829 \end{bmatrix} \quad (19)$$

> $egenvectors\_qr\_10 := \frac{Column(\ qr\_values\_10[2], 1)}{Norm(Column(\ qr\_values\_10[2], 1), 1)}, \frac{Column(\ qr\_values\_10[2], 2)}{Norm(Column(\ qr\_values\_10[2], 2), 1)}, \frac{Column(\ qr\_values\_10[2], 3)}{Norm(Column(\ qr\_values\_10[2], 3), 1)}$

$$egenvectors\_qr\_10 := \begin{bmatrix} 0.225843498173152 \\ 0.203723279642674 \\ 0.228520968571918 \\ 0.341912253633714 \end{bmatrix}, \begin{bmatrix} -0.544457955941297 \\ 0.160852337445235 \\ 0.0931738111296323 \\ 0.201515895733637 \end{bmatrix}, \begin{bmatrix} -0.0517837694907528 \\ -0.346373234356443 \\ 0.504943967259734 \\ -0.0968990291995724 \end{bmatrix} \quad (20)$$

> $egenvectors\_qr\_20 := \frac{Column(\ qr\_values\_20[2], 1)}{Norm(Column(\ qr\_values\_20[2], 1), 1)}, \frac{Column(\ qr\_values\_20[2], 2)}{Norm(Column(\ qr\_values\_20[2], 2), 1)}, \frac{Column(\ qr\_values\_20[2], 3)}{Norm(Column(\ qr\_values\_20[2], 3), 1)}$

$$egenvectors\_qr\_20 := \begin{bmatrix} 0.225840271510696 \\ 0.203724290815392 \\ 0.228521787410215 \\ 0.341913650441083 \end{bmatrix}, \begin{bmatrix} -0.544137513943673 \\ 0.148744744653993 \\ 0.109553733309202 \\ 0.197564008329257 \end{bmatrix}, \begin{bmatrix} -0.0332243372509994 \\ -0.355178791548536 \\ 0.506587144729368 \\ -0.105009726541833 \end{bmatrix} \quad (21)$$

Answer:

Eigenvalues are the same as in Eigenvector function.
The normilized vectors corresponding to the dominant eigenvalue are the same but the other eigenvectors are closed to each other but not the same.
(sign also in some of them are different)

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