Inna Williams

##

Section 12.3

##

Check written problems

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1.

c

Find the SVD of the following symmetric matrices by hand calculation, and describe geometrically the action of the matrix on the unit circle

$$A = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

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> $with(LinearAlgebra):$

>

>

>

> $A := Matrix\left(\left[\left[\frac{3}{2}, -\frac{1}{2}\right], \left[-\frac{1}{2}, \frac{3}{2}\right]\right]\right)$

$$A := \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \tag{1}$$

> $U, S, Vt := SingularValues(A, output = ['U','S','Vt'])$

$$U, S, Vt := \begin{bmatrix} -0.707106781186548 & 0.707106781186548 \\ 0.707106781186547 & 0.707106781186547 \end{bmatrix}, \begin{bmatrix} 2.00000000000000 \\ 1.00000000000000 \end{bmatrix}, \begin{bmatrix} -0.707106781186547 & 0.707106781186547 \\ 0.707106781186547 & 0.707106781186547 \end{bmatrix} \tag{2}$$

> $S := Matrix([[2, 0], [0, 1]]); U \cdot S \cdot Vt$

$$S := \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1.50000000000000 & -0.500000000000000 \\ -0.500000000000000 & 1.50000000000000 \end{bmatrix} \tag{3}$$

2.

b

$$A := \begin{bmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{bmatrix}$$

> $A := Matrix\left(\left[[6,-2], \left[8, \frac{3}{2}\right]\right]\right)$

$$A := \begin{bmatrix} 6 & -2 \\ 8 & \frac{3}{2} \end{bmatrix} \tag{4}$$

> $U, S, Vt := SingularValues(A, output = ['U','S','Vt'])$

$$U, S, Vt := \begin{bmatrix} -0.600000000000000 & -0.800000000000000 \\ -0.800000000000000 & 0.600000000000000 \end{bmatrix}, \begin{bmatrix} 10. \\ 2.50000000000000 \end{bmatrix}, \begin{bmatrix} -1. & -0. \\ 0. & 1. \end{bmatrix} \tag{5}$$

$S := Matrix([[10, 0], [0, 2.5]]); U \cdot S \cdot Vt$

$$S := \begin{bmatrix} 10 & 0 \\ 0 & 2.5 \end{bmatrix}$$

$$\begin{bmatrix} 6.00000000000000 & -2. \\ 8. & 1.50000000000000 \end{bmatrix} \tag{6}$$

different signs in the answer from wtitten problem
Check if it is correct:

> $U := Matrix\left(\left[\left[\frac{3}{5}, \frac{4}{5}\right], \left[-\frac{4}{5}, \frac{3}{5}\right]\right]\right)$

$$U := \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \tag{7}$$

> $S := Matrix\left(\left[[10, 0], \left[0, \frac{5}{2}\right]\right]\right)$

$$S := \begin{bmatrix} 10 & 0 \\ 0 & \frac{5}{2} \end{bmatrix} \tag{8}$$

> $Vt := Matrix([[1, 0], [0, 1]])$

$$Vt := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{9}$$

> $U \cdot S \cdot Vt$

$$\begin{bmatrix} 6 & 2 \\ -8 & \frac{3}{2} \end{bmatrix} \quad \textbf{(10)}$$

Both SVD from written and Maple give the same matrix A= $\begin{bmatrix} 6 & 2 \\ -8 & \frac{3}{2} \end{bmatrix}$

$$\begin{bmatrix} 6 & 2 \\ -8 & \frac{3}{2} \end{bmatrix} \quad \textbf{(11)}$$

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Section 12.4

###

1. d

Use Matlab's svd command to find the best rank-one approximation of the following matrices

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> $A := Matrix([[1, 5, 3], [2,-3, 2], [-3, 1, 1]])$

$$A := \begin{bmatrix} 1 & 5 & 3 \\ 2 & -3 & 2 \\ -3 & 1 & 1 \end{bmatrix} \quad \textbf{(12)}$$

> $U, S, Vt := SingularValues(A, output = ['U','S','Vt'])$

$$U, S, Vt := \begin{bmatrix} 0.898402133395958 & 0.438966448173354 & -0.0134931126009142 \\ -0.361733436568794 & 0.757058495583138 & 0.544069255824593 \\ 0.249043224316024 & -0.483912070156827 & 0.838931809373570 \end{bmatrix}, \quad \textbf{(13)}$$

$$\begin{bmatrix} 6.26135133093290 \\ 4.17319797039996 \\ 2.52584603855312 \end{bmatrix},$$

$$\begin{bmatrix} -0.0913851311717477 & 0.930510666638175 & 0.354681063877536 \\ 0.815877816954797 & -0.134250835932851 & 0.562423417766726 \\ -0.570957218714149 & -0.340773549918284 & 0.746914481098267 \end{bmatrix}$$

> $\boldsymbol{U \cdot Srank\_1 \cdot Vt};\ \boldsymbol{Srank\_1} := DiagonalMatrix\left(S_{1..1}, 2, 3\right)$

$$\begin{bmatrix} -0.647227628489829 & 6.59026478739581 & 2.51199928150541 \\ 0.260600309808826 & -2.65351009400633 & -1.01143363197782 \\ -0.179415931322682 & 1.82686653419382 & 0.696343404908098 \end{bmatrix}$$

(14)

$$Srank\_1 := \begin{bmatrix} 6.26135133093290 & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \tag{14}$$

> $\boldsymbol{Arank\_1 := U \cdot Srank\_1 \cdot Vt}$

$$Arank\_1 := \begin{bmatrix} -0.514060681077674 & 5.23431920388754 & 1.99515596163644 \\ 0.206981851287708 & -2.10755095445190 & -0.803331376524436 \\ -0.142501141471856 & 1.45098913190237 & 0.553070896905676 \end{bmatrix} \tag{15}$$

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Answer:

Best Rank one approximation :

$$\begin{bmatrix} -\mathbf{0.5140606810776739} & \mathbf{5.23431920388754} & \mathbf{1.995155961636438} \\ \mathbf{0.20698185128770794} & -\mathbf{2.107550954451898} & -\mathbf{0.8033313765244364} \\ -\mathbf{0.1425011414718564} & \mathbf{1.4509891319023662} & \mathbf{0.553070896905676} \end{bmatrix}$$

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2. c

Find the best rank-two approximation to the following matrices:

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> $Srank\_2 := DiagonalMatrix(S_{1..2}, 3, 3)$

$$Srank\_2 := \begin{bmatrix} 6.26135133093290 & 0. & 0. \\ 0. & 4.17319797039996 & 0. \\ 0. & 0. & 0. \end{bmatrix} \tag{16}$$

> $\boldsymbol{Arank\_2 := U \cdot Srank\_2 \cdot Vt}$

$$Arank\_2 := \begin{bmatrix} 0.980540907270313 & 4.98838591773545 & 3.02545598456845 \\ 2.78462949310486 & -2.53169700115519 & 0.973563847714099 \\ -1.79013446672211 & 1.72210344170339 & -0.582721187100628 \end{bmatrix} \tag{17}$$

Answer:

Best Rank 2 approximation is:

$$\begin{bmatrix} \mathbf{0.980540907270313} & \mathbf{4.988385917735454} & \mathbf{3.0254559845684548} \\ \mathbf{2.784629493104856} & -\mathbf{2.531697001155195} & \mathbf{0.9735638477140988} \\ -\mathbf{1.790134466722111} & \mathbf{1.7221034417033938} & -\mathbf{0.5827211871006}283 \end{bmatrix}$$

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4. b

Find the best least squares approximating plane for the following

three-dimensional vectors,
and the projections of the vectors onto the subspace:
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The best least square (12.4.2)

```
> with(VectorCalculus) :
> A := Matrix([[2,-1, 7, 1], [3, 4,-2, 1], [1, 0, 1, 0]])
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$$A := \begin{bmatrix} 2 & -1 & 7 & 1 \\ 3 & 4 & -2 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \quad \textbf{(18)}$$

```
> U, S, Vt := SingularValues(A, output = ['U','S','Vt'])
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$$U, S, Vt := \begin{bmatrix} -0.932323094927223 & -0.313172844465256 & -0.180821503574287 \\ 0.334156944734542 & -0.937229798827319 & -0.0996967425534974 \\ -0.138248988969183 & -0.153372336748329 & 0.978449867581040 \end{bmatrix}, \quad \textbf{(19)}$$

$$\begin{bmatrix} 7.76383316783266 \\ 5.14665168641866 \\ 0.484634873637294 \end{bmatrix}, [[-0.128857012121923, 0.292246217147758,$$
$$-0.944485073856352, -0.0770452091463042],$$
$$[-0.697814354065975, -0.667569239222174, -0.0915396415097822,$$
$$-0.242954588629385],$$
$$[0.655579385749605, -0.449751923553959, -0.181388457814040,$$
$$-0.578823896890521],$$
$$[0.258198889747161, -0.516397779494322, -0.258198889747161,$$
$$0.774596669241483]]$$

```
> Srank_2 := DiagonalMatrix(S[1..2], 3, 4)
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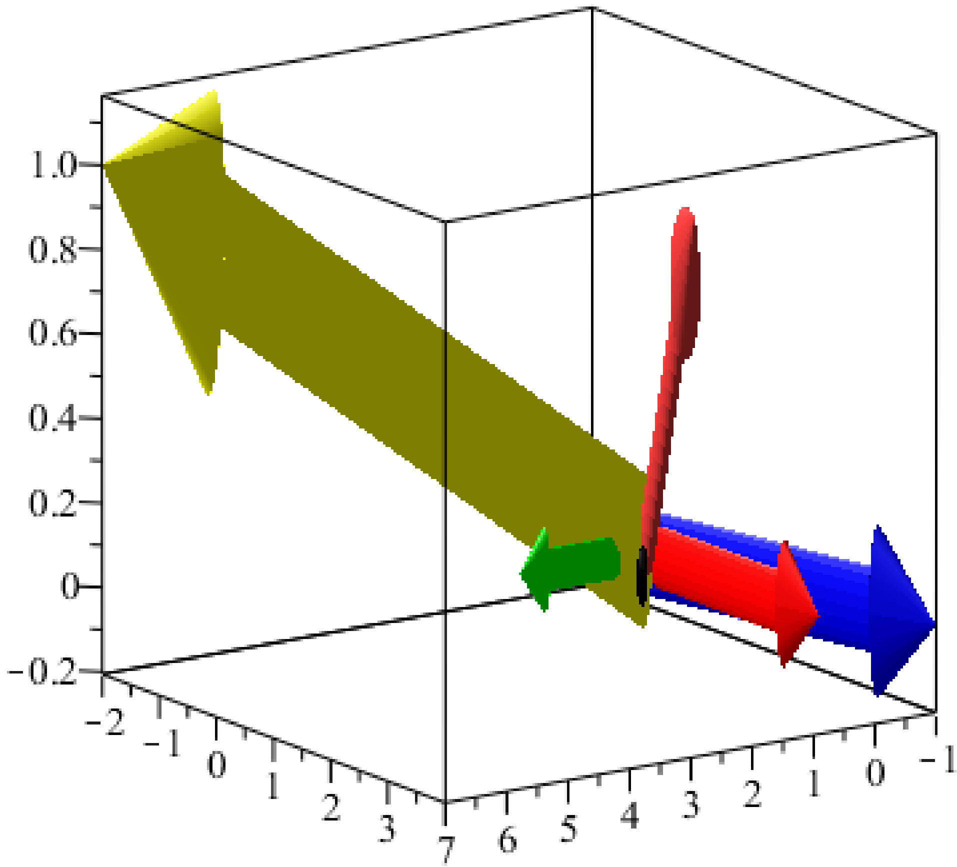
$$Srank\_2 := \begin{bmatrix} 7.76383316783266 & 0. & 0. & 0. \\ 0. & 5.14665168641866 & 0. & 0. \\ 0. & 0. & 0. & 0. \end{bmatrix} \quad \textbf{(20)}$$

```
> ARank_2 := U.Srank_2.Vt;
```

$$ARank\_2 := \quad \textbf{(21)}$$
$$[[2.05744999924839, -1.03941284340506, 6.98410449292397,$$
$$0.949276268955153],$$
$$[3.03167531334243, 3.97826955298687, -2.00876405872859, 0.972033244634240],$$
$$[0.689130202735967, 0.213268281970857, 1.08601276112764, 0.274473040777796]]$$

```
> PlotVector([⟨A[1][1], A[2][1], A[3][1]⟩, ⟨A[1][2], A[2][2], A[3][2]⟩,
    ⟨A[1][3], A[2][3], A[3][3]⟩, ⟨A[1][4], A[2][4], A[3][4]⟩,
    ⟨ARank_2[1][1], 0, 0⟩, ⟨0, ARank_2[2][1], 0⟩], color = [orange, blue, yellow, black,
    green, red]);
```



> $v1 := Vector(Column(ARank\_2, 1)); v2 := Vector(Column(ARank\_2, 2));$

$$v1 := (2.05744999924839)e_x + (3.03167531334243)e_y + (0.689130202735967)e_z$$

$$v2 := (-1.03941284340506)e_x + (3.97826955298687)e_y + (0.213268281970857)e_z \qquad (22)$$

Vectors v1 and v2 are red and green -> they are projections (2 principal components)
they are the best least square 2 dimentional subspace that is found by SVD accordintg to the section 12.4.2 on the data matrix A.
The cross product of this 2 vectors will give the vecotr n orthogonal to v1 an v2 and the plane they make
is below:

> $n := CrossProduct(v1, v2)$

$$n := (-2.09498551801817)e_x + (-1.15507961008274)e_y + (11.3362529465249)e_z \qquad (23)$$

> $plane := -2.09498551801817 \cdot x1 - 1.15507961008274 \cdot x2 + 11.3362529465249 \cdot x3 = 0$

$$plane := -2.09498551801817\, x1 - 1.15507961008274\, x2 + 11.3362529465249\, x3 = 0 \qquad (24)$$

Answer:

The best Plane is:

$$\boldsymbol{plane := -2.09498551801817\, x1 - 1.15507961008274\, x2 + 11.3362529465249\, x3 = 0}$$

The projections are:

$$\left[\begin{array}{cccc} 2.057449999248389 & -1.039412843405062 & 6.984104492923969 & 0.9492762689551526 \\ 3.0316753133424323 & 3.97826955298687 & -2.0087640587285867 & 0.97203324463424 \\ 0.6891302027359669 & 0.2132682819708573 & 1.0860127611276404 & 0.2744730407777961 \end{array}\right]$$

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