IVNA WILLIAMS

Section 9.1

1 (a) $a=2,\ b=0,\ m=5,\ x_i=(a\cdot x_{i-1}+b)\ \text{mod}\ m,\ u_i=\frac{x_i}{m}$

Start with $x_0=1$

Step 1: $x_0=1$, $u_0=1/5$
Step 2: $x_1=(2\cdot 1)\ \text{mod}\ 5=2$, $u_1=2/5$
Step 3: $x_2=(2\cdot 2)\ \text{mod}\ 5=4$, $u_2=4/5$
Step 4: $x_3=(2\cdot 4)\ \text{mod}\ 5=3$, $u_3=3/5$
Step 5: $x_4=(2\cdot 3)\ \text{mod}\ 5=1$, $u_4=1/5$ → back to step 1

⇒ **period = 4**

(b) $a=4,\ b=1,\ m=9$ $\quad x_i=[(4\cdot x_{i-1})+1]\ \text{mod}\ 9,\ u_i=\frac{x_i}{9}$

Start with $x_0=1$

Step 1: $x_0=1$, $u_0=1/9$
Step 2: $x_1=(4\cdot 1+1)\ \text{mod}\ 9=5$, $u_1=5/9$
Step 3: $x_2=(4\cdot 5+1)\ \text{mod}\ 9=3$, $u_2=3/9$
Step 4: $x_3=(4\cdot 3+1)\ \text{mod}\ 9=4$, $u_3=4/9$
Step 5: $x_4=(4\cdot 4+1)\ \text{mod}\ 9=8$, $u_4=8/9$
Step 6: $x_5=(8\cdot 4+1)\ \text{mod}\ 9=6$, $u_4=6/9$
Step 7: $x_6=(4\cdot 6+1)\ \text{mod}\ 9=7$, $u_5=7/9$
Step 8: $x_7=(4\cdot 7+1)\ \text{mod}\ 9=2$, $u_6=2/9$
Step 9: $x_8=(4\cdot 2+1)\ \text{mod}\ 9=0$, $u_7=0$
Step 10: $x_9=(4\cdot 0+1)\ \text{mod}\ 9=1$, $u_8=1/9$ → back to Step 1

⇒ **period = 9**

3 @ $y = x^2$ $0 < x < 1$, using LCG with $a = 2, b = 0, m = 5$

$x_i = (2 \cdot (x_{i-1})) / \text{mod } 5$ $\quad u_i = x_i / m = x_i / 5$

Step 1 : $x_0 = 1$, $u_0 = 1/5$
Step 2 : $x_1 = (2 \cdot 1) / \text{mod } 5 = 2$, $u_1 = 2/5$
Step 3 : $x_2 = (2 \cdot 2) / \text{mod } 5 = 4$, $u_2 = 4/5$
Step 4 : $x_3 = (2 \cdot 4) / \text{mod } 5 = 3$, $u_3 = 3/5$
Step 5 : $x_4 = (2 \cdot 3) / \text{mod } 5 = 1$, $u_4 = 1/5$ → repeat =>

period = 4 , 4 point

$x^2 = (1-0) \cdot \frac{1}{n} \sum_{i=1}^{n} f(u_i)$

Area under $x^2 = (1-0) \cdot \frac{(\frac{1}{5})^2 + (\frac{2}{5})^2 + (\frac{4}{5})^2 + (\frac{3}{5})^2}{4} = \frac{\frac{1}{25} + \frac{4}{25} + \frac{16}{25} + \frac{9}{25}}{4}$

$= \frac{30}{25 \cdot 4} = \frac{30}{100} = 0.3$

Answer : Area under $x^2 = 0.3$

(b) $a = 4, b = 1, m = 9$, $x_i = (4 \cdot x_{i-1} + 1) / \text{mod } 9$, $u_i = x_i / 9$

Step 1 : $x_0 = 1$, $u_0 = 1/9$
Step 2 : $x_1 = (4 \cdot 1 + 1) / \text{mod } 9 = 5$, $u_1 = 5/9$
Step 3 : $x_2 = (4 \cdot 5 + 1) / \text{mod } 9 = 3$, $u_2 = 3/9$
Step 4 : $x_3 = (4 \cdot 3 + 1) / \text{mod } 9 = 4$, $u_3 = 4/9$
Step 5 : $x_4 = (4 \cdot 4 + 1) / \text{mod } 9 = 8$, $u_4 = 8/9$
Step 6 : $x_5 = (4 \cdot 8 + 1) / \text{mod } 9 = 6$, $u_5 = 6/9$
Step 7 : $x_6 = (4 \cdot 6 + 1) / \text{mod } 9 = 7$, $u_6 = 7/9$
Step 8 : $x_7 = (4 \cdot 7 + 1) / \text{mod } 9 = 2$, $u_7 = 2/9$
Step 9 : $x_8 = (4 \cdot 2 + 1) / \text{mod } 9 = 0$, $u_8 = 0$
Step 10 : $x_9 = (4 \cdot 0 + 1) / \text{mod } 9 = 1$, $u_9 = 1/9$ → repeat Step 1

=> period = 9, number of points = 9, Area $= (1-0) \cdot \frac{1}{9} \sum_{i=1}^{9} x_i^2$

Area under $x^2 = (1-0) \frac{(\frac{1}{9})^2 + (\frac{5}{9})^2 + (\frac{3}{9})^2 + (\frac{4}{9})^2 + (\frac{8}{9})^2 + (\frac{6}{9})^2 + (\frac{7}{9})^2 + (\frac{2}{9})^2 + 0^2}{9} =$

$= \frac{\frac{1}{81} + \frac{4}{81} + \frac{9}{81} + \frac{16}{81} + \frac{25}{81} + \frac{36}{81} + \frac{49}{81} + \frac{64}{81} + 0}{9} (1-0) = \frac{204}{81 \cdot 9} = 0.2798$

≈ 0.28

Answer : Area under $x^2 = 0.28$