Problem 1

(b) $$\ell(c) = \int_0^{2\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$c(t) = (t \cdot \cos t,\ t \sin t)$$

$$x'(t) = (t \cdot \cos t)' = \cos t - t \cdot \sin t$$

$$y'(t) = (t \cdot \sin t)' = \sin t + t \cdot \cos t$$

$$(x'(t))^2 + (y'(t))^2 = (\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 =$$

$$= \cos^2 t - 2t\cos t \cdot \sin t + t^2 \sin^2 t + \sin^2 t + 2t \sin t \cos t + \cos^2 t =$$

$$= (\cos^2 t + \sin^2 t) + t^2(\cos^2 t + \sin^2 t) = 1 + t^2$$

$$\int_0^{2\pi} \sqrt{1+t^2}\, dt \Rightarrow \text{substitution} \quad t = \tan u,\ u = \arctan t$$

$$dt = \frac{du}{\cos^2 u} \Rightarrow \int_0^{2\pi} \sqrt{1+t^2}\, dt = \int_0^{\arctan(2\pi)} \frac{\sqrt{1+\tan^2 u}}{\cos^2 u}\, du = \int_0^{\arctan(2\pi)} \frac{du}{\cos^3 u} =$$

$$= \int_0^{\arctan(2\pi)} \sec^3 u\, du = \frac{1}{2}\Big[\sec u \cdot \tan u + \ln(\sec u + \tan u)\Big]_0^{\arctan(2\pi)} =$$

$$= \frac{1}{2}\Big[\sec(\arctan(t)) + \tan(\arctan(t)) +$$

$$+ \ln(\sec(\arctan(t)) + \tan(\arctan(t))\Big] =$$

$$= \frac{1}{2}\Big[\underbrace{\sec(\arctan(t))}_{\sqrt{1+t^2}} \cdot t + \ln(\sec(\arctan(t)) + t\Big]$$

$$= \frac{1}{2}\Big[t \cdot \sqrt{1+t^2} + \ln(t + \sqrt{1+t^2})\Big]_0^{2\pi} = \frac{1}{2}\Big[\sqrt{1+(2\pi)^2} \times 2\pi$$

$$+ \ln(\sqrt{1+(2\pi)^2} + 2\pi) - \sqrt{1+0^2} \cdot 0 + \ln(\sqrt{1+0^2} + 0)\Big] =$$

$$= \frac{1}{2}\Big[2\pi \cdot \sqrt{1+4\pi^2} + \ln(2\pi + \sqrt{1+4\pi^2})\Big] = \pi\sqrt{1+4\pi^2} + \frac{1}{2}\ln(2\pi + \sqrt{1+4\pi^2}) =$$

$$= 21.25629415$$

Answer: $\int_0^{2\pi}\sqrt{(x'(t))^2+(y'(t))^2} = \pi\sqrt{1+4\pi^2}+\frac{1}{2}\ln(2\pi+\sqrt{1+4\pi^2}) =$

$= 21.25629415$

ⓒ Simpson's rule, $b=2\pi$, $a=0$, $m=1$

$$\int_a^b f(x)dx = \frac{h}{3}\left[y_0+y_{2m}+4\sum_{i=1}^{m} y_{2i-1}+2\sum_{i=1}^{m-1} y_{2i}\right]-\frac{(b-a)\cdot h^4}{180}f^{(4)}(c)$$

$a<c<b$, $m=1$, $h=\frac{b-a}{2m}$.

$$\frac{(b-a)h^4}{180}\cdot f^4(c) = \frac{(b-a)(b-a)^4}{16\cdot m^4\cdot 180}\cdot f^4(c) = \frac{(b-a)^5}{2880\, m^4} = \frac{(2\pi)^5}{2880} =$$

$$= \frac{32\pi^5}{2880} = \frac{\pi^5}{90} \Rightarrow \text{Error Bound} = \frac{\pi^5}{90}\cdot f^4(c)\quad 0<c<2\pi$$

$$\int_0^{2\pi}\sqrt{1+t^2}\,dt = \frac{\pi}{3}\left[\sqrt{1+0^2}+\sqrt{1+4\pi^2}+4\sqrt{1+\pi^2}\right] =$$

$$= \frac{\pi}{3}\left[1+\sqrt{1+4\pi^2}+4\sqrt{1+\pi^2}\right] = \underline{21.51980325}$$

Actual error $= |21.25629415-21.51980325| = 0.2635091$

$f'(t) = t(1+t^2)^{-1/2}$

$f''(t) = (1+t^2)^{-1/2}+t\left(-\frac{1}{2}\right)(1+t^2)^{-3/2}\cdot 2t = -t^2(1+t^2)^{-3/2}+(1+t^2)^{-1/2}$

$f'''(t) = -\left(2t(1+t^2)^{-3/2}-\frac{3}{2}(1+t^2)^{-5/2}\cdot 2t\cdot t^2\right)-\frac{1}{2}(1+t^2)^{-3/2}\cdot 2t =$

$= 3t^3(1+t^2)^{-5/2}-3\cdot t(1+t^2)^{-3/2}$

$f^{(4)}(t) = 3\left(3t^2(1+t^2)^{-5/2}-\frac{5}{2}(1+t^2)^{-7/2}\cdot 2t\cdot t^3-\right.$

$\left.-1\left(-\frac{3}{2}(1+t^2)^{-5/2}\cdot 2t^3+(1+t^2)^{-3/2}\right)\right) =$

$= 18\cdot t^2(1+t^2)^{-5/2}-15t^4(1+t^2)^{-7/2}+3(1+t^2)^{-3/2}$

$|f^{(4)}(c)|_{MAX} = |f^{(4)}(0)| = 3$

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Error Bounds $= \frac{\pi^5 \cdot 3}{90} = \frac{\pi^5}{30} = 10.20065617$

Answer:

$$\int_0^{2\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 21.51980325$$

Approximation

$$\int_0^{2\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 21.25629415$$

theoretical value

Error bounds: $21.25629415 \pm 10.20065617$

$11.05563798 < 21.51980325 < 31.45695032$

Actual error $= 0.2635051 <$ Error bounds $=$

$= 10.20065617$

Problem 2 MAT 486 FINAL I.W.

ⓐ $f_1 = x^2+y+z-37$

$f_2 = x-y^2-z-5$

$f_3 = x+y+z-3$

$x_0 = (1, 1, 1)$

$$DF = \begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{vmatrix} = \begin{vmatrix} 2x & 1 & 1 \\ 1 & -2y & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

Step 1

$$DF(x_0) = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$f_1(x_0) = -34$, $f_2(x_0) = -6$, $f_3(x_0) = 0$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} s_0 \\ s_1 \\ s_2 \end{vmatrix} = \begin{vmatrix} 34 \\ 6 \\ 0 \end{vmatrix} \Rightarrow \left|\begin{array}{ccc|c} 2 & 1 & 1 & 34 \\ 2 & -2 & -1 & 6 \\ 1 & 1 & 1 & 0 \end{array}\right| = \left|\begin{array}{ccc|c} 1 & 0 & 0 & 34 \\ 0 & -3 & -2 & 6 \\ 1 & 1 & 1 & 0 \end{array}\right| = \left|\begin{array}{ccc|c} 1 & 0 & 0 & 34 \\ 0 & -3 & -2 & 6 \\ 0 & 1 & 1 & -34 \end{array}\right|$$

$$\Rightarrow \left|\begin{array}{ccc|c} 1 & 0 & 0 & 34 \\ 0 & -1 & 0 & -62 \\ 0 & 1 & 1 & -34 \end{array}\right| \Rightarrow S = \begin{Bmatrix} S_0 = 34 \\ S_2 = 62 \\ S_3 = -96 \end{Bmatrix} \quad x_1 = x_0 + S = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} + \begin{vmatrix} 34 \\ 62 \\ -96 \end{vmatrix} = \begin{vmatrix} 35 \\ 63 \\ -95 \end{vmatrix}$$

$$x_1 = \begin{vmatrix} 35 \\ 63 \\ -95 \end{vmatrix}$$

Step 2

$$DF(x_1) = \begin{vmatrix} 70 & 1 & 1 \\ 1 & -126 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$f_1(x_1) = 1225+63-95-37 = 1156$

$f_2(x_1) = 35-3969+95-5 = -3844$

$f_3(x_1) = 35+63-95-3 = 0$

$$\left|\begin{array}{ccc|c} 70 & 1 & 1 & -1156 \\ 1 & -126 & -1 & 3844 \\ 1 & 1 & 1 & 0 \end{array}\right| = \left|\begin{array}{ccc|c} 69 & 0 & 0 & -1156 \\ 0 & -127 & -2 & 3844 \\ 1 & 1 & 1 & 0 \end{array}\right| = \left|\begin{array}{ccc|c} 69 & 0 & 0 & -1156 \\ 0 & -127 & -2 & 3844 \\ 0 & 69 & 69 & 1156 \end{array}\right| =$$

$$= \left|\begin{array}{ccc|c} 69 & 0 & 0 & -1156 \\ 0 & -125 & 0 & \frac{267548}{69} \\ 0 & 2 & 2 & 1156 \cdot 2/69 \end{array}\right| = \left|\begin{array}{ccc|c} 69 & 0 & 0 & -1156 \\ 0 & -125 & 0 & 267548/69 \\ 0 & 1 & 1 & 1156/69 \end{array}\right| =$$

$$= \left|\begin{array}{ccc|c} 69 & 0 & 0 & -1156 \\ 0 & -125 & 0 & 267548/69 \\ 0 & 0 & 125 & \frac{1156 \cdot 125}{69} + \frac{267548}{69} \end{array}\right| = \left|\begin{array}{ccc|c} 69 & 0 & 0 & -1156 \\ 0 & -125 & 0 & 267548/69 \\ 0 & 0 & 125 & \frac{412048}{69} \end{array}\right|$$

$$\Rightarrow \begin{cases} S_0 = \frac{-1156}{69} = -16.75 \\ S_1 = \frac{-267548}{8625} = -31.02 \\ S_2 = \frac{412048}{69 \cdot 125} = \frac{412048}{8625} = 47.77 \end{cases}$$

$$x_2 = x_1 + S = \begin{vmatrix} 35 \\ 63 \\ -95 \end{vmatrix} + \begin{vmatrix} -16.75 \\ -31.02 \\ 47.77 \end{vmatrix} = \underline{\underline{\begin{vmatrix} 18.25 \\ 31.98 \\ -47.23 \end{vmatrix}}} = x_2$$

Answer: Step 1: $x_1 = \begin{vmatrix} 35 \\ 63 \\ -95 \end{vmatrix}$

Step 2: $x_2 = \begin{vmatrix} 18.25 \\ 31.98 \\ -47.23 \end{vmatrix}$

Problem 4 MAT 486 FINAL I.W

t	0	1	2	3
date	03/13	04/13	05/13	06/13
value	14090	14573	14701	15254

$c=0, d=0, n=4, \Delta t = \frac{d-c}{n} = \frac{4-0}{4} = 1$

$\omega = e^{-\frac{2\pi i}{4}} = e^{-\frac{\pi}{2}i} = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = 0 - i = -i$

$$\begin{vmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{vmatrix} = \frac{1}{\sqrt{4}} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{vmatrix} \times \begin{vmatrix} 14090 \\ 14573 \\ 14701 \\ 15254 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{vmatrix} \times \begin{vmatrix} 14090 \\ 14573 \\ 14701 \\ 15254 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 58618 \\ -611+681\cdot i \\ -1036 \\ -611-681i \end{vmatrix}$$

$$= \begin{vmatrix} 29309 \\ -305.5 + 340.5\cdot i \\ -518 \\ -305.5 - 340.5\cdot i \end{vmatrix} \to \text{DFT}$$

$$P_n = \frac{1}{2}\left[a_0 + 2\sum_{k=1}^{n/2}\left(a_k \cos\frac{2\pi k(t-c)}{d-c} - b_k \sin\frac{2\pi k(t-c)}{d-c}\right) + a_{n/2}\cdot\cos\left(\frac{n\pi(t-c)}{d-c}\right)\right]$$

$$P_4 = \frac{1}{2}\left[29309 - 2\cdot 305.5\cdot\cos\left(\frac{\pi}{2}\cdot t\right) - 2\cdot 340.5\cdot\sin\left(\frac{\pi}{2}\cdot t\right) - 518\cdot\cos(\pi t)\right]$$

$t = 0, 1, 2, 3$

check for $P_4(0) = \frac{1}{2}(29309 - 2\cdot 305.5 - 518) = 14090$

check for $P_4(3) = \frac{1}{2}(29309 + 2\cdot 340.5 + 518) = 15254$

Answer:

$$\boxed{P_4 = 14654.5 - 305.5\cdot\cos\left(\frac{\pi}{2}t\right) - 340.5\cdot\sin\left(\frac{\pi}{2}\cdot t\right) - 259\cdot\cos(\pi\cdot t)}$$