Math 459
Monte Carlo Simulations
Project Topic: Jackpot Pricing Euro Model
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Jackpot model Pricing

$$\ln(S(t_j)) = \ln(S(t_{j-1})) + \left(r(t_{j-1}) + \frac{\ln(1-0.15\sigma^2/2)}{0.15}\right)\Delta + \sigma\sqrt{Y_j}X_j,$$

$j = 1, ..., d$, X_j $i.i.d \sim N(0,1)$, Y_j $i.i.d. \sim Gamma(\Delta/0.15, 0.15)$ (using the shape and scale parameter definition).
And,

$$r(t_j) = r(t_{j-1}) + 0.18(0.086 - r(t_{j-1}))\Delta + 0.02\sqrt{\Delta}Z_j,$$

where Z_j $i.i.d \sim N(0,1)$.

Implementation

Code implemented using the reference class with member variables and with class methods. When object of this class is created the method initialize is called automatically. All the required constant variables initialized in this function. All the variables that are member variables also initialized when the
Constructor is called and the object of the Jackpot class is created.

The formulas will be used in the following way:

$$r(t_j) = r(t_j - 1)(1 - 0.18 \cdot \text{delta}) + 0.18 \cdot 0.096 \cdot \text{delta} + 0.02 \cdot \text{delta}^{\frac{1}{2}} \cdot Z_j$$

$$S(t_j) = e^{\left(\ln(S(t_j - 1)) + r(t_j\text{-}1) \cdot \text{delta} + \frac{dela}{0.15} \cdot \ln\left(1 - \frac{0.15 \cdot \sigma^2}{2}\right) + \sigma \cdot Y_j^{\frac{1}{2}} \cdot X_j\right)}$$

Code constants come from the formulas:

Methods

initialize()

$$const\_1 = 1 - (0.18 \cdot \text{delta})$$

$$const\_2 = 0.18 \cdot 0.086 \cdot \text{delta}$$

$$const\_3 = 0.02 \cdot \left((\text{delta})^{\frac{1}{2}} \right)$$

$$const\_4 = \frac{\text{delta}}{0.15} \cdot \ln\left(1 - \frac{0.15 \cdot \sigma^2}{2} \right)$$

this const for constant rate and constant volatility

$$const\_5 = \left(r0 + \left(\frac{\ln\left(1 - \frac{0.15 \cdot \sigma^2}{2} \right)}{0.15} \right) \right) \cdot \text{delta}$$

init()

This method initializes required matrices, such as Z_j, Y_j, X_j , X_j for the constant rate and constant sigma and calculates the following:

$$\sigma \cdot Y_j^{\frac{1}{2}} \cdot X_j \qquad 0.02 \cdot \text{delta}^{\frac{1}{2}} \cdot Z_j$$

It also initializes S_Maturity,rate_at_maturity and Discounted_payoffs

r(): Calculate matrix r used in the above modified formula. Also calculates any annualized interest rate for the fair price.

s()

Calculates the sample path

Run()

Call other methods and calculates Jackpot call option price
Calculates sample size
Calculates estimated error for the calculated sample size
Calls Method to Calculate Estimated Euro Call Fair Price
Calls Method to Calculate Variance reduction method

s_t_rate_sigma_const()
Calculates sample path for constant rate and constant volatility

Run_For_Const_RATE_SIGMA()
Calculates European call option with a constant interest
rate of r = 0.07 (and constant volatility of 13% for the jackpot model)

Calculate_Estimated_Euro_Call_Fair_Price()
Calculates estimated fair price of the European call option?

GeometricMeanAsianControlVariate()
Calculates variance reduction method using Geometric Mean Asian as Control Variant

DisplayOutput()
DisplayControlVarOutput()

Problems Output

Solve the following problems:

1. Using a time step of 1 month $d = 12$, calculate the fair price of the European call option with a strike price of \$50 that matures in on year (= 12 months = 52 weeks), with an error tolerance of \$0.05.

```
##########     Start Output     ##########
Test for d=12 n=10000
Current Sample Size =  10000
Current Step Size =  12
Jackpot Call Option Price =  4.567705
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.257518
Calculated Required Sample Size = 89053
Estimated Error =  0.135644
##########      End Output     ##########
##########     Start Output     ##########
Test for d=12 n=10000 ,  89053 ,  87789 ,  87875 ,  87200 ,  87058 ,  87605 ,  86844 ,  87455 ,  88803 ,  87921
Current Sample Size =  87921
Current Step Size =  12
Jackpot Call Option Price =  4.572218
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.217947
Calculated Required Sample Size = 87718
Estimated Error =  0.0454018
##########      End Output     ##########
Sample size =  87760.3  for d =  12
```

2. Repeat the calculation with a time step of 1 week $d = 52$.

```
##########     Start Output     ##########
Test for d=52 n=1000
Current Sample Size =  1000
Current Step Size =  52
Jackpot Call Option Price =  4.618985
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.295114
Calculated Required Sample Size = 90331
Estimated Error =  0.4320112
##########      End Output     ##########
##########     Start Output     ##########
Test for d=52 n=1000 ,  90331 ,  87308 ,  86558 ,  87515 ,  87373 ,  87845 ,  87553 ,  86757 ,  88049 ,  87198
Current Sample Size =  87198
Current Step Size =  52
Jackpot Call Option Price =  4.545891
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.214759
Calculated Required Sample Size = 87610
Estimated Error =  0.04556178
##########      End Output     ##########
Sample size =  87648.7  for d =  52
```

- What is the sample size required?
- What is the sample size required?

From the tests above for both d=12 or d=52 required size round to 88000

- What is your estimated fair price of the European call option?

```
##########     Start Output     ##########
Test for d=12 n=88000
Current Sample Size =  88000
Current Step Size =  12
Jackpot Call Option Price =  4.544387
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.198064
Calculated Required Sample Size = 87050
Estimated Error =  0.04520849
##########      End Output     ##########
```

```
##########     Start Output     ##########
Test for d=52 n=88000
Current Sample Size =  88000
Current Step Size =  52
Jackpot Call Option Price =  4.533051
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.184549
Calculated Required Sample Size = 86598
Estimated Error =  0.04509095
##########      End Output     ##########
```

- Which estimated price is higher? Time step of 1 month or time step of 1 week?
- The payoff of the European option only depends on the price at the maturity time. Should the price of the option depend on the number of time steps used? Why?

We can see from above output that the price with step of 1 month slightly higher
We will try to test for more different steps d=4,12,26,52,104,365 and compare output
To answer the question.

```
##########     Start Output     ##########
Test for d=4 different d size n=88000
Current Sample Size =  88000
Current Step Size =  4
Jackpot Call Option Price =  4.547706
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.184972
Calculated Required Sample Size = 86612
Estimated Error =  0.04509462
##########      End Output     ##########
```

```
##########     Start Output     ##########
Test for d=12 different d size n=88000
Current Sample Size =  88000
Current Step Size =  12
Jackpot Call Option Price =  4.517754
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.171042
Calculated Required Sample Size = 86148
Estimated Error =  0.04497348
##########      End Output     ##########
```

```
##########     Start Output     ##########
Test for d=26 different d size n=88000
Current Sample Size =  88000
Current Step Size =  26
Jackpot Call Option Price =  4.546785
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.229931
Calculated Required Sample Size = 88121
Estimated Error =  0.04548564
##########      End Output     ##########
```

```
##########     Start Output     ##########
Test for d=52 different d size n=88000
Current Sample Size =  88000
Current Step Size =  52
Jackpot Call Option Price =  4.534692
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.184186
Calculated Required Sample Size = 86586
Estimated Error =  0.04508779
##########      End Output     ##########
```

```
##########     Start Output     ##########
Test for d=104 different d size n=88000
Current Sample Size =  88000
Current Step Size =  104
Jackpot Call Option Price =  4.590789
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.230341
Calculated Required Sample Size = 88135
Estimated Error =  0.04548921
```

```
##########     Start Output     ##########
Test for d=365 different d sizen=88000
Current Sample Size =  88000
Current Step Size =  365
Jackpot Call Option Price =  4.586246
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.240096
Calculated Required Sample Size = 88464
Estimated Error =  0.04557405
##########      End Output     ##########
```

It seems like if d becomes very large d=104,365 shows a little larger values compare to the d =4,12,52

- How do these prices compare to the price of the European call option with a constant interest rate of $r = 0.07$ (and constant volatility of 13% for the jackpot model)?

```
##########     Start Output     ##########
Test for d=12 n=88000 const rate const variance
Current Sample Size =  88000
Current Step Size =  12
Jackpot Call Option Price =  4.559024
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.14678
Calculated Required Sample Size = 4106
Estimated Error =  0.009818432
##########      End Output     ##########
```

```
##########     Start Output     ##########
Test for d=52 n=88000 const rate const variance
Current Sample Size =  88000
Current Step Size =  52
Jackpot Call Option Price =  4.54474
Estimated Fair Price 4.547623
Sample Standard Deviation = 5.147641
Calculated Required Sample Size = 4134
Estimated Error =  0.009850937
##########      End Output     ##########
```

It seems no difference for constant rate and constant volatility

- Do you have an explanation for the above results?

- What is the computational time of your program? Can you make more efficient?

Computational complexity is **O(n x d)**
The complexity could be made more efficient if we use R each for loop
That uses parallel computing. Try to find R functions where will be no loops.

- Can you try to use some variance reduction method(s) here?

```
##########     Start Control Variate Output     ##########
Asian Geometric mean call option as control variate.
Asian Geometric mean call option price =  2.529997
Exact Asian Geometric mean call option price =  2.5326
hat_beta =  1.222872
Jackpot Call Option Price =  4.544301
Jackpot Call option MCV Price =  4.547484
Estimated Fair Price 4.547623
error_sm =  0.003321922
error_mcv =  0.0001386456
##########      End Control Variate Output     ##########
```

```
##########     Start Control Variate Output     ##########
Asian Geometric mean call option as control variate.
Asian Geometric mean call option price =  2.391126
Exact Asian Geometric mean call option price =  2.396304
hat_beta =  0.8060615
Jackpot Call Option Price =  4.541659
Jackpot Call option MCV Price =  4.545833
Estimated Fair Price 4.547623
error_sm =  0.005963855
error_mcv =  0.001790233
##########      End Control Variate Output     ##########
```

- Any interesting result? Any doubt? Any suggestion? Additional comments?

No comments