Inna Williams
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Section 5.1
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5.1.1 Computer Problem Using Maple
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1. Make a table of the error of the three-point centered-difference formula for f(0), where f (x) = sin x - cosx, with h = 10-1, . . . ,10-12, as in the table in Section 5.1.2. Draw a plot of the results. Does the minimum error correspond to the theoretical expectation?
****

```
> with(student); with(MTM) :
```

[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope, summand, trapezoid] **(1)**

```
> X0 := double(0.0) : h_start := double(0.1) : N := 12 : a := double(0.1) :
> f := x→sin(x) − cos(x);
```

$$f := x \mapsto \sin(x) - \cos(x) \quad \textbf{(2)}$$

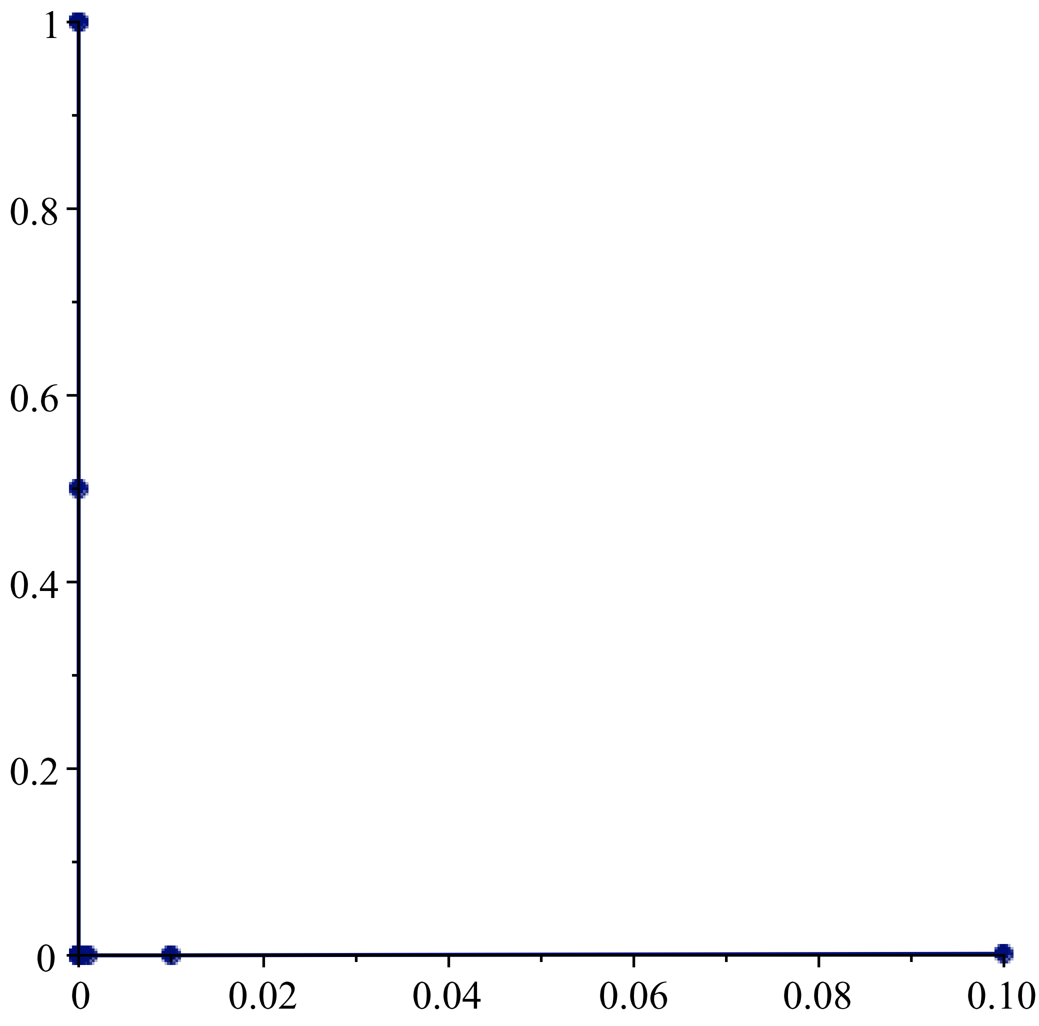
```
>
>
three_point_diff := proc (f, X0, h_start, N, a)
local i: L_h := [seq(xi, i = 1 .. N)]: L_e := [seq(xi, i = 1 .. N)];
    next_h:= h_start:
    three_pont_centered_diff := h->(double(f(X0+h))-double(f(X0-h)))
/double((2*h)):
    printf("\n          h                   value                 error\n" ) :

    printf("  ----------------------------------------------------\n")
:
          for i to N do:
          value1 := three_pont_centered_diff(next_h): L_h[i]:=next_h:
L_e[i] := abs(double((D(f))(0))- value1):
            printf("  %1.14f  %10.14f   %10.14f\n",L_h[i], value1, L_e
[i] ) :
            next_h:= next_h*a:
          end do:
   return (ListTools[Reverse](L_h),ListTools[Reverse](L_e));
end proc:
> L_h, L_e := three_point_diff(f, X0, h_start, N, a)
```

h	value	error
0.10000000000000	0.99833416700000	0.00166583300000
0.01000000000000	0.99998334500000	0.00001665500000
0.00100000000000	0.99999990000000	0.00000010000000
0.00010000000000	1.00000000000000	0.00000000000000
0.00001000000000	1.00000000000000	0.00000000000000
0.00000100000000	1.00000000000000	0.00000000000000
0.00000010000000	1.00000000000000	0.00000000000000
0.00000001000000	1.00000000000000	0.00000000000000

```
0.00000000100000  1.00000000000000   0.00000000000000
0.00000000010000  0.50000000000000   0.50000000000000
0.00000000001000  0.00000000000000   1.00000000000000
0.00000000000100  0.00000000000000   1.00000000000000
```

$L\_h, L\_e := [1.000000000\ 10^{-12}, 1.000000000\ 10^{-11}, 1.000000000\ 10^{-10}, 1.000000000\ 10^{-9}, 1.000000000\ 10^{-8}, 1.000000000\ 10^{-7}, 1.000000000\ 10^{-6}, 0.00001000000000, 0.0001000000000, 0.001000000000, 0.01000000000, 0.100000000000000006], [1., 1., 0.5000000000, 0., 0., 0., 0., 0., 0., 1.000\ 10^{-7}, 0.0000166550, 0.0016658330]$ **(3)**

> $dataplot(L\_h, L\_e)$



Answer:

Minimum Error occurs start when h = 0.0001, 0.00001, 0.000001, 0.0000001, 0.000000001

###

Section 5.2

###

5.2.1 check 1.b

****

1. Apply the composite Trapezoid Rule with m = 1, 2, and 4 panels to approximate the integral.

Compute the error by comparing with the exact value from calculus.
(and check your work with Maple);
****

> *restart*;

> *with*(*student*);

$[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare, distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox, middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope, summand, trapezoid]$ **(4)**

>

> $f := x \rightarrow \cos(x);\ y0 := 0;\ ym := \frac{\pi}{2};$

$$f := x \mapsto \cos(x)$$
$$y0 := 0$$
$$ym := \frac{\pi}{2} \quad \textbf{(5)}$$

> $theoretical\_value := \mathbf{evalf}\left(\int_{y0}^{ym} f(x)\, \mathrm{d}x\right)$

$$theoretical\_value := 1. \quad \textbf{(6)}$$

```
> approximation := proc (f, m, y0, ym, type)
   if type='trapezoid' then
     approximation := trapezoid(f(x), x=y0 .. ym, m);
  end if;
  if type='simpson' then
     approximation := simpson(f(x), x=y0 .. ym, 2· m);
  end if;
   approximation_value := evalf(approximation);
   appoximation_error := abs( theoretical_value − approximation_value);
  printf("\n       m          value        error\n");
  printf("  ---------------------------------------------------\n");
  printf("  %1.10f  %10.10f   %10.10f\n", m, approximation_value, appoximation_error)
  end proc:
```

>

>

>

> $m := 1;$ *approximation*(*f*, *m*, *y0*, *ym*,'*trapezoid*');

$$m := 1$$

```
       m             value          error
  ---------------------------------------------------
  1.0000000000   0.7853981635   0.2146018365
```

> $m := 2;$ *approximation*(*f*, *m*, *y0*, *ym*,'*trapezoid*');

$$m := 2$$

```
       m             value          error
```

```
--------------------------------------------------
2.0000000000  0.9480594490   0.0519405510
```

> $m := 4;\ approximation(f, m, y0, ym, 'trapezoid');$

$$m := 4$$

```
      m            value          error
--------------------------------------------------
4.0000000000  0.9871158012   0.0128841988
```

Answer: Maple output corresponds to the one calculated by hand

****

3. Apply the composite Simpson's Rule with m = 1, 2, and 4 panels to the integrals in Exercise 1, and report the errors. (and check your work with Maple);

> $m := 1;\ approximation(f, m, y0, ym, 'simpson');$

$$m := 1$$

```
      m            value          error
--------------------------------------------------
1.0000000000  1.0022798770   0.0022798770
```

> $m := 2;\ approximation(f, m, y0, ym, 'simpson');$

$$m := 2$$

```
      m            value          error
--------------------------------------------------
2.0000000000  1.0001345850   0.0001345850
```

> $m := 4;\ approximation(f, m, y0, ym, 'simpson');$

$$m := 4$$

```
      m            value          error
--------------------------------------------------
4.0000000000  1.0000082950   0.0000082950
```

Answer: **Answer: Maple output corresponds to the one calculated by hand**

****

5.2.3 Computer Problem

3. Use the composite Trapezoid Rule with m = 16 and 32 panels to approximate the definite integral.

> $f := x \rightarrow \exp(x^2);\ y0 := 0;\ ym := 1;$

$$f := x \mapsto e^{x^2}$$
$$y0 := 0$$
$$ym := 1 \qquad (7)$$

> $theoretical\_value := \mathbf{evalf}\left(\int_{y0}^{ym} f(x)\, \mathrm{d}x\right)$

$$theoretical\_value := 1.462651746 \qquad (8)$$

> $m := 16;\ approximation(f, m, y0, ym, 'trapezoid');$

$$m := 16$$

```
      m            value          error
--------------------------------------------------
16.0000000000  1.4644203100   0.0017685640
```

> $m := 32$; $approximation(f, m, y0, ym, 'trapezoid')$;

$$m := 32$$

```
       m            value         error
--------------------------------------------------
32.0000000000   1.4630941030   0.0004423570
```

Answer:

for m=16 integral value = 1.4644203100
for m=32 integral value = 1.4630941030

****

5.2.4

4. Apply the composite Simpson's Rule to the integrals of Computer Problem 3, using m = 16 and 32.

> $m := 16$; $approximation(f, m, y0, ym, 'simpson')$;

$$m := 16$$

```
       m            value         error
--------------------------------------------------
16.0000000000   1.4626520330   0.0000002870
```

> $m := 32$; $approximation(f, m, y0, ym, 'simpson')$;

$$m := 32$$

```
       m            value         error
--------------------------------------------------
32.0000000000   1.4626517640   0.0000000180
```

Answer:

for m=16 integral value = 1.4626520330
for m=32 integral value = 1.4626517640

we can see that simson rule has less error compare to the trapezoid rule

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