INNA WILLIAMS

0.2 | 1C | 79

$$\begin{array}{l} 79 : 2 = 39,\ \text{remainder } \boxed{1} \\ 39 : 2 = 19,\ \text{remainder } \boxed{1} \\ 19 : 2 = 9,\ \text{remainder } \boxed{1} \\ 9 : 2 = 4,\ \text{remainder } \boxed{1} \\ 4 : 2 = 2,\ \text{remainder } \boxed{0} \\ 2 : 2 = 1,\ \text{remainder } \boxed{0} \\ 1 : 2 = 0,\ \text{remainder } \boxed{1} \end{array}$$

Answer:

$(79)_{10} = (1001111)_2$

02 | 3d | 12.8

12 :

$$\begin{array}{l} 12 : 2 = 6,\ \text{remainder } \boxed{0} \\ 6 : 2 = 3,\ \text{remainder } \boxed{0} \\ 3 : 2 = 1,\ \text{remainder } \boxed{1} \\ 1 : 2 = 0,\ \text{remainder } \boxed{1} \end{array}$$

$(12)_{10} = (1100)_2$

0.8

$0.8 \cdot 2 = 1.6 = 0.6 + 1$

$0.6 \cdot 2 = 1.2 = 0.2 + 1$

$0.2 \cdot 2 = 0.4 \quad +0$

$0.4 \cdot 2 = 0.8 \quad +0$

$0.8 \cdot 2 = 1.6 = 0.6 + 1$

$0.8 \cdot 2 =$

$0.8 = .1100$

$\boxed{\text{Answer: } (12.8)_{10} = (1100.\overline{1100})_2}$

0.2 | 76 | 1011.101

$1011.101 = 2^3 \cdot 1 + 2^2 \cdot 0 + 2^1 \cdot 1 + 2^0 \cdot 1 + 2^{-1} \cdot 1 + 2^{-2} \cdot 0 + 2^{-3} \cdot 1 =$

$= 8 + 2 + 1 + \frac{1}{2} + \frac{1}{8} = 11\frac{5}{8} = \frac{93}{8} = 11.625$

Answer: $(1100.101)_2 = \left(\frac{93}{8}\right)_{10}$ or $(11.625)_{10}$

$\boxed{0.2 \mid 7c}$ $10111.\overline{01}$

$$10111.\overline{01} = (2^4 \cdot 1 + 2^3 \cdot 0 + 2^2 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1) + (0.\overline{01})_2$$

$$= 16+4+2+1+(0.\overline{01})_2 = 23+(0.\overline{01})_2$$

let $x = (0.\overline{01})_2$ $\quad 2^2 x - x = (01)_2$

$$(01)_2 = 1$$

$$2^2 x - x = 1$$

$$3x = 1 \Rightarrow x = \frac{1}{3}$$

$$(10111.\overline{01})_2 = 23\frac{1}{3} = \left(\frac{70}{3}\right)_{10}$$

$$\boxed{\text{Answer: } (10111.\overline{01})_2 = \left(\frac{70}{3}\right)_{10}}$$

$\boxed{04 \mid 1c}$ $\boxed{\frac{1}{1+x} - \frac{1}{1-x}}$

for x near 0 there is substraction of nearly equel numbers

$$\frac{1}{1+x} - \frac{1}{1-x} = \frac{1-x-1-x}{1-x^2} = \frac{-2x}{1-x^2} = \frac{2x}{x^2-1}$$

Answer: for x near 0 the alternative form will be $\boxed{\frac{2x}{x^2-1}}$

$\boxed{0.4 \mid 2}$ $x^2 + 3x - 8^{-14} = 0$

$$x_1 = \frac{1}{2}\left(-3 + \sqrt{9 + 4 \cdot 8^{-14}}\right)$$

$$x_2 = \frac{1}{2}\left(-3 - \sqrt{9 + 4 \cdot 8^{-14}}\right)$$

04.2 continue

$$x_2 = \frac{1}{2}\left(3 + \sqrt{9 + 4(2.27374 \cdot 10^{-13})}\right) = -3.00$$

for x_1 it will not work becaus it will result to 0 for 3 decimals

$$x_1 = \frac{1}{2}\left(-3 + \sqrt{9 + 4 \cdot 8^{-14}}\right) \times \frac{3 + \sqrt{9 + 4 \cdot 8^{-14}}}{3 + \sqrt{9 + 4 \cdot 8^{-14}}}$$

$$x_1 = \frac{9 + 4 \cdot 8^{-14} - 9}{2(3 + \sqrt{9 + 4 \cdot 8^{-14}})} = \frac{4 \cdot 8^{-14}}{2(3 + \sqrt{9 + 4 \cdot 8^{-14}})} =$$

$$= \frac{2 \cdot 8^{-14}}{3 + 3} \quad \frac{8^{-14}}{3} \quad \frac{2.27374 \cdot 10^{-13}}{3} = 7.58 \cdot 10^{-14}$$

Answer: $x_1 = 7.58 \cdot 10^{-14}$ $x_2 = -3.00$

MATH 485 INNA WILLIAMS

$\boxed{1.2}\ \boxed{2a}$

$$g(x) = \frac{x+6}{3x-2}$$

Solve $g(x) = x$ $\quad \frac{x+6}{3x-2} = x$

$$x+6 = x(3x-2)$$
$$x+6 = 3x^2 - 2x$$
$$3x^2 - 3x - 6 = 0$$
$$x^2 - x - 2 = 0$$
$$x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+2} = \frac{1}{2} \pm \sqrt{\frac{9}{4}} = \frac{1}{2} \pm \frac{3}{2}$$
$$x_1 = 2 \quad x_2 = -1$$

Answer: Fixed points are: $x_1 = 2$ $\quad x_2 = -1$

$\boxed{1.2 \mid 7}$

ⓐ $g(x) = (2x-1)^{1/3}$ $\quad r = 1$

$$g'(x) = \frac{2}{3}(2x-1)^{-2/3} = \frac{2}{3\sqrt[3]{(2x-1)^2}}$$

$$g'(1) = \frac{2}{3\sqrt[3]{(2 \cdot 1 - 1)^2}} = \frac{2}{3}$$

$|g'(1)| = \frac{2}{3} < 0 \Rightarrow$ that $g(x)$ is locally convergent to $r = 1$

ⓑ $g(x) = \frac{x^3+1}{2}$, $r = 1$

$$g'(x) = \frac{1}{2}(3x^2) = \frac{3}{2}x^2$$

$$g'(1) = \frac{3}{2}$$

$|g'(1)| = 1.5 > 1 \Rightarrow g(x)$ diverges from $r = 1$

$\boxed{1.2}\boxed{7}$ (c) $g(x) = \sin x + x$, $r = 0$

$g'(x) = \cos x + 1$

$|g'(0)| = |2| = 2 > 1 \Rightarrow$

$g(x)$ diverges from $r = 0$

$\boxed{1.2}\boxed{9}$ (a) $g(x) = \frac{1}{2}x^2 + \frac{1}{2}x$

Fixed points: $\frac{1}{2}x^2 + \frac{1}{2}x = x$

$\frac{1}{2}x^2 - \frac{1}{2}x = 0 \Rightarrow \frac{1}{2}x(x-1) = 0 \Rightarrow \boxed{r_1 = 0}\ \boxed{r_2 = 1}$

$g'(x) = \frac{2}{2}x + \frac{1}{2} = x + \frac{1}{2}$

$|g'(0)| = |0 + \frac{1}{2}| = \frac{1}{2} < 1 \Rightarrow$ $g(x)$ locally convergent to fixed point $r = 0$

$|g'(1)| = |1 + \frac{1}{2}| = |\frac{3}{2}| > 1 \Rightarrow g(x)$ diverges from $r = 1$

(b) $g(x) = x^2 - \frac{1}{4}x + \frac{3}{8}$

Fixed points: $x^2 - \frac{1}{4}x + \frac{3}{8} = x \Rightarrow 8x^2 - 10x + 3 = 0$

$x = \frac{5 \pm \sqrt{25 - 24}}{8} = \frac{5 \pm 1}{8} \Rightarrow \boxed{r_1 = \frac{3}{4}}\ \boxed{r_2 = \frac{1}{2}}$

$g'(x) = 2x - \frac{1}{4}$

$|g'(\frac{1}{2})| = |2 \cdot \frac{1}{2} - \frac{1}{4}| = \frac{3}{4} < 1 \Rightarrow$ $g(x)$ locally convergent to fixed point $r = \frac{1}{2}$

$|g'(\frac{3}{4})| = |2 \cdot \frac{3}{4} - \frac{1}{4}| = \frac{5}{4} > 1 \Rightarrow$ $g(x)$ diverges from fixed point $r = \frac{3}{4}$

Math 485 INNA WILLIAMS

$\boxed{1.2/12}$ $g(x) = x^2 - 0.24$

(a) $r = -0.2$

$g'(x) = 2x$

$|g'(-0.2)| = |2 \cdot (-0.2)| = 0.4 < 1$

rate of convergence $S = 0.4$ for FPI

Bisection method rate $= 0.5$

$FPI = 0.4 < 0.5 \Rightarrow$ FPI faster for $g(x)$

(b) Fixed points:

$x^2 - 0.24 = x$

$x^2 - x - 0.24 = 0$

$r = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{24}{100}} = \frac{1}{2} \pm \frac{7}{10} = 0.5 \pm 0.7$

$r_1 = 1.2 \quad r_2 = -0.2$

$|g'(1.2)| = |2 \cdot 1.2| = |2.4| = 2.4 > 1 \Rightarrow$ diverges from fixed point $r = 1.2$

Answer: Fixed points: $r_1 = 1.2$ $r_2 = -0.2$

as it was calculated in (a) $g(x)$ locally converges to fixed point $r_2 = -0.2$

$g(x)$ diverges from fixed point $r = 1.2$

$\boxed{1.2 \mid 15}$ $\quad r = \sqrt{5}$

Ⓐ $g(x) = \frac{4}{5}x + \frac{1}{x}$

$g'(x) = \frac{4}{5} - \frac{1}{x^2}$

$|g'(\sqrt{5})| = \frac{4}{5} - \frac{1}{5} = \frac{3}{5} = 0.6$

Ⓑ $g(x) = \frac{x}{2} + \frac{5}{2x}$

$g'(x) = \frac{1}{2} - \frac{5}{2x^2}$

$|g'(\sqrt{5})| = 0$

Ⓒ $g(x) = \frac{x+5}{x+1}$

$g'(x) = \frac{1}{x+1} - \frac{x+5}{(x+1)^2} = -\frac{4}{(x+1)^2}$

$|g'(\sqrt{5})| = \frac{4}{(\sqrt{5}+1)^2} = 0.382$

$0 < 0.382 < 0.6$

Answer :

Fastest		slowest
B	C	A
0	0.382	0.6

$\boxed{1.2 \mid 9}$ $g(x) = \frac{1}{2}\left(x + \frac{A}{x^2}\right) = \frac{1}{2}\left(x + A\left(x^{-2}\right)\right)$

$g'(x) = \frac{1}{2}\left(1 + (-2)A \cdot x^{-3}\right) = \frac{1}{2} - A \cdot x^{-3}$

$|g'(A^{1/3})| = \left|\frac{1}{2} - A \cdot (A^{1/3})^{-3}\right| = \left|\frac{1}{2} - A \cdot A^{-1}\right|$

$|g'(A^{1/3})| = \left|\frac{1}{2} - 1\right| = \left|\frac{-1}{2}\right| = \frac{1}{2} < 1 \Rightarrow$ FPI is locally convergent to $A^{1/3}$ with rate of convergence $\boxed{S = 0.5}$