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##

Section 1.3

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Problem 5

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5. Use (1.21) to approximate the root of f (x) = (x - 1)(x - 2)(x - 3)(x - 4) - $10^{-6} \cdot x^6$
near r = 4.
Find the error magnification factor. Use fzero to check your approximation.

$$\frac{x^6}{1000000} \quad \textbf{(1)}$$

> $f := x \rightarrow (x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4)$

$$f := x \mapsto (x-1)\,(x-2)\,(x-3)\,(x-4) \quad \textbf{(2)}$$

> $r := 4$

$$r := 4 \quad \textbf{(3)}$$

> $g := x \rightarrow x^6$

$$g := x \mapsto x^6 \quad \textbf{(4)}$$

> $e := -10^{-6}$

$$e := -\frac{1}{1000000} \quad \textbf{(5)}$$

using 1.21
$\Delta r \approx -\frac{\epsilon g(r)}{f'(r)}$, in out case r = 4

$$0.0006826666667 \approx \left(-\frac{\epsilon g(4)}{6}\right) \quad \textbf{(6)}$$

> $\Delta r := evalf\left(-\frac{e \cdot g(r)}{\mathrm{D}(f)\,(r)}\right)$

$$\Delta r := 0.0006826666667 \quad \textbf{(7)}$$

from 1.22
error magnification factor = $\left|\frac{\frac{\Delta r}{r}}{\frac{e \cdot g(r)}{g(r)}}\right| = \left|-\frac{\frac{e \cdot g(r)}{(r - f'(r))}}{e}\right| = \frac{|g(r)|}{|r \cdot f'(r)|}$

$$false \quad \textbf{(8)}$$

> $error\_magnification\_factor := evalf\left(\frac{\mathrm{abs}(g(r))}{\mathrm{abs}(r \cdot \mathrm{D}(f)\,(r))}\right)$

$$error\_magnification\_factor := 170.6666667 \quad \textbf{(9)}$$

error magnification factor =170.7 = 1.707 * 10^2.

The error modification factor of 10^2 tell us that we are going to loose 2 significan digit from input to output.

The estimated change in root : $\Delta r := 0.00068267$ **and** *actual root* $= 4 + 0.00068267$
guess acording to the 1.21 :
actual root = 4.00068267

```
> actual_root := fsolve(f(x)+e*g(x) = 0, x)
```

$$actual\_root := -1004.980173, 0.9999998333, 2.000032004, 2.999635699, 4.000682512, 994.9798230 \quad \textbf{(10)}$$

We can see that fsolve gives us the root 4.00068251
that is close to the guess acording to the 1.21 = 4.00068267 ***with 2 significan digit***
###

Section 1.4

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Problem 1

###

1. Each equation has one root. Use Newton's Method to approximate the root to eight correct decimal places.

(a) x3 = 2x + 2

```
  newton:=proc(f,X0,TOL,N)
           local i,x0;x0:=X0;
           i:=1;
           while i<=N do
                if D(f)(p0)=0 then
                   printf("Division by 0. Method failed");break;
                else
                   r:=x0-f(x0)/D(f)(x0);
                   printf("Iteration %d: %.8g\n",i,r);
                   if abs(r-x0)<TOL then
                      printf("r=%.8g, f(r)=%.8g\n",r,f(r));
                      printf("Number of iterations needed: %d",i);
return();
                   break;
                   end if;
                   i:=i+1;x0:=r;
                end if;
           end do;
           printf("The method failed after %d iterations.\n",N);
           printf("r=%.8g, f(r)=%.8g\n",r,f(r));
           return();
        end proc:
```

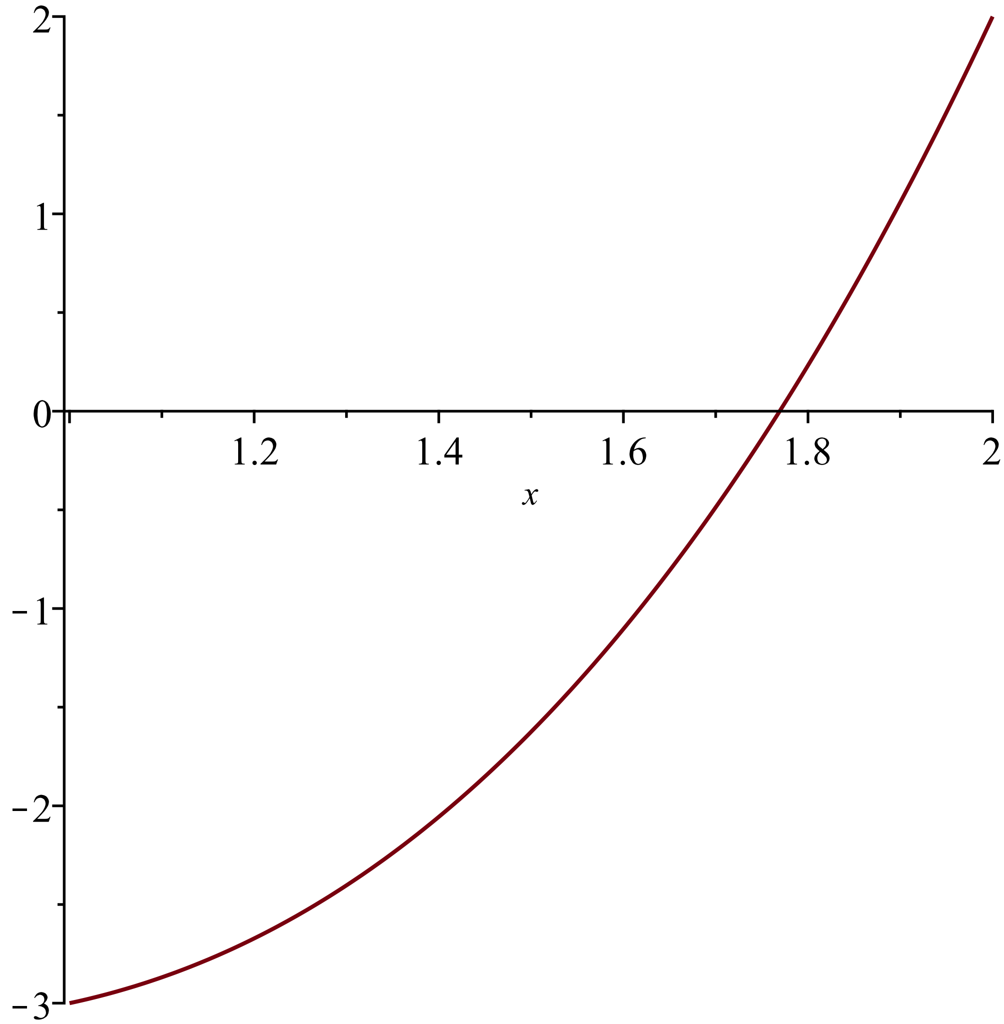
> $f := x \to x^3 - 2x - 2$

$$f := x \mapsto x^3 - 2x - 2 \quad \textbf{(11)}$$

f(0) = -2

f(1) = -3
f(2)=2
set x0 = 1

> $plot(f(x), x = 1..2)$



> $newton\left(f, 1.0, 0.5\cdot 10^{-8}, 50\right)$

```
Iteration 1: 4
Iteration 2: 2.826087
Iteration 3: 2.146719
Iteration 4: 1.8423263
Iteration 5: 1.7728476
Iteration 6: 1.7693014
Iteration 7: 1.7692924
Iteration 8: 1.7692924
r=1.7692924, f(r)=-2e-09
Number of iterations needed: 8
```

> $with(Student[Calculus1]):$
$NewtonsMethod(f(x), x = 1, iterations = 8, output = sequence);$

1, 4.000000000, 2.826086957, 2.146719014, 1.842326277, 1.772847636, 1.769301398, 1.769292354, 1.769292354 **(12)**

Answer : root = 1.76929235 to *eight decimal places*

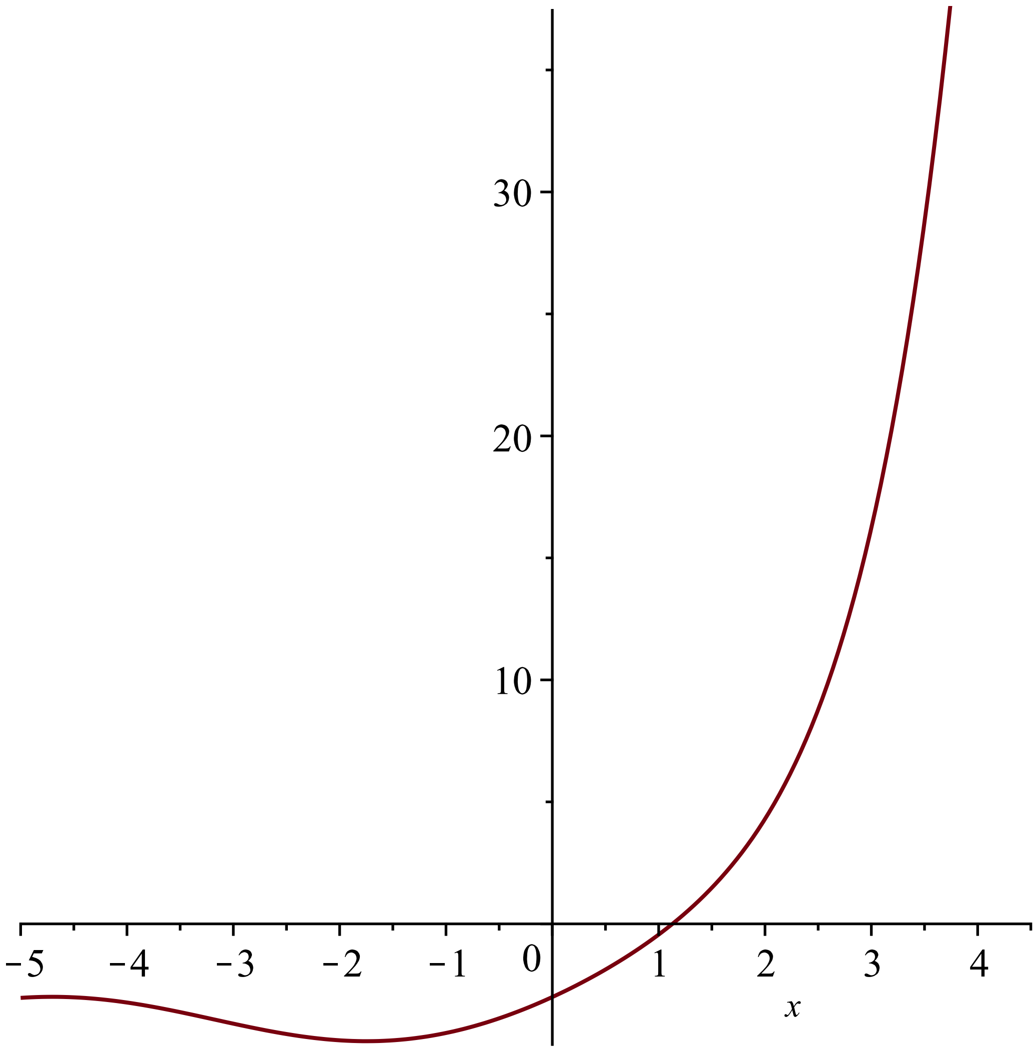
(c) e^x + sin x = 4

f(x)=e^x + sinx -4 0
initial guess x0=1
e^1 +sin1-4 = -1.264

> f := x→exp(x) + sin(x) − 4

$$f := x \mapsto e^{x} + \sin(x) - 4 \quad \textbf{(13)}$$

> plot(f(x), x=-5..5.0)



> newton(f, 1.0, 0.5·10^−8, 50)

```
Iteration 1: 1.1351038
Iteration 2: 1.1299887
Iteration 3: 1.1299805
Iteration 4: 1.1299805
r=1.1299805, f(r)=1e-09
Number of iterations needed: 4
```

> NewtonsMethod(f(x), x = 1, iterations = 8, output = sequence);

1, 1.135103827, 1.129988671, 1.129980498, 1.129980499, 1.129980499, 1.129980499, 1.129980499, 1.129980499 **(14)**

Answer: root = 1.12998050 to *eight decimal places*

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Problem 6

6. A10-cm-high cone contains 60 cm3 of ice cream, including a hemispherical scoop on top. Find the radius of the scoop to four correct decimal places.
V=(1/3)*h*Pi*r^2 + (2/3)*Pi*r^3
or f(r) = (1/3)*h*Pi*r^2 + (2/3)*Pi*r^3 - V = 0

> $h := 10$

$$h := 10 \quad (15)$$

> $V := 60$

$$V := 60 \quad (16)$$

> $f := x \to \left(\frac{1}{3}\right) \cdot h \cdot \pi \cdot x^2 + \left(\frac{2}{3}\right) \cdot \pi \cdot x^3 - V$

$$f := x \mapsto \frac{1}{3} h \pi x^2 + \frac{2}{3} \pi x^3 - V \quad (17)$$

> $NewtonsMethod(f(x), x = 1, iterations = 4, output = sequence)$

$$1, 2.742145365, 2.150541294, 2.025551307, 2.020117397 \quad (18)$$

Check if correct:

> $r := 2.0201$

$$r := 2.0201 \quad (19)$$

> $Volume := \left(\left(\frac{1}{3}\right) \cdot h \cdot \pi \cdot r^2 + \left(\frac{2}{3}\right) \cdot \pi \cdot r^3\right)$

$$Volume := 59.99950254 \quad (20)$$

Answer: Newton's Method gives in 1 interation answer.
The radius of scoop to 4 correct digital places = 2.0201 *cm*
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Problem 7

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7. Consider the function f (x) = $e^{\sin^3(x)} + x^6 - 2 \cdot x^4 - x^3 - 1$ on the interval [-2,2]. Plot the function on the interval, and find all three roots to six correct decimal places. Determine which roots converge quadratically, and find the multiplicity of the roots that converge linearly.

First guess root_0 = 0
we can see that f(0) = 0 , m = 0
f '(x) = $e^{\sin^3(x)} \cdot 3 \cdot \sin^2(x) \cdot \cos(x) + 6 \cdot x - 8 \cdot x^3 - 3 \cdot x^2$, f '(0) = 0 , $m = 1$
...

****

> $f := x \to \exp((\sin(x))^3) + x^6 - 2 \cdot x^4 - x^3 - 1$

$$f := x \mapsto e^{\sin(x)^3} + x^6 - 2 x^4 - x^3 - 1 \quad (21)$$

> $r := 0$

(22)

$$r := 0 \tag{22}$$

> $m0 := f(r)$

$$m0 := 0 \tag{23}$$

> $m1 := evalf(\mathrm{D}(f)(r))$

$$m1 := 0. \tag{24}$$

> $m2 := evalf(\mathrm{D}(\mathrm{D}(f))(r))$

$$m2 := 0. \tag{25}$$

> $m3 := evalf(\mathrm{D}(\mathrm{D}(\mathrm{D}(f)))(r))$

$$m3 := 0. \tag{26}$$

> $m4 := evalf(\mathrm{D}(\mathrm{D}(\mathrm{D}(\mathrm{D}(f))))(r))$

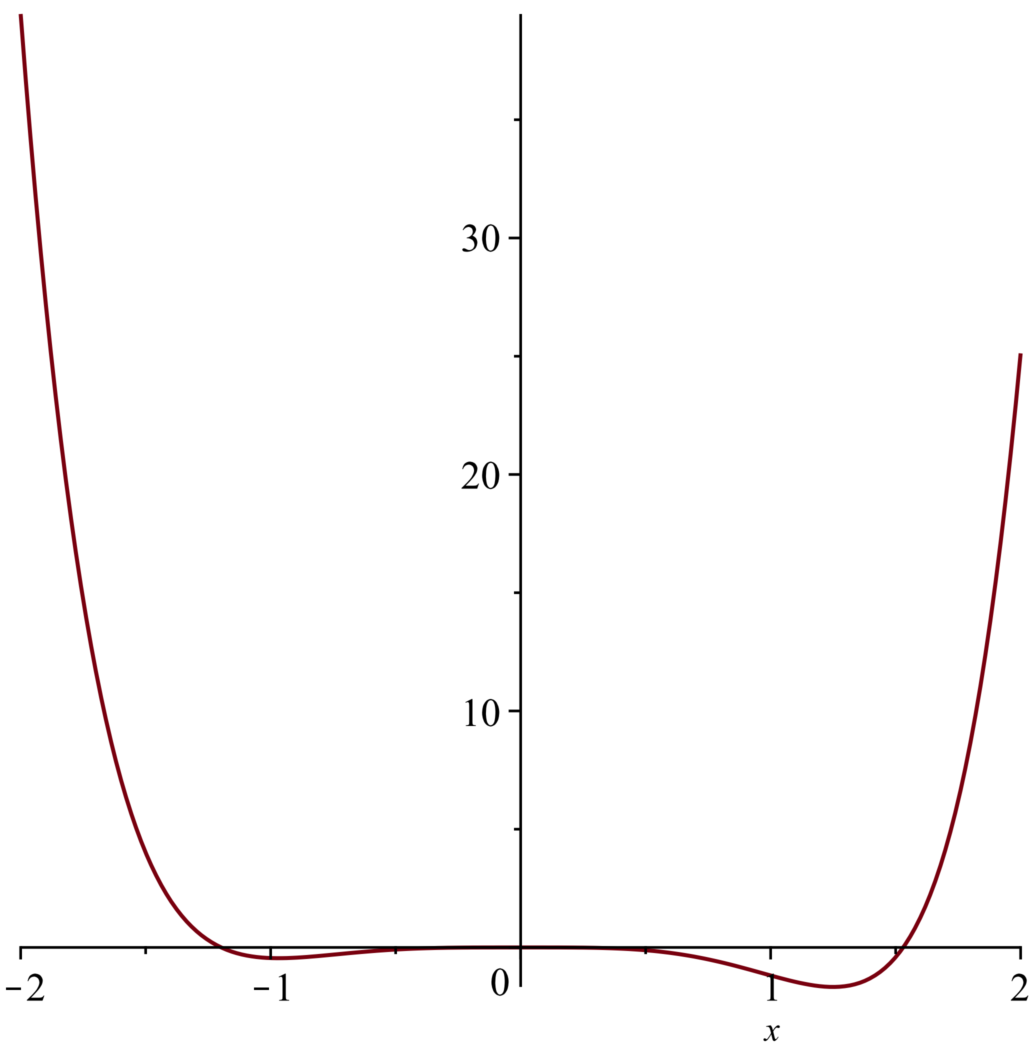
$$m4 := -48. \tag{27}$$

We can see that multiplicity of root = 0 equal m = 4

$$\boldsymbol{limit\left(\frac{(ei+1)}{ei}, i \to \text{infinity}\right) = \frac{(m-1)}{m} = \frac{(4-1)}{4} = \frac{3}{4}}$$

The root=0 converges liearly with rate of convergence S=3/4

> $plot(f(x), x = -2..2)$



The second root_1 = initial guess between (-1.5, -1) root_1 = -1.25

```
> root_1:= NewtonsMethod(f(x), x = -1.25, iterations = 4, output =
  sequence)
```

$$root\_1 := -1.25, -1.205716991, -1.197852817, -1.197623912, -1.197623722 \quad (28)$$

> $newton(f, -1.25, 0.5\cdot 10^{-6}, 50)$

```
Iteration 1: -1.205717
Iteration 2: -1.1978528
Iteration 3: -1.1976239
Iteration 4: -1.1976237
r=-1.1976237, f(r)=-2e-09
Number of iterations needed: 4
```

Newton method

root_1 = -1.197624 to 6 *decimal places*

The following comands will find multiplicity of root root_1=-**1.197624**

```
> r := -1.197624
```

$$r := -1.197624 \quad (29)$$

> $m0 := f(r)$

$$m0 := 1.367\ 10^{-6} \quad (30)$$

we can see that m0 ~ 0
let us find m1

> $m1 := evalf(D(f)(r))$

$$m1 := -4.920586765 \quad (31)$$

We can see that m1 !=0 therefore Newton's method convergers quadratically to root_1 = −1.197624

The third root_2 = initial guess between (1, 2) root_2 = 1.5

$$-1.197624 \quad (32)$$

> $root\_2 := NewtonsMethod(f(x), x = 1.5, iterations = 4, output = sequence)$

$$root\_2 := 1.5, 1.533224676, 1.530162240, 1.530133511, 1.530133508 \quad (33)$$

> $newton(f, 1.5, 0.5\cdot 10^{-6}, 50)$

```
Iteration 1: 1.5332247
Iteration 2: 1.5301622
Iteration 3: 1.5301335
Iteration 4: 1.5301335
r=1.5301335, f(r)=2e-09
Number of iterations needed: 4
```

Newton method

root_2 = 1.530134 **to 6 *decimal places***

The following comands will find multiplicity of root root_2 = 1.530134

> $r := 1.530134$

$$r := 1.530134 \quad (34)$$

> $m0 := f(r)$

$$m0 := 7.356\ 10^{-6} \quad (35)$$

we can see that m0 ~ 0
let us find m1

> $m1 := evalf(\mathrm{D}(f)(r))$

$$m1 := 14.97277594 \tag{36}$$

We can see that m1 !=0 therefore Newton's method convergers quadratically to **root_2 = 1.530134**

Answer:

3 roots are found :

root_0 = 0, converges linearly with rate = 3/4
root_1 = -1.197624, converges quadratically
root_2 = 1.530134, converges quadratically

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