INNA WILLIAMS

Section $\boxed{11.4}$

$\boxed{16}$ $x^4 - x^2 + x - 1 = 0$, $x_0 = 0$, $f'(x) = 4x^3 - 2x + 1$

$$y - f(x_0) = f'(x_0)(x - x_0), \quad \text{set } y = 0 \Rightarrow$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Step 1: $f(0) = 0 - 0 + 0 - 1 = -1$

$x_i = x_0 = 0$, $f'(0) = 0 - 0 + 1 = 1$

$$x_1 = x_0 - \frac{f(0)}{f'(0)} = 0 - \frac{-1}{1} = 1 \qquad \boxed{x_1 = 1}$$

Step 2: $f(x_1) = f(1) = 1 - 1 + 1 - 1 = 0$

$f'(x_1) = f'(1) = 4 - 2 + 1 = 3$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{0}{3} = 1 \qquad \boxed{x_2 = 1}$$

Answer: $\boxed{x_1 = 1, \quad x_2 = 1}$

$\boxed{1.4.3\,b}$ Section 1.4

$\boxed{3b}$

$f(x): 2x^4-5x^3+3x^2+x-1=0$, $r_1=-\frac{1}{2}$, $r_2=1$

$\boxed{r_2=1}$

$f(1)=2-5+3+1-1=0$ $m=0$

$f'(x)=8x^3-15x^2+6x+1$ $f'(1)=8-15+6+1=0$, $m=1$

$f''(x)=24x^2-30x+6$ $f''(1)=24-30+6=0$, $m=2$

$f'''(x)=48x-30$ $f'''(1)=48-30=18\neq 0$, $m=3$

Theorem 1.12

$$\lim_{i\to\infty}\frac{e_{i+1}}{e_i}=\frac{m-1}{m}$$

in our case $m=3$ $\Rightarrow \lim_{i\to\infty}\frac{e_{i+1}}{e_i}=\frac{3-1}{3}=\frac{2}{3}$

$\Rightarrow$ The convergence is linear $\boxed{e_{i+1}\approx\frac{2}{3}e_i}$

$r_1=-\frac{1}{2}$

$f'(-\frac{1}{2})=-\frac{27}{4}\neq 0$ $f''(-\frac{1}{2})=6+15+6=27$

$$\lim_{i\to\infty}\frac{e_{i+1}}{e_i^2}=\left|\frac{f''(r)}{2f'(r)}\right|=\left|\frac{27}{2\cdot-\frac{27}{4}}\right|=+2 \Rightarrow \boxed{e_{i+1}=e_i^2\cdot 2}$$

$\Rightarrow$ convergence for root $r=-\frac{1}{2}$ is quadratic

INNA WILLIAMS

Section $\boxed{1.4}$

$\boxed{7}$ $x^4 - 7x^3 + 18x^2 - 20x + 8 = 0$, $r = 2$

$f(2) = 16 - 56 + 72 - 40 + 8 = 0$, $m = 0$

$f'(x) = 4x^3 - 21 \cdot x^2 + 36x - 20$ $f'(2) = 32 - 84 + 72 - 20 = 0$, $m = 1$

$f''(x) = 12x^2 - 42x + 36$ $f''(2) = 48 - 84 + 36 = 0$, $m = 2$

$f'''(x) = 24x - 42$ $f'''(2) = 48 - 42 = 6 \neq 0$, $m = 3$

we have $m = 2$ then Theorem 1.11 applies with $m = 3$.

$$\lim_{i \to \infty} \frac{e_{i+1}}{e_i} = \frac{m-1}{m} = \frac{3-1}{3} = \frac{2}{3}$$

Answer: $r = 2$ is a triple root ($m = 3$) therefore Newton's method does not converge quadratically

$\boxed{\lim_{i \to \infty} \frac{e_{i+1}}{e_i} = \frac{2}{3}}$, therefore accordin Theorem 1.12 it converges lineerly

Scanned by CamScanner

Section 1.4.

$\boxed{11}$ $A^{\frac{1}{n}} = X$

$A = X^n$

$\boxed{f(x) = x^n - A = 0}$, $f(A^{1/n}) = (A^{1/n})^n - A = 0$

$f'(x) = n \cdot x^{n-1}$ $\qquad f'(A^{1/n}) = n \cdot (A^{1/n})^{n-1} = n \cdot A^{\frac{n-1}{n}} \neq 0$ if

Theorem 1.11 applies if $A \neq 0$, $n \neq 0$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{x_i^n - A}{n \cdot x_i^{n-1}} = x_i - \frac{\left(1 - \frac{A}{x_i}\right)}{n} x_i$$

$$= \frac{x_i \cdot n}{n} - \frac{x_i - \frac{A}{x_i^{n-1}}}{n} = \frac{x_i(n-1) - \frac{A}{x_i^{n-1}}}{n}$$

Answer: $\boxed{x_{i+1} = \frac{x_i(n-1) - \frac{A}{x_i^{n+1}}}{n}}$.

$f'(A^{\frac{1}{n}}) = n \cdot A^{\frac{n-1}{n}} \neq 0$ if $n \neq 0$, $A \neq 0$ and therefore Newton's Method converges quadratically ~~#~~ for

MATH 485 HW3 INNA WILLIAMS

$\boxed{\text{Section 1.3}}$

1/ (a) $r = \frac{3}{4}$ $x_a = 0.74$ $f(x) = 4x-3$

$\boxed{FE} = |r - x_c| = |\frac{3}{4} - 0.74| = \boxed{0.01}$

$\boxed{BE} = |f(x_c)| = |4 \cdot 0.74 - 3| = \boxed{0.04}$

(b) $r = \frac{3}{4}$ $x_a = 0.74$ $f(x) = (4x-3)^2$

$\boxed{FE} = |r - x_c| = |\frac{3}{4} - 0.74| = \boxed{0.01}$

$\boxed{BE} = |f(x_c)| = |(4 \cdot 0.74 - 3)^2| = \boxed{0.0016}$

(c) $r = \frac{3}{4}$ $x_a = 0.74$ $f(x) = (4x-3)^3$

$\boxed{FE} = |r - x_c| = |\frac{3}{4} - 0.74| = \boxed{0.01}$

$\boxed{BE} = |f(x_c)| = |(4 \cdot 0.74 - 3)^3| = \boxed{0.000064}$

(d) $r = \frac{3}{4}$ $x_a = 0.74$ $f(x) = (4x-3)^{1/3}$

$\boxed{FE} = |r - x_c| = |\frac{3}{4} - 0.74| = \boxed{0.01}$

$\boxed{BE} = |f(x_c)| = |(4 - 0.74 - 3)^{1/3}| = 0.34199$

$\approx \boxed{0.342}$

Answer: Forward error for a), b) c) and d) is the same and equal $\boxed{0.01}$

Backward error: (a) 0.04
(b) 0.0016
(c) 0.000064
(d) 0.342

Section 1.3

$\boxed{3}$ $r=0$ $f(x)=1-\cos x$ $x_a=0.0001=10^{-4}$

(a) Find multiplicity of root $r=0$

$f(0)=1-\cos 0=1-1=0$ $m=0$

$f'(x)=\sin x$ $f'(0)=\sin 0=0$ $m=1$

$f''(x)=\cos x$ $f''(0)=\cos 0=1$ $\boxed{m=2}$

we can see that the multiplicity of $r=0$ equal $\boxed{2}$

(b) $\boxed{FE}=|r-x_c|=|0-10^{-4}|=\boxed{0.0001}$

$\boxed{BE}=|f(x_c)|=|1-\cos(10^{-4})|=$

find Taylor approximation

$f'''(x)=-\sin x$ $f'''(0)=0$

$f^{IV}(x)=-\cos x$ $f^{IV}(0)=-1$

$1-\cos x=0+0+\frac{(x-0)^2}{2!}-\frac{(x-0)^3}{3!}\cdot 0+\frac{(x-0)^4}{4!}\cdot(-1)$

$1-\cos x=\frac{x^2}{2!}-\frac{x^4}{4!}$

$f(10^{-4})=\frac{(10^{-4})^2}{2}-\frac{(10^{-4})^4}{24}=0.5\cdot 10^{-8}-0.042\cdot 10^{-16}$

$\approx f(10^{-4})=0.5\cdot 10^{-8}=\boxed{5\cdot 10^{-9}}$

from $P_2(x)$

Answer: (a) $\boxed{\text{Multiplicity}=2}$

(b) $\boxed{\text{Forward Error}=0.0001}$ $\boxed{\text{Backward Error}=5\cdot 10^{-9}}$

Scanned by CamScanner

MATH 485 HW3 INNA WILLIAMS

$\boxed{\text{Section 1.3}}$

$\boxed{8}$ $f(x) = x^n - a x^{n-1}$, $g(x) = x^n$

ⓐ Use sensitivity formula to give prediction to $f_\varepsilon(x) = x^n - a\cdot x^{n-1} + \varepsilon x^n$ for small ε

$x^n - a\cdot x^{n-1} = 0$

$x^n(1 - a\cdot x^{-1}) = 0$, $x^n \neq 0 \Rightarrow$

$1 - a\cdot x^{-1} = 0$

$1 = \frac{a}{x} \Rightarrow x = a$, $\Rightarrow$ root $\boxed{r = a}$ is non zero root

By sensitivity formula $f_\varepsilon(x) = f(x) + \varepsilon \cdot g(x)$

$r \to f(x)$, $r + \Delta r \to f(x) + \varepsilon g(x)$ and

if $\varepsilon << f'(r)$, then $\Delta r = -\frac{\varepsilon g(r)}{f'(r)}$

$$\left\{\begin{array}{l} \Delta r = -\frac{\varepsilon \cdot g(r)}{f'(r)} \\ r = a \\ g(r) = g(a) = a^n \\ f'(x) = n\cdot x^{n-1} - a(n-1)\cdot x^{n-2} \\ f'(r=a) = n\cdot a^{n-1} - a(n-1)\cdot a^{n-2} \end{array}\right\} \Rightarrow$$

$$\Delta r = -\frac{\varepsilon\cdot g(r)}{f'(r)} = -\frac{\varepsilon \cdot a^n}{n\cdot a^n \cdot a^{-1} - a\cdot n \cdot a^{n-2} + a\cdot a^{n-2}}$$

$$= \frac{-\varepsilon a^n}{n\cdot a^n\cdot a^{-1} - \cancel{n\cdot a^{n-1}} + a^{n-1}} = \frac{-\varepsilon a^n}{a^n \cdot a^{-1}} = \boxed{-\varepsilon a}$$

$r_\varepsilon = r + \Delta r = a - \varepsilon a = a(1-\varepsilon)$

Answer: Sensitivity formula gives the prediction for nonzero root $(r=a)$ equal to $\boxed{r_\varepsilon = a(1-\varepsilon)}$

$\boxed{6}$ Find nonzero root and compare with prediction $\boxed{r_\varepsilon = a(1-\varepsilon)}$

$f_\varepsilon(x) = x^n - a x^{n-1} + \varepsilon \cdot x^n$ for small ε

$f_\varepsilon(x) = (1+\varepsilon)\cdot x^n - a x^{n-1}$

$f_\varepsilon(r_\varepsilon) = (1+\varepsilon)\cdot r_\varepsilon^{\,n} - a \cdot r_\varepsilon^{\,n-1} = 0$

$r_\varepsilon^{\,n}\left[1+\varepsilon - a\cdot r_\varepsilon^{-1}\right] = 0$

$r_\varepsilon \,!= 0$, the $1+\varepsilon - \frac{a}{r_\varepsilon} = 0 \Rightarrow$

$1+\varepsilon = \frac{a}{r_\varepsilon} \Rightarrow \boxed{r_\varepsilon = \frac{a}{1+\varepsilon}}$ → nonzero root

Let compare it with sensitivity formula approximation

$\Delta E = \frac{a}{1+\varepsilon} - a(1-\varepsilon) = \frac{a}{1+\varepsilon}\left(1-(1-\varepsilon^2)\right) = \frac{a\varepsilon^2}{1+\varepsilon}$

$\Delta E = \frac{a\,\varepsilon^2}{1+\varepsilon}$

if we find Taylor approximation for $r_\varepsilon = \frac{a}{1+\varepsilon}$; for $x_0 = 0$

$f(0) = a$

$f'(\varepsilon) = -(1+\varepsilon)^{-2} \cdot a \qquad f'(0) = -a$

$f''(\varepsilon) = 2(1+\varepsilon)^{-3} \cdot a \qquad f''(0) = 2a$

$f'''(\varepsilon) = -6(1+\varepsilon)^{-4} a \qquad f'''(0) = -6a$

$f^{IV}(\varepsilon) = 24(1+\varepsilon)^{-5} \cdot a \qquad f^{IV}(0) = 24a$

$$\frac{a}{1+\varepsilon} = a - \frac{a(\varepsilon-0)^1}{1!} + \frac{2a(\varepsilon-0)^2}{2!} - \frac{6a(\varepsilon-0)^3}{3!} + \frac{24a(\varepsilon-0)^4}{4!}$$

$$\frac{a}{1+\varepsilon} = \underbrace{a - a\varepsilon}_{} + a\varepsilon^2 - a\varepsilon^3 + a\varepsilon^4 + \ldots$$

$\parallel \Rightarrow$ to sensitivity formula approximation we calculated in $\boxed{a}$

$\boxed{r_\varepsilon = a(1-\varepsilon)}$ we can conclude that sensitivity formula is a first order approximation for the root

$\boxed{r_\varepsilon = \frac{a}{1-\varepsilon}}$