INNA WILLIAMS

Section 5.1.

$\boxed{3}$ $f'(\frac{\pi}{3})$ - ? $f(x) = \sin x$, $f'(x) = \cos x$, $f''(x) = -\sin x$

(a) $h = 0.1$

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2} \cdot f''(c) \qquad \frac{\pi}{3} < c < \frac{\pi}{3} + 0.1$$

$$f'(\frac{\pi}{3}) = \frac{\sin(\frac{\pi}{3} + 0.1) - \sin\frac{\pi}{3}}{0.1} = \frac{0.9116156 - 0.8660254}{0.1} = \boxed{0.4559019}$$

$|\text{approximation error}| = |0.4559019 - \cos\frac{\pi}{3}| = \boxed{0.044098}$

$\frac{0.1}{2} \cdot \sin(\frac{\pi}{3}) < |\text{error bounds}| < |\frac{0.1}{2} \cdot \sin(\frac{\pi}{3} + 0.1)|$

$0.04330127 < |\text{error bounds}| < 0.04558078 \Rightarrow$

$\boxed{0.04330 < 0.044098 < 0.0456}$ → containing actual approximation error as expected

Answer: $f'(\frac{\pi}{3}) = 0.4559019$

app. error $= 0.044098$

error bounds: $[0.04330, 0.0456]$

(b) $h = 0.01$

$$f'(\frac{\pi}{3}) = \frac{\sin(\frac{\pi}{3} + 0.01) - \sin\frac{\pi}{3}}{0.01} = 0.4956616 \approx 0.495662$$

$|\text{approximation error}| = |0.495662 - \cos\frac{\pi}{3}| = 0.0043384$

$|\frac{0.01}{2} \cdot \sin\frac{\pi}{3}| < |\text{error bounds}| < |\frac{0.01}{2} \cdot \sin(\frac{\pi}{3} + 0.01)|$

$0.004330127 < |\text{error bounds}| < 0.00435491$

$0.004330 < |\text{error bounds}| < 0.004355$

$0.004330 < 0.0043384 < 0.004355 \rightarrow$

containing actual approximation error as expected

Answer: $f'(\frac{\pi}{3}) = 0.495662$

app error $= 0.0043384$

error bounds: $[0.004330, 0.004355]$

(c) $h = 0.001$

$$f'(\frac{\pi}{3}) = \frac{\sin(\frac{\pi}{3}+0.001) - \sin\frac{\pi}{3}}{0.001} = 0.4995665 \approx 0.499567$$

$|\text{approximation error}| = |0.499567 - \cos\frac{\pi}{3}| = 0.0004331$

$$\left|\frac{0.001}{2} \cdot \sin\frac{\pi}{3}\right| < |\text{error bounds}| < \left|\frac{0.001}{2} \sin(\frac{\pi}{3}+0.001)\right|$$

$0.0004330127 < |\text{error bounds}| < 0.0004332625$

$0.0004330 < |\text{error bounds}| < 0.0004333$

$0.0004330 < 0.0004331 < 0.0004333 \rightarrow$

containing actual approximation error as expected

Answer: $f'(\frac{\pi}{3}) = 0.499567$

app. error $= 0.0004331$

error bounds: $[0.0004330, 0.0004333]$

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Section 5.1

$\boxed{4}$ Three point centered-difference formula

$f'(\frac{\pi}{3})$ - ? $\quad f(x)=\sin x \quad f'(x)=\cos x \quad f''(x)=-\sin x \quad f'''(x)=-\cos x$

$$f'(x)=\frac{f(x+h)-f(x-h)}{2h}-\frac{h^2}{6}\cdot f'''(c) \qquad x-h<c<x+h$$

(a) $h=0.1$

$$f'(\tfrac{\pi}{3})=\frac{\sin(\frac{\pi}{3}+0.1)-\sin(\frac{\pi}{3}-0.1)}{2\cdot 0.1}=0.4991671$$

$|\text{approximation error}| = |0.4991671-\cos\frac{\pi}{3}| = 0.0008329$

$\left|\frac{0.1^2}{2}\cdot\cos(\frac{\pi}{3}-0.1)\right| \geq$ error bounds $\geq \left|\cos(\frac{\pi}{3}+0.1)\cdot\frac{0.1^2}{6}\right|$

$0.00068507 3 <$ error bounds $< 0.0009732 67$

$0.0006851 <$ error bounds < 0.0009733

$0.000685 < 0.0008329 < 0.0009733 \rightarrow$

$\rightarrow$ containing actual approximation error as expected

Answer: $f'(\frac{\pi}{3}) = 0.4991671$

app error $= 0.0008329$

error bounds $= [0.0006851, 0.0009733]$

) $h = 0.01$

$$f'\left(\frac{\pi}{3}\right) = \frac{\sin\left(\frac{\pi}{3}+0.01\right) - \sin\left(\frac{\pi}{3}-0.01\right)}{2\cdot 0.01} = 0.499992$$

$$|\text{approximation error}| = \left|0.499992 - \cos\frac{\pi}{3}\right| = 8.3\cdot 10^{-6}$$

$$\left|\cos\left(\frac{\pi}{3}+0.01\right)\cdot\frac{0.01^2}{6}\right| < |\text{error bounds}| < \left|\cos\left(\frac{\pi}{3}-0.01\right)\cdot\frac{0.01^2}{6}\right|$$

$$0.00000819 < |\text{error bounds}| < 0.00000848$$

$$0.0000082 < |\text{error bound}| < 0.0000085$$

$$0.0000082 < 8.3\cdot 10^{-6} < 0.0000085 \rightarrow$$

containing actual approximation error

Answer: $f'\left(\frac{\pi}{3}\right) = 0.499992$

app. err $= 8.3\cdot 10^{-6}$

error bounds: $[0.0000082, 0.0000085]$

(c) $h = 0.001$

$$f'\left(\frac{\pi}{3}\right) = \frac{\sin\left(\frac{\pi}{3}+0.001\right) - \sin\left(\frac{\pi}{3}-0.001\right)}{2\cdot 0.001} = 0.49999992$$

$$|\text{approximation error}| = \left|0.49999992 - \cos\frac{\pi}{3}\right| = 8.333336\cdot 10^{-8}$$

$$\left|\cos\left(\frac{\pi}{3}+0.001\right)\cdot\frac{0.001^2}{2}\right| < \text{error bounds} < \left|\cos\left(\frac{\pi}{3}-0.001\right)\cdot\frac{0.001^2}{6}\right|$$

$$8.31890\cdot 10^{-8} < \text{error bounds} < 8.34776\cdot 10^{-8}$$

$$8.31890\cdot 10^{-8} < 8.33334\cdot 10^{-8} < 8.34776\cdot 10^{-8} \rightarrow$$

$\rightarrow$ containing actual approximation error.

Answer: $f'\left(\frac{\pi}{3}\right) = 0.49999992$, app. err $= 8.33334\cdot 10^{-8}$

error bounds: $[8.31890\cdot 10^{-8}, 8.34776\cdot 10^{-8}]$

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Section 5.1

$\boxed{7}$ According to Taylor theorem if f is twice continiously differenciable then

$$\underbrace{f(x-h) = f(x) - h\cdot f'(x) + \frac{h^2}{2}\cdot f''(c)}_{\Rightarrow}$$ where $x-h < c < x$

$$f'(x)\cdot h = f(x) - f(x-h) + \frac{h^2}{2} f''(c)$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} + \frac{h}{2} f''(c)$$

INNA WILLIAMS

Section 5.2

$\boxed{1}$ (b) TRAPEZOID Rule: $m=1,2,4$

$\int_0^{\frac{\pi}{2}} \cos x\,dx - ?$ $\quad \int_a^b f(x)\,dx = \frac{h}{2}\left(y_0+y_m+2\sum_{i=1}^{m-1} y_i\right) - \frac{(b-a)h^2}{12} f''(c)$

$\boxed{m=1}$ $\quad h = \frac{b-a}{m} = \frac{\frac{\pi}{2}-0}{1} = \frac{\pi}{2}$

$\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{2\cdot 2}\left(\cos 0 + \cos\frac{\pi}{2} + 2(0)\right) = \frac{\pi}{4}(1+0) = \boxed{\frac{\pi}{4}}$

$|\text{approximation error}| = \left|\int_0^{\pi/2} \cos x\,dx - \frac{\pi}{4}\right| = \left|1-\frac{\pi}{4}\right| = \boxed{0.214602}$

$\boxed{m=2}$ $\quad h = \frac{b-a}{m} = \frac{\frac{\pi}{2}-0}{2} = \frac{\pi}{4}$

$\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{4\cdot 2}\left(\cos 0 + \cos\frac{\pi}{2} + 2y_1\right) = \frac{\pi}{8}\left(1+0+2\overset{\cos}{\left(\frac{0+\frac{\pi}{2}}{2}\right)}\right) =$

$= \frac{\pi}{8}\left(1+2\cdot\cos\frac{\pi}{4}\right) = \frac{\pi}{8}\left(1+\frac{2\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{8}(1+\sqrt{2})}$

$|\text{approximation error}| = \left|\int_0^{\pi/2} \cos x\,dx - \frac{\pi}{8}(1+\sqrt{2})\right| =$

$= \left|1-\frac{\pi}{8}(1+\sqrt{2})\right| = \boxed{0.051941}$

$\boxed{m=4}$ $\quad h = \frac{b-a}{m} = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$

$\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{8\cdot 2}\left(\cos 0 + \cos\frac{\pi}{2} + 2(y_1+y_2+y_3)\right) = \frac{\pi}{16}\Big(1+0+$

$2\left(\cos\frac{\pi}{8}+\cos\frac{2\pi}{8}+\cos\frac{3\pi}{8}\right)\Big) = \frac{\pi}{16}\left(1+2\left(0.92388+\frac{\sqrt{2}}{2}+0.38268\right)\right)$

$= \boxed{0.987116}$

$|\text{approximation error}| = \left| \int_0^{\pi/2} \cos x\,dx - 0.987116 \right| = |1 - 0.987116|$

$= \boxed{0.012884}$

Answer:

$m = 1$ $\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{4}$, error $= 0.214602$

$m = 2$ $\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{8}(1+\sqrt{2})$, error $= 0.051941$

$m = 4$ $\int_0^{\pi/2} \cos x\,dx = 0.987116$, error $= 0.012884$

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Section 5.2

$\boxed{3}$ (6) Simpson Rule $m = 1, 2, 4.$

$\int_0^{\pi/2} \cos x\,dx - ?$ $\quad \int_a^b f(x)\,dx = \frac{h}{3}\left[y_0 + y_{2m} + 4\sum_{i=1}^{m} y_{2i-1} + 2\sum_{i=1}^{m-1} y_{2i}\right] - \frac{(b-a)\cdot h^4}{180} f^{(4)}(c)$

$\boxed{m=1}$ $\quad h = \frac{b-a}{2\cdot m} = \frac{\frac{\pi}{2}-0}{2\cdot 1} = \frac{\pi}{2\cdot 2} = \frac{\pi}{4}$

$\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{4\cdot 3}\left[\cos 0 + \cos\frac{\pi}{2} + 4y_1\right] = \frac{\pi}{12}\left[1 + 0 + 4\cdot\cos\frac{\pi}{4}\right] = \underline{1.00228}$

$|\text{approximation error}| = \left|\int_0^{\pi/2}\cos x\,dx - 1.00228\right| = |1 - 1.00228| = \underline{0.00228}$

$\boxed{m=2}$ $\quad h = \frac{b-a}{2m} = \frac{\frac{\pi}{2}-0}{2\cdot 2} = \frac{\pi}{8}$

$\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{8\cdot 3}\left[\cos 0 + \cos\frac{\pi}{2} + 4\left(\cos\frac{\pi}{8} + \cos\frac{3\pi}{8}\right) + 2\left(\cos\frac{2\pi}{8}\right)\right] =$

$= \frac{\pi}{24}\left[1 + 6.640485\right] = \frac{\pi\cdot 7.640465}{24} = \underline{1.000135}$

$|\text{approximation error}| = \left|\int_0^{\pi/2}\cos x\,dx - 1.000135\right| = |1 - 1.000135| =$

$= \underline{0.000135}$

$\boxed{m=4}$ $\quad h = \frac{b-a}{2m} = \frac{\frac{\pi}{2}-0}{2\cdot 4} = \frac{\pi}{16}$

$\int_0^{\pi/2} \cos x\,dx = \frac{\pi}{16\cdot 3}\left[\cos 0 + \cos\frac{\pi}{2} + 4\left(\cos\frac{\pi}{16} + \cos\frac{3\pi}{16} + \cos\frac{5\pi}{16} + \cos\frac{7\pi}{16}\right)\right.$

$\left. + 2\left(\cos\frac{2\pi}{16} + \cos\frac{4\pi}{16} + \cos\frac{6\pi}{16}\right)\right] = \frac{\pi}{48}\left[1 + 0 + 4(2.562915) +\right.$

$\left.+ 2(2.0136)\right] = \frac{\pi}{48}\left[1 + 14.279\right] = \frac{\pi\cdot 15.279}{48} = \underline{1.000008}$

$$|\text{approximation error}| = \left|\int_0^{\pi/2} \cos x\,dx - 1.000008\right| =$$

$$= |1 - 1.000008| = \underline{0.000008}$$

Answer:

$m=1$ $\quad \int_0^{\pi/2} \cos x\,dx = 1.00228$, error: $0{,}002280$

$m=2$ $\quad \int_0^{\pi/2} \cos x\,dx = 1.000135$, error $= 0.000135$

$m=4$ $\quad \int_0^{\pi/2} \cos x\,dx = 1.000008$, error $= 0.000008$