Section 3.1

$\boxed{1}$ (a) $(0,1), (2,3), (3,0)$

$$P_2(x) = y_1 \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + y_2 \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-3)} + y_3 \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$P_2(x) = 1 \cdot \frac{(x-2)\cdot(x-3)}{(0-2)(0-3)} + 3 \cdot \frac{(x-0)\cdot(x-3)}{(2-0)(2-3)} + 0 \cdot \frac{(x-0)\cdot(x-2)}{(3-0)(3-2)}$$

$$P_2(x) = 1 \cdot \frac{(x-2)\cdot(x-3)}{(0-2)\cdot(0-3)} + 3 \cdot \frac{(x-0)\cdot(x-3)}{(2-0)\cdot(2-3)}$$

or $P_2(x) = \frac{x^2-5x+6}{6} + \frac{-9x(x-3)}{2\cdot 3} = \frac{-8x^2+22x+6}{6} = -\frac{4}{3}x^2 + \frac{11}{3}x + 1$

Answer: $1 \cdot \frac{(x-2)\cdot(x-3)}{(0-2)\cdot(0-3)} + 3 \cdot \frac{(x-0)\cdot(x-3)}{(2-0)(2-3)}$

or $-\frac{4}{3}x^2 + \frac{11}{3}x + 1$

(b) $(-1,0), (2,1), (3,1), (5,2)$

$$P_2(x) = y_1 \frac{(x-x_2)\cdot(x-x_3)\cdot(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + y_2 \frac{(x-x_1)\cdot(x-x_3)\cdot(x-x_4)}{(x_2-x_1)\cdot(x_2-x_3)\cdot(x_2-x_4)}$$
$$+ y_3 \frac{(x-x_1)\cdot(x-x_2)\cdot(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} + y_4 \frac{(x-x_1)\cdot(x-x_2)\cdot(x-x_3)}{(x_4-x_1)\cdot(x_4-x_2)(x_4-x_3)}$$

$$P_3(x) = 0 \cdot \frac{(x-2)\cdot(x-3)\cdot(x-5)}{(-1-2)(-1-3)\cdot(-1-5)} + 1 \cdot \frac{(x+1)(x-3)\cdot(x-5)}{(2+1)(2-3)(2-5)}$$
$$+ 1 \cdot \frac{(x+1)\cdot(x-2)\cdot(x-5)}{(3+1)\cdot(3-2)(3-5)} + 2 \cdot \frac{(x+1)(x-2)\cdot(x-3)}{(5+1)\cdot(5-2)(5-3)}$$

Answer: $P_3(x) = 1 \cdot \frac{(x+1)(x-3)(x-5)}{(2+1)(2-3)(2-5)} + 1 \cdot \frac{(x+1)\cdot(x-2)\cdot(x-5)}{(3+1)\cdot(3-2)(3-5)} + 2 \cdot \frac{(x+1)(x-2)(x-3)}{(5+1)(5-2)(5-3)}$

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or $\frac{1}{24}x^3 - \frac{1}{4}x^2 + \frac{11}{24}x + \frac{3}{4}$

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$\boxed{2}$ (a) $(0,1), (2,3), (3,0)$

$$\begin{array}{llll} 0 & \boxed{1} & & \\ & & \frac{3-1}{2-0} = \boxed{1} & \\ 2 & 3 & & \frac{-3-1}{3-0} = \boxed{-\frac{4}{3}} \\ & & \frac{0-3}{3-2} = -3 & \\ 3 & 0 & & \end{array}$$

coefficients are: $1, 1, -\frac{4}{3}$

interpolated polynomial:

$$1 + 1\cdot(x-0) - \frac{4}{3}(x-0)(x-2) = 1 + x - \frac{4}{3}x(x-2) =$$

$$= 1 + x - \frac{4}{3}x^2 + \frac{8}{3}x = -\frac{4}{3}x^2 + \frac{11}{3}x + 1$$

Answer: interpolated polynomial is

$$\boxed{-\frac{4}{3}x^2 + \frac{11}{3}x + 1}$$

This polynomial agree with Lagrange interpolated polynomial we received in exersize $\boxed{1a}$

Section 3.1

$\boxed{26}$ $(-1,0), (2,1), (3,1) (5,2)$

$$\begin{array}{llllll}
-1 & \boxed{0} & & & & \\
 & & \frac{1-0}{2+1}=\boxed{\frac{1}{3}} & & & \\
2 & 1 & & \frac{0-\frac{1}{3}}{3+1}=\boxed{-\frac{1}{12}} & & \\
3 & 1 & \frac{1-1}{3-2}=0 & & \frac{\frac{1}{6}+\frac{1}{12}}{5+1}=\boxed{\frac{3}{72}}=\boxed{\frac{1}{24}} & \\
5 & 2 & \frac{2-1}{5-3}=\frac{1}{2} & \frac{\frac{1}{2}-0}{5-2}=\frac{1}{6} & &
\end{array}$$

coefficients are: $0, \frac{1}{3}, -\frac{1}{12}, \frac{1}{24}$

interpolated polinomial is:

$$0+\frac{1}{3}(x+1)-\frac{1}{12}(x+1)(x-2)+\frac{1}{24}(x+1)(x-2)(x-3)$$

$$=\frac{1}{3}x+\frac{1}{3}-\frac{1}{12}x^2-\frac{1}{12}x+\frac{2}{12}x+\frac{2}{12}+\frac{x^3}{24}-\frac{4x^2}{24}+\frac{x}{24}+\frac{1}{4}=$$

$$=\frac{x^3}{24}-\frac{4x^2}{24}+\frac{x}{24}+\frac{1}{4}+\frac{5}{12}x-\frac{x^2}{12}+\frac{1}{2}=$$

Answer $\boxed{\frac{x^3}{24}-\frac{1}{4}x^2+\frac{11}{24}x+\frac{3}{4}}$ - This polynomial

agree with Legrange interpolated polynomial we received in exersize $\boxed{16}$

INNA WILLIAMS $\boxed{5}$

Section 3.1

$\boxed{5}$ $(-2,8),(0,4),(1,2),(3,-2)$

(a)

$$\begin{array}{ll} -2 & \boxed{8} \\ & \frac{4-8}{0+2} = \boxed{-2} \\ 0 & 4 \quad\quad \frac{-2+2}{1+2} = \boxed{0} \\ & \frac{2-4}{1-0} = -2 \quad\quad \frac{0-0}{3+2} = \boxed{0} \\ 1 & 2 \quad\quad \frac{-2+2}{3-1} = 0 \\ & \frac{-2-2}{3-1} = -2 \\ 3 & -2 \end{array}$$

Coefficients are: 8, -2, 0, 0

Polynomial is: $8-2(x+2)+0\cdot(x+2)(x+0)+0\cdot(x+2)\cdot(x+0)\cdot(x+1) \Rightarrow$

$P = 8-2(x+2)$ or $= 4-2x$

Answer: The polynomial is $\boxed{4-2x}$ and it is passes through the 4 data points

(b) According to theorem 3.2 for points $(x_1,y_1)\dots(x_n,y_n)$ there exist one and only one polynomial of degree $(n-1)$ or less through n data points. Therefore there are no other degree ≤ 3 polinomial through the 4 data points given in $\boxed{5}$ but

$P_4(x) = 4-2x+A\cdot(x+2)\cdot(x+0)\cdot(x-1)\cdot(x-3)$

and it interpolates for all A such that $(\forall A \neq 0)$

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1800	$\boxed{280}$
1850	283
1900	291
2000	370

$\frac{283-280}{1850-1800} = \boxed{0.06}$

$\frac{291-283}{1900-1850} = 0.16$

$\frac{370-291}{2000-1900} = 0.79$

$\frac{0.16-0.06}{1900-1800} = \boxed{0.0010}$

$\frac{0.79-0.16}{2000-1850} = 0.0042$

$\frac{0.0042-0.0010}{2000-1800} \rightarrow \boxed{0.000016}$

coeficients are: 280, 0.06, 0.0010, 0.000016

$$\boxed{P_3(x) = 280+0.06(x-1800)+0.0010\cdot(x-1800)\cdot(x-1850)+0.000016\cdot(x-1800)\times(x-1850)\cdot(x-1900)}$$

@ $P(1950)$ - ?

$$P(1950) = 280+(0.06)\cdot(1950-1800)+0.0010\cdot(1950-1800)\cdot(1950-1850)+$$
$$+0.000016\cdot(1950-1800)\cdot(1950-1850)\cdot(1950-1900)$$

$$\boxed{P(1950) = 280+9+15+12 = 316}$$

ⓑ

$$P(2050) = 280+0.06\cdot(2050-1800)+0.0010\cdot(2050-1800)\cdot(2050-1850)+$$
$$+0.000016\cdot(2050-1800)\cdot(2050-1850)\cdot(2050-1900)$$

$$\boxed{P(2050) = 280+15+50+120 = 465}$$

Answer:

ⓐ CO_2 in 1950 was 316 ppM

ⓑ CO_2 in 2050 wos 465 ppM