INNA WILLIAMS

Section 2.7

$\boxed{1}$ c) $F(u,v) = (u^2+v^2-1,\ (u-1)^2+v^2-1)$

$$\partial F(u,v) = \begin{vmatrix} 2u & 2v \\ 2(u-1) & 2v \end{vmatrix}$$

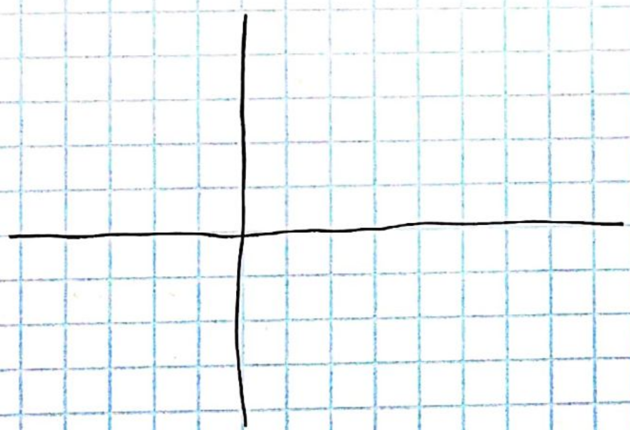
$\boxed{3}$ a) $\left.\begin{array}{l} u^2+v^2=1 \\ (u-1)^2+v^2=1 \end{array}\right\} \Rightarrow$ $v^2 = 1-u^2$

$(u-1)^2 + 1-u^2 = 1$

$u^2-2u+1+1-u^2=1$

$-2u=-1 \Rightarrow u = 1/2$

$v^2 = 1-\frac{1}{4} \Rightarrow v = \pm\frac{\sqrt{3}}{2}$



Answer: $\left(\frac{1}{2}; \frac{\sqrt{3}}{2}\right)$; $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

$\boxed{4}$ $x_0 = (1,1)$

a) $\left.\begin{array}{l} u^2+v^2=1 \\ (u-1)^2+v^2=1 \end{array}\right\}$

Jacobin $= \begin{vmatrix} 2u & 2v \\ 2(u-1) & 2v \end{vmatrix}$

Step 0: $x_0 = (1,1)$

Step 1: $f_1(1,1) = 1+1-1 = 1$

$f_2(1,1) = 0+1-1 = 0$

Jacobin $= \begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix}$

$\begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix} \times \begin{vmatrix} s_1 \\ s_2 \end{vmatrix} = \begin{vmatrix} -1 \\ 0 \end{vmatrix}$

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$$\left|\begin{matrix} 2 & 2 \\ 0 & 2 \end{matrix}\right|\begin{matrix} 1 \\ 0 \end{matrix}\Bigg| = \left|\begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}\right|\begin{matrix} -1 \\ 0 \end{matrix}\Bigg| \Big\} \Rightarrow \begin{matrix} 2S_1 = -1 \\ 2S_2 = 0 \end{matrix} \Big\} \Rightarrow S = \begin{vmatrix} -\frac{1}{2} \\ 0 \end{vmatrix}$$

$$x_1 = x_0 + S = \begin{vmatrix} 1 \\ 1 \end{vmatrix} + \begin{vmatrix} -\frac{1}{2} \\ 0 \end{vmatrix} = \boxed{\begin{vmatrix} \frac{1}{2} \\ 1 \end{vmatrix}}$$

<u>Step 2</u> $\quad x_1 = (1/2, 1)$

$$f_1\left(\frac{1}{2}, 1\right) = \frac{1}{4} + 1 - 1 = \frac{1}{4}$$

$$f_2\left(\frac{1}{2}, 1\right) = \frac{1}{4} + 1 - 1 = \frac{1}{4}$$

$$\text{Jacobian} = \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} \times \begin{vmatrix} S_1 \\ S_2 \end{vmatrix} = \begin{vmatrix} -1/4 \\ -1/4 \end{vmatrix}$$

$$\left|\begin{matrix} 1 & 2 \\ -1 & 2 \end{matrix}\right|\begin{matrix} -1/4 \\ -1/4 \end{matrix}\Bigg| = \left|\begin{matrix} 1 & 2 \\ 0 & 4 \end{matrix}\right|\begin{matrix} -1/4 \\ -1/2 \end{matrix}\Bigg| = \left|\begin{matrix} 2 & 4 \\ 0 & 4 \end{matrix}\right|\begin{matrix} -1/2 \\ -1/2 \end{matrix}\Bigg| = \left|\begin{matrix} 2 & 0 \\ 0 & 4 \end{matrix}\right|\begin{matrix} 0 \\ -\frac{1}{2} \end{matrix}\Bigg| \Rightarrow$$

$$\Rightarrow \begin{matrix} 2S_1 = 0 \\ 4S_2 = -\frac{1}{2} \end{matrix} \Big\} \Rightarrow \begin{matrix} S_1 = 0 \\ S_2 = -\frac{1}{8} \end{matrix} \Rightarrow S = \begin{vmatrix} 0 \\ -\frac{1}{8} \end{vmatrix}$$

$$x_2 = x_1 + S = \begin{vmatrix} \frac{1}{2} \\ 1 \end{vmatrix} + \begin{vmatrix} 0 \\ -\frac{1}{8} \end{vmatrix} = \boxed{\begin{vmatrix} \frac{1}{2} \\ \frac{7}{8} \end{vmatrix}}$$

Answer: (a) Step 0 $\Rightarrow$ $x_0 = (1, 1) \leftrightarrow (1, 1)$

Step 1 $\Rightarrow$ $x_1 = \left(\frac{1}{2}, 1\right) \leftrightarrow (0.5, 1)$

Step 2 $\Rightarrow$ $x_2 = \underbrace{\left(\frac{1}{2}, \frac{7}{8}\right)}$ $\leftrightarrow$ ~~(0.5, 0.875)~~

$\{0.5, 0.875\}$

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$\boxed{4b}$ $\left.\begin{matrix} u^2+4v^2=4 \\ 4u^2+v^2=4 \end{matrix}\right\} \; DF(u,v) = \begin{vmatrix} 2u & 8v \\ 8u & 2v \end{vmatrix}$

Step 0: $x_0 = (1,1)$

Step 1: $f_1(1,1) = 1+4-4 = 1$

$f_2(1,1) = 4+1-4 = 1$

$$DF(1,1) = \begin{vmatrix} 2 & 8 \\ 8 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 8 \\ 8 & 2 \end{vmatrix} \times \begin{vmatrix} s_1 \\ s_2 \end{vmatrix} = \begin{vmatrix} -1 \\ -1 \end{vmatrix}$$

$$\left|\begin{array}{cc|c} 2 & 8 & -1 \\ 8 & 2 & -1 \end{array}\right| = \left|\begin{array}{cc|c} 2 & 8 & -1 \\ 32 & 8 & -4 \end{array}\right| = \left|\begin{array}{cc|c} -30 & 0 & 3 \\ 8 & 2 & -1 \end{array}\right| = \left|\begin{array}{cc|c} -10 & 0 & 1 \\ 1 & \frac{2}{8} & -\frac{1}{8} \end{array}\right| =$$

$$= \left|\begin{array}{cc|c} -10 & 0 & 1 \\ 10 & \frac{20}{8} & -\frac{10}{8} \end{array}\right| = \left|\begin{array}{cc|c} -10 & 0 & 1 \\ 0 & \frac{20}{8} & \frac{2}{8} \end{array}\right| = \left|\begin{array}{cc|c} -10 & 0 & 1 \\ 0 & 10 & -1 \end{array}\right| \Rightarrow \begin{matrix} -10 s_1 = 1 \\ 10 s_2 = -1 \end{matrix}$$

$$\Rightarrow \left.\begin{matrix} s_1 = -\frac{1}{10} \\ s_2 = -\frac{1}{10} \end{matrix}\right\} \Rightarrow s = \begin{vmatrix} -\frac{1}{10} \\ -\frac{1}{10} \end{vmatrix} \Rightarrow x_1 = x_0 + s = \begin{vmatrix} 1 \\ 1 \end{vmatrix} + \begin{vmatrix} -\frac{1}{10} \\ -\frac{1}{10} \end{vmatrix} = \begin{vmatrix} 0.9 \\ 0.9 \end{vmatrix}$$

Step 2: $x_1 = (0.9, 0.9)$ $f_1(0.9, 0.9) = 0.81 + 4\cdot 0.81 - 4 = 0.05$

$f_2(0.9, 0.9) = 4\cdot 0.81 + 0.81 - 4 = 0.05$

$$D(F(0.9,0.9)) = \begin{vmatrix} 1.8 & 7.2 \\ 7.2 & 1.8 \end{vmatrix}$$

$$\begin{vmatrix} 1.8 & 7.2 \\ 7.2 & 1.8 \end{vmatrix} \cdot \begin{vmatrix} s_1 \\ s_2 \end{vmatrix} = \begin{vmatrix} -0.05 \\ -0.05 \end{vmatrix}$$

$$\left|\begin{array}{cc|c} 1.8 & 7.2 & -0.05 \\ 7.2 & 1.8 & -0.05 \end{array}\right| = \left|\begin{array}{cc|c} 2 & 8 & -\frac{0.5}{9} \\ 8 & 2 & -\frac{0.5}{9} \end{array}\right| = \left|\begin{array}{cc|c} 2 & 8 & -\frac{5}{90} \\ 8 & 2 & -\frac{5}{90} \end{array}\right|$$

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$$= \left|\begin{array}{cc|c} 1 & 4 & -\frac{5}{180} \\ 4 & 1 & -\frac{5}{180} \end{array}\right| = \left|\begin{array}{cc|c} 1 & 4 & -\frac{5}{180} \\ 16 & 4 & -\frac{5}{180} \end{array}\right| = \left|\begin{array}{cc|c} -15 & 0 & \frac{15}{180} \\ 4 & 1 & -\frac{5}{180} \end{array}\right| =$$

$$= \left|\begin{array}{cc|c} -1 & 0 & +1/180 \\ 4 & 1 & -5/180 \end{array}\right| = \left|\begin{array}{cc|c} -4 & 0 & \frac{4}{180} \\ 4 & 1 & -\frac{5}{180} \end{array}\right| = \left|\begin{array}{cc|c} -1 & 0 & 1/180 \\ 0 & 1 & -1/180 \end{array}\right| \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -S_1 = \frac{1}{180} \\ S_2 = -\frac{1}{180} \end{array}\right\} \Rightarrow \left.\begin{array}{l} S_1 = -\frac{1}{180} \\ S_2 = -\frac{1}{180} \end{array}\right\} \Rightarrow S = \left|\begin{array}{c} -\frac{1}{180} \\ -\frac{1}{180} \end{array}\right|$$

$$\Rightarrow X_2 = X_1 + S = \left|\begin{array}{c} \frac{9}{10} \\ \frac{9}{10} \end{array}\right| + \left|\begin{array}{c} -\frac{1}{180} \\ -\frac{1}{180} \end{array}\right| = \underline{\underline{\left|\begin{array}{c} \frac{161}{180} \\ \frac{161}{180} \end{array}\right|}}$$

Answer: Step 0 $\Rightarrow X_0 = (1, 1)$

Step 1 $\Rightarrow X_1 = \left(\frac{9}{10}, \frac{9}{10}\right) \Leftrightarrow (0.9, 0.9)$

Step 2 $\Rightarrow X_2 = \left(\frac{161}{180}, \frac{161}{180}\right) \Leftrightarrow (0.894, 0.894)$

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$\boxed{4}$ (c) $\left.\begin{array}{l} u^2-4v^2=4 \\ (u-1)^2+v^2=4 \end{array}\right\}$ $DF(u,v)=\begin{vmatrix} 2u & -8v \\ 2(u-1) & 2v \end{vmatrix}$

Step 0: $x_0 = (1,1)$

Step 1: $f_1(1,1) = 1-4-4 = -7$

$f_2(1,1) = 0+1-4 = -3$

$DF(u,v), u=1, v=1) = \begin{vmatrix} 2 & -8 \\ 0 & 2 \end{vmatrix}$

$\begin{vmatrix} 2 & -8 \\ 0 & 2 \end{vmatrix} \times \begin{vmatrix} S_1 \\ S_2 \end{vmatrix} = \begin{vmatrix} 7 \\ 3 \end{vmatrix}$

$\left|\begin{array}{cc|c} 2 & -8 & 7 \\ 0 & 2 & 3 \end{array}\right| = \left|\begin{array}{cc|c} 2 & -8 & 7 \\ 0 & 8 & 12 \end{array}\right| = \left|\begin{array}{cc|c} 2 & 0 & 19 \\ 0 & 2 & 3 \end{array}\right| \Rightarrow \left.\begin{array}{l} 2S_1 = 19 \\ 2S_2 = 3 \end{array}\right\} \Rightarrow \begin{array}{l} S_1 = \frac{19}{2} \\ S_2 = \frac{3}{2} \end{array}$

$S = \begin{vmatrix} \frac{19}{2} \\ \frac{3}{2} \end{vmatrix}$, $X_1 = X_0 + S_1 = \begin{vmatrix} 1 & \frac{19}{2} \\ 1 & \frac{3}{2} \end{vmatrix} = \begin{vmatrix} \frac{21}{2} \\ \frac{5}{2} \end{vmatrix}$

Step 2: $f_1\left(\frac{21}{2}, \frac{5}{2}\right) = \left(\frac{21}{2}\right)^2 - 4\left(\frac{5}{2}\right)^2 - 4 = \frac{325}{4}$

$f_2\left(\frac{21}{2}, \frac{5}{2}\right) = \left(\frac{21}{2} - 1\right)^2 + \left(\frac{5}{2}\right)^2 - 4 = \frac{185}{2}$

$DF(u,v), u = \frac{21}{2}, v = \frac{5}{2}) = \begin{vmatrix} 21 & -20 \\ 19 & 5 \end{vmatrix}$

$\begin{vmatrix} 21 & -20 \\ 19 & 5 \end{vmatrix} \times \begin{vmatrix} S_1 \\ S_2 \end{vmatrix} = \begin{vmatrix} -\frac{325}{4} \\ -\frac{185}{2} \end{vmatrix}$

$\left|\begin{array}{cc|c} 21 & -20 & -325/4 \\ 19 & 5 & -185/2 \end{array}\right| = \left|\begin{array}{cc|c} 21 & -20 & -\frac{325}{4} \\ 19\cdot 4 & 20 & -\frac{185\cdot 4}{2} \end{array}\right| = \left|\begin{array}{cc|c} 21+19\cdot 4 & 0 & -\frac{1805}{4} \\ 19 & 5 & -\frac{185}{2} \end{array}\right|$

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$$= \begin{vmatrix} 97\cdot 19 & 0 \\ 19\cdot 97 & 5\cdot 97 \end{vmatrix} \begin{vmatrix} -\frac{1805\cdot 19}{4} \\ -\frac{185\cdot 97}{2} \end{vmatrix} = \begin{vmatrix} 97 & 0 \\ 0 & 5\cdot 97 \end{vmatrix} \begin{vmatrix} -\frac{1805}{4} \\ -\frac{1595}{4} \end{vmatrix} =$$

$$\Rightarrow \left.\begin{array}{l} 97 S_1 = -\frac{1805}{4} \\ 5\cdot 97 \cdot S_2 = -\frac{1595}{4} \end{array}\right\} \Rightarrow \begin{array}{l} S_1 = -\frac{1805}{388} \\ S_2 = -\frac{1595}{1940} \end{array} \Rightarrow S = \begin{vmatrix} -\frac{1805}{388} \\ -\frac{1595}{1940} \end{vmatrix}$$

$$X_2 = X_1 + S = \begin{vmatrix} \frac{21}{2} \\ \frac{5}{2} \end{vmatrix} + \begin{vmatrix} -\frac{1805}{388} \\ -\frac{1595}{1940} \end{vmatrix} = \begin{vmatrix} \frac{2269}{388} \\ \frac{3255}{1940} \end{vmatrix}$$

Answer: Step:0 $X_0 = (1,1)$

Step:1 $X_1 = \begin{vmatrix} \frac{21}{2} \\ \frac{5}{2} \end{vmatrix} = \begin{vmatrix} 10.5 \\ 2.5 \end{vmatrix}$

Step:2 $X_2 = \begin{vmatrix} \frac{2269}{388} \\ \frac{3255}{940} \end{vmatrix} = \begin{vmatrix} 5.84799 \\ 1.67784 \end{vmatrix}$

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INNA WILLIAMS

Section 13.1

$\boxed{1}$ (c) $f(x) = 2x^4 + x$

$f'(x) = 8x^3 + 1 = 0 \Rightarrow x = -\frac{1}{2}$

$f''(x) = 24x^2 > 0$ - $f(x)$ is increasing function

$f(x) < 0$ if $x < -\frac{1}{2}$
$f(x) > 0$ if $x > -\frac{1}{2}$
$\Rightarrow$ therefore $x = -\frac{1}{2}$ $f(x) = -\frac{3}{8}$ - is absolute minimum

$\Rightarrow f(x)$ unimodal

$f_{min} = -\frac{3}{8}$ at $x = -\frac{1}{2}$ $\quad f\left(-\frac{1}{2}\right) = 2\cdot\left(-\frac{1}{2}\right)^4 + \left(-\frac{1}{2}\right) = -\frac{3}{8}$

$\boxed{1}$ (d) $f(x) = x - \ln x$, $x > 0$

$f'(x) = 1 - \frac{1}{x} = 0 \Rightarrow x = 1$ $\quad f(1) = 1 - \ln 1 = 1$

$f''(x) = \frac{1}{x^2} > 0 \Rightarrow f(x)$ - is increasing on $[0; \infty]$

$f(x) > 0$ if $x > 1$
$f(x) < 0$ if $0 < x < 1$
$\Rightarrow$ therefore $x = 1$ $f(x) = 1$ - is absolute minimum and occurs at $(1, 1)$

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