Numerical Analysis

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###

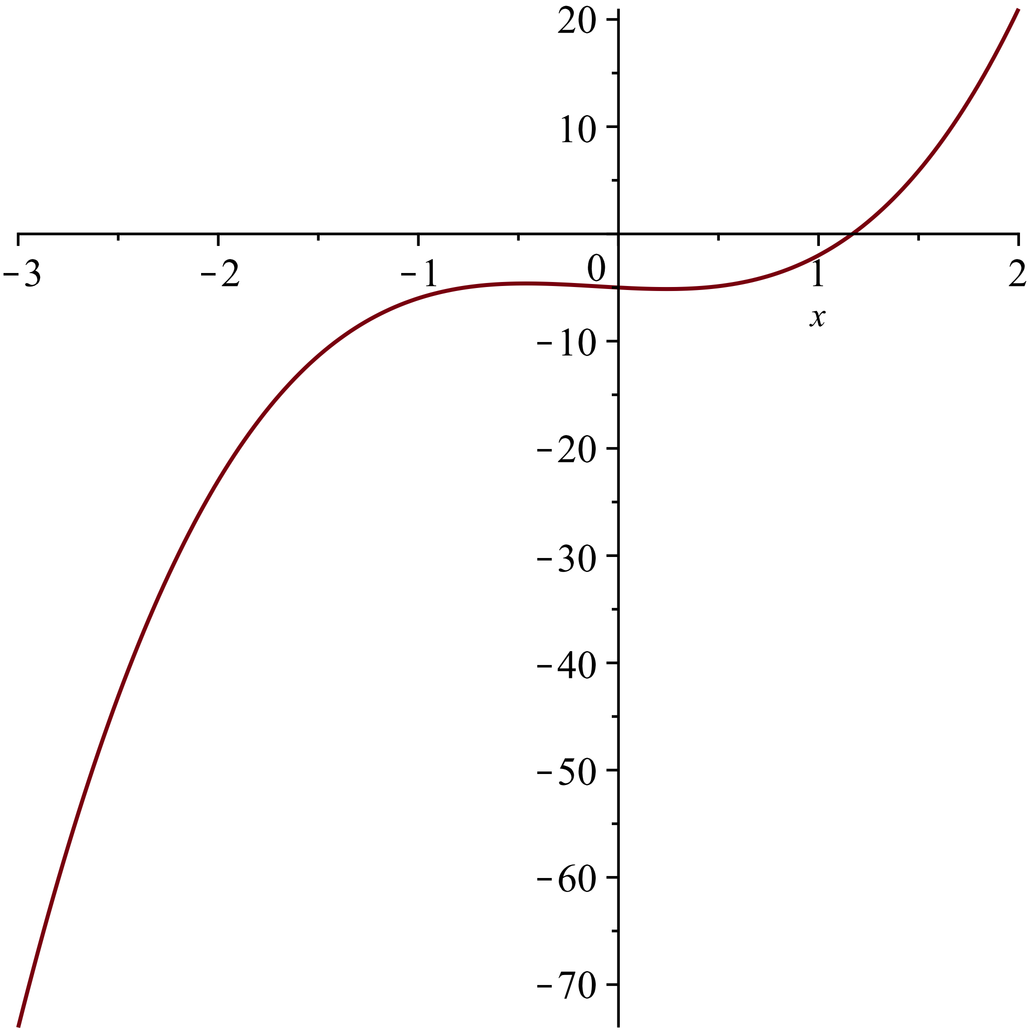
1b

```
bisect:=proc(A,B,TOL,N)    #we construct the procedure with 4 parameters
           local i,a,b;a:=A;b:=B;
           i:=0;
           while i<=N do       #use N to make sure that the loop is not infinite
                c:=(a+b)/2:    #find the midpoint of the current interval
                if f(c)=0 or (b-a)/2<TOL then
                   printf("c=%.8f, f(c)=%.8f\n",c,f(c)); #printf the approximation c and f(c) using 8 decimal places
                   printf("Number of iterations needed: %d",i);return ();
                break;
                end if;       #if not, continue with the interval halving
                if signum(f(a))*signum(f(c))<0 then
                     b:=c;
                else
                     a:=c;
                end if;
                i:=i+1;
           end do;
           printf("The method failed after %d iterations.\n",N);
           printf("c=%.8f, f(c)=%.8f\n",c,f(c));
           return();
        end proc:
```

> $f := x \rightarrow 3\cdot x^3 + x^2 - x - 5$; *#initialize the function f*

$$f := x \mapsto 3\ x^3 + x^2 - x - 5 \qquad (1)$$

> $plot(f(x), x=-3..2)$; *#plot the function f to localize the roots*



```
> bisect(0.5, 1.5, 0.5·10^-6, 1000);  #call the procedure 'bisect' to find the root in [0.5,1.5]
c=1.16972590, f(c)=-0.00000444
Number of iterations needed: 20
```

Answer: root = 1.16972590 with error in 6 decimal places found in 20 iterations on interval [0.5,1.5]

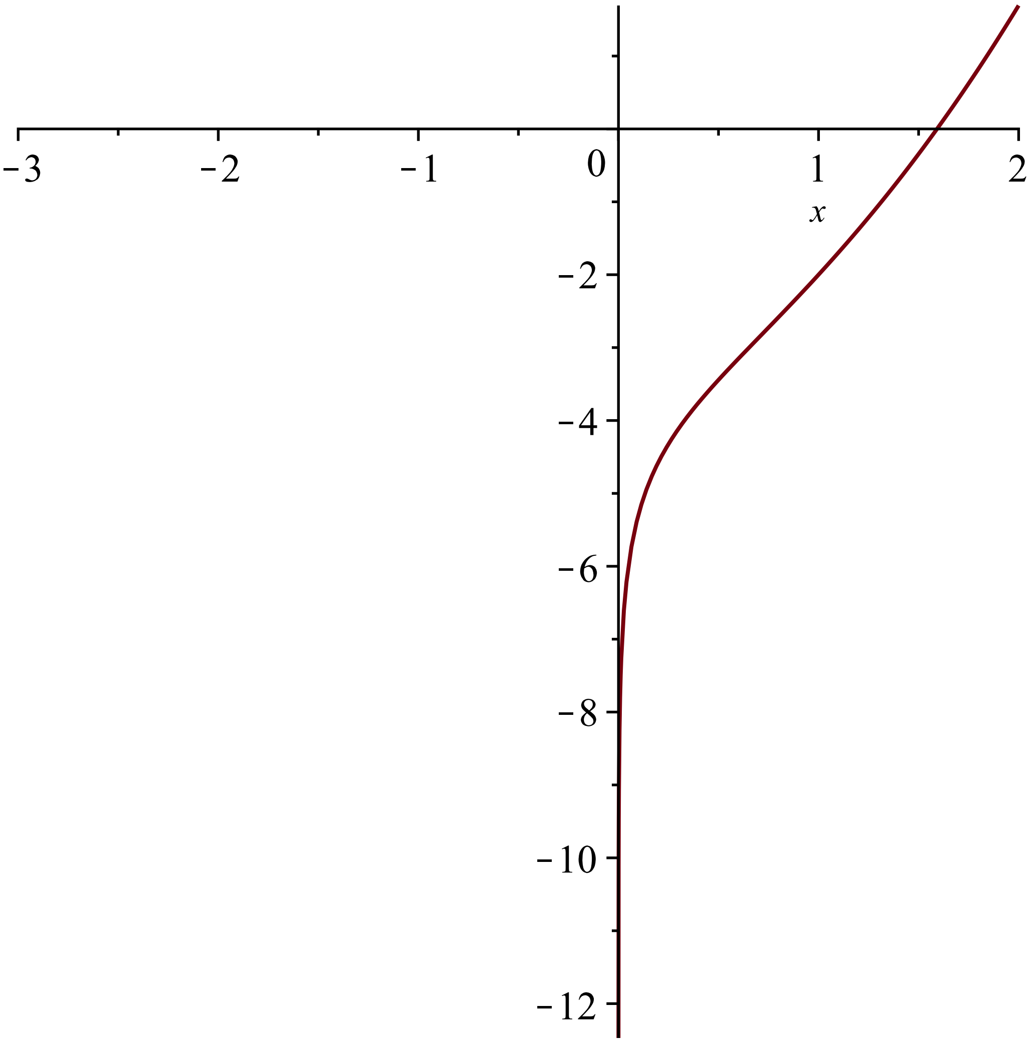
##

2c

```
> f := x→ ln(x) + x^2 − 3;  #initialize the function f
```

$$f := x \mapsto \ln(x) + x^2 - 3 \qquad \textbf{(2)}$$

```
> plot(f(x), x=-3 ..2);  #plot the function f to localize the roots
```



```
> bisect(1, 2, 0.5·10^-8, 1000); #call the procedure 'bisect' to find the root in [1,2]
c=1.59214294, f(c)=-0.00000000
Number of iterations needed: 27
```

Answer: root = 1.59214294 with error in 8 decimal places found in 27 iterations on interval [1,1]

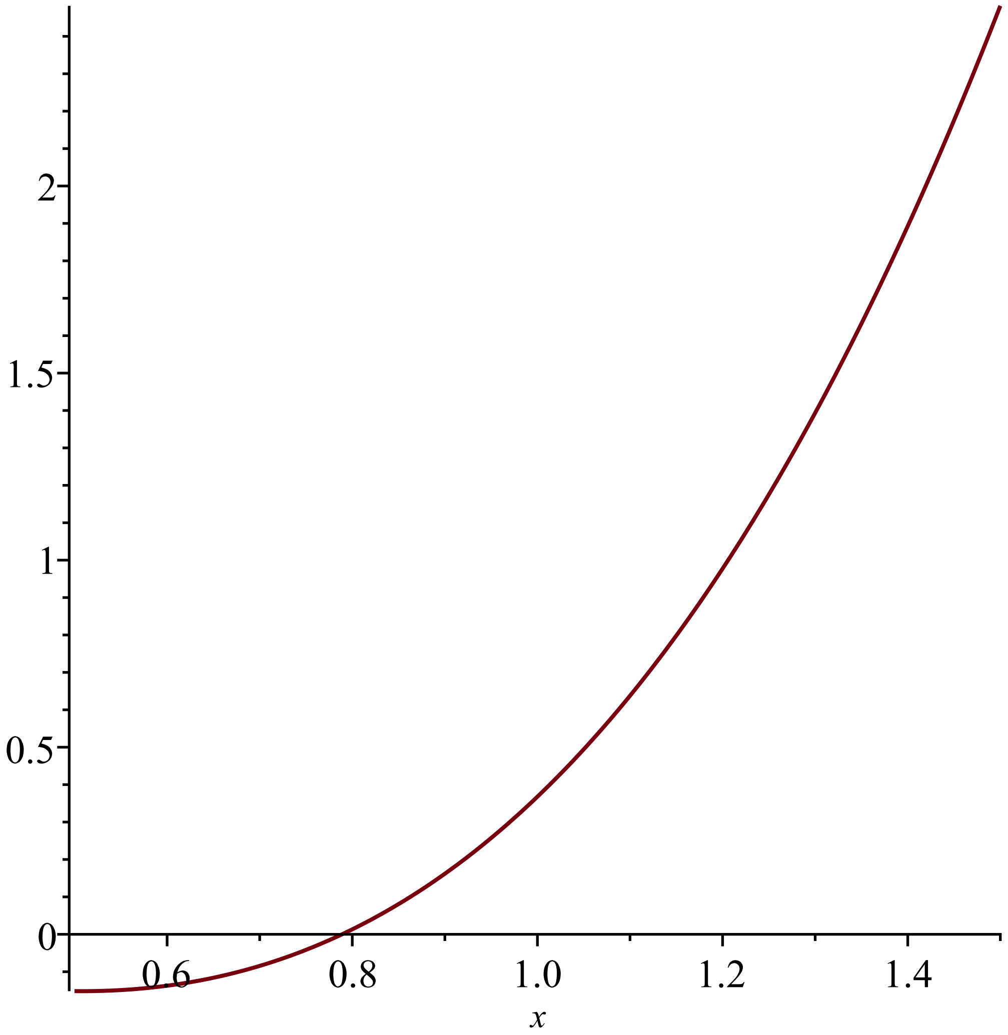
##

3b

```
> f := x→ exp(x − 2) + x^3 − x; #initialize the function f
```

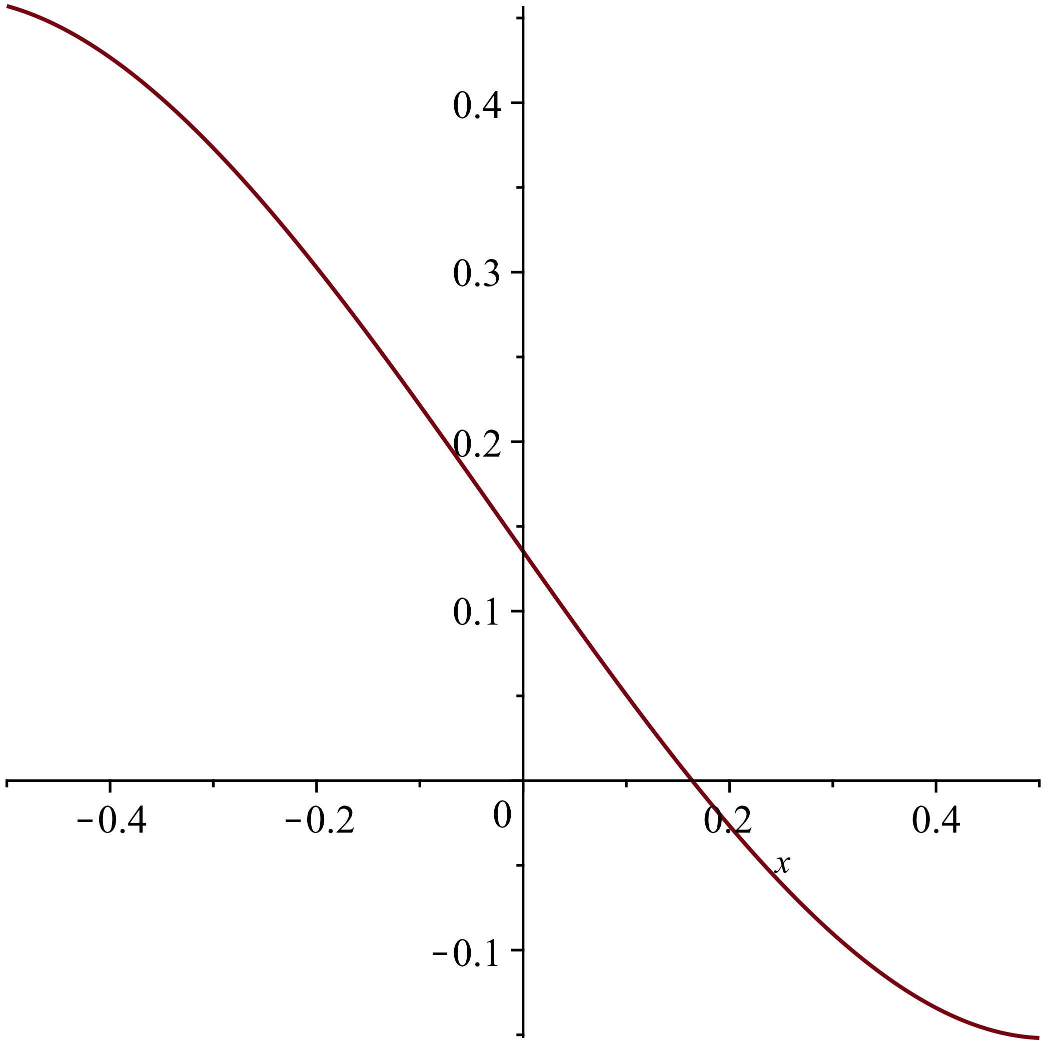
$$f := x \mapsto \mathrm{e}^{x-2} + x^3 - x \qquad \textbf{(3)}$$

```
> plot(f(x), x = 0.5 ..1.5); #plot the function f to localize the roots
```



```
> bisect(0.5, 1.5, 0.5·10^-6, 1000);  #call the procedure 'bisect' to find the root in [0.5,1.5]
c=0.78894186, f(c)=0.00000055
Number of iterations needed: 20
```

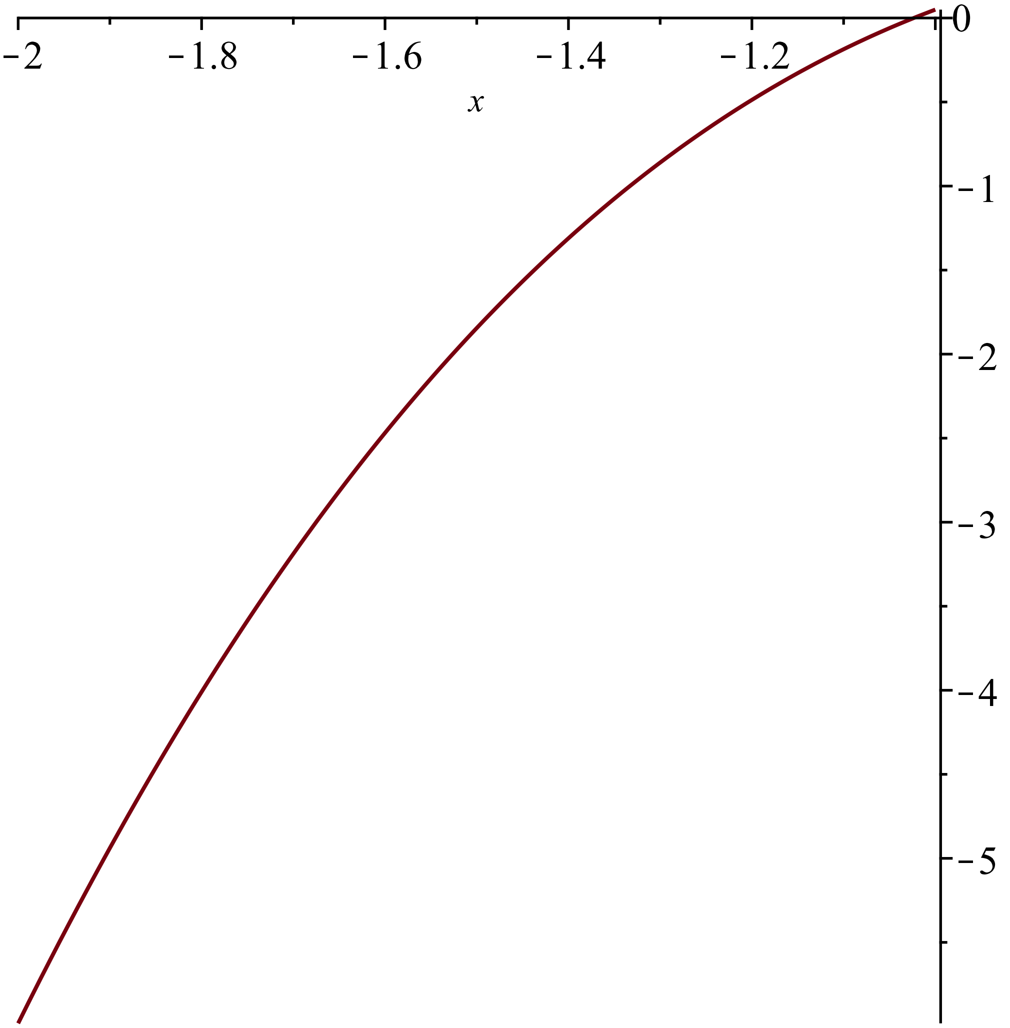
```
> plot(f(x), x=-0.5..0.5);  #plot the function f to localize the roots
```



> bisect$(-0.5, 0.5, 0.5\cdot 10^{-6},\ 1000)$; *#call the procedure 'bisect' to find the root in [-0.5,0.5]*

```
c=0.16382265, f(c)=-0.00000031
Number of iterations needed: 20
```

> *plot*$(f(x), x=-2..-1)$; *#plot the function f to localize the roots*



> bisect(-2,-1, $0.5\cdot 10^{-6}$, 1000); *#call the procedure 'bisect' to find the root in [-2,-1]*

```
c=-1.02348185, f(c)=0.00000076
Number of iterations needed: 20
```

Answer:

root = 0.58463526 with error in 6 decimal places found in 20 iterations on interval [0.5,1.5]

root = 0.21258402 with error in 6 decimal places found in 20 iterations on interval [-0.5,0.5]

root =-1.00000048 with error in 6 decimal places found in 20 iterations on interval [-2,-1]

##

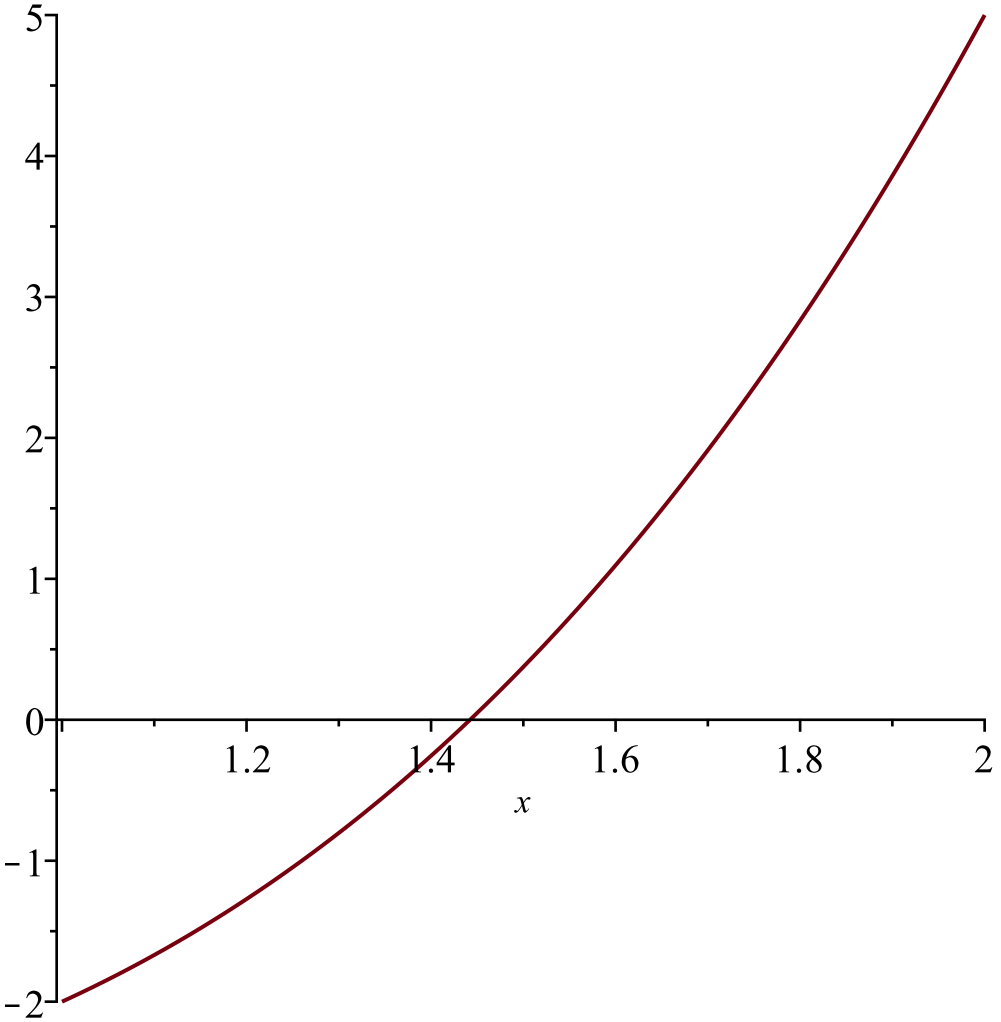
5b

Startting interval : [1, 2]

> f := x→ $x^3 - 3$; *#initialize the function f*

$$f := x \mapsto x^3 - 3 \qquad (4)$$

> *plot*(*f* (*x*), *x* = 1 ..2); *#plot the function f to localize the roots*



> bisect(1, 2, $0.5\cdot10^{-8}$, 1000); *#call the procedure 'bisect' to find the root in [0.1,1.1]*

```
c=1.44224957, f(c)=-0.00000000
Number of iterations needed: 27
```

Answer: On interval [1,2] There are 27 steps to find the cube root with 8 decimal digits and root found = 1.44224957

```
>
>
>
>
```