##

Mathematical Modeling

Inna Williams

##

Problem 10.
A psychologist wishes to conduct 2 types of studies:
Type 1 and Type 2. Each experiment requires white, gray and black rats.

Type1 :	**5 White**	**1 Gray**	**2 Black**
Type2 :	**2 White**	**3 Gray**	**2 Black**
Available :	**100 White**	**60 Gray**	**50 Black**

Type2 = 2 * Type1

How many of each type of experiment should be done to maximize the value?

Let Type1 = > x
Let Type2 => y

Solution:

Objective function : f(x,y) = x + 2y

Constrains:

x >= 0, y >= 0,
5x + 2y <= 100
x + 3y <= 60
2x + 2y <= 50

```
> 
> 
> 
> restart; with(Optimization); with(plots) :
```

$$[ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPSolve, QPSolve] \quad (1)$$

```
> obj := x + 2·y
```

$$obj := x + 2\,y \quad (2)$$

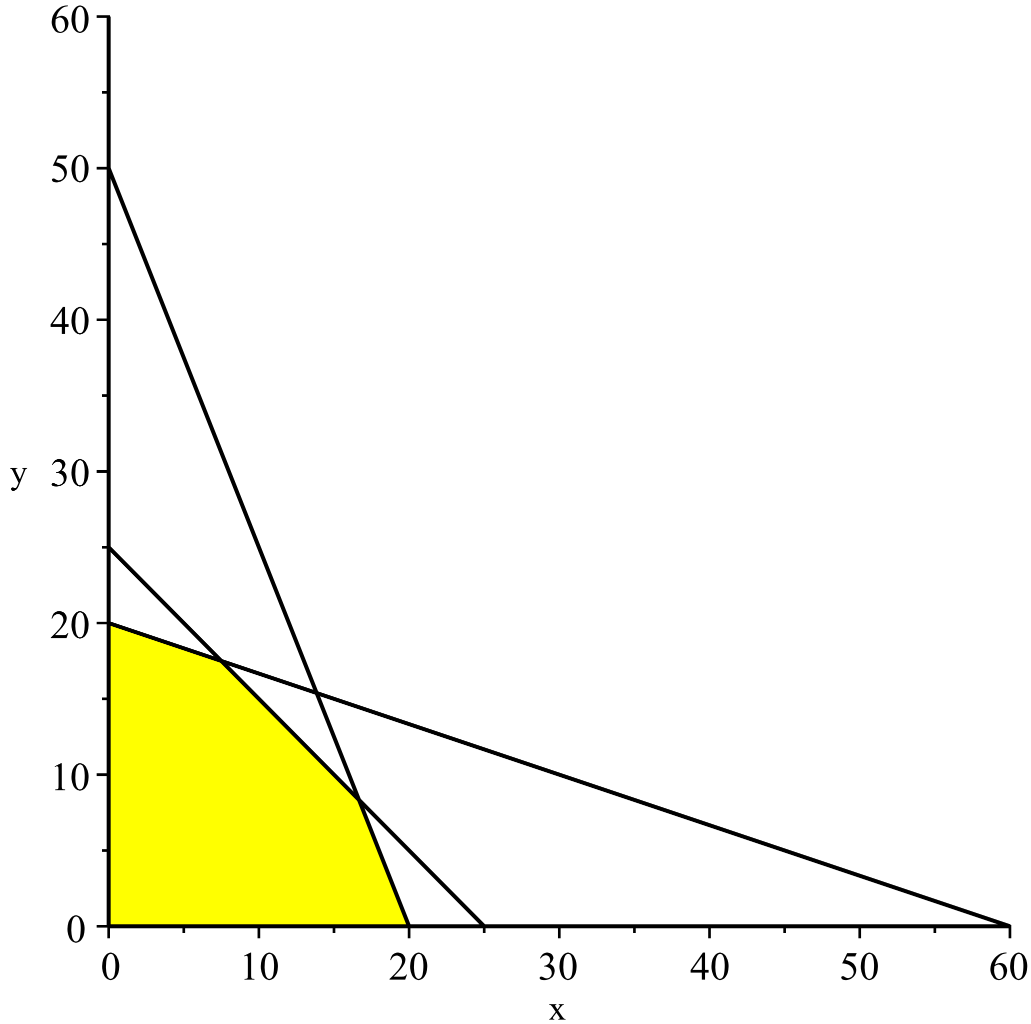
```
> constrains := [5·x + 2·y ≤ 100, x + 3·y ≤ 60, 2·x + 2·y ≤ 50, x ≥ 0, y ≥ 0]
```

$$constrains := [5\,x + 2\,y \le 100, x + 3\,y \le 60, 2\,x + 2\,y \le 50, 0 \le x, 0 \le y] \quad (3)$$

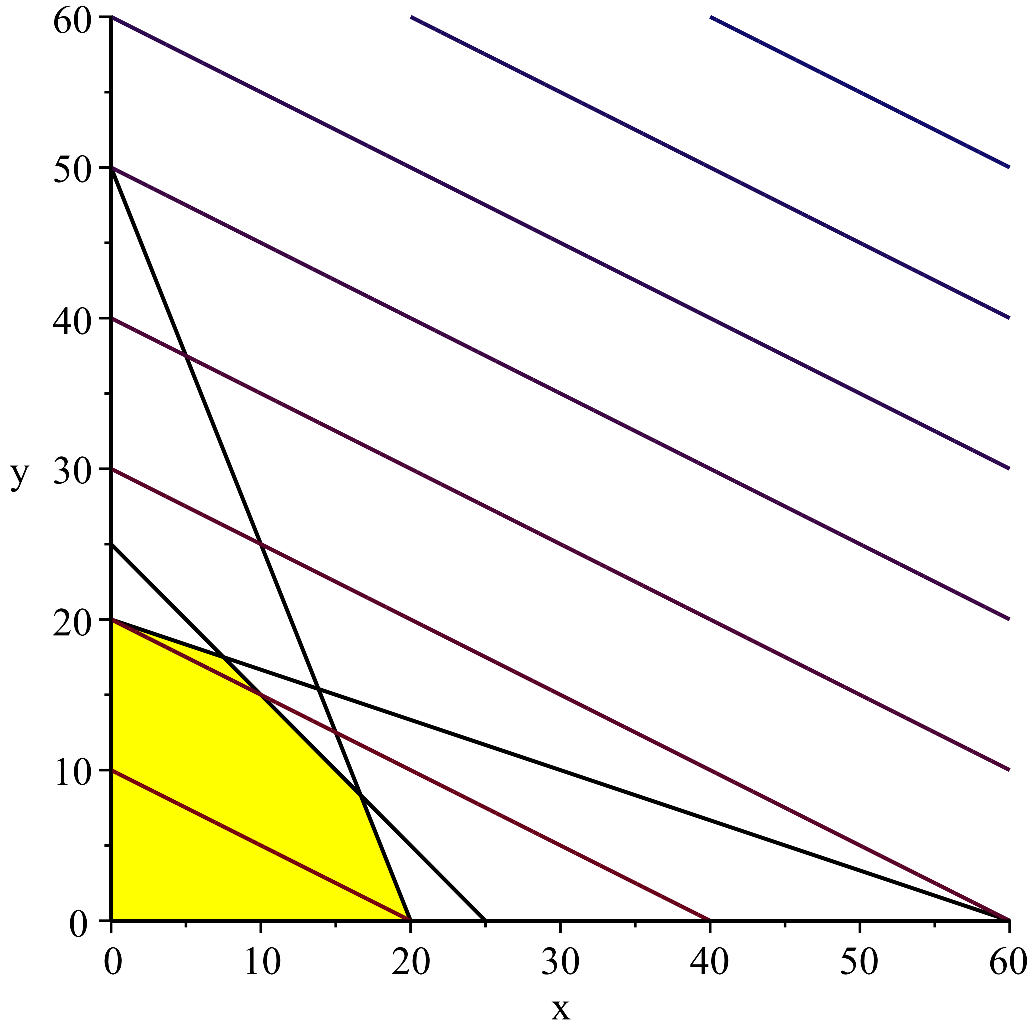
```
> LPSolve(obj, constrains, maximize, assume = integer)
```

$$[42, [x = 8, y = 17]] \quad (4)$$

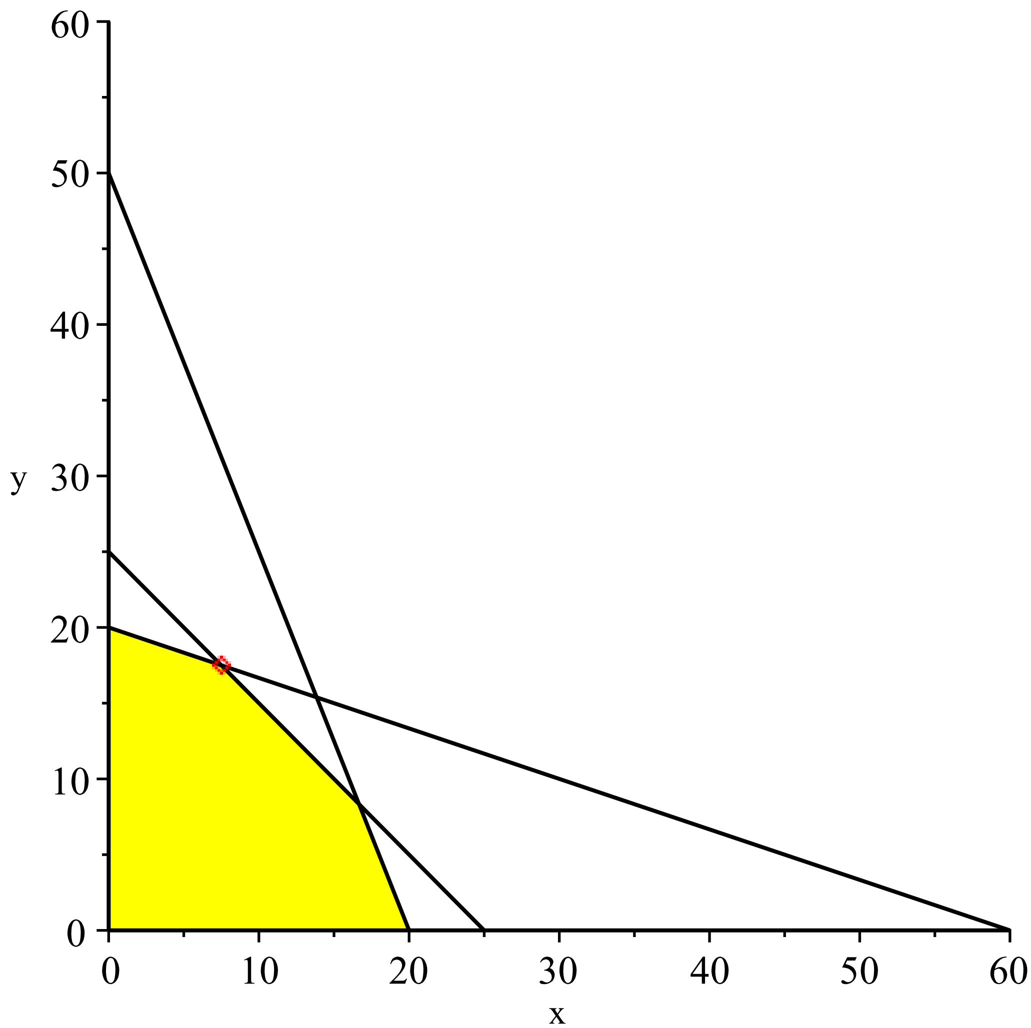
```
> p1 := inequal(constrains, x = 0 ..60, y = 0 ..60, optionsexcluded = (colour = white),
     optionsfeasible = (colour = yellow)) : display(p1)
```



> $p2 := contourplot(obj, x = 0..60, y = 0..60) : display(p1, p2)$



> $p3 := pointplot(\{[7.5, 17.5]\}, symbolsize = 13, colour = red) : display(p1, p3)$



Answer: To maximize the number of experiments it must be

Type **1 = 8,** ***y*** **= 17 ,** ***Maximum number of Experiments*** **= 8 + 2 · 17 = 42**

Constrains are satisfined:

8 >= 0

17 >= 0

5 * 8 + 2 * 17 = 74 <= 100

8 + 3*17 = 59 <= 60

2 * 8 + 2 * 17 = 42 <=50

###
###################################

Problem 11.

The King Concrete Company manufactures bags of concrete from beach and river sand.

Each pound of beach sand cost 6 cents and contain
- **4 units of fine sand**
- **3 units of coarse sand**
- **5 units of gravel**

Each pound of river sand cost 10 cents and contain
- **3 units of fine sand**
- **6 units of coarse sand**
- **12 units of gravel**

Each bag of consrete must contain
- **at least 12 units of fine sand**
- **at least 12 units of coarse sand**
- **at least 10 units of gravel**

Graphically, find the best combination o beach and river sand which will meet the minimum

requirements of the fine sand, coarse sand and gravel at the least cost and indicate the cost per pound.

Let Pound Of Beach Sand = > x

Let Pound Of River Sand = > y

Solution:

Objective function : f(x,y) = 6* x + 10*y

Constrains:

x >= 0, y >= 0,
4*x + 3*y >= 12
3*x + 6*y >= 12
5*x + 12y >= 10

> $obj := 6 \cdot x + 10 \cdot y$

$$obj := 6\,x + 10\,y \tag{5}$$

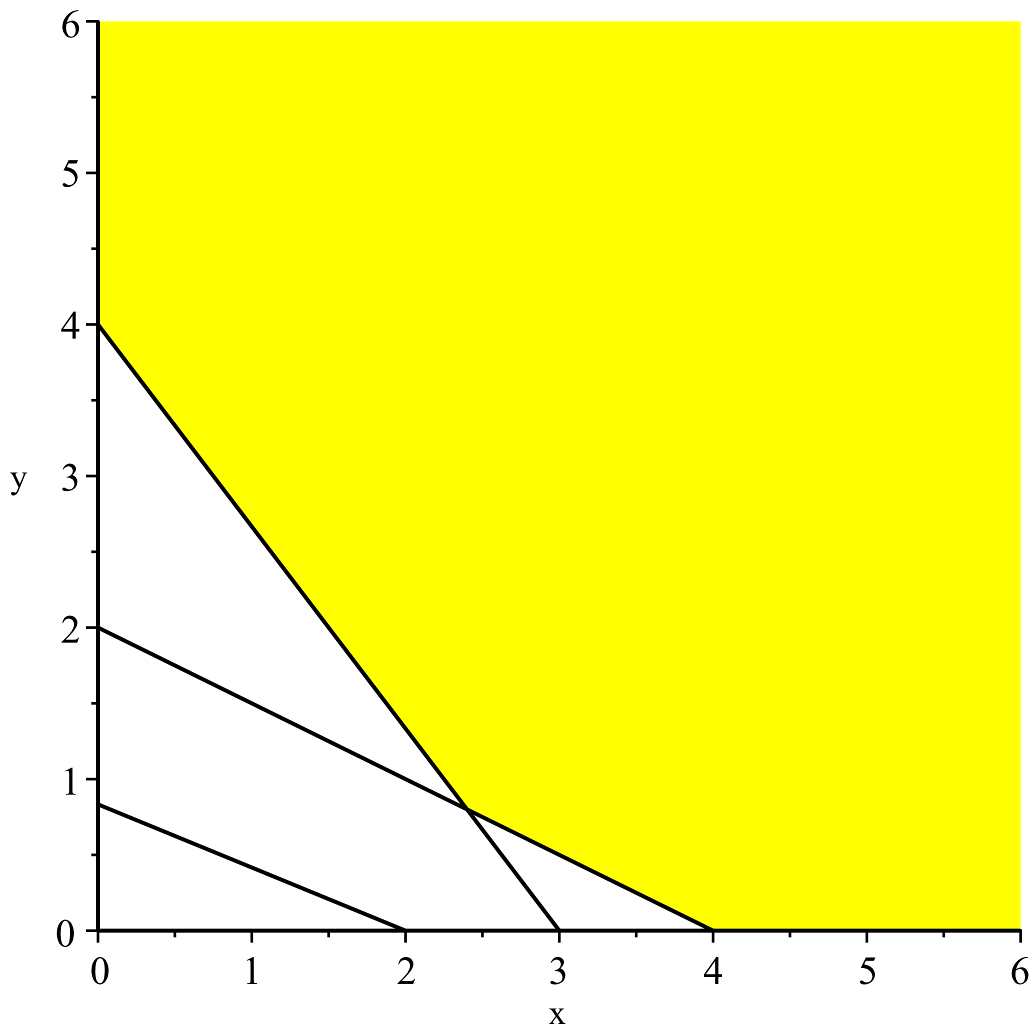
> $cons := [4 \cdot x + 3 \cdot y \geq 12, 3 \cdot x + 6 \cdot y \geq 12, 5 \cdot x + 12\,y \geq 10, x \geq 0, y \geq 0]$

$$cons := [12 \leq 4\,x + 3\,y, 12 \leq 3\,x + 6\,y, 10 \leq 5\,x + 12\,y, 0 \leq x, 0 \leq y] \tag{6}$$

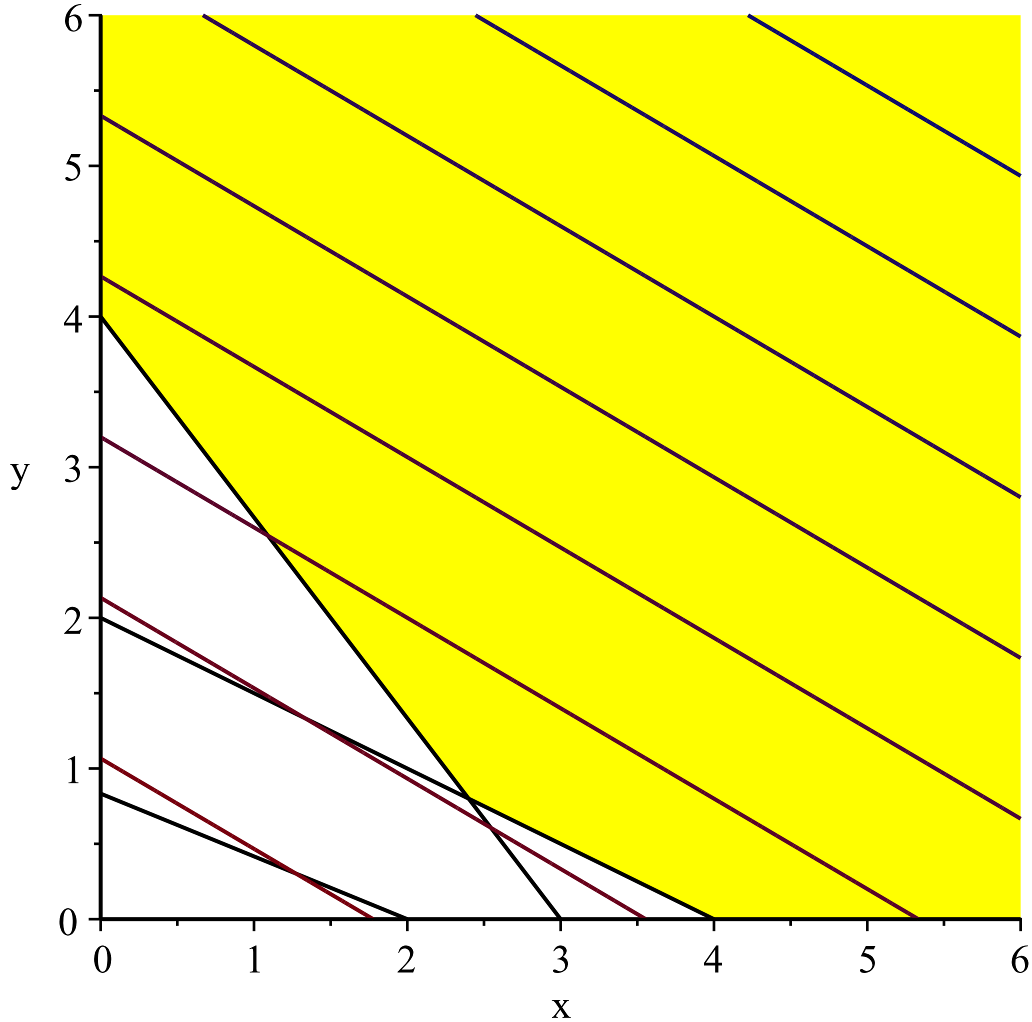
> $LPSolve(obj, cons, assume = nonnegative)$

$$[22.4000000000000, [x = 2.40000000000000, y = 0.800000000000000]] \tag{7}$$

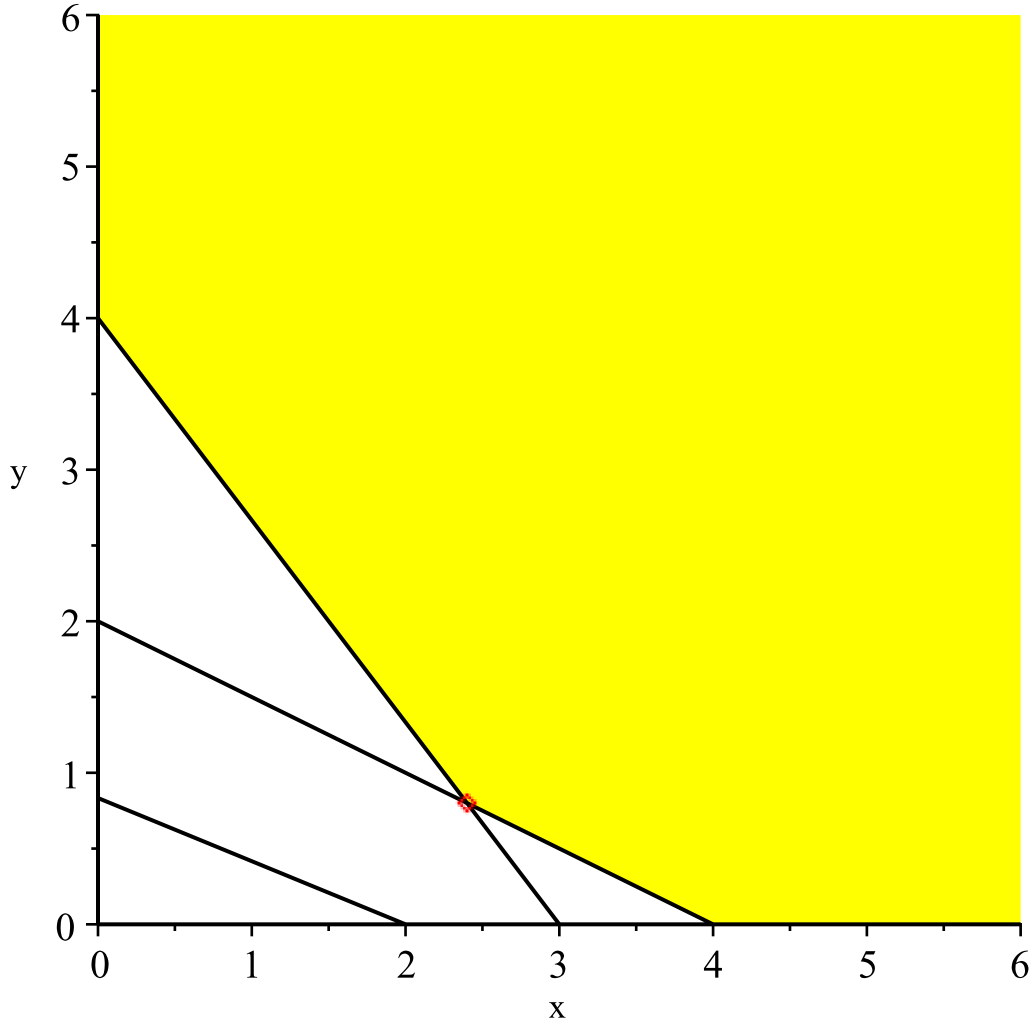
> $p1 := inequal(cons, x = 0..6, y = 0..6, optionsexcluded = (colour = white), optionsfeasible = (colour = yellow)) : display(p1)$



> $p2 := contourplot(obj, x = 0..6, y = 0..6) : display(p1, p2)$



> $p3 := pointplot(\{[2.4, 0.8]\}, symbolsize = 13, colour = red) : display(p1, p3)$



Answer:
It has to be : 2.4 pounds of the Beach sand and 0.8 pounds of the River sand
for total minimum cost = 2.4 * 6 + 10 * 0.8 = 22.4 cents.
##

Problem 7
A biologist must make a nutrient for his algae. The nutrient must contain 3 basic elements.
D, E and F.
D >= 10
E >= 12
F >= 20
The nunrient is made from 3 ingradients, 1,2 3. The quantities of the basic nutriens in this ingradients is given in a following table.
How many units of each ingradient are required to meet the biologist's needs at minimum cost

>

		Nutrients, kg per unit of ingradient		

Indradient, I unit	D >= 10	E >= 12	F >= 20	Cost, $ per unit of ingradient
1	4	3	0	4
2	1	2	4	7
3	10	1	5	5

Let ingradient 1 = x
Let ingradient 2 = y
Let ingradient 3 = z
Assume all 3 gradients must be present (x>=1,y>=1,z>=1)

Objective function :
f(x,y,z) = 4 * x + 7 * y + 5 * z
Constrains :
Assume all 3 gradients must be present
x >= 1,
y >= 1,
z >= 1,
4*x + y + 10 * z >= 10
3 * x + 2 * y + z >= 12
4 *y + 5 * z >= 20
4 * x + 7 * y + 5 * z = minimum

> $obj := 4 \cdot x + 7 \cdot y + 5 \cdot z$

$$obj := 4\,x + 7\,y + 5\,z \qquad (8)$$

> $constrains := [4 \cdot x + y + 10 \cdot z \geq 10, 3 \cdot x + 2 \cdot y + z \geq 12, 0 \cdot x + 4 \cdot y + 5 \cdot z \geq 20, x \geq 1, y \geq 1, z \geq 1]$

$$constrains := [10 \leq 4\,x + y + 10\,z, 12 \leq 3\,x + 2\,y + z, 20 \leq 4\,y + 5\,z, 1 \leq x, 1 \leq y, 1 \leq z] \qquad (9)$$

> $LPSolve(obj, constrains, assume = integer)$

$$[35, [x = 2, y = 1, z = 4]] \qquad (10)$$

Answer:
If all 3 ingradiens has to be present then
to minimize the cost it has to be :
2 units of ingradient 1
1 unit of ingradient 2
3 units of ingradient 4
Total cost = 2 *4 + 1 * 7 + 5 * 4 = $35
constrains are satisfided:
##
###############################

Problem 8:
A manufacture of sheet polyethylene has 2 plants
1 => Salt Lake City supples 120 tons per week
2 => Denver supples 140 tons per week
There are 3 distributing warehouses:
1 => Los Angeles needs 100 tons per week
2 => Chicago needs 60 tons per week
3 => New York needs 80 tons per week

Let Salt Lake City to Los Angeles shipping = x11, Salt Lake city to Chicago shipping = x12,Salt Lake city to New York shipping = x13
Let Denver to Los Angeles shipping = x21, Denver to Chicago shipping = x22, Denver to New York shipping = x23
Assume shipping not = zero from any plant

Objective function:
f(x11,x12,x13,x21,x22,x23) = 5 * x11 + 7 * x12 + 9 * x13 + 6 * x21 + 7 * x22 + 10 * x23
Constrains:
x11 >=1, x12 >=1, x13>=1,
x21 >=1, x22 >=1, x23 >=1
x11 + x12 + x13 <= 120
x21+ x22 + x23 <= 140
x11 + x21 >= 100
x12 + x22 >= 60
x13 + x23 >= 80

> $obj := 5 \cdot x11 + 7 \cdot x12 + 9 \cdot x13 + 6 \cdot x21 + 7 \cdot x22 + 10 \cdot x23$

$$obj := 5\, x11 + 7\, x12 + 9\, x13 + 6\, x21 + 7\, x22 + 10\, x23 \quad \textbf{(11)}$$

> $constrains := [x11 + x12 + x13 \leq 120, x21 + x22 + x23 \leq 140, x11 + x21 \geq 100, x12 + x22 \geq 60, x13 + x23 \geq 80, x11 \geq 1, x12 \geq 1, x13 \geq 1, x21 \geq 1, x22 \geq 1, x23 \geq 1]$

$$constrains := [x11 + x12 + x13 \leq 120, x21 + x22 + x23 \leq 140, 100 \leq x11 + x21, 60 \leq x12 + x22, 80 \leq x13 + x23, 1 \leq x11, 1 \leq x12, 1 \leq x13, 1 \leq x21, 1 \leq x22, 1 \leq x23] \quad \textbf{(12)}$$

> $LPSolve(obj, constrains, assume = integer)$

$$[1701, [x11 = 99, x12 = 1, x13 = 20, x21 = 1, x22 = 59, x23 = 60]] \quad \textbf{(13)}$$

Answer:
With asumption that no no shipping = zero($x11 \geq 1, x12 \geq 1, x13 \geq 1, x21 \geq 1, x22 \geq 1, x23 \geq 1$)
minimized cost the plants must ship :

Let Salt Lake City to Los Angeles : 99 tons
Salt Lake city to Chicago : 1 ton
Salt Lake city to New York : 20 tons
Denver to Los Angeles : 1 ton
Denver to Chicago : 59 tons
Denver to New York : 60 tons

Minimum Cost = 1701
###

Problem 10 sing simplex method

>
>
>
>
>
>