Métodos Numéricos II

Tarefa 4

Rodrigo Nogueira[473413] e Victor Torres[473741]

11 de Maio de 2022

1 Estimativa de Erro para a Regra de Milne

Utilizando-se a filosofia aberta e 2 pontos, tem-se a Regra de Milne, que define

$$\begin{aligned} I_f &= \tfrac{4h}{3}(2f(a+h) - f(a+2h) + 2f(a+3h)) \\ &= \tfrac{1}{3}(b-a)\left(2f\left(\frac{3a+b}{4}\right) - f\left(\frac{a+b}{2}\right) + 2f\left(\frac{a+3b}{4}\right)\right). \end{aligned}$$

Agora, utilizando expansões de Taylor para representar o valor da função nos pontos $a+h, a+2h$ e $a+3h$, temos

$$\begin{aligned} f(a+1h) &= f(\bar{x}) - f'(\bar{x})h + \tfrac{1}{2}f''(\bar{x})h^2 - \tfrac{1}{6}f'''(\bar{x})h^3 + \cdots, \\ f(a+2h) &= f(\bar{x}), \\ f(a+3h) &= f(\bar{x}) + f'(\bar{x})h + \tfrac{1}{2}f''(\bar{x})h^2 + \tfrac{1}{6}f'''(\bar{x})h^3 + \cdots, \end{aligned}$$

ou seja,

$$I_f = \tfrac{4h}{3}(3f(\bar{x}) + 2f''(\bar{x})h^2 + \tfrac{1}{6}f^{(\mathrm{iv})}(\bar{x})h^4 + \cdots$$

Agora, temos que $I_e = \int_a^b f(x)dx$, mas expandindo $f(x)$ na vizinhança de $\bar{x}$, podemos dizer que $x(\xi) = \bar{x} + h\xi$ e $dx = hd\xi$, daí,

$$\begin{aligned} I_e &= h\int_{-2}^{+2} (f(\bar{x}) + f'(\bar{x})(h\xi) + \tfrac{1}{2}f''(\bar{x})(h\xi)^2 + \tfrac{1}{6}f'''(\bar{x})(h\xi)^3 + \cdots)\, d\xi \\ &= h\left[f(\bar{x})\xi + \tfrac{1}{2}f'(\bar{x})h\xi^2 + \tfrac{1}{6}f''(\bar{x})h^2\xi^3 + \tfrac{1}{24}f'''(\bar{x})h^3\xi^4 + \cdots\right]_{-2}^{+2} \\ &= h(4f(\bar{x}) + \tfrac{8}{3}f(\bar{x})h^2 + \tfrac{8}{15}f^{(\mathrm{iv})}(\bar{x})h^4 + \cdots). \end{aligned}$$

Então, o erro $E = I_e - I_f$ será

$$
\begin{aligned}
E &= h(4f(\bar{x}) + \tfrac{8}{3}f''(\bar{x})h^2 + \tfrac{8}{15}f^{(\mathrm{iv})}(\bar{x})h^4 + \cdots) \\
&\quad - \tfrac{4}{3}h(3f(\bar{x}) + 2f''(\bar{x})h^2 + \tfrac{1}{6}f^{(\mathrm{iv})}(\bar{x})h^4 + \cdots) \\
&\approx \tfrac{8}{15}f^{(\mathrm{iv})}(\bar{x})h^5 - \tfrac{2}{9}f^{(\mathrm{iv})}(\bar{x})h^5 \qquad \text{(termo dominante)} \\
&= \tfrac{14}{45}f^{(\mathrm{iv})}(\bar{x})\left(\tfrac{1}{4}\Delta x\right)^5 \\
&= \tfrac{7}{23040}f^{(\mathrm{iv})}(\bar{x})(\Delta x)^5.
\end{aligned}
$$