Collinear mass for $H \to \tau\tau$

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In the measurement of $H \to \tau\tau$ (and any other physics processes with the two τ-lepton final state), there are always at least two neutrinos in the final state, due to the τ lepton decays; e.g.

- $\tau^- \to \pi^- \nu_\tau$
- $\tau^- \to \pi^- \pi^0 \nu_\tau$
- $\tau^- \to \pi^- \pi^+ \pi^- \nu_\tau$
- $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$)

etc. There is always one neutrino (ν_τ) for the hadronic τ decays, while there are two neutrinos (ν_τ and $\bar{\nu}_\ell$) for the leptonic τ decays. Therefore, depending on combination of decays of the two τ leptons, there are two to four neutrinos in $H \to \tau\tau$. A momentum brought away by these neutrinos can be only measured as the 'missing transverse momentum' (p_T^{miss}), which is the transverse component of the total neutrino momenta, as the negative of the sum of the transverse momenta of all the physics objects measured in the detector. The z-component (in the direction of the pp beams) of the missing momentum cannot be determined in the pp collider, because it is not possible to determine the initial momenta of the colliding partons.

A collinear mass is the approximated Higgs mass, assuming that the neutrino(s) from each τ lepton decay fly in the direction of the visible component of the decay daughters (e.g. sum of $\pi^-\pi^-$ for $\tau^- \to \pi^-\pi^0\nu_\tau$, ℓ^- for $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ etc.). Let us denote h for all hadrons in the hadornic τ decay, and exclusively discuss the case where both τ leptons decay hadronically.[1] In the collinear assumption, the neutrino momentum can be written as[2]

$$p_\nu^i = \left(E_\nu^i, E_\nu^i \frac{\mathbf{p}_h^i}{|\mathbf{p}_h^i|} \right), \tag{1}$$

where quantities with the suffix ν (h) correspond to the neutrino (hadrons), and the index $i (= 1, 2)$ indicates one of the τ leptons and its decay products.

The missing transverse momentum always have just two components on the transverse plane, and these two components ($p_{T,x}^{\text{miss}}, p_{T,y}^{\text{miss}}$) can be written by

$$\begin{align} p_{T,x}^{\text{miss}} &= p_{\nu,x}^1 + p_{\nu,x}^2 \tag{2} \\ &= E_\nu^1 \frac{p_{h,x}^1}{|\mathbf{p}_h^1|} + E_\nu^2 \frac{p_{h,x}^2}{|\mathbf{p}_h^2|}. \tag{3} \end{align}$$

Likewise, for the y component,

$$p_{T,x}^{\text{miss}} = E_\nu^1 \frac{p_{h,y}^1}{|\mathbf{p}_h^1|} + E_\nu^2 \frac{p_{h,y}^2}{|\mathbf{p}_h^2|}. \tag{4}$$

[1] However, the same discussion can apply to the case where only one of the τ leptons decays hadronically while its counterpart decays leptonically, or both τ leptons decay leptonically.

[2] Throughout the note, p always denotes the 4-momentum, while $\mathbf{p}$ the three-dimentional component.

By solving (3) and (4) for E_ν^1 and E_ν^2,

$$\begin{pmatrix} p_{T,x}^{\text{miss}} \\ p_{T,y}^{\text{miss}} \end{pmatrix} = \begin{pmatrix} \frac{p_{h,x}^1}{|\mathbf{p}_h^1|} & \frac{p_{h,x}^2}{|\mathbf{p}_h^2|} \\ \frac{p_{h,y}^1}{|\mathbf{p}_h^1|} & \frac{p_{h,y}^2}{|\mathbf{p}_h^2|} \end{pmatrix} \begin{pmatrix} E_\nu^1 \\ E_\nu^2 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} E_\nu^1 \\ E_\nu^2 \end{pmatrix} = \frac{1}{\frac{p_{h,x}^1 p_{h,y}^2 - p_{h,y}^1 p_{h,x}^2}{|\mathrm{p}_h^1||\mathrm{p}_h^2|}} \begin{pmatrix} \frac{p_{h,y}^2}{|\mathbf{p}_h^2|} & -\frac{p_{h,x}^2}{|\mathbf{p}_h^2|} \\ -\frac{p_{h,y}^1}{|\mathbf{p}_h^1|} & \frac{p_{h,x}^1}{|\mathbf{p}_h^1|} \end{pmatrix} \begin{pmatrix} p_{T,x}^{\text{miss}} \\ p_{T,y}^{\text{miss}} \end{pmatrix}, \tag{6}$$

therefore

$$E_\nu^1 = |\mathrm{p}_h^1| \frac{p_{T,x}^{\text{miss}} p_{h,y}^2 - p_{T,y}^{\text{miss}} p_{h,x}^2}{p_{h,x}^1 p_{h,y}^2 - p_{h,y}^1 p_{h,x}^2}, \tag{7}$$

$$E_\nu^2 = |\mathrm{p}_h^2| \frac{-p_{T,x}^{\text{miss}} p_{h,y}^1 + p_{T,y}^{\text{miss}} p_{h,x}^1}{p_{h,x}^1 p_{h,y}^2 - p_{h,y}^1 p_{h,x}^2} \tag{8}$$

The Higgs boson mass in $H \to \tau\tau$ is represented by

$$\begin{aligned} m_{\tau\tau}^2 &= (E_\tau^1 + E_\tau^2)^2 - (\mathbf{p}_\tau^1 + \mathbf{p}_\tau^2)^2 & (9) \\ &= (E_\tau^1)^2 + (E_\tau^2)^2 + 2E_\tau^1 E_\tau^2 - |\mathbf{p}_\tau^1|^2 - |\mathbf{p}_\tau^2|^2 - 2|\mathbf{p}_\tau^1||\mathbf{p}_\tau^2| \cos \Delta\phi_{\tau\tau} & (10) \\ &= 2m_\tau^2 + 2E_\tau^1 E_\tau^2 - 2|\mathbf{p}_\tau^1||\mathbf{p}_\tau^2| \cos \Delta\phi_{\tau\tau}. & (11) \end{aligned}$$

The momentum of the τ leptons from the Higgs boson decay is about 63 GeV, which is sufficiently higher than the τ lepton mass (1.8 GeV). Therefore we can consider $|\mathbf{p}_\tau^i| \sim E_\tau^i$ $(i = 1, 2)$. The angle difference $\Delta\phi_{\tau\tau}$ is the difference of the two τ leptons. Since we now consider the neutrino direction is completely aligned with the direction of the hadronic components, the τ lepton direction is completely the same as the hadronic components. The $\Delta\phi_{\tau\tau}$ is thus identical to the angle difference between the hadronic components h_1 and h_2: $\Delta\phi_{h_1 h_2}$. Using them, (11) is reformed to

$$m_{\tau\tau}^2 = 2m_\tau^2 + 2E_\tau^1 E_\tau^2 (1 - \cos \Delta\phi_{h_1 h_2}). \tag{12}$$

Let us introduce the two quantities x_1 and x_2, which denote the ratio of E_h to E_τ:

$$\begin{aligned} x_i &= \frac{E_h^i}{E_\tau^i} \ (i = 1, 2) & (13) \\ &= \frac{E_h^i}{E_h^i + E_\nu^i}. & (14) \end{aligned}$$

Using x_i, the di-τ mass is then

$$m_{\tau\tau}^2 = 2m_\tau^2 + \frac{2E_h^1 E_h^2}{x_1 x_2} (1 - \cos \Delta\phi_{h_1 h_2}). \tag{15}$$

Energy must be always greater than 0, so physical range of x_i should be $0 < x_i < 1$. Since E_h^i is measured using the detector, whether or not x_i is within or outside the physical range depends on the estimate of E_ν^i. In order for E_ν^i to be positive, for the case $i = 1$, both numerator and denominator of

Eq. (7) must be positive:

$$-p_{T,x}^{\text{miss}} p_{h,y}^2 + p_{T,y}^{\text{miss}} p_{h,x}^2 > 0 \tag{16}$$

$$p_{T,y}^{\text{miss}} p_{h,x}^2 < p_{T,x}^{\text{miss}} p_{h,y}^2 \tag{17}$$

$$\frac{p_{T,y}^{\text{miss}}}{p_{T,x}^{\text{miss}}} < \frac{p_{h,y}^2}{p_{h,x}^2}, \tag{18}$$

$$p_{h,y}^2 p_{h,x}^1 - p_{h,x}^2 p_{h,y}^1 > 0 \tag{19}$$

$$p_{h_y}^2 p_{h,x}^1 > p_{h,x}^2 p_{h,y}^1 \tag{20}$$

$$\frac{p_{h_y}^2}{p_{h,x}^2} < \frac{p_{h,y}^1}{p_{h,x}^1}. \tag{21}$$

Here, on the transverse plane, let us define

- ϕ_{miss}: angle of the p_T^{miss} vector,
- ϕ_1: angle of the p_h^1 vector,
- ϕ_2: angle of the p_h^2 vector,

and define the x- and y-axes so all the vectors are within $-\pi/2 < \phi_1, \phi_2, \phi_{\text{miss}} < \pi/2$. In this range, a tangent is monotonically increasing, therefore, from Eqs. (18) and (21) respectively,

$$\tan\phi_{\text{miss}} < \tan\phi_2 \tag{22}$$

$$\phi_{\text{miss}} < \phi_2, \tag{23}$$

$$\tan\phi_1 < \tan\phi_2 \tag{24}$$

$$\phi_1 < \phi_2. \tag{25}$$

Similarly for Eq. (8), we obtain

$$\phi_1 < \phi_{\text{miss}}, \tag{26}$$

$$\phi_1 < \phi_2, \tag{27}$$

therefore

$$\phi_1 < \phi_{\text{miss}} < \phi_2. \tag{28}$$

Another condition to have Eq. (7) positive is that both denominator and numerator negative. In this case, one obtains the relation: $\phi_2 < \phi_{\text{miss}} < \phi_1$. This is equivalent to Eq. (28).

In summary, the following relation is required: $\phi_1 < \phi_{\text{miss}} < \phi_2$. This indicates that, on the transverse plane, the direction of missing momentum must be within the vectors of the two hadronic components of the τ lepton decays.