Homework 1

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Problem 1

(a) Let A denotes the event that none has high blood lead level in a randomly selected group of 16 children.

Then $P(A) = (1 - \frac{1}{8})^{16} = 0.1181$

(b) Let B denotes the event that one child has high blood lead level in a randomly selected group of 16 children.

Then $P(B) = C_{16}^{1}\left(\frac{1}{8}\right) \times \left(\frac{7}{8}\right)^{15} = 0.2699$

(c) Let C denotes the event that two children have high blood lead level in a randomly selected group of 16 children.

Then $P(C) = C_{16}^{2}\left(\frac{1}{8}\right)^{2} \times \left(\frac{7}{8}\right)^{14} = 0.2891$

(d) Let D denotes the event that three or more have high blood lead level in a randomly selected group of 16 children.

Then $P(D) = 1 - P(A) - P(B) - P(C) = 0.3229$

Problem 2

Let X denotes the height of corn plants, then $X \sim N(145, 22)$

(a) $P(135 \le X \le 155) = P(\frac{135-145}{22} \le \frac{X-145}{22} \le \frac{155-145}{22}) = P(-\frac{5}{11} \le Z \le \frac{5}{11}) = P(Z \le \frac{5}{11}) - P(Z \le -\frac{5}{11})$

Because $pnorm(5/11) = 0.6753$ and $pnorm(-5/11) = 0.3247$,

$P(135 \le X \le 155) = 0.6753 - 0.3247 = 0.3506$

(b) According to central limit theory, $\sum_{k=1}^{16} X_k \sim N(16 \times 145, \sqrt{16} \times 22)$.

Thus, $\frac{\sum_{k=1}^{16} X_k}{16} \sim N(145, \frac{22}{\sqrt{16}}) \Rightarrow \bar{X} \sim N(145, 5.5)$ for a random sample of 16 plants.

$$P(135 \le \bar{X} \le 155) = P(\frac{135-145}{5.5} \le \frac{\bar{X}-145}{5.5} \le \frac{155-145}{5.5}) = P(-\frac{20}{11} \le Z \le \frac{20}{11}) = P(Z \le \frac{20}{11}) - P(Z \le -\frac{20}{11})$$

Because $pnorm(20/11) = 0.9655$ and $pnorm(-20/11) = 0.0345$,

$P(135 \le \bar{X} \le 155) = 0.9655 - 0.0345 = 0.9310$

(c) According to central limit theory, $\sum_{k=1}^{32} X_k \sim N(32 \times 145, \sqrt{32} \times 22)$.

Thus, $\frac{\sum_{k=1}^{32} X_k}{32} \sim N(145, \frac{22}{\sqrt{32}}) \Rightarrow \bar{X} \sim N(145, \frac{22}{\sqrt{32}})$ for a random sample of 32 plants.

$$P(135 \le \bar{X} \le 155) = P(\frac{135-145}{22/\sqrt{32}} \le \frac{\bar{X}-145}{22/\sqrt{32}} \le \frac{155-145}{22/\sqrt{32}}) = P(Z \le 2.5713) - P(Z \le -2.5713)$$

$$P(135 \le \bar{X} \le 155) = pnorm(2.5713) - pnorm(-2.5713) = 0.9899$$

Problem 3

Let A denotes the event that individuals preferred to use right hand to write.

Let B denotes the event that individuals preferred to use right foot to kick a ball.

Then $P(B) = \frac{2012+121}{2391} = 0.8921$

$P(B \mid A) = \frac{2012}{2012+142} = 0.9341$

Since $P(B) \ne P(B \mid A)$, hand and foot preferences are not independent.

Exercise 2.1

Let G denotes a child is a girl.

Let B denotes a child is a boy.

Let A denotes that at least one child is boy.

Then we have $P(GG) = P(BB) = P(GB) = P(BG) = \frac{1}{4}$

(a) The question actually asks what is $P(GB \mid A) + P(BG \mid A)$.

Using Bayes rules, we have

$$P(GB \mid A) + P(BG \mid A) = 1 - P(BB \mid A) = 1 - \frac{P(A \mid BB) \times P(BB)}{P(A)} = 1 - \frac{1/4}{3/4} = \frac{2}{3}$$

(b) Let b denotes the event that I happen to see one of his children and it is a boy.

The question actually asks what is $P(GB \mid b) + P(BG \mid b)$.

Using Bayes rules, we have

$$P(GB \mid b) + P(BG \mid b) = \frac{P(b \mid GB) \times P(GB)}{P(b)} + \frac{P(b \mid BG) \times P(BG)}{P(b)} = \frac{1/2 \times 1/4}{1/2} + \frac{1/2 \times 1/4}{1/2} = \frac{1}{2}$$

Exercise 2.2

Exercise 2.4

Exercise 2.5

Exercise 2.12