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Module 3 Homework

1) (5 points)

Suppose a random variable X has pdf as $f(x) = 2e^{-2(x-1)}$, $x > 1$. Which of the following represents $P(0 < X < 4)$? (Note: you do not need to solve for exact number).

(a) $\int_0^4 2e^{-2(x-1)} dx$;

(b) $\int_1^4 2e^{-2(x-1)} dx$;

(c) $\int_0^4 x 2e^{-2(x-1)} dx$;

(d) $\sum_{x=0}^4 2e^{-2(x-1)}$;

(e) $\int_1^{\infty} x 2e^{-2(x-1)} dx$.



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2) (10 points)

A random variable X has pdf

$$f(x) = \frac{2^x}{x!} e^{-2}, \quad x = 0, 1, 2, \dots$$

Find $P(X = 1)$.

Then find $P(-2 < X < 4)$.

Give your answers to at least four decimal places.



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3) (5 points)

If two carriers of the gene for albinism marry and have children, then each of their children has a probability of $1/4$ of being albino. Let the random variable Y denote the number of their albino children out of all 3 of their children. Then Y follows a binomial(n , p) distribution. Find the values for n and p .

$n = \underline{\hspace{2cm}}$ $p = \underline{\hspace{2cm}}$



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4) (10 points)

For Y following a binomial ($n = 3$, $p = 0.25$) distribution, compute the following:

$$P(Y \leq 2) =$$

$$E(Y) =$$

$$\text{Var}(Y) =$$

Give your answers to at least four decimal places.



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5) (20 points)

For X following a Chi-square distribution with degree of freedom $m = 3$, compute the following:

$$P(1 < X < 4) =$$

$$E(X) =$$

$$\text{Var}(X) =$$

Give your answers to at least four decimal places.

Also, use a Monte Carlo simulation with sample size $n=100,000$ to estimate $P(1 < X < 4)$. What is your Monte Carlo estimate? Does it agree with the answer above?



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6) (10 points)

Suppose X follows a Chi-square distribution with degree of freedom $m = 5$ so that $E(X) = 5$ and $\text{Var}(X) = 10$. Also, let $Y = 4X - 10$. Find $E(Y)$ and $\text{Var}(Y)$. Does Y follow a Chi-square distribution with degree of freedom $m=10$?

$$E(Y) =$$

$$\text{Var}(Y) =$$

Does Y follow a Chi-square distribution with degree of freedom $m = 10$?



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7) (20 points)

The Zyxin gene expression values are distributed according to $N(\mu = 1.6, \sigma = 0.4)$.

(a) What is the probability that a randomly chosen patient have the Zyxin gene expression values between 1 and 1.6?

(b) Use a Monte Carlo simulation of sample size $n=500,000$ to estimate the probability in part (a). Give your R code, and show the value of your estimate.

(c) What is the probability that exactly 2 out of 5 patients have the Zyxin gene expression values between 1 and 1.6?

Please show your work on how to arrive at the answer. Give your answer to at least four decimal places.



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8) (20 points)

- (a) Hand in a R script that calculates the mean and variance of two random variables $X \sim F(m=2, n=5)$ and $Y \sim F(m=10, n=5)$ from their density functions.
- (b) Use the formula in Table 3.4.1 to calculate the means and variances directly.
- (c) Run your script in (a), and check that your answers agree with those from part (b).