**Module 3 Homework**

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**Problem 1. (5 points)**

Suppose a random variable X has pdf as . Which of the following represents ?

**(a)**

**(b)**

**(c)**

**(d)**

**(e)**

**Problem 2. (10 points)**

A random variable X has pdf

, 

Find 

Then find .

f2 <- function(x) {2^x\*exp(-2)/gamma(x+1)}

print(f2(1))

[1] 0.2706706

print(f2(0)+f2(1)+f2(2)+f2(3))

[1] 0.8571235





**Problem 3. (5 points)**

If two carriers of the gene for albinism marry and have children, then each of their children has a probability of 1/4 of being albino. Let the random variable Y denote the number of their albino children out of all 3 of their children. Then Y follows a binomial(n, p) distribution. Find the values for n and p.

n=3 p=1/4

**Problem 4. (10 points)**

For Y following a binomial (n=3, p=0.25) distribution, compute the following:

**** and 

yrange = c(0:2)

print(sum(dbinom(yrange, size=3, p=0.25)))

[1] 0.984375

Yrange = c(0:3)

EY <- sum(Yrange\*dbinom(Yrange, size=3, p=0.25))

print(EY)

[1] 0.75

VarY <- sum( (Yrange-EY)^2 \* dbinom(Yrange, size=3, p=0.25))

print(VarY)

[1] 0.5625







**Problem 5. (20 points)**

For X following a Chi-square distribution with degree of freedom m=3, compute the following:

, and .

print(integrate(function(x) dchisq(x, df = 3), lower=1, upper=4)$value)

[1] 0.5397878

EX <- integrate(function(x) x\*dchisq(x, df = 3), lower=-Inf, upper=Inf)$value

print(EX)

[1] 3

VarX <- integrate(function(x) (x-EX)^2\*dchisq(x, df = 3), lower=-Inf, upper=Inf)$value

print(VarX)

[1] 6

Also, use a Monte Carlo simulation with sample size n=100,000 to estimate .

x <- rchisq(n=100000, df=3)

print( mean( (1<x) & (x<4) ) )

[1] 0.5396

It agrees the answer above.

**Problem 6. (10 points)**

Suppose X follows a Chi-square distribution with degree of freedom m = 5 so that E(X) = 5 and Var(X) = 10. Also, let Y = 4X - 10. Find E(Y) and Var (Y). Does Y follow a Chi-square distribution with degree of freedom m=10?





No, Y doesn’t follow a Chi-square distribution with degree of freedom m=10.

Problem 7. (20 points)

The Zyxin gene expression values are distributed according to

.

(a) What is the probability that a randomly chosen patient have the Zyxin

gene expression values between 1 and 1.6?

p <- integrate(function(x) dnorm(x, 1.6, 0.4), lower=1, upper=1.6)$value

print(p)

[1] 0.4331928

(b) Use a Monte Carlo simulation of sample size n=500,000 to estimate the

probability in part (a). Give your R code, and show the value of your

estimate.

x <- rnorm(n=500000, 1.6, 0.4)

print(mean((1<x)&(x<1.6)))

[1] 0.433596

(c) What is the probability that exactly 2 out of 5 patients have the Zyxin

gene expression values between 1 and 1.6?

print(dbinom(2, 5, p))

[1] 0.3417185

**8) (20 points)**

**(a)** Hand in a R script that calculates the mean and variance of two random variables X~F(m=2,n=5) and Y~F(m=10,n=5) from their density functions.

**(b)** Use the formula in Table 3.4.1 to calculate the means and variances directly.

**(c)** Run your script in (a), and check that your answers agree with those from part (b).

rm(list=ls())

m <- 2

n <- 5

print("mean and variance of X~F(m=2, n=5)")

EX<-integrate(function(x) x\*df(x, m, n), lower=0, upper=Inf)$value

print(EX)

[1] 1.666667

VarX<-integrate(function(x) (x-EX)^2\*df(x, m, n), lower=0, upper=Inf)$value

print(VarX)

[1] 13.88889

print(n/(n-2))

[1] 1.666667

print( 2\*n^2\*(m+n-2)/(m\*(n-2)^2\*(n-4)) )

[1] 13.88889

rm(list=ls())

m <- 10

n <- 5

print("mean and variance of X~F(m=10, n=5)")

EX<-integrate(function(x) x\*df(x, m, n), lower=0, upper=Inf)$value

print(EX)

[1] 1.666667

VarX<-integrate(function(x) (x-EX)^2\*df(x, m, n), lower=0, upper=Inf)$value

print(VarX)

[1] 7.222222

print(n/(n-2))

[1] 1.666667

print( 2\*n^2\*(m+n-2)/(m\*(n-2)^2\*(n-4)) )

[1] 7.222222

Answers from part(a) and part(b) match each other.