Quadratics assignment

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MPM2DE-B

Contents

1 My values

My class list number is 26. My values are therefore:

- $a = 3$
- $b = 32$
- $c = -11$
- $u = 36$
- $v = 484$
- $w = 18$

2 Distributive property

2.1 Substituting values in

$$a(bx + c)^2 = 3(32x - 11)^2$$

2.2 Expanding and simplifying

Firstly, I prefer to begin by expanding exponents. I'll start by using the distributive property to expand the square in this expression.
Knowing that by the distributive property, $(m + n)^2 = m^2 + 2mn + n^2$...

$$3(32x - 11)^2 = 3[(32x)^2 + 2(32x)(-11) + (-11)^2]$$

Simplifying that...

$$3(32x - 11)^2 = 3(1024x^2 - 704x + 121)$$

Now, I can expand this further using the distributive property by multiplying every term inside of those brackets by 3.

$$3(32x - 11)^2 = 3072x^2 - 2112x + 363\blacksquare$$

This is as simple as it gets - no like terms to collect or anything.

3 Decomposition

3.1 Substituting values in

$$ax^2 + bx + c = 3x^2 + 32x - 11$$

3.2 The decomposition process

3.2.1 Finding some appropriate values

Looking at the polynomial on our hands here, I know that there aren't any common factors to remove. I will look for a pair of values such that they sum to $b = 32$ and have a product of $a \times c = -33$.
Testing out some pairs of factors of -33, I can immediately notice that the numbers 33 and -1 will have a product of -33 and a sum of 32.

3.2.2 Working with the trinomial

Now, I can decompose the middle term of the trinomial into the values I found earlier to begin to factor it.

$$3x^2 + 32x - 11 = 3x^2 - x + 33x - 11$$

Notice that this very conveniently creates an opportunity to group some terms and begin to find some common factors.

$$(3x^2 - x) + (33x - 11) = x(3x - 1) + 11(3x - 1)$$

Rearranging this last expression a little, we find the fully factored trinomial.

$$3x^2 + 32x - 11 = (3x - 1)(x + 11)\blacksquare$$

4 Difference of squares

4.1 Substituting values in

$$ux^2 - v = 36x^2 - 484$$

4.2 Find the right method to factor this binomial

Now, you didn't explicitly tell us what factoring method to use here, but I can immediately tell that there is a very easy way to factor this binomial. This is because this binomial happens to be a difference of squares: both $36x^2$ and 484 happen to be perfect squares! We can therefore very easily factor this binomial using the difference of squares method.

4.3 Factoring the binomial with difference of squares

Knowing that the factoring of a difference of squares takes the form of $m^2 - n^2 = (m + n)(m - n)$, we can observe that from the form of the equation, $m^2 = 36x^2$ and $n^2 = 484$. We therefore know that $m = \pm\sqrt{36x^2} = \pm 6x$ and $n = \pm\sqrt{484} = \pm 22$. I will take both m and n to be positive.
Therefore, our original binomial can be factored as follows, after pulling out some common factors:

$$36x^2 - 484 = (6x + 22)(6x - 22) = 4(3x + 11)(3x - 11)\blacksquare$$

5 Completing the square

5.1 Substituting values in

$$y = ax^2 + wx + c \longrightarrow y = 3x^2 + 18x - 11$$

5.2 Completing the square for this trinomial

Might as well get our hands dirty immediately.

$$\begin{aligned}
y &= 3x^2 + 18x - 11 \\
&= 3(x^2 + 6x) - 11 \text{ (pulling } a \text{ out)} \\
&= 3(x^2 + 6x + 9 - 9) - 11 \text{ (adding an appropriate value to get a perfect square)} \\
&= 3(x^2 + 6x + 9) - 27 - 11 \text{ (pulling that value out of the bracket with dist. property)} \\
&= 3(x + 3)^2 - 38 \blacksquare \text{ (gathering the like terms on the right and simplifying the perfect square)}
\end{aligned}$$

We have now completed the square.

$$y = 3x^2 + 18x - 11 \longrightarrow \boxed{y = 3(x + 3)^2 - 38}$$