Physics review sheet

Simon Wu

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Contents

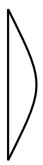
1 Optics

1.1 Lensmaker's formula

Given an object distance p, image distance q and focal length f:

$$\boxed{\frac{1}{f} = \frac{1}{p} + \frac{1}{q}}$$

1.2 Lens types



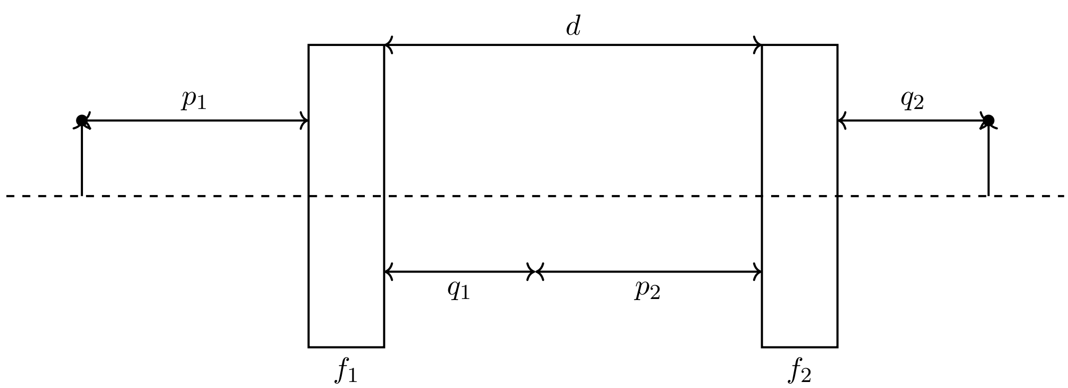
Converging Lens:
Focal point behind lens
f is positive (f more than 0)

Diverging Lens:
Focal point in front of lens
f is negative (f less than 0)

1.3 Sign convention

	Front of lens	Behind lens
f	Negative (-)	Positive (+)
p	Positive (+)	Negative (-)
q	Negative (-)	Positive (+)

1.4 Combining lenses



1.4.1 Working with multiple lenses

$$\boxed{\frac{1}{d - q_1} + \frac{1}{q_2} = \frac{1}{f_2}}$$

1.4.2 As $d \longrightarrow 0$

$$\boxed{\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2}}$$

Two lenses stuck together ($d \approx 0$) may be treated as a single lens with a new effective focal length f_{eff}.

1.5 Object and image characteristics for lenses and mirrors

1.5.1 Converging lens

Given a converging lens that has a focal point F:

Object location	Image location	Size	Orientation	Type
Beyond $2F$	Opposite, between 0 and $2F$	Smaller	Inverted	Real
At $2F$	Opposite at exactly $2F$	Same	Inverted	Real
Between $2F$ and F	Opposite beyond $2F$	Larger	Inverted	Real
At F	None	None	None	None
In front of F	Behind lens	Larger	Upright	Virtual

1.5.2 Diverging lens

The image of formed by a diverging lens will always be a **virtual image** that is **upright**, on the **same side as the object**, **smaller** than the object and **closer to the lens** than the object. As an object approaches the lens, the virtual image will approach the lens as well, and the virtual image will grow in size.

1.5.3 Converging (concave) mirror

Given a concave mirror with a center of curvature C and a focal point F exactly between the mirror and C:

Object location	Image location	Size	Orientation	Type
Beyond C	Between object and mirror	Smaller	Inverted	Real
At C	Exactly at C	Same	Inverted	Real
Between C and F	Same side beyond C	Larger	Inverted	Real
At F	None	None	None	None
In front of F	Behind mirror	Larger	Upright	Virtual

1.5.4 Diverging (Convex) mirror

The image formed by a diverging mirror will always be located **behind** the convex mirror, a **virtual** image, **upright** and **smaller** than the object.
As an object approaches the mirror, the virtual image behind the mirror will approach the mirror as well, and the virtual immage will grow in size.

1.6 Magnification

Linear magnification is based on the object and image distances p and q:

$$\boxed{M = \frac{h'}{h} = -\frac{q}{p}}$$

If $h' < 0$, the image is inverted.

1.7 Polarization

The intensity of light passing through a polarizing sheet is one half of the intensity of the light incident to the sheet:

$$\boxed{I' = \frac{I}{2}}$$

1.7.1 Malus' law

If the light then passes through another polaroid that is at an angle θ to the first polaroid, then the intensity it related to the square of the cosine of the angle:

$$\boxed{I'' = \frac{I}{2}\cos^2(\theta)}$$

1.8 Double slit interference

1.8.1 Bright fringes

Bright fringes on the screen are evenly spaced. Given wavelength of light λ, width of gap between slits d and distance from screen L:

$$\boxed{y_n = \frac{n\lambda L}{d}}$$

n is an integer $(1, 2, 3...)$

1.8.2 Bright fringes

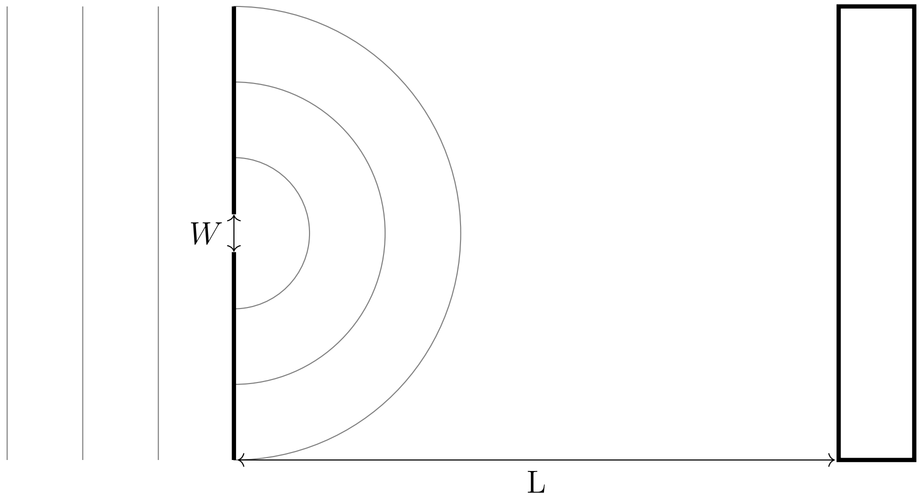
Dark fringes on the screen are also evenly spaced. Given wavelength of light λ, width of gap between slits d and distance from screen L:

$$\boxed{y_n = \left(n + \frac{1}{2}\right)\frac{\lambda L}{d}}$$

n is an integer $(1, 2, 3...)$

1.9 Single slit interference (diffraction)

Diffraction is the rounding of a wavefront as it moves through a small opening, especially an opening that is much smaller than its wavelength.



1.9.1 Bright fringes

Bright fringes on the screen are evenly spaced. Given wavelength of light λ, width of gap W and distance from screen L:

$$\boxed{y_m = \left(m + \frac{1}{2}\right)\frac{\lambda L}{W}}$$

m is an integer $(1, 2, 3...)$

1.9.2 Dark fringes

Dark fringes on the screen are also evenly spaced, and between the light fringes. Given wavelength of light λ, width of gap W and distance from screen L:

$$\boxed{y_m = \frac{m\lambda L}{W}}$$

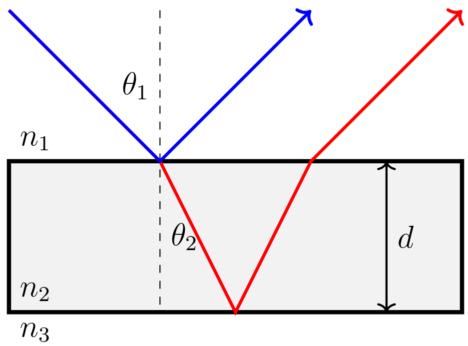
Notice that the pattern is very similar to the double-slit interference pattern, but reversed between the dark and bright fringes.

2 Mirror equation

Focal length f_m of a mirror is half of the radius of curvature, and is negative for a convex mirror. Object distance and image distance are related to the focal length of the mirror:

$$\boxed{\frac{1}{f_m} = \frac{1}{d_o} + \frac{1}{d_i}}$$

2.1 Thin-film interference



2.1.1 Path length difference

The path length difference Γ, given the angle of refraction θ_2, index of refraction n_2 and thickness d:

$$\boxed{\Gamma = 2dn_2\cos(\theta_2)}$$

2.1.2 Reflections

A reflected ray going from a fast medium to a slow medium (from lower n to higher n) has a phase shift of 180° or π when reflected.
However, a reflected ray at a boundary going between a slow medium and a fast medium (from higher n to lower n) doesn't have a phase shift.

2.1.3 Constructive and destructive interference

Consider the phase shifts when calculating the constructive and destructive interference areas. When there is a phase shift of π, for example, you need to shift everything by $\frac{1}{2}$ to account for the phase shift.
For example, considering a soap bubble, $n_1 = n_3 = n_{\text{air}} = 1$, and $n_2 = n_{\text{soap}} > 1$, so there will be a phase shift at the uppder boundary relection only.
Therefore, constructive interference occurs when Γ is a half-number multiple:

$$\boxed{2dn_{\text{soap}}\cos(\theta_2) = \left(m - \frac{1}{2}\right)\lambda}$$

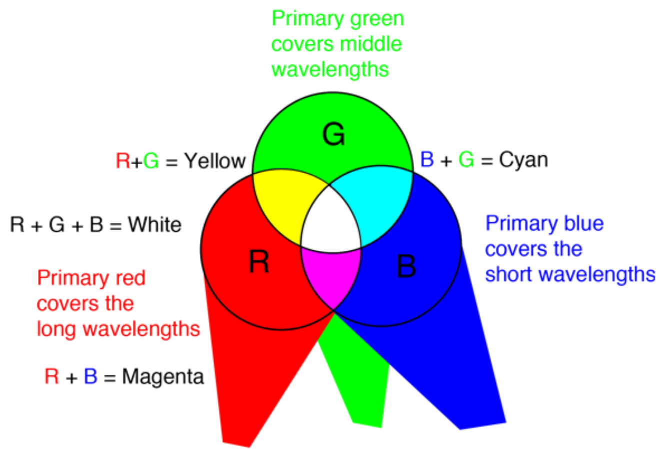
Destructive interference will occur when Γ is a whole-number multiple of the wavelength with this phase shift.

$$\boxed{2dn_{\text{soap}}\cos(\theta_2) = m\lambda}$$

2.2 Colour mixing

2.2.1 Additive colour mixing

Additive colour mixing can be conceptualized as overlapping spotlights of different colours in a dark room. It is adding light energy in different ranges of the visible spectrum. The primary colours in additive colour mixing theory are red, green and blue. Do note that this is *not* a representation of the behaviour of mixing pigments or paints.

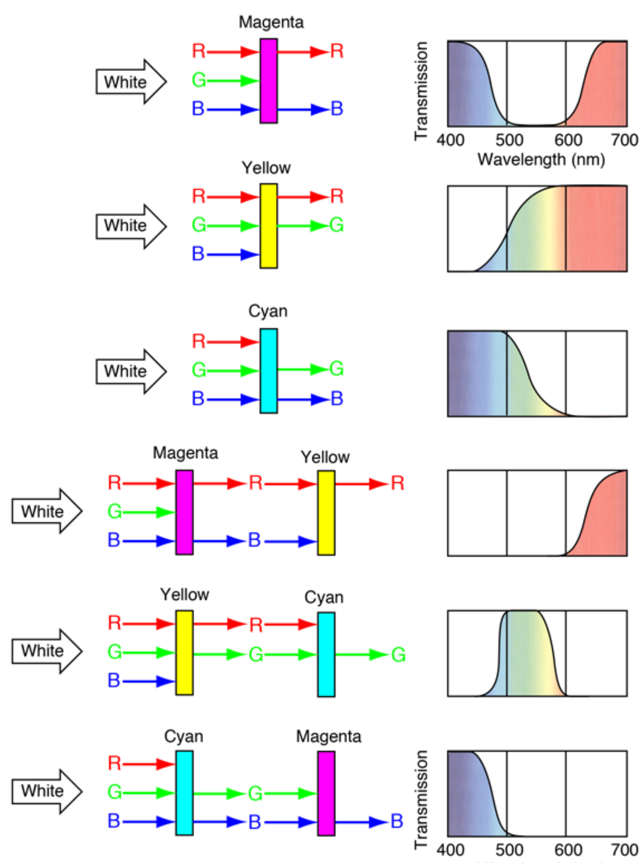


Additive color mixing with red, green and blue additive primary colors.

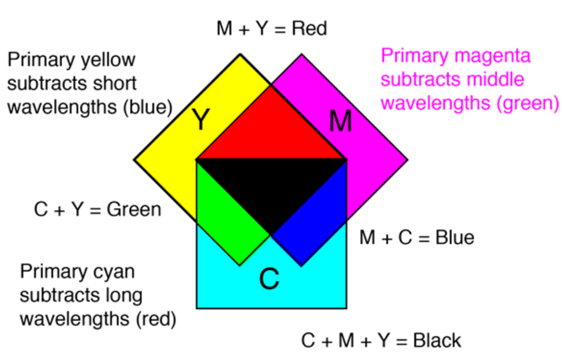
The center of the diagram represents achromatism: there is no colour there, and it could be white, black or an achromatic grey.

2.2.2 Subtractive colour mixing

Subtractive colour mixing reflects the behaviours of pigments, dyes and paints, rather than coloured lights. Paints and dyes and the like absorb specific parts of the spectrum and reflect the others, rather than add parts of the spectrum.



The primary filters generally used to conceptualize subtractive colour mixing are generally cyan, magenta and yellow. The primary additive colours can be obtained from combining two subtractive filters.



Color subtraction with idealized primary filters

3 Electricity

3.1 Current

Conventional current is the rate at which positive charges flow through a wire:

$$\boxed{I = \frac{\mathrm{d}Q}{\mathrm{d}t}}$$

It is also defined as the flux of charge density flowing past a surface (like the cross-section of a wire):

$$\boxed{I = \int \vec{J} \cdot \mathrm{d}\vec{A}}$$

3.2 Resistors

3.2.1 Resistivity

Resistivity of a material is proportional to the electric field and current density:

$$\boxed{\vec{E} = \rho \vec{J}}$$

Electric field is in newtons per coulomb, charge density in amperes per meter squared, and resistivity in ohms per meter.

3.2.2 Resistance of a conductor

Resistance of a conductor is proportional to resistivity ρ and length L and inversely proportional to cross-sectional area A:

$$\boxed{R = \rho \frac{L}{A}}$$

3.2.3 Power dissipated by a resistor

The power P dissipated by a resistor of resistance R, given a potential difference V and a current I across the resistor:

$$\boxed{P = IV = \frac{V^2}{R} = I^2 R}$$

3.2.4 Resistors in parallel

The equivalent resistance R_{eq}, given N resistors with resistances R_1, R_2 ... R_N in parallel:

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... \frac{1}{R_N}}$$

3.2.5 Resistors in series

The equivalent resistance R_{eq}, given N resistors with resistances R_1, R_2 ... R_N in series:

$$\boxed{R_{eq} = R_1 + R_2 + ... + R_N}$$

3.3 Capacitors

A parallel-plat capacitor creates a potential difference between 2 plates by transferring changes between the plates when connected to a battery. This stores energy in the capacitor that can be used later.

3.3.1 Capacitance

The plates of a capacitor will develop a charge of Q and $-Q$. The relation between the charge on the plates and the potential difference is the capacitance:

$$\boxed{C = \frac{Q}{\Delta V}} \text{ or } \boxed{Q = C\Delta V}$$

The units of capacitance are farads, which are coulombs per volt. $(F = C/V)$
If the potential difference is considered in terms of the electric field E and distance d between the plates, and the charge is considered in terms of charge density σ and area A of the plates:

$$\boxed{C = \frac{\sigma A}{Ed} = \frac{\sigma A}{(\sigma/\epsilon_0)d} = \epsilon_0 \frac{A}{d}}$$

3.3.2 Energy stored in a capacitor

Given that a capacitor has a potential difference V across it and is storing Q:

$$\boxed{U_c = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2}$$

3.3.3 Dielectric materials

A dielectric reduces the electric field within it by a factor of κ:

$$\boxed{\kappa = \frac{E_0}{E}}$$

It can also be viewed as increasing effective permittivity:

$$\boxed{\epsilon = \kappa\epsilon_0}$$

Dielectric materials amplify capacitance by a factor of κ:

$$\boxed{C = \kappa C_0}$$

3.3.4 Capacitors in parallel

The potential difference across capacitors in parallel will be the same. The equivalent capacitance C_{eq}, given N capacitors $C_1, C_2, ...C_N$ in parallel:

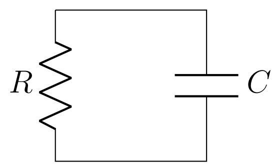
$$\boxed{C_{\text{eq}} = C_1 + C_2 + ... + C_N}$$

3.3.5 Capacitors in series

The total voltage (potential difference) across capacitors in series must be the sum of the voltages across each of them. The charges stored in each of the capacitors must also be the same! From this, given N capacitors $C_1, C_2, ...C_N$ in series:

$$\boxed{\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_N}}$$

3.3.6 Discharging a capacitor



Considering a capacitor charged to $V_c = Q_{\text{tot}}/C$ and a capacitor R: as current flows, the charge on the capacitor decreases, and the current decreases, until the capacitor is fully discharged and the current stops flowing. We can apply Kirchhoff's Voltage Law:

$$\boxed{V_c - IR = 0} \rightarrow \boxed{-\frac{Q}{C} - R\frac{\mathrm{d}Q}{\mathrm{d}t} = 0} \rightarrow \boxed{\frac{\mathrm{d}q}{\mathrm{d}Q} = -\frac{\mathrm{d}t}{RC}}$$

Solving that differential equation, you obtain the expression of charge across the capacitor, which is time-dependent:

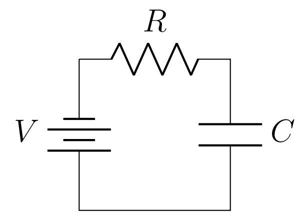
$$\boxed{Q(t) = Q_0 e^{-t/\tau}}$$

Q_0 is the initial capacitor change Q_{tot}, and $\tau = RC$, which is called the time constant. Taking the derivative of $Q(t)$ to derive the current through the circuit:

$$\boxed{I(t) = \frac{\mathrm{d}Q}{\mathrm{d}t} = I_0 e^{-t/\tau}}$$

I_0 is the initial current at $t = 0$. It is given by $Q_{\text{tot}}/\tau = Q_{\text{tot}}/RC$.

3.3.7 Charging a capacitor



Now, consider the case where there is now a battery in the circuit. We can again apply Kirchhoff's Voltage Law:

$$\boxed{V - R\frac{\mathrm{d}Q}{\mathrm{d}t} - \frac{Q}{C} = 0} \rightarrow \boxed{\frac{\mathrm{d}Q}{CV - Q} = -\frac{\mathrm{d}t}{RC}}$$

Integrating and then exponentiating:

$$\boxed{CV - Q = e^K e^{-t/RC}}$$

When $t = 0$, charge across the capacitor Q is also 0:

$$\boxed{e^K = CV = Q_{\mathrm{tot}}}$$

Therefore, we can find that the charge across the capacitor is time-dependent again:

$$\boxed{Q(t) = Q_{\mathrm{tot}}(1 - e^{-t/\tau})}$$

Once again, we can differentiate to find the current through the circuit, which is actually identical to when it is discharging:

$$\boxed{I(t) = \frac{\mathrm{d}Q}{\mathrm{d}t} = I_0 e^{-t/\tau}}$$

The time constant τ is the same as when the capacitor is being discharged. Initial current I_0 is given by $Q_{\mathrm{tot}}/\tau = V/R$, which makes sense.

3.4 Inductors

3.4.1 Inductance of a circuit

Considering a device with inductance L that is related to the number of coils N, magnetic flux ϕ_B, time t and current I:

$$\boxed{L = N\frac{\Delta\phi_B}{\Delta I} = \frac{N\phi_B}{I}}$$

The unit of inductance is the Henry (H).

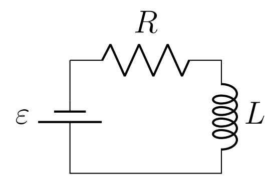
$$\boxed{1\mathrm{H} = 1\mathrm{V} \cdot \mathrm{s/A}}$$

3.4.2 Inductance of a coil

Considering the above euqation, as well as known equations for solenoid magnetic flux, the inductance of a solenoid can be determined. With a solenoid with length $l >> $ radius, N turns, volume V and with current I going through it, with $n = N/l$:

$$\boxed{L = \mu_0 n^2 V - \frac{\mu_0 N^2 A}{l}}$$

3.5 RL circuits



When a circuit has a large inductor in it, back-emf from the inductor opposes the rate of change of current. This is because current doesn't instantaneously reach its maximum value, even when only batteries and resistors are involved - that's why back-emf exists!
The back emf across the inductor coil:

$$\boxed{\varepsilon_L = -L\frac{\Delta I}{\Delta t}}$$

Back-emf falls off over time because the rate of increase of the current decreases as it approaches the maximum current. The time constant of the RL circuit, given resistance R and inductance L:

$$\boxed{\tau = \frac{L}{R}}$$

The current in an RL circuit over time, with time constant τ:

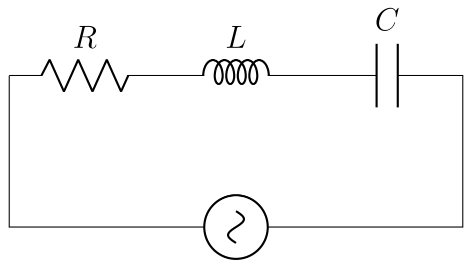
$$\boxed{I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$$

3.5.1 Energy stored by an inductor

Energy stored by an inductor is dependent on the current flowing through the circuit at that time. Given an inductor of inductance L and current flowing through the circuit of I:

$$\boxed{PE_L = \frac{1}{2}LI^2}$$

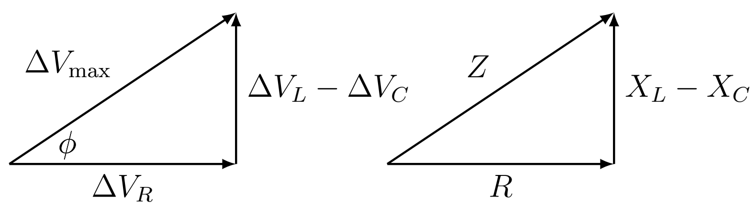
3.6 RLC circuits



In an AC circuit, the current in the circuit is the same at all points at any instant, and varies with time:

$$\boxed{i = I_{\max}\sin(2\pi f t)}$$

The voltage across each of the circuit elements isn't always in phase with the current, though. Instantaneous voltage Δv_r across the resistor is in phase with the instantaneous current, but Δv_L leads the current by 90 degrees, and the instantaneous voltage Δv_C lags the current by 90 degrees. Phase angle ϕ of the circuit elements in total, in relation to $\Delta V_{\max}$ can be conceptualized with phasor diagrams:



3.6.1 Phase angle

The total voltage across the circuit, taking into account the phases of the voltages using the phasor diagram:

$$\boxed{\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2}}$$

Phase angle ϕ has this relation:

$$\boxed{\tan(\phi) = \frac{\Delta V_L - \Delta V_C}{\Delta V_R}}$$

3.6.2 Impedance

Impedance Z of the RLC circuit can also be found from a phasor diagram:

$$\boxed{Z \equiv \sqrt{R^2 + (X_L - X_C)^2}}$$

The statement of Ohm's law for this circuit can be written in terms of impedance:

$$\boxed{\Delta V_{\max} = I_{\max}\sqrt{R^2 + (X_L - X_C)^2} = I_{\max} Z}$$

4 Magnetism

4.1 Lorentz force law

The force acting on a charged particle q in an electric field $\vec{E}$ and magnetic field $\vec{B}$ and moving with velocity $\vec{v}$:

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

4.2 Magnetic flux

Magnetic flux through a wire loop of area $\vec{A}$ in a field $\vec{B}$ that is making an angle θ perpendicular to the loop, in units of weber:

$$\boxed{\phi_B = \int \vec{B} \cdot \mathrm{d}\vec{A}}$$

Simplifies when the magnetic field is constant over the area of the wire loop:

$$\boxed{\phi_B = \vec{B} \cdot \vec{A}}$$

$$\boxed{\phi_B = BA\cos(\theta)}$$

4.2.1 Faraday's law

Faraday's law states that electromotive force is inducted in a conductor when there is a change in the number of field lines passing through it, or when it cuts across field lines. The electromotive force ε induced in a wire loop is the rate of change of flux through it.

$$\boxed{\varepsilon = -\frac{\mathrm{d}\phi_B}{\mathrm{d}t} = -\frac{\Delta\phi_B}{\Delta t}}$$

4.2.2 Lenz's law

Lenz's law states that induced current will appear in such a direction that it opposes the charge that produced it. For example, if the area of a loop of wire in a magnetic field coming out of the page is decreasing, the magnetic flux through the loop is decreasing: induced current will appear such that it counteracts that decrease, indicating that the current will be counterclockwise (inducing more field out of the page inside the loop).

4.2.3 Electromotive force in a coil

Given a coil of N turns:

$$\boxed{\varepsilon = -N\frac{\Delta\phi_B}{\Delta t}}$$

4.3 Magnetic induction from a current in a wire

4.3.1 Magnetic field at the centre of a coil

Given a coil of N turns, radius r and carrying current i:

$$\boxed{B = \frac{\mu_0 N i}{2r}}$$

4.3.2 Magnetic field on the axis of a solenoid

Given a solenoid of N turns over an axial length l:

$$\boxed{B = \frac{\mu_0 N i}{l}}$$

4.3.3 Magnetic field from a straight wire

At a distance r from a wire carrying current i, with the position subtending angles θ_1 and θ_2 from the ends of the wire"

$$\boxed{B = \frac{\mu_0 i}{4\pi r}\left(\cos(\theta_1) + \cos(\theta_2)\right)}$$

4.3.4 Magnetic field from an infinitely long wire

Given that our wire is now infinitely long, meaning that $\theta_1 \longrightarrow 0$ and $\theta_2 \longrightarrow 0$:

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

4.4 Cyclotron concepts

4.4.1 Radius of motion of a particle in a magnetic field

Assuming that a field B acts perpendicularly to the plane of orbit of a particle q and mass m moving with velocity v:

$$\boxed{r = \frac{mv}{qB}}$$

4.4.2 Angular frequency

$$\boxed{\omega = \frac{v}{r} = \frac{qB}{m}}$$

4.4.3 Cyclotron frequency

$$\boxed{f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}}$$

5 Special relativity

5.1 Rapidity

Given a a frame of reference that moves at a velocity v, and a variable β defined as $\frac{v}{c}$, and where ζ is the rapidity:

$$\boxed{\tanh(\zeta) = \beta}$$

5.1.1 Hyperbolic identities

$$\boxed{\cosh(\zeta) = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} \boxed{\sinh(\zeta) = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}} \boxed{\tanh(\zeta) = \frac{\sinh(\zeta)}{\cosh(\zeta)}}$$

$$\boxed{\cosh^2(\zeta) - \sinh^2(\zeta) = 1}$$

5.2 Length contraction

Given a speed v, and a proper length (at rest) of L:

$$\boxed{L' = \frac{L}{\cosh(\zeta)} = L\sqrt{1-\beta^2} = L\sqrt{1-\frac{v^2}{c^2}}}$$

5.3 Time dilation

Given a speed v, and a proper time (at rest) of $\Delta\tau$:

$$\boxed{\Delta\tau' = \Delta\tau\cosh(\zeta) = \frac{\Delta\tau}{\sqrt{1-\beta^2}} = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}}$$

5.4 Mass dilation

Given a rest mass m and a frame moving at v:

$$\boxed{m' = m\cosh(\zeta) = \frac{m}{\sqrt{1-\beta^2}} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}}$$

5.5 Velocity addition

Given an object moving at v_0 (rapidity ζ_0) in a frame S, and a frame S' moving at a speed v (rapidity ζ) relative to S, the speed (v_0') and rapidity (ζ_0') in S' can be found:

$$\boxed{\zeta_0' = \zeta_0 - \zeta} \text{ or } \boxed{v_0' = \frac{v_0 - v}{1-\frac{v_0 v}{c^2}}}$$

5.6 Energy

Given a particle of rest mass m_0 moving at v_0:

$$\boxed{\text{Total Energy } = E = m_0 c^2 \cosh(\zeta_0) = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = \frac{m_0 c^2}{\sqrt{1-\frac{v_0^2}{c^2}}}}$$

5.6.1 Energy at low speeds

If $\left|\frac{v_0}{c}\right| << 1$, we can use a Taylor expansion like $(1+x)^\alpha = 1 + \alpha x + O(x^2)$ to recover Newtonian kinetic energy:

$$\boxed{E = m_o c^2 + \frac{1}{2} m_0 v_0^2 + O\left(\frac{m v_0^4}{c^2}\right)}$$

The term $m_0 c^2$ is the rest energy, and $\frac{1}{2} m_0 v_0^2$ is Newtonian kinetic energy.

5.6.2 Energy in a different frame

Given a frame of rapidity ζ_0 and another frame of relative rapidity ζ:

$$\boxed{E' = m_0 c^2 \cosh(\zeta_0 - \zeta) = m_0 c^2 \cosh(\zeta_0')} \text{ where } \zeta_0' = \zeta_0 - \zeta$$

5.7 Momentum

Given a particle of rest mass m_0 moving at v_0:

$$\boxed{p = m_0 c \sinh(\zeta_0) = \frac{m_0 c \beta}{\sqrt{1-\beta^2}} = \frac{m_0 v_0}{\sqrt{1-\frac{v_0^2}{c^2}}}}$$

5.7.1 Momentum at low speeds

If $\left|\frac{v_0}{c}\right| << 1$, we can use a Taylor expansion like $(1+x)^\alpha = 1 + \alpha x + O(x^2)$ to recover Newtonian momentum:

$$\boxed{p = m_0 v_0 + O\left(\frac{m v_0^3}{c^2}\right)}$$

5.7.2 Momentum in a different frame

Given a frame of rapidity ζ_0 and another frame of relative rapidity ζ:

$$\boxed{p' = m_0 c \sinh(\zeta_0 - \zeta) = m_0 c \sinh(\zeta_0')} \text{ where } \zeta_0' = \zeta_0 - \zeta$$

5.8 Special relativity but with Lorentz factors

5.8.1 The Lorentz factor

The Lorentz factor is represented by γ.

$$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

5.8.2 Energy

The total energy E of an object, given its rest mass m:

$$\boxed{E = \gamma mc^2}$$

The rest energy that an object always possesses:

$$\boxed{E_0 = mc^2}$$

The kinetic energy of an object:

$$\boxed{K = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2}$$

Total energy of an object can also be found in relation to its momentum:

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

5.8.3 Momentum

Given that an object is moving at relativistic speeds:

$$\boxed{p = \gamma mv}$$

5.8.4 Length contraction

Given the proper length L in a frame of reference, and an observed time of L' from a moving observer in another inertial frame of reference:

$$\boxed{L' = \frac{L}{\gamma}}$$

5.8.5 Time dilation

Given the proper time t in a frame of reference, and an observed time of t' from a moving observer in another inertial frame of reference:

$$\boxed{t' = \gamma t}$$

6 Quantum mechanics

6.1 de Broglie wavelength

The de Broglie wavelength λ of a particle or photon, given its momentum p:

$$\boxed{\lambda = \frac{h}{p}}$$

h is the Planck constant, about $6.62 \times 10^{-34}\mathrm{m}^2\mathrm{kg/s}$.

6.1.1 Photons

With photons of light of frequency ν, the energy is given by:

$$E = h\nu$$

The momentum is given from inserting this into $E = pc$:

$$p = \frac{h\nu}{c}$$

Wavelength λ of the photon can therefore be written in terms of its frequency:

$$\boxed{\lambda = \frac{hc}{E}}$$

6.1.2 Particles

With particles, the wavelength lambda is given in terms of relativistic momentum:

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{\gamma m_0 v} = \frac{h}{m_0 c \sinh(\zeta_0)}}$$

λ will only be significant enough to have the particle display non-negligible wave properties if the wavelength is comparable to the size of the particle: this means that the object usually has to be really small, like an electron.

6.2 Photoelectric effect

The photoelectric effect is essentially proof that light can behave like a stream of particles instead of a wave. When light is shined on a surface, no electrons are emitted from the surface if the light is below a certain frequency. the maximum kinetic energy of the photoelectrons emitted is also independent of light intensity: it increases with increased frequency.
This means that photons are seentially knocking photoelectrons off of the surface as if they are particles colliding: raising intensity without raising frequency means that the photons are colliding in the same way, and that there are just more photons colliding and emitting for photoelectrons.

6.2.1 Maximum kinetic energy of photoelectrons

The maximum kinetic energy of a liberated photoelectron, given the frequency of light f and the work function ϕ of the metal (energy required to free the electron):

$$\boxed{KE_{\text{max}} = hf - \phi}$$

6.2.2 Cutoff frequency

The energy of the photon that strikes the metal must be larger than the work function, so that it is able to liberate a photoelectron.

$$KE_{\text{max}} = hf_c - \phi = 0$$

$$\boxed{f_c = \frac{\phi}{h}}$$

The frequency of the light photons striking the metal must be over the cutoff frequency f_c to free a photoelectron.

6.2.3 Cutoff wavelength

Given that $\lambda = \frac{c}{f}$:

$$\boxed{\lambda_c = \frac{hc}{\phi}}$$

If the wavelength of the photons hitting the metal surface is over cutoff wavelength λ_c, no photoelectron is emitted.

6.3 Electron states

6.3.1 Rydberg energy

The Rydberg unit of energy is referred to as the Ry. It is about $-13.6e\mathrm{V}$.

$$\boxed{1\ \mathrm{Ry} = -\frac{m_e e^4}{32\pi^2\epsilon_0^2\hbar^2} = -\frac{m_e e^4}{8h^2\epsilon_0^2} \approx -13.6e\mathrm{V}}$$

6.3.2 Energy of a hydrogen atom

The ground energy of a hydrogen atom is $E_0 = 1$ Ry. The energy of the hydrogen atom when the electron is at some energy state n:

$$\boxed{E_n = -\frac{E_0}{n^2}}$$

The emitted energy (in photons) when an electron goes from a higher energy state m to a lower one n:

$$\boxed{E_{\text{photon}} = E_m - E_n = -E_0\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

With atoms other than hydrogen, the energy when going from a higher state to a lower one, given the atomic number Z:

$$\boxed{E_{\text{photon}} = \frac{Z^2 m_e e^4}{8h^2\epsilon_0^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) = -13.6Z^2\left(\frac{1}{m^2} - \frac{1}{n^2}\right)e\mathrm{V}}$$

6.3.3 Rydberg constant

The Rydberg constant R is given by:

$$R = \frac{E_0}{hc} = \frac{m_e e^4}{8h^3 c\epsilon_0^2}$$

6.3.4 Wavelength of emitted photon

The inverse wavelength of the emitted photon when an atom goes from a higher state m to a lower state n:

$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

7 Thermodynamics

7.1 Linear thermal expansion

Given a length L_0 at some temperature, a change in temperature of ΔT and a coefficient of linear expansion α, the expanded length is given by:

$$\boxed{L = L_0(1 + \alpha \Delta T)}$$

7.1.1 Areal and volumetric expansion

α can also be replaced with the coefficient of areal expansion $\beta = 2\alpha$ and the coefficient of volumetric expansion $\gamma = 3\alpha$.

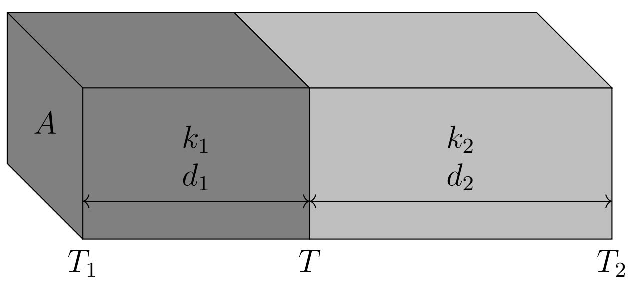
$$\boxed{A = A_0(1 + \beta \Delta T)} \text{ and } \boxed{V = V_0(1 + \gamma \Delta T)}$$

7.2 Heat transfer

Heat conduction through a slab with thickness of d, area A and thermal conductivity k_1 with one side at temperature T_1 and the other at T_2 is given by:

$$\boxed{-\frac{\mathrm{d}Q}{\mathrm{d}t} = k_1 A \left(\frac{T_1 - T_2}{d} \right)}$$

7.2.1 Slabs in series



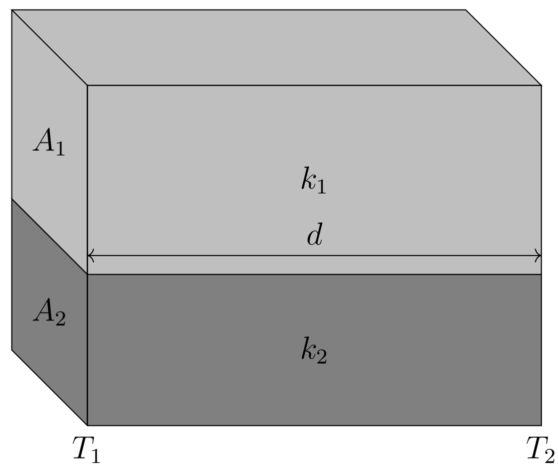
Given n slabs placed in series, with the leftmost temperature begin T_1 and rightmost temperature T_2, the equivalent conductivity of the system of slabs is given by:

$$\boxed{k_{eq} = \frac{\sum_{i=1}^{n} d_i}{\sum_{i=1}^{n} \frac{d_i}{k_i}}} \text{ With two slabs, } \boxed{k_{eq} = \frac{d_1 + d_2}{\left(\frac{d_1}{k_1}\right) + \left(\frac{d_2}{k_2}\right)}}$$

When there are two slabs, the interface temperature T is given by:

$$\boxed{T = \frac{\frac{k_1 T_1}{d_1} + \frac{k_2 T_2}{d_2}}{\frac{k_1}{d_1} + \frac{k_2}{d_2}}}$$

7.2.2 Slabs in parallel



The rate of flow of heat in a system of n parallel slabs:

$$\boxed{-\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{T_1 - T_2}{d} \sum_{i=1}^{n} k_i A_i}$$

The equivalent thermal conductivity of the system of parallel slabs:

$$\boxed{k_{eq} = \frac{\sum k_i A_i}{\sum A_i}}$$

7.3 Newton's law of cooling for convection

Given a mean temperature of θ, a room temperature of θ_0 and a constant C:

$$\boxed{-\frac{\mathrm{d}\theta}{\mathrm{d}t} = C(\theta - \theta_0)}$$

7.4 Stefan-Boltzmann formula for radiation

Given a radiator of area A, an absolute temperature T_1 of the radiator, a surrounding absolute temperature of T_2 and the Stefan-Boltzmann constant σ:

$$\boxed{-\frac{\mathrm{d}E}{\mathrm{d}t} = \sigma A \left(T_1^4 - T_2^4\right)}$$

7.5 Wien's law for blackbody radiation

Given the maximumx-intensity wavelength of blackbody radiation λ_m at absolute temperature T:

$$\boxed{\lambda_m T = b = 4 \times 10^{-3} m \cdot K}$$

7.6 Gas properties

7.6.1 Ideal gas law

The ideal gas law states that in an ideal gas, given pressure P in Pascal ($\frac{\mathrm{N}}{\mathrm{m}}$, volume V, amount of substance n (in moles), absolute temperature T and the ideal gas constant R ($R = N_A \cdot k_b$, where Avagadro's constant $N_A = 6.022 \times 10^{23}$ and the Boltzmann constant $k_b = 1.381 \times 10^{-23}\mathrm{J/K}$):

$$\boxed{PV = nRT}$$

By stating $k_b = R/N_A$ and number of molecules $N = n \cdot N_A$, ideal gas law is restated as:

$$\boxed{PV = NkT}$$

7.6.2 Gas density

Given a particuler gas with density ρ, pressure P and absolute temperature T:

$$\boxed{\frac{\rho_1 T_1}{P_1} = \frac{\rho_2 T_2}{P_2}}$$

7.6.3 Gas velocities

Most probable speed:

$$\boxed{v_p = \sqrt{\frac{2kT}{m}}}$$

Average speed:

$$\boxed{< v >= \sqrt{\frac{8kT}{\pi m}}}$$

Root-mean-square speed:

$$\boxed{\sqrt{< v^2 >} = \sqrt{\frac{8kT}{m}}}$$

7.7 Internal energy

Internal energy U, given the constant volume heat capacity c_v and absolute temperature T:

$$\boxed{U = nc_v RT}$$

c_v is $\frac{3}{2}R$ for monatomic gases, and $\frac{5}{2}R$ for diatomic gases.

7.7.1 Specific heats

Specific heat varies with the conditions that the gas is in: the heat capacity of the gas in a container that expands with heat (constant pressure) is often much higher than when it's heated in a closed container that doesn't expand (constant volume).
Constant volume specific heat is c_v, and constant pressure specific heat is c_p. The heat capacity ratio γ, given c_p and c_v:

$$\boxed{\gamma = \frac{c_p}{c_v}}$$

The heat capacity ratio γ depends on the degrees of freedom f of the molecules of a gas:

$$\boxed{\gamma = 1 + \frac{2}{f}}$$

This can be reversed as well, to determine degrees of freedom of a gas from the specific heat ratio:

$$\boxed{f = \frac{2}{\gamma - 1}}$$

For a monatomic gas with 2 degrees of freedom:

$$\boxed{\gamma_{monatomic} = \frac{5}{3}}$$

For a diatomic gas with 5 degrees of freedom at room temperature:

$$\boxed{\gamma_{diatomic} = \frac{7}{5}}$$

7.8 First law of thermodynamics

Given ΔQ is the change in heat, W is the work done by the system in a process and ΔU is the change in internal energy of the system:

$$\boxed{\Delta Q = \Delta U + W}$$

Internal energy U of a system tends to increase when energy is added as heat and tends to decrease if energy is lost as work done by the system.

7.9 Entropy

Change in entropy ΔS, given a change in heat $\mathrm{d}Q$ and a temperature T:

$$\boxed{\Delta S = \int \frac{\mathrm{d}Q}{T}}$$

7.10 Thermodynamic processes

7.10.1 Isobaric

In isobaric processes, **pressure** remains constant: $\Delta P = 0$. The work done by the system in an isobaric process is given by:

$$\boxed{W = \int P \mathrm{d}V = P(V_1 - V_2) = P \cdot \Delta V}$$

Using ideal gas law:

$$\boxed{W = P \cdot \Delta V = nR\Delta T}$$

7.10.2 Isochoric

In an isochoric process, **volume** remains constant. No pressure-volume work is done because the volume remains constant ($Q = P \cdot \Delta V$, $\Delta V = 0$). This means that any change in heat comes from change in internal energy ($\Delta Q = \Delta U$).

7.10.3 Isothermal

In an isothermal process, **temperature** remains constant. There will be no change in internal energy, so $\Delta Q = W$.

$$\boxed{W_{\text{env}} = nRT \ln \left(\frac{V_2}{V_1} \right)}$$

7.10.4 Adiabatic

In an adiabatic process, heat remains constant. This means that there will be no ΔQ, and that $\mathrm{d}U = -\mathrm{d}W$.

$$\boxed{W = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)}$$

For an ideal gas undergoing an adiabatic process (adiabatic expansion, etc.), it can also be shown that:

$$\boxed{PV^{\gamma} = \text{constant}}$$

7.11 Change in heat

Given heat capacity c, mass m and change in temperature ΔT:

$$\boxed{\Delta Q = mc\Delta T}$$

Note that c_v and c_p are in heat capacity per mol, not mass.

7.12 Heat engines

7.12.1 Thermodynamic efficiency

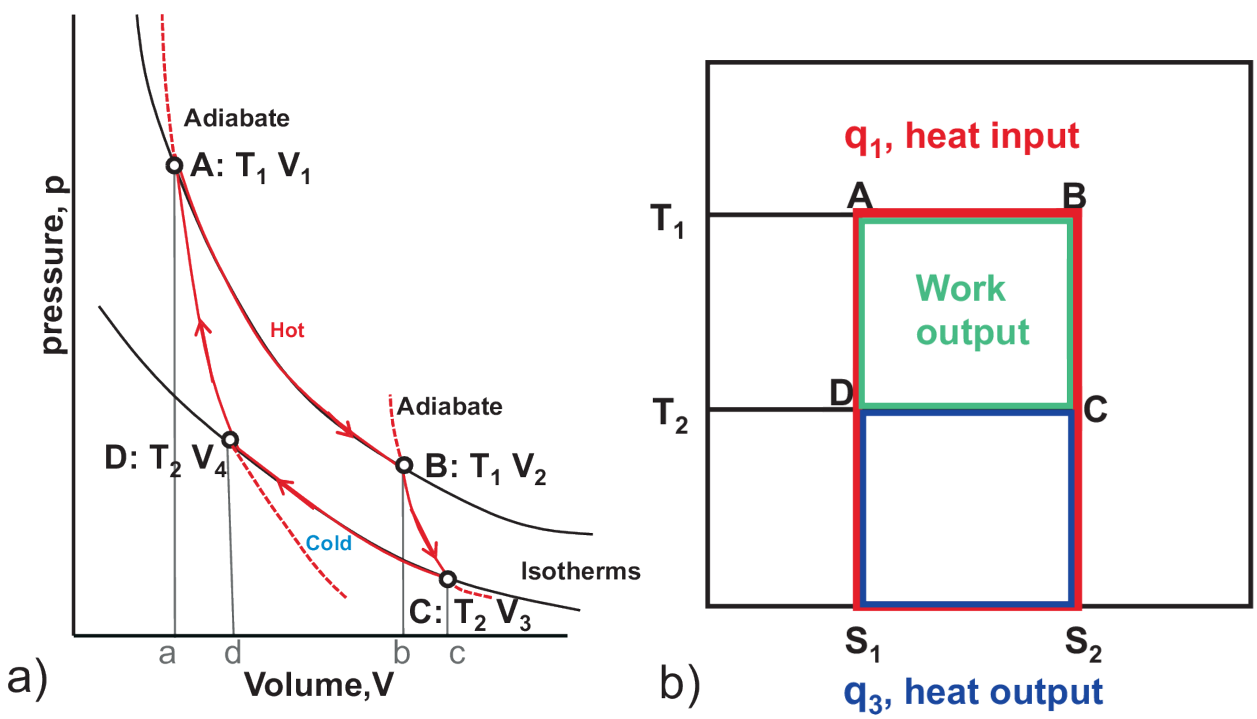
The efficiency e of a cycle:

$$e = \frac{\text{Work done by gas}}{\text{Heat put into system}} = \boxed{\frac{W}{Q_{in}}}$$

7.12.2 Carnot cycle

The Carnot cycle is the ideal thermodynamic cycle: it consists of two isothermal processes and two adiabatic processes. Given a hot resevoir temperature of T_H and a cold resevoir temperature of T_C:

$$\boxed{e = 1 - \frac{T_C}{T_H} = 1 - \frac{Q_C}{Q_H}}$$



$A \longrightarrow B$ and $C \longrightarrow D$ are isothermal processes, and $B \longrightarrow C$ and $D \longrightarrow A$ are adiabatic processes. As T_C is lowered and T_H is raised, the efficiency of the heat engine increases.

7.12.3 Sterling cycle

The Sterling cycle consists of two isothermal processes and two isochoric processes.

7.12.4 Otto cycle

The Otto cycle consists of two adiabatic processes and two isochoric processes.

8 Material properties

8.1 Stress and strain

Stress is applied force over an area.

$$\boxed{\text{Stress} = \frac{F}{A}}$$

Strain is the change in length over the original length.

$$\boxed{\text{Strain} = \frac{\Delta L}{L}}$$

8.2 Young's modulus

The Young's modulus Y, given the stress and strain:

$$\boxed{Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{A\Delta L}}$$

8.3 Shear modulus

The shear modulus η, given shear stress and shear strain ($\Delta x/y$):

$$\boxed{\eta = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{Fy}{A\Delta x}}$$

8.4 Bulk modulus

Bulk modulus K is related to the pressure increment with volume strain ($\Delta V/V$):

$$\boxed{K = \frac{\text{Pressure increment}}{\text{Volume strain}} = \frac{\Delta P}{-\Delta V/V}}$$

8.5 Poisson's ratio

Poisson's ratio relates transverse and axial strain to each other. For example, if you strain som rubber by compressing it, it will tend to stretch axially. Poisson's ratio σ is related to the ration of lateral and longitudial strain:

$$\boxed{\sigma = -\frac{\text{strain}_{\text{transverse}}}{\text{strain}_{\text{axial}}}}$$

8.6 Moduli relations

The elastic moduli are related in various ways.

$$\boxed{Y = \frac{9\eta K}{3K + \eta}}$$

$$\boxed{Y = 2\eta(1 + \sigma)}$$

$$\boxed{Y = 3K(1 - 2\sigma)}$$

$$\boxed{\sigma = \frac{2K - 2\eta}{6K + 2\eta}}$$

8.7 Radioactive decay law

The probability per unit time that a nucleus will decay is a constant, independent of time. That constant is the decay constant λ.
Given N_0 molecules with decay constant λ, the amount of molecules $N(t)$ is given by the radioactive decay law:

$$\boxed{N = N_0 e^{-\lambda t}}$$

8.7.1 Half-life

The half-life $T_{1/2}$ of a material is the amount of time it would take for half of the molecules in that material to decay. This means that:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$e^{-\lambda T_{1/2}} = \frac{1}{2}$$

$$-\lambda T_{1/2} = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$\boxed{\lambda = \frac{\ln(2)}{T_{1/2}} \approx \frac{0.693}{T_{1/2}}}$$

8.7.2 Nuclear reaction notation

The superscript is the mass number and the subscript is the atomic number. For example, with a helium nucleus of atomic number 2 and mass number 4:

$${}^{4}_{2}\mathrm{He}$$

9 Waves

9.1 Universal wave equation

Given the velocity of a wave v, its frequency f and its wavelength λ:

$$\boxed{v = f\lambda}$$

9.2 Basic wave equation

Given a wave with amplitude A, wave number k, angular frequency ω and phase ϕ:

$$\boxed{y(x,t) = A\sin(kx - \omega t + \phi)}$$

The wave number k is related to the wavelength:

$$\boxed{k = \frac{2\pi}{\lambda}}$$

The wave number and angular frequency are related to the velocity of the wave:

$$\boxed{v = \frac{\omega}{k}}$$

9.3 Intensity of a sound

Given a sound source of power P which is acting on an area A:

$$\boxed{I = \frac{P}{A} = \frac{P}{4\pi r^2}}$$

9.3.1 Decibels and intensity measurement

Soudn intensity is generally measured in units called decibels. Decibels operate on a logarithmic scale, and the decibel level is referred to with β.
Given a sound of intensity I compared to the reference intensity $I_0 = 10^{-12}\frac{W}{m^2}$:

$$\boxed{\beta = 10\log_{10}\left(\frac{I}{I_0}\right)}$$

This means that an increase of 10 decibels indicates a tenfold increase of intensity - a whole order of magnitude!

9.4 Simple harmonic motion

Simple harmonic motion occurs when the restoring force is proportional to the displacement.

$$\boxed{\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x}$$

9.5 Sound waves in pipes

9.5.1 Open pipes

When a pipe is open at both ends, there will be antinodes at both ends of the pipe.
The fundamental wavelength λ_1 is related to the length of the pipe L:

$$\boxed{\lambda_1 = 2L}$$

The fundamental frequency can also be found from this.

$$\boxed{f_1 = \frac{v}{\lambda} = \frac{v}{2L}}$$

Higher harmonics will simply have multiples of the fundamental frequency.

$$\boxed{f_n = nf_1 = \frac{nv}{2L} \text{ for } n = 1, 2, 3...}$$

$$\boxed{\lambda_n = \frac{\lambda_1}{n} = \frac{2L}{n} \text{ for } n = 1, 2, 3...}$$

9.5.2 Pipes closed on one end

A pipe closed on one end will have a node on the closed end and an antinode on the open end.
The fundamental wavelength λ_1 is related to the length of the pipe L:

$$\boxed{\lambda_1 = 4L}$$

The fundamental frequency can also be found from this.

$$\boxed{f_1 = \frac{v}{\lambda} = \frac{v}{4L}}$$

Do note, however, that a pipe with one closed end will actually have no even harmonics: there will only be 1st, 3rd, 5th, 7th harmonics and so on.

$$\boxed{f_n = nf_1 = \frac{nv}{4L} \text{ for } n = 1, 3, 5...}$$

$$\boxed{\lambda_n = \frac{\lambda_1}{n} = \frac{4L}{n} \text{ for } n = 1, 3, 5...}$$

9.6 Waves on a stretched wire

9.6.1 Wave speed

A wave on a stretched wire will travel as a certain speed v, given that it has a tension of F and a linear density μ:

$$\boxed{v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{\rho A}}}$$

9.6.2 Frequency

The fundamental frequency of the vibrations, given tension F, length L, linear density μ:

$$\boxed{f_1 = \frac{v}{\lambda} = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2L}\sqrt{\frac{F}{\rho A}}}$$

The frequency of the Nth harmonic is a multiple of the fundamental frequency:

$$\boxed{f_N = \frac{v}{\lambda_N} = \frac{N}{2L}\sqrt{\frac{F}{\mu}} = \frac{N}{2L}\sqrt{\frac{F}{\rho A}}}$$

9.7 Doppler effect

If the source of waves of frequency f is moving with velocity v_s, the observer is moving at v_o, and the speed of sound in air is v:

$$\boxed{f' = f\frac{v \pm v_o}{v \mp v_s}}$$

v_o is positive for motion towards the sound source and negative for motion away from the sound source. v_s is negative for motion towards the observer and positive for motion away from the observer.

9.8 Changing mediums

9.8.1 Fast to slow medium

If a wave moves from a fast to a slow medium, there will be an *inverted* reflected wave and an *upright* transmitted wave.

9.8.2 Slow to fast medium

If a wave moves from a slow to a fast medium, there will be an *upright* reflected and an *upright* transmitted wave.

9.9 Boundary behaviour at ends

9.9.1 Fixed end

A wave travelling along a string and hitting a fixed end will be reflected back and *inverted*.

9.9.2 Free end

A wave travelling along a string and hitting a free end will be reflected back and *upright*.

10 Fluids

10.1 Jargon

Steady flow (laminar flow) means that the velocity of the fluid at a point is always the same.
Irrotational flow means that the elements of fluid at every single point have no net angular velocity about that point, and implies the absence of eddies and vortices in the flow.
Incompressible fluid flow means that a liquid is incompressible if its density is constant.

10.2 Continuity of flow

Because of conservation of mass (nothing is created or destroyed), a steady flow has to have the same mass of fluid passing all sections in a stream or fluid per unit time.
Given points 1 and 2 in a fluid flow, where the fluid has $\rho_{1,2}$ and $v_{1,2}$, and the pipe has cross-sectional area $A_{1,2}$:

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

10.3 Bernoulli's equation

For a steady, non-viscous, incompressible flow:

$$\boxed{\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}}$$

It can also be stated as:

$$\boxed{P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}}$$

Bernoulli's equation represents the conservation of energy: the first term represents potential energy per unit mass of the liquid due to pressure, the second term represents potential energy due to gravity (e.g. due to changes in height of a tube), and the third term represents kinetic energy per unit mass of the liquid.

10.4 Torricelli's theorem

If there is a tank filled with liquid and an orifice located on the side of the tank at a depth h below the surface of the liquid, the velocity of emergence of the liquid (also known as the velocity of efflux) is:

$$\boxed{v = \sqrt{2gh}}$$

10.5 Reynold's number and stability

The stability of a fluid flow is represented by the Reynold's number R. For any liquid flowing through a pipe, there is a critical velocity where laminar flow suddenly changes into turblent flow.
Given a liquid of density ρ and viscosity η flowing at a velocity v through a pipe of diameter d:

$$\boxed{R = \frac{\rho v d}{\eta}}$$

If $R < 2200$, the flow is considered steady. If $R = 2200$, the flow is considered unstable, and if $R > 2200$, the flow is usually considered turbulent.

10.6 Poiseuille's method for determining viscosity

The volume V flowing per second of a liquid of viscosity η through a tube of radius a and length L under pressure P:

$$\boxed{V = \frac{\pi P a^4}{8\eta L}}$$

10.7 Drag force

The drag force F on a sphere of radius r is dropped into a liquid of viscosity η is proportional to the velocity of that object:

$$\boxed{F = 6\pi\eta r v}$$

10.7.1 Terminal velocity

When a sphere of radius r and density ρ_0 is dropped into an extensive liquid of density ρ and viscosity η, it will eventually approach a terminal velocity v_T due to the drag force and buoyant force:

$$\boxed{v_T = \frac{2gr^2}{9\eta}(\rho_0 - \rho)}$$

11 Astronomy

11.1 Moon phases