**1) Stochastic process**

A deterministic process: 

A stochastic process: 



A steady stochastic process: 

Consider a steady stochastic process  with its mean  and define Fourier transformation



and inverse Fourier transformation



using



The following functions can be use to characterize  instead of evaluating  itself.

I) Spectral density / Power spectrum



II) Auto-correlation function



From Eq.,



Thus the following useful relations called the Winner-Khintchine theorem is obtained.





The equations obtained by putting  in Eq. and  in Eq. are called sum rules.



and



**2) Brownian motion and the Langevin equation**

Suppose a Brownian spherical particle of radius  and mass  is moving in a liquid of viscosity  at a temporal velocity under the influence of the random force  due to thermal fluctuation and the external force  acting on the particle. The equation of motion for the Brownian particle can be given by

,

where  (Stokes friction) and . Assuming that the particle motion is over dumped , one yields the Langevin equation of the form



with

.

When the system is in equilibrium , the following results can be derived.

I) Auto-correlation function for :



II) Fluctuation dissipation theorem:



III) Einstein relation, Stokes-Einstein relation:



When the particle is driven by an external force in x-direction , the following results can be derived.

IV) Steady drift velocity:





**3) Linear response theory and the Green-Kubo formula**

I) Linear response theory (LRT):



↓



When ,





II) Application of LRT to drift velocity:

, and 



 (Green-Kubo Formula)