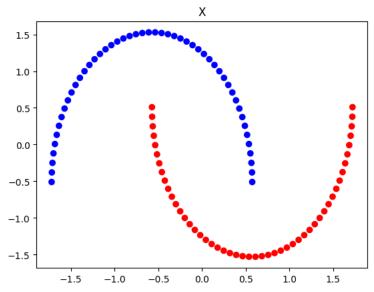
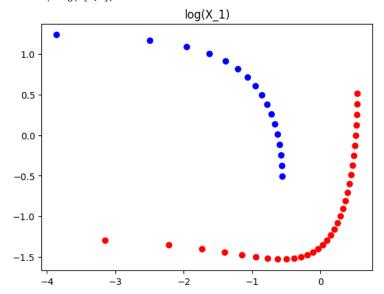
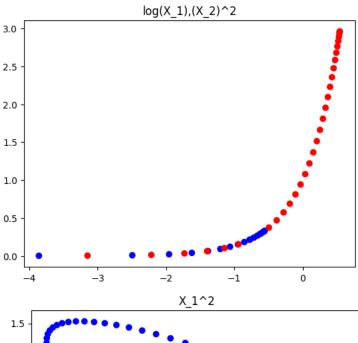
▼ Week 4 Notebook - Linear Regression, Linear Discriminant, Logistic Regression

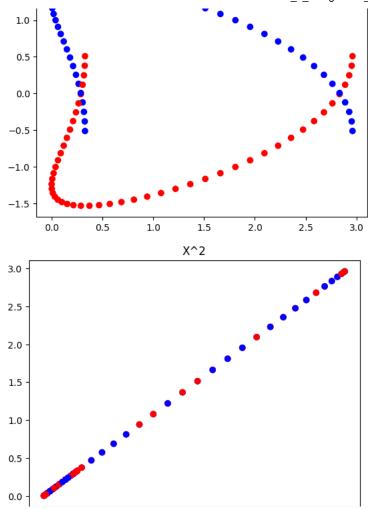
▼ Basis Functions

```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_moons, make_blobs
from sklearn import preprocessing
X, y = make_moons(n_samples=100)
\#X, y = make_blobs(n_samples=100)
X = preprocessing.StandardScaler().fit_transform(X)
plt.title('X')
plt.scatter(X[:,0],X[:,1], c=y, alpha=1, cmap=plt.cm.bwr)
plt.show()
X1 = np.log(X[:,0])
plt.title('log(X_1)')
plt.scatter(X1,X[:,1], c=y, alpha=1, cmap=plt.cm.bwr)
plt.show()
X1 = np.log(X[:,0])
X2 = np.square(X[:,0])
plt.title('log(X_1),(X_2)^2')
plt.scatter(X1,X2, c=y, alpha=1, cmap=plt.cm.bwr)
plt.show()
X1 = np.square(X[:,0])
plt.title('X_1^2')
\verb|plt.scatter(X1,X[:,1], c=y, alpha=1, cmap=plt.cm.bwr)|\\
X2 = np.square(X[:,0])
plt.title('X^2')
plt.scatter(X1,X2, c=y, alpha=1, cmap=plt.cm.bwr)
X1_mm = np.exp(-((X[:,0] - X[:,0].mean()))**2)/2
X2_mm = np.exp(-((X[:,1] - X[:,1].mean()))**2)/2
X2 = np.square(X[:,0])
plt.scatter(X1_mm,X2_mm, c=y, alpha=1, cmap=plt.cm.bwr)
plt.show()
```









Manifold Learning

Sources for this code are Rashcka <u>Kernel tricks and nonlinear dimensionality reduction via RBF kernel PCA</u> and Jake Vanderplas <u>sklearn:</u>
<u>Comparison of Manifold Learning methods</u>

```
I
       . . I
%matplotlib inline
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from sklearn import manifold, datasets
from sklearn import datasets
from sklearn.datasets.samples_generator import make_s_curve
n_points = 1000
X, color = datasets.make_s_curve(n_points, random_state=0)
n_neighbors = 10
n_{components} = 2
fig = plt.figure(figsize=(20, 15))
plt.title("Manifold Learning")
ax = fig.gca(projection='3d')
ax.scatter(X[:,\ 0],\ X[:,\ 1],\ X[:,\ 2],\ c=color,\ cmap=plt.cm.Spectral)
ax.view_init(4, -72)
plt.show()
```

```
ModuleNotFoundError
                                              Traceback (most recent call last)
     <ipython-input-12-2194d3a59b03> in <cell line: 7>()
           5 from sklearn import manifold, datasets
           6 from sklearn import datasets
     ---> 7 from sklearn.datasets.samples_generator import make_s_curve
           9 n_points = 1000
    ModuleNotFoundError: No module named 'sklearn.datasets.samples generator'
     MOTE: If your import is failing due to a missing nackage, you can
from scipy.spatial.distance import pdist, squareform
from scipy import exp
from scipy.linalg import eigh
import numpy as np
def stepwise_kpca(X, gamma, n_components):
    Implementation of a RBF kernel PCA.
   Arguments:
       X: A MxN dataset as NumPy array where the samples are stored as rows (M),
          and the attributes defined as columns (N).
        gamma: A free parameter (coefficient) for the RBF kernel.
        n_components: The number of components to be returned.
    Returns the k eigenvectors (alphas) that correspond to the k largest
        eigenvalues (lambdas).
   # Calculating the squared Euclidean distances for every pair of points
   # in the MxN dimensional dataset.
    sq_dists = pdist(X, 'sqeuclidean')
   # Converting the pairwise distances into a symmetric MxM matrix.
   mat_sq_dists = squareform(sq_dists)
   # Computing the MxM kernel matrix.
   K = exp(-gamma * mat_sq_dists)
   # Centering the symmetric NxN kernel matrix.
   N = K.shape[0]
   one_n = np.ones((N,N)) / N
    K_norm = K - one_n.dot(K) - K.dot(one_n) + one_n.dot(K).dot(one_n)
   # Obtaining eigenvalues in descending order with corresponding
    # eigenvectors from the symmetric matrix.
   eigvals, eigvecs = eigh(K_norm)
   # Obtaining the i eigenvectors (alphas) that corresponds to the i highest eigenvalues (lambdas).
   alphas = np.column_stack((eigvecs[:,-i] for i in range(1,n_components+1)))
   lambdas = [eigvals[-i] for i in range(1,n_components+1)]
    return alphas, lambdas
Y = stepwise_kpca(X, gamma=0.2, n_components=2)[0]
plt.title('Kernelized PCA, gamma = 0.2')
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral);
plt.show()
Y = stepwise_kpca(X, gamma=1, n_components=2)[0]
plt.title('Kernelized PCA, gamma = 1')
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral);
plt.show()
Y = stepwise_kpca(X, gamma=10, n_components=2)[0]
plt.title('Kernelized PCA, gamma = 10')
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral);
plt.show()
Y = manifold.LocallyLinearEmbedding(n_neighbors, n_components,
                                    eigen_solver='auto',
                                    method='standard').fit transform(X)
```

```
plt.title('Locally Linear Embedding')
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral);
Y = manifold.MDS(n_components, max_iter=100, n_init=1).fit_transform(X)
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral)
plt.title("Isomap");
Y = manifold.SpectralEmbedding(n_components=n_components,
                                n_neighbors=n_neighbors).fit_transform(X)
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral)
plt.title("Spectral Embedding/Laplacian Eigenmaps");
tsne = manifold.TSNE(n_components=n_components, init='pca', random_state=0)
Y = tsne.fit_transform(X)
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral)
plt.title("t-SNE/t-Stochastic Neighbor Estimation");
```

Simple 1D Regression (using Least-Squares)

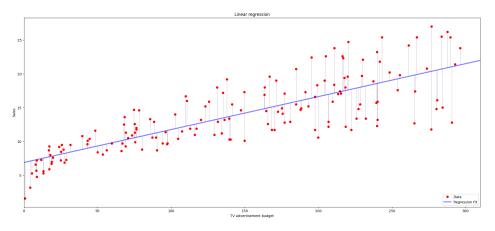
In this section we use the Advertising data from Hastie/Tibshirani book An Introduction to Statistical Learning.

```
%matplotlib inline
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn import preprocessing
from sklearn.model_selection import train_test_split
np.random.seed(47)
# If you are NOT using google colab, you need to take this part out starting from here
from google.colab import files
uploaded=files.upload()
# till here
advertising = pd.read_csv('Advertising.csv',usecols=(1,2,3,4))
X = np.array(advertising['TV']).reshape(-1,1)
y = np.array(advertising['Sales'])
advertising.head()
     Choose Files No file chosen
                                        Upload widget is only available when the cell has been executed in the
     current browser session. Please rerun this cell to enable.
     Saving Advertising.csv to Advertising (1).csv
           TV Radio Newspaper Sales
      0 230.1
                 37.8
                             69.2
                                    22.1
          44.5
                 39.3
                            45.1
                                    10.4
         17.2
                 45.9
                             69.3
                                     9.3
```

```
3 151.5
           41.3
                       58.5
                               18.5
4 180.8
           10.8
                       58.4
                               12.9
```

from pandas.plotting import scatter_matrix scatter_matrix(advertising.iloc[:160], alpha=0.7, figsize=(10, 10), diagonal='kde');

```
300
        200
      \geq
         100
       Radio
         20
         10
        100
         80
         60
       Sales
25
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.2, random_state=5)
                                                                                      Sales
from sklearn.linear_model import LinearRegression
%time regr = LinearRegression().fit(X_train,y_train)
     CPU times: user 3.55 ms, sys: 408 μs, total: 3.95 ms
     Wall time: 18 ms
from matplotlib.collections import LineCollection
y_predicted = regr.predict(X_train)
N = len(y_train)
segments = [[[X_train[i], y_train[i]], [X_train[i], y_predicted[i]]] for i in range(N)]
lc = LineCollection(segments, zorder=0)
lc.set_array(np.ones(len(y_train)))
lc.set_alpha(0.5)
lc.set_linewidths(0.5 * np.ones(len(y_train)))
fig = plt.figure(figsize=[24,10])
# plot the training data
plt.plot(X_train, y_train, 'r.', markersize=12)
# plot the prediction line
x_{in} = np.linspace(0,320,1000).reshape(-1,1)
plt.plot(x_lin, regr.predict(x_lin), color='blue',linewidth=3,alpha=0.5)
# plot the redisuals
plt.gca().add_collection(lc)
plt.xlim([0,310])
plt.xlabel('TV advertisement budget')
plt.ylabel('Sales')
plt.legend(('Data', 'Regression Fit'), loc='lower right')
plt.title('Linear regression')
# plot the regression line
plt.show()
```



```
# The mean square error
print("Residual sum of squares:", (np.mean((regr.predict(X_test) - y_test) ** 2)))
# Explained variance score: 1 is perfect prediction
print("Variance score:", regr.score(X_test, y_test))

Residual sum of squares: 12.579629630036752
Variance score: 0.4990784791512455
```

▼ 2D polynomial regression using least-squares

```
from mpl_toolkits.mplot3d.art3d import Line3DCollection
from sklearn.preprocessing import PolynomialFeatures
from sklearn.pipeline import Pipeline
X = np.array(advertising[['TV', 'Radio']])
y = np.array(advertising['Sales'])
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.2, random_state=9)
# pipeline with polynomial regression
regr = Pipeline([('poly', PolynomialFeatures(degree=1)),
                  ('linear', LinearRegression(fit_intercept=True))])
%time regr = regr.fit(X_train,y_train)
y_predicted = regr.predict(X_train)
     CPU times: user 4.04 ms, sys: 0 ns, total: 4.04 ms
     Wall time: 12 ms
# create 3d line segments for residuals
segments_a = np.concatenate((X_train,y_train.reshape(-1,1)),axis=1)
segments_b = np.concatenate((X_train,y_predicted.reshape(-1,1)),axis=1)
segments = np.array([])
for i in range(len(segments_a)):
    segments = np.append(segments,np.array([segments_a[i],segments_b[i]]))
segments = np.vsplit(segments.reshape(segments_a.shape[0]*2,segments_a.shape[1])
                     ,len(segments_a))
# create a 3D meshgrid
X1, X2 = np.mgrid[X_train[:,0].min():X_train[:,0].max():1,
                  X_train[:,0].min():X_train[:,1].max():1]
pos = np.empty(X1.shape + (2,))
pos[:, :, 0] = X1; pos[:, :, 1] = X2
Z = regr.predict(np.c_[X1.ravel(), X2.ravel()])
```

```
Z = Z.reshape(X1.shape)
fig = plt.figure(figsize=[24,10])
ax = fig.gca(projection='3d')
# plot the regression plane
ax.plot wireframe(X1, X2, Z, rstride=15, cstride=3, alpha=0.4, linewidth=1)
# plot the points
ax.scatter(X_train[:,0],X_train[:,1],y_train,c='r',s=100,alpha=1,zorder=10000)
# Create the 3D-line collection object
lc = Line3DCollection(segments)
lc.set_color('k')
lc.set_linewidth(3)
ax.add_collection3d(lc)
ax.set_xlabel('TV budget'); ax.set_xlim(X_train[:,0].min(), X_train[:,0].max())
ax.set_ylabel('Radio budget'); ax.set_ylim(X_train[:,1].min(), X_train[:,1].max())
ax.set_zlabel('Sales'); ax.set_zlim(y_train.min(), y_train.max())
plt.show()
                                               Traceback (most recent call last)
     <ipython-input-34-9fe8b7890a37> in <cell line: 2>()
          1 # create 3d line segments for residuals
     ---> 2 segments_a = np.concatenate((X_train,y_train.reshape(-1,1)),axis=1)
           3 segments_b = np.concatenate((X_{train}, y_{predicted.reshape(-1,1)}),axis=1)
           5 segments = np.array([])
     /usr/local/lib/python3.10/dist-packages/numpy/core/overrides.py in concatenate(*args,
    ValueError: all the input array dimensions for the concatenation axis must match exactly,
    but along dimension 0, the array at index 0 has size 14979 and the array at index 1 has
     size 149790
     SEARCH STACK OVERFLOW
# The mean square error
print("Residual sum of squares:",np.mean((regr.predict(X_test) - y_test) ** 2))
# Explained variance score: 1 is perfect prediction
print("Variance score:", regr.score(X_test, y_test))
     Residual sum of squares: 1.8296065752684298
     Variance score: 0.9258743223517101
```

Parametric Classification Revisited

[] L, 8 cells hidden

Logistic Regression

[] I, 6 cells hidden

Softmax Regression

[] L, 2 cells hidden

Introduction to Keras

[] l, 6 cells hidden

Programming Assignment 4: Logistic Regression on the notMIST dataset

For this assignment, we will perform logistic regression on the notMIST dataset using Keras. I have included the code to import the dataset and display the images. Since these images are black and white matrices, we flatten them to a vector so they can be used with logistic regression. I

also include the code to do the training and testing split.

Create a logistic regression model using Keras. In the first and only layer, set the only layer to:

```
model.add(Dense(output_dim=10, input_shape=[784], activation='sigmoid', W_regularizer=12(0.01)))
```

Train the model for 100 iterations (nb_epoch). Change the loss function to 'categorical_crossentropy'.

Display the loss after the final iteration and plot the training and testing loss, as above. Be sure to change the loss function label on the plot.

```
%matplotlib inline
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
from scipy.io import loadmat
# If you are NOT using google colab, you need to take this part out starting from here
from google.colab import files
uploaded=files.upload()
# till here
data = loadmat('notMNIST_small.mat')
X_temp = data['images']/255
#for i in range(X_temp.shape[2]):
X = np.empty(shape=[X_temp.shape[2]] + [784], dtype='float32')
for i in range(X_temp.shape[2]):
   X[i,:] = X_{temp}[:,:,i].flatten()
y = pd.get_dummies(data['labels']).to_numpy()
print(X_temp.shape)
print(X.shape)
print(y.shape)
X[1,:]
```

Choose Files No file chosen Upload widget is only available when the cell has been executed in the current browser session. Please rerun this cell to enable. Saving notMNIST_small.mat to notMNIST_small (1).mat (28, 28, 18724) (18724, 784) (18724, 10) array([0. , 0.00392157, 0. , 0. , 0. , 0. , 0. , 0. 0.03137255, 0.16078432, 0.38039216, 0.6509804 , 0.87058824, 0.9764706 , 0.90588236, 0.6627451 , 0.4 , 0.11764706, 0. , 0.00784314, 0. , 0. , 0.07450981. 0. , 0. , 0. , 0. , 0. , 0.07450981, 0.3647059 , 0.44313726, 0.42745098, 0.4627451 , 0.5019608 , 0.54509807, 0.6392157 , 0.77254903, 0.8627451 , 0.9607843 , , 1. , 1. , 1. , 1. 0.99607843, 1. , 0.94509804, 0.42352942, 0. 0.00392157, 0. , 0. , 0. , 0. , 0. , 0. , 0. , 0. , 0.01176471, 0.654902 , 1. , 0.9843137 , , 1. , 0.21568628, 0. , 0.00784314, 0. , 0. , 0. , 0. , 0. , 0.5882353, 0.7529412 , 1. , 0.98039216, 0.9764706 , 0.96862745, 0.96862745, 0.98039216, 0.99607843, 1. , 1. 1. , 1. , 1. , 0.99607843, 1. 1. , 1. , 1. , 0.9843137 , 1. , 0.5372549 , 0. , 0.00392157 , 0. , 0. , 0. , 0.18039216, 0.74509805, 0.96862745, 1. , 1. , 1. , 1. , 1. , 1. 0.96862745, 1. , 1. , 1. , 1. , 1. , 0.98039216, 0.9372549 , 0.85882354, 0.76862746, 0.6666667 , 0.6627451 , 0.9372549 , 1. , 0.99607843, 1. , 0.85882354, 0.00392157, , 0.99607843, 1. , 0. , 0. , 0. , 0. , 0. , 0.12 , 0. 0. , 0. , 0. , 0.1254902 , 0.2784314 , 0.3372549 , 0.3372549 , 0.24705882, 0.15294118, 0.07058824, 0.01960784, 0. , 0.01568628, 0. , 0.3764706 , , 0.99607843, 1. , 1. 1. , 1. , 0.00784314, , 0.9764706 , 0.24705882, 0. 1. , 0.00784314, , 0.00392157, , 0.01568628, 0. 0.58431375, 0. , 0. , 0.00392157, 0.00392157, , 0. 0.01176471, 0.01568628, 0.01568628, 0.01176471, 0.00784314, , 0.99607843, 0.9882353 , 1. , 1. , 1. , 0.01568628, 1. , 0.9764706 , 0.2901961 , 0. , v. , 0. , 0. , 0. , 0. v. , 0. , 0. , 0. , 0. , 0. , 0. 0. , 0. , 0. , 0. , 0. , 0. 0.00392157, 0. , 0.01960784, 0.6901961 , 1. 0.9882353 , 1. , 1. , 0.98039216, 1. 0.52156866, 0. , 0.01568628, 0. 0. , 0. , 0. , 0. , 0. , 0. , 0. , 0. , 0.36862746, 1. , 0.9882353 , 1. , 0.9882353 , 1. , 0.00392157, 0. , 0. , 0. , 0. 0. , 0. , 0. , 0. , 0. , 0. , 0.36862746, 1. , 0.01176471, 0. 0. , 0.8 , 0.07450981, 1. , 0.15294118, , 1. , 0.00784314, , 0. 0. , 0. 0. , 0.41568628, 1. 0. , 0.85490197, 0.01176471,

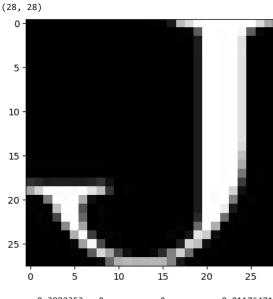
, 0.99607843, 1.

```
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          , 0.9411765 , 0.8235294 , 0.87058824, 1.
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         , 0.99607843, 0.9764706 , 0.98039216, 0.99607843,
         , 0.95686275, 0.70980394, 0.38431373, 0.12156863,
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         , 0.07450981, 0.7921569 , 1.
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          , 0.03921569, 0.41960785, 0.88235295, 1.
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1, 0.01568628 , 0. , 0.30588236 , 0.972549 ,
0.02745098, 0.
0.00784314, 0.01568628, 0.
```

print(X_temp[:,:,3].shape)
plt.imshow(X_temp[:,:,3],cmap="gray");



0.3882353 . 0. . 0. . 0.01176471. 0.01176471. plt.imshow(X_temp[:,:,4504],cmap="gray");

```
10 -

15 -

10 -

15 -

20 -

in, X_test, y_train, y_test = train_test_split(
y, y, test_size=0.2, random_state=50)

(Y, train_chara)
```

X_train, X_test, y_train, y_test =train_test_split(
 X, y, test_size=0.2, random_state=50)
print(X_train.shape)
print(y_train.shape)

(14979, 784)
 (14979, 10)

print(y_test[7])

[0 1 0 0 0 0 0 0 0 0]

```
from keras.models import Sequential
from keras.layers import Dense
from keras.regularizers import 12, 11
from tensorflow.keras.optimizers import SGD

model = Sequential()
model.add(Dense(units = 10, input_shape=[784], activation='sigmoid', kernel_regularizer=12(0.01)))
sgd = SGD(learning_rate=0.1)
model.compile(optimizer = sgd, loss='categorical_crossentropy', metrics=['accuracy'])
```

Model: "sequential_2"

model.summary()

Layer (type)	Output Shape	Param #
dense_2 (Dense)	(None, 10)	7850
Total params: 7850 (30.66 KB Trainable params: 7850 (30.66 Non-trainable params: 0 (0.00	6 KB)	

history = model.fit(X_train, y_train, batch_size = 256, epochs = 100, verbose = 0, validation_data = (X_test, y_test))

```
%matplotlib inline
import matplotlib.pyplot as plt

fig = plt.figure(figsize=(6,4))

plt.plot(history.history['loss'])

plt.plot(history.history['val_loss'], 'g--')

plt.title('Logistic Regression Model Loss')

plt.ylabel('categorical_crossentropy')

plt.xlabel('Epoch')
```

```
print("Loss after final iteration: ", history.history['val_loss'][-1])
print("Training Loss after final iteration: ", history.history['loss'][-1])
plt.show()
```

Loss after final iteration: 0.5614241361618042
Training Loss after final iteration: 0.5444937944412231

