

## Week 3

### Short Answers Assignment 3

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1. Assume the following sample is normal and estimate its mean and variance using the formulas given by maximum likelihood, show your work. [4.43, -0.67, 1.60, 2.65, 3.89, 1.93, -1.52]

To estimate the mean and variance of a normally distributed sample using maximum likelihood

I) To estimate the normal distribution of mean is

$$n = 7$$

$$\mu = (\sum X_i) / n$$

$$\mu = ((4.43 + (-0.67) + 1.60 + 2.65 + 3.89 + 1.93 + (-1.52)) / 7$$

$$\mu = 1.75857143$$

$$\mu = 1.76$$

II) To estimate the normal distribution of Variance is

$$\sigma^2 = \sum (X - \mu)^2 / N - 1$$

$$\sigma^2 = ((4.43 - 1.76)^2 + (-0.67 - 1.76)^2 + (1.60 - 1.76)^2 + (2.65 - 1.76)^2 + (3.89 - 1.76)^2 + (1.93 - 1.76)^2 + (-1.52 - 1.76)^2) / 6$$

$$\sigma^2 = 4.168$$

**Mean ( $\mu$ ) = 1.76**

**Variance ( $\sigma^2$ ) = 4.168**

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2. Assume the following sample of data is bivariate normal. Estimate the covariance matrix, mean vector, and correlation matrix. Show your work.

$$x\_1 = [3.7, -1.6, -0.6, 0.8]$$

$$x\_2 = [0.7, 1.7, 5, 7]$$

Given,

To estimate the Mean vector of the given data of bivariate normal is

$$x\_1 = [3.7, -1.6, -0.6, 0.8]$$

$$x\_2 = [0.7, 1.7, 5, 7]$$

$$x = (3.7 + (-1.6) + (-0.6) + 0.8) / 4$$
$$x = 0.58$$

$$y = (0.7 + 1.7 + 5 + 7) / 4$$
$$y = 3.6$$

$$\text{Mean vector} = [0.58 \ 3.6]$$

**Covariance Matrix:**

$$\begin{bmatrix} \text{cov}(x, x) & \text{cov}(x, y) \\ \text{cov}(y, x) & \text{cov}(y, y) \end{bmatrix}$$

$$\text{cov}(x, x) = \text{var}(x) = 1.99546$$

$$\text{cov}(y, y) = \text{var}(y) = 2.5268$$

$$\text{cov}(x, y) = \text{cov}(y, x) = -1.4525$$

$$\text{Covariance Matrix} = \begin{bmatrix} 1.99546 & -1.4525 \\ -1.4525 & 2.5268 \end{bmatrix}$$

**Correlation Matrix:**

$$\begin{bmatrix} 1 & r_{xy} \\ r_{yx} & 1 \end{bmatrix}$$

$$r_{xy} = -5.81 / \sqrt{15.9275 \times 25.54} = 0.288$$

$$r_{yx} = 0.288$$

$$S_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2$$

$$S_{yy} = \sum_{i=1}^n (y_i - \bar{y})^2$$

$$S_{xy} = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

$$\text{Correlation Matrix} = \begin{bmatrix} 1 & 0.288 \\ 0.288 & 1 \end{bmatrix}$$

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**3. What would cause our estimate of a bivariate normal distribution to have an inclining slope? When would the distribution be taller or wider in one axis than another?**

If  $X = aU + bV$  and  $Y = cU + dV$  can be used to represent them, then  $X$  and  $Y$  are considered jointly normal.

So, when there is a negative slope. The estimate of the bivariate normal distribution has an inclined plane.

**4. What entries of a covariance matrix would we use for Gaussian Naive Bayes?**

The naive Bayes Gaussian classifier utilizes the assumption that the  $x$  variables are Gaussian and independent, which means that the covariance matrix is diagonal.

**5. Say we have the following corpus of documents:**

**Document\_1:** love great great = Class\_good

**Document\_2:** good great good okay = Class\_good

**Document\_3:** terrible terrible bad bad okay = Class\_bad

**Document\_4:** terrible bad terrible = Class\_bad

**Document\_5:** okay bad dismal - Class\_bad

**What is the class of the following document? Show your work.**

**Document\_?:** love great terrible bad

$n(\text{love})=n(\text{bad})=(\text{great})=n(\text{terrible})$

$n(\text{bad})=4,$

$n(\text{great}) = 3,$

$n(\text{terrible}) = 4,$

$n(\text{love})=1$

The above document shows that bad and terrible occur most.

Class\_bad.

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#### 6. What does PCA do? How do we interpret a scree graph?

Data analysis and machine learning can use a dimensionality reduction method known as principal component analysis (PCA). Its main objective is to convert high-dimensional data into a lower-dimensional representation while maintaining as much of the original variance is affordable. PCA does this by locating and extracting the principal components, which are linear combinations of the original characteristics that describe the most important sources of variation in the data.

A scree graph (is also called a scree plot) is a graphical tool used to help finding how many principal components we can retain during PCA. It's very helpful when determining how many dimensions to maintain when your data's dimensions are decreased.