6. Piezo pickups

Around 1880, the French physicist Pierre Curie discovered the **piezo-effect**: as crystal sheets of special material are deformed, an electric voltage occurs on their surface. If we mount a small plate of crystal to a guitar top or to a guitar bridge, the vibrations of the guitar lead to deformations of the crystal and therefore to electrical signals. In contrast to the principles of the electro-magnetic transducer, it is not the original string oscillation that is tapped into, but the effect of the latter onto the section of the guitar that can vibrate. This has the advantage that the vibrations of non-magnetic strings can also be captured. However, piezo pickup and magnetic pickup differ not only in the transducer principle but also in their transmission function: the magnetic pickup captures the string velocity (particle velocity v) at one location (single-coil pickup), or at two locations (humbucking pickup) – with position-dependent comb-filter effects (Chapter 2.8). Conversely, the piezo pickup converts the force acting within the bridge (bridge-insert pickup), or it captures the movement of a small area of the guitar top (top-mounted pickup). In the 1960's, top-mounted pickups manufactured by Barcus-Berry started to capture the market as retrofit kits, while in particular the Ovation company fitted their guitars ex-factory with bridge-insert pickups. By now, some solid-body guitars are also feature a piezo pickup as alternative or as supplement to magnetic pickups.

6.1 The piezo-effect

As external forces act on special crystals, an electrical **polarization** results in addition to the deformation – this is due to shifts in the charges located in the material. The descriptive piezo-electric material parameters in fact have a tensor-characteristic, since both the mechanical and the electrical tensions act in three dimensions. For the guitar pickup, however, a simplified description will suffice. A scalar material-characteristic connects both force and electrical voltage, and particle velocity and current with each other in the sense of a two-port (quadripole) electro-mechanical transducer.

Today, most piezo sensors are manufactured from artificially polarized ferro-electric crystal mixtures (**lead zirconium titanate**) that can be optimally matched to the specific applications by suitable doting and composition. **PVDF-foils** (polyvinylidine fluoride) are also deployed. The piezo ceramics are formed from mixed crystals, and need to be polarized after manufacture (sintering, sanding, metalizing) at high temperature by a strong electric DC-field. Over the years, this polarization will decrease again – but only to a relatively insignificant extent so that long-term stability is, as a rule, very good. Thermal or mechanical overload may lead to substantial deterioration of the transmission behavior, but such irreversible changes must not be feared after the sensor has been mounted in place.

The **transducer constant** of a piezo pickup depends on the geometry of the piezo sheet (surface S, thickness h) and on the material-specific piezo-constant e. For an unobstructed thickness-mode transducer, we obtain simple correspondences between the mechanical and electrical quantities that are operating in parallel [3]:

$$\underline{F} = \alpha \cdot \underline{U}; \qquad \underline{I} = \alpha \cdot \underline{v}; \qquad \alpha = e \cdot S / h$$ Transducer equations

The voltage $\underline{U}$ generated at the crystal is proportional to the force $\underline{F}$, the current $\underline{I}$ is proportional to the velocity $\underline{v}$; the transducer constant α is the coefficient of proportionality. In terms of manufacture, the piezo constant e can be trimmed over a wide range; typical values are e = 20 ... 50 N/Vm. As a first value for orientation, α is 1 N/V.

Moreover, the piezo crystal does not only convert mechanical quantities into electrical ones but it also includes mechanical and electrical elements. As a simplification, we need to consider the **stiffness** s on the mechanical side, and the **capacitance** C on the electrical side. It is not surprising that a small sheet of glass-like hardness and a modulus of elasticity of around $5 \cdot 10^{10}$ Pa features a high stiffness. However, a significant share of the stiffness is caused by the electromechanical coupling: we obtain from the capacitance equation $I = C \cdot dU/dt$, via the transducer equations:

$$\alpha \cdot v = C_K \cdot \dot{F} / \alpha \;\rightarrow\; \alpha^2 = C_K \cdot \dot{F} / v = C_K \cdot s_C \;\rightarrow\; s_C = \alpha^2 / C_K$$

The stiffness s_C (caused electrically and converted to the mechanical side) contributes significantly to the overall stiffness that results from the sum of the *crystal stiffness* s_k and the *capacitance stiffness* s_c: $s = s_K + s_C$. This sum also shows up when considering the **energy scenario**: when compressing a small crystal sheet, potential field-energy is stored (even without the piezo-effect). The piezo-effect causes an electrical voltage – and thus potential electrical energy – to appear across the crystal sheet (a dielectric with high dielectric constant). This potential electrical field-energy is supplied by the mechanical side and generates a load to the mechanical source just like an additional spring.

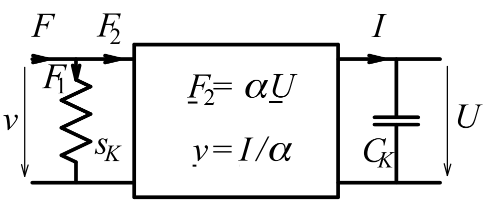


Fig. 6.1: Equivalent circuit of the piezo-transducer. The driving force $\underline{F}$ is divided up into $\underline{F}_1$ and $\underline{F}_2$.

Fig. 6.1 shows the electro-mechanical equivalent circuit diagram. The partial forces $\underline{F}_1$ and $\underline{F}_2$ act in parallel; the representation in the figure (a scalar flow-diagram of the forces) disregards directions in space. As we short the output (U = 0), the input force of the transducer becomes zero, as well, (F_2 = 0), and s_K now is the sole active spring. Literature designates this special load case with a superscript E (clamped field-strength E). The **modulus of elasticity** measured under these conditions is symbolically termed with E^{E} (the superscript E is no mathematical exponent here). Given this, we find for s_K:

$$s_K = E^{\mathrm{E}} \cdot S / h$$ Crystal stiffness

Typical values are: $E^{\mathrm{E}} = 5 \cdot 10^{10}$ Pa … $8 \cdot 10^{10}$ Pa (modulus of elasticity without piezo-effect).

The modulus of elasticity E^{E} describes the mechanical elasticity of the piezo-material for full decoupling, as it is achievable for any value of α if the electrodes on the piezo sheet are shorted. For electrical measurements, on the other hand, the decoupling occurs for every value of α if any movement is prevented. As a thought experiment: for $v = 0$, the secondary current of the transducer is zero, and therefore only the capacitance of the crystal C_K remains. In literature, this special case is designated with a superscript S (clamped mechanical relative deformation S). Typical values for the relative dielectric constant are $\varepsilon_r^{\mathrm{S}} = 1000 \ldots 4000$. Given the above, the capacitance of the crystal C_K can be calculated:

$$C_K = \varepsilon_0 \cdot \varepsilon_r^{\mathrm{S}} \cdot S / h\,; \qquad \varepsilon_0 = 8{,}85 \cdot 10^{-12}\ \mathrm{As/Vm} \qquad \text{Crystal-capacitance}$$

For regular (unclamped) operation, we measure – on the electrical side – two capacitances connected in parallel: the *crystal-capacitance* C_K, and the *spring capacitance* C_s caused by the mechanical side:

$$C_s = \alpha^2 / s_K \qquad \text{Spring-capacitance}$$

In summary: For electrical no-load, i.e. the open-circuit situation with only C_K having an effect on the electrical side, we measure two stiffnesses on the mechanical side: $s = s_K + s_C$. For mechanical no-load with only s_K having an effect on the mechanical side, we measure two capacitances on the electrical side: $C = C_K + C_s$. The reactive load that is transformed by the transducer to the respective other side is: $C_s / C_K = s_C / s_K \approx 50...100\%$. Given high-grade electro-mechanical or mechano-electrical linkage♣, we may assume $C_s \approx C_K$; depending on the piezo material, lower values are possible, as well.

6.2 Electrical loading

An open-circuit connection at the transducer represents an **idealization** that does not really exist in this form. For a piezo-electric guitar pickup, the electrical side is loaded via the cable (acting as a capacitance) and the input impedance of the amplifier, while on the mechanical side, the bridge and the strings need to be considered. Let us assume as a first approach an imprinted force $\underline{F}$ to be the mechanical source. In order to calculate the output voltage, the simplest approach is to transform both this force and the crystal stiffness onto the electrical side [3]:

$$\underline{U} = \underline{F} / \alpha \qquad C_s = \alpha^2 / s_K \qquad \text{Transformed quantities}$$

This transformation yields a purely electrical network that may be investigated using the known approaches for network analysis. Of particular importance is the effect of the electrical load impedance on the **transmission function** $\underline{H}$. The input impedance of a guitar amplifier typically amounts to 1 MΩ; relative to this, a line input can be of much lower impedance (e.g. 50 kΩ). The capacitive internal impedance of the pickup forms – on cooperation with the input impedance of the amplifier – a first-order high-pass (**Fig. 6.2**).

♣ Linkage-factor: $k^2 = W_{\mathrm{mech}} / W_{\Sigma} = C_s / (C_s + C_K) = 0{,}3 \ldots \underline{0{,}5}$.

Seen from the side of the load resistance, the two capacitances are connected in parallel, and they therefore are added up to calculate the cutoff-frequency of the high-pass (3-dB-frequency): $C = C_K + C_s$.

$$f_g = \frac{1}{2\pi R(C_K + C_s)} = \frac{1}{2\pi RC}$$ Cutoff-frequency of the high-pass

With C = 1,5 nF and R = 1 MΩ, we obtain f_g = 106 Hz, which is a value matching the frequency range of the guitar. For R = 50 kΩ (line input), the cutoff frequency would rise to 2,1 kHz, corresponding to a complete loss of the lows. Even less suitable would be a microphone input: its input impedance usually amounts to only about 2 kΩ.

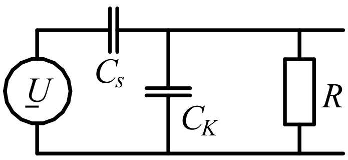


Fig. 6.2: Equivalent circuit of the transducer

For an **active** piezo pickup, the input impedance of the guitar amplifier is immaterial – a battery-operated preamplifier with low output impedance is integrated into the guitar, and makes for a problem-free connection to line inputs. A **passive** piezo system, however, does not include a preamplifier, and the pickup-signal needs to be fed to the guitar amplifier using a shielded cable. High-quality cables represent merely a capacitive load (Chapter 9.4), and therefore increase C_K. On the one hand, this has the effect of a broadband signal attenuation (capacitive divider), on the other hand it decreases the cutoff frequency of the high-pass. The ubiquitous assumption that a **long cable** would attenuate in particular the treble range does not hold for the piezo pickup – the internal impedance of the latter does not have a resistive character but a capacitive one.

6.3 The piezo-transducer as a sensor

In its operation as a sensor, the guitar pickup converts mechanical input signals (string vibrations) into electrical output signals. This represents the normal case; an operation as actor – which is also possible – is of interest only in the context of metrology (Chapter 6.5). According to the idealization used so far, the piezo transducer works as force-to-voltage converter. It captures in particular the AC-component of the bearing force the strings cause, and generates a correspondingly proportional voltage. The bearing force does not act directly onto the piezo crystal, though – a **pickup housing**, with its masses and springs, represents a mechanical filter. The effects of this filter will be investigated from a metrology-point-of-view in the following, using an Ovation pickup (Fig. 6.3) as example.

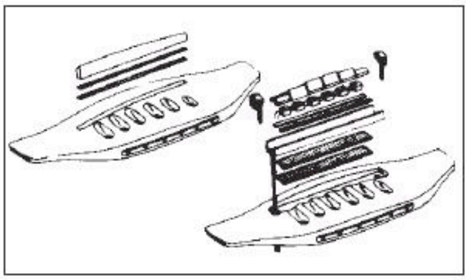


Fig. 6.3: Two different piezo pickups (Ovation).

For the **measurements**, the guitar was laid – within its opened case – on a stone table (with a weight of 250 kg). An electromechanical shaker (B&K 4810) generated translational vibrations; force and acceleration could be measured with an impedance-measurement head (B&K 8001). A small chisel blade was screwed into the impedance head and set onto the saddle-piece of the bridge, and the load for the pickup during the measurements was 2 MΩ. **Fig. 6.4** shows the measured frequency-dependence of the transmission factor, with the corresponding ideal curve of a 2[nd]-order low-pass indicated as a dashed line. The resonance frequency is located at about 3,9 kHz, the Q-factor is about 18.

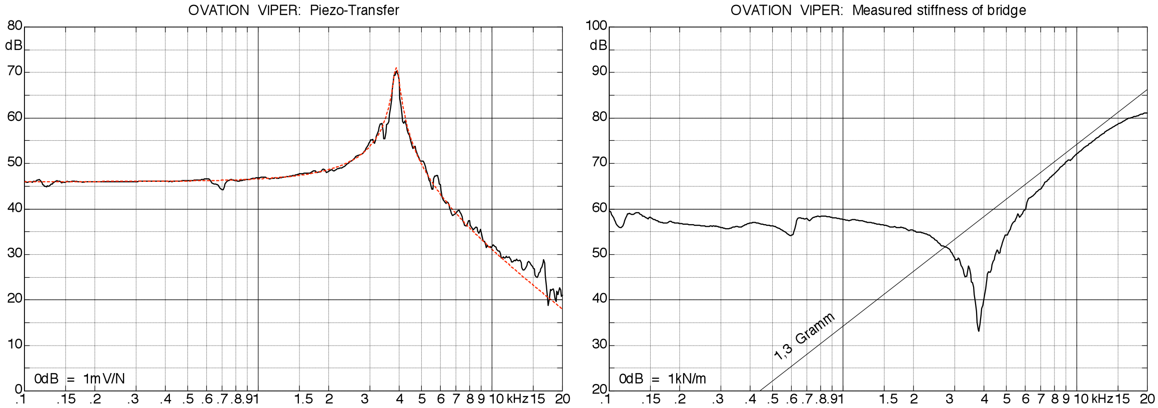
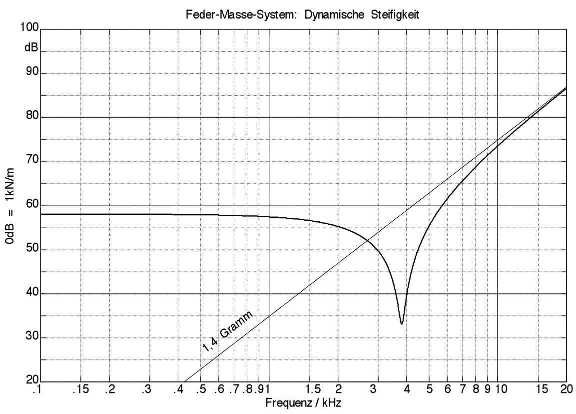
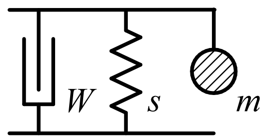


Fig. 6.4: Transmission (left) and dynamic stiffness (right) of the piezo pickup (Ovation).

The shown transmission behavior must not be interpreted as the “frequency response” of the guitar. In the left graph of Fig. 6.4, we see the frequency dependence of the 20-fold logarithm of the quotient U/F; with U = voltage at the piezo and F = measured force. However, the force measured in the impedance head does not correspond exactly to the force at the bridge; rather, a small additional mass is also measured – this mass is due to the mounting plate of the impedance head towards the load. In other words: during the measurement, there is a small **additional mass** of 1,4 gram located on the bridge, and the effect of this mass is also measured. Together with the stiffness of the bridge, the additional mass generates a resonance at 3,9 kHz. To confirm this hypothesis, the **right-hand graph** of Fig. 6.4 shows the quotient of the measured force F and the measured deflection x (again in the usual dB-scaling i.e. the 20-fold logarithm). For f > 100 Hz, we can nicely recognize the behavior of a mass-spring system, with its idealization shown in **Fig. 6.5**. As a simplification, the guitar bridge acts as a stiffness (about 800 kN/m), and together with the additional mass contributed by the impedance head, it forms a resonance at 3.9 kHz.





W=1,9Ns/m; s=800kN/m; m=1,4g.

Fig. 6.5: Dynamic stiffness F/x of an ideal spring-mass system.

The resonance at 3.9 kHz results from the cooperation of the stiffness s_R of the bridge pieces and the additional mass m_0 generated by the end plate of the impedance head and the chisel blade. The exact value of m_0 can easily be measured applying a no-load condition of the impedance head: **Fig. 6.6** correspondingly shows the magnitude of the complex quotient $\underline{F}/\underline{a}$. For an ideal mass, a frequency-independent graph would have to result; any deviations are effects of structural resonances (impedance head and cable).

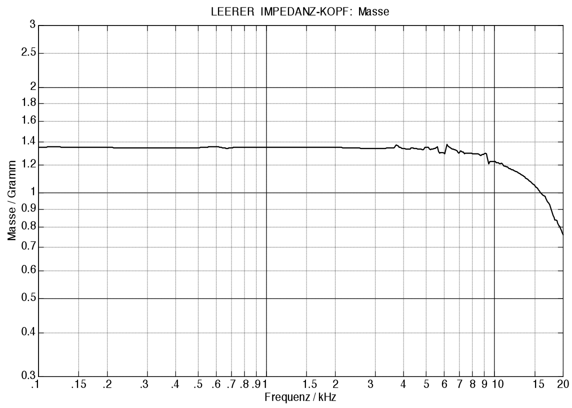


Abb. 6.6: Magnitude of the co-vibrating complex mass $\underline{F}\,/\,\underline{a}$ of the impedance head (incl. chisel). "LEERER IMPEDANZ-KOPF: Masse" = empty impedance head: mass; "Frequenz" = frequency

Assuming time-invariance (not reachable to a full 100%), any artifacts of the measurement head may largely be **compensated**: first, the force- and acceleration-signals are digitally recorded with highest possible quality for the empty measurement head. In a second step, the corresponding analytical signals $\underline{F}$ and $\underline{a}$ are generated using a Hilbert transform. The quotient of $\underline{F}$ and $\underline{a}$, suitable averaged, yields the complex mass $\underline{m}_0 = \underline{F}/\underline{a}$. In order to achieve the compensation when the head is under load, $\underline{m}_0$ is simply subtracted from the measured $\underline{F}/\underline{a}$-quotient. **Fig. 6.7** depicts the results achieved with this compensation. As transmission factor, we get a value of 0,2 V/N (frequency-independent, as a 1st-order approximation), for the bridge-stiffness, about 800 kN/m result.

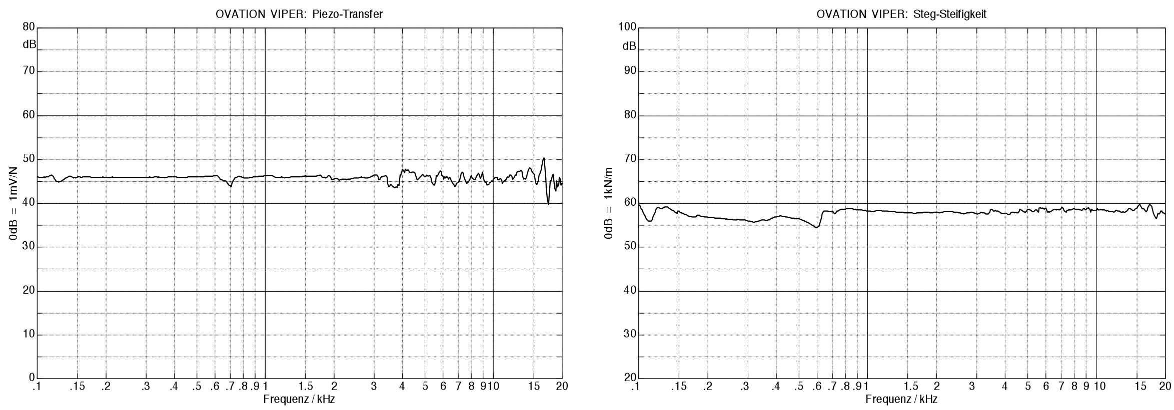


Fig. 6.7: Measurements with compensation of the measurement-head artifacts. Transmission factor (left), bridge stiffness ("Steg-Steifigkeit", right). "Frequenz" = frequency.

In summary: as expected, the investigated piezo-pickup (Ovation EA-68) operates as a force→voltage-converter with a transmission coefficient of 0,2 V/N. The guitar bridge acts as stiffness of about 800 kN/m. For the simple model, these values are frequency-independent; the 3,9-kHz-resonance is and artifact caused by the drive. A closer check reveals small structural resonances not reflected via the simple model.

Fig. 6.8 condenses the results obtained so far into an equivalent circuit diagram. On the electrical side of the two-port network of the transducer, we see the capacitance of the crystal, while the mechanical side holds the stiffness of the crystal s_K and the stiffness of the bridge pieces s_R. The bridge piece is a plastic piece shaped like a pitched roof; it sits on top of the crystal and represents the connection to the string. The stiffness of the bridge piece is smaller than that of the crystal by about three orders of magnitude. Due to this relationship, changes in the electrical load (e.g. shorts) cannot be measured at the mechanical pickup-input (at F) – these changes do cause a difference in the input impedance of the transducer (at F_2), but they are completely insignificant relative to s_R: when connecting two spring in series, the softer one dominates. For the same reason, changes in the mechanical loading (e.g. the mass of the shaker to be connected at the far right in Fig. 6.8) cannot be measured on the electrical side: the very stiff spring s_K dominates since in a parallel arrangement of two springs, the softer spring has little impact on the overall stiffness.

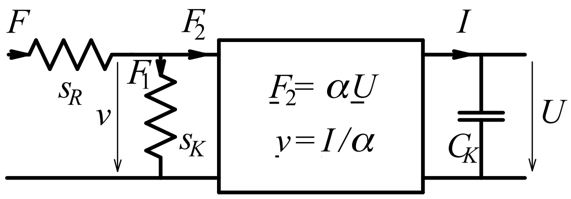


Abb. 6.8: Equivalent circuit diagram of the piezo pickup (Ovation EA-68).

However, with the measured transmission coefficient T_{UF} = U/F = 0,2 V/N, and with the pickup capacitance C = 1,45 nF, we have merely two conditions for the three variables s_K, α, and C_K at our disposal; only s_R is fully defined by the resonance at 3,9 kHz and the mass of the measuring head. Sind there is no supplementary condition available without invasive (and therefore undesirable) action, the ratio $C_s/C_K = s_C/s_K$ was **arbitrarily** taken to be 50%. Given this, we can nevertheless define input and output impedance, as well as the transfer function within the framework of the limits of the model – uncertainty remains only when calculating back to the material parameters. Such an uncertainty, however, exists anyway, since the distributions in space of the mechanical tensions in the piezo and in the bridge pieces is unknown.

The following calculations are based on the assumption that there is no single crystal strip across the whole bridge, but that there is a small, square crystal plate beneath each string, connected with its neighbor by two contact wires. As a mechanical excitation of one bridge piece occurs, only *one single* crystal plate will generate an electrical signal, and the other five crystal plates have the effect of an electric load. For such a string-specific crystal plate, the calculation yields:

$$\alpha = 0{,}28 \text{ N/V}, \qquad C_K = 161 \text{ pF}, \qquad s_K = 9{,}7 \cdot 10^8 \text{ N/m}, \qquad s_R = 7{,}6 \cdot 10^5 \text{ N/m}.$$

Using these data, the model proposed in Fig. 6.8 can explain the following measured quantities: the electrical impedance (as a pure capacitance), the mechanical impedance (as a pure spring), the impedance-behavior shown in Fig. 6.5 as a mass is set onto the piezo), and the frequency-independent transmission behavior. The resonance peaks seen in Fig. 6.7 are not modeled. We will see how resilient this model is as the direction of the signal flow is reversed, i.e. as the sensor is turned into an actor when we apply an electrical voltage. For this case, the above model (using the same parameters) needs to be able to explain the measurement results (Chapter 6.5).

 Translated by Tilmann Zwicker

6.4 Reciprocity

The theory of two-port networks describes the relationships between the input- and output-quantities of linear, time-independent systems using **two-port matrices**. For the electrical two-port system, the input signals applied to the input connectors (port 1) are input voltage U_1 and input current I_1; correspondingly, output voltage U_2 and output current I_2 are found at the output connectors. Connecting these four quantities, **two-port equations** may be defined:

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \qquad \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \cdot \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

Impedance matrix ***Z***
Admittance matrix ***Y***

$$\begin{pmatrix} U_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ U_2 \end{pmatrix} \qquad \begin{pmatrix} I_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \cdot \begin{pmatrix} U_1 \\ I_2 \end{pmatrix}$$

Hybrid matrix ***H***
Inverse hybrid matrix ***G***

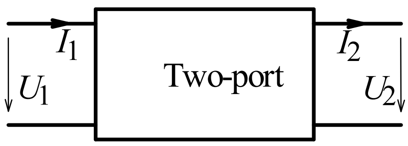


Fig. 6.9: Electrical two-port with technical directions of the reference arrows. In the general case, all quantities are complex.

Besides these four matrices, we may also define the chain matrix (***A***) and the inverse chain matrix (***B***); theses are, however, not required here. Each matrix fully describes the two-port, and the elements of one matrix may be recalculated from the elements of every other matrix. In the general case, the four matrix elements are independent of each other. However, for passive RLCT-two-ports (consisting exclusively of resistors, inductors, capacitors and transformers), only three degrees of freedom remain [e.g. 7], so that two matrix elements are dependent on each other in a simple fashion. Such two-port networks are called transmission-symmetric or **reciprocal**. Using so-called technical♣ reference arrows [7], the following holds for reciprocal two-ports:

$$Z_{12} = -Z_{21}, \quad Y_{12} = -Y_{21}, \quad H_{12} = H_{21}, \quad G_{12} = G_{21}$$

Conditions of reciprocity

Plugging these conditions into the two-port equations and generating, with $U = 0$ or $I = 0$, a rogue starting state, simple relationships between the operation in the forward and the reverse directions result.

$U_1/I_2 = Z_{12}$ $(I_1 = 0)$ $Z_{21} = U_2/I_1$ $(I_2 = 0)$	$I_1/U_2 = Y_{12}$ $(U_1 = 0)$ $Y_{21} = I_2/U_1$ $(U_2 = 0)$
$U_1/U_2 = H_{12}$ $(I_1 = 0)$ $H_{21} = I_2/I_1$ $(U_2 = 0)$	$I_1/I_2 = G_{12}$ $(U_1 = 0)$ $G_{21} = U_2/U_1$ $(I_2 = 0)$

As an **example**: The retroaction $H_{12} = U_1 / U_2$ identified for primary open circuit ($I_1 = 0$) corresponds to the current amplification $H_{21} = I_2 / I_1$ established for secondary short circuit ($U_2 = 0$).

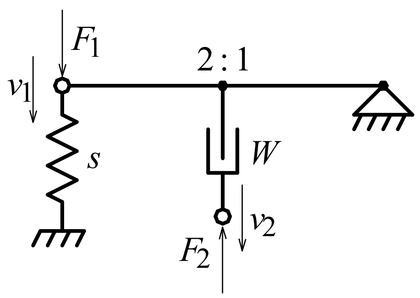
♣ Besides the technical reference-arrow system, applying the **symmetric reference-arrow system** would be just as justifiable; in that case, the direction of the output current would be reversed, and in the conditions of reciprocity, all signs would be reversed. The symmetric reference-arrow system is not used in this book.

In more general terms: knowledge of the transmission behavior in one direction (e.g. from pole-pair 1 to pole-pair 2, or from port 1 to port 2) enables us to determine the transmission behavior into the reverse direction (from port 2 to port 1).

A pickup is not a purely electrical two-port, but this does not stand in the way of a consideration of reciprocity: given the transducer equations $\underline{F} = \alpha \cdot \underline{U}$ and $\underline{I} = \alpha \cdot \underline{v}$, the mechanical signal quantities may be recalculated into the electrical signal quantities, and using $\underline{Z}_{el} = \underline{Z}_{mech} / \alpha^2$, the system quantities may be recalculated [3], such that the electromechanical transducer two-port is changed into a purely electrical reciprocal two-port. As a rogue connector-loading, $v = 0$ and $F = 0$ could theoretically be defined, with only $F = 0$ being practically significant; the total immobility is not obtainable precisely enough.

Of the four conditions of reciprocity, the equality of the G-parameters is best suitable to calculate the piezo pickup. Defining the connector pair designated with 1 as electrical side, and the pair designated with 2 as the mechanical side, the boundary conditions are: from $U_1 = 0$, $F = 0$ results (no external force, bridge pieces without load), and $I_2 = 0$ implies an electrical open circuit (the usual piezo-connection). Using the reciprocity-conditions, the force→voltage-transmission we are looking for can be deduced from the voltage→velocity-transmission (measureable via Laser-vibrometer).

As an **example** for the reciprocity conditions, let us examine an ideal lever to which a spring and a dampening resistance are mounted (**Fig. 6.10**). The reference system used here is an option – other reference systems are possible, as well.



$s = \underline{F}/\underline{x} = j\omega \underline{F}/\underline{v}$ ideal spring (Hooke)

$W = \underline{F}/\underline{v}$ ideal friction (Stokes)

small deflections ≈ linear operation.

Fig. 6.10: Mechanical 2-port purely as example; no direct bearing to the piezo pickup.

The boundary conditions relating to $G_{12} = G_{21}$ are reformulated (using $\underline{F} = \alpha \cdot \underline{U}$ and $\underline{I} = \alpha \cdot \underline{v}$) into a first operational state with mechanical open-circuit ($F_1 = 0$), and into a second operational state with fixed output ($v_2 = 0$). This yields:

$$\frac{\underline{v}_2}{\underline{v}_1} = \frac{2s}{j\omega W} + \frac{1}{2} \quad \text{for } F_1 = 0 \qquad \text{and} \qquad \frac{\underline{F}_1}{\underline{F}_2} = \frac{2s}{j\omega W} + \frac{1}{2} \quad \text{for } v_2 = 0$$

In the 1st case the excitation happens at the right-hand connector (index 2), and the velocity-ratio is determined; in the 2nd case excitation occurs at the left-hand connector, and the force-ratio is determined. In both cases the same transmission function results – but only for the specified load conditions, and not for all load-types.

The system specified above can be reformulated as an equivalent electrical system using the algorithms of electromechanical analogies [e.g. 3]. From the possible choices FI- and FU-analogies, we select the latter because it corresponds to the physical transducer principles present in a piezo transducer. For the network-structure, however, we need to consider that not an isomorphic but a **duality**-network results (the flow-quantity force is mapped to the potential-quantity voltage [3]). The FU-analogy converts a spring into a capacitance, a friction-resistance into an electrical resistor, and a lever into an ideal (inductance-free) transformer that can also transmit DC. For a lever, the ratio of the lever arms (and thus v_1/v_2) is stated; a transformer, however, has – corresponding to the duality – a *voltage* ratio, and thus the transformation ratio is inverted (**Fig. 6.11**).

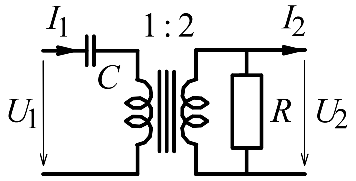


Fig. 6.11: Electrical equivalent circuit diagram derived via FU-analogy from Fig. 6.10. Equal velocity results in equal current, i.e. series connection; equal force results in equal voltage, i.e. parallel connection (dual network)

For the two load conditions $U_1 = 0$ and $I_2 = 0$, a transmission function can be defined for the above; the same result is obtained with the conversion $C = \alpha^2/s$ and $R = W/\alpha^2$:

$$\frac{\underline{I}_2}{\underline{I}_1} = \frac{2}{j\omega RC} + \frac{1}{2} \quad \text{for } U_1 = 0 \qquad \text{and} \qquad \frac{\underline{U}_1}{\underline{U}_2} = \frac{2}{j\omega RC} + \frac{1}{2} \quad \text{for } I_2 = 0 \qquad \Diamond$$

Contrary to a purely mechanical system (Fig. 6.10) or a purely electrical system (Fig. 6.11), the **piezo pickup** represents an electromechanical system. For such a system, the relations of reciprocity hold, too. In the ideal transducer (**Fig. 6.12**), we immediately note that the transmission factors are the same: for primary open-circuit ($F_1 = 0$), we have $I_2 / v_1 = \alpha$, and for secondary open-circuit, the transducer factor $F_1 / U_2 = \alpha$ results. This sameness even holds for every load condition in the ideal transducer – and therefore naturally also for the boundary conditions of the reciprocity-relations.

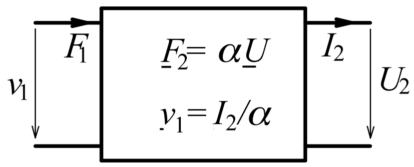


Fig. 6.12: Ideal piezo transducer

As opposed to an ideal piezo transducer, the real piezo transducer includes mechanical and electrical components that are to be connected as a mechanical two-port on the left side and as an electrical two-port on the right side. The two-ports are of reciprocal character as far as they merely include masses, springs, friction-resistances and levers, or inductances, capacitances, resistors and transformers, respectively, and the overall system is then reciprocal, as well. A corresponding two-port ladder-network is shown in **Fig. 6.13** – it may, of course, again be consolidated into a single two-port. Given the boundary conditions mentioned above, we obtain, for F_1 / U_2 and I_2 / v_1, the same coefficient of proportionality. However, the latter does not correspond anymore to the transducer constant α, but depends (in a possibly complicated fashion) on the frequency. This dependency can be measured relatively easily in the $I_2 \rightarrow v_1$-operation, and may be carried over to the $F_1 \rightarrow U_2$-operation (that is more difficult to measure).

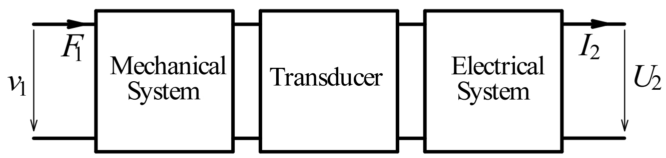


Fig. 6.13: Real piezo transducer

For the $I_2 \rightarrow v_1$-operation, a generator with a low-impedance AC-output is connected to the electrical input of the pickup. Since the pickup represents (in good approximation) a purely capacitive electric load independent of its mechanical loading, it is easy to make the connection from the electrical voltage $\underline{U}_2$ to the current $\underline{I}_2$ ($\underline{I}_2 = j\omega C \underline{U}_2$). This current→velocity transmission-coefficient T_{vI} corresponds to the force→voltage transmission-coefficient T_{UF} for the electrical open-circuit:

$$\frac{\underline{U}_2}{\underline{F}_1} = \frac{\underline{v}_1}{\underline{I}_2} = \frac{\underline{v}_1}{j\omega C \cdot \underline{U}_2} = \frac{\underline{x}_1}{C \cdot \underline{U}_2} \qquad T_{UF} \text{ for } I_2 = 0, \quad T_{vI} \text{ for } F_1 = 0$$

The oscillation-velocity v_1 can be determined e.g. with a Laser-vibrometer; due to the small values to be measured, suitable averaging is mandatory.

6.5 Operation as an actor

Piezo-electric materials convert in both directions: mechanical quantities into electrical ones (operation as sensor), and electrical quantities into mechanical ones (operation as actor). As an electric AC-voltage is connected to the electrical connectors of the pickup, the bridge piece vibrates up and down … a bit. A very small bit, actually: merely a few nanometers. We could not find out at which voltage the crystal was going to receive irreversible damage, and therefore the following measurements were carried out with a RMS-voltage of 10 V – no recognizable damage occurred there. During the measurement, the Ovation guitar was placed in its case, and the vibration velocity was measured using a **laser-vibrometer** (Polytec). Based on the equivalent circuit diagram shown in Fig. 6.8, we would expect, for a mass-free bridge piece (idealization), a frequency-independent *displacement*, if a frequency independent voltage is imprinted. However, the vibrometer – based on the Doppler effect – measures the vibration velocity as its source-quantity, and therefore the measurement grows more difficult with decreasing frequency. Nevertheless, using sufficiently narrow-band filters makes coherent results possible also in the low frequency domain (**Fig. 6.14**). Both the actor- and the sensor-measurements show, as a 1st-order approximation, a frequency-independent transmission factor, although there are smallish frequency peaks – these are mainly caused by the guitar and not by the measuring process.

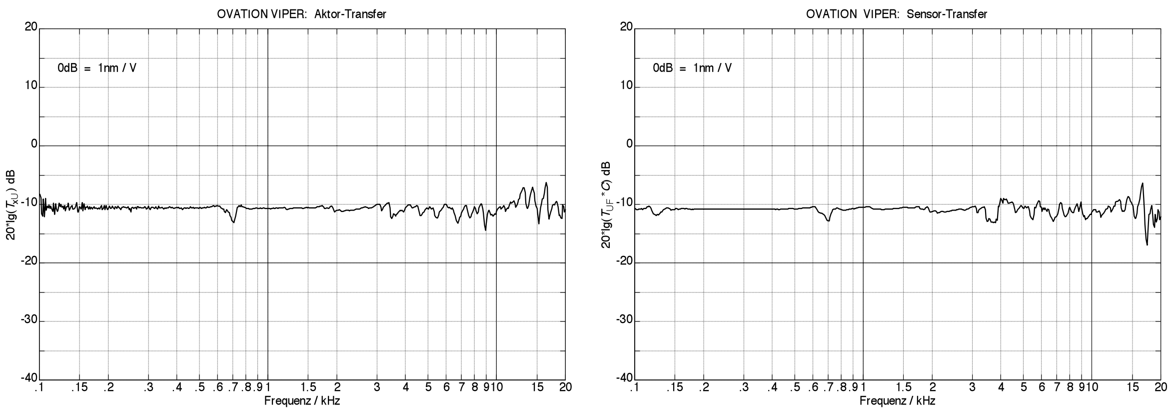


Fig. 6.14: Transmission factor G_{xU} as measured with the laser-vibrometer (left). For comparison, the corresponding sensor-measurement is shown on the right: $T_{xU} = C \cdot T_{UF}$. The correspondence is impressive. "Frequenz" = frequency; "Aktor" = actor.

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The actor-measurement gives us a voltage→deflection transmission coefficient of 0,29 nm/V. For the selected generator voltage (10 V_{eff}), this implies a deflection of 2,9 nm, and a velocity of only 1,8 μm/s at 100 Hz, and it also means that the laser-vibrometer generates as little as 0,36 mV (for 0,2 Vs/mm). This small voltage of the signal is clearly below the **intrinsic noise** of the laser-vibrometer, and an exact measurement requires deployment of a frequency-selective tracking-filter (**Fig. 6.15**). Alternatively to the process applied here (**Hilbert-transform**, complex quotient, block-averaging), we could also calculate the complex quotient of two DFT-spectra – however, the path via the Hilbert-transform is more conducive for the logarithmic frequency axis we employ.

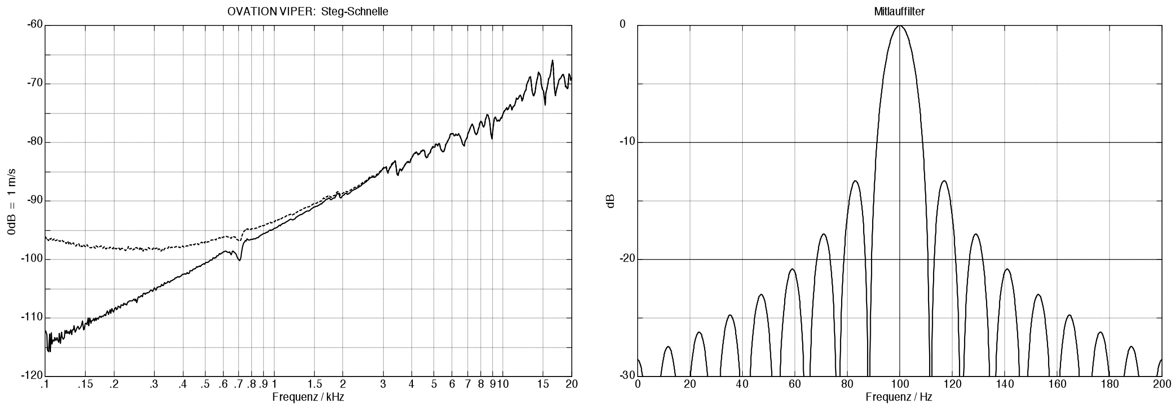


Fig. 6.15: Measured velocity w/out (---) and with tracking filter (left); filter characteristic at 100 Hz (right).

The **comparison** of operations as sensor and actor (Fig. 6.14) indicates that the mass-compensation elaborated in Chapter 6.3 delivers exact results, and that our modeling of the transducer is suitable as a first approximation. The noise-effects appearing in the low-frequency range with the laser measurement can easily by reduced by extending the averaging-window – thus we have at our disposal two equivalent processes for f < 3 kHz. In the higher frequency range, the two approaches show smallish differences that can be attributed to an imperfect mass-compensation, and to small differences in the measurement position.

Many of the high-frequency resonance peaks can be traced to structural resonances of the roof-shaped string pieces and the strings. Still, the **lower side of the pickup** merits some consideration because it forms the reference system relative to which the upper side of the piezo (oriented towards the string) vibrates. The lower side of the pickup is formed from a u-shaped aluminum rail laid on top of shims (made from Pertinax) that are placed on the routed-out guitar body. At least this is the situation with the factory-fitted instrument. For the measurements described above, strips of corrugated paper replaced the shims; any misgivings that the absorption would be unduly increased were in fact unfounded. On the contrary, the original shims resulted in a small dip in the frequency response of the transmission at around 5 kHz that could be attributed to a resonance in the u-rail. It appears the strips of corrugated paper made for a better contact and therefore a better dampening of this resonance. Note, though, that axiom "the sound is in the ear of the beholder" does always hold. In its left-hand section, **Fig. 6.16** shows the transmission factor G_{xU} for actor-operation with the original shims, and on the right the displacement of the u-rail measured for the same drive-level. We clearly see that the u-rail starts to vibrate more strongly in the range around 5 kHz. The special shape of these vibrations was not further investigated since the effort would have been unreasonable.

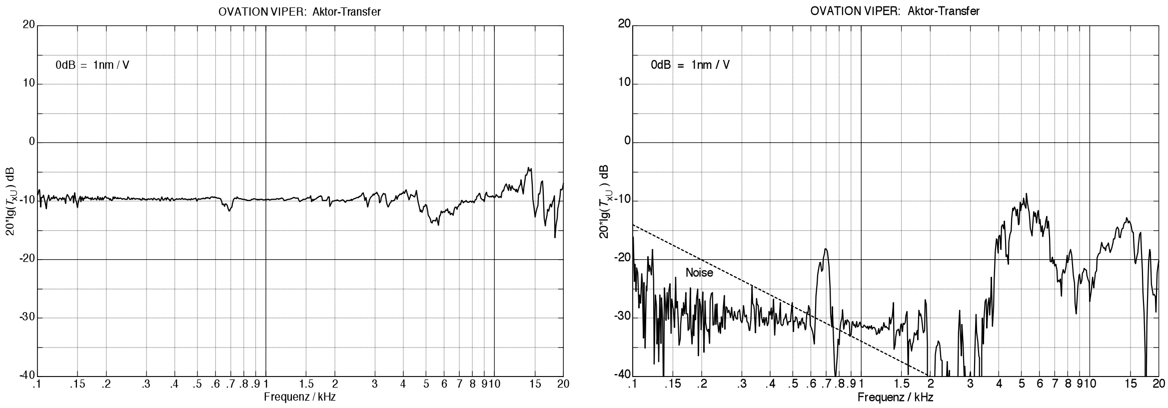


Fig. 6.16: Transmission factor incl. the Pertinax-shims: bridge pieces (left), u-rail (right). The piezo-pickup was driven from a low-impedance source for both measurements. We see a resonance of the guitar top around 700 Hz,; around 5 kHz and 15 kHz, a resonance of the u-rail carrying the piezo crystal occurs.
“Frequenz” = frequency; “Aktor” = actor.

Besides the solid-body Viper (EA-68, a rather uncommon design for an Ovation guitar), a more typical steel string acoustic (**Adamas SMT**) was also analyzed. The “mid-depth bowl”-designated Lyrachord-body carries a laminated top of birch and carbon-fiber layers; a piezo-pickup is built into the bridge. **Fig. 6.17** shows the comparison between sensor- and actor-operation; again there is good correspondence. Measuring in the actor-operation proved to be somewhat more difficult than for the Ovation EA-68, because the vibration-happy top was excited by ambient noise – especially in the range of the 160-Hz-resonance.

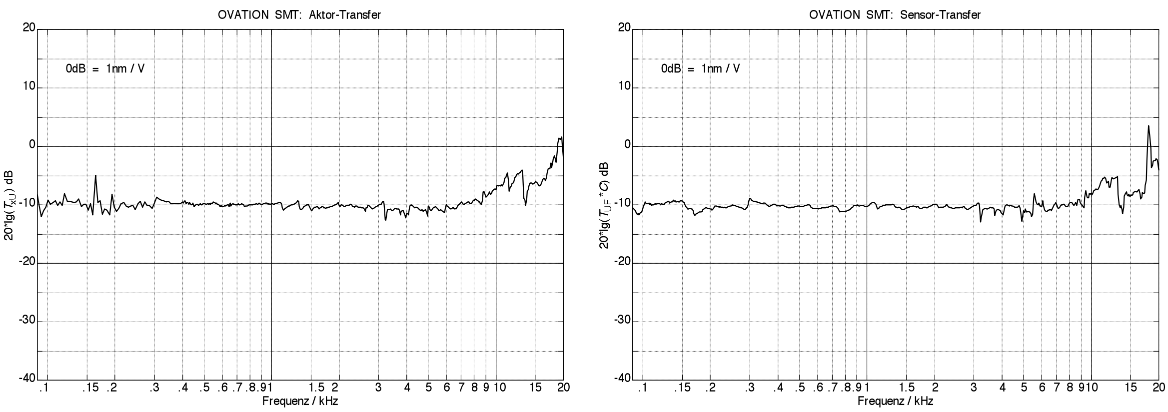


Fig. 6.17: Transmission factor of the Adamas SMT. Actor-measurement (left), sensor-measurement (right).
C = 450pF. “Frequenz” = frequency; “Aktor” = actor

Compared to the Viper, two significant differences show up: the capacitance of the SMT-pickup only amounts to 450 pF (re. 1,45 nF for the Viper), and the transmission behavior of the SMT includes a boost in the range of the highest octave. Operating the pickup in conjunction with the FET-preamp built into the respective guitar, a small difference is also revealed in the low-frequency range: the Viper-piezo sees a load of 500 kΩ resulting in a high-pass with a cutoff-frequency of 220 Hz. Conversely, the SMT-piezo is connected to a preamp with an input-impedance of 2 MΩ, yielding a lower cutoff-frequency of 177 Hz. The above measurements do, however, not show any high-pass behavior: this does not occur in the actor-operation as a matter of principle, and it was computed out for the sensor-operation.

6.6 The pickup in isolation

Its internal build does not exclusively determine the transmission behavior of the piezo pickup – the surface it is positioned on also weighs in. Measuring the pickup without an abutting surface, i.e. without guitar body, therefore represents an obvious step. We must, however, not expect to find an ideal, frequency-independent transmission factor. Rather, the contrary will be the case: in isolated condition, strong Eigen-vibrations may possibly happen that only receive attenuation by mounting the pickup. **Fig. 6.18** depicts laser-measurements of the Viper-pickup (Ovation EA-68) taken out of the guitar and clamped in a bench vise in two different ways.

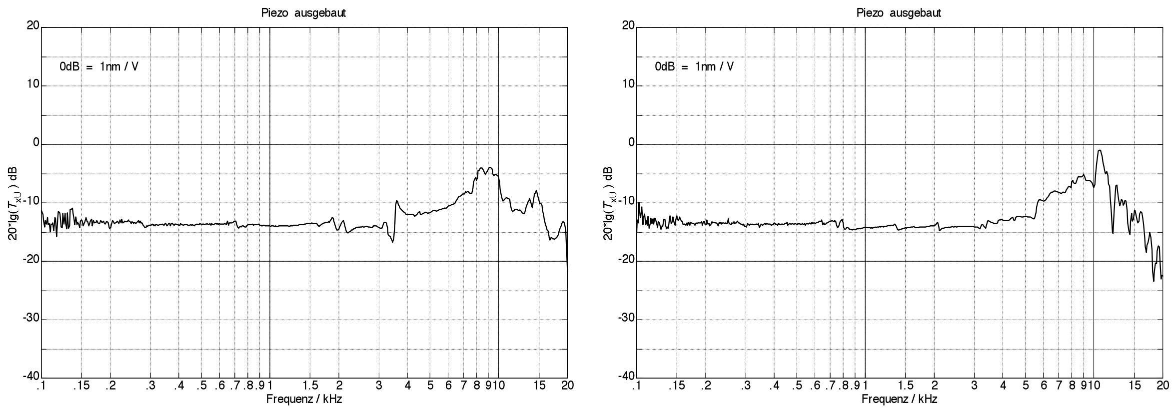


Abb. 6.18: Transmission factor of the Viper-pickup taken out of the guitar and clamped in a bench vise. “Ausgebaut” = de-mounted i.e. in isolation.

This type of mounting is not very meaningful and difficult to reproduce. As an alternative, the pickup was laid on top of a fist-sized brass-block and firmly braced with 2 to 6 steel wires (**Fig. 6.19**). Depending on the contact between u-rail and brass-block, we obtain a big variety of transmission characteristics – and therefore the measurement of the pickup mounted to the guitar is indeed the most suitable one.

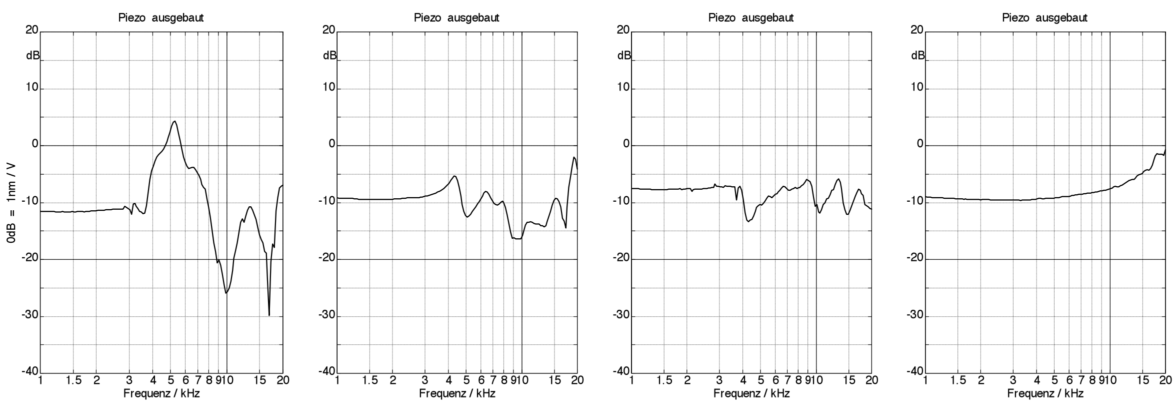


Fig. 6.19: Transmission factor of the Viper-pickup taken from the guitar and braced to a brass-block. “ausgebaut” = de-mounted i.e. in isolation. “Frequenz” = frequency

A curiosity on the side: the u-rail had a paper label (type, serial number) glued to its lower side. Due to this, part of the rail had no mechanical contact to the base anymore. Genius or ignorance?

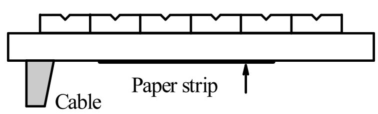


Fig. 6.20: Viper-pickup

6.7 Noise

For every piezo pickup cooperating with a battery-powered preamplifier, we encounter conflicting goals between noise interference and life-time of the battery: the smaller the drain-current of the involved field-effect transistor, the longer the battery keeps – but the higher also the noise-voltage density e_n, and thus the preamp noise. However, as a rule we are not sailing in very critical waters here, and we can opt in favor of the staying power of the battery.

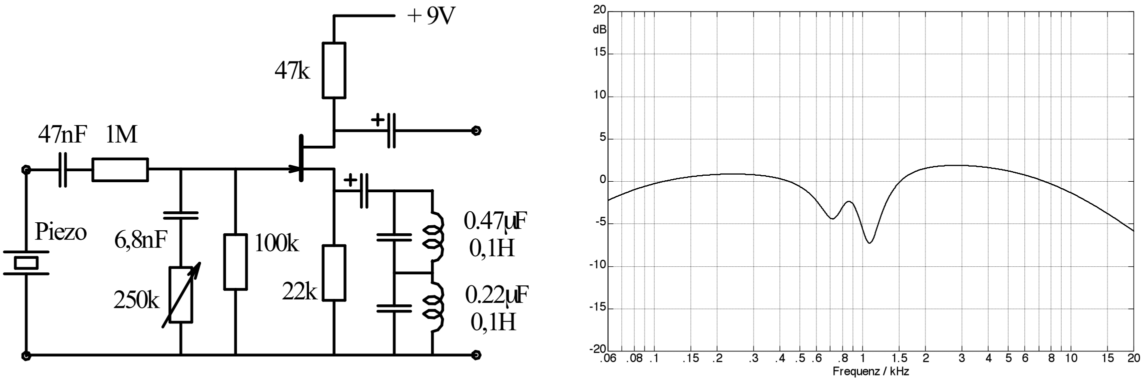
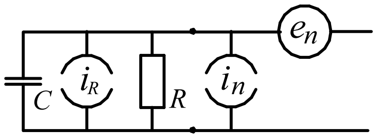


Fig. 6.21: Schematic and gain of an early Ovation-preamplifier. The adjustable 250-kΩ-resistor serves as tone control, at the output we find a volume potentiometer of 100-kΩ (not included in the drawing).

Fig. 6.21 shows the schematic of a battery-powered FET-amplifier as it was deployed in the first-generation **Ovation** guitars. Given a quiescent current of about 0,1 mA, a battery lifetime of about 4000 h may be expected (for Alkaline batteries). The two resonance circuits connected to the source cause an attenuation of the mids that – according to the manufacturer – influence the sound advantageously. Since in this configuration, the FET reaches a voltage-gain factor of about 20 and could easily be overdriven, the signal from the piezo needs to be correspondingly reduced by a voltage divider at the input. In the 2-kHz-frequency-range that is important to our hearing, we therefore find an effective resistance of 67 kΩ at the FET-input that causes – with 33 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ – so much noise that the FET-noise itself may be disregarded. The multiplication of the noise-voltage density mentioned above with the square-root of the bandwidth of the 1/3rd -octave at 2 kHz, and with the voltage gain gives us a noise voltage of 14 µV at the circuit output – that ain't really much of a low-noise design, but it's not that bad either given a maximum obtainable signal voltage of 1 – 2 V.

Later Ovation preamps distinguish themselves from the model discussed above by a somewhat lower noise, and by distinctly higher current consumption. For example, the **quiescent current** of the Ovation Viper described in the preceding chapters amounts to about 1,2 mA, and the SMT even draws as much as 4,6 mA from the 9V-battery. On the other hand, we should not be silent about the versatile equalizer built into these models that also requires power. If we want to look into the data of integrated operational amplifiers (OP), we need to direct our attention first to their voltage supply: typically, an OP is operated with ±15 V; battery-operation with ±4,5 V or even as low as ±3,5 V is often but not always possible. Moreover it needs to be noted that some OP-data deteriorate relative to the datasheets if the supply voltage is reduced.

In order to calculate the amplifier-noise it is conducive to transform all noise sources to the input of the amplifier and to indicate them as so-called equivalent input-noise sources. In this respect, datasheets for OPs specify the **noise voltage density** e_n and the **noise current density** i_n. A straightforward equivalent-circuit for noise (**Fig. 6.22**) considers the piezo-capacitance C (1,5 nF), the input impedance R (1 MΩ) of the amplifier, the noise voltage density e_n (42 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$) of the amplifier, and the noise current density i_n ($10\,\mathrm{fA}/\sqrt{\mathrm{Hz}}$) of the amplifier. For this first example, the noise densities were taken from the datasheet of a FET-OP (TL061) that is suitable for battery operation (current drain: 0,25 mA).



$$i_R = \sqrt{4kT/R}$$

$$\sqrt{4kT} = 1{,}3 \cdot 10^{-10}\,\sqrt{\mathrm{W/Hz}}$$

Fig. 6.22: Equivalent circuit for noise of a piezo pickup with connected amplifier.

As an approximation, all noise processes are statistically independent, and their effects need to by added using a Pythagorean summation. Relative to the noise current density i_R caused by a resistor R, the noise current density i_n of a typical FET-OP is insignificant. In conjunction with R and i_R, the capacitance C forms a low-pass with a cutoff frequency at around 150 Hz. Therefore, the capacitance of the piezo pickup increasingly shorts the resistor noise toward higher frequencies. The noise generated in the kHz-range that is significant for our hearing is predominantly caused by e_n. For the TL071, the resistor noise (i_R) dominates in the low-frequency range, while for the TL061, the OP itself makes a considerable contribution, as well (flicker noise).

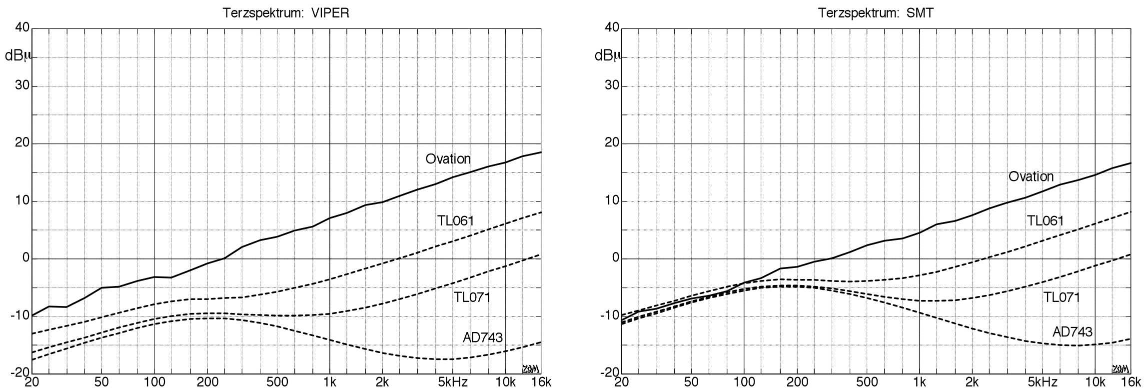


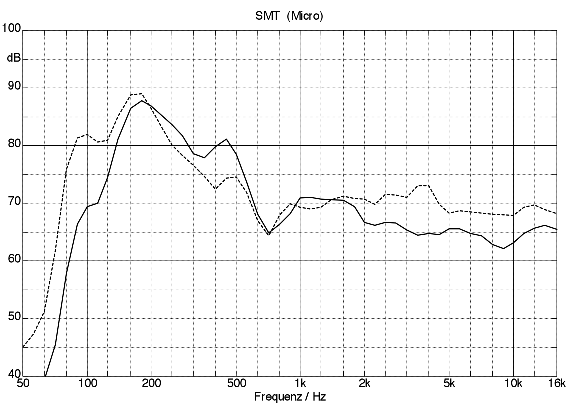
Fig. 6.23: 1/3rd-octave spectrum (= "Terzspektrum") of the piezo-preamp output. 0 dBµ ⇒ 1 µV. Ovation-Viper (left), Ovation-SMT (right). Voltage gain factor = 0,5. Ovation: measurement; FET-OP: calculation.

Fig. 6.23 depicts measurement and calculation in comparison: the **TL061** is well suitable, and with 0,25 mA supply current, a battery life of 1600 h should be possible. Even at only 7 V supply voltage, the output can deliver 1,7 V_{eff}. The **TL071** requires, at 2 mA, clearly more current, but it is less noisy in the high-frequency range. Significantly less noise if generated by the **AD743**, but this OP draws 10 mA and should be operated from a mains power supply. The degree of suffering that we encounter here, however, is not that terrible that it would push us too much towards using especially low-noise amps: when playing loudly, the pickup delivers about 1 V, and therefore an adequate signal-to-noise ration is reachable even with the Ovation preamp – in particular since the guitar itself generates interference noise, too.

6.8 Piezo pickup vs. microphone

An acoustic guitar fitted with a piezo pickup offers two possibilities for recording: airborne sound via a microphone, and structure-borne sound via the piezo. Since the sound radiated by the guitar body is not particularly loud, a recoding microphone needs to be placed as close as possible to the guitar – which a) obliges the guitarist to maintain substantial "positional discipline", and b) generates the permanent fear that in a moment of negligence he/she might ram that one-of-a-kind pre-war Adirondack-fir-top into the mike. Indeed, these shortcomings led to the development of the piezo pickup in the first place: a pickup enabling the player of an acoustic to move around. It sounds differently, though. The preceding chapters have shown that the piezo pickup can deliver the full frequency range of the human hearing with good quality. Since non-linearities (harmonic distortion) and noise do not show up as quality-degrading factors, either, the piezo pickup in principle allows for a sound recording of very high quality. Nevertheless, clearly noticeable differences compared to the radiated airborne-sound remain – this is due to the lack of the transmission filters (formed by the guitar body and neck). The piezo pickup converts the alternating force fed (perpendicularly to the guitar top) to the bridge into an electrical voltage. Conversely, the microphone converts the airborne sound radiated by the guitar top (and the neck, bottom and sides) into an electrical voltage, and therefore measures a different quantity. The latter also does have its source in the force at the bridge (amongst other forces), but depends on it via highly complicated functions.

In order to capture the differences between piezo- and microphone-recording in a practice-oriented manner, an Ovation guitar (Adamas SMT) was recorded in the anechoic chamber using a **measuring microphone** (B&K 4190), and in parallel the piezo-signal was recorded without any filtering. The free-field-equalized microphone generates an objective reference, independent of any treble boost as it is common in studio-work. There are myriad possibilities to position the microphone – not all were tried. For stage-work, often a microphone is pointed to the rear section of the guitar top, supplemented by a second microphone aiming at the neck/body-transition. Arrows mark these positions in **Fig 6.24**; in addition, the correspondingly measured 1/3rd-octave spectra are shown (1/3rd-octave wide filters, measured with overlap at 1/6th-octave distance). A chord-playing guitarist activated the guitar-strings. In both spectra, we recognize a strong emphasis in the frequency range below 500 Hz; this corresponds to the auditory impression. For music productions, an equalizer would be called in to perform suitable attenuation, but for the measurements, filtering was dispensed with.



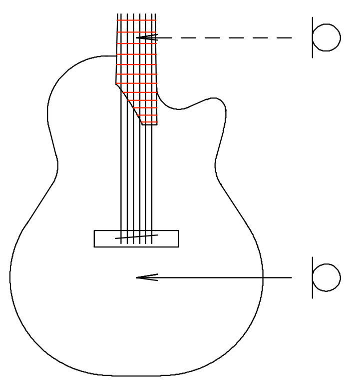


Fig. 6.24: Measured 1/3rd-octave spectra (SPL re. 20 μPa) and microphone positions (at 12 cm distance). Arbitrarily fingered chords, strings struck with a pick in quick succession. "Mikro" = microphone ("mike"), "Frequenz" = frequency.

 Translated by Tilmann Zwicker

Fig. 6.25 shows the 1/3rd-octvae spectrum of the piezo signal that is comparable to Fig. 6.24. It emphasizes the mids but is, in its even curve, well suited for further processing. The right-hand section of the figure highlights the difference between microphone- and piezo-recording: between 100 Hz and 500 Hz, resonances of the guitar top generate a strong emphasis in the microphone-spectrum, and around 700 Hz we find a minimum in the radiation which is, according to statements by manufacturers, desirable.

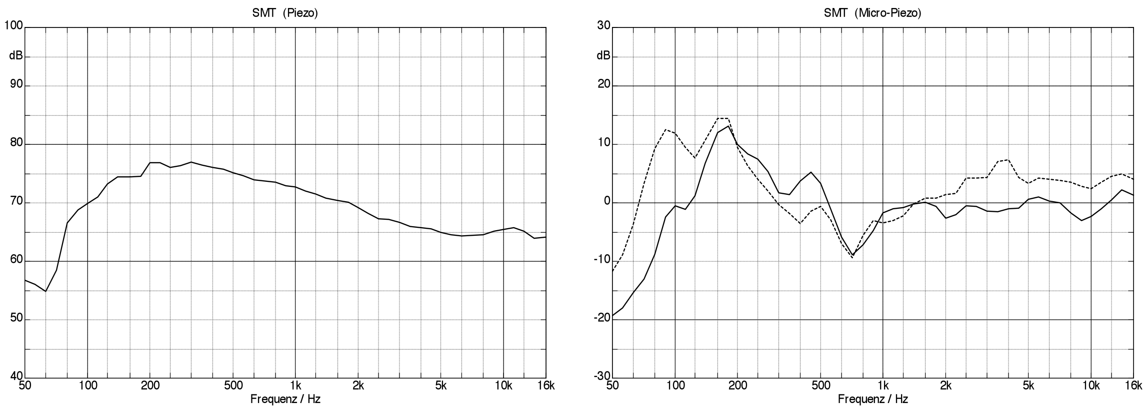


Fig. 6.25: 1/3rd-octave spectra of the piezo signal (left); 0 dB arbitrarily set for a curve comparable to the one in Fig. 6.24. Right: difference "mike-spectrum minus piezo-spectrum" (2 mike positions). "Micro" = mike

Differences in the 1/3rd-octave levels of ±15 dB indicate considerable differences in sound – **listening tests** immediately confirm this. However, as soon as we start to filter one of the two signals with an equalizer such that one spectrum approaches the other, the audible differences become weaker, and limit themselves to minor effects that might possibly also disappear if we could only edit the EQ-curve subtly enough. In fact, as long as we model the guitar as a linear, time-invariant system, this should indeed (better) be the case: the two transmission functions from source (string) to destination (piezo and microphone) are each describable by a fractionally rational function that an equalizer can emulate at least approximately. Non-linear effects are certainly also involved in the generation of sound – but in the investigated example, their impact obviously was rather small.

Consequently, the signals from piezo and microphone are similar to a high degree, and can be, respectively, converted into each other via (a not entirely trivial) filtering. This does not deny that significant portions of the sound energy may be fed to the neck by the nut (or the frets), and radiated there or after transmission to the guitar body – something the pickup integrated into the bridge would not sense. Still, both the sound-flux through the nut/frets and through the bridge have the string vibration as their common source, and are mathematically connected via a fractionally rational function. Conversely, the direction of the string-vibration has a more complicated effect: if a fretboard-normal 110-Hz-eigen-oscillation and a fretboard-parallel 111-Hz-eigen-oscillation jointly act onto the bridge, they might both lead to a radiation of sound. The piezo in the bridge will mainly capture the fretboard-normal component, though. In theory, one signal could again be derived from the other by filtering, but the required Q-factors of the filter stand in the way of a realization. Presumably, one of the main reasons for remaining differences is found here – but the latter are indeed weak.

In summary: neither the piezo-signal nor the microphone signal would be mixed down in the studio in their raw version; given suitable filtering, however, both form a good basis for further processing. Whether one chooses the piezo signal or that from the microphone, or uses a mix of both – that decision needs to be taken based on artistic reasoning.

6.9 Microphonics

Piezo pickups do not only react to vibrations caused be the guitar strings (desirable), but also to vibrations generated by the airborne sound impinging on the guitar (not desired). A sound-wave arriving at the top of the guitar sets the bridge (and thus also the pickup) in motion. Since due to its mass- and spring-loading, the small piezo plate will not join in with this movement in an identical fashion, and a change in thickness generating a corresponding piezo-voltage will result. If the latter is amplified and reproduced by a loudspeaker positioned close-by, a loud feedback-howl might happen. The following investigations will target the quantitative description of this sensitivity to airborne sound (termed "microphonics").

In systems theory, the term **feedback** designates a signal path back to the input; this may be in-phase (positive feedback) or with opposite phase (negative feedback). In the studio- or stage-environment, the term "feedback" usually indicates that the feedback loop has already reached self-excitation, and oscillation occurs in the form of howling or whistling noises. Self-excitation requires a value of equal to or larger than one for the magnitude of the loop gain, and a phase of 0°. For example: a microphone generates a voltage of 50 mV at an SPL of 1 Pa, and a loudspeaker fed from this microphone generates, at the location of the microphone, an SPL of 1 Pa from an operation with 10 V input voltage. If the microphone voltage is amplified by a factor of 200, feedback howling could start (given a matching phase shift).

In practice, reaching self-excitation depends on many details: the directivity of microphone and loudspeaker, the filters, the transmission function of the room. With the **Ovation** Adamas-SMT guitar as test-object, the following will exemplarily illustrate the difference in feedback-sensitivity between operation with a microphone, and operation with the pickup. Other guitars, other speakers and other rooms will lead to similar but of course not identical scenarios.

If an acoustic guitar is to be captured and amplified live via a microphone, we will try to position the microphone as close as possible to the guitar. In this setting, the top of the guitar will act as a reflector that may, depending on the circumstances, direct the impinging sound in an unfavorable manner to the microphone. **Fig. 6.26** depicts a setup as it was put together in the anechoic chamber for orientation measurements. A loudspeaker radiated sound to a guitar that had a measuring microphone (B&K 4190) set up in front of it. The right hand graph shows the comb-filer effects caused by the reflections.

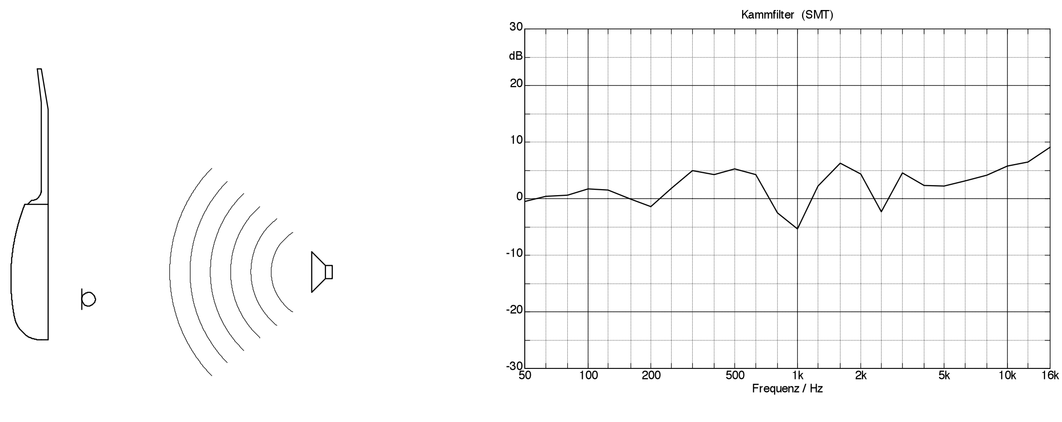


Fig. 6.26: Experimental setup in the anechoic chamber; comb-filter-effect due to reflections. "Kammfilter" = comb-filter", "Frequenz" = frequency.

At low frequencies, the dimensions of the guitar are small compared to the wavelength, and the reflections are insignificant. Around 200 Hz, the guitar acts as an absorber – and at mid-range frequencies we see the typical **comb-filter** peaks. Due to the irregular shape of the guitar, the diffraction waves have slightly different delay times, with the result being that at high frequencies we find an even curve. The increase in the two highest octaves is due to the microphone directionality: nominally, the 4190 is omni-directional, but at high frequencies it does feature increasing beaming – as do all ½"-microphones. With studio-microphones having stronger directionality, the corresponding effects will show up already in the mid-frequency range. Such microphones may be able to attenuate the sound coming from the loudspeaker, but not the sound reflected by the guitar. This example is meant to highlight *that* the guitar as reflector can distinctly deteriorate the resilience against feedback. The degree of this deterioration (and its frequency-dependency) will be a function of the given microphone-distance and –directionality, and of the specific guitar- and microphone-geometry.

To arrive at a statement regarding the sensitivity to feedback in the piezo pickup, we first need to take care that the microphone and the piezo lead to the same sound. For this reason, the microphone signal was filtered such that the same transmission function was achieved with both microphone and piezo. In a second step, we introduced a slight mid-cut, combined with a slight treble boost – adjusted according to artistic criteria. Both via microphone, and via piezo, the sound of the guitar corresponded to the taste of the guitarist. Now the guitar was subjected to the sound emitted by the loudspeaker according to Fig. 6.26, and the loop gain was measured (**Fig. 6.27**). The origin of the ordinate was assigned to the maximum of the airborne sound.

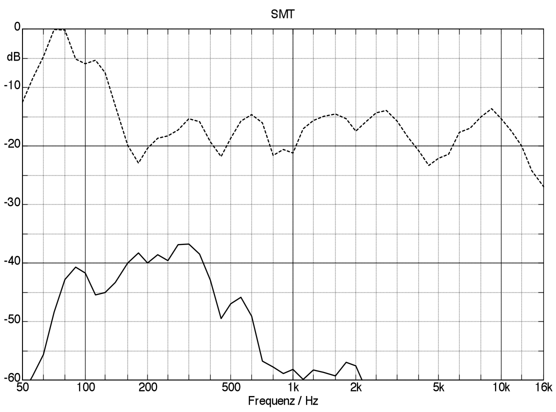


Fig. 6.27: loop-gain for microphone-use (---) and for piezo-use (——). Both transmission paths equalized for the same sound. “Frequenz” = frequency

Fig. 6.27 shows a high loop-gain in the bass range when using the microphone pickup – this will quickly lead to howling feedback as the volume-control is turned up. Clearly, the filtering used here is unsuitable for live-conditions, and it will be mandatory to attenuate the bass – and make do with a more “slender” sound. Not that the latter would be unusable; it is just less full than the sound heard from the unamplified guitar. Compared to using the microphone with the attenuated bass, the piezo offers – in this example – an advantage of 20 – 25 dB. This is the degree to which the level generated by the loudspeaker can be increased before feedback occurs. As already mentioned, this value is for orientation – in every individual case, many parameters will determine the onset of feedback. In any case, however, the differences are so pronounced that a piezo bridge-pickup will always drastically reduce the danger of feedback.

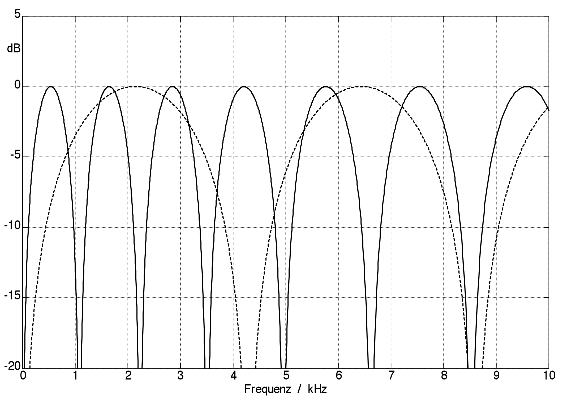
6.10 Differences to the magnetic pickup

Compared to guitars that use one or more magnetic pickups as transducers for the sound, guitars fitted with a piezo pickup sound different (when operated with an amplifier). There are mainly two reasons for this: the transmission range of the pickup, and its position on the guitar. The lower cutoff-frequency of the transmission is sufficiently low for both transducer-types; the upper limit, however, differs: it is 20 kHz for the piezo pickup but only 2 – 5 kHz for your regular magnetic pickup. It would not be a particular problem to radically change the treble response: without any capacitive loading, the magnetic pickup can deliver signals up to 15 kHz as well, and conversely an increase in the mass of the bridge for the piezo pickup can reduce its upper cutoff frequency. However, the piezo pickup typically will "give more treble" than the magnetic pickup.

The magnetic pickup is **positioned** about 3 – 15 cm away from the bridge – mounting it *within* the bridge is disadvantageous because here the string-velocity that needs to be captured is almost zero. The piezo pickup, on the other hand, can only be mounted in the bridge (if we disregard transducers stuck to the guitar top as they have become less important). According to the theory of linear time-invariant systems [6], every signal exciting the strings can be interpreted as the sum of super-positioned impulses. Each of these impulses runs along the strings as a wave (Chapter 2) and is reflected at the bridge and at the nut (or fret). Within *one* round (1 period) it therefore passes the position of the magnetic pickup *twice*. The delay time between both passes results from twice the distance between pickup and bridge, divided by the phase-velocity of the transversal waves♣. Since the transversal wave captured by the magnetic pickup is reflected at the bridge with opposite phase, the double-sampling acts like a filtering with sine-magnitude frequency response (comb-filter):

$$\underline{H} = 1 - e^{-2p\tau} = (e^{p\tau} - e^{-p\tau}) \cdot e^{-p\tau} \quad \rightarrow \quad |\underline{H}| = 2\sin(\omega\tau) \qquad \text{Comb-filter}$$

In this equation, τ stands of the (single) travel time between pickup and bridge. Disregarding any dispersion-effects, the zeroes of the comb-filter lie at the integer multiples of $f_0 = f_G \cdot M/d$, with f_G = fundamental frequency of the string, M = scale, d = distance between pickup and bridge (**Fig. 6.28**). Due to the dispersive propagation of the transversal waves, we get a spreading of the zeroes towards high frequencies (Chapter 1.3).



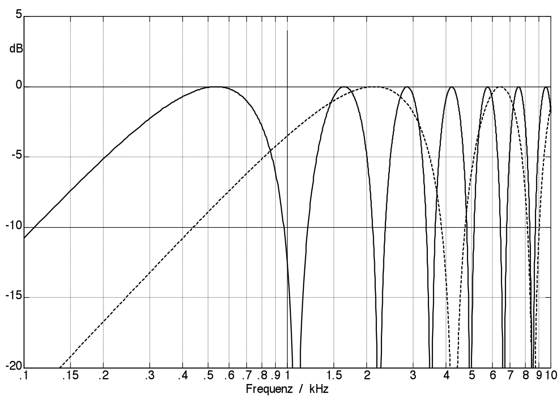


Fig. 6.28: Normalized comb-filter frequency responses. E_2-string (——), E_4-string (----); d = 5 cm, M = 65 cm. "Frequenz" = frequency.

♣ We could also use twice the distance between pickup and nut/fret here; the results would be different at first but can be reformulated into an equivalent model..

Because misunderstandings can easily happen when we term the transmission coefficient of a magnetic pickup as “comb-filter-like”, let’s be precise: the transmission coefficient of velocity→voltage is of a low-pass characteristic if “velocity” indicates the local string velocity over the magnet of the pickup. With respect to the velocity of the transversal *wave*, the mentioned comb-filter comes into play in addition to the low-pass characteristic (low-pass and comb-filter serially connected), if one oscillation period is observed as the timeframe. For the steady state (very long time-window, no dampening), this frequency-continuous transmission function is to be sampled at the locations of frequencies of the partials (frequency-discretization).

For the **piezo pickup**, it is force at the bridge rather than the velocity that forms the input signal to the transducer – though of course for both the initial source is the wave travelling on the string. To compare with the magnetic pickup, it is conducive to specify the same input signal for both transducers, for example the (particle) velocity of the transversal wave. Since the wave-impedance is real, we have proportionality between the velocity of the propagating wave and the respective force; and because the force-wave is reflected at the bearing with the same phase, the bearing-force is also proportional to the velocity of the transversal wave. We arrive at the following **conclusion**: starting from the transversal-wave velocity (the particle velocity of a propagating transversal wave), the piezo pickup mounted in the bridge practically transmits independently of the frequency. If the electrical load-impedance requires it, we may need to consider a high-pass with a cutoff-frequency of about 200 Hz in some circumstances (Chapter 6.5). Conversely, the magnetic pickup will generate string-specific comb-filtering. The further the pickup is located away from the bridge, the closer the “prongs” of the comb-filter will be (**Fig. 6.29**).

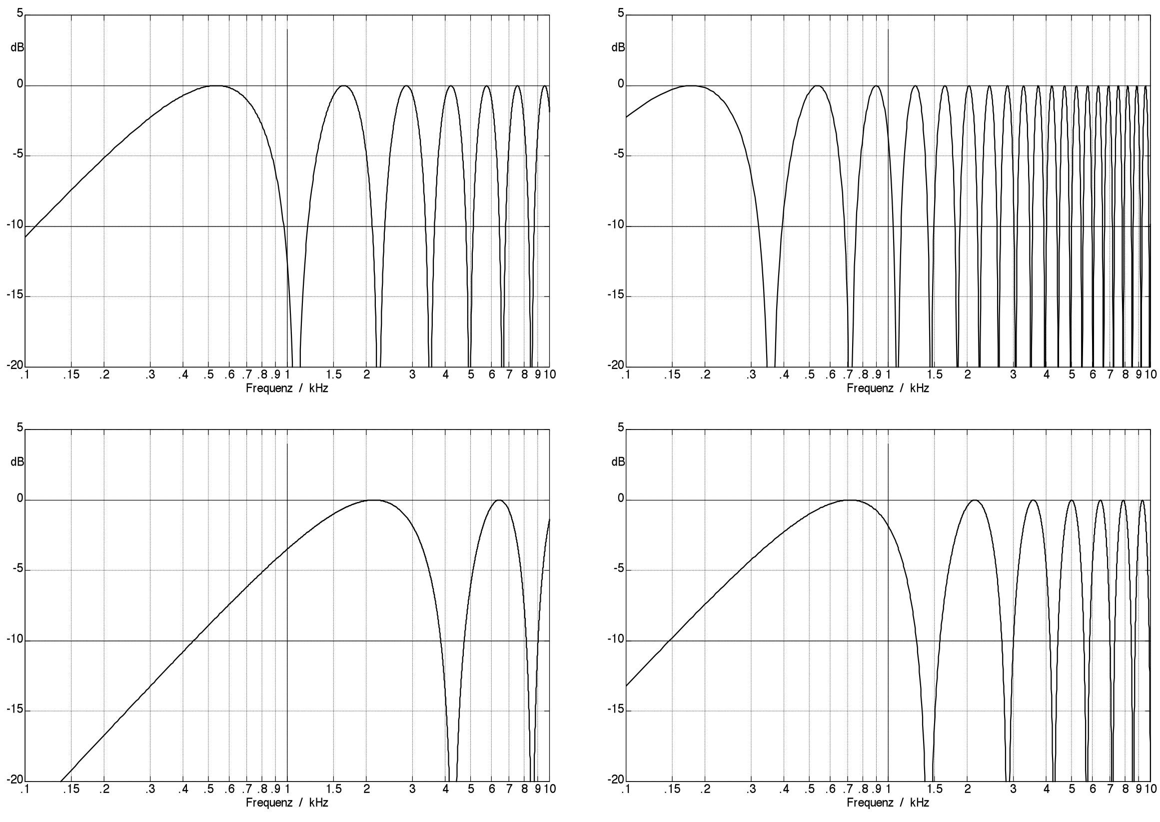


Abb. 6.29: Normalized frequency responses of the comb-filter. E_2-string (top), E_4-string (bottom); M = 65 cm. d = 5 cm (left), d = 15 cm (right). “Frequenz” = frequency.

Supplement to Chapter 6: piezo-electric equations of state

The description of piezo-electric energy conversion uses generally accepted **formula symbols** that, however, often have a different implication in other frameworks. The symbol *S*, for example, elsewhere often stands for a surface area; for the piezo crystal, however, it designates the relative deformation (strain). The letter *E* finds much use as indicator for field strength, but also represents energy, or the modulus of elasticity. The letter *d* may serve to designate a diameter, or a thickness – or the piezo-modulus. The following table lists the formula symbols required to describe piezo-electric transducers. In order to avoid ambiguities, the definition of some of these quantities is limited to the present special chapter.

Symbol	Unit	Designation		Symbol	Unit	Designation
A	m^2	(Surface-) Area		L	H	Inductance
C	F	Capacitance		m	kg	Mass
d	m / V	Piezo-modulus		Q	As	Electrical charge
D	As / m^2	Dielectric displacement		R	Ω	Electrical resistance
e	N / Vm	Piezo-force constant		s^E, s^D	m^2 / N	Elasticity-coefficient♣
E^E, E^D	N / m^2	Modulus of elasticity♣		S	1	Relative deformation
E	V / m	Electric fields-strength		t	s	Time
F	N	Force		T	N / m^2	Mechanical stress
g	Vm / N	Piezo-voltage constant		U	V	Electrical voltage
h	V/m	-		W	Ws	Energy
k	1	Coupling factor		α	N / V	Transducer constant
l	m	Length		ε^S , ε^T	F / m	Dielectric constant♣

In the indexing, the three **coordinates in space** (x, y, z) are replaced by the numbers 1, 2, 3 – in agreement with common convention, the direction perpendicular to the vibrating surface of a thickness-mode oscillating block is indexed with the index 3 (**Fig. 6A.1**). The thickness-mode oscillator is the transducer type most often found in pickups; in the following only this type will be discussed. Designated here with *l*, the thickness typically amounts 0.2 … 1 mm in many cases, while the surface *A* will be about 0,1 … 1 cm^2. Except for the thickness-mode oscillator, there are also flexural oscillators, planary-mode oscillators, shear-mode oscillators, and more types. The thickness-mode oscillator is sometimes also called longitudinal oscillator, or thickness-oscillator with longitudinal effect – the terms are not consistent.

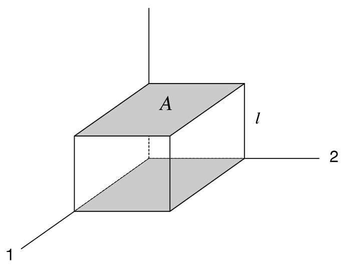


Fig. 6A.1: Piezo-crystal with directional definitions. Top and bottom are metalized and have the surface *A* each; between the surfaces the electrical field strength is formed. The height (thickness) of the crystal is *l*. For the thickness-mode oscillator shown here, the movement (and force) occurs in the vertical direction indexed with 3. The electrical fields run correspondingly in parallel.

♣ The superscript letters designate special load conditions – this will be elaborated on later.

The piezo-crystal shown in Fig. 6A.1 is subjected to the force F_3 in the vertical direction, and a vertical mechanical stress $T_3 = F_3 / A$ results. This, however, is the **first particular case** that needs to be specially designated: *only* one single mechanical loading is to be present – without any electrical loading. The electrical voltage therefore needs to be set to zero via a short across the electrodes (the metalized connecting surfaces). Because, due to this short, the electrical field strength is zero, this special condition is designated with a superscript E – which must not be seen as a mathematical power. Electrically shorted, the crystal reacts as if the piezo-effect did not exist, i.e. for static loading the crystal merely reacts like a spring:

$$S_3 = \Delta l / l = T_3 / E_{33}^E = T_3 \cdot s_{33}^E ; \quad T_3 = F_3 / A \qquad \text{Hooke's law}$$

In this equation, S_3 designates the relative change in length often termed ε in mechanics. However, since ε is also the symbol for the dielectric constant (that is required here, as well), we term – in electro-mechanics – the relative length-change with S. Specifically, we term it with S_3 because it is directed vertically in the direction indexed with 3. The mechanical stress occurring in the vertical direction is designated with T_3 (normally in mechanics the symbol would be σ). E_{33}^E stands for the modulus of elasticity without piezo-effect, i.e. for $E = 0$. The double-indexing (33) is required in the framework of general considerations, because electrical vectors (first number) and mechanical vectors (second number) can occur in different directions. For the present considerations, however, we have a limitation to the vertical direction (see figure), so that the indexing could be dispensed with. Even though, we will keep it in order to maintain conformity to literature. The superscript E is not a mathematical power but a reference to the boundary condition of the electrical field strength: $E = 0$. The formula symbol E (for the modulus of elasticity) can easily be mixed up with an E designating the field strength; we therefore normally use, rather than the modulus of elasticity, its inverse s with the same indexing. Thus, s_{33}^E does not represent a stiffness here, but it indicates the elasticity-coefficient in the direction 3 for the field strength set to zero. While this nomenclature requires getting used to, it is also found in datasheets (e.g. Siemens piezo-ceramics).

To summarize: if we disable the piezo-effect by electrical shorting, we obtain – for static mechanical loading – a **crystal acting like a spring** in the direction 3, having an elasticity coefficient s_{33}^E, the (purely mechanical) behavior of which is described by Hooke's law.

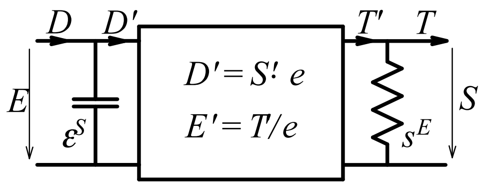


Fig. 6A.2: The piezo-crystal under static loading. Contrary to Chapter 6.1, the area-specific and length-specific quantities are given here. Quadripole arrows in the technical direction. $E = E'$, $S = S'$.

We must now account for the fact that the electrical side will of course not always be shorted. For the **second particular case** with purely electrical loading that is looked into now, any contribution from the mechanical side is prevented by the condition $S = 0$. This mechanical short circuit, also called "firmly-braked condition", necessitates complete rigidity on the mechanical side: the relative change in length (change in thickness) needs to be zero. Conceptually, this can be achieved with an infinitely stiff crystal.

It is customary to indicate this lack of mechanical deformation (S = 0) in the dielectric constant ε by a superscript S:

$$D_3 = Q/A = \varepsilon_{33}^S \cdot E_3 = \varepsilon_{33}^S \cdot U/l = C_K \cdot U/A \qquad \text{Capacitor-equation}$$

In summary: if we disable the piezo-effect via a mechanical short, we obtain – for static loading in the direction 3 – a **capacitor** with the dielectric constant ε_{33}^S; the (purely electric) behavior of this capacitor is described by the capacitor equation.

In the third and last step, we drop the particular conditions of the purely mechanical or purely electrical loading; we now arrive at the general operation by superposition of the two particular cases. The *ideal* piezo-electrical transducer-process (as shown in **Fig. 6A.2** by the rectangle) connects electrical and mechanical quantities:

$$D' = S' \cdot e_{33} \qquad E' = T' / e_{33} \qquad \text{Differential transducer-equations}$$

Using the reference arrows defined in Fig 6A.2, the two node-conditions read:

$$T' = T + T_s \qquad D' = D - D_\varepsilon \qquad \text{Node-equations}$$

The two-pole equations of the storage-elements connect flow- and potential-quantities:

$$T_s = S / s_{33}^E \qquad D_\varepsilon = E \cdot \varepsilon_{33}^S \qquad \text{Two-pole-equations}$$

From this, we can deduce the system of equations for the general operational case:

$$\begin{aligned} D &= D_\varepsilon + D' = E \cdot \varepsilon_{33}^S + S \cdot e_{33} \\ T &= T' - T_s = E \cdot e_{33} - S / s_{33}^E \end{aligned} \qquad \begin{pmatrix} D \\ T \end{pmatrix} = \begin{pmatrix} \varepsilon_{33}^S & e_{33} \\ e_{33} & -1/s_{33}^E \end{pmatrix} \cdot \begin{pmatrix} E \\ S \end{pmatrix}$$

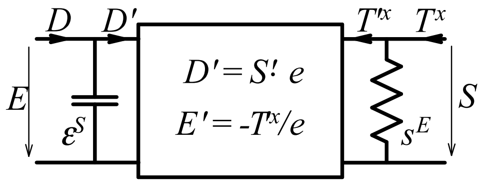
The matrix shown on the right maps the two potential-quantities (E, S) onto two flux-quantities (D, T). It may be interpreted as conductivity-matrix, and transformed to the chain-matrix; at the same time new piezo-coefficients (d, e, g, h) are defined, as well:

$$\begin{aligned} E &= S / (e_{33} \cdot s_{33}^E) + T / e_{33} \\ D &= S \cdot \left(e_{33} + \varepsilon_{33}^S / (e_{33} \cdot s_{33}^E) \right) + T \cdot \varepsilon_{33}^S / e_{33} \end{aligned} \qquad \begin{pmatrix} E \\ D \end{pmatrix} = \begin{pmatrix} 1/d & 1/e \\ 1/g & 1/h \end{pmatrix} \cdot \begin{pmatrix} S \\ T \end{pmatrix} = \mathbf{A} \cdot \begin{pmatrix} S \\ T \end{pmatrix}$$

$$d = e_{33} \cdot s_{33}^E ; \qquad e = e_{33} ; \qquad 1/g = e_{33} + \varepsilon_{33}^S / (e_{33} \cdot s_{33}^E) ; \qquad h = e_{33} / \varepsilon_{33}^S ;$$

The chain-matrix **A** connects both quadripole input-quantities (E, D) to both output quantities (S, T). Its determinant $[\det(\mathbf{A}) = 1/dh - 1/ge = -1]$ is negative because we have a gyratorical mapping here: the ideal transducer maps the potential quantities (E', S') onto the flux-quantities (D', T') [3]. The correspondingly specified signs are in agreement with the four-pole-theory, but in contradiction to the piezo-parameters normally stated in datasheets – these parameters are based on old IEEE-recommendations.

Using the **definition of the algebraic sign** as it is customary in datasheets, the determinant of the chain matrix will be +1, and not -1 (as it would be specific for a gyrator). Now, there are several possibilities to invert the signs in a series connection of three quadripoles. The conversion from the technical direction of the arrows to the symmetrical direction is easy to interpret. This simply corresponds to reversal of the direction of the forces – a direction that requires special attention anyway. Reversing the direction of the forces also reverses the reference direction of the perpendicular stress T specific to the surface-area – the reversed variant of which is termed T^x in the following (Fig. 6A.3).



$$\begin{pmatrix} E \\ D \end{pmatrix} = \begin{pmatrix} 1/d & -1/e \\ 1/g & -1/h \end{pmatrix} \cdot \begin{pmatrix} S \\ T^x \end{pmatrix}$$

Fig. 6A.3: The piezo crystal under static stress. Flow-arrow at the output reversed relative to Fig. 6A.2).

The quadripole equivalent-circuit as shown in Fig. 6A.3 uses the symmetric arrow-direction (as it is common in quadripole theory for the **X**-, **Y**-, **H**- and **G**-matrix) for the definition of the chain-matrix. This is not the usual approach but it is a way to arrive at compatibility with the datasheets. The individual mappings are as follows:

$$\begin{pmatrix} E \\ D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \varepsilon_{33}^S & 1 \end{pmatrix} \cdot \begin{pmatrix} E' \\ D' \end{pmatrix} \qquad \begin{pmatrix} E' \\ D' \end{pmatrix} = \begin{pmatrix} 0 & -1/e \\ e & 0 \end{pmatrix} \cdot \begin{pmatrix} S' \\ T'^x \end{pmatrix} \qquad \begin{pmatrix} S' \\ T'^x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1/s_{33}^E & 1 \end{pmatrix} \cdot \begin{pmatrix} S \\ T^x \end{pmatrix}$$

Consolidating the three matrices into an overall matrix via a multiplication, we get:

$$\begin{pmatrix} E \\ D \end{pmatrix} = \begin{pmatrix} \dfrac{1}{e_{33}s_{33}^E} & \dfrac{-1}{e_{33}} \\ \dfrac{\varepsilon_{33}^S}{e_{33}s_{33}^E} + e_{33} & \dfrac{-\varepsilon_{33}^S}{e_{33}} \end{pmatrix} \cdot \begin{pmatrix} S \\ T^x \end{pmatrix} = \begin{pmatrix} 1/d & -1/e \\ 1/g & -1/h \end{pmatrix} \cdot \begin{pmatrix} S \\ T^x \end{pmatrix} \quad \Leftrightarrow \quad \begin{pmatrix} D \\ T^x \end{pmatrix} = \begin{pmatrix} \varepsilon_{33}^S & e_{33} \\ -e_{33} & 1/s_{33}^E \end{pmatrix} \cdot \begin{pmatrix} E \\ S \end{pmatrix}$$

In this representation, the determinant of the chain-matrix is +1, as is customary in the datasheets. The piezo-parameters can be converted as follows:

$$d = e \cdot s^E = g \cdot \varepsilon^T; \qquad e = d/s^E = h \cdot \varepsilon^S; \qquad g = h \cdot s^D = d/\varepsilon^T; \qquad h = g \cdot c^D = e/\varepsilon^S;$$

$$s^E - s^D = d^2/\varepsilon^T = g^2 \cdot \varepsilon^T = d \cdot g; \qquad \varepsilon^T - \varepsilon^S = d^2/s^E = e^2 \cdot s^E = d \cdot e$$

The superscript letters in these formulas refer to setting the respective quantity to zero, i.e. for example ε^T = dielectric constant for zero-ed (mechanical) perpendicular stress T. However, this always refers to external quantities: in Fig. 6A.3, this would be $T^x = 0$ and not $T'^x = 0$, and $D = 0$, not $D' = 0$. For the other signal quantities, this distinction is not necessary because of $E = E'$ and $S = S'$. The subscript indices need to be included if the orientation in space is to be specified: for example d_{33} (thickness-mode oscillator), d_{15} (thickness-shear-mode oscillator), d_{25} (surface-shear-mode oscillator).

Besides the description of the system with differential quantities (referring to length and area), there is also an integral (macroscopic, global) representation in which U, I, v, F are used rather than E, D, S, T. The electrical field strength E is the length-specific electrical voltage U, the mechanical stress T is the area-specific force F:

$$E = U / l \qquad T = F / A \qquad I = A \cdot j\omega D \qquad v = l \cdot j\omega S$$

For the other two quantities, a time derivative (or an integration, respectively) is required – in the spectral representation, this corresponds to a multiplication with (or, respectively, a division by) $j\omega$. All signals are complex, as is always the case in general signal theory; the under-strike often is dispensed with:

$$I = dQ/dt \;\Leftrightarrow\; \underline{I} = A \cdot j\omega \underline{D} \qquad v = d\xi/dt = l \cdot dS/dt \;\Leftrightarrow\; \underline{v} = l \cdot j\omega \underline{S}$$

The microscopic description using differential quantities looks at the scenario of static stress ($f = 0$). Here, a velocity – the vibration- (particle-) velocity – would be of little help, though, and therefore its integral referring to the length (the relative deformation S) is used. Correspondingly, the current strength I (which is zero in the case of a static load) is replaced by its integral referring to the area, i.e. the charge density (displacement density) D. This static load condition is, however, less relevant for the practical deployment: the electrical resistances cannot be increased indefinitely, and therefore there is always some current flowing that leads to recharging processes. For this reason, calculations in practice mostly use U, I, v, F. Applying integral notation, the equations for the ideal piezo transducer read:

$$F' = U' \cdot \alpha \qquad v' = I' / \alpha \qquad \alpha = eA / l \qquad \text{Integral notation}$$

The algebraic signs are oriented towards Fig. 6A.2, and therefore do not correspond to datasheet-conventions. The apostrophes express that the ideal transducer-effect is referred to – without any contributions by mechanical or electrical two-poles. A block diagram is shown in **Fig. 6A.4** – contrary to Fig 6A.2 it now includes the integral quantities. The two-pole parameters change correspondingly:

$$C^v = \frac{A \cdot \varepsilon^S}{l} \qquad n^U = \frac{l}{A \cdot s^E} \qquad C = Q/U \qquad n = \xi/F$$

For the “firmly-braked”, mechanically fixed case ($S = 0$, or $v = 0$), the dielectric constant ε^S becomes the capacitance C^v, for the electrical short-circuit case ($E = 0$, or $U = 0$) the elasticity constant s^E becomes the spring-compliance n^U.

Cave!: in this chapter, s^E does not designate the stiffness (= force / deflection) but the elasticity coefficient (= 1 / modulus of elasticity)!

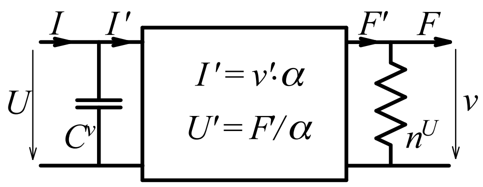


Fig. 6A.4: Block-diagram of the piezo transducer. The capacitance shown in the figure is the crystal-capacitance from Fig. 6.1 B ($C^v = C_K$), the shown compliance is reciprocal to the stiffness of the crystal: $n^U = 1 / s_K$.

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