Exercise-8.1

3. Determine the inverse z-transformation for the following:

a. $X(z) = \dfrac{1}{1+\frac{1}{4}z^{-1}}$; $|z| > \frac{1}{4}$

$X(z) = \dfrac{1}{1-az^{-1}}$

$x[n] = a^n u[n]$

$$z^{-1}\left\{X(z)\right\} = z^{-1}\left\{\frac{1}{1-(-\frac{1}{4})z^{-1}}\right\} \quad ; \quad |z| > |-\tfrac{1}{4}|$$

$$\Rightarrow x[n] = \left(-\frac{1}{4}\right)^n u[n]$$

b. $X(z) = \dfrac{1-\frac{1}{2}z^{-1}}{1+\frac{2}{5}z^{-1}-\frac{3}{5}z^{-2}}$

$$\Rightarrow X(z) = \frac{1-\frac{1}{2z}}{1+\frac{2}{5z}-\frac{3}{5z^2}}$$

$$\Rightarrow X(z) = \frac{\frac{2z-1}{2z}}{\frac{5z^2+2z-3}{5z^2}}$$

$$\Rightarrow X(z) = \frac{2z-1}{2z} \times \frac{5z^2}{5z^2+5z-3z-3}$$

$$\Rightarrow X(z) = \frac{5z(2z-1)}{2(z+1)(5z-3)}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{5(2z-1)}{2(z+1)(5z-3)} \quad \text{--- (i)}$$

$$\frac{2z-1}{(z+1)(5z-3)} = \frac{A}{z+1} + \frac{B}{5z-3}$$

$$\therefore A = \frac{-3}{-8} = \frac{3}{8}, \quad B = \frac{\frac{6}{5}-1}{\frac{3}{5}+1} = \frac{\frac{1}{5}}{\frac{8}{5}} = \frac{1}{8}$$

From (i) $\Rightarrow$

$$\frac{X(z)}{z} = \frac{5}{2}\left[\frac{3}{8}\frac{1}{z+1} + \frac{1}{8}\frac{1}{5z-3}\right]$$

$$\Rightarrow X(z) = \frac{5}{2}\left[\frac{3}{8}\frac{z}{z+1} + \frac{1}{8}\frac{z}{5z-3}\right]$$

$$\Rightarrow X(z) = \frac{15}{16}\frac{z}{z(1+z^{-1})} + \frac{5}{16}\frac{z}{5z(1-\frac{3}{5}z^{-1})}$$

$$= \frac{15}{16}\frac{1}{1-(-1)z^{-1}} + \frac{1}{16}\frac{1}{1-\frac{3}{5}z^{-1}}$$

$z^{-1}\left\{\dfrac{1}{1-az^{-1}}\right\} = a^n u[n]$

$$\therefore x[n] = \frac{15}{16}(-1)^n u[n] + \frac{1}{16}\left(\frac{3}{5}\right)^n u[n]$$

3. (c) $X(z) = \dfrac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{9}z^{-2}}$; $|z| > \frac{1}{3}$

$$\Rightarrow X(z) = \frac{1+\frac{1}{3}z^{-1}}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

$$\Rightarrow X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$\therefore x[n] = \left(\frac{1}{3}\right)^n u[n]$$

4. b, d, e $\rightarrow$ H.W

4. (b) $X(z) = \dfrac{1}{(1+.5z^{-1})(1-.5z^{-1})(1-z^{-1})}$; $|z| > 1$

$$\frac{1}{(1+.5z^{-1})(1-.5z^{-1})(1-z^{-1})} = \frac{A}{1+.5z^{-1}} + \frac{B}{1-.5z^{-1}} + \frac{C}{1-z^{-1}}$$

$$A = \frac{1}{(1+.5\times 2)(1+2)} = \frac{1}{6}$$

$$B = \frac{1}{3.(-1)} = -\frac{1}{3}$$

$$C = \frac{1}{\frac{3}{2}\cdot\frac{1}{2}} = \frac{4}{3}$$

$$x(z) = \frac{1}{6}\,\frac{1}{1-(-.5)z^{-1}} - \frac{1}{3}\,\frac{1}{1-.5z^{-1}} + \frac{4}{3}\,\frac{1}{1-z^{-1}}$$

$$\Rightarrow x[n] = \frac{1}{6}(-.5)^n u[n] - \frac{1}{3}(.5)^n u[n] + \frac{4}{3}u[n]$$

$$\mathcal{Z}\{x[n+1]\} = z\left[X(z) - x[0]\right]$$

$$\mathcal{Z}\{x[n+2]\} = z^2\left[X(z) - x[0] - \frac{x[1]}{z}\right]$$

$$\mathcal{Z}\{x[n-1]\} = z^{-1}\left[X(z) + x[-1]z\right]$$

$$\mathcal{Z}\{x[n-2]\} = z^{-2}\left[X(z) + x[-1]z + x[-2]z^2\right]$$

8.2 ; 6 (a, b, c) → H.W

Use Z-transform to determine $y[n]$; $\boxed{n \geq 0}$

Solving difference equation

a. $6y[n] - 5y[n-1] + y[n-2] = \frac{1}{4^n}u[n]$; $y[-1] = 0$, $y[-2] = 0$

$$\Rightarrow 6Y(z) - 5z^{-1}\left[Y(z) + y[-1]z\right] + z^{-2}\left[Y(z) + y[-1]z + y[-2]z^2\right] = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\Rightarrow 6Y(z) - 5z^{-1}Y(z) + z^{-2}Y(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\Rightarrow Y(z)\left[6 - 5z^{-1} + z^{-2}\right] = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\Rightarrow Y(z)\left[6 - \frac{5}{z} + \frac{1}{z^2}\right] = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\Rightarrow Y(z)\frac{6z^2 - 5z + 1}{z^2} = \frac{1}{1 - \frac{1}{4z}}$$

$$\Rightarrow Y(z)\frac{6z^2 - 3z - 2z + 1}{z^2} = \frac{4z}{4z-1}$$

$$\Rightarrow Y(z) \cdot \frac{3z(2z-1) - 1(2z-1)}{z^2} = \frac{4z}{4z-1}$$

$$\Rightarrow Y(z) = \frac{4z}{4z-1} \cdot \frac{z^2}{(3z-1)(2z-1)}$$

$$\Rightarrow Y(z) = \frac{4z^3}{(4z-1)(3z-1)(2z-1)}$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{4z^2}{(4z-1)(3z-1)(2z-1)}$$

$$\frac{4z^2}{(4z-1)(3z-1)(2z-1)} = \frac{A}{4z-1} + \frac{B}{3z-1} + \frac{C}{2z-1}$$

$$A = \frac{4 \cdot \frac{1}{16}}{\left(\frac{3}{4}-1\right)\left(\frac{1}{2}-1\right)} = \frac{\frac{1}{4}}{-\frac{1}{4} \cdot \left(-\frac{1}{2}\right)} = \frac{\frac{1}{4}}{\frac{1}{8}} = 2$$

$$B = \frac{4 \cdot \frac{1}{9}}{\left(\frac{4}{3}-1\right)\left(\frac{2}{3}-1\right)} = \frac{\frac{4}{9}}{-\frac{1}{3} \cdot \frac{1}{3}} = \frac{\frac{4}{9}}{-\frac{1}{9}} = -4$$

$$C = \frac{4 \cdot \frac{1}{4}}{1 \cdot \left(\frac{3}{2}-1\right)} = \frac{1}{\frac{1}{2}} = 2$$

$$\frac{Y(z)}{z} = 2 \cdot \frac{1}{4z-1} - 4\frac{1}{3z-1} + 2\frac{1}{2z-1}$$

$$\Rightarrow Y(z) = 2 \cdot \frac{z}{4z\left(1-\frac{1}{4}z^{-1}\right)} - 4\frac{z}{3z\left(1-\frac{1}{3}z^{-1}\right)} + 2\frac{z}{2z\left(1-\frac{1}{2}z^{-1}\right)}$$

$$\Rightarrow Y(z) = \frac{1}{2}\frac{1}{1-\frac{1}{4}z^{-1}} - \frac{4}{3}\frac{1}{1-\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$u[n] = \begin{cases} 0; & n < 0 \\ 1; & n \geq 0 \end{cases}$$

$$y[n] = \frac{1}{2}\left(\frac{1}{4}\right)^n u[n] - \frac{4}{3}\left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[n]$$

$$\Rightarrow y[n] = \frac{1}{2}\left(\frac{1}{4}\right)^n - \frac{3}{5}\left(\frac{1}{3}\right)^n + \left(\frac{1}{2}\right)^n \quad [\because n \geq 0;\ u[n] = 1]$$

Ans