REVIEW ON THE LAST CLASS

Capacitor

$$\boxed{C = \frac{Q}{V}}$$

C = farads (F)
Q = coulombs (C) **(10.5)**
V = volts (V)

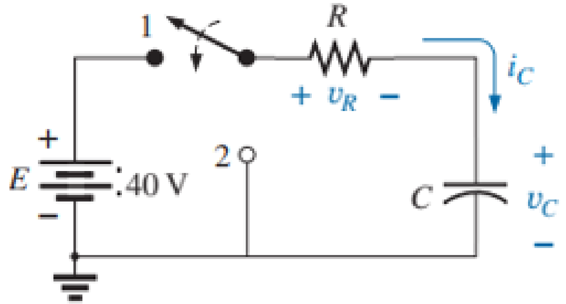
$$\boxed{Q = CV}$$ (coulombs, C) **(10.6)**

$$\boxed{C = \epsilon\frac{A}{d}}$$

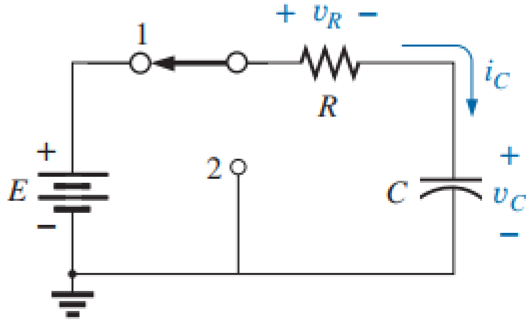
C = farads (F)
ϵ = permittivity (F/m) **(10.9)**
A = m^2
d = m

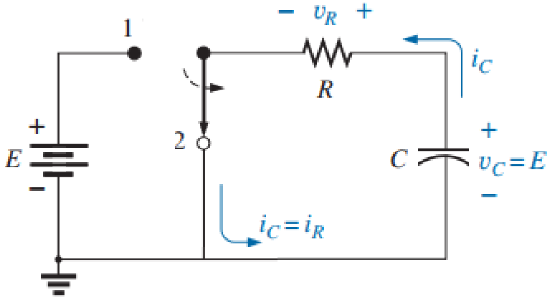
$$\boxed{C = \epsilon_o\epsilon_r\frac{A}{d}}$$ (farads, F) **(10.10)**

$$\boxed{C = 8.85 \times 10^{-12}\epsilon_r\frac{A}{d}}$$ (farads, F) **(10.11)**



$$\tau = RC$$





Charging Mode:

$$v_C = E(1 - e^{-t/\tau})$$

$$i_C = \frac{E}{R}e^{-t/\tau}$$

$$v_R = Ee^{-t/\tau}$$

Discharging Mode:

$$v_C = Ee^{-t/\tau} = -v_R$$

$$i_C = -\frac{E}{R}e^{-t/\tau}$$

Chapter 11

Inductors

Inductor

Iron-core

Air-core

Inductor:

An inductor is a passive element that is constructed simply raping the wire on air or iron core. The inductor is also called **coil** or **choke**.

Inductance:

Inductance is a measure the ability of an inductor to store energy in its magnetic field as well as to oppose the rate of change of current (*di*/*dt*). Unit of capacitance is **Henry** (H).

The higher the inductance of an inductor, the greater the amount of stored magnetic energy on the coil or winding.

Type of Inductors

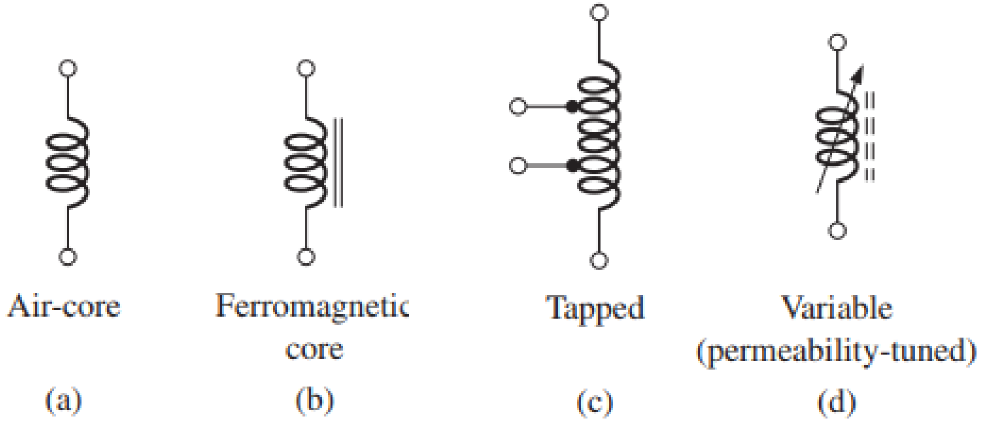
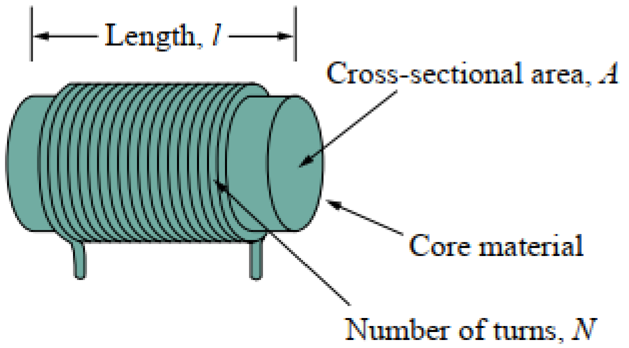


FIG. 11.20 *Inductor (coil) symbols.*

Inductance Based on Physical Dimension

Equation of inductance:



Typical form of an inductor.

$$L = \frac{\mu N^2 A}{l} \qquad (11.6)$$

μ = permeability (Wb/A · m)
N = number of turns (t)
A = m^2
l = m
L = henries (H)

$$L = \frac{\mu_o \mu_r N^2 A}{l} \text{ (henries, H)} \qquad \textbf{(11.7)}$$

$$\mu_o = 4\pi \times 10^{-7} \text{ (Wb/A.m)} \qquad \textbf{(11.4)}$$

μ_r is called the relative **permeability**.

Let, L_o is the value of inductance considering air core and C is the value of inductance for any other metal core, then we have

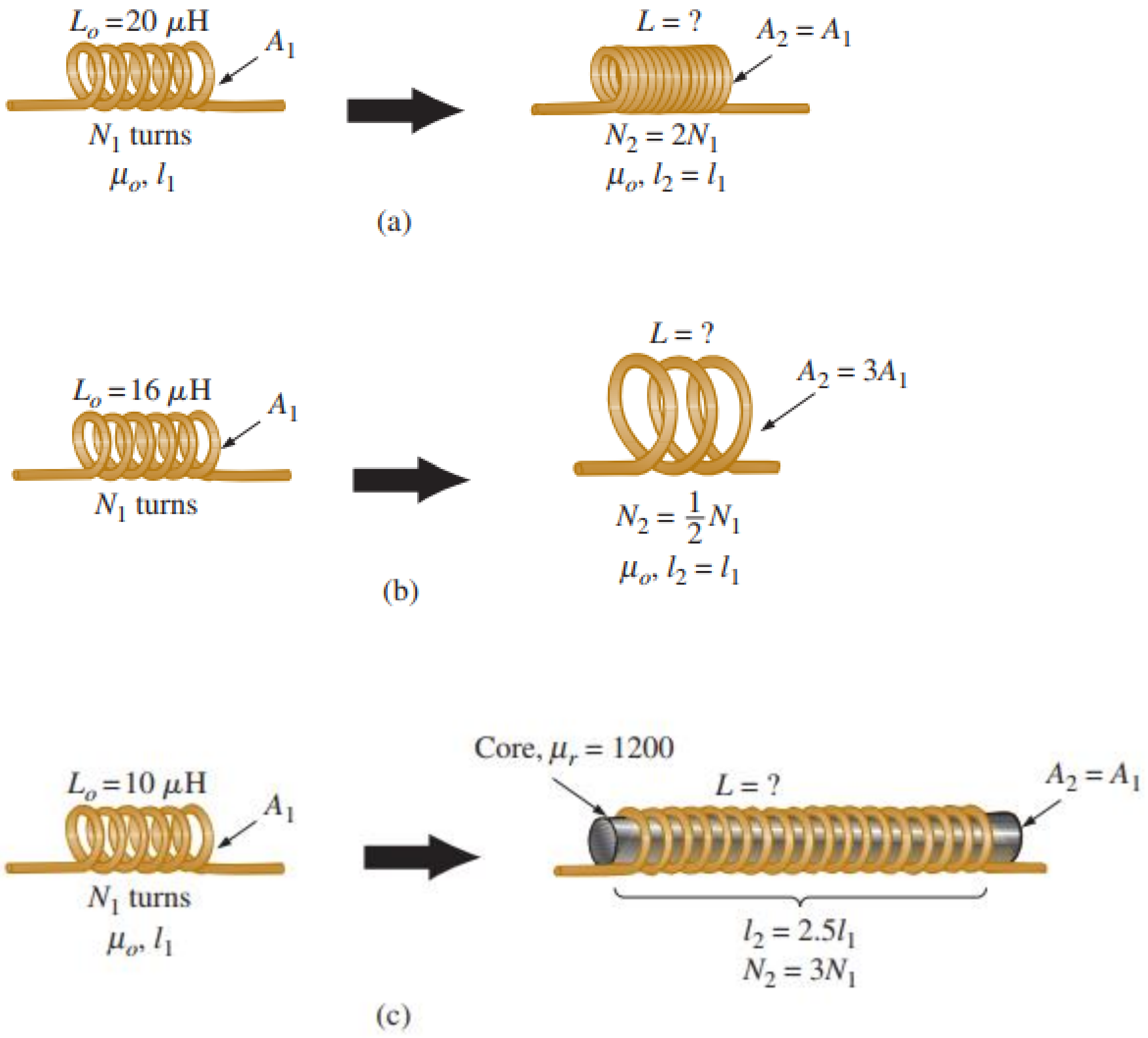
$$L = \mu_r L_o \text{ (henries, H)} \qquad \textbf{(11.8)}$$

$$\text{where, } L_o = \frac{\mu_o N^2 A}{l} \text{ (henries, H)}$$

Relative permeability of metals.

Material	Relative permeability, μ_r	Material	Relative permeability, μ_r
Aluminum	1.00000065	Nickel	50–600
Cobalt	60	Palladium	1.0008
Copper	0.999994	Permalloy 45	2500
Ferrite (NiZn)	16–640	Platinum	1.000265
Gold	0.999998	Silver	0.99999981
Iron	5000–6000	Steel	100–40000
Lead	0.999983	Superconductors	0
Magnesium	1.00000693	Supermalloy	100000
Manganese	1.000125	Tungsten	1.000068
Mumetal	20000–1000000	Wood (dry)	0.99999942

EXAMPLE 11.2 In Fig. 11.19, if each inductor in the left column is changed to the type appearing in the right column, find the new inductance level. For each change, assume that the other factors remain the same.



Solutions:

a. The only change was the number of turns, but it is a squared factor, resulting in

$$L = (2)^2 L_o = (4)(20\ \mu\text{H}) = \mathbf{80\ \mu H}$$

b. In this case, the area is three times the original size, and the number of turns is 1/2. Since the area is in the numerator, it increases the inductance by a factor of three. The drop in the number of turns reduces the inductance by a factor of $(1/2)^2 = 1/4$. Therefore,

$$L = (3)\left(\frac{1}{4}\right)L_o = \frac{3}{4}(16\ \mu\text{H}) = \mathbf{12\ \mu H}$$

c. Both μ and the number of turns have increased, although the increase in the number of turns is squared. The increased length reduces the inductance. Therefore,

$$L = \frac{(3)^2(1200)}{2.5}L_o = (4.32 \times 10^3)(10\ \mu\text{H}) = \mathbf{43.2\ mH}$$

EXAMPLE 11.1 For the air-core coil in Fig. 11.18:

a. Find the inductance.
b. Find the inductance if a metallic core with $\mu_r = 2000$ is inserted in the coil.

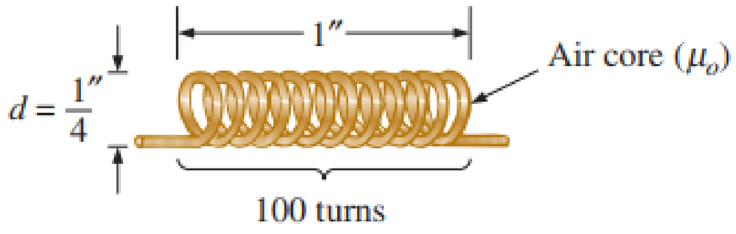


FIG. 11.18 *Air-core coil for Example 11.1.*

Solution: We know that, 39.37 inch = 1 meter

a. $d = \frac{1}{4}\cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right) = 6.35\text{ mm}$ $\quad A = \frac{\pi d^2}{4} = \frac{\pi(6.35\text{ mm})^2}{4} = 31.7\ \mu\text{m}^2$ $\quad l = 1\ \cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right) = 25.4\text{ mm}$

$$L = 4\pi \times 10^{-7}\frac{\mu_r N^2 A}{l} = 4\pi \times 10^{-7}\frac{(1)(100\text{ t})^2(31.7\ \mu\text{m}^2)}{25.4\text{ mm}} = \mathbf{15.68\ \mu H}$$

b. Eq. (11.8): $\quad L = \mu_r L_o = (2000)(15.68\ \mu\text{H}) = \mathbf{31.36\ mH}$

Practice Problem 1 ~ 5 [P504]

Problems [P504]

2. For the inductor in Fig. 11.82, find the inductance L in henries.

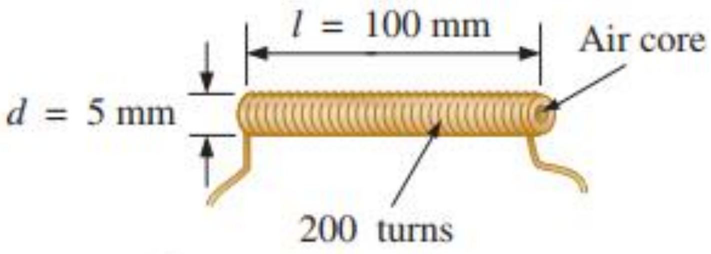


FIG. 11.82 *Problems 2 and 3.*

Solution:

$$A = \frac{\pi d^2}{4} = \frac{\pi(5\text{ mm})^2}{4} = 19.63 \times 10^{-6}\text{ m}^2$$

$$L = \frac{N^2 \mu A}{\ell} = \frac{(200\text{ t})^2(4\pi \times 10^{-7})(19.63 \times 10^{-6}\text{ m}^2)}{100\text{ mm}} = \mathbf{9.87\ \mu H}$$

3. Repeat Problem 2 with $l = 1.6$ in., $d = 0.2$ in., and a ferromagnetic core with $\mu_r = 500$.

Solution:

$$d = 0.2\ \cancel{\text{in.}}\left[\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right] = 5.08\text{ mm}$$

$$A = \frac{\pi d^2}{4} = \frac{(\pi)(5.08\text{ mm})^2}{4} = 20.27 \times 10^{-6}\text{ m}^2$$

$$\ell = 1.6\ \cancel{\text{in.}}\left(\frac{1\text{ m}}{39.37\ \cancel{\text{in.}}}\right) = 40.64\text{ mm}$$

$$L = \frac{N^2 \mu_r \mu_o A}{\ell} = \frac{(200\text{ t})^2(500)(4\pi \times 10^{-7})(20.27 \times 10^{-6}\text{ m}^2)}{40.64\text{ mm}} = \mathbf{12.54\ mH}$$

Different Types of Inductors

Applications of Inductor:

1. Limit rate of change of current
2. Power Supplies
3. Transformers
4. Radios
5. Televisions (TVs)
6. Radars
7. Electric Heater
8. Electric motors and generators

TRANSIENTS IN Inductive NETWORKS

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.

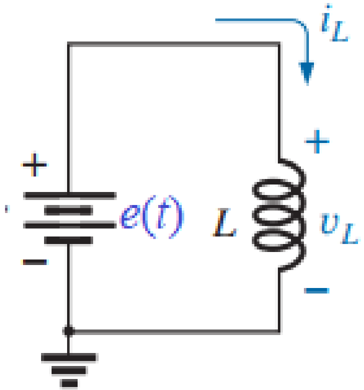
$$v_L = L\frac{di_L}{dt} \qquad \textbf{(11.12)}$$

By integrating Eq. (11.12) in both sides we have:

$$i_L = \frac{1}{L}\int_{t_0}^{t} v_L\,dt + i_L(t_0) \qquad \textbf{(11.12.1)}$$

Energy storage by an inductor can be calculated as follows:

$$W_L = \frac{1}{2}Li_L^2 \quad [\text{J}] \qquad \textbf{(11.12.2)}$$



Effect of Opening a Switch in Series with an Inductor with a Steady-State Current

In *R-L* circuits, the energy is stored in the form of a magnetic field established by the current through the coil. If the series *R-L* circuit in Fig. 11.41 reaches steady state conditions and the switch is quickly opened, a ***spark*** will occur across the contacts due to the rapid change in current from a maximum value to zero amperes. ***The change in current establishes a high voltage drop across the coil*** that, in conjunction with the applied voltage *E*, appears across the points of the switch.

The solution is to use a network like that in Fig. 11.42.

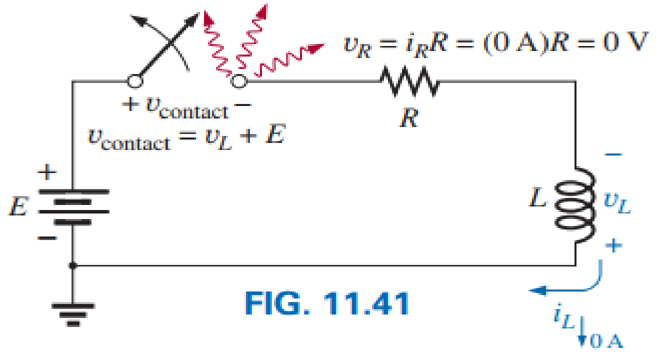


FIG. 11.41

Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

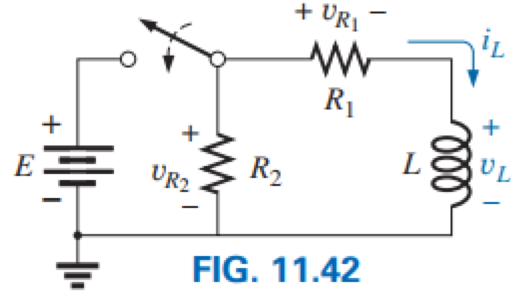
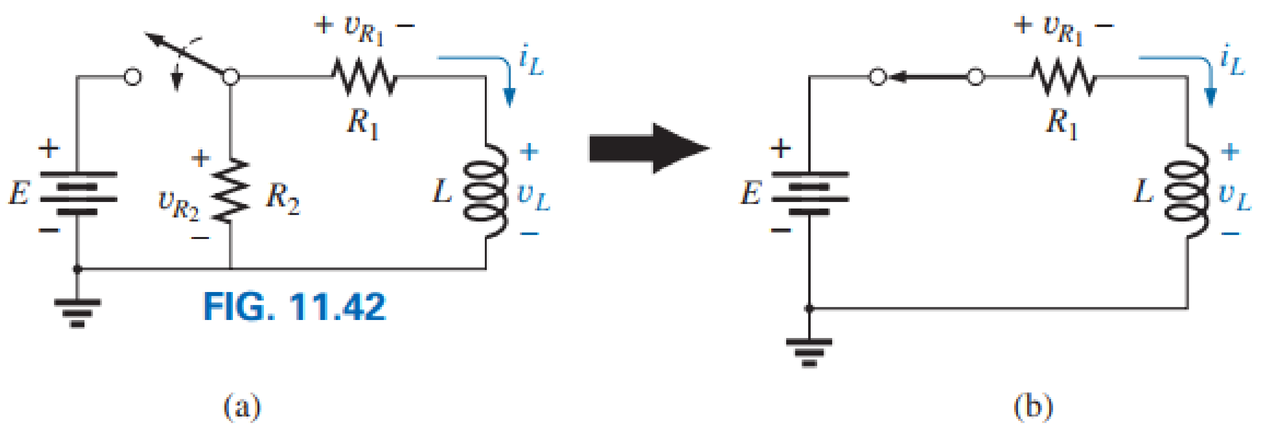


FIG. 11.42

11.5 *R-L* TRANSIENTS: THE STORAGE PHASE



Initiating the storage phase for an inductor by closing the switch.

Charging phase of Fig. (b): According to KVL we have:

$$v_{R1} + v_L = E$$

$$R_1 i_L + L\frac{di_L}{dt} = E$$

$$L\frac{di_L}{dt} + R_1 i_L = E \qquad (i)$$

The solution of Eq. (*i*) is as follows:

$$i_L = \frac{E}{R_1}\left(1 - e^{-t/\tau}\right) \quad \text{(ampere, A)} \qquad \textbf{(11.13)}$$

Substitute Eq. (11.13) into Eq. (11.12), we have:

$$v_L = Ee^{-t/\tau} \quad \text{(volt, V)} \qquad \textbf{(11.15)}$$

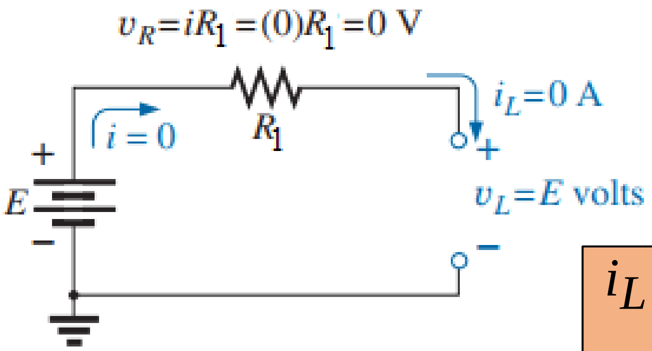
The voltage drop across the resistor will be:

$$v_{R1} = E\left(1 - e^{-t/\tau}\right) \quad \text{(volt, V)} \qquad \textbf{(11.16)}$$

The quantity τ is called **time constant**, which is given by:

$$\tau = \frac{L}{R} = \frac{L}{R_1} \quad \text{(second, s)} \qquad \textbf{(11.14)}$$

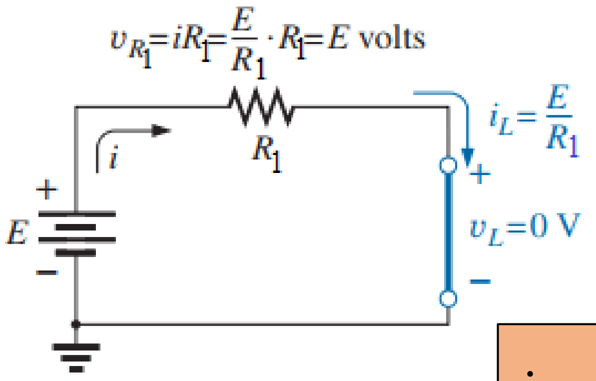
At time $t = 0$ (at the instant of switch is closed), from Eqs. (11.13) and (11.15) we have:



$$i_L = 0 \ \ \textbf{(L is open)}$$

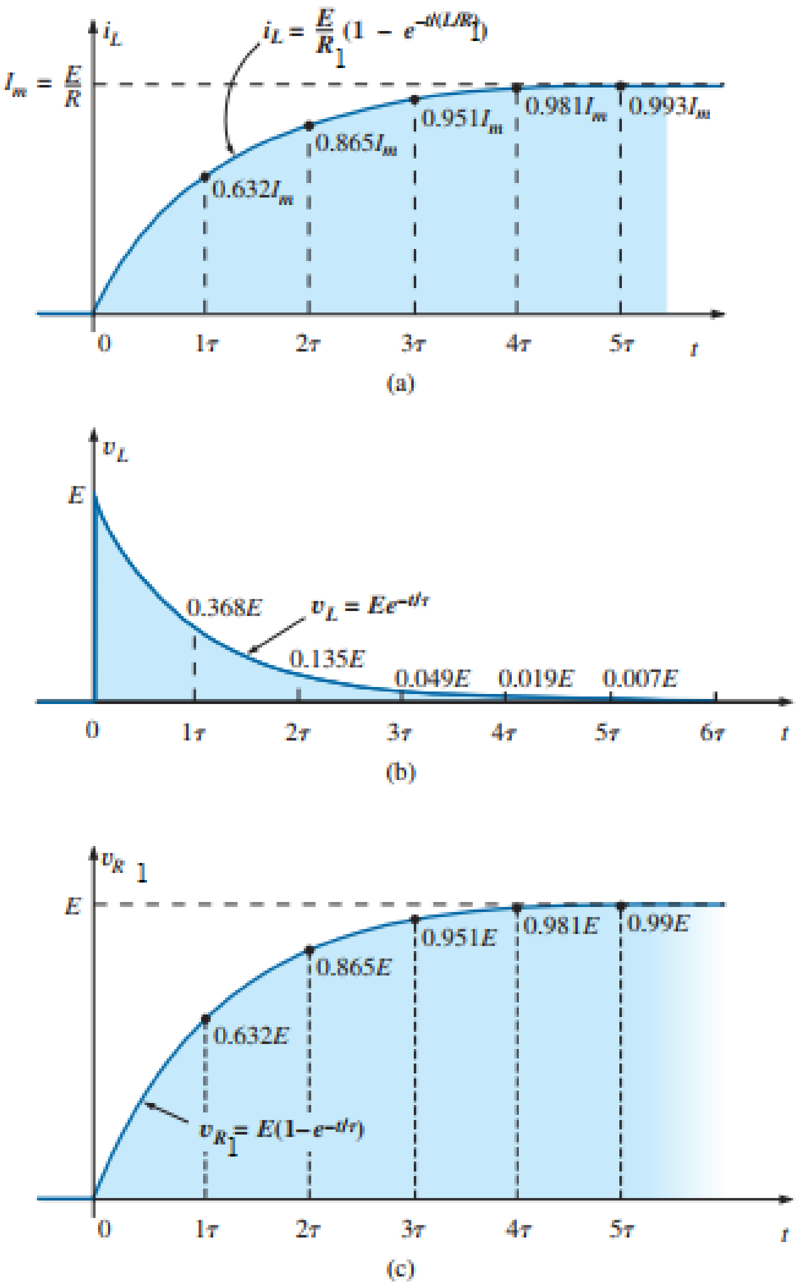
$$v_L = E \qquad v_{R1} = 0$$

At time $t = \infty$ (steady-state condition), from Eqs. (11.13) and (15.15) we have:



$$i_L = \frac{E}{R_1} \qquad v_{R1} = E$$

$$v_L = 0 \ \ \textbf{(L is shorted)}$$



It has been observed from graphs of Figures (a) and (b) that at $t = 5\tau$, the inductor current becomes almost equal to its maximum value and the voltage becomes almost zero.

The transient or storage phase of an inductor has essentially ended after five time constants.

After five time constants, circuit in steady state.

EXAMPLE 11.3 Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit in Fig. 11.36 if the switch is closed at $t = 0$ s. Sketch the resulting curves.

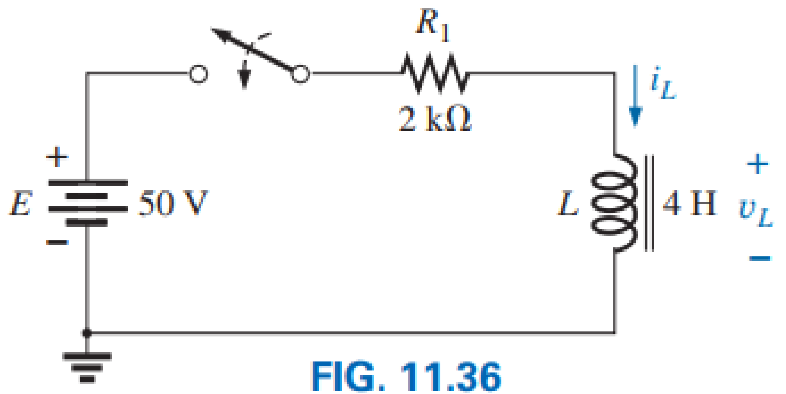


FIG. 11.36
Series R-L circuit for Example 11.3.

Solution: First, the time constant is determined: $\tau = \dfrac{L}{R_1} = \dfrac{4\ \text{H}}{2\ \text{k}\Omega} = 2\ \text{ms}$

Then the maximum or steady-state current is

$$I_m = \frac{E}{R_1} = \frac{50\ \text{V}}{2\ \text{k}\Omega} = 25 \times 10^{-3}\text{A} = 25\ \text{mA}$$

Substituting into Eq. (11.13): $\boldsymbol{i_L = 25\ \text{mA}\ (1 - e^{-t/2\text{ms}})}$

Using Eq. (11.15): $\boldsymbol{v_L = 50\ \text{V}e^{-t/2\text{ms}}}$

The resulting waveforms appear in Fig. 11.37.

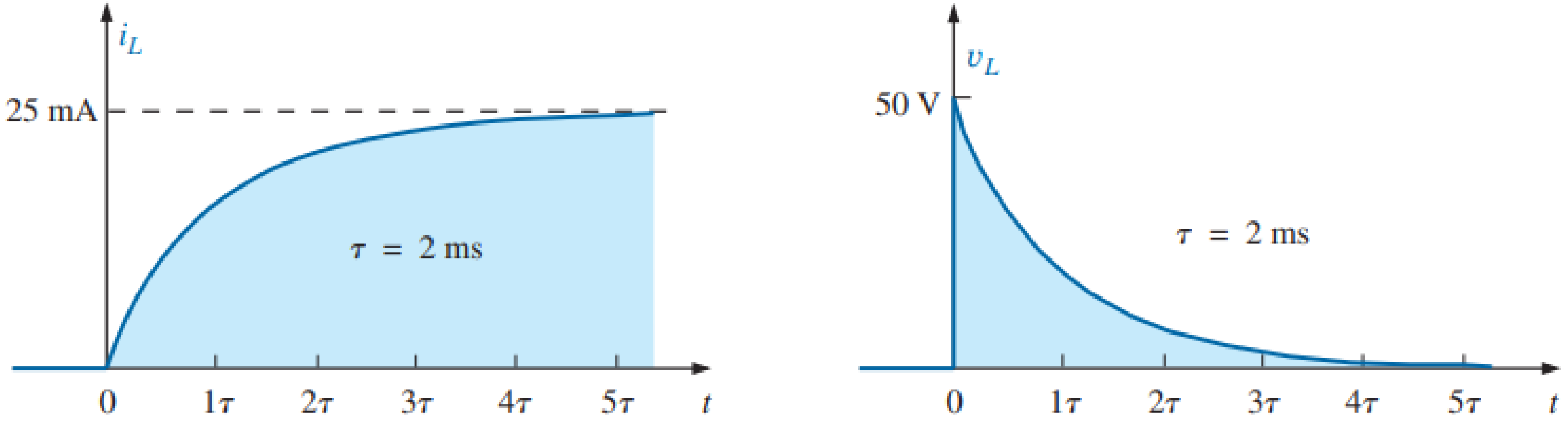


FIG. 11.37 *i_L and v_L for the network in Fig. 11.36.*

11.7 *R-L* TRANSIENTS: THE RELEASE PHASE

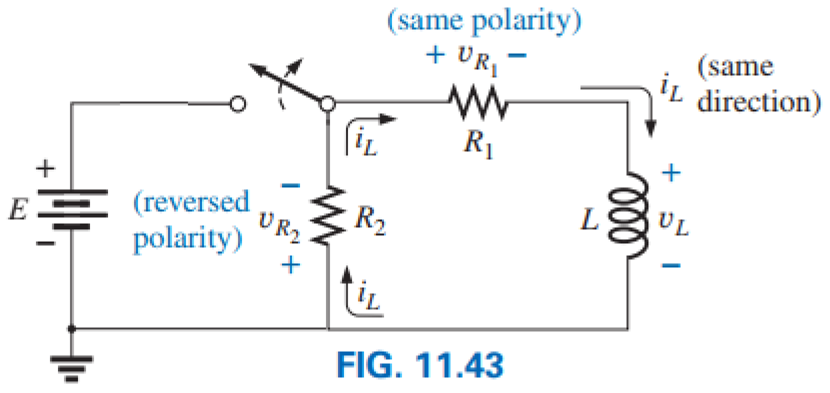


FIG. 11.43
Network in Fig. 11.42 the instant the switch is opened.

According to KVL we have:

$$v_L = -(v_{R1} + v_{R2}) = -(R_1 + R_2)i_L$$

$$L\frac{di_L}{dt} + (R_1 + R_2)i_L = 0 \qquad (ii)$$

At the moment of switch open, $i_{L0} = \dfrac{E}{R_1}$ thus:

$$v_{L0} = -V_i \qquad V_i = (R_1 + R_2)i_{L0} = \left(1 + \frac{R_2}{R_1}\right)E$$

The inductor current exponentially decays from the initial value of i_{L0} to the final value of zero. Thus the solution of Eq. (ii) yields:

$$i_L = \frac{E}{R_1}e^{-t/\tau'} \quad \text{(ampere, A)} \qquad \mathbf{(11.21)}$$

$$\tau' = \frac{L}{R_1 + R_2} \quad \text{(second, s)} \qquad \mathbf{(11.14.1)}$$

Substitute Eq. (11.21) into Eq. (11.12), we have:

$$v_L = -V_i e^{-t/\tau'} \quad \text{(volt, V)} \qquad \mathbf{(11.20)}$$

The voltage drop across the resistor R_1 will be:

$$v_{R1} = Ee^{-t/\tau'} \quad \text{(volt, V)} \qquad \mathbf{(11.22)}$$

The voltage drop across the resistor R_2 will be:

$$v_{R2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} \quad \text{(volt, V)} \qquad \mathbf{(11.22)}$$

EXAMPLE 11.5 Resistor R_2 was added to the network in Fig. 11.36 as shown in Fig. 11.44.

a. Find the mathematical expressions for i_L, υ_L, υ_{R_1}, and υ_{R_2} for five time constants of the storage phase.
b. Find the mathematical expressions for i_L, υ_L, υ_{R_1}, and υ_{R_2} if the switch is opened after five time constants of the storage phase.
c. Sketch the waveforms for each voltage and current for both phases covered by this example. Use the defined polarities in Fig. 11.43.

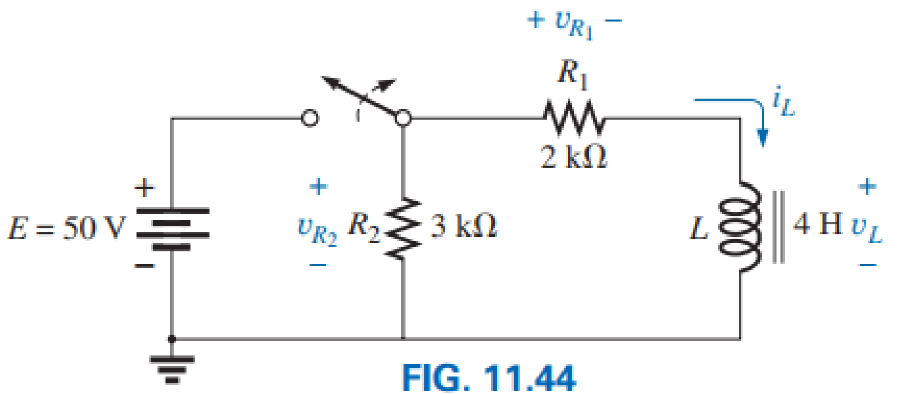


FIG. 11.44

Defined polarities for υ_{R_1}, υ_{R_2}, υ_L, and current direction for i_L for Example 11.5.

Solution:

(a) First, the time constant is determined: $\tau = \dfrac{L}{R_1} = \dfrac{4\text{ H}}{2\text{ k}\Omega} = 2\text{ ms}$

Then the maximum or steady-state current is $I_m = \dfrac{E}{R_1} = \dfrac{50\text{ V}}{2\text{ k}\Omega} = 25 \times 10^{-3}\text{A} = 25\text{ mA}$

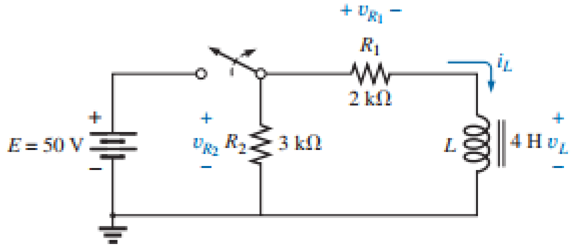
Substituting into Eq. (11.13): $i_L = \mathbf{25\ mA\ (1 - e^{-t/2ms})}$

Using Eq. (11.15): $\upsilon_L = \mathbf{50\ Ve^{-t/2ms}}$

$$\upsilon_{R_1} = \mathbf{50\ V}(1 - e^{-t/2\mathrm{ms}})$$
$$\upsilon_{R_2} = E = \mathbf{50\ V}$$

b. $\tau' = \dfrac{L}{R_1 + R_2} = \dfrac{4\text{ H}}{2\text{ k}\Omega + 3\text{ k}\Omega} = \dfrac{4\text{ H}}{5 \times 10^3\,\Omega}$

$= 0.8 \times 10^{-3}\text{ s} = 0.8\text{ ms}$



By Eqs. (11.19) and (11.20):

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\text{ k}\Omega}{2\text{ k}\Omega}\right)(50\text{ V}) = 125\text{ V}$$

and $$v_L = -V_i e^{-t/\tau'} = \mathbf{-125\ Ve^{-t/-0.8ms}}$$

By Eq. (11.21):

$$I_m = \frac{E}{R_1} = \frac{50\text{ V}}{2\text{ k}\Omega} = 25\text{ mA}$$

and $$i_L = I_m e^{-t/\tau'} = \mathbf{25\ mAe^{-t/0.8ms}}$$

By Eq. (11.22):

$$v_{R_1} = Ee^{-t/\tau'} = \mathbf{50\ Ve^{-t/0.8ms}}$$

By Eq. (11.23):

$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'} = -\frac{3\text{ k}\Omega}{2\text{ k}\Omega}(50\text{ V})e^{-t/\tau'} = \mathbf{-75\ Ve^{-t/0.8\ ms}}$$

c. See Fig. 11.45:

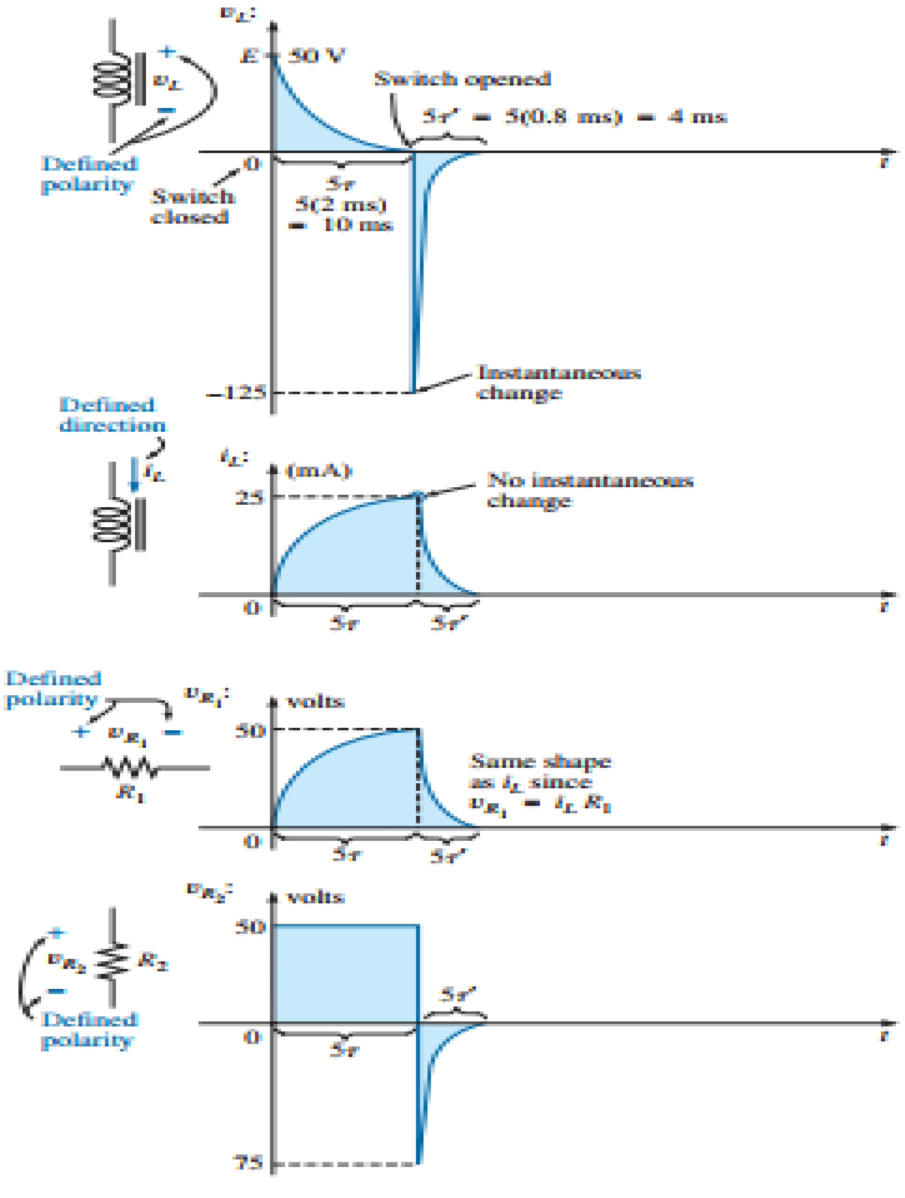


FIG. 11.45
The various voltages and the current for the network in Fig. 11.44.

Problem 12 [P505]:

12. For the circuit in Fig. 11.84:

a. Determine the time constant.

b. Write the mathematical expression for the current i_L after the switch is closed.

c. Repeat part (b) for v_L and v_R.

d. Determine i_L and v_L at one, three, and five time constants.

e. Sketch the waveforms of i_L, v_L, and v_R.

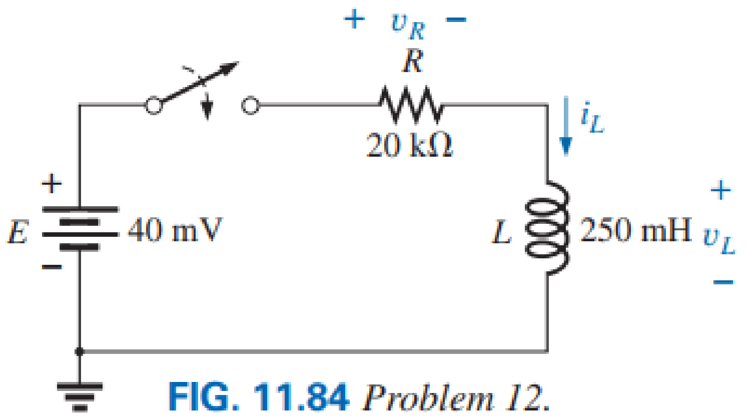


FIG. 11.84 *Problem 12.*

Solution:

$$(a)\ \tau = \frac{L}{R} = \frac{250\,\text{mH}}{20\,\text{k}\Omega} = \mathbf{12.5\ \mu s}$$

$$(b)\ i_L = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$$

$$= \frac{40\,\text{mV}}{20\,\text{k}\Omega}\left(1 - e^{-t/12.5\mu s}\right) = \mathbf{2\,\mu A\left(1 - e^{-t/12.5\mu s}\right)}$$

$$(c)\ v_L = Ee^{-t/\tau} = \mathbf{40\,mV e^{-t/12.5\mu s}}$$

$$v_R = Ri_L = E\left(1 - e^{-t/\tau}\right) = \mathbf{40\,mV\left(1 - e^{-t/12.5\mu s}\right)}$$

$$(d)\ i_L(t = \tau) = 2\mu\text{A}\left(1 - e^{-\tau/\tau}\right)$$

$$= 2\mu\text{A}\left(1 - e^{-1}\right) = \mathbf{1.26\,\mu A}$$

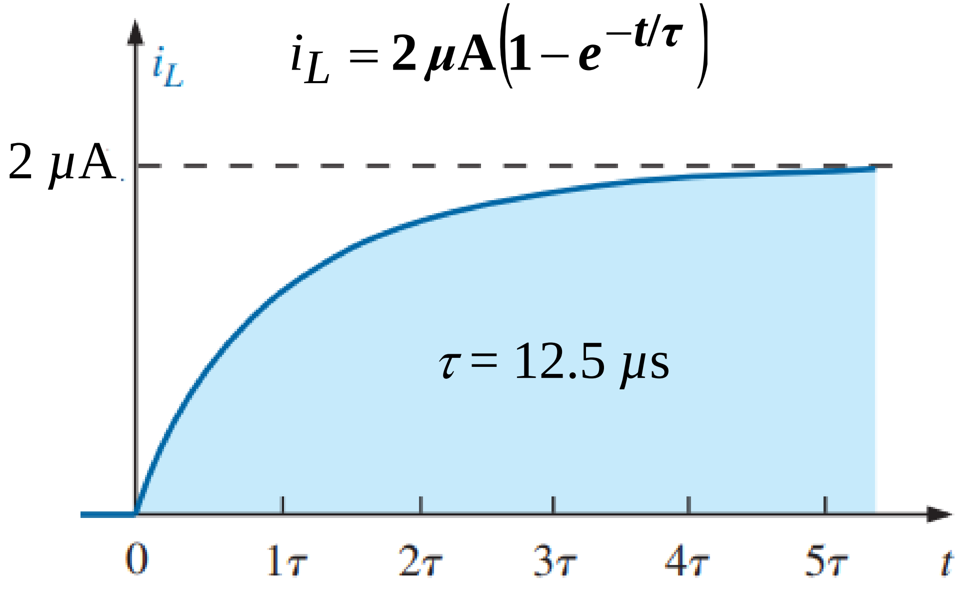
$$i_L(t = 3\tau) = 2\mu\text{A}\left(1 - e^{-3}\right) = \mathbf{1.9\,\mu A}$$

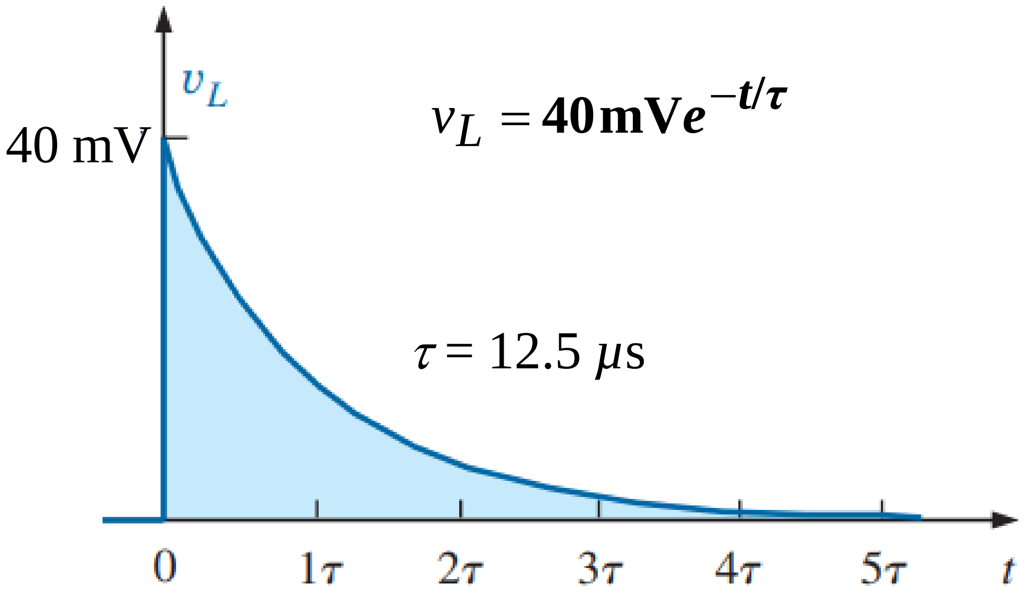
$$i_L(t = 5\tau) = 2\mu\text{A}\left(1 - e^{-3}\right) = \mathbf{1.99\,\mu A}$$

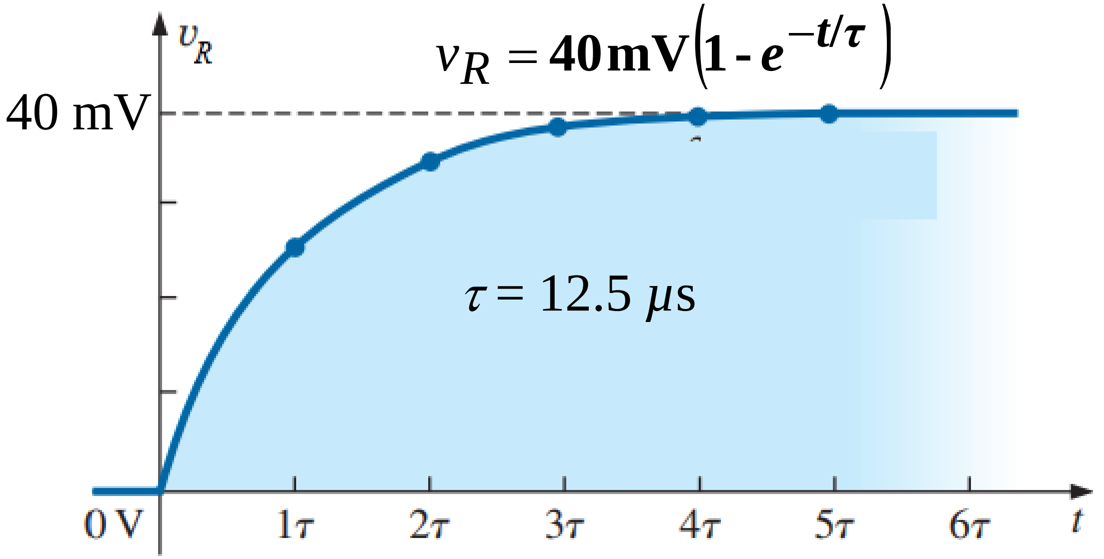
$$v_L(t = \tau) = 40\text{mV}\left(e^{-t/\tau}\right) = 40\text{mV}\left(e^{-1}\right) = \mathbf{14.72\,V}$$

$$v_L(t = 3\tau) = 40\text{mV}\left(e^{-3}\right) = \mathbf{1.99\,V}$$

$$v_L(t = 5\tau) = 40\text{mV}\left(e^{-5}\right) = \mathbf{0.27\,V}$$


i_L
$i_L = 2\,\mu\mathbf{A}\left(1 - e^{-t/\tau}\right)$
2 μA
τ = 12.5 μs
0
1τ
2τ
3τ
4τ
5τ
t


v_L
40 mV
$v_L = 40\,\mathrm{mV}e^{-t/\tau}$
τ = 12.5 μs
0
1τ
2τ
3τ
4τ
5τ
t


v_R
$v_R = 40\,\mathrm{mV}\left(1 - e^{-t/\tau}\right)$
40 mV
τ = 12.5 μs
0 V
1τ
2τ
3τ
4τ
5τ
6τ
t

Problem 20 [P505]:

20. For the network in Fig. 11.92:

a. Determine the mathematical expressions for the current i_L and the voltage v_L following the closing of the switch.

b. Repeat part (a) if the switch is opened at $t = 1\ \mu s$.

c. Sketch the waveforms of parts (a) and (b) on the same set of axes.

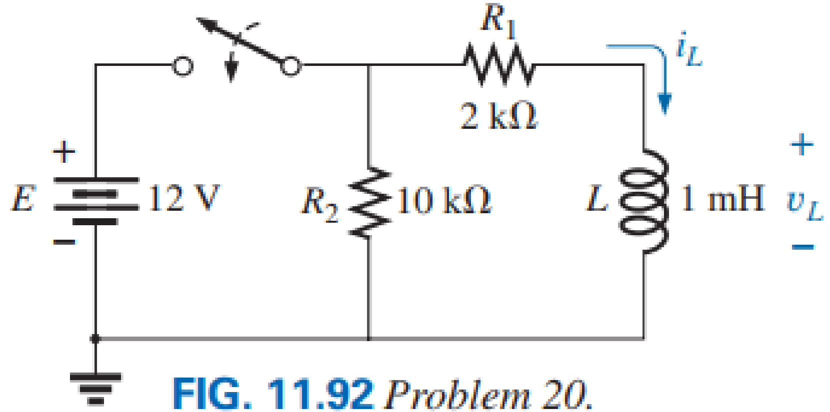


FIG. 11.92 *Problem 20.*

Solution:

$$(a)\ \tau = \frac{L}{R_1} = \frac{1\text{mH}}{2\text{k}\Omega} = \mathbf{0.5\ \mu s}$$

$$i_L = \frac{E}{R}\left(1 - e^{-t/\tau}\right) = \frac{12\text{V}}{2\text{k}\Omega}\left(1 - e^{-t/0.5\mu s}\right)$$

$$= \mathbf{6mA\left(1 - e^{-t/0.5\mu s}\right)}$$

$$v_L = Ee^{-t/\tau} = \mathbf{12Ve^{-t/0.5\mu s}}$$

(b) At $t = 1\ \mu s$,

$$\tau' = \frac{L}{R_1 + R_2} = \frac{1\text{mH}}{12\text{k}\Omega} = \mathbf{8.33\ ns}$$

$$i_L = 6\text{mA}\left(1 - e^{-t/0.5\mu s}\right) = 6\text{mA}\left(1 - e^{-1\mu s/0.5\mu s}\right) = 5.19\text{mA}$$

$$I_m = 5.19\text{mA}$$

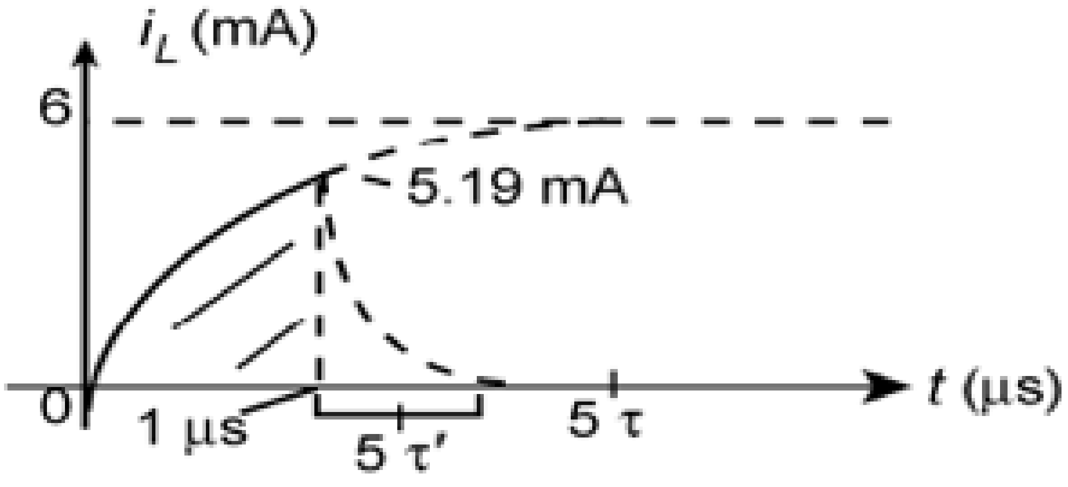
$$i_L = I_m e^{-t/\tau} = \mathbf{5.19mAe^{-t/8.33ns}}$$

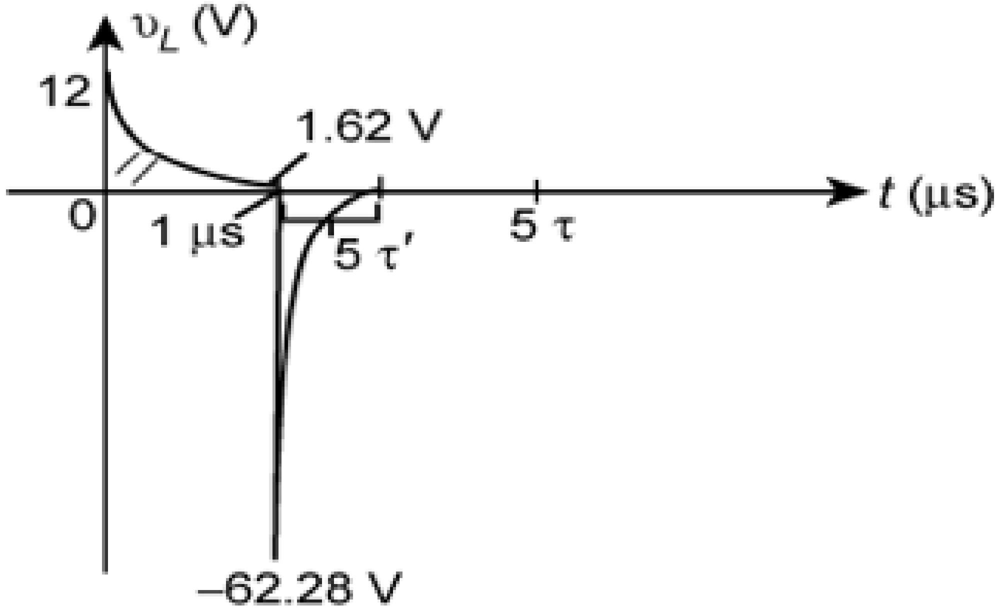
$$v_L = 12\text{V}e^{-t/0.5\mu s} = 12\text{V}e^{-1\mu s/0.5\mu s} = 1.62\text{V}$$

$$V_i = (R_1 + R_2)I_m = (5.19\text{mA})(12\text{k}\Omega) = 62.28\text{ V}$$

$$v_L = -V_i e^{-t/\tau} = \mathbf{62.28Ve^{-t/8.33ns}}$$

c.





Practice Problem 19 ~ 21 [P506]

11.11 INDUCTORS IN SERIES AND IN PARALLEL

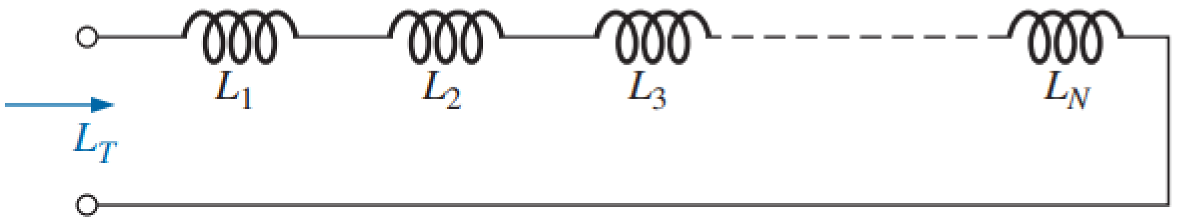


FIG. 11.56
Inductors in series.

$$L_T = L_1 + L_2 + L_3 + \ldots\ldots + L_N \qquad \textbf{(11.30)}$$

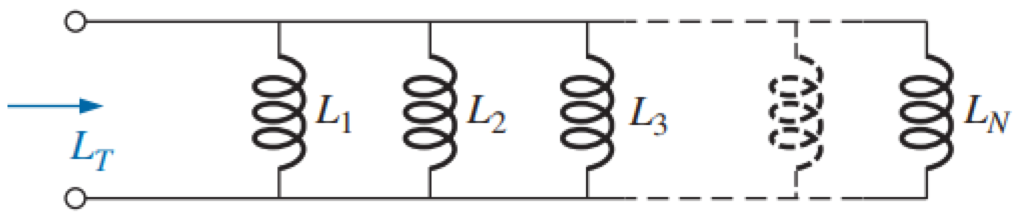
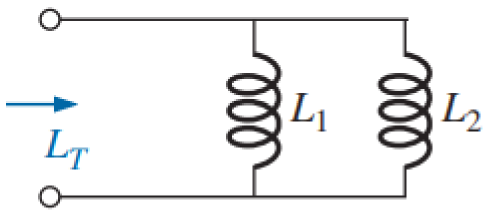


FIG. 11.57
Inductors in parallel.

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots + \frac{1}{L_N} \qquad \textbf{(11.31)}$$

When two inductors are connected in parallel:

$$L_T = \frac{L_1 L_2}{L_1 + L_2} \qquad \textbf{(11.32)}$$



35. Find the total inductance of the circuits in Fig. 11.106.

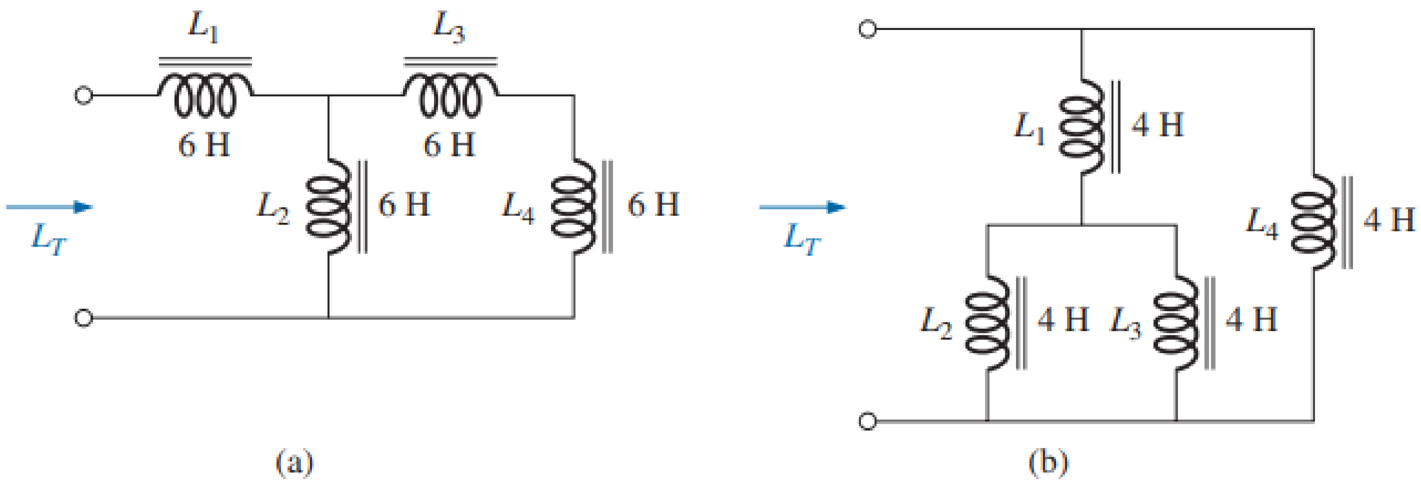


FIG. 11.106 *Problem 35.*

(a) L_T = 6+[6//(6+6)] = 6 + [6//12] = 6 + 4 = **10 H**

(b) L_T = 4//[4+(4//4)] = 4//[4+2] = 4//6 = **2.4 H**