Review Last Class

APPLICATION OF COMPLEX NUMBERS IN AC CIRCUIT

Instantaneous Form (Time Domain) Equation:

$$e(t) = E_m sin(\omega t + \theta_e)\ \text{V}$$
$$v(t) = V_m sin(\omega t + \theta_v)\ \text{V}$$
$$i(t) = I_m sin(\omega t + \theta_i)\ \text{A}$$

Phasor Form (Polar Form) Equation:

$$\boldsymbol{E} = \vec{E} = E_{rms}\angle\theta_e = E\angle\theta_e\ \text{V}$$
$$\boldsymbol{V} = \vec{V} = V_{rms}\angle\theta_v = V\angle\theta_v\ \text{V}$$
$$\boldsymbol{I} = \vec{I} = I_{rms}\angle\theta_i = I\angle\theta_i\ \text{A}$$

Rectangular Form (Cartesian Form) Equation:

$$\boldsymbol{E} = \vec{E} = E_r + jE_i\ \ \text{V}$$
$$\boldsymbol{V} = \vec{V} = V_r + jV_i\ \ \text{V}$$
$$\boldsymbol{I} = \vec{I} = I_r + jI_i\ \ \text{A}$$

IMPEDANCE

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = Z\angle\theta_z = R + jX\ \ \Omega$$

$$Z = \frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}} = \frac{V}{I}\ \ \Omega$$

$$\theta_z = \theta_v - \theta_i$$

$$X_L = \omega L = 2\pi f L\ \ [\Omega]$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \ [\Omega]$$

ADMITTANCE

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = Y\angle\theta_y = G + jB\ \text{S}$$

$$Y = \frac{1}{Z} = \frac{I_m}{V_m} = \frac{I_{rms}}{V_{rms}} = \frac{I}{V}\ \ \text{S}$$

$$\theta_y = -\theta_z = \theta_i - \theta_v$$

$$G = \frac{1}{R} = Y cos\theta_y\ \ \ \text{S}$$

$$B = \frac{1}{X} = Y sin\theta_y\ \ \ \text{S}$$

$$B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L}\ \ [\text{S}]$$

$$B_C = \frac{1}{X_C} = \omega C = 2\pi f C\ \ [\text{S}]$$

EXAMPLE The supply voltage and impedance of a circuit are $v(t)$ = 282.84cos314t V and $\boldsymbol{Z}$ = 20Ω∠60°. Find the current $i(t)$.

Solution: Converting voltage from cosine to sine, we have: $v(t)$ = 282.84sin(314t+90°) V.

Now, V_m = 282.84 V, θ_v = 90° and Z = 20 Ω, θ_z = 60°

We know that: $Z = \dfrac{V_m}{I_m}$ $\qquad \theta_z = \theta_v - \theta_i$

$$I_m = \frac{V_m}{Z} = \frac{282.84}{20} = 14.142 \text{ A}$$

$$\theta_i = \theta_v - \theta_z = 90° - 60° = 30°$$

Thus, $i(t)$ = 14.142sin(314t + 30°) A

EXAMPLE The supply current and impedance of a circuit are $i(t)$ = 15sin377t V and $\boldsymbol{Z}$ = 17.32 + j10 Ω. Find the voltage $v(t)$.

Solution: Converting impedance from Cartesian to Polar form:

$$\boldsymbol{Z} = 17.32 + j10\ \Omega = 20\Omega\angle 30°$$

Now, I_m = 15 V, θ_i = 0° and Z = 20 Ω, θ_z = 30°

We know that: $Z = \dfrac{V_m}{I_m}$ $\qquad \theta_z = \theta_v - \theta_i$

$$V_m = ZI_m = 20 \times 15 = 300 \text{ V}$$

$$\theta_v = \theta_i + \theta_z = 0° + 30° = 30°$$

Thus, $v(t)$ = 300sin(377t + 30°) V

POWER CALCULATION IN AC CIRCUIT

Instantaneous Power [$p(t)$]

Let, instantaneous voltage and current are:

$$v(t) = V_m \sin(\omega t + \theta_v) \quad [\text{V}]$$

$$i(t) = I_m \sin(\omega t + \theta_i) \quad [\text{A}]$$

By shifting the angle θ_i the instantaneous voltage and current are:

$$v(t) = V_m \sin(\omega t + \theta) \quad [\text{V}]$$

$$i(t) = I_m \sin \omega t \quad [\text{A}]$$

where, $\theta = \theta_Z = (\theta_v - \theta_i)$

and θ is called **Power Factor Angle**.

The instantaneous power is as follows:

$$p(t) = v(t)i(t) = V_m \sin(\omega t + \theta) I_m \sin \omega t \quad [\text{W}]$$

After simplification, the instantaneous power can be written as:

$$p(t) = \underbrace{P(1 - \cos 2\omega t)}_{\textbf{Active Power}} + \underbrace{Q \sin 2\omega t}_{\textbf{Reactive Power}}$$

where, $P = \dfrac{V_m I_m}{2} \cos\theta \ \text{W}$ $\quad Q = P_x = \dfrac{V_m I_m}{2} \sin\theta \ \text{Var}$

$$P = V_{rms} I_{rms} \cos\theta = VI \cos\theta \ \text{W} \quad \textbf{(14.13)}$$

$$Q = V_{rms} I_{rms} \sin\theta = VI \sin\theta \ \text{Var} \quad \textbf{(19.12)}$$

$$\because \ V = \frac{V_m}{\sqrt{2}} \quad I = \frac{I_m}{\sqrt{2}}$$

The first term [***P*(1 – cos2*ωt*)**] in the preceding equation is called **instantaneous real** [or **true** or **active** or **wattfull** or **useful**] **power**.
The unit of instantaneous real power is watt [**W**].

The second term [***Q*sin2*ωt***] in the preceding equation is called **instantaneous reactive volt-ampere** or **instantaneous reactive** [or **imaginary** or **wattless** or **useless** or **quadrature**] **power**.
The unit of instantaneous reactive power is volt-ampere reactive [**Var**].

Power (or Average or Real or Active or True or Wattfull or Usefull Power)

The average value can be obtained by:

$$P_{ave} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T [P(1-\cos 2\omega t) + Q\sin 2\omega t]dt = P = \frac{V_m I_m}{2}\cos\theta = VI\cos\theta \quad \text{W}$$

The average power is also called ***real***/***active***/***true***/***wattfull***/***usefull power or*** **simply** ***Power***. The **unit** of real power is **watt**. The real power is measured by **wattmeter**.

Real power converts from electrical energy to other form of energy. This is happened in resistive circuit.

Reactive or Imaginary or Quadrature or Wattless or Useless Power (or Reactive Volt-Ampere)

The **peak or maximum value of instantaneous reactive power** (or instantaneous reactive volt-ampere) is called the ***reactive***/***imaginary/quadrature***/***wattless***/***useless power*** (or ***reactive volt-ampere***). The **unit** of reactive power is called **var (reactive volt-ampere)**. The reactive power is measured by **varmeter**. It is given by:

$$Q = P_x = \frac{V_m I_m}{2}\sin\theta \quad [\text{var}] = VI\sin\theta \quad [\text{var}]$$

Reactive power is used for storing energy. This is happened in inductive and capacitive circuit.

Reactive power is **positive** (for **Inductive load**) or **negative** (for **Capacitive load**).

Apparent Power or Volt-Ampere

Apparent power is the product of the rms value of voltage and the rms value of current.

The **unit** of apparent power is called **VA (volt-ampere)**.

$$S = \sqrt{P^2 + Q^2} \quad [\text{VA}] \; = \frac{V_m I_m}{2} \quad [\text{VA}]$$

$$= V_{rms} I_{rms} = VI \quad [\text{VA}]$$

Power Factor

Cosine θ ($\cos\theta$) which is a factor, by which volt-amperes are multiplied to give power, is called power factor. Power factor is always **positive**. Power factor can be given by:

$$pf = F_p = \cos\theta = \cos\theta_z = \frac{P}{S} \qquad 0 \le pf \le 1$$

Unity Power Factor: If $\theta = \theta_z = \theta_v - \theta_i = 0^o$ the **power factor is 1** which is called **unity power factor**.

Lagging Power Factor: If $\theta = \theta_z = \theta_v - \theta_i > 0^o$ then **current lags** voltage which is called **lagging power factor**.

Leading Power Factor: If $\theta = \theta_z = \theta_v - \theta_i < 0^o$ then **current leads** voltage which is called **leading power factor**.

Reactive Factor

Sine θ ($\sin\theta$) which is a factor, by which volt-amperes are multiplied to give reactive power, is called reactive factor.

Reactive factor may be **positive** (for Inductive load) or **negative** (for Capacitive load). Reactive factor can be given by:

$$rf = F_q = \sin\theta = \sin\theta_z = \frac{Q}{S} \qquad -1 \le rf \le 1$$

Complex Power

Voltage and Current in Cartesian Form

$$\boldsymbol{V} = V\angle\theta_v = V\cos\theta_v + jV\sin\theta_v = V_r + jV_i$$

$$V_r = V\cos\theta_v; \qquad V_i = V\sin\theta_v$$

$$\boldsymbol{I} = I\angle\theta_i = I\cos\theta_i + jI\sin\theta_i = I_r + jI_i$$

$$I_r = I\cos\theta_i; \qquad I_i = I\sin\theta_i$$

Real or Active or Average Power

$$P = VI\cos\theta = VI\cos(\theta_v - \theta_i)$$
$$= VI\cos\theta_v\cos\theta_i + VI\sin\theta_v\sin\theta_i = V_rI_r + V_iI_i$$

Reactive or Imaginary or Quadrature Power

$$Q = VI\sin\theta = VI\sin(\theta_v - \theta_i)$$
$$= VI\sin\theta_v\cos\theta_i - VI\cos\theta_v\sin\theta_i = V_iI_r - V_rI_i$$

Complex Power by Conjugate Current

Using the previous equations of P and Q, the complex power can be written as follows:

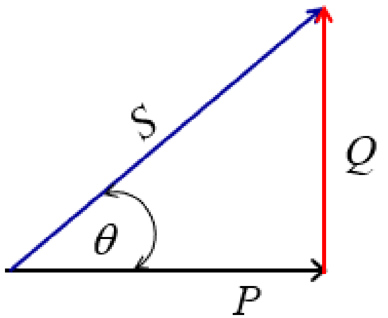
$$\boldsymbol{S} = P + jQ = (V_rI_r + V_iI_i) + j(V_iI_r - V_rI_i)$$

$$\boldsymbol{S} = P + jQ = (V_r + jV_i)(I_r - jI_i) = \boldsymbol{VI}^* = S\angle\theta_s$$

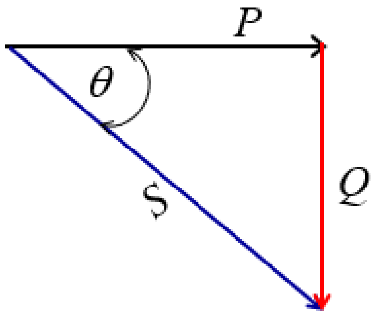
$$P = \mathrm{Re}[\boldsymbol{S}] = \mathrm{Re}[\boldsymbol{VI}^*]; \qquad Q = \mathrm{Im}[\boldsymbol{S}] = \mathrm{Im}[\boldsymbol{VI}^*]$$

Power Triangle

Graphical representation of **active power**, **reactive power**, and **apparent power** in a complex plane is called power triangle.



When Q is Positive



When Q is Negative

EXAMPLE Determine the power factor, the reactive factor and indicate whether power factor is leading or lagging for the following input voltage and current pairs of a network:

(***a***) $v(t) = 150\sin(377t + 70°)$ V

$i(t) = 3\sin(377t + 10°)$ A

(***b***) $v(t) = 100\sin(314t - 50°)$ V

$i(t) = 12\sin(314t + 40°)$ A

(***c***) $v(t) = 120\sin(157t + 30°)$ V

$i(t) = 8\cos(157t - 110°)$ A

(***d***) $v(t) = -80\cos(200t + 60°)$ V

$i(t) = 5\sin(200t - 30°)$ A

Solution: (***a***) Here, $\theta_v = 70^o$ and $\theta_i = 10^o$, thus

$$\theta = \theta_v - \theta_i = 70^o - 10^o = 60^o$$

$$pf = \cos\theta = \cos(60°) = \mathbf{0.5}\ \textbf{lagging}$$

$$rf = \sin\theta = \sin(60°) = \mathbf{0.866}$$

(***b***) Here, $\theta_v = -50^o$ and $\theta_i = 40^o$, thus

$$\theta = \theta_v - \theta_i = -50^o - (40^o) = -90^o$$

$$pf = \cos\theta = \cos(-90°) = \mathbf{0}\ \textbf{leading power factor}$$

$$rf = \sin\theta = \sin(-90°) = \mathbf{-1}$$

(***c***) $i(t) = 8\cos(157t - 110°) = 8\sin(157t + 90° - 150°)$ A

$i(t) = 8\sin(157t - 60°)$ A

Here, $\theta_v = 30^o$ and $\theta_i = -60^o$, thus

$$\theta = \theta_v - \theta_i = 30^o - (-60^o) = 90^o$$

$$pf = \cos\theta = \cos(90°) = \mathbf{0}\ \textbf{lagging power factor}$$

$$rf = \sin\theta = \sin(90°) = \mathbf{1}$$

(***d***) $v(t) = -80\cos(200t + 60°) = 80\sin(200t - 90° + 60°)$ V

$v(t) = 80\sin(200t - 30°)$ V

Here, $\theta_v = -30^o$ and $\theta_i = -30^o$, thus

$$\theta = \theta_v - \theta_i = -30^o - (-30^o) = 0^o$$

$$pf = \cos\theta = \cos(0°) = \mathbf{1}\ \textbf{unity power factor}$$

$$rf = \sin\theta = \sin(0°) = \mathbf{0}$$

EXAMPLE The supply voltage and current of a circuit are $v(t) = 100\sin(314t + 80°)$ V and $i(t) = 12\sin(377t + 50°)$ A.

(**a**) Calculate the power factor, the reactive factor and comment on the power factor.
(**b**) Calculate the power, the reactive power and the apparent power delivered by source.
(**c**) Write the instantaneous power equation.
(**d**) Draw the power triangle.

Solution: (***a***) Here, $\theta_v = 80^o$ and $\theta_i = 50^o$, thus

$$\theta = \theta_v - \theta_i = 80^o - 50^o = 30^o$$

$$pf = \cos\theta = \cos(30^o) = \mathbf{0.866}$$
$$rf = \sin\theta = \sin(30^o) = \mathbf{0.5}$$

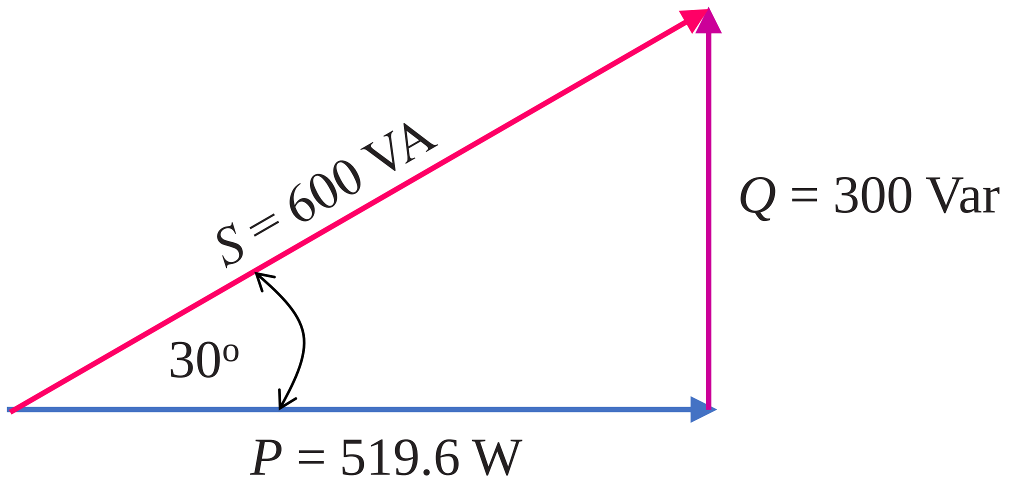
(**b**) $P = \frac{V_m I_m}{2}\cos\theta = VI\cos\theta = \frac{100\times 12}{2}\times 0.866 = \mathbf{519.6\ W}$

$$Q = \frac{V_m I_m}{2}\sin\theta = VI\sin\theta = \frac{100\times 12}{2}\times 0.5 = \mathbf{300\ Var}$$

$$S = \frac{V_m I_m}{2} = VI = \frac{100\times 12}{2} = \mathbf{600\ VA}$$

(***c***) $p(t) = P(1 - \cos 2\omega t) + Q\sin 2\omega t$ W
$= 519.6(1-\cos 754t) + 300\sin 754t$ W

(***d***) Power triangle:



EXAMPLE The supply voltage and current of a circuit are $\boldsymbol{V} = 150\text{V}\angle 50°$ and $\boldsymbol{I} = 5\text{A}\angle 110°$.

(**a**) Calculate the complex power an represent it in both polar and cartesian forms.

(**b**) From the result of (a), find the real power, the reactive power and the apparent power.

(**c**) Calculate the power factor and the reactive factor and make commend on power factor.

(**d**) Write the instantaneous power equation for the 400 rad/s of source voltage.

(**e**) Draw the power triangle.

Solution: (***a***) $\boldsymbol{S} = \boldsymbol{VI}^* = (150\text{V}\angle 50°)(5\text{A}\angle 110°)^*$

$$= (150\text{V}\angle 50°)(5\text{A}\angle -110°)$$

$$= 750\text{VA}\angle -60°$$

$$= 375 - j649.52 \text{ VA} = P + jQ$$

(***b***) From (a) we have:

$P = 375$ W, $Q = -\,649.52$ Var and $S = 750$ VA

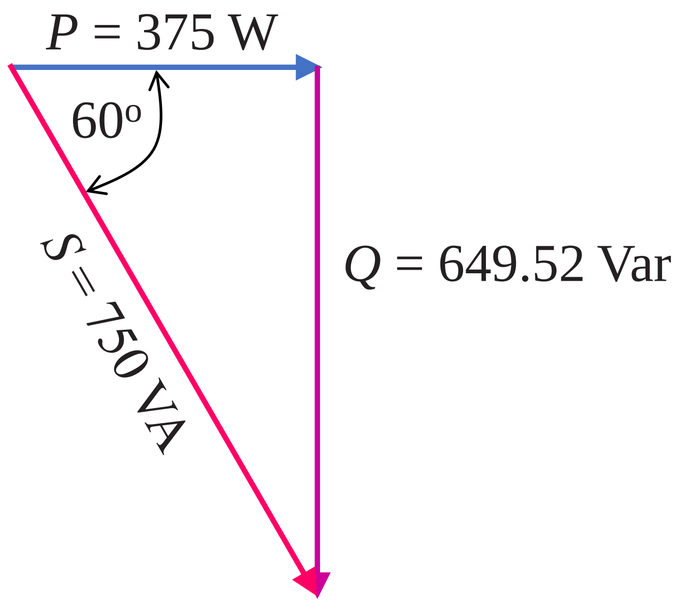
(***c***) $$pf = \frac{P}{S} = \frac{375\ W}{750\ VA} = \mathbf{0.5\ Leading}$$

$$rf = \frac{Q}{S} = \frac{-649.52\ Var}{750\ VA} = \mathbf{-0.866}$$

(***d***) $p(t) = P(1 - \cos 2\omega t) + Q\sin 2\omega t$ W

$$= 375(1 - \cos 800t) - 649.52\sin 800t \text{ W}$$

(***e***) Power triangle:

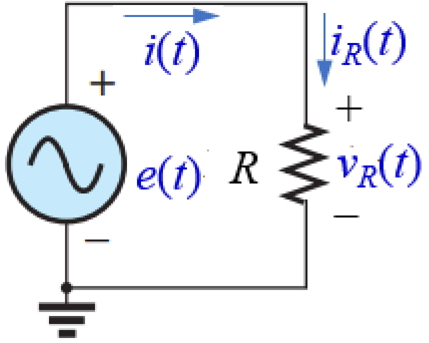


Theory Related To

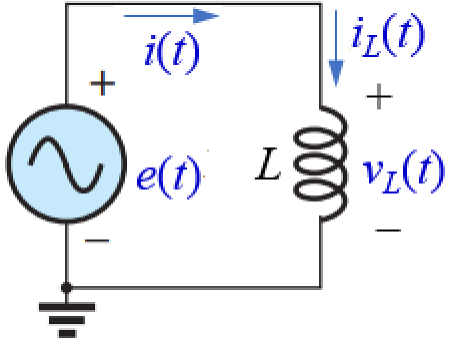
Pure Resistive, Pure Inductive and Pure Capacitive Circuits

Based on Instantaneous Equations

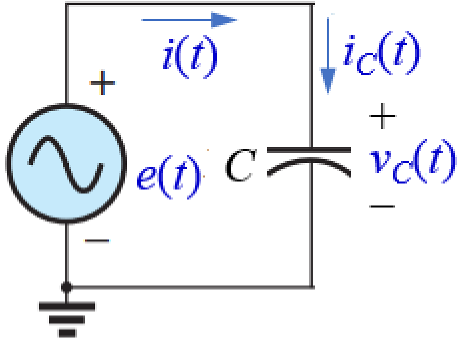
PURE RESISTIVE CIRCUIT



PURE INDUCTIVE CIRCUIT



PURE CAPACITIVE CIRCUIT



Instantaneous or Transient or Time-domain Voltage and Current Relation

$$v_R(t) = Ri_R(t) \qquad i_R(t) = \frac{v_R(t)}{R}$$

$$v_R(t) = e(t) \qquad i_R(t) = i(t)$$

$$v_L(t) = L\frac{di_L(t)}{dt} \qquad i_L(t) = \frac{1}{L}\int v_L(t)dt$$

$$v_L(t) = e(t) \qquad i_L(t) = i(t)$$

$$v_C(t) = \frac{1}{C}\int i_C(t)dt \qquad i_C(t) = C\frac{di_C(t)}{dt}$$

$$v_C(t) = e(t) \qquad i_C(t) = i(t)$$

Let, the input is $e(t) = E_m\sin(\omega t + \theta_e)$V; according to KVL and KCL, we have:

$$i(t) = \frac{E_m}{R}\sin(\omega t + \theta_e)$$

$$i(t) = \frac{E_m}{L}\int \sin(\omega t + \theta_e)dt$$

$$= \frac{E_m}{X_L}\sin(\omega t + \theta_e - 90°)$$

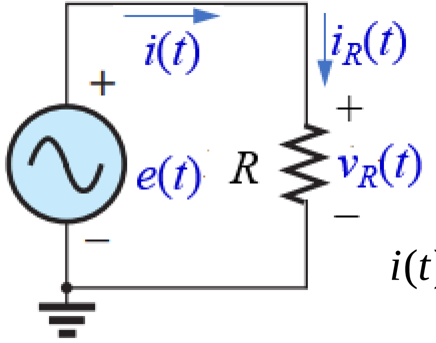
where, $X_L = \omega L = 2\pi f L \ \Omega$

$$i(t) = C\frac{d}{dt}[E_m\sin(\omega t + \theta_e)]$$

$$= \frac{E_m}{X_C}\sin(\omega t + \theta_e + 90°)$$

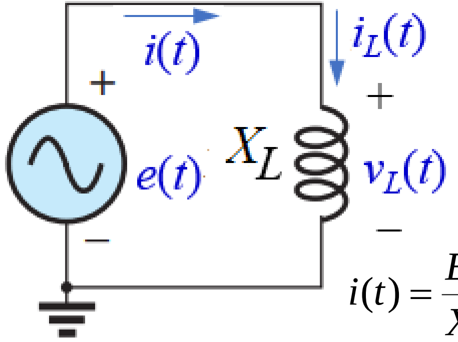
where, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} \ \Omega$

PURE RESISTIVE CIRCUIT



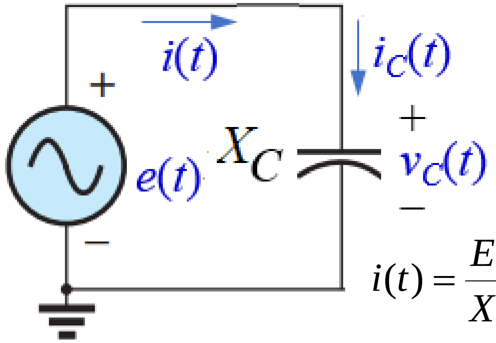
$$i(t) = \frac{E_m}{R}\sin(\omega t + \theta_e)$$

PURE INDUCTIVE CIRCUIT



$$i(t) = \frac{E_m}{X_L}\sin(\omega t + \theta_e - 90°)$$

PURE CAPACITIVE CIRCUIT



$$i(t) = \frac{E_m}{X_C}\sin(\omega t + \theta_e + 90°)$$

Compare the obtained current equation with general current equation of $i(t) = I_m\sin(\omega t + \theta_i)$ A, we have

$$I_m = \frac{E_m}{R} = \frac{V_{Rm}}{R}; \qquad \theta_i = \theta_e$$

$$E_m = V_{Rm} = RI_{Rm} = RI_m$$

$$\theta_{vR} = \theta_{iR}$$

V_{Rm}: Peak value of resistor voltage
I_{Rm}: Peak value of resistor current

$$I_m = \frac{E_m}{X_L}; \qquad \theta_i = \theta_e - 90°$$

$$E_m = V_{Lm} = X_L I_{Lm} = X_L I_m$$

$$\theta_e = \theta_i + 90° \qquad \theta_{vL} = \theta_{iL} + 90°$$

V_{Lm}: Peak value of inductor voltage
I_{Lm}: Peak value of inductor current

$$I_m = \frac{E_m}{X_C}; \qquad \theta_i = \theta_e + 90°$$

$$E_m = V_{Cm} = X_C I_{Cm} = X_C I_m$$

$$\theta_e = \theta_i - 90° \qquad \theta_{vC} = \theta_{iC} + 90°$$

V_{Cm}: Peak value of capacitor voltage
I_{Cm}: Peak value of capacitor current

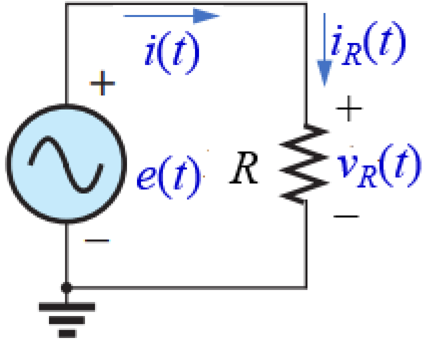
Impedance Magnitude and Impedance Angle

$$Z = \frac{E_m}{I_m} = R; \qquad \theta_z = \theta_e - \theta_i = 0°$$

$$Z = \frac{E_m}{I_m} = X_L; \qquad \theta_z = \theta_e - \theta_i = 90°$$

$$Z = \frac{E_m}{I_m} = X_C; \qquad \theta_z = \theta_e - \theta_i = -90°$$

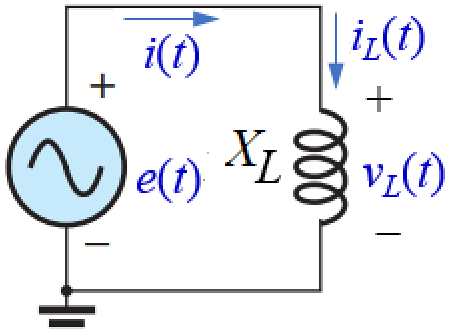
PURE **RESISTIVE** CIRCUIT



$Z = R$

$\theta_Z = 0°$

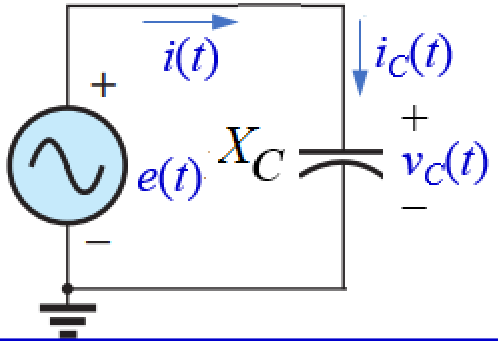
PURE **INDUCTIVE** CIRCUIT



$Z = X_L$

$\theta_Z = 90°$

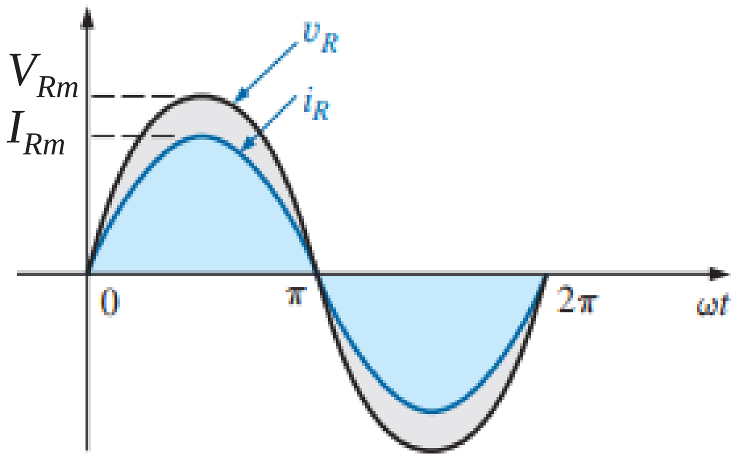
PURE **CAPACITIVE** CIRCUIT



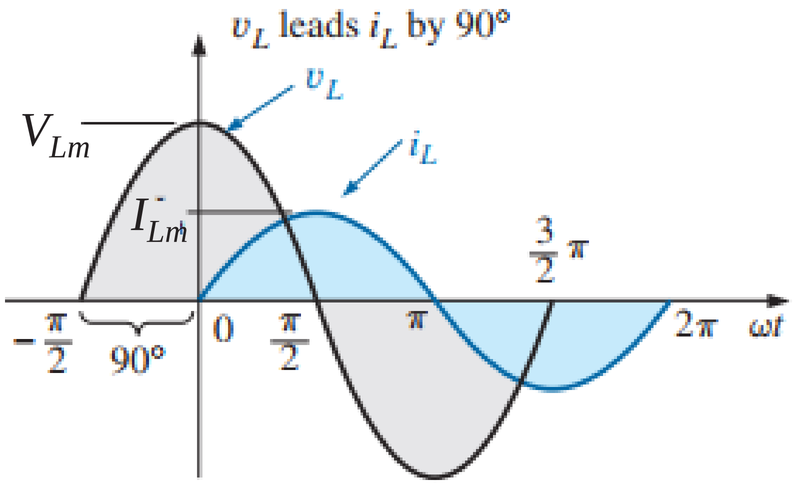
$Z = X_C$

$\theta_Z = -90°$

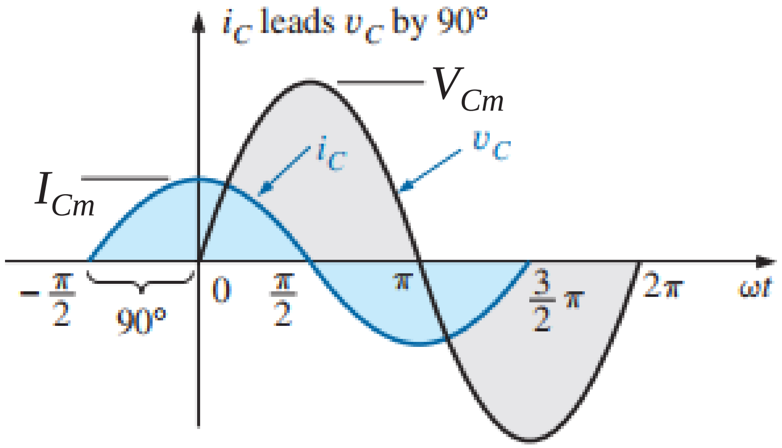
Phase Relation Between Voltage and Current



The phase difference $v_R(t)$ and $i_R(t)$ is **0º**. $v_R(t)$ and $i_R(t)$ are **in phase**.



The phase difference $v_L(t)$ and $i_L(t)$ is **90º**. $v_L(t)$ **leads** and $i_L(t)$ or $i_L(t)$ **lags** $v_L(t)$.



The phase difference $v_C(t)$ and $i_C(t)$ is **90º**. $v_C(t)$ **lags** and $i_C(t)$ or $i_C(t)$ **leads** $v_C(t)$.

X_L VERSUS FREQUENCY (f) CURVE

Inductive reactance is **directly proportional** to frequency ($X_L \propto f$) so the inductive reactance versus frequency curve is a straight line with slope equal to $2\pi L$.

If frequency **decreases**, inductive reactance will be **decreases**.

f frequency **increases**, inductive reactance will be **increases**.

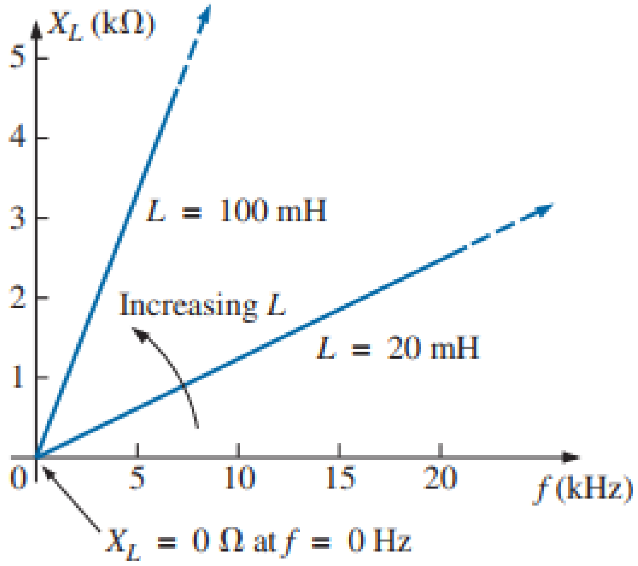


FIG. 14.20 X_L *versus frequency.*

Inductive Reactance With DC Supply

For DC voltage the **frequency is zero** so inductive reactance with DC supply is ***zero*** that means inductor behave as a ***short-circuit*** with DC input.

X_C VERSUS FREQUENCY (f) CURVE

Capacitive reactance is **inversely proportional** to frequency ($X_C \propto 1/f$) so the capacitive reactance versus frequency curve is a *rectangular hyperbola*.

If frequency **decreases**, capacitive reactance will be **increases**.

If frequency **increases**, capacitive reactance will be **decreases**.

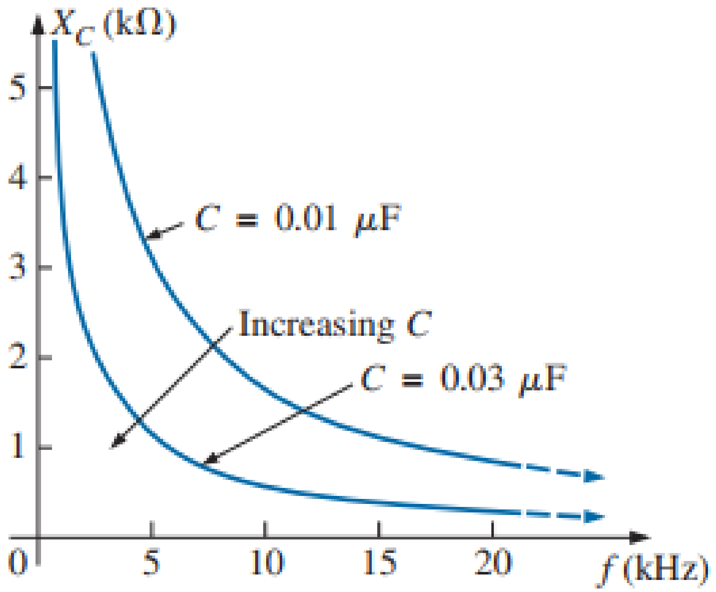


FIG. 14.22 X_C *versus frequency.*

Capacitive Reactance With DC Supply

For DC voltage the frequency is zero so capacitive reactance with DC supply is ***infinity*** that means capacitor behave as an ***open-circuit*** with DC input.

EXAMPLE 14.1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for $v(t)$ and $i(t)$.

$$v(t) = 100\sin 377t \text{ V}$$

Solution: (a) Given, $V_m = 100$ V, $\theta_v = 0^o$, $\omega = 377$ rad/s and $R = 10\ \Omega$

For a resistive circuit, we know that

$$I_m = \frac{V_m}{R} \qquad \theta_i = \theta_v$$

thus $$I_m = \frac{100\text{V}}{10\Omega} = 10\text{ A} \qquad \theta_i = \theta_v = 0°$$

The sinusoidal expression of current is:

$$\boldsymbol{i(t) = 10\sin 377t \text{ A}}$$

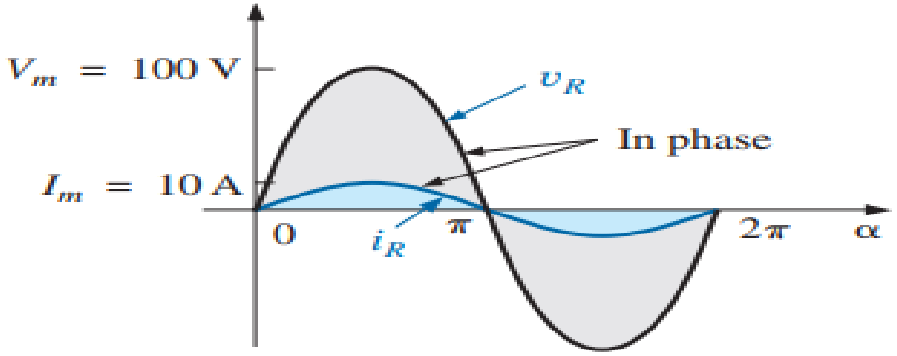


FIG. 14.13 *Example 14.1(a).*

EXAMPLE 14.2 The current through a 5 Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for:

$$i(t) = 40\sin(377t + 30°) \text{ A}.$$

Solution:

(a) Given, $I_m = 40$ A, $\theta_i = 30^o$, $\omega = 377$ rad/s and $R = 5\ \Omega$

We know that $I_m = \dfrac{V_m}{R}$; $\quad \theta_i = \theta_v$

thus $$V_m = RI_m = (5\Omega)(40\text{ A}) = 200\text{ V}$$
$$\theta_v = \theta_i = 30°$$

The sinusoidal expression of voltage is:

$$\boldsymbol{v(t) = 200\sin(377t + 30°) \text{ V}}$$

Practice Book Problems [Ch. 14] 4 and 5

EXAMPLE 14.3(a) The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil for $i(t) = 10\sin 377t$ A. Sketch the curves for $v(t)$ and $i(t)$.

Solution: (a) Given, $I_m = 10$ A, $\theta_i = 0^o$, $\omega = 377$ rad/s and $L = 0.1$ H

We know that, $X_L = \omega L = (377\,\text{rad/s})(0.1\,\text{H}) = 37.3\ \Omega$

We know that $I_m = \dfrac{V_m}{X_L}$; $\quad \theta_i = \theta_v - 90°$

thus $V_m = X_L I_m = (37.7\,\Omega)(10\,\text{A}) = 377\ \text{A}$

$\theta_v = \theta_i + 90° = 90°$

The sinusoidal expression of voltage is:

$$\boldsymbol{v(t) = 377\sin(377t + 90°)\ \text{V}}$$

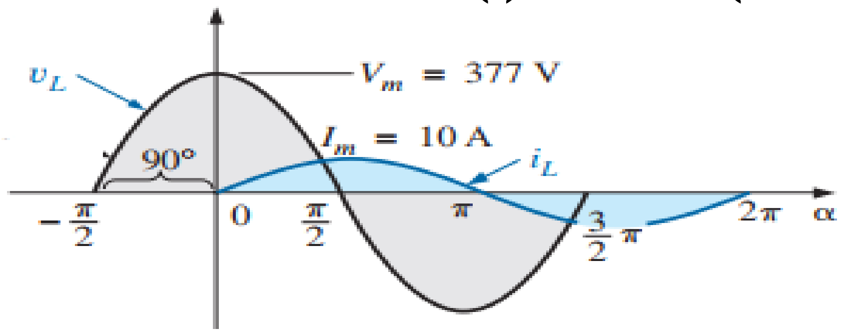


FIG. 14.15 *Example 14.3(a).*

EXAMPLE 14.4 The voltage across a 0.5 H coil is provided. Find the sinusoidal expression for the current through the coil for $v(t) = 100\sin(20t + 30°)$ V.

Solution: (a) Given, $V_m = 100$ V, $\theta_v = 30°$, and $\omega = 20$ rad/s and $L = 0.5$ H

$X_L = \omega L = (20\,\text{rad/s})(0.5\,\text{H}) = 10\ \Omega$

We know that $I_m = \dfrac{V_m}{X_L}$; $\quad \theta_i = \theta_v - 90°$

thus $I_m = \dfrac{100\text{V}}{10\Omega} = 10\,\text{A} \qquad \theta_i = 30° - 90° = -60°$

The sinusoidal expression of current is:

$$\boldsymbol{i(t) = 10\sin(20t - 60°)\ \text{A}}$$

Practice Book Problems [Ch. 14] 6 and 12

EXAMPLE 14.5 The voltage across a 1 μF capacitor is provided. Find the sinusoidal expression for the current through the capacitor for $v(t) = 30\sin 400t$ V. Sketch the curves for $v(t)$ and $i(t)$.

Solution: (a) Given, $V_m = 30$ V, $\theta_v = 0°$, $\omega = 400$ rad/s and $C = 1$ μF $= 1\times10^{-6}$ F

We know that, $$X_C = \frac{1}{\omega C} = \frac{1}{(400\,\text{rad/s})(1\times10^{-6}\,\text{F})} = \frac{10^6}{400}\,\Omega = 2500\,\Omega$$

We know that $$I_m = \frac{V_m}{X_C}; \qquad \theta_i = \theta_v + 90°$$

thus $$I_m = \frac{30\text{V}}{2500\Omega} = 0.012\,\text{A} = 12\text{ mA}$$

$$\theta_i = 0° + 90° = 90°$$

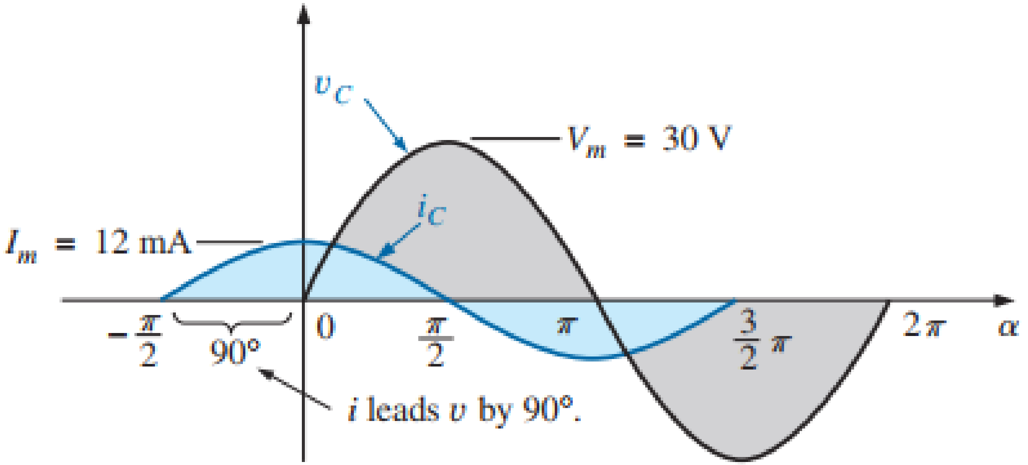


FIG. 14.17 *Example 14.5.*

The sinusoidal expression of current is:

$$\boldsymbol{i(t) = 12\sin(20t + 90°)\text{ mA} = 12\times10^{-3}\sin(20t + 90°)\text{ A}}$$

EXAMPLE 14.6 The current through a 100 μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor for $i(t) = 40\sin(500t + 60°)$ A. Sketch the curves for $v(t)$ and $i(t)$.

Solution: (a) Given, $I_m = 40$ A, $\theta_i = 60°$, $\omega = 500$ rad/s and $C = 100\ \mu\text{F} = 100\times10^{-6}$ F

We know that, $$X_C = \frac{1}{\omega C} = \frac{1}{(500\,\text{rad/s})(100\times10^{-6}\,\text{F})} = \frac{10^6}{5\times10^4}\,\Omega = \frac{10^2}{5}\,\Omega = 20\,\Omega$$

We know that $$I_m = \frac{V_m}{X_C}; \qquad \theta_i = \theta_v + 90°$$

thus $$V_m = X_C I_m = (20\,\Omega)(40\,\text{A}) = 800\,\text{V}$$

$$\theta_v = \theta_i - 90° = 60° - 90° = -30°$$

The sinusoidal expression of voltage is: $\boldsymbol{v(t) = 800\sin(500t - 30°)\ \text{V}}$

Practice Book Problems [Ch. 14] 13 and 19

EXAMPLE 14.7 For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of C, L, or R if sufficient data are provided (Fig. 14.18):

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 40°)$

b. $v = 1000 \sin(377t + 10°)$
   $i = 5 \sin(377t - 80°)$

c. $v = 500 \sin(157t + 30°)$
   $i = 1 \sin(157t + 120°)$

d. $v = 50 \cos(\omega t + 20°)$
   $i = 5 \sin(\omega t + 110°)$

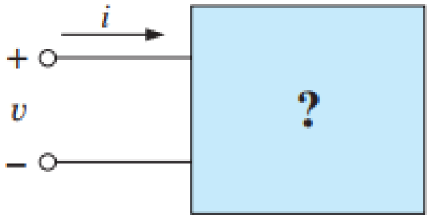


FIG. 14.18 *Example 14.7.*

Solutions:

a. Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{100\ \text{V}}{20\ \text{A}} = \mathbf{5\ \Omega}$$

b. Since v *leads* i by 90°, the element is an *inductor*, and

$$X_L = \frac{V_m}{I_m} = \frac{1000\ \text{V}}{5\ \text{A}} = 200\ \Omega$$

so that $X_L = \omega L = 200\ \Omega$ or

$$L = \frac{200\ \Omega}{\omega} = \frac{200\ \Omega}{377\ \text{rad/s}} = \mathbf{0.53\ H}$$

c. Since i *leads* v by 90°, the element is a *capacitor*, and

$$X_C = \frac{V_m}{I_m} = \frac{500\ \text{V}}{1\ \text{A}} = 500\ \Omega$$

so that $X_C = \dfrac{1}{\omega C} = 500\ \Omega$ or

$$C = \frac{1}{\omega 500\ \Omega} = \frac{1}{(157\ \text{rad/s})(500\ \Omega)} = \mathbf{12.74\ \mu F}$$

d. $v = 50 \cos(\omega t + 20°) = 50 \sin(\omega t + 20° + 90°)$
   $= 50 \sin(\omega t + 110°)$

Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{50\ \text{V}}{5\ \text{A}} = \mathbf{10\ \Omega}$$

Practice Book Problems [Ch. 14] 20 and 21

Theory Related To

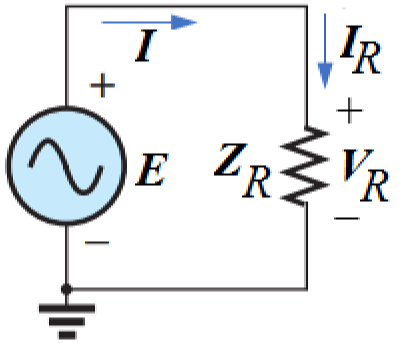
Pure Resistive, Pure Inductive and Pure Capacitive Circuits

Based on Complex or Phasor Algebra

PURE RESISTIVE CIRCUIT | PURE INDUCTIVE CIRCUIT | PURE CAPACITIVE CIRCUIT

Circuit in Phasor Notation

Pure Resistive Circuit



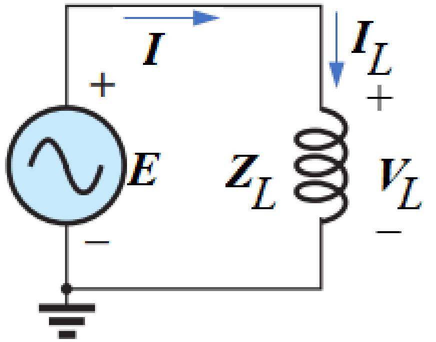
$$\mathbf{V}_R = \mathbf{E}$$
$$\mathbf{I}_R = \mathbf{I}$$
$$\mathbf{Z} = \mathbf{Z}_R = \frac{\mathbf{V}_R}{\mathbf{I}_R}$$

$$\mathbf{V}_R = V_R\angle\theta_{vR} \qquad \mathbf{I}_R = I_R\angle\theta_{iR}$$

$$\mathbf{Z}_R = Z_R\angle\theta_{zR} \qquad Z_R = \frac{V_R}{I_R} = R$$

V_R: rms value of resistor voltage
I_R: rms value of resistor current

Pure Inductive Circuit



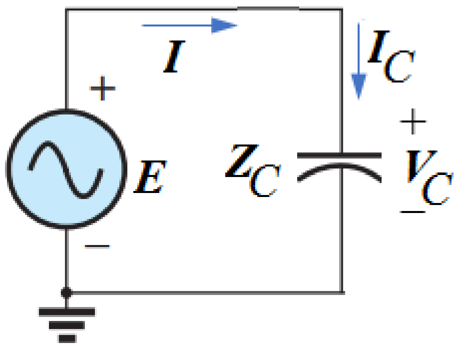
$$\mathbf{V}_L = \mathbf{E}$$
$$\mathbf{I}_L = \mathbf{I}$$
$$\mathbf{Z} = \mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_L}$$

$$\mathbf{V}_L = V_L\angle\theta_{vL} \qquad \mathbf{I}_L = I_L\angle\theta_{iL}$$

$$\mathbf{Z}_L = Z_L\angle\theta_{zL} \qquad Z_L = \frac{V_L}{I_L} = X_L$$

V_L: rms value of inductor voltage
I_L: rms value of inductor current

Pure Capacitive Circuit



$$\mathbf{V}_C = \mathbf{E}$$
$$\mathbf{I}_C = \mathbf{I}$$
$$\mathbf{Z} = \mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}_C}$$

$$\mathbf{V}_C = V_C\angle\theta_{vC} \qquad \mathbf{I}_C = I_C\angle\theta_{iC}$$

$$\mathbf{Z}_C = Z_C\angle\theta_{zC} \qquad Z_C = \frac{V_C}{I_C} = X_C$$

V_C: rms value of capacitor voltage
I_C: rms value of capacitor current

Impedance in Both Polar Form and Cartesian or Rectangular Form

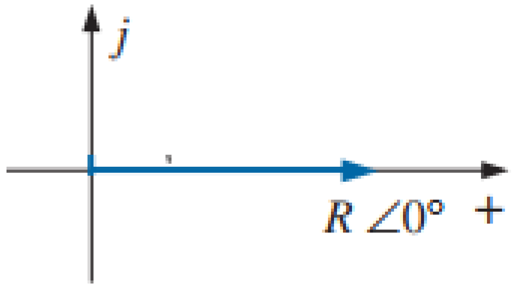
$$\mathbf{Z} = \mathbf{Z}_R = \frac{\mathbf{V}_R}{\mathbf{I}_R}\ \Omega$$
$$\mathbf{Z} = \mathbf{Z}_R = R\angle 0^\circ\ \Omega = R + j0\ \Omega$$

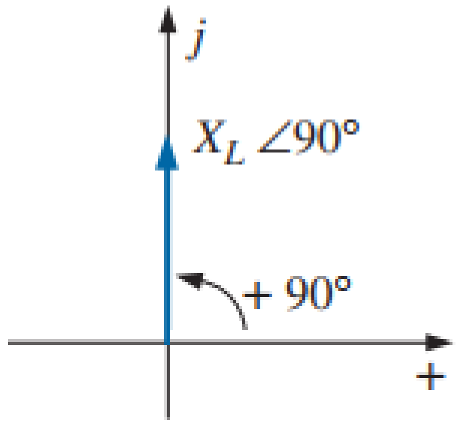
$$\mathbf{Z} = \mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_L}\ \Omega$$
$$\mathbf{Z} = \mathbf{Z}_L = X_L\angle 90^\circ\ \Omega = 0 + jX_L\ \Omega$$

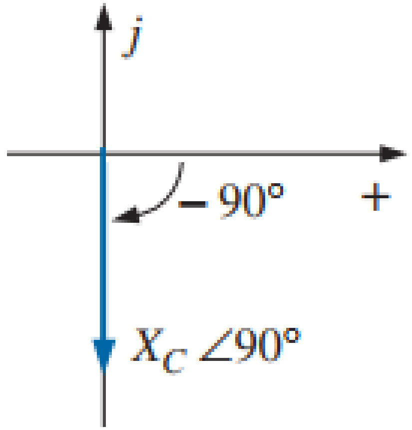
$$\mathbf{Z} = \mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}_C}\ \Omega$$
$$\mathbf{Z} = \mathbf{Z}_C = X_C\angle -90^\circ\ \Omega = 0 - jX_C\ \Omega$$

PURE RESISTIVE CIRCUIT | PURE INDUCTIVE CIRCUIT | PURE CAPACITIVE CIRCUIT

Impedance Diagram







Admittance in Both Polar Form and Cartesian or Rectangular Form

Pure Resistive Circuit

$$\boldsymbol{Y} = \boldsymbol{Y}_R = \frac{\boldsymbol{I}_R}{\boldsymbol{V}_R} = \frac{1}{\boldsymbol{Z}_R}$$

$$\boldsymbol{Y} = \boldsymbol{Y}_R = G\angle 0^\circ \ \text{S} = G + j0 \ \text{S}$$

where, $G = \dfrac{1}{R}$ S

Pure Inductive Circuit

$$\boldsymbol{Y} = \boldsymbol{Y}_L = \frac{\boldsymbol{I}_L}{\boldsymbol{V}_L} = \frac{1}{\boldsymbol{Z}_L}$$

$$\boldsymbol{Y} = \boldsymbol{Y}_L = B_L\angle -90^\circ \ \text{S} = 0 - jB_L \ \text{S}$$

where, $B_L = \dfrac{1}{X_L}$ S

Pure Capacitive Circuit

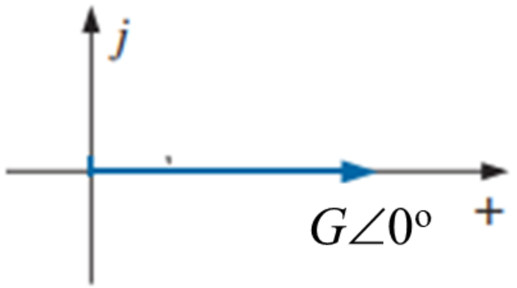
$$\boldsymbol{Y} = \boldsymbol{Y}_C = \frac{\boldsymbol{I}_C}{\boldsymbol{V}_C} = \frac{1}{\boldsymbol{Z}_C}$$

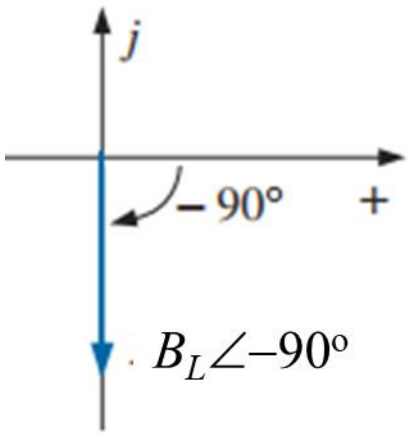
$$\boldsymbol{Y} = \boldsymbol{Y}_C = B_C\angle 90^\circ \ \text{S} = 0 + jB_C \ \text{S}$$

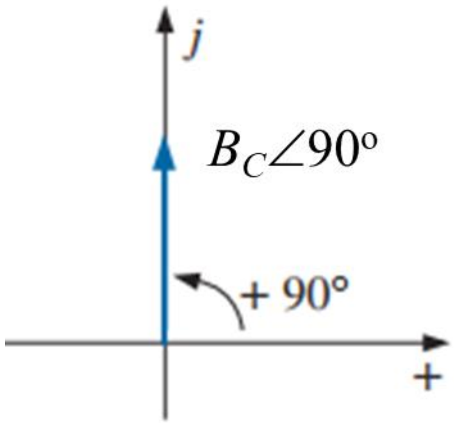
where, $B_C = \dfrac{1}{X_C}$ S

PURE RESISTIVE CIRCUIT	PURE INDUCTIVE CIRCUIT	PURE CAPACITIVE CIRCUIT

Admittance Diagram







Phasor Diagram