REVIEW ON THE LAST CLASS

KIRCHHOFF'S CURRENT LAW (KCL)

OPEN CIRCUIT AND SHORT CIRCUIT

SERIES AND PARALLEL CIRCUIT

Problem 12 [Ch 7]: For the network in Fig. 7.72:

a. Determine the current I_1.

b. Calculate the currents I_2 and I_3.

c. Determine the voltage levels V_a and V_b.

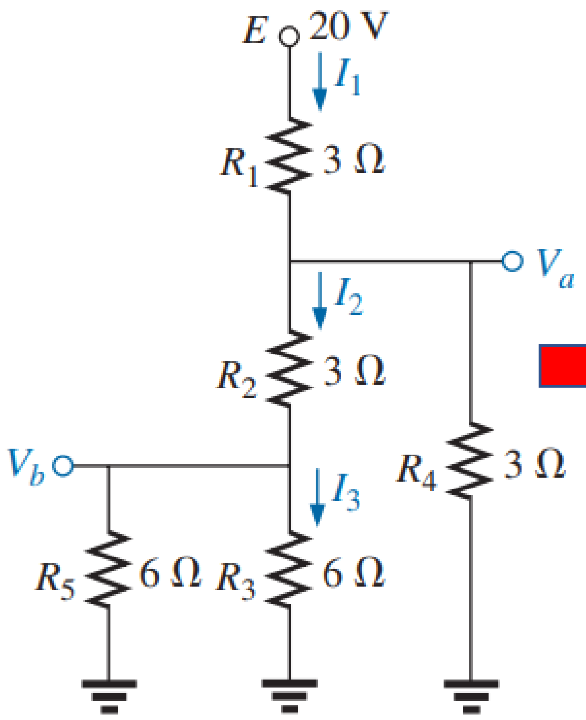
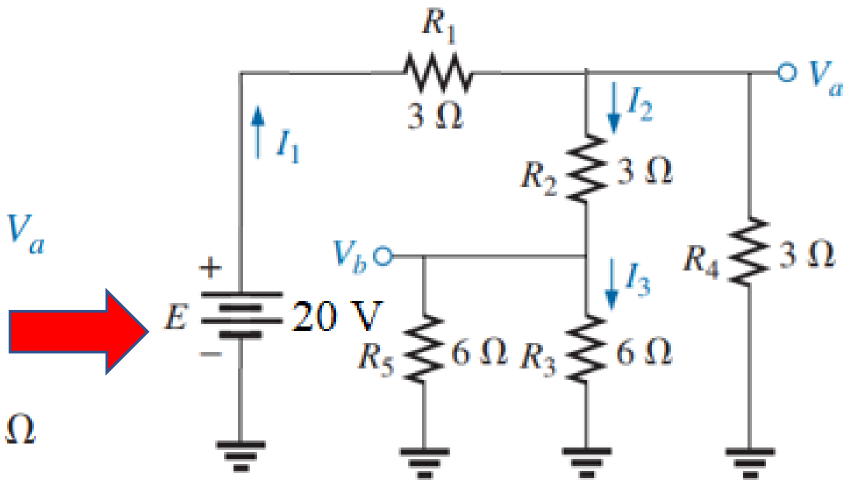


FIG. 7.72
Problem 12.



$$R_b = R_5 \,//\, R_6 = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_{b2} = R_b + R_2 = 3\ \Omega + 3\ \Omega = 6\ \Omega$$

$$R_a = \frac{R_{b2}R_4}{R_{b2}+R_4} = 2\ \Omega$$

$$R_T = R_1 + R_a = 3\ \Omega + 2\ \Omega = 5\ \Omega$$

$$I_1 = \frac{E}{R_T} = \frac{20\ \text{V}}{5\ \Omega} = \mathbf{4\ A}$$

$$V_a = \frac{R_a}{R_T}E = \frac{2\ \Omega}{5\ \Omega} \times 20\ \text{V} = \mathbf{8\ V}$$

$$I_2 = \frac{V_a}{R_{b2}} = \frac{8\ \text{V}}{6\ \Omega} = \mathbf{1.333\ A}$$

$$V_b = I_2 R_b = 1.333\ \text{A} \times 3\ \Omega \cong \mathbf{4\ V}$$

$$I_3 = \frac{V_b}{R_3} = \frac{I_2}{2} = \frac{1.333\ \text{A}}{2} = \mathbf{0.667\ A}$$

Problem 10 **[Ch 7]** Determine the unknown voltage (V) and current (I) for the following network.

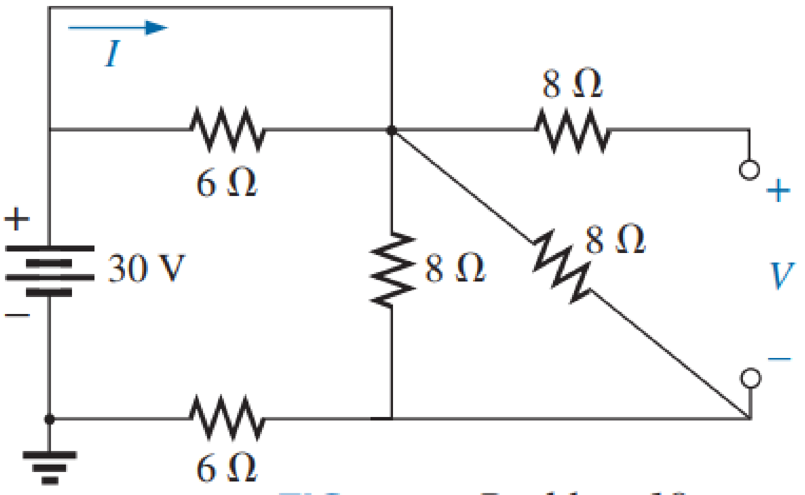
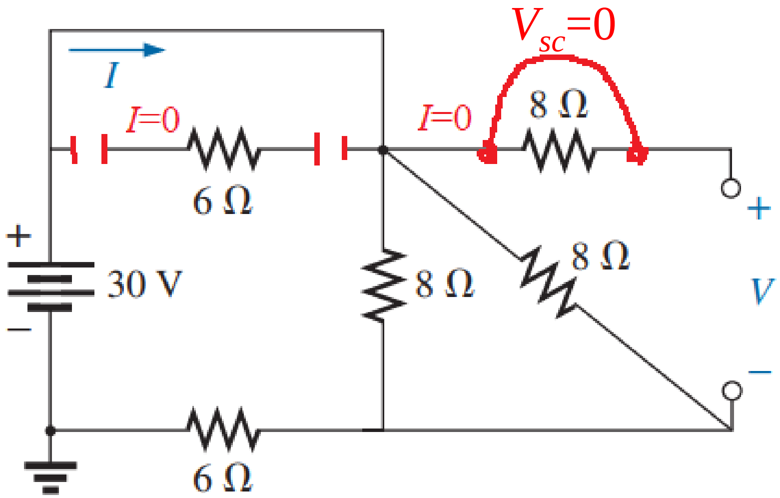
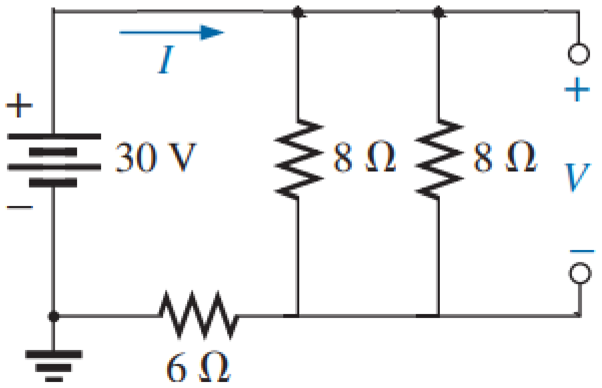
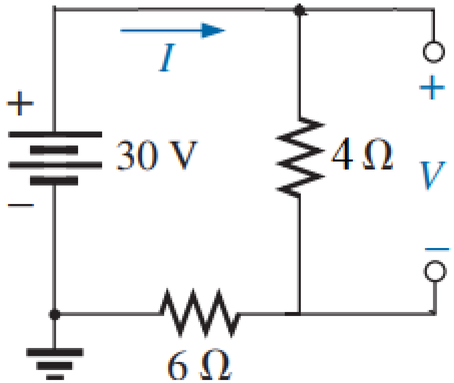


FIG. 7.78 *Problem 18.*





$$R_{8//8} = \frac{8\ \Omega}{2} = 4\ \Omega$$



$$I = \frac{30\ \text{V}}{4\ \Omega + 6\ \Omega} = \mathbf{3\ A}$$

$$V = 3\ \text{A} \times 4\ \Omega = \mathbf{12\ V}$$

Problem 26 For the ladder network in Fig. 7.86: ***a***. Determine R_T. ***b***. Calculate I. ***c***. Find I_8. d. Power consumed by R_6 resistance. e. Power delivered by the 2 V supply.

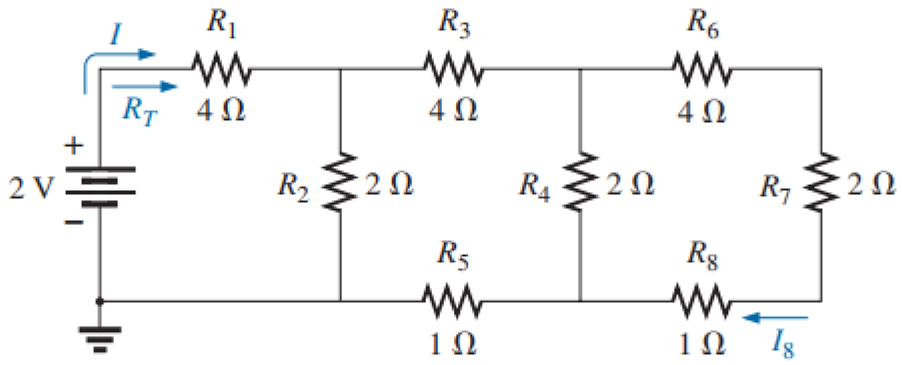
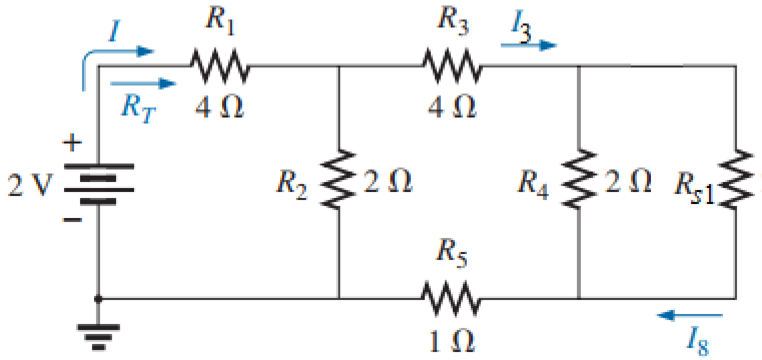
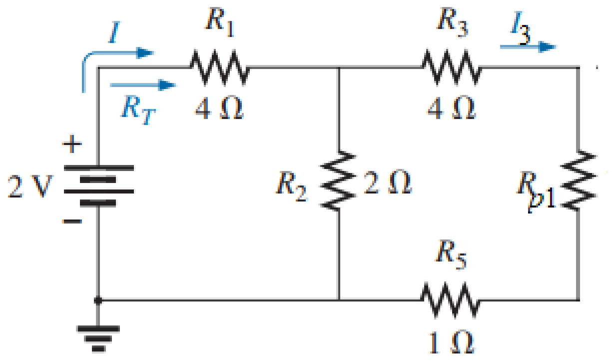


FIG. 7.86 *Problem 26.*

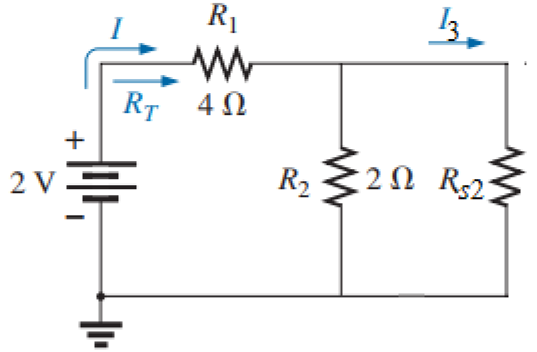
$$R_{s1} = R_6 + R_7 + R_8 = 7\ \Omega$$



$$R_{p1} = \frac{R_{s1}R_4}{R_{s1} + R_4} = 1.56\ \Omega$$



$$R_{s2} = R_3 + R_{p1} + R_5 = 6.56\ \Omega$$



$$R_{p2} = \frac{R_{s2}R_2}{R_{s2} + R_2} = 1.53\ \Omega$$

$$R_T = R_{p2} + R_1 = \mathbf{5.53\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{20\text{ V}}{5.53\ \Omega} = \mathbf{361.66\,mA}$$

$$I_3 = \frac{R_2}{R_2 + R_{s2}} I = \mathbf{84.50\,mA}$$

$$I_8 = \frac{R_4}{R_4 + R_{s1}} I_3 = \mathbf{18.78\,mA}$$

$$P_6 = I_8^2 R_6 = (18.78 \times 10^{-3})^2 \times 4\Omega$$
$$= \mathbf{1.41\,mW}$$

$$P_E = EI = (2\text{ V}) \times (361.66 \times 10^{-3})$$
$$= \mathbf{723.32\,mW}$$

Problem 28. [Ch. 7] For the multiple ladder configuration in Fig. 7.88:

a. Determine I. **b.** Calculate I_4. **c.** Find I_6. **d.** Find I_{10}.

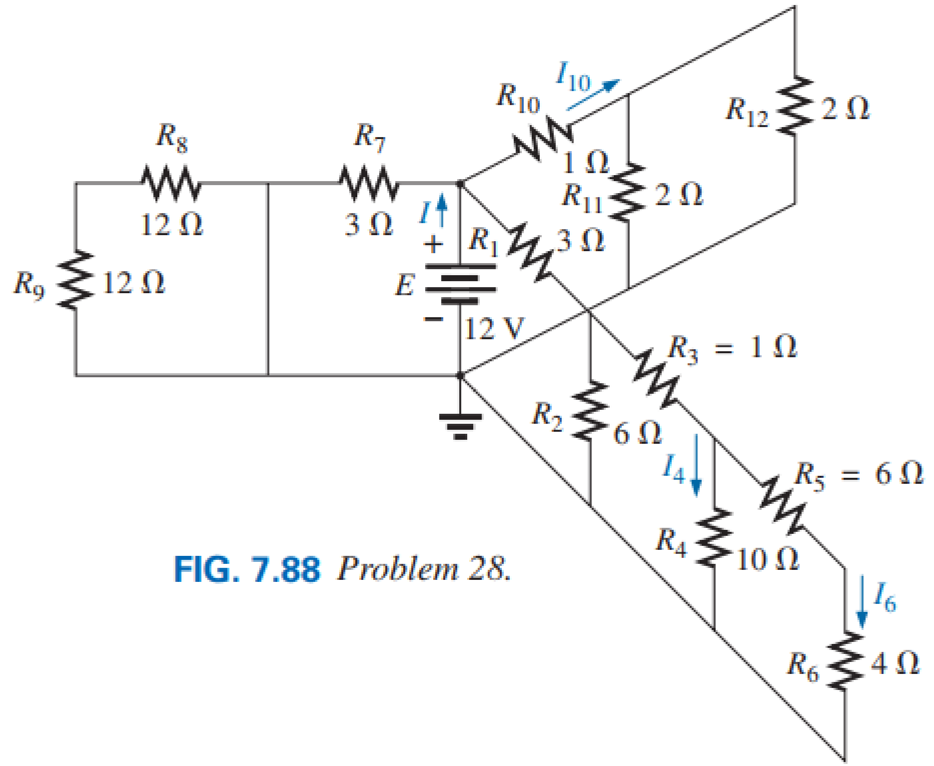


FIG. 7.88 *Problem 28.*

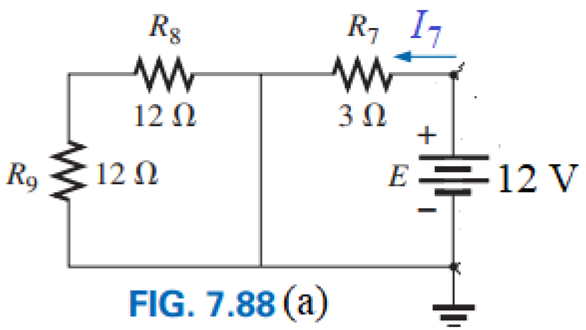


FIG. 7.88 (a)

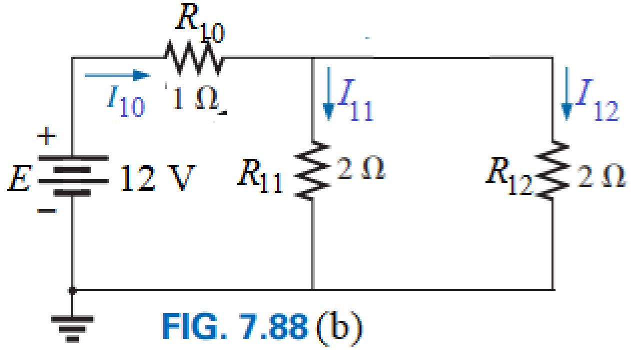


FIG. 7.88 (b)

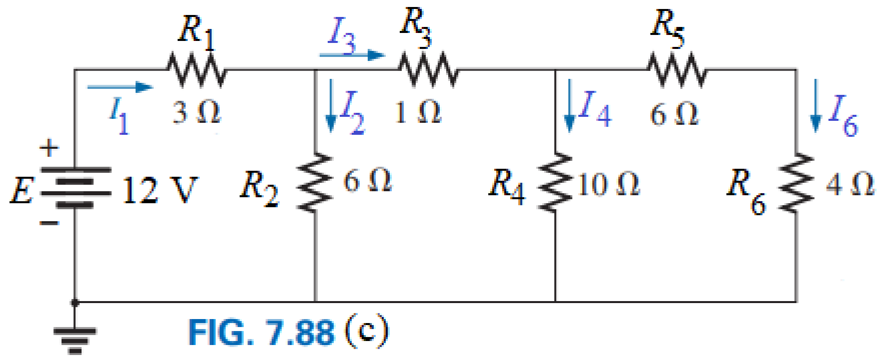


FIG. 7.88 (c)

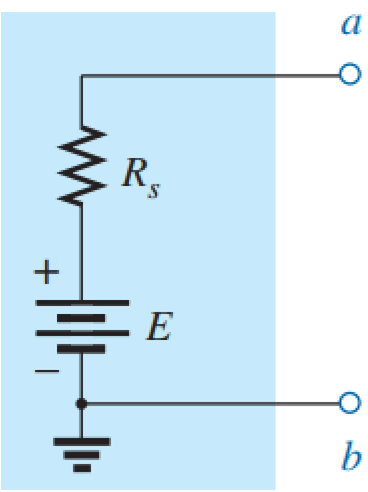
Chapter 8

Methods of Analysis (DC)

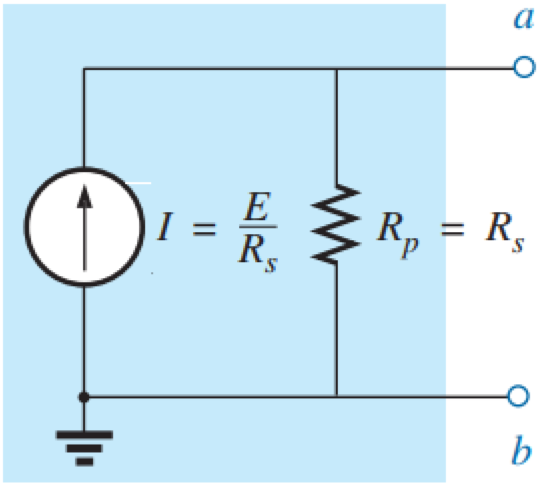
And Selected Topics (DC)

8.3 SOURCE CONVERSIONS

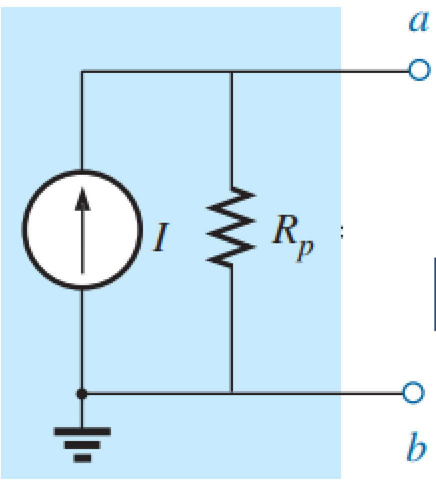
Voltage Source Convert to Current Source



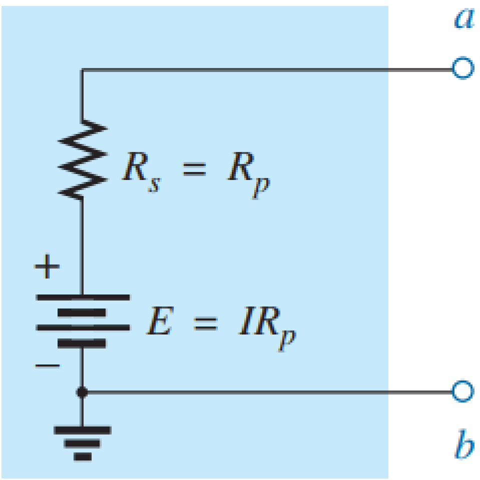
$$I = \frac{E}{R_S} \qquad R_p = R_S$$



Current Source Convert to Voltage Source



$$E = IR_p \qquad R_S = R_p$$



13. Convert the voltage sources in Fig. 8.114 to current sources.

a. Find the voltage V_{ab} and the polarity of points a and b.

b. Find the magnitude and direction of the current I_3.

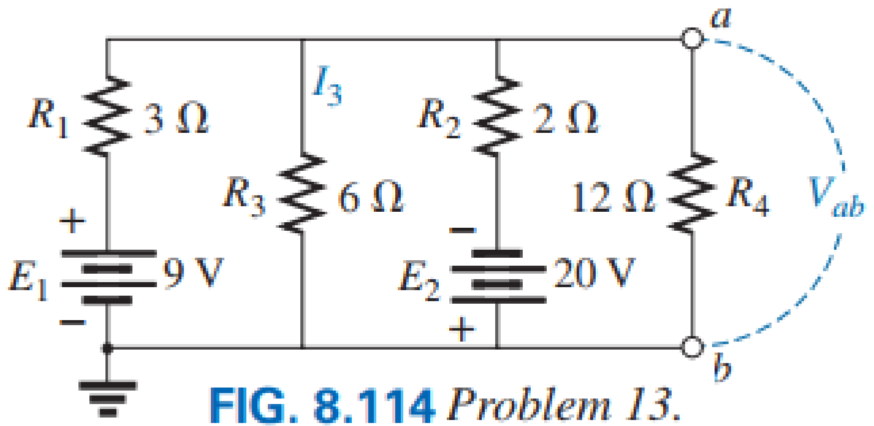
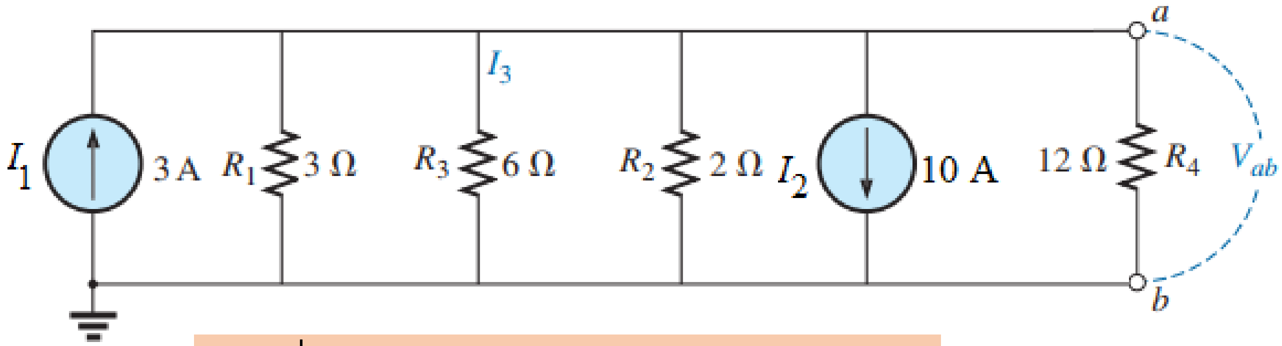
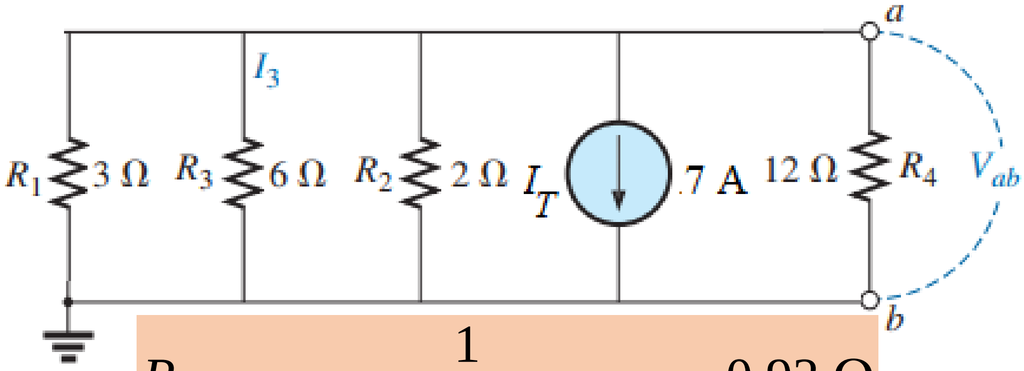


FIG. 8.114 *Problem 13.*

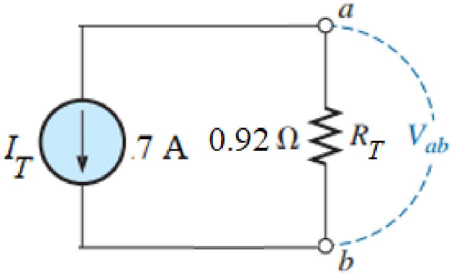
$$I_1 = \frac{E_1}{R_1} = \frac{9\text{ V}}{3\ \Omega} = 3\text{ A} \qquad I_2 = \frac{E_2}{R_2} = \frac{20\text{ V}}{2\ \Omega} = 10\text{ A}$$



$$I_T \downarrow = I_2 - I_1 = 10\text{ A} - 3\text{ A} = 7\text{ A}$$



$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_6} + \frac{1}{R_2} + \frac{1}{R_4}} = 0.92\ \Omega$$



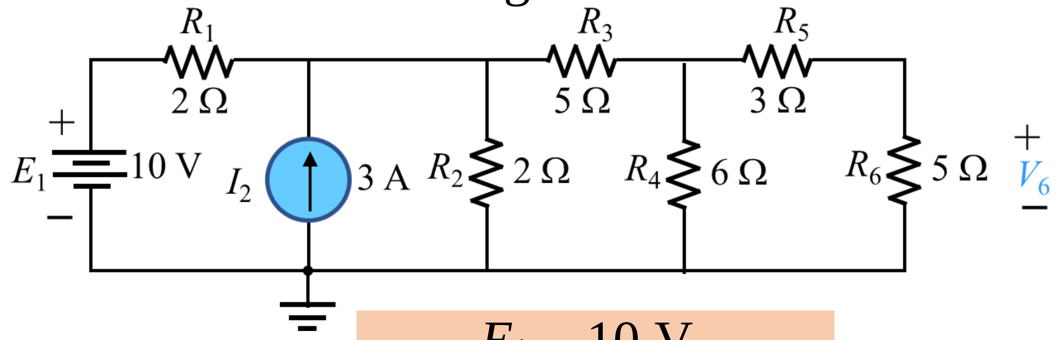
$$V_{ab} = -I_T R_T = \mathbf{-6.44\ V}$$

$$I_3 \uparrow = \frac{-V_{ab}}{R_3} = \frac{6.44\text{ V}}{6\ \Omega} = \mathbf{1.07\ A}$$

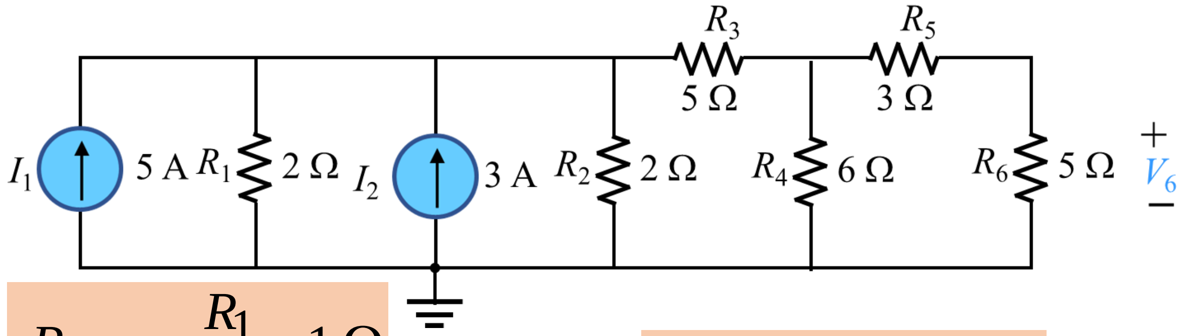
Practice Book [Ch 8]

Problem: 7 ~ 10 and 14

EXAMPLE 8.3.1 Using the source conversion find the value of voltage V_6.

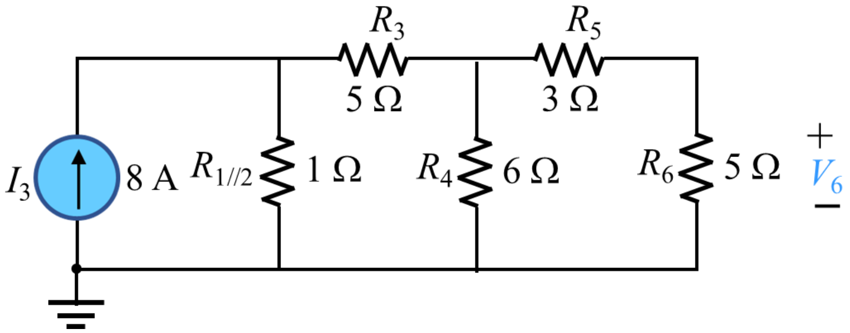


$$I_1 = \frac{E_1}{R_1} = \frac{10\ \text{V}}{2\ \Omega} = 5\ \text{A}$$

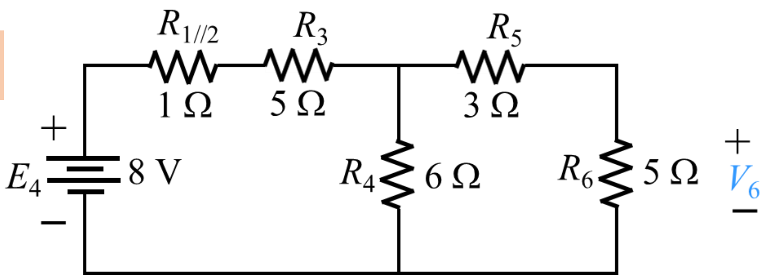


$$R_{1//2} = \frac{R_1}{2} = 1\ \Omega$$

$$I_3 = I_1 + I_2 = 8\ \text{A}$$

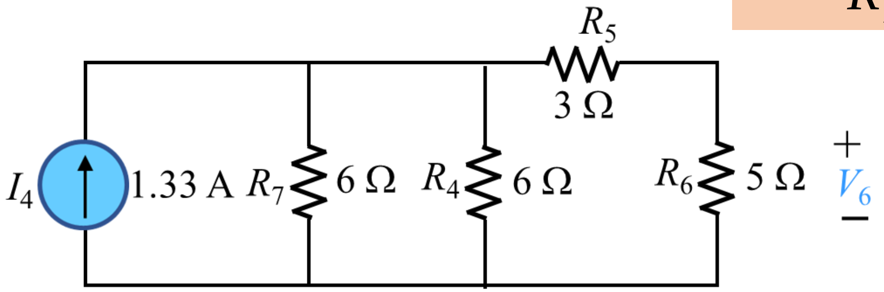


$$E_4 = I_3 R_{1//2} = 8\ \text{V}$$



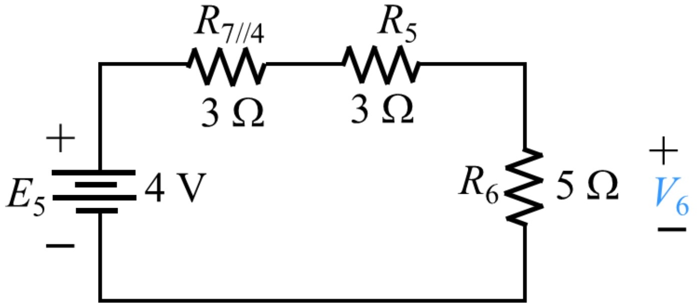
$$R_7 = R_{1//2} + R_3 = 6\ \Omega$$

$$I_4 = \frac{E_4}{R_7} = 1.33\ \text{A}$$



$$R_{7//4} = \frac{R_7}{2} = 3\ \Omega$$

$$E_5 = I_4 R_{7//4} = 4\ \text{V}$$



$$V_6 = \frac{R_6}{R_{7//4} + R_5 + R_6} E_5$$
$$= 1.1\ \text{V}$$

Solution of Equations
For
Mesh Analysis
and
Nodal Analysis

Equation Solution with Two variables *x* and *y*

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

$$\begin{array}{ccccc} \text{Col. } x & \text{Col. } y & Var & = & Const.\,Col. \end{array}$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Calculate the Determinants using *x* and *y* columns:

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_2$$

Calculate the Determinants (which is used to calculate the value of *x*) *replacing x column by constant column* and *y* columns:

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$$

Calculate the Determinants (which is used to calculate the value of *y*) *x* column and *replacing y column by constant column*:

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - a_2c_2$$

Now, *x* and *y* can be calculated as follows:

$$x = \frac{D_x}{D} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_2}$$

$$y = \frac{D_y}{D} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_2}$$

Equation Solution with Three Variables x, y, and z

$$a_1x+b_1y+c_1z=d_1$$
$$a_2x+b_2y+c_2z=d_2$$
$$a_3x+b_3y+c_3z=d_3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Calculate the Determinants using *x*, *y* and *z* columns:

$$D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Calculate the Determinants (which is used to calculate the value of *x*) *replacing x column by constant column*, *y* and *z* columns:

$$D_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

Calculate the Determinants (which is used to calculate the value of *y*) *x* column, *replacing y column by constant column*: and *z* columns :

$$D_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

Calculate the Determinants (which is used to calculate the value of *z*) *x* column, *x* column and *replacing z column by constant column*:

$$D_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Now, *x*, and *y* and *z* can be calculated as follows:

$$x=\frac{D_x}{D} \qquad y=\frac{D_y}{D} \qquad z=\frac{D_z}{D}$$

$$\begin{array}{cccccc} (+) & (+) & (+) & (-) & (-) & (-) \end{array}$$

$$D = \left|\begin{array}{ccc} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{array}\right| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{array}$$

$$D = a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - c_1b_2a_3 - a_1c_2b_3 - b_1a_2c_3$$
$$= a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - (c_1b_2a_3 + a_1c_2b_3 + b_1a_2c_3)$$

$$\begin{array}{cccccc} (+) & (+) & (+) & (-) & (-) & (-) \end{array}$$

$$D_y = \left|\begin{array}{ccc} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{array}\right| \begin{array}{cc} a_1 & d_1 \\ a_2 & d_2 \\ a_3 & d_3 \end{array}$$

$$D_y = a_1d_2c_3 + d_1c_2a_3 + c_1a_2b_3 - c_1d_2a_3 - a_1c_2d_3 - d_1a_2c_3$$
$$= a_1d_2c_3 + d_1c_2a_3 + c_1a_2b_3 - (c_1d_2a_3 + a_1c_2d_3 + d_1a_2c_3)$$

$$\begin{array}{cccccc} (+) & (+) & (+) & (-) & (-) & (-) \end{array}$$

$$D_x = \left|\begin{array}{ccc} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{array}\right| \begin{array}{cc} d_1 & b_1 \\ d_2 & b_2 \\ d_3 & b_3 \end{array}$$

$$D_x = d_1b_2c_3 + b_1c_2d_3 + c_1d_2b_3 - c_1b_2d_3 - d_1c_2b_3 - b_1d_2c_3$$
$$= d_1b_2c_3 + b_1c_2d_3 + c_1d_2b_3 - (c_1b_2d_3 + d_1c_2b_3 + b_1d_2c_3)$$

$$\begin{array}{cccccc} (+) & (+) & (+) & (-) & (-) & (-) \end{array}$$

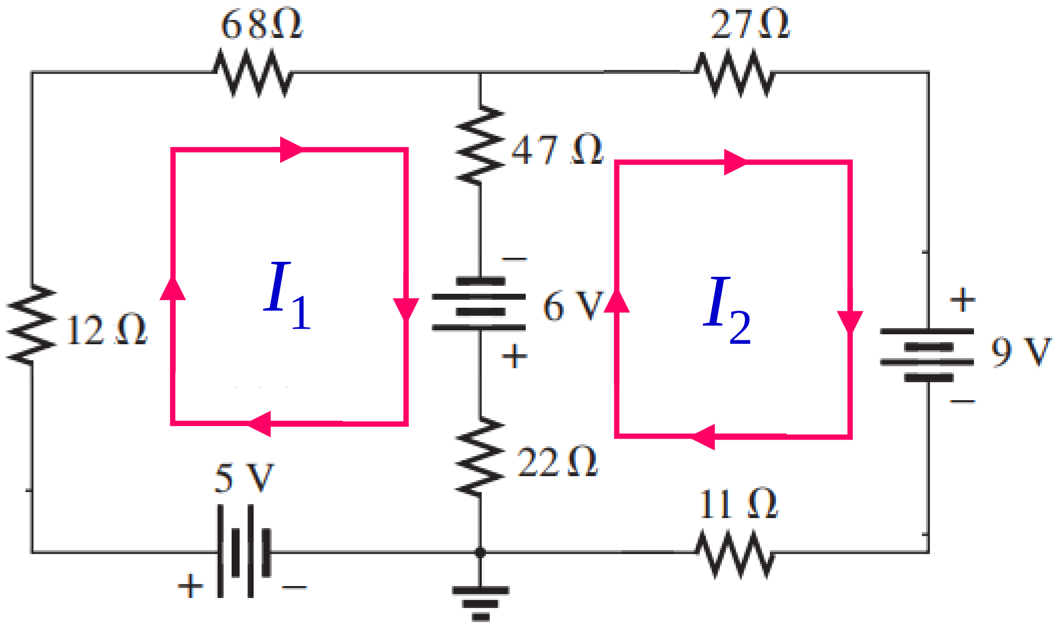
$$D_z = \left|\begin{array}{ccc} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{array}\right| \begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{array}$$

$$D_z = a_1b_2d_3 + b_1d_2a_3 + d_1a_2b_3 - d_1b_2a_3 - a_1d_2b_3 - b_1a_2d_3$$
$$= a_1b_2d_3 + b_1d_2a_3 + d_1a_2b_3 - (d_1b_2a_3 + a_1d_2b_3 + b_1a_2d_3)$$

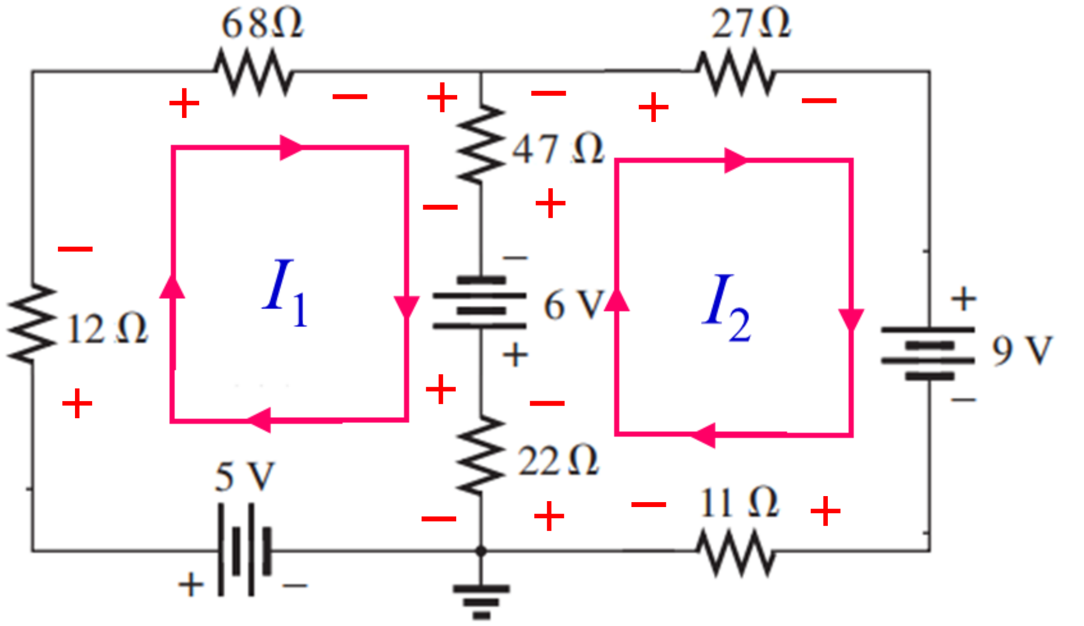
8.7 MESH ANALYSIS

8.7 MESH ANALYSIS

Step 1: Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current.



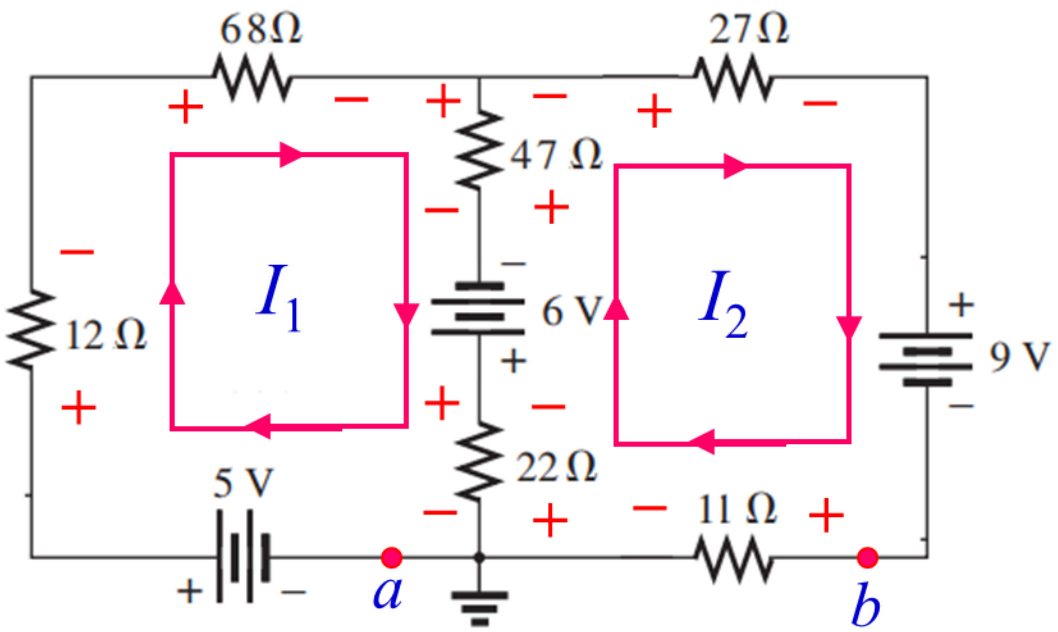
Step 2: 2. Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop.



Step 3: Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. [Consider voltage is positive if current entering through negative terminal of an elements and voltage is negative if current entering through positive terminal of an elements]

a. If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.

b. The polarity of a voltage source is unaffected by the direction of the assigned loop currents.



Loop 1: Start from point *a*

$$5\text{V} - 12I_1 - 68I_1 - 47(I_1 - I_2) + 6\text{V} - 22(I_1 - I_2) = 0$$

Loop 2: Start from point *b*

$$-11I_2 - 22(I_2 - I_1) - 6\text{V} - 47(I_2 - I_1) - 11I_2 - 9\text{V} = 0$$

Simplify Equations:

$$149I_1 - 69I_2 = 11\text{V}$$

$$-69I_1 + 107I_2 = -15\text{V}$$

Step 4: Solve the resulting simultaneous linear equations for the assumed loop currents.

$$\begin{bmatrix} 149 & -69 \\ -69 & 107 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 11\text{V} \\ -15\text{V} \end{bmatrix}$$

$$D = \begin{vmatrix} 149 & -69 \\ -69 & 107 \end{vmatrix} = 149 \times 107 - (-69)(-69) = 11182$$

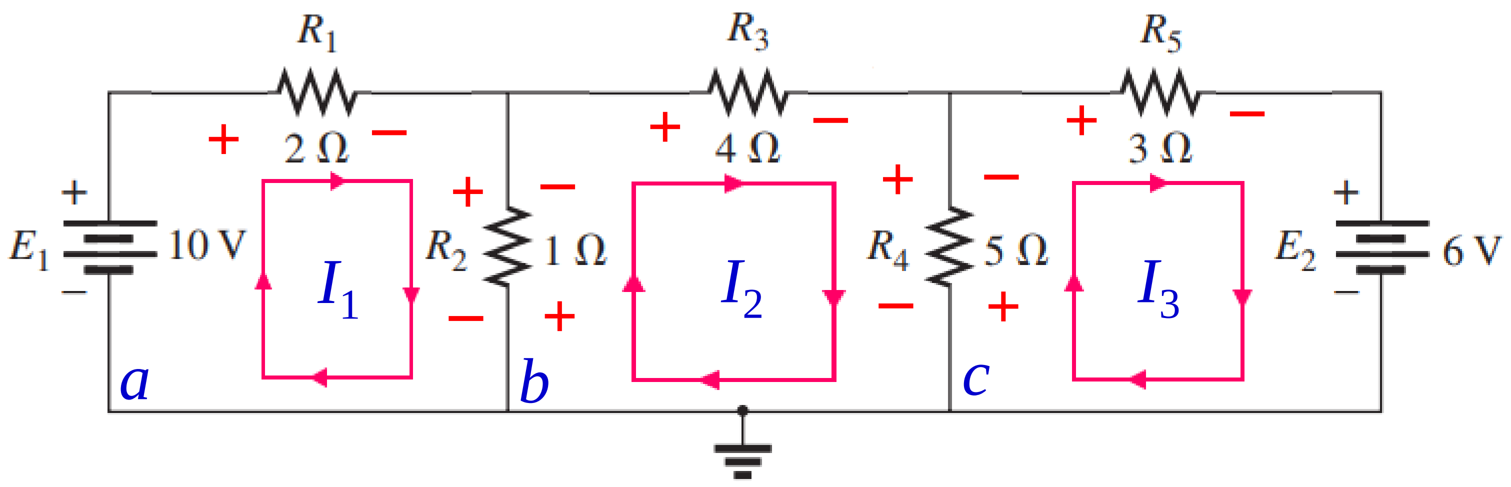
$$D_1 = \begin{vmatrix} 11\text{V} & -69 \\ -15\text{V} & 107 \end{vmatrix} = 11\text{V} \times 107 - (-15\text{V})(-69) = 142$$

$$D_2 = \begin{vmatrix} 149 & 11\text{V} \\ -69 & -15\text{V} \end{vmatrix} = 149 \times (-15\text{V}) - (-69)(11\text{V}) = -1476$$

$$I_1 = \frac{D_1}{D} = \frac{142}{11182} = \mathbf{12.7\ mA}$$

$$I_2 = \frac{D_2}{D} = \frac{-1476}{11182} = \mathbf{-131.99\ mA}$$

EXAMPLE 8.7.1 (**a**) Write the mesh equations for each loop of the networks. (**b**) Using determinants, solve for the loop currents. (**b**) Find the current of each branch.



Simplify Equations:

$$(R_1+R_2)I_1-R_2I_2=E_1$$

$$-R_2I_1+(R_2+R_3+R_4)I_2-R_4I_3=0$$

$$-R_4I_2+(R_4+R_5)I_3=-E_2$$

Loop 1: Start from point *a*

$$E_1-R_1I_1-R_2(I_1-I_2)=0$$

Loop 2: Start from point *b*

$$-R_2(I_2-I_1)-R_3I_2-R_4(I_2-I_3)=0$$

Loop 3: Start from point *c*

$$-R_4(I_2-I_3)-R_5I_3-E_2=0$$

Putting the values of resistances and voltages:

$$3I_1-I_2=10\text{V}$$

$$-I_1+10I_2-5I_3=0\text{V}$$

$$-5I_2+8I_3=-6\text{V}$$

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 10 & -5 \\ 0 & -5 & 8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\text{V} \\ 0\text{V} \\ 6\text{V} \end{bmatrix}$$

$$D = \begin{vmatrix} 3 & -1 & 0 \\ -1 & 10 & -5 \\ 0 & -5 & 8 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} 10\text{V} & -1 & 0 \\ 0\text{V} & 10 & -5 \\ -6\text{V} & -5 & 8 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} 3 & 10\text{V} & 0 \\ -1 & 0\text{V} & -5 \\ 0 & -6\text{V} & 8 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} 3 & -1 & 10\text{V} \\ -1 & 10 & 0\text{V} \\ 0 & -5 & -6\text{V} \end{vmatrix}$$

$$D = \begin{vmatrix} 3 & -1 & 0 \\ -1 & 10 & -5 \\ 0 & -5 & 8 \end{vmatrix} \begin{matrix} 3 & -1 \\ -1 & 10 \\ 0 & -5 \end{matrix}$$

$$\begin{aligned} D &= (3)(10)(8) + (-1)(-5)(0) + (0)(-1)(-5) \\ &\quad - (0)(10)(0) - (3)(-5)(-5) - (-1)(-1)(8) \\ &= 240 - 75 - 8 = 157 \end{aligned}$$

$$D_1 = \begin{vmatrix} 10\text{V} & -1 & 0 \\ 0\text{V} & 10 & -5 \\ -6\text{V} & -5 & 8 \end{vmatrix} \begin{matrix} 10\text{V} & -1 \\ 0\text{V} & 10 \\ -6\text{V} & -5 \end{matrix}$$

$$\begin{aligned} D_1 &= (10)(10)(8) + (-1)(-5)(-6) + (0)(0)(-5) \\ &\quad - (0)(10)(-6) - (10)(-5)(-5) - (-1)(0)(8) \\ &= 800 - 30 - 250 = 520 \end{aligned}$$

$$D_2 = \begin{vmatrix} 3 & 10\text{V} & 0 \\ -1 & 0\text{V} & -5 \\ 0 & -6\text{V} & 8 \end{vmatrix} \begin{matrix} 3 & 10\text{V} \\ -1 & 0\text{V} \\ 0 & -6\text{V} \end{matrix}$$

$$\begin{aligned} D_2 &= (3)(0)(8) + (10)(-5)(0) + (0)(-1)(-6) \\ &\quad - (0)(0)(0) - (3)(-5)(-6) - (10)(-1)(8) \\ &= -90 + 80 = -10 \end{aligned}$$

$$D_3 = \begin{vmatrix} 3 & -1 & 10\text{V} \\ -1 & 10 & 0\text{V} \\ 0 & -5 & -6\text{V} \end{vmatrix} \begin{matrix} 3 & -1 \\ -1 & 10 \\ 0 & -5 \end{matrix}$$

$$\begin{aligned} D_3 &= (3)(10)(-6) + (-1)(0)(0) + (10)(-1)(-5) \\ &\quad - (10)(10)(0) - (3)(0)(-5) - (-1)(-1)(-6) \\ &= -180 + 50 + 6 = -124 \end{aligned}$$

$$I_1 = \frac{D_1}{D} = \frac{520}{157} = \mathbf{3.31\ A}$$

$$I_2 = \frac{D_2}{D} = \frac{-10}{157} = \mathbf{-0.0637\ A} \quad \text{or} \quad \mathbf{-63.7\ mA}$$

$$I_3 = \frac{D_3}{D} = \frac{-124}{157} = \mathbf{-0.79\ A} \quad \text{or} \quad \mathbf{-790\ mA}$$

Check or Justification of Results:

$$\begin{aligned} 3I_1 - I_2 &= 10\text{V} \\ -I_1 + 10I_2 - 5I_3 &= 0\text{V} \\ -5I_2 + 8I_3 &= -6\text{V} \end{aligned}$$

$$\begin{aligned} 3I_1 - I_2 &= 3\times 3.31 - 0.0637 = 10\text{V} \\ -I_1 + 10I_2 - 5I_3 &= -3.31 + 10\times 0.0637 - 5\times 0.79 = 0\text{V} \\ -5I_2 + 8I_3 &= -5\times 0.0637 + 8\times 0.79 = -6\text{V} \end{aligned}$$

(Justified)

8.8 MESH ANALYSIS (FORMAT APPROACH)

In closed path, according to KVL:

Summation of Voltage Drop = Summation of Voltage Rise

$$\Sigma_{\circlearrowright} V_{\mathrm{drops}} = \Sigma_{\circlearrowright} V_{\mathrm{rises}}$$

Positive if current entering through positive terminal
Negative if current entering through negative terminal

Positive if current entering through negative terminal
Negative if current entering through positive terminal

EXAMPLE 8.16 Write the mesh equations for the network in Fig. 8.40, and find the current through the 8 Ω and 7 Ω resistors.

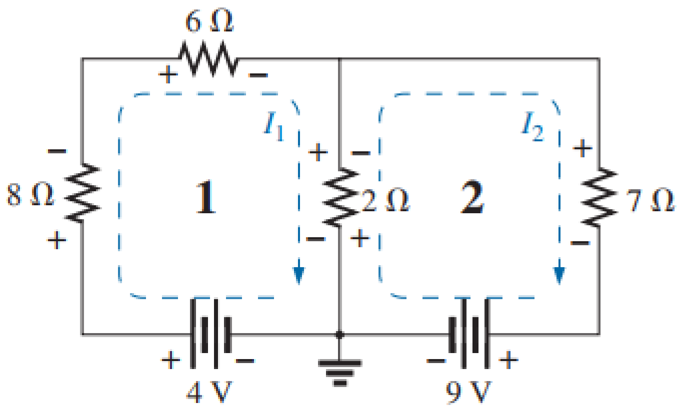


FIG. 8.40 *Example 8.16.*

Loop 1: $(8+6+2)I_1 - 2I_2 = 4$

Loop 2: $(2+7)I_2 - 2I_1 = -9$

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

$$D = \begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix} = 16\times9-(-2)\times(-2) = 140$$

$$D_1 = \begin{vmatrix} 4 & -2 \\ -9 & 9 \end{vmatrix} = 4\times9-(-2)\times(-9) = 18$$

$$D_2 = \begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix} = 16\times(-9)-(-2)\times4 = -136$$

$$I_1 = I_{8\Omega} = \frac{D_1}{D} = \frac{18}{140} = \mathbf{0.13\ A}$$

$$I_2 = I_{7\Omega} = \frac{D_2}{D} = \frac{-136}{140} = \mathbf{-0.97\ A}$$

EXAMPLE 8.16 Write the mesh equations for the network in Fig. 8.32, and find the branch currents.

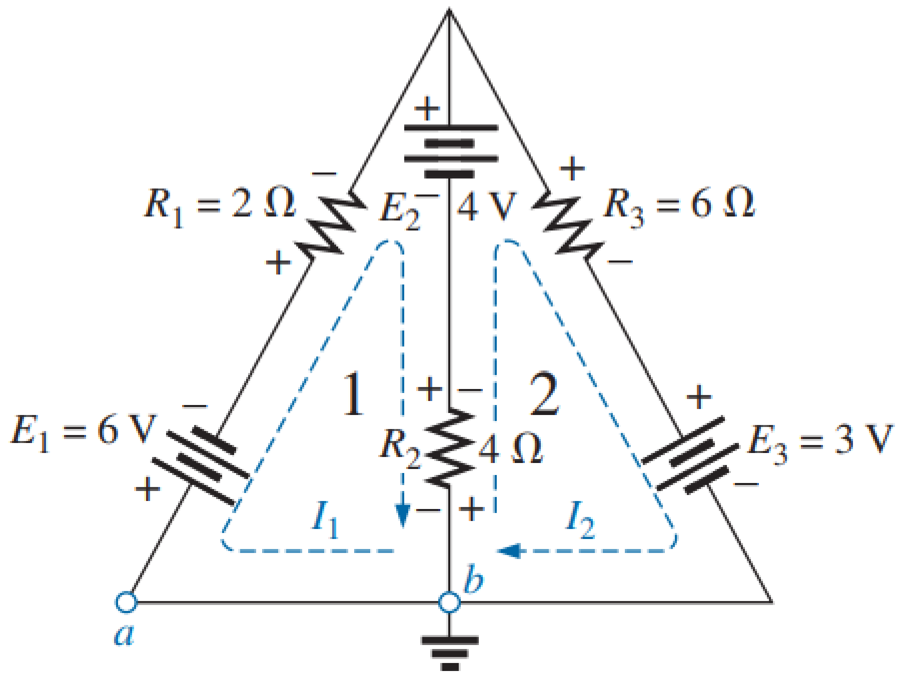


FIG. 8.32 *Example 8.13.*

Loop 1: $(2+4)I_1 - 4I_2 = -6-4$

Loop 2: $(4+6)I_2 - 4I_1 = 4-3$

$$6I_1 - 4I_2 = -10$$
$$-4I_1 + 10I_2 = 1$$

$$D = \begin{vmatrix} 6 & -4 \\ -4 & 10 \end{vmatrix} = 6\times 10 - (-4)\times(-4) = 44$$

$$D_1 = \begin{vmatrix} -10 & -4 \\ 1 & 10 \end{vmatrix} = (-10)\times 10 - (1)\times(-4) = -96$$

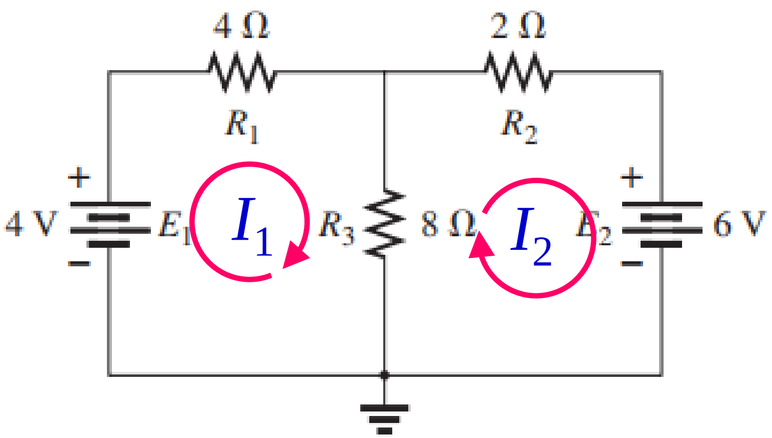
$$D_2 = \begin{vmatrix} 6 & -10 \\ -4 & 1 \end{vmatrix} = 6\times 1 - (-4)\times(-10) = -34$$

$$I_1 = I_{2\Omega} = \frac{D_1}{D} = \frac{-96}{44} = \mathbf{-2.18\ A}$$

$$I_2 = I_{6\Omega} = \frac{D_2}{D} = \frac{-34}{44} = \mathbf{-0.77\ A}$$

$$I_4 = I_1 - I_2 = -2.18\text{ A} - (-0.77\text{ A}) = \mathbf{-1.41\ A}$$

Write the mesh/loop equations for the following networks.



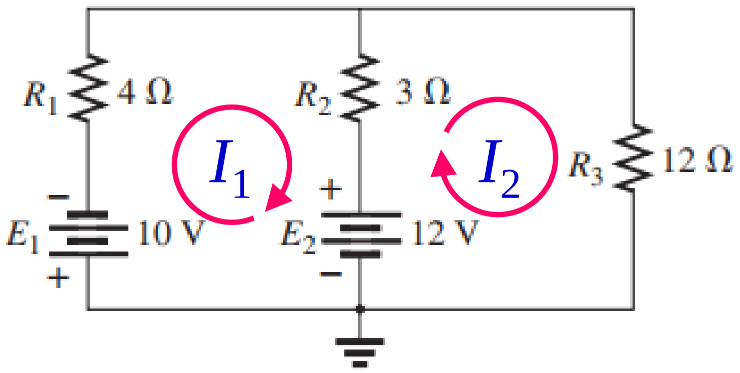
Loop 1:

$$(4+8)I_1 - 8I_2 = 4$$

Loop 2:

$$(8+2)I_2 - 8I_1 = -6$$

$$12I_1 - 8I_2 = 4$$
$$-8I_1 + 10I_2 = -6$$



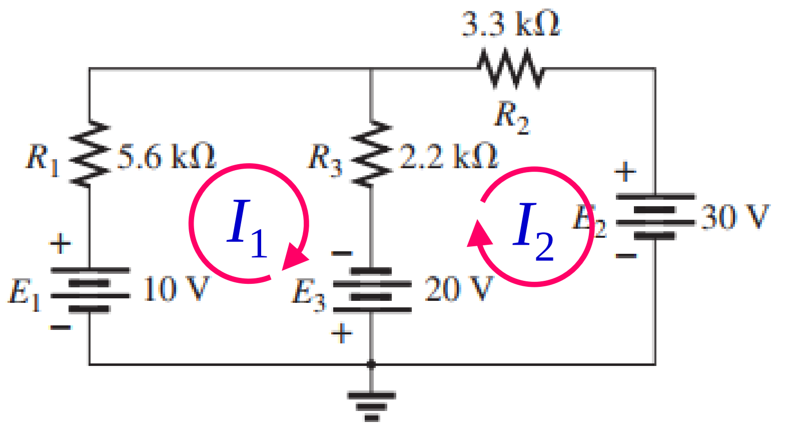
Loop 1:

$$(4+3)I_1 - 3I_2 = -10-12$$

Loop 2:

$$(3+12)I_2 - 3I_1 = 12$$

$$7I_1 - 3I_2 = -22$$
$$-3I_1 + 15I_2 = 12$$



Loop 1:

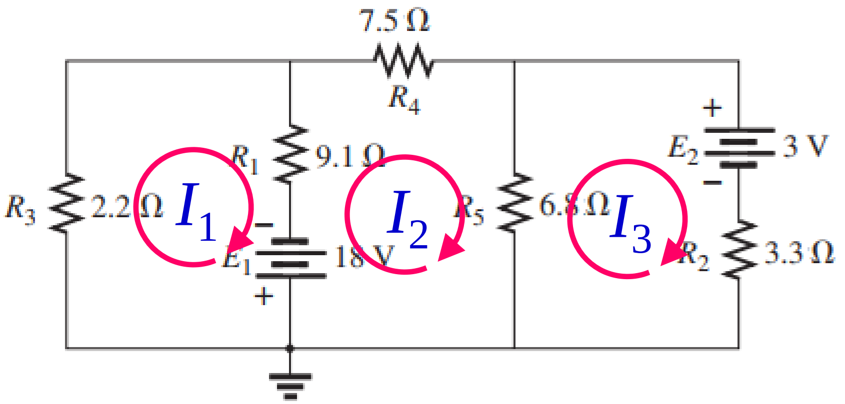
$$(5.6+2.2)(\text{k}\Omega)I_1 - 2.2(\text{k}\Omega)I_2 = 10+20$$

Loop 2:

$$(2.2+3.3)(\text{k}\Omega)I_2 - 2.2(\text{k}\Omega)I_1 = -20-30$$

$$(7.8\text{k}\Omega)I_1 - (2.2\text{k}\Omega)I_2 = 30$$
$$-(2.2\text{k}\Omega)I_1 + (5.5\text{k}\Omega)I_2 = -50$$

Write the mesh/loop equations for the following networks.

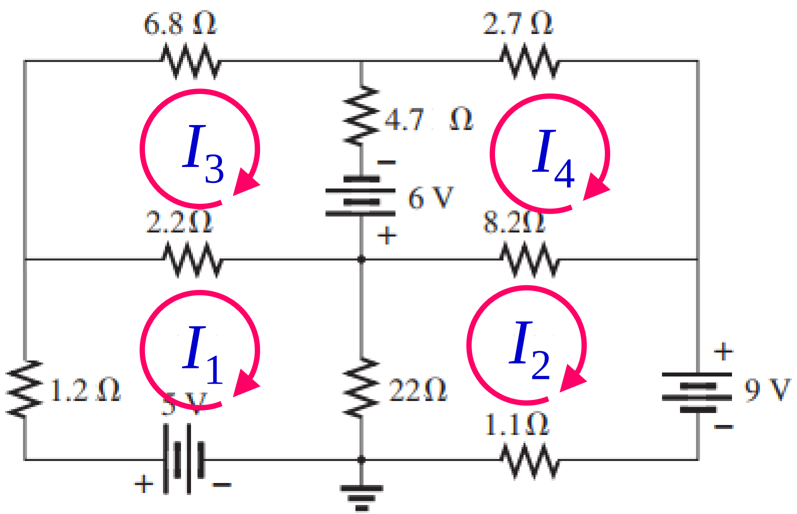


Loop 1: $(2.2+9.1)I_1 - 9.1I_2 = 18$

Loop 2: $(9.1+7.5+6.8)I_2 - 9.1I_1 - 6.8I_3 = -18$

Loop 3: $(6.8+3.3)I_3 - 6.8I_2 = -3$

$$11.3I_1 - 9.1I_2 = 18$$
$$-9.1I_1 + 23.4I_2 - 6.8I_3 = -18$$
$$-6.8I_2 + 10.1I_3 = -3$$



Loop 1: $(1.2+2.2+22)I_1 - 22I_2 - 2.2I_3 = 5$

Loop 2: $(22+8.2+1.1)I_2 - 22I_1 - 8.2I_4 = -18$

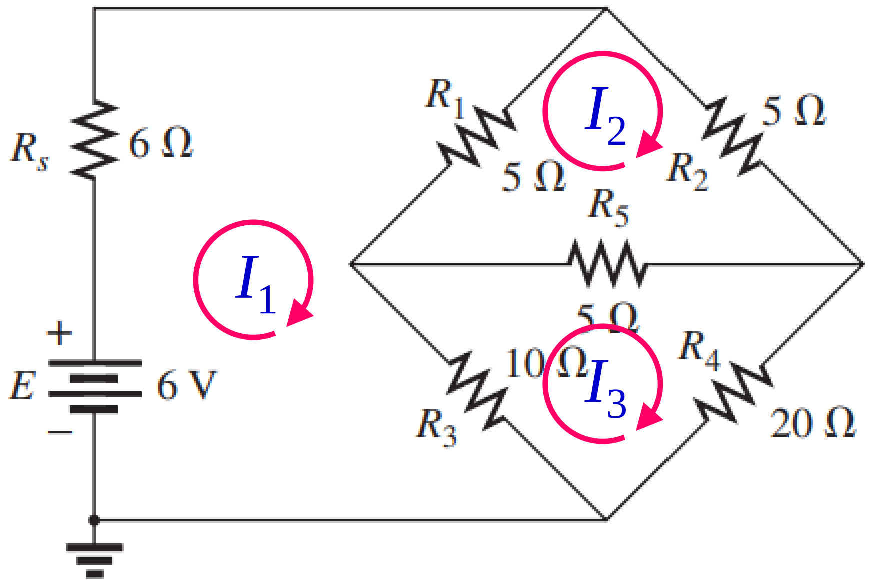
Loop 3: $(2.2+6.8+4.7)I_3 - 2.2I_1 - 4.7I_4 = 6$

Loop 4: $(4.7+2.7+8.2)I_4 - 8.2I_2 - 4.7I_3 = -6$

$$25.4I_1 - 22I_2 - 2.2I_3 = 5$$
$$-22I_1 + 31.3I_2 - 8.2I_4 = -18$$
$$-2.2I_1 + 13.7I_3 - 4.7I_4 = 6$$
$$-8.2I_2 - 4.7I_3 + 15.6I_4 = -6$$

Practice Book [Ch 8] Problem: 28 ~ 34

Write the mesh/loop equations for the following networks.



Loop 1: $(6+5+10)I_1 - 5I_2 - 10I_3 = 6$

Loop 2: $(5+5+5)I_2 - 5I_1 - 5I_3 = 0$

Loop 3: $(10+5+20)I_3 - 10I_1 - 5I_2 = 0$

8.9 NODAL ANALYSIS (GENERAL APPROACH)

Steps of Nodal Analysis:

1. Determine the number of nodes within the network.
2. Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.
3. Apply Kirchhoff's current law (KCL) at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law (KCL).

   Each node is to be treated as a separate entity, independent of the application of KCL to the other nodes.
4. Solve the resulting equations for the nodal voltages.

EXAMPLE 8.20 Apply nodal analysis to the network in Fig. 8.49.

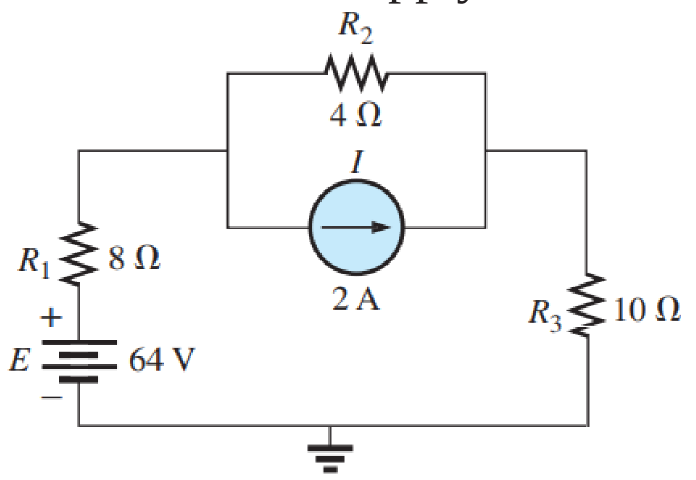
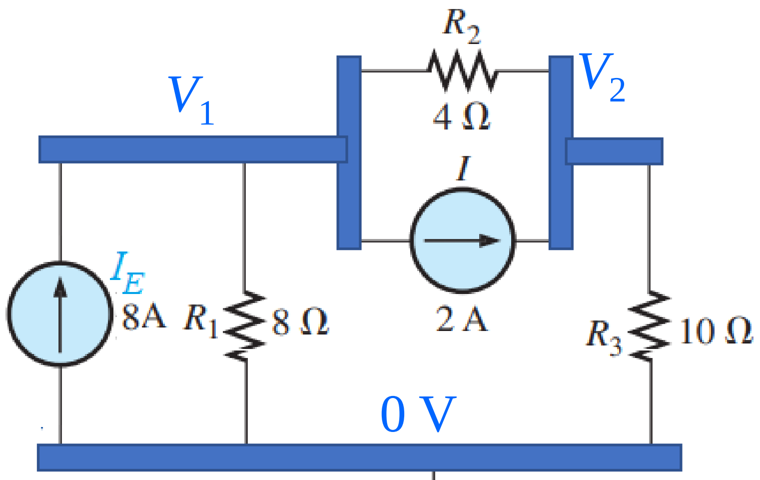


FIG. 8.49 *Example 8.20.*

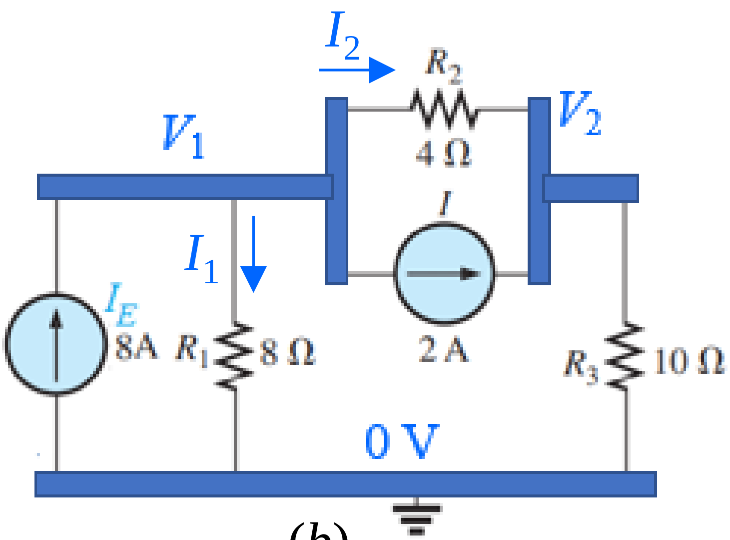
Solution: Convert the voltage sources to current sources as shown in Figure (a).



(*a*)

Step 1 and 2: The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:



(*b*)

$$I_1 + I_2 + I = I_E$$

$$I_1 + I_2 = I_E - I$$

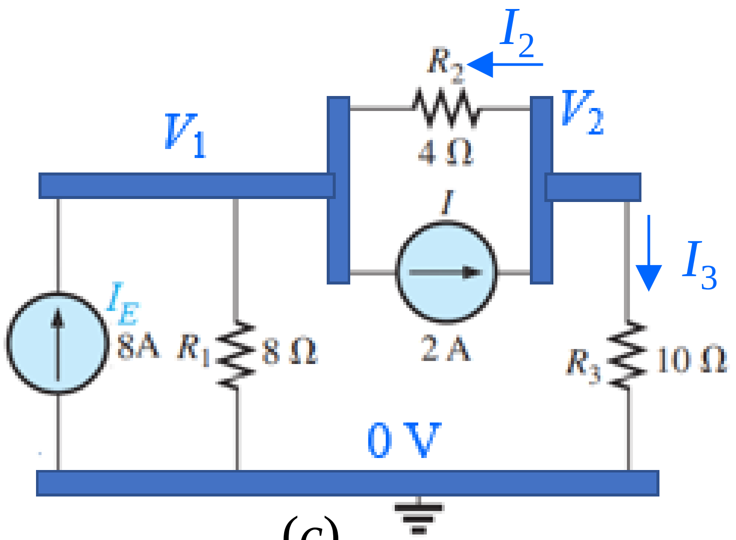
$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = I_E - I$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 - \left(\frac{1}{R_2}\right)V_2 = I_E - I$$

$$\left(\frac{1}{8} + \frac{1}{4}\right)V_1 - \left(\frac{1}{4}\right)V_2 = 8 - 2$$

$$3V_1 - 2V_2 = 48$$

For node V_2, the currents are defined as shown in the following Figure (c) and Kirchhoff's current law is applied:



(c)

$$I_2 + I_3 = I$$

$$\left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_2 - \left(\frac{1}{R_2}\right)V_1 = I$$

$$\left(\frac{1}{4} + \frac{1}{10}\right)V_2 - \left(\frac{1}{4}\right)V_1 = 2$$

$$7V_2 - 5V_1 = 40$$

$$3V_1 - 2V_2 = 48$$

$$-5V_2 + 7V_2 = 40$$

$$D = \begin{vmatrix} 3 & -2 \\ -5 & 7 \end{vmatrix} = 21 - 10 = 11$$

$$D_1 = \begin{vmatrix} 48 & -2 \\ 40 & 7 \end{vmatrix} = 336 + 80 = 416$$

$$D_2 = \begin{vmatrix} 3 & 48 \\ -5 & 40 \end{vmatrix} = 120 + 240 = 360$$

$$V_1 = \frac{D_1}{D} = \frac{416}{11} = \mathbf{37.82\ V}$$

$$V_2 = \frac{D_2}{D} = \frac{360}{11} = \mathbf{32.72\ V}$$

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\text{ V} - 37.82\text{ V}}{8\ \Omega} = \mathbf{3.27\ A}$$

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.73\text{ V}}{10\ \Omega} = \mathbf{3.27\ A}$$

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.82\text{ V} - 32.73\text{ V}}{4\ \Omega} = \mathbf{1.27\ A}$$

EXAMPLE 8.21 Determine the nodal voltages for the network in Fig. 8.54.

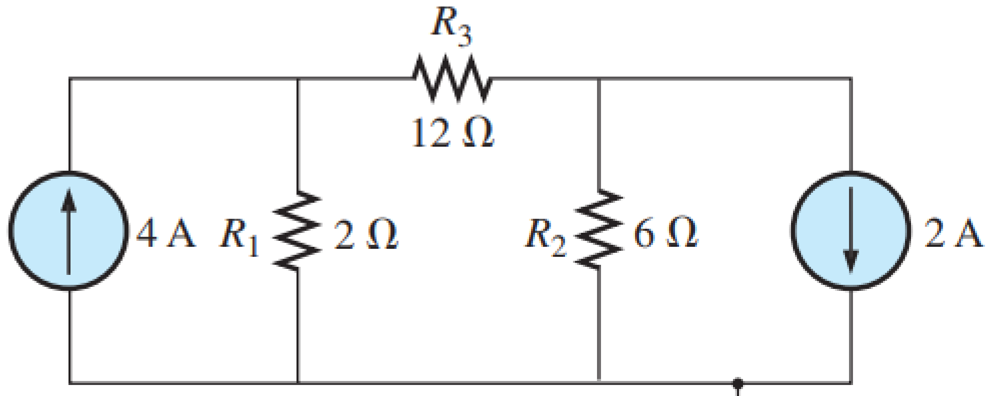
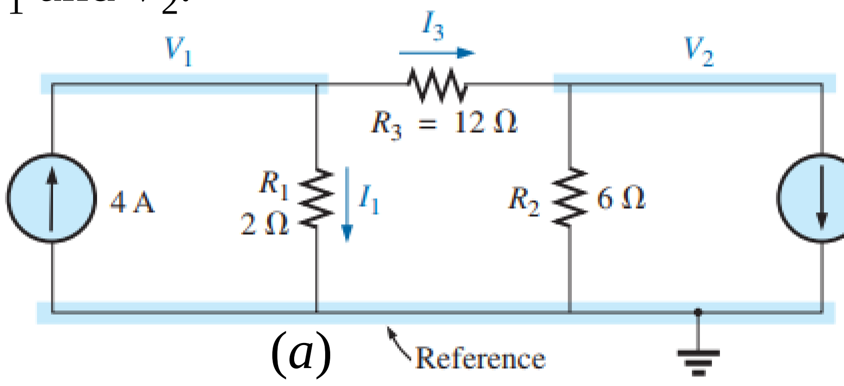


FIG. 8.54 *Example 8.21.*

Step 1 and 2: The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

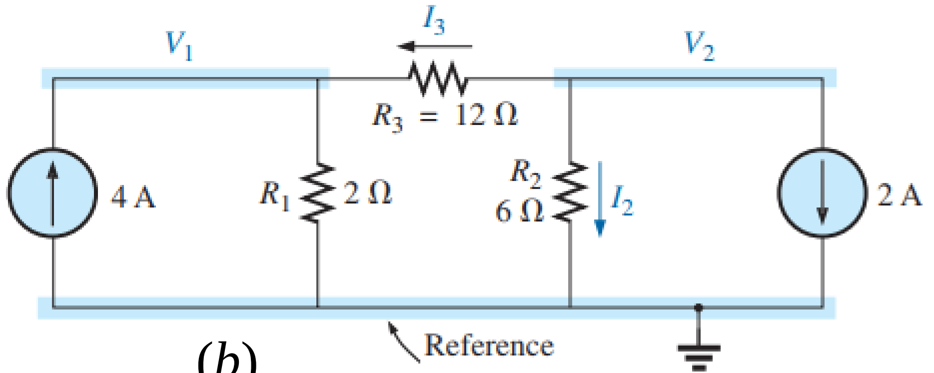


(*a*)

Step 3: For node V_1, the currents are defined as shown in the following Figure (a) and Kirchhoff's current law is applied:

$$I_1 + I_3 = 4 \qquad \left(\frac{1}{2}+\frac{1}{12}\right)V_1 - \left(\frac{1}{12}\right)V_2 = 4 \qquad 7V_1 - V_2 = 48$$

For node V_2, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:

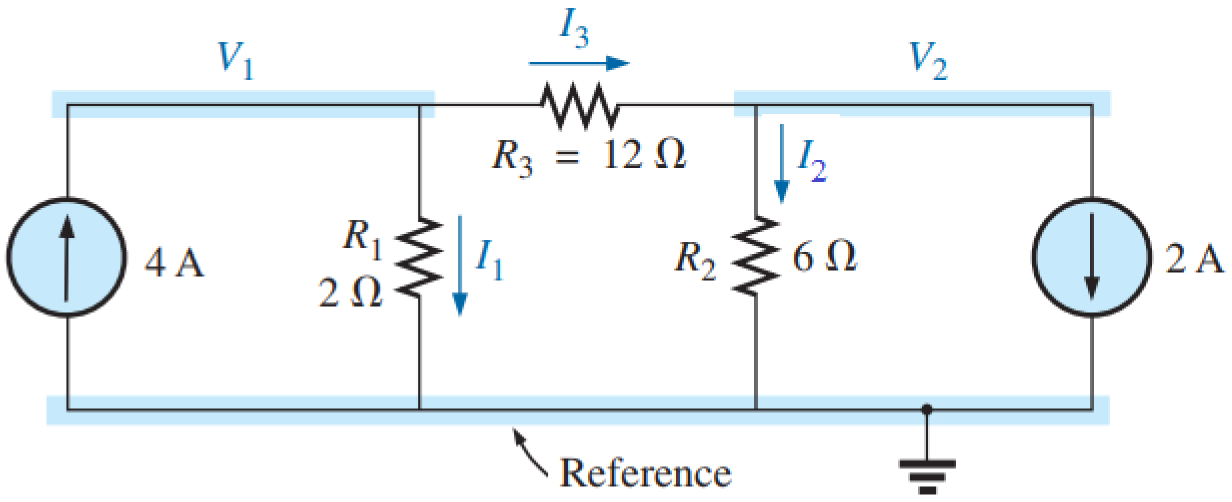


(*b*)

$$I_2 + I_3 = -2$$

$$\left(\frac{1}{12}+\frac{1}{6}\right)V_2 - \left(\frac{1}{12}\right)V_1 = -2 \qquad 3V_2 - V_1 = -24$$

$$7V_1 - V_2 = 48$$
$$-V_1 + 3V_2 = -24$$



$$D = \begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix} = 21 - 1 = 20$$

$$D_1 = \begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix} = 144 - 24 = 120$$

$$D_2 = \begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix} = -168 + 48 = -120$$

$$V_1 = \frac{D_1}{D} = \frac{120}{20} = \mathbf{6\ V}$$

$$V_2 = \frac{D_2}{D} = \frac{-120}{20} = \mathbf{-20\ V}$$

Here, $V_1 > V_1$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\text{ V} - (-6\text{ V})}{12\ \Omega} = \frac{12\text{ V}}{12\ \Omega} = \mathbf{1\ A}$$

EXAMPLE 8.24 Find the voltage across the 3 Ω resistor in Fig. 8.61 by nodal analysis.

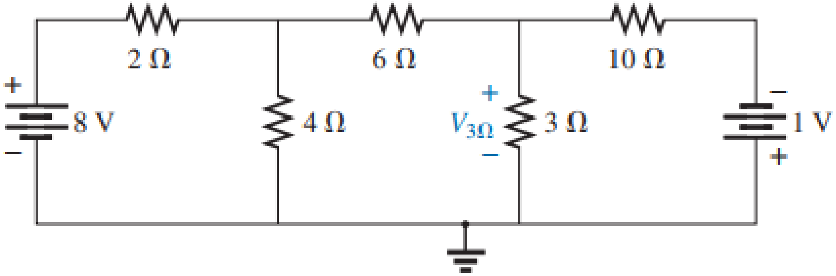
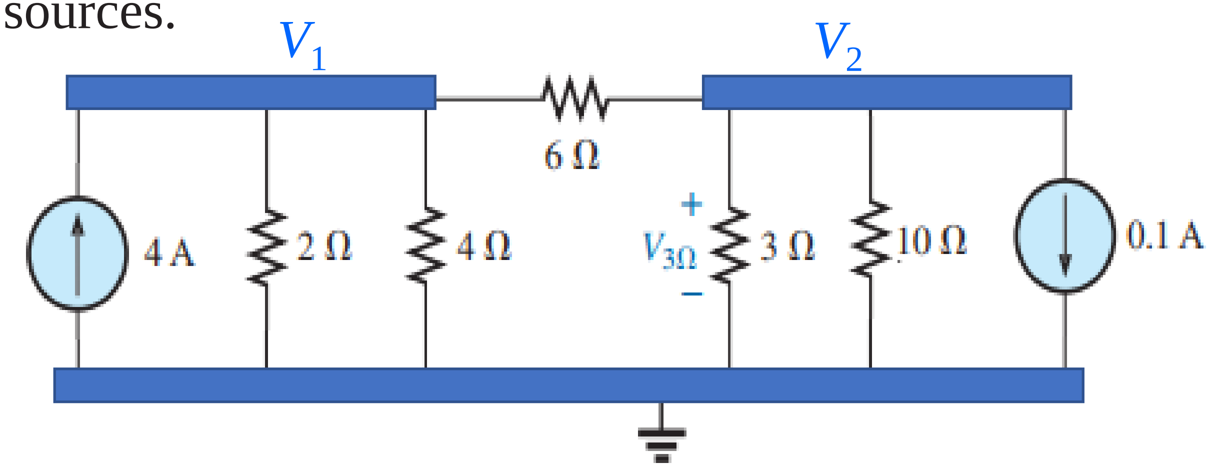


FIG. 8.61 *Example 8.24.*

Solution: First convert two voltage sources to current sources.



Step 1 and 2: The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_1-\left(\frac{1}{6}\right)V_2=4 \qquad 11V_1-2V_2=48$$

For node V_2:

$$\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{10}\right)V_2-\left(\frac{1}{6}\right)V_1=-0.1 \qquad 18V_2-5V_1=-3$$

Simplified form:

$$11V_1-2V_2=48$$
$$-5V_1+18V_2=-3$$

$$D=\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}=198-10=188$$

$$11V_1 - 2V_2 = 48$$

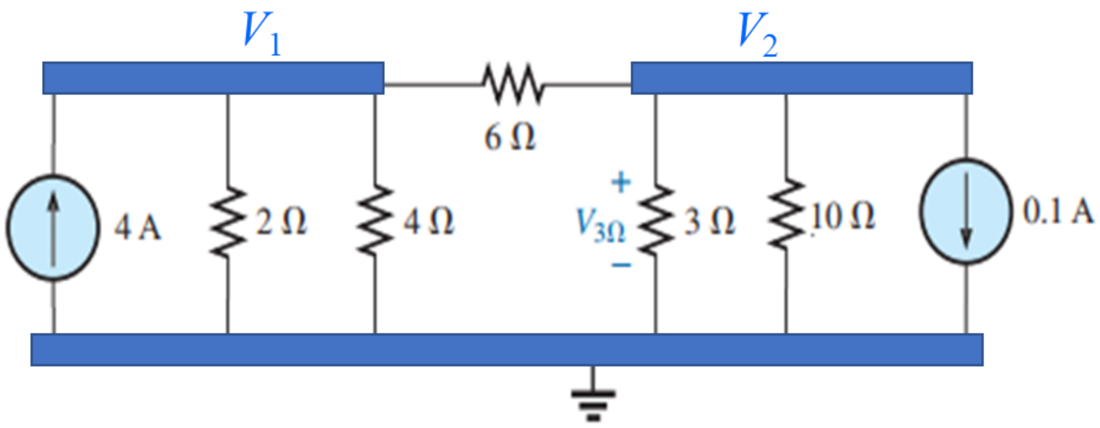
$$-5V_1 + 18V_2 = -3$$

$$D_1 = \begin{vmatrix} 48 & -2 \\ -3 & 18 \end{vmatrix} = 864 - 6 = 858$$

$$D_2 = \begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix} = -33 + 240 = 207$$

$$V_1 = \frac{D_1}{D} = \frac{858}{188} = \mathbf{4.56\ V}$$

$$V_2 = V_{3\Omega} = \frac{D_2}{D} = \frac{207}{188} = \mathbf{1.1\ V}$$



Current of branches:

$$I_{2\Omega} = \frac{V_1}{2\Omega} = \frac{4.56\,\text{V}}{2\Omega} = \mathbf{2.28\ A}$$

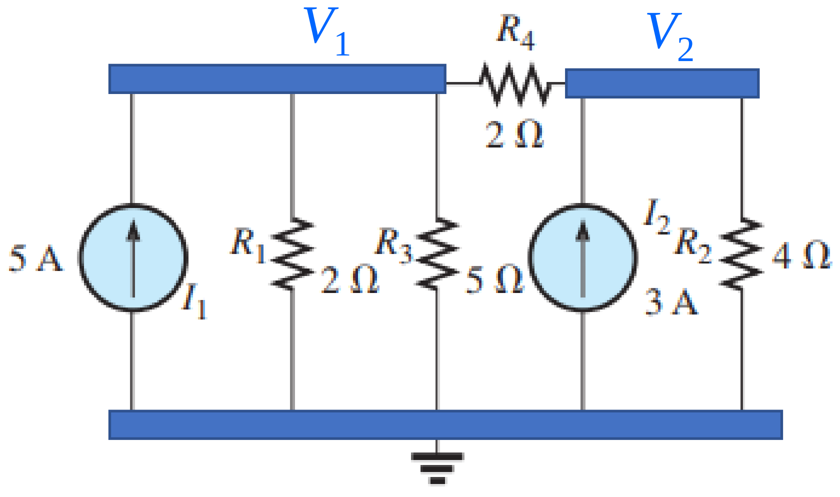
$$I_{4\Omega} = \frac{V_1}{4\Omega} = \frac{4.56\,\text{V}}{4\Omega} = \mathbf{1.14\ A}$$

$$I_{6\Omega} = \frac{V_1 - V_2}{6\Omega} = \frac{4.56\,\text{V} - 1.1\text{V}}{6\Omega} = \mathbf{0.577\ A}$$

$$I_{3\Omega} = \frac{V_2}{3\Omega} = \frac{1.1\text{V}}{3\Omega} = \mathbf{0.367\ A}$$

$$I_{10\Omega} = \frac{V_2}{10\Omega} = \frac{1.1\text{V}}{10\Omega} = \mathbf{0.11\ A}$$

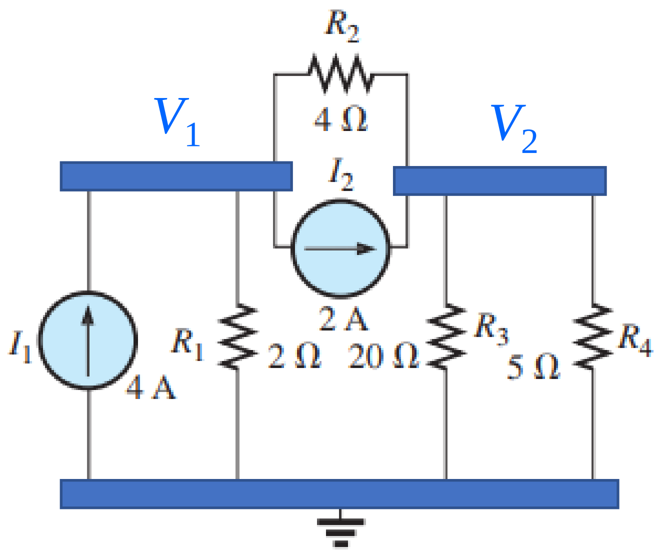
Write the nodal equations for the following networks.



(a)

$$\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{2}\right)V_1-\left(\frac{1}{2}\right)V_2=5 \qquad 12V_1-5V_2=50$$

$$\left(\frac{1}{2}+\frac{1}{4}\right)V_2-\left(\frac{1}{2}\right)V_1=3 \qquad 3V_2-2V_1=6$$

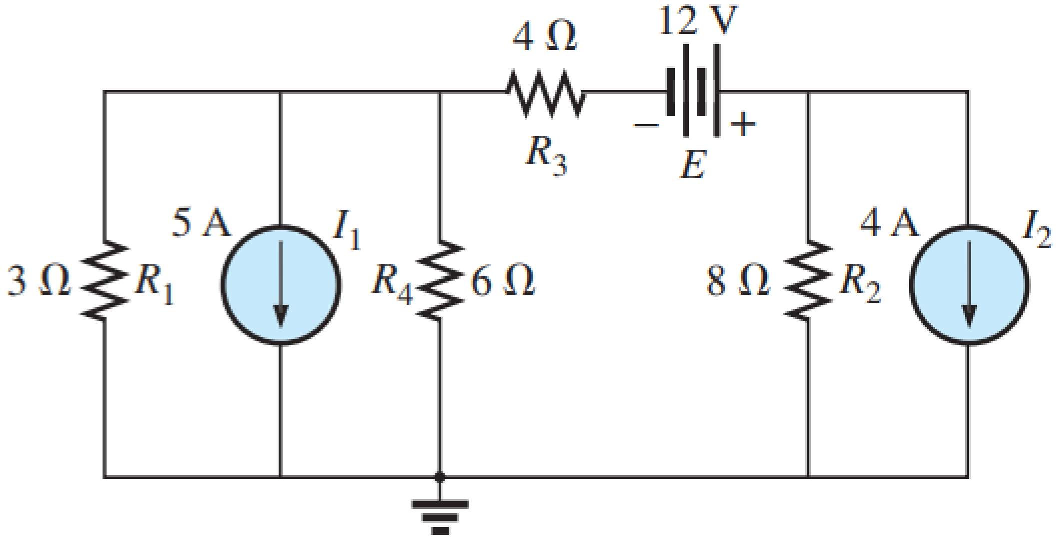


(b)

$$\left(\frac{1}{2}+\frac{1}{4}\right)V_1-\left(\frac{1}{4}\right)V_2=4-2 \qquad 3V_1-V_2=8$$

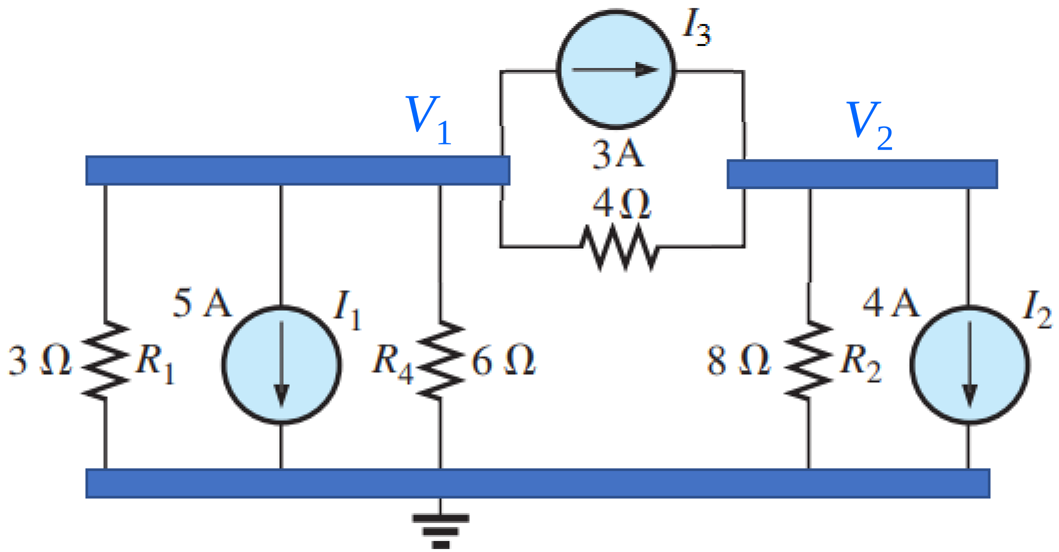
$$\left(\frac{1}{4}+\frac{1}{20}+\frac{1}{5}\right)V_2-\left(\frac{1}{4}\right)V_1=2 \qquad 10V_2-5V_1=40$$

Write the nodal equations for the following networks.

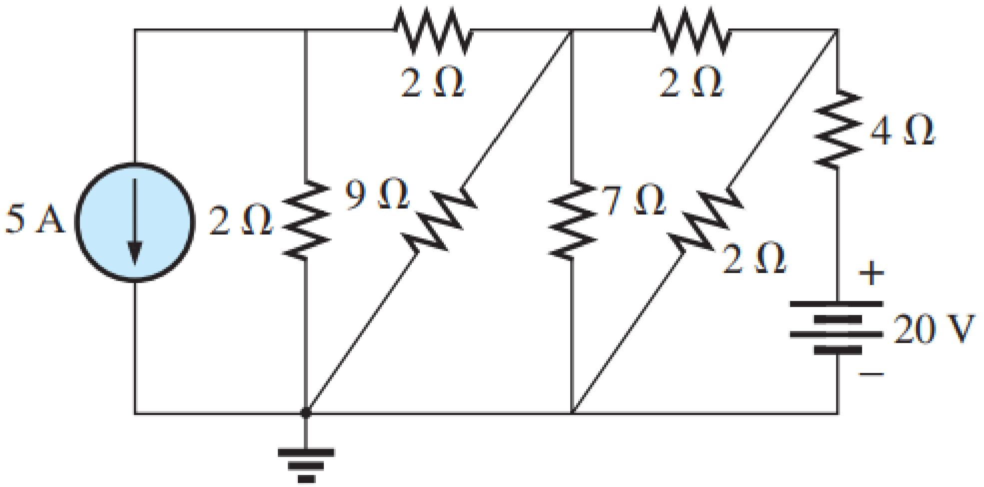


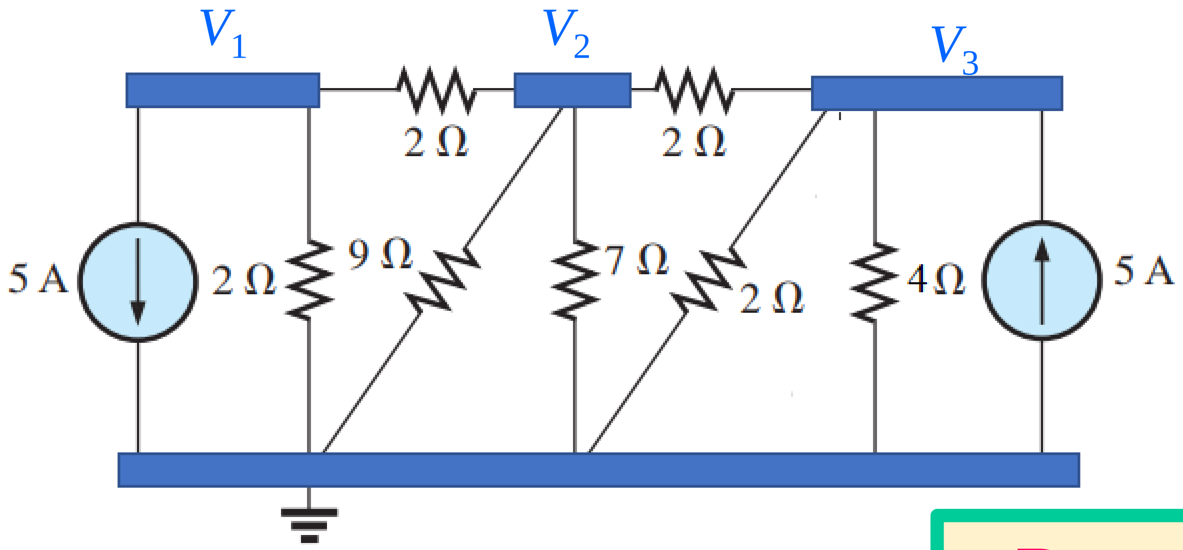
$$\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{4}\right)V_1-\left(\frac{1}{4}\right)V_2=-5-3 \qquad 9V_1-3V_2=-48$$

$$\left(\frac{1}{4}+\frac{1}{8}\right)V_2-\left(\frac{1}{4}\right)V_1=3-4 \qquad 3V_2-2V_1=-8$$



Write the nodal equations for the following networks.





$$\left(\frac{1}{2}+\frac{1}{2}\right)V_1-\left(\frac{1}{2}\right)V_2=-5 \qquad 2V_1-V_2=-10$$

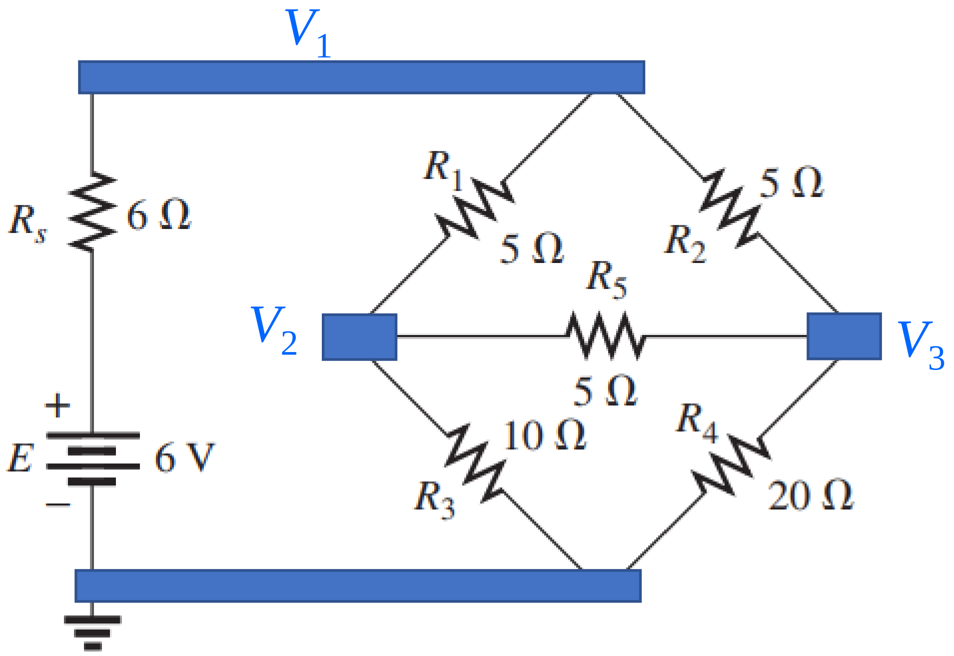
$$\left(\frac{1}{2}+\frac{1}{9}+\frac{1}{7}+\frac{1}{2}\right)V_2-\left(\frac{1}{2}\right)V_1-\left(\frac{1}{2}\right)V_3=0$$

$$158V_2-63V_1-63V_3=0$$

$$\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{4}\right)V_3-\left(\frac{1}{2}\right)V_2=5 \qquad 5V_2-2V_2=20$$

Practice Book [Ch 8] Problem: 41 ~ 44

Write the nodal equations for the following networks.



$$I_E = \frac{E}{R_S} = \frac{6\text{ V}}{6\ \Omega} = 1\text{ A}$$

Node V_1 : $$\left(\frac{1}{R_S}+\frac{1}{R_1}+\frac{1}{R_2}\right)V_1-\left(\frac{1}{R_1}\right)V_2-\left(\frac{1}{R_2}\right)V_3=I_E$$

Node V_2 : $$\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right)V_2-\left(\frac{1}{R_1}\right)V_1-\left(\frac{1}{R_5}\right)V_3=0$$

Node V_3 : $$\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)V_3-\left(\frac{1}{R_2}\right)V_1-\left(\frac{1}{R_5}\right)V_2=0$$