Review on the Last Class

Instantaneous value: $e(t)$, $v(t)$, $i(t)$, $p(t)$ etc.

Peak or Crest value: E_m, V_m, I_m

Peak-to-peak value: $E_{p\text{-}p}$, $V_{p\text{-}p}$, $I_{p\text{-}p}$

Period: T [s]

Frequency: f [Hz]

Angular Frequency: ω [rad/s]

Phase angle: [θ_e, θ_v and θ_i]

Initial Angle: [θ_{e0}, θ_{v0} and θ_{i0}]

Angle Difference: Voltage Angle – Current Angle

Phase Difference: | Angle Difference |

Phase Relation: [in phase, leading, lagging]

$$\boxed{f = \frac{1}{T}\ \text{Hz}}$$

$$\boxed{\omega = \frac{\alpha}{t}\ \text{rad/s}}$$

$$\boxed{\omega = \frac{2\pi}{T}\ \text{rad/s} = 2\pi f\ \text{rad/s}}$$

The instantaneous or time domain equation

$$e(t) = E_m \sin(\alpha + \theta_e)\ \text{V} = E_m \sin(\omega t + \theta_e)\ \text{V}$$

$$v(t) = V_m \sin(\alpha + \theta_v)\ \text{V} = V_m \sin(\omega t + \theta_v)\ \text{V}$$

$$i(t) = I_m \sin(\alpha + \theta_i)\ \text{A} = I_m \sin(\omega t + \theta_i)\ \text{A}$$

Initial Angle = – Phase Angle

Angle Difference= Voltage angle – Current Angle = 0^o
Phase Relation: $v(t)$ and $i(t)$ are **in phse**.

Angle Difference= Voltage angle – Current Angle > 0^o
Phase Relation: $v(t)$ **leads** $i(t)$, or $i(t)$ **lags** $v(t)$

Angle Difference= Voltage angle – Current Angle < 0^o
Phase Relation: $v(t)$ **lags** $i(t)$, or $i(t)$ **leads** $v(t)$

$$\cos\alpha = \sin(\alpha + 90^o)$$

$$-\cos\alpha = \sin(\alpha - 90^o)$$

$$-\sin\alpha = \sin(\alpha \pm 180^o)$$

Problem 01: The supply voltage of an electrical load is $e(t) = 100\sin(120\pi t+100^o)$ V.

(***i***) Identify the peak value, the angular frequency and the phase angle.

(***ii***) Calculate the peak-to-peak value, the frequency and the time period.

Problem 02: The supply voltage and current of an electrical load are: $e(t) = 100\sin(157t+50^o)$ V and

$i(t) = -10\cos(157t + 120^o)$ A

Find the phase difference and phase relation between $e(t)$ and $i(t)$.

Average or Mean Value
Effective or Root Mean Square (rms) Value

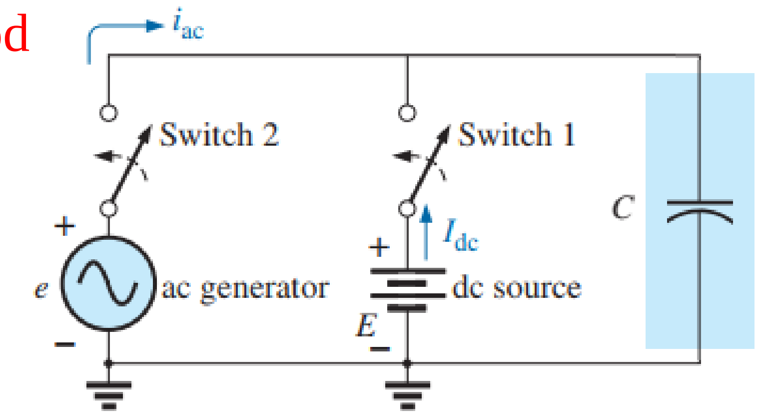
Average Value or Mean Value

Average Value: The **average value** of an alternating current is expressed by that dc current which transfers across any circuit the same ***charge as*** is transferred by that alternating current during the same time.

In case of symmetrical waveform, the average value over a full cycle is zero. Thus, **the average value is calculated over half-cycle for symmetrical waveform**. But **for asymmetrical waveform, the average value is calculated over a full cycle**.

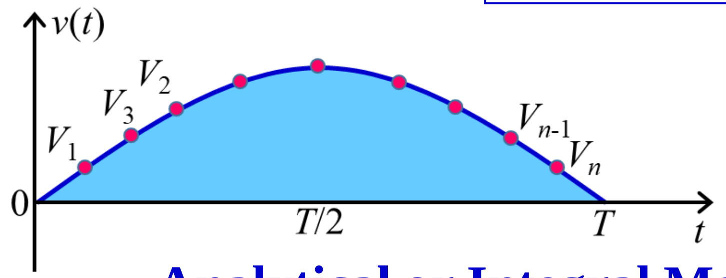
Average value can be calculated by the following methods:

- Graphical Method
- Analytical or Integral Method



Graphical Method:

$$V_{ave} = \frac{V_1 + V_2 + V_3 + \ldots + V_n}{n}$$

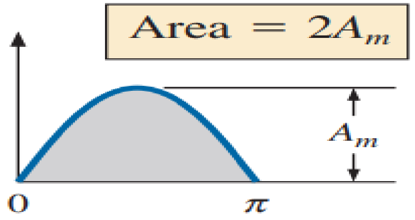


Analytical or Integral Method:

For asymmetrical wave:

$$\text{Average Value} = \frac{\text{Area under the curve in one cycle}}{\text{Duration of one cycle}}$$

$$I_{ave} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{2\pi}\int_0^{2\pi} i(\theta)d\theta$$



For symmetrical wave:

$$\text{Average Value} = \frac{\text{Area under the curve in half-cycle}}{\text{Duration of half-cycle}}$$

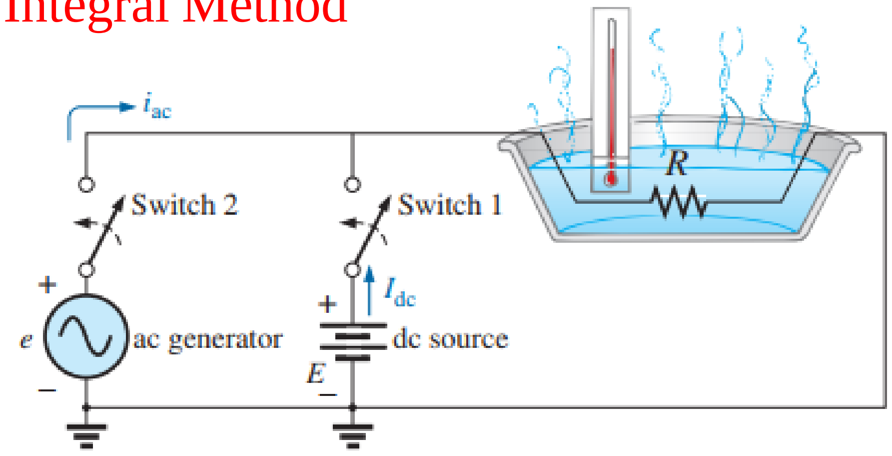
$$I_{ave} = \frac{1}{T/2}\int_0^{T/2} i(t)dt = \frac{1}{\pi}\int_0^{\pi} i(\theta)d\theta$$

Root Mean Square (RMS) or Effective Value

RMS or Effective Value: The **effective or RMS value** of an alternating current is given by that dc current which, when flowing through a given circuit for a given time, produces the same amount of **heat** as produced by the alternating current, when flowing through the same circuit for the same time.

RMS value can be calculated by the following methods:

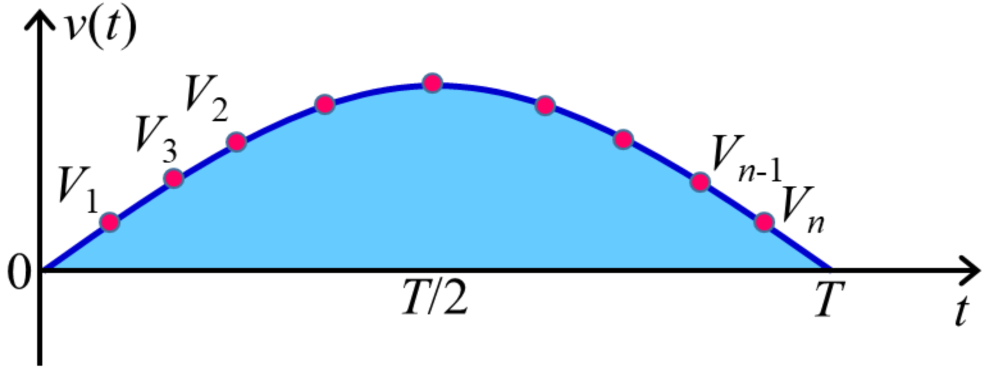
- Graphical Method
- Analytical or Integral Method



$$P_{dc} = I_{dc}^2 R = \frac{V_{dc}^2}{R}$$

$$P_{ac} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

$$P_{ac} = P_{dc}$$



Graphical Method:

$$V = V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + \ldots + V_n^2}{n}}$$

Analytical or Integral Method:

$$I_{\text{rms}} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}} \quad (13.31)$$

$$I_{\text{rms}} = \sqrt{\frac{\text{area }(i^2(t))}{T}} \quad (13.32)$$

Average Value and RMS Value for Sinewave

$$I_{ave} = \frac{\pi}{2} I_m = 0.637 I_m$$

$$E_{ave} = \frac{\pi}{2} E_m = 0.637 E_m$$

$$V_{ave} = \frac{\pi}{2} V_m = 0.637 V_m$$

$$I = I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$E = E_{rms} = \frac{E_m}{\sqrt{2}} = 0.707 E_m$$

$$V = V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

$$I_m = \frac{2}{\pi} I_{ave} = \frac{I_{ave}}{0.637}$$

$$E_m = \frac{2}{\pi} E_{ave} = \frac{E_{ave}}{0.637}$$

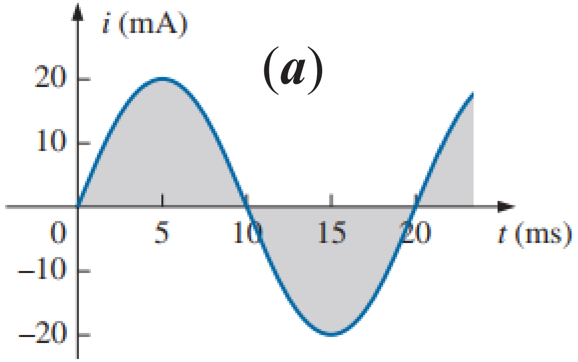
$$V_m = \frac{2}{\pi} V_{ave} = \frac{V_{ave}}{0.637}$$

$$I_m = \sqrt{2} I_{rms} = \frac{I_{rms}}{0.707}$$

$$E_m = \sqrt{2} E_{rms} = \frac{E_{rms}}{0.707}$$

$$V_m = \sqrt{2} V_{rms} = \frac{V_{rms}}{0.707}$$

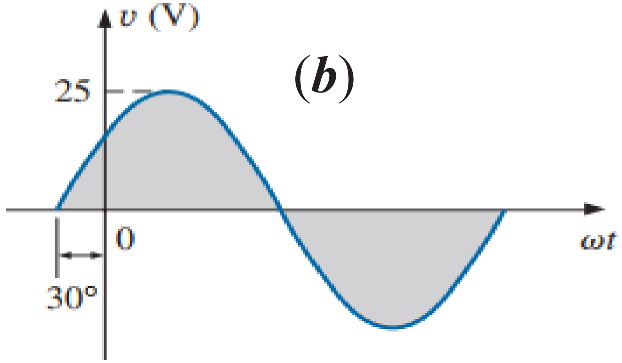
EXAMPLE 13.20 Find the average value and rms values for the following sinusoidal waveforms.



Here, $I_m = 20$ mA

$$I_{ave} = 0.637 \times 20\text{mA} = \mathbf{12.74\ mA}$$

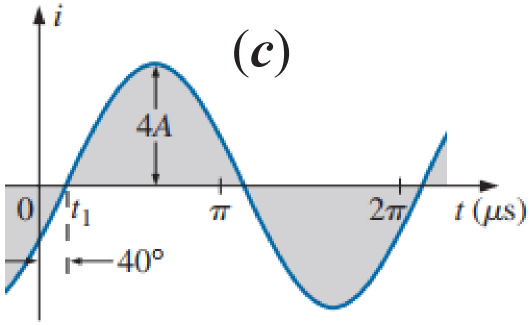
$$I = I_{rms} = 0.707 \times 20\text{mA} = \mathbf{14.14\ mA}$$



Here, $V_m = 25$ V

$$V_{ave} = 0.637 \times 25\text{V} = \mathbf{15.93\ V}$$

$$V = V_{rms} = 0.707 \times 25\text{V} = \mathbf{17.68\ V}$$



Here, $I_m = 4$ A

$$I_{ave} = 0.637 \times 4\text{A} = \mathbf{2.55\ A}$$

$$I = I_{rms} = 0.707 \times 4\text{A} = \mathbf{2.83\ A}$$

EXAMPLE 13.21 The 120 V dc source in Fig. 13.59(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. 13.59(b)] is to deliver the same power to the load.

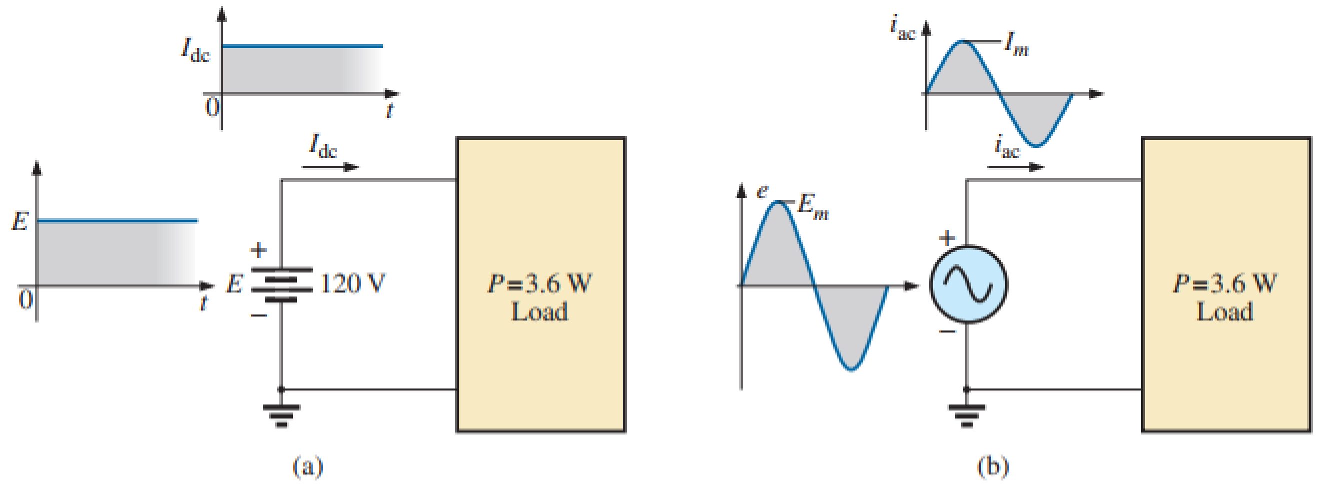


FIG. 13.59 *Example 13.21.*

Solution: $P_{dc} = V_{dc} I_{dc}$

and $$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

$$I_m = \sqrt{2} I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\text{ mA}}$$

$$E_m = \sqrt{2} E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\text{ V}}$$

EXAMPLE 13.20.1 Find the average value and rms values for the following sinusoidal waveforms:

(*a*) $i(t) = 10\sin(\omega t + 30°)$ A
(*b*) $v(t) = 150\cos(\omega t - 60°)$ A
(*c*) $i(t) = -12\cos(\omega t + 80°)$ μA
(*d*) $v(t) = -200\sin(\omega t - 120°)$ mV

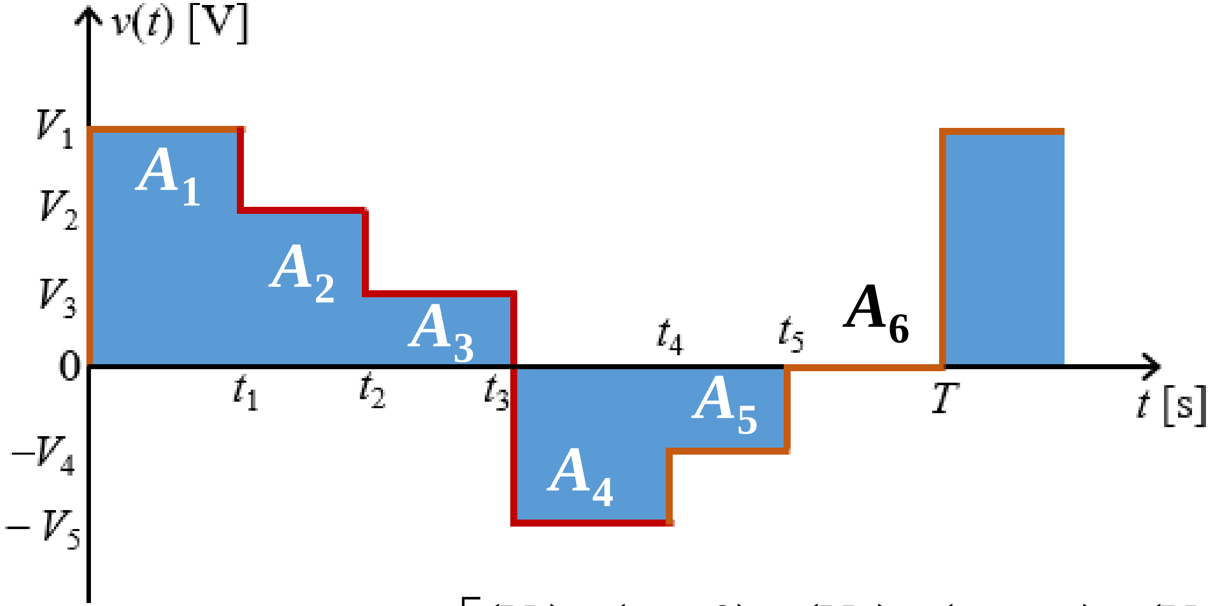
$$(a)\ I_{ave} = 0.637 \times 10\text{A} = \mathbf{6.37\ A}$$
$$I = I_{rms} = 0.707 \times 10\text{A} = \mathbf{7.07\ A}$$

$$(b)\ V_{ave} = 0.637 \times 150\text{V} = \mathbf{95.55\ V}$$
$$V = V_{rms} = 0.707 \times 150\text{V} = \mathbf{106.05\ V}$$

$$(c)\ I_{ave} = 0.637 \times 12\mu\text{A} = \mathbf{7.64\ \mu A}$$
$$I = I_{rms} = 0.707 \times 12\mu\text{A} = \mathbf{8.48\ \mu A}$$

$$(d)\ V_{ave} = 0.637 \times 200\text{mV} = \mathbf{127.4\ mV}$$
$$V = V_{rms} = 0.707 \times 200\text{mV} = \mathbf{141.1\ mV}$$

Average Value and RMS Value for Rectangular Waveform



$$A_1 = (V_1) \times (t_1 - 0) \qquad A_2 = (V_2) \times (t_2 - t_1)$$

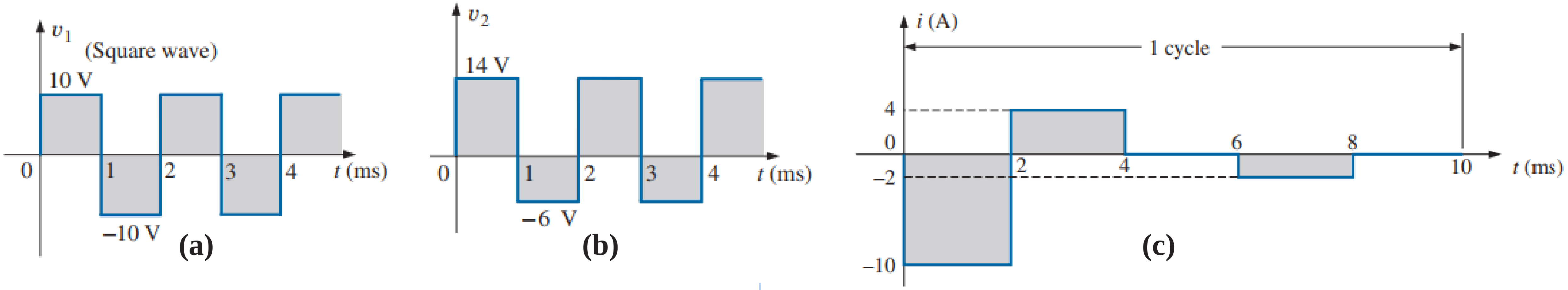
$$A_3 = (V_3) \times (t_3 - t_2) \qquad A_4 = (-V_5) \times (t_4 - t_3)$$

$$A_5 = (-V_4) \times (t_5 - t_4) \qquad A_6 = (0) \times (T - t_5)$$

$$V_{ave} = \frac{\begin{bmatrix} (V_1) \times (t_1 - 0) + (V_2) \times (t_2 - t_1) + (V_3) \times (t_3 - t_2) + (-V_5) \times (t_4 - t_3) \\ + (-V_4) \times (t_5 - t_4) + (0) \times (T - t_5) \end{bmatrix}}{T}$$

$$V_{rms} = \sqrt{\frac{\begin{bmatrix} (V_1)^2 \times (t_1 - 0) + (V_2)^2 \times (t_2 - t_1) + (V_3)^2 \times (t_3 - t_2) + (-V_5)^2 \times (t_4 - t_3) \\ + (-V_4)^2 \times (t_5 - t_4) + (0)^2 \times (T - t_5) \end{bmatrix}}{T}}$$

EXAMPLE 13.14 Determine the average value, the rms value for the following waveforms.



(a) (b) (c)

(a) $V_{ave} = \dfrac{(10)\times(1-0)+(-10)(2-1)}{2} = 0$

$$V_{rms} = \sqrt{\frac{(10)^2\times(1-0)+(-10)^2(2-1)}{2}} = \mathbf{10\ V}$$

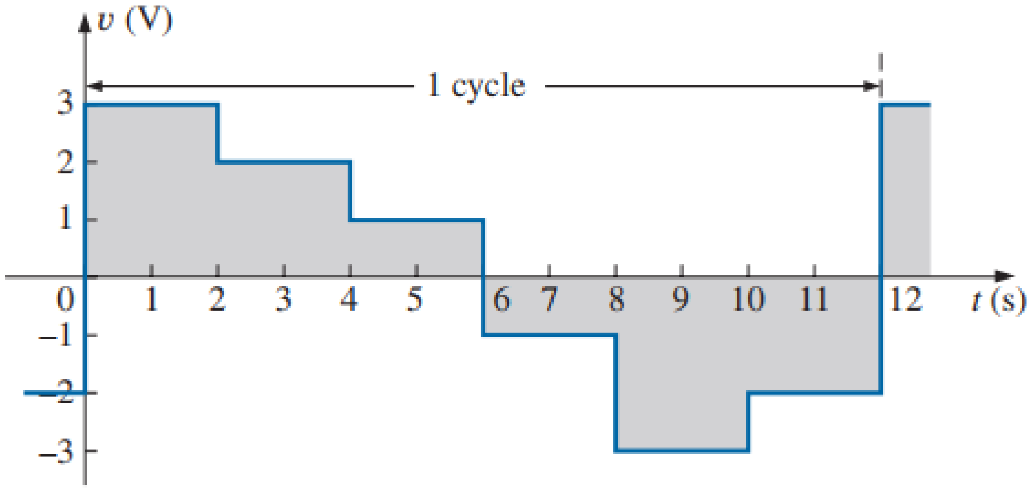
(b) $V_{ave} = \dfrac{(14)\times(1-0)+(-6)(2-1)}{2} = \mathbf{4\,V}$

$$V_{rms} = \sqrt{\frac{(14)^2\times(1-0)+(-6)^2(2-1)}{2}} = \mathbf{10.77\ V}$$

(c) $I_{ave} = \dfrac{(-10)\times 2+(4)\times 2+(-2)\times 2}{10} = \mathbf{-1.6\,A}$

$$I_{rms} = \sqrt{\frac{(-10)^2\times 2+(4)^2\times 2+(-2)^2\times 2}{10}} = \mathbf{4.9\,A}$$

EXAMPLE 13.14.1 Determine the average value, the rms value for the following waveforms. Also, determine the average power consumption if the voltage applied across 10 ohm resistance.



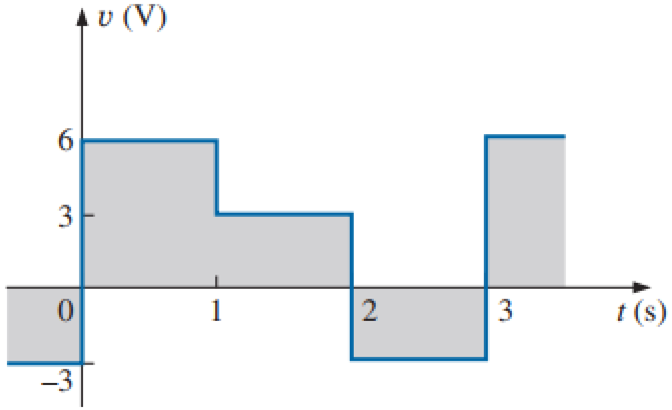
$$V_{ave} = \frac{(3)\times 2+(2)\times 2+(1)\times 2+(-1)\times 2+(-3)\times 2+(-2)\times 2}{12}$$

$$= \mathbf{0\,V}$$

$$V_{rms} = \sqrt{\frac{(3)^2\times 2+(2)^2\times 2+(1)^2\times 2+(-1)^2\times 2+(-3)^2\times 2+(-2)^2\times 2}{12}}$$

$$= \mathbf{2.16\,V}$$

$$P_{ave} = \frac{V_{rms}^2}{R} = \frac{(2.16)^2}{10} = \mathbf{0.466\,W}$$



$$V_{ave} = \frac{(6)\times 1+(3)\times 1+(-3)\times 1}{3} = \mathbf{2\,V}$$

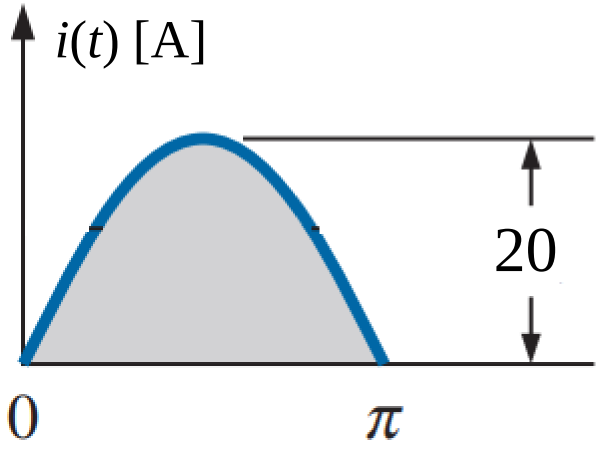
$$V_{rms} = \sqrt{\frac{(6)^2\times 1+(3)^2\times 1+(-3)^2\times 1}{3}}$$

$$= \mathbf{4.24\,V}$$

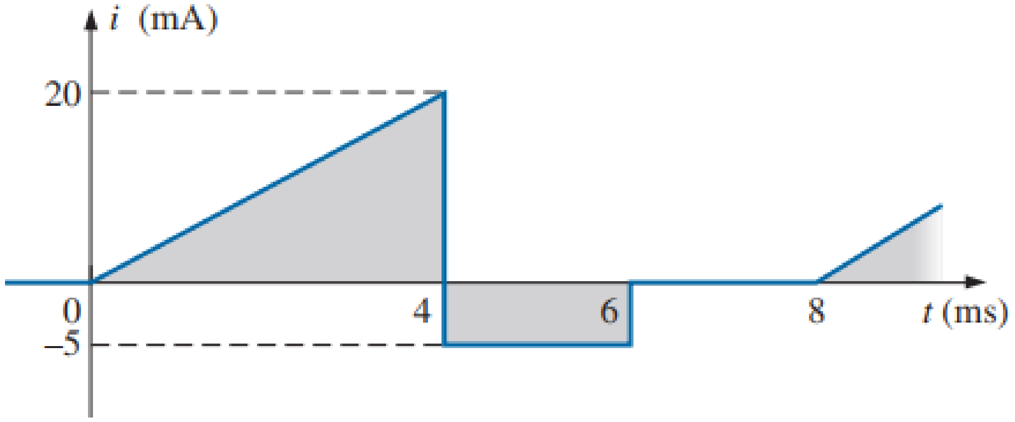
$$P_{ave} = \frac{V_{rms}^2}{R} = \frac{(4.24)^2}{10} = \mathbf{1.78\,W}$$

EXAMPLE 13.14.1 Determine the average value for the following waveforms.

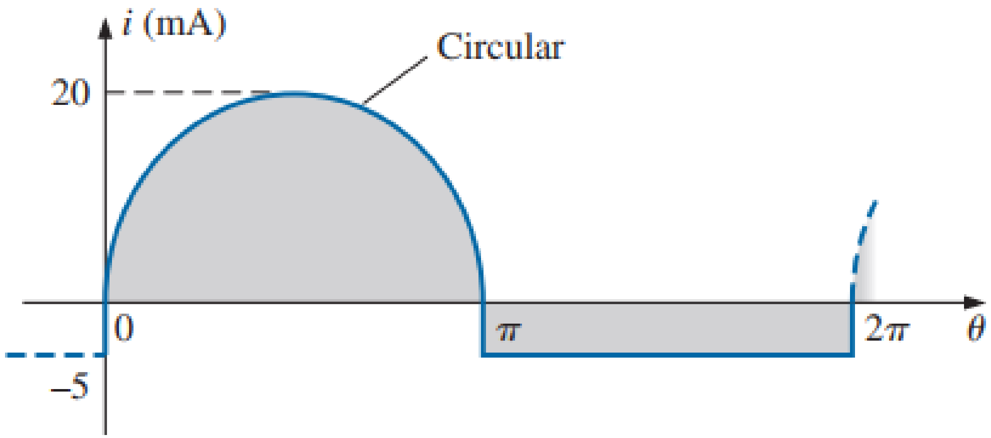
We know that the area of half-cycle of sine wave = 2 × Peak Value



$$V_{ave} = \frac{Area}{Duration} = \frac{2\times 20}{\pi} = \mathbf{12.74\,A}$$



$$V_{ave} = \frac{(1/2)\times 20\times 4 + (-5)\times 2}{8} = \mathbf{3.87\,mA}$$



Practice Problem 37 ~ 46 [Ch. 13]

$$V_{ave} = \frac{2\times 20 + (-5)\times \pi}{2\pi} = \mathbf{3.87\,mA}$$

Application of Complex Number in AC Circuit

Vector Quantities Represent by Complex Number :

1. Magnitude **2.** Direction

Phasor Quantities Represent by Complex Number:

1. Magnitude (RMS value for voltage and current)
2. Direction (Phase angle)
3. Continuously change with respect to time [such as sine and cosine waves)

Complex Number can be represented by three different ways:

1. Polar or Phasor form
2. Cartesian or Rectangular form
3. Exponential form

14.7 RECTANGULAR FORM:

$$\boxed{\boldsymbol{C} = X + jY \qquad (14.17)}$$

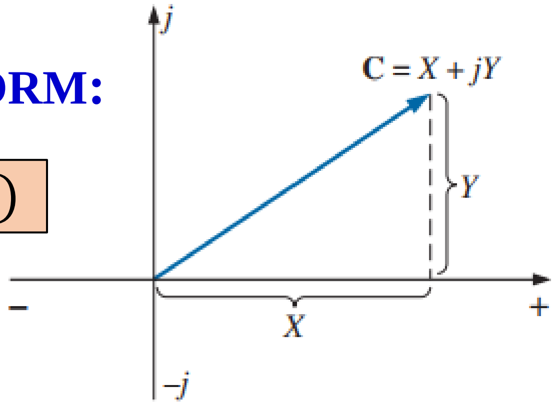
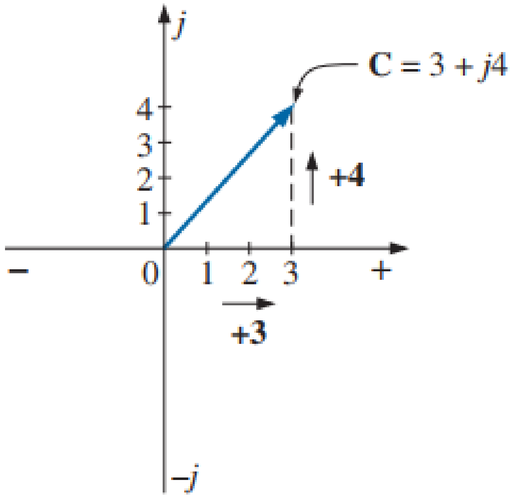
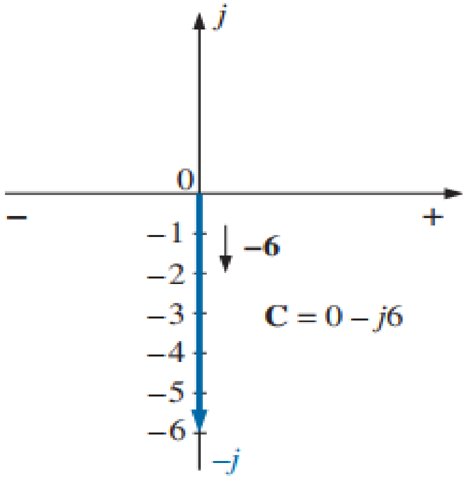


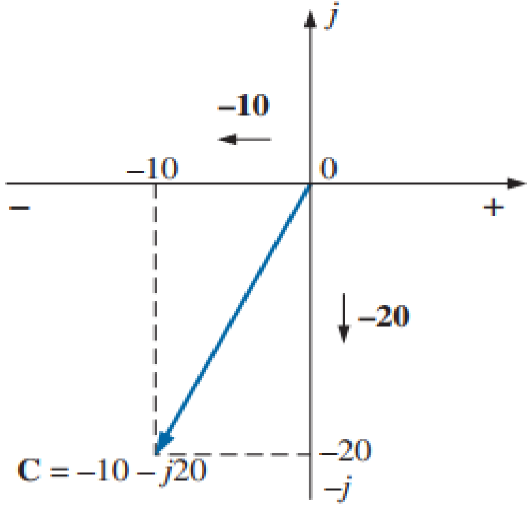
FIG. 14.39 *Defining the rectangular form.*

EXAMPLE 14.13 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 3 + j4$ b. $\mathbf{C} = 0 - j6$ c. $\mathbf{C} = -10 - j20$







$$\boxed{j = \sqrt{-1}} \qquad (14.24)$$

$$\boxed{j^2 = -1} \qquad (14.25)$$

$$\boxed{\frac{1}{j} = -j} \qquad (14.26)$$

14.8 POLAR OR PHASOR FORM:

$$\boxed{C = Z\angle\theta \qquad (\mathbf{14.18})}$$

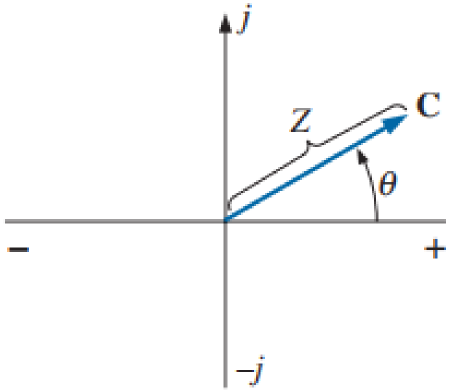


FIG. 14.43 *Defining the polar form.*

$$\boxed{C = -Z\angle\theta = -Z\angle\theta \pm 180^\circ \; (\mathbf{14.19})}$$

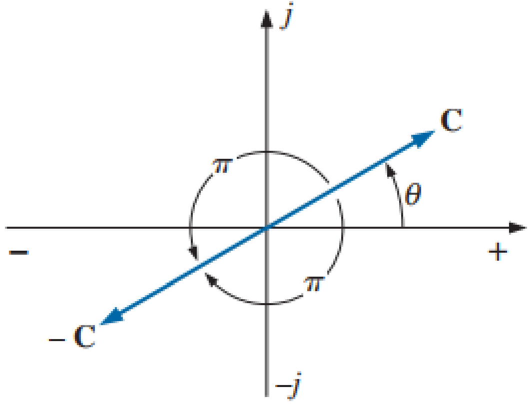
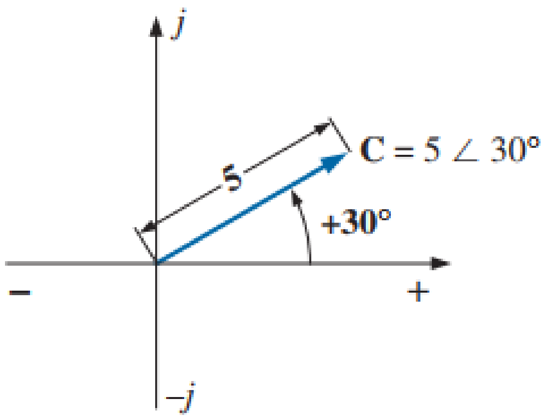
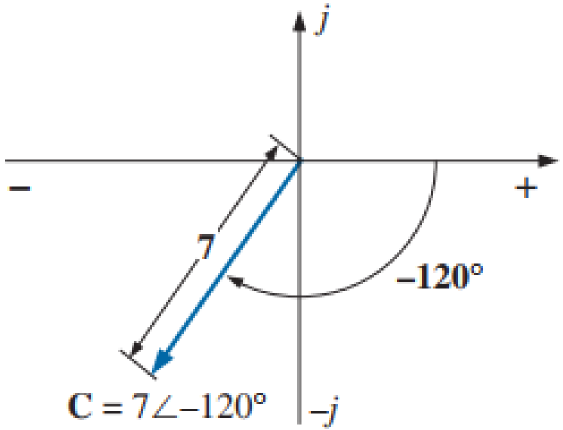


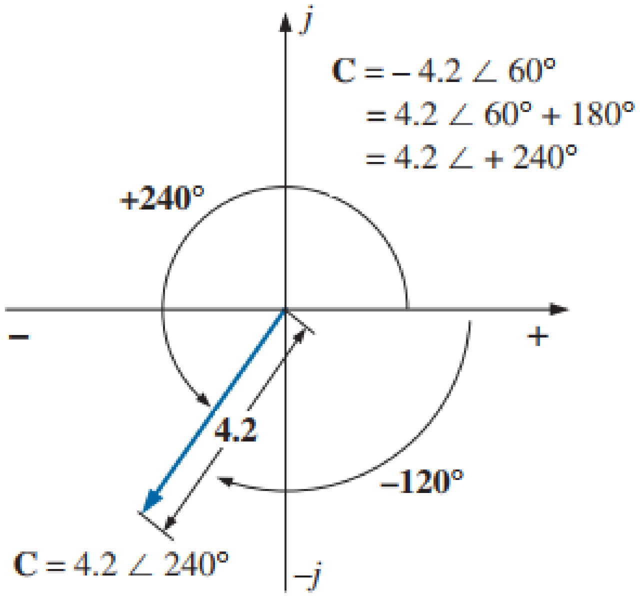
FIG. 14.44
Demonstrating the effect of a negative sign on the polar form.

EXAMPLE 14.14 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 5\angle 30^\circ$ b. $\mathbf{C} = 7\angle -120^\circ$ c. $\mathbf{C} = -4.2\angle 60^\circ$







14.9 CONVERSION BETWEEN FORMS

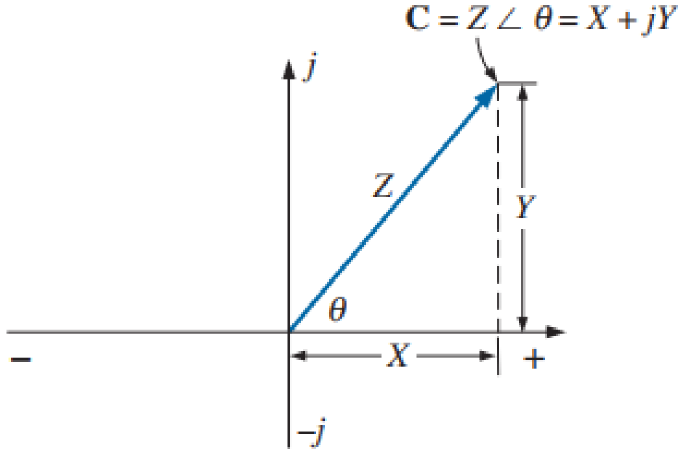


FIG. 14.48 *Conversion between forms.*

Rectangular to Polar

$$\boxed{Z = \sqrt{X^2 + Y^2}} \qquad \textbf{(14.20)}$$

$$\boxed{\theta = \tan^{-1}\frac{Y}{X}} \qquad \textbf{(14.21)}$$

Polar to Rectangular

$$\boxed{X = Z\cos\theta} \qquad \textbf{(14.22)}$$

$$\boxed{Y = Z\sin\theta} \qquad \textbf{(14.23)}$$

EXAMPLE 14.15 Convert the following from rectangular to polar form:

$$\mathbf{C} = 3 + j4 \qquad \text{(Fig. 14.49)}$$

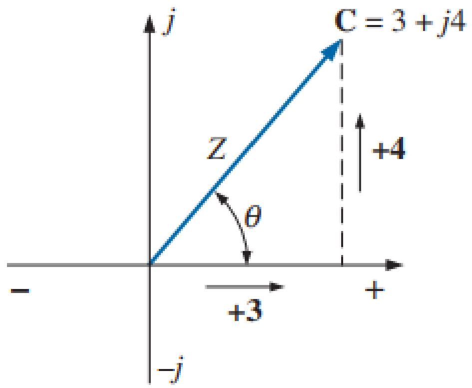


FIG. 14.49 *Example 14.15.*

Solution: $Z = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{25} = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

and $\quad \mathbf{C = 5\angle 53.13°}$

EXAMPLE 14.16 Convert the following from polar to rectangular form:

$$\mathbf{C} = 10\angle 45° \qquad \text{(Fig. 14.50)}$$

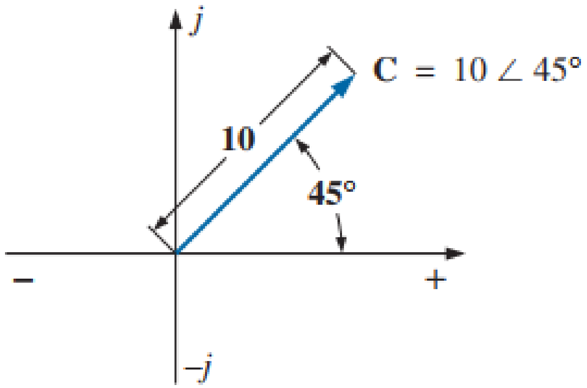


FIG. 14.50 *Example 14.16.*

Solution: $X = 10\cos 45°$

$= (10)(0.707)$

$= 7.07$

$Y = 10\sin 45°$

$= (10)(0.707)$

$= 7.07$

and $\quad \mathbf{C = 7.07 + j7.07}$

14.10 MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

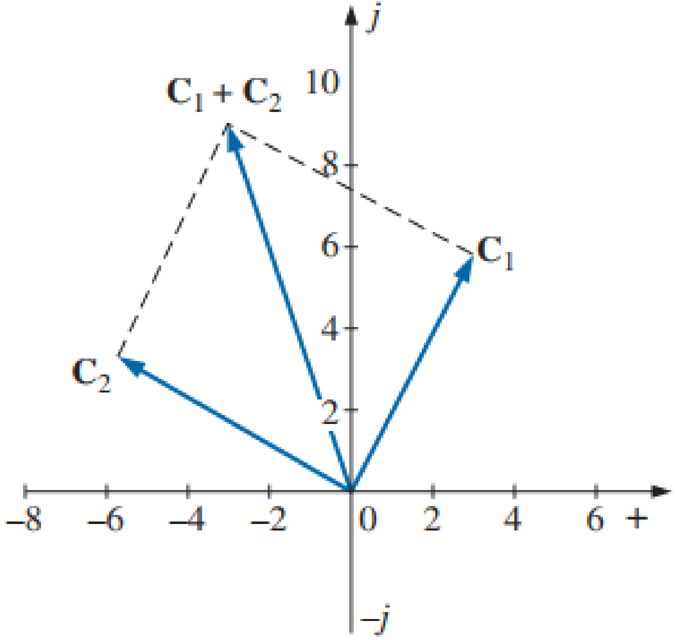
Addition

$\mathbf{C}_1 = \pm X_1 \pm jY_1$ and $\mathbf{C}_2 = \pm X_2 \pm jY_2$

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)} \qquad (14.27)$$

EXAMPLE 14.19 Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

Solutions: $\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$



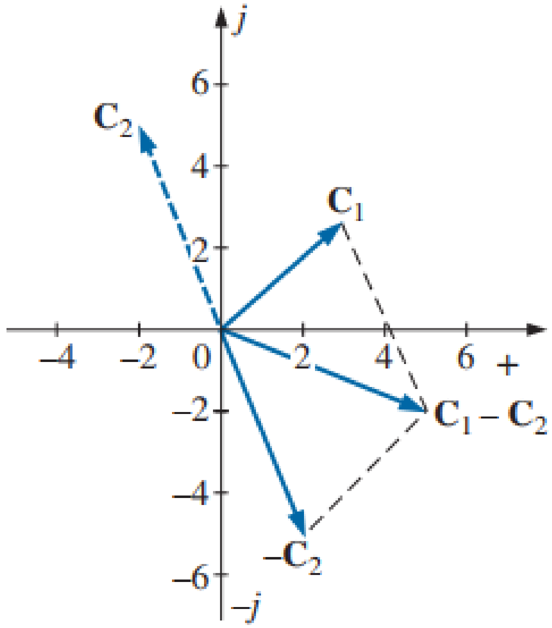
Subtraction

$\mathbf{C}_1 = \pm X_1 \pm jY_1$ and $\mathbf{C}_2 = \pm X_2 \pm jY_2$

$$\boxed{\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_1 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]} \qquad (14.28)$$

EXAMPLE 14.20 Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.

Solutions: $\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$



Multiplication

$\mathbf{C}_1 = X_1 + jY_1$ and $\mathbf{C}_2 = X_2 + jY_2$

then $\mathbf{C}_1 \cdot \mathbf{C}_2$:

$$\begin{array}{r} X_1 + jY_1 \\ \underline{X_2 + jY_2} \\ X_1X_2 + jY_1X_2 \\ \underline{+ jX_1Y_2 + j^2Y_1Y_2} \\ X_1X_2 + j(Y_1X_2 + X_1Y_2) + Y_1Y_2(-1) \end{array}$$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)} \qquad \textbf{(14.29)}$$

Division

$\mathbf{C}_1 = X_1 + jY_1$ and $\mathbf{C}_2 = X_2 + jY_2$

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{(X_1 + jY_1)(X_2 - jY_2)}{(X_2 + jY_2)(X_2 - jY_2)}$$

$$= \frac{(X_1X_2 + Y_1Y_2) + j(X_2Y_1 - X_1Y_2)}{X_2^2 + Y_2^2}$$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}} \qquad \textbf{(14.31)}$$

Multiplication in Polar Form:

$\mathbf{C}_1 = Z_1 \angle\theta_1$ and $\mathbf{C}_2 = Z_2 \angle\theta_2$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1Z_2 \angle\theta_1 + \theta_2} \qquad \textbf{(14.30)}$$

Division in Polar Form:

$\mathbf{C}_1 = Z_1 \angle\theta_1$ and $\mathbf{C}_2 = Z_2 \angle\theta_2$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2} \angle\theta_1 - \theta_2} \qquad \textbf{(14.32)}$$

EXAMPLE 14.23

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = 5 \angle 20°$ and $\mathbf{C}_2 = 10 \angle 30°$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = 2 \angle -40°$ and $\mathbf{C}_2 = 7 \angle +120°$

Solutions:

a. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (5 \angle 20°)(10 \angle 30°) = (5)(10) \underline{/20° + 30°} = \mathbf{50 \angle 50°}$

b. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (2 \angle -40°)(7 \angle +120°) = (2)(7) \underline{/-40° + 120°}$
$= \mathbf{14 \angle +80°}$

EXAMPLE 14.25

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15 \angle 10°$ and $\mathbf{C}_2 = 2 \angle 7°$.

b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8 \angle 120°$ and $\mathbf{C}_2 = 16 \angle -50°$.

Practice Problem 39 ~ 49 [Ch. 14]

Solutions:

a. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{15 \angle 10°}{2 \angle 7°} = \frac{15}{2} \underline{/10° - 7°} = \mathbf{7.5 \angle 3°}$$

b. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{8 \angle 120°}{16 \angle -50°} = \frac{8}{16} \underline{/120° - (-50°)} = \mathbf{0.5 \angle 170°}$$

APPLICATION OF COMPLEX NUMBERS IN AC CIRCUIT

Instantaneous Form (Time Domain) Equation:

$$e(t) = E_m sin(\omega t + \theta_e)\ \text{V}$$
$$v(t) = V_m sin(\omega t + \theta_v)\ \text{V}$$
$$i(t) = I_m sin(\omega t + \theta_i)\ \text{A}$$

Phasor Form (Polar Form) Equation:

$$\boldsymbol{E} = \vec{E} = E_{rms}\angle\theta_e = E\angle\theta_e\ \text{V}$$
$$\boldsymbol{V} = \vec{V} = V_{rms}\angle\theta_v = V\angle\theta_v\ \text{V}$$
$$\boldsymbol{I} = \vec{I} = I_{rms}\angle\theta_i = I\angle\theta_i\ \text{A}$$

Rectangular Form (Cartesian Form) Equation:

$$\boldsymbol{E} = \vec{E} = E_r + jE_i\ \ \text{V}$$
$$\boldsymbol{V} = \vec{V} = V_r + jV_i\ \ \text{V}$$
$$\boldsymbol{I} = \vec{I} = I_r + jI_i\ \ \text{A}$$

EXAMPLE 14.27 Convert the following from the time to (***i***) the phasor domain, and (***ii***) the rectangular domain.

Time Domain	Phasor Domain	Rectangular Domain
(***a***) $v(t) = 70.7\sin(\omega t - 60^o)$ V	$\vec{V} = (0.707\times70.7)\text{V}\angle{-60^o} = \mathbf{50V\angle{-60^o}}$	$\boldsymbol{V} = 25 - j43.3$ V
(***b***) $i(t) = 21.21\cos(\omega t + 20^o)$ A $= 21.21\sin(\omega t + 110^o)$ A	$\vec{I} = (0.707\times21.21)\text{A}\angle110^o = \mathbf{15A\angle110^o}$	$\boldsymbol{I} = 15[\cos(110^o)+j\sin(110^o)]$ $= -5.13 + j14.1$ A
(***c***) $e(t) = -\ 200\cos\omega t$ V $= 200\sin(\omega t - 90^o)$ V	$\vec{E} = (0.707\times200)\text{V}\angle{-90^o} = \mathbf{141.42V\angle{-90^o}}$	$\boldsymbol{E} = 141.42[\cos(-90^o)+j\sin(-90^o)]$ $= 0 - j141.42$ V
(***d***) $i(t) = -4.5\sin(\omega t + 30^o)$ A $= 4.5\sin(\omega t - 150^o)$ A $= 4.5\sin(\omega t + 210^o)$ A	$\vec{I} = (0.707\times4.5)\text{A}\angle{-150^o} = \mathbf{3.18A\angle{-150^o}}$ $\vec{I} = (0.707\times4.5)\text{A}\angle210^o = \mathbf{3.18A\angle210^o}$	$\boldsymbol{I} = 3.18[\cos(210^o)+j\sin(210^o)]$ $= -2.75 - j1.59$ A

EXAMPLE 14.27.1 Convert the following from Cartesian form to (***i***) the phasor domain, and (***ii***) the instantaneous form for 50 Hz.

Rectangular Form	Phasor Form	Instantaneous Form
(***a***) $\vec{V} = 25 - j43.3$ V RMS value: 50 V Phase Angle: –60º Peak Value: 70.7 V	$V = \sqrt{25^2 + (-43.3)^2} = 50$ V $\theta_v = \tan^{-1}\left[\frac{-43.3}{25}\right] = -60^o$ $\boldsymbol{V} = \mathbf{50V\angle{-60^o}}$	$\omega = 2\pi\times 50 = 314$ rad/s $v(t) = (\sqrt{2})\times 50\sin(314t - 60^o)$ V $= \mathbf{70.7\sin(314\mathit{t} - 60^o)\ V}$
(***b***) $\vec{E} = j150$ V RMS value: 150 V Phase Angle: 90º Peak Value: 212.13 V	$E = \sqrt{0^2 + 150^2} = 150$ V $\theta_e = \tan^{-1}\left[\frac{150}{0}\right] = 90^o$ $\boldsymbol{E} = \mathbf{150V\angle 90^o}$	$e(t) = (\sqrt{2})\times 150\sin(314t + 90^o)$ V $= \mathbf{212.13\sin(314\mathit{t} + 90^o)\ V}$ $= \mathbf{212.13\cos 314\mathit{t}\ V}$
(***d***) $\vec{I} = -j5$ A RMS value: 5 A Phase Angle: –90º Peak Value: 7.07 A	$I = \sqrt{0^2 + (-5)^2} = 5$ A $\theta_i = \tan^{-1}\left[\frac{-5}{0}\right] = -90^o$ $\boldsymbol{I} = \mathbf{5A\angle{-90^o}}$	$i(t) = (\sqrt{2})\times 5\sin(314t - 90^o)$ A $= \mathbf{7.07\sin(314\mathit{t} - 90^o)\ A}$ $= \mathbf{-\,7.07\cos 314\mathit{t}\ A}$
(***e***) $\vec{V} = -100$ V RMS value: 100 V	$\boldsymbol{V} = \mathbf{100V\angle{\pm 180^o}}$ Phase Angle: ± 180º Peak Value: 141.42 V	$v(t) = (\sqrt{2})\times 100\sin(314t \pm 180^o)$ V $= \mathbf{141.42\sin(314\mathit{t} \pm 180^o)\ V}$ $= \mathbf{-\,141.42\sin 314\mathit{t}\ V}$