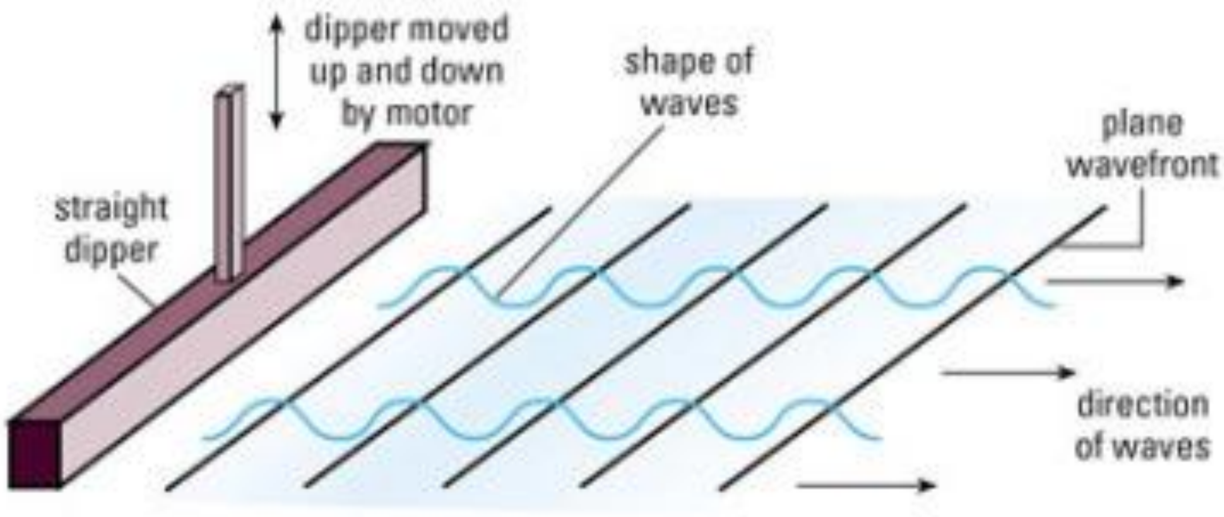
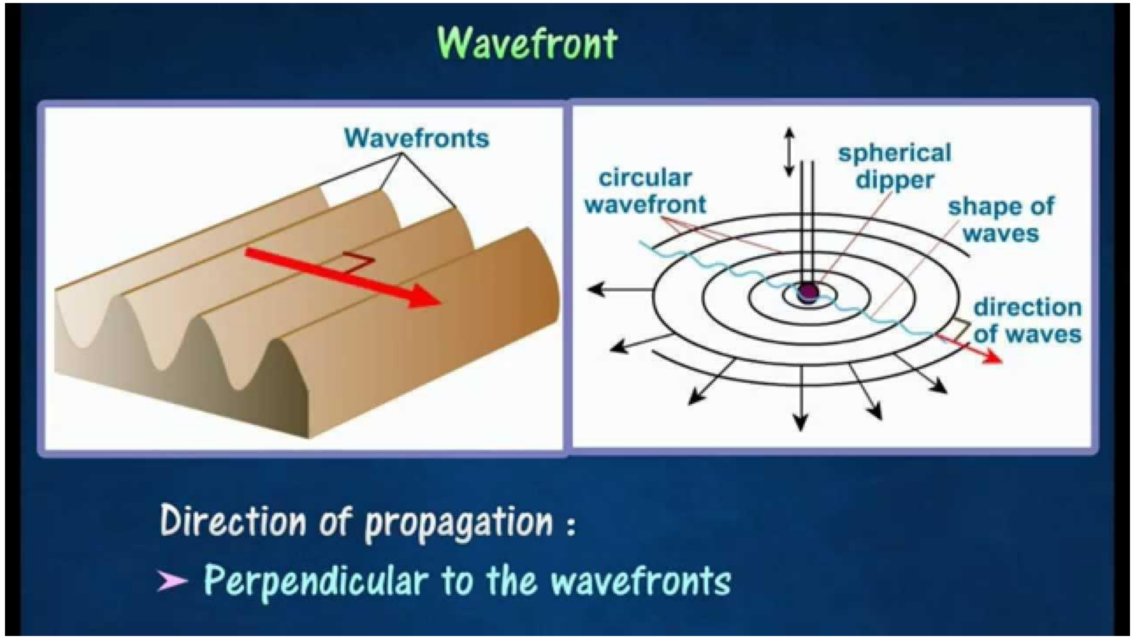
Lecture 22

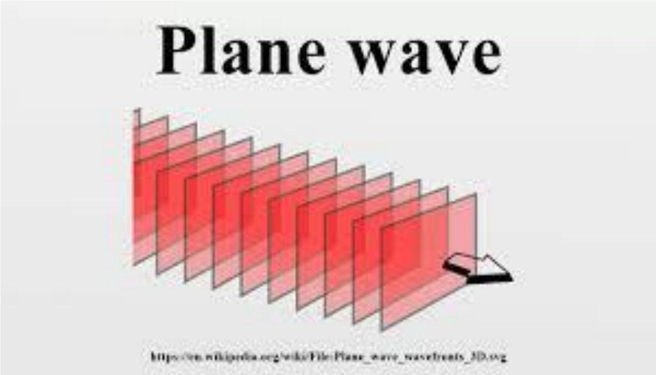
Wave front: In physics the wave front of a time-varying field is the set of all points where the wave has the same phase.





Lecture 22

A plane wave is a constant-frequency wave whose wave fronts are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.

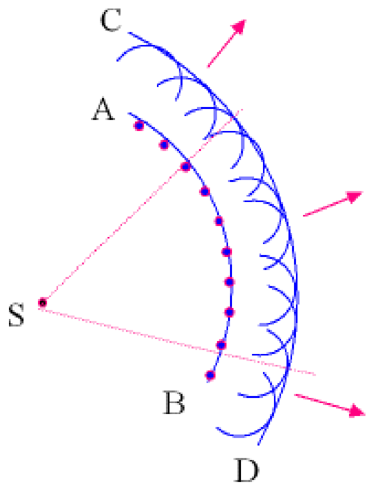


Circular wave on the water surface generated by a small ball oscillating in the vertical direction.

Light as a Wave

Huygens’ wave theory is based on a geometrical construction that allows us to tell where a given wavefront will be at any time in the future if we know its present position. **Huygens’ principle** is:

All points on a wavefront serve as point sources of spherical secondary wavelets. After a time *t*, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.



Huygens' Principle:
Each wavefront is the envelope of the wavelets. Each point on a wavefront acts as an independent source to generate wavelets for the next wavefront. AB and CD are two wavefronts.

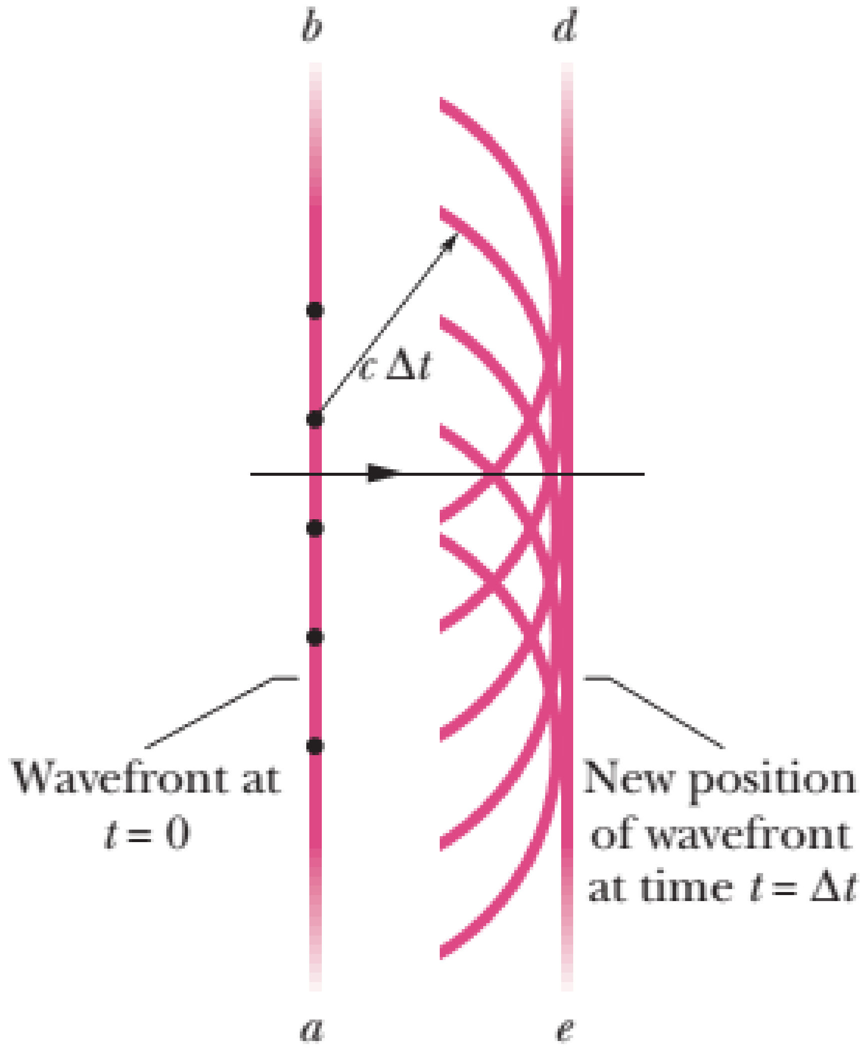
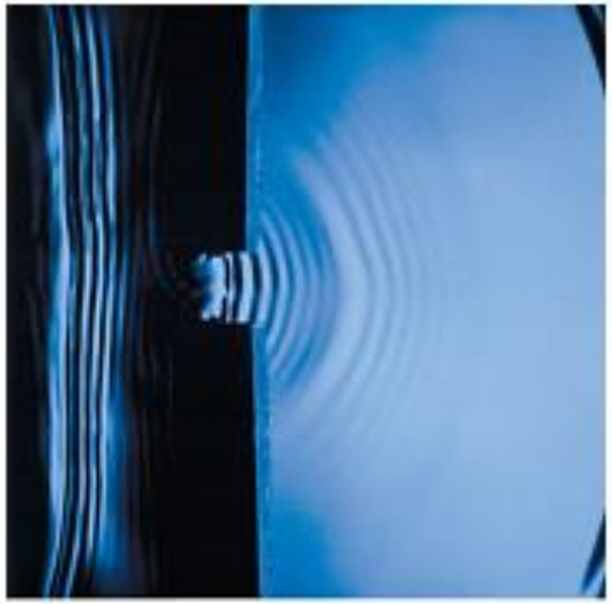


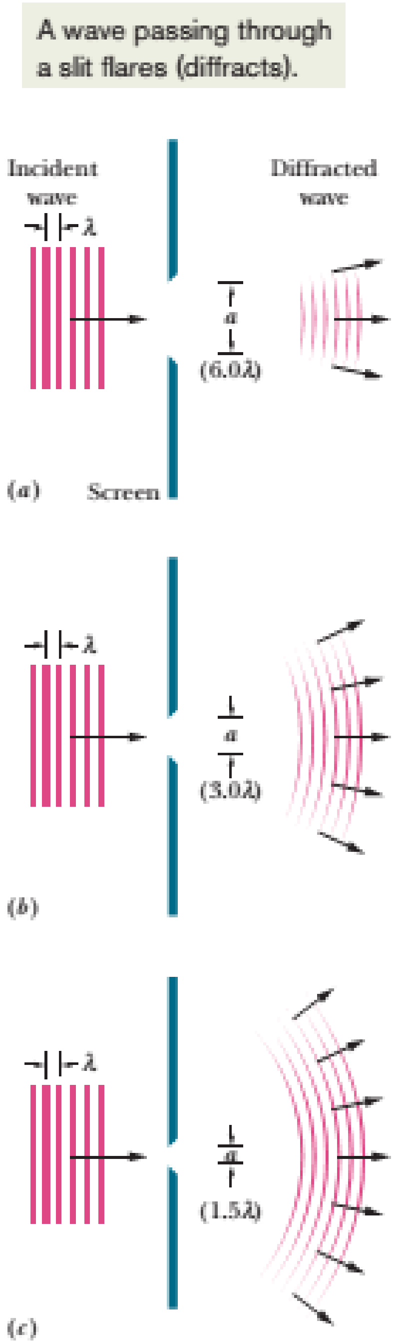
Figure 35-2 The propagation of a plane wave in vacuum, as portrayed by Huygens’ principle.

Diffraction

If a wave encounters a barrier that has an opening of dimensions similar to the wavelength, the part of the wave that passes through the opening will flare (spread) out—will *diffract*—into the region beyond the barrier. The flaring is consistent with the spreading of wavelets in the Huygens construction of Fig. 35-2. Diffraction occurs for waves of all types, not just light waves; Fig. 35-6 shows the diffraction of water waves traveling across the surface of water in a shallow tank.



George Resch/Fundamental Photographs

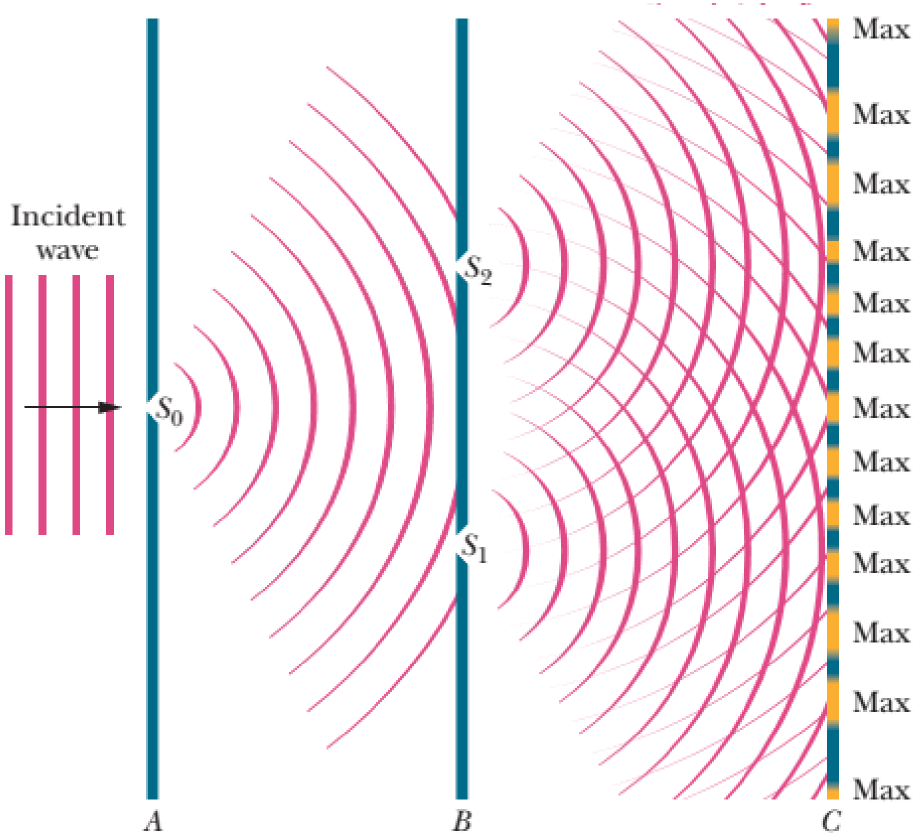


35-2 YOUNG'S INTERFERENCE EXPERIMENT:

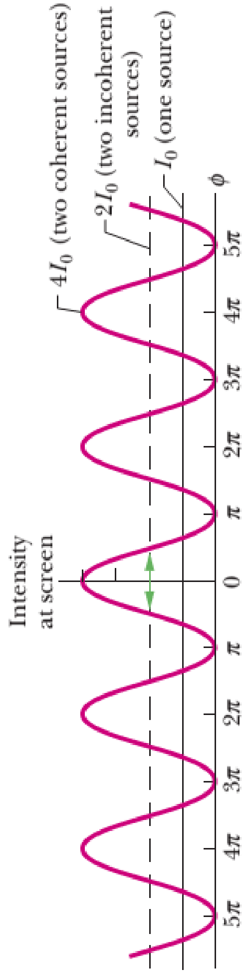
To form Interference, the incident light satisfy two conditions:

(1) Monochromatic source: Light consists of one colour or one wavelength.

(2) Coherent source: Plane waves from the monochromatic source maintain a constant phase relation. If two waves are out of phase, this phase difference must not change with time.



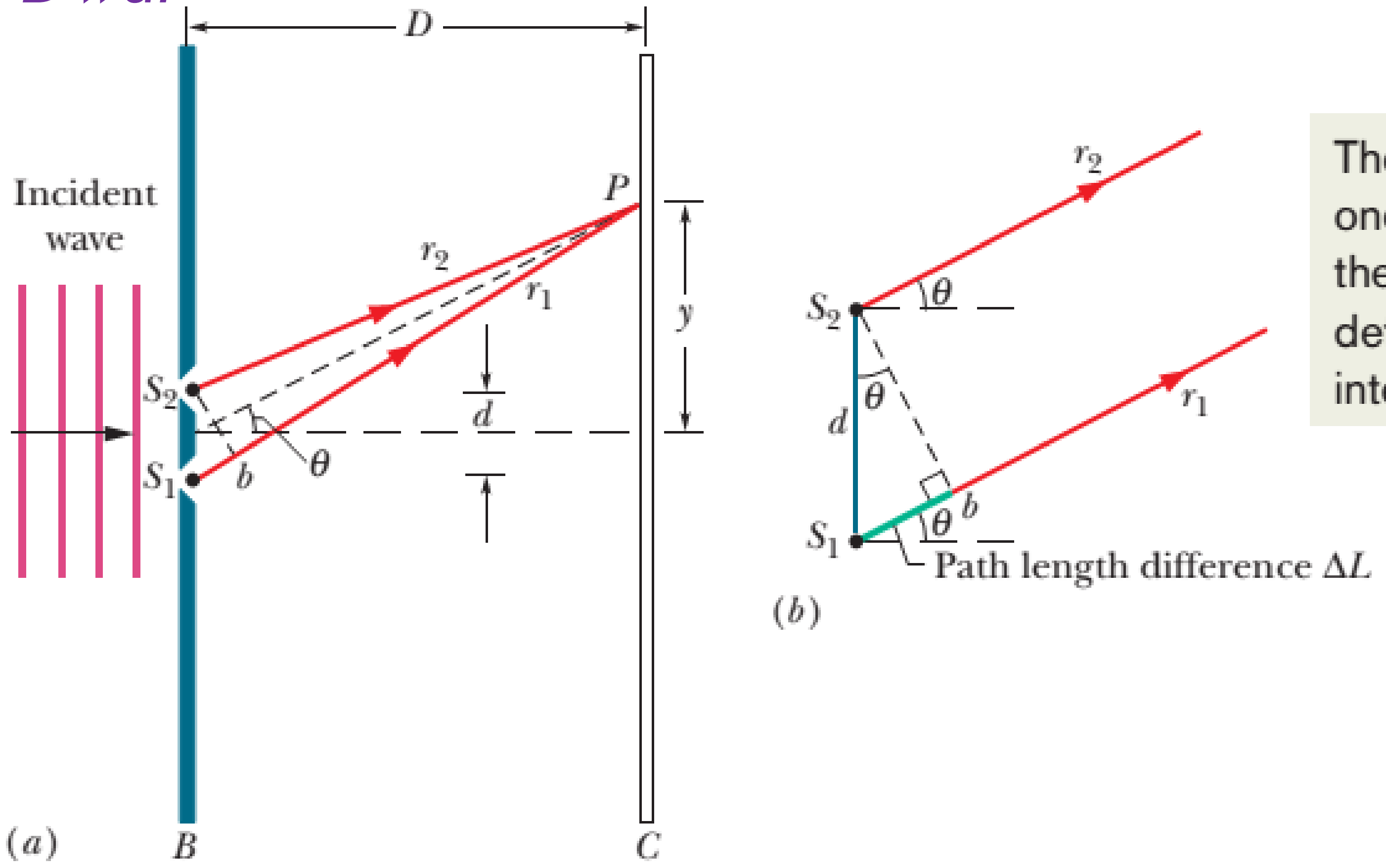
diffracted wavs diffracted wavs interference



Locating the Fringes:

Path Length Difference: The phase difference between two waves can change if the waves travel paths of different lengths.

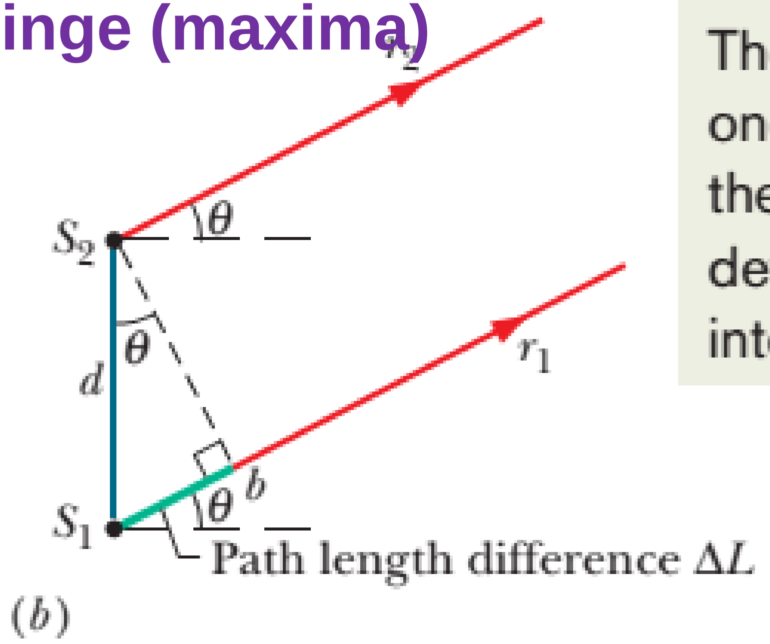
What appears at each point on the viewing screen in a Young's double-slit interference experiment is determined by the path length difference ΔL of the rays reaching that point. *If $D \gg d$.*



The ΔL shifts one wave from the other, which determines the interference.

Condition for maximum and minimum:

In phase (Constructive interference): bright fringe (maxima)



The ΔL shifts one wave from the other, which determines the interference.

$$\sin\theta = \frac{\Delta L}{d}$$

Path length difference, $\Delta L = d \sin\theta$

$\Delta L = 0, 2\frac{\lambda}{2}, 4\frac{\lambda}{2}, 6\frac{\lambda}{2}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$d \sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots\ldots\ldots$

$d \sin\theta = m\lambda$ for $m = 0, 1, 2, 3 \ldots$ (maxima-bright fringe)

Out of phase (Destructive interference): dark fringe (minima)

$\Delta L = 1\frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots\ldots\ldots\ldots\ldots$

$d \sin\theta = (m + \frac{1}{2})\lambda$ for $m = 0, 1, 2, 3 \ldots$ (minima-dark fringe)

Find the angle to any fringe:

bright fringe: d sinθ = mλ   for m = 0, 1, 2, 3 . . .

(1) m = 0: central maximum

$d\sin\theta = (0)\lambda$ $\sin\theta = 0$ $\theta = \sin^{-1} 0$ $\theta = 0$

(2) m = 1: first bright fringe/ first maxima

$d\sin\theta = 1\lambda$ $\sin\theta = \frac{\lambda}{d}$ $\theta = \sin^{-1}(\frac{\lambda}{d})$

(3) m = 2: second bright fringe/ second maxima

$d\sin\theta = 2\lambda$ $\sin\theta = \frac{2\lambda}{d}$ $\theta = \sin^{-1}(\frac{2\lambda}{d})$

dark fringe: $d\sin\theta = (m + \frac{1}{2})\lambda$ for m = 0, 1, 2, 3 . . .

(1) m = 0: first dark fringe/ first minima

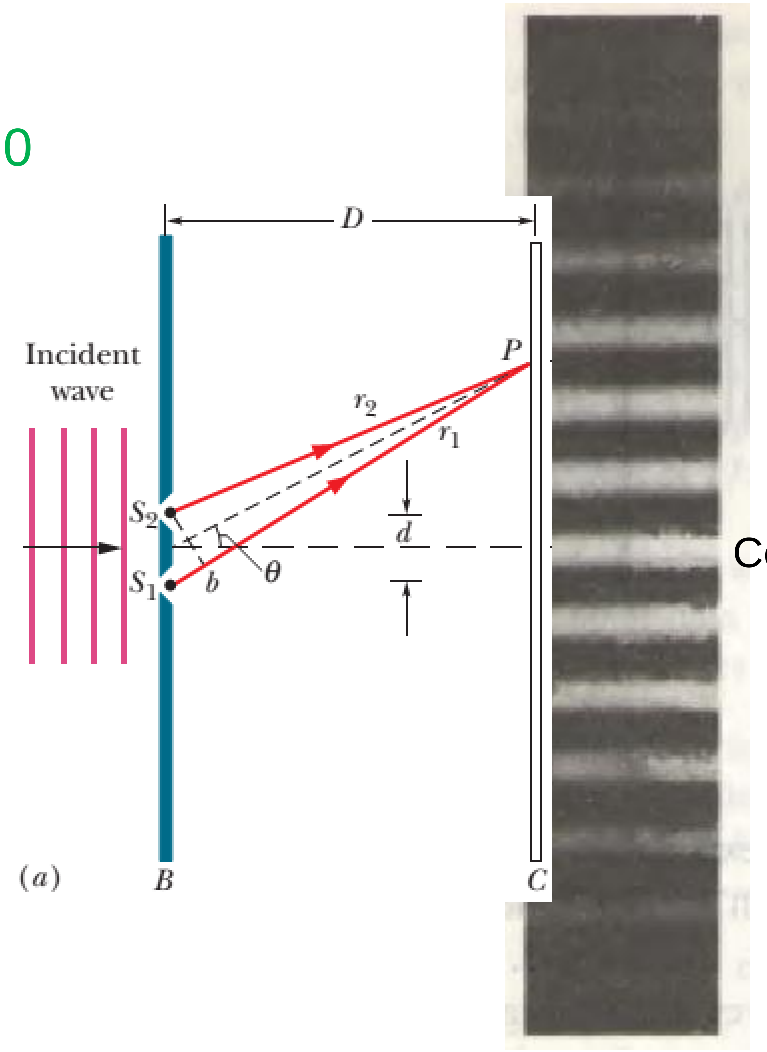
$d\sin\theta = (0 + \frac{1}{2})\lambda$ $\sin\theta = \frac{\lambda}{2d}$ $\theta = \sin^{-1}(\frac{\lambda}{2d})$

(2) m = 1: second dark fringe/ second minima

$d\sin\theta = (1 + \frac{1}{2})\lambda$ $d\sin\theta = (\frac{3\lambda}{2})$ $\theta = \sin^{-1}(\frac{3\lambda}{2d})$

(3) m = 2: third dark fringe/ third minima

$d\sin\theta = (2 + \frac{1}{2})\lambda$ $d\sin\theta = (\frac{5\lambda}{2})$ $\theta = \sin^{-1}(\frac{5\lambda}{2d})$



Central maximum

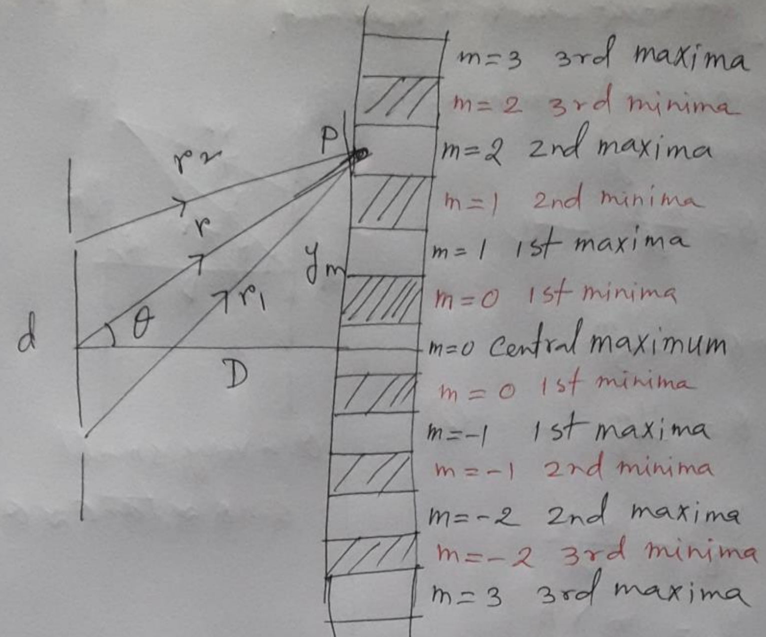


Fig: Young's double slit experiment

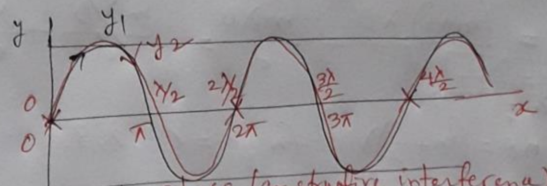


Fig: In phase (constructive interference)

Maxima:

$\Delta L = d\sin\theta = 0, 2\frac{\lambda}{2}, 4\frac{\lambda}{2}, 6\frac{\lambda}{2}$

$d\sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots$

$d\sin\theta = m\lambda$ for $m = 0, 1, 2, 3, \ldots$

Minima:

$\Delta L = d\sin\theta = 1\frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots$

$d\sin\theta = (m+\frac{1}{2})\lambda$ for $m = 0, 1, 2, 3, \ldots$

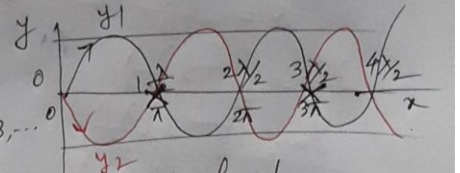


Fig: ~~out~~ out of phase (destructive interference)

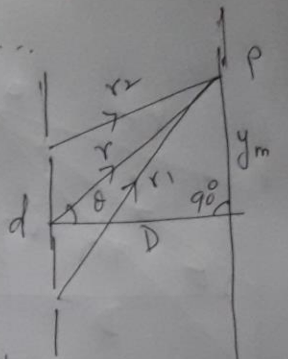
T4: Derive the positions of mth maxima and minima for Young's double-slit interference experiment.

Maxima:

$d\sin\theta = m\lambda$ for $m = 0,1,2,3,\ldots$

$\sin\theta = \frac{m\lambda}{d}$

$\theta = \frac{m\lambda}{d}$ —— (1)



$\tan\theta = \frac{y_m}{D}$

$\theta = \frac{y_m}{D}$ —— (2)

θ is very small

$\sin\theta \approx \tan\theta \approx \theta$

$\therefore \frac{y_m}{D} = \frac{m\lambda}{d}$

$$\boxed{y_m = \frac{m\lambda D}{d}}$$

$\therefore y_{m+1} = \frac{(m+1)\lambda D}{d}$

$\Delta y = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d} = \cancel{\frac{m\lambda D}{d}} + \frac{\lambda D}{d} - \cancel{\frac{m\lambda D}{d}} = \frac{\lambda D}{d}$

$$\boxed{\Delta y = \frac{\lambda D}{d}}$$

Minima:

$d\sin\theta = (m+\tfrac{1}{2})\lambda$ for $m = 0,1,2,3,\ldots$

$\sin\theta = (m+\tfrac{1}{2})\frac{\lambda}{d}$

$\theta = (m+\tfrac{1}{2})\frac{\lambda}{d}$ —— (1)

$\tan\theta = \frac{y_m}{D}$

$\theta = \frac{y_m}{D}$ —— (2)

$\frac{y_m}{D} = (m+\tfrac{1}{2})\frac{\lambda}{d}$

$$\boxed{y_m = (m+\tfrac{1}{2})\frac{\lambda D}{d}}$$

$y_{m+1} = (m+\tfrac{1}{2}+1)\frac{\lambda D}{d}$

$\Delta y = (m+\tfrac{1}{2}+1)\frac{\lambda D}{d} - (m+\tfrac{1}{2})\frac{\lambda D}{d}$

$= \cancel{(m+\tfrac{1}{2})\frac{\lambda D}{d}} + \frac{\lambda D}{d} - \cancel{(m+\tfrac{1}{2})\frac{\lambda D}{d}}$

$$\boxed{\Delta y = \frac{\lambda D}{d}}$$

20. Monochromatic green light, of wavelength 550 nm, illuminates two parallel narrow slits 7.70 mm apart. Calculate the angular deviation (θ in Fig. 35-10) of the third-order bright fringe (a) in radians and (b) in degrees.

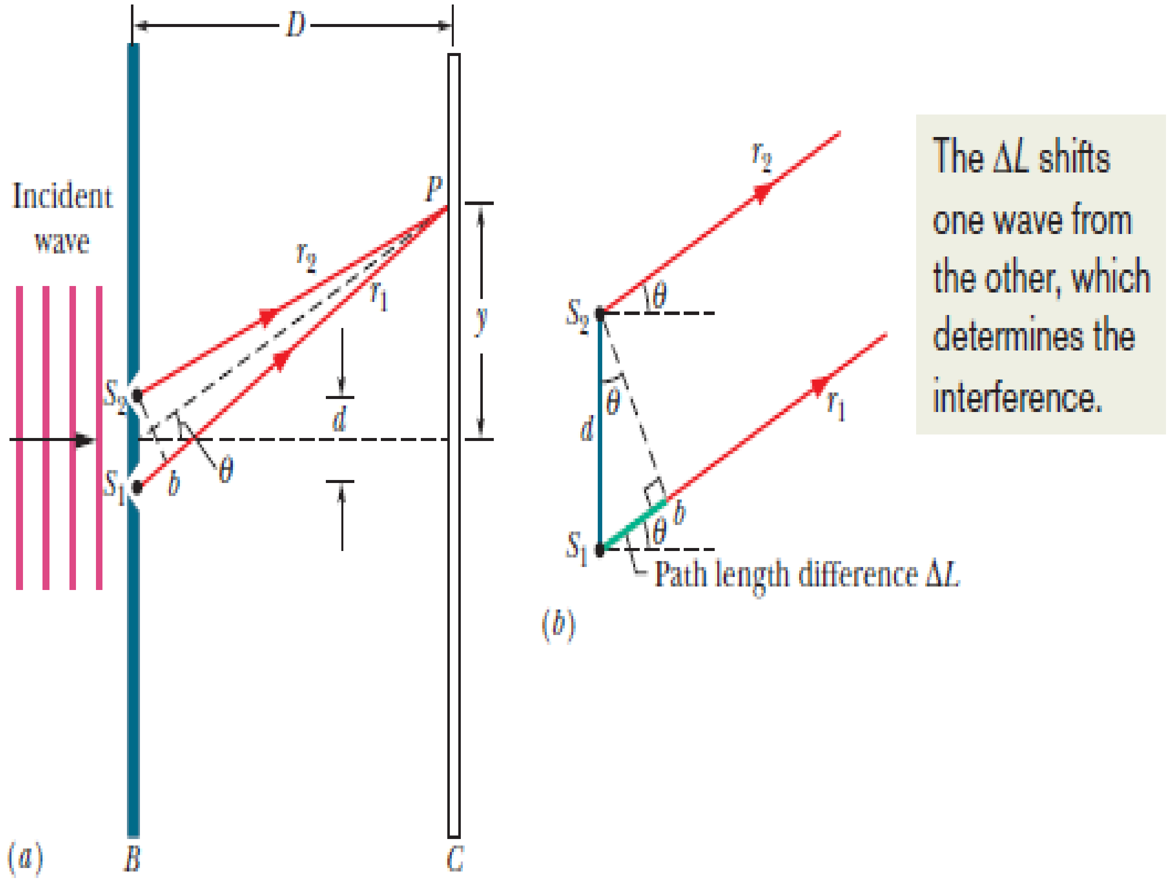


Figure 35-10 (*a*) Waves from slits S_1 and S_2 (which extend into and out of the page) combine at *P*, an arbitrary point on screen *C* at distance *y* from the central axis. The angle *θ* serves as a convenient locator for *P*. (*b*) For $D \gg d$, we can approximate rays r_1 and r_2 as being parallel, at angle *θ* to the central axis.

$\lambda = 550\,nm = 550 \times 10^{-9}\,m$

$d = 7.70\,\mu m = 7.70 \times 10^{-6}\,m$

(a) θ in rad = ?

For a third-order bright fringe, $m = 3$

$d\sin\theta = m\lambda$

or, $d\sin\theta = 3\lambda$

or, $\sin\theta = \frac{3\lambda}{d}$

or, $\theta = \sin^{-1}\left(\frac{3\lambda}{d}\right)$

$= \sin^{-1}\left(\frac{3 \times 550 \times 10^{-9}}{7.70 \times 10^{-6}}\right)$

$= \sin^{-1}\left(\frac{3 \times 550}{7.70} \times 10^{-3}\right)$

$= \sin^{-1}\left(214.286 \times 10^{-3}\right)$

$\boxed{\theta = 0.216\,rad}$ Ans.

(b) θ in degrees = ?

$\pi\,rad = 180^\circ$

$1 \text{ "} = \frac{180}{\pi}$

$0.216\,rad = \frac{180 \times 0.216}{\pi}$

$\boxed{\therefore \theta = 12.4^\circ}$ Ans.

93. If the distance between the first and tenth minima of a double-slit pattern is 18.0 mm and the slits are separated by 0.150 mm with the screen 50.0 cm from the slits, what is the wavelength of the light used?

for minima or dark fringe:

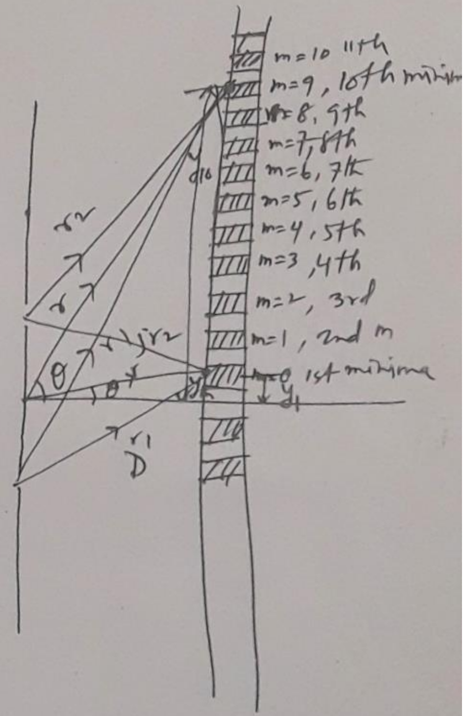
$$d\sin\theta = \left(m+\frac{1}{2}\right)\lambda \; ; \; m = 0,1,2,3,\dots$$

$$\sin\theta = \frac{\left(m+\frac{1}{2}\right)\lambda}{d}$$

$$\theta = \left(m+\frac{1}{2}\right)\frac{\lambda}{d} \quad \text{--- (1)}$$

$$\tan\theta = \frac{y_m}{D}$$

$$\theta = \frac{y_m}{D} \quad \text{--- (2)}$$



$$\frac{y_m}{D} = \left(m+\frac{1}{2}\right)\frac{\lambda}{d}$$

$$y_m = \left(m+\frac{1}{2}\right)\frac{\lambda D}{d} \quad \text{--- (3)}$$

1st minima, $m = 0$

$$y_1 = \left(0+\frac{1}{2}\right)\frac{\lambda D}{d} = \frac{1}{2}\frac{\lambda D}{d}$$

10th minima $m = 9$

$$y_{10} = \left(9+\frac{1}{2}\right)\frac{\lambda D}{d} = \frac{19}{2}\cdot\frac{\lambda D}{d}$$

$$\Delta y = y_{10} - y_1 = \left(\frac{19}{2}-\frac{1}{2}\right)\frac{\lambda D}{d} = \frac{\cancel{18}^{\,9}}{\cancel{2}}\frac{\lambda D}{d} = 9\frac{\lambda D}{d}$$

$$\lambda = \frac{\Delta y\, d}{9D} = \frac{18\times10^{-3}\,(0.150\times10^{-3})}{9\,(0.50)}$$

$\Delta y = 18$ mm $= 18\times10^{-3}$ m

$d = 0.150$ mm $= 0.150\times10^{-3}$ m

$D = 50$ cm $= 0.50$ m

$$\lambda = \frac{2.7\times10^{-6}}{4.5} = 0.6\times10^{-6}\ \text{m}$$

$$= 600\times10^{-6-3}\ \text{m} = 600\times10^{-9}\ \text{m} = 600\ \text{nm} \quad \text{Ans.}$$

Additional problem:
Sample problem 35.02, page: 1057