Lecture 11

Chapter 20: **Entropy and the second law of thermodynamics**

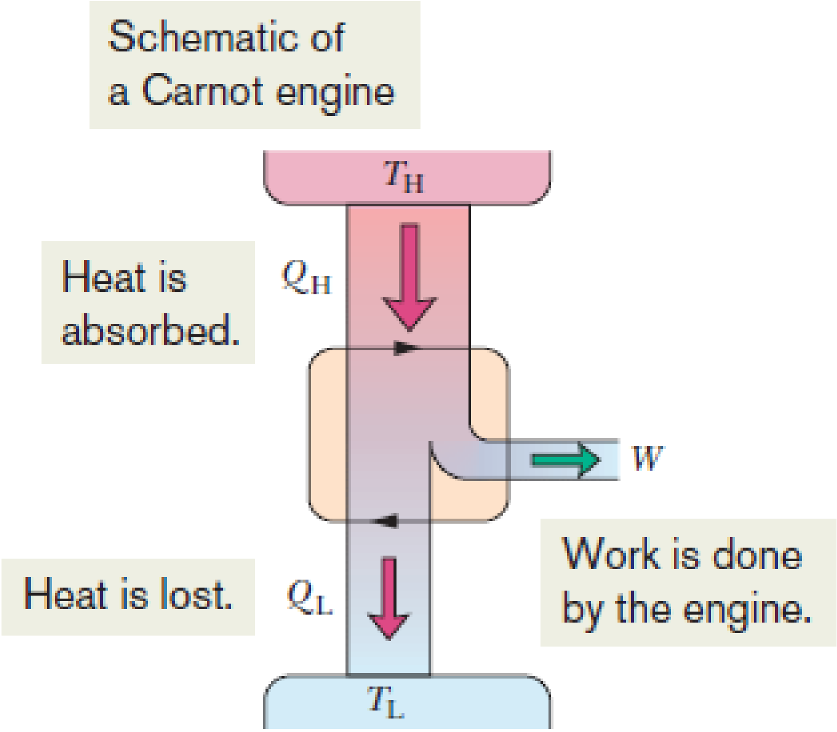
The Work:

In a Carnot engine, the working substance completes reversible cycles.

For a complete cycle of the working substance, the net internal energy change,

$$\Delta E_{int} = 0$$

In each cycle of a Carnot engine, the heat Q_H is transferred to the working substance from the high temperature reservoir T_H and the heat Q_L is transferred from the working substance to the low temperature reservoir T_L.



So, the net heat transfer per cycle,

$$Q = |Q_H| - |Q_L|$$

The first law of thermodynamics for the Carnot cycle,

$$\Delta E_{int} = Q - W$$

$$0 = Q - W$$

$$W = Q$$

$$W = |Q_H| - |Q_L|$$

This is the net work done by a Carnot engine during a cycle.

Entropy Changes:

There are two isothermal processes in each cycle of a Carnot engine.

During the isothermal expansion, the working substance absorbs heat $|Q_H|$ at temperature T_H.

The increase in entropy, $\Delta S_H = \frac{+|Q_H|}{T_H}$

Again during the isothermal compression, the working substance releases heat $|Q_L|$ at constant temperature T_L.

The decrease in entropy, $\Delta S_L = \frac{-|Q_L|}{T_L}$

Thus, the net entropy change per cycle,

$$\Delta S = \Delta S_{\mathrm{H}} + \Delta S_{\mathrm{L}}$$

$$\Delta S = \frac{+|Q_H|}{T_H} + \frac{-|Q_L|}{T_L}$$

For a complete cycle, $\Delta S = 0$ (cause entropy is a state function)

$$0 = \frac{+|Q_H|}{T_H} + \frac{-|Q_L|}{T_L}$$

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$$

$$\frac{|Q_L|}{|Q_H|} = \frac{T_L}{T_H}$$

Efficiency of a Carnot Engine:

Thermal efficiency of any engine is defined as,

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we provide}} = \frac{|W|}{|Q_H|}$$

$$\varepsilon = \frac{|Q_H| - |Q_L|}{|Q_H|}$$

$$\varepsilon = 1 - \frac{|Q_L|}{|Q_H|} \quad \text{[any engine]}$$

$$\varepsilon = 1 - \frac{T_L}{T_H} \quad \text{[Carnot engine]}$$

$[T_L < T_H]$ Using , $\frac{|Q_L|}{|Q_H|} = \frac{T_L}{T_H}$

Because $T_L < T_H$, the efficiency of Carnot engine is less than unity or less than 100%. Thus only a part of the extracted heat is available to do work and the rest is delivered to the low temperature reservoir.

23. A Carnot engine whose low-temperature reservoir is at 17 ^{0}C has an efficiency of 40%. By how much should the temperature of the high-temperature reservoir be increased to increase the efficiency to 50%?

Solution:

Given,

$$T_L = 17^0 C = 290\ K$$

$$Initial\ efficiency, \varepsilon_c = 40\%$$

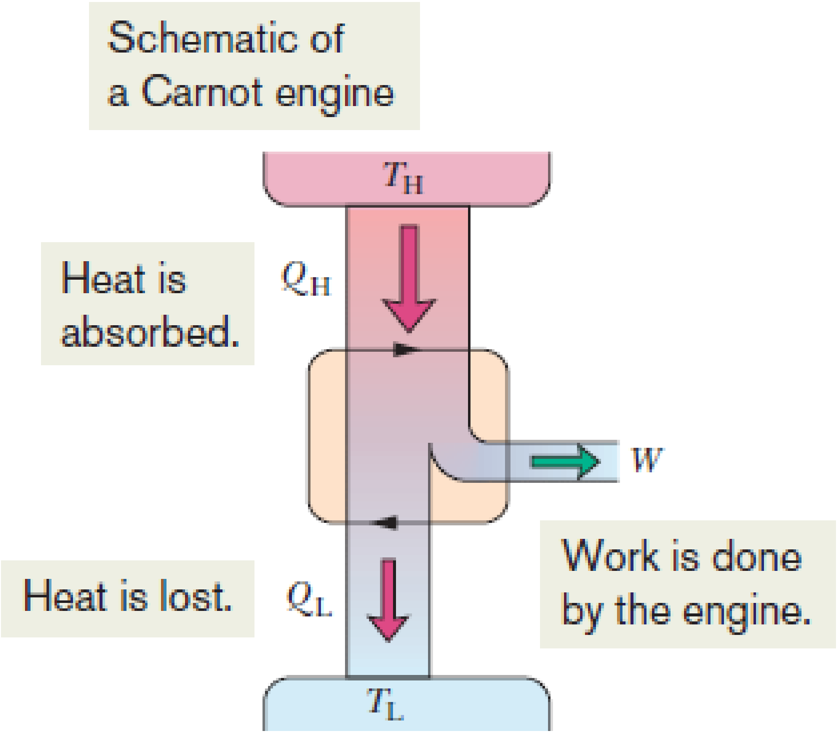
$$Final\ efficiency, \varepsilon_c' = 50\%$$

$$\Delta T_H = ?$$

For the initial state,

$$\varepsilon_c = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow 40\% = 1 - \frac{T_L}{T_H}$$



$$\Rightarrow \frac{T_L}{T_H} = 1 - 0.40$$

$$\therefore T_H = 483.33\ K$$

For the final state,

$$\varepsilon_c' = 1 - \frac{T_L}{T_{H'}}$$

$$\Rightarrow 50\% = 1 - \frac{T_L}{T_H'}$$

$$\Rightarrow \frac{T_L}{T_H'} = 1 - 0.50$$

$$\therefore T_H' = 580\ K$$

So the increased temperature of the high temperature reservoir,

$$\Delta T_H = T_H' - T_H$$

$$= (580 - 483.33)\ K$$

$$= 96.67\ K$$

24. A Carnot engine absorbs 52 kJ as heat and exhausts 36 kJ as heat in each cycle. Calculate (a) the engine's efficiency and (b) the work done per cycle in kilojoules.

Solution:

Given,

$$|Q_H| = 52\ kJ = 52 \times 10^3\ J$$

$$|Q_L| = 36\ kJ = 36 \times 10^3\ J$$

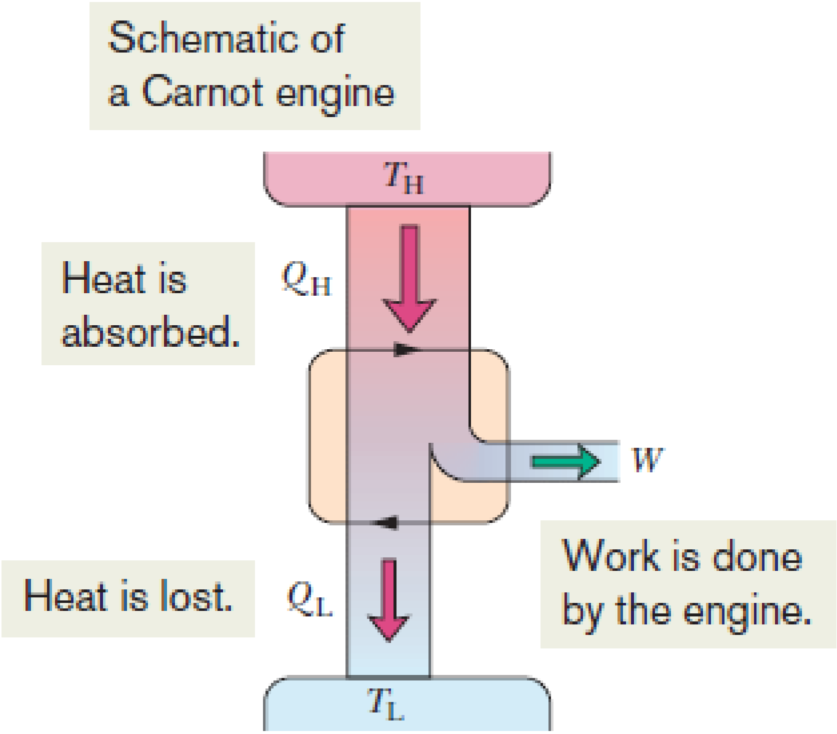
$$(a)\varepsilon_c = ?$$

We know,

$$\varepsilon_c = \left(1 - \frac{|Q_L|}{|Q_H|}\right) \times 100\%$$

$$= \left(1 - \frac{36 \times 10^3}{52 \times 10^3}\right) \times 100\%$$

$$= 30.77\ \%$$



$(\boldsymbol{b})\boldsymbol{W} = ?$

We know

$$\boldsymbol{W} = |\boldsymbol{Q_H}| - |\boldsymbol{Q_L}|$$

$$= \mathbf{52}\ \boldsymbol{kJ} - \mathbf{36}\ \boldsymbol{kJ}$$

$$\boldsymbol{W} = \mathbf{16}\ \boldsymbol{kJ}$$