REVIEW ON THE LAST CLASS

SUPERPOSITION THEOREM

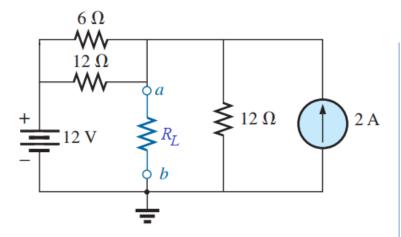
Thevenins Theorem

Nortons Theorem

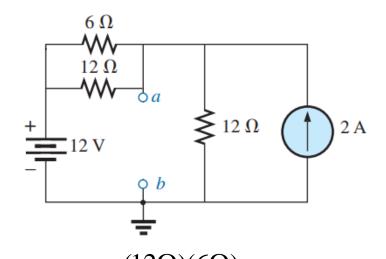
Maximum Power Transfer Theorem



Example: For following network:

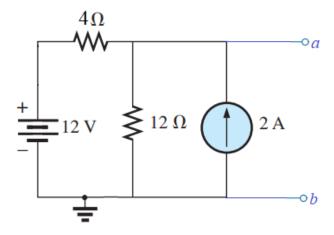


Step 1 and Step 2:

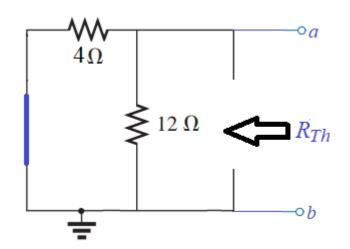


- (a) Find the value of R_L for maximum power to R_L .
- (b) Determine the maximum power to R_L for each network.

Redraw the circuit



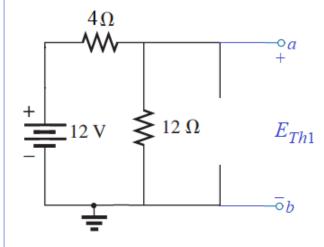
Step 3: R_{Th} calculation



$$R_{Th} = \frac{(12\Omega)(4\Omega)}{12\Omega + 4\Omega} = 3\Omega$$

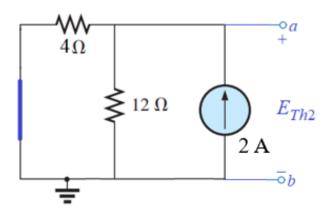
Step 4: E_{Th} calculation

Consider 12 V:



$$E_{Th1} = \frac{(12\Omega)(12V)}{12\Omega + 4\Omega} = 9V$$

Consider 2 A:

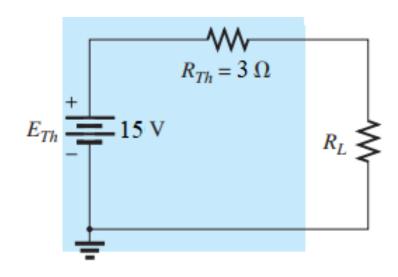


$$R_{p2} = \frac{(12\Omega)(4\Omega)}{12\Omega + 4\Omega} = 3\Omega$$

$$E_{Th2} = 2A \times R_{p2} = 2A \times 3\Omega = 6V$$

$$E_{Th} = E_{Th1} + E_{Th2} = 9V + 6V = 15V$$

Step 5: Draw the Thévenin equivalent circuit



$$R_L = R_{Th} = 3\Omega$$

$$P_{\text{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(15\text{V})^2}{4\times3\Omega} = 18.75 \text{ W}$$

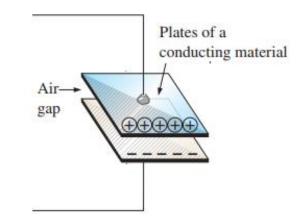
Chapter 10 Capacitors



10.4 Capacitor

Capacitor:

A capacitor is a passive element that is constructed simply of two conducting surfaces or plats separated by the air gap or insulator or dielectric materials. Capacitor is also called condenser.



V = E

 $-\varrho$

Capacitance:

Capacitance is a measure the ability of a capacitor to store charge on its plates as well as to oppose the rate of change of voltage (dv/dt). Unit of capacitance is **Farad** (F).

The higher the capacitance of a capacitor, the greater the amount of charge stored on the plates for the same applied voltage.

Relationship among the applied voltage (E=V), the charge on the plates (Q), and the capacitance level (C):

Capacitance is the ratio of the charge (Q) in one plate of a capacitor to the voltage difference (V) between the two plates.

$$C = \frac{Q}{V}$$

$$C = \text{farads}(F)$$

$$Q = \text{coulombs}(C)$$
 (10.5)

$$V = \text{volts}(V)$$

$$Q = CV$$

Problem 3 [P453] Find the capacitance of a parallel plate capacitor if 1200 μC of charge are deposited on its plates when 10 V are applied across the plates.

Solution:
$$Q = 1200 \,\mu\text{C} = 1200 \times 10^{-6} \,\mu\text{C}, V = 10 \,\text{V}$$

We know that:

$$C = \frac{Q}{V} = \frac{1200 \times 10^{-6} \text{ C}}{10 \text{ V}} = 120 \times 10^{-6} \text{ F or } 120 \text{ }\mu\text{F}$$

Problem 4 [P453] How much charge is deposited on the plates of a 0.15 μF capacitor if 45 V are applied across the capacitor?

Solution:
$$C = 0.15 \mu F = 0.15 \times 10^{-6} \mu F$$
, $V = 45 \text{ V}$

We know that:
$$C = \frac{Q}{V}$$

:
$$Q = CV = (0.15 \times 10^{-6} \text{ C})(45 \text{ V}) = 6.75 \times 10^{-6} \text{ C} \text{ or } 6.75 \text{ }\mu\text{C}$$

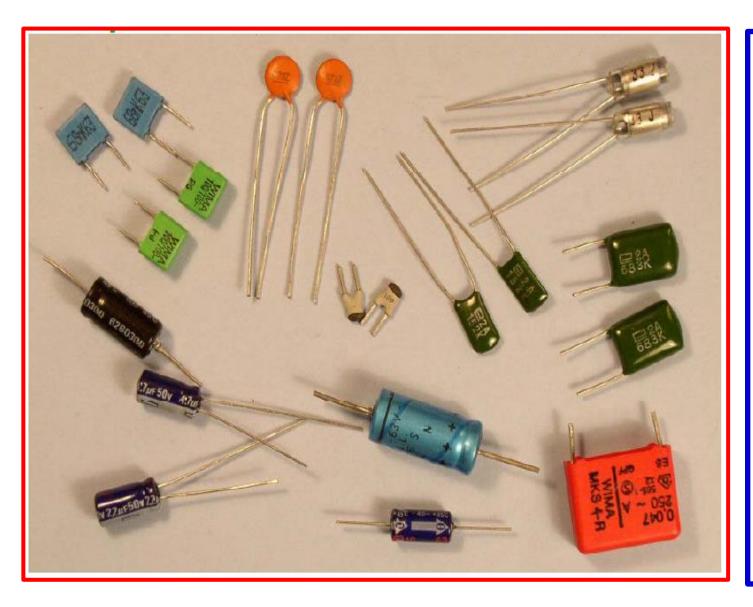
$$C = \frac{Q}{V}$$

$$Q = CV$$

Practice Example 10.1



Different Types of Capacitors



Applications of Capacitor:

- 1. Limit rate of change of voltage
- 2. Radio Receiver
- 3. Block dc
- 4. Pass ac
- 5. Shift phase
- 6. Store energy
- 7. Start motors
- 8. Suppress noise
- 9. Filter Circuits
- 10. Improve power factor

Capacitance Based on Physical Dimension

Equation of Capacitance:

$$C = \epsilon \frac{A}{d}$$

$$C = \text{farads (F)}$$

$$\epsilon = \text{permittivity (F/m)}$$

$$A = m^2$$

$$d = m$$

$$(10.9)$$

$$C = \epsilon_o \epsilon_r \frac{A}{d} \qquad \text{(farads, F)} \tag{10.10}$$

$$C = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d}$$
 (farads, F) (10.11)

 $\epsilon_0 = 8.85 \times 10^{-12}$ [F/m] and ϵ_r is called the relative **permittivity**.



FIG. 10.11 (*a*) *fixed*; (*b*) *variable*.

Let, C_o is the value of capacitance considering air as a dielectric material and C is the value of capacitance for any other dielectric material, then we have

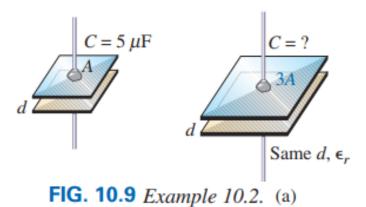
$$C = \epsilon_r C_o \tag{10.12}$$

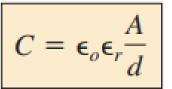
$$C_o = \epsilon_o \frac{A}{d}$$
 (farads, F)

TABLE 10.1 *Relative permittivity (dielectric constant)* ϵ , of various dielectrics.

Dielectric	ϵ_r (Average Values)
Vacuum	1.0
Air	1.0006
Teflon®	2.0
Paper, paraffined	2.5
Rubber	3.0
Polystyrene	3.0
Oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite®	7.0
Aluminum oxide	7
Glass	7.5
Tantalum oxide	30
Ceramics	20-7500
Barium-strontium titanite (ceramic)	7500.0

EXAMPLE 10.2 In Fig. 10.9(a) and (b), if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.

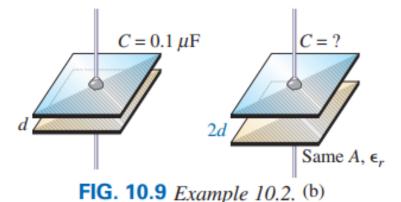




Solutions:

a. In Fig. 10.9(a), the area has increased by a factor of three, providing more space for the storage of charge on each plate. Since the area appears in the numerator of the capacitance equation, the capacitance increases by a factor of three. That is,

$$C = 3(C_0) = 3(5 \mu F) = 15 \mu F$$



b. In Fig. 10.9(b), the area stayed the same, but the distance between the plates was increased by a factor of two. Increasing the distance reduces the capacitance level, so the resulting capacitance is onehalf of what it was before. That is,

$$C = \frac{1}{2}(0.1 \,\mu\text{F}) = 0.05 \,\mu\text{F}$$

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EXAMPLE 10.2 In Fig. 10.9(c) and (d), if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.

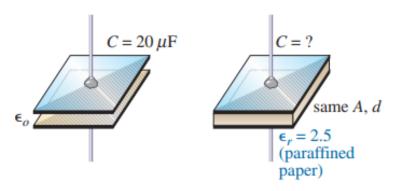


FIG. 10.9 *Example 10.2.* (c)

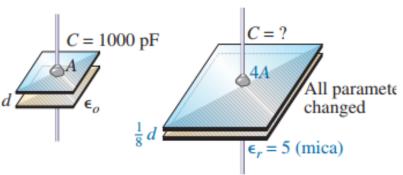


FIG. 10.9 *Example 10.2.*(d)

Practice Problem 14 [P453]

c. In Fig. 10.9(c), the area and the distance between the plates were maintained, but a dielectric of paraffined (waxed) paper was added between the plates. Since the permittivity appears in the numerator of the capacitance equation, the capacitance increases by a factor determined by the relative permittivity. That is,

$$C = \epsilon_r C_o = 2.5(20 \ \mu\text{F}) = 50 \ \mu\text{F}$$

d. In Fig. 10.9(d), a multitude of changes are happening at the same time. However, solving the problem is simply a matter of determining whether the change increases or decreases the capacitance and then placing the multiplying factor in the numerator or denominator of the equation. The increase in area by a factor of four produces a multiplier of four in the numerator, as shown in the equation below. Reducing the distance by a factor of 1/8 will increase the capacitance by its inverse, or a factor of eight. Inserting the mica dielectric increases the capacitance by a factor of five. The result is

$$C = (5)\frac{4}{(1/8)}(C_o) = 160(1000 \text{ pF}) = 0.16 \,\mu\text{F}$$

10.5 TRANSIENTS IN CAPACITIVE NETWORKS

If the applied voltage is change with respect to time the storage charge and current are also change in a capacitor. The changeable voltage, current and charge are represented by small letter $i.e.\ v,\ i$ and q.

Eq. (10.6) can be written as follows where voltage (v) and charge (q) are changing with respect to time:

$$q = Cv$$
 (10.6.1)

Similarly, Eq. (2.5) can be written as follows where current (i) and charge (q) are changing with respect to time:

$$i = \frac{dq}{dt}$$

$$(2.5.1)$$

$$e(t) \quad C$$

$$v_{C}$$

Combining Eq. (10.6.1) and Eq. (2.5.1) we have:

$$i_C = C \frac{dv_C}{dt}$$
 (10.5.1)

By integrating Eq. (10.5.1) in both sides we have:

$$v_C = \frac{1}{C} \int_{t_0}^t i_C dt + v_C(t_0)$$
 (10.5.2)

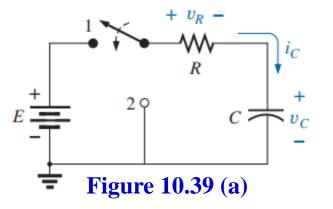
Energy storage by a capacitor can be calculated as follows:

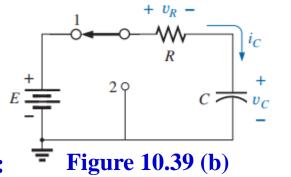
$$W_C = \frac{1}{2}Cv_C^2$$
 [J] (10.5.3)

Charging and Discharging in a Capacitor

In Figure 10.39(a):

Charging Mode: When the switch is in position 1 as shown in Figure 10.39 (b), $E \equiv$ capacitor is stored the charge. **Discharging Mode**: When the switch is in **position 2 as** shown in Figure 10.39 (c), capacitor is released the charge through the resistor.

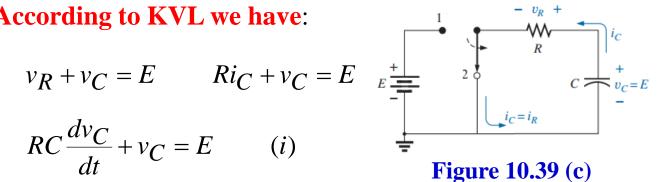




Charging Mode of Fig. 10.39(a): **According to KVL we have:**

$$v_R + v_C = E$$
 $Ri_C + v_C = E$

$$RC\frac{dv_C}{dt} + v_C = E \qquad (i)$$



The solution of Eq. (i) is as follows:

$$v_C = E(1 - e^{-t/\tau})$$
 (volt, V) (10.13)

Substitute Eq. (10.13) into Eq. (10.5.1), we have:

$$i_C = \frac{E}{R}e^{-t/\tau}$$
 (ampere, A) (10.15)

The voltage drop across the resistor will be:

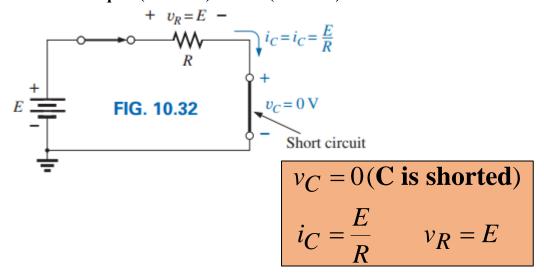
$$v_R = Ee^{-t/\tau}$$
 (volt, V) (10.16)

The quantity τ is called **time constant**, which is given by: $\tau = RC \quad \text{(second, s)} \qquad \textbf{(10.14)}$

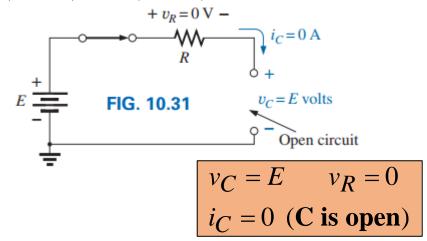
Prove that the unit of $\tau = RC$ is seconds.

$$RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{Q}{I}\right) = \left(\frac{It}{I}\right) = t \text{ (seconds)}$$

At time t = 0 (at the instant of switch is closed), from Eqs. (10.13) and (10.15) we have:



At time $t = \infty$ (steady-state condition), from Eqs. (10.13) and (10.15) we have:



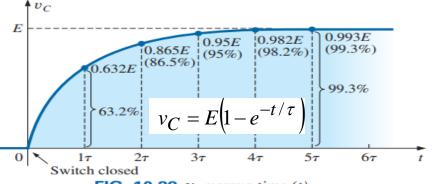
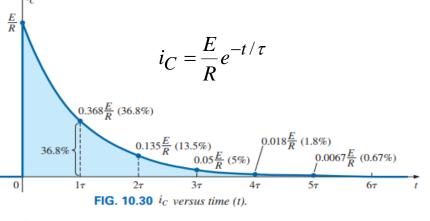
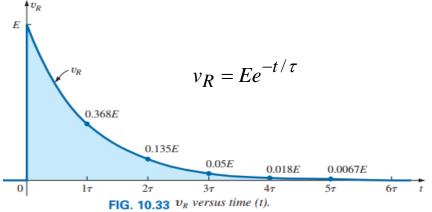


FIG. 10.29 v_C versus time (t).





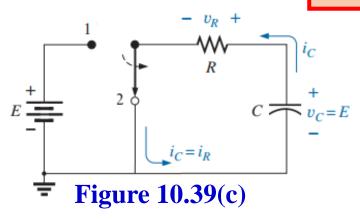
It has been observed from graphs 10.29 **Figures** and 10.30 that at $t = 5\tau$, the capacitor voltage becomes almost equal to supply voltage the and becomes current almost zero.

The transient charging phase of a capacitor has essentially ended five after time constants.

After five time constants, circuit in steady state.



Discharging Phase



According to KVL we have:

$$v_R = v_C$$
 $Ri_C = E$ $RC\frac{dv_C}{dt} = E$ (ii)

The solution of Eq. (ii) is as follows:

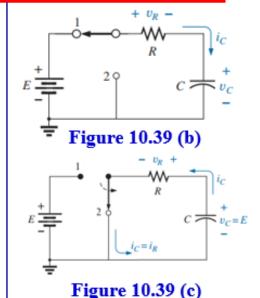
$$v_C = Ee^{-t/\tau}$$
 (volt, V) (10.17)

Substitute Eq. (10.17) into Eq. (10.5.1), we have:

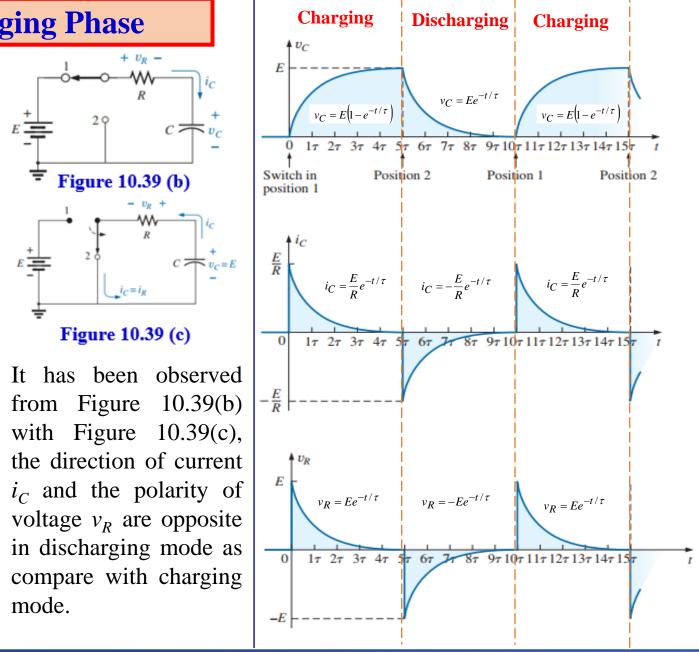
$$i_C = \frac{E}{R} e^{-t/\tau}$$
 (ampere, A) (10.19)

The voltage drop across the resistor will be:

$$v_R = Ee^{-t/\tau}$$
 (volt, V) (10.20)



It has been observed from Figure 10.39(b) with Figure 10.39(c), the direction of current i_C and the polarity of voltage v_R are opposite in discharging mode as compare with charging mode.



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EXAMPLE 10.6 For the circuit in Fig. 10.35:

a. Find the mathematical expression for the transient behavior of v_C , i_C and v_R if the switch is closed at t = 0 s.

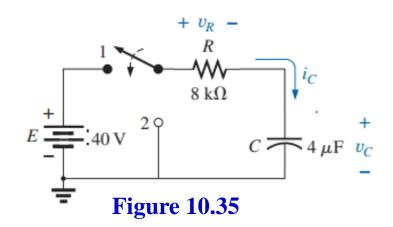
b. Plot the waveform of v_C versus the time constant of the network.

c. Plot the waveforms of i_C and v_R versus the time constant of the network.

d. What is the value of v_C at t = 20 ms?

e. On a practical basis, how much time must pass before we can assume that the charging phase has passed?

f. When the charging phase has passed, how much charge is sitting on the plates?



Solution: Given, E = 40 V, $R = 8 \times 10^3 \Omega$, and $C = 4 \times 10^{-6} \text{ F}$,

. The time constant of the network is

$$\tau = RC = (8 \text{ k}\Omega)(4 \mu\text{F}) = 32 \text{ ms}$$

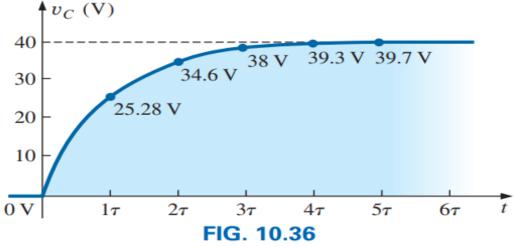
resulting in the following mathematical equations:

$$v_C = E(1 - e^{-t/\tau}) = 40 \text{ V}(1 - e^{-t/32\text{ms}})$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40 \text{ V}}{8 \text{ k}\Omega}e^{-t/32\text{ms}} = 5 \text{ mA}e^{-t/32\text{ms}}$$

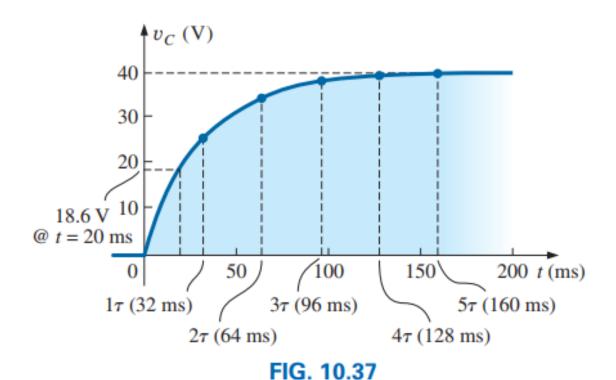
$$v_R = Ee^{-t/\tau} = 40 \text{ V}e^{-t/32\text{ms}}$$

b. The resulting plot appears in Fig. 10.36.



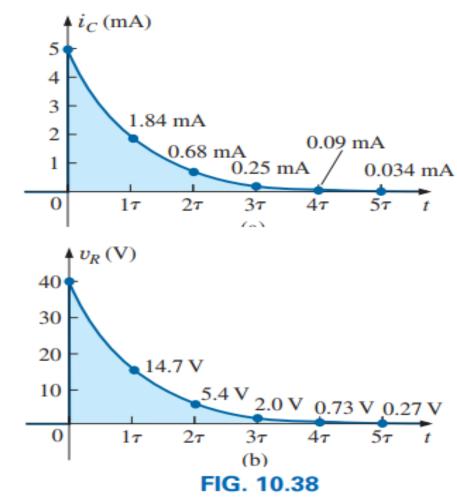
 v_C versus time for the charging network in Fig. 10.35.

c. The horizontal scale will now be against time rather than time constants, as shown in Fig. 10.37. The plot points in Fig. 10.37 were taken from Fig. 10.36.



Plotting the waveform in Fig. 10.36 versus time (t).

d. Both plots appear in Fig. 10.38.



 i_C and v_R for the charging network in Fig. 10.36.

e. Substituting the time t = 20 ms results in the following for the exponential part of the equation:

$$e^{-t/\tau} = e^{-20\text{ms/32ms}} = e^{-0.625} = 0.535$$
 (using a calculator)
so that $v_C = 40 \text{ V} (1 - e^{-t/32\text{ms}}) = 40 \text{ V} (1 - 0.535)$
= $(40 \text{ V})(0.465) = 18.6 \text{ V}$ (as verified by Fig. 10.37)

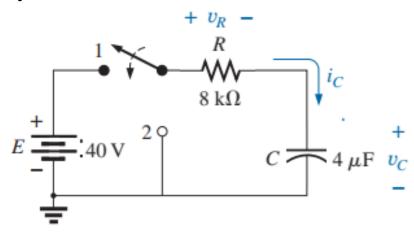
f. Assuming a full charge in five time constants results in

$$5\tau = 5(32 \text{ ms}) = 160 \text{ ms} = 0.16 \text{ s}$$

g. Using Eq. (10.6):

$$Q = CV = (4 \mu F)(40 V) = 160 \mu C$$

EXAMPLE 10.7 Plot the waveforms for v_C and i_C resulting from switching between contacts 1 and 2 in the following figure every five time constants.



Solution: Given, E = 40 V, $R = 8 \times 10^3 \Omega$, and $C = 4 \times 10^{-6}$ F

The time constant of the network is

$$\tau = RC = (8 \text{ k}\Omega)(4 \mu\text{F}) = 32 \text{ ms}$$

Charging Mode:

$$v_C = E(1 - e^{-t/\tau}) = 40 \text{ V}(1 - e^{-t/32\text{ms}})$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40 \text{ V}}{8 \text{ k}\Omega}e^{-t/32\text{ms}} = 5 \text{ mA}e^{-t/32\text{ms}}$$

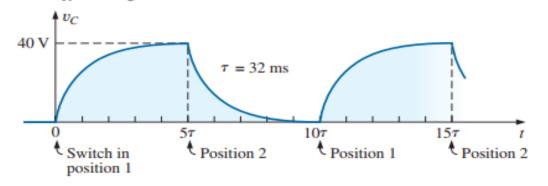
$$v_R = Ee^{-t/\tau} = 40 \text{ V}e^{-t/32\text{ms}}$$

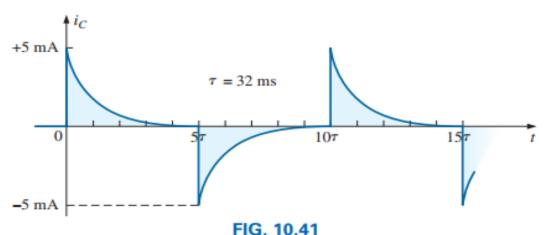
Discharging Mode:

$$v_C = Ee^{-t/\tau} = 40 \text{ V}e^{-t/32\text{ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau} = \frac{40 \text{ V}}{8 \text{ k}\Omega}e^{-t/32\text{ms}} = -5 \text{ mA}e^{-t/32\text{ms}}$$

$$v_R = v_C = 40 \text{ V}e^{-t/32\text{ms}}$$

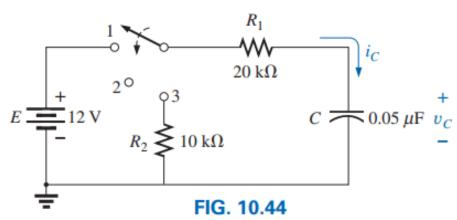




 v_C and i_C for the network in Fig. 10.39(a) with the values in Example 10.6.

EXAMPLE 10.8 For the circuit in Fig. 10.44:

- a. Find the mathematical expressions for the transient behavior of the voltage v_C and the current i_C if the capacitor was initially uncharged and the switch is thrown into position 1 at t = 0 s.
- b. Find the mathematical expressions for the voltage v_C and the current i_C if the switch is moved to position 2 at t = 10 ms. (Assume that the leakage resistance of the capacitor is infinite ohms; that is, there is no leakage current.)
- c. Find the mathematical expressions for the voltage v_C and the current i_C if the switch is thrown into position 3 at t = 20 ms.
- d. Plot the waveforms obtained in parts (a)–(c) on the same time axis using the defined polarities in Fig. 10.44.



Network to be analyzed in Example 10.8.

Solutions:

a. Charging phase:

$$\tau = R_1 C = (20 \text{ k}\Omega)(0.05 \mu\text{F}) = 1 \text{ ms}$$

$$\upsilon_C = E(1 - e^{-t/\tau}) = \mathbf{12 \text{ V}}(\mathbf{1} - e^{-t/1\text{ms}})$$

$$i_C = \frac{E}{R_1}e^{-t/\tau} = \frac{12 \text{ V}}{20 \text{ k}\Omega}e^{-t/1\text{ms}} = \mathbf{0.6 \text{ mA}}e^{-t/1\text{ms}}$$

b. Storage phase: At 10 ms, a period of time equal to 10τ has passed, permitting the assumption that the capacitor is fully charged. The result is that both v_C and i_C will remain at a fixed value of

$$v_C = 12 \text{ V}$$
$$i_C = 0 \text{ A}$$

c. Discharge phase (using 20 ms as the new t = 0 s for the equations): The new time constant is

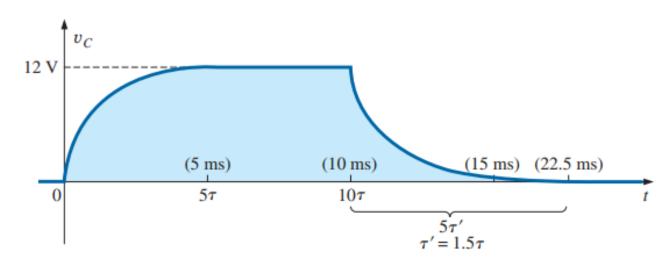
$$\tau' = RC = (R_1 + R_2)C = (20 \text{ k}\Omega + 10 \text{ k}\Omega)(0.05 \,\mu\text{F}) = 1.5 \text{ ms}$$

$$v_C = Ee^{-t/\tau'} = \mathbf{12} \, \mathbf{V}e^{-t/1.5\text{ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau'} = -\frac{E}{R_1 + R_2}e^{-t/\tau'}$$

$$= -\frac{12 \, \text{V}}{20 \, \text{k}\Omega + 10 \, \text{k}\Omega} e^{-t/1.5\text{ms}} = -\mathbf{0.4} \, \mathbf{mA}e^{-t/1.5\text{ms}}$$

d. See Fig. 10.45.



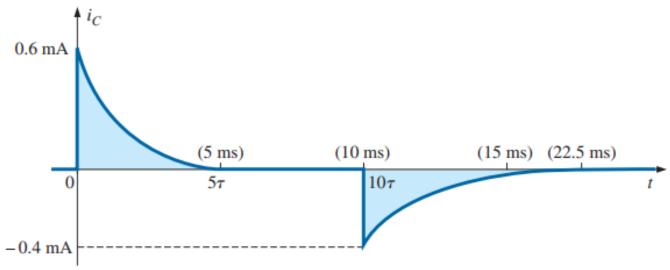
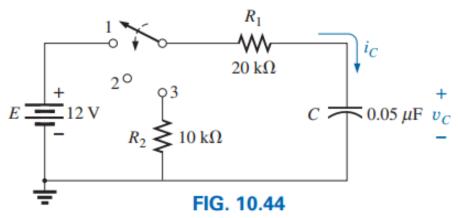


FIG. 10.45 v_C and i_C for the network in Fig. 10.44.

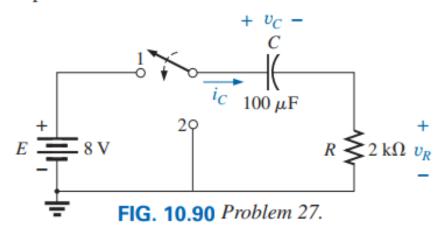


Network to be analyzed in Example 10.8.

$$\tau = R_1 C = (20 \text{ k}\Omega)(0.05 \mu\text{F}) = 1 \text{ ms}$$

 $\tau' = RC = (R_1 + R_2)C = (20 \text{ k}\Omega + 10 \text{ k}\Omega)(0.05 \mu\text{F}) = 1.5 \text{ ms}$

- **27.** For the *R-C* circuit in Fig. 10.90, composed of standard values:
 - **a.** Determine the time constant of the circuit when the switch is thrown into position 1.
 - **b.** Find the mathematical expression for the voltage across the capacitor and the current after the switch is thrown into position 1.
 - **c.** Determine the magnitude of the voltage v_C and the current i_C the instant the switch is thrown into position 2 at t = 1 s.
 - **d.** Determine the mathematical expression for the voltage v_C and the current i_C for the discharge phase.
 - **e.** Plot the waveforms of v_C and i_C for a period of time extending from 0 to 2 s from when the switch was thrown into position 1.



Solution: Given, E = 8 V, $R = 2 \times 10^3 \Omega$, and $C = 100 \times 10^{-6}$ F

(a)
$$\tau = RC = (2 \times 10^3)(100 \times 10^{-6}) = 200 \text{ ms}$$

(b)
$$v_C = E\left(1 - e^{-t/\tau}\right) = 8V\left(1 - e^{-t/200\text{ms}}\right)$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{8V}{2k\Omega} = 4\text{mA}e^{-t/200\text{ms}}$$

(c)
$$e^{-t/\tau} = e^{-1s/200\text{ms}} = e^{-5} = 0.0067$$

 $v_C = 8V(1 - e^{-t/200\text{ms}}) = 8V \times (1 - 0.0067) = 7.95V$
 $i_C = 4\text{mA}e^{-t/200\text{ms}} = 4\text{mA} \times 0.0067 = 0.0268\text{mA}$

(d)
$$v_C = Ee^{-t/\tau} = 8Ve^{-t/200\text{ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau} = -\frac{8V}{2k\Omega} = -4\text{mA}e^{-t/200\text{ms}}$$

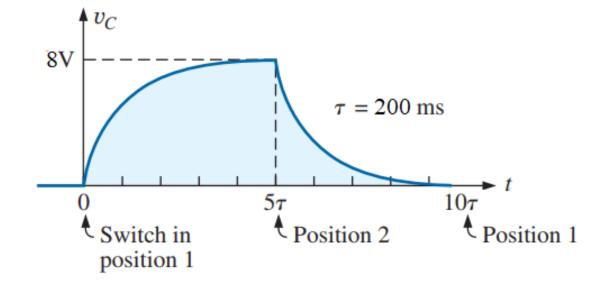
21

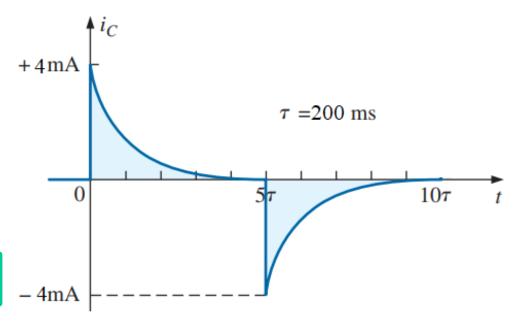
e. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 2 s from when the switch was thrown into position 1.

$$\tau = 200 \, \mathrm{ms}$$

$$1s = 5\tau$$

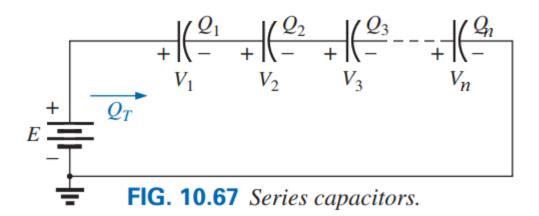
$$2s=10\tau$$





Practice Problems 21 ~ 30 [Ch. 10]

10.11 CAPACITORS IN SERIES AND IN PARALLEL



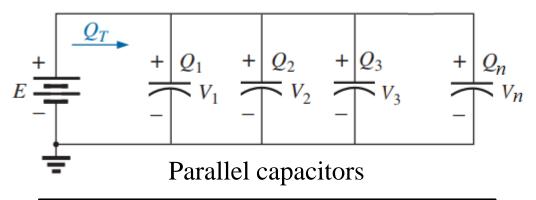
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$
 (10.29)

When two capacitors are connected in series:

$$C_{T} = \frac{C_{1}C_{2}}{C_{1} + C_{2}}$$

$$C_{T} = \frac{C_{1}C_{2}}{C_{1} + C_{2}}$$

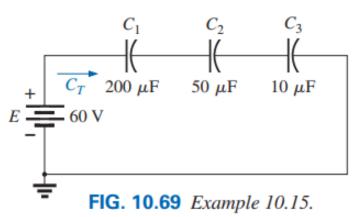
$$(10.30)$$



$$C_T = C_1 + C_2 + C_3 + \dots + C_n$$
 (10.33)

EXAMPLE 10.15 For the circuit in Fig. 10.69:

- a. Find the total capacitance.
- b. Determine the charge on each plate.
- c. Find the voltage across each capacitor.



Solutions:

a.
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{200 \times 10^{-6} \,\text{F}} + \frac{1}{50 \times 10^{-6} \,\text{F}} + \frac{1}{10 \times 10^{-6} \,\text{F}}$$

$$= 0.005 \times 10^6 + 0.02 \times 10^6 + 0.1 \times 10^6$$

$$= 0.125 \times 10^6$$
and
$$C_T = \frac{1}{0.125 \times 10^6} = 8 \,\mu\text{F}$$

b.
$$Q_T = Q_1 = Q_2 = Q_3$$

= $C_T E = (8 \times 10^{-6} \text{ F}) (60 \text{ V}) = 480 \ \mu\text{C}$

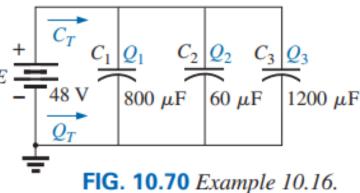
c.
$$V_1 = \frac{Q_1}{C_1} = \frac{480 \times 10^{-6} \,\mathrm{C}}{200 \times 10^{-6} \,\mathrm{F}} = \mathbf{2.4 \, V}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{480 \times 10^{-6} \,\mathrm{C}}{50 \times 10^{-6} \,\mathrm{F}} = \mathbf{9.6 \, V}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{480 \times 10^{-6} \,\mathrm{C}}{10 \times 10^{-6} \,\mathrm{F}} = \mathbf{48.0 \, V}$$
and $E = V_1 + V_2 + V_3 = 2.4 \,\mathrm{V} + 9.6 \,\mathrm{V} + 48 \,\mathrm{V} = \mathbf{60 \, V}$ (checks)

EXAMPLE 10.16 For the network in Fig. 10.70:

- a. Find the total capacitance.
- Determine the charge on each plate.
- c. Find the total charge.



Solutions:

a.
$$C_T = C_1 + C_2 + C_3 = 800 \,\mu\text{F} + 60 \,\mu\text{F} + 1200 \,\mu\text{F} = 2060 \,\mu\text{F}$$

b.
$$Q_1 = C_1 E = (800 \times 10^{-6} \,\text{F})(48 \,\text{V}) = 38.4 \,\text{mC}$$

 $Q_2 = C_2 E = (60 \times 10^{-6} \,\text{F})(48 \,\text{V}) = 2.88 \,\text{mC}$
 $Q_3 = C_3 E = (1200 \times 10^{-6} \,\text{F})(48 \,\text{V}) = 57.6 \,\text{mC}$

c.
$$Q_T = Q_1 + Q_2 + Q_3 = 38.4 \text{ mC} + 2.88 \text{ mC} + 57.6 \text{ mC} = 98.88 \text{ mC}$$