REVIEW ON THE LAST CLASS

Ohm’s Law

Power and Tellegen’s Theorem

Energy

Efficiency

Basic Elements of a Circuit

DEFINING DIRECTION OF CURRENT

DEFINING POLARIES OF VOLTAGE DROP

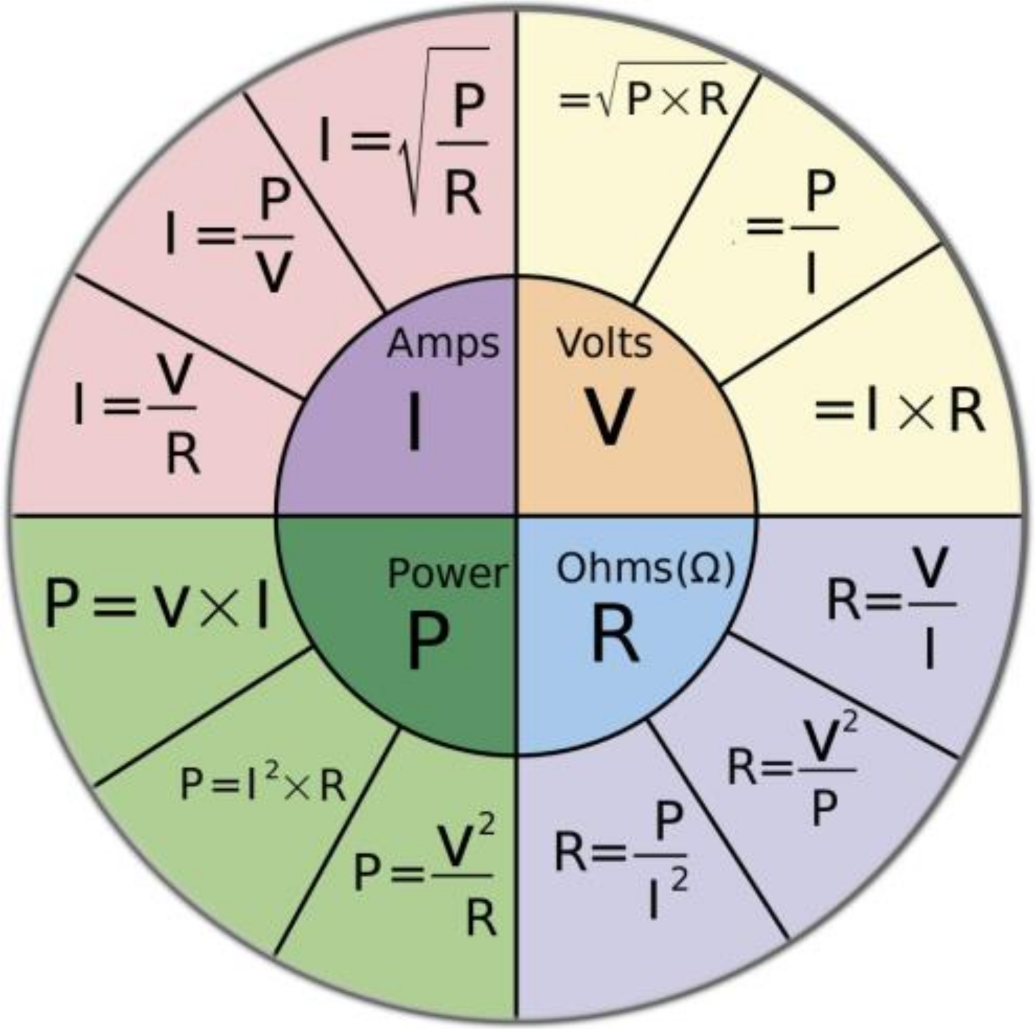
Passive Sign Convention of Power

Branch, Junction Point, Node and Mesh (or Loop)

Two Elements are in Series

Total resistance of Series Resistance

Remember



$$P=\frac{W}{t} \qquad W=Pt$$

$$\%\eta=\frac{P_o}{P_i}\times 100\%=\frac{W_o}{W_i}\times 100\%$$

$$\eta_{total}=\eta_1.\eta_2.\eta_3......\eta_n$$

1 hp = 746 W

For Series Circuit:

$$R_T=R_1+R_2+R_3+......+R_n$$

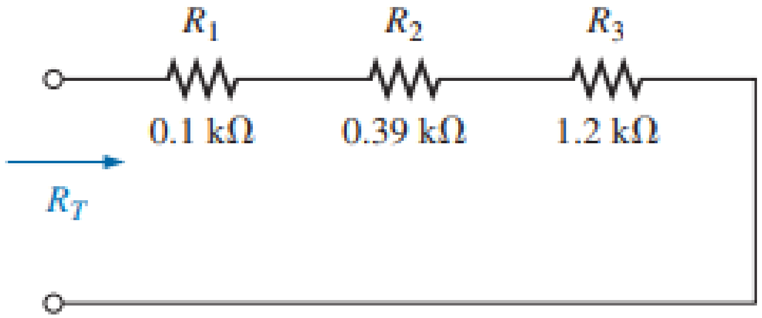
$$R_T>R_1;\ R_T>R_2;\ R_T>R_3;\ ....R_T>R_n$$

If $R_1=R_2=R_3=......=R_n=R$ then

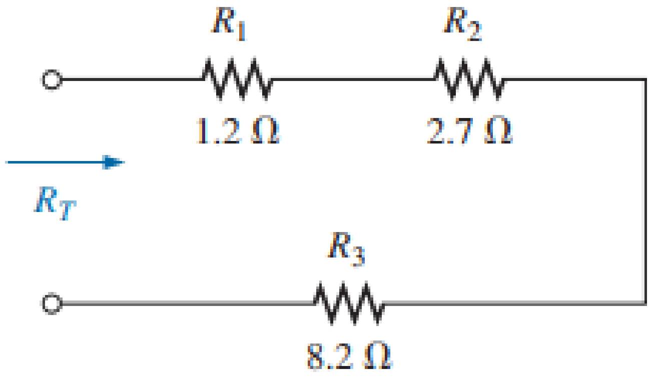
$$R_T=nR$$

Test Your Knowledge

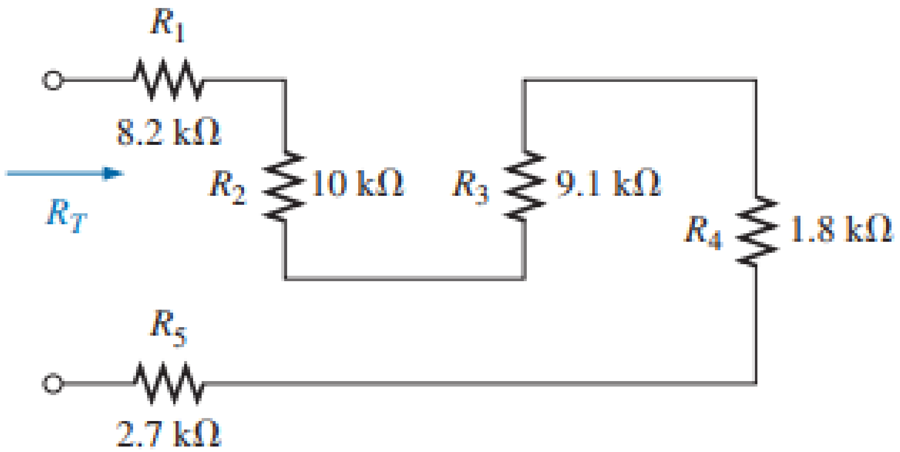
2. Find the total resistance R_T for each configuration in Fig. 5.86. Note that only standard resistor values were used.



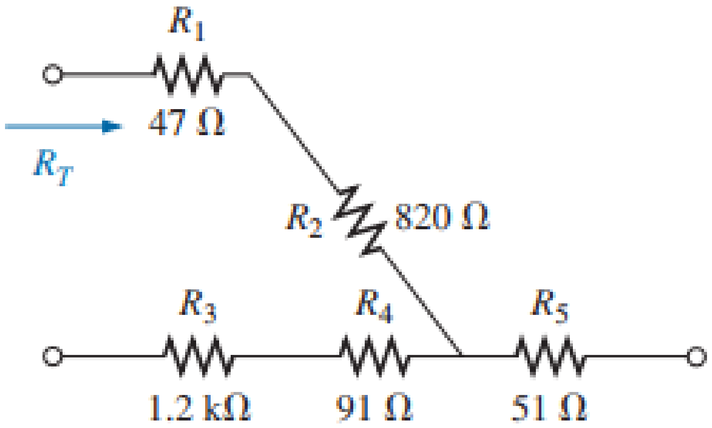
$R_T = 0.1\ \text{k}\Omega + 0.39\ \text{k}\Omega + 1.2\ \text{k}\Omega = \mathbf{1.69\ k\Omega}$



$R_T = 1.2\ \Omega + 2.7\ \Omega + 8.2\ \Omega = \mathbf{12.1\ \Omega}$



$R_T = 8.2\ \text{k}\Omega + 10\ \text{k}\Omega + 9.1\ \text{k}\Omega + 1.8\ \text{k}\Omega + 2.7\ \text{k}\Omega = \mathbf{31\ k\Omega}$



$R_T = 47\ \Omega + 820\ \Omega + 91\ \Omega + 1200\ \Omega = \mathbf{2158\ \Omega}$

FIG. 5.86 *Problem 2.*

Test Your Knowledge

46. a. If a house is supplied with 120 V, 100 A service, find the maximum power capability.

b. Can the homeowner safely operate the following loads at the same time?

5 hp motor
3000 W clothes dryer
2400 W electric range
1000 W steam iron

c. If all the appliances are used for 2 hours, how much energy is converted in kWh?

(**a**) $P = VI = 120\text{ V} \times 100\text{ A} = \mathbf{12000\ W}$ or **12 kW**

(**b**) Total Current: $I_T = [(5\times746\text{W})+3000\text{W}+2400\text{W}+1000\text{W}]/120\text{V} = 84.41\text{ A}$
Since the total current is less than 100 A the homeowner can safely operate all the loads at the same time.

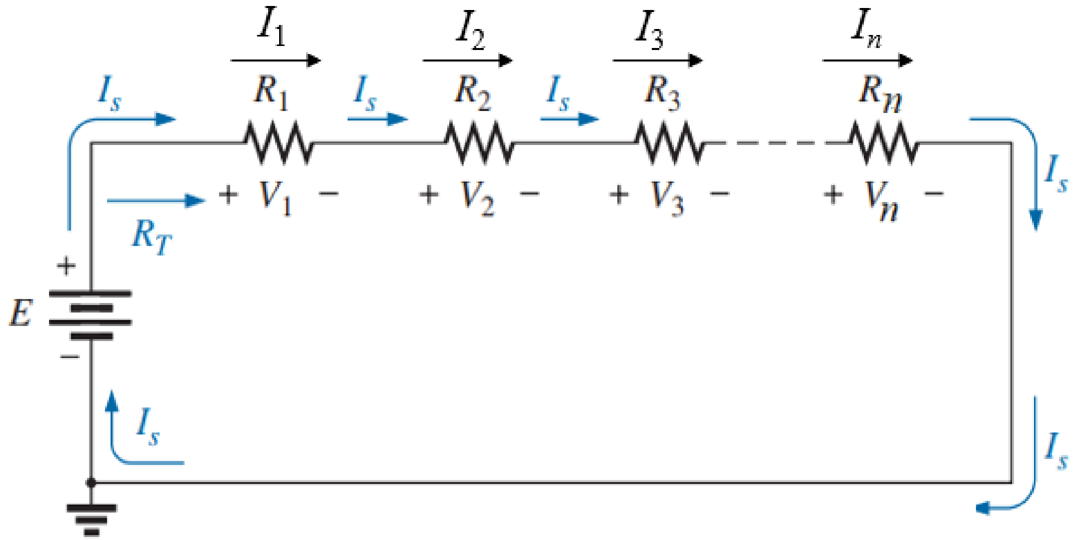
(**c**) Energy $= [(5\times746\text{W})+3000\text{W}+2400\text{W}+1000\text{W}]\times[2\text{h}]/1000 = \mathbf{20.26\ kWh}$

5.3 Series Circuit

A **series circuit** is one in which several resistances are connected one after the other.

Current in a series circuit: The current is the same at every point in a series circuit, that is as following equation for the following circuit.

$$\boxed{I_1 = I_2 = I_3 = \ldots\ldots = I_n = I_s}$$



Current Calculation of a series circuit: According ohm's law, the current of the following circuit is as follows:

$$\boxed{I_s = \frac{E}{R_T}} \quad \text{(A)} \quad \textbf{(5.3)}$$

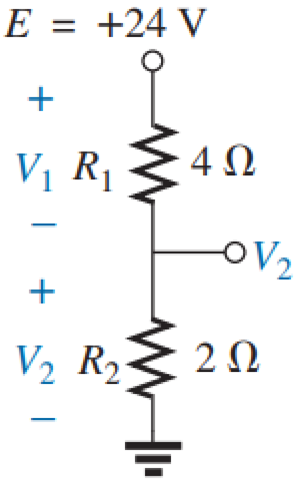
Voltage drop across individual resistance in a series circuit : According ohm's law, the of the following circuit is as follows:

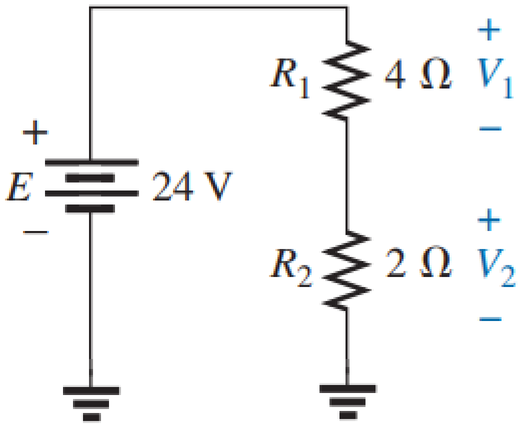
$$\begin{array}{ll} V_1 = I_1R_1 = I_sR_1 & V_2 = I_2R_2 = I_sR_2 \\ V_3 = I_3R_3 = I_sR_3 \;\; \ldots\ldots\ldots & V_n = I_nR_n = I_nR_2 \end{array} \quad \text{(V)} \quad \textbf{(5.4)}$$

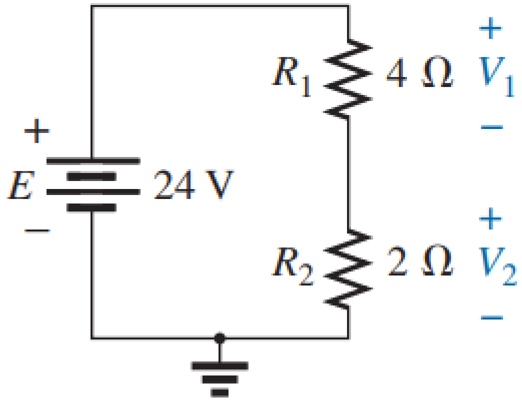
If $R_1 = R_2 = R_3 = \ldots\ldots = R_n = R$

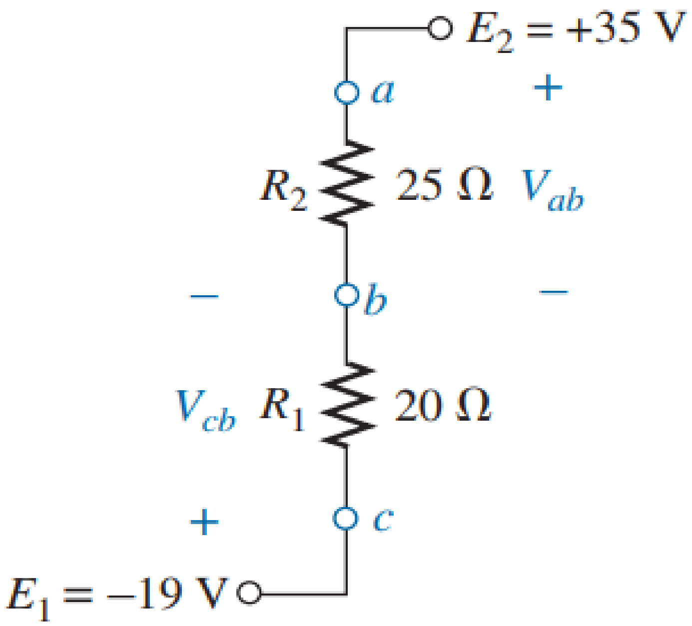
$$\boxed{V_1 = V_2 = V_3 = \ldots. = V_n = \frac{E}{n}}$$

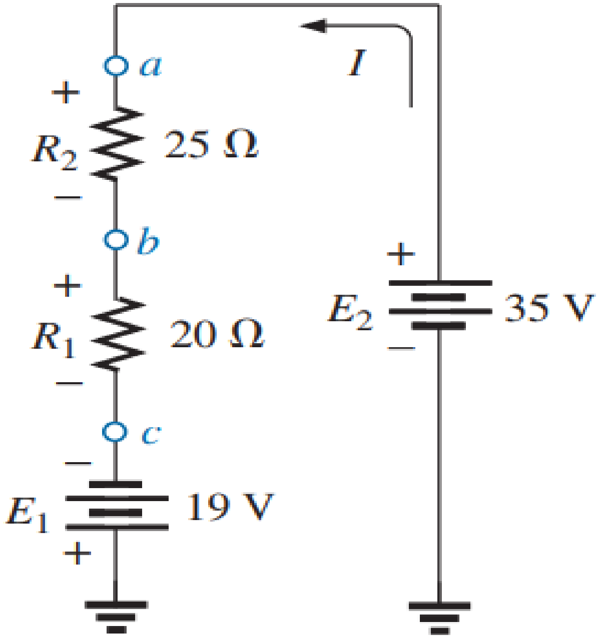
Different Ways to Sketch the Same Series Circuit

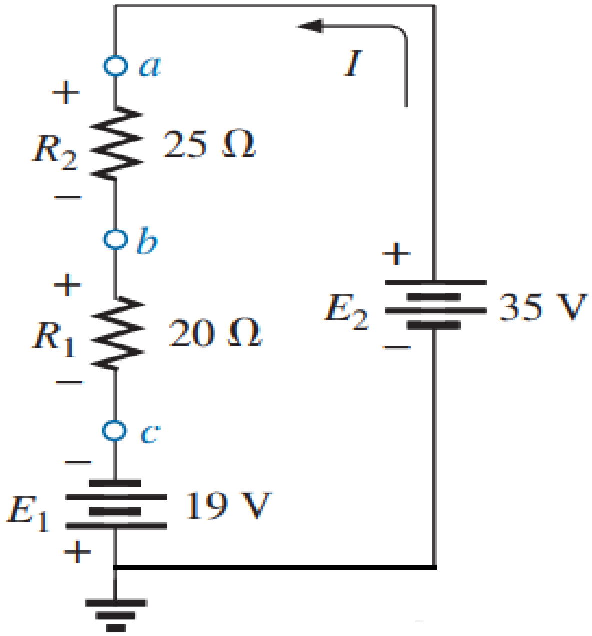












EXAMPLE 5.4 For the series circuit in Fig. 5.15:

a. Find the total resistance R_T.
b. Calculate the resulting source current I_s.
c. Determine the voltage across each resistor.

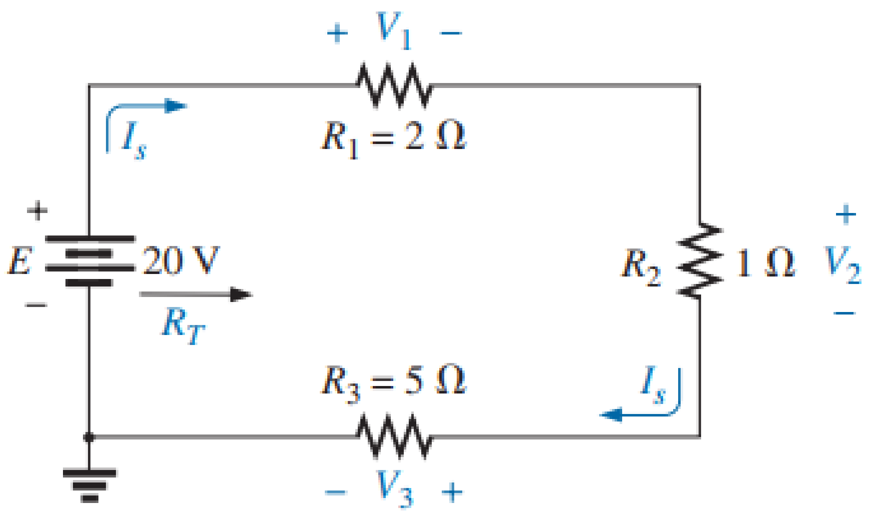


FIG. 5.15
Series circuit to be investigated in Example 5.4.

Solutions:

a. $$R_T = R_1 + R_2 + R_3$$
$$= 2\ \Omega + 1\ \Omega + 5\ \Omega$$
$$R_T = \mathbf{8\ \Omega}$$

b. $$I_s = \frac{E}{R_T} = \frac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$$

c. $$V_1 = I_1R_1 = I_sR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$$
$$V_2 = I_2R_2 = I_sR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$$
$$V_3 = I_3R_3 = I_sR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$$

EXAMPLE 5.6 Given R_T and I_3, calculate R_1 and E for the circuit in Fig. 5.18.

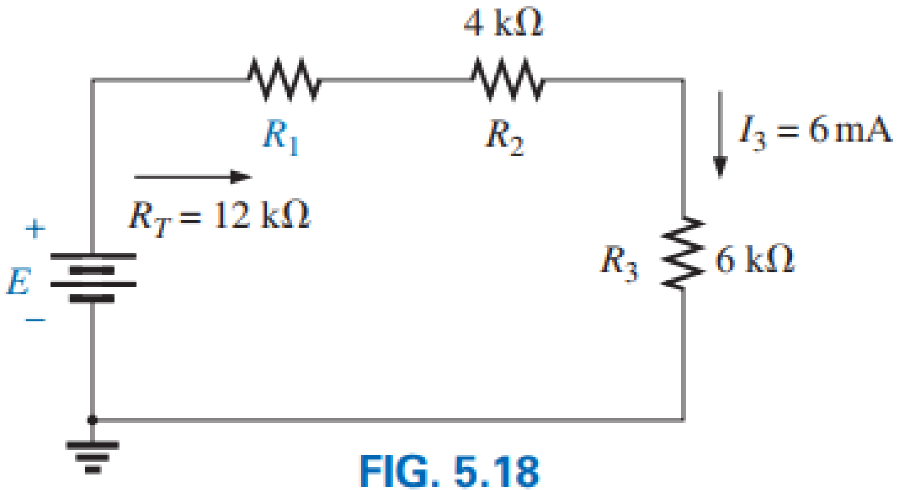


FIG. 5.18

Series circuit to be analyzed in Example 5.6.

Solution: Since we are given the total resistance, it seems natural to first write the equation for the total resistance and then insert what we know.

$$R_T = R_1 + R_2 + R_3$$

We find that there is only one unknown, and it can be determined with some simple mathematical manipulations. That is,

$$12\text{ k}\Omega = R_1 + 4\text{ k}\Omega + 6\text{ k}\Omega = R_1 + 10\text{ k}\Omega$$

and $\quad 12\text{ k}\Omega - 10\text{ k}\Omega = R_1$

so that $\quad R_1 = \mathbf{2\text{ k}\Omega}$

The dc voltage can be determined directly from Ohm's law.

$$E = I_s R_T = I_3 R_T = (6\text{ mA})(12\text{ k}\Omega) = \mathbf{72\text{ V}}$$

Practice Book Problem [5.3 Series Circuit] Problems: 7 to 11

5.7 Voltage Division in Series Circuit
Voltage Divider Rule (VDR)

Voltage drop across individual resistance in a series circuit : According ohm's law, the of the following circuit is as follows:

$$V_1 = I_s R_1 = \left(\frac{E}{R_T}\right) R_1 = \frac{R_1}{R_T} E \qquad \textbf{(5.7.1)}$$

$$V_2 = I_s R_2 = \left(\frac{E}{R_T}\right) R_2 = \frac{R_2}{R_T} E \qquad \textbf{(5.7.2)}$$

$$V_3 = I_s R_3 = \left(\frac{E}{R_T}\right) R_3 = \frac{R_3}{R_T} E \qquad \textbf{(5.7.3)}$$

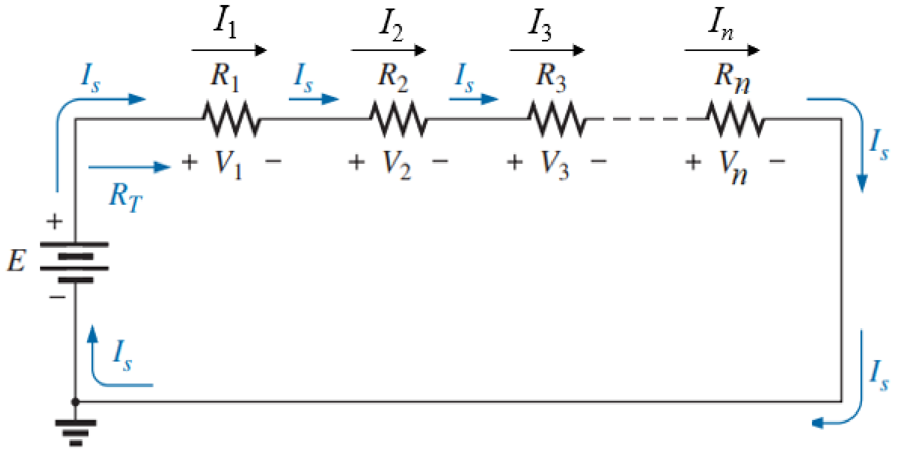
.

$$V_n = I_s R_n = \left(\frac{E}{R_T}\right) R_n = \frac{R_n}{R_T} E \qquad \textbf{(5.7.n)}$$

Based on the Eq. (5.7.1) to (5.7.n), a general equation can be written as follows:

$$\boxed{V_x = R_x \frac{E}{R_T} = \frac{R_x}{R_T} E \qquad \textbf{(5.10)}}$$

Voltage Divider Rule (VDR): The voltage across a resistor in a series circuit is equal to the value of that resistor (*R_x*) times the total applied voltage (*E*) divided by the total resistance (*R_T*) of the series configuration.



EXAMPLE 5.16 Using the voltage divider rule, determine voltages V_1, V_2 and V_3 for the series circuit in **Fig. 5.38**.

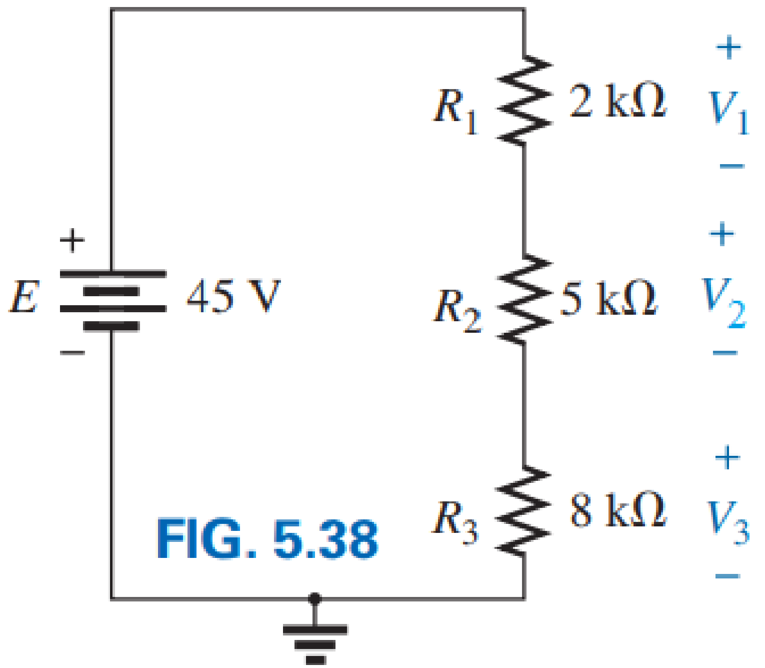


FIG. 5.38

Solution:

$$R_T = R_1 + R_2 + R_3$$
$$= 2\text{ k}\Omega + 5\text{ k}\Omega + 8\text{ k}\Omega$$
$$= 15\text{ k}\Omega$$

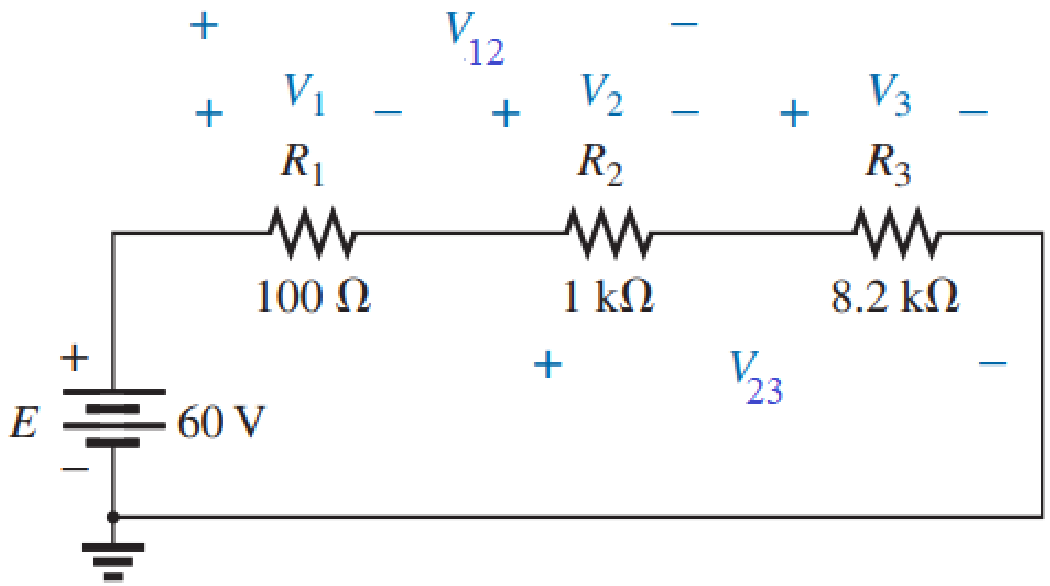
$$V_1 = R_1\frac{E}{R_T} = 2\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{6\text{ V}}$$

$$V_2 = R_2\frac{E}{R_T} = 5\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{15\text{ V}}$$

$$V_3 = R_3\frac{E}{R_T} = 8\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{24\text{ V}}$$

Practice Book Problem [5.7 Voltage Divider Rule] Problems: 24, 25 and 26

EXAMPLE 5.7.1: Using the voltage divider rule, determine voltages V_1, V_2, V_3, V_{12} and V_{23} for the following series circuit.



Solution:
$$R_T = R_1 + R_2 + R_3$$
$$= 100\ \Omega + 1\ \text{k}\Omega + 8.2\ \text{k}\Omega = 9.3\ \text{k}\Omega$$

$$V_1 = R_1 \frac{E}{R_T} = 100\ \Omega \left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{0.65\ V}$$

$$V_2 = R_2 \frac{E}{R_T} = 1\ \text{k}\Omega \left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{6.45\ V}$$

$$V_3 = R_3 \frac{E}{R_T} = 8.2\ \text{k}\Omega \left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{52.9\ V}$$

V_{12} is the voltage drop across the series combination of R_1 and R_2 and V_{23} is the voltage drop across the series combination of R_2 and R_3.

$$R_{12} = R_1 + R_2 = 100\ \Omega + 1\ \text{k}\Omega = 1.1\ \text{k}\Omega$$
$$R_{23} = R_2 + R_3 = 1\ \text{k}\Omega + 8.2\ \text{k}\Omega = 9.2\ \text{k}\Omega$$

$$V_{12} = R_{12} \frac{E}{R_T} = 1.1\ \text{k}\Omega \left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{7.1\ V}$$

$$V_{23} = R_{23} \frac{E}{R_T} = 9.2\ \text{k}\Omega \left(\frac{60\ \text{V}}{9.3\ \text{k}\Omega}\right) = \mathbf{59.35\ V}$$

5.6 KIRCHHOFF'S VOLTAGE LAW (KVL)

Statement:

(**1**) The algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.

OR

(**2**) The sum of the applied or supplied or rise voltage of a series dc circuit will equal the sum of the voltage drops of the circuit.

According to Statement (1):

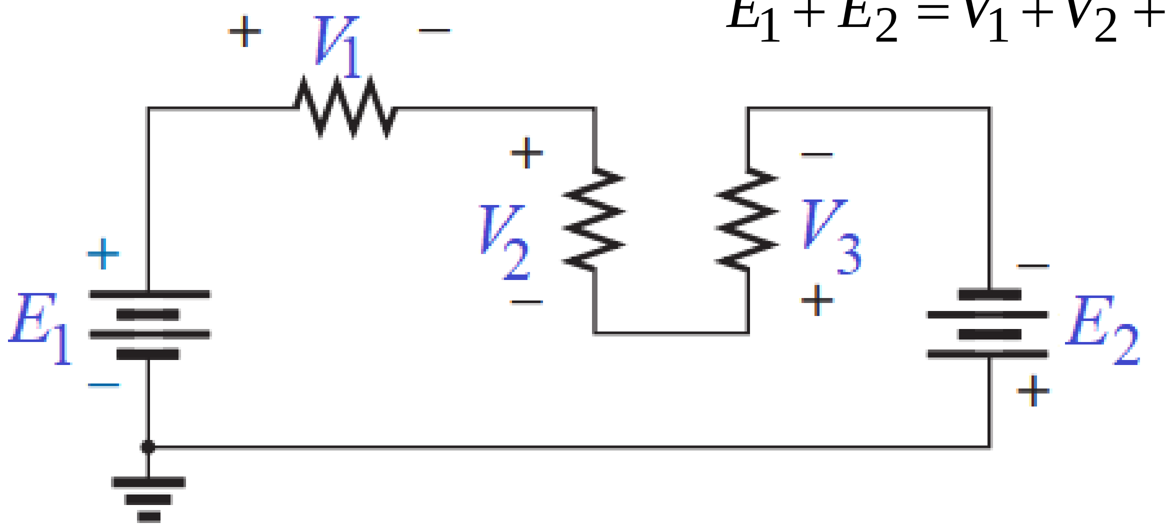
$$\boxed{\Sigma_{\circlearrowright} V = 0} \qquad \text{(Kirchhoff's voltage law in symbolic form)} \quad \textbf{(5.8)}$$

$$E_1 - V_1 - V_2 - V_3 + E_2 = 0$$

According to Statement (2):

$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}} \quad \textbf{(5.9)}$$

$$E_1 + E_2 = V_1 + V_2 + V_3$$



EXAMPLE 5.8 Use Kirchhoff's voltage law to determine the unknown voltage (V_1) for the circuit in Fig. 5.27.

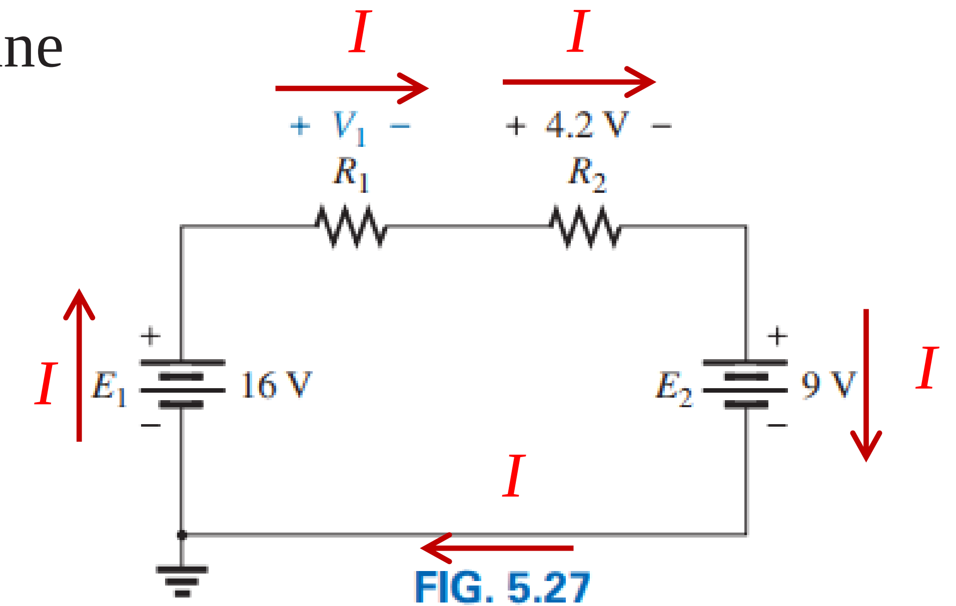


FIG. 5.27
Series circuit to be examined in Example 5.8.

Solution: Let considered current (I) make closed path by flowing in clockwise (CW) direction.

I is entering through **negative** terminal of E_1.

Now, let E_1 is **positive**.

I is entering through **positive** terminal of V_1.

So V_1 is **negative**.

I is entering through **positive** terminal of 4.2 V.

So 4.2 V is **negative**.

I is entering through **positive** terminal of E_2.

So E_2 is **negative**.

According to KVL we have:

$$+E_1 - V_1 - 4.2\text{V} - E_2 = 0$$

$$V_1 = E_1 - 4.2\text{V} - E_2$$

$$V_1 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$

$$= \mathbf{2.8\ V}$$

EXAMPLE 5.9 Determine the unknown voltage for the circuit in Fig. 5.28.

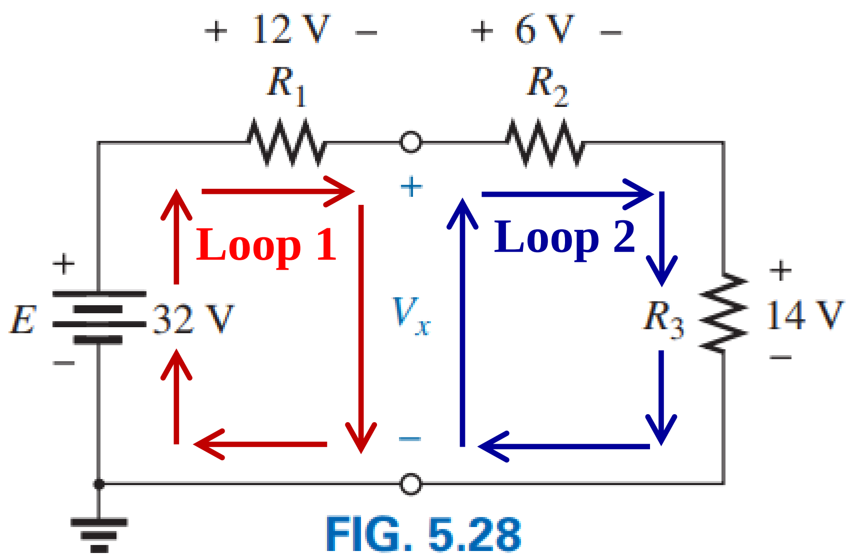


FIG. 5.28

Solution: This problem can be solved by two ways.

First way: Apply KVL around a path, including the source E and R_1 (as **Loop 1**).

$$+E_1 - 12V - V_x = 0$$

$$V_x = E_1 - 12V = 22V - 12V = \mathbf{20\ V}$$

Second way: Apply KVL around a path, including the R_2 and R_3 (as **Loop 2**).

$$+V_x - 6V - 14V = 0$$

$$V_x = 6V + 14V = \mathbf{20\ V}$$

EXAMPLE 5.6.1 Determine the unknown voltages for the following circuit.

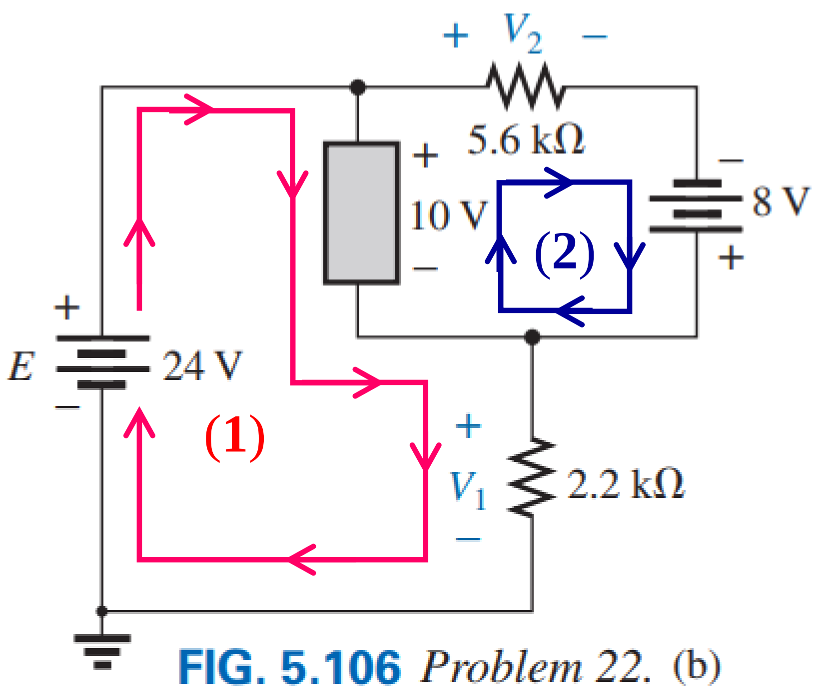


FIG. 5.106 *Problem 22.* (b)

Solution: Apply KVL in **Loop 1**:

$$+E_1 - 10\text{V} - V_1 = 0$$

$$V_1 = E_1 - 10\text{V} = 24\text{V} - 10\text{V} = \mathbf{14\ V}$$

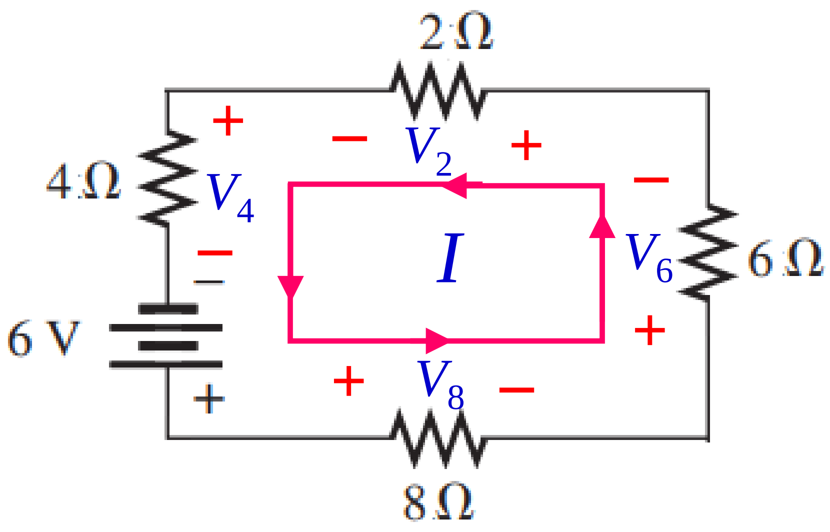
Apply KVL in **Loop 2**:

$$+E_2 + 10\text{V} - V_2 = 0$$

$$V_1 = E_2 + 10\text{V} = 8\text{V} + 10\text{V} = \mathbf{18\ V}$$

Practice Book Problem [SECTION 5.6 Kirchhoff's Voltage Law] Problems: 20 to 23

Using KVL, write the Loop Equation of the following Circuit:



$$6\text{ V} - V_8 - V_6 - V_2 - V_4 = 0$$

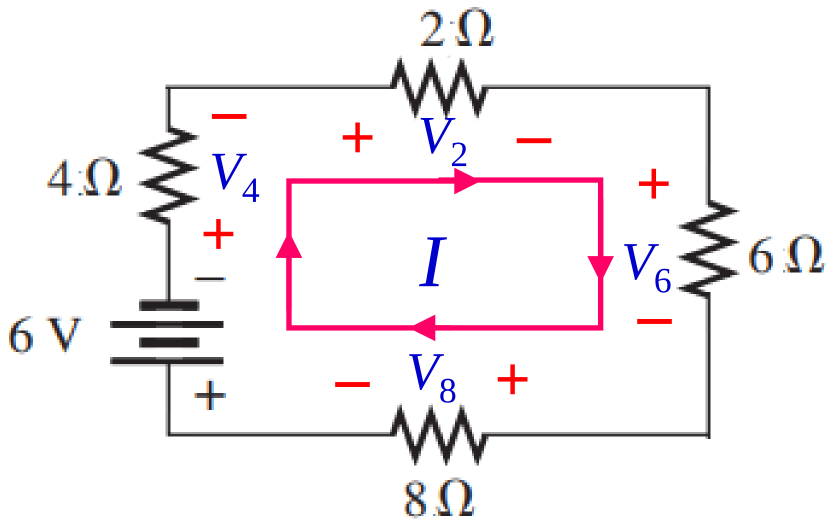
$$V_8 + V_6 + V_2 + V_4 = 6\text{ V}$$

$$8I + 6I + 2I + 4I = 6\text{ V}$$

$$(8+6+2+4)I = 6\text{ V}$$

$$20I = 6\text{ V}$$

Consider voltage is positive if current entering through negative terminal of an elements and voltage is negative if current entering through positive terminal of an elements.



$$6\text{ V} + V_4 + V_2 + V_6 + V_8 = 0$$

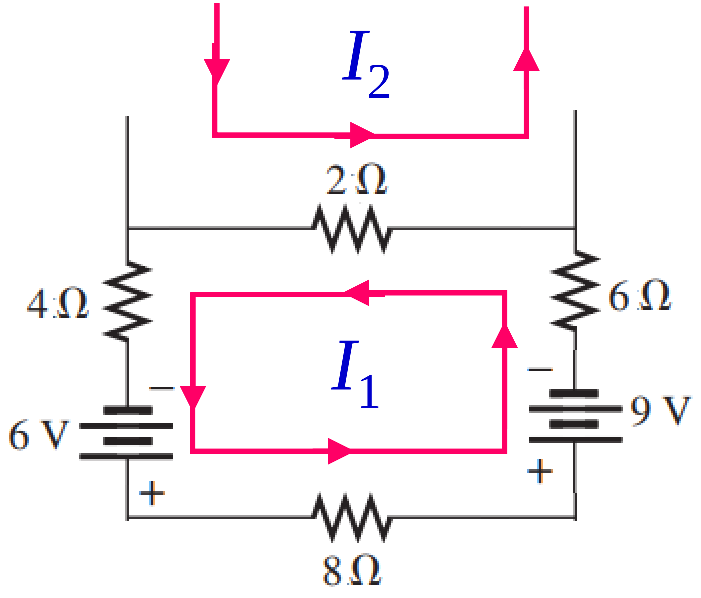
$$V_4 + V_2 + V_6 + V_8 = -6\text{ V}$$

$$4I + 2I + 6I + 8I = -6\text{ V}$$

$$(4+2+6+8)I = -6\text{ V}$$

$$20I = -6\text{ V}$$

Using KVL, write the Loop Equation of the following Circuit:



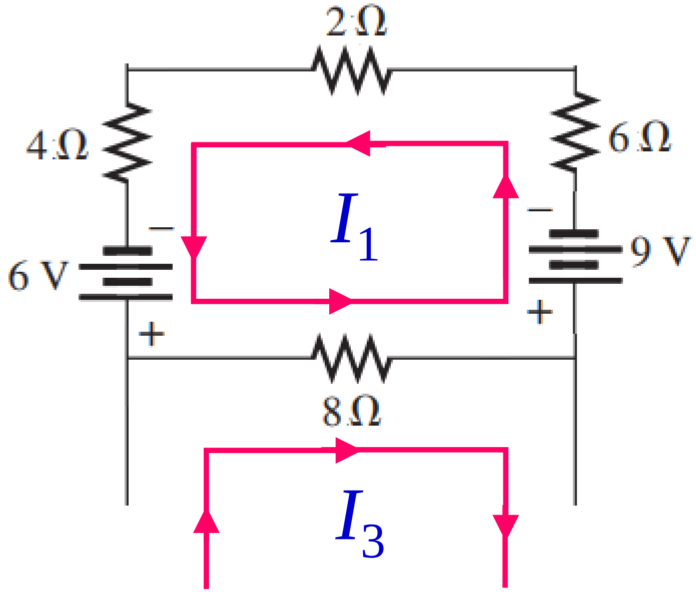
$$6\text{ V} - 8I_1 - 9\text{ V} - 6I_1 - 2(I_1 - I_2) - 4I_1 = 0$$

$$8I_1 + 6I_1 + 2(I_1 - I_2) + 4I_1 = 6\text{ V} - 9\text{ V}$$

$$(8+6+2+4)I_1 - 2I_2 = 6\text{ V} - 9\text{ V}$$

$$(8+6+2+4)I_1 - 2I_2 = -3\text{ V}$$

If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which KVL is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.



$$6\text{ V} - 8(I_1 + I_3) - 9\text{ V} - 6I_1 - 2I_1 - 4I_1 = 0$$

$$8(I_1 + I_3) + 6I_1 + 2I_1 + 4I_1 = 6\text{ V} - 9\text{ V}$$

$$(8+6+2+4)I_1 + 8I_3 = 6\text{ V} - 9\text{ V}$$

$$(8+6+2+4)I_1 + 8I_3 = -3\text{ V}$$

5.4 Power Distribution in a Series Circuit

In any electrical system, the power supplied or applied or delivered will equal the power dissipated or absorbed or consumed.

$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}} \quad \textbf{(5.5)}$$

$$\boxed{P_E = EI_s} \qquad \text{(watts, W)} \quad \textbf{(5.6)}$$

$$\begin{aligned}
P_1 &= V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} = V_1 I_s = I_s^2 R_1 \\
P_2 &= V_2 I_2 = I_2^2 R_2 = \frac{V_2^2}{R_2} = V_2 I_s = I_s^2 R_2 \\
P_3 &= V_3 I_3 = I_3^2 R_3 = \frac{V_3^2}{R_3} = V_3 I_s = I_s^2 R_3
\end{aligned} \qquad \text{(W)} \quad \textbf{(5.7)}$$

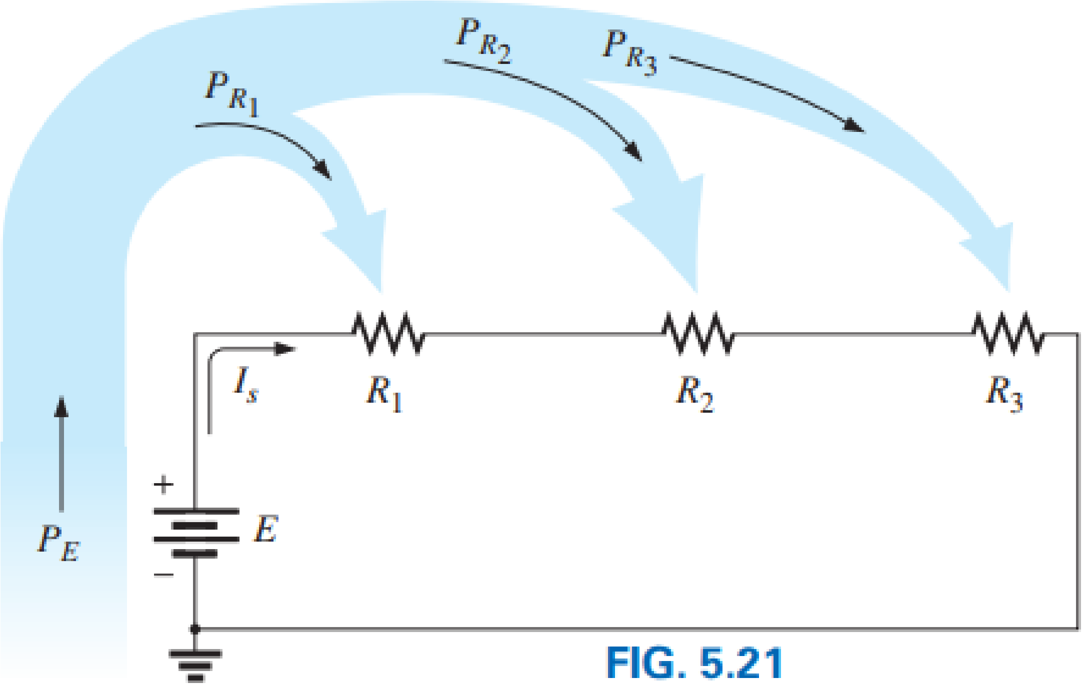


FIG. 5.21
Power distribution in a series circuit.

EXAMPLE 5.7 For the series circuit in Fig. 5.22 (all standard values):

a. Determine the total resistance R_T.
b. Calculate the current I_s.
c. Determine the voltage across each resistor.
d. Find the power supplied by the battery.
e. Verify Kirchhoff's Voltage Law (KVL).
f. Determine the power dissipated by each resistor.
g. Comment on whether the total power supplied equals the total power dissipated.

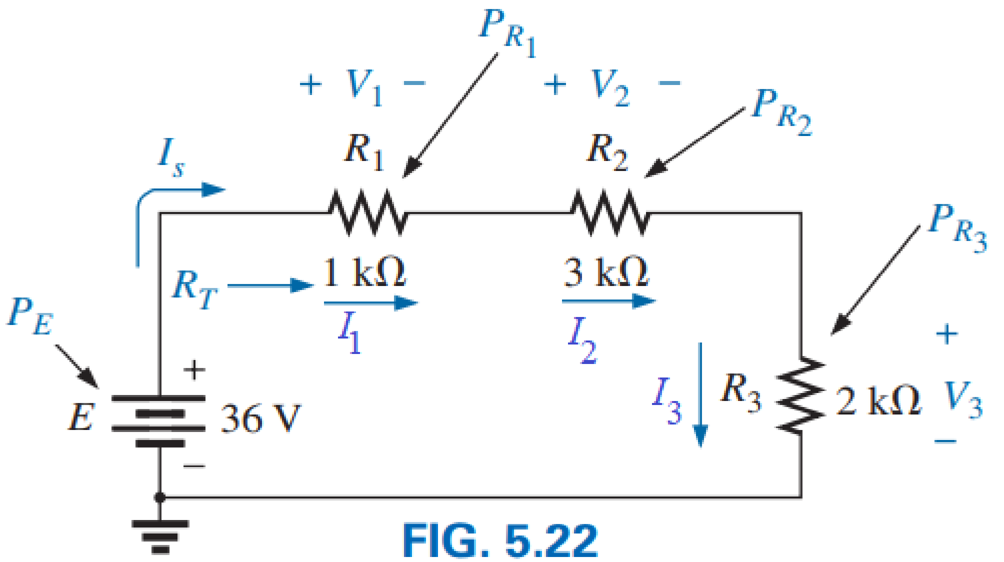


FIG. 5.22

Series circuit to be investigated in Example 5.7.

Solution:

$$(a)\quad R_T = R_1 + R_2 + R_3 = 1\ \text{k}\Omega + 3\ \text{k}\Omega + 2\ \text{k}\Omega = \mathbf{6\ k\Omega}$$

$$(\mathbf{b})\quad I_s = I_1 = I_2 = I_3 = \frac{E}{R_T} = \frac{36\,\text{V}}{6\ \text{k}\Omega} = \mathbf{6\ mA}$$

$$(\mathbf{c})\quad V_1 = R_1\frac{E}{R_T} = I_1R_1 = I_sR_1 = (6\,\text{mA})(1\ \text{k}\Omega) = \mathbf{6\ V}$$

$$V_2 = R_2\frac{E}{R_T} = I_2R_2 = I_sR_2 = (6\,\text{mA})(3\ \text{k}\Omega) = \mathbf{18\ V}$$

$$V_3 = R_3\frac{E}{R_T} = I_3R_3 = I_sR_3 = (6\,\text{mA})(2\ \text{k}\Omega) = \mathbf{12\ V}$$

$$(\mathbf{d})\quad V_1 + V_2 + V_3 = 6\,\text{V} + 18\,\text{V} + 12\,\text{V} = \mathbf{36\ V} = E\ \ [\textbf{Verified}]$$

$$(\mathbf{e})\quad P_E = I_s^2R_T = \frac{E^2}{R_T} = EI_s = (36\ \text{V})(6\ \text{mA}) = \mathbf{216\ mW}$$

EXAMPLE 5.7 For the series circuit in Fig. 5.22 (all standard values):

f. Determine the power dissipated by each resistor.

g. Comment on whether the total power supplied equals the total power dissipated.

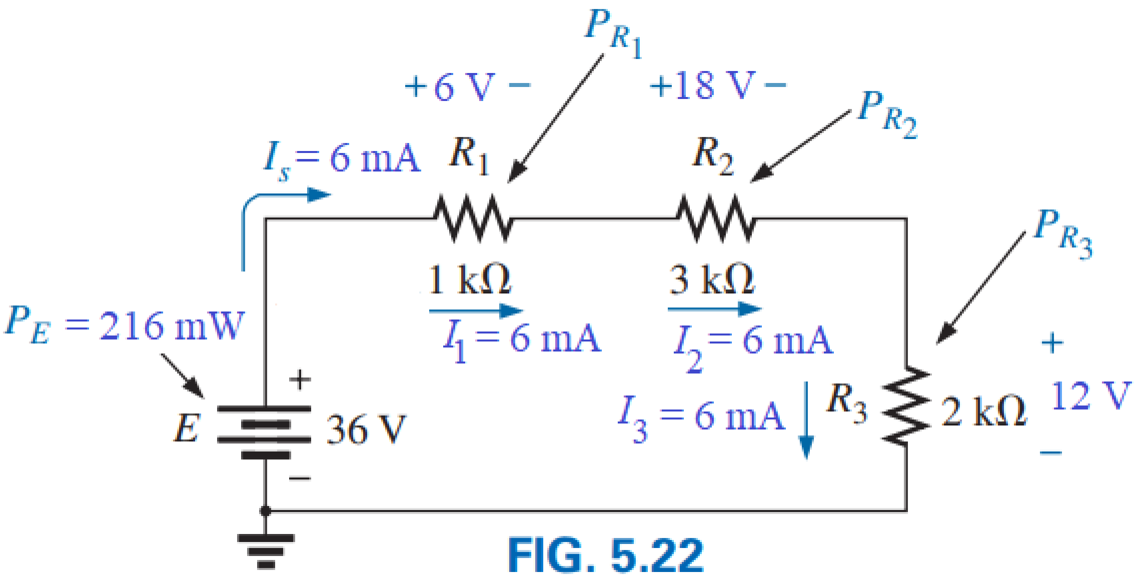


FIG. 5.22

Series circuit to be investigated in Example 5.7.

Solution:

$$\text{(e)}\quad P_{R1} = I_1^2 R_1 = I_s^2 R_1 = V_1 I_1 = \frac{V_1^2}{R_1} = V_1 I_s = (6\text{ V})(6\text{ mA}) = \mathbf{36\ mW}$$

$$P_{R2} = I_s^2 R_2 = V_2 I_2 = V_2 I_s = \frac{V_2^2}{R_2} = I_2^2 R_2 = (6\text{ mA})^2 (3\text{ k}\Omega) = \mathbf{108\ mW}$$

$$P_{R3} = I_3^2 R_3 = I_s^2 R_3 = V_3 I_3 = V_3 I_s = \frac{V_3^2}{R_3} = \frac{(12\text{ V})^2}{2\text{ k}\Omega} = \mathbf{72\ mW}$$

$$\text{(f)}\quad P_E = P_{R1} + R_{R2} + R_{R3}$$
$$= 36\text{ mW} + 108\text{ mW} + 72\text{ mW}$$
$$= \mathbf{216\ mW}\ \ \text{(Checked)}$$

Practice Book Problem [5.4 Power Distribution] Problems: 12 to 16

Single-Subscript Notation of Voltage

A single-subscript notation of voltage used to provide the voltage at a point with respect to ground.

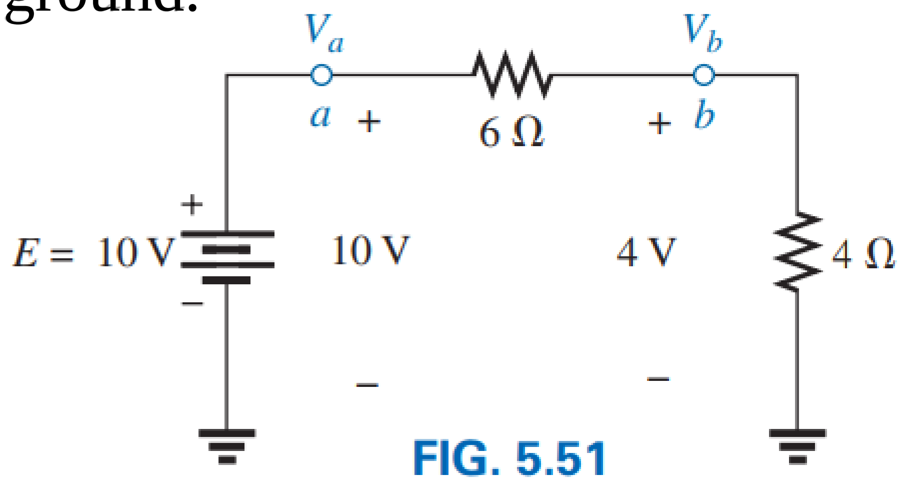


FIG. 5.51

Defining the use of single-subscript notation for voltage levels.

In Fig. 5.51:

V_a is the voltage from point ***a*** to ***ground***. In this case, it is obviously 10 V since it is right across the source voltage E. i.e. $V_a = E = 10$ V.

V_b is the voltage from point ***b*** to ***ground***. Because it is directly across the 4 Ω resistor, $V_b = 4$ V.

Here, $V_{ab} = V_a - V_b = 6$ V $\quad V_{ba} = V_b - V_a = -6$ V

EXAMPLE 5.25 Determine V_{ab}, V_{cb}, and V_c for the network in Fig. 5.59.

Solution:

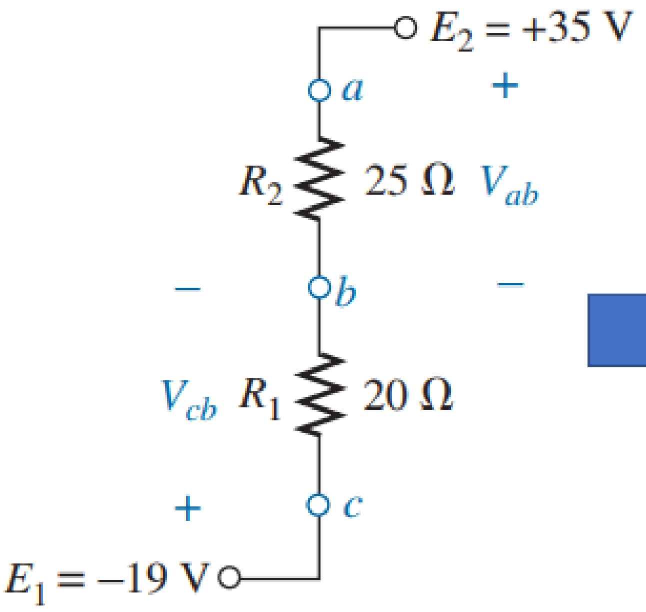
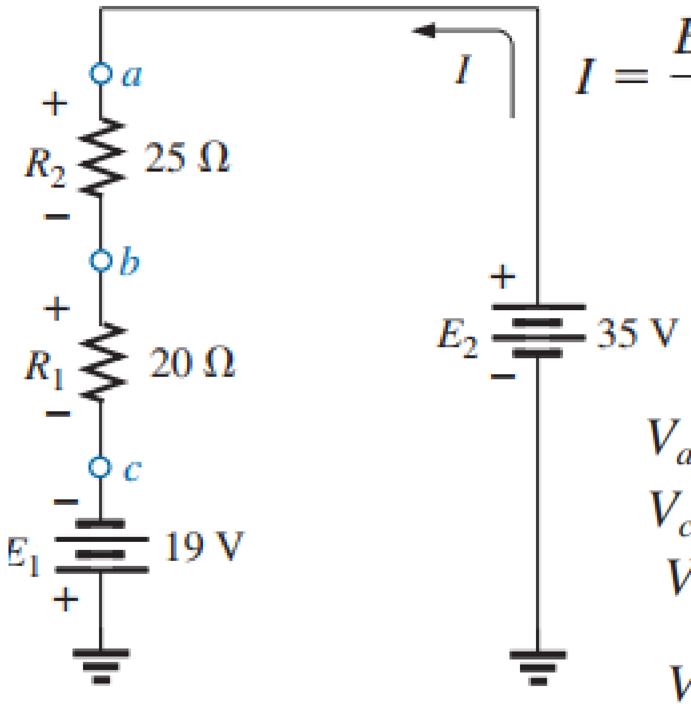


FIG. 5.59 *Example 5.25.*

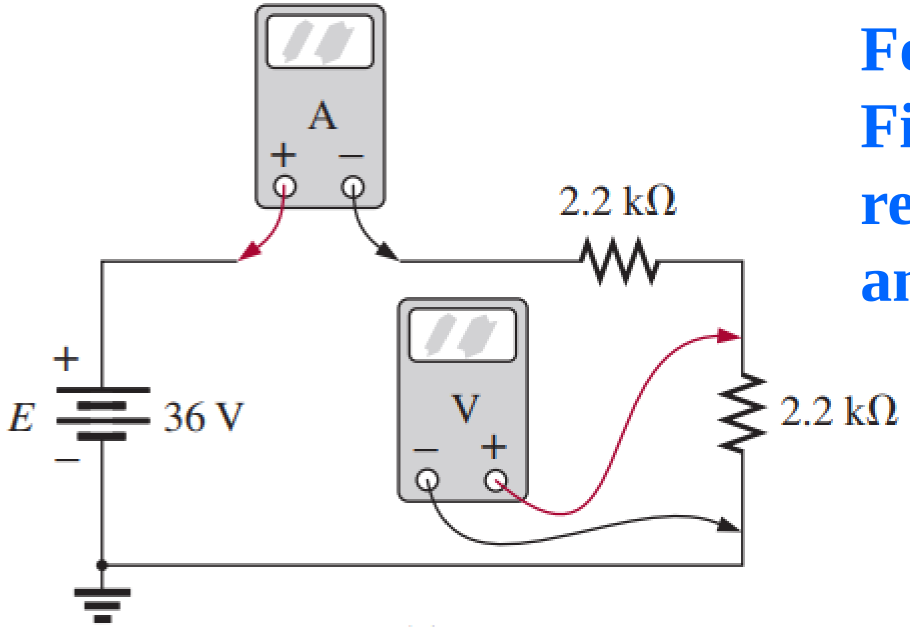


$$I = \frac{E_1 + E_2}{R_T} = \frac{19\text{ V} + 35\text{ V}}{45\ \Omega} = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

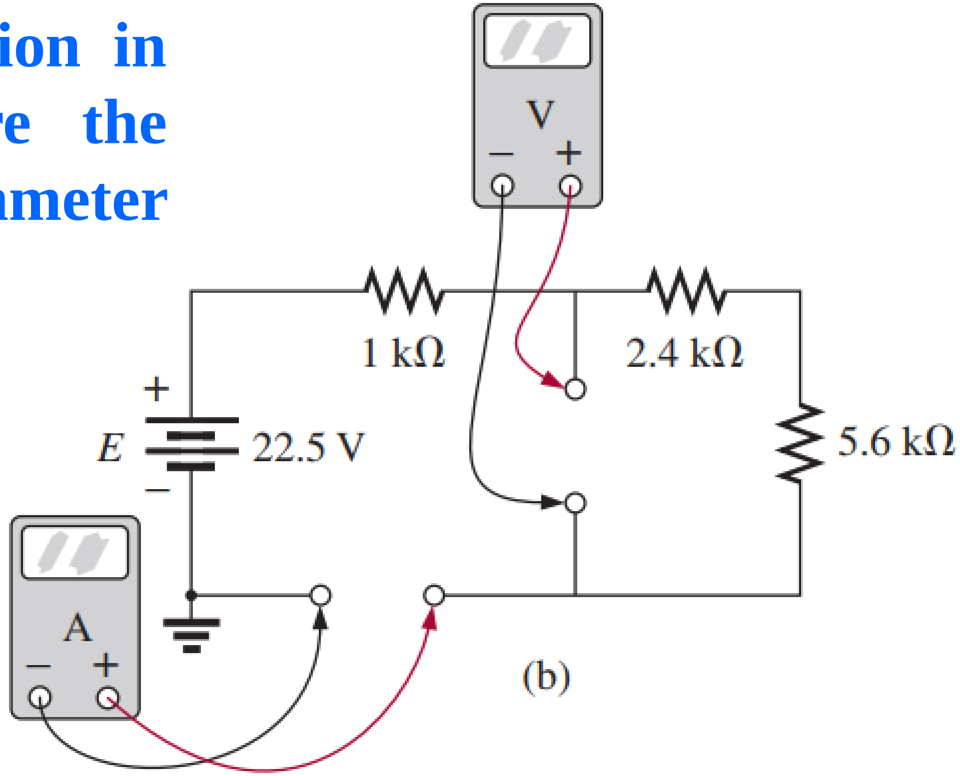
$$V_{ab} = IR_2 = (1.2\text{ A})(25\ \Omega) = \mathbf{30\ V}$$
$$V_{cb} = -IR_1 = -(1.2\text{ A})(20\ \Omega) = \mathbf{-24\ V}$$
$$V_c = E_1 = \mathbf{-19\ V}$$

$V_{ab} = \mathbf{30\ V}$ $\quad V_{cb} = \mathbf{-24\ V}$ $\quad V_c = \mathbf{-19\ V}$

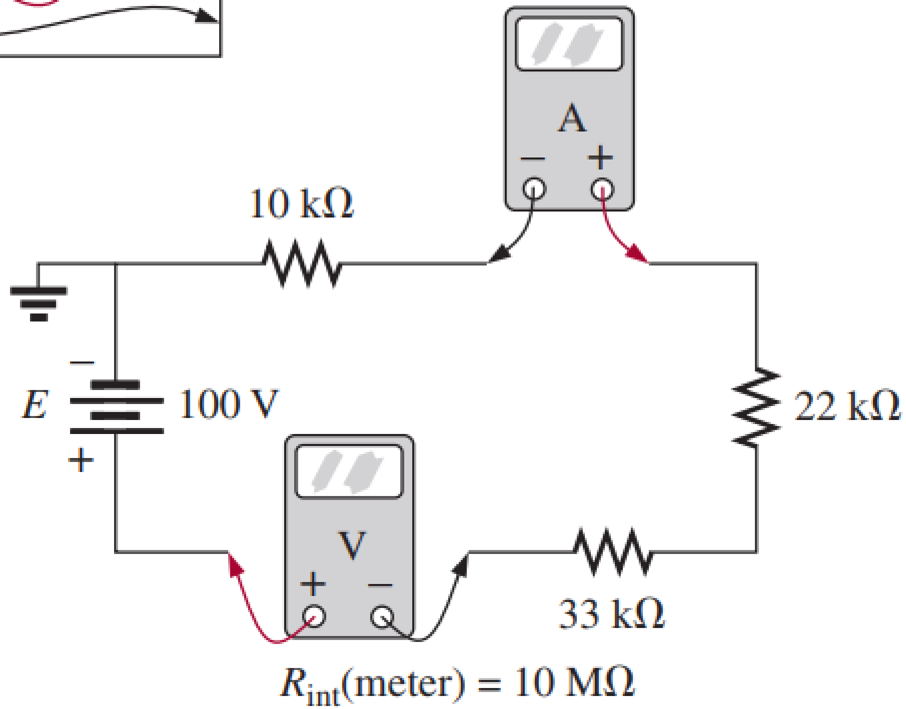
For each configuration in Fig. 5.95, what are the readings of the ammeter and the voltmeter?



(a)



(b)



(c)

Chapter 6

Parallel DC Circuit

Two Elements are in Parallel

Two **elements**, **branches**, or **circuits** are in parallel:

1. They have **two points in common**.
2. The **currents are not the same** through the two parallel elements.
3. The **voltage are the same across** the two parallel elements.

Figure 1: R_1 and R_2 are parallel since they have two common points *a* and *b*.

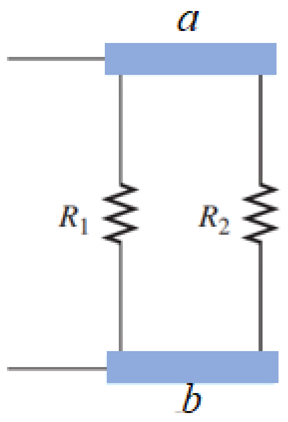


Figure 1

Figure 2: R_1 and R_2 are parallel since they have two common points *a* and *b*. R_3 is series with the parallel combination of R_1 and R_2.

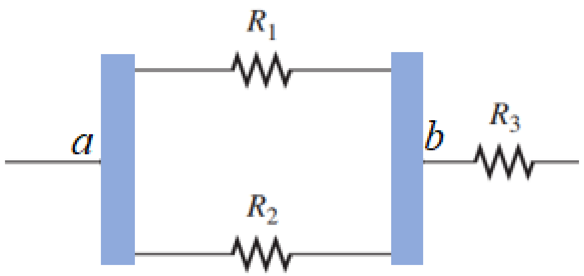


Figure 2

Figure 3: R_1 and R_2 are in series. The series combination of R_1 and R_2 are parallel with R_3

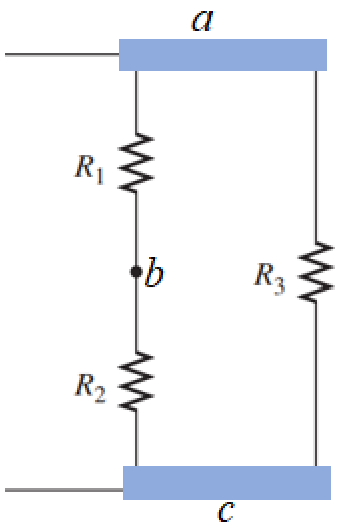
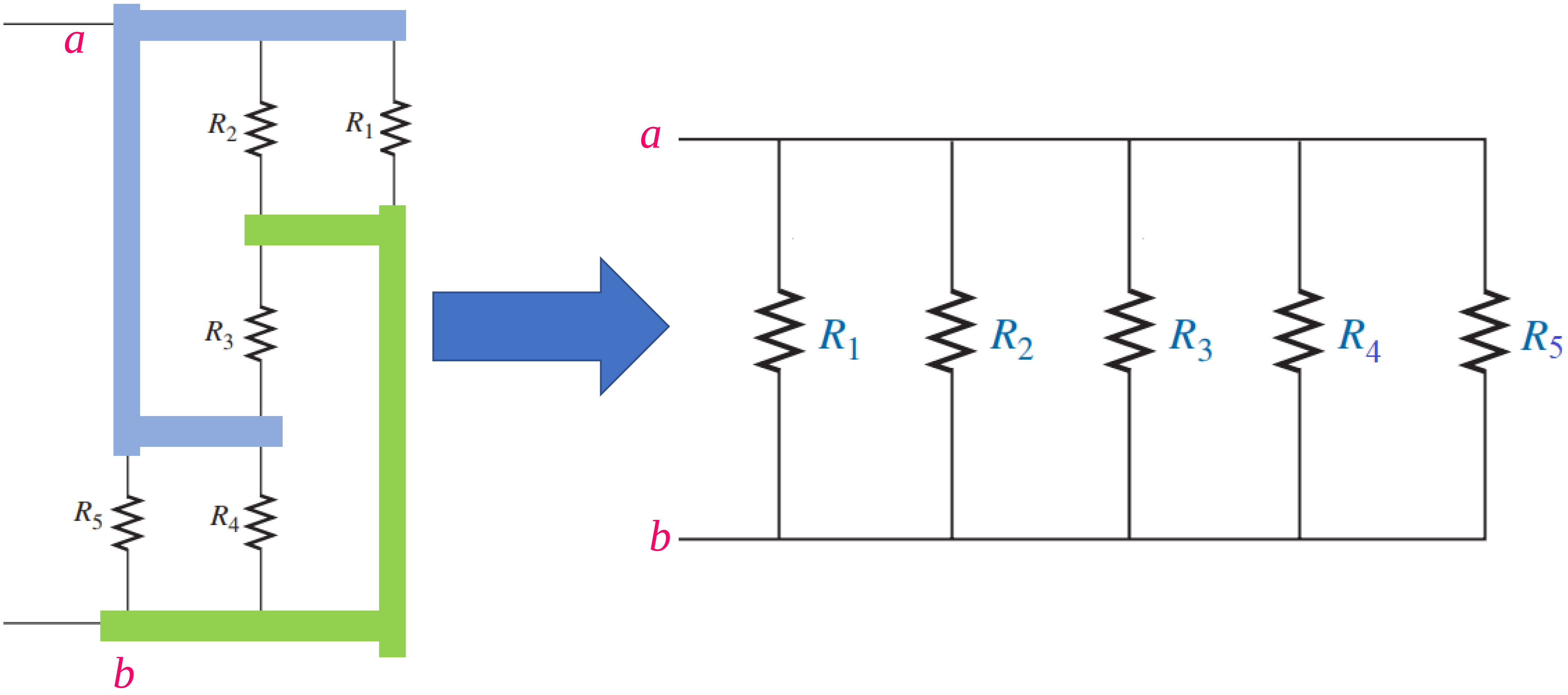


Figure 3

Identify the Combination of Connection for the Following Circuit



6.2 Parallel Resistance

- The **total or net or effective or equivalent conductance** of a parallel configuration is **the sum of the value of individual conductance**, that is Eq. (**6.2**).
- The reciprocal of the total or effective or net or equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances, that is Eq. **(6.1)**. Eq. (**6.3**) can be obtained from Eq. (6.1).

$$\boxed{G_T = G_1 + G_2 + G_3 + \cdots + G_N} \quad \text{(siemens, S)} \quad \textbf{(6.2)}$$

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \quad \textbf{(6.1)}$$

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}} \quad \textbf{(6.3)}$$

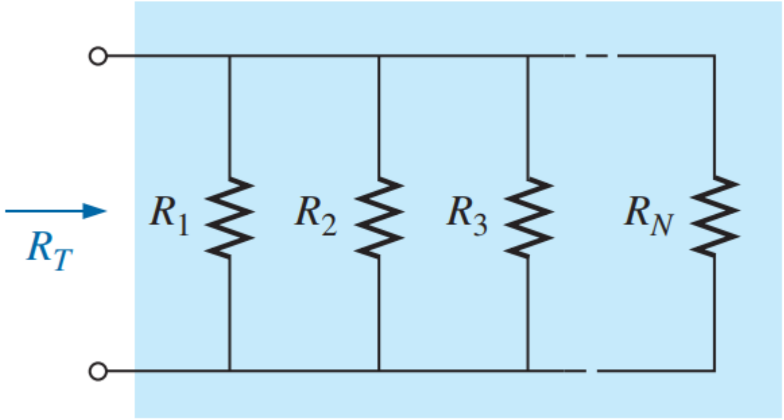


FIG. 6.3 *Parallel combination of resistors.*

- The total or equivalent resistance is the smallest of all the individual resistances. that is Eq. **(6.1.1)**.
- The total resistance of n resistors of the same value in parallel is simply ***divide the value of one of the resistors by the number of resistor in parallel***; that is as Eq. (**6.4**)

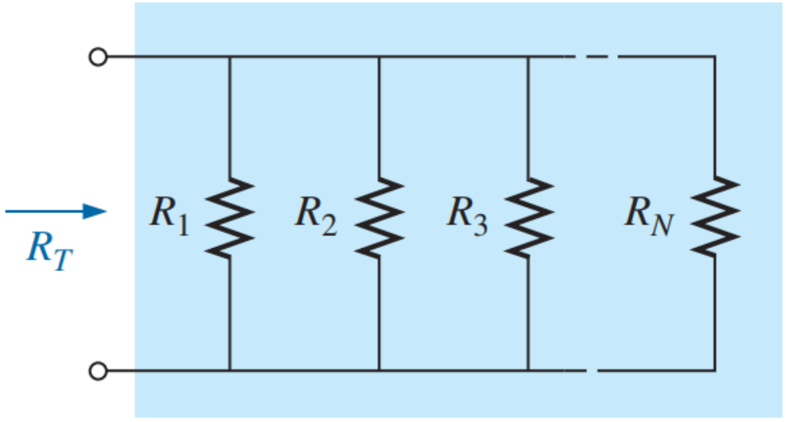


FIG. 6.3 *Parallel combination of resistors.*

$$\boxed{R_T < R_1;\ \ R_T < R_2; \ldots\ldots, R_T < R_N} \quad \textbf{(6.1.1)}$$

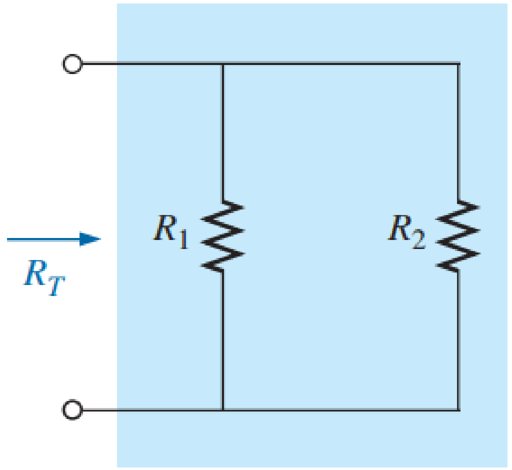
$$\text{If } R_1 = R_2 = R_3 = \ldots\ldots = R_N = R \qquad R_N = \frac{R}{N} \quad \textbf{(6.4)}$$

Special Case: Two Parallel Resistors

The total resistance of two parallel resistors is simply the product of their values divided by their sum.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore\ R_T = \frac{R_1 R_2}{R_1 + R_2} \quad \textbf{(6.5)}$$



EXAMPLE 6.1

a. Find the total conductance of the parallel network in Fig. 6.4.

b. Find the total resistance of the same network using (***i***) the results of part (a), (***ii***) using Eq. (6.3) and (***iii***) using Eq. (6.5).

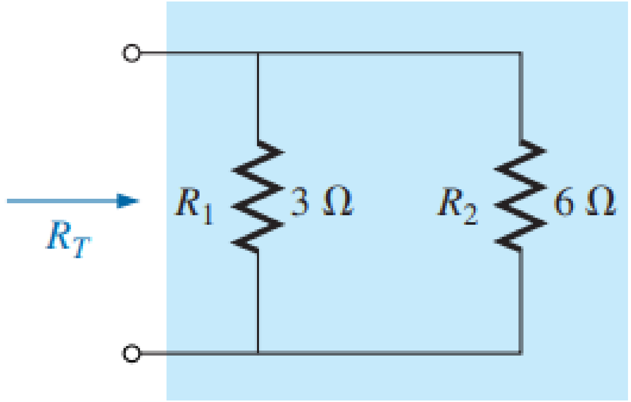


FIG. 6.4

Solution:

a. $G_1 = \dfrac{1}{R_1} = \dfrac{1}{3\Omega} = 0.333\text{ S}, \qquad G_2 = \dfrac{1}{R_2} = \dfrac{1}{6\Omega} = 0.167\text{ S}$

$$G_T = G_1 + G_2 = 0.333\text{ S} + 0.167\text{ S} = \mathbf{0.5\ S}$$

b. (***i***) $R_T = \dfrac{1}{G_T} = \dfrac{1}{0.5\text{ S}} = \mathbf{2\ \Omega}$

(***ii***) Applying Eq. (6.3):

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{1}{\dfrac{1}{3\Omega} + \dfrac{1}{6\Omega}}$$

$$= \frac{1}{0.333\text{ S} + 0.167\text{ S}} = \frac{1}{0.5\text{ S}} = \mathbf{2\ \Omega}$$

(***iii***) Applying Eq. (6.5):

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega} = \frac{18\Omega}{9\Omega} = \mathbf{2\ \Omega}$$

EXAMPLE 6.3 Find the total resistance of the configuration in Fig. 6.7.

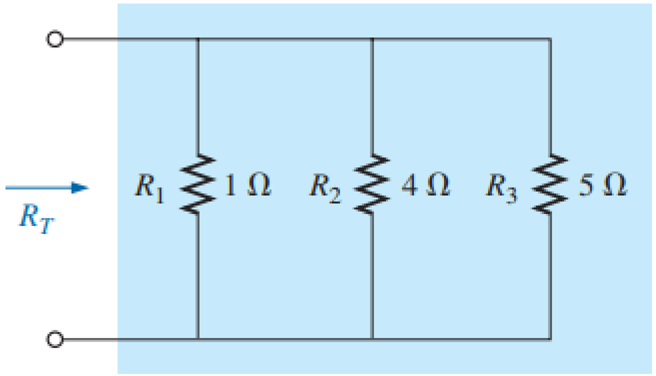


FIG. 6.7

Solution: Method 1

$$G_1 = \frac{1}{R_1} = \frac{1}{1\Omega} = 1.0\text{ S},\qquad G_2 = \frac{1}{R_2} = \frac{1}{4\Omega} = 0.25\text{ S},\qquad G_3 = \frac{1}{R_3} = \frac{1}{5\Omega} = 0.2\text{ S}$$

$$G_T = G_1 + G_2 + G_3 = 1.0\text{ S} + 0.25\text{ S} + 0.2\text{ S} = 1.45\text{ S}\qquad R_T = \frac{1}{G_T} = \frac{1}{1.45\text{ S}} = \mathbf{0.69\ \Omega}$$

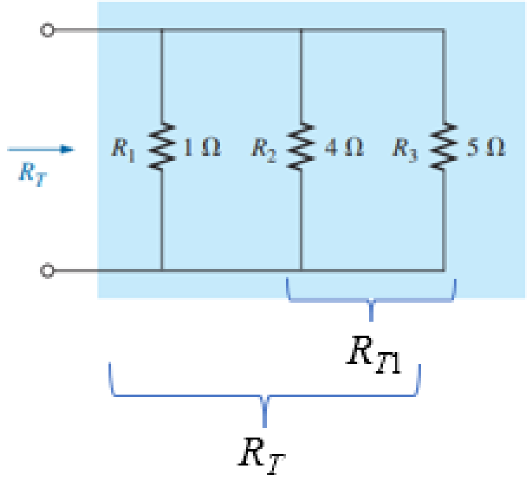
Method 2: Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{1\Omega}+\frac{1}{4\Omega}+\frac{1}{5\Omega}} = \frac{1}{1.0\text{ S}+0.25\text{ S}+0.2\text{ S}} = \frac{1}{1.45\text{ S}} \approx \mathbf{0.69\ \Omega}$$

Method 2: Applying Eq. (6.5):

$$R_{T1} = \frac{R_2R_3}{R_2+R_3} = \frac{(4\ \Omega)(5\ \Omega)}{4\ \Omega + 5\ \Omega} \cong 2.22\ \Omega$$

$$R_T = \frac{R_1R_{T1}}{R_1+R_{T1}} = \frac{(1\ \Omega)(2.22\ \Omega)}{1\ \Omega + 2.22\ \Omega} \cong \mathbf{0.69\ \Omega}$$



EXAMPLE 6.5 Find the total resistance of the parallel resistors in (*i*) Fig. 6.9 and (*ii*) Fig. 6.11.

Solution: In both figures the individual resistance value are equal.

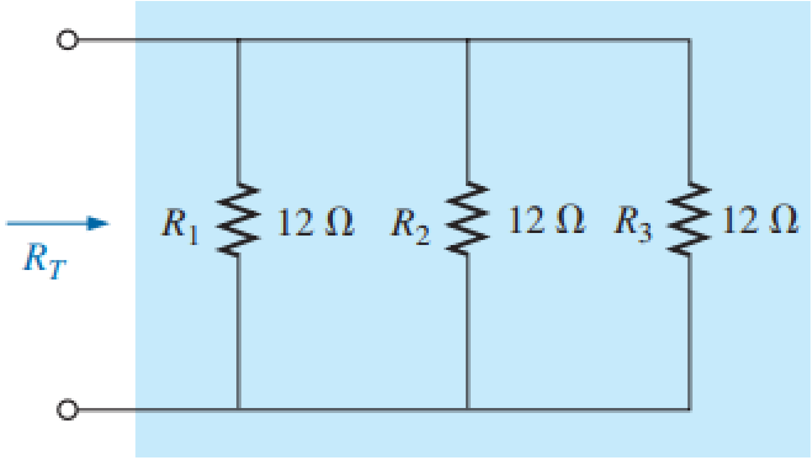


FIG. 6.9

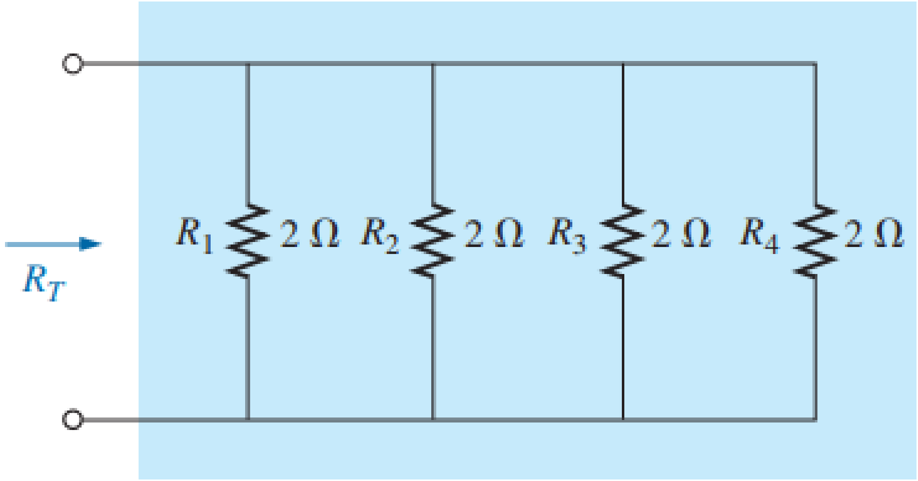


FIG. 6.11

For Fig. 6.9:

$$R_1 = R_2 = R_3 = R = 12\ \Omega$$

Total resistance: Applying Eq. (6.4):

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

For Fig. 6.11:

$$R_1 = R_2 = R_3 = R_4 = R = 2\ \Omega$$

Total resistance: Applying Eq. (6.3):

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Problem 9 [P. 235] Determine R_1 for the network in Fig. 6.80

Practice Example 6.10 and 6.11

Solution: Here, all resistance are connected in parallel.

Let, $R_2 = 24\ \Omega$, $R_3 = 24\ \Omega$, and $R_4 = 120\ \Omega$

We know that, $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_1} = \frac{1}{R_T} - \left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right]$$

$$= \frac{1}{10\ \Omega} - \left[\frac{1}{24\ \Omega} + \frac{1}{24\ \Omega} + \frac{1}{120\ \Omega}\right]$$

$$= 0.1\ \text{S} + 0.042\ \text{S} + 0.042\ \text{S} + 0.008\ \text{S} = 0.192\ \text{S}$$

$$R_1 = \frac{1}{0.192\ \text{S}} = \mathbf{5.21\ \Omega}$$

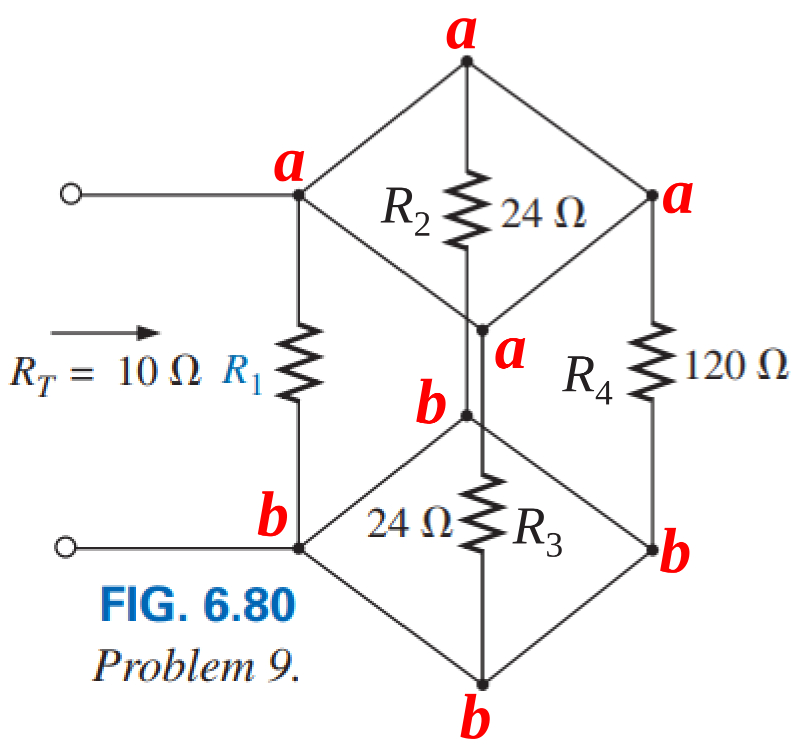
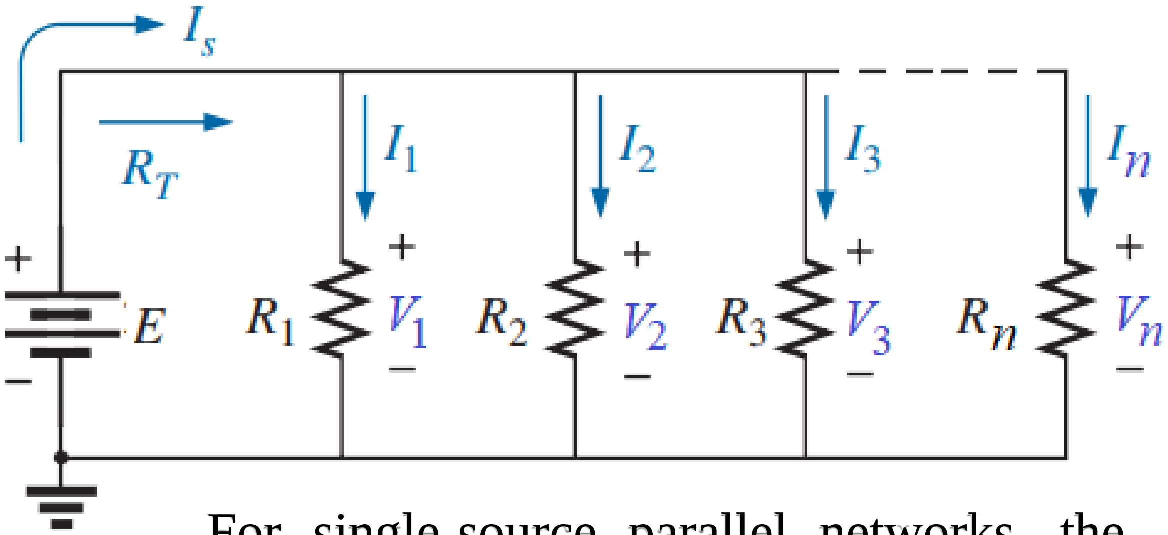


FIG. 6.80
Problem 9.

Practice Book Problem [SECTION 6.2 Parallel Resistors] Problems: 1, 3, 4, 5, 7 and 8

6.3 Parallel Circuit

A **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to from a junction point while the remaining ends are also joined to form another junction point.



Voltage in a parallel circuit: the voltage is always the same across parallel elements.

$$V_1 = V_2 = V_3 = \ldots = V_n = E \qquad \textbf{(6.6)}$$

Current Calculation of a Parallel circuit: According ohm's law, the current of the following circuit is as follows:

$$I_s = \frac{E}{R_T} \qquad \textbf{(6.7)}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}; \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}; \quad I_n = \frac{V_n}{R_n} = \frac{E}{R_n} \qquad \textbf{(6.8)}$$

For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.

$$I_s = I_1 + I_2 + I_3 \ldots . I_n \qquad \textbf{(6.9)}$$

If $R_1 = R_2 = R_3 = \ldots\ldots = R_n = R$

$$I_1 = I_2 = I_3 = \ldots . = I_n = \frac{I_s}{n}$$

EXAMPLE 6.13 For the parallel network in Fig. 6.23.

a. Find the total resistance.

b. Calculate the source current.

c. Determine the current through each branch.

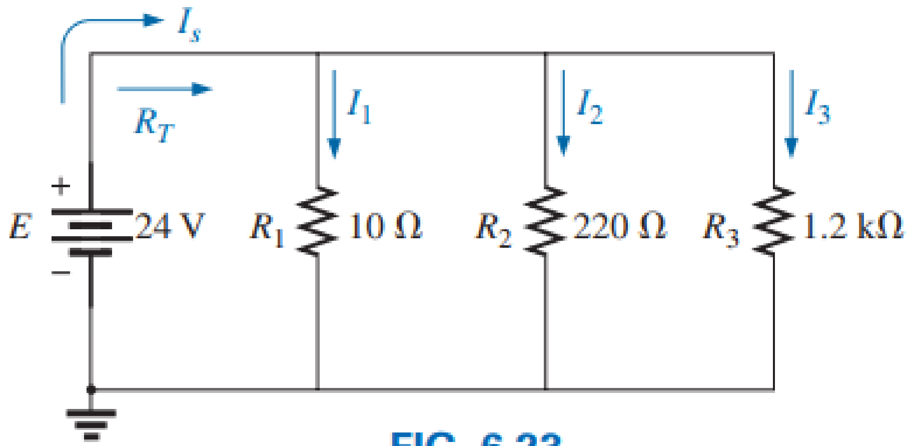


FIG. 6.23
Parallel network for Example 6.13.

Solution:

a. Applying Eq. (6.3):

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{10\ \Omega}+\frac{1}{220\ \Omega}+\frac{1}{1.2\ \text{k}\Omega}}$$

$$= \frac{1}{100\times 10^{-3}+4.545\times 10^{-3}+0.833\times 10^{-3}} = \frac{1}{105.38\times 10^{-3}}$$

$$R_T = \mathbf{9.49\ \Omega}$$

b. Using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{9.49\ \Omega} = \mathbf{2.53\ A}$$

c. Applying Ohm's law:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{24\ \text{V}}{10\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{24\ \text{V}}{220\ \Omega} = \mathbf{0.11\ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{24\ \text{V}}{1.2\ \text{k}\Omega} = \mathbf{0.02\ A}$$

EXAMPLE 6.14 Given the information provided in Fig. 6.24.

a. Determine R_3.
b. Find the applied voltage E.
c. Find the source current I_s.
d. Find I_2.

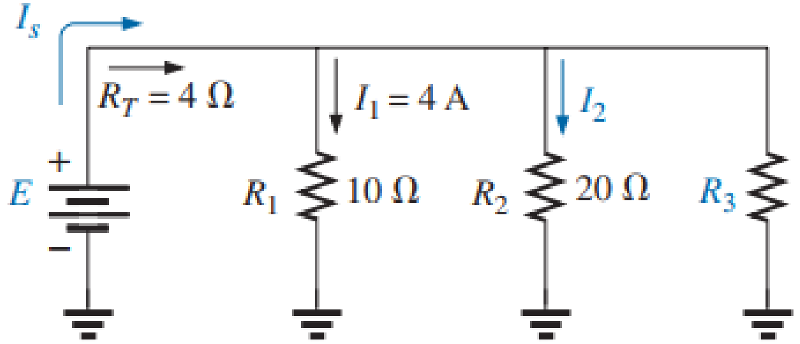


FIG. 6.24
Parallel network for Example 6.14.

Solutions:

a. Applying Eq. (6.1): $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Substituting: $$\frac{1}{4\ \Omega} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{R_3}$$

so that $$0.25\ \text{S} = 0.1\ \text{S} + 0.05\ \text{S} + \frac{1}{R_3}$$

and $$0.25\ \text{S} = 0.15\ \text{S} + \frac{1}{R_3}$$

with $$\frac{1}{R_3} = 0.1\ \text{S}$$

and $$R_3 = \frac{1}{0.1\ \text{S}} = \mathbf{10\ \Omega}$$

b. Using Ohm's law:

$$E = V_1 = I_1R_1 = (4\ \text{A})(10\ \Omega) = \mathbf{40\ V}$$

c. $$I_s = \frac{E}{R_T} = \frac{40\ \text{V}}{4\ \Omega} = \mathbf{10\ A}$$

d. Applying Ohm's law:

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40\ \text{V}}{20\ \Omega} = \mathbf{2\ A}$$

Practice Book Problem [SECTION 6.3 Parallel Circuits] Problems: 10 to 17

6.6 Current Division in Parallel Circuit
Current Divider Rule (CDR)

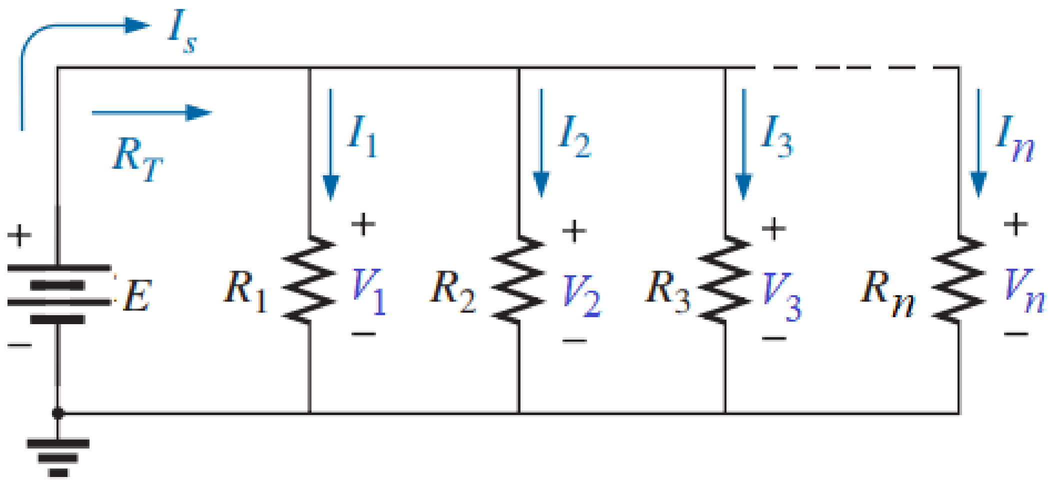
Current pass through individual resistance in a series circuit : According ohm's law, the of the following circuit is as follows:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{R_T}{R_1} I_s = \frac{G_1}{G_T} I_s \qquad (6.14.1)$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{R_T}{R_2} I_s = \frac{G_2}{G_T} I_s \qquad (6.14.2)$$

. . .

$$I_n = \frac{V_n}{R_n} = \frac{E}{R_n} = \frac{R_T}{R_n} I_s = \frac{G_n}{G_T} I_s \qquad (6.14.n)$$

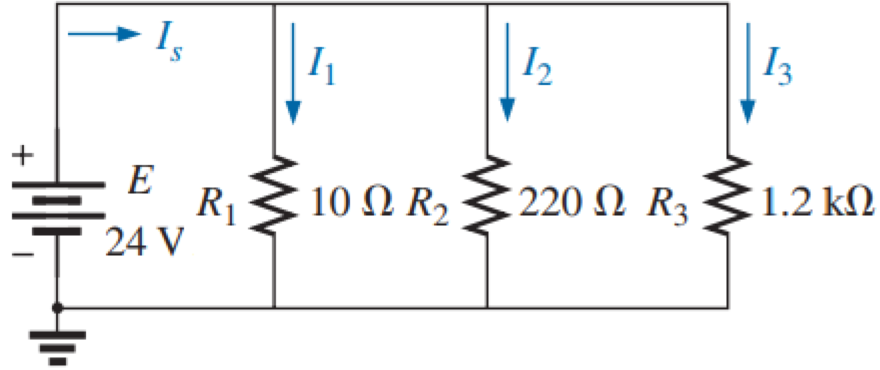


Based on the Eq. (6.14.1) to (16.14.n), a general equation can be written as follows:

$$I_x = \frac{R_T}{R_x} I_s = \frac{G_x}{G_T} I_s \qquad \textbf{(6.8)}$$

Current Divider Rule (CDR): The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.

EXAMPLE 6.6.1 For the following parallel network as shown in following figure. Using current divider rule, determine the current through each branch.



Solution:

$$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{1}{10\Omega}+\frac{1}{220\Omega}+\frac{1}{1.2\times10^3\,\Omega}$$

$$=105.38\times10^{-3}\,\mathrm{S}$$

$$R_T=\frac{1}{105.38\times10^{-3}\,\mathrm{S}}=9.49\ \Omega$$

$$I_s=\frac{E}{R_T}=\frac{24\ \mathrm{V}}{9.49\ \Omega}=2.53\ \mathrm{A}$$

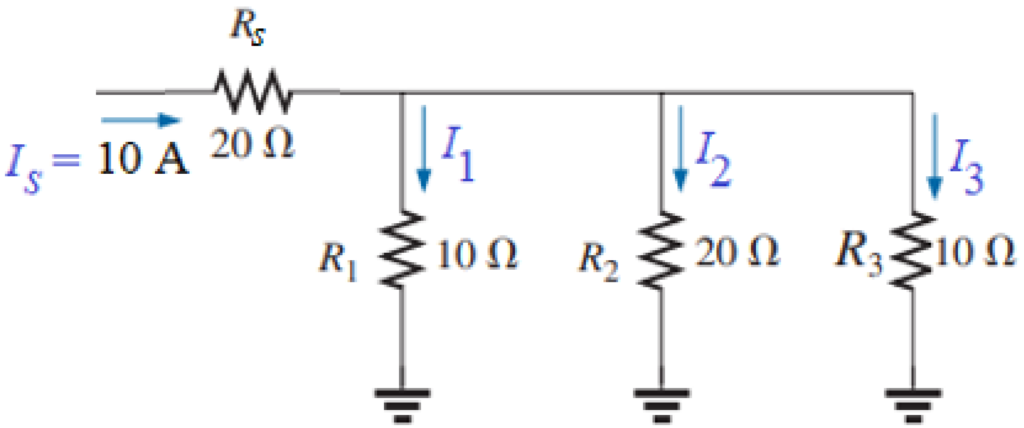
Applying current divider rule (CDR) Eq. (6.8):

$$I_1=\frac{R_T}{R_1}I_s=\frac{9.49\ \Omega}{10\ \Omega}\times2.53\ \mathrm{A}=\mathbf{2.4\ A}$$

$$I_2=\frac{R_T}{R_2}I_s=\frac{9.49\ \Omega}{220\ \Omega}\times2.53\ \mathrm{A}=\mathbf{0.11\ A}$$

$$I_3=\frac{R_T}{R_3}I_s=\frac{9.49\ \Omega}{1.2\times10^3\ \Omega}\times2.53\ \mathrm{A}=\mathbf{0.02\ A}$$

EXAMPLE 6.6.2 For the following parallel network as shown in following figure. Using current divider rule, determine the current through each branch.



Solution:

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{10\Omega}+\frac{1}{20\Omega}+\frac{1}{10\Omega}} = 4\ \Omega$$

Applying current divider rule CDR) Eq. (6.8):

$$I_1 = \frac{R_T}{R_1} I_s = \frac{4\ \Omega}{10\ \Omega} \times 10\ \text{A} = \mathbf{4\ A}$$

$$I_2 = \frac{R_T}{R_2} I_s = \frac{4\ \Omega}{20\ \Omega} \times 10\ \text{A} = \mathbf{2\ A}$$

$$I_3 = \frac{R_T}{R_3} I_s = \frac{4\ \Omega}{10\ \Omega} \times 10\ \text{A} = \mathbf{4\ A}$$

Since R_1 and R_3 are equal, the current I_1 and I_3 must be equal.

Since R_2 is twice R_1 or R_3, the current I_2 must be one-half I_1 or I_3.

Practice Book Problem [SECTION 6.6 CDR] Problems: 29 to 34