Lecture 5

Chapter 19: The Kinetic Theory of Gases

19-1 Avogadro's number, N_A:

One mole of a substance contains N_A (Avogadro's number) elementary units (usually atoms or molecules).

1 mole of a substance contains, $N_A = 6.023 \times 10^{23}$ elementary units

Amedeo Avogadro suggested that all gases occupy the same volume under the same conditions of temperature and pressure. All gases contain the same number of atoms or molecules.

1 mole of He contains, $N_A = 6.023 \times 10^{23}$ atoms

n mole of He contains total number of atoms, $N = nN_A$ atoms

1 mole of O_2 contains, $N_A = 6.023 \times 10^{23}$ molecules

n mole of O_2 contains, total number of molecules, $N = nN_A$ molecules

One molar mass M of any substance is the mass of one mole of the substance.

1 molar mass, $M = mN_A$ where m is the mass of 1 atom or molecule

n molar mass (sample mass), $M_{sam} = mN = m(nN_A) = n(mN_A) = nM$

$$\frac{M_{sam}}{M} = \frac{n(mNA)}{mN_A}$$

$$n = \frac{M_{sam}}{M}$$

19-2 Ideal gases:

Boyle's Law describes the inverse proportional relationship between pressure and volume at a constant temperature and a fixed amount of gas. This law came from a manipulation of the Ideal Gas Law.

$p \propto 1/V$ ---------------------------------(1)

Charles's Law describes the directly proportional relationship between the volume and temperature (in Kelvin) of a fixed amount of gas, when the pressure is held constant.

$V \propto T$ ---(2)

By combining two equations and the fact $V \propto n$, we can write the ideal gas equation

$pV=nRT$ [R in terms of mole]

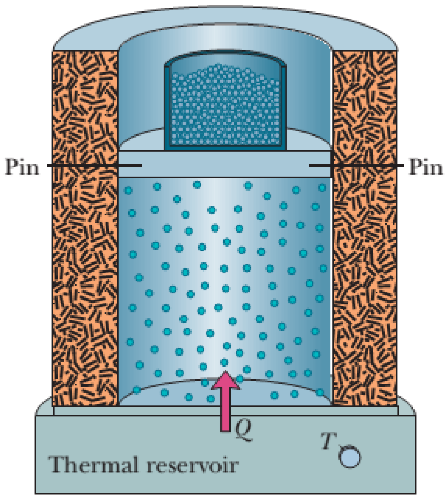
Boltzmann constant, $k = R/N_A$ $N_A k = R$ $\frac{N}{n} k = R$ $nR = Nk$ $[N = nN_A]$

$pV=NkT$ [k in terms of molecule]

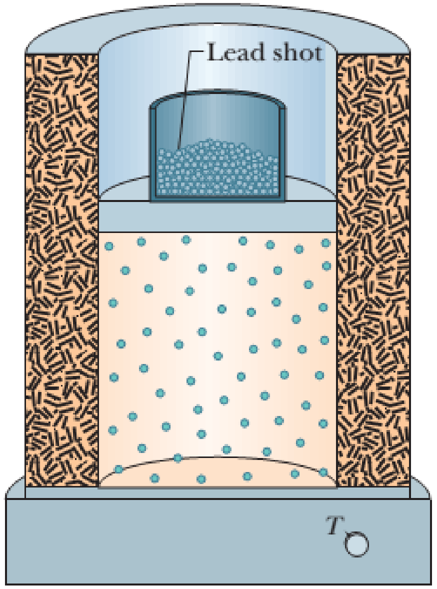
Ideal gas: It governs the macroscopic properties. We can deduce many properties of the ideal gas in a simple way. Although there is no such thing in nature as a truly ideal gas, all real gases approach the ideal state at low enough densities their molecules are far enough apart that they do not interact with one another.

19-2 Work done by an ideal gas at constant temperature:

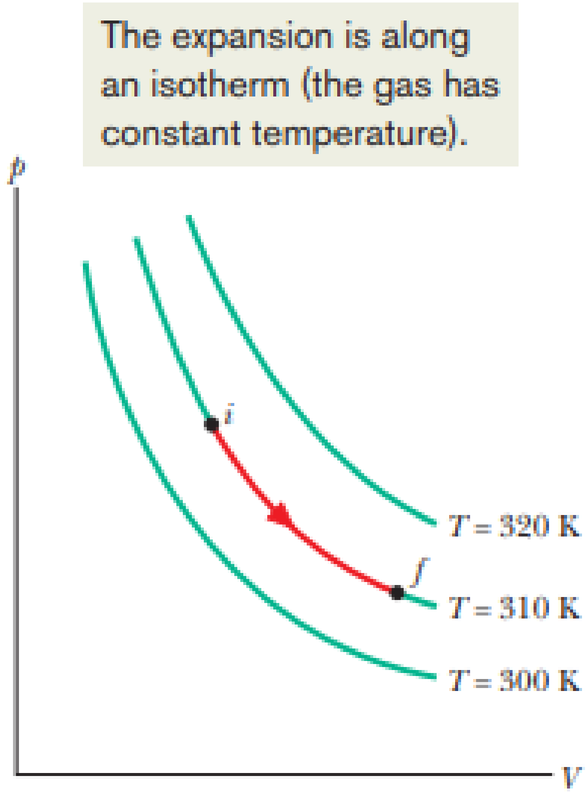
Suppose that we allow the ideal gas to expand from an initial volume V_i to a final volume V_f while we keep the temperature T of the gas constant. Such a process, at constant temperature, is called an isothermal expansion (and the reverse is called an isothermal compression).



$i(p, T, V)$



$f(p+\Delta p, T, V+\Delta V)$



This is a general expression for the work done during any change in volume of any gas.

$$W = \int_{V_i}^{V_f} p\, dV$$

For an ideal gas, pV = nRT

$$p = \frac{nRT}{V}$$

$$W = \int_{V_i}^{V_f} \frac{nRT}{V} dV$$

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$ [As T = constant, nRT = constant]

$$W = nRT\, [\ln V]_{V_i}^{V_f}$$

$$W = nRT\, [\ln V_f - \ln V_i]$$

$$W = nRT \ln \frac{V_f}{V_i}$$ [Isothermal process for an ideal gas]

19-2 Work done at constant volume:

If the volume of the gas is constant, then the work done is as follows:

W = pΔV = p(V-V) = 0

W = 0

19-2 Work done at constant pressure:

If the volume changes while the pressure *p* of the gas is held constant, then the work done is as follows:

$$W = \int_{V_i}^{V_f} p \, dV = p \int_{V_i}^{V_f} dV = p \, [V]_{V_i}^{V_f} = p \, [V_f - V_i]$$

$$W = p\Delta V$$

4. A quantity of ideal gas at 10.0 ^{0}C and 100 kPa occupies a volume of 2.50 m^3. (a) How many moles of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature is raised to 30.0 ^{0}C, how much volume does the gas occupy? Assume no leaks.

Solution:

(a) $p_iV_i = nRT_i$ -------------------(1)

$$n = \frac{p_i V_i}{RT_i} = \frac{1\times10^5 \times2.50}{8.31 \times283} = \textbf{106 mol}$$

(b) $p_fV_f = nRT_f$ ....(2)

(2) / (1)

$$V_f = \frac{V_i\, P_i T_f}{P_f T_i} = \frac{1 \times 10^5 \times 2.50 \times 303.15}{3 \times 10^5 \times 283} = \mathbf{0.89\ m^3}$$

OR

$p_fV_f = nRT_f$

7. Suppose 1.80 mol of an ideal gas is taken from a volume of 3.00 m^3 to a volume of 1.50 m^3 via an isothermal compression at 30 ^{0}C. (a) How much energy is transferred as heat during the compression, and (b) is the transfer *to* or *from* the gas?

Solution:

(a) $\Delta E_{int} = Q - W$

$$E_{int} = \frac{3}{2}nRT$$

$$\Delta E_{int} = \frac{3}{2}nR\Delta T$$

$$\Delta E_{int} = \frac{3}{2}nR(T\text{-}T)$$

[Isothermal process, T= constant]

$$\Delta E_{int} = \frac{3}{2}nR(0)$$

$$\Delta E_{int} = 0$$

$$0 = Q - W$$

$$Q = W$$

$$W = nRT\ln\left(\frac{V_f}{V_i}\right) = 1.80 \times 8.314 \times 303\ln\left(\frac{1.50}{3.00}\right) = -3140\ J$$

Q = W

Q = -3140 J

(b) The heat is transferred from the gas.