# Chapter-4
# Complex Numbers

$z = a + i\,b \rightarrow$ Imaginary part $\boxed{Im\{z\}} = b$

Real part $\boxed{Re\{z\} = a}$

$\boxed{x^2 = -1}$

$\Rightarrow x = \boxed{\sqrt{-1}}$ $i$

$\sqrt{-5} = \sqrt{5(-1)}$
$= \sqrt{5}\sqrt{-1}$
$= \sqrt{5}\, i$

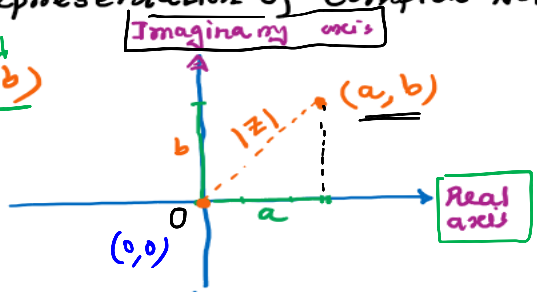
$\boxed{i^2 = -1}$

$i = \sqrt{-1} \Rightarrow i^2 = -1$

If real part of $z$ is $0$ then $z = ib$, it is called as pure imaginary number.

## Graphical Representation of Complex Number / Argand diagram:

$z = a + ib$ / $(a, b)$




$|z|$ represents the distance between the complex no. & the origin.

Conjugates: $z = a+ib \xrightarrow{\text{conjugate}} \bar{z} = a - ib$

$-1 + i \xrightarrow{\text{conjugate}} -1 - i$

Absolute value / Modulus: $\text{mod } z = |z| = \sqrt{a^2+b^2}$

## Powers of Imaginary Unit:

$i^0 = 1$, $i^1 = i$, $i^2 = -1$, $i^3 = i^2 \cdot i$
$= -1 \cdot i$
$= -i$

$i^4 = i^3 \cdot i$
$= -i \cdot i$
$= -i^2$
$= 1$

$i^n = \{1, -1, i, -i\}$

$i^6 = i^3 \cdot i^3 = -i \cdot (-i) = i^2 = -1$

$\boxed{6065}$

$i^{6065} = i$

$i^{4n} = (i^4)^n = \boxed{1}$

$i^{4n+1} = \boxed{i^{4n}} \cdot i = \boxed{i}$

$i^{4n+2} = i^{4n} \cdot i^2 = 1 \cdot (-1) = -1$

$i^{4n+3} = i^{4n} \cdot i^3 = 1 \cdot (-i) = -i$

$i^{497} = i^{4 \times 124 + 1} = i$

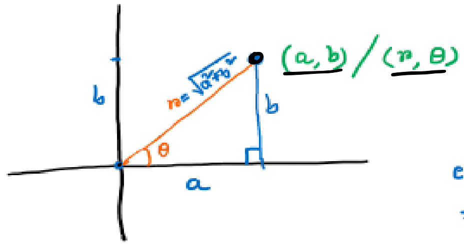
$i^{3058} = -1$

Rectangular form of Complex number:

$a+ib$

Polar form of complex Number: $(r, \theta)$

Polar form of complex number: $re^{i\theta}$




$\cos\theta = \frac{a}{r}$

$\Rightarrow a = \boxed{r\cos\theta}$

$\sin\theta = \frac{b}{r}$

$\Rightarrow b = \boxed{r\sin\theta}$

$$z = a+ib$$
$$= r\cos\theta + i\,r\sin\theta$$
$$= r(\cos\theta + i\sin\theta)$$
$$= r\boxed{e^{i\theta}}$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

Rectangular form: $z = a+ib$ ; $a = r\cos\theta$ , $b = r\sin\theta$

Polar form: $z = re^{i\theta}$ ; $r = \sqrt{a^2+b^2}$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

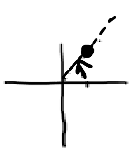
Argument of a complex number:

$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$z = 1+i$ $\boxed{(1,1)}$

$r = \sqrt{2}$

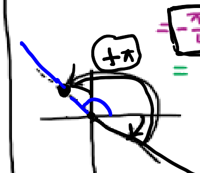
$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \boxed{\frac{\pi}{4}}$ ✓



$z = -1+i$ $(-1,1)$

$r = \sqrt{2}$

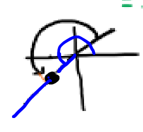
$\theta = \tan^{-1}\left(\frac{1}{-1}\right) = -\tan^{-1}(1) = \boxed{-\frac{\pi}{4}} + \pi = \frac{3\pi}{4}$ ✓




$z = -1-i$ $(-1,-1)$

$r = \sqrt{2}$

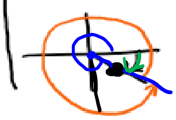
$\theta = \tan^{-1}\left(\frac{-1}{-1}\right) = \tan^{-1}(1) = \boxed{\frac{\pi}{4}} + \pi = \frac{5\pi}{4}$



$z = 1-i$ $(1,-1)$

$r = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = -\tan^{-1}(1) = \boxed{-\frac{\pi}{4}} + 2\pi = \boxed{\frac{7\pi}{4}}$



Conversion of complex number from rectangular to polar coordinate system:

1. If it's in $1^{st}$ quadrant: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$
2. If it's in $2^{nd}$ quadrant: $\theta = -\tan^{-1}\left(\frac{b}{a}\right) + \pi$
3. If it's in $3^{rd}$ quadrant: $\theta = \tan^{-1}\left(\frac{b}{a}\right) + \pi$
4. If it's in $4^{th}$ quadrant: $\theta = -\tan^{-1}\left(\frac{b}{a}\right) / -\tan^{-1}\left(\frac{b}{a}\right) + 2\pi$

where a and b are real and imaginary part respectively, considering without sign.

If you wish you may avoid this part.

$\boxed{z = -1+\sqrt{3}\,i}$ → convert into polar form

$re^{i\theta}$

$a = 1,\ b = \sqrt{3}$

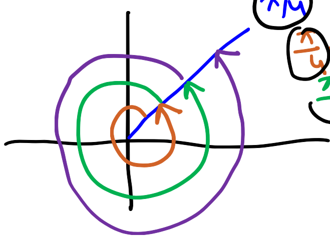
$\theta = -\tan^{-1}\left(\frac{\sqrt{3}}{1}\right) + \pi$

$= -\frac{\pi}{3} + \pi = \boxed{\frac{2\pi}{3}}$

$r = 2$

$$\boxed{z = 2e^{i\frac{2\pi}{3}}}$$

## Principle Argument:

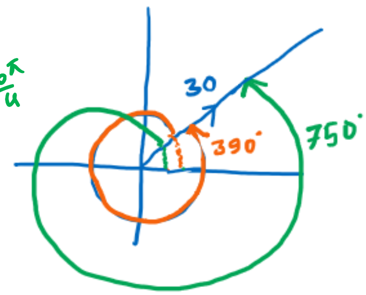


$\frac{\pi}{4}$

$\frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$

$\frac{\pi}{4} + 2 \cdot 2\pi = \frac{16\pi}{4}$

$\frac{\pi}{4} + 3 \cdot 2\pi = \frac{25\pi}{4}$




$\boxed{30°}$

$390° = \boxed{30°} + 360°$

$750° = \boxed{30°} + 2 \times 360$

$$\boxed{0 \leq \theta < 2\pi}$$

## Calculating principle argument from argument:

$\underbrace{\text{principle argument}}_{Arg\, z}$ from $\underbrace{\text{argument}}_{arg\, z}$

$$arg\, z = Arg\, z + 2n\pi \quad ; \quad n \in \mathbb{Z}$$

$\mathbb{Z}$ → set of integers numbers $\{0, \pm 1, \pm 2, \ldots\}$

**Example:** $arg\, z = \frac{35\pi}{2}$ ; Find it's principle argument

$$arg\, z = Arg\, z + 2n\pi$$

$$\Rightarrow Arg\, z = arg\, z - 2n\pi$$

$$= \frac{35\pi}{2} - 16\pi \quad ; \quad n = \boxed{8}$$

$$= \frac{3\pi}{2}$$

$0 \leq \theta < 2\pi$

$arg\, z = \frac{7\pi}{2}$ ; $\quad arg\, z = -\frac{5\pi}{2}$