# REVIEW ON THE LAST CLASS

**SOURCE CONVERSIONS**

**MESH ANALYSIS**

**NODAL ANALYSIS**

# 8.9 NODAL ANALYSIS (GENERAL APPROACH)

**EXAMPLE 8.21** Determine the nodal voltages for the network in Fig. 8.54.

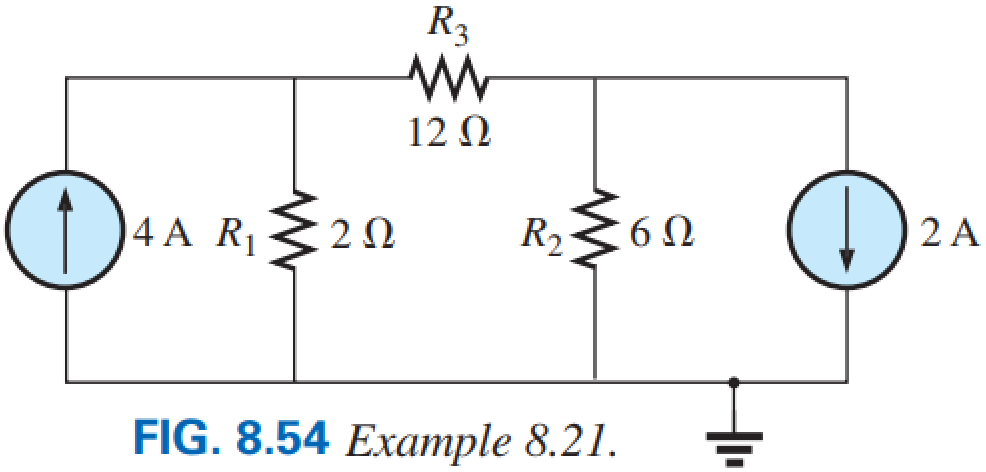
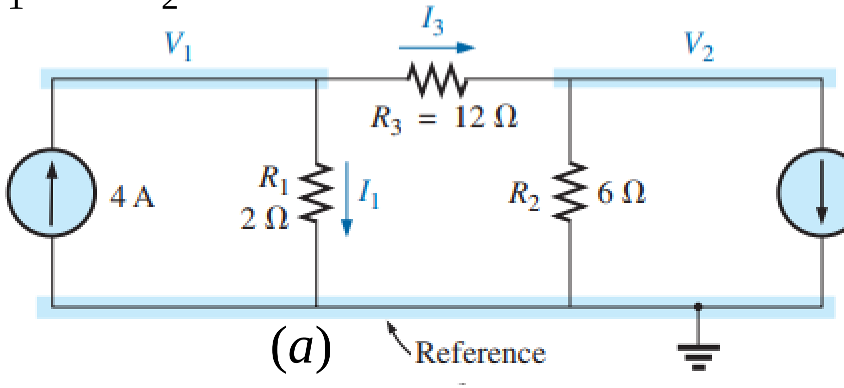



**FIG. 8.54** *Example 8.21.*

**Step 1 and 2:** The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as $V_1$ and $V_2$.

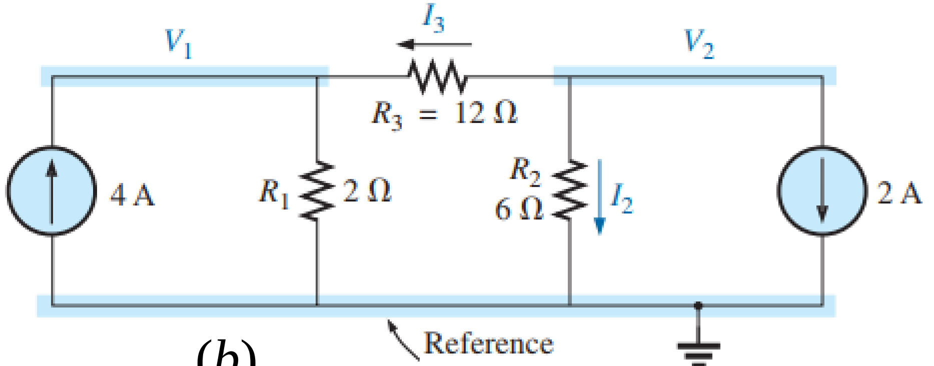



(a)

**Step 3:** For node $V_1$, the currents are defined as shown in the following Figure (a) and Kirchhoff's current law is applied:

$$I_1 + I_3 = 4 \qquad \left(\frac{1}{2}+\frac{1}{12}\right)V_1 - \left(\frac{1}{12}\right)V_2 = 4 \qquad 7V_1 - V_2 = 48$$

For node $V_2$, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:

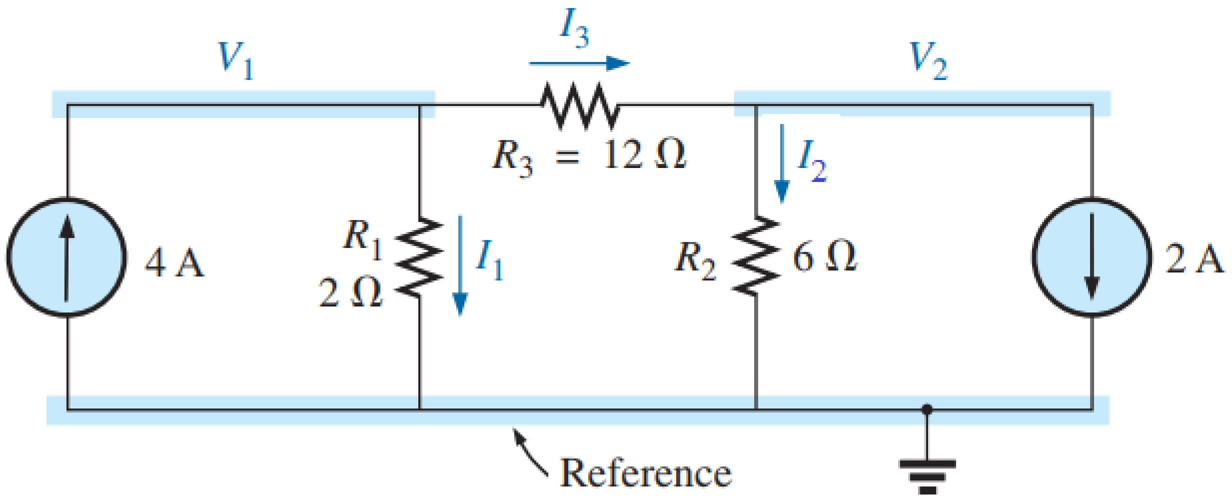



(b)

$$I_2 + I_3 = -2$$

$$\left(\frac{1}{12}+\frac{1}{6}\right)V_2 - \left(\frac{1}{12}\right)V_1 = -2 \qquad 3V_2 - V_1 = -24$$

$$7V_1 - V_2 = 48$$
$$-V_1 + 3V_2 = -24$$




$$D = \begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix} = 21 - 1 = 20$$

$$D_1 = \begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix} = 144 - 24 = 120$$

$$D_2 = \begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix} = -168 + 48 = -120$$

$$V_1 = \frac{D_1}{D} = \frac{120}{20} = \mathbf{6\ V}$$

$$V_2 = \frac{D_2}{D} = \frac{-120}{20} = \mathbf{-6\ V}$$

**Here,** $V_1 > V_2$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\ \text{V}}{2\ \Omega} = \mathbf{3\ A}$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\ \text{V}}{6\ \Omega} = \mathbf{1\ A}$$

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\ \text{V} - (-6\ \text{V})}{12\ \Omega} = \frac{12\ \text{V}}{12\ \Omega} = \mathbf{1\ A}$$

**EXAMPLE 8.24** Find the voltage across the 3 Ω resistor in Fig. 8.61 by nodal analysis.

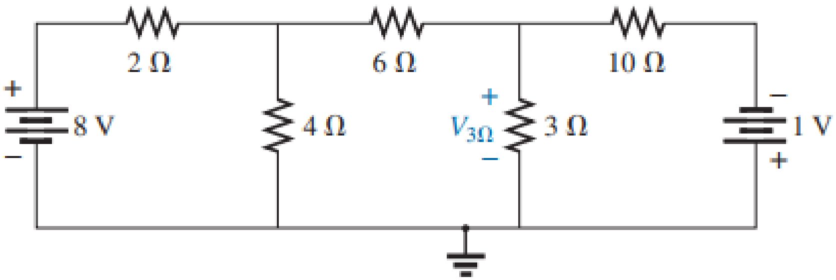
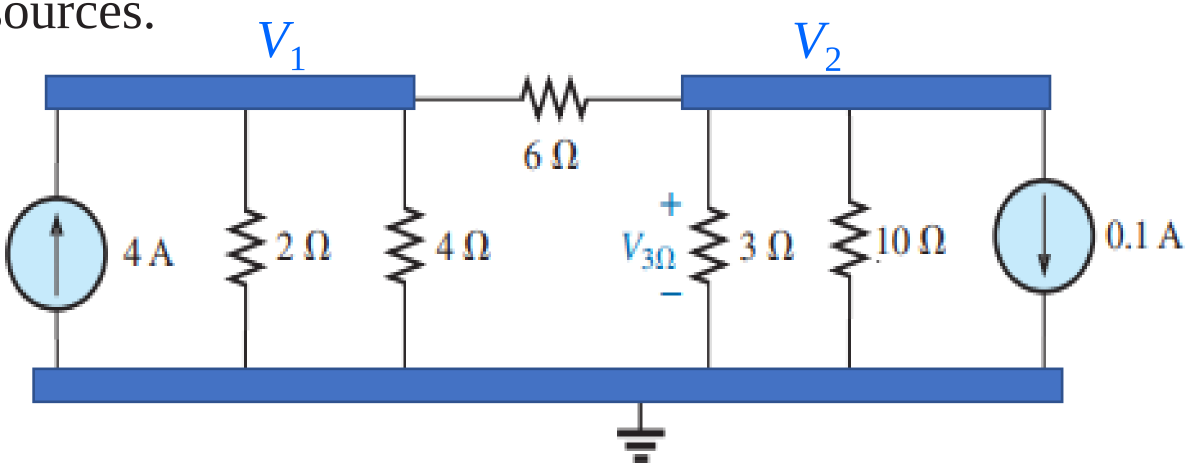



**FIG. 8.61** *Example 8.24.*

**Solution:** First convert two voltage sources to current sources.




**Step 1 and 2:** The network has three nodes with the bottom node defined as the reference node (at ground potential, or zero volts), and the other nodes as $V_1$ and $V_2$.

**Step 3:** For node $V_1$, the currents are defined as shown in the following Figure (b) and Kirchhoff's current law is applied:

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_1-\left(\frac{1}{6}\right)V_2=4 \qquad 11V_1-2V_2=48$$

For node $V_2$:

$$\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{10}\right)V_2-\left(\frac{1}{6}\right)V_1=-0.1 \qquad 18V_2-5V_1=-3$$

Simplified form:

$$11V_1-2V_2=48$$
$$-5V_1+18V_2=-3$$

$$D=\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}=198-10=188$$

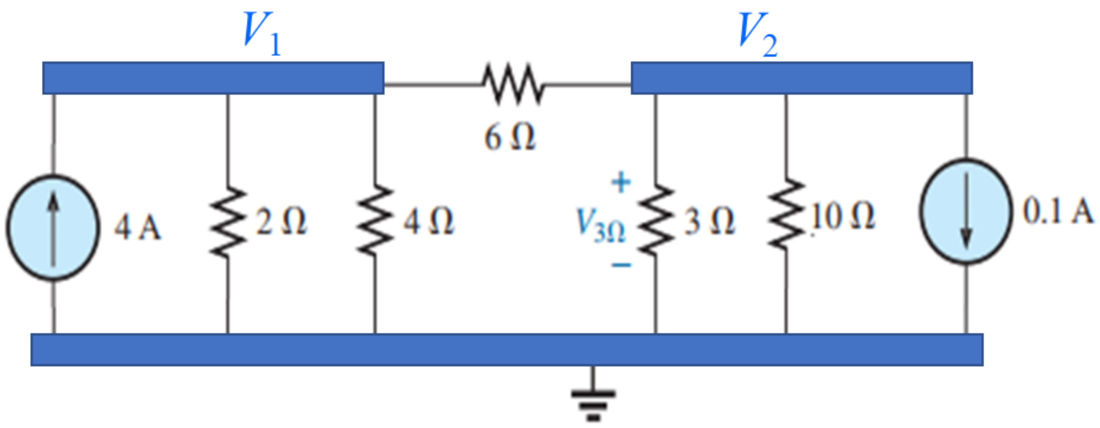
$$11V_1 - 2V_2 = 48$$
$$-5V_1 + 18V_2 = -3$$

$$D_1 = \begin{vmatrix} 48 & -2 \\ -3 & 18 \end{vmatrix} = 864 - 6 = 858$$

$$D_2 = \begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix} = -33 + 240 = 207$$

$$V_1 = \frac{D_1}{D} = \frac{858}{188} = \mathbf{4.56\ V}$$

$$V_2 = V_{3\Omega} = \frac{D_2}{D} = \frac{207}{188} = \mathbf{1.1\ V}$$




**Current of branches:**

$$I_{2\Omega} = \frac{V_1}{2\Omega} = \frac{4.56\,\text{V}}{2\Omega} = \mathbf{2.28\ A}$$

$$I_{4\Omega} = \frac{V_1}{4\Omega} = \frac{4.56\,\text{V}}{4\Omega} = \mathbf{1.14\ A}$$

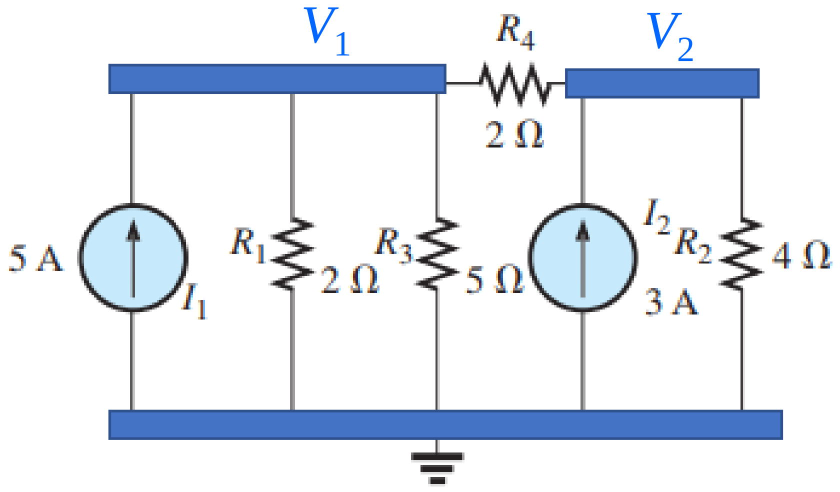
$$I_{6\Omega} = \frac{V_1 - V_2}{6\Omega} = \frac{4.56\,\text{V} - 1.1\text{V}}{6\Omega} = \mathbf{0.577\ A}$$

$$I_{3\Omega} = \frac{V_2}{3\Omega} = \frac{1.1\text{V}}{3\Omega} = \mathbf{0.367\ A}$$

$$I_{10\Omega} = \frac{V_2}{10\Omega} = \frac{1.1\text{V}}{10\Omega} = \mathbf{0.11\ A}$$

Write the nodal equations for the following networks.

**Practice Book [Ch 8] Problem: 41 ~ 44**



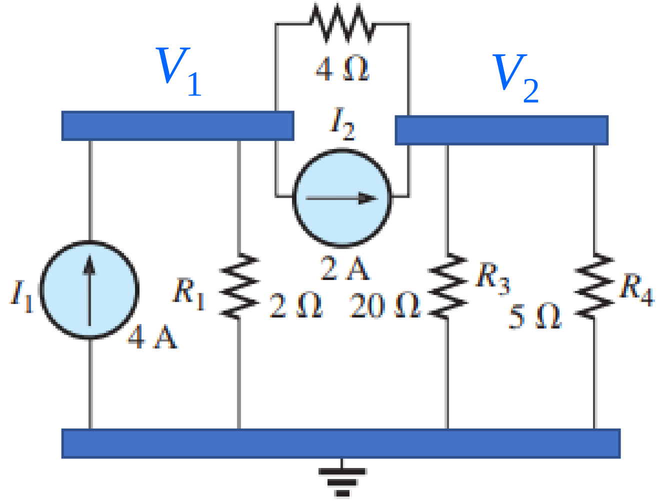

(a)

$$\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{2}\right)V_1-\left(\frac{1}{2}\right)V_2=5$$

$$\left(\frac{5+2+5}{10}\right)V_1-\left(\frac{1}{2}\right)V_2=5 \qquad 12V_1-5V_2=50$$

$$\left(\frac{1}{2}+\frac{1}{4}\right)V_2-\left(\frac{1}{2}\right)V_1=3$$

$$\left(\frac{2+1}{4}\right)V_2-\left(\frac{1}{2}\right)V_1=3 \qquad 3V_2-2V_1=6$$




(b)

$$\left(\frac{1}{2}+\frac{1}{4}\right)V_1-\left(\frac{1}{4}\right)V_2=4-2$$

$$\left(\frac{2+1}{4}\right)V_1-\left(\frac{1}{4}\right)V_2=2 \qquad 3V_1-V_2=8$$

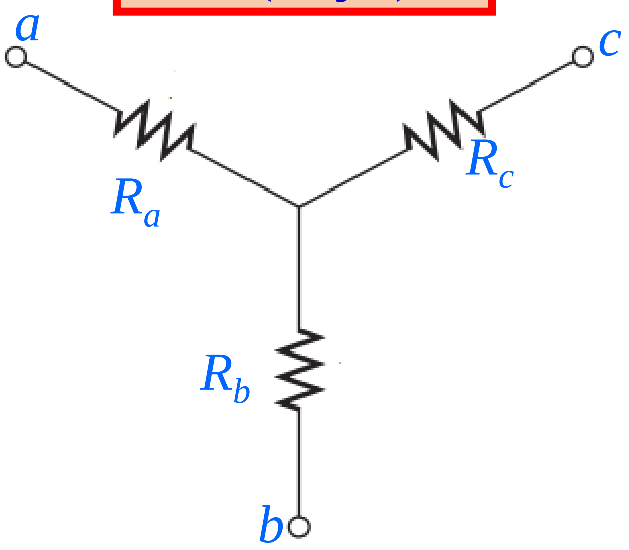
$$\left(\frac{1}{4}+\frac{1}{20}+\frac{1}{5}\right)V_2-\left(\frac{1}{4}\right)V_1=2$$

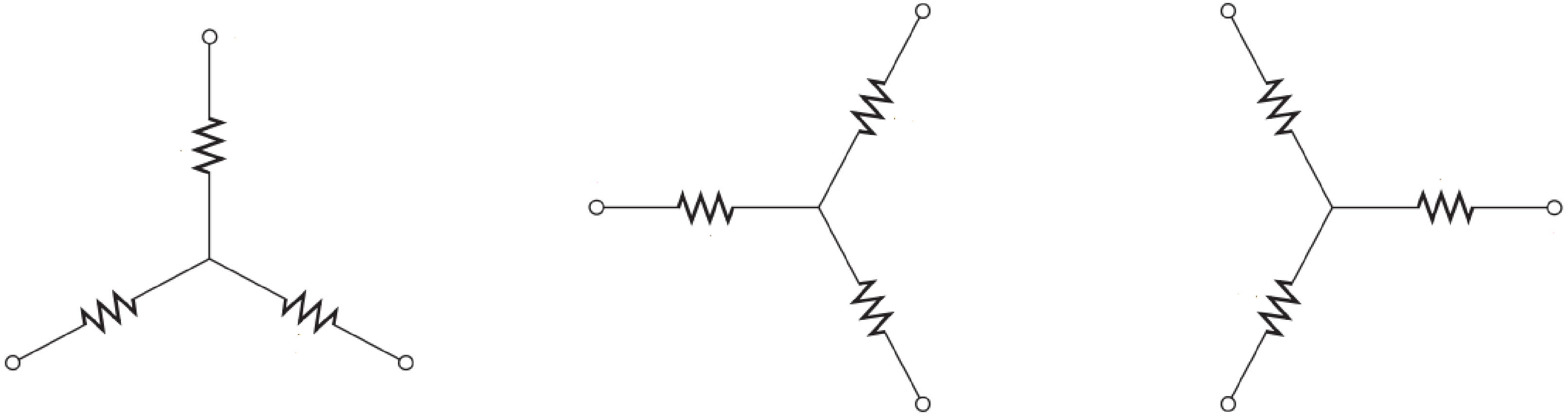
$$\left(\frac{5+1+4}{20}\right)V_2-\left(\frac{1}{4}\right)V_1=2 \qquad 10V_2-5V_1=40$$

# 8.12 Y–Δ (T–Π) and Δ–Y (Π–T) CONVERSIONS

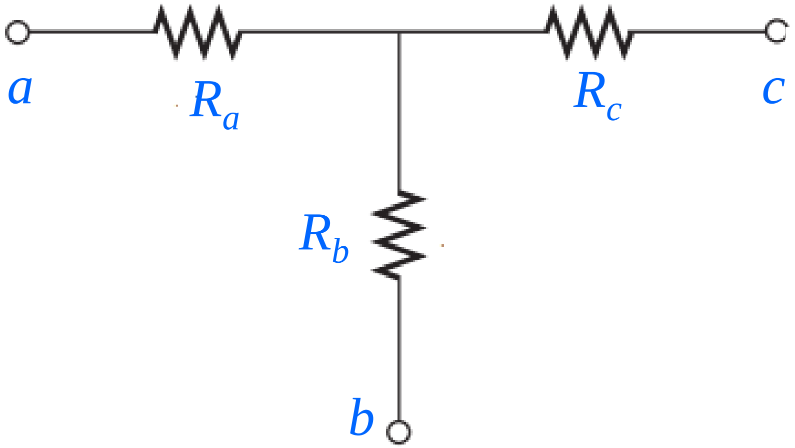
Circuit configurations are often encountered in which the resistors do **not appear to be in series or parallel**. Under these conditions, it may be necessary to convert the circuit from one form to another [*i*.*e*. convert from **Y/T to Δ/Π or Δ/Π to Y/T**] to solve for any unknown quantities

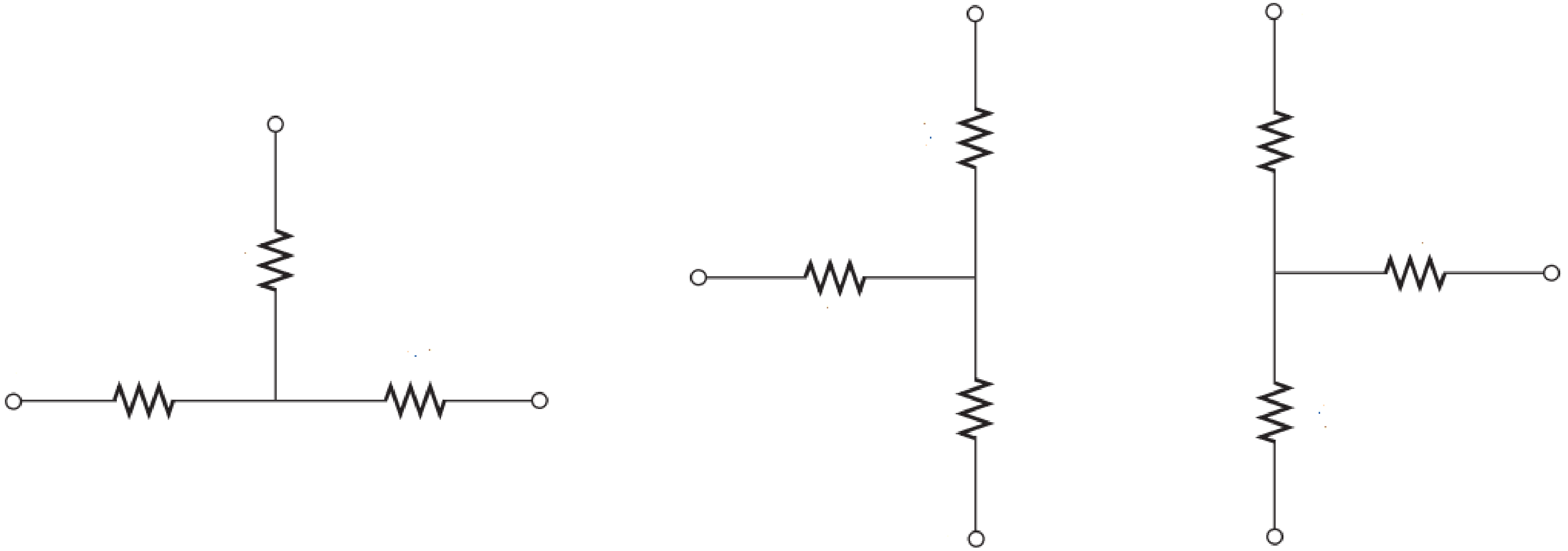
## Y (Wye)



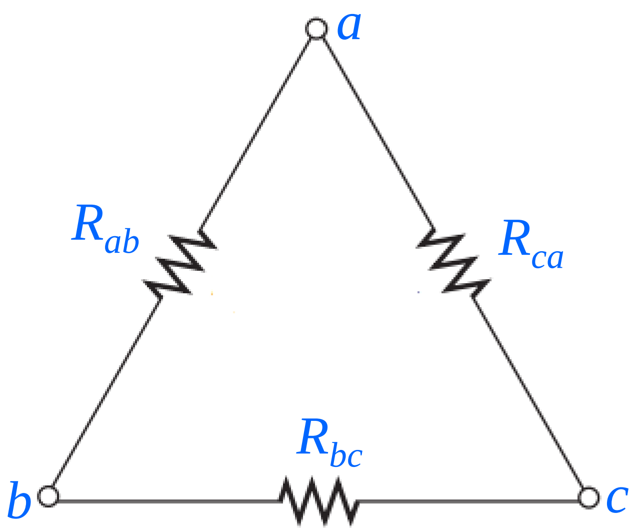



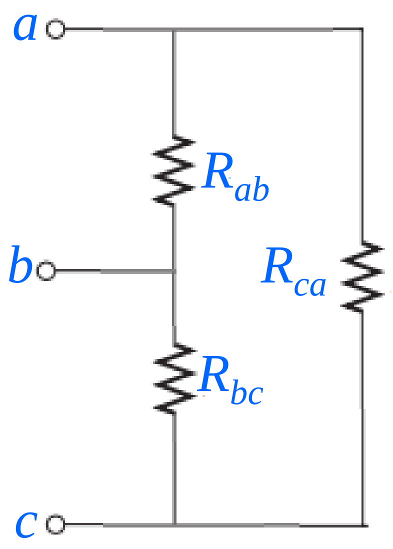
## T (Tee)

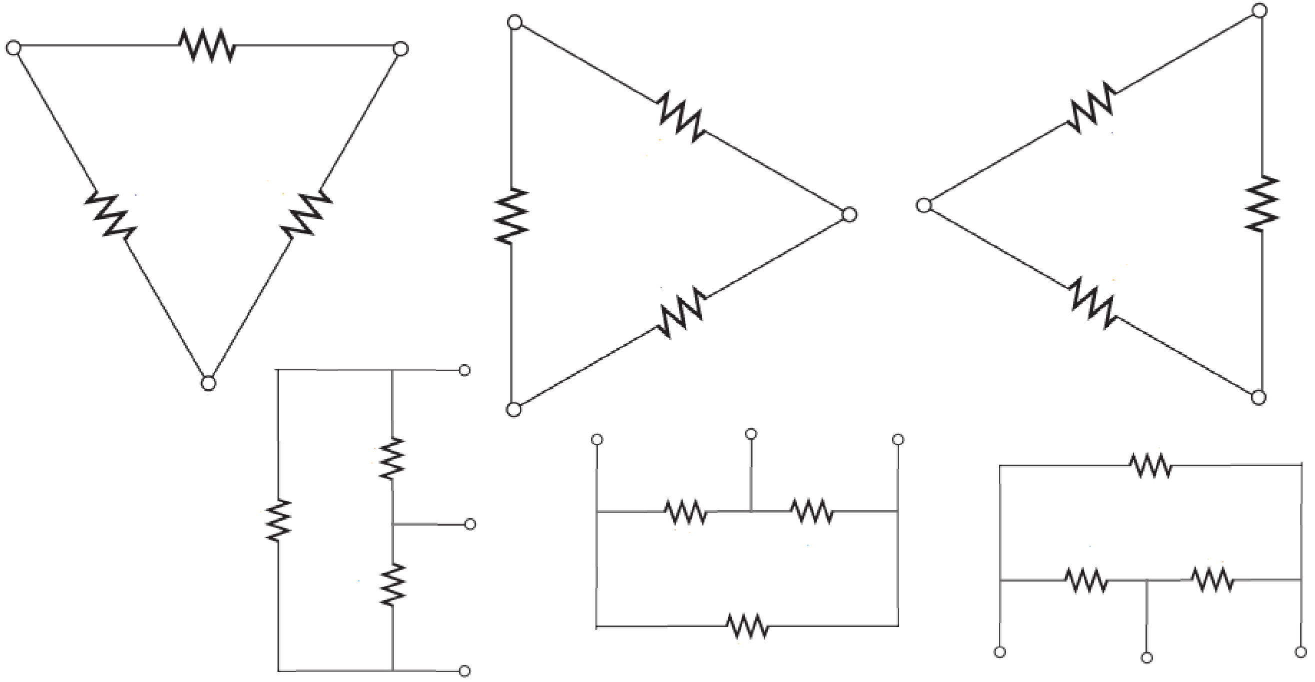


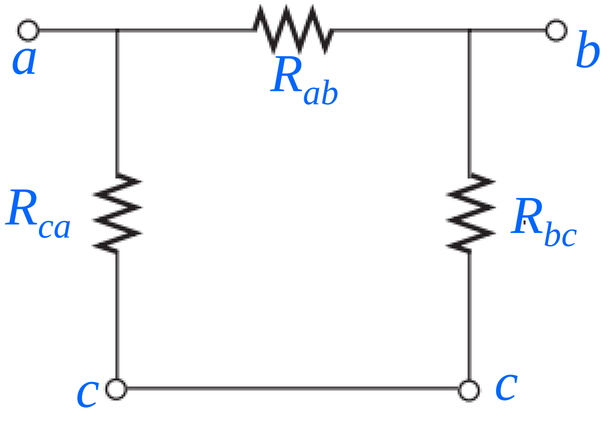
## Δ (Delta)

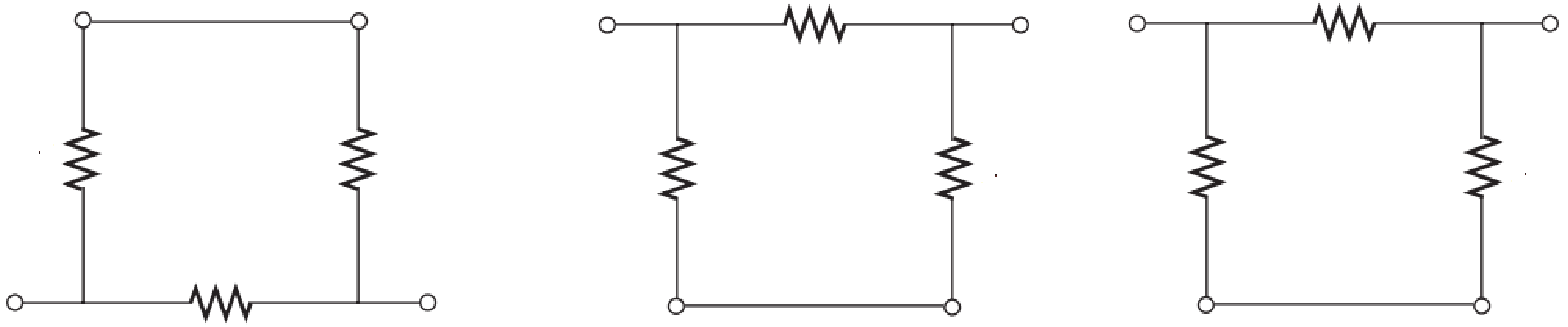


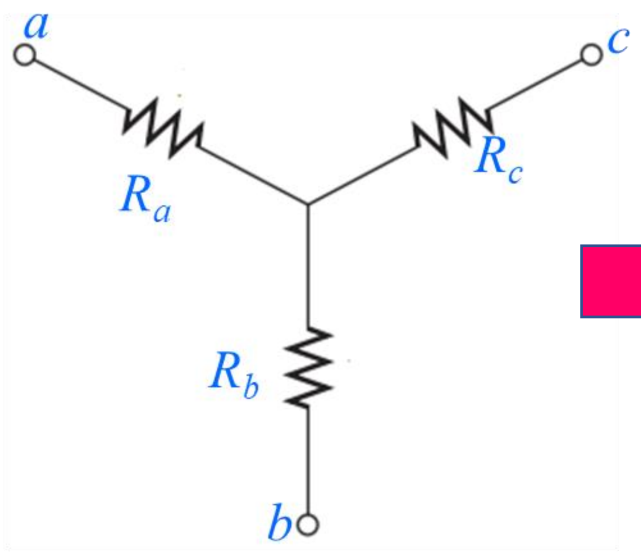



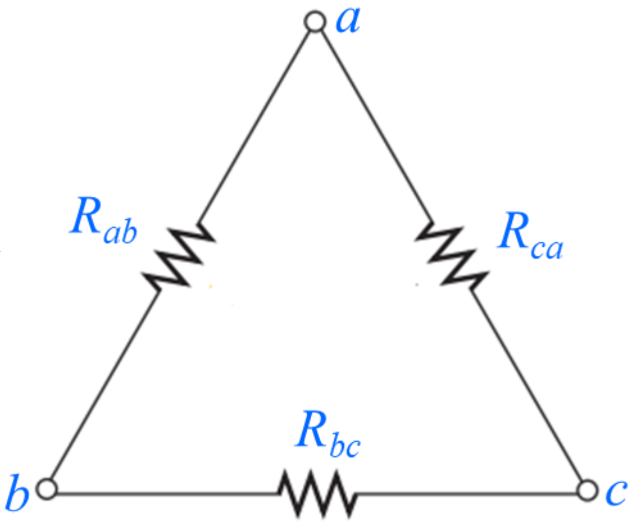
## Π (Pai)



# Conversion from Y (T) to Δ (Π)




Here, $R_a$, $R_b$ and $R_c$ are known.

Calculate $R_{ab}$, $R_{bc}$ and $R_{ca}$

$$R_{abc} = R_a R_b + R_b R_c + R_c R_a$$

$$R_{ab} = \frac{R_{abc}}{R_c} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

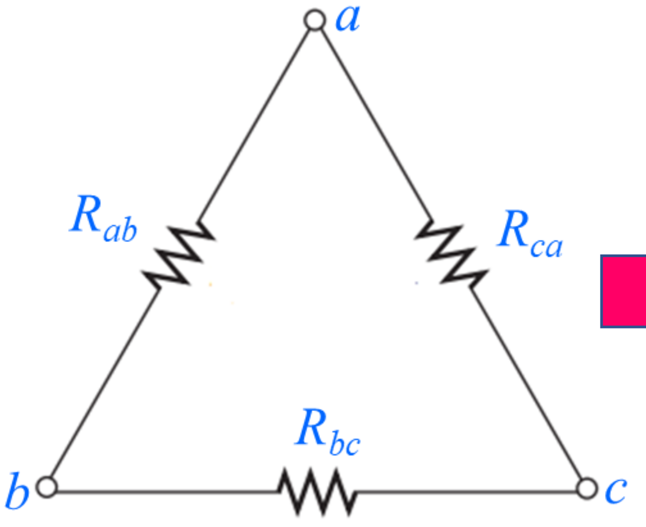
$$R_{bc} = \frac{R_{abc}}{R_a} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

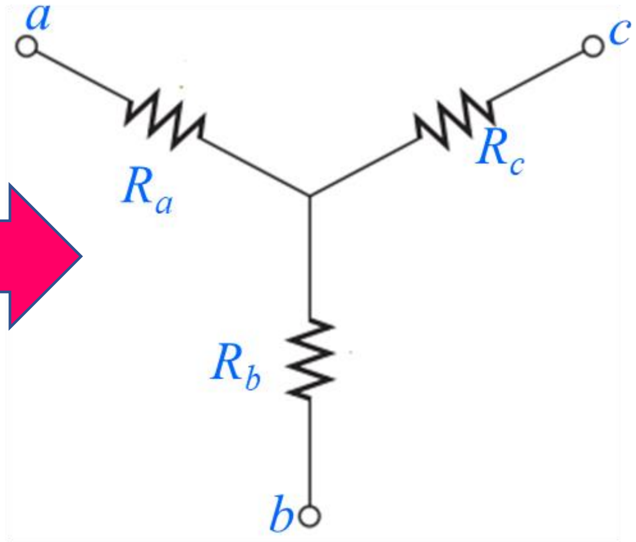
$$R_{ca} = \frac{R_{abc}}{R_b} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

If $R_a = R_b = R_c = R_Y$ then $R_{ab} = R_{bc} = R_{ca} = R_\Delta = 3R_Y$

**For derivation of these equations go through Eq. (8.3a) to (8.4 c)**

# Conversion from Δ (Π) to Y (T)




Here, $R_{ab}$, $R_{bc}$ and $R_{ca}$ are known.

Calculate $R_a$, $R_b$ and $R_c$

$$R_{abc} = R_{ab} + R_{bc} + R_{ca}$$

$$R_a = \frac{R_{ab}R_{ca}}{R_{abc}} = \frac{R_{ab}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

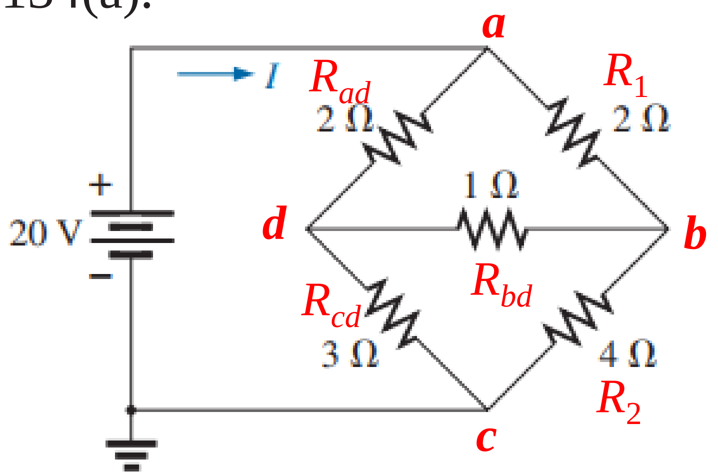
$$R_b = \frac{R_{bc}R_{ab}}{R_{abc}} = \frac{R_{bc}R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_c = \frac{R_{ca}R_{bc}}{R_{abc}} = \frac{R_{ca}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

If $R_{ab} = R_{bc} = R_{ca} = R_\Delta$ then $R_{ab} = R_{bc} = R_{ca} = R_Y = \dfrac{R_\Delta}{3}$

**For derivation of these equations go through Eq. (8.5a) to (8.5c)**

**Problem 51(a) [P. 342].** Using a Δ-Y or Y-Δ conversion, find the current $I$ in each of the networks in Fig. 8.134(a).




**Solution:** First, marked four nodes ($a$, $b$, $c$, and $d$) in the circuit.

Let, $R_{ad}$ = 2 Ω, $R_{bd}$ = 1 Ω, $R_{cd}$ = 3 Ω, $R_1$ = 2 Ω, and $R_2$ = 4 Ω

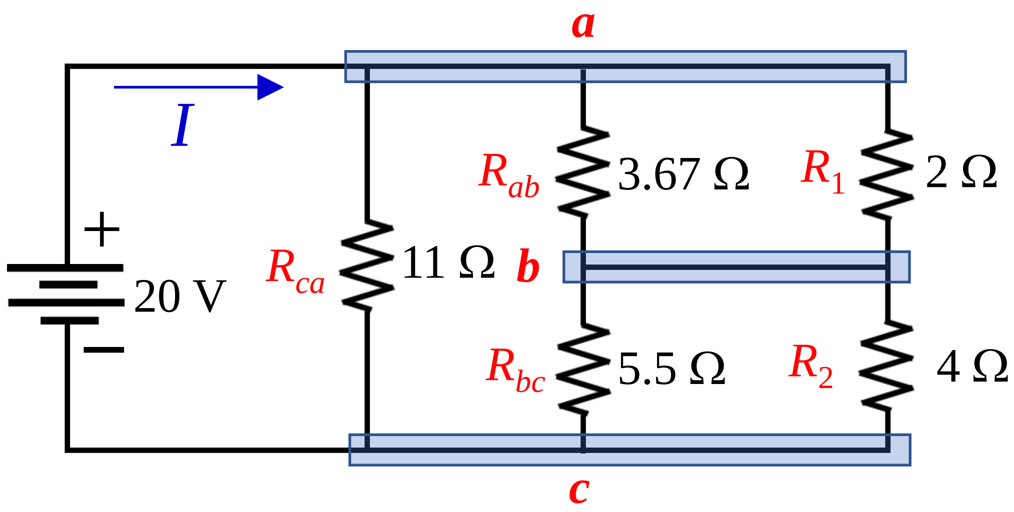
Here, $R_{ad}$, $R_{bd}$, and $R_{cd}$ made Y connection where $d$ is common point. So, this Y connection is going to convert Δ connection.

$$R_{abc} = R_{ad}R_{bd} + R_{bd}R_{cd} + R_{cd}R_{ad}$$
$$= (2\Omega)(1\Omega) + (1\Omega)(3\Omega) + (3\Omega)(2\Omega)$$
$$= 2\Omega + 3\Omega + 6\Omega = 11\Omega$$

$$R_{ab} = \frac{R_{abc}}{R_{cd}} = \frac{11\Omega}{3\Omega} = 3.67\,\Omega$$

$$R_{bc} = \frac{R_{abc}}{R_{ad}} = \frac{11\Omega}{2\Omega} = 5.5\Omega$$

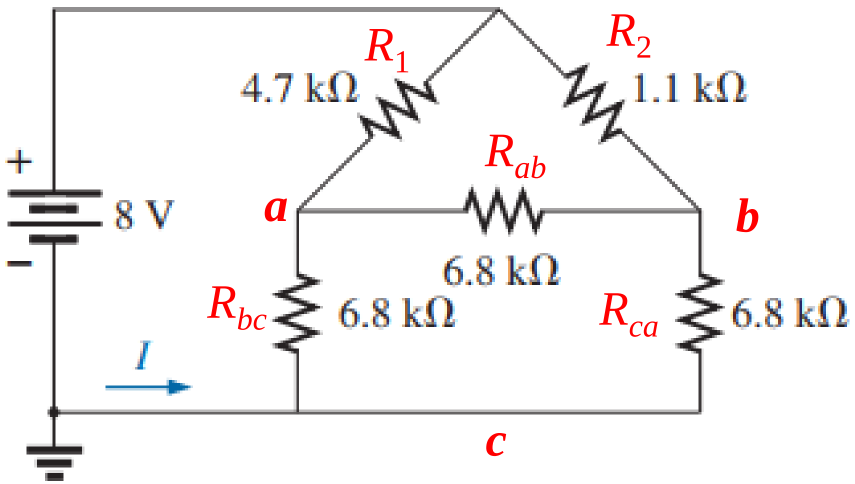
$$R_{ca} = \frac{R_{abc}}{R_{bd}} = \frac{11\Omega}{1\Omega} = 11\Omega$$




$$
\begin{aligned}
R_T &= (R_{ca}) // \{(R_{ab} // R_1) + (R_{bc} // R_2)\} \\
&= (11\Omega) // \{(3.67\Omega // 2\Omega) + (5.5\Omega // 4\Omega)\} \\
&= (11\Omega) // \{1.29\Omega + 2.32\Omega\} \\
&= (11\Omega) // \{3.61\Omega\} \\
&= 2.72\ \Omega
\end{aligned}
$$

$$
I = \frac{E}{R_T} = \frac{20\,\text{V}}{2.72\,\Omega} = \mathbf{7.35\,V}
$$

**Problem 51(b) [P. 342].** Using a Δ-Y or Y-Δ conversion, find the current $I$ in each of the networks in Fig. 8.134(b).




**Solution:** First, marked four nodes ($a$, $b$, and $c$) in the circuit.

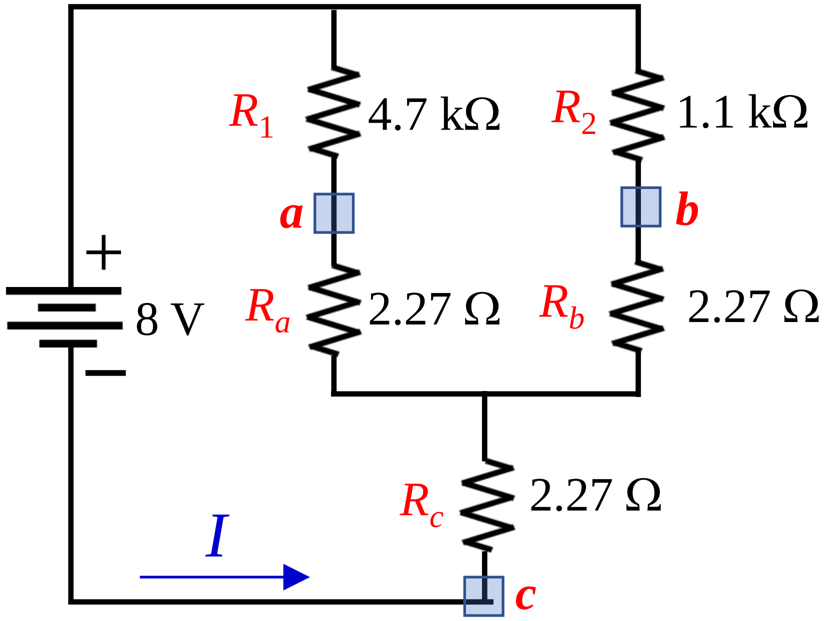
Let, $R_1$=4.7 kΩ, $R_2$=1.1 kΩ, $R_{ab}$=$R_{bc}$=$R_{ca}$= 6.8 kΩ

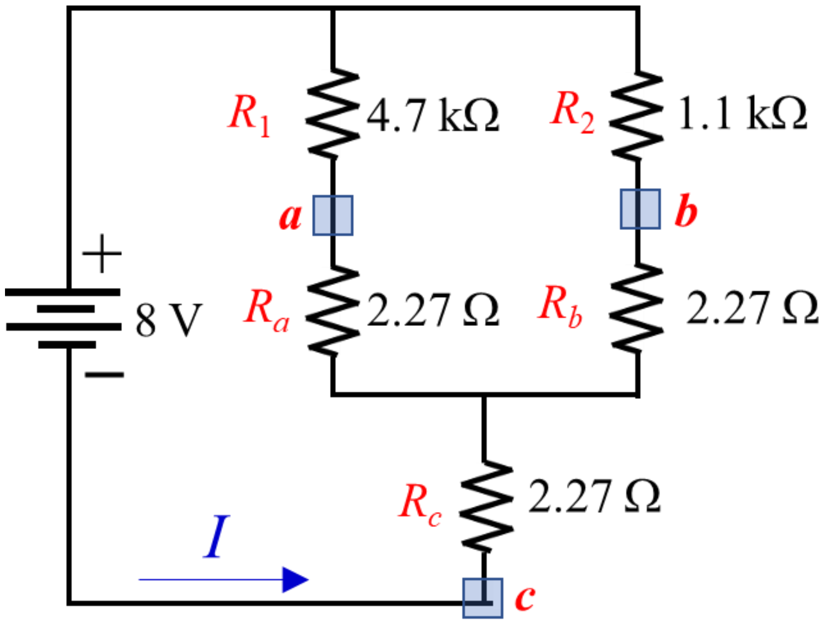
Here, $R_{ab}$, $R_{bc}$, and $R_{ca}$ made Δ connection. So, this Δ connection is going to convert Y connection.

Here, $R_{ab} = R_{bc} = R_{ca} = R_\Delta = 6.8\ \text{k}\Omega$

so, $R_a = R_b = R_c = R_Y = \dfrac{R_\Delta}{3}$

$$= \frac{6.8\ \text{k}\Omega}{3} = 2.27\ \text{k}\Omega$$






$$
\begin{aligned}
R_T &= [(R_1 + R_a)//(R_2 + R_b)] + R_c \\
&= [(4.7\text{k}\Omega + 2.27\text{k}\Omega)//(1.1\text{k}\Omega + 2.27\text{k}\Omega)] + 2.27\text{k}\Omega \\
&= [(6.97\text{k}\Omega)//(3.37\text{k}\Omega)] + 2.27\text{k}\Omega \\
&= 2.27\text{k}\Omega + 2.27\text{k}\Omega \\
&= 4.54\text{k}\Omega
\end{aligned}
$$

$$I = \frac{E}{R_T} = \frac{8\,\text{V}}{4.54\ \text{k}\Omega} = \mathbf{1.76\ \ mA}$$

**Practice Book [Ch 8] Problem: 51 ~ 56**

# 8.11 BRIDGE NETWORKS

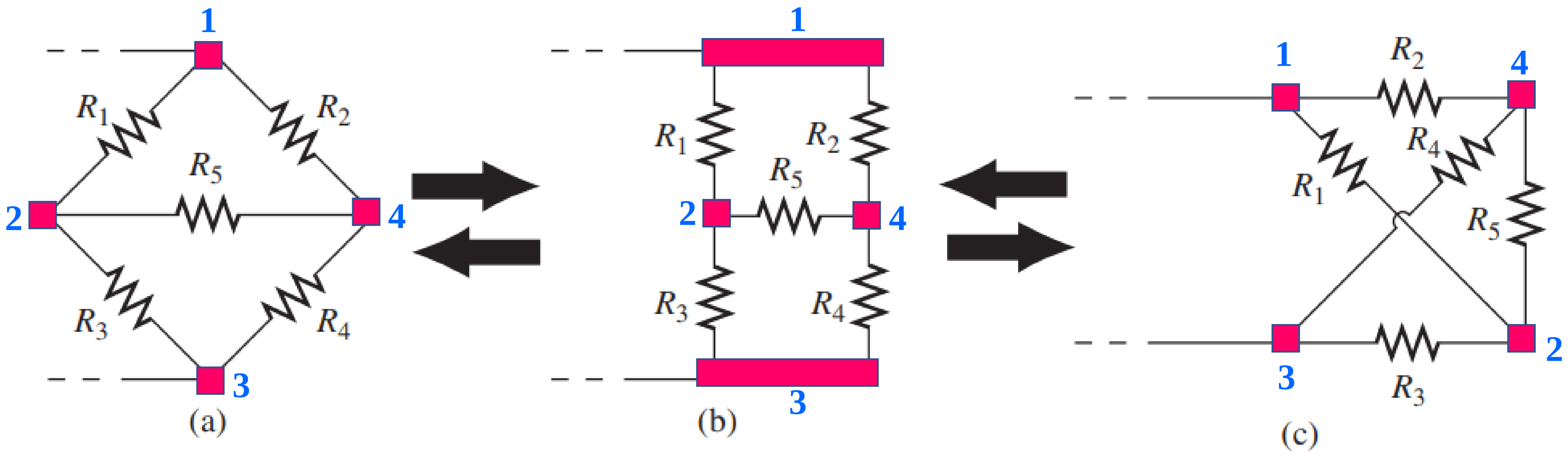
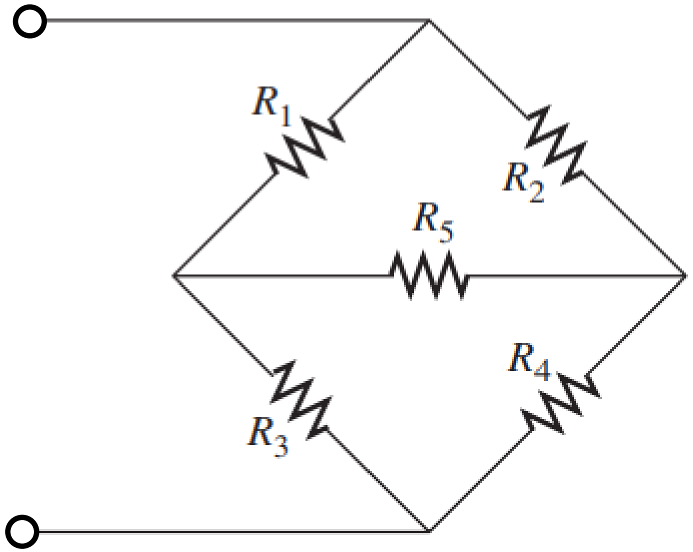


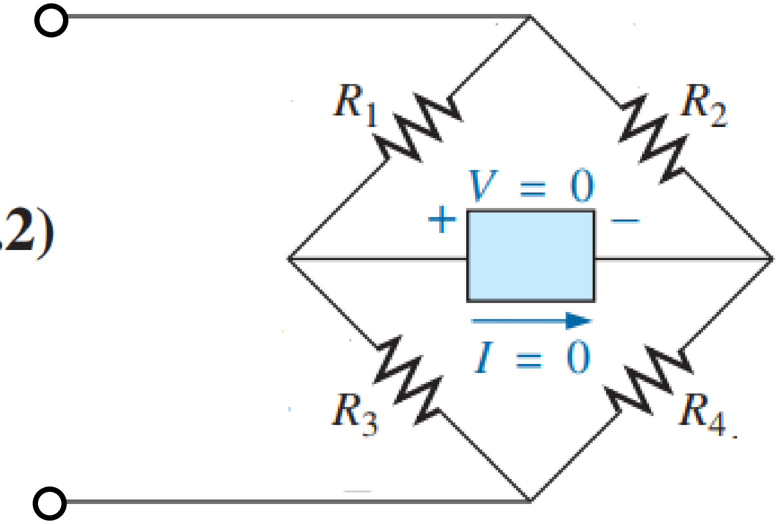

**FIG. 8.69** *Various formats for a bridge network.*

# Balanced Bridge Network



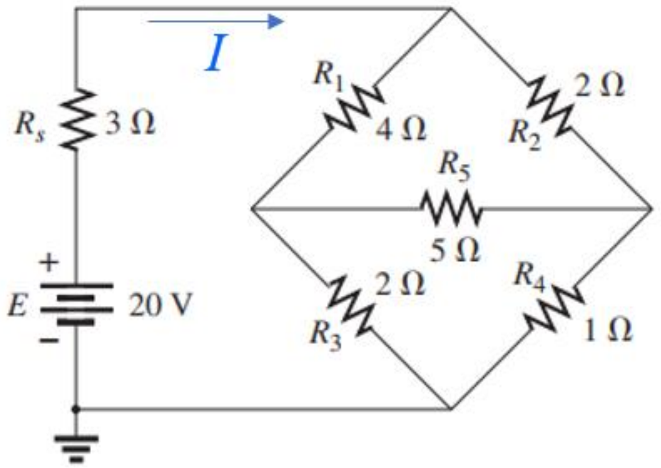

If the ratio of $R_1$ to $R_3$ is equal to that of $R_2$ to $R_4$, the bridge is balanced, and $I = 0$ A or $V = 0$ V.

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \qquad \textbf{(8.2)}$$

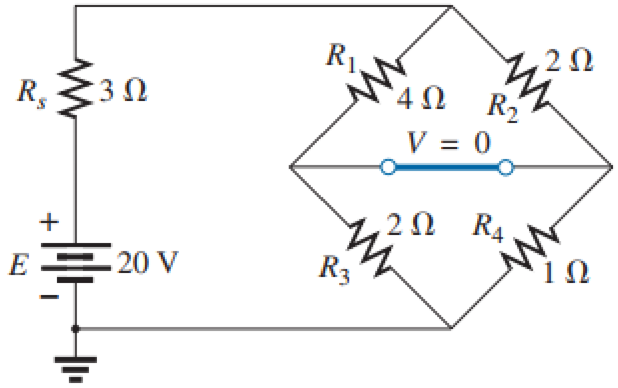


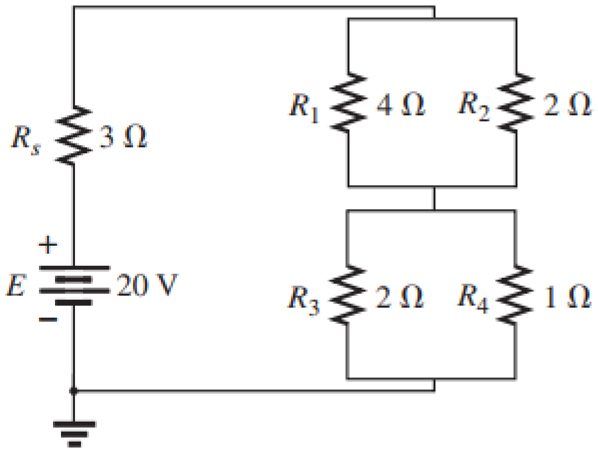

**If this condition is not satisfied, to solve the bridge circuit use the other techniques.**

**Example 8.11.1:** Find the current $I$ for the following network.



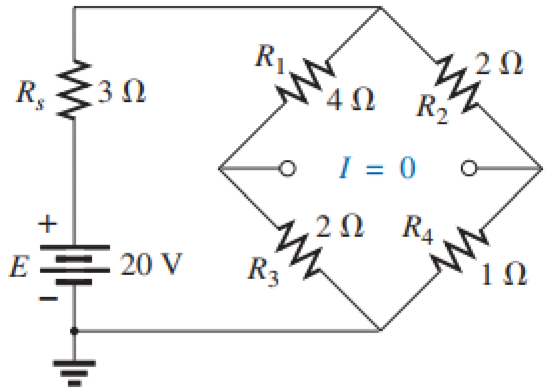

**Solution:** Here: $\frac{R_1}{R_3} = \frac{4\Omega}{2\Omega} = 2$ and $\frac{R_2}{R_4} = \frac{2\Omega}{1\Omega} = 2$ so $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

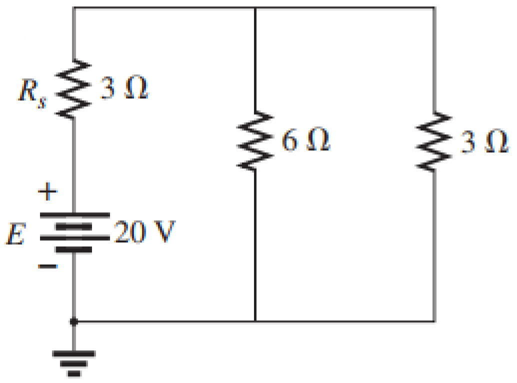



$$R_T = 3\Omega + (4\Omega // 2\Omega) + (2\Omega // 1\Omega) = 5\Omega$$

$$I = \frac{E}{R_T} = \frac{20\text{ V}}{5\Omega} = \mathbf{4\,A}$$




$$R_T = 3\Omega + (6\Omega // 3\Omega) = 5\Omega$$

$$I = \frac{E}{R_T} = \frac{20\text{ V}}{5\Omega} = \mathbf{4\,A}$$

# Chapter 9

## Network Theorems (DC)

# 9.2 SUPERPOSITION THEOREM

## Statement of Superposition Theorem

**The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.**

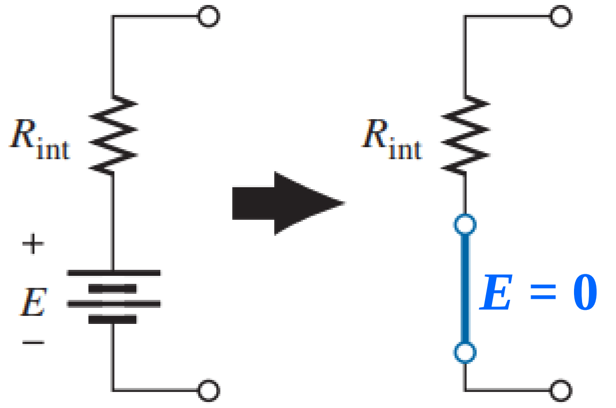
## Steps to Apply Superposition Theorem

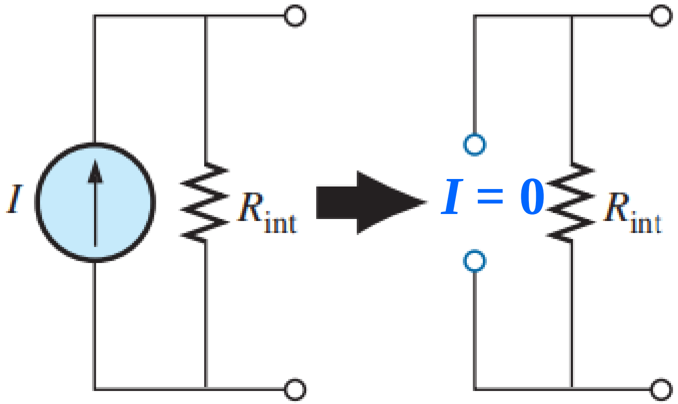
**Step 1:** Select a single source acting alone. Short the other voltage sources to make voltage is zero and open the current sources to make current is zero, if internal impedances are not known. If known, replace them by their internal impedances.

**Step 2:** Find the current through or the voltage across the required element, due to the source under consideration, using a suitable simplification technique.

**Step 3:** Repeat the above two steps for all the sources.

**Step 4:** Add all the individual effects produced by individual sources, to obtained the total current in or voltage across the element.



**EXAMPLE 9.1** Using the superposition theorem, determine current $I_1$ for the network in Fig. 9.2.

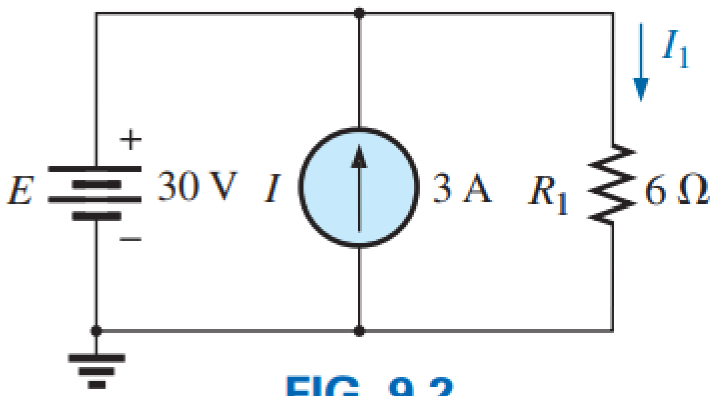
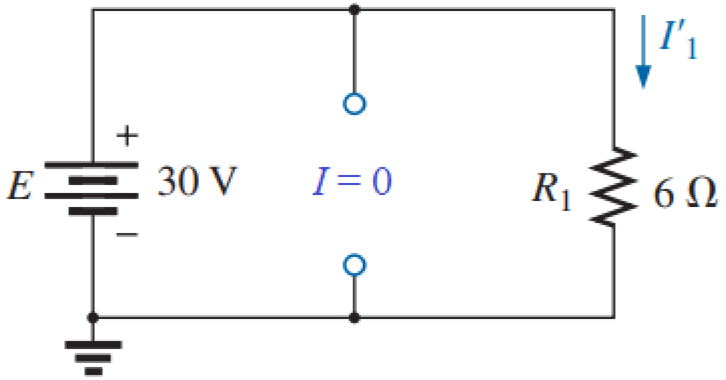


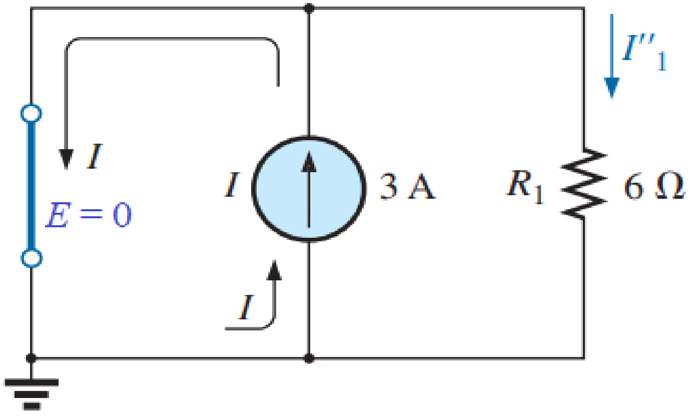

FIG. 9.2

**Solution**: Consider $E$ then and $I$ = 0 A (**open**).




$$I'_1 = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = 5\text{ A}$$

Consider $I$ then and $E$ = 0 V (**shorted**).




$$I''_1 = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\text{ A}$$

According to Superposition Theorem:

$$I_1 = I'_1 + I''_1 = 5\text{ A} + 0\text{ A} = \mathbf{5\ A}$$