

Lecture 19

The Principle of Superposition for Waves

Suppose that **two waves** travel simultaneously along the **same stretched string**. Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone.

The displacement of the string when the waves overlap is then the **algebraic sum**

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$

Overlapping waves **algebraically add** to produce a resultant wave (or net wave)

Overlapping waves **do not** in any way alter the travel of each other.

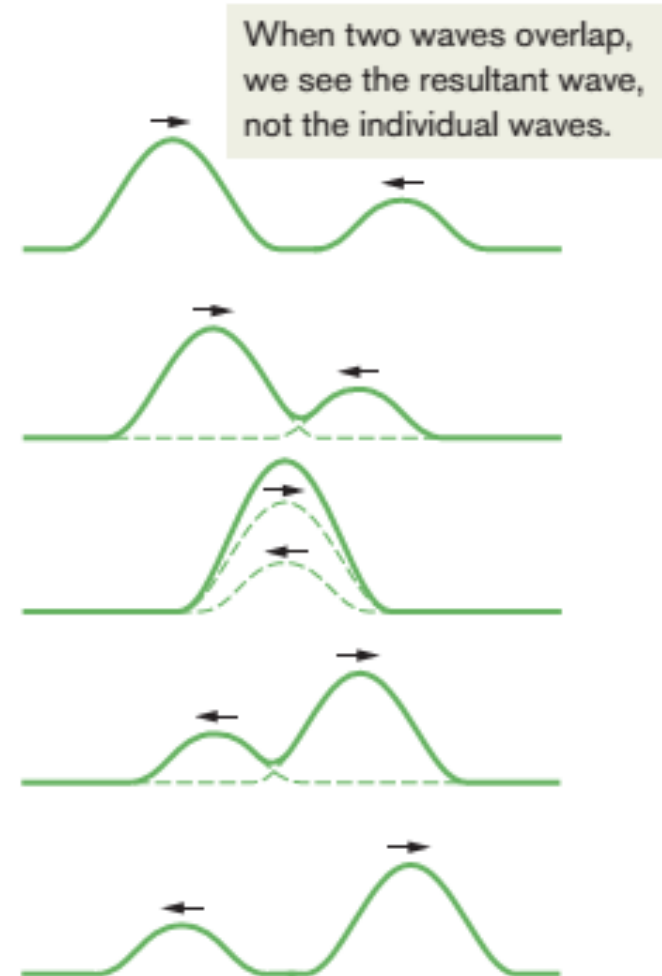


Figure 16-12 A series of snapshots that show two pulses traveling in opposite directions along a stretched string. The superposition principle applies as the pulses move through each other.

Interference of Waves

Suppose we send two sinusoidal waves of the same wavelength and amplitude in the **same direction** along a stretched string.

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx - \omega t + \varphi)$$

ω (f), k (λ), y_m , v are same

Superposition principle, $y'(x, t) = y_1(x, t) + y_2(x, t)$

$$y'(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \varphi)$$

$$= y_m \{ \sin(kx - \omega t) + \sin(kx - \omega t + \varphi) \}$$

$$= y_m \left\{ 2 \sin \left(\frac{kx - \omega t + kx - \omega t + \varphi}{2} \right) \cos \left(\frac{kx - \omega t - kx + \omega t - \varphi}{2} \right) \right\}$$

$$= 2y_m \sin \left\{ \frac{2(kx - \omega t) + \varphi}{2} \right\} \cos \left(\frac{-\varphi}{2} \right)$$

$$= 2y_m \sin \left\{ (kx - \omega t) + \frac{\varphi}{2} \right\} \cos \left(\frac{\varphi}{2} \right)$$

$$y'(x, t) = [2y_m \cos \left(\frac{\varphi}{2} \right)] \sin \left(kx - \omega t + \frac{\varphi}{2} \right) \quad [\text{traveling waves}]$$

Resultant displacement = $y'(x, t)$

Amplitude = $[2y_m \cos \left(\frac{\varphi}{2} \right)]$

Oscillating term = $\sin \left(kx - \omega t + \frac{\varphi}{2} \right)$

If two sinusoidal waves of the same amplitude and wavelength **travel in the same direction** along a stretched string, they interfere to produce a resultant sinusoidal wave **traveling in that direction**.

Interfering waves: $y_1(x, t) = y_m \sin(kx - \omega t)$

$$y_2(x, t) = y_m \sin(kx - \omega t + \varphi)$$

Resultant wave: $y'(x, t) = [2y_m \cos(\frac{\varphi}{2})] \sin(kx - \omega t + \frac{\varphi}{2})$

The resultant wave differs from the interfering waves in two respects:

(1) its phase constant is $\frac{\varphi}{2}$ and (2) its amplitude is $y'_m = [2y_m \cos(\frac{\varphi}{2})]$

(1) If $\varphi = 0$ rad (0°): fully constructive interference

$$y'(x, t) = [2y_m \cos(\frac{0}{2})] \sin(kx - \omega t + \frac{0}{2}) \}$$

$$= [2y_m \cos 0] \sin(kx - \omega t)$$

$$y'(x, t) = 2y_m \sin(kx - \omega t) \quad \text{[greatest amplitude]}$$

(2) If $\varphi = \pi$ rad (180°): fully destructive interference

$$y'(x, t) = [2y_m \cos(\frac{\pi}{2})] \sin(kx - \omega t + \frac{\pi}{2})$$

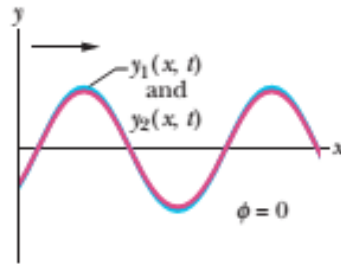
$$= [2y_m (0)] \sin(kx - \omega t + \frac{\pi}{2})$$

$$y'(x, t) = 0$$

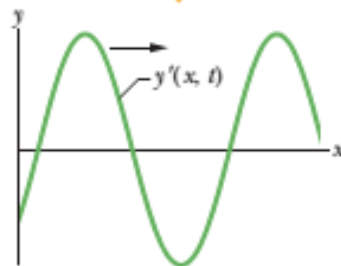
(3) If $\phi = \frac{2\pi}{3}$ rad (120°): intermediate interference

$$\begin{aligned}y'(x, t) &= [2y_m \cos(\frac{2\pi}{3})] \sin \{kx - \omega t + (\frac{3}{2})\} \\&= 2y_m \cos(\frac{\pi}{3}) \sin(kx - \omega t + \frac{\pi}{3}) \\&= 2y_m (\frac{1}{2}) \sin(kx - \omega t + \frac{\pi}{3}) \\y'(x, t) &= y_m \sin(kx - \omega t + \frac{\pi}{3})\end{aligned}$$

Being exactly in phase, the waves produce a large resultant wave.

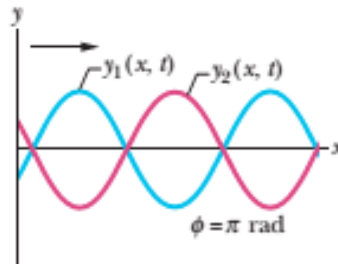


(a)

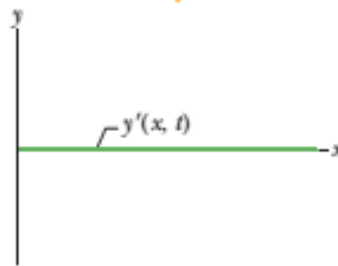


(d)

Being exactly out of phase, they produce a flat string.

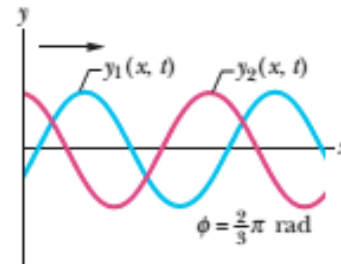


(b)

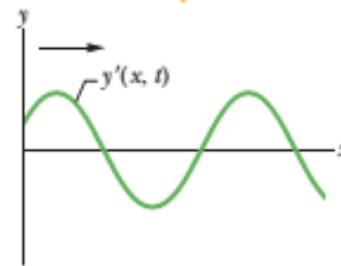


(e)

This is an intermediate situation, with an intermediate result.



(c)



(f)

32. What phase difference between two identical traveling waves, moving in the same direction along a stretched string, results in the combined wave having an amplitude 1.50 times that of the common amplitude of the two combining waves? Express your answer in (a) degrees, (b) radians, and (c) wavelengths.

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx - \omega t + \varphi)$$

Superposition principle, $y'(x, t) = y_1(x, t) + y_2(x, t)$

$$y'(x, t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \varphi)$$

$$= y_m \{ \sin(kx - \omega t) + \sin(kx - \omega t + \varphi) \}$$

$$= y_m \left\{ 2 \sin \left(\frac{kx - \omega t + kx - \omega t + \varphi}{2} \right) \cos \left(\frac{kx - \omega t - kx + \omega t - \varphi}{2} \right) \right\}$$

$$= 2y_m \sin \left\{ \frac{2(kx - \omega t) + \varphi}{2} \right\} \cos \left(\frac{kx - \omega t - kx + \omega t - \varphi}{2} \right)$$

$$= 2y_m \sin \left\{ (kx - \omega t) + \frac{\varphi}{2} \right\} \cos \left(-\frac{\varphi}{2} \right)$$

$$y'(x, t) = [2y_m \cos(\frac{\varphi}{2})] \sin(kx - \omega t + \frac{\varphi}{2})$$

$$(a) [2y_m \cos(\frac{\varphi}{2})] = 1.50 y_m$$

$$\cos(\frac{\varphi}{2}) = 1.50/2$$

$$\cos\left(\frac{\varphi}{2}\right) = 0.75$$

$$\frac{\varphi}{2} = \cos^{-1}(0.75)$$

$$\frac{\varphi}{2} = 41.41$$

$$\varphi = 2(41.41)$$

$$\varphi = 82.82^\circ \quad \text{Ans.}$$

$$(b) \frac{\varphi}{2} = \cos^{-1}(0.75 \text{ rad})$$

$$\frac{\varphi}{2} = 0.7227 \text{ rad}$$

$$\varphi = 1.45 \text{ rad} \quad \text{Ans.}$$

$$(c) 2\pi \text{ rad} = \lambda$$

$$1 \text{ rad} = \left(\frac{\lambda}{2\pi}\right)$$

$$1.45 \text{ rad} = 1.45 \left(\frac{\lambda}{2\pi}\right) = 0.23\lambda \quad \text{Ans.}$$

Additional problem:

Sample problem 16.04, page: 461