Lecture 7

47. The temperature of 2.00 mol of an ideal monatomic gas is raised 15.0 K at constant volume. What are (a) the work *W* done by the gas, (b) the energy transferred as heat *Q*, (c) the change ΔE_{int} in the internal energy of the gas, and (d) the change ΔK in the average kinetic energy per atom?

Solution:

Here, n = 2.00 mol

$\Delta T = 15$ K

$\Delta V = V - V = 0$

$k = \frac{R}{N_A} = \frac{8.31\ J/mol-K}{6.023 \times 10^{23}/mol} = 1.38 \times 10^{-23}\ J/K$

(a) $W = p\Delta V = p(0) = 0$

(b) $Q = nC_V\Delta T = n\,(\frac{3}{2})R\,\Delta T = 2.00\,(\frac{3}{2})8.31\,(15) = 373.95$ J

$[C_V = (\frac{3}{2})R]$

$[R = 8.31\ J/mol - K]$

(c) $\Delta E_{int} = Q - W = 373.95 - 0 = 373.95$ J

(d) $\Delta K = (\frac{3}{2})k\Delta T = (\frac{3}{2})(1.38 \times 10^{-23})15 = 31.05 \times 10^{-23}$ J

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48. When 20.9 J was added as heat to a particular ideal gas, the volume of the gas changed from 50.0 cm^3 to 100 cm^3 while the pressure remained at 1.00 atm. (a) By how much did the internal energy of the gas change? If the quantity of gas present was 2.00×10^{-3} mol, find (b) C_p and (c) C_V.

Solution:

Here, Q = +20.9 J

$\Delta V = (100 - 50)\ cm^3 = 50\ cm^3 = 50(10^{-2}\ m)^3 = 50 \times 10^{-6}\ m^3$

$p = 1.00\ atm = 1 \times 10^5$ Pa

$n = 2.00 \times 10^{-3}$ mol

(a) $\Delta E_{int} = Q - W = Q - p\,\Delta V = 20.9 - 1 \times 10^5(50 \times 10^{-6}) = 20.9 - 5.0 = 15.9$ J

(b) $Q = nC_p\Delta T$ [p = constant]

$C_p = \frac{Q}{n\Delta T}$

Ideal gas law, $pV = nRT$

$$\frac{\Delta}{\Delta T}(pV) = \frac{\Delta}{\Delta T}(nRT)$$

$$p\frac{\Delta V}{\Delta T} = nR\frac{\Delta T}{\Delta T}$$

$$p\frac{\Delta V}{\Delta T} = nR$$

$$p\Delta V = nR\,\Delta T$$

$$\frac{p\Delta V}{R} = n\,\Delta T$$

$$C_p = \frac{Q}{\frac{p\Delta V}{R}}$$

$$C_p = \frac{QR}{p\Delta V} = \frac{20.9(8.31)}{1\text{x}10^5(50\text{x}10^{-6})} = \frac{173.68}{5.0} = 34.74 \text{ J/mol-K}$$

(c) $C_p - C_V = R$

$C_V = C_p - R = 34.74 - 8.31 = 26.43$ J/mol-K

Lecture 8

54. We know that for an adiabatic process pVγ = a constant. Evaluate "a constant" for an adiabatic process involving exactly 2.0 mol of an ideal gas passing through the state having exactly p = 1.0 atm and T = 300 K. Assume a diatomic gas whose molecules rotate but do not oscillate.

Solution:

Here, n = 2 mol

$p = 1.0$ atm $= 1.0\text{x}10^5$ Pa

$T = 300$ K

pV^γ = constant

Diatomic gas whose molecules rotate but do not oscillate, f = 3+2 = 5

$$C_V = (\tfrac{f}{2})R = (\tfrac{5}{2})R$$

$$C_p - C_V = R$$

$$C_p = C_V + R = (\tfrac{5}{2})R + R = (\tfrac{7}{2})R$$

$$\gamma = \frac{C_P}{C_v} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} = 1.4$$

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pV^γ = constant

$a = pV^\gamma$ [Ideal gas law, $pV = nRT$]

$a = p(\frac{nRT}{p})^\gamma$ [$V = \frac{nRT}{p}$]

$$= 1.0\text{x}10^5\{\frac{2(8.31)(300)}{1.0\times10^5}\}^{1.4}$$

$$= 1.0\text{x}10^5\{0.04986\}^{1.4}$$

$a = 1.5\text{x}10^3\ Nm^{2.2}$

Unit of a $= pV^\gamma = \frac{F}{A}V^\gamma = (\frac{N}{m^2})(m^3)^\gamma = N(\frac{m^{3\gamma}}{m^2}) = Nm^{3\gamma-2} = Nm^{3(1.4)-2} = Nm^{4.2-2.0}$

$a = Nm^{2.2}$

55. A certain gas occupies a volume of 4.3 L at a pressure of 1.2 atm and a temperature of 310 K. It is compressed adiabatically to a volume of 0.76 L. Determine (a) the final pressure and (b) the final temperature, assuming the gas to be an ideal gas for which γ = 1.4

Solution:

Here, $V_i = 4.3$ L

$p_i = 1.2$ atm $= 1.2\text{x}10^5$ Pa

$T_i = 310$ K

$V_f = 0.76$ L

$\gamma = 1.4$

(a) pV^γ = constant

$p_iV_i^\gamma = p_fV_f^\gamma$

$$p_f = \frac{p_iV_i^\gamma}{V_f^\gamma} = p_i\left(\frac{V_i^\gamma}{V_f^\gamma}\right) = p_i\left(\frac{V_i}{V_f}\right)^\gamma = 1.2\text{x}10^5\left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4} = 1.2\text{x}10^5(11.3166) = 1.36\text{x}10^6\text{ Pa}$$

(b) $TV^{\gamma-1}$ = constant

$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

$$T_f = \frac{T_iV_i^{\gamma-1}}{V_f^{\gamma-1}} = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 310\left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4-1} = 310(2.00) = 620\text{ K}$$

62. An ideal diatomic gas, with rotation but no oscillation, undergoes an adiabatic compression. Its initial pressure and volume are1.20 atm and 0.200 m3 . Its final pressure is 2.40 atm. How much work is done by the gas?

Solution:

Given:

$\gamma = 1.40$

$p_i = 1.20$ atm $= 1.20 \times 10^5$ Pa

$V_i = 0.200$ m^3

$p_f = 2.40$ atm $= = 2.40 \times 10^5$ Pa

W =?

$$W = \frac{p_iV_i - p_fV_f}{\gamma - 1}$$

V_f = ?

$p_iV_i^\gamma = p_fV_f^\gamma$

$$\frac{p_i}{p_f} = \left(\frac{V_f^\gamma}{V_i^\gamma}\right)$$

$$\frac{p_i}{p_f} = \left(\frac{V_f}{V_i}\right)^\gamma$$

$$\sqrt[\gamma]{\frac{p_i}{p_f}} = \sqrt[\gamma]{\left(\frac{V_f}{V_i}\right)^\gamma}$$

$$\sqrt[\gamma]{\frac{p_i}{p_f}} = \left(\frac{V_f}{V_i}\right)$$

$$V_f = V_i\sqrt[\gamma]{\frac{p_i}{p_f}}$$

$$V_f = (0.200)\sqrt[1.40]{\frac{1.20\times10^5}{2.40\times10^5}}$$

$$V_f = (0.200)\,(0.5)^{0.714}$$

$$V_f = (0.200)(0.6096)$$

$$V_f = 0.122\text{ m}^3$$

$$W = \frac{p_iV_i - p_fV_f}{\gamma - 1}$$

$$W = \frac{(1.20 \times 10^5)(0.200) - (2.40 \times 10^5)(0.122)}{1.40 - 1}$$

$W = -1.32 \times 10^4$ J (Ans.)

Lecture 9

Sample Problem 20.02: Suppose 1.0 mol of nitrogen gas is confined to the left side of the container of Fig. 20-1a. You open the stopcock, and the volume of the gas doubles. What is the entropy change of the gas for this irreversible process?

Solution:

Here, n = 1 mol

$V_i = V$

$V_f = 2V$

$T_i = T_f$

$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\frac{T_f}{T_i}$$

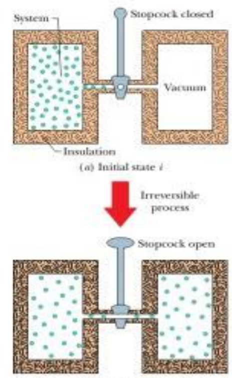
$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\frac{T_f}{T_f}$$

$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln1 \qquad \Delta S = nRln\frac{V_f}{V_i} + 0$$

$$\Delta S = nRIn\frac{V_f}{V_i} = 1.0(8.31)In\frac{2V}{V}$$

$$= 1.0(8.31)In\,2$$

$$= 5.76\,J/K$$



(a) Initial state i

(b) Final state f

2. An ideal gas undergoes a reversible isothermal expansion at 77.0 °C, increasing its volume from 1.30 L to 3.40 L. The entropy change of the gas is 22.0 J/K. How many moles of gas are present?

Solution:

Here, $T_i = T_f = 77^0C = (77 + 273)\ K = 350\ K$ $\quad V_i = 1.30\ L \quad V_f = 3.40\ L$

$\Delta S = 22.0\ J/K$

$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\frac{T_f}{T_i}$$

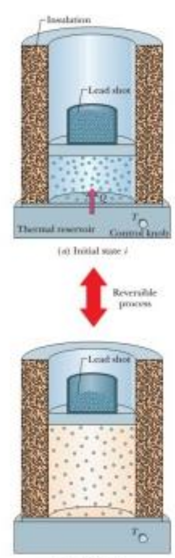
$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\frac{350}{350}$$

$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\,1, \quad \Delta S = nRln\frac{V_f}{V_i}$$

$$n = \frac{\Delta S}{RIn\frac{V_f}{V_i}}$$

$$n = \frac{22}{8.31(In(\frac{3.4L}{1.3L}))}$$

n = 2.754 mol



(a) Initial state i

(b) Final state f

Lecture 10

3. A 2.50 mol sample of an ideal gas expands reversibly and isothermally at 360 K until its volume is doubled. What is the increase in entropy of the gas?

Solution:

$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\frac{T_f}{T_i}$$

$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\frac{360}{360}$$

$$\Delta S = nRln\frac{V_f}{V_i} + nC_v ln\,1$$

$$\Delta S = nRIn\frac{V_f}{V_i}$$

$$\Delta S = 2.5\times 8.31\times In\frac{2V}{V} = 2.5\times 8.31\times In2$$

$\Delta S = 14.4$ J/K **(Answer)**

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4. How much energy must be transferred as heat for a reversible isothermal expansion of an ideal gas at 132 ^{0}C if the entropy of the gas increases by 46.0 J/K?

Solution:

$$\Delta S = \int_i^f \frac{dQ}{T} = \frac{1}{T}\int_i^f dQ$$

$$\Delta S = \frac{Q}{T}$$

Q = $\Delta S \times$T $\quad$ [$\Delta S = 46$ J/K]

Q = 46x408 $\quad$ [T = (132 +273) K= 408 K]

Q = 1.88 $\times 10^6$ J $\quad$ (Answer)

Lecture 11

23. A Carnot engine whose low-temperature reservoir is at 17 ⁰C has an efficiency of 40%. By how much should the temperature of the high-temperature reservoir be increased to increase the efficiency to 50%?

Solution:

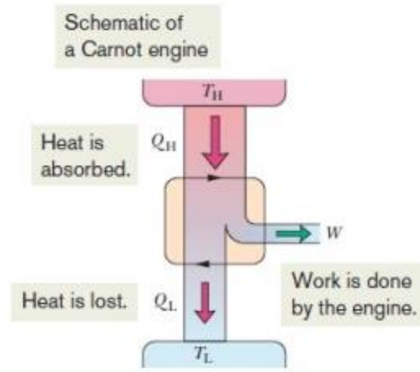
Given,

$T_L = 17^0C = 290\ K$

$Initial\ efficiency, \varepsilon_c = 40\%$

$Final\ efficiency, \varepsilon_c' = 50\%$

$\Delta T_H = ?$



For the initial state,

$$\varepsilon_c = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow 40\% = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow \frac{T_L}{T_H} = 1 - 0.40$$

$$\therefore T_H = 483.33\ K$$

For the final state,

$$\varepsilon_c' = 1 - \frac{T_L}{T_{H'}}$$

$$\Rightarrow 50\% = 1 - \frac{T_L}{T_H'}$$

$$\Rightarrow \frac{T_L}{T_H'} = 1 - 0.50$$

$$\therefore T_H' = 580\ K$$

So the increased temperature of the high temperature reservoir,

$$\Delta T_H = T_H' - T_H$$

$$= (580 - 483.33)\ K$$

$$= 96.67\ K$$

24. A Carnot engine absorbs 52 kJ as heat and exhausts 36 kJ as heat in each cycle. Calculate (a) the engine's efficiency and (b) the work done per cycle in kilojoules.

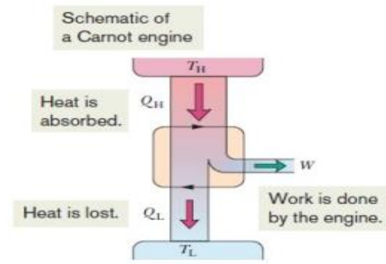
Solution:

Given,

$|Q_H| = 52\ kJ = 52 \times 10^3\ J$

$|Q_L| = 36\ kJ = 36 \times 10^3\ J$

$(a)\varepsilon_c = ?$



We know,

$$\varepsilon_c = \left(1 - \frac{|Q_L|}{|Q_H|}\right) \times 100\%$$

$$= \left(1 - \frac{36 \times 10^3}{52 \times 10^3}\right) \times 100\%$$

$$= 30.77\ \%$$

$(b) W = ?$

We know

$$W = |Q_H| - |Q_L|$$

$$= 52\ kJ - 36\ kJ$$

$$W = 16\ kJ$$

Lecture 12

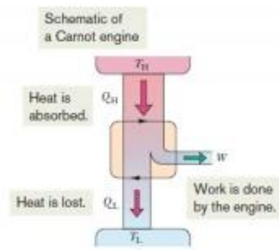
25. A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by 75.0 C^0. What is the temperature of the (a) lower-temperature and (b) higher-temperature reservoir?

Given,

Efficiency, $\varepsilon_c = 22.0\% = 0.22$

Difference in temperature, $T_H - T_L = 75C^0 = 75\ K$

$$(T_H+273) - (T_L+273) = 75K$$



$(a)\ T_L = ?$

We know,

$$\varepsilon_c = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow 22\% = 1 - \frac{T_L}{75 + T_L} \qquad [as \quad T_H = 75 + T_L]$$

$$\Rightarrow 0.22 = \frac{75 + T_L - T_L}{75 + T_L}$$

$$\Rightarrow 0.22 = \frac{75}{75 + T_L}$$

$$\Rightarrow 0.22(75 + T_L) = 75$$

$$\therefore T_L = 266\ \mathrm{K}$$

$(b)\ T_H = ?$

We have,

$$T_H - T_L = 75\ K$$

$$T_H - 266 = 75$$

$$\therefore T_H = 341\ \mathrm{K}$$

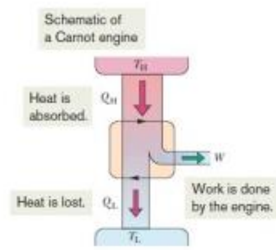
27. A Carnot engine operates between 235 ^{0}C and 115 ^{0}C, absorbing 6.30x10^4 J per cycle at the higher temperature. (a) What is the efficiency of the engine? (b) How much work per cycle is this engine capable of performing?

Given,

$T_H = 235^0C = 508\ K$

$T_L = 115^0C = 388\ K$

$Q_H = 6.3 \times 10^4\ J$



$(a)\ \varepsilon_c = ?$

We know,

$$\varepsilon_c = \left(1 - \frac{T_L}{T_H}\right)$$

$$= \left(1 - \frac{388}{508}\right)$$

$$= 0.2362 = 23.62\%$$

The work done per cycle,

$$W = \varepsilon_c\,|Q_H| \qquad \left[\varepsilon_c = \frac{W}{|Q_H|}\right]$$

$$= 0.2362 \times (6.3 \times 10^4)$$

$$W = 1.48 \times 10^4\ J$$