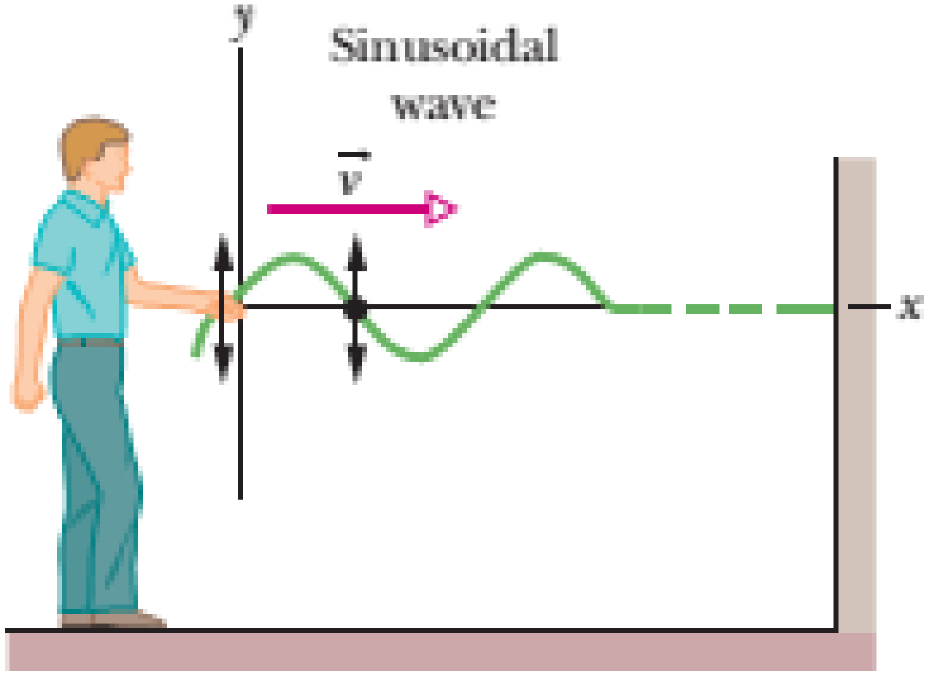
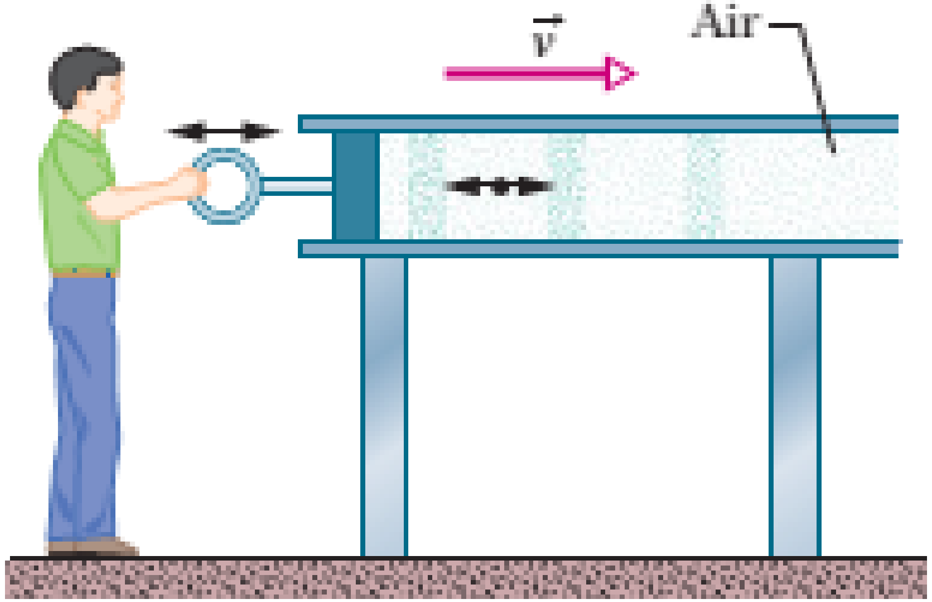
Lecture 17: Waves

Transverse wave:
Vibration of particles of the string perpendicular to the velocity of the propagation of wave



Longitudinal wave:
Vibration of the particle of air parallel to velocity of the propagation of wave



Sinusoidal Function:

Imagine a sinusoidal wave like that of Fig. 16-1b traveling in the positive direction of an x axis. As the wave sweeps through succeeding elements (that is, very short sections) of the string, the elements oscillate parallel to the y axis. At time t, the displacement y of the element located at position x is given by

Wave function:

$$y(x,t) = y_m \sin(kx - \omega t) \qquad [+x\ axis]$$

$$y(x,t) = y_m \sin(kx + \omega t) \qquad [-\text{ x axis}]$$

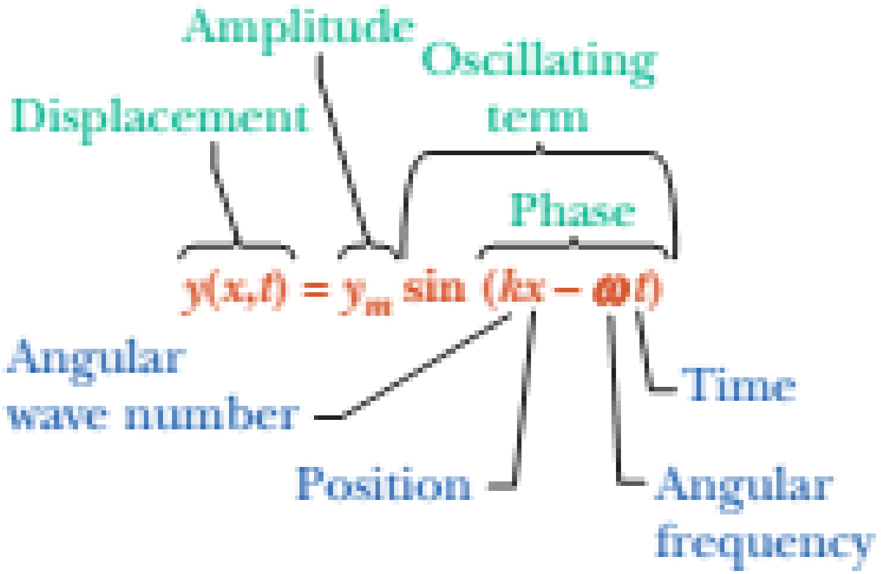
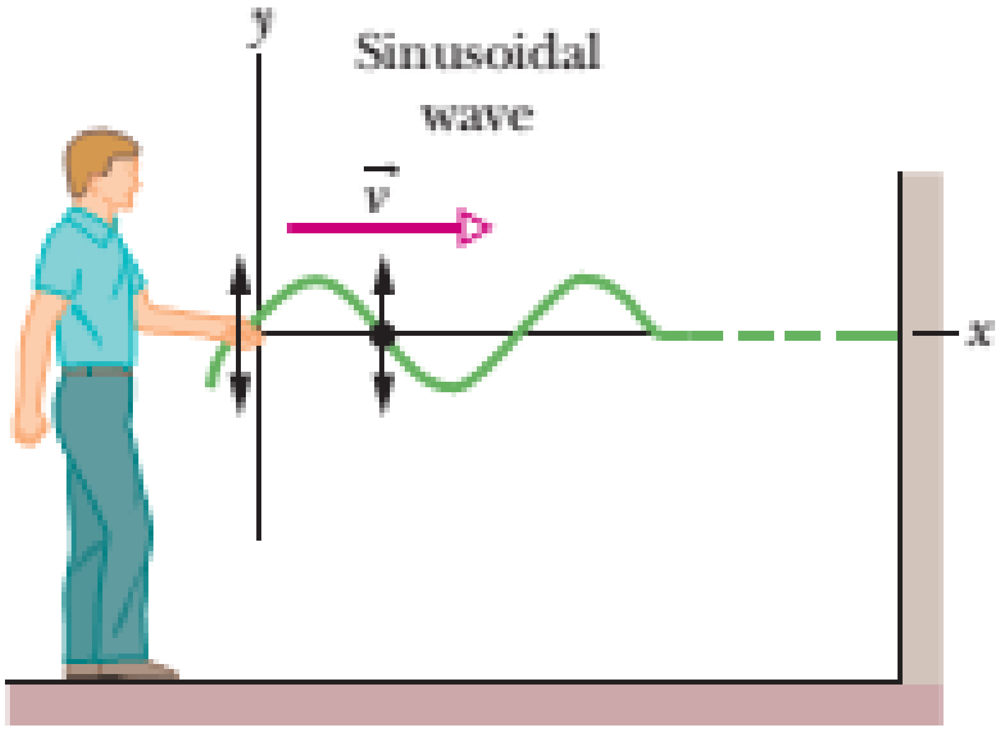


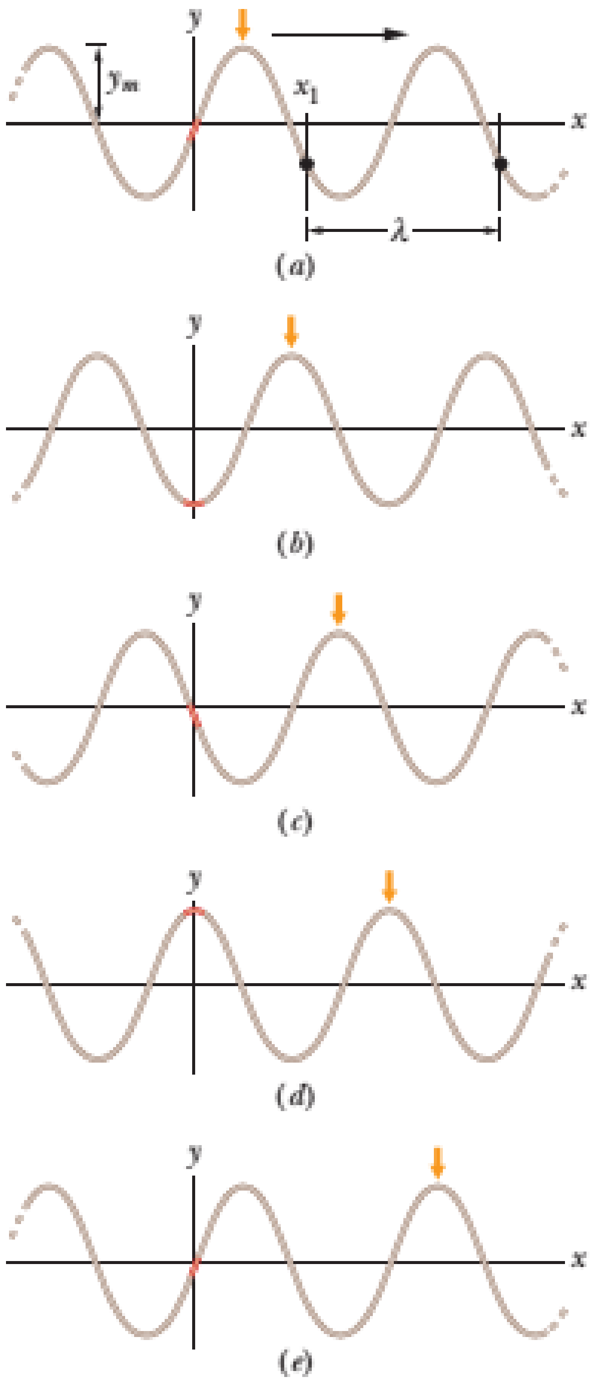
Fig. Transverse sinusoidal wave

Because this equation is written in terms of position x, it can be used to find the displacements of all the elements of the string as a function of time. Thus, it can tell us the shape of the wave at any given time.



Watch this spot in this series of snapshots.

Figure 16-4 Five “snapshots” of a string wave traveling in the positive direction of an x axis. The amplitude y_m is indicated. A typical wavelength λ measured from an arbitrary position x_1, is also indicated.



(i) Prove that $k = \frac{2\pi}{\lambda}$

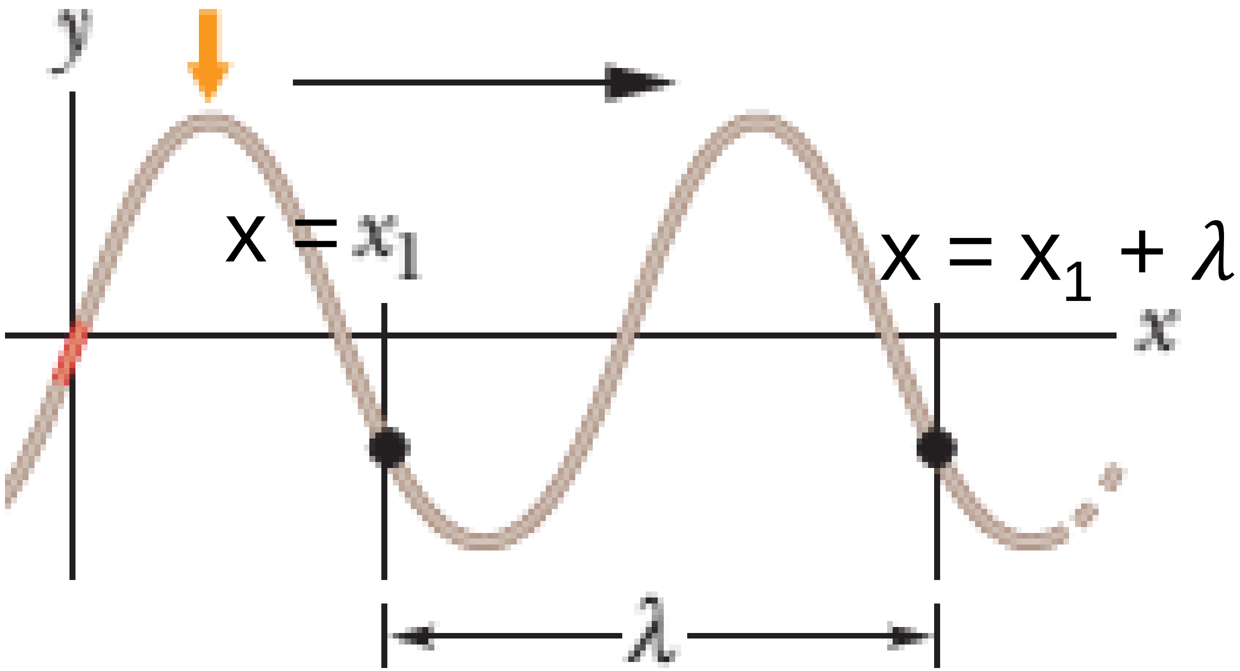
$$y(x,t) = y_m \sin(kx - \omega t)$$

The wavelength λ of a wave is the distance (parallel to the direction of the wave's travel) between repetitions of the shape of the wave (or wave shape).

At $t = 0$

$$y(x,0) = y_m \sin\{kx - \omega(0)\} = y_m \sin kx$$

The displacement y is the same at both ends of this wavelength at $x = x_1, x = x_1 + \lambda$



$$y(x_1, 0) = y_m \sin kx_1$$

$$y(x_1 + \lambda, 0) = y_m \sin\{k(x_1 + \lambda)\}$$

$$y(x_1 + \lambda, 0) = y_m \sin(kx_1 + k\lambda)$$

$$y(x_1, 0) = y(x_1 + \lambda, 0)$$

$$y_m \sin kx_1 = y_m \sin(kx_1 + k\lambda)$$

A sine function begins to repeat itself when its angle (or argument) is increased by $k\lambda = 2\pi\ rad$

$$k = \frac{2\pi}{\lambda}$$

SI unit of k = rad/m

(ii) *Prove that* $\omega = \frac{2\pi}{T}$

Fig. shows a graph of the displacement y versus time t at a certain position along the string, taken to be x = 0.

$$y(x,t) = y_m \sin(kx - \omega t)$$

$$x = 0$$

$$y(0,t) = y_m \sin\{k(0) - \omega t\}$$

$$y(0,t) = -y_m \sin \omega\, t$$

We define the period of oscillation T of a wave to be the time any string element takes to move through one full oscillation.

The displacement y is the same at both ends of this time period at $t = t_1, t = t_1 + T$.

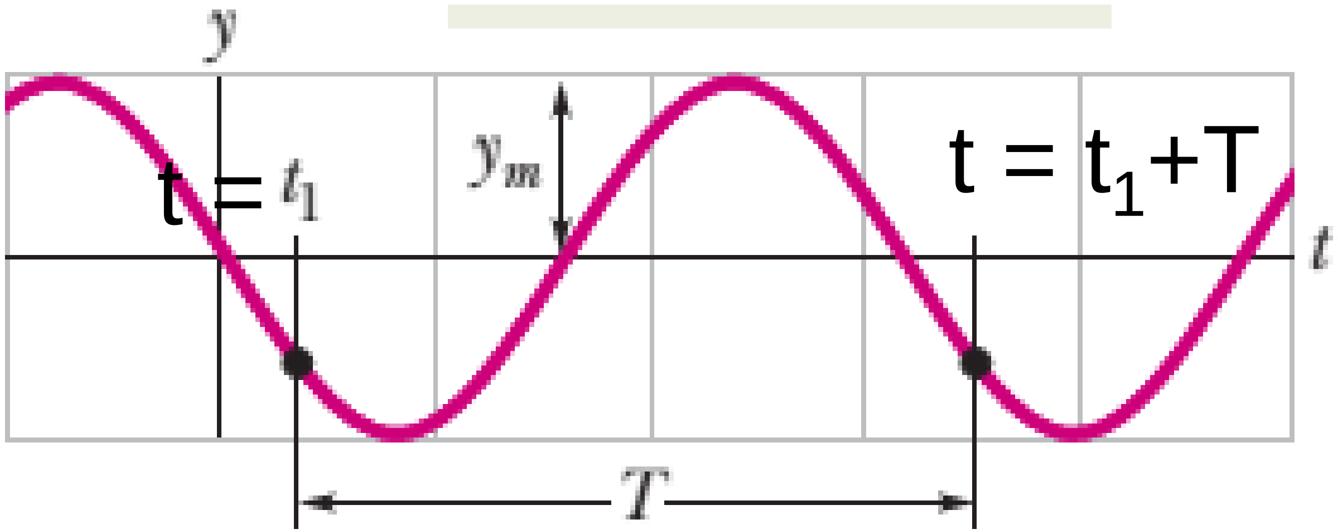
$$y(0,t_1) = -y_m \sin \omega\, t_1$$

$$y(0,t_1 + T) = -y_m \sin\{\omega(t_1 + T)\}$$

$$y(0,t_1 + T) = -y_m \sin(\omega t_1 + \omega T)$$

$$y(0,t_1) = y(0,t_1 + T)$$

$$-y_m \sin \omega t_1 = -y_m \sin(\omega t_1 + \omega T)$$



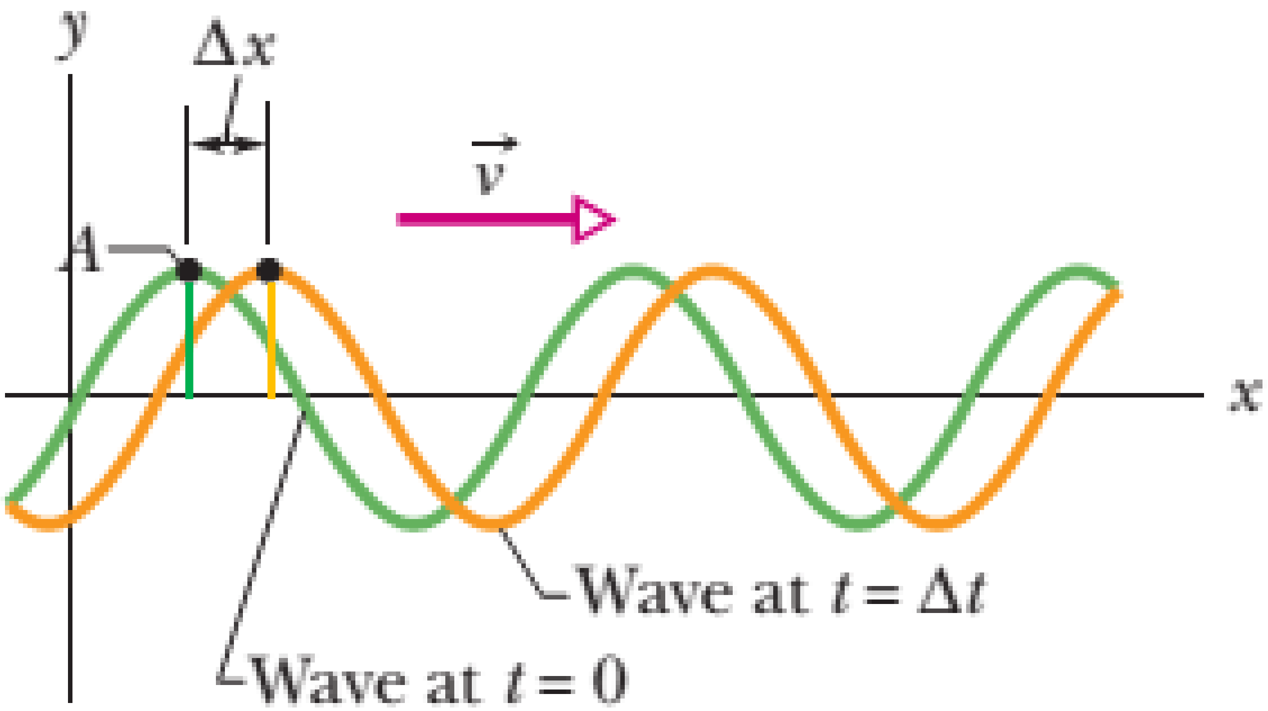
This can be true only if $\omega T = 2\pi\, rad$

$$\omega = \frac{2\pi}{T}$$

SI unit of ω = rad/s

(iii) Prove that $v = \frac{+\omega}{k}$

The wave is traveling in the positive direction of x.



$$y(x,t) = y_m \sin(kx - \omega t)$$

If point A retains its displacement as it moves, the phase giving it that displacement must remain a constant:

$$phase = kx - \omega t = constant$$

$$kx \uparrow - \omega t \uparrow = constant$$

This phase (argument) is constant but both x and t are changing. In fact, as t increases, x must also, to keep the argument constant. This confirms that the wave pattern is moving in the positive direction of x.

To find the wave speed v, we take a derivative of phase $= kx - \omega t =$ constant with respect to t.

$$\frac{d}{dt}(kx - \omega t) = \frac{d}{dt}(constant)$$

$$k\frac{dx}{dt} - \omega\frac{dt}{dt} = 0$$

$$kv - \omega = 0$$

$$kv = \omega$$

$$v = \frac{+\omega}{k}$$

The plus sign verifies that the wave is indeed moving in the positive direction of x.

$$v = \frac{\omega}{k} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{2\pi}{T}\left(\frac{\lambda}{2\pi}\right) = \frac{\lambda}{T} = f\lambda$$

The equation $\frac{\lambda}{T}$ tells us that the wave speed is one wavelength per period; the wave moves a distance of one wavelength in one period of oscillation.

(iv) Prove that $v = \frac{-\omega}{k}$

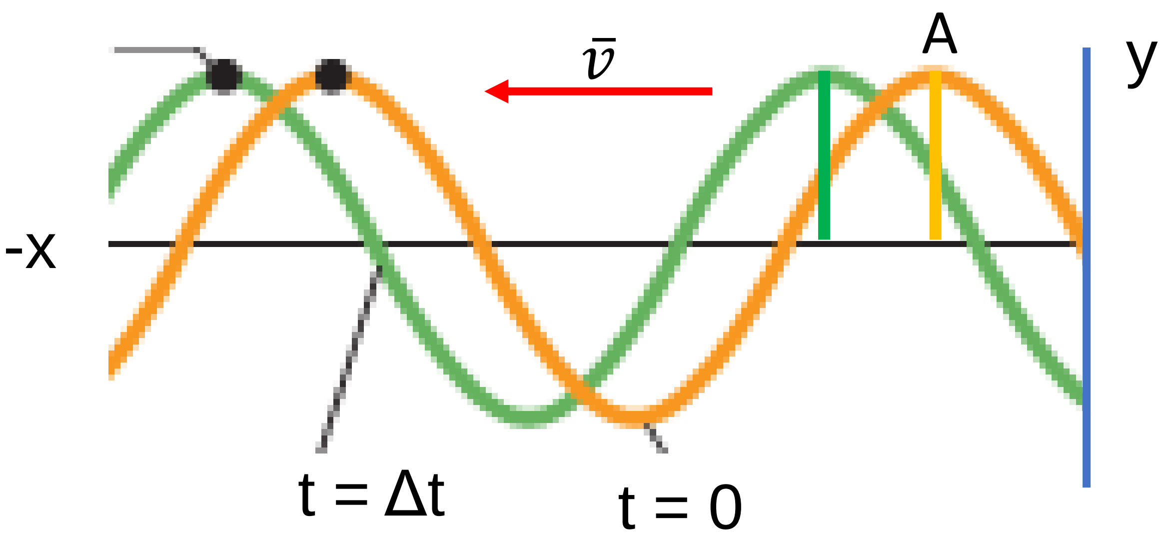
The wave is traveling in the negative direction of x.

$$y(x,t) = y_m \sin(kx - \omega t)$$

$$t = -t$$

$$y(x,t) = y_m \sin(kx + \omega t)$$

If point A retains its displacement as it moves, the phase giving it that displacement must remain a constant:



$$phase = kx + \omega t = constant$$

$$kx \downarrow + \omega t \uparrow = constant$$

The x decrease with time. Thus, a wave is traveling in the negative direction of x.

To find the wave speed v, we take a derivative of phase $= kx + \omega t =$ constant with respect to t.

$$\frac{\mathrm{d}}{\mathrm{d}t}(kx + \omega t) = \frac{\mathrm{d}}{\mathrm{d}t}(constant)$$

$$\mathrm{k}\frac{\mathrm{dx}}{\mathrm{dt}} + \omega\frac{\mathrm{dt}}{\mathrm{dt}} = 0$$

$$kv + \omega = 0$$

$$kv = -\omega$$

$$v = \frac{-\omega}{k}$$

The minus sign verifies that the wave is indeed moving in the negative direction of x.

1. If a wave y(x, t) = (6.0 mm) sin(kx + (600 rad/s)t + φ) travels along a string, how much time does any given point on the string take to move between displacements y = + 2.0 mm and y = - 2.0 mm?

$$+0.002\text{ m} = 0.006\text{ m }\sin(kx + 600\, t_1 + \psi)$$

$$\sin(kx + 600\, t_1 + \psi) = \frac{0.002}{0.006}$$

$$\sin(kx + 600\, t_1 + \psi) = \frac{1}{3}$$

$$(kx + 600\, t_1 + \psi) = \sin^{-1}(\frac{1}{3}) ----[1]$$

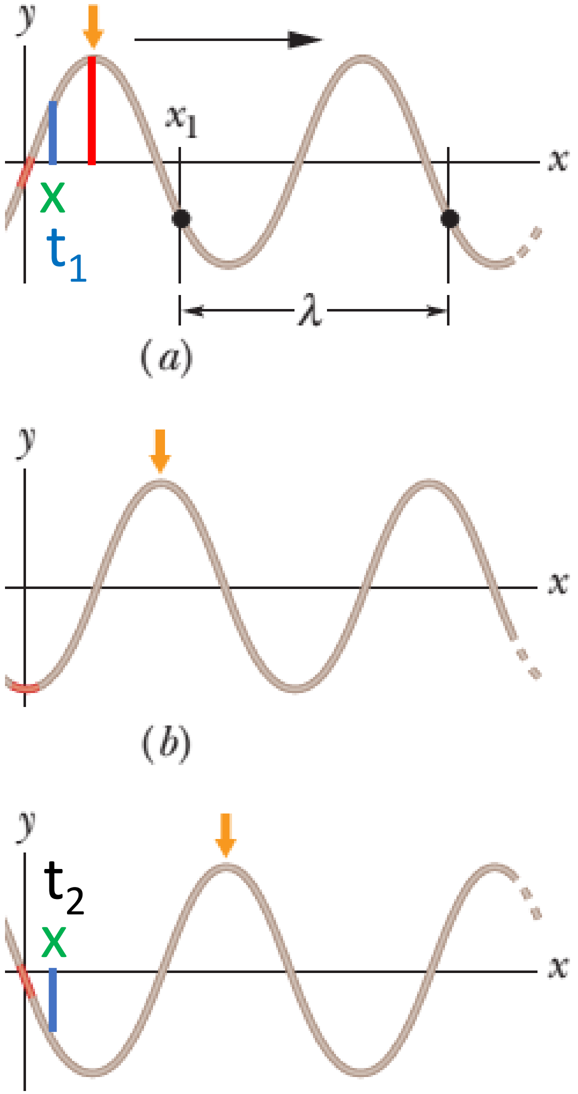
$$-0.002\text{ m} = 0.006\text{ m }\sin(kx + 600\, t_2 + \psi)$$

$$\sin(kx + 600\, t_2 + \psi) = \frac{-0.002}{0.006}$$

$$\sin(kx + 600\, t_2 + \psi) = \frac{-1}{3}$$

$$(kx + 600\, t_2 + \psi) = \sin^{-1}(\frac{-1}{3}) ----[2]$$

$$[1] - [2]$$



$$(kx + 600\,t_1 + \psi) - (kx + 600\,t_2 + \psi) = \sin^{-1}(\frac{1}{3}) - \sin^{-1}(\frac{-1}{3})$$

$$kx + 600\,t_1 + \psi - kx - 600\,t_2 - \psi = \sin^{-1}(\frac{1}{3}) + \sin^{-1}(\frac{1}{3})$$

$$600\,t_1 - 600\,t_2 = 2\sin^{-1}(\frac{1}{3})$$

$$600\,(t_1 - t_2) = 2\sin^{-1}(\frac{1}{3})$$

$$(t_1 - t_2) = \frac{2}{600}\sin^{-1}(\frac{1}{3})$$

$$t = \frac{1}{300}\sin^{-1}(\frac{1}{3}\ rad)$$

t = 0.001133 s [Ans]

5. A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero is 0.170 s. What are (a) the period and (b) frequency? (c) The wavelength is 1.40 m; what is the wave speed?

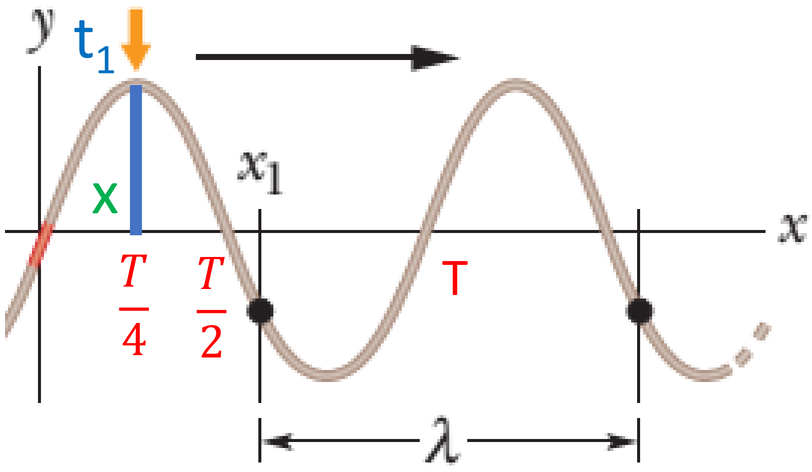
(a) $t_1 - t_2 = T/4 = 0.170$ s

$T = 4\,(0.170)$ s $= 0.680$ s

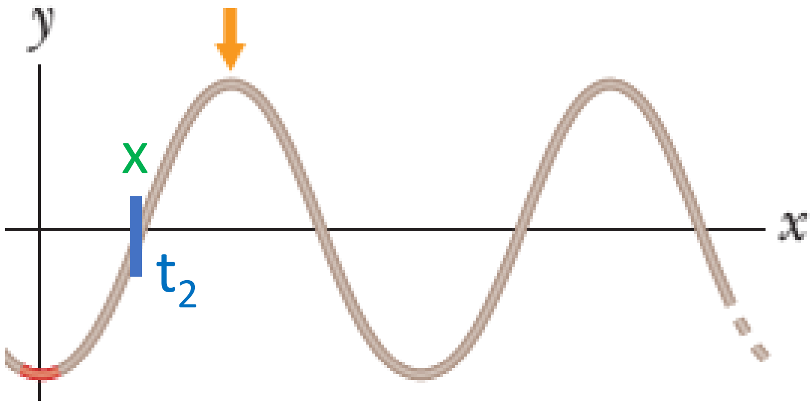
(b) $f = 1/T = (1/\,0.680)$ Hz $= 1.47$ Hz

(c) $\lambda = 1.40$ m

$v = f\lambda = 1.47\,(1.40)$ m/s $= 2.06$ m/s



(a)



Additional problem:

Sample problem 16.02, page:451