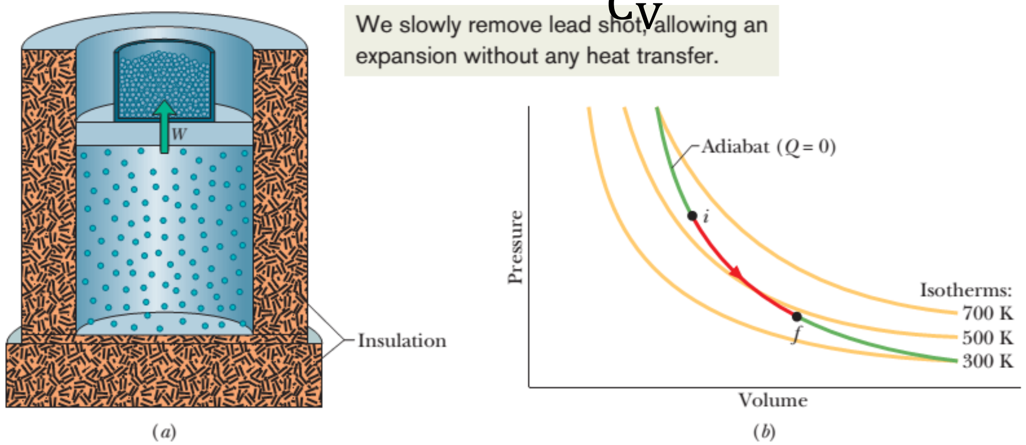
Lecture 8

Chapter 19: The Kinetic Theory of Gases

19-9 Adiabatic expansion of an ideal gas: pV^{γ} = a constant , where $\gamma = \frac{C_p}{C_V}$

adiabatic process, Q = 0



Suppose that you remove some shots from the piston, allowing the ideal gas to push the piston and the remaining shots upward and thus to increase the volume by a differential amount dV. Since the volume change is tiny, we may assume that the pressure p of the gas on the piston is constant during the change. The work dW done by the gas during the volume increase is equal to W = p dV.

1st law of thermodynamics, $\Delta E_{int} = Q - W$

$$nC_V\,\Delta T = 0 - p\,dV$$

$$n\Delta T = -\frac{p\,dV}{C_V}$$

$[\Delta E_{int} = Q - W = nC_V\,\Delta T - p\Delta V = nC_V\,\Delta T - p(V - V) = nC_V\,\Delta T - p(0) = nC_V\,\Delta T]$

Ideal gas equation, pV = nRT

$$\frac{d}{dT}(pV) = \frac{d}{dT}(nRT)$$

$$\left[\frac{d}{dx}(uv) = u\,\frac{dv}{dx} + v\,\frac{du}{dx}\right]$$

$$p\,\frac{dV}{dT} + V\,\frac{dp}{dT} = nR\,\frac{dT}{dT}$$

$$\frac{p\,dV + V\,dp}{dT} = nR$$

$$\frac{p\,dV + V\,dp}{R} = n\,dT$$

$$\frac{p\,dV + V\,dp}{R} = -\,\frac{p\,dV}{C_V}$$

$$\left[\,n\,dT = -\,\frac{p\,dV}{C_V}\,\right]$$

$$p\,dV + V\,dp = -\left(\frac{R}{C_V}\right) p\,dV$$

$$[C_p - C_V = R]$$

$$p\,dV + V\,dp = -\left(\frac{C_p - C_V}{C_V}\right) p\,dV = -\left(\frac{C_p}{C_V} - 1\right) p\,dV = -(\gamma - 1)\, p\,dV = -\gamma\, p\,dV + p\,dV$$

$$V\,dp = -\gamma\, p\,dV$$

$$\frac{dp}{p} = -\gamma\,\frac{dV}{V}$$ [divided by pV]

$$\int \frac{dp}{p} = -\int \gamma\,\frac{dV}{V} = -\gamma \int \frac{dV}{V}$$

$$\ln p + C_1 = -\gamma \ln V + C_2$$

$$\ln p + \gamma \ln V = C$$

$$\ln p + \ln V^{\gamma} = C$$

$$\ln (pV^{\gamma}) = C$$

$$e^{\ln (pV^{\gamma})} = e^{C}$$

pV^{γ} = a constant [adiabatic expansion or compression]

$p_i V_i^{\gamma} = p_f V_f^{\gamma}$ [from an initial state, *i* to a final state, *f*]

19-9 $TV^{\gamma-1}$ = constant for an adiabatic process:

For an adiabatic process, pV^{γ} = constant

To write an equation for an adiabatic process in terms of *T* and *V*, we use the ideal gas equation to eliminate *p*

Ideal gas equation, pV = nRT

$$p = \frac{nRT}{V}$$

$$\left(\frac{nRT}{V}\right) V^{\gamma} = \text{constant}$$

$$T\left(\frac{V^{\gamma}}{V^{1}}\right) = \frac{\text{constant}}{nR}$$

[n and R are constants]

$$TV^{\gamma-1} = \text{constant}$$

When the gas goes from an initial state *i* to a final state *f*: $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$

19-9 *Work done for an ideal gas in an adiabatic process*: $W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$

$$W = \int_{V_i}^{V_f} p\,dV = = \int_{V_i}^{V_f} \frac{a}{V^\gamma} dV = a \int_{v_i}^{v_f} V^{-\gamma}\, dV = a\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f}$$

Adiabatic process of an ideal gas: $pV^\gamma = a$

$$p = \frac{a}{V^\gamma}$$

$$W = \frac{a}{-\gamma+1}\left[V^{-\gamma+1}\right]_{V_i}^{V_f} = \frac{a}{-\gamma+1}\left(V_f^{-\gamma+1} - V_i^{-\gamma+1}\right)$$

$$p_i V_i^{\gamma} = p_f V_f^{\gamma} = a$$

$$W = \frac{aV_f^{-\gamma+1} - aV_i^{-\gamma+1}}{-\gamma+1} = \frac{p_f V_f^{\gamma} V_f^{-\gamma+1} - p_i V_i^{\gamma} V_i^{-\gamma+1}}{-\gamma+1} = \frac{p_f V_f^{\gamma-\gamma+1} - p_i V_i^{\gamma-\gamma+1}}{-\gamma+1}$$

$$W = \frac{p_f V_f - p_i V_i}{-\gamma+1} = \frac{-(p_i V_i - p_f V_f)}{-(\gamma-1)}$$

$$W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

54. We know that for an adiabatic process pV^γ = a constant. Evaluate “a constant” for an adiabatic process involving exactly 2.0 mol of an ideal gas passing through the state having exactly $p = 1.0$ atm and $T = 300$ K. Assume a diatomic gas whose molecules rotate but do not oscillate.

Solution:

Here, n = 2 mol

p = 1.0 atm = 1.0×10^5 Pa

T = 300 K

pV^{γ} = constant

Diatomic gas whose molecules rotate but do not oscillate, f = 3+2 = 5

$C_V = \left(\frac{f}{2}\right)R = \left(\frac{5}{2}\right)R$

$C_p - C_V = R$

$C_p = C_V + R = \left(\frac{5}{2}\right)R + R = \left(\frac{7}{2}\right)R$

$$\gamma = \frac{c_P}{c_v} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} = 1.4$$

pV^{γ} = constant

[Ideal gas law, pV = nRT]

$a = pV^{\gamma}$

$a = p(\frac{nRT}{p})^{\gamma}$

$[V = \frac{nRT}{p}]$

$= 1.0\text{x}10^5\{\frac{2(8.31)(300)}{1.0\times10^5}\}^{1.4}$

$= 1.0\text{x}10^5\{0.04986\}^{1.4}$

a = $1.5\text{x}10^3$ $Nm^{2.2}$

Unit of a = $pV^{\gamma} = \frac{F}{A}V^{\gamma} = (\frac{N}{m^2})(m^3)^{\gamma} = N(\frac{m^{3\gamma}}{m^2}) = Nm^{3\gamma-2} = Nm^{3(1.4)-2} = Nm^{4.2-2.0}$

a = $Nm^{2.2}$

55. A certain gas occupies a volume of 4.3 L at a pressure of 1.2 atm and a temperature of 310 K. It is compressed adiabatically to a volume of 0.76 L. Determine (a) the final pressure and (b) the final temperature, assuming the gas to be an ideal gas for which $\gamma = 1.4$.

Solution:

Here, $V_i = 4.3$ L

$p_i = 1.2$ atm $= 1.2\text{x}10^5$ Pa

$T_i = 310$ K

$V_f = 0.76$ L

$\gamma = 1.4$

(a) pV^{γ} = constant

$p_iV_i^{\gamma} = p_fV_f^{\gamma}$

$$p_f = \frac{p_iV_i^{\gamma}}{V_f^{\gamma}} = p_i\left(\frac{V_i^{\gamma}}{V_f^{\gamma}}\right) = p_i\left(\frac{V_i}{V_f}\right)^{\gamma} = 1.2\text{x}10^5\left(\frac{4.3\text{ L}}{0.76\text{ L}}\right)^{1.4} = 1.2\text{x}10^5(11.3166) = 1.36\text{x}10^6\text{ Pa}$$

(b) $TV^{\gamma-1}$ = constant

$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

$T_f = \dfrac{T_iV_i^{\gamma-1}}{V_f^{\gamma-1}} = T_i\left(\dfrac{V_i}{V_f}\right)^{\gamma-1} = 310\left(\dfrac{4.3\ \mathrm{L}}{0.76\ \mathrm{L}}\right)^{1.4-1} = 310(2.00) = 620\ \mathrm{K}$

62. An ideal diatomic gas, with rotation but no oscillation, undergoes an adiabatic compression. Its initial pressure and volume are1.20 atm and 0.200 m^3 . Its final pressure is 2.40 atm. How much work is done by the gas?

Solution:

Given:

$\gamma = 1.40$

$p_i = 1.20 \text{ atm} = 1.20 \times 10^5$ Pa

$V_i = 0.200 \text{ m}^3$

$p_f = 2.40 \text{ atm} = = 2.40 \times 10^5$ Pa

W =?

$$W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

$V_f = ?$

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$

$$\frac{p_i}{p_f} = \left(\frac{Vf^{\gamma}}{Vi^{\gamma}}\right)$$

$$\frac{p_i}{p_f} = \left(\frac{V_f}{V_i}\right)^{\gamma}$$

$$\sqrt[\gamma]{\frac{p_i}{p_f}} = \sqrt[\gamma]{\left(\frac{V_f}{Vi}\right)^{\gamma}}$$

$$\sqrt[\gamma]{\frac{p_i}{p_f}} = \left(\frac{V_f}{V_i}\right)$$

$$V_f = V_i \sqrt[\gamma]{\frac{p_i}{p_f}}$$

$$V_f = (0.200) \sqrt[1.40]{\frac{1.20 \times 10^5}{2.40 \times 10^5}}$$

$$V_f = (0.200)\ (0.5)^{0.714}$$

$$V_f = (0.200)(0.6096)$$

$$V_f = 0.122 \text{ m}^3$$

$$W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

$$W = \frac{(1.20 \times 10^5)(0.200) - (2.40 \times 10^5)(0.122)}{1.40 - 1}$$

$$W = -1.32 \times 10^4 \text{ J} \quad \text{(Ans.)}$$