Review Last Class

$$v(t) = V_m \sin(\omega t + \theta_v) \quad [\mathrm{V}]$$

$$i(t) = I_m \sin(\omega t + \theta_i) \quad [\mathrm{A}]$$

$$p(t) = P(1 - \cos 2\omega t) + Q \sin 2\omega t \quad [\mathrm{W}]$$

$$\theta = \theta_z = \theta_s = \theta_v - \theta_i$$

$$S = VI = \sqrt{P^2 + Q^2} \quad [\mathrm{VA}]$$

$$P = VI\cos\theta = S\cos\theta = \sqrt{S^2 - Q^2} \quad [\mathrm{W}]$$

$$Q = VI\sin\theta = S\sin\theta = \sqrt{S^2 - P^2} \quad [\mathrm{Var}]$$

$$pf = F_p = \cos\theta = \cos\theta_z = \frac{P}{S} \qquad 0 < F_p < 1$$

$$rf = F_q = \sin\theta = \sin\theta_z = \frac{Q}{S} \qquad -1 < F_q < 1$$

$$\vec{V} = \boldsymbol{V} = V_{rms}\angle\theta_v = V\angle\theta_v = V_r + jV_i \quad [\mathrm{V}]$$

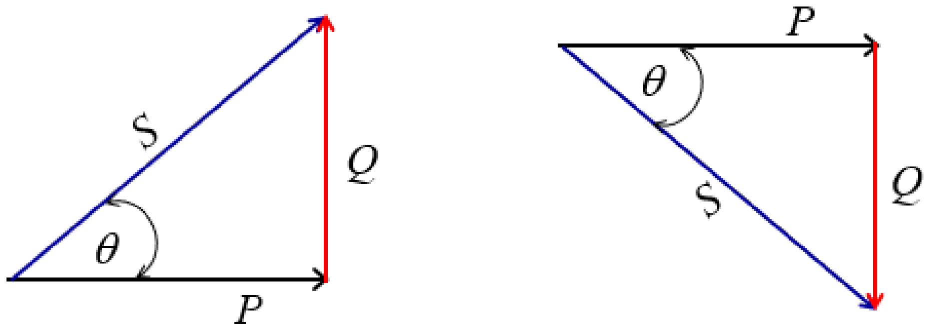
$$\vec{I} = \boldsymbol{I} = I_{rms}\angle\theta_i = I\angle\theta_i = I_r + jI_i \quad [\mathrm{A}]$$

$$P = V_r I_r + V_i I_i \quad [\mathrm{W}]$$

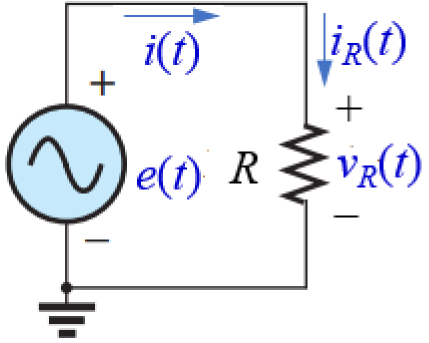
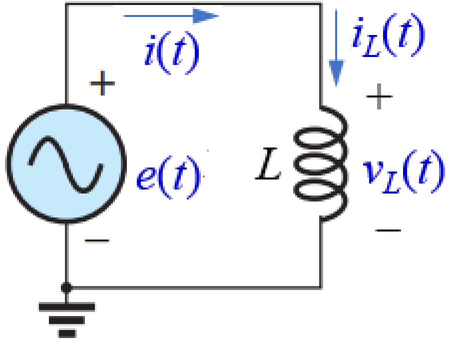
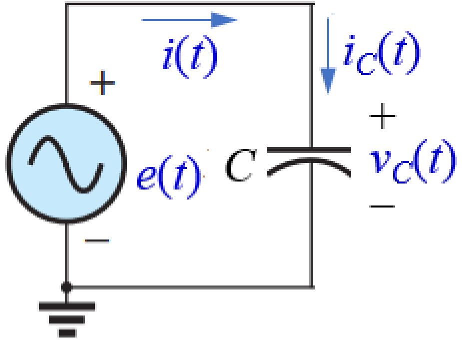
$$Q = V_i I_r - V_r I_i \quad [\mathrm{W}]$$

$$\vec{S} = \boldsymbol{S} = \boldsymbol{V}(\boldsymbol{I})^* = P + jQ = S\angle\theta_s$$

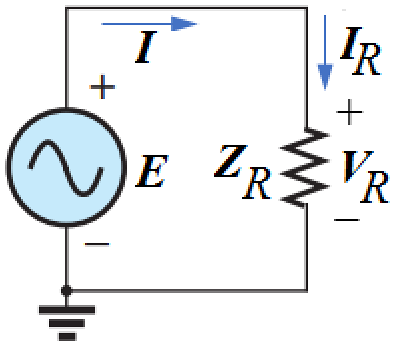
Power Triangle



When Q is Positive **When Q is Negative**

PURE RESISTIVE CIRCUIT	**PURE INDUCTIVE CIRCUIT**	**PURE CAPACITIVE CIRCUIT**
		
$i(t)=i_R(t)=\frac{E_m}{R}\sin(\omega t+\theta_e)$	$i(t)=i_L(t)=\frac{E_m}{X_L}\sin(\omega t+\theta_e-90°)$ where, $X_L=\omega L=2\pi fL\ \Omega$	$i(t)=i_C(t)=\frac{E_m}{X_C}\sin(\omega t+\theta_e+90°)$ where, $X_C=\frac{1}{\omega C}=\frac{1}{2\pi fC}\ \Omega$
$Z=\frac{E_m}{I_m}=R;\quad \theta_z=\theta_e-\theta_i=0°$	$Z=\frac{E_m}{I_m}=X_L;\quad \theta_z=\theta_e-\theta_i=90°$	$Z=\frac{E_m}{I_m}=X_C;\quad \theta_z=\theta_e-\theta_i=-90°$
The phase difference $v_R(t)$ and $i_R(t)$ is **0º**. $v_R(t)$ and $i_R(t)$ are **in phase**.	The phase difference $v_L(t)$ and $i_L(t)$ is **90º**. $v_L(t)$ **leads** and $i_L(t)$ or $i_L(t)$ **lags** $v_L(t)$.	The phase difference $v_C(t)$ and $i_C(t)$ is **90º**. $v_C(t)$ **lags** and $i_C(t)$ or $i_C(t)$ **leads** $v_C(t)$.

PURE RESISTIVE CIRCUIT



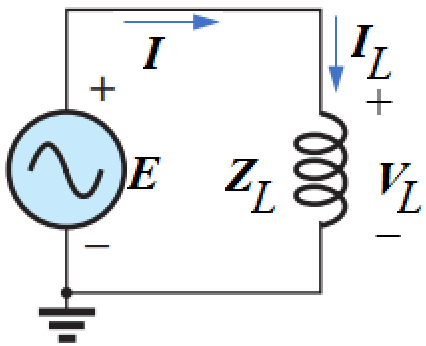
$$V_R = E$$
$$I_R = I$$
$$Z = Z_R = \frac{V_R}{I_R}$$

$$Z = Z_R = R\angle 0^\circ\ \Omega = R + j0\ \Omega$$

$$Y = Y_R = \frac{I_R}{V_R} = \frac{1}{Z_R}$$
$$Y = Y_R = G\angle 0^\circ\ \mathrm{S} = G + j0\ \mathrm{S}$$

where, $G = \dfrac{1}{R}\quad \mathrm{S}$

PURE INDUCTIVE CIRCUIT



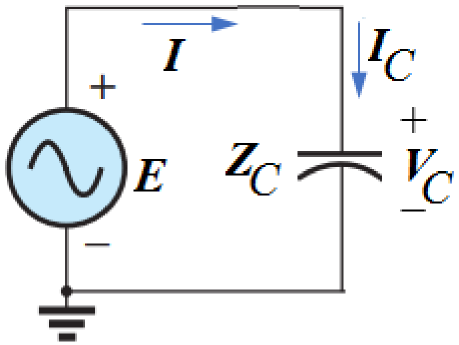
$$V_L = E$$
$$I_L = I$$
$$Z = Z_L = \frac{V_L}{I_L}$$

$$Z = Z_L = X_L\angle 90^\circ\ \Omega = 0 + jX_L\ \Omega$$

$$Y = Y_L = \frac{I_L}{V_L} = \frac{1}{Z_L}$$
$$Y = Y_L = B_L\angle -90^\circ\ \mathrm{S} = 0 - jB_L\ \mathrm{S}$$

where, $B_L = \dfrac{1}{X_L}\quad \mathrm{S}$

PURE CAPACITIVE CIRCUIT



$$V_C = E$$
$$I_C = I$$
$$Z = Z_C = \frac{V_C}{I_C}$$

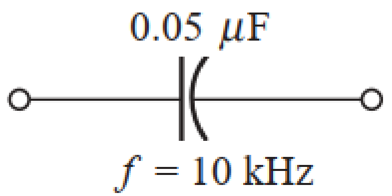
$$Z = Z_C = X_C\angle -90^\circ\ \Omega = 0 - jX_C\ \Omega$$

$$Y = Y_C = \frac{I_C}{V_C} = \frac{1}{Z_C}$$
$$Y = Y_C = B_C\angle 90^\circ\ \mathrm{S} = 0 + jB_C\ \mathrm{S}$$

where, $B_C = \dfrac{1}{X_C}\quad \mathrm{S}$

Problem 1 [Ch. 15] Express the impedances of following figure in both polar and rectangular forms.

$$\boldsymbol{Z} = 6.8\Omega\angle 0^\circ = 6.8\ \Omega$$

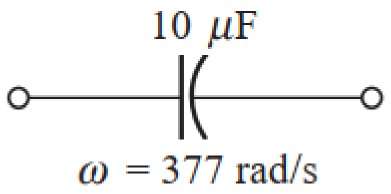


$$\omega = 2\pi \times 10 \times 10^3 = 62.8 \text{ krad/s}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(62.8\times 10^3 \text{rad/s}\right)\left(0.05\times 10^{-6}\text{ F}\right)}$$

$$= 318.47\ \Omega$$

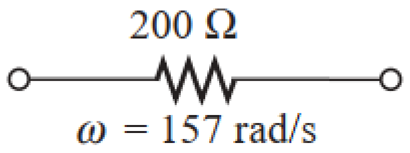
$$\boldsymbol{Z} = 318.47\Omega\angle -90^\circ = -j318.47\ \Omega$$



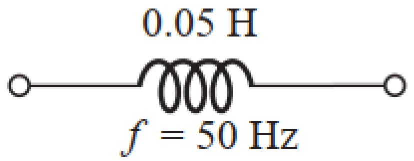
$$X_C = \frac{1}{\omega C} = \frac{1}{\left(377\text{rad/s}\right)\left(10\times 10^{-6}\text{ F}\right)}$$

$$= 265.25\ \Omega$$

$$\boldsymbol{Z} = 265.25\Omega\angle -90^\circ = -j265.25\ \Omega$$



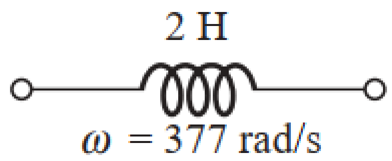
$$\boldsymbol{Z} = 200\Omega\angle 0^\circ = 200\ \Omega$$



$$\omega = 2\pi \times 50 = 314 \text{ rad/s}$$

$$X_L = \omega L = \left(314\text{rad/s}\right)\left(0.05\text{H}\right) = 15.17\ \Omega$$

$$\boldsymbol{Z} = 15.17\Omega\angle 90^\circ = j15.17\ \Omega$$

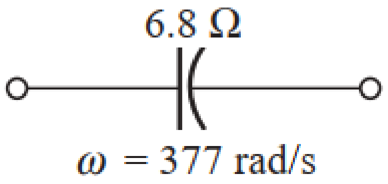


$$X_L = \left(377\text{ rad/s}\right)\left(2\text{H}\right) = 754\ \Omega$$

$$\boldsymbol{Z} = 754\Omega\angle 90^\circ = j754\ \Omega$$

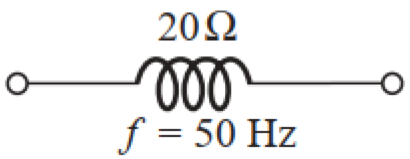
Problem 1 [Ch. 15]

(a) Express the impedances of following figure in both polar and rectangular forms.
(b) Calculate the value of inductor and capacitor



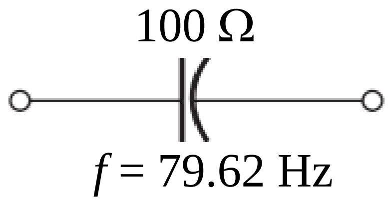
$$\boldsymbol{Z} = 6.8\Omega\angle -90^\circ = -j6.8\ \Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{(377\text{rad/s})(6.8\Omega)} = 390.1\ \mu\text{F}$$



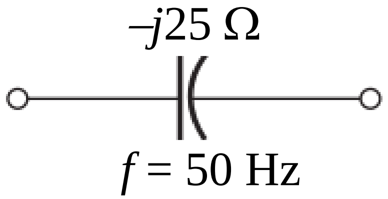
$$\boldsymbol{Z} = 20\Omega\angle 90^\circ = j20\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{20\Omega}{2\pi\times 50} = 63.69\ \text{mH}$$



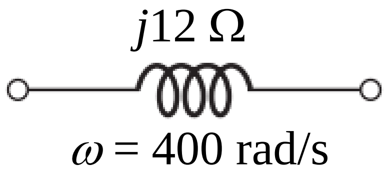
$$\boldsymbol{Z} = 100\Omega\angle -90^\circ = -j100\ \Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi(79.62\,\text{Hz})(100\Omega)} = 20\ \mu\text{F}$$



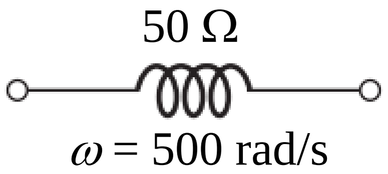
$$\boldsymbol{Z} = 25\Omega\angle -90^\circ = -j25\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\times(50\,\text{Hz})(25\Omega)} = 127.39\ \mu\text{F}$$



$$\boldsymbol{Z} = 12\Omega\angle 90^\circ = j12\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{12\Omega}{400} = 30\ \text{mH}$$



$$\boldsymbol{Z} = 50\Omega\angle 90^\circ = j50\ \Omega$$

$$L = \frac{X_L}{\omega} = \frac{50\Omega}{500} = 100\ \text{mH}$$

EXAMPLE 15.1 Using complex algebra, find the current i for the circuit of Fig. 15.2. Sketch the waveforms of v and i.

Solution: $v = 100 \sin \omega t \Rightarrow$ phasor form $\mathbf{V} = 70.71 \text{ V} \angle 0°$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V \angle \theta}{R \angle 0°} = \frac{70.71 \text{ V} \angle 0°}{5 \ \Omega \angle 0°} = 14.14 \text{ A} \angle 0°$$

$$i = \sqrt{2}(14.14) \sin \omega t = \mathbf{20 \sin \omega t}$$

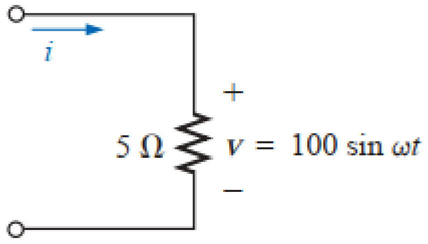


FIG. 15.2 *Example 15.1.*

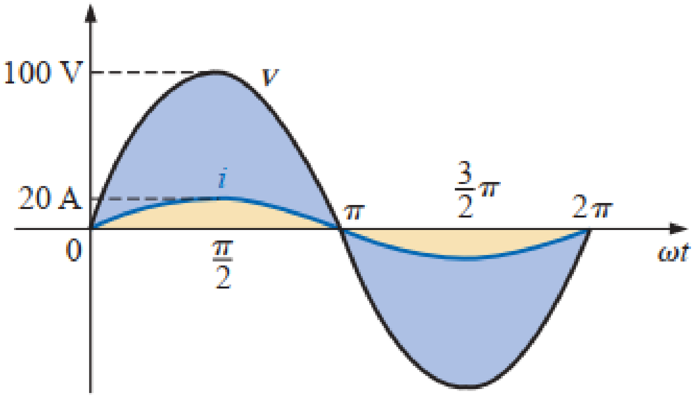
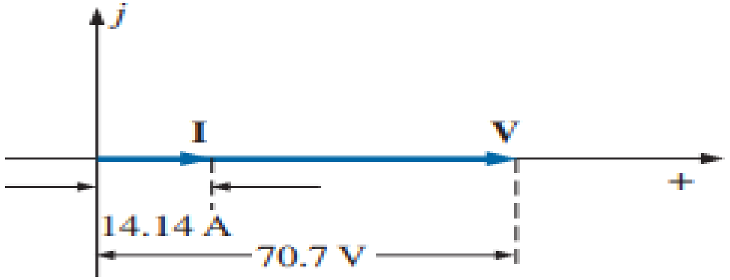


FIG. 15.3 *Waveforms for Example 15.1.*



Phasor diagrams

EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit of Fig. 15.4. Sketch the waveforms of v and i.

Solution: $i = 4 \sin(\omega t + 30°) \Rightarrow$ phasor form $\mathbf{I} = 2.828 \text{ A} \angle 30°$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (2.828 \text{ A} \angle 30°)(2 \ \Omega \angle 0°)$$
$$= 5.656 \text{ V} \angle 30°$$

and $\quad v = \sqrt{2}(5.656) \sin(\omega t + 30°) = \mathbf{8.0 \sin(\omega t + 30°)}$

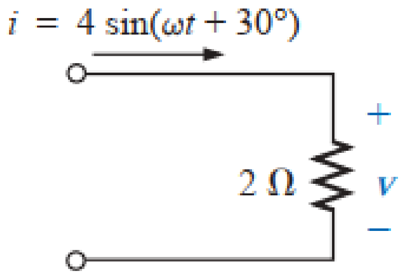


FIG. 15.4 *Example 15.2.*

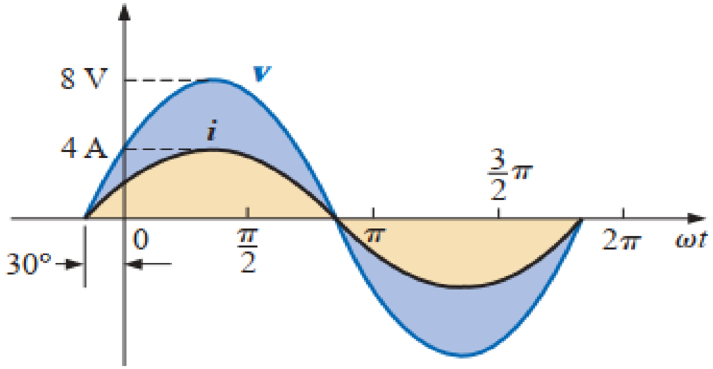
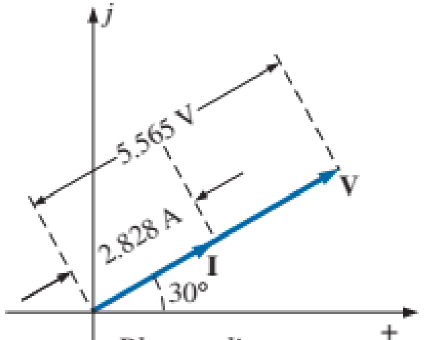


FIG. 15.5 *Waveforms for Example 15.2.*



Phasor diagrams

EXAMPLE 15.3 Using complex algebra, find the current i for the circuit in Fig. 15.8. Sketch the v and i curves.

Solution: Note Fig. 15.9:

$$v = 24 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 16.968 \text{ V} \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V \angle \theta}{X_L \angle 90^\circ} = \frac{16.968 \text{ V} \angle 0^\circ}{3\ \Omega \angle 90^\circ} = 5.656 \text{ A} \angle -90^\circ$$

and $$i = \sqrt{2}(5.656) \sin(\omega t - 90^\circ) = \mathbf{8.0 \sin(\omega t - 90^\circ)}$$

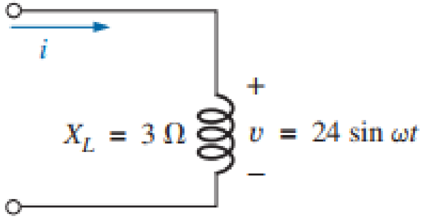


FIG. 15.8 *Example 15.3.*

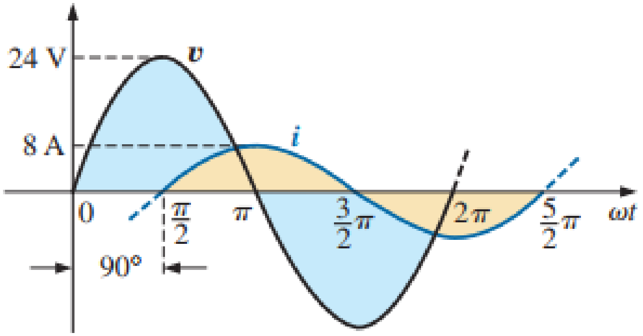
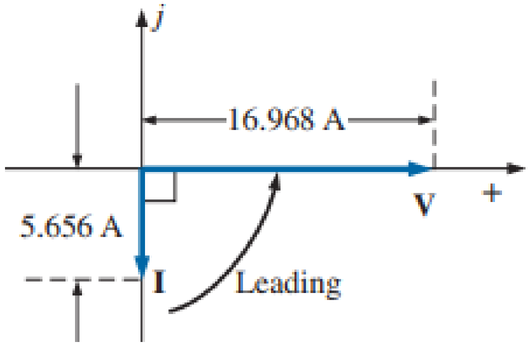


FIG. 15.9
Waveforms for Example 15.3.



Phasor diagrams

EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit in Fig. 15.10. Sketch the v and i curves.

Solution: Note Fig. 15.11:

$$i = 5 \sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 3.535 \text{ A} \angle 30^\circ$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90^\circ) = (3.535 \text{ A} \angle 30^\circ)(4\ \Omega \angle +90^\circ)$$
$$= 14.140 \text{ V} \angle 120^\circ$$

and $$v = \sqrt{2}(14.140) \sin(\omega t + 120^\circ) = \mathbf{20 \sin(\omega t + 120^\circ)}$$

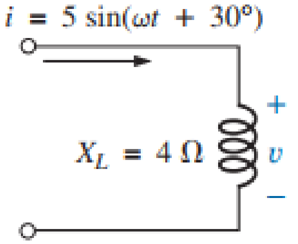
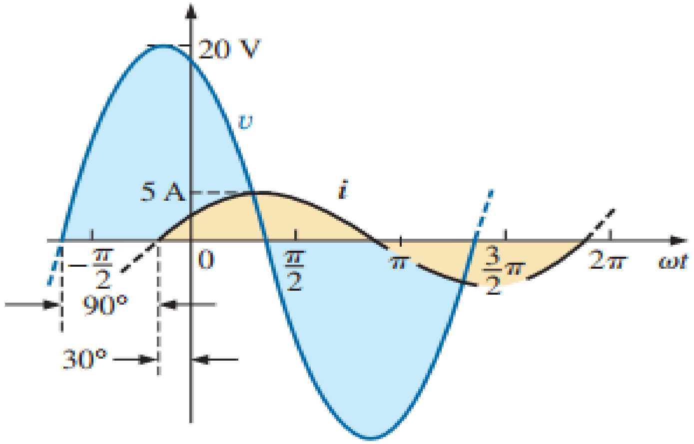
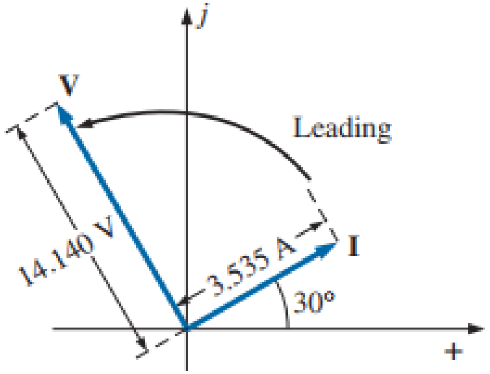


FIG. 15.10 *Example 15.4.*



5.11 *Waveforms for Example 15.4.*



Phasor diagrams

EXAMPLE 15.5 Using complex algebra, find the current i for the circuit in Fig. 15.14. Sketch the υ and i curves.

Solution: Note Fig. 15.15:

$$\upsilon = 15 \sin \omega t \Rightarrow \text{phasor notation } \mathbf{V} = 10.605 \text{ V} \angle 0°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_C} = \frac{V \angle \theta}{X_C \angle -90°} = \frac{10.605 \text{ V} \angle 0°}{2\ \Omega \angle -90°} = 5.303 \text{ A} \angle 90°$$

and $\quad i = \sqrt{2}(5.303) \sin(\omega t + 90°) = \mathbf{7.5 \sin(\omega t + 90°)}$

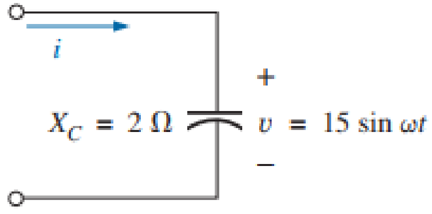


FIG. 15.14 *Example 15.5.*

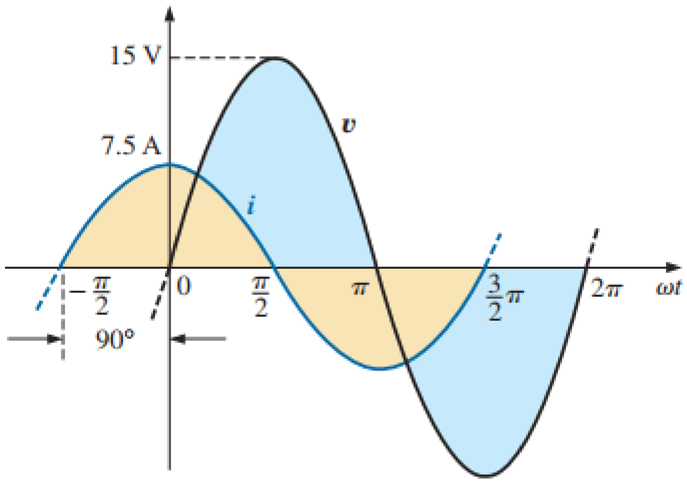
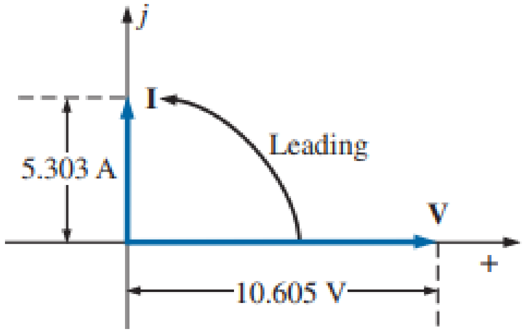


FIG. 15.15 *Waveforms for Example 15.5.*



Phasor diagrams

EXAMPLE 15.6 Using complex algebra, find the voltage υ for the circuit in Fig. 15.16. Sketch the υ and i curves.

Solution: Note Fig. 15.17:

$$i = 6 \sin(\omega t - 60°) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242 \text{ A} \angle -60°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (4.242 \text{ A} \angle -60°)(0.5\ \Omega \angle -90°)$$
$$= 2.121 \text{ V} \angle -150°$$

and $\quad \upsilon = \sqrt{2}(2.121) \sin(\omega t - 150°) = \mathbf{3.0 \sin(\omega t - 150°)}$

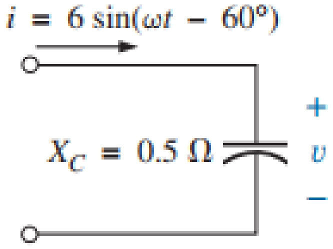


FIG. 15.16 *Example 15.6.*

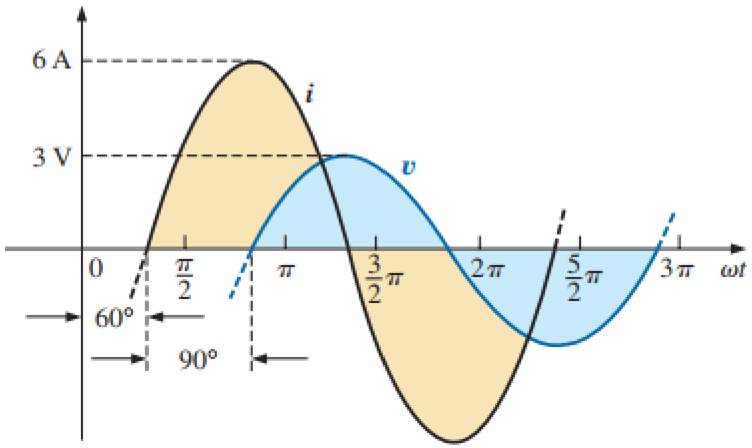
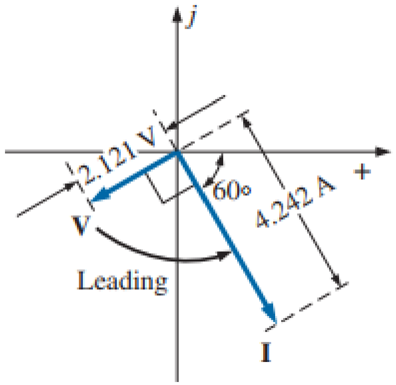


FIG. 15.17 *Waveforms for Example 15.6.*



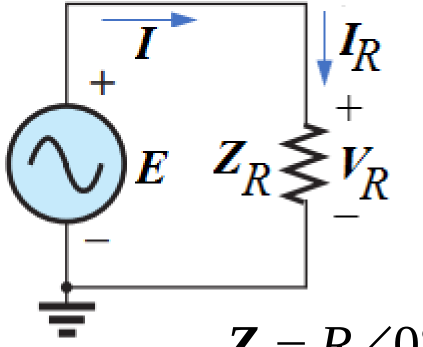
Phasor diagrams

Practice Book Problems
[Ch. 15] 2 and 3

Power and Energy Related Theory for

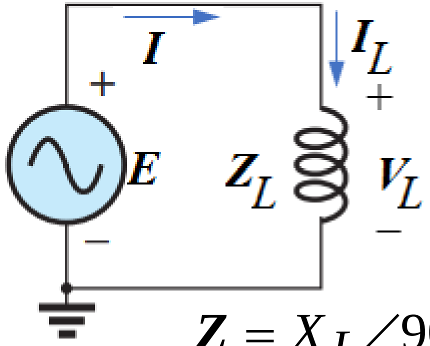
Pure Resistive, Pure Inductive and Pure Capacitive Circuits

PURE RESISTIVE CIRCUIT



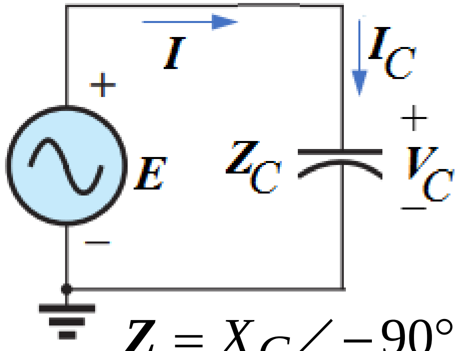
$$I_R = I$$
$$V_R = E$$
$$Z_R = Z$$
$$Z = R\angle 0°\ \Omega = R\ \Omega$$

PURE INDUCTIVE CIRCUIT



$$I_L = I$$
$$V_L = E$$
$$Z_L = Z$$
$$Z = X_L\angle 90°\ \Omega = jX_L\ \Omega$$

PURE CAPACITIVE CIRCUIT



$$I_C = I$$
$$V_C = E$$
$$Z_C = Z$$
$$Z = X_C\angle -90°\ \Omega = -jX_C\ \Omega$$

Power Factor, reactive Factor, Real Power, Reactive Power, Apparent Power, Instantaneous power

Resistive:

$$pf = F_p = \cos\theta = \cos 0° = 1$$
$$rf = F_q = \sin\theta = \sin 0° = 0$$

Unity Power Factor

$$P = V_R I_R = I_R^2 R = \frac{V_R^2}{R}$$
$$Q = 0$$
$$S = V_R I_R$$
$$p_R(t) = V_R I_R (1 - \cos 2\omega t)\ \text{W}$$

Inductive:

$$pf = F_p = \cos\theta = \cos 90° = 0$$
$$rf = F_q = \sin\theta = \sin 90° = 1$$

Zero Lagging Power Factor

$$P = 0$$
$$Q_L = V_L I_L = I_L^2 X_L = \frac{V_L^2}{X_L}$$
$$S = V_R I_R$$
$$p_L(t) = V_L I_L \sin 2\omega t\ \text{W}$$

Capacitive:

$$pf = F_p = \cos\theta = \cos(-90°) = 0$$
$$rf = F_q = \sin\theta = \sin(-90°) = -1$$

Zero Leading Power Factor

$$P = 0$$
$$Q_C = -V_C I_C = -I_C^2 X_C = \frac{V_C^2}{X_C}$$
$$S = V_R I_R$$
$$p_C(t) = -V_C I_C \sin 2\omega t\ \text{W}$$

PURE RESISTIVE CIRCUIT

$$p_R(t) = V_R I_R (1 - \cos 2\omega t) \text{ W}$$

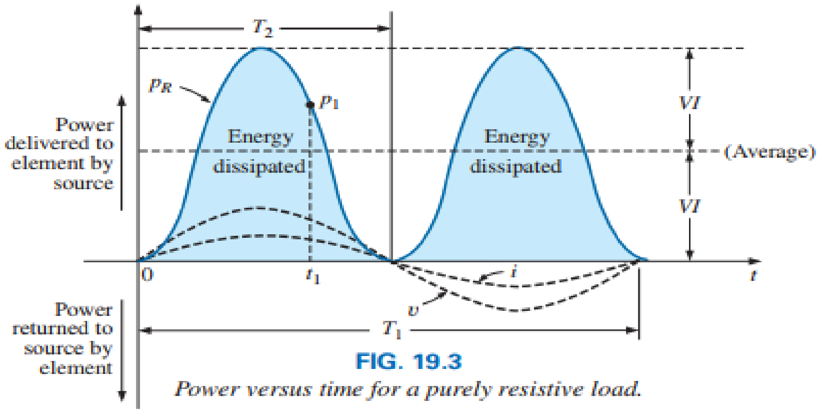


FIG. 19.3
Power versus time for a purely resistive load.

PURE INDUCTIVE CIRCUIT

$$p_L(t) = V_L I_L \sin 2\omega t \text{ W}$$

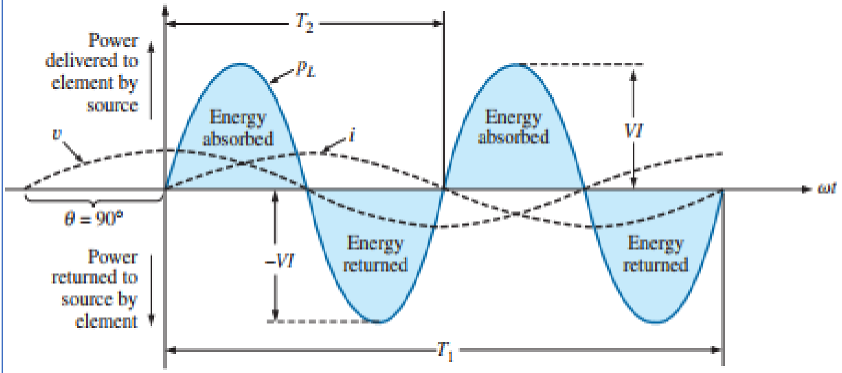


FIG. 19.9
The power curve for a purely inductive load.

PURE CAPACITIVE CIRCUIT

$$p_C(t) = -V_C I_C \sin 2\omega t \text{ W}$$

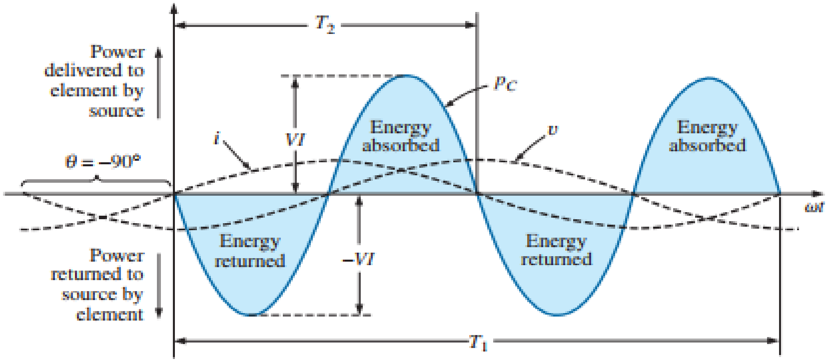


FIG. 19.13
The power curve for a purely capacitive load.

T_1 = Period of input voltage or current; T_2 = Period of power curve

Energy Dissipation and Energy Stored Calculation by using the Equation of: $W = Pt$ J

The energy **dissipated** by the resistor (W_R) over **one full cycle**:

$$W_R = (V_R I_R)T \quad [\text{J}] \qquad \textbf{(19.4)}$$

$$W_R = \frac{V_R I_R}{f} \quad [\text{J}] \qquad \textbf{(19.5)}$$

$$W_R = 2\pi \frac{V_R I_R}{\omega} \quad [\text{J}] \qquad \textbf{(19.5.1)}$$

The energy **stored** by the **inductor** (W_L) and **capacitor** (W_C) during the positive portion of the cycle (Fig. 19.9 and Fig. 19.13) is equal to that returned during the negative portion and can be determined **over half cycle** using the following equation:

$$W_L = \frac{V_{Lm} I_{Lm}}{2\omega} = \frac{V_L I_L}{\omega} \quad [\text{J}]$$

$$W_L = \frac{1}{2} L I_{Lm}^2 = L I_L^2 \quad [\text{J}] \quad \textbf{(19.18)}$$

$$W_C = \frac{V_{Cm} I_{Cm}}{2\omega} = \frac{V_C I_C}{\omega} \quad [\text{J}]$$

$$W_C = \frac{1}{2} C V_{Cm}^2 = C V_C^2 \quad [\text{J}] \quad \textbf{(19.26)}$$

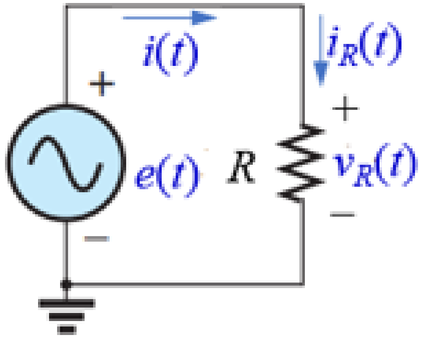
EXAMPLE The voltage $e(t) = 100\sin(314t+60^o)$ V is applied across in the following circuits.

(*a*) Write the instantaneous equation of current.

(*b*) Find the power factor, the reactive factor, the power, the reactive power, the apparent power.

(*c*) Write the instantaneous power equation.

(*d*) Find the energy dissipated by the resistor over one full cycle of the input voltage and the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve.



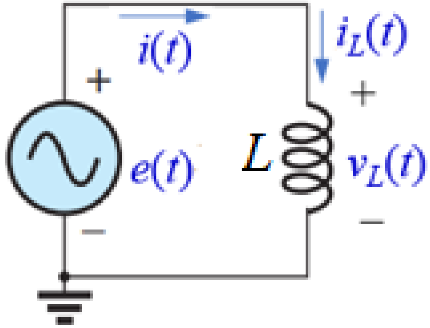
$R = 10\ \Omega$

$$I_m = \frac{V_m}{R} = \frac{100\,\text{V}}{10\,\Omega} = 10\ \text{A}$$

Here, $i(t)$ and $e(t)$ are in phase

$$\theta_i = \theta_e = 60^\circ$$

$i(t)$ = 10sin(314t + 60°) A



$L = 31.85$ mH

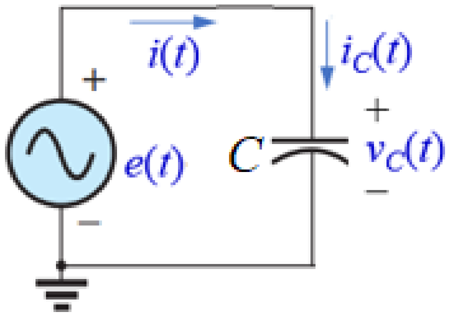
$$X_L = (314\,\text{rad/s})(31.85\,\text{H}) = 10\ \Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100\,\text{V}}{10\,\Omega} = 10\ \text{A}$$

Here, $i(t)$ lags $e(t)$ by 90°

$$\theta_i = \theta_e - 90^\circ = 60^\circ - 90^\circ = -30^\circ$$

$i(t)$ = 10sin(314t - 30°) A



$C = 318.47\ \mu$F

$$X_C = \frac{1}{(314\,\text{rad/s})(318.47\times10^{-6}\,\text{F})} = 10\ \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{100\,\text{V}}{10\,\Omega} = 10\ \text{A}$$

Here, $i(t)$ leads $e(t)$ by 90°

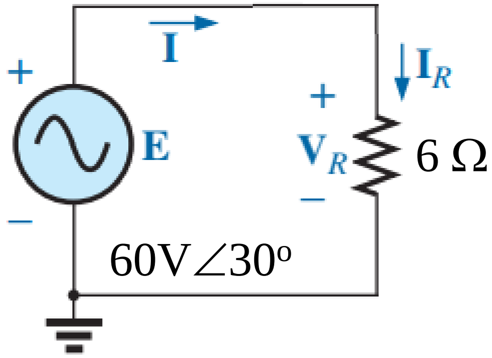
$$\theta_i = \theta_e + 90^\circ = 60^\circ + 90^\circ = 150^\circ$$

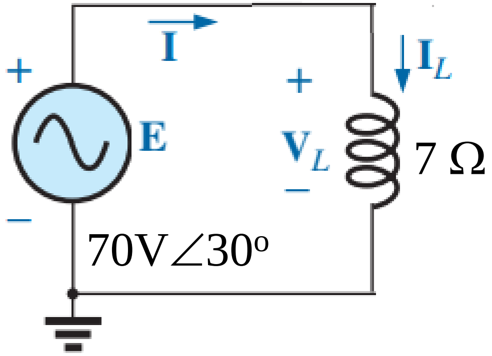
$i(t)$ = 10sin(314t + 150°) A

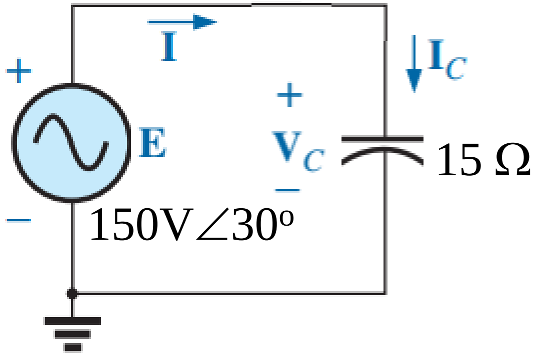
(**b**) Find the power factor, the reactive factor, the power, the reactive power, the apparent power.
(**c**) Write the instantaneous power equation.
(**d**) Find the energy dissipated by the resistor over one full cycle of the input voltage and the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve.

$R = 10\ \Omega \quad I_m = 10\text{ A} \quad \theta_i = 60°$	$L = 31.85\text{ mH} \quad X_L = 10\ \Omega$ $I_m = 10\text{ A} \quad \theta_i = 150°$	$C = 318.47\ \mu\text{F} \quad X_C = 10\ \Omega$ $I_m = 10\text{ A} \quad \theta_i = -30°$
$\theta = \theta_z = \theta_v - \theta_i = 0°$ $pf = \cos\theta = \cos\theta_z = \cos(0°) = 1$ $rf = \sin\theta = \sin\theta_z = \sin(0°) = 0$	$\theta = \theta_z = \theta_v - \theta_i = 90°$ $pf = \cos\theta = \cos\theta_z = \cos(90°) = 0$ $rf = \sin\theta = \sin\theta_z = \sin(90°) = 1$	$\theta = \theta_z = \theta_v - \theta_i = -90°$ $pf = \cos\theta = \cos\theta_z = \cos(-90°) = 0$ $rf = \sin\theta = \sin\theta_z = \sin(-90°) = -1$
$S = \frac{V_m I_m}{2} = \frac{(100\text{V})(10\text{A})}{2} = 500\text{ VA}$ $P = S\cos\theta = (500\text{VA})\cos(0°) = 500\text{ W}$ $Q = S\sin\theta = (500\text{VA})\sin(0°) = 0\text{ Var}$ $p(t) = 500(1 - \cos 628t)\text{ W}$	$S = \frac{V_m I_m}{2} = \frac{(100\text{V})(10\text{A})}{2} = 500\text{ VA}$ $P = S\cos\theta = (500\text{VA})\cos(90°) = 0\text{ W}$ $Q = S\sin\theta = (500\text{VA})\sin(90°) = 500\text{ Var}$ $p(t) = 500\sin 628t\text{ W}$	$S = \frac{V_m I_m}{2} = \frac{(100\text{V})(10\text{A})}{2} = 500\text{ VA}$ $P = S\cos\theta = (500\text{VA})\cos(-90°) = 0\text{ W}$ $Q = S\sin\theta = (500\text{VA})\sin(-90°) = -500\text{ Var}$ $p(t) = -500\sin 628t\text{ W}$
$W_R = \frac{V_R I_R}{f} = \frac{V_{Rm} I_{Rm}}{2f}$ $= \frac{(100\text{V})(10\text{V})}{2 \times 50\text{Hz}} = 10\text{ J}$	$W_L = \frac{V_L I_L}{\omega} = \frac{1}{2} L I_m^2$ $= \frac{1}{2}(31.85 \times 10^{-3}\text{H})(10\text{A})^2 = 1.59\text{ J}$	$W_C = \frac{V_C I_C}{\omega} = \frac{1}{2} C V_m^2$ $= \frac{1}{2}(318.47 \times 10^{-6}\text{F})(100\text{V})^2 = 1.59\text{ J}$

EXAMPLE The voltage ***E*** with 50 Hz frequency applied across in the following circuits.
(*a*) Find the power factor, the reactive factor, the power, the reactive power, the apparent power.
(*b*) Write the instantaneous power equation.
(*c*) Find the energy dissipated by the resistor over one full cycle of the input voltage and the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve.







Solution:

$$\boldsymbol{Z} = \boldsymbol{Z}_R = 6\Omega\angle 0° = 6\Omega$$

$$\boldsymbol{I} = \boldsymbol{I}_R = \frac{\boldsymbol{V}_R}{\boldsymbol{Z}_R} = \frac{\boldsymbol{E}}{\boldsymbol{Z}} = \frac{60\text{V}\angle 30°}{6\Omega} = 10\text{A}\angle 30°$$

$$pf = \cos\theta = \cos\theta_Z = \cos(0°) = 1$$

$$rf = \sin\theta = \sin\theta_Z = \sin(0°) = 0$$

$$\boldsymbol{Z} = \boldsymbol{Z}_L = 7\Omega\angle 90° = j7\ \Omega$$

$$\boldsymbol{I} = \boldsymbol{I}_L = \frac{\boldsymbol{V}_L}{\boldsymbol{Z}_L} = \frac{\boldsymbol{E}}{\boldsymbol{Z}} = \frac{70\text{V}\angle 30°}{7\Omega\angle 90°} = 10\text{A}\angle -60°$$

$$pf = \cos\theta = \cos\theta_Z = \cos(90°) = 0$$

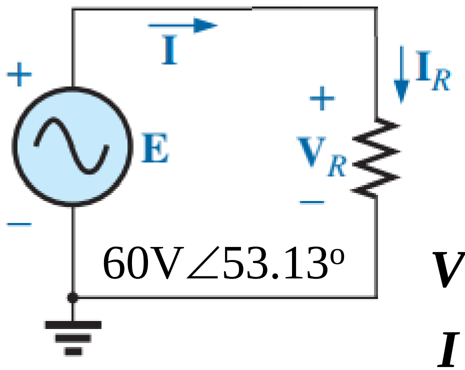
$$rf = \sin\theta = \sin\theta_Z = \sin(90°) = 1$$

$$\boldsymbol{Z} = \boldsymbol{Z}_C = 15\Omega\angle -90° = -j15\ \Omega$$

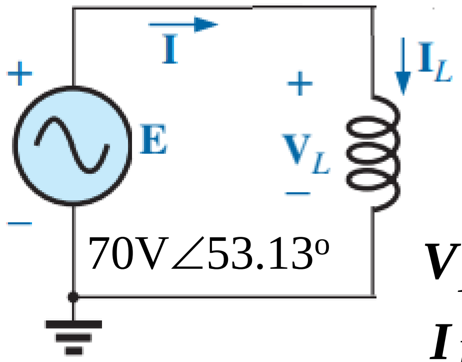
$$\boldsymbol{I} = \boldsymbol{I}_C = \frac{\boldsymbol{V}_C}{\boldsymbol{Z}_C} = \frac{\boldsymbol{E}}{\boldsymbol{Z}} = \frac{150\text{V}\angle 30°}{15\Omega\angle -90°} = 10\text{A}\angle 120°$$

$$pf = \cos\theta = \cos\theta_Z = \cos(-90°) = 0$$

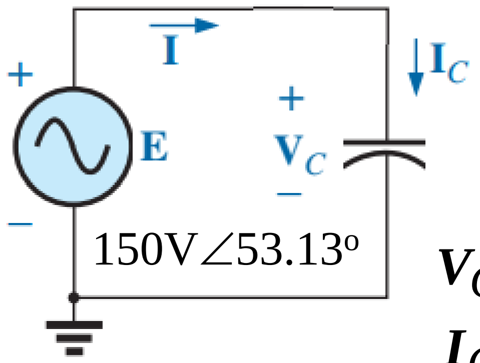
$$rf = \sin\theta = \sin\theta_Z = \sin(-90°) = -1$$



$$V_R = 60\text{V}\angle 30^\circ$$
$$I_R = 10\text{A}\angle 30^\circ$$



$$V_L = 70\text{V}\angle 30^\circ$$
$$I_L = 10\text{A}\angle -60^\circ$$



$$V_C = 150\text{V}\angle 30^\circ$$
$$I_C = 10\text{A}\angle 120^\circ$$

$$S = VI = (60\text{V})(10\text{A}) = 600\ \text{VA}$$

$$P = VI\cos\theta_Z = (60\text{V})(10\text{A})\cos(0^\circ)$$
$$= 600\ \text{W}$$

$$Q = VI\sin\theta_Z = (60\text{V})(10\text{A})\sin(0^\circ)$$
$$= 0\ \text{Var}$$

$$\omega = 2\pi f = 2\pi\times 50 = 314\ \text{rad/s}$$

$$p(t) = 600(1-\cos 628t)\ \text{W}$$

$$W_R = \frac{V_R I_R}{f} = \frac{(60\text{V})(10\text{V})}{50\,\text{Hz}} = 12\ \text{J}$$

$$S = VI = (70\text{V})(10\text{A}) = 700\ \text{VA}$$

$$P = VI\cos\theta_Z = (70\text{V})(10\text{A})\cos(90^\circ)$$
$$= 0\ \text{W}$$

$$Q = VI\sin\theta_Z = (70\text{V})(10\text{A})\sin(90^\circ)$$
$$= 700\ \text{Var}$$

$$p(t) = 700\sin 628t\ \text{W}$$

$$W_L = \frac{V_L I_L}{\omega} = \frac{(70\text{V})(10\text{V})}{314\ \text{rad/s}} = 2.23\ \text{J}$$

$$S = VI = (150\text{V})(10\text{A}) = 1500\ \text{VA}$$

$$P = VI\cos\theta_Z = (150\text{V})(10\text{A})\cos(-90^\circ)$$
$$= 0\ \text{W}$$

$$Q = VI\sin\theta_Z = (150\text{V})(10\text{A})\sin(-90^\circ)$$
$$= -1500\ \text{Var}$$

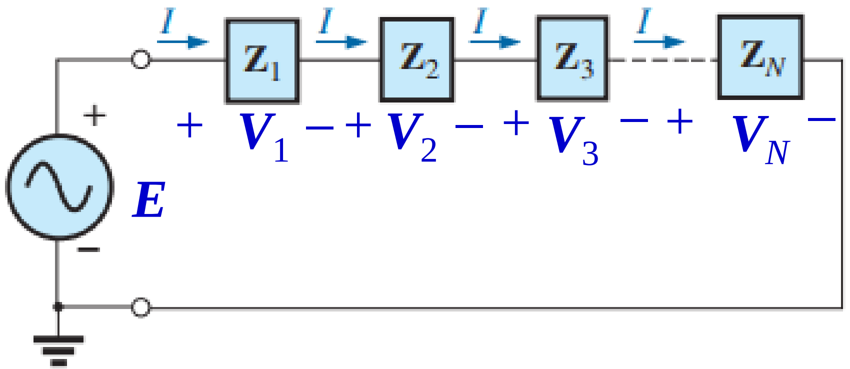
$$p(t) = -1500\sin 628t\ \text{W}$$

$$W_C = \frac{V_C I_C}{\omega} = \frac{(150\text{V})(10\text{V})}{314\,\text{rad/s}} = 4.78\ \text{J}$$

Chapter 15

Series Circuits

15.3 SERIES CONFIGURATION



The total impedance of a series configuration is the sum of the individual impedances:

$$Z_T = Z_1 + Z_2 + Z_3 + \ldots\ldots + Z_N \qquad \textbf{(15.4)}$$

$$I = \frac{E}{Z_T} = EY_T \qquad E = IZ_T = \frac{I}{Y_T}$$

$$V_1 = I_1 Z_1 = IZ_1 \qquad V_2 = I_2 Z_2 = IZ_2$$

$$V_3 = I_3 Z_3 = IZ_3 \qquad V_N = I_N Z_N = IZ_N$$

If $Z_1 = Z_2 = Z_3 = \ldots\ldots = Z_n = Z_s$

$$Z_T = N \times Z_N \quad V_1 = V_2 = V_3 = \ldots. = V_N = IZ_s = \frac{E}{N}$$

Voltage Divider Rule (VDR)

The voltage across an impedance in a series circuit is equal to the value of that impedance (Z_x) times the total applied voltage (E) divided by the total impedance (Z_T) of the series configuration.

$$V_x = \frac{Z_x}{Z_T} E = \frac{Y_T}{Y_x} E$$

Kirchhoff's Voltage Law (KVL)

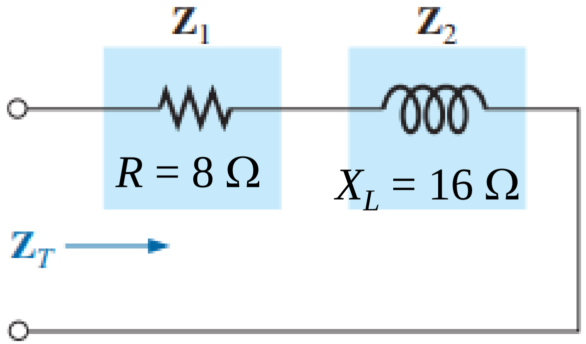
(1) The algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.

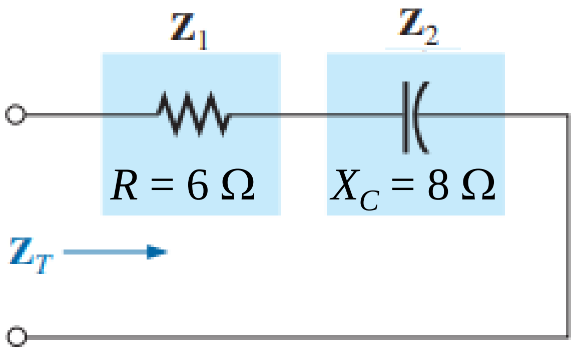
$$\sum_{\text{closed path}} V = 0$$

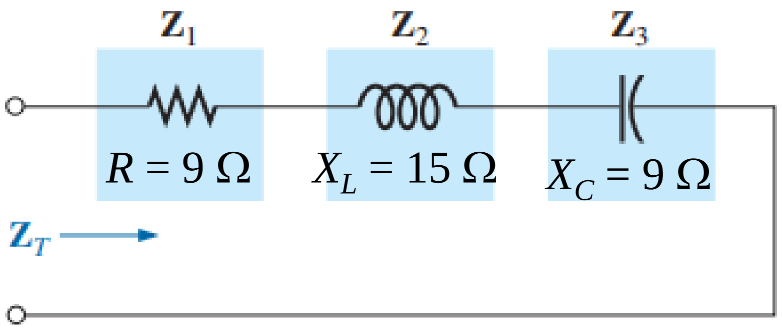
(2) The sum of the applied or supplied or rise voltage of a series circuit will equal the sum of the voltage drops of the circuit.

$$\sum_{\text{closed path}} V_{rise} = \sum_{\text{closed path}} V_{drop}$$

EXAMPLE For the following circuits, find the total impedance and draw the impedance diagram.



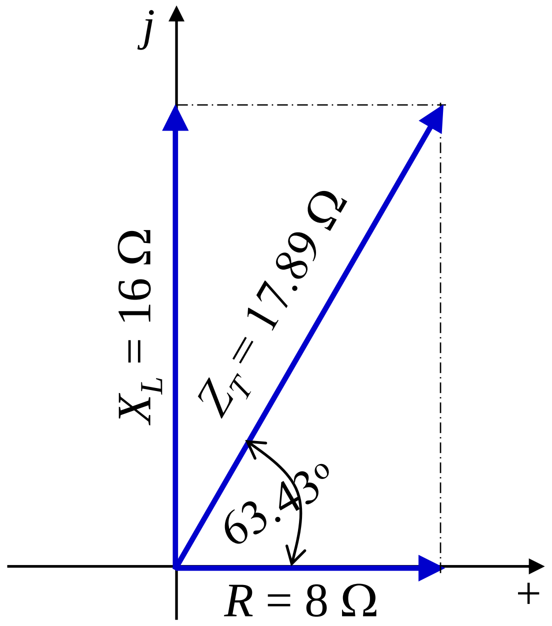


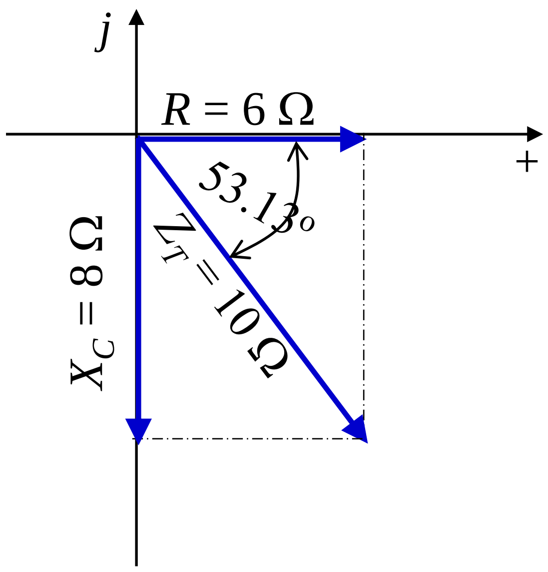


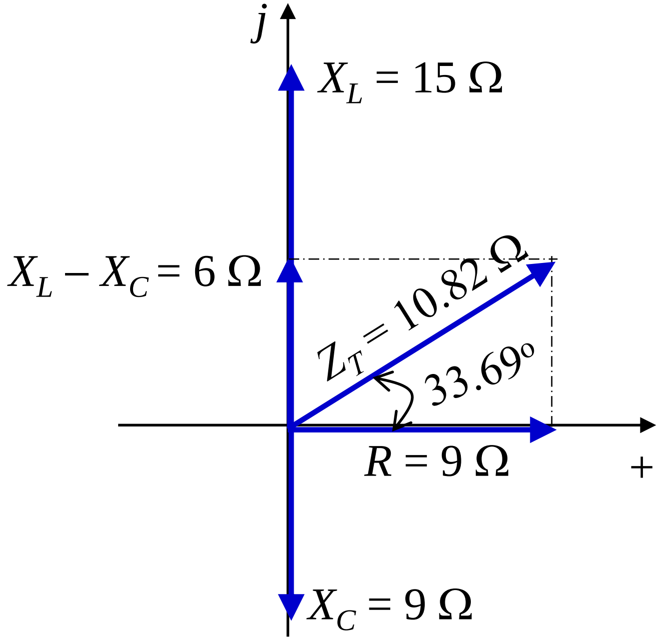
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = R + jX_L$$
$$= 8 + j16 = \mathbf{17.89\Omega\angle 63.43^\circ}$$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = R - jX_C$$
$$= 6 - j8 = \mathbf{10\Omega\angle -53.13^\circ}$$

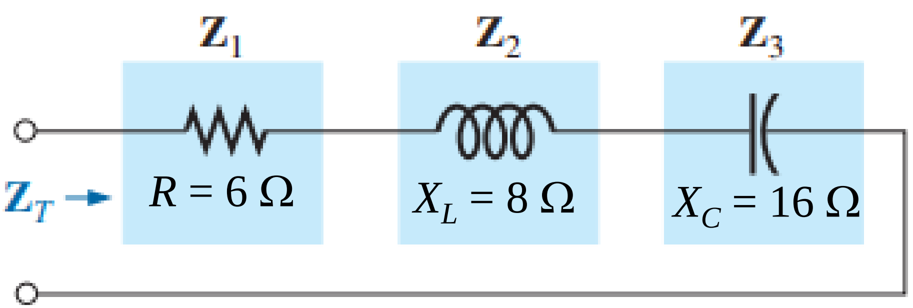
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R + jX_L - jX_C$$
$$= R + j(X_L - X_C) = 9\Omega + j(15\Omega - 9\Omega)$$
$$= 9\Omega + j6\Omega = \mathbf{10.82\Omega\angle 33.69^\circ}$$

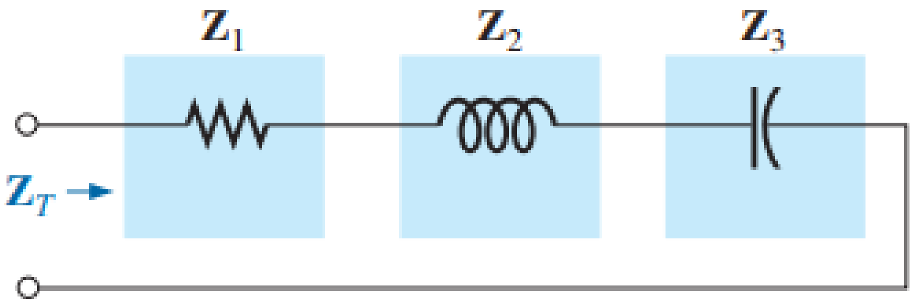






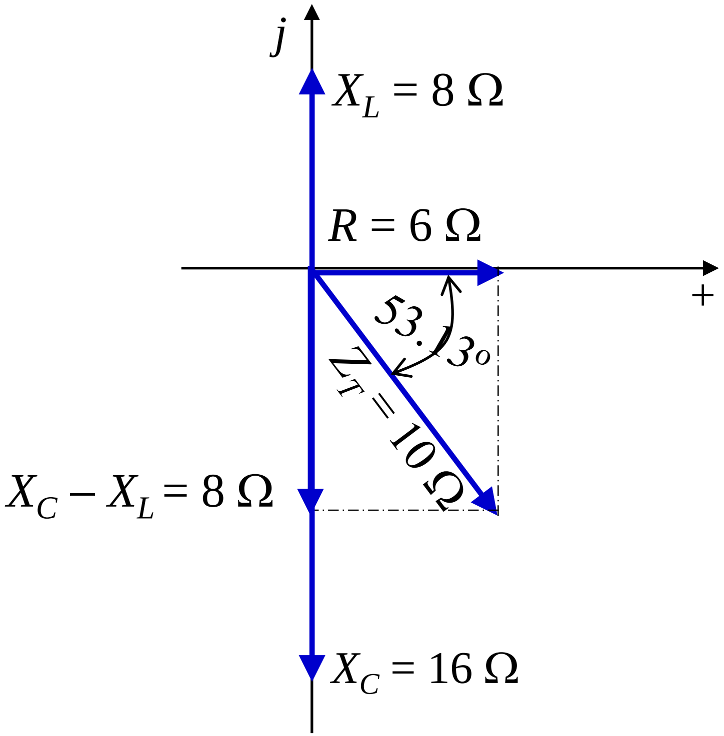
EXAMPLE For the following circuits, fnd the total impedance and draw the impedance diagram.





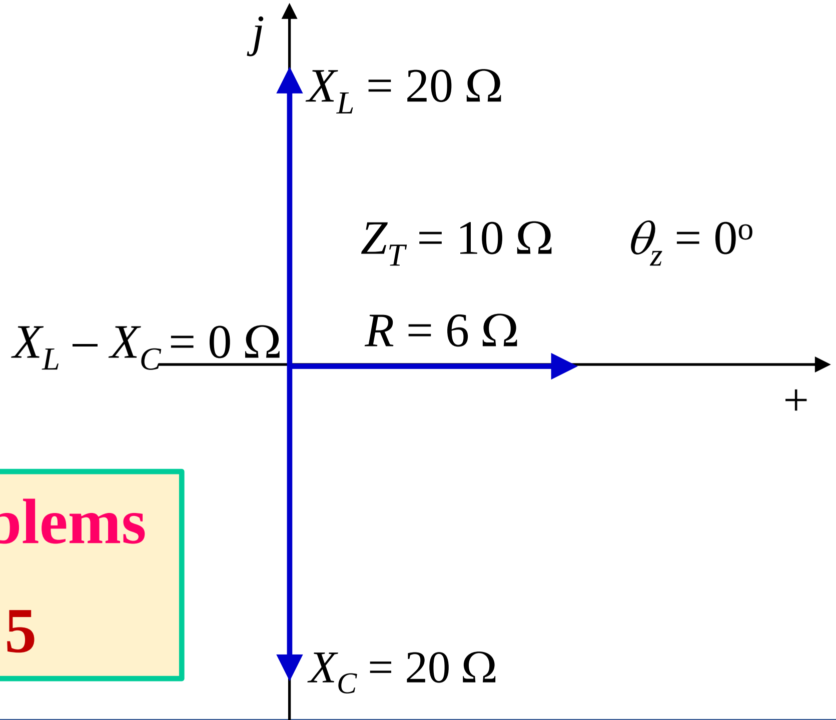
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R + jX_L - jX_C = R + j(X_L - X_C)$$
$$= 6\Omega + j(8\Omega - 16\Omega) = 6\Omega - j8\Omega = \mathbf{10\Omega\angle -53.13^\circ}$$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R + jX_L - jX_C = R + j(X_L - X_C)$$
$$= 10\Omega + j(20\Omega - 20\Omega) = 10\Omega - j0\Omega = \mathbf{10\Omega\angle 0^\circ}$$

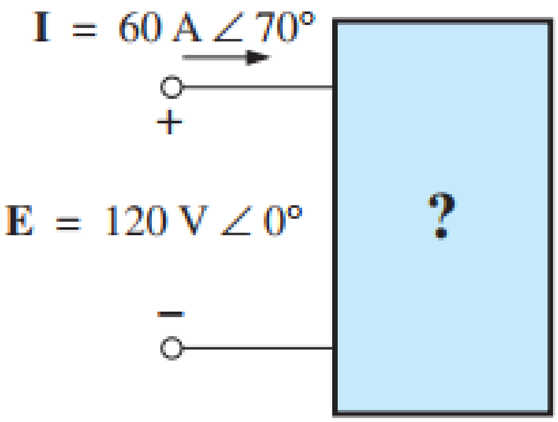


Practice Book Problems

[Ch. 15] 4 and 5



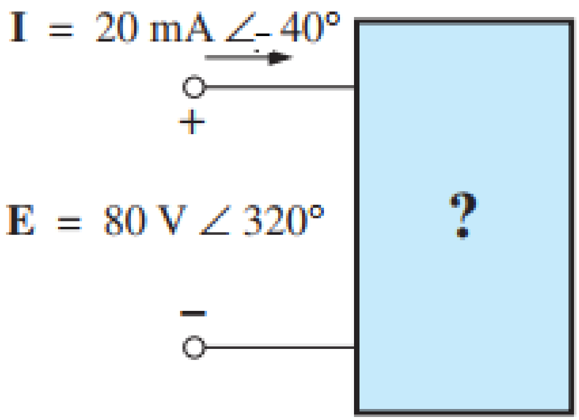
Problem 6 [Ch. 15] Find the type and impedance in ohms of the series circuit elements that must be in the closed container in Fig. 15.125 for the indicated voltages and currents to exist at the input terminals. (Find the simplest series circuit that will satisfy the indicated conditions.)



(a)

$\theta_z = \theta_v - \theta_i = 0^o - 70^o = -70^o$

Since $\theta_z < 0^o$ so voltage lags current. The circuit is *R–C* series circuit or *R–L –C* series circuit with $X_C > X_L$.



(b)

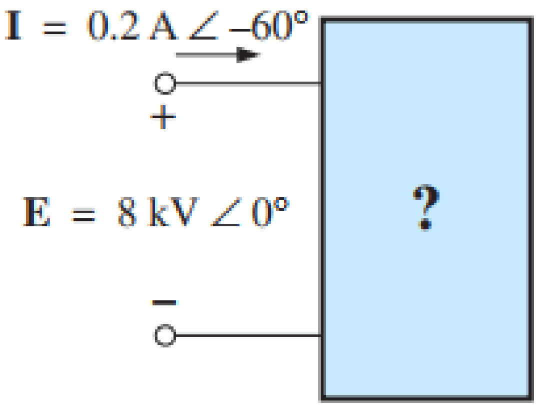
Practically, $-90^o \leq \theta_z \leq 90^o$

$\theta_v = 320^o - 360^o = -40^o$

$\theta_z = \theta_v - \theta_i = -40^o - (-40^o) = 0^o$

Since $\theta_z = 0^o$ so voltage and current are in phase.

The circuit is pure resistive or *R–L –C* series circuit with $X_L = X_C$.



(c)

$\theta_z = \theta_v - \theta_i = 0^o - (-60^o) = 60^o$

Since $\theta_z > 0^o$ so voltage leads current. The circuit is *R–L* series circuit or *R–L –C* series circuit with $X_L > X_C$.

R-*L* Series Circuit

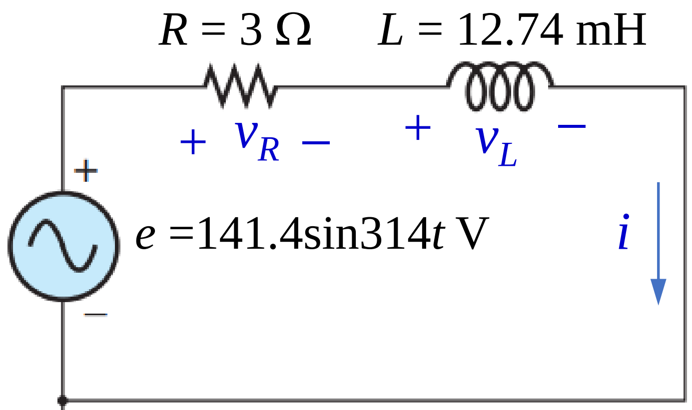


Fig 15.26 *Series R-L circuit*

$$X_L = \omega L = 314\times(12.47\times 10^{-3}) = 4\ \Omega$$

$$\boldsymbol{E} = (0.707\times 141.4)\angle 0^{o} = 100\text{V}\angle 0^{o}$$

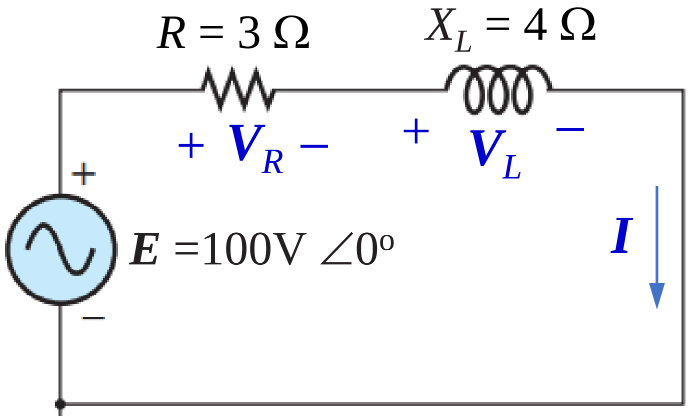


Fig 15.27 *Phasor notation of Fig. 15.26*

R-L Series Circuit

Impedance

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 = R + jX_L$$
$$= 3 + j4 = \mathbf{5\Omega\angle 53.13°}$$

Impedance Diagram

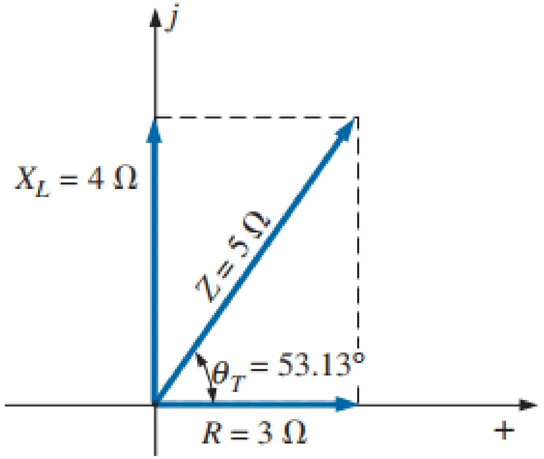


FIG. 15.28 *Impedance diagram*

Current

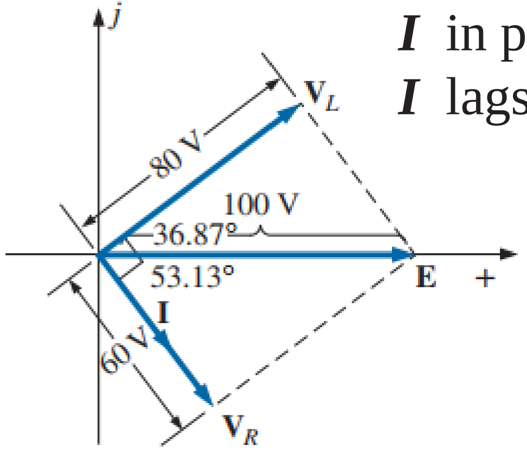
$$\boldsymbol{I} = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \frac{100\text{V}\angle 0°}{5\Omega\angle 53.13°}$$
$$= 20\text{A}\angle -53.13°$$

V_R and V_L

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20\ \text{A}\ \angle -53.13°)(3\ \Omega\ \angle 0°)$$
$$= \mathbf{60\ V\ \angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20\ \text{A}\ \angle -53.13°)(4\ \Omega\ \angle 90°)$$
$$= \mathbf{80\ V\ \angle 36.87°}$$

Phasor or Vector Diagram



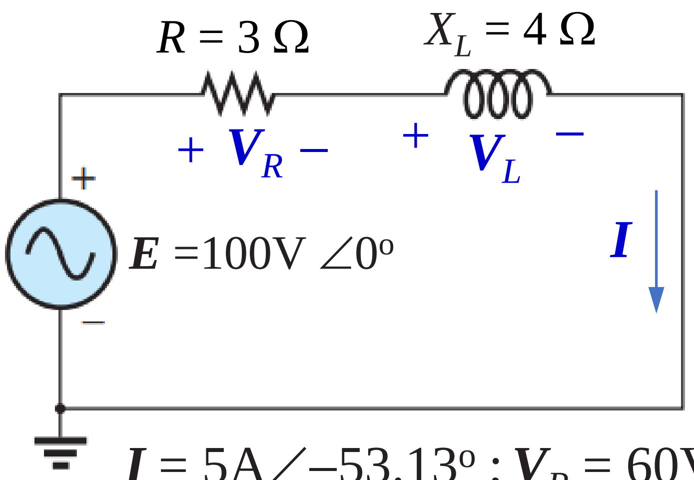
$\boldsymbol{I}$ in phase with $\boldsymbol{V}_R$

$\boldsymbol{I}$ lags with $\boldsymbol{V}_L$ by 90°

FIG. 15.29 *Phasor diagram*

KVL:

$$\boldsymbol{E} = \boldsymbol{V}_R + \boldsymbol{V}_L$$
$$= 60\text{V}\angle -53.13^{o} + 80\text{V}\angle 36.87^{o}$$
$$= 100\text{V}\ \angle 0^{o}$$



I = 5A∠–53.13° ; V_R = 60V∠–53.13°;
V_L = 80V∠36.87°

Power Factor and Reactive Factor

$pf = (R/Z_T) = \cos\theta_z = \cos(53.13^o)$ **= 0.6 Lagging**
$rf = (X_L/Z_T) = \sin\theta_z = \sin(53.13^o)$ **= 0.8**

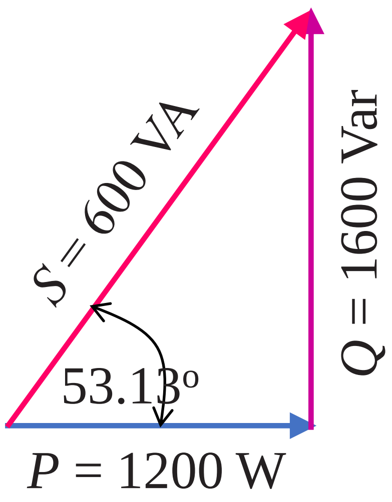
Power [Total watts]

$P_E = EI\cos\theta_z = 100\times20\cos(53.13^o)$ **= 1200 W**
$P_R = I^2R = (20A)^2 \times 3\Omega$ **= 1200 W**

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 100\times20\sin(53.13^o)$ **= 1600 Var**
$Q_L = I^2X_L = (20A)^2 \times 4\Omega$ **= 1600 Var**

Power Triangle



Apparent Power [volt-ampere]

$S_E = EI = 100\times20$ **= 2000 VA**
$S_Z = I^2Z = (20A)^2 \times 5\Omega$ **= 2000 VA**

Instantaneous Power Equation

$p(t) = P(1 - \cos2\omega t) + Q\sin2\omega t$ W
$= 1200(1-\cos628t) + 1600\sin628t$ W

Instantaneous Current and Voltages Equation

$i(t) = (\sqrt{2}\times20)\sin(314t - 53.13^o)$ A
$= 28.28\sin(314t - 53.13^o)$ A

$v_R(t) = (\sqrt{2}\times60)\sin(314t - 53.13^o)$ V
$= 84.85\sin(314t - 53.13^o)$ V

$v_L(t) = (\sqrt{2}\times80)\sin(314t + 36.87^o)$ V
$= 113.14\sin(314t + 36.87^o)$ V

Practice Book Problems [Ch. 15] 7

R-*C* Series Circuit

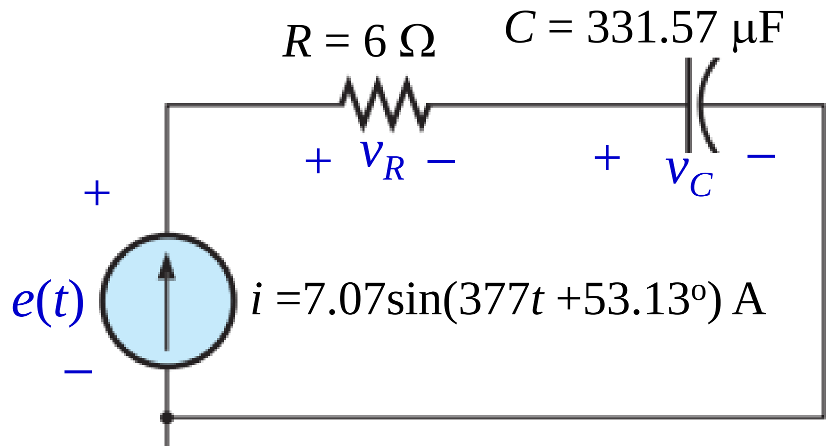


Fig 15.30 *Series R-C circuit*

$$X_C = \frac{1}{377 \times \left(331.57 \times 10^{-6}\right)} = 8\ \Omega$$

$$I = (0.707 \times 7.07)\angle 53.13^o = 5A\angle 53.13^o$$

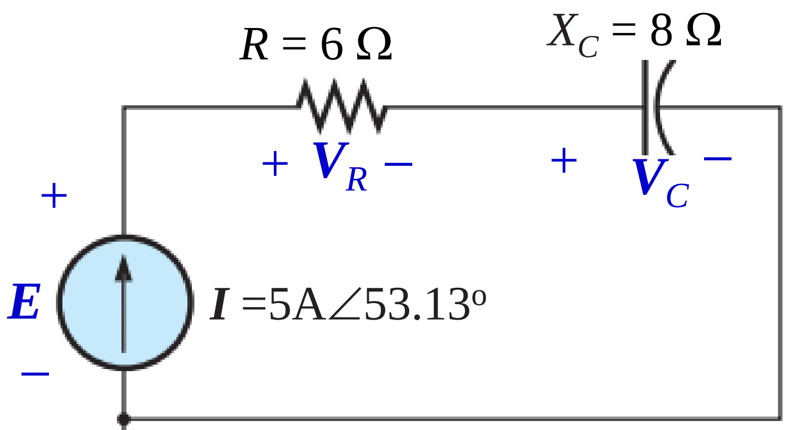


Fig 15.31 *Phasor notation of Fig. 15.30*

R-C Series Circuit

Impedance

$$Z_T = Z_1 + Z_2 = R - jX_C$$

$$= 6 - j8 = \mathbf{10\Omega\angle -53.13°}$$

Impedance Diagram

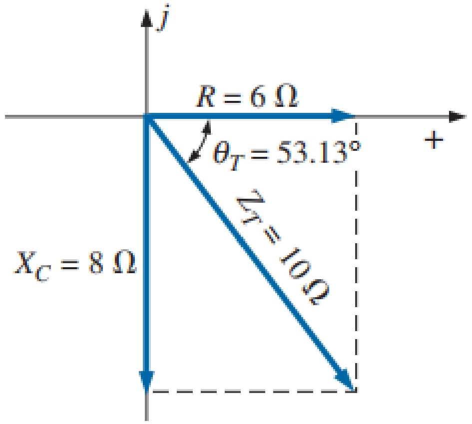


FIG. 15.32 *Impedance diagram*

Voltage

$$E = IZ_T$$

$$= (5V\angle 53.13°)(10\Omega\angle -53.13°)$$

$$= 50V\angle 0°$$

V_R and V_C

$$\mathbf{V}_R = \mathbf{IZ}_R = (I\angle\theta)(R\angle 0°)$$
$$= (5\text{ A}\angle 53.13°)(6\ \Omega\angle 0°) = \mathbf{30\text{ V}\angle 53.13°}$$

$$\mathbf{V}_C = \mathbf{IZ}_C = (I\angle\theta)(X_C\angle -90°)$$
$$= (5\text{ A}\angle 53.13°)(8\ \Omega\angle -90°) = \mathbf{40\text{ V}\angle -36.87°}$$

Phasor or Vector Diagram

I in phase with V_R

I leads with V_L by 90º

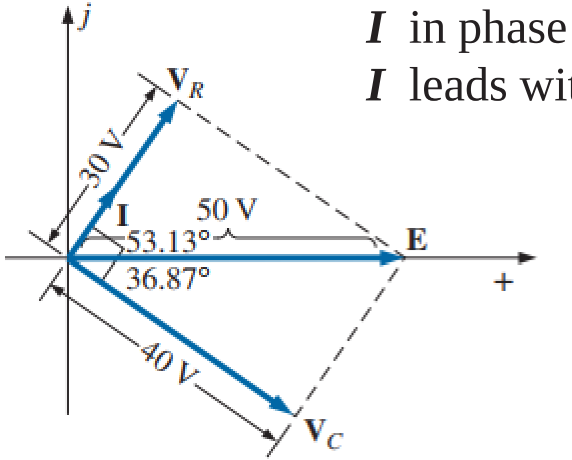


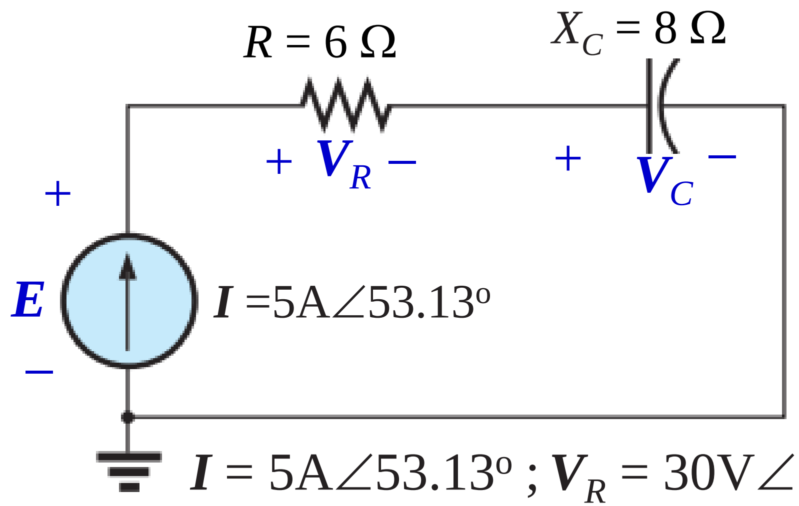
FIG. 15.33 *Phasor diagram*

KVL:

$$E = V_R + V_C$$

$$= 30V\angle 53.13^o + 40V\angle -36.87^o$$

$$= 50V\angle 0^o$$



I = 5A∠53.13º ; V_R = 30V∠53.13º;
V_C = 40V∠–36.87º

Power Factor and Reactive Factor

$pf = (R/Z_T) = \cos\theta_z = \cos(-53.13^o)$ **= 0.6 Leading**
$rf = (X_L/Z_T) = \sin\theta_z = \sin(-53.13^o)$ **= –0.8**

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times5\cos(-53.13^o)$ **= 150 W**
$P_R = I^2R = (5A)^2 \times 6\Omega$ = **= 150 W**

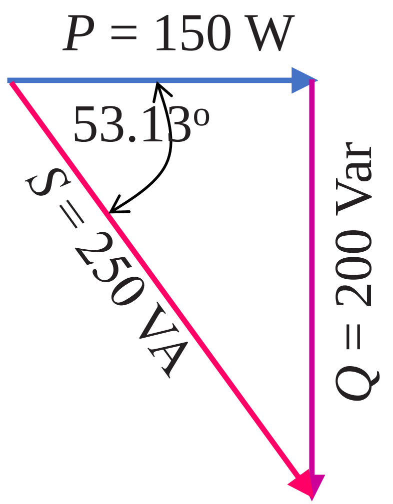
Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times5\sin(-53.13^o)$ **= – 200 Var**
$Q_C = -I^2X_C = -(5A)^2 \times 8\Omega$ = **– 200 Var**

Apparent Power [volt-ampere]

$S_E = EI = 50\times5$ **= 250 VA**
$S_Z = I^2Z = (5A)^2 \times 10\Omega$ = **250 VA**

Power Triangle



Instantaneous Power Equation

$p(t) = P(1 - \cos2\omega t) + Q\sin2\omega t$ W
$= 150(1 - \cos754t) + 200\sin754t$ W

Instantaneous or Time Domain Current and Voltages Equation

$e(t) = (\sqrt{2}\times 50)\sin(377t)$ V = $70.7\sin(314t - 53.13^o)$ V

$v_R(t) = (\sqrt{2}\times 30)\sin(377t + 53.13^o)$ V = $42.43\sin(314t - 53.13^o)$ V

$v_C(t) = (\sqrt{2}\times 40)\sin(377t - 36.87^o)$ V = $56.57\sin(377t - 36.87^o)$ V

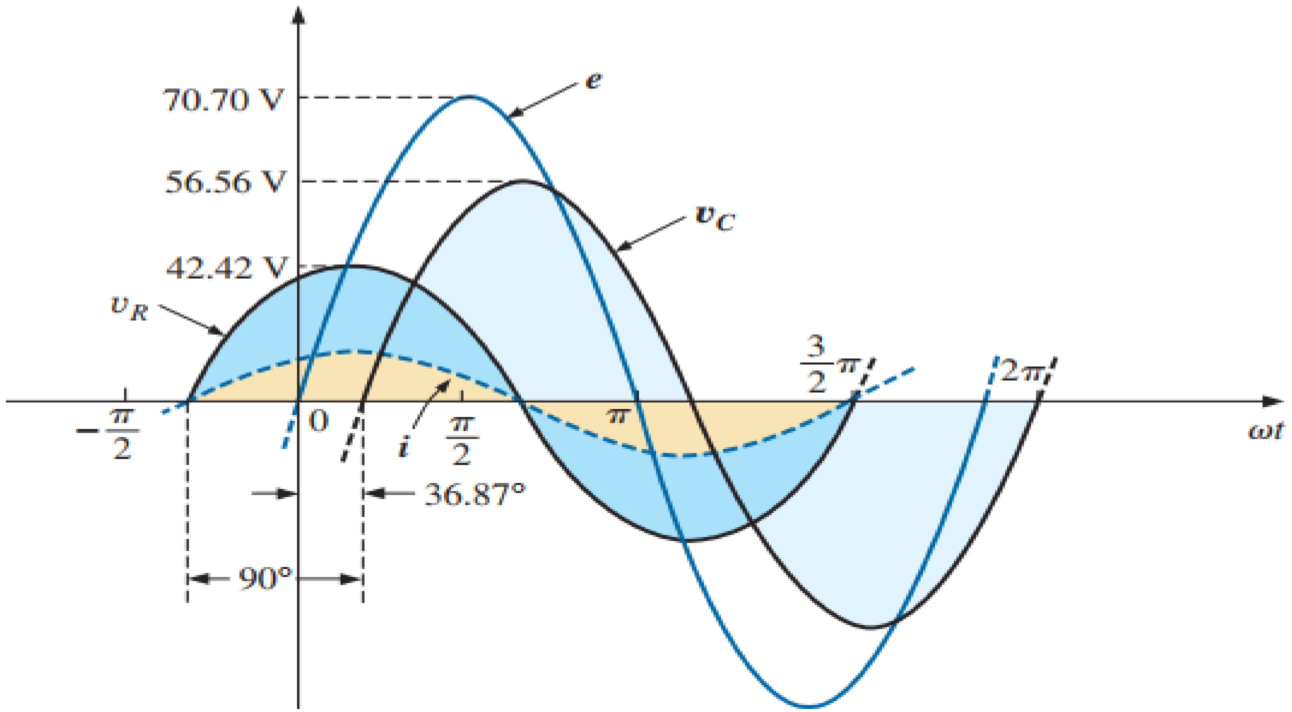


FIG. 15.34
Waveforms for the series R-C circuit in Fig. 15.30.

Practice Book Problems [Ch. 15] 8 and 9

R-L-C Series Circuit

R-L-C Series Circuit

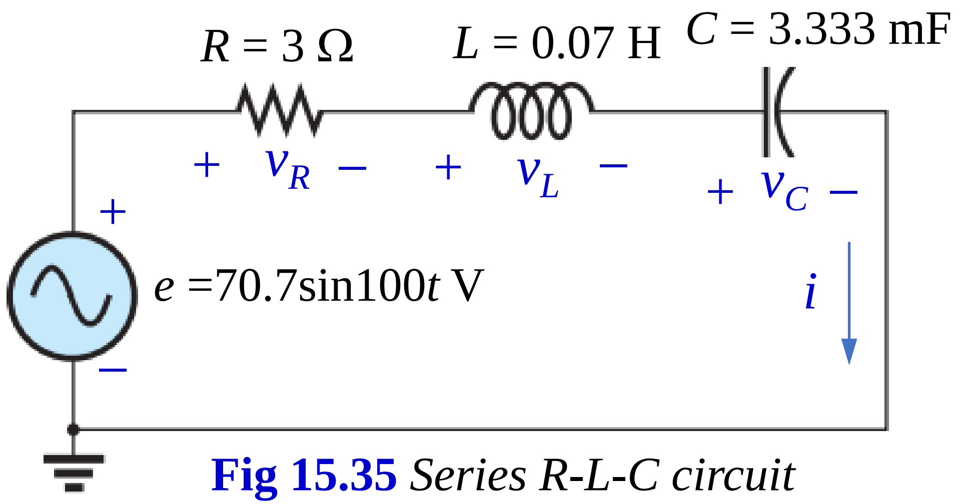


Fig 15.35 *Series R-L-C circuit*

$$X_L = \omega L = 100 \times 0.07 = 7\ \Omega$$

$$X_C = \frac{1}{100 \times \left(3.333 \times 10^{-3}\right)} = 3\ \Omega$$

$$\boldsymbol{E} = (0.707 \times 70.7)\angle 0^{\circ} = 50\text{V}\angle 0^{\circ}$$

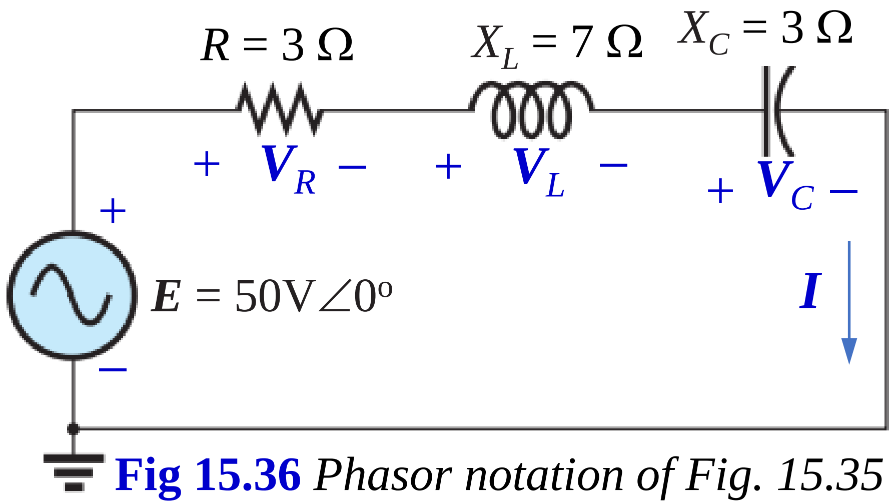


Fig 15.36 *Phasor notation of Fig. 15.35*

Impedance

$$\begin{aligned} \boldsymbol{Z}_T &= \boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \boldsymbol{Z}_3 = R + jX_L - jX_C \\ &= R + j\left(X_L - jX_C\right) = 3 + j(7 - j3) \\ &= 3 + j4 = \mathbf{5\Omega\angle 53.13^{\circ}} \end{aligned}$$

Current

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\ \text{V}\ \angle 0^{\circ}}{5\ \Omega\ \angle 53.13^{\circ}} = \mathbf{10\ A\ \angle\ -53.13^{\circ}}$$

Impedance Diagram

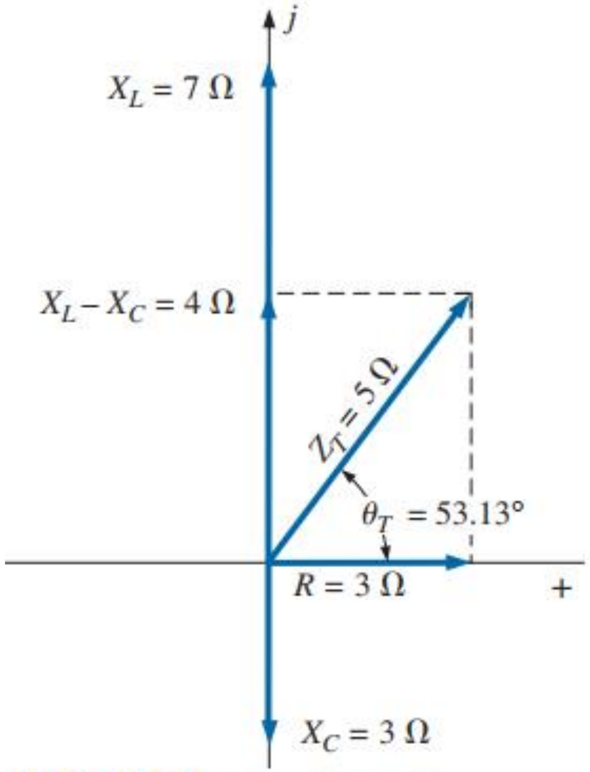


FIG. 15.37 *Impedance diagram*

V_R, V_L and, V_C

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle 0^\circ) = (10\text{ A}\angle -53.13^\circ)(3\ \Omega\angle 0^\circ)$$
$$= \mathbf{30\ V\angle -53.13^\circ}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\angle\theta)(X_L\angle 90^\circ) = (10\text{ A}\angle -53.13^\circ)(7\ \Omega\angle 90^\circ)$$
$$= \mathbf{70\ V\angle 36.87^\circ}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\angle\theta)(X_C\angle -90^\circ) = (10\text{ A}\angle -53.13^\circ)(3\ \Omega\angle -90^\circ)$$
$$= \mathbf{30\ V\angle -143.13^\circ}$$

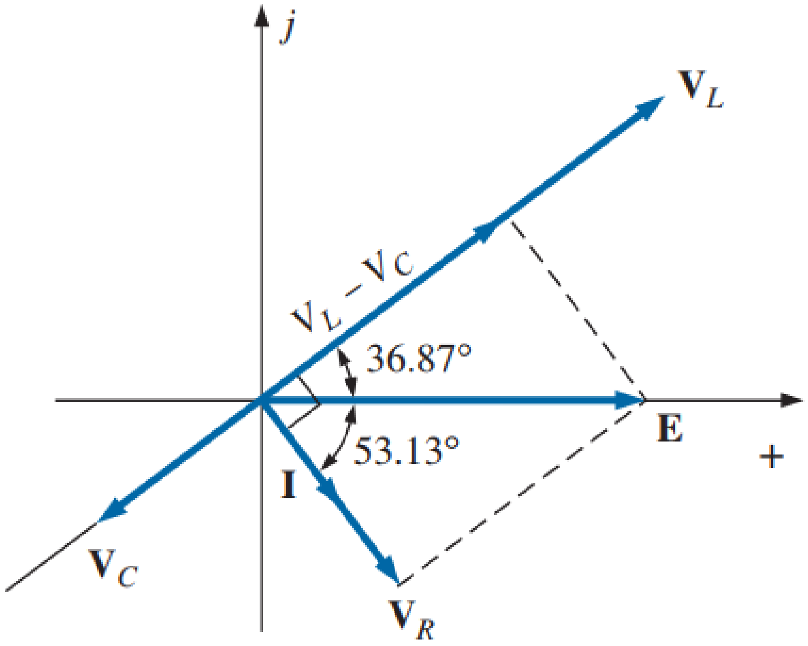


FIG. 15.38

Phasor diagram for the series R-L-C circuit in Fig. 15.35.

KVL:

$$E = V_R + V_C + V_L$$
$$= 30\text{V}\angle{-53.13^o} + 70\text{V}\angle{-36.87^o} + 30\text{V}\angle{-143.13^o}$$
$$= 50\text{V}\angle 0^o$$

Power Factor and Reactive Factor

$pf = (R/Z_T) = \cos\theta_z = \cos(53.13^o) = \mathbf{0.6\ Lagging}$

$rf = (X_L - X_C)/Z_T = \sin\theta_z = \sin(53.13^o) = \mathbf{0.8}$

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times 10\cos(53.13^o) = \mathbf{300\ W}$

$P_R = I^2R = (10\text{A})^2 \times 3\Omega = \mathbf{300\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times 10\sin(53.13^o) = \mathbf{400\ Var}$

$Q_L = I^2X_L = (10\text{A})^2 \times 7\Omega = \mathbf{700\ Var}$

$Q_C = -I^2X_C = -(10\text{A})^2 \times 3\Omega = -\mathbf{300\ Var}$

$Q = Q_L + Q_C = 700 - 300 = \mathbf{400\ Var}$

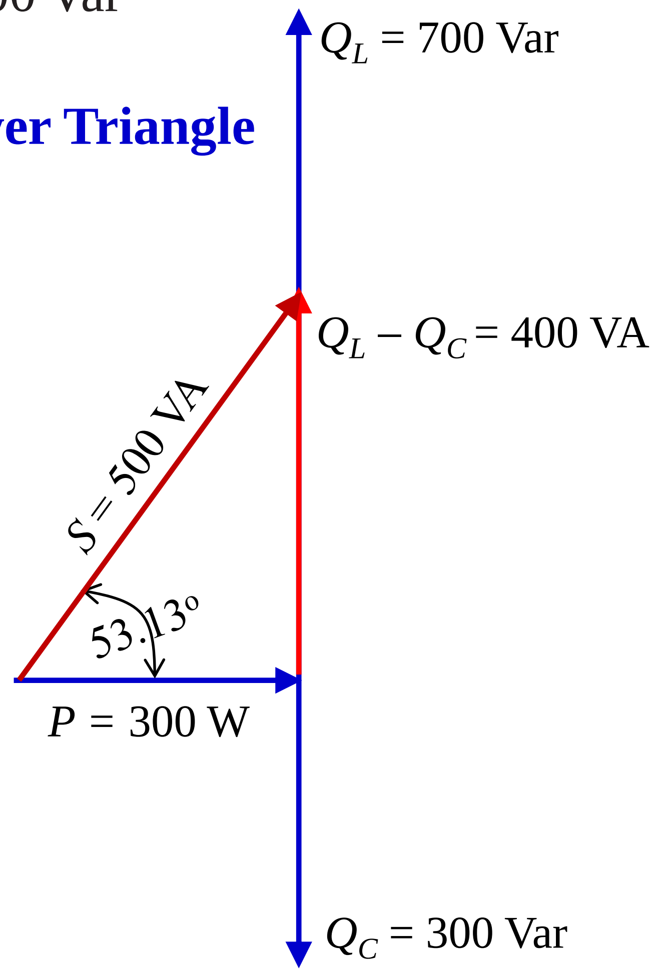
P = 300 W;

Q_L = 700 Var;

Q_C = – 300 Var;

Q = 400 Var

Power Triangle



Apparent Power [volt-ampere]

$S_E = EI = 50 \times 10 = \mathbf{500\ VA}$

$S_L = I^2 Z = (10\text{A})^2 \times 5\Omega = \mathbf{500\ VA}$

Energy dissipated by the resistor over one full cycle of the input voltage

$$W_R = \frac{V_R I_R}{f} = 2\pi \frac{V_R I_R}{\omega} = 2\pi \frac{30\text{V} \times 10\text{A}}{100\,\text{rad/s}} = 18.84\text{ J}$$

Energy stored in, or returned by, the inductor over one half-cycle of the power curve

$$W_L = \frac{V_L I_L}{\omega} = \frac{70\,\text{V} \times 10\text{A}}{100\,\text{rad/s}} = 7\ \text{J}$$

Energy stored in, or returned by, the capacitor over one half-cycle of the power curve

$$W_C = \frac{V_C I_C}{\omega} = \frac{30\,\text{V} \times 10\text{A}}{100\,\text{rad/s}} = 3\ \text{J}$$

Instantaneous Power Equation

$$p(t) = P(1 - \cos 2\omega t) + Q\sin 2\omega t \text{ W} = 300(1 - \cos 200t) + 400\sin 200t \text{ W}$$

Instantaneous or Time Domain Current and Voltages Equation

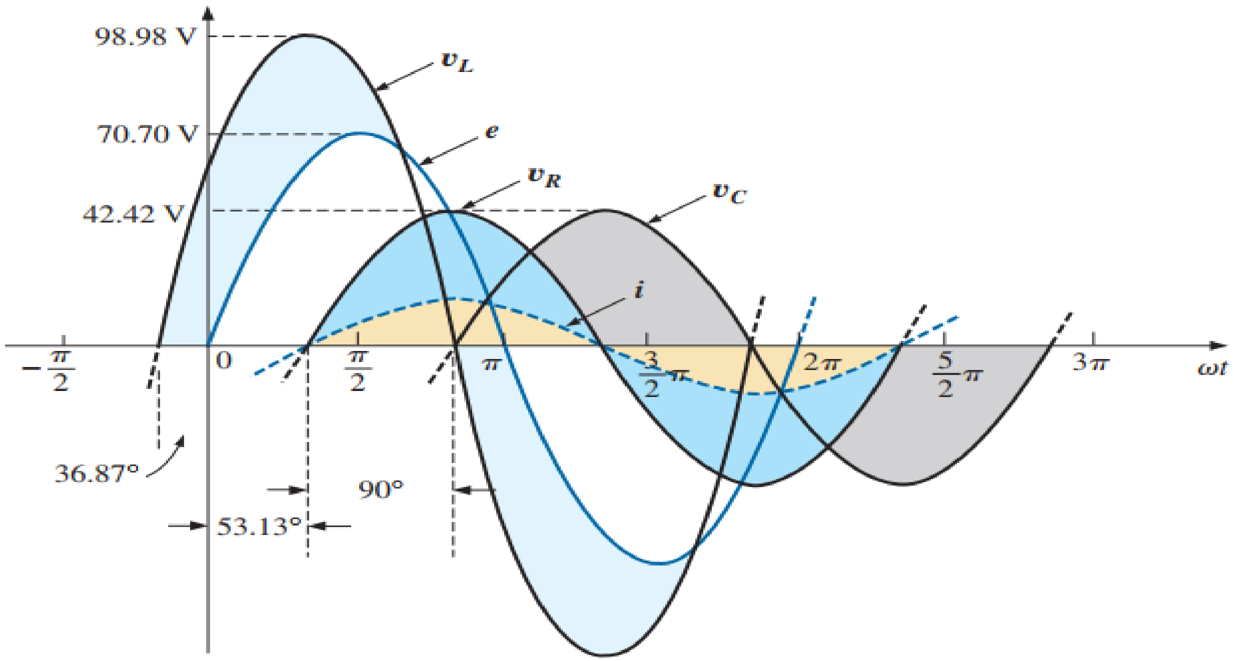
$$i = \sqrt{2}(10)\sin(\omega t - 53.13°) = \mathbf{14.14\sin(\omega t - 53.13°)}$$

$$v_R = \sqrt{2}(30)\sin(\omega t - 53.13°) = \mathbf{42.42\sin(\omega t - 53.13°)}$$

$$v_L = \sqrt{2}(70)\sin(\omega t + 36.87°) = \mathbf{98.98\sin(\omega t + 36.87°)}$$

$$v_C = \sqrt{2}(30)\sin(\omega t - 143.13°) = \mathbf{42.42\sin(\omega t - 143.13°)}$$

Practice Book Problems
[Ch. 15] 10 and 11



Voltage Divider Rule (VDR)

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit in Fig. 15.40.

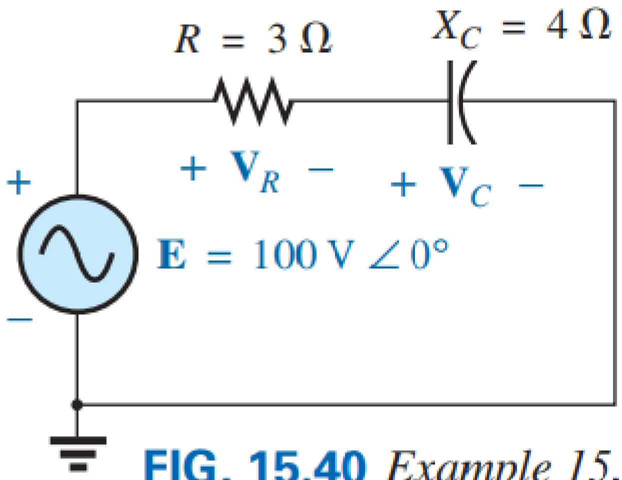


FIG. 15.40 *Example 15.9.*

Solution:

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega \angle -90°)(100\ \text{V} \angle 0°)}{4\ \Omega \angle -90° + 3\ \Omega \angle 0°} = \frac{400 \angle -90°}{3 - j4}$$

$$= \frac{400 \angle -90°}{5 \angle -53.13°} = \mathbf{80\ V \angle -36.87°}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega \angle 0°)(100\ \text{V} \angle 0°)}{5\ \Omega \angle -53.13°} = \frac{300 \angle 0°}{5 \angle -53.13°}$$

$$= \mathbf{60\ V \angle +53.13°}$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages $\mathbf{V}_R$, $\mathbf{V}_L$, $\mathbf{V}_C$, and $\mathbf{V}_1$ for the circuit in Fig. 15.41.

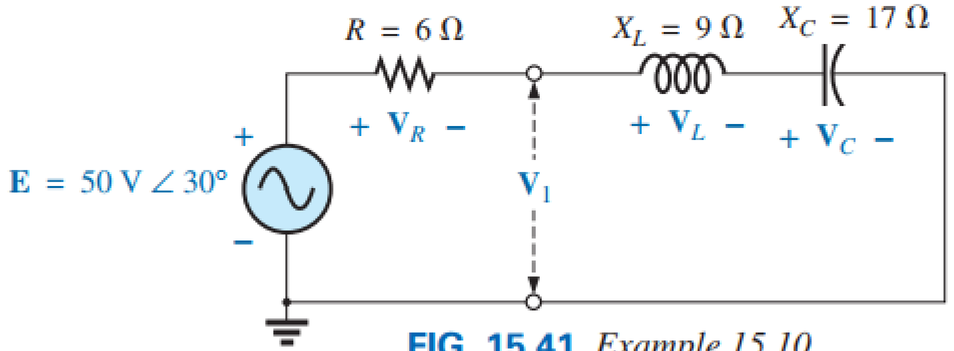


FIG. 15.41 *Example 15.10.*

Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega \angle 0^\circ)(50\ \text{V} \angle 30^\circ)}{6\ \Omega \angle 0^\circ + 9\ \Omega \angle 90^\circ + 17\ \Omega \angle -90^\circ}$$

$$= \frac{300 \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300 \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300 \angle 30^\circ}{10 \angle -53.13^\circ} = \mathbf{30\ V \angle 83.13^\circ}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90^\circ)(50\ \text{V} \angle 30^\circ)}{10\ \Omega \angle -53.13^\circ} = \frac{450\ \text{V} \angle 120^\circ}{10 \angle -53.13^\circ}$$

$$= \mathbf{45\ V \angle 173.13^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega \angle -90^\circ)(50\ \text{V} \angle 30^\circ)}{10\ \Omega \angle -53.13^\circ} = \frac{850\ \text{V} \angle -60^\circ}{10 \angle -53^\circ}$$

$$= \mathbf{85\ V \angle -6.87^\circ}$$

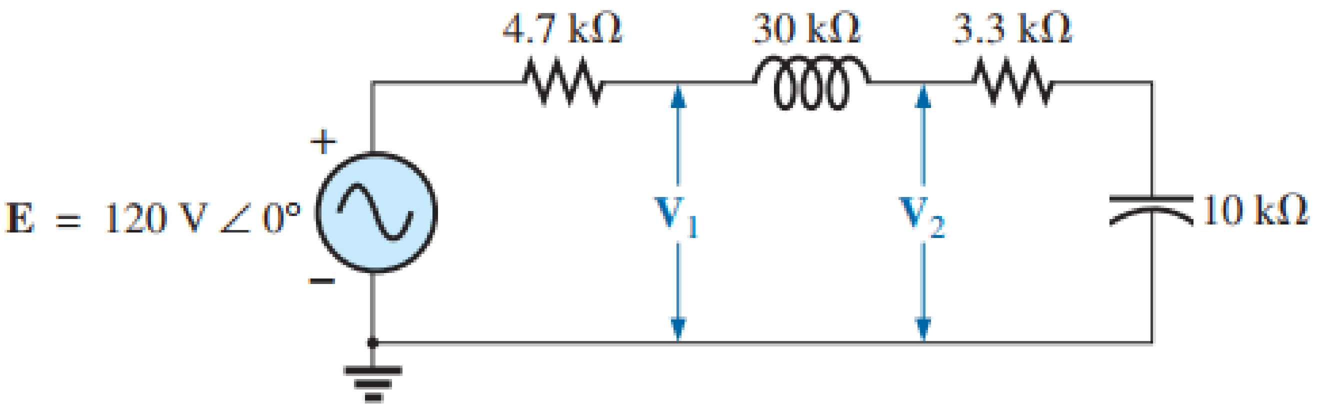
$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90^\circ + 17\ \Omega \angle -90^\circ)(50\ \text{V} \angle 30^\circ)}{10\ \Omega \angle -53.13^\circ}$$

$$= \frac{(8 \angle -90^\circ)(50 \angle 30^\circ)}{10 \angle -53.13^\circ}$$

$$= \frac{400 \angle -60^\circ}{10 \angle -53.13^\circ} = \mathbf{40\ V \angle -6.87^\circ}$$

Practice Book Problems
[Ch. 15] 15 and 16

Problem 16(b) Calculate the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ for the circuits in Fig. 15.135 in phasor form using the voltage divider rule.



$$\boldsymbol{E} = 120\text{V}\angle 0° = 120\,\text{V}$$

$$\boldsymbol{Z}_T = (4.7 + j30 + 3.3 - j10)\,\text{k}\Omega = (8 + j20)\,\text{k}\Omega$$

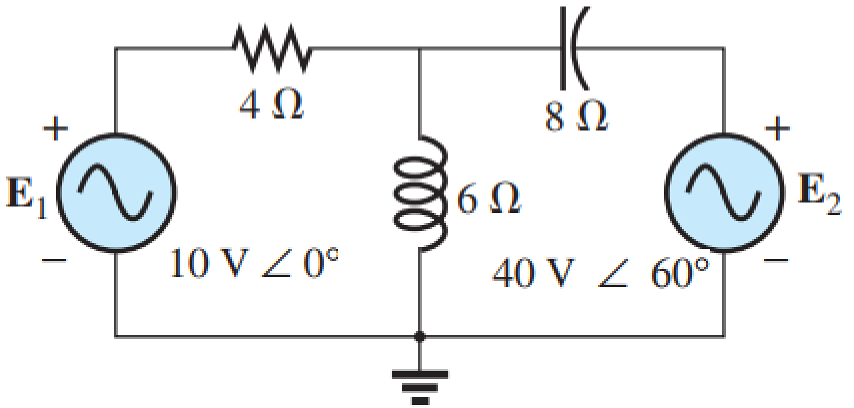
Let, $\boldsymbol{Z}_1 = (j30 + 3.3 - j10)\,\text{k}\Omega = (3.3 + j20)\,\text{k}\Omega$

$$\boldsymbol{Z}_2 = (3.3 - j10)\,\text{k}\Omega$$

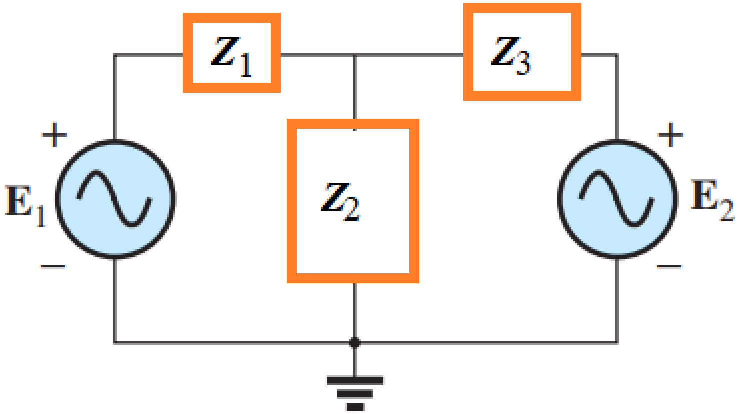
$$\boldsymbol{V}_1 = \frac{\boldsymbol{Z}_1}{\boldsymbol{Z}_T}\boldsymbol{E} = \frac{(3.3 + j20)\,\text{k}\Omega}{(8 + j20)\,\text{k}\Omega}(120\text{V})$$
$$= 110.28 + j24.31\ \text{V}$$
$$= 112.93\text{V}\angle 12.43°$$

$$\boldsymbol{V}_2 = \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_T}\boldsymbol{E} = \frac{(3.3 - j10)\,\text{k}\Omega}{(8 + j20)\,\text{k}\Omega}(120\text{V})$$
$$= -44.9 - j37.76\ \text{V}$$
$$= 58.67\text{V}\angle -139.937°$$

Write the mesh equations [**Application of KVL**].

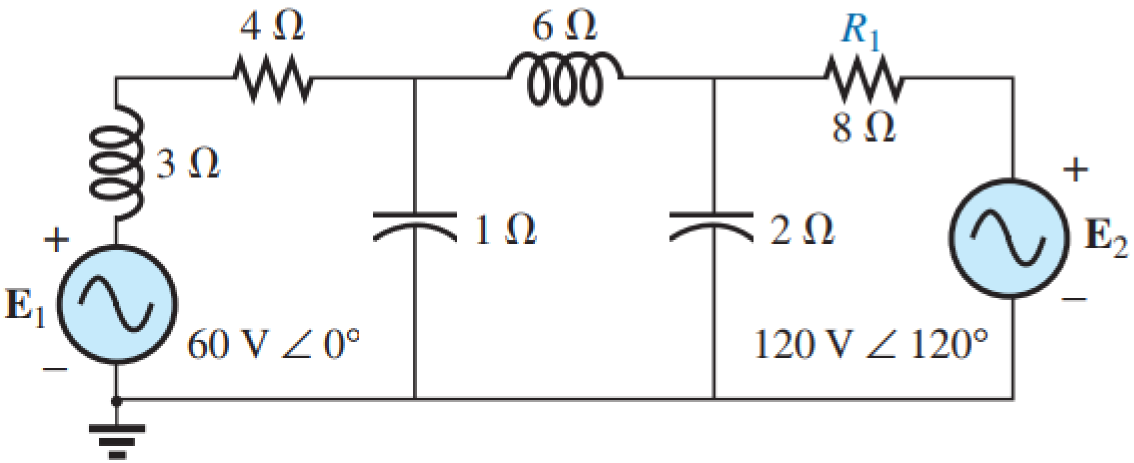


Let, $Z_1 = 4\ \Omega$, $Z_2 = j6\ \Omega$, $Z_3 = -j6\ \Omega$,



$$(Z_1 + Z_2)I_1 - Z_2I_2 = E_1$$

$$-Z_2I_1 + (Z_2 + Z_3)I_2 = -E_2$$



$$(j3 + 4 - j1)I_1 - (-\ j1)\ I_2 = E_1$$

$$-(-\ j1)\ I_1 + (-j1 + j6 - j2)I_2 - (-j2)\ I_3 = 0$$

$$-(-j2)I_2 + (-j2 + 8)I_3 = -E_2$$