MID-POINT LINE PLOTTING ALGORITHM

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INTRODUCTION

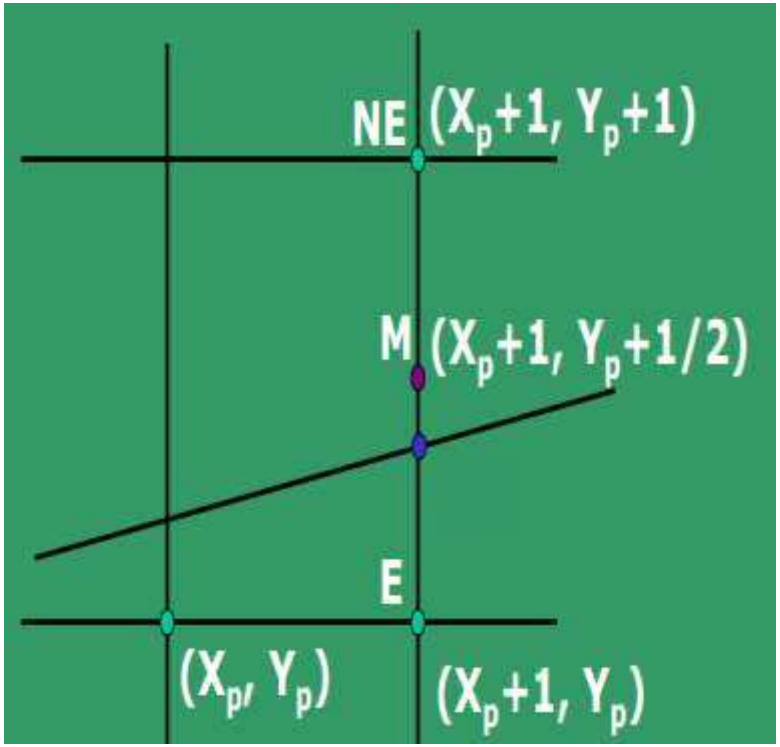
The Midpoint line algorithm is an incremental line plotting algorithm i.e. at each step we make incremental calculations based on preceding step to find next y value, in order to form a close approximation to a straight line between two points.

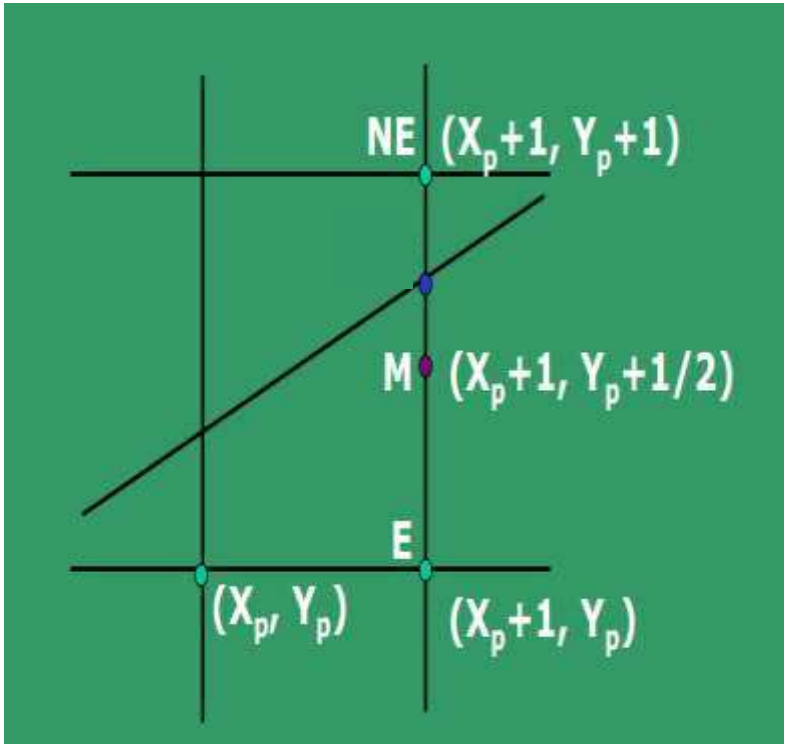
ADVANTAGES

- It chooses the pixels closest to the line with accuracy, consistency and straightness.
- It is very simple and requires only integer data and simple arithmetic.
- It avoids division and multiplication and thus avoid truncate errors.

BASIS OF ALGORITHM

- Given the previous pixel P, there are two candidates for the next pixel closest to the line, E and NE .
- If the M is above the line, choose E. If M is below the line, choose NE.





Previous pixel **Choices for current pixel**

Previous pixel **Choices for current pixel**

DERIVATION

Assumptions:

Two end points of a line : (x_0, y_0) and (x_1, y_1)

Also, $x_0 < x_1$

Since, we are sampling in x-direction, so, for the next pixel , x-coordinate will be x_{p+1} i.e. $x_p + 1$ and correspondingly we will calculate the value of y-coordinate .

The implicit equation of a line is:

$F(x, y) = a\,x + b\,y + c$ …….(1)

$dx = x_1 - x_0$

$dy = y_1 - y_0$

Slope intercept form a line is : **$y = m\,x + B$**

$$y = (dy/dx)\,x + B$$

$$F(x,y) = (x)dy - (y)dx + Bdx \qquad \textbf{……(2)}$$

Comparing (1) and(2) we get,

$a = dy$, $b = -dx$ and $c = Bdx$

- For all points on the line, the solution to **F(x, y)** is 0.
- For all points above the line **F(x, y)** result in a negative number
- For all points below **F(x, y)** result in a positive number.

This relationship is used to determine the relative position of M.

$$M=(x_{p+1}, y_{p+1/2})$$

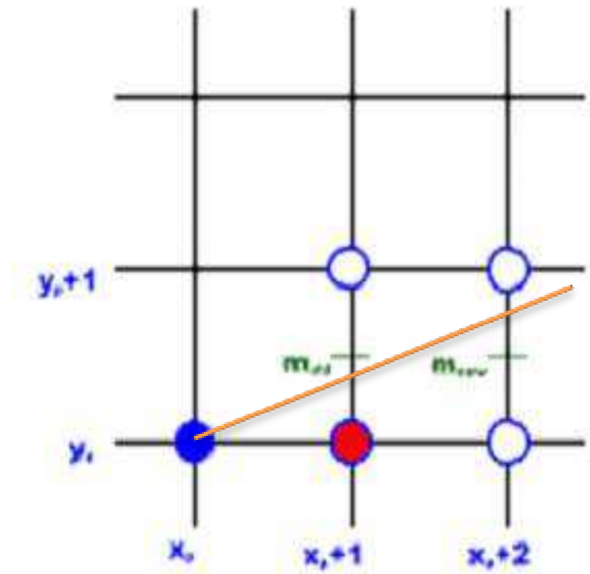
$$d= F(M)= F(x_{p+1}, y_{p+1/2})$$

- The sign of the decision variable '**d**' is used to make the midpoint determination for all remaining pixels.

  - If **d** is negative, the midpoint is above the line and E is chosen i.e. (x_{p+1}, y_p) will be plotted.
  - If **d** is positive, the midpoint is below the line and NE is chosen, i.e. we will plot (x_{p+1}, y_{p+1}).

As the algorithm progresses from pixel to pixel, **d** is calculated with one of two pre-calculated values based on the E/NE decision.

Case 1: If E is chosen (d<0)



dnew= $F(x_p +2, y_p+1/2)$

$= a(x_p +2) + b(y_p+1/2) +c$

dold $= a(x_p+1) + b(y_p+1/2)+c$

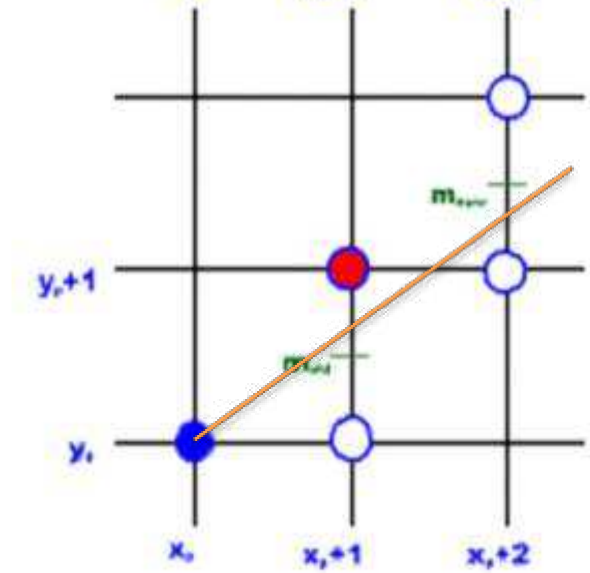
Δd = dnew -dold

$= a(x_p +2)- a(x_p+1)+ b(y_p+1/2)- b(y_p+1/2)+c-c$

$= a(x_p) +2a -a(x_p) -a = a.$

Therefore, dnew = dold + dy

Case 2: If NE is chosen (d>0)



dnew= $F(x_p +2, y_p+3/2)$

$= a(x_p +2) + b(y_p+3/2) +c$

dold $= a(x_p+1) + b(y_p+1/2)+c$

Δd = dnew -dold

$= a(x_p +2)- a(x_p+1)+ b(y_p+3/2)- b(y_p+1/2)+c-c$

$= a(x_p) +2a -a(x_p) -a+ b(y_p) +3/2b - b(y_p) -1/2b$

$= a+b$

Therefore, dnew = dold + dy-dx

Derivation for calculating the initial value for do

$d0 = F(x_0+1\ ,\ y_0+1/2)$

$= a(x_0+1) + b(y_0+1/2) + c$

$= ax_0 + by_0 + c + a + b/2$

$= F(x_0, y_0) + a + b/2$

$= 0 + a + b/2 \qquad (\ a = dy,\ b = -dx)$

Therefore, $d0 = dy - dx/2$

ALGORITHM (|M|<1)

1) Input (x0,y0) and (x1,y1)
2) Calculate dy and dx
3) d= dy-(dx/2)
4) x= x0 and y=y0
5) Plot(x , y)
6) While(x<x1)
7) x=x+1
8) If(d<0)
9) d=d+dy
10) else
11) d=d+dy-dx
12) y=y+1
13) Plot(x,y)

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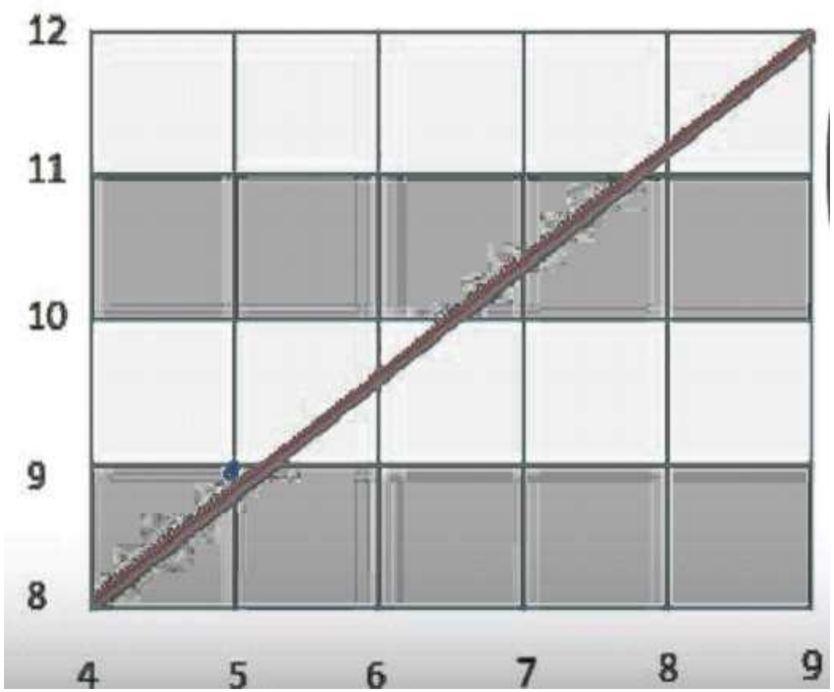
EXAMPLE



Draw a line from (4,8) to (9,12) and plot the points accordingly.

Initially:
(x,y)=(4,8)
(x1,y1)=(9,12)
dy=(y1-y0)= (12-8)= 4
dx= (x1-x0)=(9-4)= 5
Now, the first decision variable
(d0)= dy- dx/2
= 4-5/2
= 1.5

As d0 >0 , NE is chosen and the next pixel to be plotted will be
(x+1,y+1) i.e. **(5,9)**
-> d1= d0+ (dy-dx)
= 1.5+ 4-5 = 0.5
As d1 >0 , again NE is chosen and the next pixel to be plotted will be
(x+1,y+1) i.e.**(6,10)**

-> d2=d1+ dy-dx

= 0.5+4-5 = -0.5

As d2 <0 , E is chosen and the next pixel to be plotted will be

(x+1,y) i.e. **(7,10)**

-> d3= d2+dy

= -0.5 + 4 = 3.5

As d3 >0 , NE is chosen and the next pixel to be plotted will be (x+1,y+1) i.e. **(8,11)**

-> d4= d3+dy-dx

= 3.5+ 4-5 = 2.5

As d4 >0 , NE is chosen and the next pixel to be plotted will be (x+1,y+1) i.e. **(9,12)**

Now as we have reached our second end point i.e.

(x1,y1)= (9,12) ,we will stop the procedure.

Therefore, the plotted points on the grids will be **(5,9) ,(6,10), (7,10) ,(8,11) and (9,12).**

THANK YOU!