Review Last Class

Solution of Series-Parallel Circuit

Power Factor Correction (PFC) Or Power Factor Improvement (PFI)

Source Transformation Theory

Y (T) to Δ (Π) Conversion and Δ (Π) to Y (T) Conversion

Superposition Theorem

Chapter 18

Networks Theorem (AC)

EXAMPLE 18.4 For the network in Fig. 18.12, determine the sinusoidal expression for the voltage υ_3 using superposition.

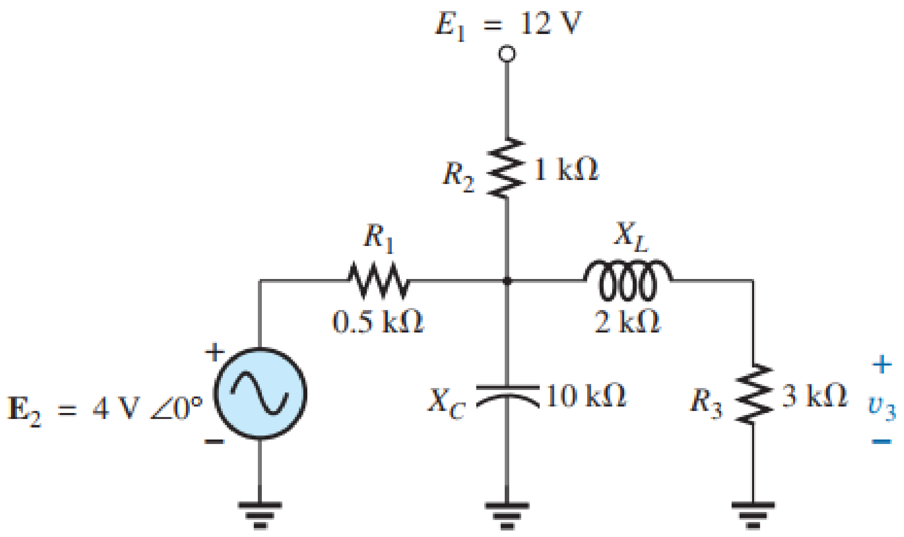


FIG. 18.12 *Example 18.4.*

Solution: For the dc analysis, the capacitor can be replaced by an open-circuit equivalent, and the inductor by a short-circuit equivalent. The result is the network in Fig. 18.13.

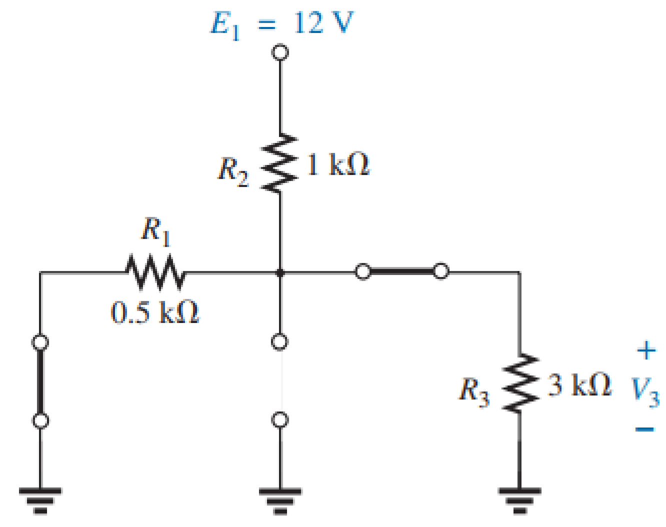


FIG. 18.13
Determining the effect of the dc voltage source E_1 on the voltage υ_3 of the network in Fig. 18.12.

The resistors R_1 and R_3 are then in parallel, and the voltage V_3 can be determined using the voltage divider rule:

$$R' = R_1 \parallel R_3 = 0.5\text{ k}\Omega \parallel 3\text{ k}\Omega = 0.429\text{ k}\Omega$$

and
$$V_3 = \frac{R'E_1}{R' + R_2} = \frac{(0.429\text{ k}\Omega)(12\text{ V})}{0.429\text{ k}\Omega + 1\text{ k}\Omega} = \frac{5.148\text{ V}}{1.429}$$

$$V_3 \cong \mathbf{3.6\ V}$$

For the ac analysis, the dc source is set to zero and the network is redrawn, as shown in Fig. 18.14.

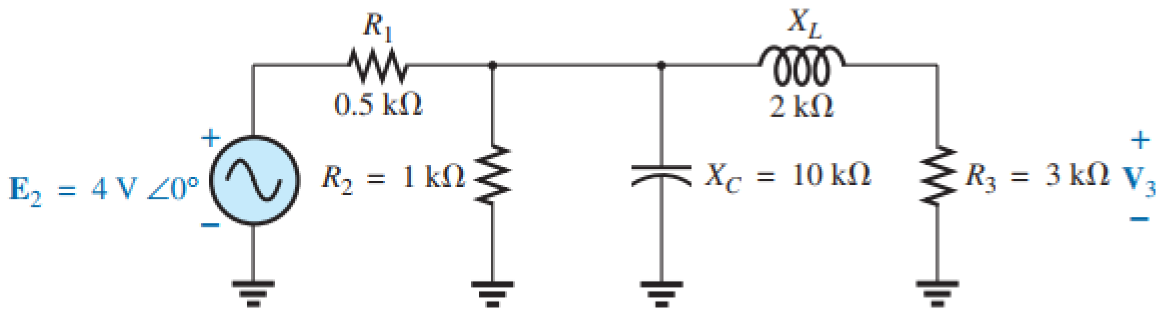


FIG. 18.14

Redrawing the network in Fig. 18.12 to determine the effect of the ac voltage source $\mathbf{E}_2$.

The block impedances are then defined as in Fig. 18.15, and series-parallel techniques are applied as follows:

$$\mathbf{Z}_1 = 0.5\text{ k}\Omega \angle 0°$$

$$\mathbf{Z}_2 = (R_2 \angle 0° \parallel (X_C \angle -90°)$$

$$= \frac{(1\text{ k}\Omega \angle 0°)(10\text{ k}\Omega \angle -90°)}{1\text{ k}\Omega - j\,10\text{ k}\Omega} = \frac{10\text{ k}\Omega \angle -90°}{10.05 \angle -84.29°}$$

$$= 0.995\text{ k}\Omega \angle -5.71°$$

$$\mathbf{Z}_3 = R_3 + j\,X_L = 3\text{ k}\Omega + j\,2\text{ k}\Omega = 3.61\text{ k}\Omega \angle 33.69°$$

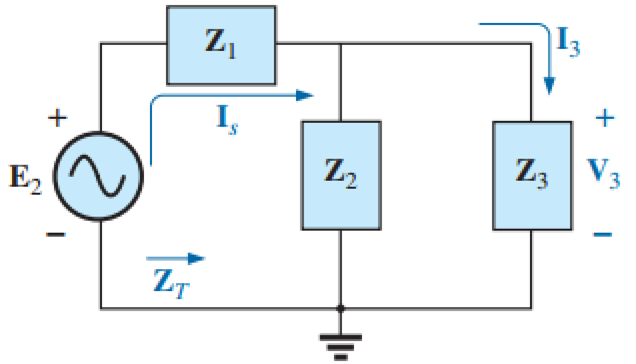


FIG. 18.15

Assigning the subscripted impedances to the network in Fig. 18.14.

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3$$

$$= 0.5\text{ k}\Omega + (0.995\text{ k}\Omega \angle -5.71°) \parallel (3.61\text{ k}\Omega \angle 33.69°)$$

$$= 1.312\text{ k}\Omega \angle 1.57°$$

$$\mathbf{I}_s = \frac{\mathbf{E}_2}{\mathbf{Z}_T} = \frac{4\text{ V} \angle 0°}{1.312\text{ k}\Omega \angle 1.57°} = 3.05\text{ mA} \angle -1.57°$$

Current divider rule:

$$\mathbf{I}_3 = \frac{\mathbf{Z}_2\mathbf{I}_s}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(0.995\text{ k}\Omega \angle -5.71°)(3.05\text{ mA} \angle -1.57°)}{0.995\text{ k}\Omega \angle -5.71° + 3.61\text{ k}\Omega \angle 33.69°}$$

$$= 0.686\text{ mA} \angle -32.74°$$

$$\begin{aligned}\mathbf{V}_3 &= (I_3 \angle\theta)(R_3 \angle 0^\circ)\\ &= (0.686\text{ mA} \angle -32.74^\circ)(3\text{ k}\Omega \angle 0^\circ)\\ &= \mathbf{2.06\text{ V} \angle -32.74^\circ}\end{aligned}$$

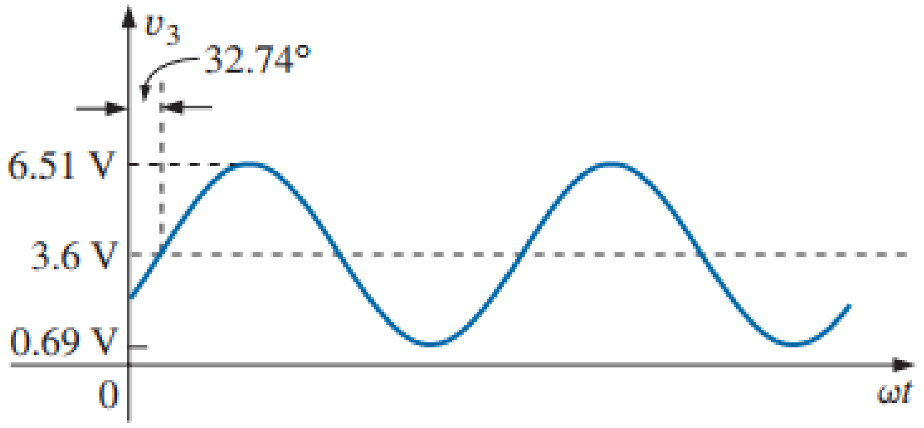


FIG. 18.17

The resultant voltage v_3 for the network in Fig. 18.12.

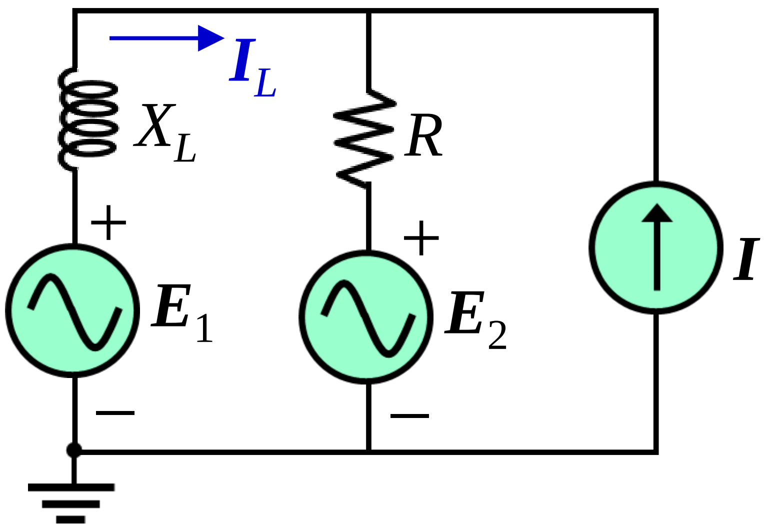
The total solution:

$$\begin{aligned}v_3 &= v_3\text{ (dc)} + v_3\text{ (ac)}\\ &= 3.6\text{ V} + 2.06\text{ V} \angle -32.74^\circ\\ v_3 &= \mathbf{3.6 + 2.91\sin(\omega t - 32.74^\circ)}\end{aligned}$$

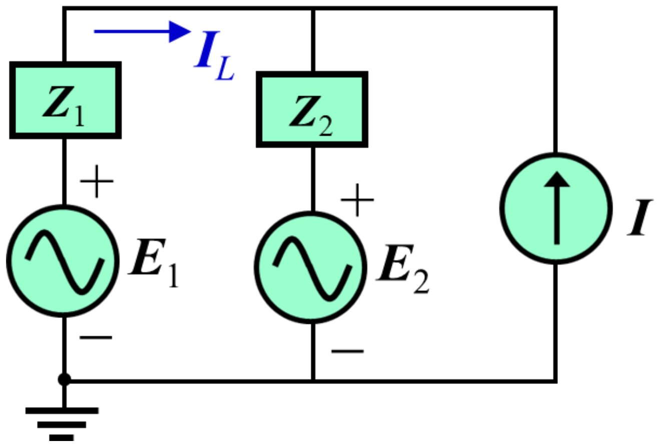
The result is a sinusoidal voltage having a peak value of 2.91 V riding on an average value of 3.6 V, as shown in Fig. 18.17.

Practice Book Remaining Examples And Problem 1- 5 [Ch. 18]

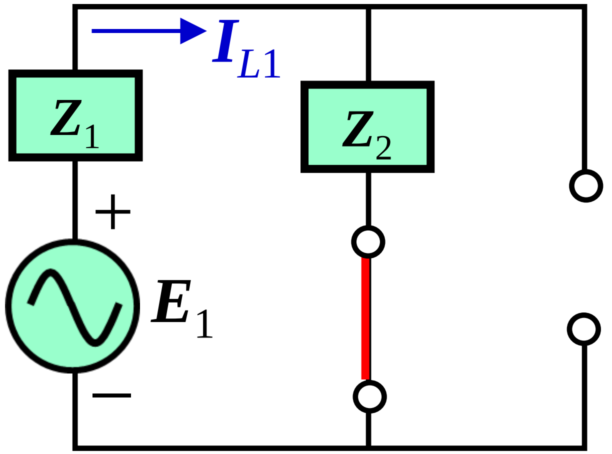
PROBLEM 2(a): Using superposition, find the current $\boldsymbol{I}_L$ in Fig. 18.6, where, R = 3 Ω; X_L = 4 Ω; $\boldsymbol{E}_1 = 20\text{V}\angle 0^\circ$; $\boldsymbol{E}_2 = 120\text{V}\angle 90^\circ$; $\boldsymbol{I} = 0.5\text{A}\angle 60^\circ$.



Let, $\boldsymbol{Z}_1 = jX_L = j4\ \Omega$; $\boldsymbol{Z}_2 = R = 3\ \Omega$;

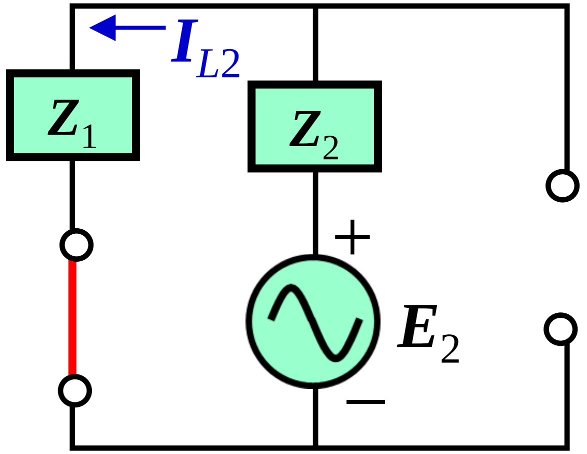


Consider $\boldsymbol{E}_1 = 20\text{V}\angle 0^\circ$:



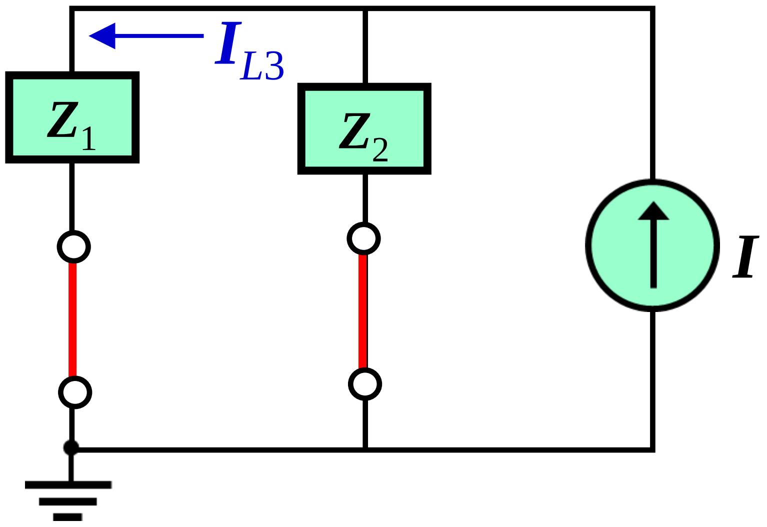
$$\boldsymbol{I}_{L1} = \frac{\boldsymbol{E}_1}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = \frac{20\text{V}\angle 0^\circ}{3 + j4\Omega}$$
$$= 2.4 - j3.2\ \text{A}$$
$$= 4\text{A}\angle -53.13^\circ$$

Consider $\boldsymbol{E}_2 = 120\text{V}\angle 90^\circ$:



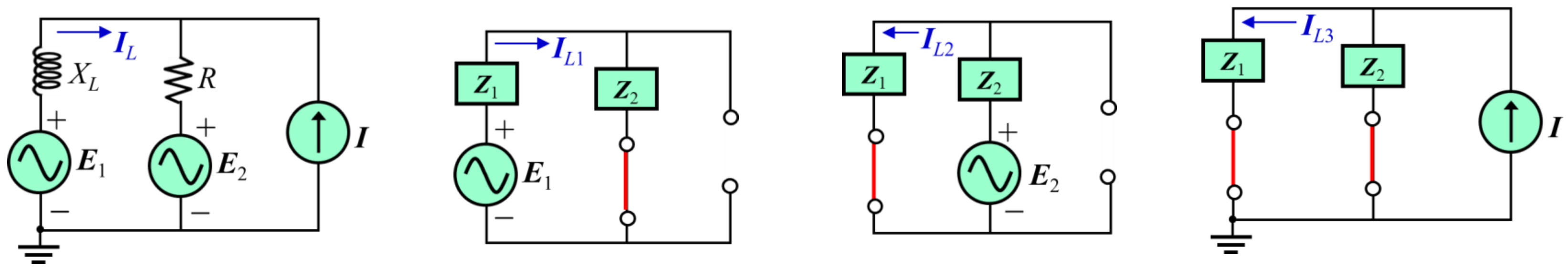
$$\boldsymbol{I}_{L2} = \frac{\boldsymbol{E}_2}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = \frac{120\text{V}\angle 90^\circ}{3 + j4\Omega}$$
$$= -24 + j18\ \text{A}$$
$$= 30\text{A}\angle 143.13^\circ$$

Consider $\boldsymbol{I} = 0.5\text{A}\angle 60^{\circ}$:



$$\boldsymbol{I}_{L3} = \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2}\boldsymbol{I} = \frac{(0.5\text{A}\angle 60^{\circ})(3\ \Omega)}{3 + j4\Omega}$$
$$= 0.3 + j0.03\ \text{A}$$
$$= 0.3\text{A}\angle 5.71^{\circ}$$

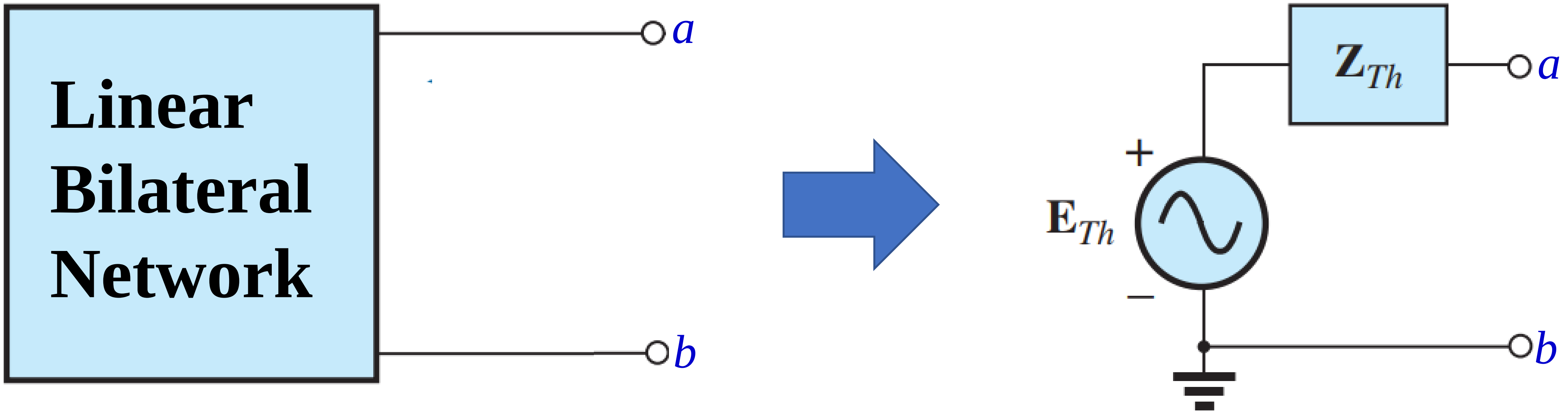
According to Superposition Theorem:



$$\boldsymbol{I}_L = \boldsymbol{I}_{L1} - \boldsymbol{I}_{L2} - \boldsymbol{I}_{L3} = 26.1 - j2.23\ \text{A} = 33.64\text{A}\angle -39.13^{\circ}$$

Thevenin's Theorem

Statement: **Any two-terminal, linear bilateral network can be replaced by an equivalent circuit consisting of a voltage source and a series impedance, as shown in the following figure.**



Z_{Th}: Thevenin's equivalent impedance
E_{Th}: Thevenin's equivalent voltage

Steps to Apply Thevenin's Theorem

Step 1: Remove that portion of the network where the Thévenin equivalent circuit is found.

Step 2: Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

Step 3: Calculate Z_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

Step 4: Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open circuit potential between the two terminals marked in step 2.)

Step 5: Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Step 6: Do the remaining required calculation

EXAMPLE 18.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 18.24.

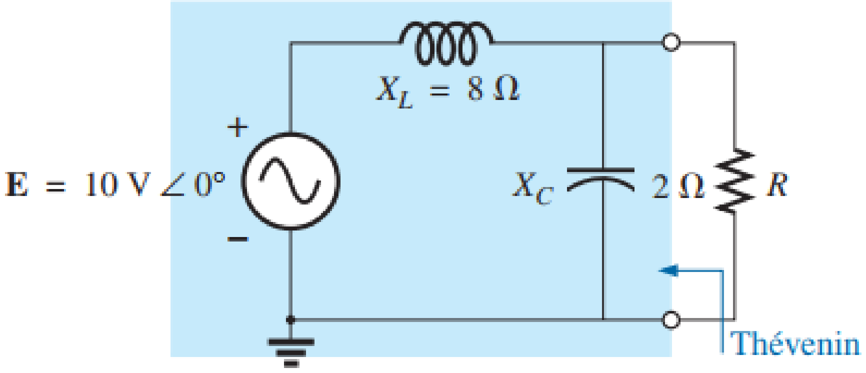


FIG. 18.24 *Example 18.7.*

Solution: $\mathbf{Z}_1 = j X_L = j\,8\ \Omega \quad \mathbf{Z}_2 = -j X_C = -j\,2\ \Omega$

Steps 1 and 2 (Fig. 18.25):

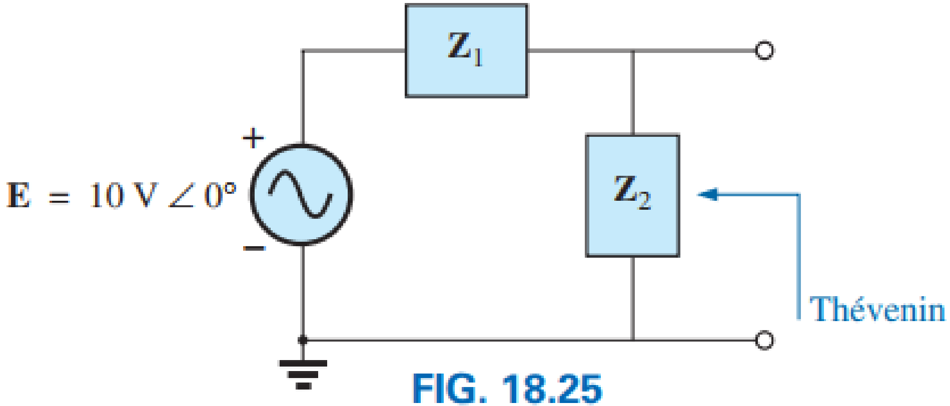


FIG. 18.25

Step 3 (Fig. 18.26):

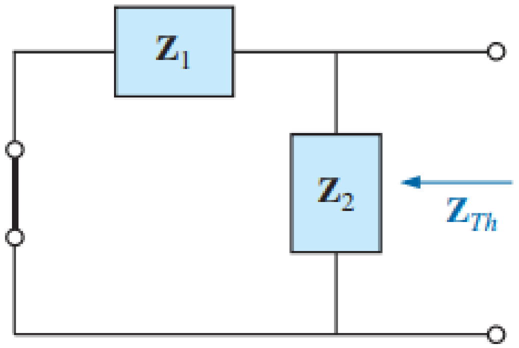


FIG. 18.26

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\ \Omega)(-j\,2\ \Omega)}{j\,8\ \Omega - j\,2\ \Omega} = \mathbf{2.67\ \Omega \angle -90°}$$

Step 4 (Fig. 18.27):

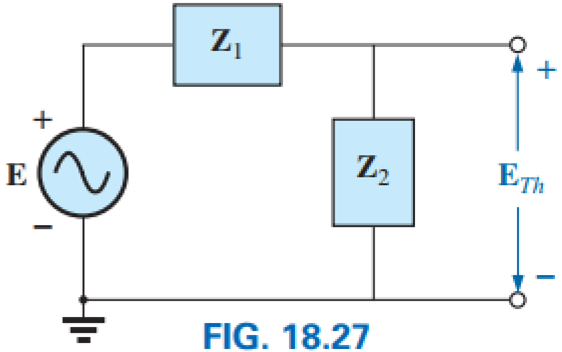


FIG. 18.27

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(-j\,2\ \Omega)(10\ \text{V})}{j\,8\ \Omega - j\,2\ \Omega} = \frac{-j\,20\ \text{V}}{j\,6} = \mathbf{3.33\ V \angle -180°}$$

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.28.

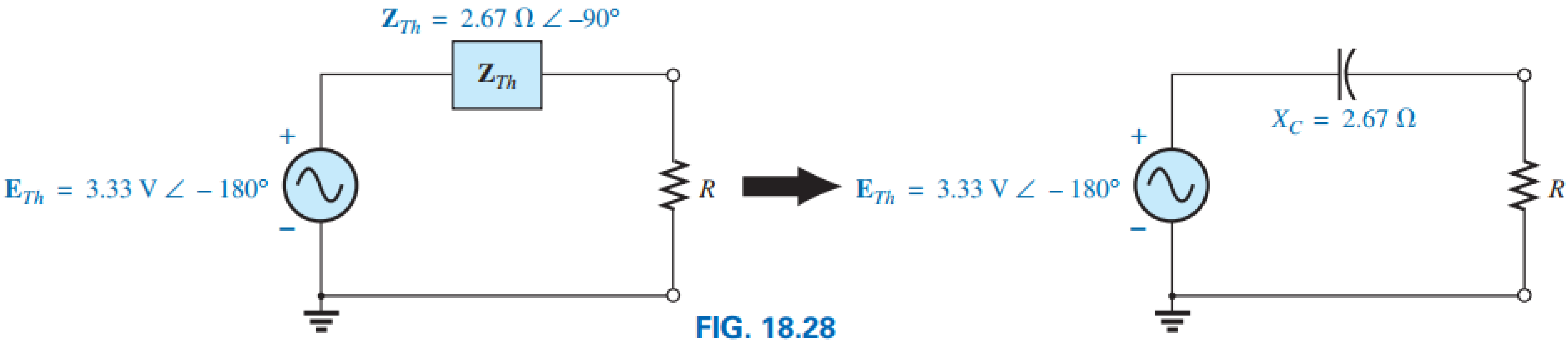


FIG. 18.28
The Thévenin equivalent circuit for the network in Fig. 18.24.

EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch *a-a′* in Fig. 18.29.

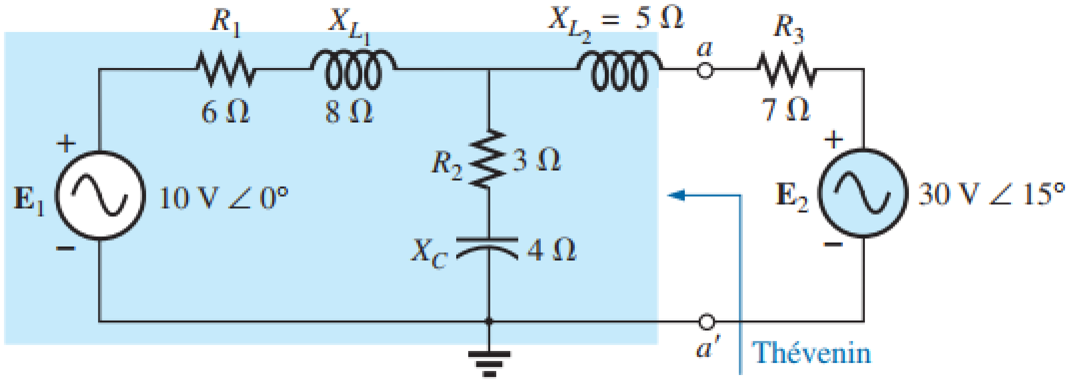


FIG. 18.29 *Example 18.8.*

Solution:

Steps 1 and 2 (Fig. 18.30): Note the reduced complexity with subscripted impedances:

$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 6\ \Omega + j\,8\ \Omega \qquad \mathbf{Z}_2 = R_2 - jX_C = 3\ \Omega - j\,4\ \Omega$$

$$\mathbf{Z}_3 = +jX_{L_2} = j\,5\ \Omega$$

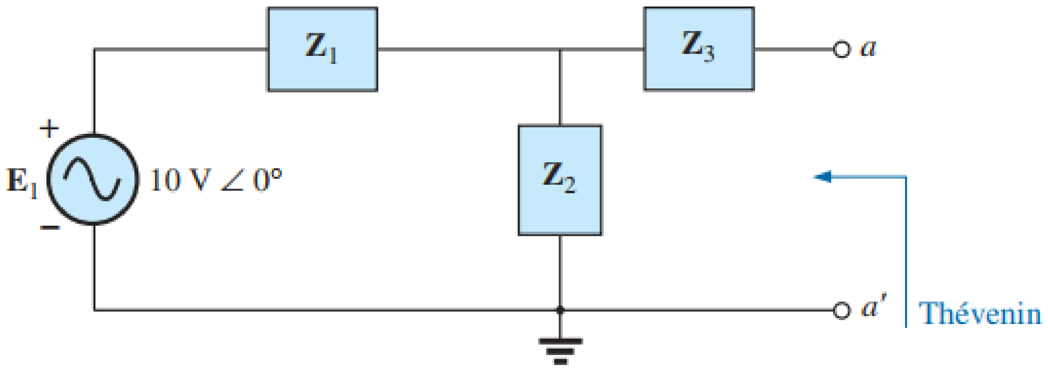


FIG. 18.30

Assigning the subscripted impedances for the network in Fig. 18.29.

Step 3 (Fig. 18.31):

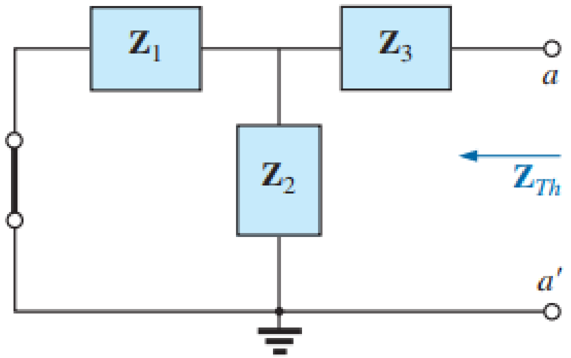


FIG. 18.31

$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = j\,5\ \Omega + \frac{(10\ \Omega \angle 53.13^\circ)(5\ \Omega \angle -53.13^\circ)}{(6\ \Omega + j\,8\ \Omega) + (3\ \Omega - j\,4\ \Omega)}$$

$$\mathbf{Z}_{Th} = \mathbf{4.64\ \Omega + j\,2.94\ \Omega = 5.49\ \Omega \angle 32.36^\circ}$$

Step 4 (Fig. 18.32): Since a-a' is an open circuit, $\mathbf{I}_{Z_3} = 0$. Then $\mathbf{E}_{Th}$ is the voltage drop across $\mathbf{Z}_2$:

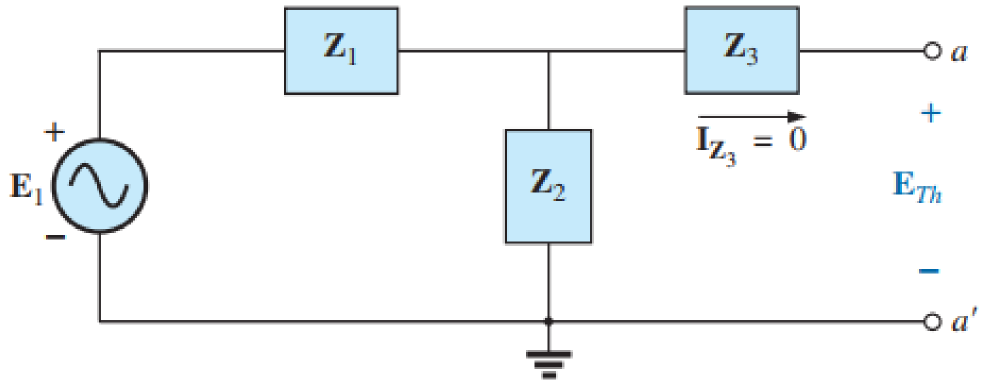


FIG. 18.32

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \quad \text{(voltage divider rule)}$$

$$= \frac{(5\ \Omega \angle -53.13^\circ)(10\ \text{V} \angle 0^\circ)}{9.85\ \Omega \angle 23.96^\circ}$$

$$\mathbf{E}_{Th} = \frac{50\ \text{V} \angle -53.13^\circ}{9.85 \angle 23.96^\circ} = \mathbf{5.08\ V \angle -77.09^\circ}$$

Practice Book Remaining Examples
And
Problem 12- 16 [Ch. 18]

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.33.

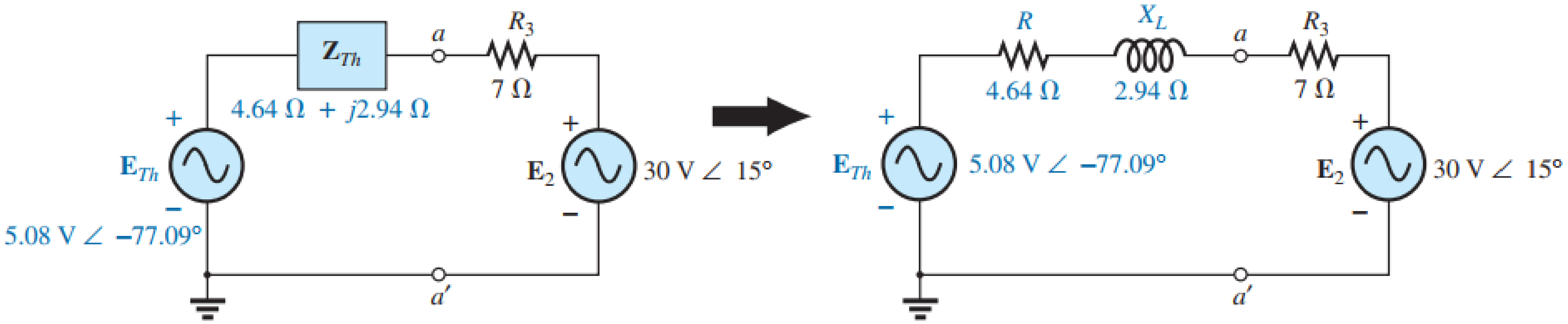
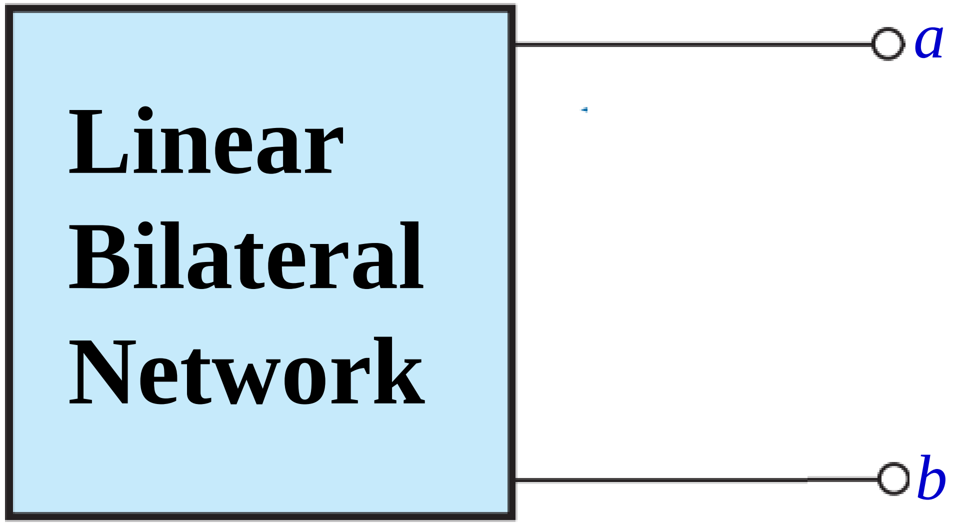
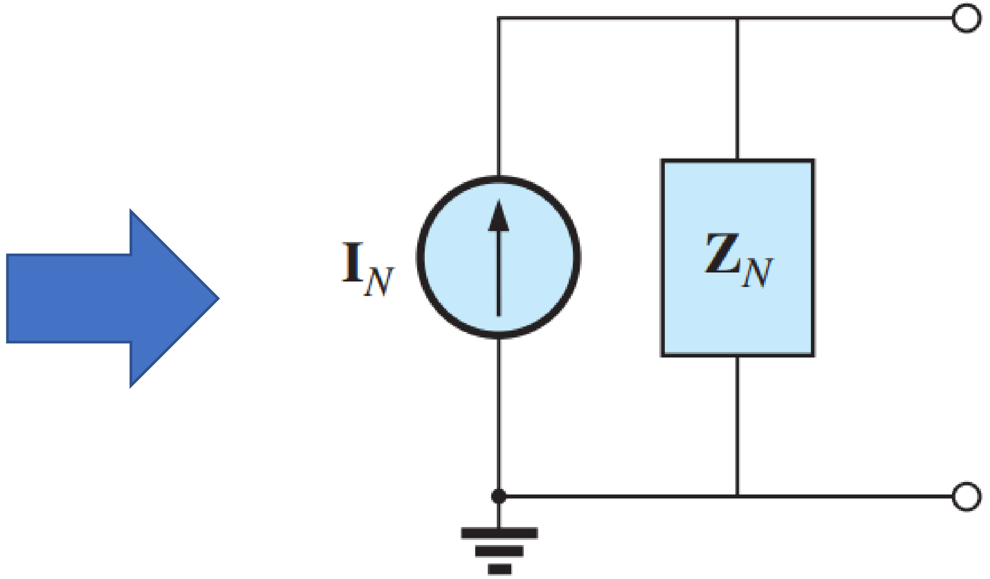


FIG. 18.33 *The Thévenin equivalent circuit for the network in Fig. 18.29.*

Norton's Theorem

Statement: Any two-terminal, linear bilateral network can be replaced by an equivalent circuit consisting of a **current source** and a **parallel impedance**, as shown in the following figure.





Z_N: Norton's equivalent impedance

I_N: Norton's equivalent current

Steps to Apply Norton's Theorem

Step 1: Remove that portion of the network where the Thévenin equivalent circuit is found.

Step 2: Mark the terminals of the remaining two-terminal network.

Step 3: Calculate Z_N by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

Step 4: Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

Step 5: Draw the Norton's equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Step 6: Do the remaining required calculation

EXAMPLE 18.14 Determine the Norton equivalent circuit for the network external to the 6 Ω resistor in Fig. 18.62.

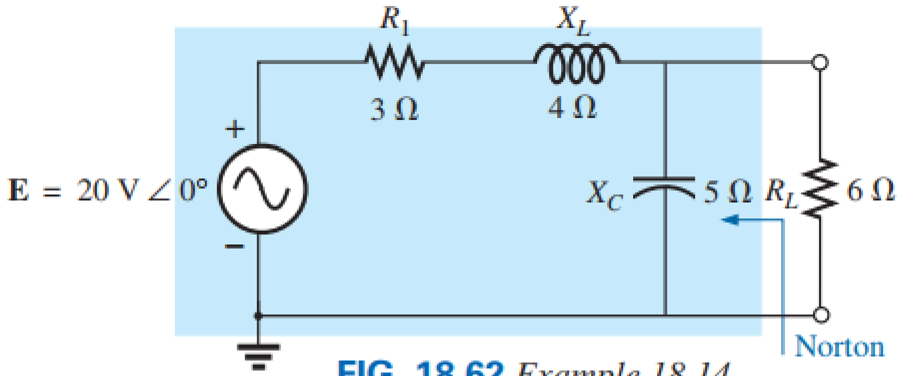


FIG. 18.62 *Example 18.14.*

Solution: $\mathbf{Z}_1 = R_1 + j\,X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega\ \angle 53.13^\circ$

$\mathbf{Z}_2 = -j\,X_C = -j\,5\ \Omega$

Steps 1 and 2 (Fig. 18.63):

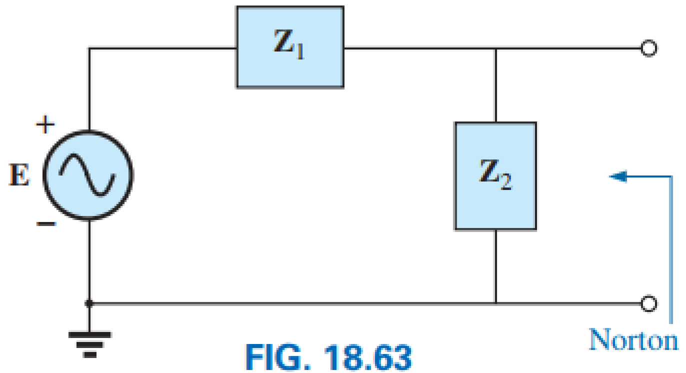


FIG. 18.63

Step 3 (Fig. 18.64):

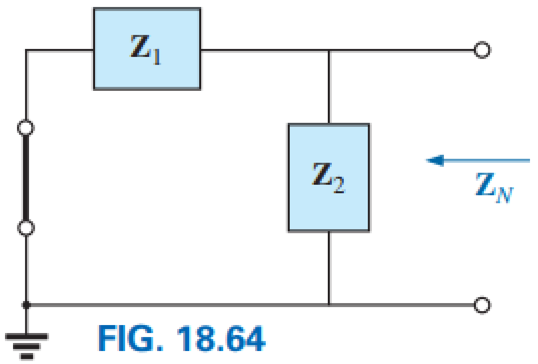


FIG. 18.64

$$\mathbf{Z}_N = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$$= \frac{(5\ \Omega\ \angle 53.13^\circ)(5\ \Omega\ \angle -90^\circ)}{3\ \Omega + j\,4\ \Omega - j\,5\ \Omega}$$

$$= 7.91\ \Omega\ \angle -18.44^\circ$$

$$= \mathbf{7.50\ \Omega - \mathit{j}\,2.50\ \Omega}$$

Step 4 (Fig. 18.65):

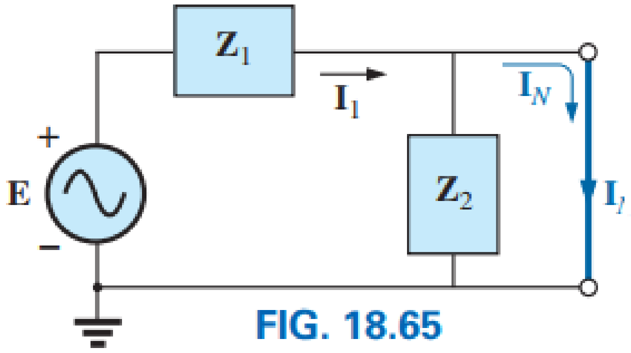


FIG. 18.65

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1}$$

$$= \frac{20\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ}$$

$$= \mathbf{4\ A\ \angle -53.13^\circ}$$

Step 5: The Norton equivalent circuit is shown in Fig. 18.66.

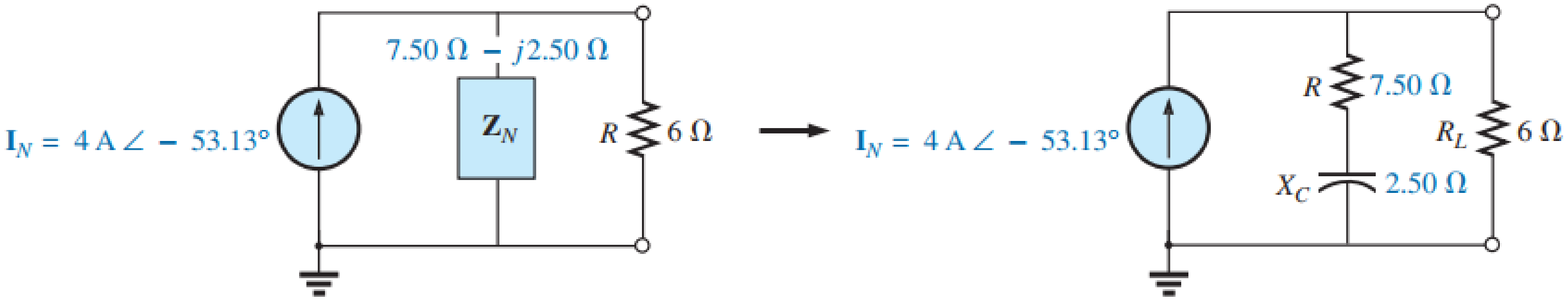


FIG. 18.66

The Norton equivalent circuit for the network in Fig. 18.62.

EXAMPLE 18.15 Find the Norton equivalent circuit for the network external to the 7 Ω capacitive reactance in Fig. 18.67.

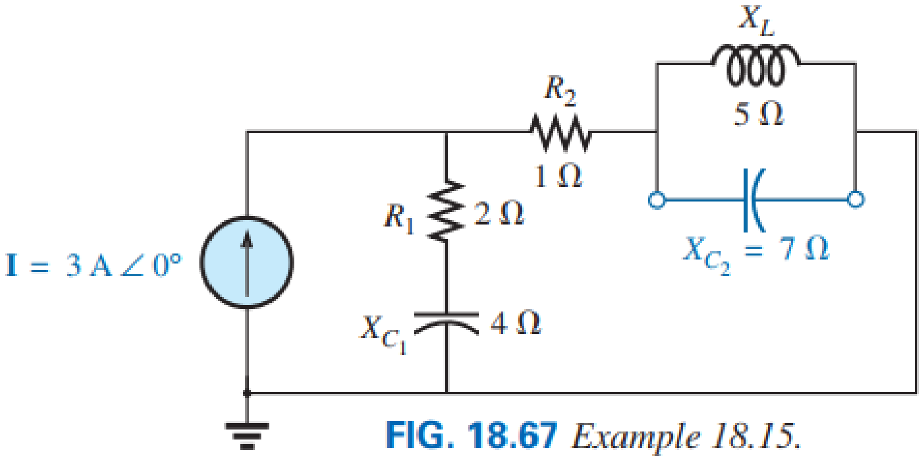


FIG. 18.67 *Example 18.15.*

Solution:

$$\mathbf{Z}_1 = R_1 - j\,X_{C_1} = 2\ \Omega - j\,4\ \Omega$$
$$\mathbf{Z}_2 = R_2 = 1\ \Omega$$
$$\mathbf{Z}_3 = +j\,X_L = j\,5\ \Omega$$

Steps 1 and 2 (Fig. 18.68):

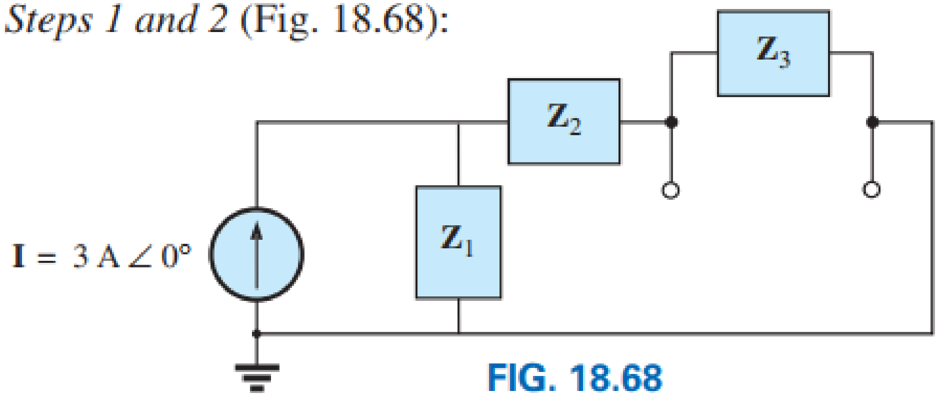


FIG. 18.68

Step 3 (Fig. 18.69):

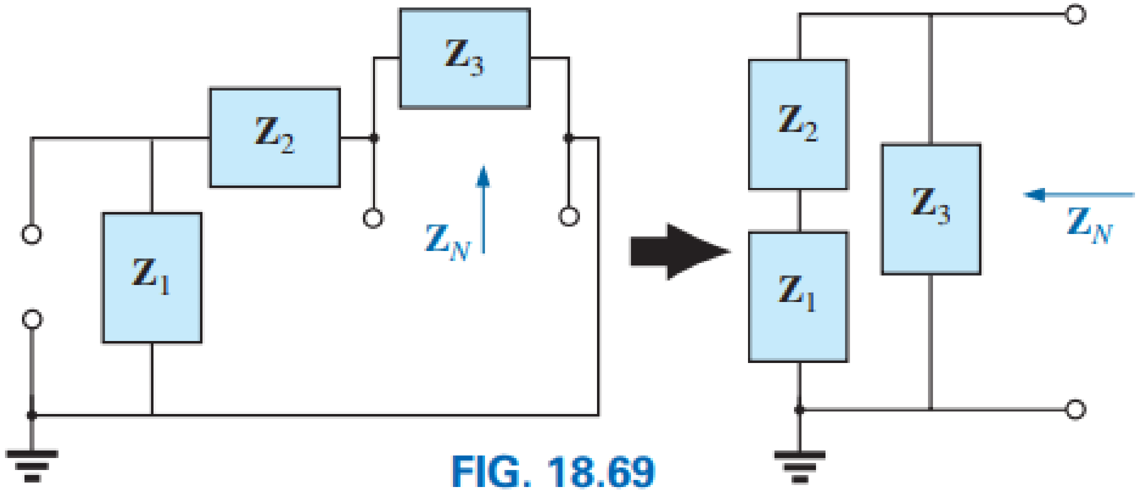


FIG. 18.69

$$\mathbf{Z}_1 + \mathbf{Z}_2 = 2\ \Omega - j\,4\ \Omega + 1\ \Omega$$
$$= 3\ \Omega - j\,4\ \Omega = 5\ \Omega\ \angle -53.13^\circ$$

$$\mathbf{Z}_N = \frac{\mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2)}{\mathbf{Z}_3 + (\mathbf{Z}_1 + \mathbf{Z}_2)}$$
$$= \frac{(5\ \Omega\ \angle 90^\circ)(5\ \Omega\ \angle -53.13^\circ)}{j\,5\ \Omega + 3\ \Omega - j\,4\ \Omega} = \frac{25\ \Omega\ \angle 36.87^\circ}{3 + j\,1}$$
$$= \frac{25\ \Omega\ \angle 36.87^\circ}{3.16\ \angle +18.43^\circ}$$

$$\mathbf{Z}_N = 7.91\ \Omega\ \angle 18.44^\circ = \mathbf{7.50\ \Omega + \mathit{j}\,2.50\ \Omega}$$

Step 4 (Fig. 18.71):

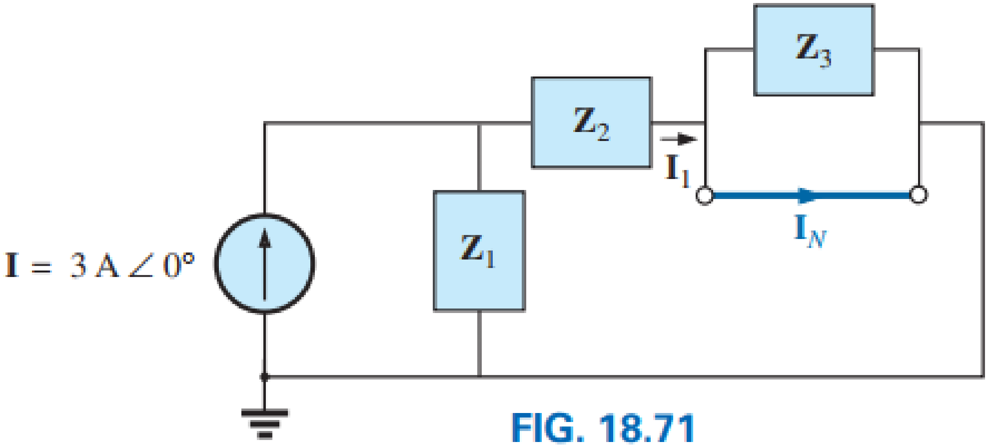


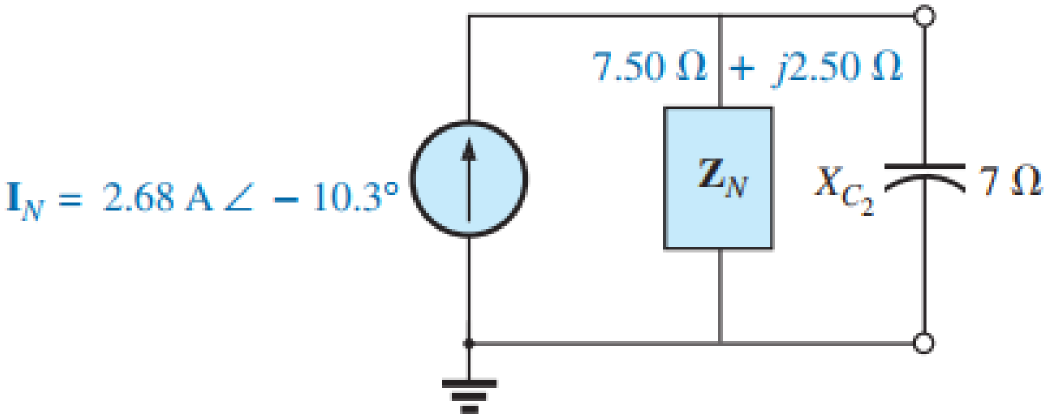
FIG. 18.71

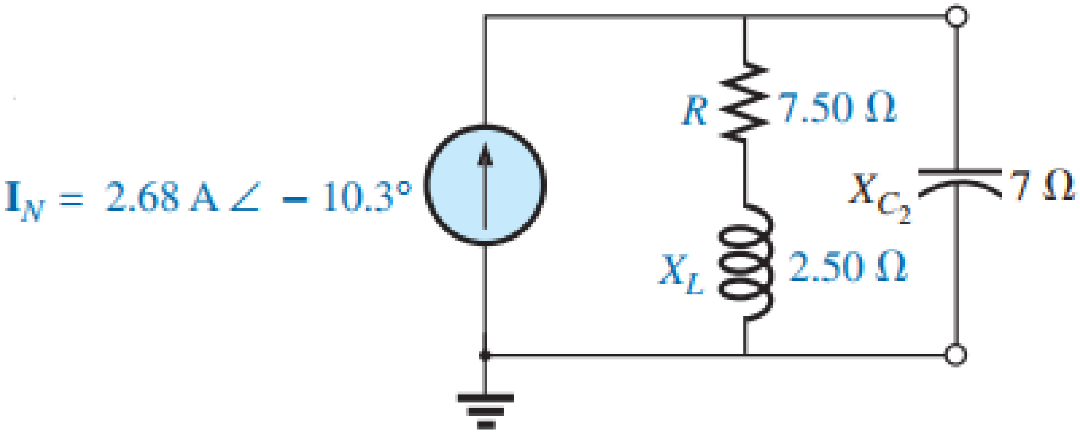
$$I_N = I_1 = \frac{Z_1 I}{Z_1 + Z_2} \quad \text{(current divider rule)}$$

$$= \frac{(2\ \Omega - j\,4\ \Omega)(3\ \text{A})}{3\ \Omega - j\,4\ \Omega} = \frac{6\ \text{A} - j\,12\ \text{A}}{5\ \angle -53.13^\circ} = \frac{13.4\ \text{A}\ \angle -63.43^\circ}{5\ \angle -53.13^\circ}$$

$$\mathbf{I_N = 2.68\ A\ \angle -10.3^\circ}$$

Step 5: The Norton equivalent circuit is shown in Fig. 18.72.





Practice Book Remaining Examples
And
Problem 26- 29 [Ch. 18]

Maximum Power Transfer or Impedance Matching Theorem

Statement: Maximum power will be delivered to a load when the load impedance is the complex conjugate of the Thévenin impedance across its terminals.

If, $\mathbf{Z}_L = R \pm jX$ and $\mathbf{Z}_{Th} = R_{Th} \pm jX_{Th}$

Then, according to maximum power transfer theorem:

$$\mathbf{Z}_L = R \pm jX = \left(\mathbf{Z}_{Th}\right)^* = R_{Th} \mp jX_{Th}$$

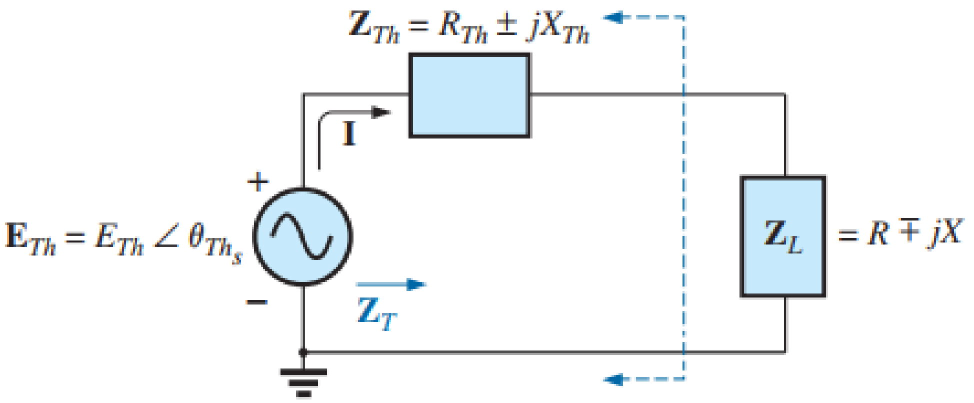


FIG. 18.82
Conditions for maximum power transfer to $\mathbf{Z}_L$.

$$\boxed{Z_L = Z_{Th} \text{ and } \theta_L = -\theta_{Th_Z}} \qquad (18.16)$$

$$\boxed{R_L = R_{Th} \quad \text{and} \quad \pm j\,X_{\text{load}} = \mp j\,X_{Th}} \qquad (18.17)$$

$$\boxed{\mathbf{Z}_T = 2R} \qquad (18.18)$$

$$\boxed{F_p = 1} \quad \text{(maximum power transfer)} \qquad (18.19)$$

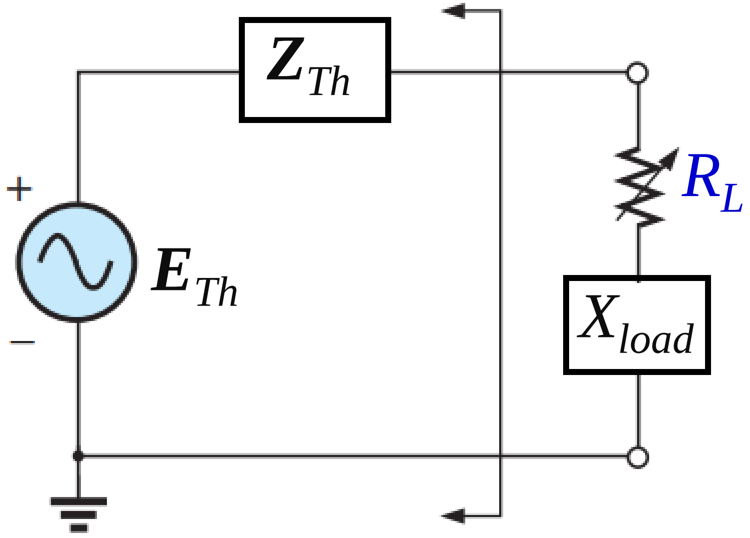
$$\boxed{P_{\max} = \frac{E_{Th}^2}{4R}} \qquad (18.20)$$

Special Situation: If the load resistance is adjustable but the magnitude of the load reactance cannot be set equal to the magnitude of the Thévenin reactance, then the maximum power that can be delivered to the load will occur when the load reactance is made as close to the Thévenin reactance as possible, and the load resistance is set to the following value:

$$\boxed{R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2}} \qquad \textbf{(18.21)}$$

$$\boxed{P = E_{Th}^2/4R_{\text{av}}} \qquad \textbf{(18.22)}$$

$$\boxed{R_{\text{av}} = \frac{R_{Th} + R_L}{2}} \qquad \textbf{(18.23)}$$



EXAMPLE 18.19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

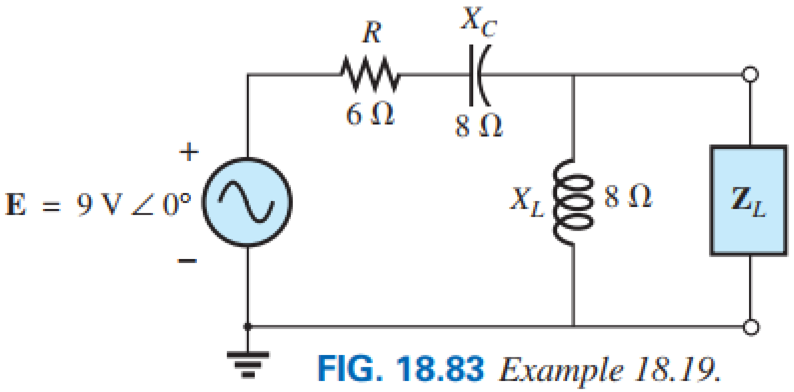


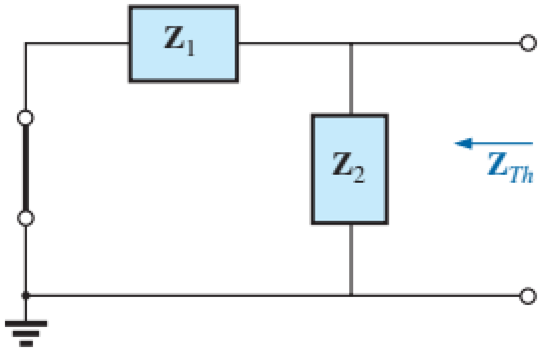
FIG. 18.83 *Example 18.19.*

Solution:

$$\mathbf{Z}_1 = R - jX_C = 6\ \Omega - j\,8\ \Omega = 10\ \Omega\ \angle -53.13^\circ$$

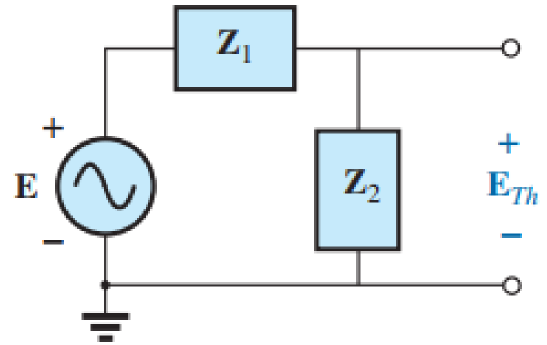
$$\mathbf{Z}_2 = +jX_L = j\,8\ \Omega$$

Determine $\mathbf{Z}_{Th}$ [Fig. 18.84(a)]:



$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\ \Omega\ \angle -53.13^\circ)(8\ \Omega\ \angle 90^\circ)}{6\ \Omega - j\,8\ \Omega + j\,8\ \Omega} = 13.33\ \Omega\ \angle 36.87^\circ = 10.66\ \Omega + j\,8\ \Omega$$

To find the maximum power, we must first find $\mathbf{E}_{Th}$ [Fig. 18.84(b)], as follows:



$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega\ \angle 90^\circ)(9\ \text{V}\ \angle 0^\circ)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V}\ \angle 90^\circ}{6\ \angle 0^\circ} = 12\ \text{V}\ \angle 90^\circ$$

According to maximum power transfer theorem:

$$\mathbf{Z}_L = 13.3\ \Omega\ \angle -36.87^\circ = \mathbf{10.66\ \Omega - j\,8\ \Omega}$$

Maximum power received by load:

$$P_{\max} = \frac{E_{Th}^2}{4R} = \frac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \frac{144}{42.64} = \mathbf{3.38\ W}$$

EXAMPLE 18.21 For the network in Fig. 18.90:

a. Determine the value of R_L for maximum power to the load if the load reactance is fixed at 4 Ω.
b. Find the power delivered to the load under the conditions of part (a).
c. Find the maximum power to the load if the load reactance is made adjustable to any value, and compare the result to part (b) above.

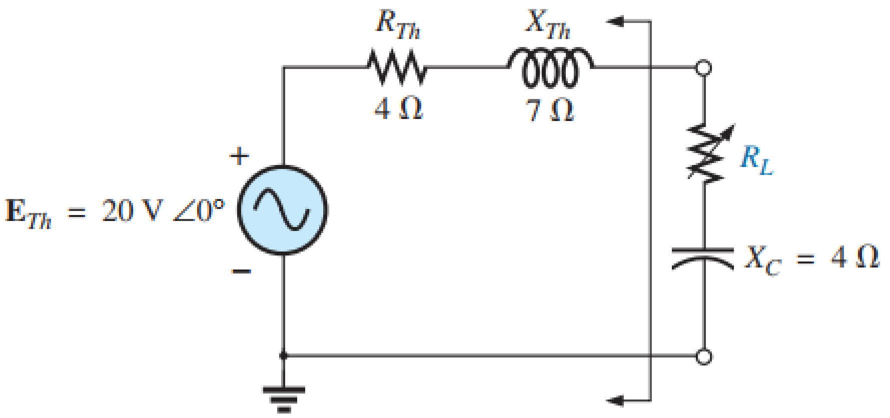


FIG. 18.90 *Example 18.21.*

Solutions:

a. Eq. (18.21):

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{\text{load}})^2}$$
$$= \sqrt{(4\ \Omega)^2 + (7\ \Omega - 4\ \Omega)^2}$$
$$= \sqrt{16 + 9} = \sqrt{25}$$
$$R_L = \mathbf{5\ \Omega}$$

b. Eq. (18.23):

$$R_{av} = \frac{R_{Th} + R_L}{2} = \frac{4\ \Omega + 5\ \Omega}{2}$$
$$= \mathbf{4.5\ \Omega}$$

Eq. (18.22):

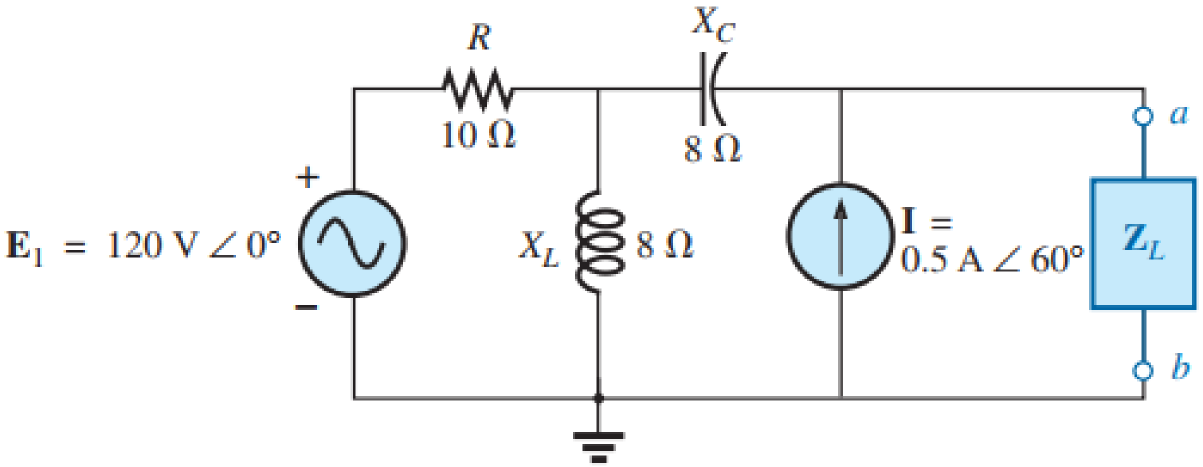
$$P = \frac{E_{Th}^2}{4R_{av}}$$
$$= \frac{(20\ \text{V})^2}{4(4.5\ \Omega)} = \frac{400}{18}\ \text{W}$$
$$\cong \mathbf{22.22\ W}$$

c. For $\mathbf{Z}_L = 4\ \Omega - j\,7\ \Omega$,

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(20\ \text{V})^2}{4(4\ \Omega)}$$
$$= \mathbf{25\ W}$$

exceeding the result of part (b) by 2.78 W.

PROBLEM: Find the load impedance Z_L for the networks in following Figure for maximum power to the load, and find the maximum power to the load.



Solution: $Z_1 = R\ \Omega = \ 10\ \Omega$; $Z_2 = jX_L\ \Omega = \ j8\ \Omega$;
$Z_3 = -jX_C\ \Omega = -j8\ \Omega$;

Step 1 and **Step 2**:

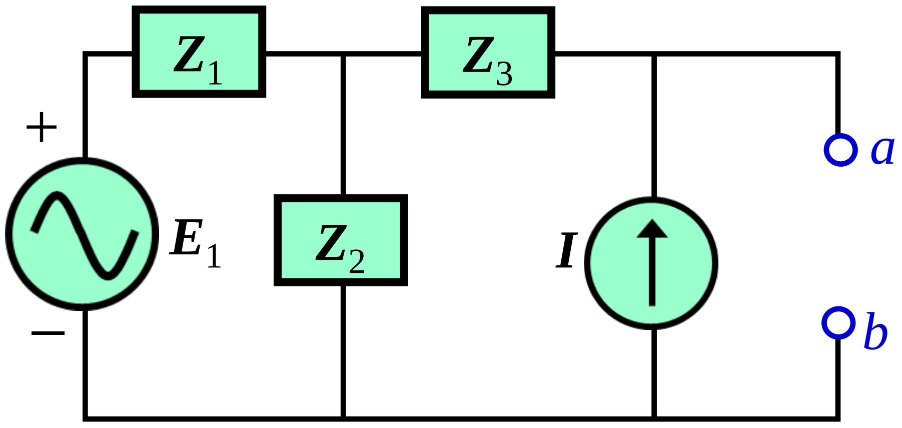


Fig. (a)

Step 3: Z_{Th} calculation

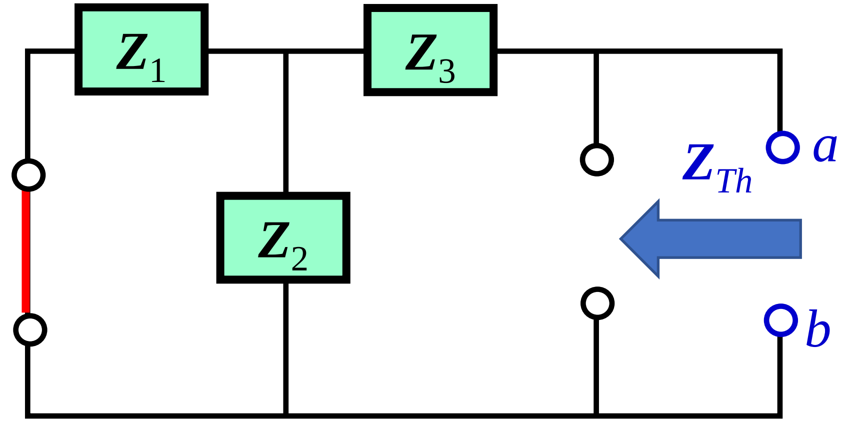


Fig. (b)

$$Z_{Th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} = 3.9 - j3.12\ \ \Omega = 5\Omega\angle -38.66°$$

Step 4: $\boldsymbol{E}_{Th}$ calculation

Since there are two sources, Thevenin's voltage can be calculated by using Superposition Theorem.

Considering $\boldsymbol{E}_1$:

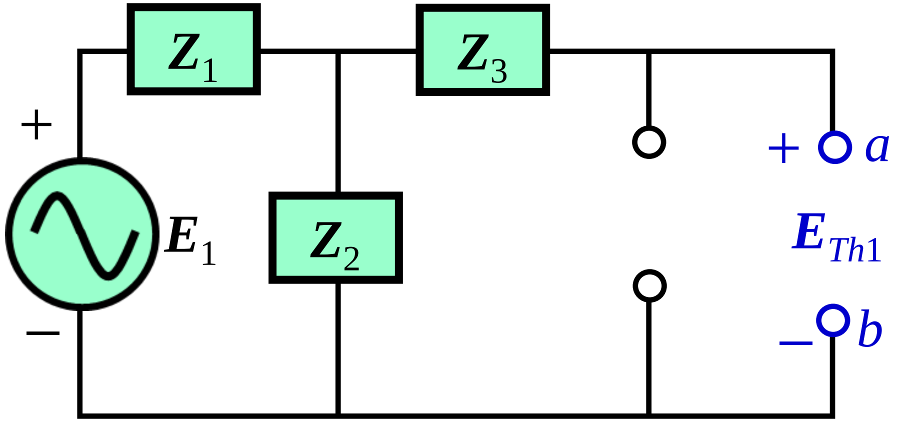


Fig. (c)

$$\boldsymbol{E}_{Th1} = \frac{\boldsymbol{Z}_2\boldsymbol{E}_1}{\boldsymbol{Z}_1+\boldsymbol{Z}_2} = 46.83 + j58.54\ \Omega$$
$$= 74.96\Omega\angle 51.34^\circ$$

Considering $\boldsymbol{I}$:

$$\boldsymbol{Z}_{T2} = \boldsymbol{Z}_3 + \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2}{\boldsymbol{Z}_1+\boldsymbol{Z}_2}$$
$$= 3.9 - j3.12\ \ \Omega$$
$$= 5\Omega\angle -38.66^\circ$$

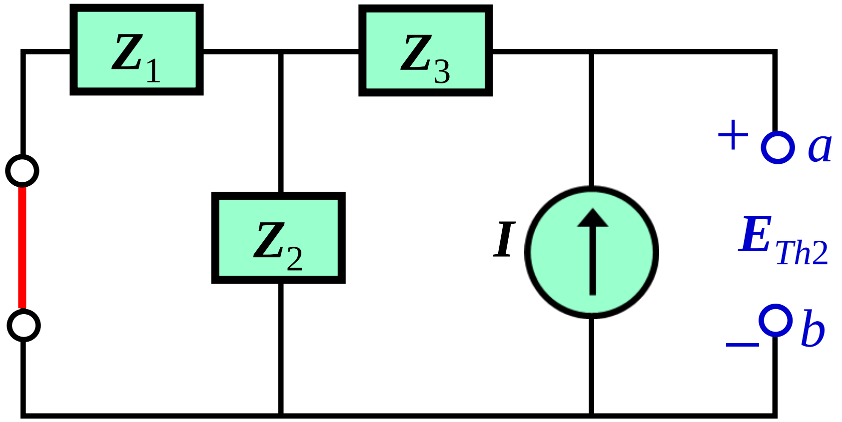
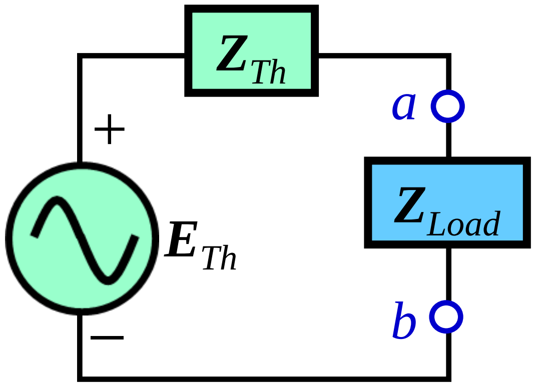


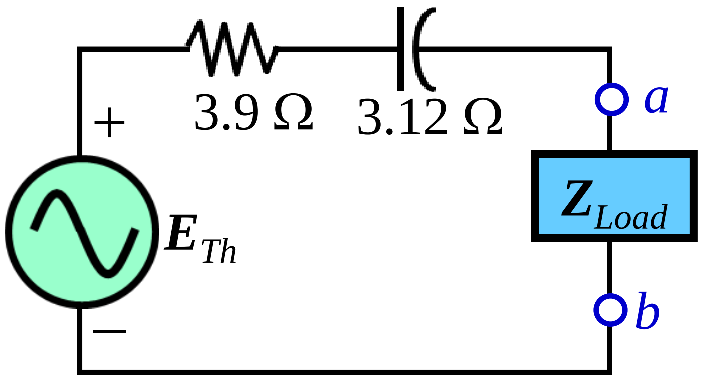
Fig. (d)

$$\boldsymbol{E}_{Th2} = \boldsymbol{Z}_{T2}\boldsymbol{I} = 2.32 + j0.897\ \text{V} = 2.49\text{V}\angle 21.16^\circ$$

According Superposition Theorem:

$$\boldsymbol{E}_{Th} = \boldsymbol{E}_{Th1} + \boldsymbol{E}_{Th2} = 49.15 + j59.43\ \text{V} = 77.12\text{V}\angle 50.41^\circ$$





According to maximum power transfer theorem:

$$\mathbf{Z}_L = (\mathbf{Z}_{Th})^* = (3.9 - j3.12)^* = 3.9 + j3.12\ \Omega$$

Maximum power received by load:

$$P_{\max} = \frac{E_{Th}^2}{4R_{Th}}$$

$$= \frac{(77.12\text{V})^2}{4 \times 3.9\Omega}$$

$$= \mathbf{381.25\ W}$$

Practice Book Remaining Examples
And
Problem 39, 40, 45 and 46 [Ch. 18]

Chapter 24

Poly-phase System

Poly-Phase Generator

An ac generator designed to **develop a single sinusoidal voltage** for each rotation of the shaft (rotor) is referred to as a **single-phase ac generator**.

If the number of coils on the rotor is increased in a specified manner, the result is a **polyphase ac generator**, which **develops more than one ac phase voltage** per rotation of the rotor.

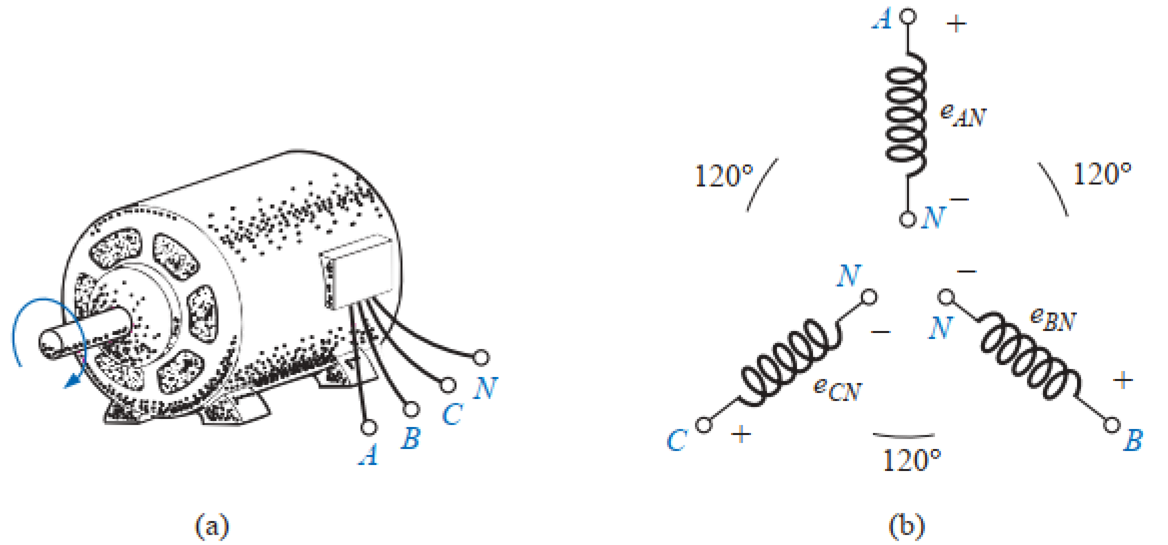


FIG. 22.1

(a) Three-phase generator; (b) induced voltages of a three-phase generator.

For a balanced three phase source, the peak value of voltage $e_{AN}(t)$, $e_{BN}(t)$, and $e_{CN}(t)$ are equal and the phase displacement from each other is 120°.

$$e_{AN}(t) = E_m \sin \omega t$$

$$e_{BN}(t) = E_m \sin(\omega t - 120°)$$

$$e_{CN}(t) = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$$

Let, the rms value of voltages $e_{AN}(t)$, $e_{BN}(t)$, and $e_{CN}(t)$ is E_p then these voltage are in phasor form as folows:

$$\boldsymbol{E}_{AN} = E_p \angle 0°$$

$$\boldsymbol{E}_{BN} = E_p \angle -120°$$

$$\boldsymbol{E}_{CN} = E_p \angle -240° = E_p \angle 120°$$

where, $E_p = \dfrac{1}{\sqrt{2}} E_m$

$$= 0.707 E_m$$

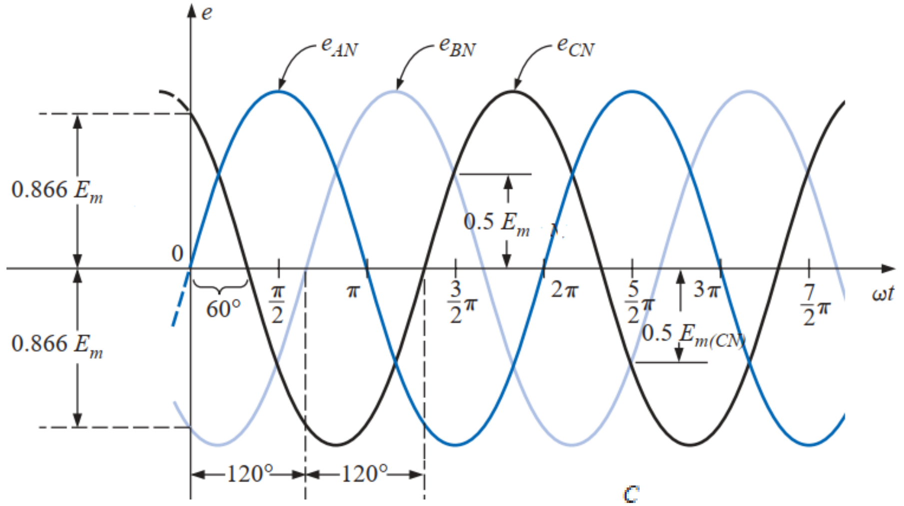
In a balanced system, **at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is *zero***. That means:

$$e_{AN}(t) + e_{BN}(t) + e_{CN}(t) = 0 \qquad \boldsymbol{E}_{AN} + \boldsymbol{E}_{CN} + \boldsymbol{E}_{CN} = 0$$

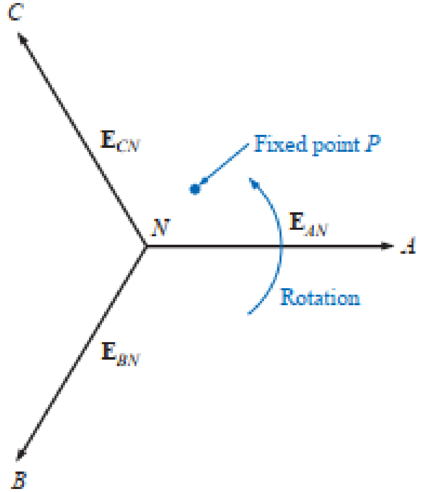
Phase Sequence or Phase Order

There are two phase sequence or phase orders:

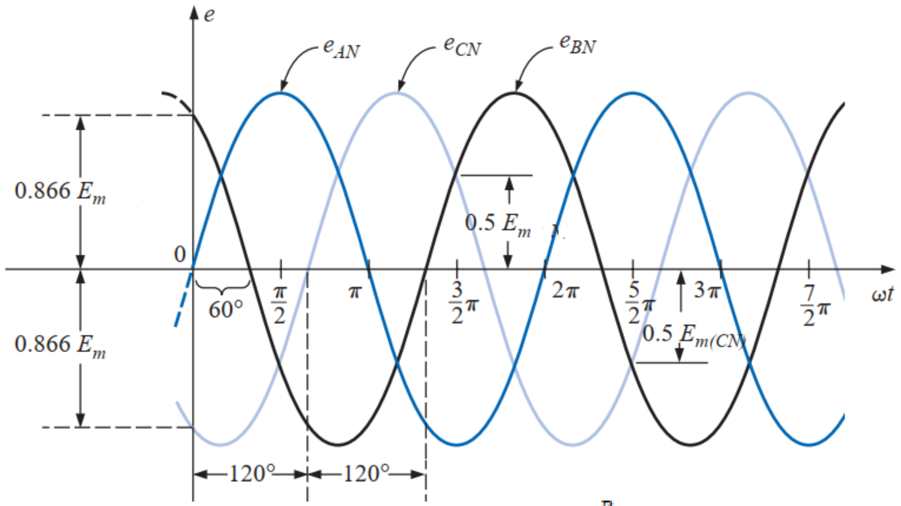
(***a***) ***ABC–sequence*** [*B* lags *A* by 120° and *C* lags *B* by 120° that means *C* lags *A* by 240°]



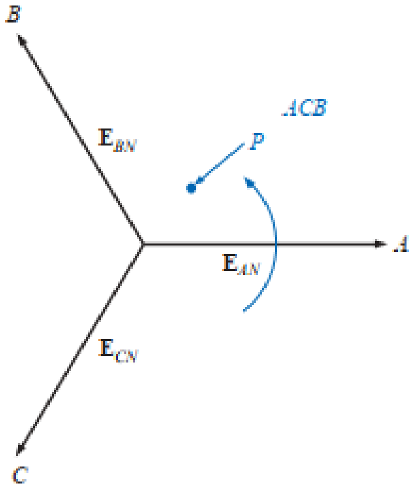
$$\boldsymbol{E}_{AN} = E_p\angle 0^\circ$$
$$\boldsymbol{E}_{BN} = E_p\angle -120^\circ$$
$$\boldsymbol{E}_{CN} = E_p\angle -240^\circ = E_p\angle 120^\circ$$



(***b***) ***ACB–sequence*** [*C* lags *A* by 120° and *B* lags *C* by 120° that means *B* lags *A* by 240°]



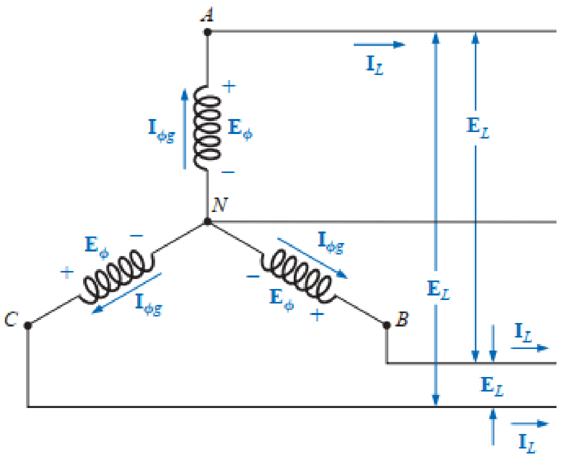
$$\boldsymbol{E}_{AN} = E_p\angle 0^\circ$$
$$\boldsymbol{E}_{CN} = E_p\angle -120^\circ$$
$$\boldsymbol{E}_{BN} = E_p\angle -240^\circ = E_p\angle 120^\circ$$

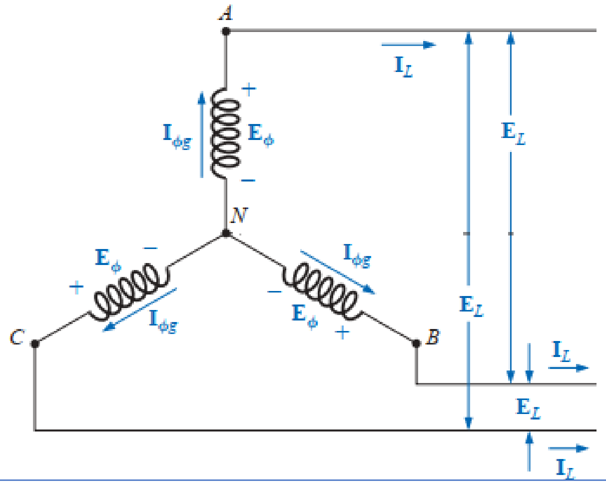


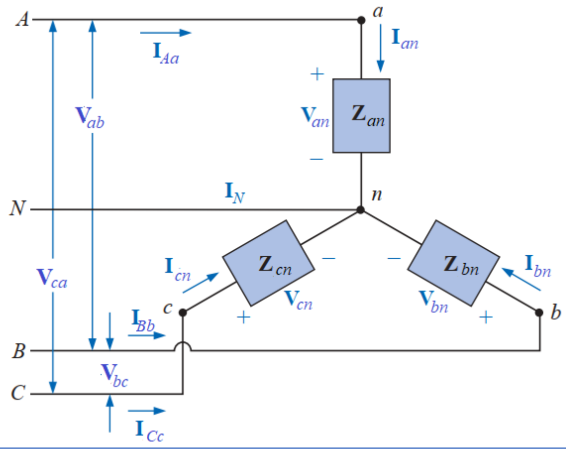
Connection of Three-Phase System

Three phase system can be connected two different ways:

(*a*) **Star** or **Y** (Wye) or **T** (Tee) connection





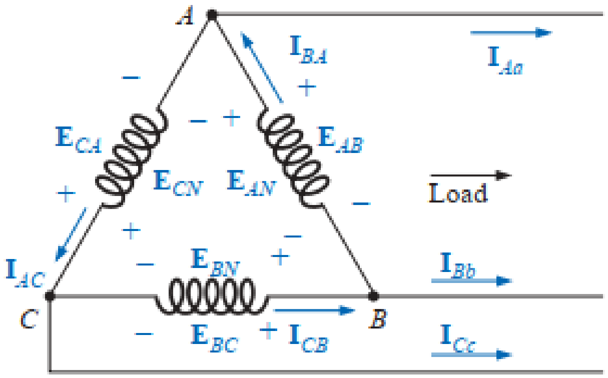


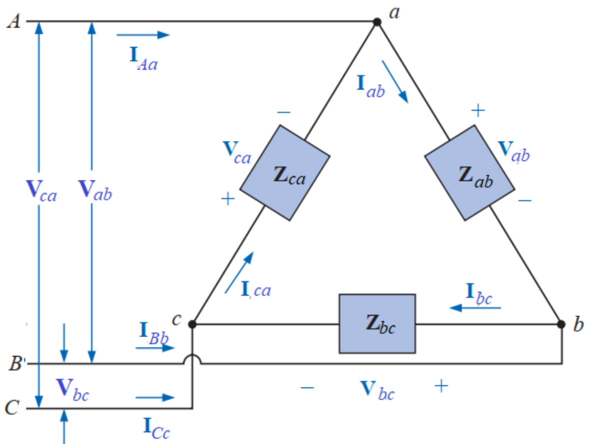
Balanced source voltages are equal in magnitude and are out of phase with each other by 120º.

$$Z_{an} = Z_{bn} = Z_{cn} = Z_Y$$

$$Z_Y = Z\angle\theta_z = R \pm jX$$

(*b*) **Mesh** or **Δ** (delta) or **Π** (pai) connection



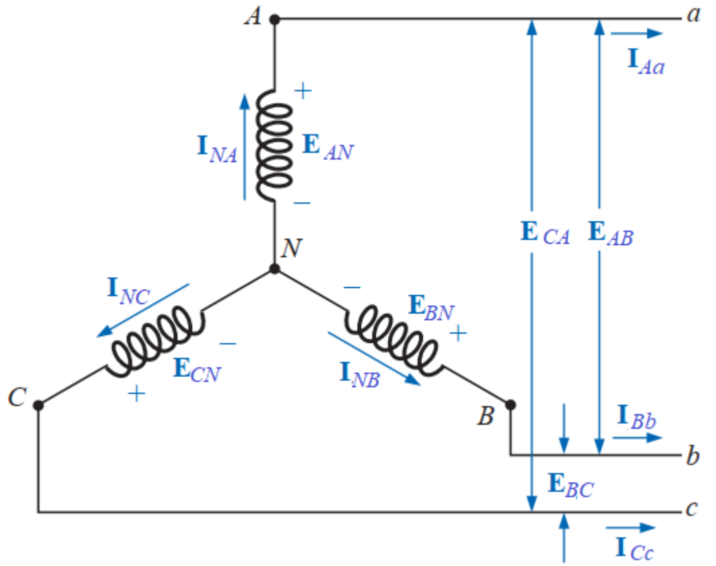


$$Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta$$

$$Z_\Delta = Z\angle\theta_z = R \pm jX$$

A balanced load is one which the phase impedance are equal in magnitude and in phase (also, equal in real part and equal in imaginary part).

Star or **Y** (Wye) or **T** (Tee) connection



Phase Voltages: $\mathbf{E}_{AN}$, $\mathbf{E}_{BN}$ and $\mathbf{E}_{CN}$

Line Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Phase Currents: $\mathbf{I}_{NA}$, $\mathbf{I}_{NB}$ and $\mathbf{I}_{NC}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

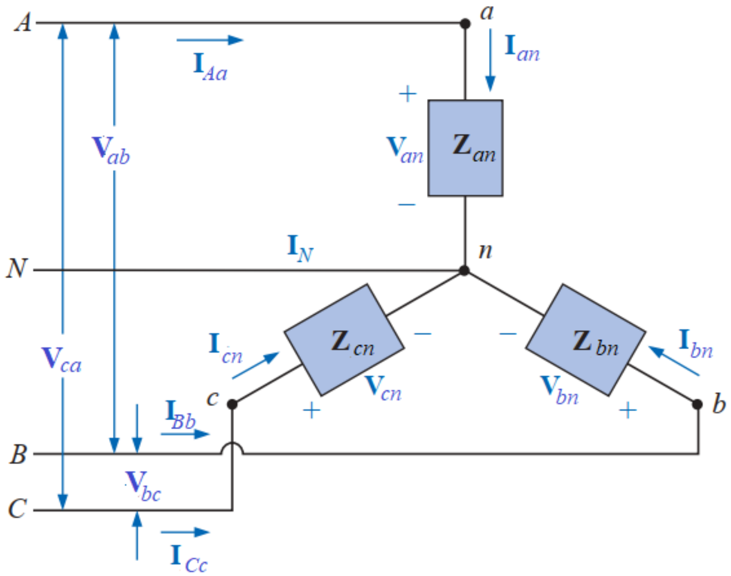
Line Currents = Phase Currents

$\mathbf{I}_{Aa} = \mathbf{I}_{NA}$; $\mathbf{I}_{Bb} = \mathbf{I}_{NB}$; and $\mathbf{I}_{Cc} = \mathbf{I}_{NC}$

Line Voltage ≠ Phase Voltage

$$\boldsymbol{E}_{AB} = \boldsymbol{E}_{AN} - \boldsymbol{E}_{BN}$$
$$\boldsymbol{E}_{BC} = \boldsymbol{E}_{BN} - \boldsymbol{E}_{CN}$$
$$\boldsymbol{E}_{CA} = \boldsymbol{E}_{CN} - \boldsymbol{E}_{AN}$$

$$\boldsymbol{V}_{ab} = \boldsymbol{V}_{an} - \boldsymbol{V}_{bn}$$
$$\boldsymbol{V}_{bc} = \boldsymbol{V}_{bn} - \boldsymbol{V}_{cn}$$
$$\boldsymbol{V}_{ca} = \boldsymbol{V}_{cn} - \boldsymbol{V}_{an}$$



Phase Voltages: $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$

Line Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Phase Currents: $\mathbf{I}_{an}$, $\mathbf{I}_{bn}$ and $\mathbf{I}_{cn}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Currents = Phase Currents

$\mathbf{I}_{Aa} = \mathbf{I}_{an}$; $\mathbf{I}_{Bb} = \mathbf{I}_{bn}$; and $\mathbf{I}_{Cc} = \mathbf{I}_{cn}$

Let,

V_P and E_P: RMS value of phase voltage

V_L and E_L: RMS value of line voltage

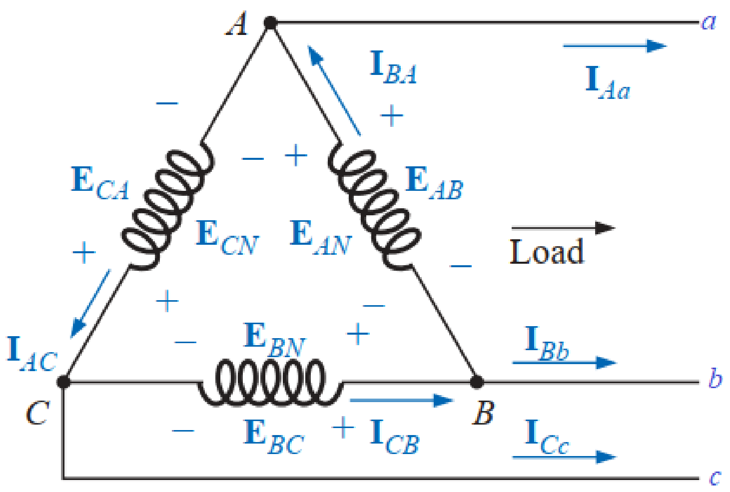
I_P: RMS value of phase current

I_L: RMS value of line current

$$E_L = \sqrt{3}E_P \qquad V_L = \sqrt{3}V_P$$

$$I_L = I_P$$

Mesh or Δ (delta) or Π (pai) connection



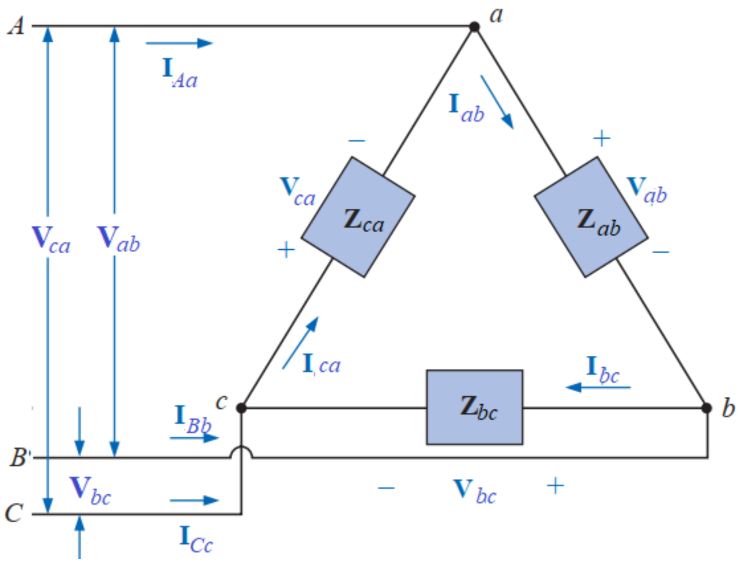
Phase Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Line Voltages: $\mathbf{E}_{AB}$, $\mathbf{E}_{BC}$ and $\mathbf{E}_{CA}$

Phase Currents: $\mathbf{I}_{BA}$, $\mathbf{I}_{AC}$ and $\mathbf{I}_{CB}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Voltage = Phase Voltage



Phase Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Line Voltages: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$

Phase Currents: $\mathbf{I}_{ab}$, $\mathbf{I}_{bc}$ and $\mathbf{I}_{ca}$

Line Currents: $\mathbf{I}_{Aa}$, $\mathbf{I}_{Bb}$ and $\mathbf{I}_{Cc}$

Line Voltage = Phase Voltage

Line Current ≠ Phase Current

$$\boldsymbol{I}_{Aa} = \boldsymbol{I}_{BA} - \boldsymbol{I}_{AC}$$
$$\boldsymbol{I}_{Bb} = \boldsymbol{I}_{CB} - \boldsymbol{I}_{BA}$$
$$\boldsymbol{I}_{Cc} = \boldsymbol{I}_{AC} - \boldsymbol{I}_{CB}$$

$$\boldsymbol{I}_{Aa} = \boldsymbol{I}_{ab} - \boldsymbol{I}_{ca}$$
$$\boldsymbol{I}_{Bb} = \boldsymbol{I}_{bc} - \boldsymbol{I}_{ab}$$
$$\boldsymbol{I}_{Cc} = \boldsymbol{I}_{ca} - \boldsymbol{I}_{bc}$$

Let,

V_P and E_P: RMS value of phase voltage

V_L and E_L: RMS value of line voltage

I_P: RMS value of phase current

I_L: RMS value of line current

$$I_L = \sqrt{3} I_P$$

$$E_L = E_P \qquad V_L = V_P$$

Power Calculation

Instantaneous Power Equation: $p(t) = \frac{3}{2} E_m I_m \cos\theta = 3 E_p I_p \cos\theta$ [W]

V_m and E_m: Peak value of phase voltage
I_m : Peak value of phase current
V_P and E_P: RMS value of phase voltage
V_L and E_L: RMS value of line voltage
I_P: RMS value of phase current
I_L: RMS value of line current

For Y – Connection:

$$\theta = \theta_z = \theta_{e(an)} - \theta_{i(an)} = \theta_{e(bn)} - \theta_{i(bn)} = \theta_{e(cn)} - \theta_{i(cn)}$$

For Δ – Connection:

$$\theta = \theta_z = \theta_{e(ab)} - \theta_{i(ab)} = \theta_{e(bc)} - \theta_{i(bc)} = \theta_{e(ca)} - \theta_{i(ca)}$$

Source Side

$$pf = \cos\theta \qquad rf = \sin\theta$$

$$S = 3E_P I_P = \sqrt{3} E_L I_L$$

$$P = 3E_P I_P \cos\theta = \sqrt{3} E_L I_L \cos\theta = S\cos\theta$$

$$Q = 3E_P I_P \sin\theta = \sqrt{3} E_L I_L \sin\theta = S\sin\theta$$

Load Side

$$S = 3I_P^2 Z \qquad P = 3I_P^2 R \qquad Q_L = 3I_P^2 X_L$$

$$Q_C = -3I_P^2 X_C \qquad Q = Q_L + Q_C$$

$$pf = \frac{P}{S} \qquad rf = \frac{Q}{S}$$

Example: The line voltage and line current of a three-phase system are 440 V and 40 A. Calculate the phase voltage and phase current for (***i***) star-connection, and (***ii***) mesh-connection.

Solution: Given: V_L =440 V, I_L = 40 A

For Star connection $I_P = I_L = 40 \text{ A}$

$$V_P = \frac{V_L}{\sqrt{3}} = 254 \text{ V}$$

For Mesh connection $V_P = V_L = 440 \text{ V}$

$$I_P = \frac{I_L}{\sqrt{3}} = 23.1 \text{ A}$$

Example 4.1.6: The phase voltage and phase current of a three-phase system are 400 V and 20 A. Calculate the line voltage and line current for (***i***) star or Wye-connection, and (***ii***) mesh or Delta-connection.

Solution: Given: V_P =400 V, I_P = 20 A, n=3

Star or Wye connection

$$I_L = I_P = 20 \text{ A}$$

$$V_L = \sqrt{3}V_P = \sqrt{3} \times 400 = 692.82\ V$$

Mesh or Delta connection

$$V_L = V_P = 400 \text{ V}$$

$$I_L = \sqrt{3}I_P = \sqrt{3} \times 20 = 34.64 \text{ A}$$

Example: A 500 volts three-phase supply is connected with a Δ-connected load having R = 18 Ω and X_C = 24 Ω in series in per phase. Calculate (***i***) the real power, (***ii***) the reactive power, (***iii***) the apparent, (***iv***) the power factor and (***v***) the reactive factor.

Solution: Given, $V_L = 500 \text{ V}$ $\quad \mathbf{Z}_\Delta = \mathbf{Z}_{ab} = \mathbf{Z}_{bc} = \mathbf{Z}_{ca} = 18 - j24 = 30\angle -53.13° \quad \Omega$

$$Z_p = 30\ \Omega \qquad \theta = \theta_z = -53.13°$$

$$V_p = V_L = 500 \text{ V} \qquad I_p = \frac{V_p}{Z_p} = \frac{500}{30} = 16.67 \text{ A} \qquad I_L = \sqrt{3} I_p = \sqrt{3} \times 16.67 = 28.87 \text{ A}$$

$$P = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 500 \times 28.87 \times \cos(-53.13°) = 15001.33 \text{ W}$$

$$Q = 3V_p I_p \sin\theta = \sqrt{3} V_L I_L \sin\theta = \sqrt{3} \times 500 \times 28.87 \times \sin(-53.13°) = -20001.7 \text{ Var}$$

$$S = 3V_p I_p = \sqrt{3} V_L I_L = \sqrt{3} \times 500 \times 28.87 = 25002.15 \text{ VA}$$

$$pf = \cos\theta = \cos(-53.13°) = 0.6 \qquad rf = \sin\theta = \sin(-53.13°) = -0.8$$

Example: A three-phase Y-connected motor draws 5.6 kW at a power factor of 0.8 lagging when the line voltage is 220 V. Determine (***i***) the line current and (***ii***) the impedance of the motor.

Solution: Given, P = 5.6 kW = 5600 W; $pf = \cos\theta_z = \cos\theta = 0.8$ lagging, $V_L = 220$ V

Since $P = \sqrt{3} V_L I_L \cos\theta = 5600$ W

$$I_L = \frac{P}{\sqrt{3} V_L \cos\theta} = \frac{5600}{\sqrt{3} \times 220 \times 0.8} = 18.37 \text{ A}$$

For Y-connection:

$$I_p = I_L = 18.37 \text{ A} \qquad V_L = \sqrt{3} V_p \qquad V_p = \frac{1}{\sqrt{3}} V_L = \frac{1}{\sqrt{3}} \times 220 = 127.02 \text{ V}$$

The magnitude of impedance is: $Z_p = \dfrac{V_p}{I_p} = \dfrac{127.02}{18.37} = 6.91\ \Omega$

Since power factor is lagging, we have $\theta_z = \theta = \cos(pf) = \cos(0.8) = 36.87°$

The impedance is: $\mathbf{Z} = Z_p \angle\theta_z = 6.91\Omega\angle 36.87° = 5.53 + j4.15\ \Omega$