Fig. 2-12 The Mandelbrot set

Julia Sets

Now if we set z to some fixed non-zero value and vary x_0 across the complex plane, we obtain a set of non-divergence numbers (values of x_0 that do not diverge under the given transformation) that form a Julia set. Different z values lead to different Julia sets. The image in Fig. 2-13 is produced by making slight modifications to the pseudo-code for the Mandelbrot set. It shows the Julia set defined by $z = -0.74543 + 0.11301i$ with $-1.2 \le x_0.\text{real} \le 1.2$, $-1.2 \le x_0.\text{imag} \le 1.2$, and $N = 128$.

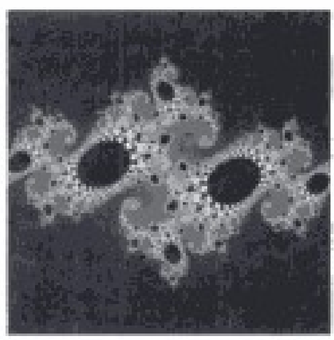

Fig. 2-13 A Julia set.

Solved Problems

2.1 What is the resolution of an image?

SOLUTION

The number of pixels (i.e., picture elements) per unit length (e.g., inch) in the horizontal as well as vertical direction.

2.2 Compute the size of a 640×480 image at 240 pixels per inch.

SOLUTION

$640/240$ by $480/240$ or $2\frac{2}{3}$ by 2 inches.

Copyrighted material

2.3 Compute the resolution of a 2×2 inch image that has 512×512 pixels.

SOLUTION

$512/2$ or 256 pixels per inch.

2.4 What is an image's aspect ratio?

SOLUTION

The ratio of its width to its height, measured in unit length or number of pixels.

2.5 If an image has a height of 2 inches and an aspect ratio of 1.5, what is its width?

SOLUTION

width $= 1.5 \times$ height $= 1.5 \times 2 = 3$ inches.

2.6 If we want to resize a 1024×768 image to one that is 640 pixels wide with the same aspect ratio, what would be the height of the resized image?

SOLUTION

height $= 640 \times 768/1024 = 480$.

2.7 If we want to cut a 512×512 sub-image out from the center of an 800×600 image, what are the coordinates of the pixel in the large image that is at the lower left corner of the small image?

SOLUTION

$[(800 - 512)/2, (600 - 512)/2] = (144, 44)$.

2.8 Sometimes the pixel at the upper left corner of an image is considered to be at the origin of the pixel coordinate system (a left-handed system). How to convert the coordinates of a pixel at (x, y) in this coordinate system into its coordinates (x', y') in the lower-left-corner-as-origin coordinate system (a right-handed system)?

SOLUTION

$(x', y') = (x, m - y - 1)$ where m is the number of pixels in the vertical direction.

2.9 Find the CMY coordinates of a color at $(0.2, 1, 0.5)$ in the RGB space.

SOLUTION

$(1 - 0.2, 1 - 1, 1 - 0.5) = (0.8, 0, 0.5)$.

2.10 Find the RGB coordinates of a color at $(0.15, 0.75, 0)$ in the CMY space.

SOLUTION

$(1 - 0.15, 1 - 0.75, 1 - 0) = (0.85, 0.25, 1)$.

2.11 If we use direct coding of RGB values with 2 bits per primary color, how many possible colors do we have for each pixel?

Copyrighted material

SOLUTION

$2^2 \times 2^2 \times 2^2 = 4 \times 4 \times 4 = 64.$

2.12 If we use direct coding of RGB values with 10 bits per primary color, how many possible colors do we have for each pixel?

SOLUTION

$2^{10} \times 2^{10} \times 2^{10} = 1024^3 = 1073,741,824 > 1$ billion.

2.13 The direct coding method is flexible in that it allows the allocation of a different number of bits to each primary color. If we use 5 bits each for red and blue and 6 bits for green for a total of 16 bits per pixel, how many possible simultaneous colors do we have?

SOLUTION

$2^5 \times 2^5 \times 2^6 = 2^{16} = 65,536.$

2.14 If we use 12-bit pixel values in a lookup table representation, how many entries does the lookup table have?

SOLUTION

$2^{12} = 4096.$

2.15 If we use 2-byte pixel values in a 24-bit lookup table representation, how many bytes does the lookup table occupy?

SOLUTION

$2^{16} \times 24/8 = 65,536 \times 3 = 196,608.$

2.16 True or false: fluorescence is the term used to describe the light given off by a phosphor after it has been exposed to an electron beam. Explain your answer.

SOLUTION

False. Phosphorescence is the correct term. Fluorescence refers to the light given off by a phosphor while it is being exposed to an electron beam.

2.17 What is persistence?

SOLUTION

The duration of phosphorescence exhibited by a phosphor.

2.18 What is the function of the control electrode in a CRT?

SOLUTION

Regulate the intensity of the electron beam.

2.19 Name the two methods by which an electron beam can be bent?

SOLUTION

Electrostatic deflection and magnetic deflection.

Copyrighted material

2.20 What do you call the path the electron beam takes when returning to the left side of the CRT screen?

SOLUTION

Horizontal retrace.

2.21 What do you call the path the electron beam takes at the end of each refresh cycle?

SOLUTION

Vertical retrace.

2.22 What is the pitch of a color CRT?

SOLUTION

The distance between the center of the phosphor dot patterns on the inside of the display screen.

2.23 Why do many color printers use black pigment?

SOLUTION

Color pigments (cyan, magenta, and yellow) are relatively more expensive and it is technically difficult to produce high-quality black using several color pigments.

2.24 Show that with an $n \times n$ pixel grid, where each pixel can take on m intensity levels, we can approximate $n \times n \times (m-1)+1$ overall intensity levels.

SOLUTION

Since the $n \times n$ pixels can be set to a non-zero intensity value one after another to produce $n \times n$ overall intensity levels, and there are $m-1$ non-zero intensity levels for the individual pixels, we can approximate a total of $n \times n \times (m-1)$ non-zero overall intensity levels. Finally we need to add one more overall intensity level that corresponds to zero intensity (all pixels off).

2.25 Represent the grid patterns in Fig. 2-8 with a dither matrix.

SOLUTION

$$\begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$$

2.26 What are the error propagation formulas for a top-to-bottom and right-to-left scanning order in the Floyd–Steinberg error diffusion algorithm?

SOLUTION

$$\begin{aligned} S(x-1, y) &= S(x-1, y) + ae \\ S(x+1, y-1) &= S(x+1, y-1) + be \\ S(x, y-1) &= S(x, y-1) + ce \\ S(x-1, y-1) &= S(x-1, y-1) + de \end{aligned}$$

2.27 What is RLE?

SOLUTION

RLE stands for run-length encoding, a technique for image data compression.

Copyrighted material

2.28 Follow the illustrative example in the text to reconstruct the string that has been compressed to "981435" using RLE.

SOLUTION

"8888888884555"

2.29 If an 8-bit gray scale image is stored uncompressed in sequential memory or in an image file in left-to-right and bottom-to-top pixel order, what is the offset or displacement of the byte for the pixel at (x, y) from the beginning of the memory segment or the file's image data section?

SOLUTION

offset $= y \times n + x$ where n is the number of pixels in the horizontal direction.

2.30 What if the image in Prob. 2.29 is stored in left-to-right and top-to-bottom order?

SOLUTION

offset $= (m - y - 1)n + x$ where n and m are the number of pixels in the horizontal and vertical direction, respectively.

2.31 Develop a pseudo-code segment to initialize a 24-bit 256-entry lookup table with gray-scale values.

SOLUTION

```
int i, rgb[3];
for (i = 0; i < 256; i++) {
   rgb[0] = rgb[1] = rgb[2] = i;
   setEntry(i, rgb);
}
```

2.32 Develop a pseudo-code segment to swap the red and green components of all colors in a 256-entry lookup table.

SOLUTION

```
int i, x, rgb[3];
for (i = 0; i < 256; i++) {
   getEntry(i, rgb);
   x = rgb[0];
   rgb[0] = rgb[1];
   rgb[1] = x;
   setEntry(i, rgb);
}
```

2.33 Develop a pseudo-code segment to draw a rectangular area of $w \times h$ (in number of pixels) that starts at (x, y) using color rgb.

SOLUTION

```
int i, j;
setColor(rgb);
for ( j = y; j < y + h; j++)
   for (i = x; i < x + w; i++) setPixel(i, j);
```

Copyrighted material

2.34 Develop a pseudo-code segment to draw a triangular area with the three vertices at (x, y), $(x, y+t)$, and $(x+t, y)$, where integer $t \geq 0$, using color *rgb*.

SOLUTION

```
int i, j;
setColor(rgb);
for ( j = y; j <= y + t; j++)
   for (i = x; i <= x + y + t - j; i++) setPixel(i, j);
```

2.35 Develop a pseudo-code segment to reset every pixel in an image that is in the 24-bit 256-entry lookup table representation to its complementary color.

SOLUTION

```
int i, rgb[3];
for (i = 0; i < 256; i++) {
   getEntry(i, rgb);
      rgb[0] = 255 - rgb[0];
      rgb[1] = 255 - rgb[1];
      rgb[2] = 255 - rgb[2];
      setEntry(i, rgb);
   }
```

2.36 What if the image in Prob. 2.35 is in the 24-bit true color representation?

SOLUTION

```
int i, j, rgb[3];
for ( j = 0; j < height; j++)
   for (i = 0; i < width; i++) {
   getPixel(i, j, rgb);
      rgb[0] = 255 - rgb[0];
      rgb[1] = 255 - rgb[1];
      rgb[2] = 255 - rgb[2];
      setPixel(i, j, rgb);
   }
```

2.37 Calculate the sum and product of $0.5 + 2.0i$ and $1.0 - 1.0i$.

SOLUTION

$$(0.5 + 1.0) + (2.0 + (-1.0))i = 1.5 + 1.0i$$
$$(0.5 \times 1.0 - 2.0 \times (-1.0)) + (0.5 \times (-1.0) + 2.0 \times 1.0)i = 2.5 + 1.5i$$

2.38 Calculate the square of the two complex numbers in Prob. 2.37.

SOLUTION

$$(0.5^2 - 2.0^2) + 2 \times 0.5 \times 2.0i = -3.75 + 2.0i$$
$$(1.0^2 - (-1.0)^2) + 2 \times 1.0 \times (-1.0)i = 0.0 - 2.0i$$

2.39 Show that $1 + 254 \times \text{count}/N$ provides a proportional mapping from count in $[0, N]$ to c in $[1, 255]$.

Copyrighted material

SOLUTION

Proportional mapping means that we want

$$(c-1)/(255-1) = (\text{count}-0)/(N-0)$$

Hence $c = 1 + 254 \times \text{count}/N$.

2.40 Modify the pseudo code for visualizing the Mandelbrot set to visualize the Julia sets.

SOLUTION

```
int i, j, count;
float delta = (Re_max – Re_min)/width;
for (i = 0, x.real = Re_min; i < width; i++, x.real+ = delta)
  for ( j = 0, x.imag = Im_min; j < height; j++, x.imag+ = delta) {
    count = 0;
    while (|x| ≤ 2.0 && count < N) {
      compute x = x² + z;
      count++;
    }
    if (|x| ≤ 2.0) setColor(0);
    else setColor(1 + 254*count/N);
    setPixel(i, j);
  }
```

2.41 How to avoid the calculation of square root in an actual implementation of the algorithms for visualizing the Mandelbrot and Julia sets?

SOLUTION

Test for $|x|^2 \leq 4.0$ instead of $|x| \leq 2.0$.

Supplementary Problems

2.42 Can a 5 by $3\frac{1}{2}$ inch image be presented at 6 by 4 inch without introducing geometric distortion?

2.43 Refering to Prob. 2.42, what if the original is $5\frac{1}{4}$ by $3\frac{1}{2}$ inch?

2.44 Given the portrait image of a person, describe a simple way to make the person look more slender.

2.45 An RGB color image can be converted to a gray-scale image using the formula 0.299R + 0.587G + 0.114B for gray levels (see Chap. 11, Sec. 11.1 under "The NTSC YIQ Color Model"). Assuming that getPixel(x, y, rgb) now reads pixel values from a 24-bit input image and setPixel(x, y, i) assigns pixel values to an output image that uses a gray-scale lookup table, develop a pseudo-code segment to convert the input image to a gray-scale output image.

Copyrighted material