Engineering Management Transportation Model

AIM OF TRANSPORTATION MODEL

■ To find out the **optimum transportation schedule** keeping in mind the cost of transportation to be **minimized**.

THE MODEL REQUIRES FEW DATA ELEMENTS

- Origin of Supply
- Destination
- Unit Cost of Shipping (Per Unit Cost)

Transportation Model is a special case of LPP (Linear Programming Problem) in which the main objective is to transport a product from various sources various destinations at total minimum costs.

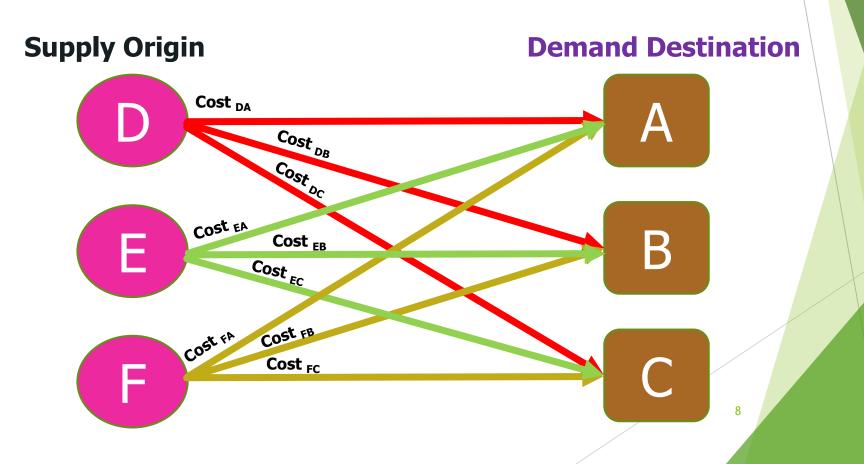
- In transportation models, the sources and destinations at each source and destination are also known.
- It is desired to find the best arrangement for transportation such that the transportation cost is minimum.

As it is a model, we have to make some assumptions.

These assumptions are:

- Items are homogeneous
- Shipping costs per unit are the same, no matter the quantity delivered
- Only one route is choosen between the origin and destination

- For example, consider three companies (Company 1, Company 2, and Company 3) which produces mobile phones and are located in different regions.
- □ Similarly, consider three cities (namely A, B & C) where the mobile phones are transported.
- ☐ The companies where mobile phones are available are known as sources and the cities where mobile phones are transported are called destinations.



- Let, Company 1 produces a1 units,
 Company 2 produces a2 units, and
 Company 3 produces a3 units,
- Let, demand in City A is b1 units, demand in City B is b2 units, and demand in City C is c3 units

The cost of transportation from each source to destination is given in table.

DESTINATIONS						
SOURCES		CITY A	CITY B	CITY C	SUPPLY	
	COMPANY 1	C _{1A}	C _{1B}	C _{1C}	a1	
	COMPANY 2	C _{2A}	C _{2B}	C _{2C}	a2	
	COMPANY 3	C _{3A}	C _{3B}	C _{3C}	a3	
	DEMAND	b1	b2	b3	$\Sigma a = \Sigma b$	

The transportation of mobile phones should be done in such a way, that the total transportation cost is minimum.

TYPES OF TRANSPORTATION PROBLEMS

There are two types of transportation problems.

i) Balanced Transportation Problems

The sum of supply and sum of demand are same.

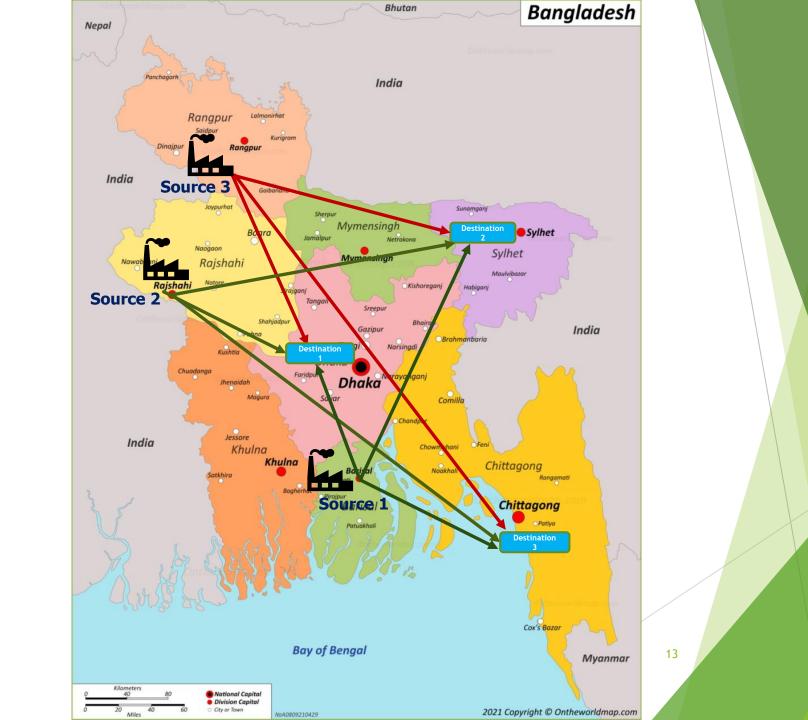
 Σ Supply = Σ Demand

ii) Unbalanced Transportation Problems

The sum of supply and sum of demand are different.

 Σ Supply $\neq \Sigma$ Demand

- i. North-West Corner Method
- ii. Least Cost Method



NORTH-WEST CORNER METHOD

i. North-West Corner Method

The North West Corner Method is one of the methods to obtain a basic feasible solution of the transportation problems (special case of LPP).

We will now see how to apply this very simple method to a transportation problem.

We will study steps of this method while applying it in the problem itself.

i. North-West Corner Method

Sample Problem# 1

A mobile phone manufacturing company has three branches located in three different regions, say Barishal, Rajshahi, and Rangpur.

The company has to transport mobile phones to three destinations, say Dhaka, Sylhet and Chittagong.

The availability from Barishal, Rajshahi, and Rangpur is 40, 60 and 70 units respectively.

The demand at Dhaka, Sylhet, and Chittagong are 70, 40 and 60 respectively.

The transportation cost is shown in the matrix below (in Tk.).

Use the **North-West Corner Method** to find a **basic feasible solution (BFS)**.

i. North-West Corner Method

Sample Problem# 1

DESTINATIONS							
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY		
	Barishal	4	5	1	40		
	Rajshahi	3	4	6	60		
	Rangpur	6	2	8	70		
	DEMAND	70	40	60	170		

i. North-West Corner Method

Sample Problem# 1

Solution

Step 1: Balance the problem

Balance the problem meaning we need to check that if; Σ Supply = Σ Demand

If this holds true, then we will consider the given problem as a balanced problem.

i. North-West Corner Method

Sample Problem# 1

Solution

Step 2: Start allocating from North-West corner cell

We will start the allocation from the left hand top most corner (north-west) cell in the matrix and make allocation based on availability and demand.

Now, verify the smallest among the availability (Supply) and requirement (Demand), corresponding to this cell.

The smallest value will be allocated to this cell and check out the difference in supply and demand, representing that supply and demand are fulfilled, as shown below.

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS						
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY	
	Barishal	4 (40)	5	1	40 (0)	
	Rajshahi	3	4	6	60	
	Rangpur	6	2	8	70	
	DEMAND	70 (30)	40	60		

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS							
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY		
	Barishal	4 (40)	5	4	40 (0)		
	Rajshahi	3	4	6	60		
	Rangpur	6	2	8	70		
	DEMAND	70 (30)	40	60			

i. North-West Corner Method

Sample Problem# 1

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

As we have fulfilled the availability or requirement for that row or column respectively, remove that row or column and prepare a new matrix, as shown below.

DESTINATIONS						
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY	
	Rajshahi	3	4	6	60	
	Rangpur	6	2	8 22	70	
	DEMAND	30	40	60		

i. North-West Corner Method

Sample Problem# 1

Solution

Step 4: Repeat the procedure until all the allocations are over

Repeat the same procedure of allocation of the new North-west corner so generated and check based on the smallest value as shown below, until all allocations are over.

DESTINATIONS						
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY	
	Rajshahi	3 (30)	4	6	60 <mark>(30)</mark>	
	Rangpur	6	2	8	70	
	DEMAND	30 (0)	40	60	20	

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS						
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY	
	Rajshahi	3 (30)	4	6	60 (30)	
	Rangpur	6	2	8	70	
	DEMAND	30 (0)	40	60	24	

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS							
		Sylhet Chitt		SUPPLY			
COLIDCEC	Rajshahi	4 (30)	6	30 (0)			
SOURCES	Rangpur	2	8	70			
	DEMAND	40 (10)	60	25			

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS							
		Sylhet	Chittagong	SUPPLY			
COLIDCEC	Rajshahi	4 (30)	6	30 (0)			
SOURCES	Rangpur	2	8	70			
	DEMAND	40 (10)	60	26			

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS							
		Sylhet	Chittagong	SUPPLY			
SOURCES	Rangpur	2 (10)	8 (60)	70 (60) (0)			
	DEMAND	10 (0)	60 (0)				

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS							
		Sylhet	Chittagong	SUPPLY			
SOURCES	Rangpur	2 (10)	8 (60)	(0)			
	DEMAND	(0)	(0)				

i. North-West Corner Method

Sample Problem# 1

Solution

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

Once all allocations are over, prepare the table with all allocations marked and calculate the transportation cost as follows.

i. North-West Corner Method

Sample Problem# 1

Solution

Step 5: After all the allocations are over, write the allocations and calculate

the transportation cost

DESTINATIONS							
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY		
	Barishal	4	5	1	40		
	Rajshahi	3	4	6	60		
	Rangpur	6	2	8	70		
	DEMAND	70	40 30	60	170		

i. North-West Corner Method

Sample Problem# 1

Solution

DESTINATIONS							
		Dhaka	Sylhet	Chittagong	SUPPLY		
SOURCES	Barishal	4 (40)	5	1	40		
	Rajshahi	3 (30)	4 (30)	6	60		
	Rangpur	6	2 (10)	8 (60)	70		
	DEMAND	70	40	60	170		

Total Transportation Costs = $(4\times40)+(3\times30)+(4\times30)+(2\times10)+(8\times60)$ = Tk. 870/-

LEAST COST METHOD

ii. Least Cost Method

Sample Problem# 2

A mobile phone manufacturing company has three branches located in three different regions, say Barishal, Rajshahi, and Rangpur.

The company has to transport mobile phones to three destinations, say Dhaka, Sylhet and Chittagong.

The availability from Barishal, Rajshahi, and Rangpur is 40, 60 and 70 units respectively.

The demand at Dhaka, Sylhet, and Chittagong are 70, 40 and 60 respectively.

The transportation cost is shown in the matrix below (in Tk.).

Use the **Least Cost Method** to find a **basic feasible solution (BFS)**.

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS								
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY			
	Barishal	4	5	1	40			
	Rajshahi	3	4	6	60			
	Rangpur	6	2	8	70			
	DEMAND	70	40	60	170			

ii. Least Cost Method

Sample Problem# 2

Solution

Step 1: Balance the problem

The given Transportation problem is balanced as the Σ Supply = Σ Demand

ii. Least Cost Method

Sample Problem# 2

Solution

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand

DESTINATIONS							
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY		
	Barishal	4	5	1 (40)	40 (0)		
	Rajshahi	3	4	6	60		
	Rangpur	6	2	8	70		
	DEMAND	70	40	60 <mark>(20)</mark>			

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS							
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY		
	Barishal	4	5	1 (40)	40 (0)		
	Rajshahi	3	4	6	60		
	Rangpur	6	2	8	70		
	DEMAND	70	40	60 (20)			

ii. Least Cost Method

Sample Problem# 2

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix.

DESTINATIONS							
		Dhaka	Sylhet	Chittagong	SUPPLY		
SOURCES	Rajshahi	3	4	6	60		
	Rangpur	6	2	8	70		
	DEMAND	70	40	20	38		

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS							
		Dhaka	Sylhet	Chittagong	SUPPLY		
SOURCES	Rajshahi	3 4		6	60		
	Rangpur	6	2 (40)	8	70 (30)		
	DEMAND	70	40 (0)	20	39		

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS							
		Dhaka	Sylhet	Chittagong	SUPPLY		
SOURCES	Rajshahi	3	4	6	60		
	Rangpur	6	2 (40)	8	70 (30)		
	DEMAND	70	40 (0)	20			

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS							
SOURCES		Dhaka	Chittagong	SUPPLY			
	Rajshahi	3	6	60			
	Rangpur	Rangpur 6		30			
	DEMAND	70	20				

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS							
SOURCES		Dhaka	Chittagong	SUPPLY			
	Rajshahi 3 (60)		6	60 (<mark>0</mark>)			
	Rangpur	6	8	30			
	DEMAND	70 (10)	20				

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS							
SOURCES		Dhaka	Chittagong	SUPPLY			
	Rajshahi 3 (60)		6	60 (0)			
	Rangpur	6	8	30			
	DEMAND	70 (10)	20				

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS							
		Dhaka	Chittagong	SUPPLY			
SOURCES	Rangpur	6	8	30			
	DEMAND	10	20				

ii. Least Cost Method

Sample Problem# 2

DESTINATIONS							
		Dhaka	Chittagong	SUPPLY			
SOURCES	Rangpur	6 (10)	8 (20)	30 (20) (0)			
	DEMAND	10 (0)	20 (0)				

ii. Least Cost Method

Sample Problem# 2

Solution

Step 5: After all allocations are over, write the allocations and calculate the transportation costs.

ii. Least Cost Method

Sample Problem# 2

Solution

DESTINATIONS							
SOURCES		Dhaka	Sylhet	Chittagong	SUPPLY		
	Barishal	4 5		1 (40)	40		
	Rajshahi	3 (60)	4	6	60		
	Rangpur	6 (10)	2 (40)	8 (20)	70		
	DEMAND	70	40	60	170		

Total Transportation Costs = $(1\times40)+(3\times60)+(6\times10)+(2\times40)+(8\times20)$ = Tk. 520/-

Unbalanced Transportation Problems

Solve the following problem for BFS using Least Cost Method

DESTINATIONS							
		Α	В	С	SUPPLY		
	Р	5	1	7	20		
SOURCES	Q	7	4	6	70		
	R	3	2	5	25		
	DEMAND	65	30	50			

Solve the following problem for BFS using Least Cost Method

Sample Problem# 4

Solution

Step 1: Balance the problem

The given Transportation problem is unbalanced as the Σ Supply $\neq \Sigma$ Demand

As the Σ Supply = 115 and the Σ Demand = 145

DESTINATIONS							
SOURCES		Α	В	С	SUPPLY		
	Р	5	1	7	20		
	Q	7	4	6	70		
	R	3	2	5	25		
	DEMAND	65	30	50	145//115		

Solution

Step 1: Balance the problem

Therefore, Dummy Row D_1'' is added with the supply of 30 units in order to balance the Transport problem.

DESTINATIONS						
		Α	В	С	SUPPLY	
SOURCES	Р	5	1	7	20	
	Q	7	4	6	70	
	R	3	2	5	25	
	D_1	0	0	0	30	
	DEMAND	65	30	50	145	

Solution

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand

DESTINATIONS							
		Α	В	С	SUPPLY		
	Р	5	1 (20)	7	20 (0)		
SOURCES	Q	7	4	6	70		
	R	3	2	5	25		
	D_1	0	0	0	30		
	DEMAND	65	30 (10)	50	<u></u>		

Solution

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand

DESTINATIONS						
		Α	В	С	SUPPLY	
COURCEC	P	5	1 (20)	7	20 -(0)	
	Q	7	4	6	70	
SOURCES	R	3	2	5	25	
	D_1	0	0	0	30	
	DEMAND	65	30 (10)	50		

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

DESTINATIONS						
		Α	В	С	SUPPLY	
	Q	7	4	6	70	
SOURCES	R	3	2	5	25	
	D_1	0	0	0	30	
	DEMAND	65	10	50		

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

DESTINATIONS						
SOURCES		Α	В	С	SUPPLY	
	Q	7	4	6	70	
	R	3	2 (10)	5	25 (15)	
	D_1	0	0	0	30	
	DEMAND	65	10 (0)	50		

Solution

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

DESTINATIONS						
SOURCES		Α	B	С	SUPPLY	
	Q	7	4	6	70	
	R	3	2 (10)	5	25 (15)	
	D_1	0	0	0	30	
	DEMAND	65	10 (0)	50		

Solution

DESTINATIONS							
		Α	С	SUPPLY			
	Q 7		6	70			
SOURCES	R	3 (15)	5	15 (0)			
	D_1	0	0	30			
	DEMAND	65 (50)	50				

Solution

DESTINATIONS						
SOURCES		Α	С	SUPPLY		
	Q 7		6	70		
	R	3 (15)	5	15 (0)		
	D_1	0	0	30		
	DEMAND	65 (50)	50			

Solution

DESTINATIONS						
SOURCES		Α	С	SUPPLY		
	Q	7	6 (50)	70 (20)		
	D_1	0	0	30		
	DEMAND	50	50 (0)			

Solution

DESTINATIONS						
COURCEC		Α	E	SUPPLY		
	Q	7	6 (50)	70 (20)		
SOURCES	D_1	0	0	30		
	DEMAND	50	50 (0)			

Solution

DESTINATIONS						
		Α	SUPPLY			
COURCES	Q	7 (20)	20 (0)			
SOURCES	D_1	0 (30)	30 (0)			
	DEMAND	50 (30) (0)				

Solution

Step 5: After all allocations are over, write all the allocations and calculate the transportation cost.

DESTINATIONS							
		Α	В	С	SUPPLY		
	Р	5	1 (20)	7	20		
COURCEC	Q	7 (20)	4	6 (50)	70		
SOURCES	R	3 (15)	2 (10)	5	25		
	D_1	0 (30)	0	0	30		
	DEMAND	65	30	50	145		

Total Transportation Costs = $(1\times20)+(7\times20)+(6\times50)+(3\times15)+(2\times10)+(0\times30)$ = Tk. 525/-

NOTE

If the sum of demand is less than the sum of supply, then we will have to add a "Dummy Column".

DESTINATIONS									
SOURCES		Α	В	С	SUPPLY				
	Р	5	1	7	65				
	Q	7	4	6	30				
	R	3	2	5	50				
	DEMAND	20	70	25	115//145				

NOTE

In order to balance the Transportation problem, we have to add "0" in the "Dummy Column" having demand of 30

DESTINATIONS									
SOURCES		Α	В	С	D_1	SUPPLY			
	Р	5	1	7	0	65			
	Q	7	4	6	0	30			
	R	3	2	5	0	50			
	DEMAND	20	70	25	30	145//145			

Further, we can solve this by using any of the discussed method (NWCM, LCM or VAM)

PRACTICE PROBLEM...

Problem...

Explanation:

Given three sources O1, O2 and O3 and four destinations D1, D2, D3 and D4.

For the sources O1, O2 and O3, the supply is 300, 400 and 500 respectively.

The destinations D1, D2, D3 and D4 have demands 250, 350, 400 and 200 respectively.

Based on the matrix given in the next slide, calculate the Transportation Costing for **North-West Corner Method** and **Least Cost Method**, and comment on which transportation method is ideal to be chosen?

PRACTICE PROBLEM...

Problem...

DESTINATIONS									
SOURCES		D1	D2	D3	D4	SUPPLY			
	Q1	3	1	7	4	300			
	Q2	2	6	5	9	400			
	Q3	8	3	3	2	500			
	DEMAND	250	350	400	200				

END OF THE CHAPTER...

PRACTICE PROBLEM...

Problem...

DESTINATIONS									
		D1	D2	D3	D4	SUPPLY			
	Q1	3	3001	7	4	300			
SOURCES	Q2	2502	6	1505	9	400			
	Q3	8	503	2503	2002	500			
	DEMAND	250	350	400	200				

EXERCISE: NORTH-WEST CORNER METHOD

A dairy firm has three plants located throughout a state. Daily milk production at each plant is as follows:

Plant 1: 6 million litres; Plant 2: 1 million litres and Plant 3: 10 million litres

Each Day the firm must fulfil the needs of its distribution centres. Minimum requirement at each centre is as follows:

Distribition centre 1: 7 million litres

Distribition centre 2: 5 million litres

Distribition centre 3: 3 million litres

Distribition centre 4: 2 million litres

Cost of shipping one million litres of milk from each plant to each distribution centre is given in the following table in hundred of BDT.

	DC1	DC2	DC3	DC4
Plant 1	2	3	11	7
Plant 2	1	0	6	1
Plant3	5	8	15	9

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Find its initial basic feasible solution by North-West Corner method.

EXERCISE: NORTH-WEST CORNER METHOD

- Start with the north-west (upper left) corner cell of the transportation matrix. Compare the supply of source 1 (S_1) with the demand of destination 1 (D_1).
- \gt S₁ > D₁, set X₁₁ = D₁ and proceed horizontally to cell (1,2)
- \rightarrow If $S_1 = D_1$, set $X_{11} = D_1$ and proceed diagonally to cell (2,2)
- \rightarrow If $S_1 < D_1$, set $X_{11} = S_1$ and proceed vertically to cell (2,1)
- Continue the procedure, step by step, away from the north-west corner cell till an allocation is made in the south-east corner cell.

	DC1		DC2		DC3		DC4	
Plant 1	6	2		3		11		7
Plant 2	1	1		0		6		1
Plant 3		5	5	8	3	15	2	9

EXERCISE: NORTH-WEST CORNER METHOD

- It can be easily seen that the proposed solution is a feasible solution since all the supply and requirement constraints are fully satisfied.
- ► The transportation cost with the solution is

EXERCISE: LEAST COST METHOD

A dairy firm has three plants located throughout a state. Daily milk production at each plant is as follows:

Plant 1: 6 million litres; Plant 2: 1 million litres and Plant 3: 10 million litres

Each Day the firm must fulfil the needs of its distribution centres. Minimum requirement at each centre is as follows:

Distribition centre 1: 7 million litres

Distribition centre 2: 5 million litres

Distribition centre 3: 3 million litres

Distribition centre 4: 2 million litres

Cost of shipping one million litres of milk from each plant to each distribution centre is given in the following table in hundred of BDT.

	DC1	DC2	DC3	DC4
Plant 1	2	3	11	7
Plant 2	1	0	6	1
Plant3	5	8	15	9

16-72

Find its initial basic feasible solution by Least Cost method.

EXERCISE: LEAST COST METHOD

- Here, the lowest cell is (2,2) and maximum possible allocation (meeting supply and requirement positions) is made here. Evidently, maximum feasible allocation I cell (2, 2) is 1 m litres. This meets the supply position of plant 2. Therefore, row 2 is crossed out, indicating that no allocations are to be made in cells (2,1), (2,3), and (2,4).
- ► The next lowest cell (excluding the cells in row 2) is (1,1) and maximum possible allocation (meeting supply and requirement positions) of 6 is made here. Now row 1 is crossed out.
- ▶ Next lowest cell in row 3 is (3,1) and allocation is 1 is done here.
- Likewise allocations of 4, 2, and 3 are done in cells (3,2), (3,4) and (3,3) respectively.

	DC1		DC2		DC3		DC4	
Plant 1	6	2		3		11		7
Plant 2		1	1	0		6		1
Plant3	1	5	4	8	3	15	2	9

EXERCISE: LEAST COST METHOD

- It can be easily seen that the proposed solution is a feasible solution since all the supply and requirement constraints are fully satisfied.
- ► The transportation cost with the solution is

$$Z = BDT (2X6 + 0X1 + 5X1 + 8X4 + 15X3 + 9X2) X 100$$

$$= BDT (12+0+5+32+45+18) \times 100$$

$$= BDT 11, 200/00$$