Lecture 23: Intensity

Incoherent Sources: **If the phase difference between the waves emitted by the sources varies randomly then the sources are called incoherent sources.**

Coherent Sources: **If the phase difference between the waves emitted by the sources remains constant then the sources are called coherent sources.**

Intensity in Double-Slit Interference:

Let us assume, at point P the electric field components of the waves from the two slits are

$$E_1 = E_0 \sin \omega t$$

$$E_2 = E_0 \sin (\omega t + \varphi)$$

Where, ω= angular frequency of the waves.

φ= phase difference between the waves.

E_0= amplitude of the waves.

In Fig-1, the waves with components E_1 and E_2 are represented by phasors of magnitude E_0 at time t that rotate around the origin at angular speed ω.

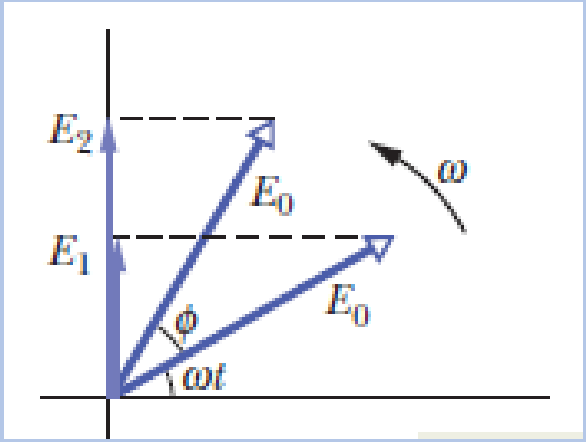


Fig-1

In Fig-2, the vector addition of the two phasors gives the phasor representing the resultant wave, with amplitude E and phase constant $\beta = \frac{\varphi}{2}$.

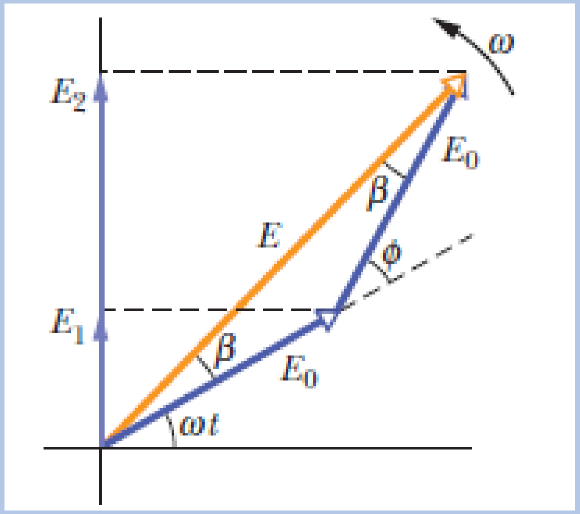


Fig-2

$$E^2 = E_0^2 + E_0^2 + 2E_0.E_0 cos\varphi$$

$$\Rightarrow E^2 = 2E_0^2 + 2E_0^2\, cos\varphi$$

$$\Rightarrow E^2 = 2E_0^2(1 + cos\varphi)$$

$$\Rightarrow E^2 = 2E_0^2.2cos^2\frac{\varphi}{2}$$

$$\therefore E^2 = 4E_0^2\, cos^2\frac{\varphi}{2} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Now, we know that the intensity of an electromagnetic wave is proportional to the square of its amplitude. Therefore, if I_0 is the intensity of the individual waves and I is the intensity of the resultant wave then,

$$\frac{I}{I_0} = \frac{E^2}{E_0^2}$$

$$\Rightarrow \frac{I}{I_0} = \frac{4E_0^2 \, cos^2\frac{\varphi}{2}}{E_0^2}$$

$$\therefore I = 4I_0 cos^2\frac{\varphi}{2} \dots\dots\dots\dots\dots (2)$$

Again the phase difference,

$$\varphi = \frac{2\pi}{\lambda} \times path\ difference$$

$$\boxed{\therefore \varphi = \frac{2\pi d}{\lambda}\ sin\theta}$$

Condition for Intensity Maxima and Minima

From equation (2), we can see that intensity maxima will occur when,

$$\frac{\varphi}{2} = m\pi$$

$$\Rightarrow \varphi = 2m\pi$$

$$\Rightarrow \frac{2\pi d}{\lambda}\, sin\theta = 2m\pi$$

$$\therefore\ d sin\theta = m\lambda \qquad [m = 0, 1, 2, 3, \ldots.]$$

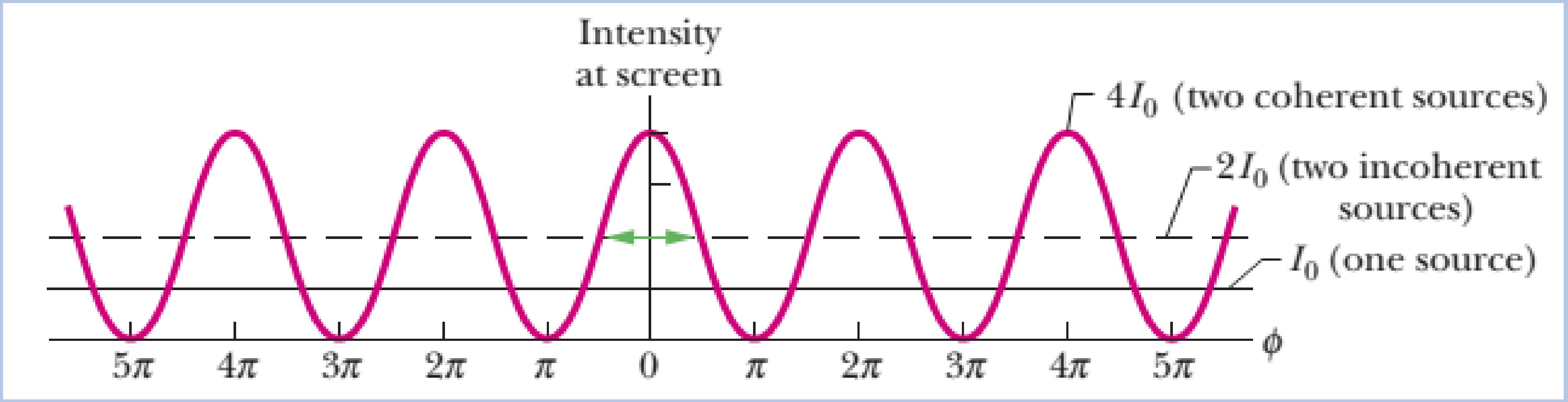
Again the intensity minima will occur when,

$$\frac{\varphi}{2} = \left(m + \frac{1}{2}\right)\pi$$

$$\Rightarrow \varphi = 2\left(m + \frac{1}{2}\right)\pi$$

$$\Rightarrow \frac{2\pi d}{\lambda}\, sin\theta = 2\left(m + \frac{1}{2}\right)\pi$$

$$\therefore\ dsin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad [m = 0, 1, 2, 3, \dots.]$$



Intensity
at screen
4I_0 (two coherent sources)
2I_0 (two incoherent sources)
I_0 (one source)
ϕ
5π
4π
3π
2π
π
0
π
2π
3π
4π
5π

Sample Problem 35.02. **In a double-slit interference pattern, what is the distance on screen C between adjacent maxima near the center of the interference pattern? The wavelength λ of the light is 546 nm, the slit separation d is 0.12 mm, and the slit–screen separation D is 55 cm.**

Given,

The wavelength of the light is $\lambda = 546\ nm = 546 \times 10^{-9} m$

The slit separation $d = 0.12\ mm = 0.12 \times 10^{-3} m$

The slit–screen separation $D = 55\ cm = 0.55 m$

The position of the mth maxima, $y_m = \frac{m\lambda D}{d}$

And the position of the next maxima, $y_{m+1} = \frac{(m+1)\lambda D}{d}$

So, the distance between the adjacent maxima, $\Delta y = Y_{m+1} - Y_m$

$$\Delta y = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d}$$

$$\Delta y = \frac{m\lambda D + \lambda D - m\lambda D}{d}$$

$$\Delta y = \frac{\lambda D}{d} = \frac{(546 \times 10^{-9}) \times 0.55m}{0.12 \times 10^{-3}}$$

$$\therefore \Delta y = 2.50 \times 10^{-3} m$$

29. Two waves of the same frequency have amplitudes 1.00 and 2.00. They interfere at a point where their phase difference is 60.0°. What is the resultant amplitude?

Given: The amplitude of the waves, E1 = 1 and E2 =2

Phase difference = 60.0°.

The resultant amplitude, $\boldsymbol{E} = ?$

$$\boldsymbol{E = \sqrt{E_1^2 + E_2^2 + 2E_1.E_2 cos\varphi}}$$

$$\boldsymbol{\Rightarrow E = \sqrt{1^2 + 2^2 + 2.1.2\ cos\ 60^0}}$$

$$\boldsymbol{\therefore E = 2.65}$$

29. Two waves of the same frequency have amplitudes 1.00 and 2.00. They interfere at a point where their phase difference is 60.0°. What is the resultant amplitude?

Note: The following slides are the intensity theory without phasor. Now it is your choice to follow one of them.

35-3: Intensity in Double-Slit Interference

bright fringe: $d \sin\theta = m\lambda$ for $m = 0, 1, 2, 3 \ldots$

dark fringe: $d \sin\theta = (m + \frac{1}{2})\lambda$ for $m = 0, 1, 2, 3 \ldots$

These two equations tell how to locate the maxima and minima of the double slit interference pattern on screen as a function of the angle.

Here we wish to derive an expression for the intensity of the fringes as a function of the angle. Suppose that the waves emerged from the slits are coherent sinusoidal plane waves. Electric field components of the light waves of point P are not in phase and vary with time as

$$\boldsymbol{E_1 = E_0 \, sin \, \omega t}$$

$$\boldsymbol{E_2 = E_0 \, sin \, (\omega t + \varphi)}$$

For simplicity, we have chosen the point P to be the origin so that the kx dependence in the wave function is eliminated.

Resultant wave: superposition principle

$$E = E_1 + E_2$$

$$= E_0 \sin \omega t + E_0 \sin(\omega t + \varphi)$$

$$= E_0 \{\sin \omega t + \sin(\omega t + \varphi)\}$$

$$= E_0 \{\sin \omega t + \sin(\omega t + \varphi)\}$$

$$= E_0 \{2 \sin\left(\frac{\omega t + \omega t + \varphi}{2}\right) \cos\left(\frac{\omega t - \omega t - \varphi}{2}\right)\}$$

$$= 2E_0 \sin\left(\frac{2\omega t + \varphi}{2}\right) \cos\left(\frac{-\varphi}{2}\right)$$

$$= 2E_0 \sin\left(\omega t + \frac{\varphi}{2}\right) \cos\left(\frac{\varphi}{2}\right)$$

$$E = \left[2E_0 \cos\left(\frac{\varphi}{2}\right)\right] \sin\left(\omega t + \frac{\varphi}{2}\right)$$

Amplitude of the resultant wave = $2E_0 \cos\left(\frac{\varphi}{2}\right)$

oscillating term = $\sin\left(\omega t + \frac{\varphi}{2}\right)$

$$E_m = 2E_0 \cos(\frac{\varphi}{2})$$

$$E_m^2 = (2E_0 \cos\frac{\varphi}{2})^2$$

$$E_m^2 = 4E_0^2 \cos^2\frac{\varphi}{2}$$

Intensity of an electromagnetic wave is proportional to the square of its amplitude.

The resultant wave with amplitude E_m has an intensity I that is proportional to E_m^2 .

$$I \propto E_m^2 \quad \text{[resultant wave]}$$

$$I = kE_m^2 \quad \dots\dots\dots\dots(1) \quad [k = \text{constant}]$$

Each wave with amplitude E_o has an intensity I_o that is proportional to E_0^2 .

$$I_0 \propto E_0^2 \quad \text{[each wave]}$$

$$I_0 = kE_0^2 \quad \dots\dots\dots\dots(2) \quad [k = \text{constant}]$$

(1) / (2)

$$\frac{I}{I_0} = \frac{kE_m^2}{kE_0^2} = \frac{E_m^2}{E_0^2} = \frac{4E_0^2\ \cos^2\frac{\varphi}{2}}{E_0^2} = 4\cos^2\frac{\varphi}{2}$$

$$I = 4I_0\cos^2\frac{\varphi}{2}$$

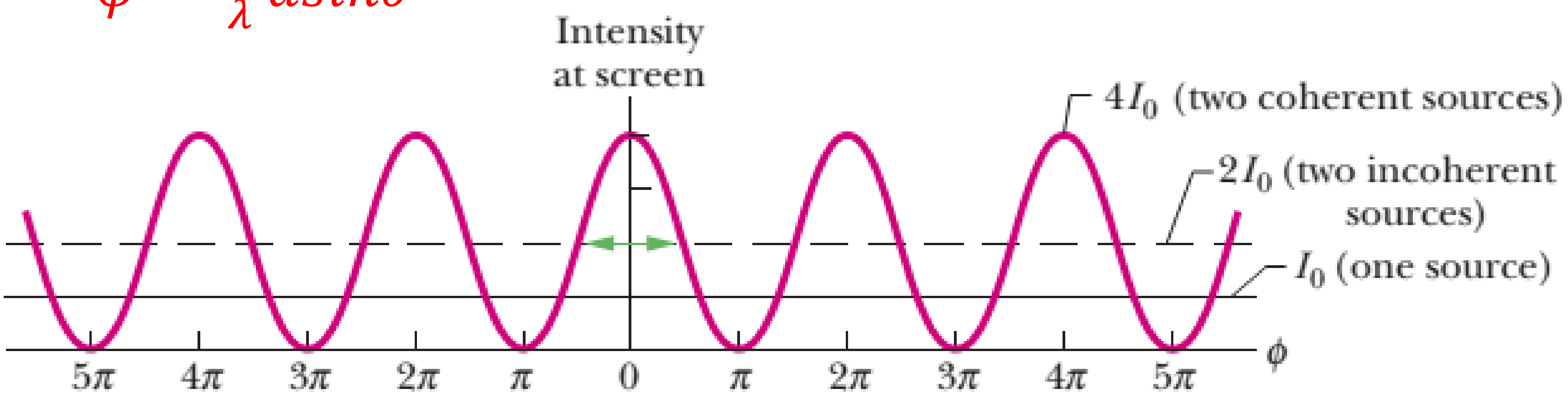
Phase difference φ: The phase difference φ is associate with path difference, $S_1b = d\sin\theta$ as $E_2 = E_0 \sin(\omega t + \varphi)$.

If $S_1b = \lambda$, then $\varphi = 2\pi$

$$\frac{\varphi}{S_1b} = \frac{2\pi}{\lambda}$$

$$\frac{\varphi}{d\sin\theta} = \frac{2\pi}{\lambda}$$

$$\varphi = \frac{2\pi}{\lambda}d\sin\theta$$



Phase difference φ :

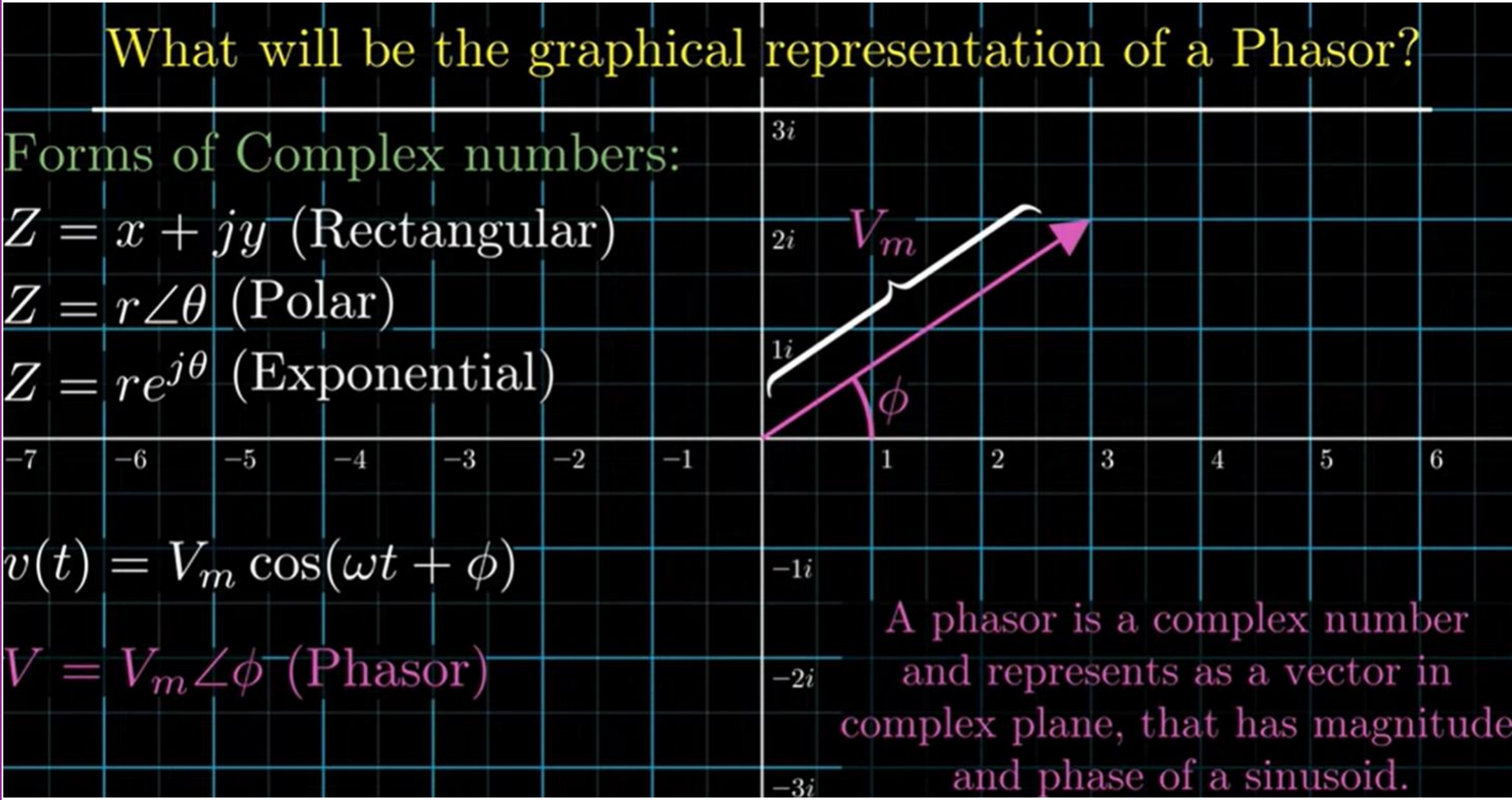
The phase difference φ is associate with path difference, $S_1b = d sin\theta$ as $E_2 = E_0\, sin\,(\omega t + \varphi)$.

If $S_1b = \lambda$, then $\varphi = 2\pi$

$$\frac{\varphi}{S_1 b} = \frac{2\pi}{\lambda}$$

$$\frac{\varphi}{d sin\theta} = \frac{2\pi}{\lambda}$$

$$\varphi = \frac{2\pi}{\lambda} d sin\theta$$


What will be the graphical representation of a Phasor?
Forms of Complex numbers:
$Z = x + jy$ (Rectangular)
$Z = r\angle\theta$ (Polar)
$Z = re^{j\theta}$ (Exponential)
$v(t) = V_m \cos(\omega t + \phi)$
$V = V_m\angle\phi$ (Phasor)
V_m
ϕ
3i
2i
1i
−1i
−2i
−3i
−7
−6
−5
−4
−3
−2
−1
1
2
3
4
5
6
A phasor is a complex number and represents as a vector in complex plane, that has magnitude and phase of a sinusoid.

Phasors

A complex number that represents the amplitude and phase of a sinusoid is called as PHASOR.