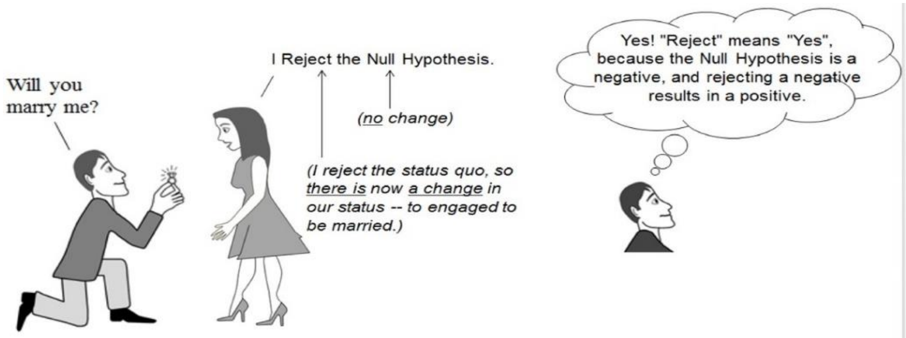
MAT 3103: Computational Statistics and Probability

Chapter 10: Test of Hypothesis



Hypothesis test:

A hypothesis test is a formal way to make a decision based on statistical analysis. A hypothesis test has the following general steps:

- Set up two contradictory hypotheses. One represents our assumption.
- Perform an experiment to collect data.
- Analyze the data using the appropriate distribution.
- Decide if the experimental data contradicts the assumption or not.
- Translate the decision into a clear, non-technical conclusion.

We will build up all the pieces we need, then put them together to test hypothesis.

Parameter: It is the unknown constant characteristic of the population observations. Function of population, population mean (μ), population variance (σ^2) are parameters.

Statistic: It is the function of sample observations. Sample mean ($\overline{x}$), sample variance (s^2) are statistic.

Hypothesis: A hypothesis is a statement about one or more of parameter(s) of a population which we want to verify on the basis of information contained in a sample.

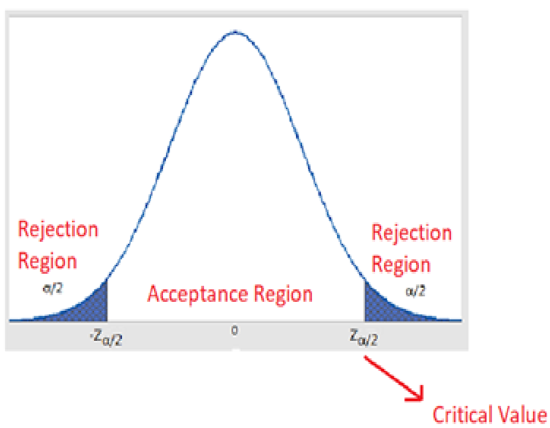
Example: Internet server claims that computer users in **AIUB** spend on the average 15 hours per week on browsing. We conduct a survey based on a sample of 250 users to arrive at a correct decision. Here, the server's claim is referred to as a hypothesis.

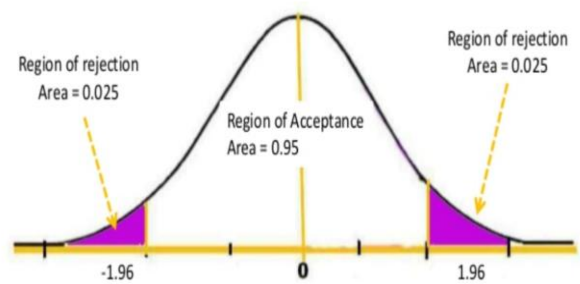
Null hypothesis: It is a statement which tells us that no difference exits between the parameter and the statistic, i.e., given the test scores of two random samples , does one group differ from the other? A possible null hypothesis is, H_0: $\mu_1 = \mu_2$, [μ_1 = mean of population 1, μ_2 = mean of population 2].

Alternative hypothesis: The alternative hypothesis is the logical opposite of the null hypothesis. The rejection of a null hypothesis leads to the acceptance of the alternative hypothesis, i.e., Possible alternative hypothesizes are, H_1: $\mu_1 \neq \mu_2$, or, H_1: $\mu_1 > \mu_2$ or, H_1: $\mu_1 < \mu_2$.

Test statistic: It is the function of sample observations which is used to verify the null hypothesis.

Level of significance: It is the probability with which we want to risk rejecting the null hypothesis even though it is true. We denote it by α; usually α = 0.05.





Acceptance region: If the value of the test statistic falls into the probability space of the distribution of the test statistic and lead us to accept the null hypothesis then the probability space is called the acceptance region.

Critical Region: The probability space in which the test statistic falls and leads us to reject the null hypothesis is called critical region or rejection region. In the given figure the critical and acceptance region are shown.

If $Z \geq Z_{\alpha/2}$, H_0 is rejected in favor of H_1.

If $-Z \leq -Z_{\alpha/2}$, H_0 is rejected in favor of H_1.

If $|Z| \geq Z_{\alpha/2}$, H_0 is rejected in favor of H_1.

Type I error: It is the probability of rejecting the null hypothesis when the null hypothesis is true.

Type II error: It is the probability of accepting the null hypothesis when the null hypothesis is false.

Example: Consider a defendant in a trial. The null hypothesis is "defendant is not guilty;" the alternate is "defendant is guilty." A Type I error would correspond to convicting an innocent person; a Type II error would correspond to setting a guilty person free.

		Reality	
		Not guilty	Guilty
Verdict	Guilty	**Type I Error:** Innocent goes jail	Correct Decision
	Not guilty	Correct Decision	**Type II Error:** Guilty goes free

Test of hypothesis:

It is the statistical process of verifying the null hypothesis using any test statistic. The steps are:

- ⇨ State the null hypothesis, H_0.
- ⇨ State the alternative hypothesis, H_1.
- ⇨ Choose the level of significance, α.
- ⇨ Select an appropriate test statistic.
- ⇨ Calculate the value of the test statistic.
- ⇨ Determine the critical region.
- ⇨ Reject H_0 if the value of the test statistics falls in the critical region; otherwise accept H_0.

Test regarding one mean:

Let $x_1, x_2, \dots, x_n$ be n sample observations drawn independently from a population with mean μ and variance σ^2. Let us assume that the sample observations follow normal distribution, i.e. $x \sim N(\mu, \sigma^2)$. The problem is to test the null hypothesis H_0: $\mu = \mu_0$ against H_1: $\mu \neq \mu_0$.

Assumptions:

1. σ^2 is unknown and n is small ($n < 30$), the test statistic is: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim t_{n-1}$
2. σ^2 is unknown and n is large ($n \geq 30$), the test statistic is: $z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim N(0,1)$
3. σ^2 is known (n is small or large), the test statistic is: $z = \frac{\bar{x}-\mu_0}{\sigma_0/\sqrt{n}} \sim N(0,1)$

Example 10.1: A random sample of 10 persons is selected and their level of education (in completed years of schooling) is recorded. The sample observations are, x: $5, 2, 0, 4, 16, 14, 10, 10, 6, 5$. Do you think that the average schooling years of the persons in population is 5?

Solution: Let, $x \sim N\,(\mu, \sigma^2)$, σ^2 is unknown.

We need to test, H_0: $\mu = \mu_0 = 5$ vs H_1: $\mu \neq \mu_0$.

$\bar{x} = \frac{1}{n}\sum x = \frac{72}{10} = 7.2,\ s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right] = \frac{1}{9}(758 - \frac{72^2}{10}) = 26.62.$

Test statistic: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{7.2-5}{5.16/\sqrt{10}} = 1.35$. Since $|t| < t_9 = 2.262$. So H_0 is accepted.

Hence, we can conclude that the average schooling years can be considered as 5.

Test of equality of two means:

Example 10.2: The number of computer graduates coming out from two different universities A and B are employed in different organizations to do job related to computer.

University	Number of graduates employed in computer related job
A	x_1 : $18, 16, 15, 20, 18, 15, 12$
B	x_2 : $20, 14, 12, 22, 16, 14, 15, 10, 12, 18, 10$

Do you think that the employment facility for both the universities is similar?

Solution: Let, $x_1 \sim N(\mu_1, \sigma_1^2)$, $x_2 \sim N(\mu_2, \sigma_2^2)$. Also, let $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

We need to test, H_0: $\mu_1 = \mu_2$ vs H_1: $\mu_1 \neq \mu_2$.

Both $n_1 = 7$ and $n_2 = 11$ are small (< 30) and σ^2 is not known.

Test statistic: $t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim t_{(n_1-1)+(n_2-1)}$.

$\overline{x_1} = \frac{1}{n_1}\sum x_1 = \frac{114}{7} = 16.29$	$s_1^2 = \frac{1}{n_1-1}\left[\sum x_1^2 - \frac{(\sum x_{1i})^2}{n_1}\right] = \frac{1}{6}(1898 - \frac{114^2}{7}) = 6.905$
$\overline{x_2} = \frac{1}{n_2}\sum x_2 = \frac{163}{11} = 14.82$	$s_2^2 = \frac{1}{n_2-1}\left[\sum x_2^2 - \frac{(\sum x_{2i})^2}{n_2}\right] = \frac{1}{10}(2569 - \frac{163^2}{11}) = 15.364$
$t = \frac{16.29 - 14.82}{\sqrt{12.192\left(\frac{1}{7}+\frac{1}{11}\right)}} = 0.87$	$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{(n_1-1)+(n_2-1)} = \frac{6(6.905) + 10(15.364)}{16}$ 12.192
Since $\lvert t \rvert < t_{16} = 2.12$, H_0 is accepted. Employment facility for students of both universities is same.	

Note	If both n_1 and n_2 are large ($\geq$ 30), then test statistic is: $z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} \sim N(0,1)$

Test of equality of several means:

Example 10.3: There are 25 computers in an office. These computers are supplied by 5 companies in lots of 5. During working hours each computer fails to work for some time (in hour). The failure time of different computers are shown below:

$Company$	$Failure\ time\ (in\ hour)\ of\ computers\ (y_{ij})$					$Total(y_i)$	$Mean(\overline{y_i})$
A	1.5	1	0.5	2	3	8	1.6
B	1	0.5	0.2	0.5	0.5	2.7	0.54
C	0	2	2	0	0	4	0.8
D	2	2.5	1	2	3	10.5	2.1
E	3	3	2	2.5	2.5	13	2.6

Are the computers of different companies alike in failure time?

Solution: We need to test: H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ vs H_1: At least one of them doesn't hold.

$Correction\ Term, CT$ $= \frac{G^2}{n} = \frac{38.2^2}{25} = 58.37$	$Total\ Sum\ of\ Squares, TSS$ $= \sum\sum {y_{ij}}^2 - CT$ $= 85.04 - 58.3 = 26.67$
$SS\ (Company) = \sum \frac{{y_i}^2}{n_i} - CT$ $= \frac{366.54}{5} - 58.37 = 14.94$	$SS\ (Error) = TSS - SS\ (Company)$ $= 26.67 - 14.94$ $= 11.73$

ANOVA TABLE					
Sources of variation	df	SS	$MS = \frac{SS}{df}$	F	$F_{4,20}$
$Company$	$k - 1 = 5 - 1 = 4$	14.94	$\frac{14.94}{4} = 3.73$	6.43	2.87
$Error$	$n - k = 25 - 5 = 20$	11.73	$\frac{11.73}{20} = 0.58$		
$Total$	$n - 1 = 25 - 1 = 24$	$\lvert F \rvert > F_{4,20} = 2.87, H_0 is\ rejected$			
Hence, the computers of different companies are not similar in respect of failure time.					

Test regarding one proportion:

Let $X_1, X_2, \dots, X_N$ be the values of the variable X observed from N population units, where

$$X_i = \begin{cases} 1, if\ i-th\ unit\ possesses\ some\ characteristic\ under\ study \\ 0,\ otherwise \end{cases}$$

Let, A = number of units in the **population** who possess a particular characteristic

a = number of **sample** units possessing the characteristic under study.

So, $P = \frac{A}{N}$ = proportion of **population** units who possess a particular characteristic.

$p = \frac{a}{n}$ = proportion of **sample** units possessing the characteristic.

The problem is to test, H_0: $\boldsymbol{P} = P_0$ vs H_1: $\boldsymbol{P} \neq P_0$.

Test statistic, $z = \frac{p - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} \sim N\,(0, 1)$.

Example 10.4: A sample of 15 students is selected from a group of 100 students and their grade in SSC examination is recorded as follows:

Students	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Grade	B	C	A	D	B	C	D	A	B	C	D	B	C	C	D

Do you think that 10% students get grade A?

Solution: We need to test, H_0: $\boldsymbol{P} = P_0 = 0.10$ vs H_1: $\boldsymbol{P} \neq P_0$.

$$Now, p = \frac{a}{n} = \frac{2}{15} = 0.13.$$

Test statistic: $z = \frac{p - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.13 - 0.10}{\sqrt{\frac{0.10 X 0.90}{15}}} = 0.39$

Since $|z| < 1.96$, H_0 is accepted. It can be considered that 10% students got grade A.

Test regarding two proportions:

Example 10.5: Is the severity of the drug problem in high school the same for boys and girls? 85 boys and 70 girls were questioned and 34 of the boys and 14 of the girls admitted to having tried some sort of drug. What can be concluded at the 0.05 level?

Solution: We need to test $H_0: P_1 = P_2$ vs $H_1: P_1 \neq P_2$.

Test statistic, $z = \frac{p_1 - p_2}{\sqrt{PQ(\frac{1}{n_1}+\frac{1}{n_2})}} \sim N(0,1);\quad P = \frac{a_1+a_2}{n_1+n_2},\ Q = 1 - P.$

$$p_1 = \frac{34}{85} = 0.4,\ p_2 = \frac{14}{70} = 0.2,\ p = \frac{48}{155} = 0.31,\ q = 0.69$$

$$z = \frac{0.4 - 0.2}{\sqrt{(0.31)(0.69)(\frac{1}{85}+\frac{1}{70})}} = 2.68$$

Since $|z|$> 1.96, H_0 is rejected. We conclude that gender does make a difference for drug use.

Test regarding several proportions:

Example 10.6: The following are the number of defective computers of different laboratories:

Laboratories	L_1	L_2	L_3	L_4	Total
No. of defective computers (O_i)	8	15	5	12	40

Are the proportions of defective computers of different laboratories similar?

Solution: We need to test $H_0: P_1 = P_2 = P_3 = P_4$ vs H_1: At least one of them doesn't hold. Test Statistic,

$$\chi^2 = \Sigma \frac{O_i^2}{E_i} - n$$

$$= \frac{1}{10}[8^2 + 15^2 + 5^2 + 12^2] - 40 = 45.8 - 40 = 5.8$$

$$E_i = \frac{n}{k} = \frac{40}{4} = 10$$

Since $\chi^2 < \chi^2_{k-1} = \chi^2_3 = 7.81$, H_0 is accepted. Hence, the proportions of defective computers of different laboratories are similar.

Test of independence:

Let us consider that a researcher has n units in a sample. The sample observations are classified according to qualitative characters, say A and B as follows:

A \ B	B	Not B	$Total\ (R_i)$
A	$O_{11} = a$	$O_{12} = b$	$R_1 = a + b$
Not A	$O_{21} = c$	$O_{22} = d$	$R_2 = c + d$
Total (C_j)	$C_1 = a + c$	$C_2 = b + d$	$n = R_1 + R_2 + C_1 + C_2$

Here, a, b, c, d are observed frequencies in different cells. $n = \sum\sum O_{ij}$

O_{ij}= observation of i^{th} row and j^{th} column recorded from the experiment.

$E_{ij}=\frac{R_i C_j}{n}$ = expected frequency corresponding to i^{th} row and j^{th}column under H_0

We need to test, H_0 : The characters A and B are independent vs H_1: They are not independent.

Test statistic: $\chi^2 = \sum\sum \frac{O_{ij}^2}{E_{ij}} - n \sim \chi^2_{(r-1)(c-1)}$. Here, r = no. of rows and c = no. of columns.

For r = 2 and c = 2, $\chi^2 = \frac{n(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)} \sim \chi^2_1$.

Example 10.7: 150 computer graduates are interviewed and are classified according to their result and job satisfaction. Do you think that the graduates with good result are satisfied with their job?

Result	Job satisfaction		Total
	Yes	No	
Good	22	58	80
Not good	20	50	70
Total	42	108	150

Solution: H_0 : Job satisfaction does not depend on good result

H_1: The good result and job satisfaction are associated

Test statistic: $\chi^2 = \frac{\mathrm{n(ad-bc)^2}}{\mathrm{(a+b)(a+c)(b+d)(c+d)}} = \frac{150(22X50-22X58)^2}{80X70X42X108} = 0.02$

Since $\chi^2 < \chi_1^2 = 3.84$, H_0 is accepted. So, job satisfaction does not depend on good result.

Example 10.8: The following are number of emails of different organizations, where emails are classified according to local and foreign emails. The classified results are shown below:

Origin of emails	Emails of organizations (O_{ij})				
	O_1	O_2	O_3	O_4	Total
Local	8	22	25	20	75
Foreign	17	73	25	10	125
Total	25	95	50	30	200

Is there any association between origin of emails and organizations?

Solution: H_0 : Origin of emails does not depend on organization

H_1 : Origin of emails depends on organization.

Origin of emails	Emails of organizations (E_{ij})				
	O_1	O_2	O_3	O_4	Total
Local	9.375	35.625	18.75	11.25	75
Foreign	15.625	59.375	31.25	18.75	125
Total	25	95	50	30	200

Test statistic: $\chi^2 = \sum\sum \frac{O_{ij}^2}{E_{ij}} - n = \frac{8^2}{9.375} + \frac{22^2}{35.625} + \cdots + \frac{10^2}{18.75} - 200 = 222.88 - 200 = 22.88$

Since $\chi^2 > \chi_3^2 = 7.815$, $H_0$is rejected. Hence, origin of emails depends on organization.

Hints	$E_{11} = \frac{R_1 C_1}{n} = \frac{75 \times 25}{200} = 9.375$

Exercise 10

10.1 Among 157 African-American men, the mean systolic blood pressure was 146 mm Hg with a standard deviation of 27. We wish to know if on the basis of these data, we may conclude that the mean systolic blood pressure for a population of African-American is 140.

10.2 Are the proportions of road accidents similar in various highways of Bangladesh?

Highways	1	2	3	4	Total
No. of road accidents (O_i)	50	42	32	82	206

10.3 Is there any association between subjects taught and job satisfaction?

Job satisfaction	Subjects taught (O_{ij})					Total
	BBA	EEE	CS	CSE	SE	
Yes	12	22	18	15	20	87
No	18	32	30	15	25	120
Total	30	54	48	30	45	207

10.4 Are the proportions of female students similar in various departments of AIUB?

Departments	1	2	3	4	Total
No. of female students (O_i)	250	450	150	150	1000

10.5 For a sample of size 36, $\sum x = 761.6$, $\sum x^2 = 16125.5$. Is the population mean 21?

10.6 A company claims that its batteries have a mean life of 100 hours. You try to verify this for a sample of size 21 with mean 97 hours and variance 9 hours.

10.7 Out of 25 students, 8 are female. Is the overall proportion of female students 0.40 in AIUB?

10.8 A researcher claims that 10-year old children watch 6.6 hours of TV daily on average. You try to verify this for a sample of size 100 with mean 6.1 hours and standard deviation 2.5 hours.

10.9 Is the probation problem the same for boys and girls at AIUB? CGPA of 100 boys and 125 girls were randomly checked. 25 boys and 18 girls were under probation.

10.10 The information about daily temperature (in ^{0} Celsius) of two months are as:

Month 1	n_1 = 31	$\sum x_1$ = 1032	${s_1}^2$= 1.41
Month 2	n_2 = 30	$\sum x_2$ = 1035	${s_2}^2$ = 1.09

Do you think that the temperature of both the months are similar?

10.11 Is high blood pressure associated with heart problem?

Blood Pressure	Heart Problem	
	Yes	No
High	150	120
Not High	122	158

10.12 A group of students are classified by their residential origin and full attention to learning:

Residential origin	Full attention		Total
	Yes	No	
Rural	138	64	202
Urban	64	84	148
Total	202	148	350

Is there any association between origin and full attention?

10.13 Are the computers of different companies alike in failure time?

Company	Failure time (in hour) of computers (x_{ij})					
A	120	122	110	125	118	$\sum\sum x_{ij}$= 2167
B	100	105	108	110	100	
C	95	90	80	90	85	$\sum\sum {x_{ij}}^2$=238943
D	125	130	114	125	115	

10.14 The information about salary of two different institutions (in taka) are as:

Institution 1	n_1 = 15	$\sum x_1$ = 16875	$s_1{}^2$= 5625
Institution 2	n_2 = 20	$\sum x_2$ = 26500	$s_2{}^2$ = 50625

Do you think that the salary information of both institutions are similar?

10.15 A toothpick manufacturer wants every box to contain exactly (on average) 500 toothpicks. In a random sample of 25 boxes, mean is 498 toothpicks and standard deviation is 9 toothpicks.

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17

Sample MCQs

1. A quality control specialist took a random sample of n = 10 pieces of gum and measured their thickness and found the mean 7.6 and standard deviation 0.10. Do you think that the mean thickness of the spearmint gum it produces is 7.5?

a) Reject the null hypothesis
b) Accept the null hypothesis
c) Not concluded
d) None of the above

2. The information about daily temperature (in ° Celsius) of two months are as:

Month 1	$n_1 = 31$	$\Sigma x_1 = 2042$	$s_1^2 = 2.31$
Month 2	$n_2 = 30$	$\Sigma x_2 = 2045$	$s_2^2 = 2.19$

Do you think that the temperature of both the months are similar?

a) Null hypothesis is accepted
b) Null hypothesis is rejected
c) Both a and b
d) None of the above

3. The information about daily temperature (in ° Celsius) of two cities of different days are as:

City 1	$n_1 = 11$	$\Sigma x_1 = 204$	$s_1^2 = 4$
City 2	$n_2 = 15$	$\Sigma x_2 = 209$	$s_2^2 = 5$

Do you think that the temperature of both the cities are similar?

a) Null hypothesis is accepted
b) Null hypothesis is rejected
c) Both a and b
d) None of the above

4. Is gender independent of education level? A random sample of 395 people were surveyed, and each person was asked to report the highest education level they obtained. The data that resulted from the survey is summarized in the following table:

	High School	**Bachelors**	**Masters**	**Ph.d.**	**Total**
Female	60(50.89)	54(49.87)	46(50.38)	41(49.87)	201
Male	40(49.11)	44(48.13)	53(48.62)	57(48.13)	194
Total	100	98	99	98	395

a) Null hypothesis is accepted
b) Null hypothesis is rejected
c) Both a and b
d) None of the above