Find all values of z for which $z^3 + 2 - i2\sqrt{3} = 0$ and also locate these values in complex plane.

$x+iy \to re^{i\theta}$

$r = \sqrt{x^2+y^2}$

$r = 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right)$

$= -\tan^{-1}(\sqrt{3})$

$= -\frac{\pi}{3} + \pi$

$= \frac{2\pi}{3}$

$-2 + i2\sqrt{3} = 4e^{i\frac{2\pi}{3}}$

$r = \sqrt{(-2)^2 + (2\sqrt{3})^2}$

$= \sqrt{16}$

$= 4$

$$\boxed{\begin{aligned}\theta &= -\tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) + \pi \\ &= -\frac{\pi}{3} + \pi \\ &= \frac{2\pi}{3}\end{aligned}}$$

$z^3 + 2 - i2\sqrt{3} = 0$

$\Rightarrow z^3 = -2 + i2\sqrt{3}$

polar form $\to$ $\Rightarrow z^3 = 4e^{i\frac{2\pi}{3}}$ ✓

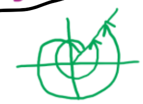
$\Rightarrow z_n^3 = 4e^{i\left(\boxed{\frac{2\pi}{3}} + 2n\pi\right)}$; $n \in \mathbb{Z}$

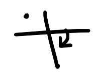
$\Rightarrow z_n^3 = 4e^{i\frac{2\pi + 6n\pi}{3}}$

$\Rightarrow z_n^3 = 4e^{i(6n+2)\frac{\pi}{3}}$

$\Rightarrow \boxed{z_n = 4^{\frac{1}{3}} e^{i(6n+2)\frac{\pi}{9}}}$ → General Solution

where $n \in \mathbb{Z}$





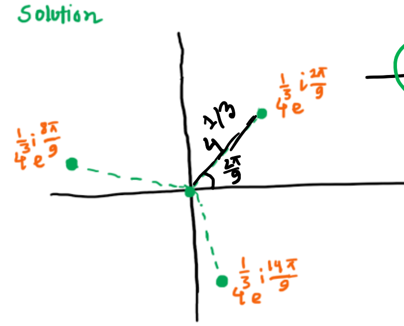
$360°/2\pi$

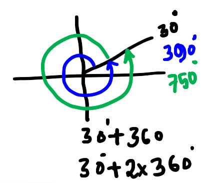
$2 \times 360°/2\pi$

$n=0, \quad z_0 = 4^{\frac{1}{3}} e^{i\frac{2\pi}{9}}$

$n=1, \quad z_1 = 4^{\frac{1}{3}} e^{i\frac{8\pi}{9}}$

$n=2, \quad z_2 = 4^{\frac{1}{3}} e^{i\frac{14\pi}{9}}$

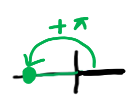




Find all values of z and locate the values in complex plane from $z^4 = -81$

$r = \sqrt{(-81)^2 + 0}$

$r = 81$



$\theta = \tan^{-1}\left(\frac{0}{-81}\right)$

$= \boxed{0} + \pi$

$= \pi$ ✓

$z^4 = -81$

$\Rightarrow z^4 = 81e^{i(\pi)}$

$\Rightarrow z_n^4 = 81e^{i(\pi + 2n\pi)}$

$\Rightarrow z_n^4 = 81e^{i(2n+1)\pi}$

$\Rightarrow z_n = (81)^{\frac{1}{4}} e^{i(2n+1)\frac{\pi}{4}}$

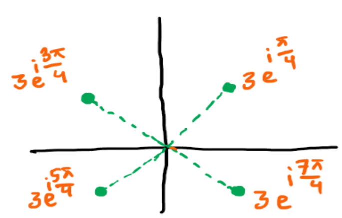
$\Rightarrow z_n = 3e^{i(2n+1)\frac{\pi}{4}}$; $n \in \mathbb{Z}$

$n=0, \quad z_0 = 3e^{i\frac{\pi}{4}}$

$n=1, \quad z_1 = 3e^{i\frac{3\pi}{4}}$

$n=2, \quad z_2 = 3e^{i\frac{5\pi}{4}}$

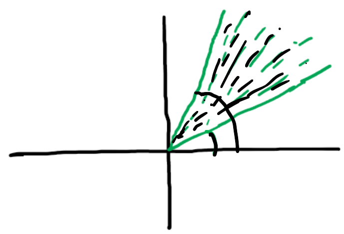
$n=3, \quad z_3 = 3e^{i\frac{7\pi}{4}}$



exercise: 6 (a - d)

Exercise set 7: a, b, f

7.(f) $\frac{\pi}{6} \le \arg z \le \frac{\pi}{3}$



7.(b) $1 < |z-2+i| \le 3$

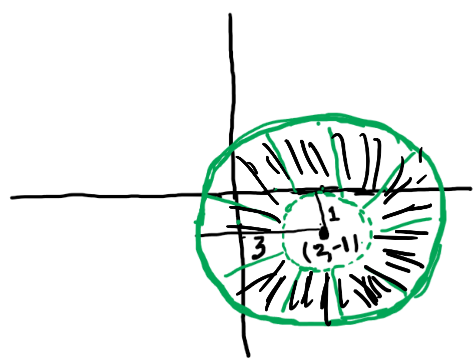
$|z| = |x+iy| = \sqrt{x^2+y^2}$

$\Rightarrow 1 < |(x+iy)-2+i| \le 3$

$\Rightarrow 1 < |(x-2)+i(y+1)| \le 3$

$\Rightarrow 1 < \sqrt{(x-2)^2+(y+1)^2} \le 3$

$\Rightarrow 1^2 < (x-2)^2+(y+1)^2 \le 3^2$



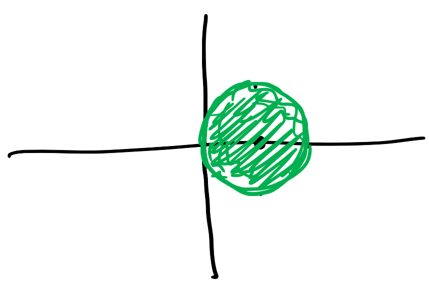
$|z-1| \le 1$

$\Rightarrow |(x+iy)-1| \le 1$

$\Rightarrow |(x-1)+iy| \le 1$

$\Rightarrow \sqrt{(x-1)^2+y^2} \le 1$

$\Rightarrow (x-1)^2+y^2 \le 1$



Find principle argument of $z = \dfrac{(-1-i)^{12}}{(3+\sqrt{3}i)^6(-2+2\sqrt{3}i)^3}$

$-1-i$: $r=\sqrt{2}$, $\theta = \tan^{-1}\left(\frac{-1}{-1}\right) = \frac{\pi}{4}+\pi = \frac{5\pi}{4}$

$-1-i = \sqrt{2}\,e^{i\frac{5\pi}{4}}$

$3+\sqrt{3}i$: $r = 2\sqrt{3}$, $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

$3+\sqrt{3}i = 2\sqrt{3}\,e^{i\frac{\pi}{6}}$

$-2+2\sqrt{3}i$: $r=4$, $\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right) = -\tan^{-1}(\sqrt{3}) = -\frac{\pi}{3}+\pi = \frac{2\pi}{3}$

$-2+2\sqrt{3}i = 4\,e^{i\frac{2\pi}{3}}$

$$z = \frac{\left(\sqrt{2}e^{i\frac{5\pi}{4}}\right)^{12}}{\left(2\sqrt{3}e^{i\pi/6}\right)^6\left(4e^{i\frac{2\pi}{3}}\right)^3}$$

$$= \frac{\cancel{2^6}\, e^{i15\pi}}{\cancel{2^6}\,3^3 e^{i\pi}\cdot 2^6 e^{i2\pi}}$$

$$= \frac{1}{3^3 2^6} e^{i\boxed{12\pi}}$$

$\arg z = 12\pi$

$\arg z = \operatorname{Arg} z + 2n\pi$; $n \in \mathbb{Z}$

$\Rightarrow \operatorname{Arg} z = \arg z - 2n\pi$

$= 12\pi - 10\pi$; $n=5$

$= 2\pi$

or,

$12\pi - 12\pi$; $n=6$

$= 0$

$12\pi / 0$ (Ans)