Lecture 24: Diffraction

General Equation of Diffraction Minima by a Single-Slit:

Let us consider the diffraction pattern of plane waves of light of wavelength λ that are diffracted by a single long, narrow slit of width a in an otherwise opaque screen *B*, as shown in the figure-1.

When the diffracted light reaches viewing screen *C*, waves from different points within the slit undergo interference and produce a diffraction pattern of bright and dark fringes on the screen.

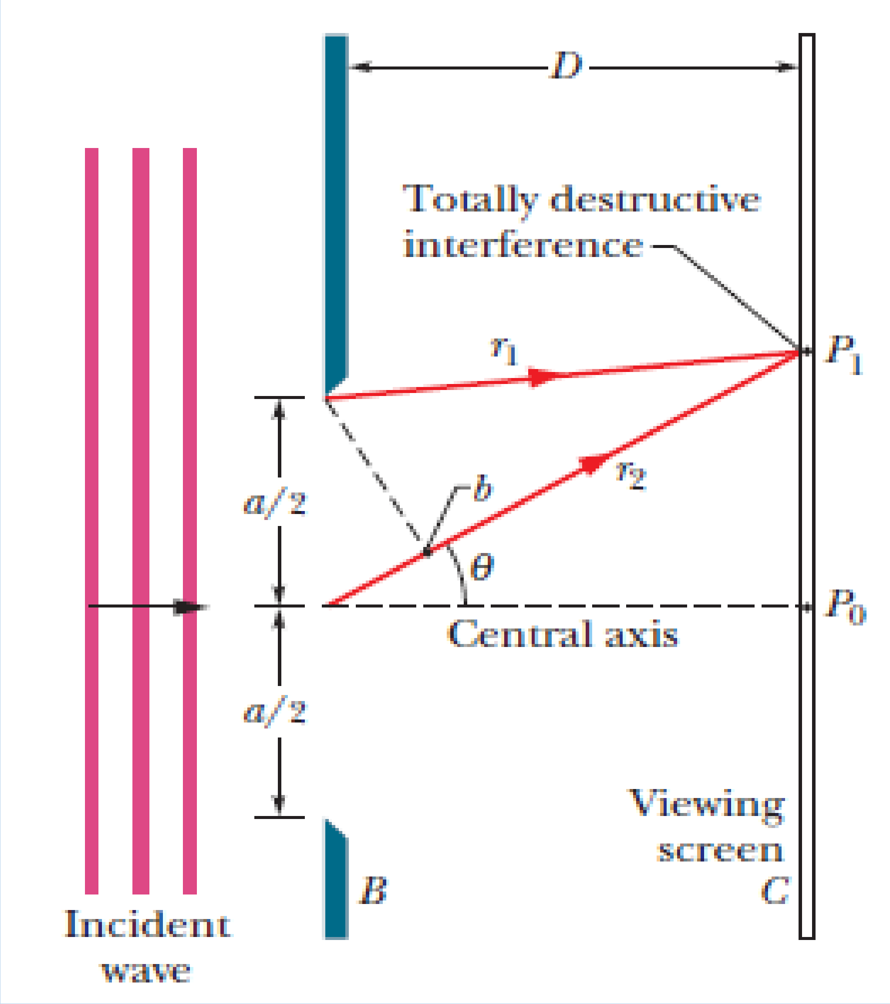

figure-1

To locate the first minima at point P_1, we mentally divide the slit into two *zones* of equal widths *a*/2. Then we extend to P_1 a light ray r_1 from the top point of the top zone and a light ray r_2 from the top point of the bottom zone.

These two rays must be out of phase by $\frac{\lambda}{2}$ when they reach P_1; this phase difference is due to their path length difference.

For $D >> a$ in figure-2 the path length difference between rays r_1 and r_2 is then equal to $\frac{a}{2}\sin\theta$.

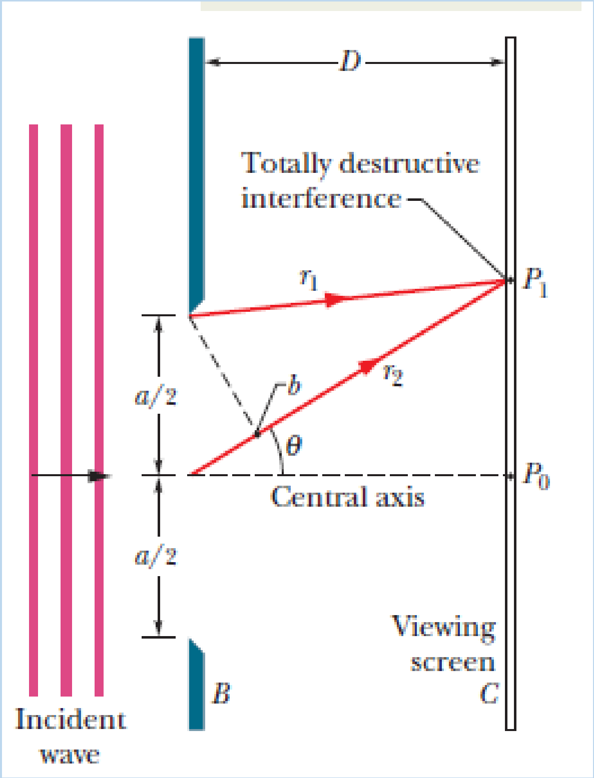

figure-1

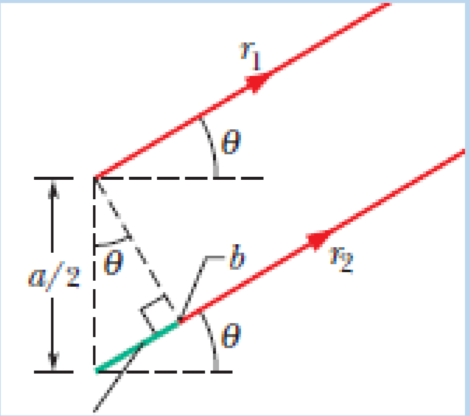

figure-2

Thus for the first minima, $\frac{a}{2} \sin\theta = \frac{\lambda}{2}$

$$\Rightarrow a \sin\theta = \lambda$$

For the second minima at point P_2, we now divide the slit into *four* zones of equal widths *a*/4, as shown in figure-3. We then extend rays r_1, r_2, r_3 and r_4 from the top points of the zones to point P_2.

All of these adjacent rays must be out of phase by $\frac{\lambda}{2}$ when they reach P_2; this phase difference is due to their path length difference.

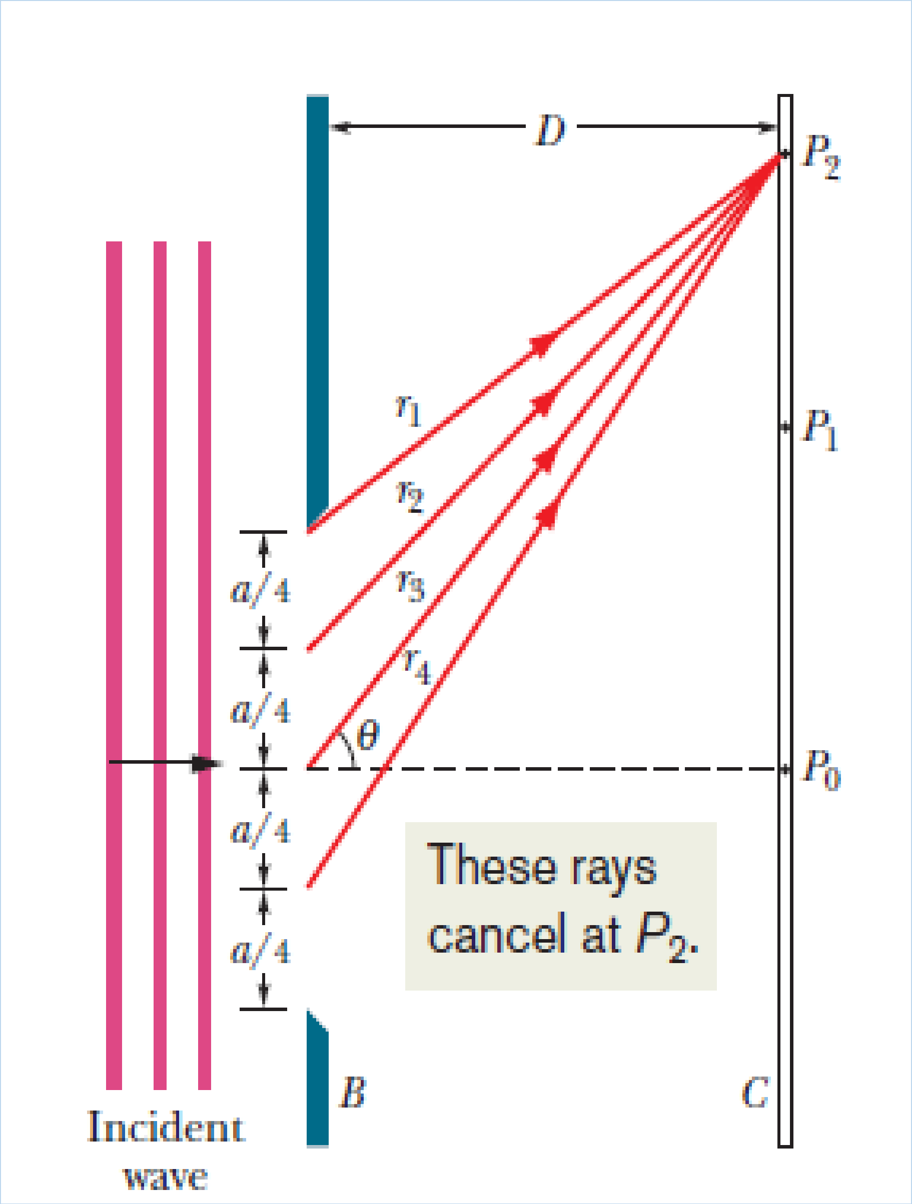

figure-3

For $D >> a$ in figure-4, the path length difference between the adjacent rays is equal to $\frac{a}{4}\sin\theta$.

Thus for the first minima, $$\frac{a}{4}\sin\theta = \frac{\lambda}{2}$$

$$\Rightarrow a\sin\theta = 2\lambda$$

Similarly, other minima can be located in the diffraction pattern by splitting up the slit into more zones of equal width.

Thus the general equation of diffraction Minima is,

$$a\sin\theta = m\lambda \qquad [\, m = 1, 2, 3 \ldots\ldots\ldots \,]$$

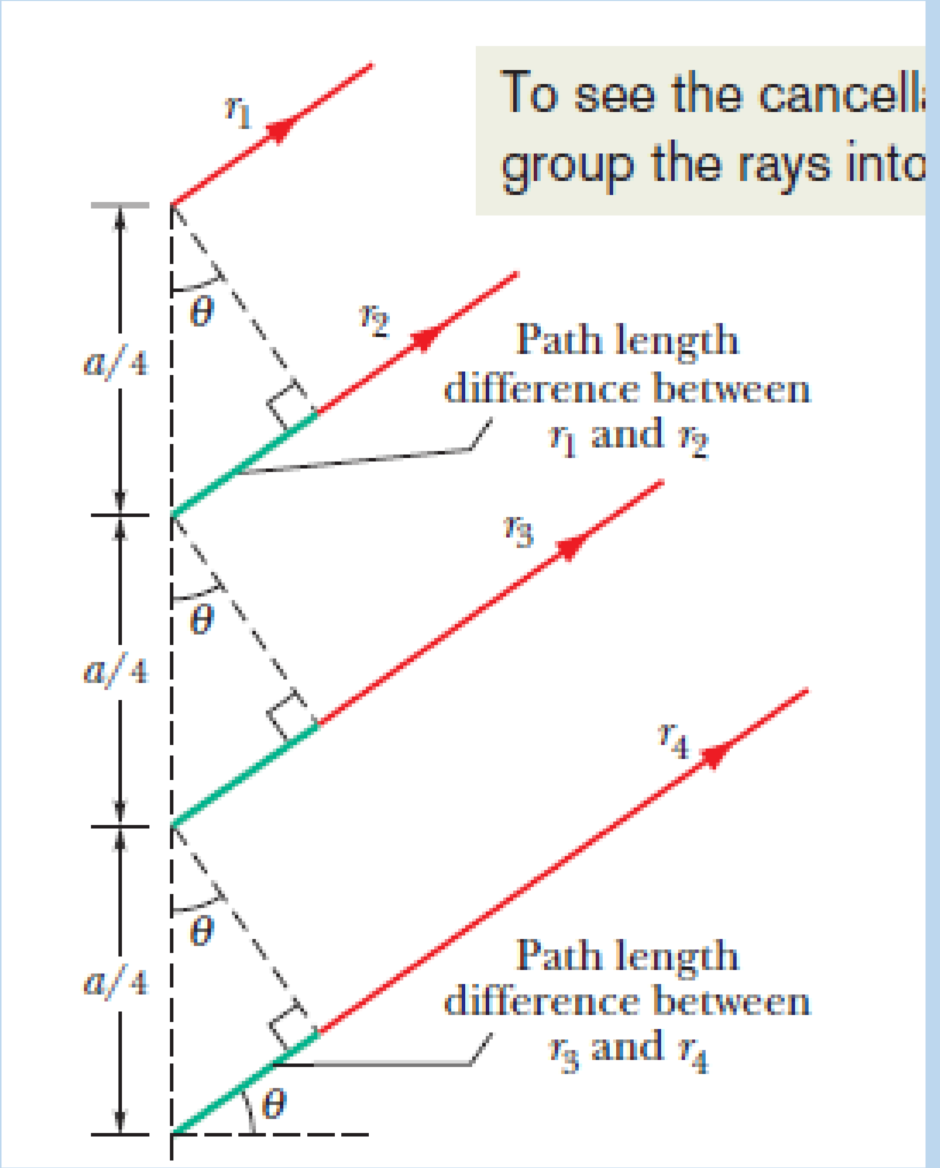

figure-4

2. What must be the ratio of the slit width to the wavelength for a single slit to have the first diffraction minimum at $\theta = 45.0°$?

$Given,\quad \theta = 45.0°$

For the 1st minima, $m = 1$

$\frac{a}{\lambda} = ?$

$$a\,sin\theta = m\lambda$$

$$\Rightarrow \frac{a}{\lambda} = \frac{m}{sin\theta}$$

$$\Rightarrow \frac{a}{\lambda} = \frac{1}{sin\,45^o}$$

$$\boxed{\therefore \frac{a}{\lambda} = \sqrt{2}}$$

7. Light of wavelength 633 nm is incident on a narrow slit. The angle between the first diffraction minimum on one side of the central maximum and the first minimum on the other side is 1.20°. What is the width of the slit?

$Given,$ *Wavelength,* $\lambda = 633\ nm = 633 \times 10^{-9} m$

For the 1st minima, $m = 1$

Angle between two diffraction minima, $2\theta = 1.2°$

$\therefore \theta = 0.6°$

Width of the slit, $a = ?$

$$a\ sin\theta = m\lambda$$

$$\Rightarrow a = \frac{m\lambda}{sin\theta}$$

$$\Rightarrow a = \frac{1(633 \times 10^{-9})}{sin\, 0.6}$$

$$\therefore a = 60448 \times 10^{-9} m$$

Additional problem:

Sample problem 36.01, page: 1085