$z = \frac{(1+i)^2}{1-i}$;

a. convert it into a+ib form

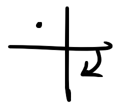
b. Convert it into polar coordinate form

c. Find it's argument & principle argument.

a. $z = \frac{(1+i)^2}{1-i}$

$$= \frac{(1+i)^3}{(1-i)(1+i)} = \frac{1+i^3+3i+3i^2}{1-i^2} = \frac{1-i+3i-3}{2} = \frac{-2+2i}{2} = -1+i$$

b. $r = \sqrt{2}$, $\theta = \tan^{-1}\left(\frac{1}{-1}\right) = -\tan^{-1}(1) = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$



$$\sqrt{2}\, e^{i\frac{3\pi}{4}}$$

c. $\arg z = \frac{3\pi}{4}$; principle argument $= \frac{3\pi}{4}$

Chapter - 5

$f(z) = \ln z$

$z=0$ singular point

$f' = \frac{1}{z}$

Analytic Function: differentiable everywhere at a particular domain.

Singular point: where $f'(z)$ does not exist.

Necessary Condition for $f(z)$ to be analytic:

If $z = x+iy$ and $f(z) = u(x,y) + i\,v(x,y)$ satisfies Cauchy-Riemann (C-R) equations.

$u_x = v_y$ and $u_y = -v_x$ (Rectangular form)

$u_x = \frac{\partial u}{\partial x}$, $v_y = \frac{\partial v}{\partial y}$

then,

$$\boxed{f'(z) = u_x + i v_x}$$

If $z = re^{i\theta}$ then $f(z) = u(r,\theta) + i\,v(r,\theta)$

$u_r = \frac{1}{r} v_\theta$ and $v_r = -\frac{1}{r} u_\theta$ (Polar form)

then,

$$\boxed{f'(z) = e^{-i\theta}(u_r + i v_r)}$$

$f(z) = z^2$

$= (x+iy)^2$

$= x^2 + 2\cdot x\cdot iy + i^2y^2$

$= (x^2 - y^2) + i(2xy)$

$\quad u(x,y) + i\,v(x,y)$

$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$

$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$

$f(z) = e^{2z}$.

i. Find $u(x,y)$ and $v(x,y)$

ii. Prove that $f(z)$ satisfies C-R equations.

iii. Hence find $f'(z)$

$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$

i. $f(z) = e^{2(x+iy)}$

$= e^{2x} \cdot e^{2iy}$

$= e^{2x}(\cos 2y + i \sin 2y)$

$= e^{2x}\cos 2y + i\, e^{2x}\sin 2y$

$u(x,y) = e^{2x}\cos 2y, \quad v(x,y) = e^{2x}\sin 2y$

ii. $u_x = 2\cos 2y\, e^{2x}, \quad v_y = 2e^{2x}\cos 2y \Rightarrow u_x = v_y$

$u_y = -2e^{2x}\sin 2y, \quad v_x = 2\sin 2y\, e^{2x} \Rightarrow u_y = -v_x$

So, satisfies C-R equations.

iii. $f'(z) = \boxed{u_x + i v_x}$

$= 2\cos 2y\, e^{2x} + i\, 2\sin 2y\, e^{2x}$

$= 2e^{2x}(\cos 2y + i \sin 2y)$

$= 2e^{\boxed{2x}}\, e^{i\boxed{2y}}$

$= 2 \cdot e^{2(x+iy)}$

$= 2e^{2z}$

$z^{3/2} \to$ polar form

$z^{-5} \to$ polar form

$f(z) = z^9$

$$= (re^{i\theta})^9$$
$$= r^9 e^{i9\theta}$$
$$= r^9(\cos 9\theta + i\sin 9\theta)$$
$$= r^9\cos 9\theta + i\, r^9 \sin 9\theta$$

$u(r,\theta) = r^9 \cos 9\theta$, $v(r,\theta) = r^9 \sin 9\theta$

$u_r = 9\cos 9\theta\, r^8$, $v_\theta = 9 r^9 \cos 9\theta \Rightarrow u_r = \frac{1}{r} v_\theta$

$u_\theta = -9 r^9 \sin 9\theta$ $v_r = 9\sin 9\theta \cdot r^8 \Rightarrow v_r = -\frac{1}{r} u_\theta$

$$f'(z) = e^{-i\theta}(u_r + i v_r) = e^{-i\theta}(9\cos 9\theta\, r^8 + i 9 \sin 9\theta\, r^8)$$
$$= e^{-i\theta}\, 9 r^8 (\cos 9\theta + i \sin 9\theta)$$
$$= e^{-i\theta}\, 9 r^8 e^{i9\theta}$$
$$= 9 \cdot r^8 e^{i8\theta}$$
$$= 9 (re^{i\theta})^8$$
$$= 9 z^8$$

$$\frac{1}{z^3} = z^{-3} = (re^{i\theta})^{-3}$$
$$= r^{-3} e^{-i3\theta}$$
$$= r^{-3}(\cos 3\theta - i \sin 3\theta)$$
$$= r^{-3}\cos 3\theta - i\, r^{-3} \sin 3\theta$$

$$Re\left\{\frac{1}{z^3}\right\} = r^{-3}\cos 3\theta$$
$$Im\left\{\frac{1}{z^3}\right\} = -r^{-3}\sin 3\theta$$

$\theta = \tan^{-1}(\frac{0}{1})$

$= 0$

$$z^4 = 1 \Rightarrow z^4 = e^{i \cdot 0}$$
$$\Rightarrow z_n^4 = e^{i(0 + 2n\pi)}$$
$$\Rightarrow z_n = e^{i 2n \cdot \frac{\pi}{4}}$$

$$\boxed{e^{-i\theta} = \cos\theta - i\sin\theta}$$