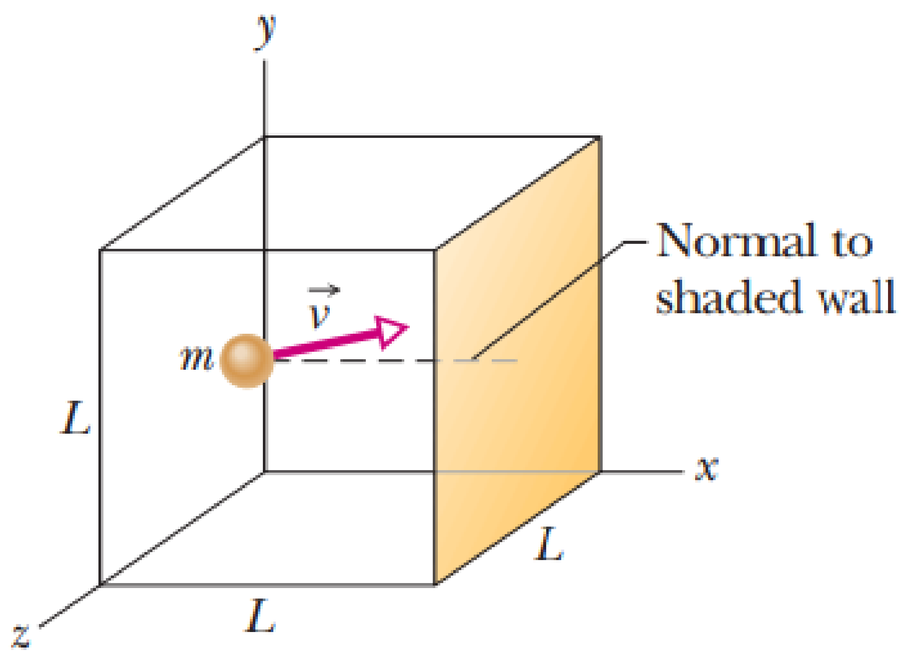
Lecture 6

Chapter 19: The Kinetic Theory of Gases

19-3 Pressure, temperature and rms speed :

Let n moles of an ideal gas be confined in a cubical box of volume $V = L^3$ at temperature T.

The molecules of gas in the box are moving in all directions and with various speeds and consider only elastic collisions with the walls.



Assumptions of Kinetic theory of gases:

The simplest kinetic model is based on the assumptions that:

(1) the gas is composed of a large number of identical molecules moving in random directions, separated by distances that are large compared with their size;

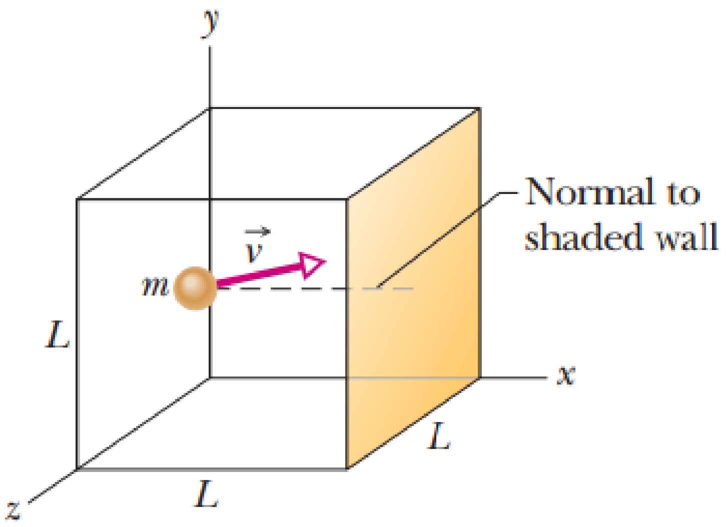
(2) the molecules undergo perfectly elastic collisions (no energy loss) with each other and with the walls of the container, but otherwise do not interact; and

(3) the transfer of kinetic energy between molecules is heat

One molecule: 1D (x-axis)

$$p = \frac{F_x}{A}$$

$$p = \frac{\frac{\Delta p_x}{\Delta t}}{A}$$



$$\Delta\overrightarrow{p_x} = \overrightarrow{p_{xf}} - \overrightarrow{p_{xi}} = \mathrm{m}(-v_x) - \mathrm{m}(+v_x) = -mv_x - \mathrm{m}v_x = -2mv_x$$

$$\Delta p_x = 2mv_x ,$$

However, according to the law of conservation of momentum in one dimension, momentum is transferred to the wall by molecule

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t = \frac{2\mathrm{L}}{v_x}$$

$$A = L^2$$

$$p = \frac{\frac{\Delta p_x}{\Delta t}}{A} = \frac{\Delta p_x}{\Delta t}\left(\frac{1}{A}\right) = \frac{2mv_x}{\frac{2L}{v_x}}\left(\frac{1}{L^2}\right) = 2mv_x\left(\frac{v_x}{2L}\right)\left(\frac{1}{L^2}\right) = \frac{mv_x^2}{L^3} = \frac{mv_x^2}{V}$$

N molecules: 1D (x-axis) N = nN_A and M = mN_A

$$p = \frac{m\,v_x^2}{V} = \frac{m}{V}\left(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \cdots + v_{xN}^2\right)$$

$$p = \frac{m\,N}{V}\left(\frac{v_{x1}^2+v_{x2}^2+v_{x3}^2 + \cdots + v_{xN}^2}{N}\right)$$

$$p = \frac{m\,\mathrm{n}N_A}{V}\left(\frac{v_{x1}^2+v_{x2}^2+v_{x3}^2 + \cdots + v_{xN}^2}{N}\right)$$

$$p = \frac{\mathrm{n}(\mathrm{m}N_A)}{V}\left(v_x^2\right)_{avg}$$

$$p = \frac{nM}{V}\left(v_x^2\right)_{avg}$$

This tells us how the pressure of the gas (a purely macroscopic quantity) depends on the speed of the molecules (a purely microscopic quantity).

N molecules (not an ideal gas): 3D (x, y, z - axes)

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$(v^2)_{avg} = (v_x^2)_{avg} + (v_y^2)_{avg} + (v_z^2)_{avg}$$

$$[v_x = v_y = v_z]$$

$$(v^2)_{avg} = (v_x^2)_{avg} + (v_x^2)_{avg} + (v_x^2)_{avg}$$

$$(v^2)_{avg} = 3(v_x^2)_{avg}$$

$$(v_x^2)_{avg} = \frac{(v^2)_{avg}}{3}$$

$$p = \frac{nM}{V}\frac{(v^2)_{avg}}{3}$$

$$p = \frac{nM}{3V}(v^2)_{avg}$$

$$p = \frac{nM}{3V}v_{rms}^2$$

$$v_{rms}^2 = \frac{3pV}{nM}$$

$$v_{rms} = \sqrt{\frac{3nRT}{nM}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

$$v_{rms}^2 = (v^2)_{avg}$$

Ideal gas, $pV = nRT$

v_{rms} = microscopic property

T = macroscopic property

This is the relation between the rms speed of a microscopic property and the temperature of a macroscopic property.

$$(v_x^2)_{avg} + (v_x^2)_{avg} + (v_x^2)_{avg} = \left(\frac{v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \cdots \qquad + v_{xN}^2}{N}\right)$$

$$\left(\frac{v_{y1}^2 + v_{y2}^2 + v_{y3}^2 + \cdots \qquad + v_{xN}^2}{N}\right) + \left(\frac{v_{z1}^2 + v_{z2}^2 + v_{z3}^2 + \cdots \qquad + v_{xN}^2}{N}\right)$$

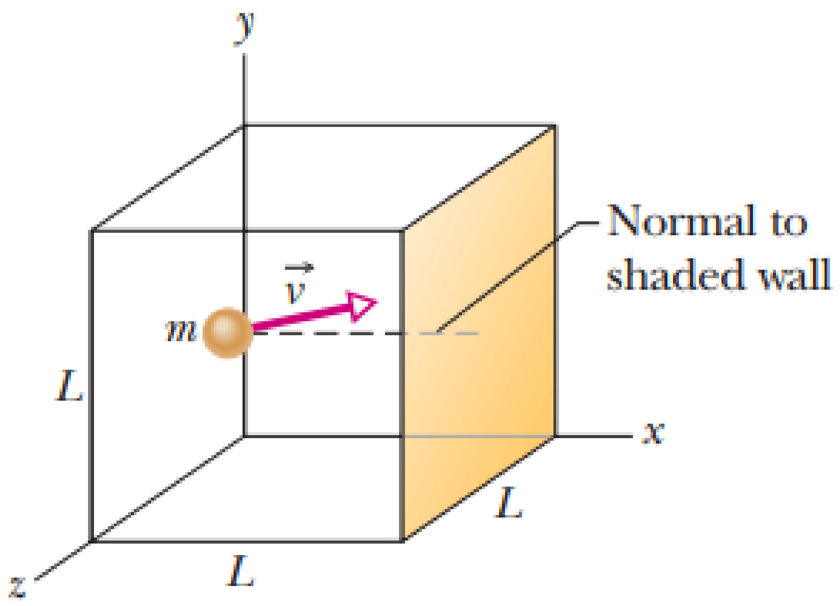
$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$v_{rms} = \sqrt{}(v^2)_{avg}$$

$$v_{rms}^2 = (v^2)_{avg}$$

19-4 Translational kinetic energy :

We consider a single molecule of an ideal gas as it moves around in the box but we now assume that its speed changes when it collides with other molecules. Its translational kinetic energy at any instant $\frac{1}{2}mv^2$. Its *average* translational kinetic energy over the time,



$$K_{avg} = \left(\tfrac{1}{2}mv^2\right)_{avg} = \tfrac{1}{2}m(v^2)_{avg} = \tfrac{1}{2}mv_{rms}^2,$$

$$K_{avg} = \left(\tfrac{1}{2}m\right)\frac{3RT}{M}.$$

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

$$v_{rms}^2 = (v^2)_{avg}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$K_{avg} = \left(\tfrac{1}{2}m\right)\frac{3RT}{M}.$$

$M = mN_A$

$$\frac{m}{M} = \frac{1}{N_A}$$

$$K_{avg} = \frac{3RT}{2N_A}.$$

$$\boxed{K_{avg} = \tfrac{3}{2}kT.}$$

Boltzmann constant, $k = R/N_A$

The average kinetic energy of an atom is a function of temperature.

Problem 18:

The temperature and pressure in the Sun's atmosphere are 2.00x10^6 K and 0.0300 Pa. Calculate the rms speed of free electrons (mass 9.11x10^{-31} kg) there, assuming they are an ideal gas.

Solution:

M = mN_A = **9.11x10^{-31}** (6.023X10^{23})= 5.49×10^{-7} kg

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 2.00 \times 10^6}{5.49 \times 10^{-7}}} = 9.53 \times 10^6 \text{ m/s}$$

$$v_{rms} = 95{,}00000 \text{ m/s}$$

Problem 25

Determine the average value of the translational kinetic energy of the molecules of an ideal gas at temperatures (a) 0.00 ^{0}C and (b) 100 ^{0}C. What is the translational kinetic energy per mole of an ideal gas at (c) 0.00 ^{0}C and (d) 100 ^{0}C?

Solution:

$$\text{(a) } K_{avg} \text{ per molecule} = \left(\frac{3}{2}\right) kT = \frac{3}{2}\left(\frac{R}{N_A}\right) T = \frac{3}{2} \times \frac{8.314}{6.022 \times 10^{23}} (273.0) = 5.654 \times 10^{-21} \text{J}$$

$$\text{(b) } K_{avg} \text{ per molecule} = \left(\frac{3}{2}\right) kT = \frac{3}{2}\left(\frac{R}{N_A}\right) T = \frac{3}{2} \times \frac{8.314}{6.022 \times 10^{23}} (373.0) = 7.724 \times 10^{-21} \text{J}$$

The unit mole may be thought of as a (large) collection: 6.02 × 10^23 molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

(c) K_{avg} per mole $= \mathrm{K}_{avg} N_A = 5.654 \times 10^{-21}$ x 6.022×10^{23} = 3405 J

(d) K_{avg} per mole $= \mathrm{K}_{avg} N_A = 7.724\ \mathrm{J} \times 10^{-21}$ x 6.022×10^{23} = 4651 J