Welcome to Final Term [Fall 2021-22 Semester]

Final-Term Evaluation

Attendance	**: 10 Marks**
Quiz	**: 20 marks**
[Mainly Two (2) Quizzes and Two (2) Make up Quizzes]	
Assignments	**: 20 marks**
Viva	**: 10 marks**
Home Exam [MCQ/FB/TF in Final-Term Full Syllabus]	**: 20 marks**
Home Exam [Math on Final-Term Full Syllabus]	**: 20 marks**
Total	**: 100 marks**

Final-Term Course Descriptions

AC Circuits:

- Phasor Algebraic, Phasor or Vector Diagram.
- *R* branch, *L* branch, *C* branch, *RL* series/Parallel, *RC* series/parallel, *RLC* series/parallel, series-parallel circuits with AC source: Equation of instantaneous voltage, current and power; Total impedances/admittance calculation; power factor, reactive factor, real power, reactive power, apparent power calculation; and Application of VDR, CDR, KVL and KCL
- Source Conversion, Mesh Analysis, Nodal Analysis, Wye–Delta (Y-Δ) or Tee–Pai (T-Π) conversion and Delta–Wye (Δ-Y) or Pai – Tee (Π - T)
- Super position Theorem, Thevenin's Theorem, Norton's Theorem and Maximum Power Transfer Theorem

Electrical Machine:

- **Basic Theories for Electrical Machines**: Electromagnetism, Flemings hand rules, Transformer,
- **DC Machines**: DC generator and DC motor,
- **AC Machines**: Induction motor, Synchronous generator or Alternator, Synchronous Motor, Single phase induction Motor
- **Special Machines**: Stepper Motor, Universal Motor, Servo Motor, Permanent-magnet Synchronous motor, hysteresis motor, Reluctance motor, Linear motor

Review on AC Circuit

Instantaneous value: $e(t)$, $v(t)$, $i(t)$, $p(t)$ etc.

Peak or Crest value: E_m, V_m, I_m

Peak-to-peak value: $E_{p\text{-}p}$, $V_{p\text{-}p}$, $I_{p\text{-}p}$

Period: T [s]

Frequency: f [Hz]

Angular Frequency: ω [rad/s]

Phase angle: [θ_e, θ_v and θ_i]

Initial Angle: [θ_{e0}, θ_{v0} and θ_{i0}]

Angle Difference: Voltage Angle – Current Angle

Phase Difference: | Angle Difference |

Phase Relation: [in phase, leading, lagging]

$$\boxed{f = \frac{1}{T}\ \text{Hz}}$$

$$\boxed{\omega = \frac{\alpha}{t}\ \text{rad/s}}$$

$$\boxed{\begin{aligned}\omega &= \frac{2\pi}{T}\ \text{rad/s}\\ &= 2\pi f\ \text{rad/s}\end{aligned}}$$

The instantaneous or time domain equation

$$e(t) = E_m\sin(\alpha + \theta_e)\ \text{V} = E_m\sin(\omega t + \theta_e)\ \text{V}$$

$$v(t) = V_m\sin(\alpha + \theta_v)\ \text{V} = V_m\sin(\omega t + \theta_v)\ \text{V}$$

$$i(t) = I_m\sin(\alpha + \theta_i)\ \text{A} = I_m\sin(\omega t + \theta_i)\ \text{A}$$

Initial Angle = – Phase Angle

Angle Difference= Voltage angle – Current Angle = 0^o
Phase Relation: $v(t)$ and $i(t)$ are **in phse**.

Angle Difference= Voltage angle – Current Angle > 0^o
Phase Relation: $v(t)$ **leads** $i(t)$, or $i(t)$ **lags** $v(t)$

Angle Difference= Voltage angle – Current Angle < 0^o
Phase Relation: $v(t)$ **lags** $i(t)$, or $i(t)$ **leads** $v(t)$

$$\cos\alpha = \sin(\alpha + 90^o)$$
$$-\cos\alpha = \sin(\alpha - 90^o)$$
$$-\sin\alpha = \sin(\alpha \pm 180^o)$$

Average Value or Mean Value

For asymmetrical wave:

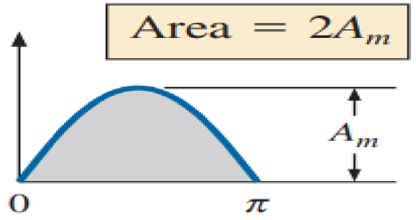
$$\text{Average Value} = \frac{\text{Area under the curve in one cycle}}{\text{Duration of one cycle}}$$

$$I_{ave} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{2\pi}\int_0^{2\pi} i(\theta)d\theta$$

For symmetrical wave:

$$\text{Average Value} = \frac{\text{Area under the curve in } half-\text{cycle}}{\text{Duration of } half-\text{cycle}}$$

$$I_{ave} = \frac{1}{T/2}\int_0^{T/2} i(t)dt = \frac{1}{\pi}\int_0^{\pi} i(\theta)d\theta$$



RMS or Effective Value

Analytical or Integral Method:

$$I_{rms} = \sqrt{\frac{\int_0^T i^2(t)\, dt}{T}} \qquad \textbf{(13.31)}$$

$$I_{rms} = \sqrt{\frac{\text{area } (i^2(t))}{T}} \qquad \textbf{(13.32)}$$

$$I_{ave} = \frac{\pi}{2} I_m = 0.637 I_m$$

$$E_{ave} = \frac{\pi}{2} E_m = 0.637 E_m$$

$$V_{ave} = \frac{\pi}{2} V_m = 0.637 V_m$$

$$I = I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$E = E_{rms} = \frac{E_m}{\sqrt{2}} = 0.707 E_m$$

$$V = V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

Chapter 14

The Basic Elements and Phasors

Phasor Algebra/Complex Number

Vector Quantities Represent by Complex Number :

1. Magnitude **2.** Direction

Phasor Quantities Represent by Complex Number:

1. Magnitude (RMS value for voltage and current)
2. Direction (Phase angle)
3. Continuously change with respect to time [such as sine and cosine waves)

Complex Number can be represented by three different ways:

1. Polar or Phasor form
2. Cartesian or Rectangular form
3. Exponential form

14.7 RECTANGULAR FORM:

$$\boxed{\boldsymbol{C} = X + jY \qquad (\mathbf{14.17})}$$

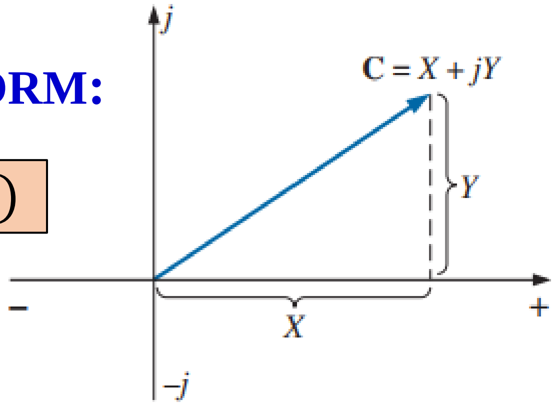


FIG. 14.39 *Defining the rectangular form.*

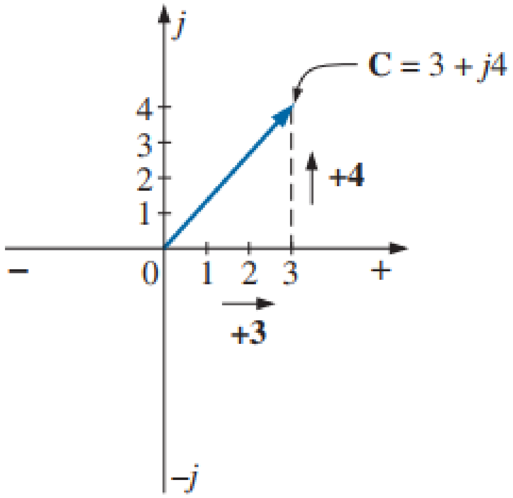
$$\boxed{j = \sqrt{-1}} \qquad (14.24)$$

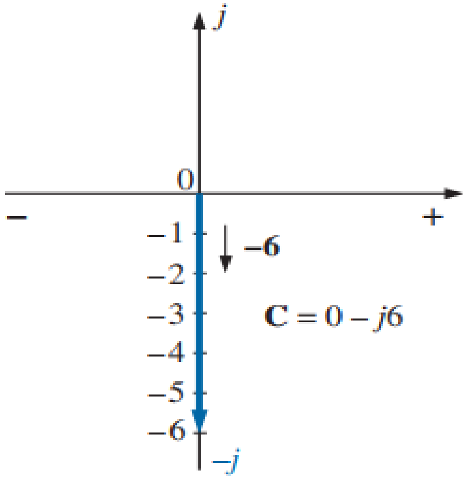
$$\boxed{j^2 = -1} \qquad (14.25)$$

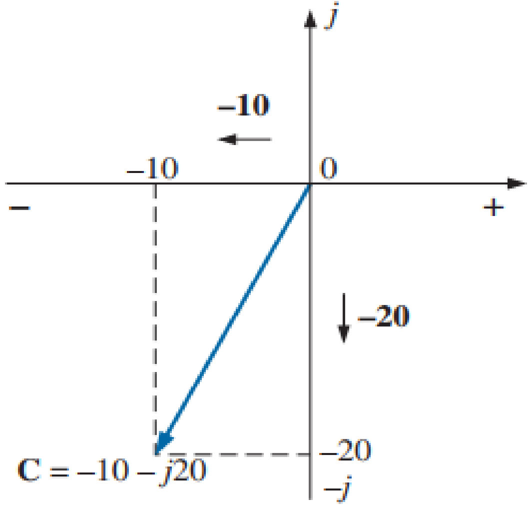
$$\boxed{\frac{1}{j} = -j} \qquad (14.26)$$

EXAMPLE 14.13 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 3 + j4$ b. $\mathbf{C} = 0 - j6$ c. $\mathbf{C} = -10 - j20$







14.8 POLAR OR PHASOR FORM:

$$\boldsymbol{C = Z\angle\theta} \qquad \mathbf{(14.18)}$$

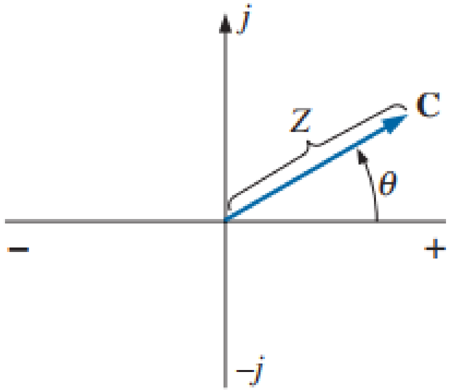


FIG. 14.43 *Defining the polar form.*

$$\boldsymbol{C} = -Z\angle\theta = -Z\angle\theta \pm 180° \ \mathbf{(14.19)}$$

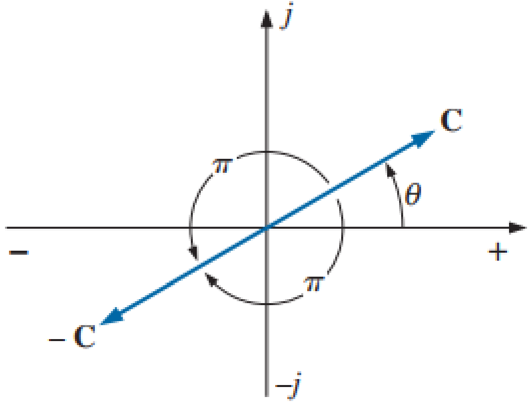
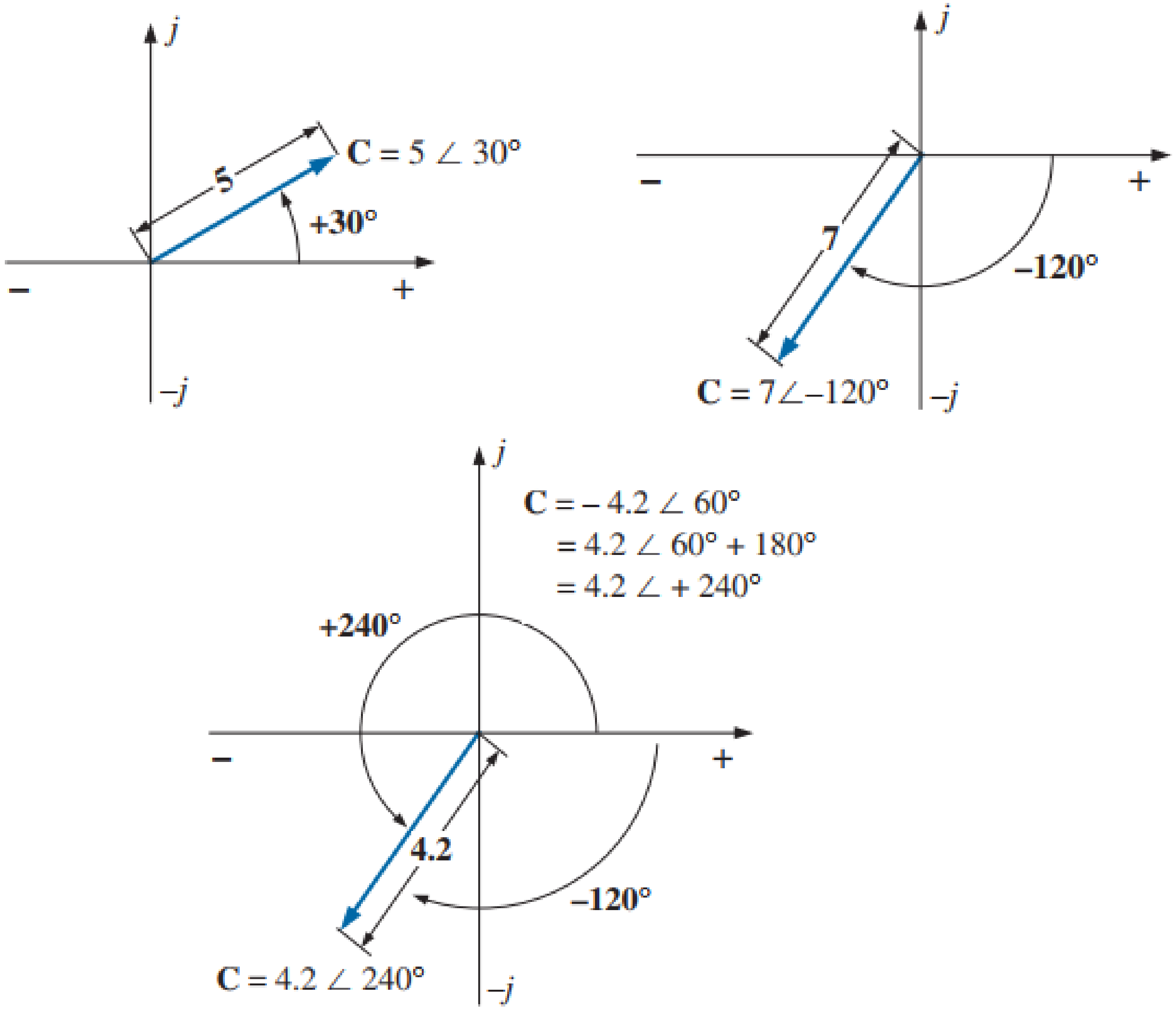


FIG. 14.44
Demonstrating the effect of a negative sign on the polar form.

EXAMPLE 14.14 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 5\ \angle 30°$ b. $\mathbf{C} = 7\ \angle{-120°}$ c. $\mathbf{C} = -4.2\ \angle 60°$



14.9 CONVERSION BETWEEN FORMS

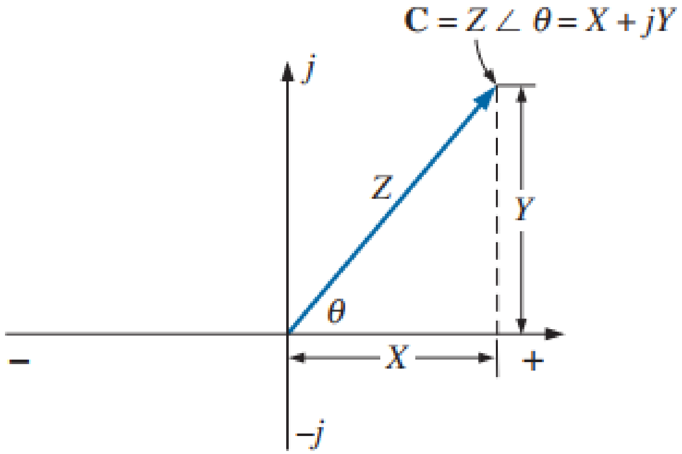


FIG. 14.48 *Conversion between forms.*

Rectangular to Polar

$$\boxed{Z = \sqrt{X^2 + Y^2}} \qquad \textbf{(14.20)}$$

$$\boxed{\theta = \tan^{-1}\frac{Y}{X}} \qquad \textbf{(14.21)}$$

Polar to Rectangular

$$\boxed{X = Z\cos\theta} \qquad \textbf{(14.22)}$$

$$\boxed{Y = Z\sin\theta} \qquad \textbf{(14.23)}$$

EXAMPLE 14.15 Convert the following from rectangular to polar form:

$$\mathbf{C} = 3 + j4 \qquad \text{(Fig. 14.49)}$$

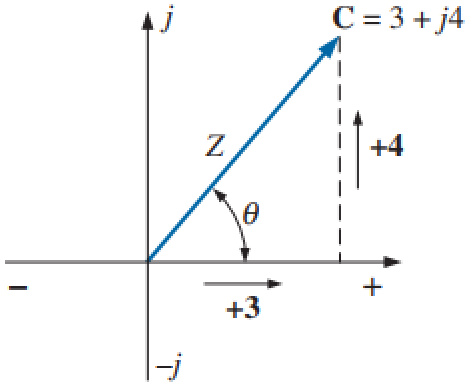


FIG. 14.49 *Example 14.15.*

Solution:

$$Z = \sqrt{(3)^2 + (4)^2}$$
$$= \sqrt{25} = 5$$
$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

and $\mathbf{C = 5\angle 53.13°}$

EXAMPLE 14.16 Convert the following from polar to rectangular form:

$$\mathbf{C} = 10\,\angle 45° \qquad \text{(Fig. 14.50)}$$

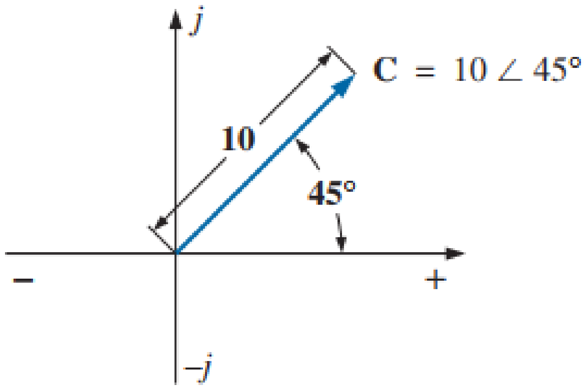


FIG. 14.50 *Example 14.16.*

Solution:

$$X = 10\cos 45°$$
$$= (10)(0.707)$$
$$= 7.07$$
$$Y = 10\sin 45°$$
$$= (10)(0.707)$$
$$= 7.07$$

and $\mathbf{C = 7.07 + \mathit{j}7.07}$

14.10 MATHEMATICAL OPERATIONS WITH COMPLEX NUMBERS

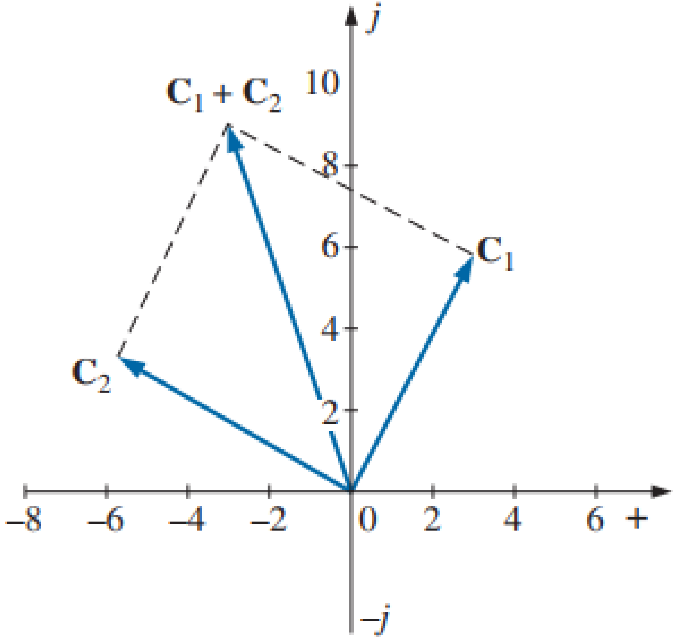
Addition

$\mathbf{C}_1 = \pm X_1 \pm jY_1$ and $\mathbf{C}_2 = \pm X_2 \pm jY_2$

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)} \qquad (14.27)$$

EXAMPLE 14.19 Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

Solutions: $\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$



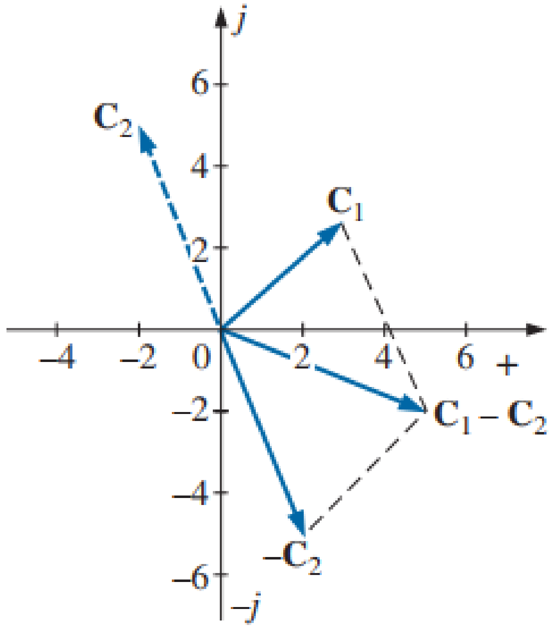
Subtraction

$\mathbf{C}_1 = \pm X_1 \pm jY_1$ and $\mathbf{C}_2 = \pm X_2 \pm jY_2$

$$\boxed{\mathbf{C}_1 - \mathbf{C}_2 = [\pm X_1 - (\pm X_2)] + j[\pm Y_1 - (\pm Y_2)]} \qquad (14.28)$$

EXAMPLE 14.20 Subtract $\mathbf{C}_2 = -2 + j5$ from $\mathbf{C}_1 = +3 + j3$.

Solutions: $\mathbf{C}_1 - \mathbf{C}_2 = [3 - (-2)] + j(3 - 5) = \mathbf{5 - j2}$



Multiplication

$\mathbf{C}_1 = X_1 + jY_1$ and $\mathbf{C}_2 = X_2 + jY_2$

then $\mathbf{C}_1 \cdot \mathbf{C}_2$:

$$
\begin{array}{r}
X_1 + jY_1 \\
\underline{X_2 + jY_2} \\
X_1X_2 + jY_1X_2 \\
\underline{+ jX_1Y_2 + j^2Y_1Y_2} \\
X_1X_2 + j(Y_1X_2 + X_1Y_2) + Y_1Y_2(-1)
\end{array}
$$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)} \qquad \mathbf{(14.29)}$$

Division

$\mathbf{C}_1 = X_1 + jY_1$ and $\mathbf{C}_2 = X_2 + jY_2$

$$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{(X_1 + jY_1)(X_2 - jY_2)}{(X_2 + jY_2)(X_2 - jY_2)}$$

$$= \frac{(X_1X_2 + Y_1Y_2) + j(X_2Y_1 - X_1Y_2)}{X_2^2 + Y_2^2}$$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}} \qquad \mathbf{(14.31)}$$

Multiplication in Polar Form:

$\mathbf{C}_1 = Z_1 \angle\theta_1$ and $\mathbf{C}_2 = Z_2 \angle\theta_2$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1Z_2 \angle \theta_1 + \theta_2} \qquad \mathbf{(14.30)}$$

Division in Polar Form:

$\mathbf{C}_1 = Z_1 \angle\theta_1$ and $\mathbf{C}_2 = Z_2 \angle\theta_2$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2} \angle \theta_1 - \theta_2} \qquad \mathbf{(14.32)}$$

EXAMPLE 14.23

a. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = 5 \angle 20°$ and $\mathbf{C}_2 = 10 \angle 30°$

b. Find $\mathbf{C}_1 \cdot \mathbf{C}_2$ if $\mathbf{C}_1 = 2 \angle -40°$ and $\mathbf{C}_2 = 7 \angle +120°$

Solutions:

a. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (5 \angle 20°)(10 \angle 30°) = (5)(10)\ \underline{/20° + 30°} = \mathbf{50 \angle 50°}$

b. $\mathbf{C}_1 \cdot \mathbf{C}_2 = (2 \angle -40°)(7 \angle +120°) = (2)(7)\ \underline{/-40° + 120°}$
$= \mathbf{14 \angle +80°}$

EXAMPLE 14.25

a. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 15 \angle 10°$ and $\mathbf{C}_2 = 2 \angle 7°$.
b. Find $\mathbf{C}_1/\mathbf{C}_2$ if $\mathbf{C}_1 = 8 \angle 120°$ and $\mathbf{C}_2 = 16 \angle -50°$.

Practice Problem 39 ~ 49 [Ch. 14]

Solutions:

a. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{15 \angle 10°}{2 \angle 7°} = \frac{15}{2}\ \underline{/10° - 7°} = \mathbf{7.5 \angle 3°}$$

b. $$\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{8 \angle 120°}{16 \angle -50°} = \frac{8}{16}\ \underline{/120° - (-50°)} = \mathbf{0.5 \angle 170°}$$

APPLICATION OF COMPLEX NUMBERS IN AC CIRCUIT

Instantaneous Form (Time Domain) Equation:

$$e(t) = E_m sin(\omega t + \theta_e)\ \text{V}$$
$$v(t) = V_m sin(\omega t + \theta_v)\ \text{V}$$
$$i(t) = I_m sin(\omega t + \theta_i)\ \text{A}$$

Phasor Form (Polar Form) Equation:

$$\boldsymbol{E} = \vec{E} = E_{rms}\angle\theta_e = E\angle\theta_e\,\text{V}$$
$$\boldsymbol{V} = \vec{V} = V_{rms}\angle\theta_v = V\angle\theta_v\,\text{V}$$
$$\boldsymbol{I} = \vec{I} = I_{rms}\angle\theta_i = I\angle\theta_i\,\text{A}$$

Rectangular Form (Cartesian Form) Equation:

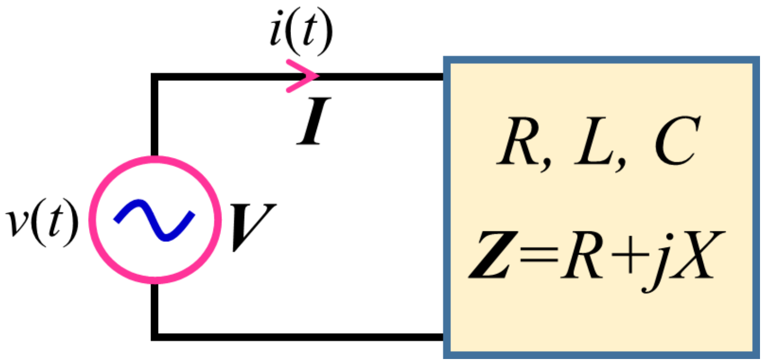
$$\boldsymbol{E} = \vec{E} = E_r + jE_i\ \ \text{V}$$
$$\boldsymbol{V} = \vec{V} = V_r + jV_i\ \ \text{V}$$
$$\boldsymbol{I} = \vec{I} = I_r + jI_i\ \ \text{A}$$

EXAMPLE 14.27 Convert the following from the time to (***i***) the phasor domain, and (***ii***) the rectangular domain.

Time Domain	Phasor Domain	Rectangular Domain
(***a***) $v(t) = 70.7\sin(\omega t - 60^o)$ V	$\vec{V} = (0.707\times70.7)\text{V}\angle{-60^o} = \mathbf{50V\angle{-60^o}}$	$\boldsymbol{V} = 25 - j43.3$ V
(***b***) $i(t) = 21.21\cos(\omega t + 20^o)$ A $= 21.21\sin(\omega t + 110^o)$ A	$\vec{I} = (0.707\times21.21)\text{A}\angle110^o = \mathbf{15A\angle110^o}$	$\boldsymbol{I} = 15[\cos(110^o)+j\sin(110^o)]$ $= -5.13 + j14.1$ A
(***c***) $e(t) = -\ 200\cos\omega t$ V $= 200\sin(\omega t - 90^o)$ V	$\vec{E} = (0.707\times200)\text{V}\angle{-90^o} = \mathbf{141.42V\angle{-90^o}}$	$\boldsymbol{E} = 141.42[\cos(-90^o)+j\sin(-90^o)]$ $= 0 - j141.42$ V
(***d***) $i(t) = -4.5\sin(\omega t + 30^o)$ A $= 4.5\sin(\omega t - 150^o)$ A $= 4.5\sin(\omega t + 210^o)$ A	$\vec{I} = (0.707\times4.5)\text{A}\angle{-150^o} = \mathbf{3.18A\angle{-150^o}}$ $\vec{I} = (0.707\times4.5)\text{A}\angle210^o = \mathbf{3.18A\angle210^o}$	$\boldsymbol{I} = 3.18[\cos(210^o)+j\sin(210^o)]$ $= -2.75 - j1.59$ A

EXAMPLE 14.27.1 Convert the following from Cartesian form to (***i***) the phasor domain, and (***ii***) the instantaneous form for 50 Hz.

Rectangular Form	Phasor Form	Instantaneous Form
(***a***) $\vec{V} = 25 - j43.3$ V RMS value: 50 V Phase Angle: –60º Peak Value: 70.7 V	$V = \sqrt{25^2 + (-43.3)^2} = 50$ V $\theta_v = \tan^{-1}\left[\frac{-43.3}{25}\right] = -60^o$ $\boldsymbol{V = 50V\angle{-60^o}}$	$\omega = 2\pi\times50 = 314$ rad/s $v(t) = (\sqrt{2})\times50\sin(314t - 60^o)$ V $= \boldsymbol{70.7\sin(314t - 60^o)}$ **V**
(***b***) $\vec{E} = j150$ V RMS value: 150 V Phase Angle: 90º Peak Value: 212.13 V	$E = \sqrt{0^2 + 150^2} = 150$ V $\theta_e = \tan^{-1}\left[\frac{150}{0}\right] = 90^o$ $\boldsymbol{E = 150V\angle{90^o}}$	$e(t) = (\sqrt{2})\times150\sin(314t + 90^o)$ V $= \boldsymbol{212.13\sin(314t + 90^o)}$ **V** $= \boldsymbol{212.13\cos314t}$ **V**
(***d***) $\vec{I} = -j5$ A RMS value: 5 A Phase Angle: –90º Peak Value: 7.07 A	$I = \sqrt{0^2 + (-5)^2} = 5$ A $\theta_i = \tan^{-1}\left[\frac{-5}{0}\right] = -90^o$ $\boldsymbol{I = 5A\angle{-90^o}}$	$i(t) = (\sqrt{2})\times5\sin(314t - 90^o)$ A $= \boldsymbol{7.07\sin(314t - 90^o)}$ **A** $= \boldsymbol{-7.07\cos314t}$ **A**
(***e***) $\vec{V} = -100$ V RMS value: 100 V	$\boldsymbol{V = 100V\angle{\pm180^o}}$ Phase Angle: ± 180º Peak Value: 141.42 V	$v(t) = (\sqrt{2})\times100\sin(314t \pm 180^o)$ V $= \boldsymbol{141.42\sin(314t \pm 180^o)}$ **V** $= \boldsymbol{-141.42\sin314t}$ **V**



IMPEDANCE

Impedance: Impedance is the ratio of **voltage** to **current**.

Impedance opposes the flow of current.

Impedance represent by **Z**. Its unit is **ohm** (Ω).

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{V_{rms}\angle\theta_v}{I_{rms}\angle\theta_i} = \frac{V\angle\theta_v}{I\angle\theta_i} = \frac{V}{I}\angle(\theta_v - \theta_i) = Z\angle\theta_z = R + jX \;\; \Omega$$

Magnitude of Impedance: $Z = \frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}} = \frac{V}{I} \quad \Omega$

Angle of Impedance: $\theta_z = \theta_v - \theta_i$

Resistance (Real Part of Impedance): $R = Z\cos\theta_z \quad \Omega$

Reactance (Imaginary Part of Impedance): $X = Z\sin\theta_z \quad \Omega$

Practically, $-90^{o} \leq \theta_z \leq 90^{o}$

Reactance is the property of inductor and capacitor to oppose the flow of current. The are two reactance in electrical circuit: (***i***) **inductive reactance** (X_L), and (***ii***) **capacitive reactance** (X_C).

Inductive Reactance:

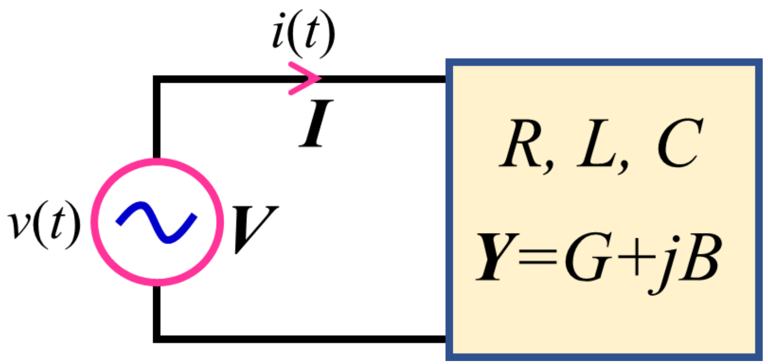
$$X_L = \omega L = 2\pi f L \;\; [\Omega] \qquad X_L \propto f$$

Capacitive Reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \;\; [\Omega] \qquad X_C \propto \frac{1}{f}$$

Impedance (***Z***) is not a phasor quantity because for a circuit it is constant. That means impedance does not change with respect to time.

ADMITTANCE



Admittance (Y) is also not a phasor quantity.

Admittance: Admittance is the ratio of **current** to **voltage**. ***Admittance*** is **reciprocal of impedance**.
Admittance is a measure of how well an ac circuit will *admit*, or allow, current to flow in the circuit.
Admittance represent by ***Y***. Its unit is **Siemens** (S).

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{I_{rms}\angle\theta_i}{V_{rms}\angle\theta_v} = \frac{I\angle\theta_i}{V\angle\theta_v} = \frac{I}{V}\angle(\theta_i - \theta_v)$$
$$= Y\angle\theta_y = G + jB \text{ S}$$

Magnitude of Admittance: $Y = \frac{1}{Z} = \frac{I_m}{V_m} = \frac{I_{rms}}{V_{rms}} = \frac{I}{V} \quad \text{S}$

Angle of Admittance: $\theta_y = -\theta_z = \theta_i - \theta_v$

Conductance (Real Part of admittance):

$$G = \frac{1}{R} = Y\cos\theta_y \qquad \text{S}$$

Susceptance (Imaginary Part of admittance):

$$B = \frac{1}{X} = Y\sin\theta_y \qquad \text{S}$$

Susceptance is the property of inductor and capacitor to help the flow of current. The are two susceptance in electrical circuit: (***i***) **inductive susceptance** (B_L), and (***ii***) **capacitive susceptance** (B_C).

Inductive Susceptance :

$$B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L} \text{ [S]} \qquad B_L \propto \frac{1}{f}$$

Capacitive Susceptance :

$$B_C = \frac{1}{X_C} = \omega C = 2\pi f C \text{ [S]} \qquad B_C \propto f$$

EXAMPLE The supply voltage and current of a circuit are $v(t) = 100\sin 314t$ V and $i(t) = 15\cos(314t–120^o)$ A.

(***a***) Find (***i***) the magnitude of impedance, (***ii***) the angle of impedance, (***iii***) the value of resistance, and (***iv***) the value of reactance.

(***b***) Find (***i***) the magnitude of admittance, (***ii***) the angle of admittance, (***iii***) the value of conductance, and (***iv***) the value of susceptance.

(***c***) Write the impedance and admittance in both polar and cartesian or rectangular form.

Solution: Converting current from cosine to sine, we have: $i(t) = 15\sin(314t–30^o)$ A.

Now, $V_m = 100$ V, $I_m = 15$ A, $\theta_v = 0^o$ and $\theta_i = –30^o$

$$(\boldsymbol{a})\ (\boldsymbol{i})\ \ Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100\text{V}}{15\text{A}} = \mathbf{6.67\ \Omega}$$

$$(\boldsymbol{ii})\ \ \theta_Z = \theta_v - \theta_i = 0° - (-30°) = \mathbf{30}°$$

$$(\boldsymbol{iii})\ \ R = Z\cos\theta_Z = 6.67 \times \cos(30°) = \mathbf{5.78\ \ \Omega}$$

$$(\boldsymbol{iv})\ \ X = Z\sin\theta_Z = 6.67 \times \sin(30°) = \mathbf{3.34\ \ \Omega}$$

$$(\boldsymbol{b})\ (\boldsymbol{i})\ \ Y = \frac{1}{Z} = \frac{I}{V} = \frac{I_m}{V_m} = \frac{15\text{A}}{100\text{V}} = \mathbf{0.15\ S}\ \text{ or } \mathbf{150\,mS}$$

$$(\boldsymbol{ii})\ \ \theta_y = -\theta_Z = \theta_i - \theta_v = -30° - 0° = -\mathbf{30}°$$

$$(\boldsymbol{iii})\ \ G = Y\cos\theta_y = 150 \times \cos(-30°) = \mathbf{129.9\ \ mS}$$

$$(\boldsymbol{iv})\ \ B = Y\sin\theta_y = 150 \times \sin(-30°) = -\mathbf{75\ \ mS}$$

$$(\boldsymbol{c})\ \boldsymbol{Z} = \vec{Z} = 6.67\Omega\angle 30°$$

$$\boldsymbol{Z} = \vec{Z} = 5.78 + j3.34\ \Omega$$

$$\boldsymbol{Y} = \vec{Y} = 150\text{mS}\angle -30°$$

$$\boldsymbol{Y} = \vec{Y} = 129.9 + j75\ \text{mS}$$

EXAMPLE The supply voltage and current of a circuit are $\boldsymbol{V} = 200\text{V}\angle 90^{o}$ and $\boldsymbol{I} = 10\text{A}\angle 30^{o}$.

(***a***) Find the impedance and admittance in both polar and cartesian or rectangular form.

(***b***) Find (***i***) the magnitude of impedance, (***ii***) the angle of impedance, (***iii***) the value of resistance, and (***iv***) the value of reactance.

(***c***) Find (***i***) the magnitude of admittance, (***ii***) the angle of admittance, (***iii***) the value of conductance, and (***iv***) the value of susceptance.

Solution:

$$(\boldsymbol{a})\ \boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{200\text{V}\angle 90^\circ}{10\text{V}\angle 30^\circ} = 20\Omega\angle 60^\circ = 10 + j17.32\ \ \Omega$$

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Y}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{10\text{V}\angle 30^\circ}{200\text{V}\angle 90^\circ} = 005\text{S}\angle -60^\circ = 0.025 + j0.0433\ \ \text{S} = 25 + j43.3\ \ \text{mS}$$

$$(\boldsymbol{b})\ (\boldsymbol{i})\ \ Z = 20\quad \Omega; \qquad (\boldsymbol{ii})\ \ \theta_Z = 60^\circ; \qquad (\boldsymbol{iii})\ R = 10\ \Omega; \qquad (\boldsymbol{iv})\ \ X = 17.32\ \Omega$$

$$(\boldsymbol{c})\ (\boldsymbol{i})\ \ Y = 0.05\ \text{S}; \qquad (\boldsymbol{ii})\ \ \theta_y = -60^\circ; \qquad (\boldsymbol{iii})\ G = 25\ \text{mS}; \qquad (\boldsymbol{iv})\ \ B = 43.3\ \text{mS}$$

EXAMPLE The supply voltage and impedance of a circuit are $v(t)$ = 282.84cos314t V and $\mathbf{Z} = 20\Omega\angle 60^{o}$. Find the current $i(t)$.

Solution: Converting voltage from cosine to sine, we have: $v(t) = 282.84\sin(314t+90^{o})$ V.

Now, $V_m = 282.84$ V, $\theta_v = 90^{o}$ and $Z = 20\ \Omega$, $\theta_z = 60^{o}$

We know that: $Z = \dfrac{V_m}{I_m}$ $\qquad \theta_z = \theta_v - \theta_i$

$$I_m = \frac{V_m}{Z} = \frac{282.84}{20} = 14.142 \text{ A}$$

$$\theta_i = \theta_v - \theta_z = 90° - 60° = 30°$$

Thus, $i(t) = 14.142\sin(314t + 30^{o})$ A

EXAMPLE The supply current and impedance of a circuit are $i(t)$ = 15sin377t V and $\mathbf{Z} = 17.32 + j10\ \Omega$. Find the voltage $v(t)$.

Solution: Converting impedance from Cartesian to Polar form:

$$\mathbf{Z} = 17.32 + j10\ \Omega = 20\Omega\angle 30°$$

Now, $I_m = 15$ V, $\theta_i = 0^{o}$ and $Z = 20\ \Omega$, $\theta_z = 30^{o}$

We know that: $Z = \dfrac{V_m}{I_m}$ $\qquad \theta_z = \theta_v - \theta_i$

$$V_m = ZI_m = 20 \times 15 = 300 \text{ V}$$

$$\theta_v = \theta_i + \theta_z = 0° + 30° = 30°$$

Thus, $v(t) = 300\sin(377t + 30^{o})$ V