Review Last Class

For Pure Resistive Circuit, Pure Inductive Circuit and Pure Capacitive Circuit

Power Factor, Reactive Factor, Power or Active Power, Reactive Power, Apparent Power

Energy Dissipation by resistor and Energy Storage by inductor and capacitor

For Series [*RL* Series, *RC* Series, *RLC* Series] Circuit

Impedance calculation, Impedance Diagram

Supply voltage or current Calculation

Voltage Drop in Different elements Calculation

Instantaneous equation of current and different voltage drop

Phasor or Vector Diagram

Power Factor, Reactive Factor,

Power or Active Power, Reactive Power, Apparent Power

Instantaneous Power equation

Power Triangle

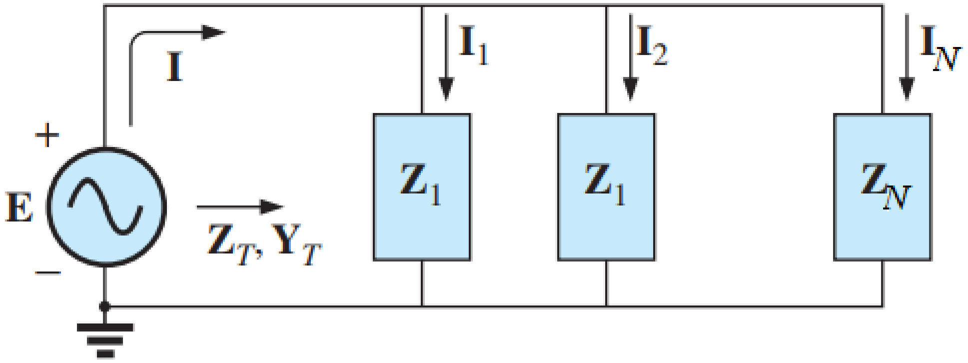
Voltage Divider Rule (VDR)

Kirchhoff's Voltage Law (KVL)

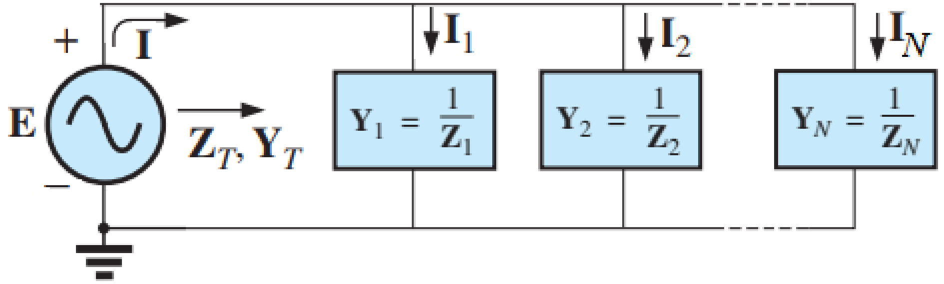
Chapter 15

Parallel Circuits

Parallel Configuration



$$Y_1 = \frac{1}{Z_1}; \quad Y_2 = \frac{1}{Z_2}; \ldots\ldots \quad Y_N = \frac{1}{Z_N}$$



The **total admittance** of a parallel configuration is the sum of the individual admittances:

$$Y_T = Y_1 + Y_2 + \ldots\ldots + Y_N \qquad \textbf{(15.16)}$$

The **total impedance** of a parallel configuration can be calculated as follows:

$$Z_T = \frac{1}{Y_T}$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots + \frac{1}{Z_N} \qquad \textbf{(15.17)}$$

$$Z_T = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots + \frac{1}{Z_N}} \qquad \textbf{(15.18)}$$

For **two impedance in parallel** :

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad \textbf{(15.19)}$$

Current

$$I = \frac{E}{Z_T} = EY_T$$

$$I_1 = \frac{E}{Z_1} = EY_1$$

$$I_2 = \frac{E}{Z_2} = EY_2$$

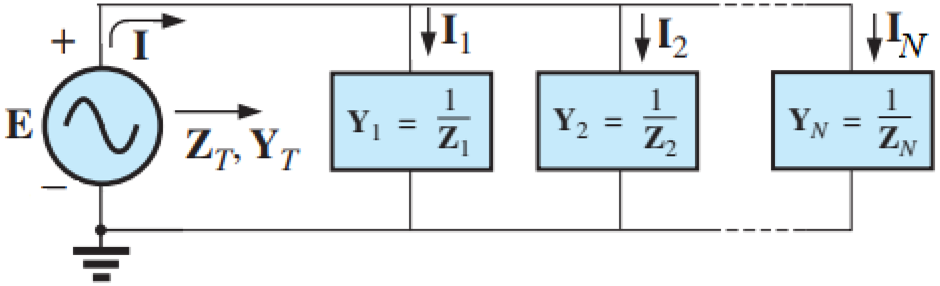
$$I_N = \frac{E}{Z_N} = EY_N$$

If $Z_1 = Z_2 = \ldots. = Z_N = Z_p$

$$Y_1 = Y_2 = \ldots. = Y_N = Y_p$$

$$Y_T = N \times Y_p \qquad Z_T = \frac{Z_p}{N}$$

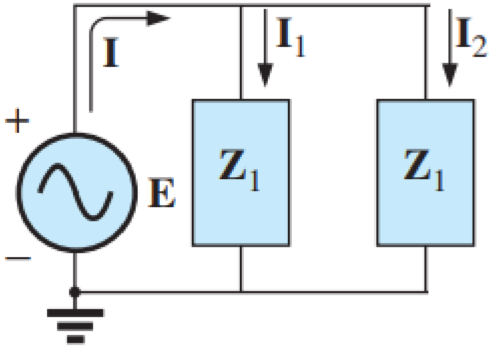
$$I_1 = I_2 = .. = I_N = \frac{I}{N}$$



Current Divider Rule (CDR)

The current flows through an admittance in a parallel circuit is equal to the value of that admittance ($\boldsymbol{Y_x}$) times the total current ($\boldsymbol{I}$) divided by the total admittance ($\boldsymbol{Y_T}$) of the parallel configuration.

$$\boldsymbol{I}_x = \frac{\boldsymbol{Y}_x}{\boldsymbol{Y}_T}\boldsymbol{I} = \frac{\boldsymbol{Z}_T}{\boldsymbol{Z}_x}\boldsymbol{I}$$



$$\boldsymbol{I}_1 = \frac{\boldsymbol{Z}_2}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2}\boldsymbol{I}$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{Z}_1}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2}\boldsymbol{I}$$

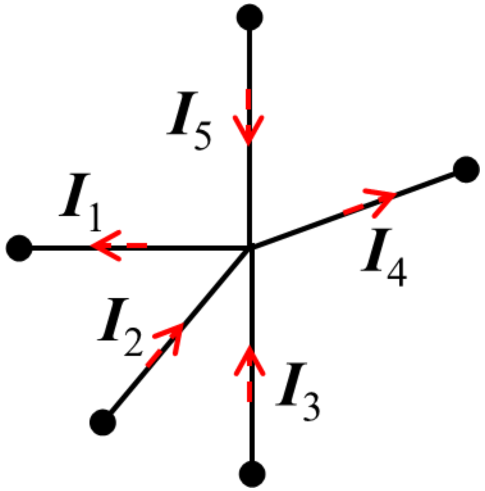
Kirchhoff's Current Law (KCL)

(1) The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

$$\sum \boldsymbol{I}_{entering} - \sum \boldsymbol{I}_{leaving} = 0$$

(2) The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

$$\sum \boldsymbol{I}_{entering} = \sum \boldsymbol{I}_{leaving}$$

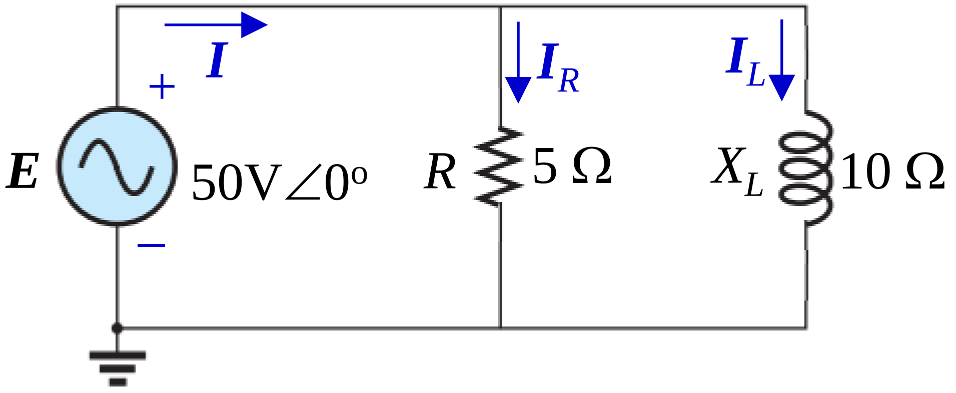


(1) $(\boldsymbol{I}_2 + \boldsymbol{I}_3 + \boldsymbol{I}_5) - (\boldsymbol{I}_1 + \boldsymbol{I}_4) = 0$

(2) $\boldsymbol{I}_2 + \boldsymbol{I}_3 + \boldsymbol{I}_5 = \boldsymbol{I}_1 + \boldsymbol{I}_4$

R-L Parallel Circuit

R-L Parallel Circuit



Admittance

$$Z_R = 5\Omega\angle 0° = 5\ \Omega \qquad Z_L = 10\Omega\angle 90° = j10\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{10\Omega\angle 90°} = 0.1\text{S}\angle -90° = -j0.1\ \text{S}$$

$$Y_T = Y_R + Y_L = 0.2\ \text{S} - j0.1\ \text{S} = 0.224\text{S}\angle -26.57°$$

Admittance Diagram

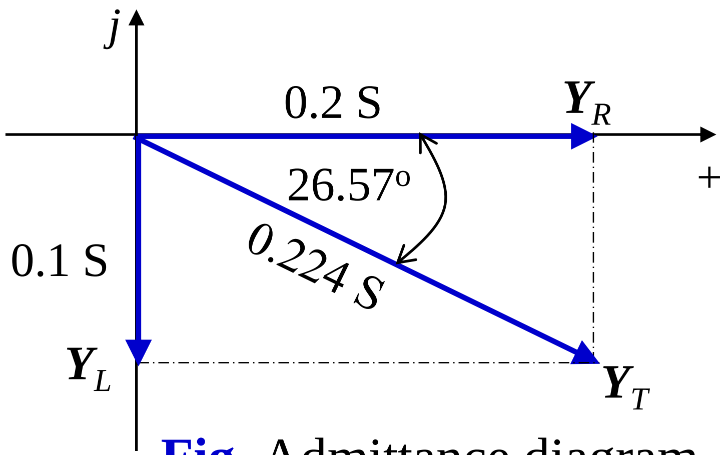


Fig. Admittance diagram

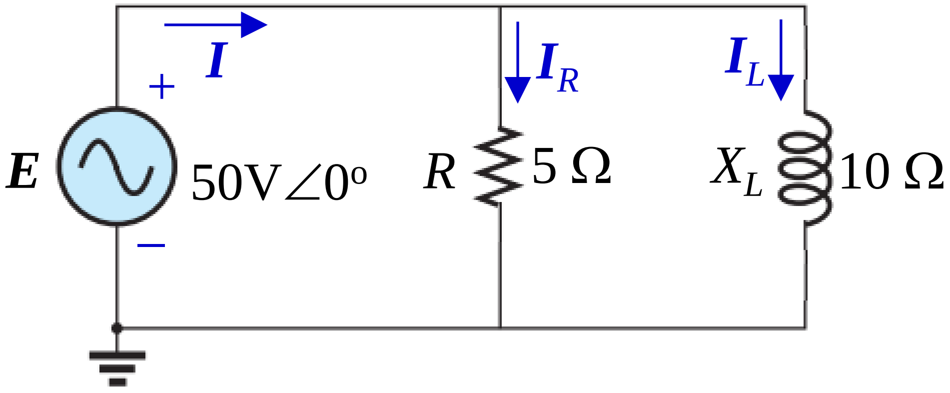
Impedance

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.224\text{S}\angle -26.57°} = 4.46\Omega\angle 26.57° \cong 4 + j2\Omega$$

$$Z_T = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L}} = \frac{1}{\frac{1}{5\Omega} + \frac{1}{j10\Omega}} = 4 + j2\Omega = 4.47\Omega\angle 26.57$$

$$Z_T = \frac{Z_R Z_L}{Z_R + Z_L} = \frac{(5\Omega)(j10\Omega)}{5\Omega + j10\Omega} = 4 + j2\ \Omega = 4.47\Omega\angle 26.57°$$

R-L Parallel Circuit



Current

$$I = \frac{E}{Z_T} = EY_T = \frac{50\text{V}\angle 0^\circ}{4.47\Omega\angle 26.57^\circ}$$
$$= 11.18\text{A}\angle -26.57^\circ$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{50\text{V}\angle 0^\circ}{5\Omega\angle 0^\circ} = 10\text{A}\angle 0^\circ$$

$$I_L = \frac{E}{Z_L} = EY_L = \frac{50\text{V}\angle 0^\circ}{10\Omega\angle 90^\circ} = 5\text{A}\angle -90^\circ$$

KCL:

$$I_R + I_L = 10\text{A}\angle 0^\circ + 5\text{A}\angle -90^\circ = 11.18\text{A}\angle -26.57^\circ = I$$

Phasor Diagram

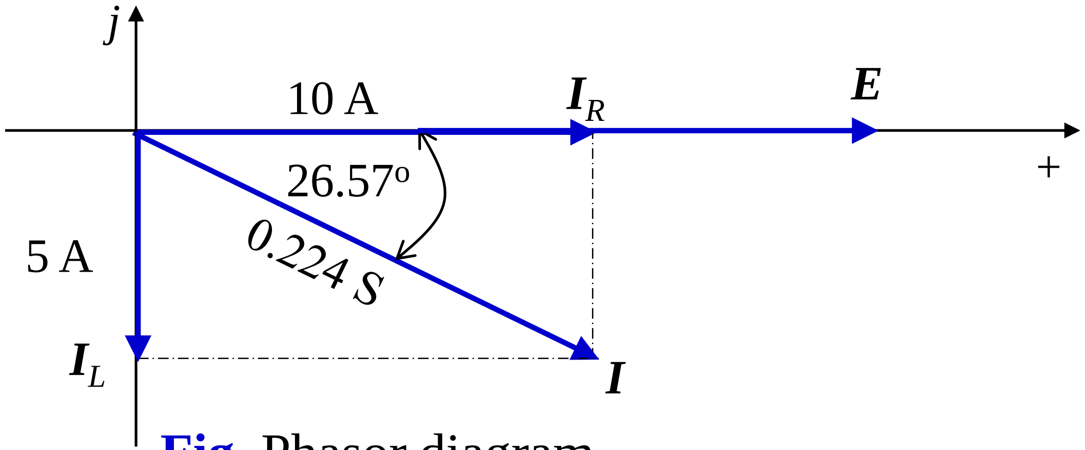
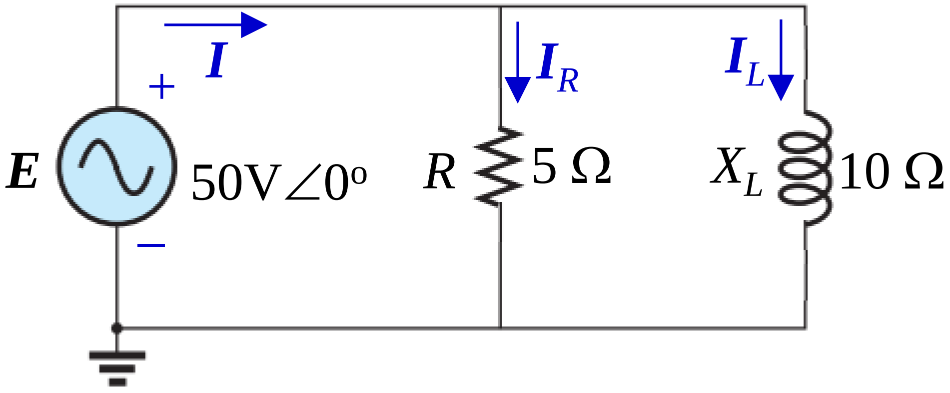


Fig. Phasor diagram

Practice Solution of Fig. 15.68 [Ch. 15], Problems 28 and 30

R-L Parallel Circuit



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(26.57^o)$ **= 0.894 lagging**

$rf = (B_L/Y_T) = \sin\theta_z = \sin(26.57^o)$ **= 0.447**

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times11.18\cos(26.57^o)$ **= 500.19 W**

$P_R = I_R^2R = (E^2/R) = (50V)^2/5\Omega$ = **500 W**

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times11.18\sin(26.57^o)$ **= 250.1 Var**

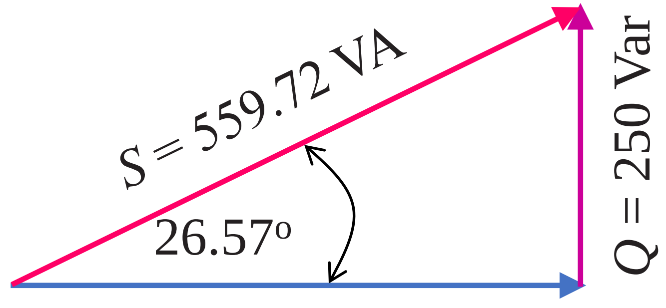
$Q_L = I_L^2X_L = (E^2/X_L) = (50V)^2/10\Omega$ = **250 Var**

Apparent Power [volt-ampere]

$S_E = EI = 50\times11.18$ **= 559.5 VA**

$S_Z = I^2Z = (E^2/Z) = (50V)^2/4.47\Omega$ = **559.72 VA**

Power Triangle



Instantaneous Equation

$p(t) = 500(1 - \cos2\omega t) + 250\sin2\omega t$ W

$e(t) = (\sqrt{2}\times50)\sin\omega t$ V

$i(t) = (\sqrt{2}\times11.18)\sin(\omega t - 26.57^\circ)$ A

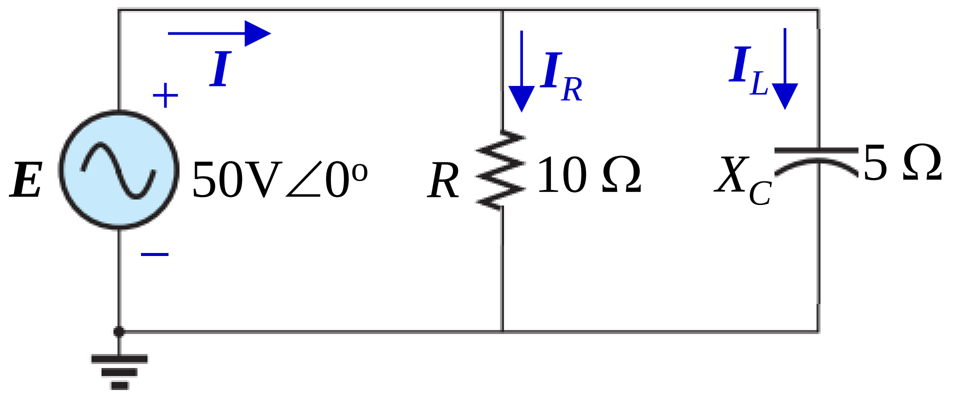
$i_R(t) = (\sqrt{2}\times10)\sin\omega t$ A

$i_L(t) = (\sqrt{2}\times5)\sin(\omega t - 90^\circ)$ A

Practice Solution of Fig. 15.68 [Ch. 15], Problems 28 and 30

R-*C* Parallel Circuit

R-C Parallel Circuit



Admittance

$$Z_R = 10\Omega\angle 0° = 10\ \Omega$$

$$Z_C = 5\Omega\angle -90° = -j5\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{10\Omega\angle 0°} = 0.1\text{S}\angle 0° = 0.1\ \text{S}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{5\Omega\angle -90°} = 0.2\text{S}\angle 90° = j0.2\ \text{S}$$

$$Y_T = Y_R + Y_C = 0.1\ \text{S} + j0.2\ \text{S}$$

$$= 0.224\text{S}\angle 63.43°$$

Admittance Diagram

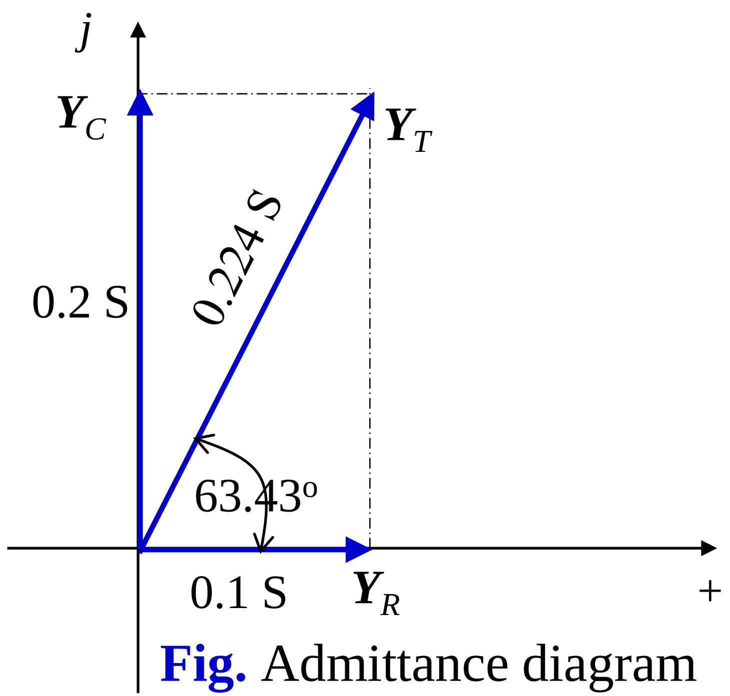


Fig. Admittance diagram

Impedance

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.224\text{S}\angle 63.43°}$$

$$= 4.46\Omega\angle -63.43°$$

$$\cong 2 - j4\Omega$$

$$Z_T = \frac{Z_R Z_C}{Z_R + Z_C}$$

$$= \frac{(10\Omega)(j5\Omega)}{10\Omega - j5\Omega}$$

$$= 2 - j4\Omega$$

$$= 4.472\Omega\angle -63.43°$$

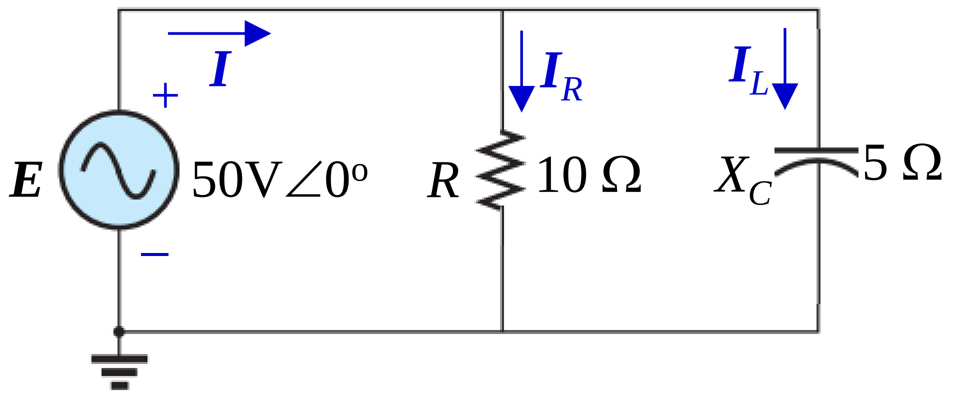
$$Z_T = \frac{1}{\dfrac{1}{Z_R} + \dfrac{1}{Z_C}}$$

$$= \frac{1}{\dfrac{1}{10\Omega} + \dfrac{1}{-j5\Omega}}$$

$$= 2 - j4\Omega$$

$$= 4.472\Omega\angle -63.43°$$

R-C Parallel Circuit



Current

$$I = \frac{E}{Z_T} = EY_T = \frac{50\text{V}\angle 0^\circ}{4.47\Omega\angle -63.43^\circ}$$

$$= 11.18\text{A}\angle 63.43^\circ$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{50\text{V}\angle 0^\circ}{10\Omega\angle 0^\circ} = 5\text{A}\angle 0^\circ$$

$$I_C = \frac{E}{Z_C} = EY_C = \frac{50\text{V}\angle 0^\circ}{5\Omega\angle -90^\circ} = 10\text{A}\angle 90^\circ$$

KCL:

$$I_R + I_C = 5\text{A}\angle 0^\circ + 10\text{A}\angle 90^\circ = 11.18\text{A}\angle 63.43^\circ = I$$

Phasor Diagram

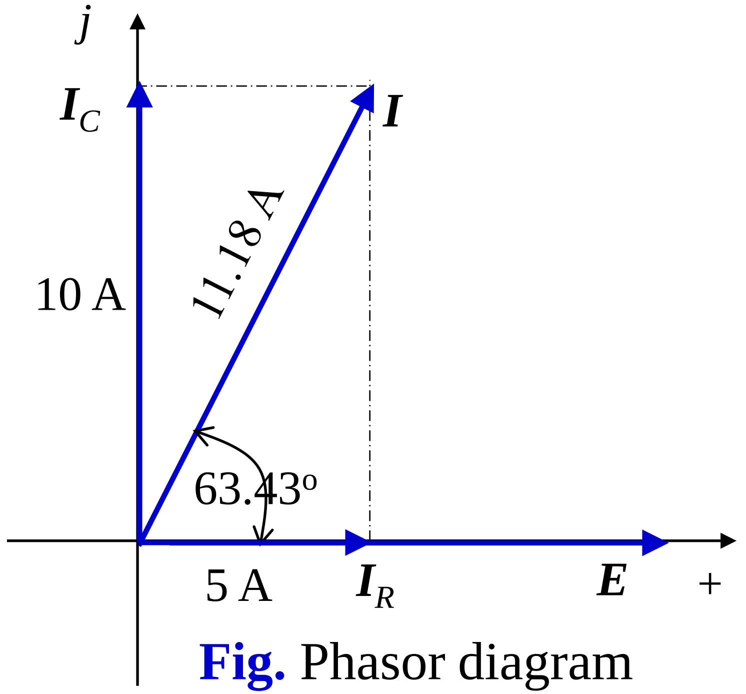
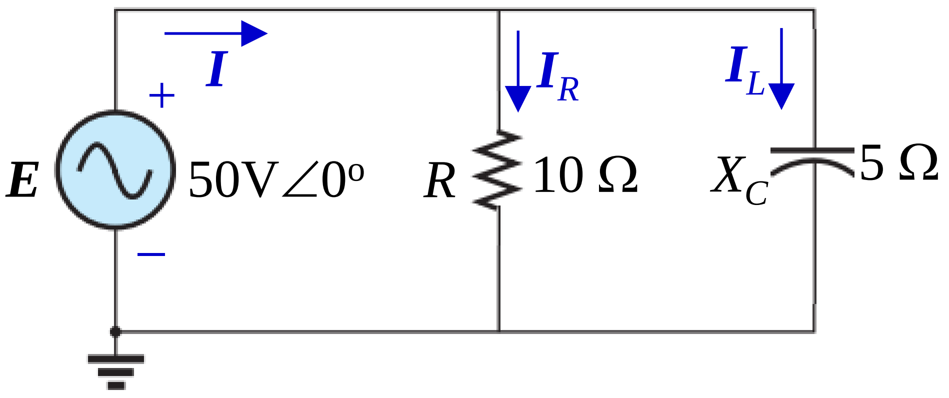


Fig. Phasor diagram

Practice Solution of Fig. 15.72 [Ch. 15], Problem 29

R-C Parallel Circuit



Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(-63.43^o)$ **= 0.447 leading**

$rf = (B_L/Y_T) = \sin\theta_z = \sin(-63.43^o)$ **= – 0.894**

Power [Total watts]

$P_E = EI\cos\theta_z = 50\times11.18\cos(-63.43^o)$ **= 250.1 W**

$P_R = I_R^2R = (E^2/R) = (50V)^2/10\Omega$ **= 250 W**

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 50\times11.18\sin(-63.43^o)$ **= – 500.19 Var**

$Q_C = -I_C^2X_C = -(E^2/X_C) = -(50V)^2/5\Omega$ **= – 500 Var**

Apparent Power [volt-ampere]

$S_E = EI$

$= 50\times11.18$ VA

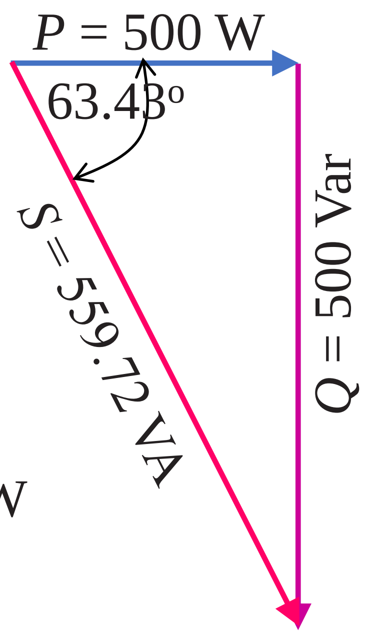
= 559.5 VA

$S_Z = I^2Z = (E^2/Z)$

$= (50V)^2/4.47\Omega$

= 559.72 VA

Power Triangle



Instantaneous Equation

$p(t) = 250(1 - \cos 2\omega t) - 500\sin 2\omega t$ W

$$e(t) = \left(\sqrt{2}\times50\right)\sin\omega t \text{ V}$$

$$i(t) = \left(\sqrt{2}\times11.18\right)\sin\left(\omega t + 63.43°\right) \text{ A}$$

$$i_R(t) = \left(\sqrt{2}\times5\right)\sin\omega t \text{ A}$$

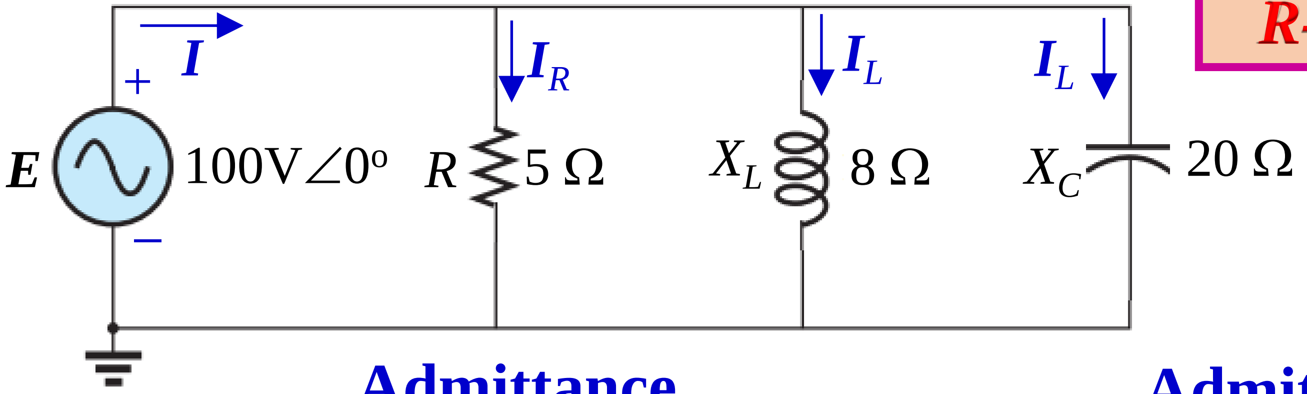
$$i_C(t) = \left(\sqrt{2}\times10\right)\sin\left(\omega t + 90°\right) \text{ A}$$

Practice Solution of Fig. 15.72 [Ch. 15], Problem 29

R-L-C Parallel Circuit

Example 1

R-L-C **Parallel Circuit 1**



Admittance

$$Z_R = 5\Omega\angle 0° = 5\ \Omega \qquad Z_L = 8\Omega\angle 90° = j8\ \Omega$$

$$Z_C = 20\Omega\angle -90° = -j20\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{8\Omega\angle 90°} = 0.125\text{S}\angle -90° = -j0.125\ \text{S}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{20\Omega\angle -90°} = 0.05\text{S}\angle 90° = j0.05\ \text{S}$$

$$Y_T = Y_R + Y_L + Y_C = 0.2\ \text{S} - j0.125\ \text{S} + j0.05\ \text{S}$$

$$= 0.2\ \text{S} - j0.075\ \text{S} = 0.214\text{S}\angle -20.56°$$

Admittance Diagram

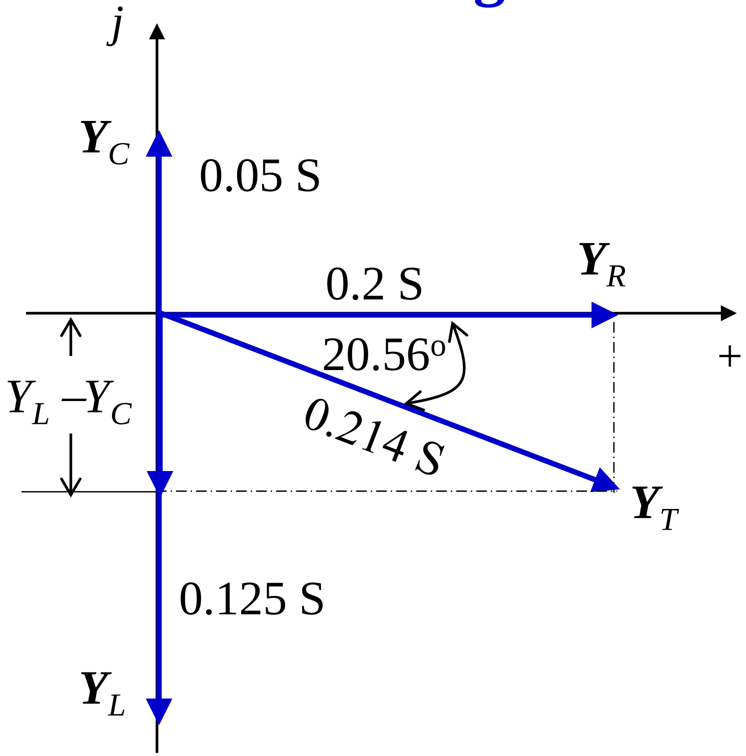


Fig. Admittance diagram

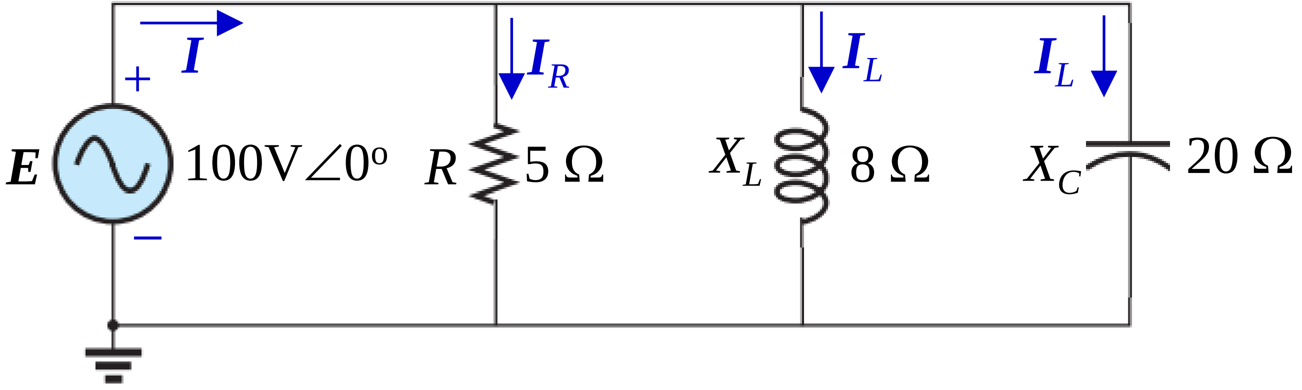
Impedance

$$Z_T = \frac{1}{\dfrac{1}{Z_R} + \dfrac{1}{Z_L} + \dfrac{1}{Z_C}}$$

$$= \frac{1}{\dfrac{1}{5\Omega} + \dfrac{1}{j8\Omega} + \dfrac{1}{-j20\Omega}}$$

$$= 4.38 + j1.64\Omega$$

$$= 4.68\Omega\angle 20.56°$$



Current

$$I = \frac{E}{Z_T} = EY_T = \frac{100\text{V}\angle 0^\circ}{4.68\Omega\angle 20.56^\circ} = 21.37\text{A}\angle -20.56^\circ$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{100\text{V}\angle 0^\circ}{5\Omega\angle 0^\circ} = 20\text{A}\angle 0^\circ$$

$$I_L = \frac{E}{Z_L} = EY_L = \frac{100\text{V}\angle 0^\circ}{8\Omega\angle 90^\circ} = 12.5\text{A}\angle -90^\circ$$

$$I_C = \frac{E}{Z_C} = EY_C = \frac{100\text{V}\angle 0^\circ}{20\Omega\angle -90^\circ} = 5\text{A}\angle 90^\circ$$

KCL:

$$I_R + I_L + I_C = 20\text{A}\angle 0^\circ + 12.5\text{A}\angle -90^\circ + 5\text{A}\angle 90^\circ$$
$$= 21.37\text{A}\angle -20.56^\circ = I$$

Phasor Diagram

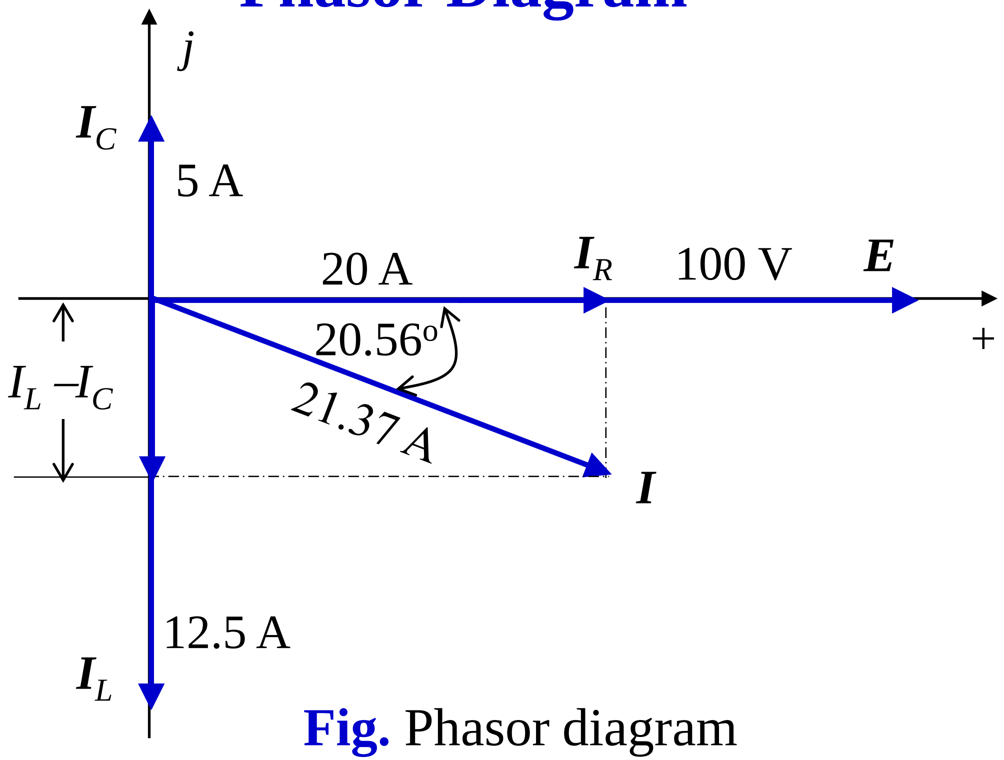
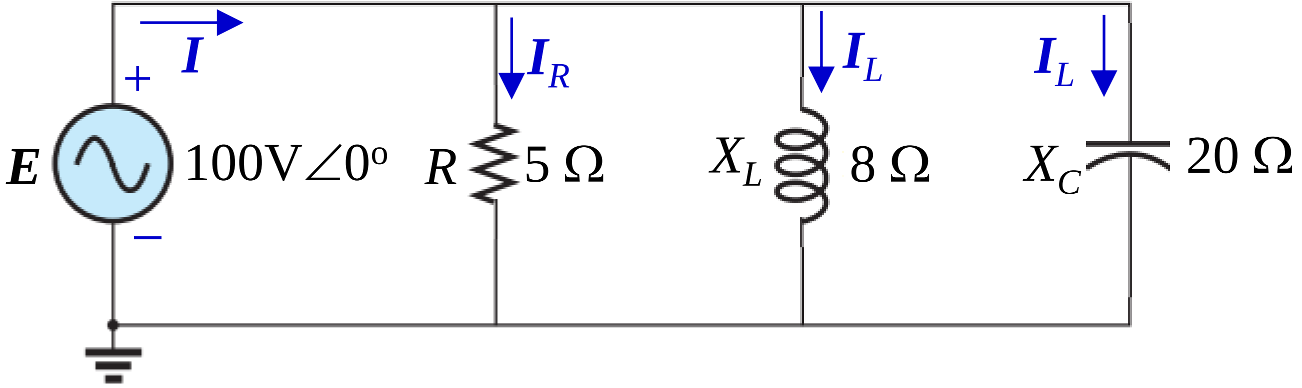


Fig. Phasor diagram

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32



Apparent Power [volt-ampere]

$S_E = EI = 100 \times 21.37 = \mathbf{2137\ VA}$

$S_Z = I^2Z = (E^2/Z) = (100V)^2/4.68\Omega = \mathbf{2137.75\ VA}$

Power Triangle

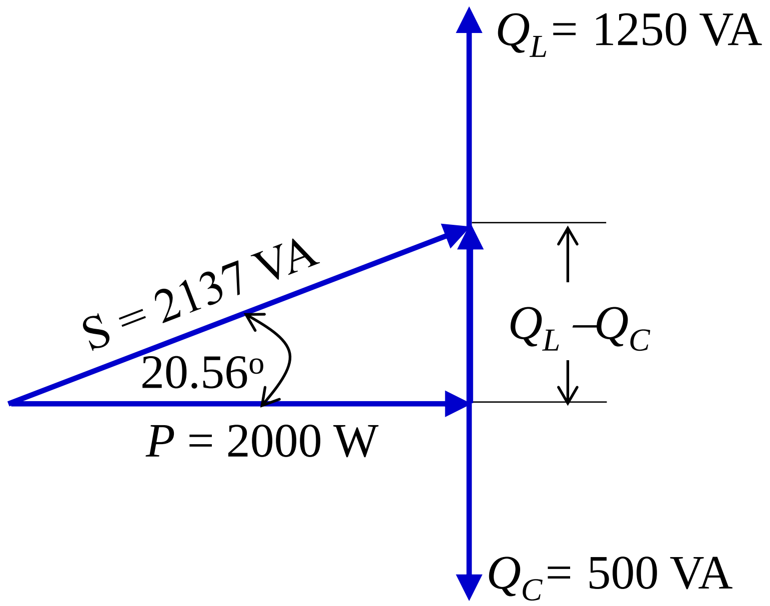


Fig. Admittance diagram

Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(20.56^o) = \mathbf{0.936\ lagging}$

$rf = (B/Y_T) = \sin\theta_z = \sin(20.56^o) = \mathbf{0.351}$

Power [Total watts]

$P_E = EI\cos\theta_z = 100 \times 21.37\cos(20.56^o) = \mathbf{2000.23\ W}$

$P_R = I_R^2R = (E^2/R) = (100V)^2/5\Omega = \mathbf{2000\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 100 \times 21.37\sin(20.56^o) = \mathbf{750.09\ Var}$

$Q_L = I_L^2X_L = (E^2/X_L) = (100V)^2/8\Omega = \mathbf{1250\ Var}$

$Q_C = -I_C^2X_C = -(E^2/X_C) = -(100V)^2/20\Omega = \mathbf{-500\ Var}$

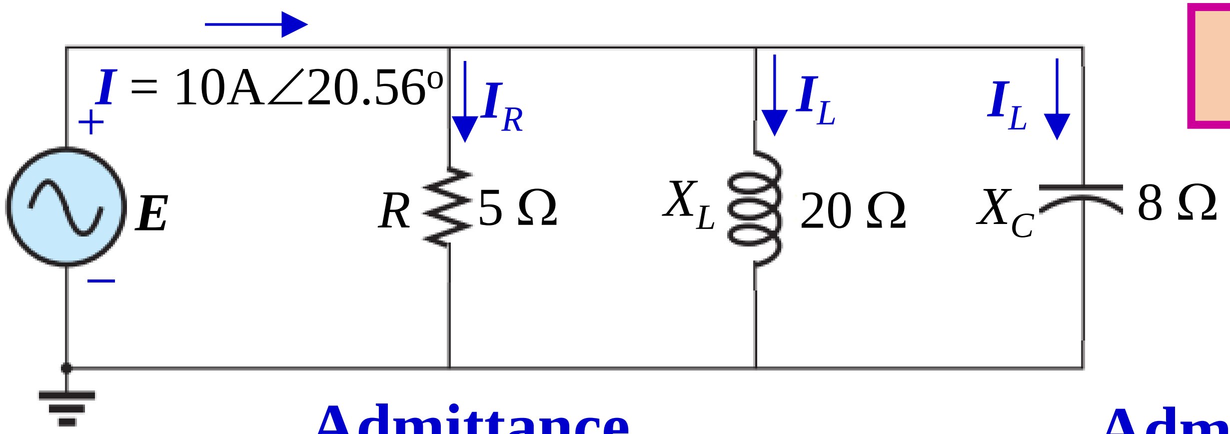
$Q = Q_L + Q_C = \mathbf{750\ Var}$

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32

R-*L*-*C* Parallel Circuit

Example 2

R-L-C Parallel Circuit 2



Admittance

$$Z_R = 5\Omega\angle 0° = 5\ \Omega \qquad Z_L = 20\Omega\angle 90° = j20\ \Omega$$

$$Z_C = 8\Omega\angle -90° = -j8\ \Omega$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\Omega\angle 0°} = 0.2\text{S}\angle 0° = 0.2\ \text{S}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{20\Omega\angle 90°} = 0.05\text{S}\angle -90° = -j0.05\ \text{S}$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{8\Omega\angle -90°} = 0.125\text{S}\angle 90° = j0.125\ \text{S}$$

$$Y_T = Y_R + Y_L + Y_C = 0.2\ \text{S} - j0.05\ \text{S} + j0.125\ \text{S}$$
$$= 0.2\ \text{S} + j0.075\ \text{S} = 0.214\text{S}\angle 20.56°$$

Admittance Diagram

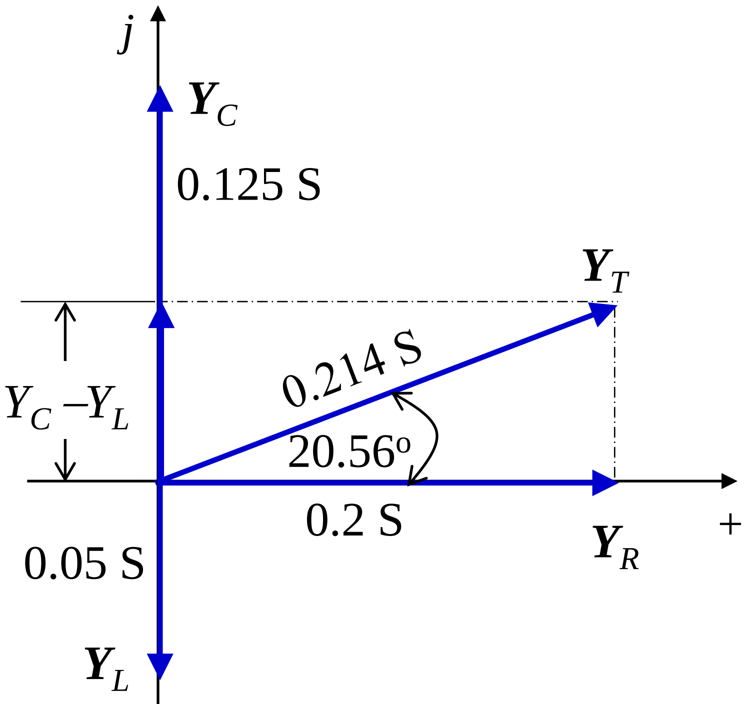


Fig. Admittance diagram

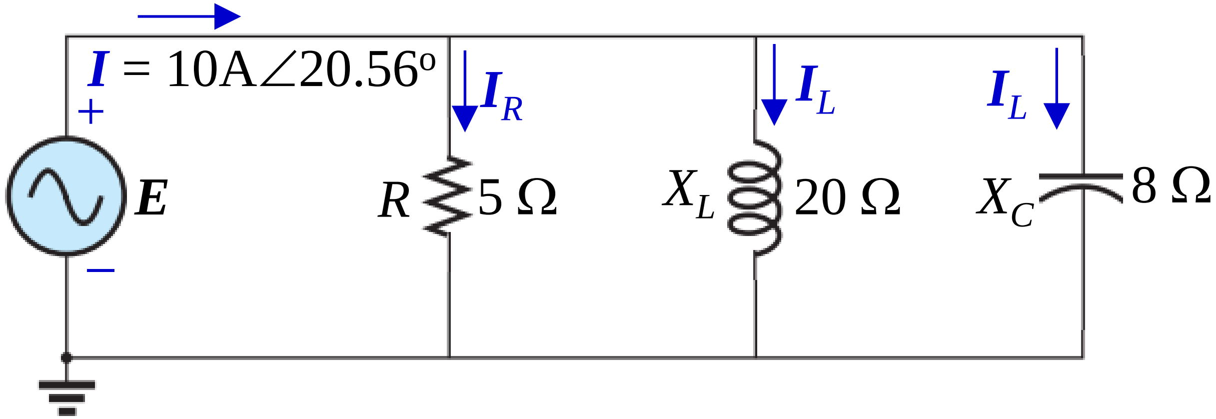
Impedance

$$Z_T = \frac{1}{\dfrac{1}{Z_R} + \dfrac{1}{Z_L} + \dfrac{1}{Z_C}}$$

$$= \frac{1}{\dfrac{1}{5\Omega} + \dfrac{1}{-j8\Omega} + \dfrac{1}{j20\Omega}}$$

$$= 4.38 - j1.64\Omega$$

$$= 4.68\Omega\angle -20.56°$$



Current

$$E = IZ_T = \frac{I}{Y_T} = \frac{10\text{A}\angle 20.56^\circ}{0.214\text{S}\angle -20.56^\circ} = 46.73\text{V}\angle 0^\circ$$

$$I_R = \frac{E}{Z_R} = EY_R = \frac{46.73\text{V}\angle 0^\circ}{5\Omega\angle 0^\circ} = 9.35\text{A}\angle 0^\circ$$

$$I_L = \frac{E}{Z_L} = EY_L = \frac{46.73\text{V}\angle 0^\circ}{20\Omega\angle 90^\circ} = 2.34\text{A}\angle -90^\circ$$

$$I_C = \frac{E}{Z_C} = EY_C = \frac{46.73\text{V}\angle 0^\circ}{8\Omega\angle -90^\circ} = 5.84\text{A}\angle 90^\circ$$

KCL:

$$I_R + I_L + I_C = 9.35\text{A}\angle 0^\circ + 2.34\text{A}\angle -90^\circ + 5.84\text{A}\angle 90^\circ$$
$$= 10\text{A}\angle 20.56^\circ = I$$

Phasor Diagram

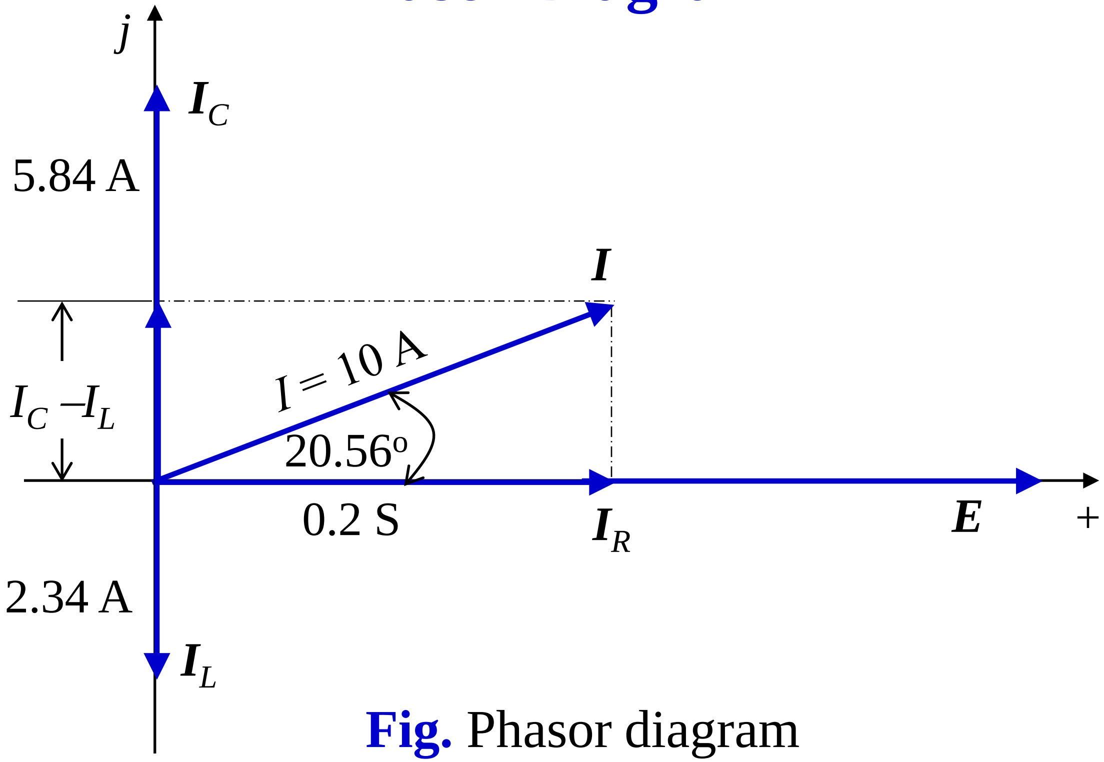
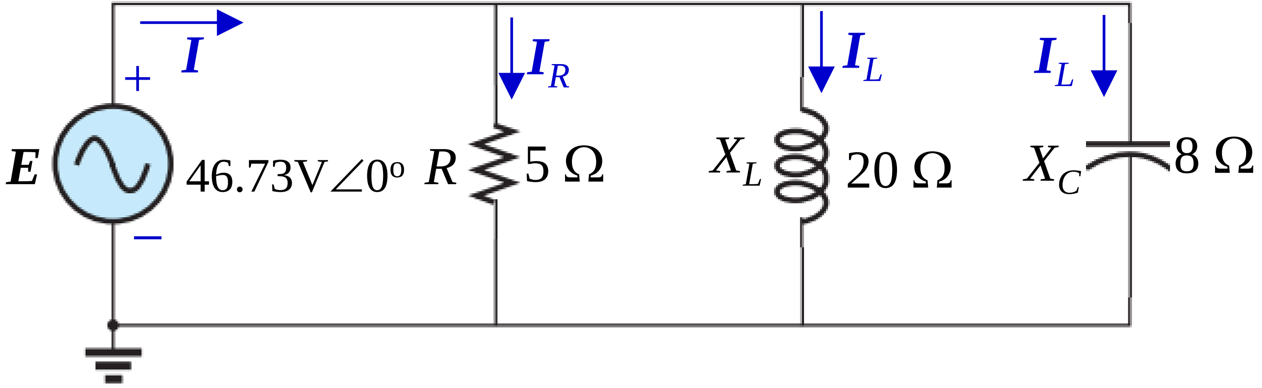


Fig. Phasor diagram



Apparent Power [volt-ampere]

$S_E = EI = 46.73\times 10 = \mathbf{467.3\ VA}$

$S_Z = I^2 Z = (E^2/Z) = (46.73\text{V})^2/4.68\Omega = \mathbf{468\ VA}$

Power Factor and Reactive Factor

$pf = (G/Y_T) = \cos\theta_z = \cos(-20.56^o) = \mathbf{0.351\ leading}$

$rf = (B/Y_T) = \sin\theta_z = \sin(-20.56^o) = \mathbf{-0.936}$

Power [Total watts]

$P_E = EI\cos\theta_z = 46.73\times 10\cos(-20.56^o) = \mathbf{437.39\ W}$

$P_R = I_R^2 R = (E^2/R) = (46.73\text{ V})^2/5\Omega = \mathbf{437.11\ W}$

Reactive Power [volt-ampere reactive]

$Q_E = EI\sin\theta_z = 46.73\times 21.37\sin(-20.56^o) = \mathbf{-164.02\ Var}$

$Q_L = I_L^2 X_L = (E^2/X_L) = (46.73\text{V})^2/20\Omega = \mathbf{109.51\ Var}$

$Q_C = -I_C^2 X_C = -(E^2/X_C) = -(46.73\text{V})^2/8\Omega = \mathbf{-272.84\ Var}$

$Q = Q_L + Q_C = \mathbf{-163.33\ Var}$

Power Triangle

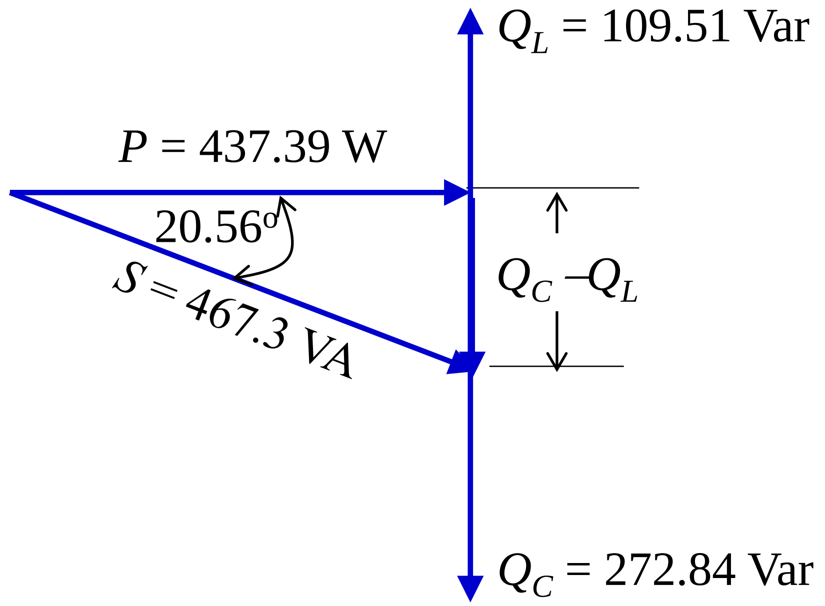


Fig. Phasor diagram

Practice Solution of Fig. 15.77 [Ch. 15], Problem 31 to 32

Problem: For the following network:

(a) Find the conductance, susceptance and admittance for each branch.

(b) Determine the total admittance, impedance and the source current $\boldsymbol{I}_s$.

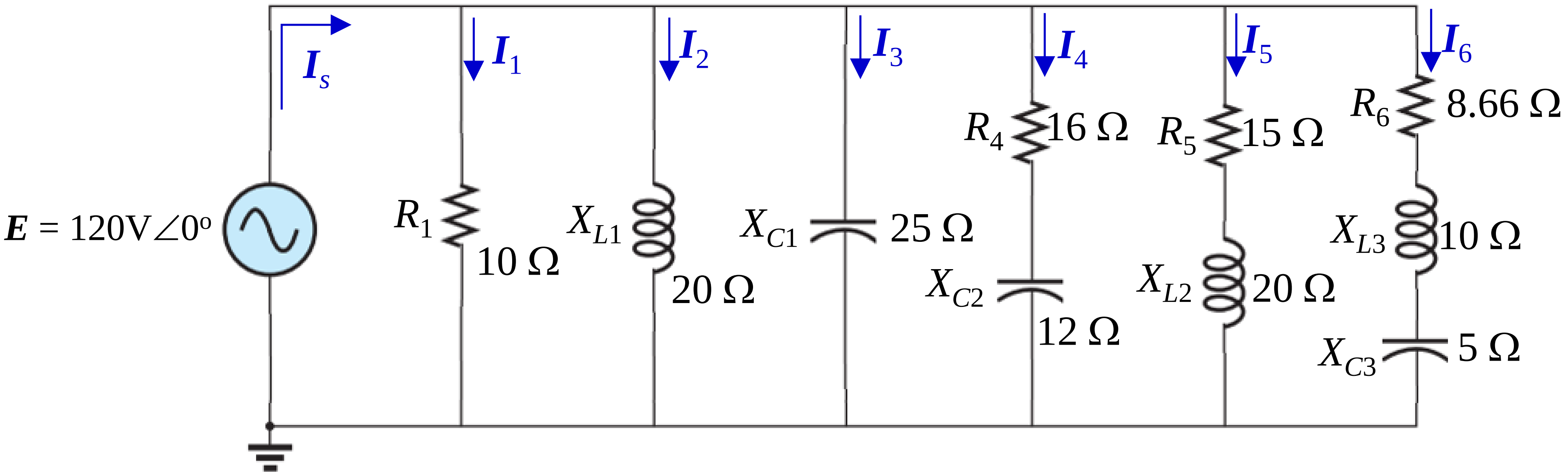
(c) Using the current divider rule, determine the currents $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$, $\boldsymbol{I}_4$, $\boldsymbol{I}_5$, and $\boldsymbol{I}_6$.

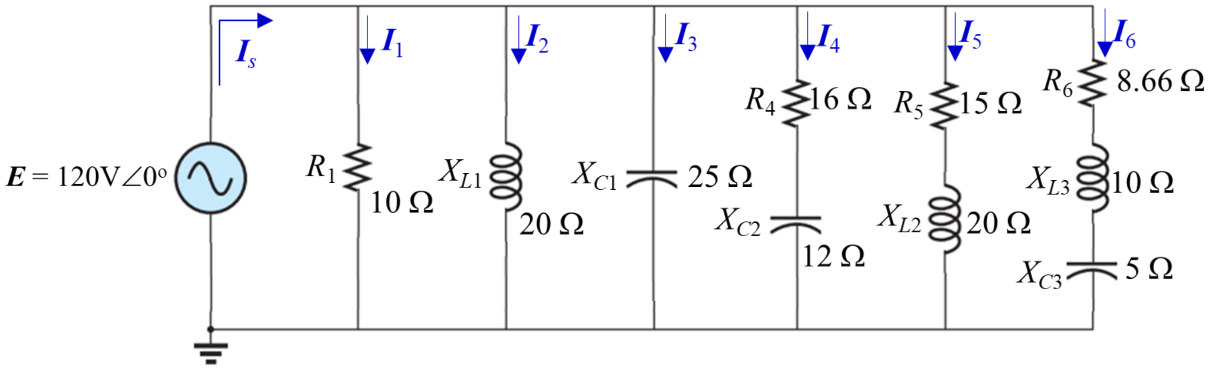
(d) Determine the power consumption each resistor.

(e) Determine the reactive power consumption each inductor.

(f) Determine the reactive power supply by each capacitor.

(g) Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor.





For	Conductance [S]	Susceptance [S]
Z_1 or Y_1	0.1	0
Z_2 or Y_2	0	0.05
Z_3 or Y_3	0	0.04
Z_4 or Y_4	0.04	0.03
Z_5 or Y_5	0.024	0.032
Z_6 or Y_6	0.087	0.05

Solution: Let,

$$Z_1 = 10\ \Omega$$
$$Z_2 = j20\ \Omega$$
$$Z_3 = -j25\ \Omega$$
$$Z_4 = 16 - j12\ \Omega$$
$$Z_5 = 15 + j20\ \Omega$$
$$Z_6 = 8.66 + j10 - j5\ \Omega$$
$$= 8.66 + j5\ \Omega$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{10\ \Omega} = 0.1\ \text{S}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{j20\ \Omega} = -j0.05\ \text{S}$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{-j25\ \Omega} = j0.04\ \text{S}$$

$$Y_4 = \frac{1}{Z_4} = \frac{1}{16 - j12\ \Omega} = 0.04 + j0.03\ \text{S}$$

$$Y_5 = \frac{1}{Z_5} = \frac{1}{15 + j20\ \Omega} = 0.024 - j0.032\ \text{S}$$

$$Y_6 = \frac{1}{Z_6} = \frac{1}{8.66 + j5\ \Omega} = 0.087 + j0.05\ \text{S}$$

$$Y_T = Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6$$
$$= 0.251 - j0.062\ \text{S} = 0.26\text{S}\angle -13.87^\circ$$

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.26\text{S}\angle -13.87^\circ} = 3.87\text{S}\angle 13.87^\circ$$

$$I_s = \frac{E}{Z_T} = \frac{120\text{V}\angle 0^\circ}{3.87\Omega\angle 13.87^\circ}$$
$$= 30.1 - j7.44\ \text{A} = 31.01\text{A}\angle -13.87^\circ$$

$Y_1 = 0.1\text{ S}$ $\quad Y_2 = -j0.05\text{ S}$ $\quad Y_3 = j0.04\text{ S}$ $\quad Y_T = 0.233 - j0.031\text{ S} = 0.24\text{S}\angle -7.58°$

$Y_4 = 0.04 + j0.03\text{ S}$ $\quad Y_5 = 0.006 - j0.001\text{ S}$ $\quad Y_6 = 0.087 + j0.05\text{ S}$ $\quad I_s = 27.99 - j3.73\text{ A} = 28.24\text{A}\angle -7.58°$

(c) Using the current divider rule, determine the currents $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$, $\boldsymbol{I}_4$, $\boldsymbol{I}_5$, and $\boldsymbol{I}_6$.

$$I_1 = \frac{Z_T}{Z_1} I_s = \frac{Y_1}{Y_T} I_s = \frac{0.1\text{ S}}{0.251 - j0.062\text{ S}}(30.1 - j7.44) = 12\text{ A} = 12\text{A}\angle 0°$$

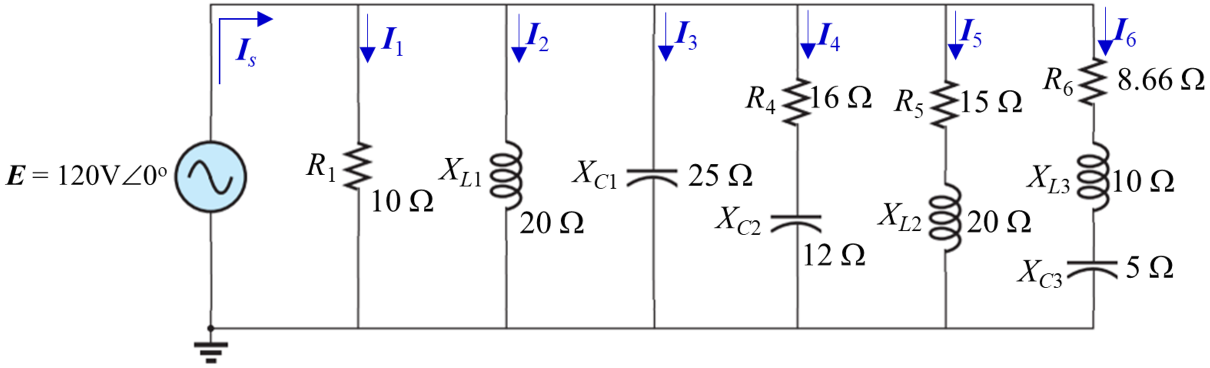
$$I_2 = \frac{Z_T}{Z_2} I_s = \frac{Y_2}{Y_T} I_s = \frac{-j0.05\text{ S}}{0.251 - j0.062\text{ S}}(30.1 - j7.44) = -j6\text{A} = 6\text{A}\angle -90°$$

$$I_3 = \frac{Z_T}{Z_3} I_s = \frac{Y_3}{Y_T} I_s = \frac{-j0.04\text{ S}}{0.251 - j0.062\text{ S}}(30.1 - j7.44) = j4.8\text{ A} = 4.8\text{A}\angle 90°$$

$$I_4 = \frac{Z_T}{Z_4} I_s = \frac{Y_4}{Y_T} I_s = \frac{0.04 + j0.03\text{ S}}{0.251 - j0.062\text{ S}}(30.1 - j7.44) = 4.8 + j3.6\text{ A} = 6\text{A}\angle 36.81°$$

$$I_5 = \frac{Z_T}{Z_5} I_s = \frac{Y_5}{Y_T} I_s = \frac{0.024 - j0.032\text{ S}}{0.251 - j0.062\text{ S}}(30.1 - j7.44) = 2.88 - j3.84\text{ A} = 4.8\text{A}\angle -53.13°$$

$$I_6 = \frac{Z_T}{Z_6} I_s = \frac{Y_6}{Y_T} I_s = \frac{0.087 - j0.05\text{ S}}{0.251 - j0.062\text{ S}}(30.1 - j7.44) = 10.43 - j6\text{ A} = 12\text{A}\angle -30°$$



$\boldsymbol{I}_1 = 12\text{A}\angle 0°$ $\boldsymbol{I}_2 = 6\text{A}\angle -90°$

$\boldsymbol{I}_3 = 4.8\text{A}\angle 90°$ $\boldsymbol{I}_4 = 6\text{A}\angle 36.81°$

$\boldsymbol{I}_5 = 4.8\text{A}\angle -53.13°$ $\boldsymbol{I}_6 = 12\text{A}\angle -30°$

(d) Determine the power consumption each resistor.

$$P_{R1} = I_1^2 R_1 = (12\text{A})^2 \times 10\Omega = 1440 \text{ W}$$

$$P_{R4} = I_4^2 R_4 = (6\text{A})^2 \times 16\Omega = 576 \text{ W}$$

$$P_{R5} = I_5^2 R_5 = (4.8\text{A})^2 \times 15\Omega = 345.6 \text{ W}$$

$$P_{R6} = I_6^2 R_6 = (12\text{A})^2 \times 8.66\Omega = 1253.28 \text{ W}$$

(e) Determine the reactive power consumption each inductor.

$$Q_{L1} = I_2^2 X_{L1} = (6\text{A})^2 \times 20\Omega = 720 \text{ Var}$$

$$Q_{L2} = I_5^2 X_{L2} = (4.8\text{A})^2 \times 20\Omega = 460.8 \text{ Var}$$

$$Q_{L3} = I_6^2 X_{L3} = (12\text{A})^2 \times 10\Omega = 1440 \text{ Var}$$

(e) Determine the reactive power consumption each capacitor.

$$Q_{C1} = -I_3^2 X_{C1} = (4.8\text{A})^2 \times 25\Omega = -576 \text{ Var}$$

$$Q_{C2} = I_4^2 X_{C2} = (6\text{A})^2 \times 12\Omega = -432 \text{ Var}$$

$$Q_{C3} = I_6^2 X_{C3} = (12\text{A})^2 \times 5\Omega = -720 \text{ Var}$$

(g) Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor.

Total number of watts:

$$P_R = P_{R1} + P_{R2} + P_{R3} + P_{R4} = 3614.88 \text{ W}$$

$$P_E = EI\cos\theta = 120 \times 31.01 \times \cos(13.87°) = 3612.54 \text{ W}$$

Total number of volt-amperes reactive:

$$Q_L = Q_{L1} + Q_{L2} + Q_{L3} = 2628.01 \text{ Var}$$

$$Q_C = Q_{C1} + Q_{C2} + Q_{C3} = -1731.6 \text{ Var}$$

$$Q = Q_L + Q_C = 896.41 \text{ Var}$$

$$Q_E = EI\sin\theta = 120 \times 31.01 \times \sin(13.87°) = 892.68 \text{ Var}$$

Total number of volt-amperes:

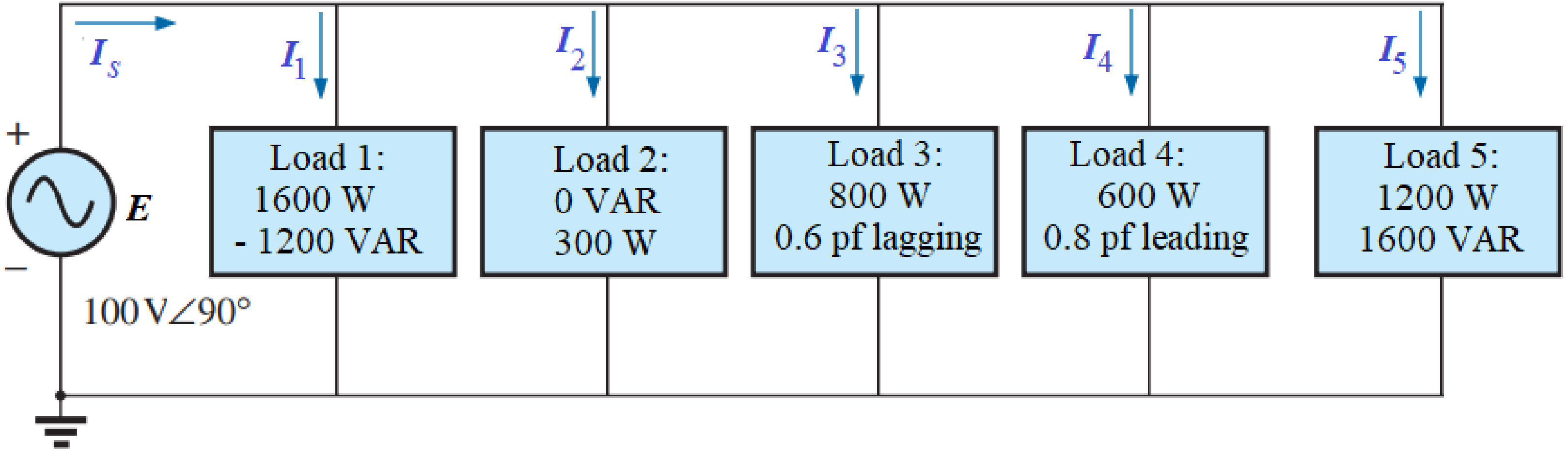
$$S_Z = I^2 Z = (31.01\text{A})^2 \times 3.87\Omega = 3721.2 \text{ VA}$$

$$S_E = EI = 120 \times 31.01 = 3721.47 \text{ VA}$$

Power factor:

$$pf = \cos\theta = \cos(13.87°) = 0.97$$

Problem: For the following network: Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor.



Solution:

Load 3: Power factor lagging, so circuit is reactive power is positive.

$$\theta_{z3} = cos^{-1}[0.6] = 53.13°$$

$$Q_3 = Ptan\theta_{z3} = 800 \times tan(53.13°) = 1066.67 \text{ VAR}$$

Load 4: Power factor leading, so circuit is reactive power is negative.

$$\theta_{z4} = -cos^{-1}[0.8] = -36.87°$$

$$Q_4 = Ptan\theta_{z4} = 600 \times tan(-36.87°) = -450 \text{ VAR}$$

$$P_T = 1600 \text{ W} + 300 \text{ W} + 800 \text{ W} + 600 \text{ W} + 1200 \text{ W} = 4500 \text{ W}$$

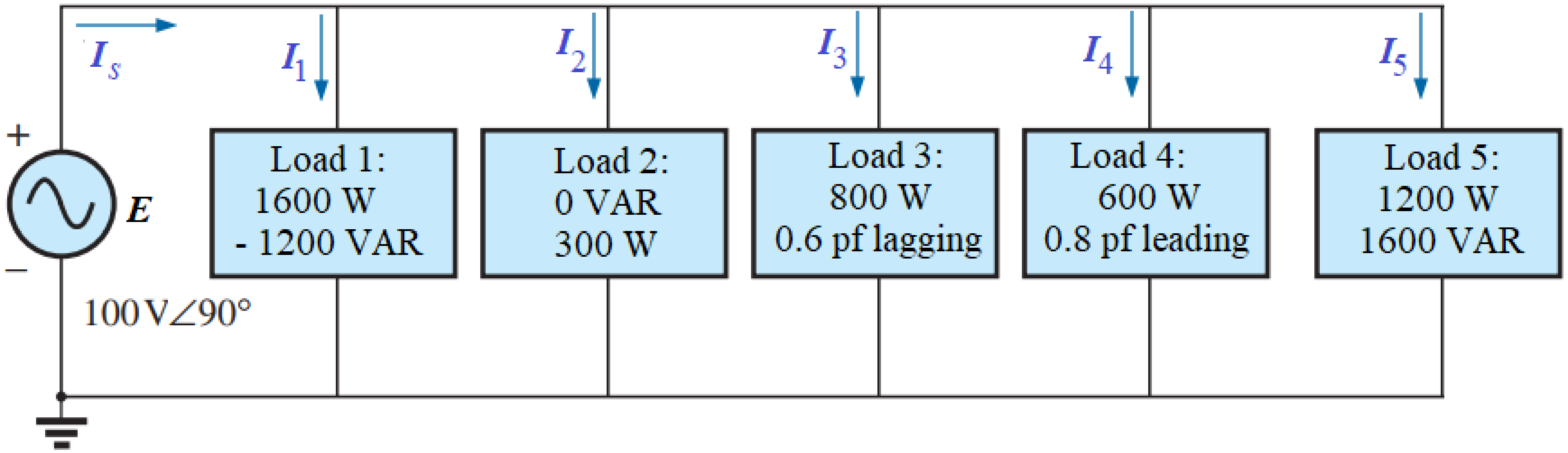
$$Q_T = -1200 \text{ VAR} + 1066.67 \text{ VAR} - 450 \text{ VAR} + 1600 \text{ VAR} = 1016.67 \text{ VAR}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = 4613.41 \text{ VA}$$

$$pf = \frac{P_T}{S_T} = 0.975$$

Problem: For the following network: Determine the currents $\boldsymbol{I}_s$, $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$, $\boldsymbol{I}_4$, $\boldsymbol{I}_5$, and $\boldsymbol{I}_6$.

Solution: Here,
E = 100 V and $\theta_e = 90^o$.



Load 1: Reactive power is negative, so circuit is capacitive and impedance angle is negative.

$$\theta_{z1} = tan^{-1}\left[\frac{Q}{P}\right] = tan^{-1}\left[\frac{-1200}{1600}\right] = -36.87°$$

$$\theta_{i1} = \theta_e - \theta_{z1} = 90° - (-36.87°) = 126.87°$$

$$I_1 = \frac{P}{Ecos\theta_{z1}} = \frac{1600}{100 \times \cos(-36.87°)} = 20 \text{ A}$$

$$\boldsymbol{I_1 = 20A\angle 126.87° = -12 + j\,16\text{ A}}$$

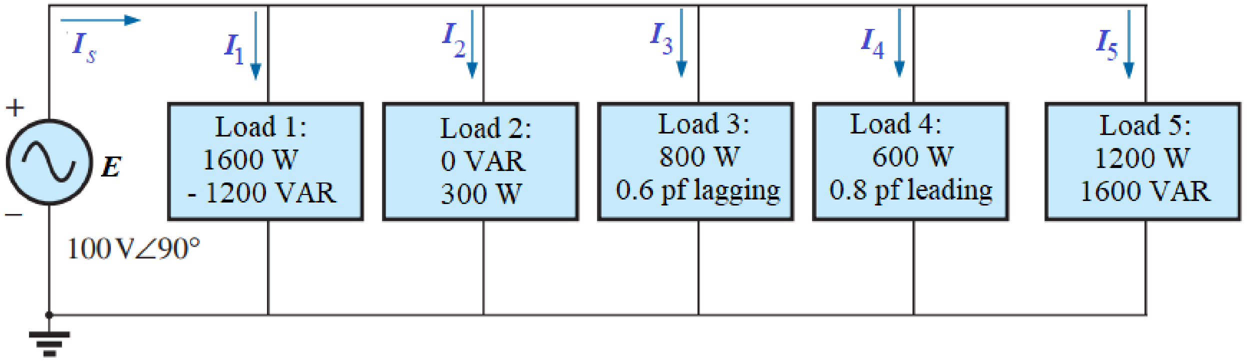
Load 2: Reactive power is zero, so circuit is resistive and voltage and current are in phase.

$$\theta_{z2} = 0°$$

$$\theta_{i2} = \theta_e - \theta_{z1} = 90° - 0° = 90°$$

$$I_2 = \frac{P}{Ecos\theta_{z2}} = \frac{300}{100 \times \cos(0°)} = 3 \text{ A}$$

$$\boldsymbol{I_2 = 3A\angle 90° = j3\text{ A}}$$



Load 3: Power factor lagging, so circuit is inductive and impedance angle is positive.

$$\theta_{z3} = cos^{-1}[0.6] = 53.13°$$

$$\theta_{i3} = \theta_e - \theta_{z3} = 90° - 53.13° = 36.87°$$

$$I_3 = \frac{P}{Ecos\theta_{z3}} = \frac{800}{100 \times 0.6} = 13.13\text{ A}$$

$$\boldsymbol{I_3 = 13.13A\angle 36.87° = 10.66 + j\,8\text{ A}}$$

Load 4: Power factor leading, so circuit is capacitive and impedance angle is negative.

$$\theta_{z4} = -cos^{-1}[0.8] = -36.87°$$

$$\theta_{i4} = \theta_e - \theta_{z4} = 90° - (-36.87°) = 126.87°$$

$$I_4 = \frac{P}{Ecos\theta_{z4}} = \frac{600}{100 \times 0.8} = 7.5\text{ A}$$

$$\boldsymbol{I_4 = 7.5A\angle 126.87° = -4.5 + j\,6\text{ A}}$$

Load 5: Reactive power is positive, so circuit is inductive and impedance angle is positive.

$$\theta_{z5} = tan^{-1}\left[\frac{Q}{P}\right] = tan^{-1}\left[\frac{1600}{1200}\right] = 53.13°$$

$$\theta_{i5} = \theta_e - \theta_{z5} = 90° - -53.13° = 36.87°$$

$$I_5 = \frac{P}{Ecos\theta_{z5}} = \frac{1200}{100 \times \cos(53.13°)} = 20\text{ A}$$

$$\boldsymbol{I_5 = 20A\angle 36.87° = 16 + j\,12\text{ A}}$$

$$\boldsymbol{I_s = I_1 + I_2 + I_3 + I_4 + I_5 = 10.16 + j45}$$
$$\boldsymbol{= 46.13\angle 77.28°}$$

Chapter 16

Series-Parallel Circuits

EXAMPLE 16.1: For the network in Fig. 16.1:

(*a*) Calculate $\boldsymbol{Z}_T$. (*b*) Determine $\boldsymbol{I}_s$.

(*c*) Calculate V_R, V_C and V_L. (*d*) Find the $\boldsymbol{I}_C$ and $\boldsymbol{I}_L$.

(*e*) Compute the power delivered.

(*e*) Find power factor (F_p) of the network.

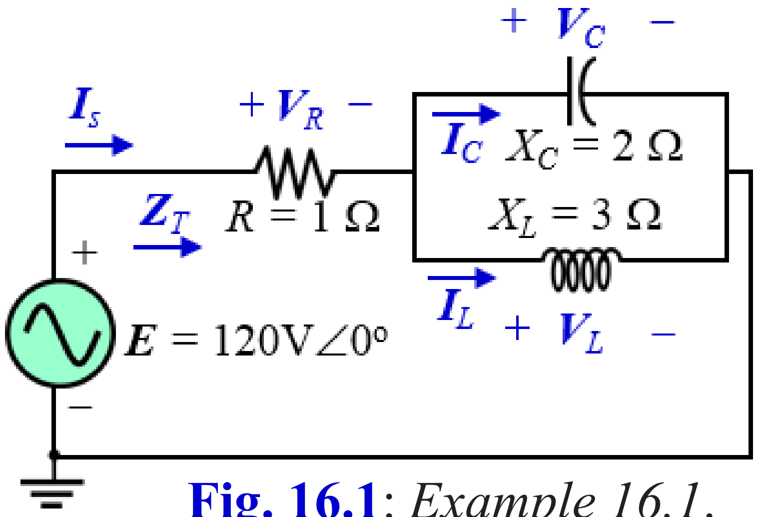


Fig. 16.1: *Example 16.1.*

Solution: (a) Let, $\boldsymbol{Z}_1 = 1\ \Omega = 1\Omega\angle 0^\circ$; $\boldsymbol{Z}_2 = -j2\ \Omega = 2\Omega\angle -90^\circ$; $\boldsymbol{Z}_3 = j3\ \Omega = 3\Omega\angle 90^\circ$;

Fig. 16.1(a) shows the redrawing circuit of Fig. 16.1.

$$\boldsymbol{Z}_4 = \frac{\boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_2+\boldsymbol{Z}_3} = \frac{(-j2)(j3)}{(-j2)+(j3)} = -j6\ \Omega = 6\Omega\angle -90^\circ$$

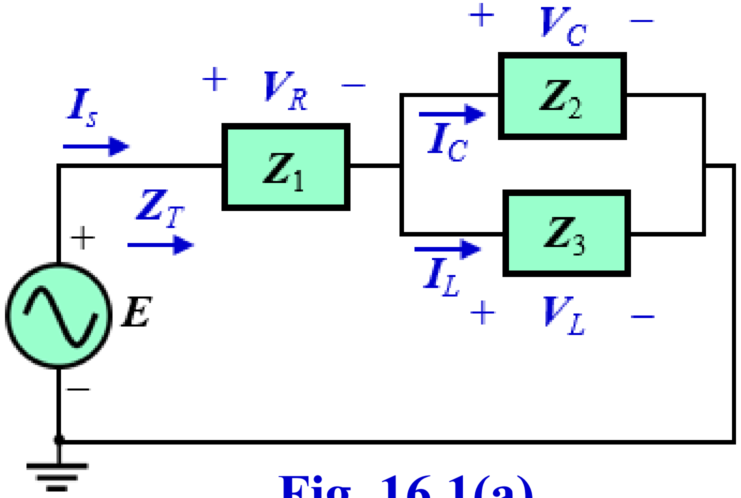


Fig. 16.1(a)

Fig. 16.1(b) shows the redrawing circuit of Fig. 16.1(a).

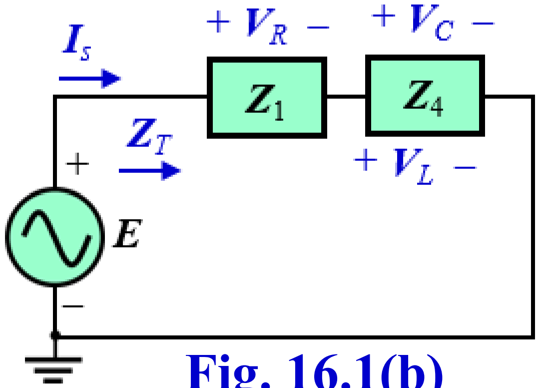


Fig. 16.1(b)

$$\boldsymbol{Z}_T = \boldsymbol{Z}_1 + \boldsymbol{Z}_4 = 1 - j6\ \Omega$$
$$= \mathbf{6.08\Omega\angle -80.54^\circ}$$

$$(\boldsymbol{b})\ \boldsymbol{I}_s = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \frac{120\text{V}\angle 0^\circ}{6.08\Omega\angle -80.54^\circ}$$
$$= 19.74\text{A}\angle 80.54^\circ$$

(**c**) Referring to Fig. 16.1(b), we have.

$$\boldsymbol{V}_R = \boldsymbol{I}_s\boldsymbol{Z}_1 = (19.74\text{A}\angle 80.54°)(1\Omega\angle 0°) = \mathbf{19.74}V\angle\mathbf{80.54°}$$

$$\boldsymbol{V}_C = \boldsymbol{V}_L = \boldsymbol{I}_s\boldsymbol{Z}_4 = (19.74\text{A}\angle 80.54°)(6\Omega\angle -90°) = \mathbf{118.44}\text{V}\angle -\mathbf{9.46°}$$

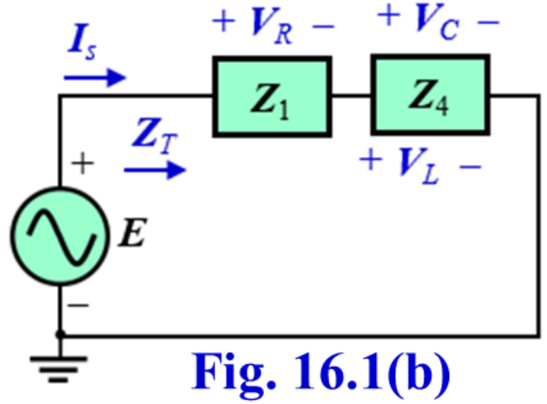


Fig. 16.1(b)

(**d**) Referring to Fig. 16.1(b), we have.

$$\boldsymbol{I}_C = \frac{\boldsymbol{V}_C}{\boldsymbol{Z}_C} = \frac{118.44\text{V}\angle -9.46°}{2\Omega\angle -90°} = \mathbf{59.22A\angle 80.54°}$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{V}_L}{\boldsymbol{Z}_L} = \frac{118.44\text{V}\angle -9.46°}{3\Omega\angle 90°} = \mathbf{39.48A\angle -99.46°}$$

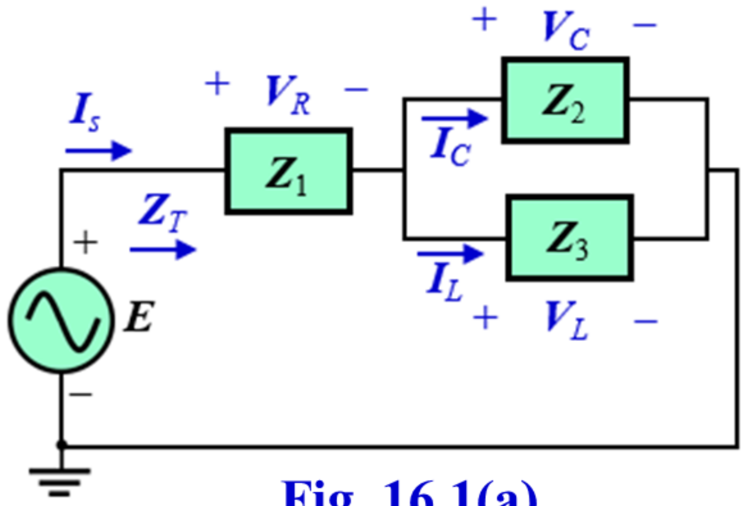


Fig. 16.1(a)

$(\boldsymbol{e})\ P_{del} = I_s^2 R = (19.74)^2\,(1\Omega) = \mathbf{389.67\ W}$

$(\boldsymbol{f})\ pf = F_p = cos\theta = \cos(80.54°) = \mathbf{0.164\ Leading}$

EXAMPLE 16.3: For the network in Fig. 16.5:

(***a***) Calculate the total impedance $\boldsymbol{Z}_T$ and the current $\boldsymbol{I}_s$.

(***b***) Calculate the voltage $\boldsymbol{V}_C$ using the voltage divider rule.

(***c***) Calculate the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ using the current divider rule.

(***d***) Calculate the power consumption by R, the reactive power consumption by L and the reactive power supplied by C.

(***e***) Calculate the apparent power, the power and the reactive power delivered by source.

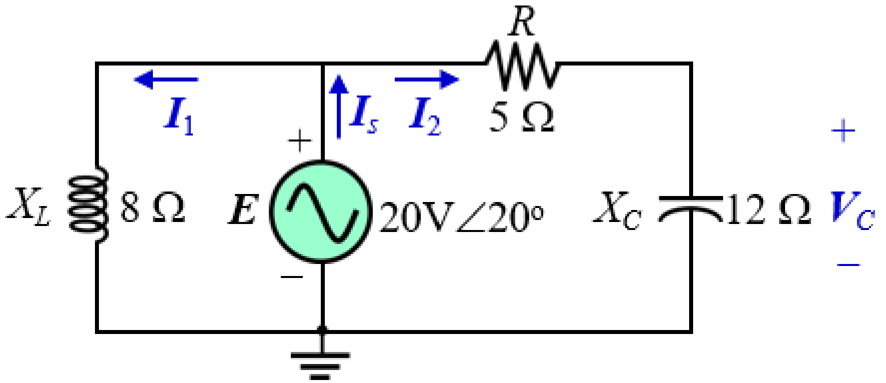


Fig. 16.5: *Example 16.3.*

Solution: (a) Let, $\boldsymbol{Z}_1 = 5\ \Omega = 5\Omega\angle 0^\circ$;

$\boldsymbol{Z}_2 = -j12\ \Omega = 12\Omega\angle -90^\circ$;

$\boldsymbol{Z}_3 = j8\ \Omega = 8\Omega\angle 90^\circ$;

Fig. 16.5(a) shows the redrawing circuit of Fig. 16.5.

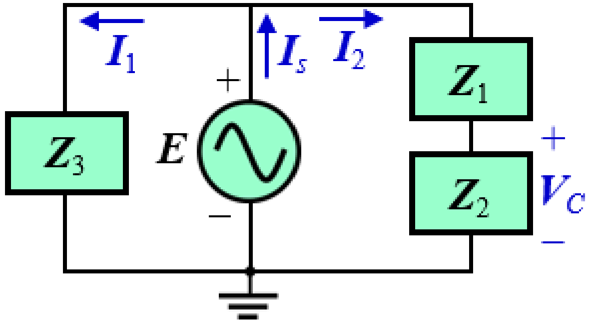


Fig. 16.5(a)

$\boldsymbol{Z}_4 = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 = 5 - j12\ \Omega = 13\Omega\angle -67.38^\circ$

Fig. 16.5(b) shows the redrawing circuit of Fig. 16.5(a).

$$\boldsymbol{Z}_T = \frac{\boldsymbol{Z}_3\boldsymbol{Z}_4}{\boldsymbol{Z}_3+\boldsymbol{Z}_4} = \frac{(j8)(5-j3)}{(j8)+(5-j3)}$$

$$= 7.8 + j14.24\ \Omega$$

$$= 16.24\Omega\angle 61.29^\circ$$

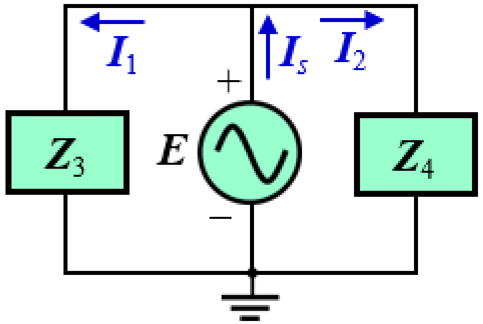


Fig. 16.5(b)

$$\boldsymbol{I}_s = \frac{\boldsymbol{E}}{\boldsymbol{Z}_T} = \frac{20\text{V}\angle 20^\circ}{16.24\Omega\angle 61.29^\circ}$$

$$= 1.23\text{A}\angle 40.29^\circ$$

(***b***) Calculate the voltage V_C using the voltage divider rule.

Referring to Fig. 16.5(a), we have.

$$\boldsymbol{V}_C = \frac{\boldsymbol{Z}_2\boldsymbol{E}}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = \frac{(12\Omega\angle -90°)(20\text{V}\angle 20°)}{5 - j12}$$
$$= 18.46\text{V}\angle -2.62°$$

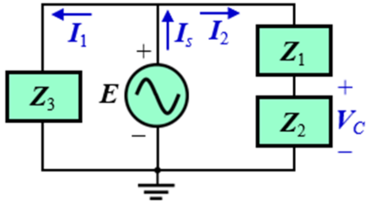


Fig. 16.5(a)

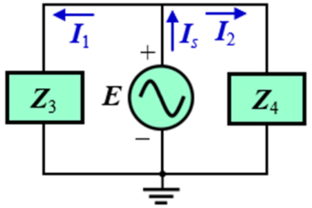


Fig. 16.5(b)

(***c***) Calculate the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ using the current divider rule.

Referring to Fig. 16.5(b), we have.

$$\boldsymbol{I}_1 = \frac{\boldsymbol{Z}_4\boldsymbol{I}_s}{\boldsymbol{Z}_3 + \boldsymbol{Z}_4} = \frac{(5 - j12\ \Omega)(1.23\text{A}\angle 40.29°)}{(j8) + (5 - j12)}$$
$$= 0.87 - j2.35\text{ A} = 2.51\text{A}\angle -68.68°$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{Z}_3\boldsymbol{I}_s}{\boldsymbol{Z}_3 + \boldsymbol{Z}_4} = \frac{(j8\ \Omega)(1.23\text{A}\angle 40.29°)}{(j8) + (5 - j12)}$$
$$= 0.06 + j1.54\text{ A} = 1.54\text{A}\angle 87.77°$$

(***d***) Calculate the power consumption by R, the reactive power consumption by L and the reactive power supplied by C.

$$P_R = I_2^2 R = (1.54\text{A})^2 \times (5\Omega) = 11.86\text{ W}$$

$$Q_L = I_1^2 X_L = (2.51\text{A})^2 \times (8\Omega) = 50.4\text{ VAR}$$

$$Q_C = -I_2^2 X_C = -(1.54\text{A})^2 \times (12\Omega) = -28.46\text{ VAR}$$

(***e***) Calculate the apparent power, the power and the reactive power delivered by source.

$$S = EI_s = (20\text{V}) \times (1.23\text{A}) = 24.6\text{ VA}$$

$$P = EI_s cos\theta = EI_s\cos\theta_z$$
$$= (20\text{V}) \times (1.23\text{A})\cos(61.29°)$$
$$= 11.82\text{ W}$$

$$Q = EI_s sin\theta = EI_s\sin\theta_z$$
$$= (20\text{V}) \times (1.23\text{A})\sin(61.29°)$$
$$= 21.58\text{ VAR}$$

EXAMPLE 16.6 For the network in Fig. 16.12:

a. Determine the current **I**.
b. Find the voltage **V**.

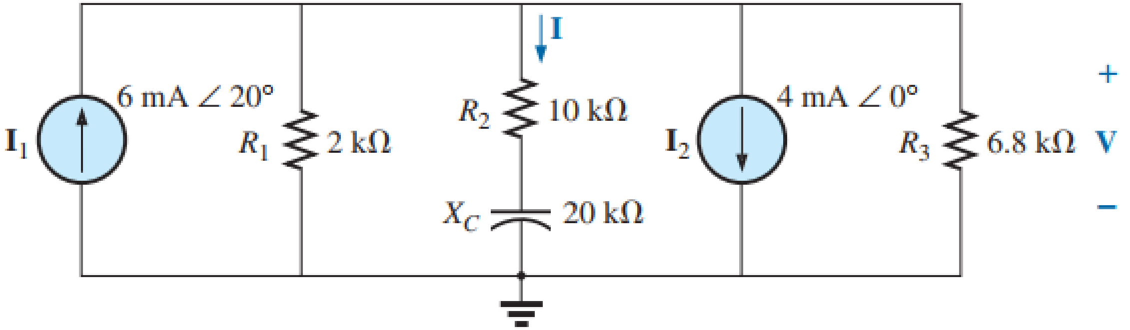


FIG. 16.12 *Example 16.6.*

Solutions:

a. The rules for parallel current sources are the same for dc and ac networks. That is, the equivalent current source is their sum or difference (as phasors). Therefore,

$$\begin{aligned}\mathbf{I}_T &= 6\text{ mA}\angle 20^\circ - 4\text{ mA}\angle 0^\circ\\ &= 5.638\text{ mA} + j\,2.052\text{ mA} - 4\text{ mA}\\ &= 1.638\text{ mA} + j\,2.052\text{ mA}\\ &= 2.626\text{ mA}\angle 51.402^\circ\end{aligned}$$

Redrawing the network using block impedances results in the network in Fig. 16.13 where

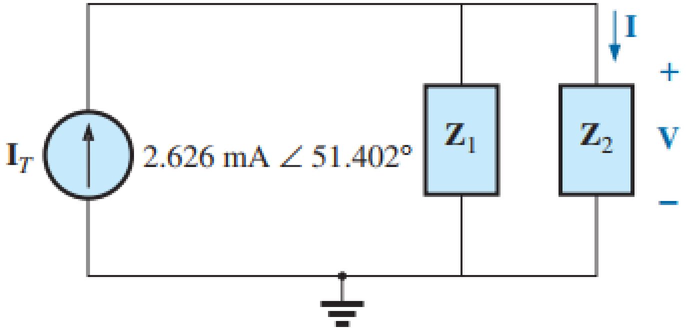


FIG. 16.13
Network in Fig. 16.12 following the assignment of the subscripted impedances.

$$\mathbf{Z}_1 = 2\text{ k}\Omega\angle 0^\circ \parallel 6.8\text{ k}\Omega\angle 0^\circ = 1.545\text{ k}\Omega\angle 0^\circ$$

and $$\mathbf{Z}_2 = 10\text{ k}\Omega - j\,20\text{ k}\Omega = 22.\,361\text{ k}\Omega\angle -63.435^\circ$$

Note that **I** and **V** are still defined in Fig. 16.13.

Calculate the currents ***I***.

Current divider rule:

$$\mathbf{I} = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(1.545\ \text{k}\Omega\ \angle 0°)(2.626\ \text{mA}\ \angle 51.402°)}{1.545\ \text{k}\Omega + 10\ \text{k}\Omega - j\,20\ \text{k}\Omega}$$

$$= \frac{4.057\ \text{A}\ \angle 51.402°}{11.545 \times 10^3 - j\,20 \times 10^3} = \frac{4.057\ \text{A}\ \angle 51.402°}{23.093 \times 10^3\ \angle -60.004°}$$

$$= \mathbf{0.18\ mA\ \angle 111.41°}$$

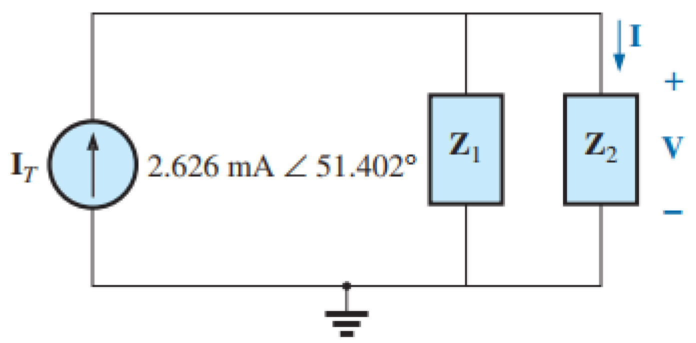


FIG. 16.13
Network in Fig. 16.12 following the assignment of the subscripted impedances.

(***b***) Calculate the voltage ***V***.

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_2$$
$$= (0.176\ \text{mA}\ \angle 111.406°)(22.36\ \text{k}\Omega\ \angle -63.435°)$$
$$= \mathbf{3.94\ V\ \angle 47.97°}$$