Chapter 13

Sinusoidal Alternating Waveforms

WAVEFORMS

Waveform: The path traced by a quantity, such as the voltage, current etc. as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

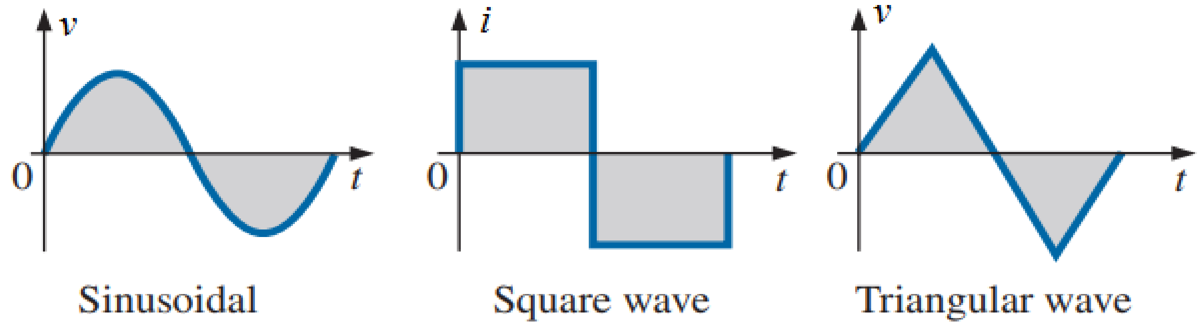
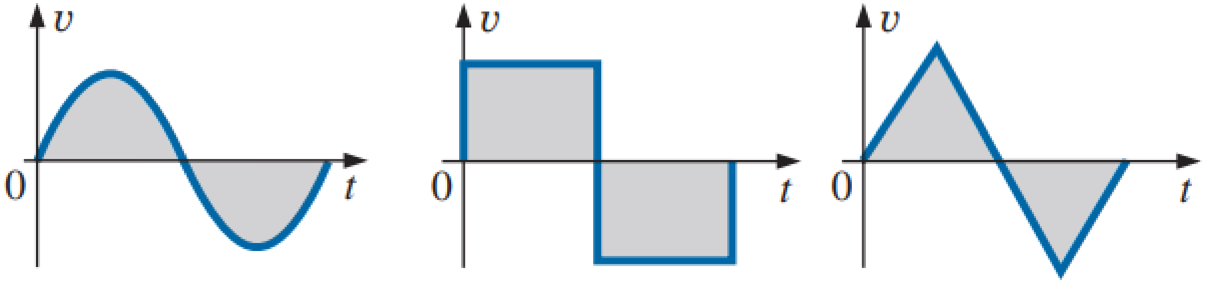


FIG. 13.1 *Alternating waveforms.*

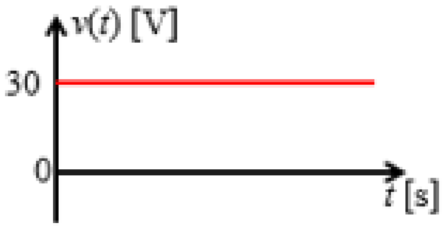
(1) Waveform Based on the Following Sine Rule



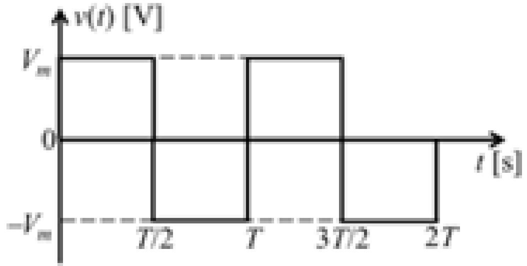
Sinusoidal

Non-sinusoidal

(2) Waveform Based on Direction of Propagation

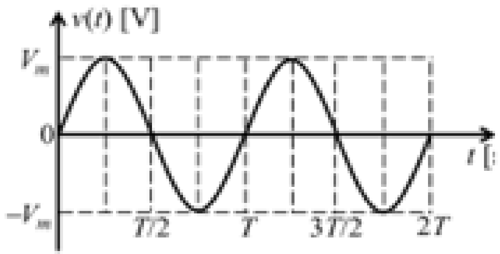


Unidirectional

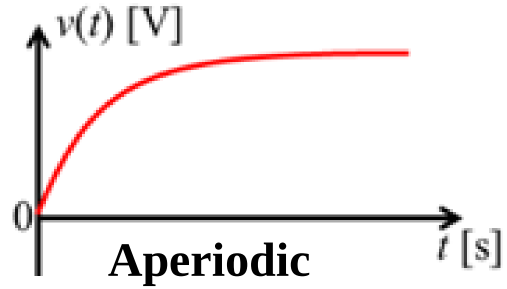


Bidirectional

(3) Waveform Based on Periodicity

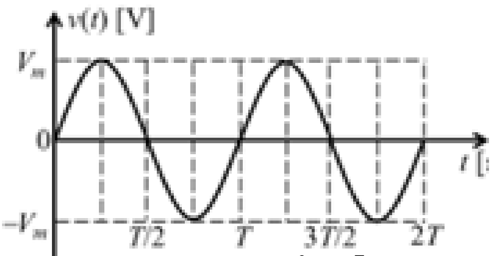


Periodic

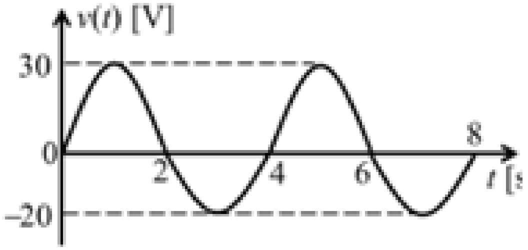


Aperiodic

(4) Waveform Based on the Shape



Symmetrical

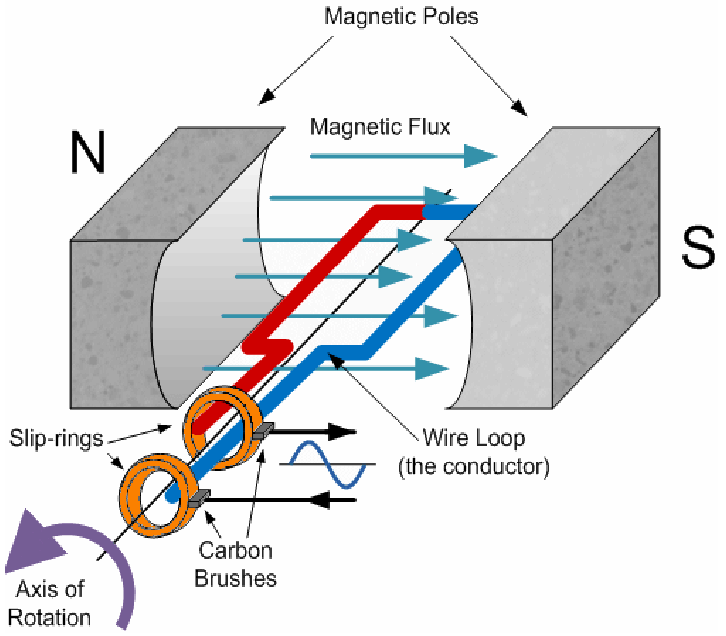


Asymmetrical

GENERATION OF SINE WAVEFORM

Faradays Law of Electromagnetic Induction:

Faradays First Law: Whenever a **conductor** is placed in a **varying magnetic field** an electromotive force (EMF) is induced or produced or developed across the conductor (called as **induced emf**), and if the conductor is a closed circuit, then induced current flows through it.



(a) Simple construction of a generator

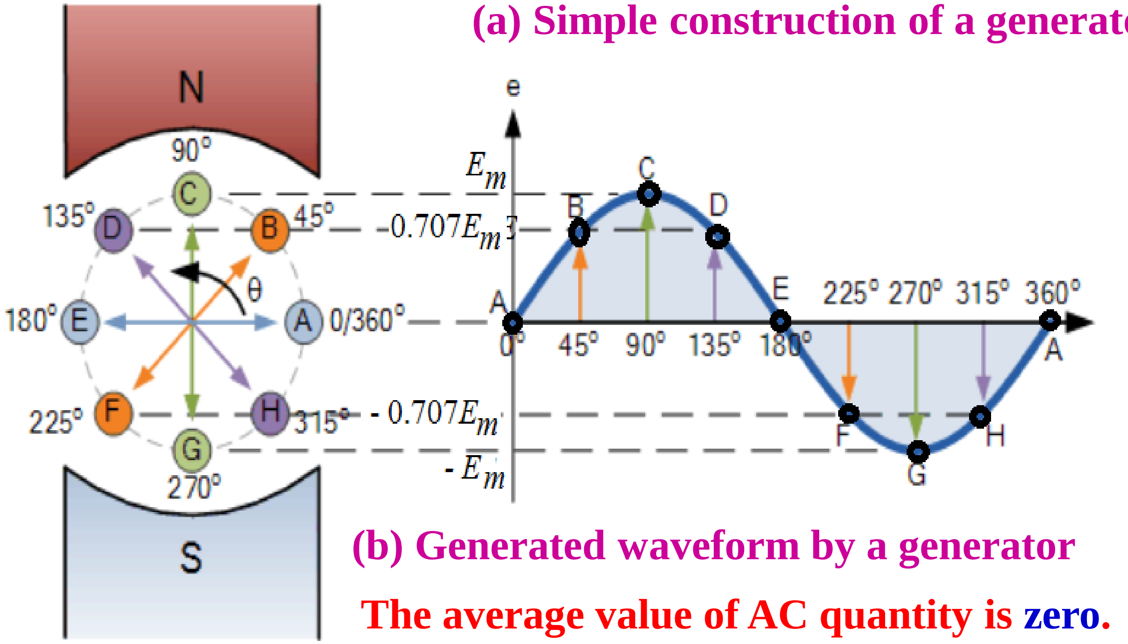
Faradays Second Law: The magnitude of induced emf is equal to the rate of change of flux linkages ($d\phi/dt$) with the coil. So, induced emf: $e = -N(d\phi/dt)$ V.

Magnitude of Induced voltage affected by **flux density** (B), the **effective length of conductor** (l), the **conductor velocity** (v) and sine of angle between **flux line and the direction of motion of conductor**.

$$e(t) = Blv\sin\alpha \text{ V} = E_m\sin\alpha \text{ V}$$

where, $E_m = Blv$ is constant.

$$\boxed{\text{So, } e(t) \propto \sin\alpha \text{ V}}$$



(b) Generated waveform by a generator

The average value of AC quantity is zero.

SOME DEFINITION RELATED TO WAVEFORMS

Instantaneous Value: The magnitude of a waveform at any instant of time is called instantaneous value. Instantaneous value is denoted by lowercase letters [such as e for sources of voltage, v for the voltage drop across a load, i for the current pass through a load and p for the power]. In **Fig. 13.3**, $e_1 = 64.7$ V and $e_2 = -56.5$ V are instantaneous value of e at time $t_1 = 0.06$ s and $t_2 = 0.25$ s.

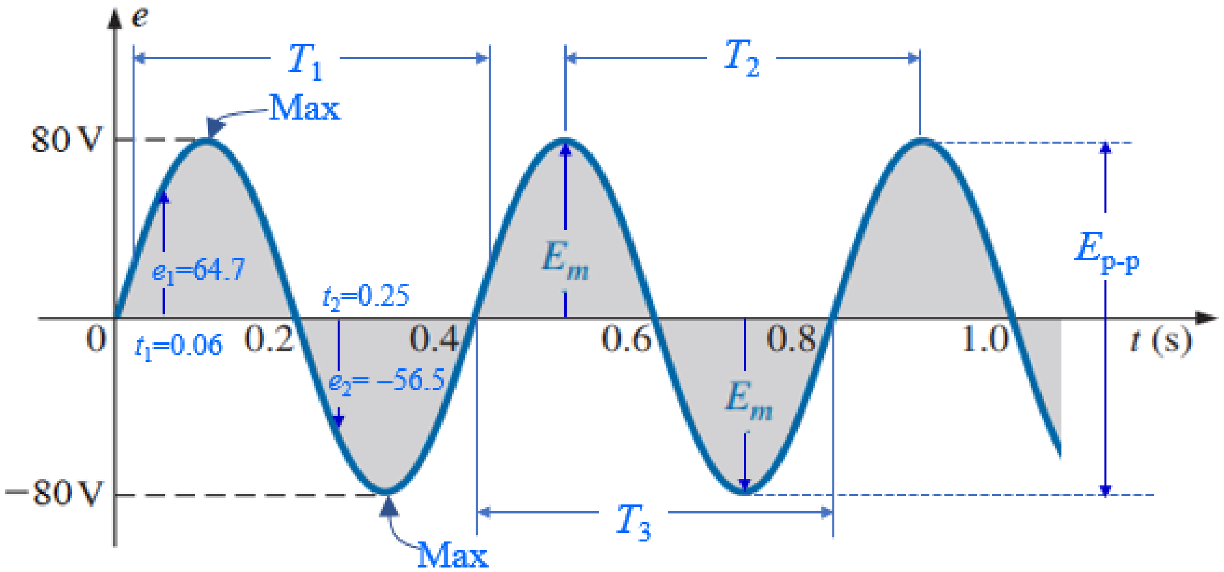


Fig. 13.3 Important parameters for a sinusoidal voltage.

Peak or Crest or Amplitude or Maximum Value: The maximum instantaneous value attained by an alternating quantity during positive and negative half-cycle is called its ***amplitude*** or ***peak*** or ***crest*** or ***maximum*** value. Peak value is denoted by uppercase letters [such as E_m, V_m and I_m]. In **Fig. 13.3**, $E_m = 80$ V peak value of e.

Peak-to-Peak Value: The full voltage between positive and negative peaks of the waveform, that is, the difference between the positive peak and the negative peak.
Peak-to-Peak value is denoted by uppercase letters [such as $E_{p\text{-}p}$, $V_{p\text{-}p}$ and $I_{p\text{-}p}$]. In **Fig. 13.3**, $E_{p\text{-}p} = 80\text{ V} - (-80\text{ V}) = 160$ V peak-to-peak value of e.

Periodic Waveform: A waveform that continually repeats itself after the same time interval. The waveform in Fig. 13.3 is a periodic waveform.

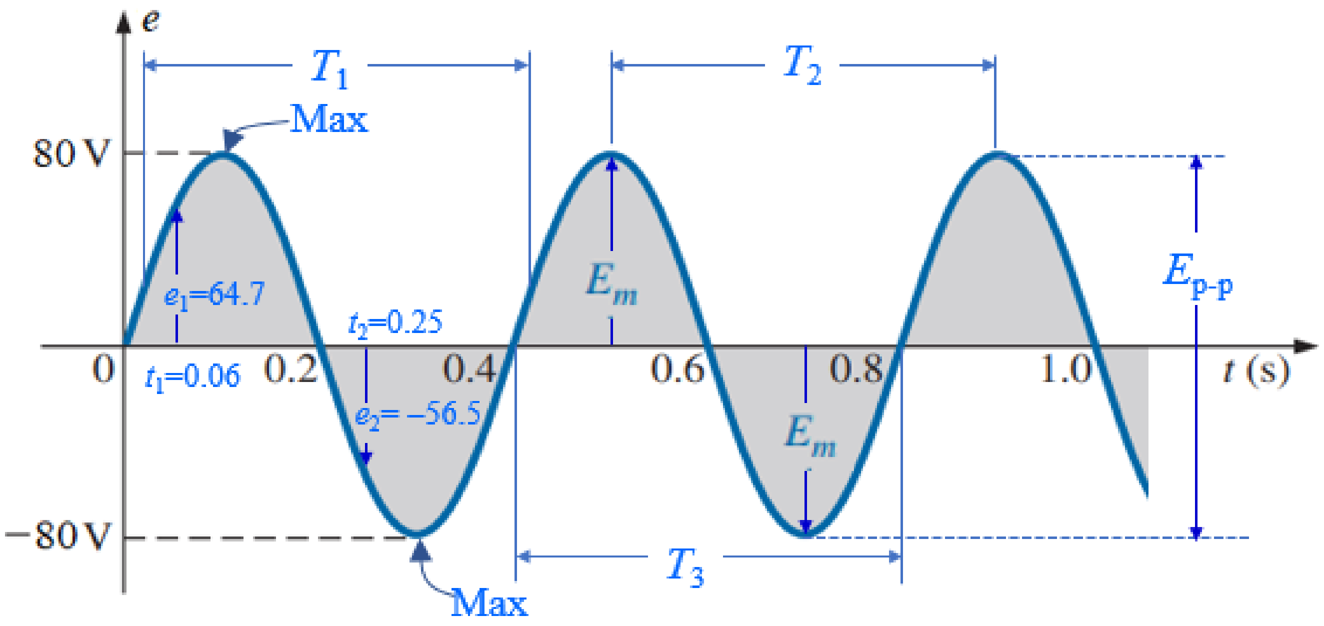


Fig. 13.3 Important parameters for a sinusoidal voltage.

Cycle: The portion of a waveform that continually repeats itself after the same time interval is called a cycle.

The cycles within T_1, T_2, and T_3 in Fig. 13.3 may appear different in Fig. 13.4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

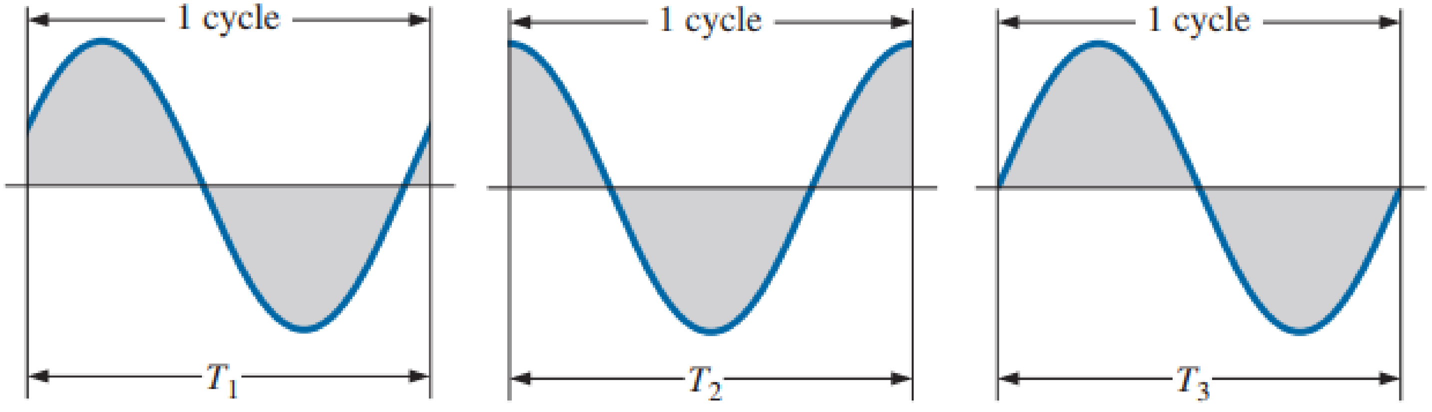


FIG. 13.4 *Defining the cycle and period of a sinusoidal waveform.*

Period (T): The time taken by an alternating quantity to complete its one cycle is known as time period.

It is denoted by T seconds. After every seconds, the cycle of an alternating quantity repeats.

Frequency (*f*): The number of cycles completed by an alternating quantity per second is known as frequency.

It is denoted by *f* and it is measured in **cycle/second** which is known as **Hertz**, denoted by **Hz**.

In the world, some countries (**Bangladesh**, India, Argentina, Australia, Combodia etc.) use **50 Hz** and some countries (USA, Canada, Brazil, Haiti, Eucador, Colombia, Costa Rica etc.) use **60 Hz** as power frequency.

The frequency of **DC voltage is zero (0) Hz**.

Relation between Frequency and Time Period: Frequency is reciprocal of time period.

$$f = \frac{1}{T}\ \text{Hz}$$

$$T = \frac{1}{f}\ \text{s}$$

Audio Frequencies	: 15 Hz ~ 20 kHz
Radio Frequencies	: 3 kHz ~ 300 GHz
Infrared Frequencies	: More than 300 GHz

Relation Between Degree and Radian:

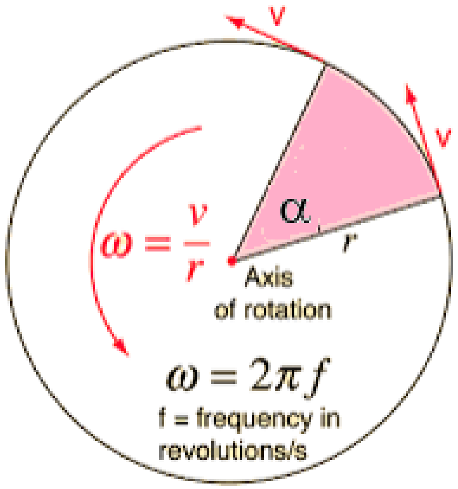
$$\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees}) \qquad (13.6)$$

$$\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians}) \qquad (13.7)$$

Angular Frequency or Velocity (*ω*): The rate of change of angular position is called angular frequency or angular velocity. Unit of angular frequency is **rad/s**.

$$\omega = \frac{\alpha}{t}\ \text{rad/s}$$

$$\alpha = \omega t\ \ [\text{rad}]$$



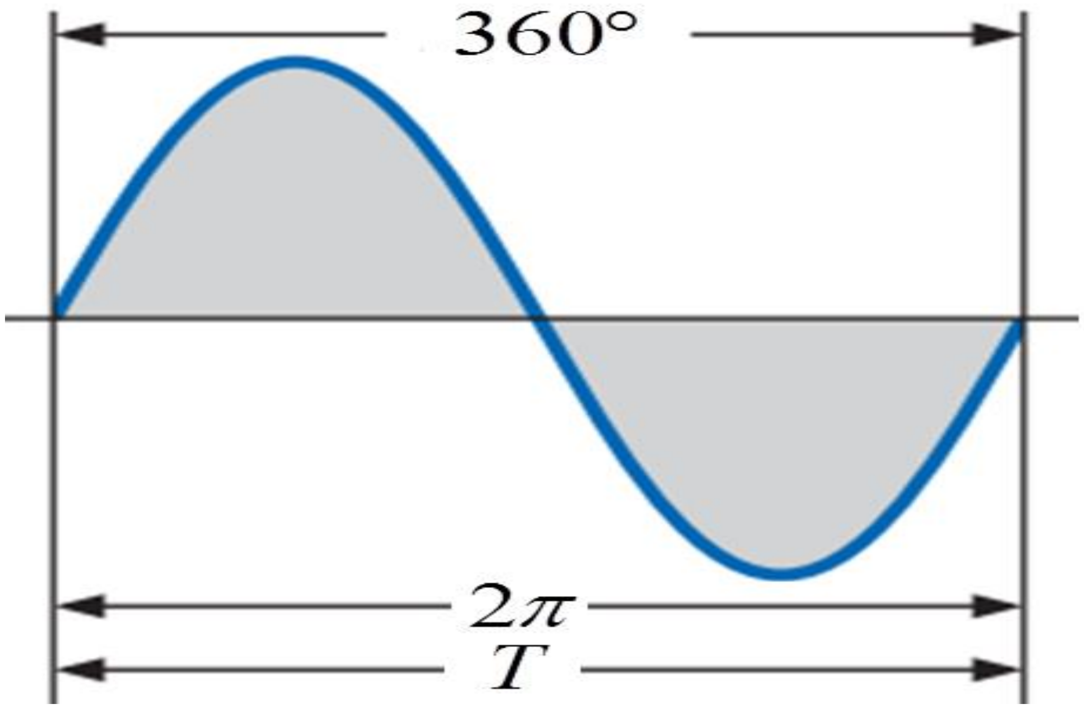
$$t = \frac{\alpha\,[\text{rad.}]}{\omega}\ \text{s} = \frac{\alpha\,[\text{deg.}]}{\omega} \times \frac{\pi}{180^{o}}\ \text{s}$$

Relation among Angular Frequency, Frequency and Time Period:

$$\omega = \frac{2\pi}{T}\ \text{rad/s} = 2\pi f\ \text{rad/s}$$

$$T = \frac{2\pi}{\omega}\ \text{s}$$

$$f = \frac{\omega}{2\pi}\ \text{Hz}$$



Alternating Current/Voltage:

Alternating current/voltage rises from zero to a maximum value in one direction and decreases back to zero. It then rises to the same maximum value in the opposite direction and again decreases to zero. These values are repeated again and again at equal intervals of time.

The average value of AC quantity is zero.

It is seen from the following figure that the abscissa (***x*-axis**) of alternating voltage and current can be represent in terms of time (t), angle (α) in deg., or angle (α) in rad.

EXAMPLE 13.2 Find the period of periodic waveform with a frequency of

a. 60 Hz. b. 1000 Hz.

Solutions: a. $T = \dfrac{1}{f} = \dfrac{1}{60\text{ Hz}} \cong 0.01667\text{ s or } \mathbf{16.67\text{ ms}}$

b. $T = \dfrac{1}{f} = \dfrac{1}{1000\text{ Hz}} = 10^{-3}\text{ s} = \mathbf{1\text{ ms}}$

EXAMPLE 13.4 Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution: $\omega = 2\pi f = (2\pi)(60\text{ Hz}) \cong \mathbf{377\text{ rad/s}}$

EXAMPLE 13.5 Determine the frequency and period of the sine wave having an angular frequency of 500 rad/s.

Solution: Since $\omega = 2\pi/T$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \mathbf{12.57\text{ ms}}$$

and $$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3}\text{ s}} = \mathbf{79.58\text{ Hz}}$$

EXAMPLE 13.6 Given $\omega = 200$ rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

***Solution*:** We know that, $\alpha = \omega t$, thus $t = \dfrac{\alpha}{\omega}$ s

However α must be converted to rad., since is ω rad/s.

$$\alpha\,[\text{rad.}] = \frac{\pi}{180^o} \times 90^o\text{ s} = 1.57\text{ rad}$$

$$t = \frac{\alpha}{\omega} = \frac{1.57\text{ rad}}{200\text{ rad/s}} = \frac{\pi}{400}\text{ s} = \mathbf{7.85\text{ ms}}$$

EXAMPLE 13.7 Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

***Solution*:** We know that, $\alpha = \omega t$ or

$$\alpha = 2\pi f t = (2\pi)(60\text{ Hz})(5 \times 10^{-3}\text{ s}) = \mathbf{1.885\text{ rad}}$$

If not careful, you might be tempted to interpret the answer as 1.885°. However,

$$\alpha\,(^\circ) = \frac{180^\circ}{\pi\,\cancel{\text{rad}}}(1.885\,\cancel{\text{rad}}) = \mathbf{108^\circ}$$

Practice Problem 4 ~ 16 [P582]

General Format of Mathematical Equation for the Sinusoidal Voltage or Current

Induced EMF:

$$e(t) = E_m \sin\alpha \text{ V} = E_m \sin\omega t \text{ V}$$

$$= E_m \sin 2\pi f t \text{ V} = E_m \sin\frac{2\pi}{T}t \text{ V}$$

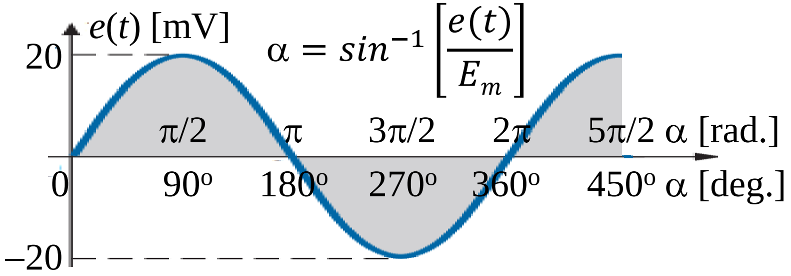
Example: $e(t) = 20\sin 31.4t$ mV

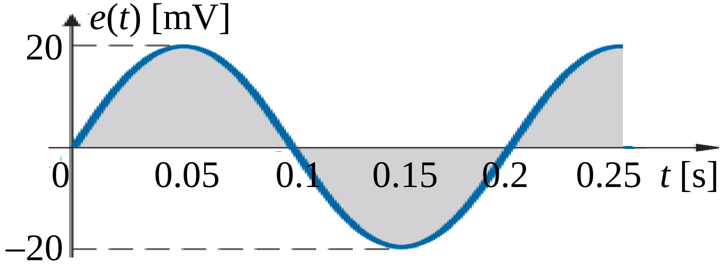
$E_m = 20$ mV; $\omega = 31.4$ rad/s

$f = (31.4 \text{ rad/s})/2\pi = 5$ Hz

$T = 1/(5 \text{ Hz}) = 0.2$ s

$$\alpha = \sin^{-1}\left[\frac{e(t)}{E_m}\right]$$





Voltage Drop:

$$v(t) = V_m \sin\alpha \text{ V} = V_m \sin\omega t \text{ V}$$

$$= V_m \sin 2\pi f t \text{ V} = V_m \sin\frac{2\pi}{T}t \text{ V}$$

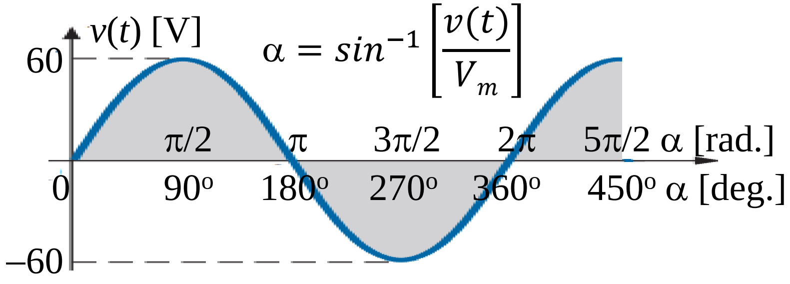
Example: $v(t) = 60\sin 100\pi t$ V

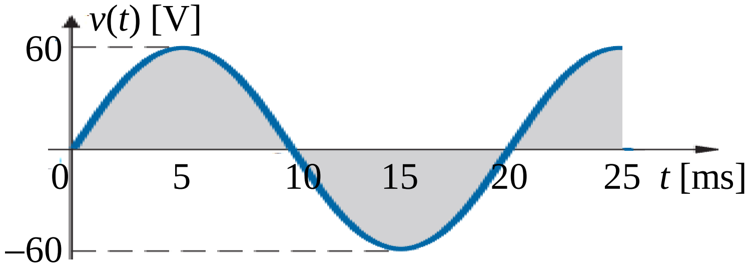
$V_m = 60$ V; $\omega = 100\pi$ rad/s

$f = (100\pi \text{ rad/s})/2\pi = 50$ Hz

$T = 1/(50 \text{ Hz}) = 0.02$ s or 20 ms

$$\alpha = \sin^{-1}\left[\frac{v(t)}{V_m}\right]$$





Current:

$$i(t) = I_m \sin\alpha \text{ A} = I_m \sin\omega t \text{ A}$$

$$= I_m \sin 2\pi f t \text{ A} = I_m \sin\frac{2\pi}{T}t \text{ A}$$

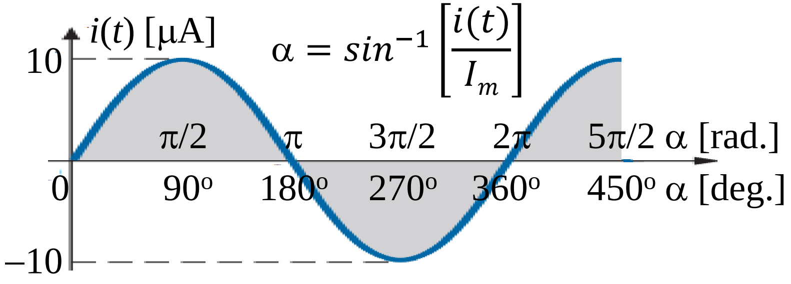
Example: $i(t) = 10\sin 261.67t$ μA

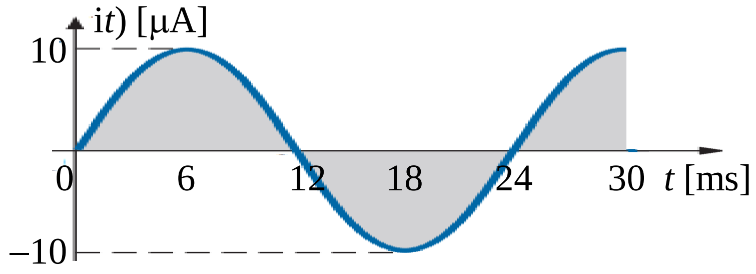
$I_m = 10$ μA; $\omega = 261.67$ rad/s

$f = (261.67 \text{ rad/s})/2\pi = 41.67$ Hz

$T = 1/(41.67 \text{ Hz}) = 24$ ms

$$\alpha = \sin^{-1}\left[\frac{i(t)}{I_m}\right]$$





EXAMPLE 13.8 Given, $e(t) = 5\sin\alpha$ V, determine $e(t)$ at (a) $\alpha = 40^o$, (b) $\alpha = 0.8\pi$, and (c) $t = 13$ ms if the angular frequency is 314 rad/s.

Solution: For $\alpha = 40^o$,

$$e(t) = 5\sin(40^o) = 5 \times (0.6428) = \mathbf{3.21\ V}$$

For $\alpha = 0.8\pi = (180^o/\pi)\times(0.8\pi) = 144^o$

$$e(t) = 5\sin(144^o) = 5\times(0.6428) = \mathbf{3.21\ V}$$

For $t = 13$ ms and 314 rad/s,

$$\alpha = \omega t = (314)(13\times10^{-3}) \text{ rad.}$$

$$= (314)\times(13\times10^{-3})\times(180^o/\pi) \text{ deg.} = 234^o$$

$$e(t) = 5\sin(234^o) = \mathbf{-4.05\ V}$$

EXAMPLE 13.9

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.
b. Determine the time at which the magnitude is attained.

Solution: (a) According to question we can write,

$$v(t) = 10\sin\alpha = 4 \text{ V}$$

$$\alpha_1 = \sin^{-1}\left[\frac{4\text{ V}}{10\text{ V}}\right] = \sin^{-1}[0.4] = \mathbf{23.58^o}$$

However, Fig.13.19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

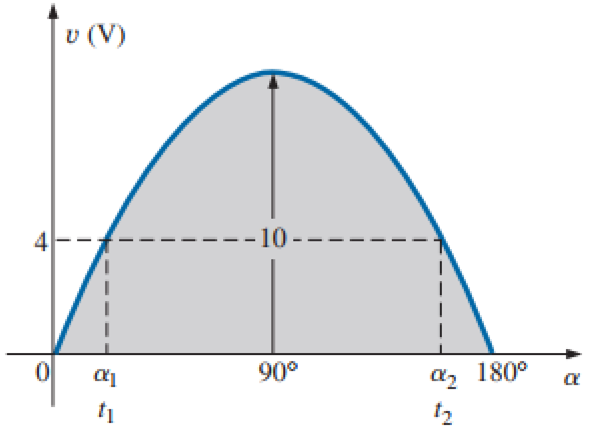


FIG. 13.19 *Example 13.9.*

$$\alpha_2 = 180^o - \alpha_1 = 180^o - 23.58^o = \mathbf{156.42^o}$$

b. Since $\alpha = \omega t$, and so $t = \alpha/\omega$. However, α must be in radians. Thus,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(23.578^\circ) = 0.412 \text{ rad}$$

and

$$t_1 = \frac{\alpha}{\omega} = \frac{0.412 \text{ rad}}{377 \text{ rad/s}} = \mathbf{1.09\ ms}$$

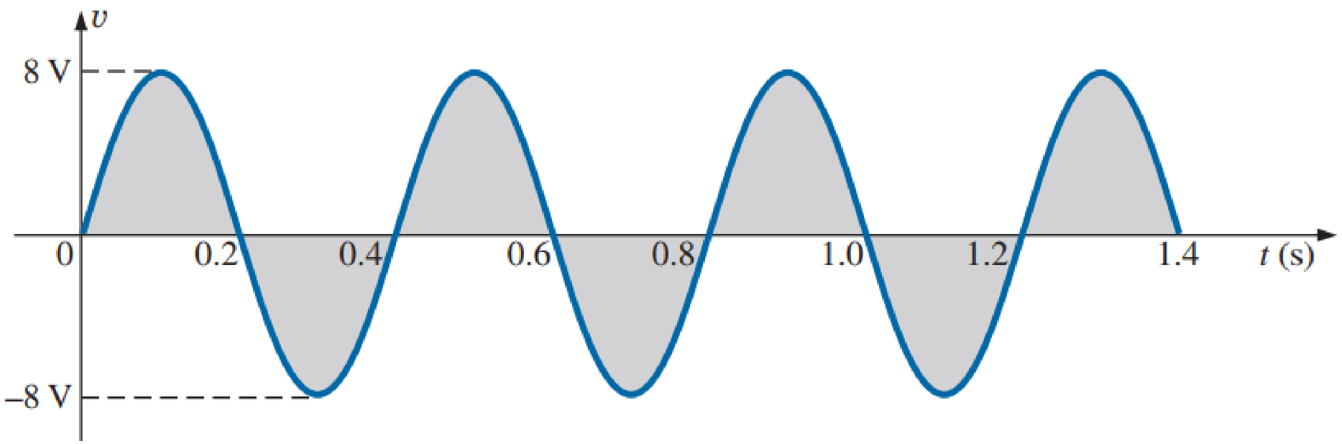
For the second intersection,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73 \text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73 \text{ rad}}{377 \text{ rad/s}} = \mathbf{7.24\ ms}$$

EXAMPLE 13.1 For the sinusoidal waveform in Fig. 13.7.

a. What is the peak value?
b. What is the instantaneous value at 0.3 s and 0.6 s?
c. What is the peak-to-peak value of the waveform?
d. What is the period of the waveform?
e. How many cycles are shown?
f. What is the frequency of the waveform?
g. What is the value of angular frequency of the waveform?
h. Write the instantaneous equation the waveform.



Solution:

(a) Peak Value: V_m = **8 V**

(b) Instantaneous value at 0.3 s is **–8V**
Instantaneous value at 0.6 s is **0 V**

(c) Peak-to-peak Value: $V_{p\text{-}p} = 2V_m$ = **16 V**

(d) Period: T = **0.4 s**

(e) Number of cycle: (1.4s/0.4s) = **3.5 cycles**

(f) Frequency: $f = 1/T$ = 1/(0.4 s) = **2.5 Hz**

(g) Angular Frequency: $\omega = 2\pi f$ = **15.7 rad/s**

(h) Instantaneous Equation:

$v(t) = \mathbf{8V\sin\alpha}\ \mathbf{V} = \mathbf{8V\sin 15.7}t\ \mathbf{V}$

1. For the sinusoidal waveform in Fig. 13.80:
   a. What is the peak value?
   b. What is the instantaneous value at 15 ms and at 20 ms?
   c. What is the peak-to-peak value of the waveform?
   d. What is the period of the waveform?
   e. How many cycles are shown?
   f. What is the frequency of the waveform?
   g. What is the value of angular frequency of the waveform?
   h. Write the instantaneous equation of the waveform.

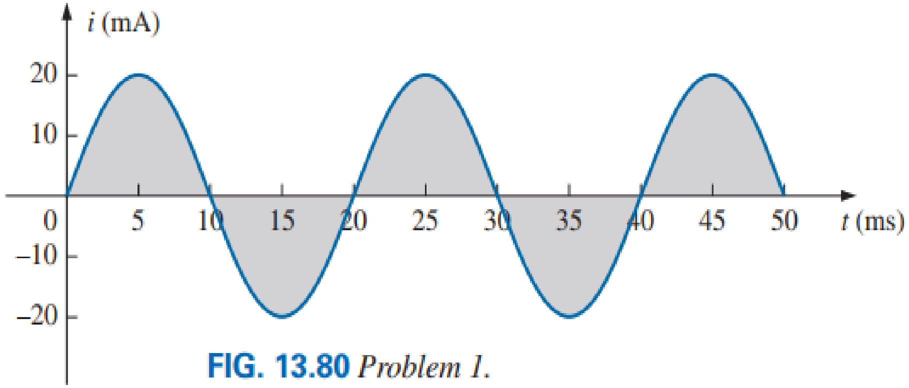


FIG. 13.80 *Problem 1.*

a. **20 mA**
b. 15 ms: **–20 mA**, 20 ms: **0 mA**
c. **40 mA**
d. **20 ms**
e. **2.5 cycles**
f. **50 Hz**
g. **314 rad/s**
h. $i(t) = 20\text{mA}\sin\alpha = 20\text{mA}\sin 314t$

2. For the square-wave signal in Fig. 13.81:
   a. What is the peak value?
   b. What is the instantaneous value at 5 μs and at 11 μs?
   c. What is the peak-to-peak value of the waveform?
   d. What is the period of the waveform?
   e. How many cycles are shown?

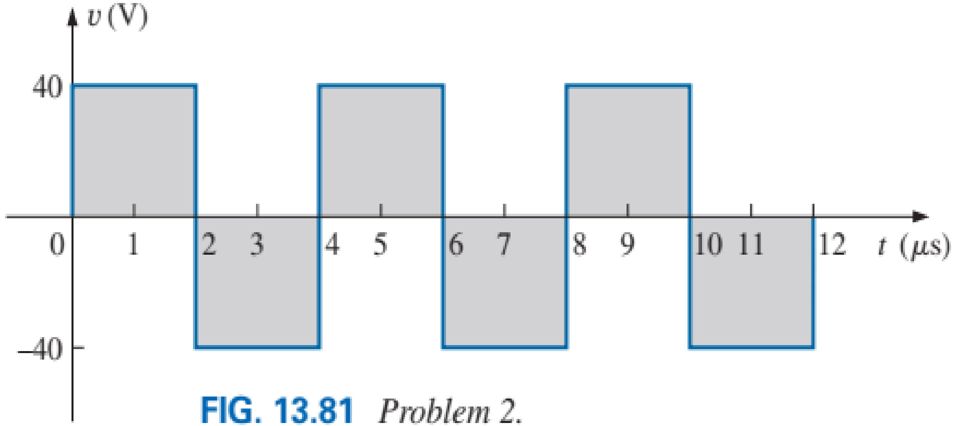


FIG. 13.81 *Problem 2.*

2. a. **40 V**
   b. 5 μs: **40 V**, 11 μs: **–40 V**
   c. **80 V**
   d. **4 μs**
   e. **3 cycles**

3. For the periodic waveform in Fig. 13.82:
   a. What is the peak value?
   b. What is the instantaneous value at 3 μs and at 9 μs?
   c. What is the peak-to-peak value of the waveform?
   d. What is the period of the waveform?
   e. How many cycles are shown?

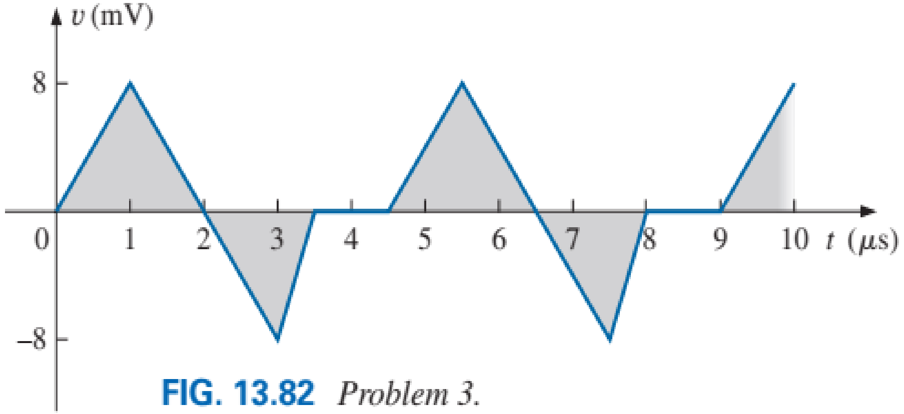


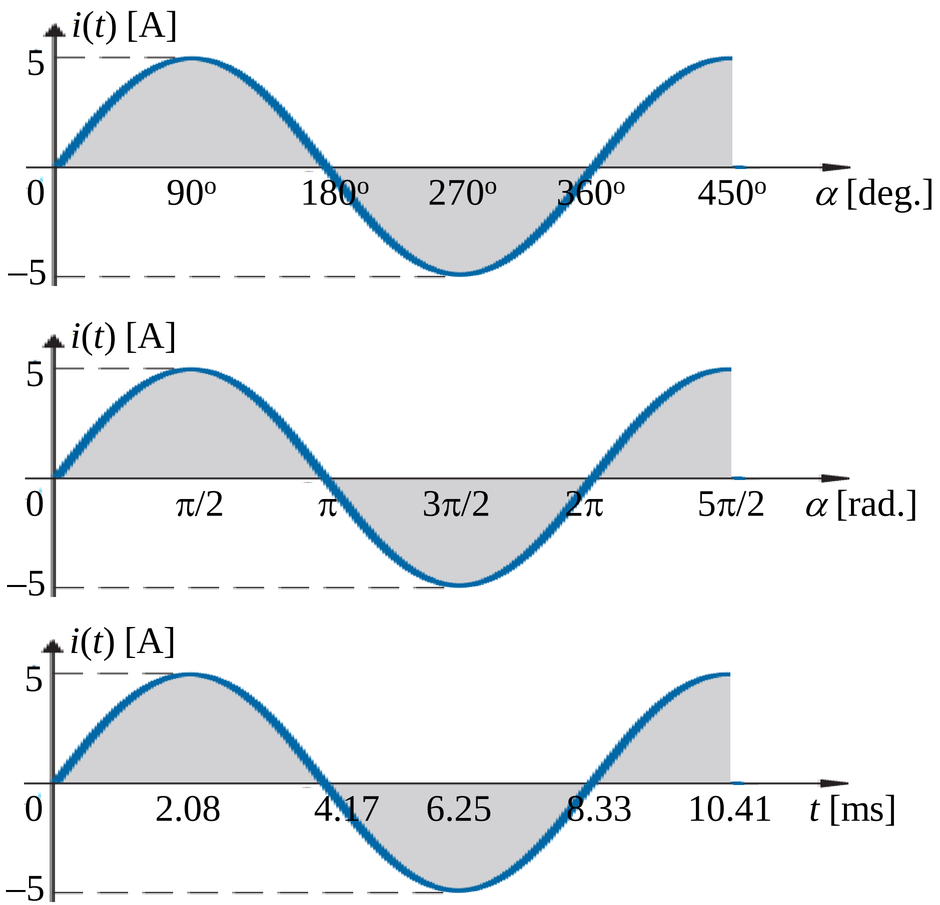
FIG. 13.82 *Problem 3.*

3. a. **8 mV**
   b. 3 μs: **–8 mV**, 9 μs: **0 mV**
   c. **16 mV**
   d. **4.5 μs**
   e. $\dfrac{10\ \mu\text{S}}{4.5\ \mu\text{S/cycle}}$ **= 2.22 cycles**

PROBLEM 18 [P582] Sketch $i(t) = 5\sin 754t$ A with the abscissa: **a.** angle in degrees, **b.** angle in radians, and **c.** time in seconds.

Solution: Here, $I_m = 5$ A, $\omega = 754$ rad/s.

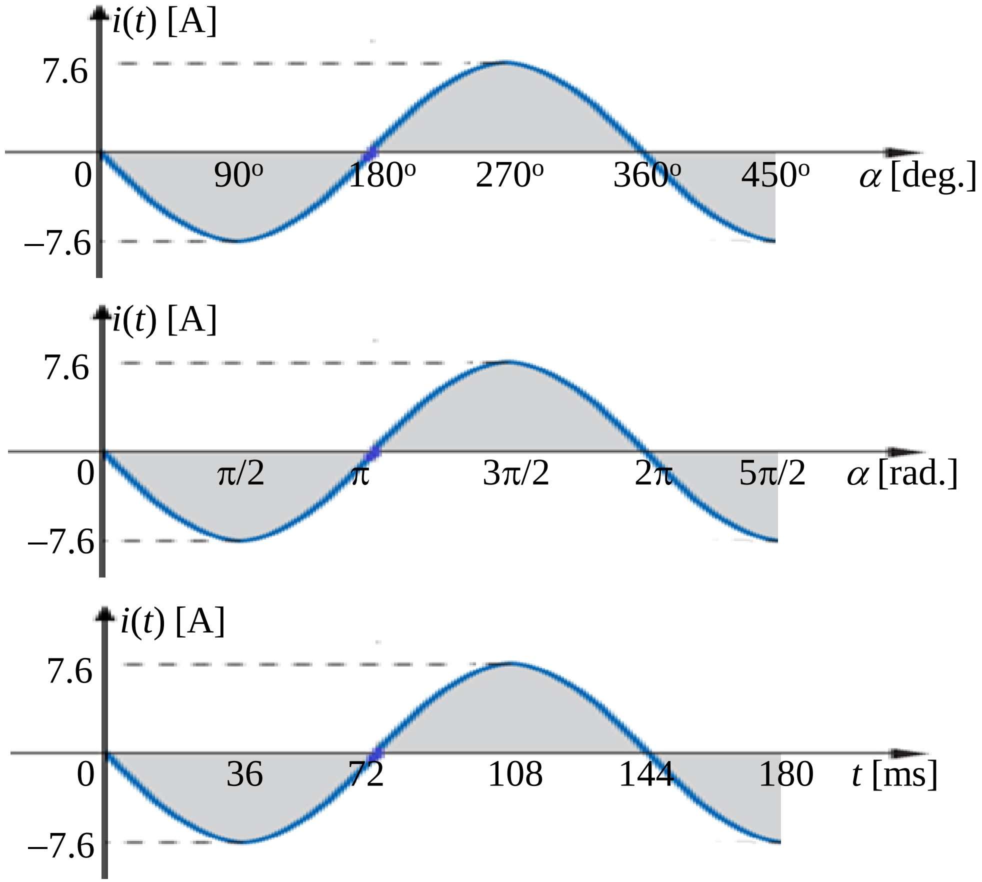
$$T = 2\pi/\omega = 2\pi/754 = 8.33 \text{ ms}$$



PROBLEM 19 [P582] Sketch $i(t) = -7.6\sin 43.6t$ A with the abscissa: **a.** angle in degrees, **b.** angle in radians, and **c.** time in seconds.

Solution: Here, $I_m = 7.6$ A, $\omega = 43.6$ rad/s.

$$T = 2\pi/\omega = 2\pi/43.6 = 144 \text{ ms}$$



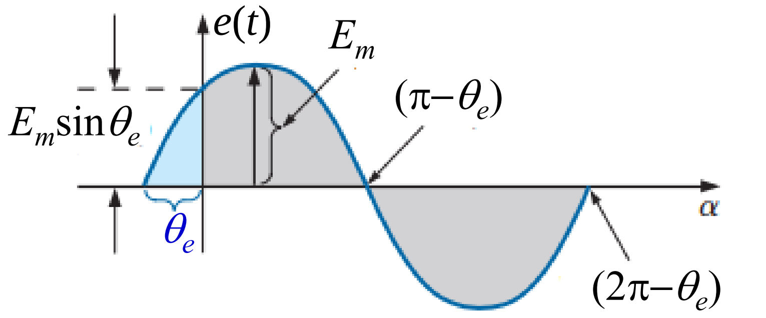
Practice Problem 17 ~ 24 [P582]

Phase Difference and Phase Relation Between Two Waveforms

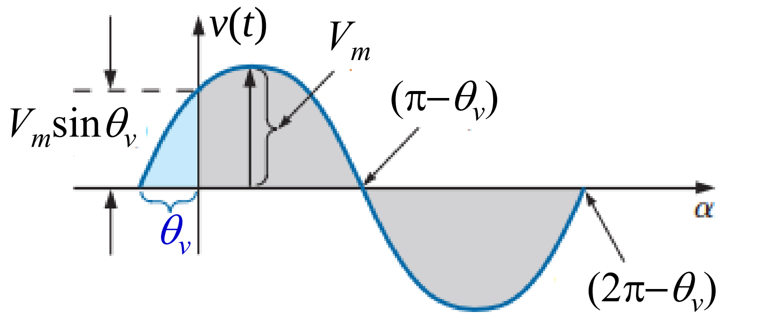
Mathematical Equation for the Sinusoidal when start Before or After 0°

If the waveform passes through the horizontal axis (y-axis) with a *positive-going* (increasing with time) slope *before* 0°, as shown in the following Figures, the expression is

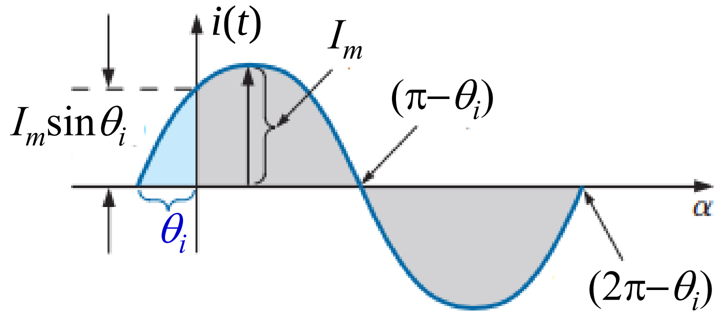
$e(t)=E_m\sin(\alpha+\theta_e)$ V $=\ E_m\sin(\omega t+\theta_e)$ V



$v(t)=V_m\sin(\alpha+\theta_v)$ V $=\ V_m\sin(\omega t+\theta_v)$ V

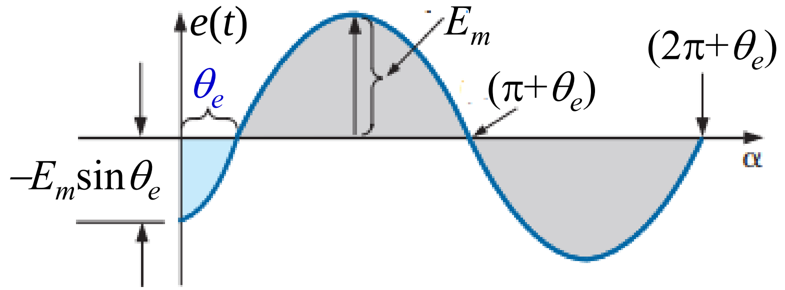


$i(t)=I_m\sin(\alpha+\theta_i)$ V $=\ I_m\sin(\omega t+\theta_i)$ A

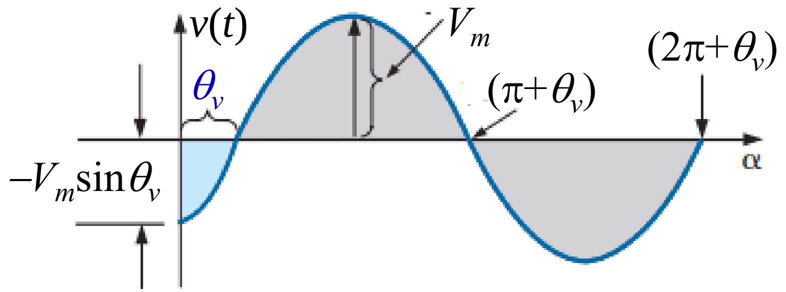


If the waveform passes through the horizontal axis (y-axis) with a *positive-going* (increasing with time) slope *after* 0°, as shown in the following Figures, the expression is

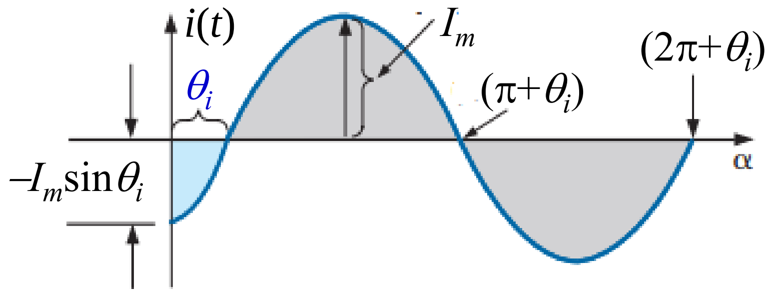
$e(t)=E_m\sin(\alpha-\theta_e)$ V $=\ E_m\sin(\omega t-\theta_e)$ V



$v(t)=V_m\sin(\alpha-\theta_v)$ V $=\ V_m\sin(\omega t-\theta_v)$ V



$i(t)=I_m\sin(\alpha-\theta_i)$ V $=\ I_m\sin(\omega t-\theta_i)$ A



Phase Angle and Initial Angle

In generally, the instantaneous or time domain equation can be written as follows:

$$e(t) = E_m\sin(\alpha + \theta_e)\ \text{V} = E_m\sin(\omega t + \theta_e)\ \text{V}$$

$$v(t) = V_m\sin(\alpha + \theta_v)\ \text{V} = V_m\sin(\omega t + \theta_v)\ \text{V}$$

$$i(t) = I_m\sin(\alpha + \theta_i)\ \text{A} = I_m\sin(\omega t + \theta_i)\ \text{A}$$

θ_e, θ_v and θ_i are called phase angle.

Initial Angle: The angle from which a waveform started with a *positive-going* (increasing with time) slope is called initial angle.

Initial Angle = – Phase Angle

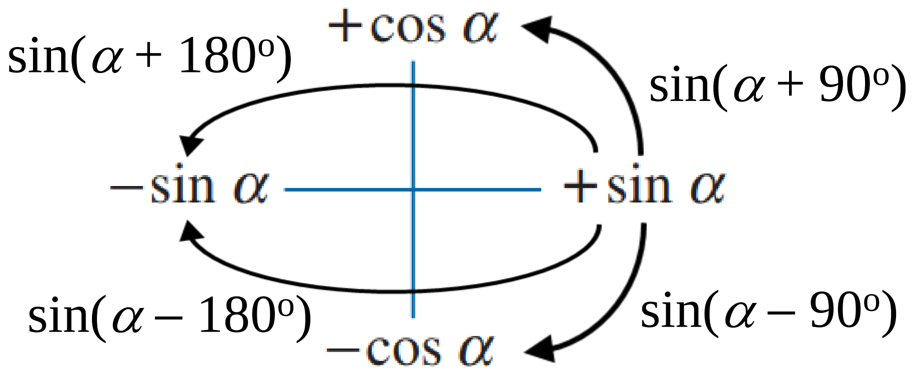
$\theta_{e0} = -\,\theta_e$; $\theta_{v0} = -\,\theta_v$; $\theta_{i0} = -\,\theta_i$;

where, θ_{e0}, θ_{v0} and θ_{i0} are initial angle.

Remember: To identify the phase angle from an equation, at first +cos, – cos and – sin convert to +sin.

If the waveform is given as **cosα**, **–cosα**, and **–sinα** must be convert to positive sine wave that means **sinα** using the following relations:

$$\cos\alpha = \sin(\alpha + 90^o)$$
$$-\cos\alpha = \sin(\alpha - 90^o)$$
$$-\sin\alpha = \sin(\alpha \pm 180^o)$$



EXAMPLE Identify the phase angle and find the initial angle for the following waveforms:

(**a**) $e(t) = 20\sin\omega t$ V

Phase angle, $\theta_e = \mathbf{0^o}$

Initial angle, $\theta_{e0} = -\theta_e = \mathbf{0^o}$

(**b**) $v(t) = 100\cos\omega t$ V

$v(t) = 100\sin(\omega t + 90^o)$ V

Phase angle, $\theta_v = \mathbf{90^o}$

Initial angle, $\theta_{v0} = -\theta_v = \mathbf{-90^o}$

(**c**) $i(t) = -2\cos\omega t$ A

$i(t) = 2\sin(\omega t - 90^o)$ A

Phase angle, $\theta_i = \mathbf{-90^o}$

Initial angle, $\theta_{i0} = -\theta_i = \mathbf{90^o}$

(**d**) $v(t) = -150\sin\omega t$ V

$v(t) = 100\sin(\omega t \pm 180^o)$ V

Phase angle, $\theta_v = \pm\mathbf{180^o}$

Initial angle, $\theta_{v0} = -\theta_v = \mp\mathbf{180^o}$

(**e**) $v(t) = 60\cos(\omega t - 20^o)$ V

$v(t) = 60\sin(\omega t - 20^o + 90^o)$ V $= 60\sin(\omega t + 70^o)$ V

Phase angle, $\theta_v = \mathbf{70^o}$

Initial angle, $\theta_{v0} = -\theta_v = \mathbf{-70^o}$

(**f**) $e(t) = -160\cos(\omega t - 20^o)$ V

$e(t) = 160\sin(\omega t - 20^o - 90^o)$ V $= 160\sin(\omega t - 110^o)$ V

Phase angle, $\theta_e = \mathbf{-110^o}$

Initial angle, $\theta_{e0} = -\theta_e = \mathbf{110^o}$

(**g**) $i(t) = -8\sin(\omega t + 80^o)$ A

$i(t) = 8\sin(\omega t + 80^o \pm 180^o)$ A

$= 8\sin(\omega t + 260^o)$ A

$= 8\sin(\omega t - 100^o)$ A

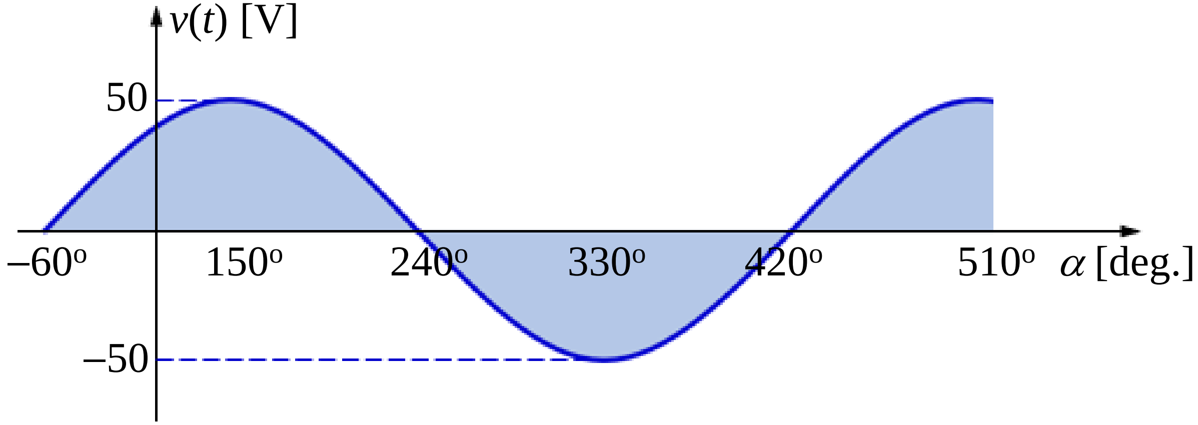
Phase angle or, $\theta_i = \mathbf{260^o}$ or $\mathbf{-100^o}$

Initial angle, $\theta_{i0} = -\theta_i = \mathbf{-260^o}$ or $\mathbf{100^o}$

EXAMPLE Sketch $v(t) = 50\sin(261.67t + 60^{\circ})$ V with the abscissa:

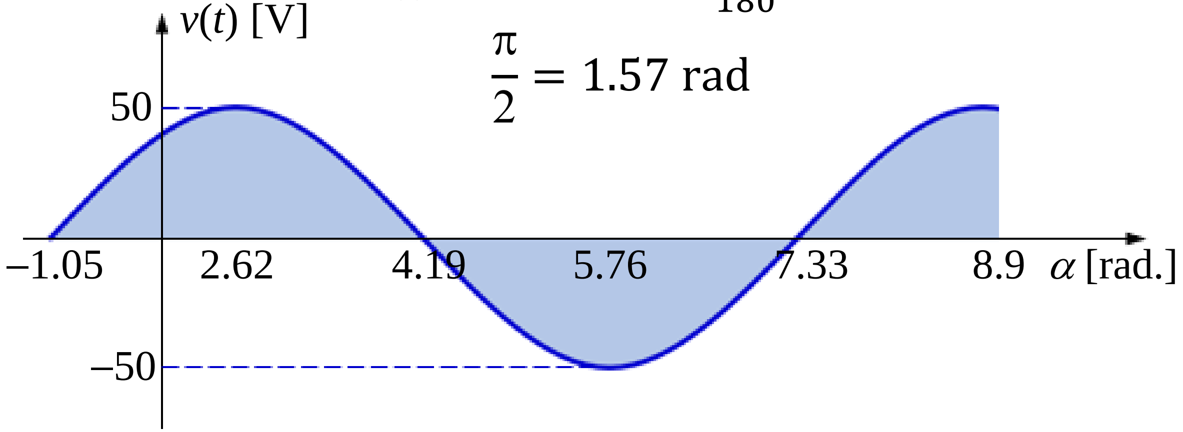
a. angle in degrees, **b.** angle in radians, and **c.** time in seconds.

Solution: Here, $V_m = 50$ V, $\omega = 261.67$ rad/s, $\theta_{v0} = -\theta_v = -60^{\circ}$ $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{261.67} = 24$ ms



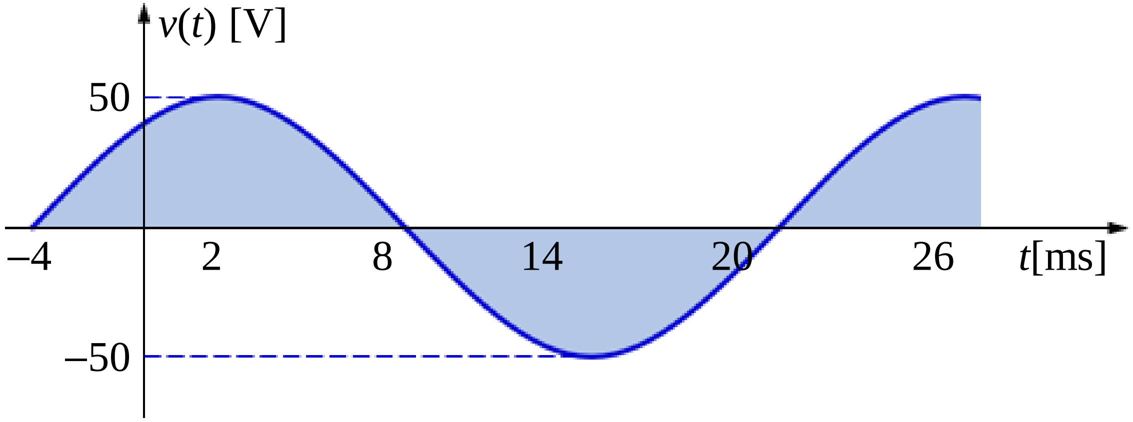
$$\theta_{v0}\,(\text{rad.}) = -60° \times \frac{\pi}{180°} = -1.05 \text{ rad}$$

$$\frac{\pi}{2} = 1.57 \text{ rad}$$



$$t_0 = \frac{\theta_{v0}}{\omega} \times \frac{\pi}{180°} = \frac{-60°}{261.67} \times \frac{\pi}{180°} = -4 \text{ ms}$$

$$\frac{T}{4} = 6 \text{ ms}$$

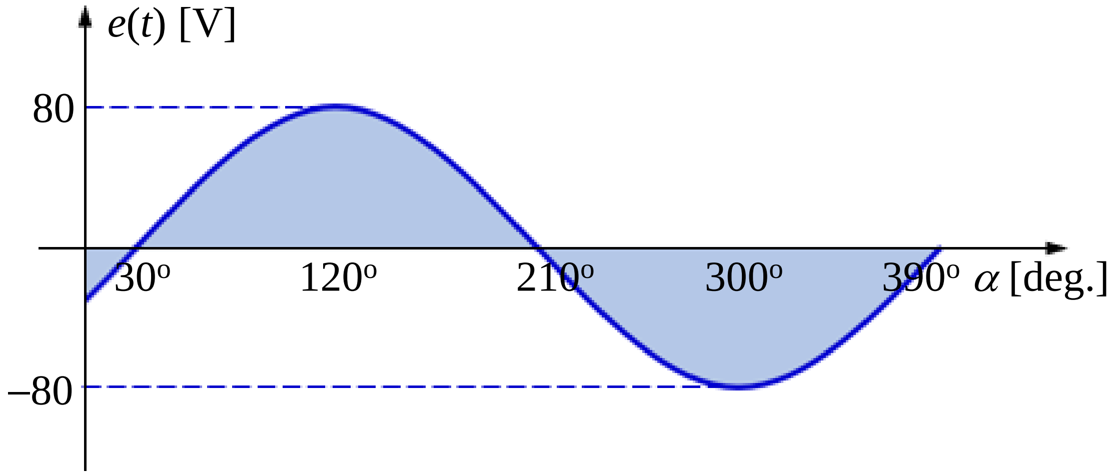


EXAMPLE Sketch $e(t) = 80\sin(157t - 30^o)$ V with the abscissa:

a. angle in degrees, **b.** angle in radians, and **c.** time in seconds.

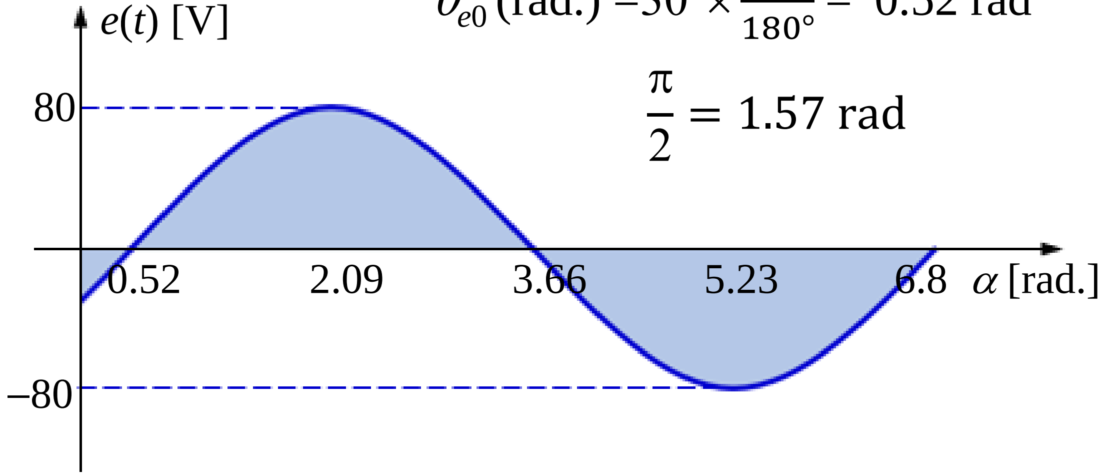
Solution: Here, $E_m = 80$ V, $\omega = 157$ rad/s, $\theta_{e0} = -\theta_e = 30^o$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{157} = 40 \text{ ms}$$



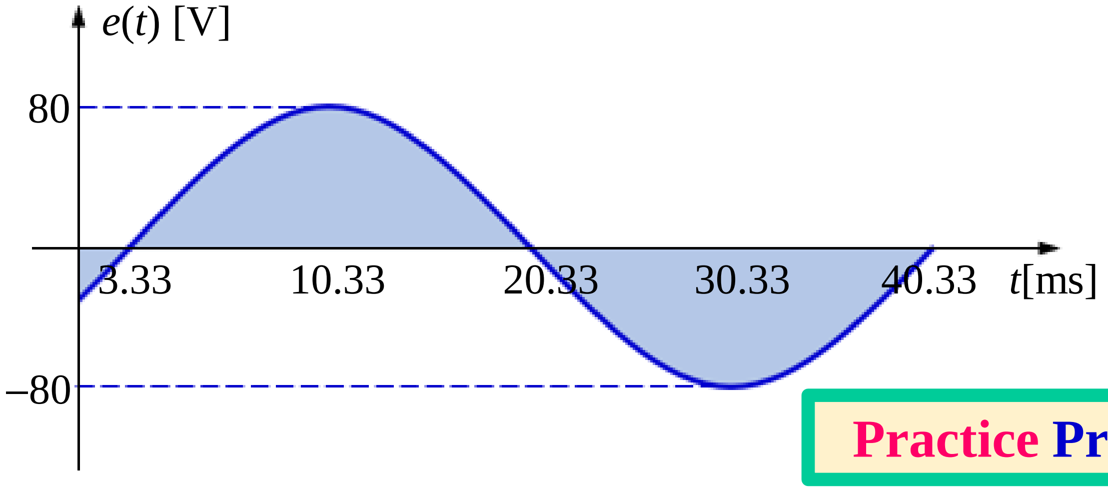
$$\theta_{e0}\,(\text{rad.}) = 30° \times \frac{\pi}{180°} = 0.52 \text{ rad}$$

$$\frac{\pi}{2} = 1.57 \text{ rad}$$



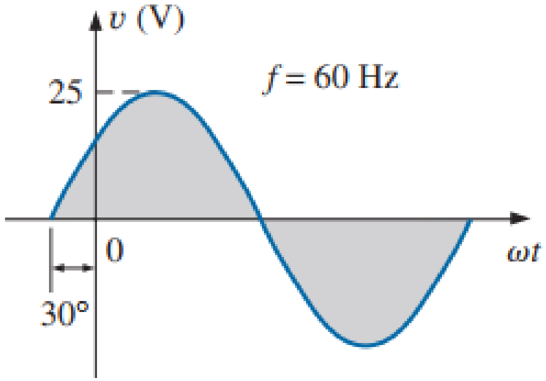
$$t_0 = \frac{\theta_{e0}}{\omega} \times \frac{\pi}{180°} = \frac{30°}{157} \times \frac{\pi}{180°} = 3.33 \text{ ms}$$

$$\frac{T}{4} = 10 \text{ ms}$$



Practice Problem 25 ~ 26 [P582]

PROBLEM 27 and 28 [P583] Write the analytical expression for the waveforms in Fig. 13.84 with the phase angle in degrees.

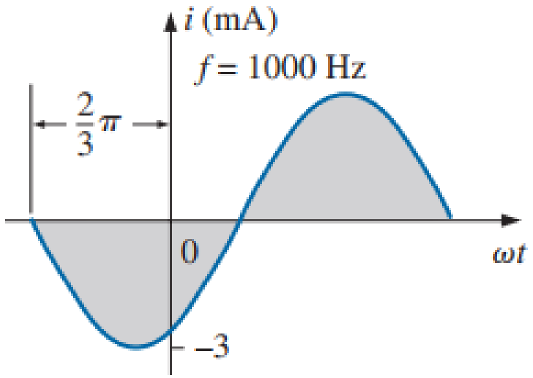


$V_m = 25$ V

$\omega = 2\pi\times 60 \approx 377$ rad/s

$\theta_e = -\ \theta_{e0} = -(-\ 30^o) = 30^o$

$\boldsymbol{v(t) = 25\sin(377t + 30^o)}$ **V**



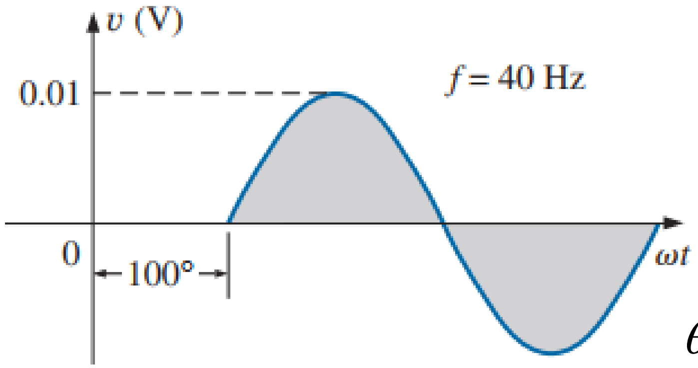
$I_m = 3$ mA

$\omega = 2\pi\times 1000 \approx 6280$ rad/s

$\theta_i = -\ \theta_{i0} = -(-\frac{2\pi}{3}) = \frac{2\pi}{3}$

$\theta_i = \frac{2\pi}{3} \times \frac{180°}{\pi} = 120°$

$\boldsymbol{i(t) = -\ 3\sin(6280t + 120^o)}$ **mA**

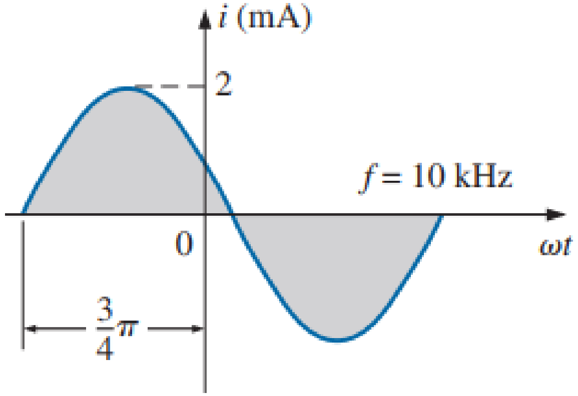


$V_m = 0.01$ V

$\omega = 2\pi\times 40 \approx 251.2$ rad/s

$\theta_e = -\ \theta_{e0} = -(100^o) = -\ 100^o$

$\boldsymbol{v(t) = 0.01\sin(251.2t - 100^o)}$ **V**



$I_m = 2$ mA

$\omega = 2\pi\times 10000 \approx 62800$ rad/s

$\theta_i = -\ \theta_{i0} = -(-\frac{3\pi}{4}) = \frac{3\pi}{4}$

$\theta_i = \frac{3\pi}{4} \times \frac{180°}{\pi} = 135°$

$\boldsymbol{i(t) = 2\sin(62800t + 135^o)}$ **mA**

Phase Difference and Phase Relation Between Two Waveforms

Conditions to find the Phase difference, Time difference and phase relation between two waveforms:

a) They must be same type waveforms
b) Their frequency or angular frequency or time period must be same

Angle Difference = Voltage angle – Current Angle
Phase Difference = | Angle Difference |

Case I:

Angle Difference= Voltage angle – Current Angle = 0^o
Phase Relation: $v(t)$ and $i(t)$ are **in phse**.
The starting points of $v(t)$ and $i(t)$ are same.

Case II:

Angle Difference= Voltage angle – Current Angle > 0^o
Phase Relation: $v(t)$ **leads** $i(t)$, or $i(t)$ **lags** $v(t)$
$v(t)$ **starts before** $i(t)$, or $i(t)$ **starts after** $v(t)$

Case III:

Angle Difference= Voltage angle – Current Angle < 0^o
Phase Relation: $v(t)$ **lags** $i(t)$, or $i(t)$ **leads** $v(t)$
$v(t)$ **starts after** $i(t)$, or $i(t)$ **starts before** $v(t)$

EXAMPLE 13.12 (a) What is (*i*) the phase difference, and (*ii*) the phase relationship between the sinusoidal waveforms of the following sets? (a) $v(t) = 10\sin(\omega t + 30°)$ V; $i(t) = 5\sin(\omega t + 70°)$ A

Solution: (*i*) Angle Difference = (+30°) – (+70°) = – 40°

Phase Difference = $|-40°|$ = **40°**

(*ii*) Angle Difference < 0°; So,

***v*(*t*) lags *i*(*t*) by 40º, or *i*(*t*) leads *v*(*t*) by 40º**

It is seen from Fig. 13.31 that *v*(*t*) starts before *i*(*t*) thus:

***v*(*t*) lags *i*(*t*) by 40º, or *i*(*t*) leads *v*(*t*) by 40º**

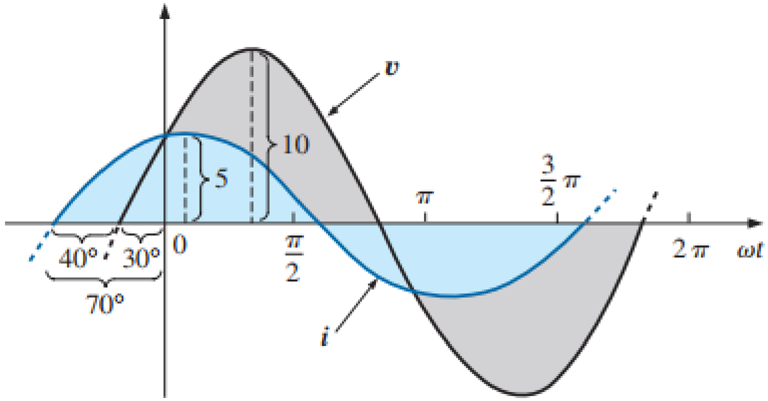


FIG. 13.31 *Example 13.12(a): i leads v by 40°.*

EXAMPLE 13.12 (b) What is (*i*) the phase difference, and (*ii*) the phase relationship between the sinusoidal waveforms of the following sets? (b) $i(t) = -15\cos(\omega t + 150°)$ A; $v(t) = 10\sin(\omega t - 20°)$ V

Solution: $i(t) = -15\cos(\omega t + 150°)$ A $= 15\sin(\omega t + 150° - 90°)$ A $= 15\sin(\omega t + 60°)$ A

(*i*) Angle Difference = (–20°) – (+60°) = – 80°

Phase Difference = $|-80°|$ = **80°**

(*ii*) Angle Difference < 0°; So,

***v*(*t*) lags *i*(*t*) by 80º, or *i*(*t*) leads *v*(*t*) by 80º**

It is seen from Fig. 13.32 that *v*(*t*) starts before *i*(*t*) thus:

***v*(*t*) lags *i*(*t*) by 80º, or *i*(*t*) leads *v*(*t*) by 80º**

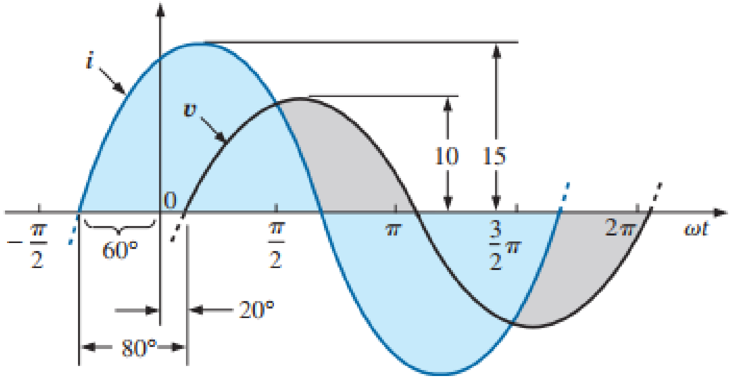


FIG. 13.32 *Example 13.12(b): i leads v by 80°.*

EXAMPLE 13.12 (c) What is (*i*) the phase difference, and (*ii*) the phase relationship between the sinusoidal waveforms of the following sets? (c) $i(t) = 2\cos(\omega t + 10°)$ A; $v(t) = 3\sin(\omega t - 10°)$ V

Solution: $i(t) = 2\cos(\omega t + 10°)\text{ A} = 2\sin(\omega t + 10° + 90°)\text{ A} = 2\sin(\omega t + 100°)$ A

(*i*) Angle Difference = (– 10°) – (+100°) = – 110°

Phase Difference = $|-110°|$ = **110°**

(*ii*) Angle Difference < 0°; So,

$v(t)$ lags $i(t)$ by 110º, or $i(t)$ leads $v(t)$ by 110º

It is seen from Fig. 13.33 that $v(t)$ starts before $i(t)$ thus:

$v(t)$ lags $i(t)$ by 110º, or $i(t)$ leads $v(t)$ by 110º

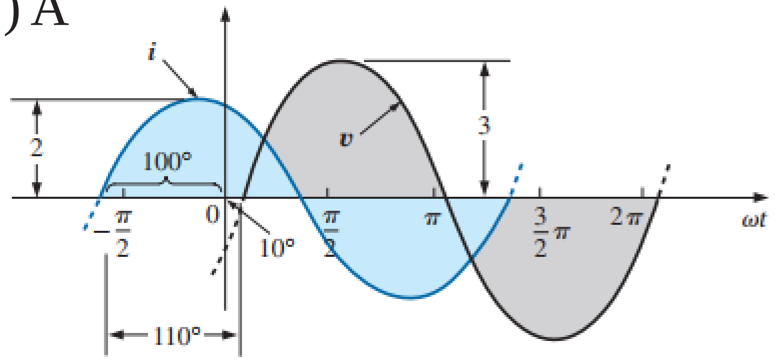


FIG. 13.33 *Example 13.12(c): i leads v by 110°.*

EXAMPLE 13.12 (d) What is (*i*) the phase difference, and (*ii*) the phase relationship between the sinusoidal waveforms of the following sets? (d) $i(t) = -2\cos(\omega t - 60°)$ A; $v(t) = 3\sin(\omega t - 150°)$ V

Solution: $i(t) = -2\cos(\omega t - 60°)\text{ A} = 2\sin(\omega t - 60° - 90°)\text{ A} = 2\sin(\omega t - 150°)$ A

(*i*) Angle Difference = (–150°) – (–150°) = 0°

Phase Difference = $|0°|$ = **0°**

(*ii*) Angle Difference = 0°; So,

$v(t)$ and $i(t)$ are in phase

It is seen from Fig. 13.32 that $v(t)$ and $i(t)$ start at the same time thus:

$v(t)$ and $i(t)$ are in phase

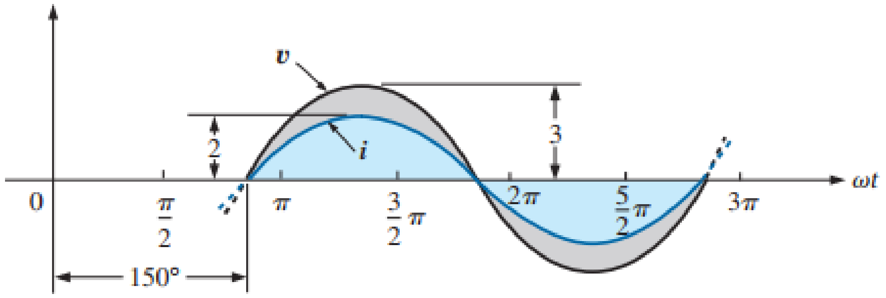


FIG. 13.35 *Example 13.12(e): v and i are in phase.*

EXAMPLE 13.12 (e) What is (*i*) the phase difference, and (*ii*) the phase relationship between the sinusoidal waveforms of the following sets? (e) $v(t) = 2\sin(\omega t + 10°)$ V; $i(t) = -\sin(\omega t + 30°)$ A

Solution: $i(t) = -\sin(\omega t + 30°)\text{ A} = \sin(\omega t + 30° - 180°)\text{ A} = \sin(\omega t - 150°)\text{ A}$

(*i*) Angle Difference = (10°) – (– 100°) = 160°

Phase Difference = $|-160°|$ = **160°**

(*ii*) Angle Difference > 0°; So,

***v*(*t*) leads *i*(*t*) by 160º, or *i*(*t*) lags *v*(*t*) by 160º**

It is seen from Fig. 13.33 that *v*(*t*) starts before *i*(*t*) thus:

***v*(*t*) lags *i*(*t*) by 110º, or *i*(*t*) leads *v*(*t*) by 110º**

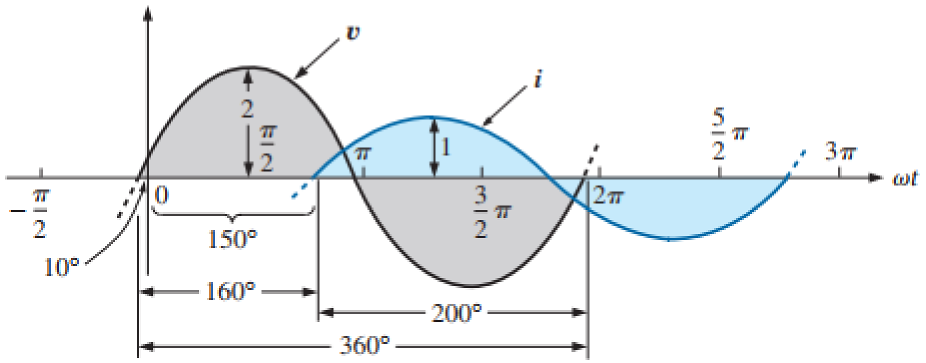


FIG. 13.34 *Example 13.12(d): v leads i by 160°.*

Another Answer: $i(t) = -\sin(\omega t + 30°)\text{ A} = \sin(\omega t + 30° + 180°)\text{ A} = \sin(\omega t + 210°)\text{ A}$

(*i*) Angle Difference = (10°) – (+210°) = – 200°

Phase Difference = $|-200°|$ = **200°**

Practice Problem 29 ~ 33 [P583]

(*ii*) Angle Difference < 0°; So,

***v*(*t*) lags *i*(*t*) by 200º, or *i*(*t*) leads *v*(*t*) by 200º**