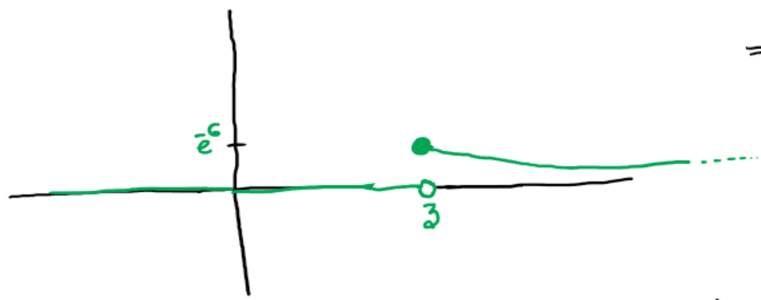
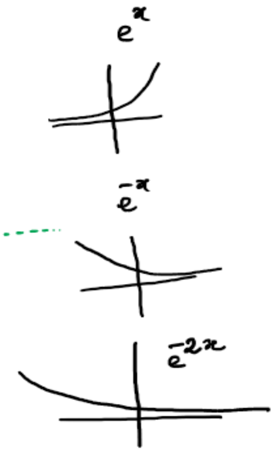
24. $f(t) = e^{-2t} \cdot u(t-3)$

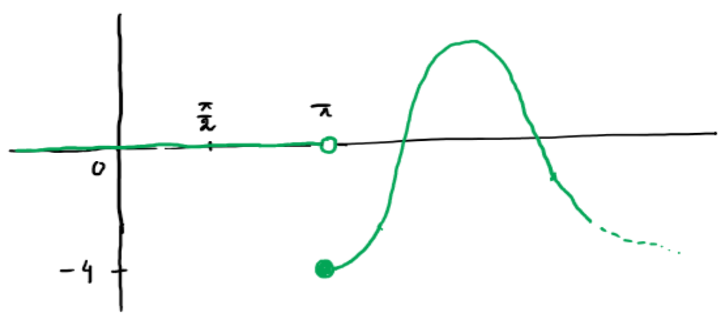
$$= \begin{cases} 0 & ; t-3<0 \\ e^{-2t} & ; t-3 \geq 0 \end{cases} = \begin{cases} 0 ; & t<3 \\ e^{-2t} ; & t \geq 3 \end{cases}$$

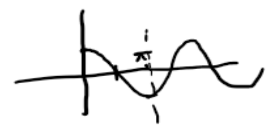




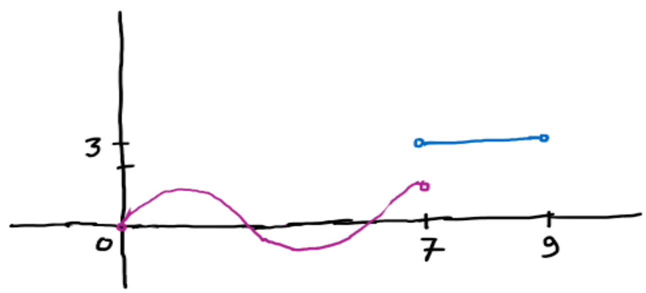
25. $f(t) = 4\cos t \cdot u(t-\pi)$

$$= \begin{cases} 0 & ; t-\pi<0 \\ 4\cos t & ; t-\pi \geq 0 \end{cases} = \begin{cases} 0 ; & t<\pi \\ 4\cos t ; & t \geq \pi \end{cases}$$





28.



$f(t) = \sin t$

$f(t) = 3$

$$f(t) = \begin{cases} \sin t ; & 0<t<7 \\ 3 ; & 7<t<9 \end{cases}$$

$$= \sin t \left[u(t) - u(t-7)\right] + 3\left[u(t-7) - u(t-9)\right]$$

$$= \sin t \cdot u(t) + (3-\sin t) \cdot u(t-7) - 3 \cdot u(t-9)$$

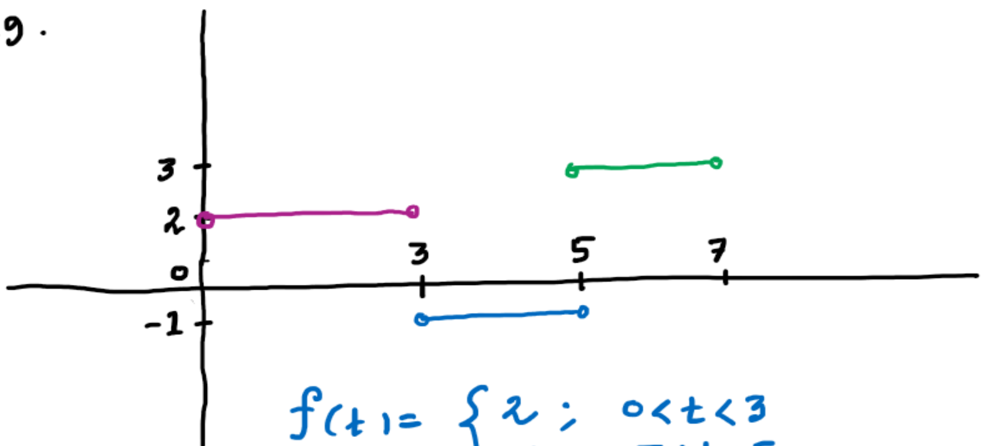
$$F(s) = \mathcal{L}\{\sin t \cdot u(t)\} + \mathcal{L}\{(3-\sin t) \cdot u(t-7)\} - 3\,\mathcal{L}\{u(t-9)\}$$

$$= \mathcal{L}\{\sin t\} + e^{-7s}\mathcal{L}\{3 - \sin(t+7)\} - 3\frac{e^{-9s}}{s}$$

$$= \frac{1}{s^2+1} + e^{-7s}\mathcal{L}\{3 - \sin t \cos 7 - \cos t \sin 7\} - 3\frac{e^{-9s}}{s}$$

$$= \frac{1}{s^2+1} + e^{-7s}\left[\frac{3}{s} - \cos 7 \cdot \frac{1}{s^2+1} - \sin 7 \cdot \frac{s}{s^2+1}\right] - 3\frac{e^{-9s}}{s}$$

29.



$$f(t) = \begin{cases} 2 \; ; & 0 < t < 3 \\ -1 \; ; & 3 < t < 5 \\ 3 \; ; & 5 < t < 7 \end{cases}$$

$$= 2\left[u(t) - u(t-3)\right] + (-1)\left[u(t-3) - u(t-5)\right. $$
$$\left. + 3\left[u(t-5) - u(t-7)\right]\right.$$

$$= 2\,u(t) - 3\,u(t-3) + 4\,u(t-5) - 3\,u(t-7)$$

$$F(s) = 2\frac{1}{s} - 3\frac{e^{-3s}}{s} + 4\frac{e^{-5s}}{s} - 3\frac{e^{-7s}}{s}$$

Chapter 2

Inverse Laplace Trans.

$\mathcal{L}\{f(t)\} = F(s)$ $\qquad$ $\mathcal{L}^{-1}\{F(s)\} = f(t)$

- $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$
- $\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$
- $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$
- $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$
- $\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at$
- $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$
- $\mathcal{L}^{-1}\left\{\frac{a}{s^2-a^2}\right\} = \sinh at$

$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$

$\Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$

- $\mathcal{L}^{-1}\left\{\frac{4}{s-2} - \frac{s}{s^2-16} + \frac{4}{s^2-4}\right\}$ $\quad 2^2$

$= 4 \cdot e^{2t} - \cosh 4t + 2 \sinh 2t$

$\mathcal{L}\{e^{at} f(t)\} = F(s-a) \to \mathcal{L}^{-1}\{F(s)\}$

$\mathcal{L}^{-1}\{F(s-a)\} = e^{at} \cdot \boxed{f(t)} = e^{at}\,\mathcal{L}^{-1}\{F(s)\}$

First translation Property:

$F(s-3)$ $\qquad$ $\mathcal{L}^{-1}\{F(s-a)\} = e^{at} \cdot \mathcal{L}^{-1}\{F(s)\}$ $\quad\Big|\quad$ $\mathcal{L}^{-1}\{F(s+a)\} = e^{-at}\,\mathcal{L}^{-1}\{F(s)\}$

✓ $\mathcal{L}^{-1}\left\{\frac{1}{(s-3)^4}\right\} = e^{3t}\,\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} = e^{3t} \cdot \frac{t^3}{3!}$ $\quad a$

$F(s-3) = \frac{1}{(s-3)^4}$

$F(s) = \frac{1}{s^4}$

✓ $\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2-3^2}\right\} = e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2-3^2}\right\}$

$F(s+2)$ $\qquad = \frac{1}{3} e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{3}{s^2-3^2}\right\}$

$= \frac{1}{3} e^{-2t} \sinh(3t)$.

$F(s+2) = \frac{1}{(s+2)^2-3^2}$

$\to \mathcal{L}^{-1}\left\{\frac{a}{s^2-a^2}\right\} = \sinh(at)$

✓ $\mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+4^2}\right\} = \mathcal{L}^{-1}\left\{\boxed{\frac{(s+2)-2}{(s+2)^2+4^2}}\right\} \to F(s+2)$

$= e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{s-2}{s^2+4^2}\right\}$

$= e^{-2t}\left[\mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2}\right\} - \mathcal{L}^{-1}\left\{\frac{2}{s^2+4^2}\right\}\right]$

$= e^{-2t}\left[\mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{4}{s^2+4^2}\right\}\right]$

$= e^{-2t}\left[\cos 4t - \frac{1}{2}\sin 4t\right]$

$F(s+2) = \frac{(s+2)-2}{(s+2)^2+4^2}$

s

$\mathcal{L}^{-1}\{F(s+2)\}$

$= e^{-2t}\,\mathcal{L}^{-1}\{F(s)\}$

✓ $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+19}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+2\cdot s\cdot 2+4+15}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+15}\right\}$ ✓

$= \mathcal{L}^{-1}\left\{\frac{(s+2)-2}{(s+2)^2+15}\right\}$

$= e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{s-2}{s^2+15}\right\}$

$= e^{-2t}\left[\mathcal{L}^{-1}\left\{\frac{s}{s^2+(\sqrt{15})^2}\right\} - 2\frac{1}{\sqrt{15}}\mathcal{L}^{-1}\left\{\frac{\sqrt{15}}{s^2+(\sqrt{15})^2}\right\}\right]$

$= e^{-2t}\left[\cos(\sqrt{15}\,t) - \frac{2}{\sqrt{15}}\sin(\sqrt{15}\,t)\right]$

Problem Set 2.1 (1 - 12) HW

numerator

Partial Fraction:

1. $\frac{x^2+2}{(x+3)(x+5)} = \frac{A}{x+3} + \frac{B}{x+5}$ unrepeated case

Denominator

2. $\frac{x^2+2}{(x+3)^2(x+5)^3} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+5} + \frac{D}{(x+5)^2} + \frac{E}{(x+5)^3}$ Repeated Case

(coefficient. variable + constant)

3. $\frac{x^2+2}{(x^2+3)(x^2+x+1)} = \frac{\boxed{Ax+B}}{x^2+3} + \frac{Cx+D}{x^2+x+1}$ type complex & irrational root

quadratic expression $(\underline{ax^2+bx+c})$

$\left.\begin{matrix} a=3 \\ b=0 \\ c=1 \end{matrix}\right\} 3x^2+1$

$\left.\begin{matrix} a=2 \\ b=3 \\ c=4 \end{matrix}\right\} 2x^2+3x+4$

$x^2+3=0$

$\Rightarrow x^2=-3$

$\Rightarrow x^2 = 3i^2$

$\Rightarrow x = \boxed{\pm\sqrt{3}\, i}$

$x^2-2=0$

$\Rightarrow x = \boxed{\pm\sqrt{2}}$

$(x^2+5x+6)=0$

$\Rightarrow (x+3)(x+2)=0$

$\Rightarrow x = -3, -2$

Real roots

Combination type:

$$\frac{2x+3}{(x^2+3)^2(x+5)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{(x^2+3)^2} + \frac{E}{x+5}$$

Exercise

$\boxed{2.1}$

9. $$\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2-16}\right\} = e^{2t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2-4^2}\right\}$$

$$= e^{2t}\cosh(4t) \qquad \cosh x = \frac{e^{x}+e^{-x}}{2}$$

$$= e^{2t}\,\frac{e^{4t}+e^{-4t}}{2}$$

$$= \frac{e^{6t}+e^{-2t}}{2} = \frac{e^{-2t}}{2}+\frac{e^{6t}}{2}$$

11. $$\mathcal{L}^{-1}\left\{\frac{5s-7}{s^2-6s+25}\right\} = \mathcal{L}^{-1}\left\{\frac{5(s-3)+8}{(s-3)^2+16}\right\}$$

$$=5\,\mathcal{L}^{-1}\left\{\frac{s-3}{(s-3)^2+4^2}\right\} + 2\,\mathcal{L}^{-1}\left\{\frac{4}{(s-3)^2+4^2}\right\}$$

$$=5\,e^{3t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2}\right\} + 2\cdot e^{3t}\,\mathcal{L}^{-1}\left\{\frac{4}{s^2+4^2}\right\}$$

$$= 5e^{3t}\cos(4t) + 2e^{3t}\sin(4t)$$

$$= 5e^{3t}\left(\cos 4t + \frac{2}{5}\sin 4t\right)$$

Type Unrepeated factor:

$$\frac{x^2+2}{(x+3)(x-6)} = \frac{A}{x+3} + \frac{B}{x-6}$$

$\boxed{ax^2+bx+c}$

↳ quadratic expression

Type repeated factor:

$$\frac{x^2+2}{(x+3)^2(x-6)^3} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-6} + \frac{D}{(x-6)^2} + \frac{E}{(x-6)^3}$$

Type complex or irrational factors:

$$\frac{x^2+2}{(x^2+1)(x^2+4x+1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4x+1}$$

↳ Type complex or irrational factors

$(x^2+4x+1)=0$

$\Rightarrow x = -2+\sqrt{3}, -2-\sqrt{3}$

$x^2+9=0$

$\Rightarrow x = 3i, -3i$

$x^2+5x+6=0$

$\Rightarrow x = -3, -2$

$(x+3)(x+2)=0$

$\mathcal{L}^{-1}\left\{\frac{s+1}{s(s+3)(s-4)}\right\}$

$$\frac{s+1}{s(s+3)(s-4)} = \frac{A}{\boxed{s}} + \frac{B}{\boxed{s+3}} + \frac{C}{\boxed{s-4}}$$

$\boxed{s=0}$

$\boxed{s+3=0}$

$\Rightarrow s=-3$

$\boxed{s-4=0}$

$\Rightarrow s=4$

$$A = \frac{0+1}{3\cdot(-4)} = -\frac{1}{12}$$

$$B = \frac{-2}{(-3)(-7)} = -\frac{2}{21}$$

$$C = \frac{5}{4\cdot 7} = \frac{5}{28}$$

$$\frac{s+1}{s(s+3)(s-4)} = -\frac{1}{12}\frac{1}{s} - \frac{2}{21}\frac{1}{s+3} + \frac{5}{28}\frac{1}{s-4}$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{s+1}{s(s+3)(s-4)}\right\} = -\frac{1}{12}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{2}{21}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{5}{28}\mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= -\frac{1}{12} - \frac{2}{21}e^{-3t} + \frac{5}{28}e^{4t}$$

$$ax^2 + bx + c = 2x^2 + 3x + 4$$

$a = 2$
$b = 3$
$c = 4$

$$\mathcal{L}^{-1}\left\{\frac{s^2+s+2}{(s-1)^2(s-2)}\right\}$$

$$\frac{s^2+s+2}{\boxed{(s-1)^2(s-2)}} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s-2}$$

$$As^2 - 3As + 2A + Bs - 2B + Cs^2 - 2Cs + C$$

$$\Rightarrow s^2+s+2 = A(s-1)(s-2) + B(s-2) + C(s-1)^2$$

$$\Rightarrow s^2+s+2 = (A+C)s^2 + (-3A+B-2C)s + (2A-2B+C)$$

Equating coefficients from both sides,

$$\left\{\begin{aligned} A \quad + C &= 1 \\ -3A+B-2C &= 1 \\ 2A-2B+C &= 2 \end{aligned}\right\} \quad A = -7,\ B = -4,\ C = 8$$

$$\frac{s^2+s+2}{(s-1)^2(s-2)} = \frac{-7}{s-1} + \frac{-4}{(s-1)^2} + \frac{8}{s-2}$$

first translation property

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{s^2+s+2}{(s-1)^2(s-2)}\right\} = -7\,\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - 4\,\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\} + 8\,\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= -7e^t - 4e^t\,\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + 8e^{2t}$$

$$= -7e^t - 4\boxed{e^t t} + 8e^{2t}$$

$$\frac{1}{(s-1)^2} = e^t\,\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$

$$= e^t\frac{t}{1!} = e^t \cdot t$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)(s-1)}\right\}$$

$$\frac{s}{(s^2+4)(s-1)} = \frac{As+B}{s^2+4} + \frac{C}{s-1}$$

$$\Rightarrow \quad s = (As+B)(s-1) + C(s^2+4)$$

$$\Rightarrow \quad s = (A+C)s^2 + (-A+B)s + (-B+4C)$$

Equating coefficients from both sides,

$$\left.\begin{array}{rl} A \quad + C &= 0 \\ -A + B \quad &= 1 \\ -B + 4C &= 0 \end{array}\right\} A = -\frac{1}{5},\ B = \frac{4}{5},\ C = \frac{1}{5}$$

$$\frac{s}{(s^2+4)(s-1)} = \frac{-\frac{1}{5}s+\frac{4}{5}}{s^2+4} + \frac{\frac{1}{5}}{s-1}$$

$$= -\frac{1}{5}\,\frac{s}{s^2+4} + \frac{4}{5\times 2}\,\frac{2}{s^2+2^2} + \frac{1}{5}\,\frac{1}{s-1}$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)(s-1)}\right\} = -\frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{2}{s^2+2^2}\right\} + \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$= -\frac{1}{5}\cos 2t + \frac{2}{5}\sin 2t + \frac{1}{5}e^{t}$$

problem set 22 (13- 19)

$$\mathcal{L}\{f(t)\cdot u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$$

$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

Inverse Laplace associated with Unit Step Function:

$u(t-2)/u_2(t)$

- $\mathcal{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} = u(t-a)$
- $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\cdot u(t-a)$

$u(t-a)/u_a(t)$

$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s}\right\} = u(t-2) \leftarrow$

$$= \begin{cases} 0 \; ; \; t<2 \\ 1 \; ; \; t\geq 2 \end{cases}$$

$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2}\right\} = f(t-1)\cdot u(t-1)$

$$= \boxed{(t-1)}\cdot u(t-1)$$

$$= \begin{cases} 0 \; ; \; t<1 \\ (t-1) \; ; \; t\geq 1 \end{cases}$$

$$\boxed{F(s) = \frac{1}{s^2}}$$

$F(s) = \frac{1}{s^2}$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$

$\therefore f(t) = t$

$\Rightarrow f(t-1) = (t-1)$

$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\} = f(t-\pi)\cdot u(t-\pi)$

$$= \boxed{-\sin t}\cdot u(t-\pi)$$

$$F(s) = \frac{1}{s^2+1}$$

$$= \begin{cases} 0 \; ; \; t<\pi \\ -\sin t \; ; \; t\geq\pi \end{cases}$$

$F(s) = \frac{1}{s^2+1}$

$f(t) = \boxed{\sin t}$

$f(t-\pi) = \sin(t-\pi)$

$= \sin\{-(\pi-t)\}$

$= -\sin(\pi-t)$

$= -\sin\left(2\cdot\frac{\pi}{2} - t\right)$

$= -\sin t$

problem set 2.2

(24-28)

$$\sin\left(n\times\frac{\pi}{2}\pm\theta\right)$$

$$\sin(-\theta) = -\sin\theta$$