

Comparison of ordered linked list, binary search tree and balanced binary search tree.

Ordered linked list

Linked list (LL) is a data structure that includes a sequence of connected nodes. Every node consists of two parts. Stored data and a pointer to the address of the next node. This approach causes that to get value from the 3rd node, one must go through the first and second node thus time complexity of accessing nodes is equal to $O(n)$ (linear). There is no possibility to get value faster (e. g. by random choose). This structure also limits traversing backward. Every element stores only the address of the next node therefore, the address of the previous node is unknown and unreachable.

But the Ordered Linked List has a lot of pros. The most important is that it is always sorted. Specific inserting function allows you to keep order without sorting the list after every insert. In contrast to normal arrays, LL allows fast insertion and removal of values in particular places without reallocating and reorganization of the entire database. Second one is that it is very easy to understand and implement in a code. Also there is no need to know how big finally this list will be. These advantages make it one of the most common data structures used in the IT world.

Before presenting all graphs we want to explain some important issues.

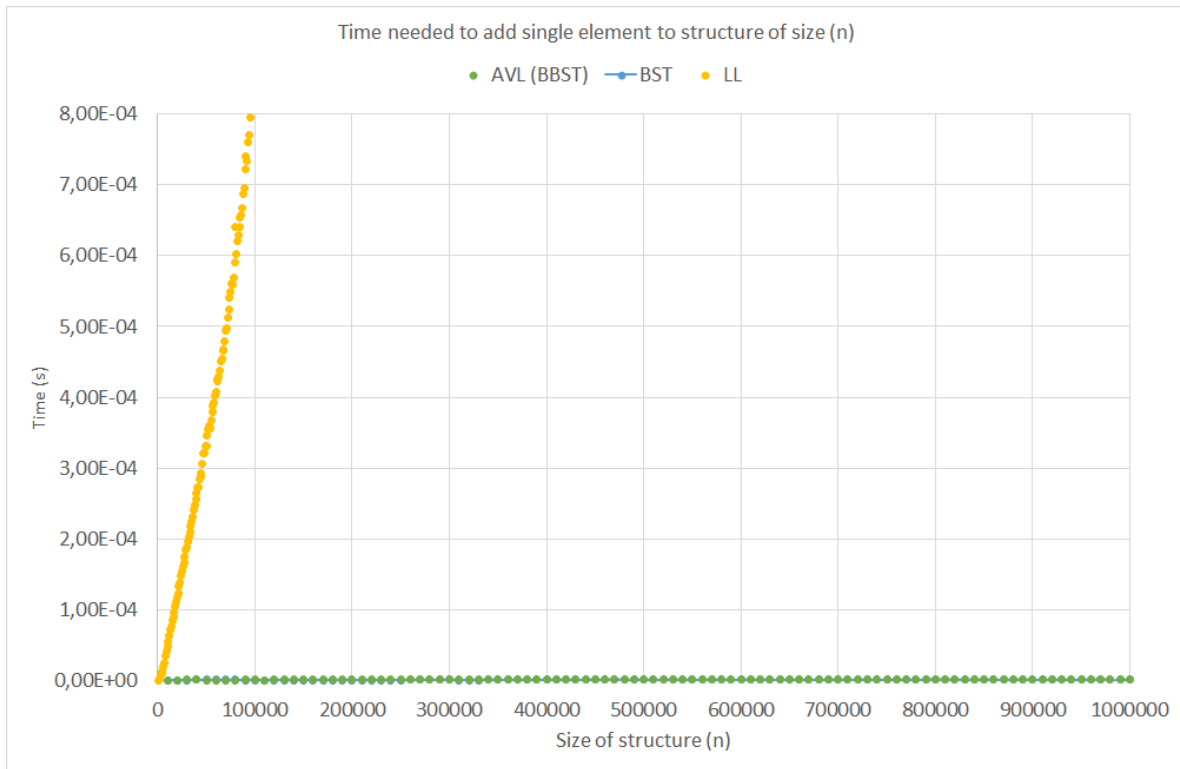
Remark 1: Adding means insertion of elements to the ordered database in such a way that structure will stay ordered. It's very important because unordered addition of the element on the first place in the linked list has complexity equal to 1 (constant).

Remark 2: In analysis below all added to the structures data was unordered before use. Searching through and removing elements from the data structures was also totally random.

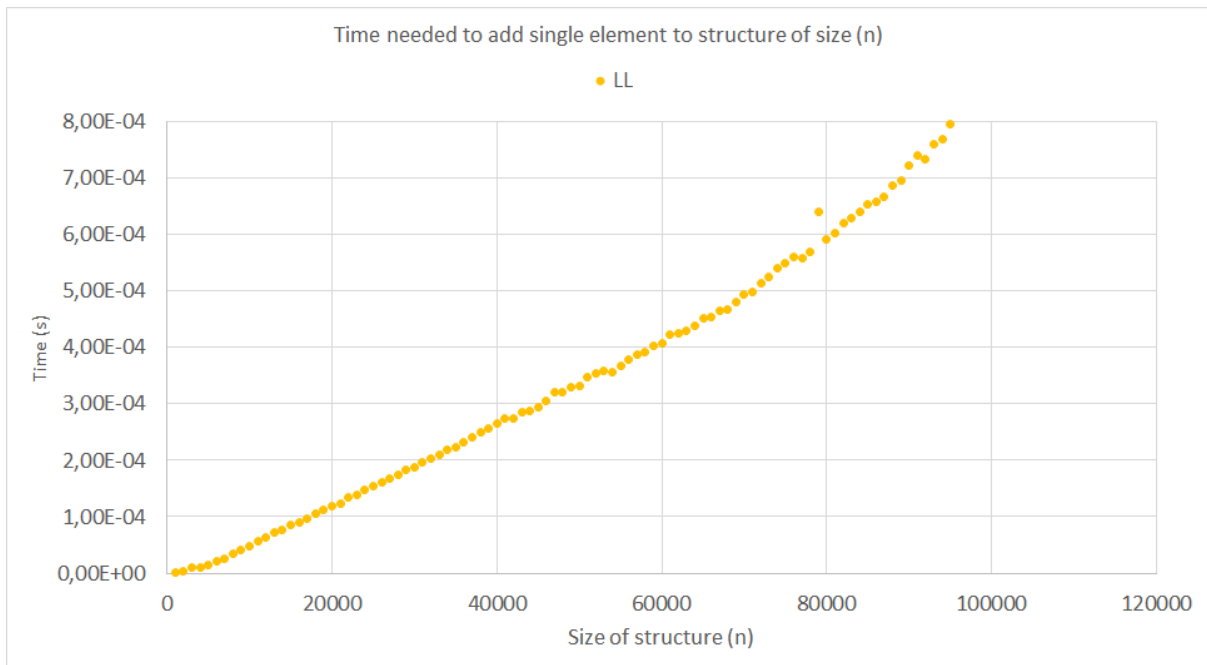
Remark 3: In case of tree data structures, we've used databases of size 1 000 000, with unique indices. All results of our test are a mean value of 10 runs with different databases. Measurements were taken every 10 000 elements.

Remark 4: Databases for testing Linked List were size 100 000 because testing bigger instances would make no difference in our graphs and would take much time to compute. In this case measurements were made every 1 000 elements. Results are also mean of 10 runs with different databases.

On the chart below the ordered linked list was compared to binary search trees (balanced and unbalanced).



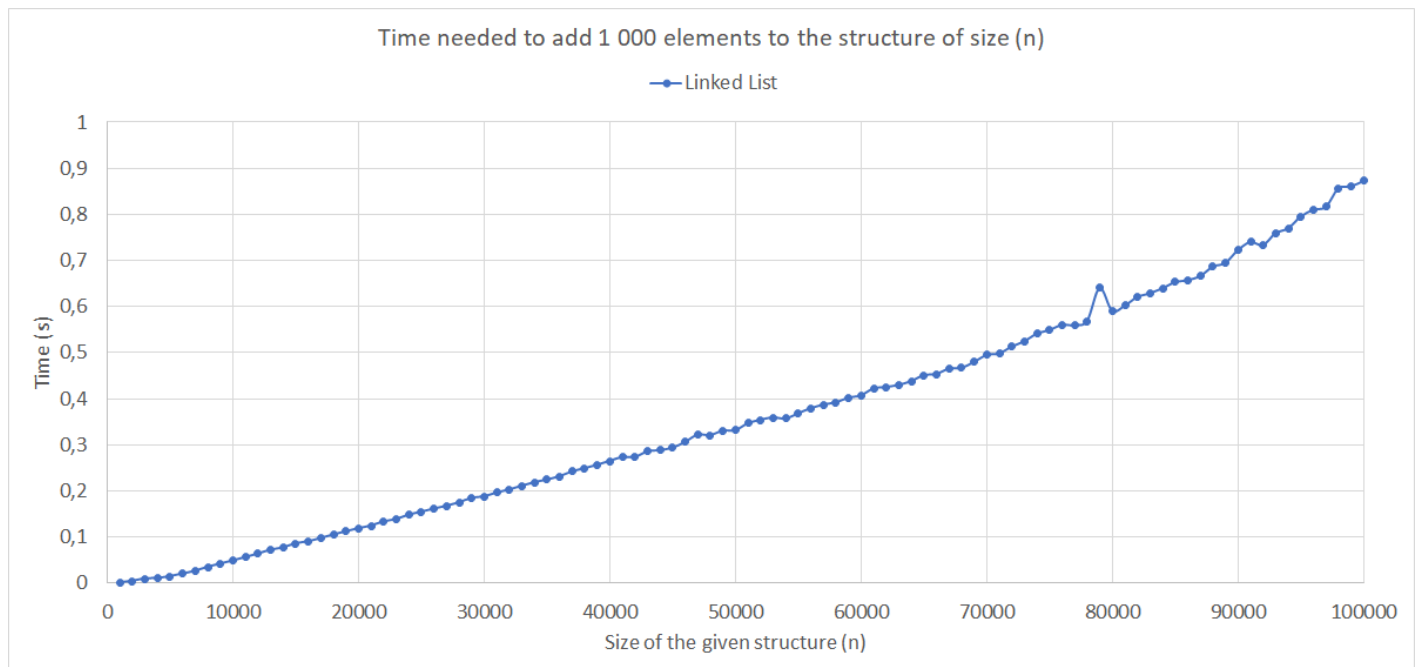
And the same graph with Ordered linked list separately.



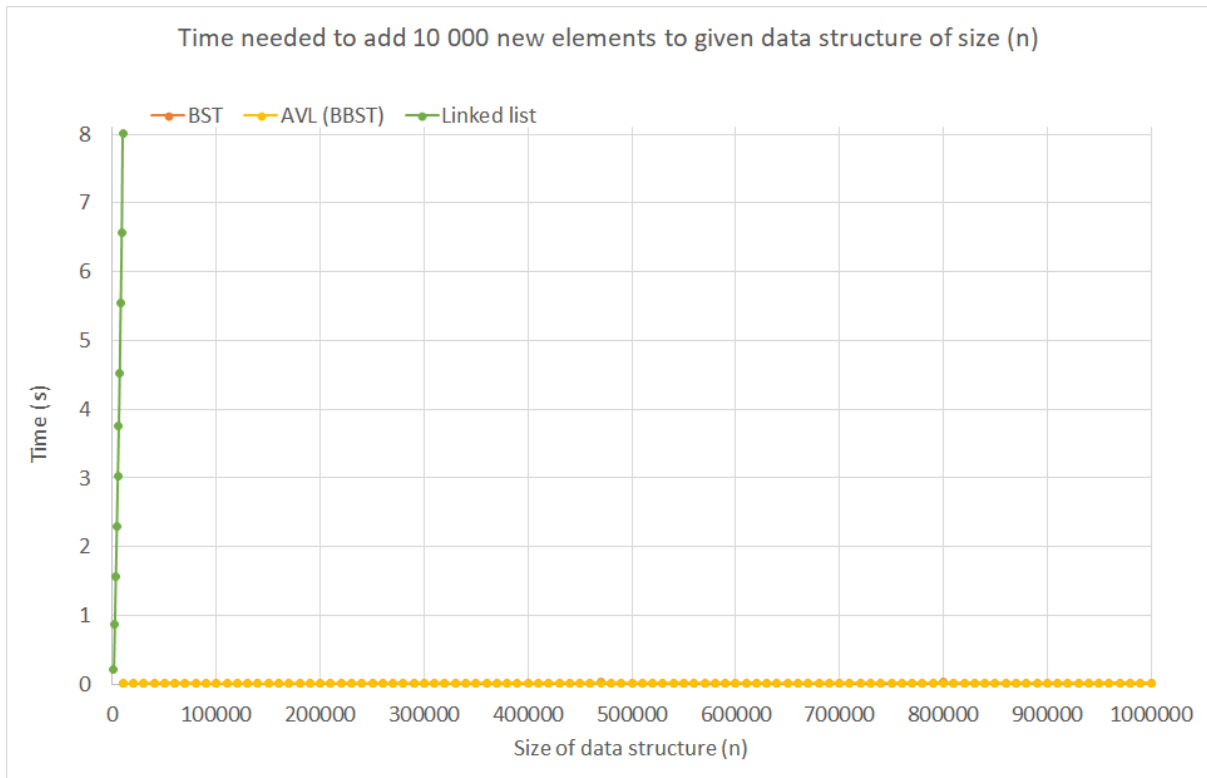
On the first graph, the sorted linked list is significantly less efficient than BST and AVL. That's because in contrast to trees there is only one path that every node follows until the algorithm finds a proper place for it. In case of trees, data is stored in separate branches,

which simply shorten the path to the searched place. Line of the linked list on the chart above is straight (a linear function) so we can conclude the complexity of sorted addition of elements equals to $O(n)$ where n is the size of the database - which was expected, since every new node in the worst case has to be compared with all nodes of given structure. Worth mentioning is the fact that addition of the sorted element to the ordered linked list consists of two actions. Searching the right place to insert node (linear complexity) and adding i.e. changing values of the pointers in neighboring nodes (constant complexity). So summarizing, the complexity of this function is equal to complexity of search time plus constant ($O(n) + O(1)$).

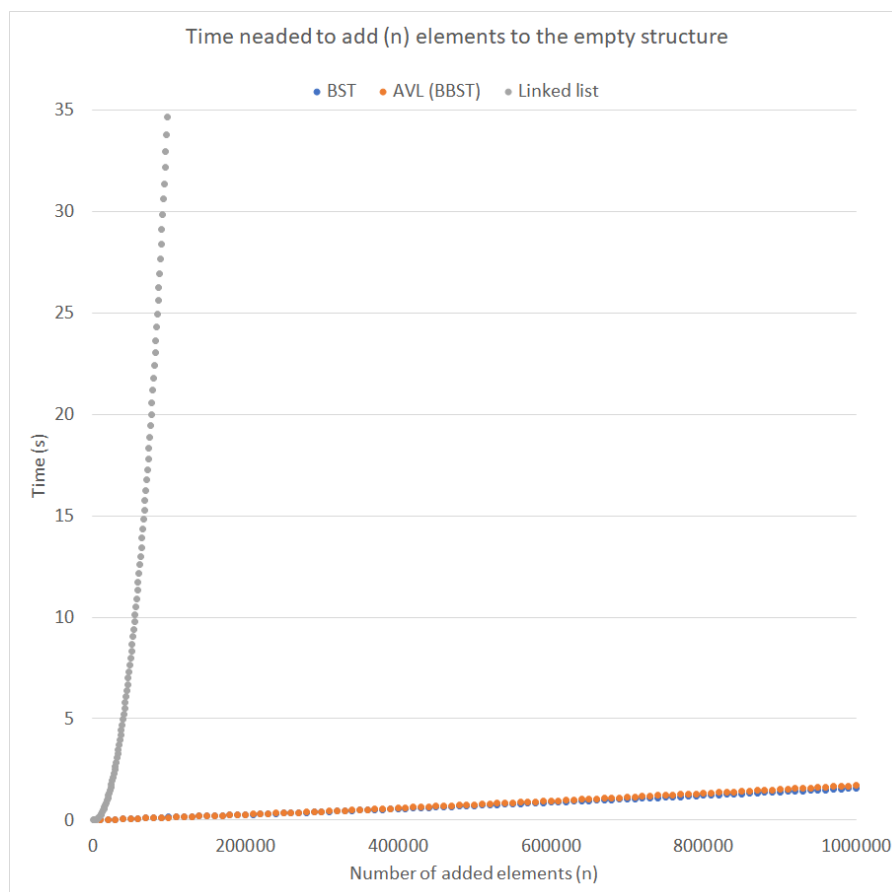
Next two graphs show how much time is needed to add 1 000 (first graph) and 10 000 (second graph) new elements to the particular structures. First consist of only an ordered list.



The second one is a comparison of the list and the trees.

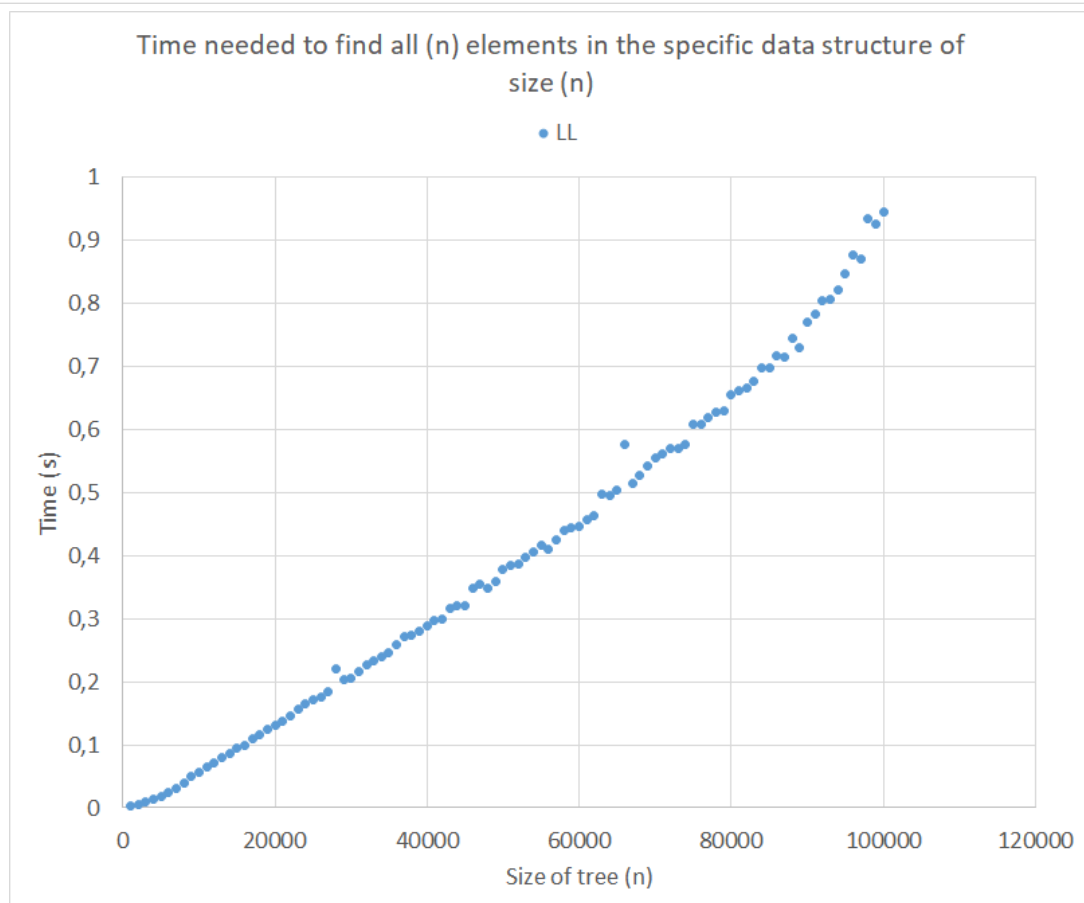
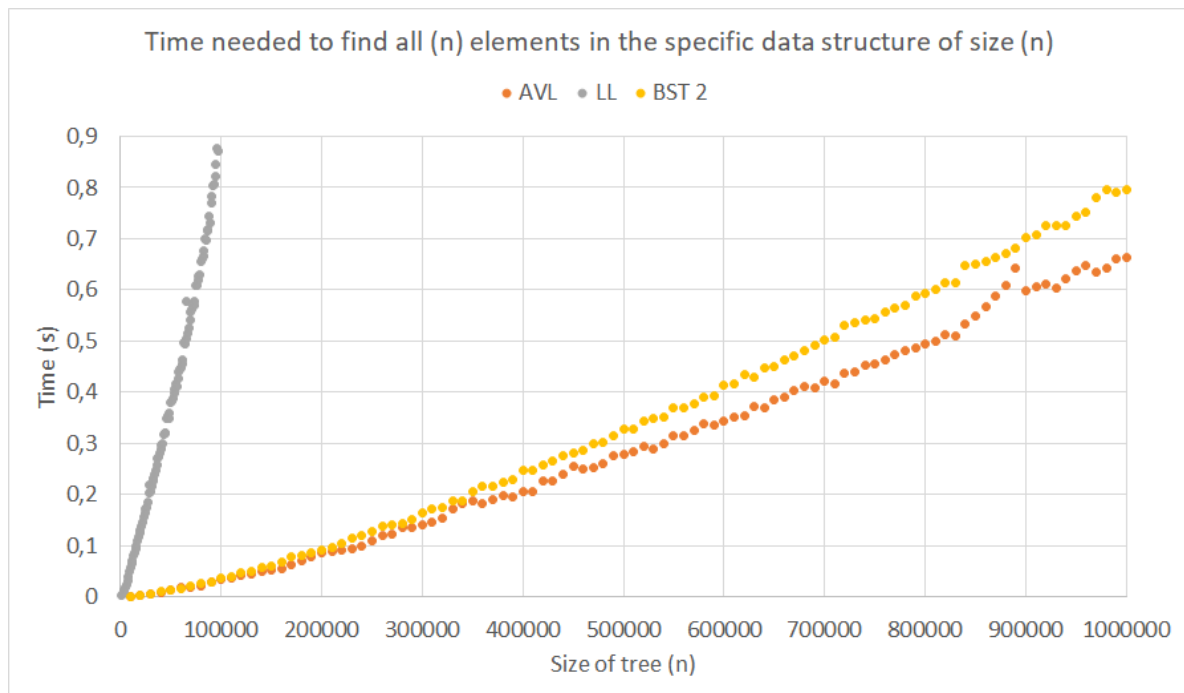


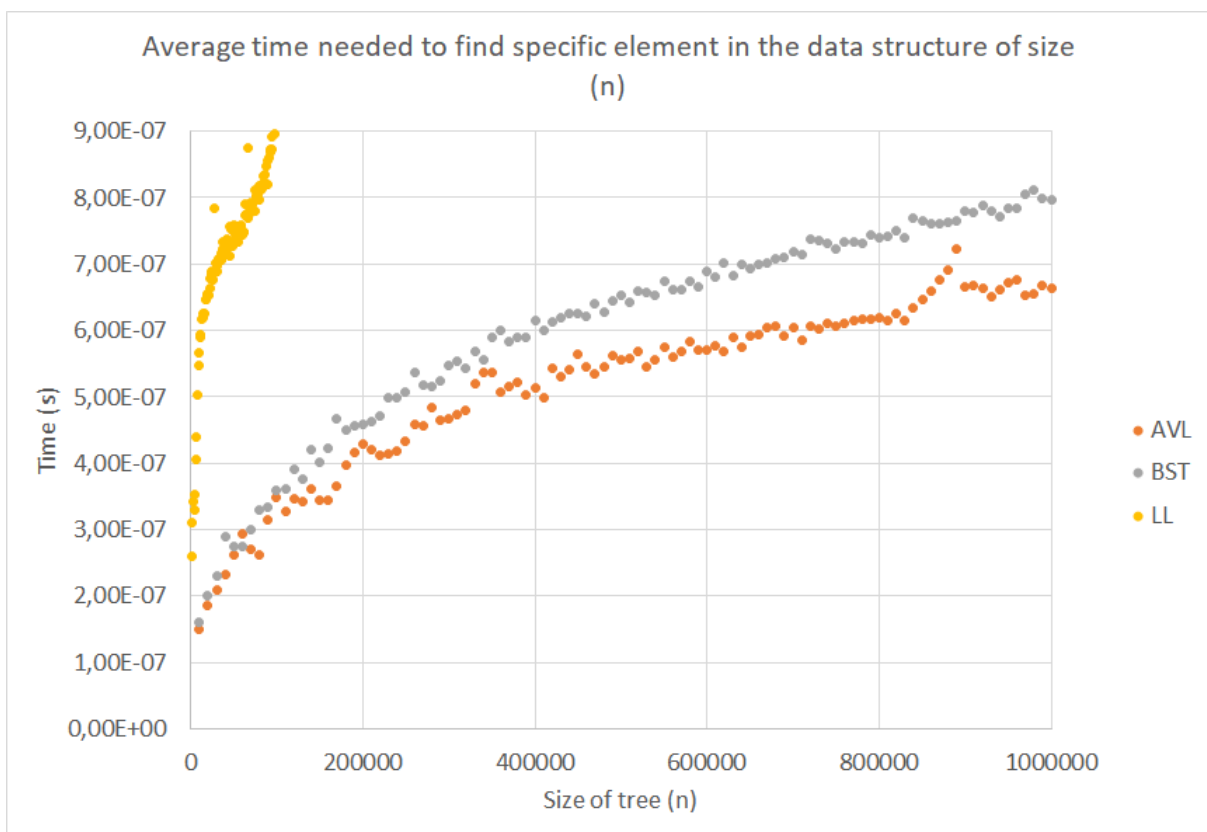
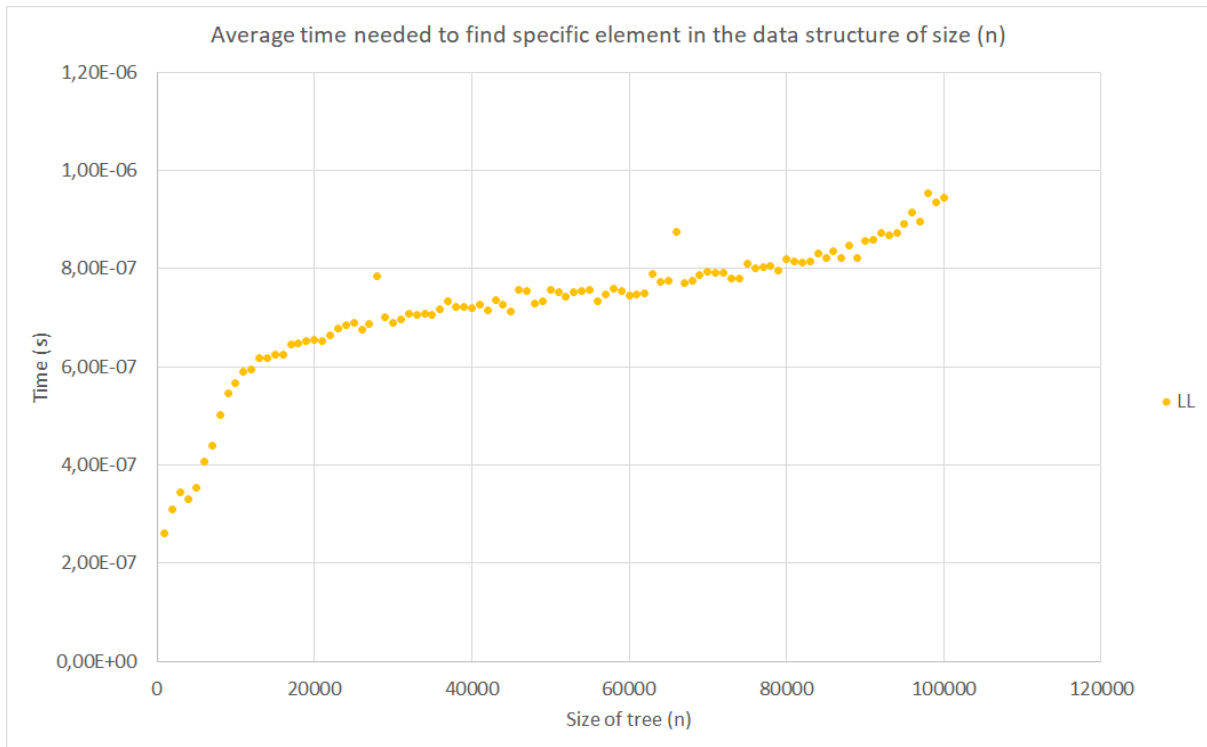
The last graph related to addition shows time needed to add n elements to the empty structure. To compute complexity of this action we have to multiply complexity of addition single element by number of elements, thus binary search tree has $O(n \log n)$ and ordered linked list leads to n^2 .



Searching

Graphs which represent time needed to search one element in an ordered linked list are pretty similar to charts with addition and that's because to insert the node in a proper way we have to first find the right place so basically addition function involves searching. It causes that complexity of this is the same as the inserting function and it's equal to $O(n)$. In comparison to BST linked list is highly inefficient.

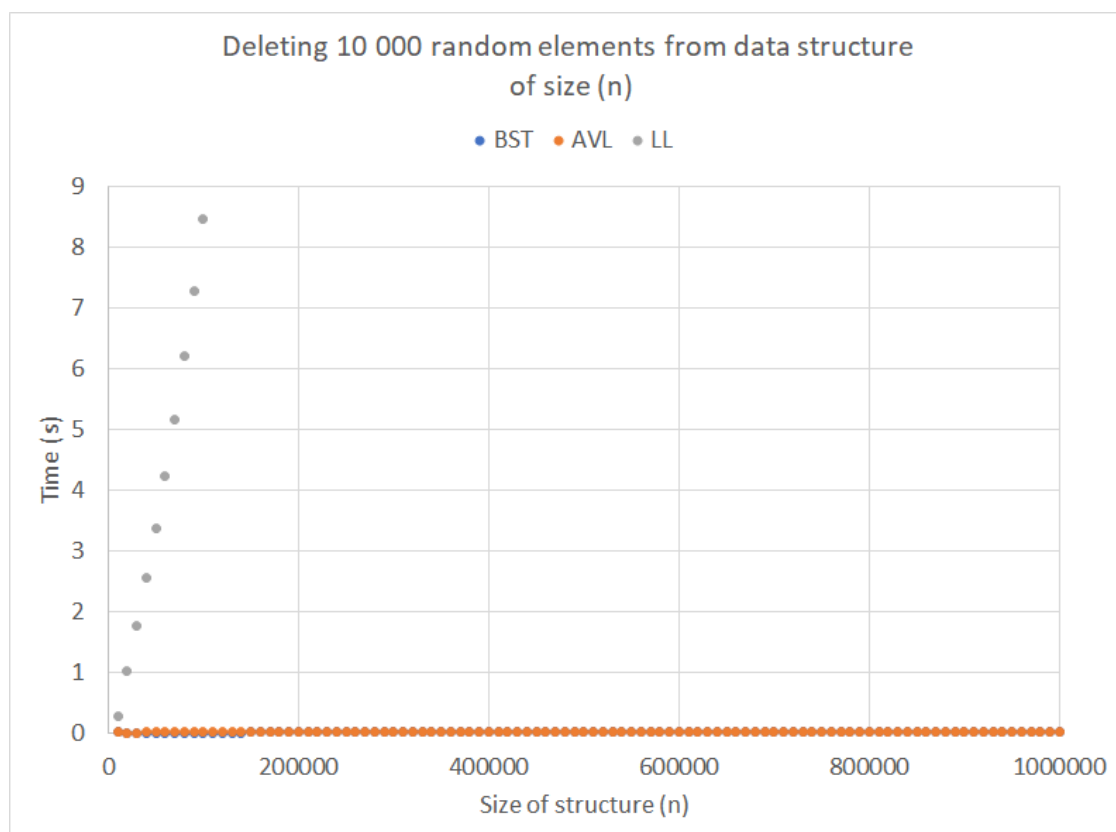
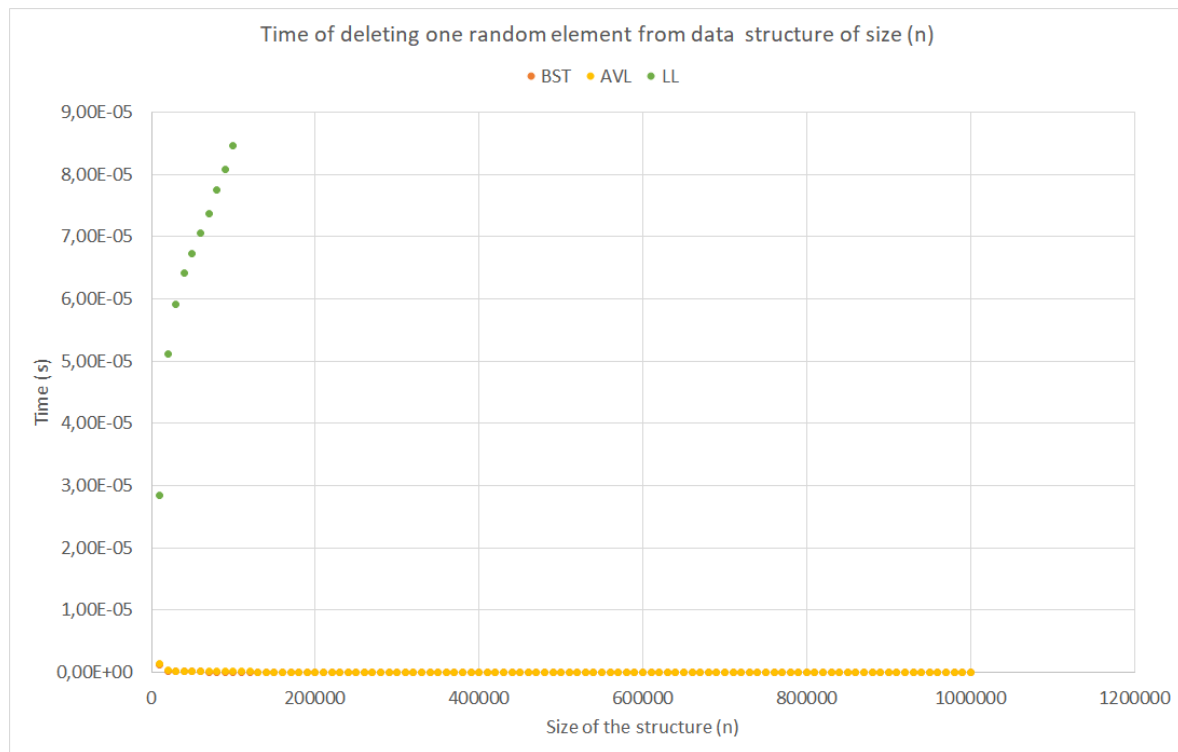




Deletion

As well as addition, the removal algorithm uses the searching part first. So it is expected that it has similar time needed to perform and what follows from that it has similar complexity $O(n)$. Also as in previous cases deleting is drastically slower than BST.

Situation changes if the address of the node that has to be removed is known, then complexity isn't dependent on size of database and is equal to 1 (constant).



Binary Search Tree (BST) and Balanced Binary Search Tree (Adelson-Velsky and Landis tree - AVL):

BST and BBST is a data structure which was modelled to be fast. It takes the advantage from the easy rule: smaller values go to the left branch, and rest to the right branch, and this condition is considered in every node. In that way, while considering the perfect tree the length of the path to every single node is less than, or equal to $\log_2(n)$, where n is the size of the structure. For the sake of illustration, in the case of a database of size 1 000 000, length of the path to the last node in the linked list is equal to 1 000 000, and in a well distributed binary tree it is non longer than 20 steps. Seems that this structure has no downsides, but there is a dark side of this way of keeping data.

In the case of standard BST, performance of this structure is strictly related to the order in which the data was added / inserted, because if data is sorted, then suddenly BST becomes Ordered Linked List which diametrically decreases speed of adding / searching / deleting nodes.

When it comes to code implementation, a tree is constructed of nodes (structures) which contains some data, and two pointers to the next two childrens (smaller valued - left and bigger valued - right nodes). In the standard version of BST there is no way to iterate the other way than from the root node. Basic operations implemented to BST:

Adding - this operation is pretty easy. Just simply follow the path checking if the value to be added is bigger or smaller than nodes on the path. Complexity of such an operation is equal to $O(\log_2(n))$ where n is the size of the structure.

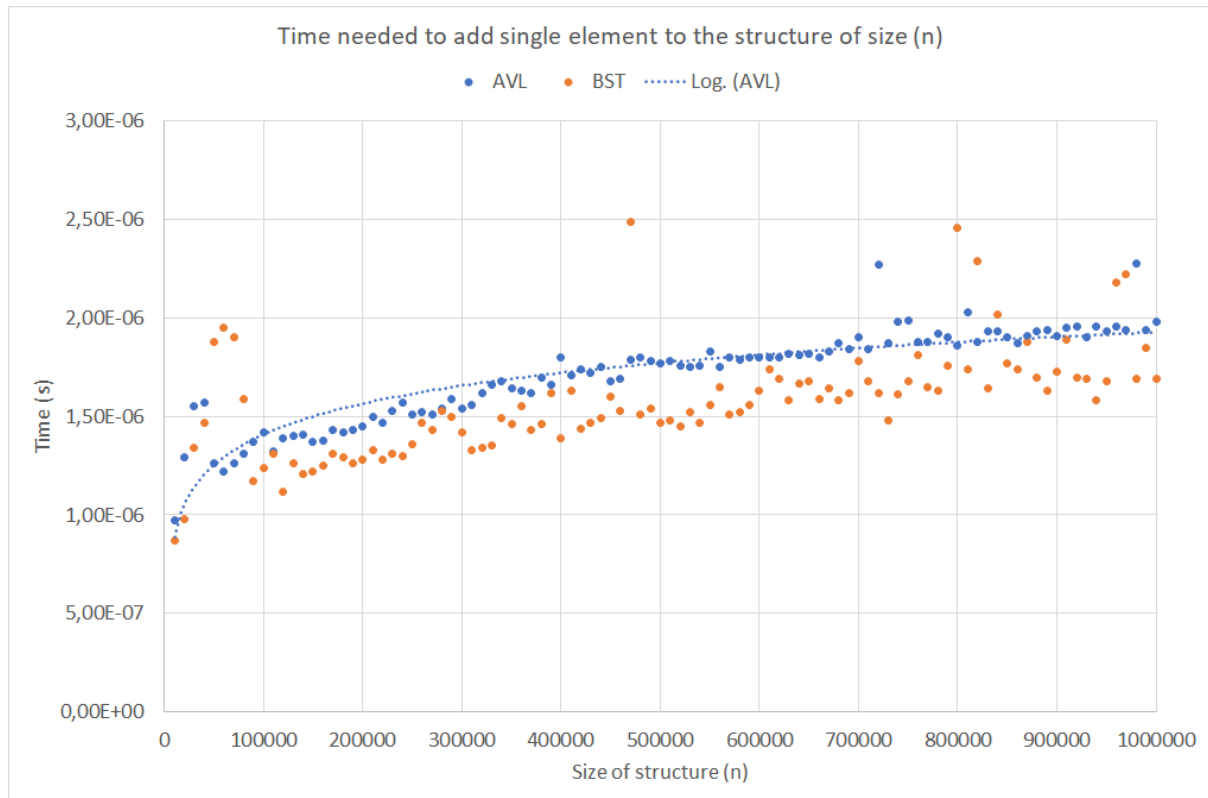
Deletion - this kind of data structure allows us to remove any node we want, but implementation is not that easy. Programs have to know if the node to be deleted has children, and how many. In case of no children or one child, it can be simply deleted and the previous node will have an updated pointer to the considered child. When there are 2 children, the node to be deleted needs to be replaced by a node which is bigger, but as close as possible to the value of the node to be deleted. Complexity of such an operation in the worst case is equal to $O(n)$ (BST becomes LL), and the average complexity is equal to $O(\log_2(n))$ where n is the size of the structure.

Searching is usually a recursive function, which searches from the beginning of the tree to the expected node. Complexity of searching in the worst case is equal to $O(n)$ (BST becomes LL), and the average complexity is equal to $O(\log_2(n))$ where n is the size of the structure.

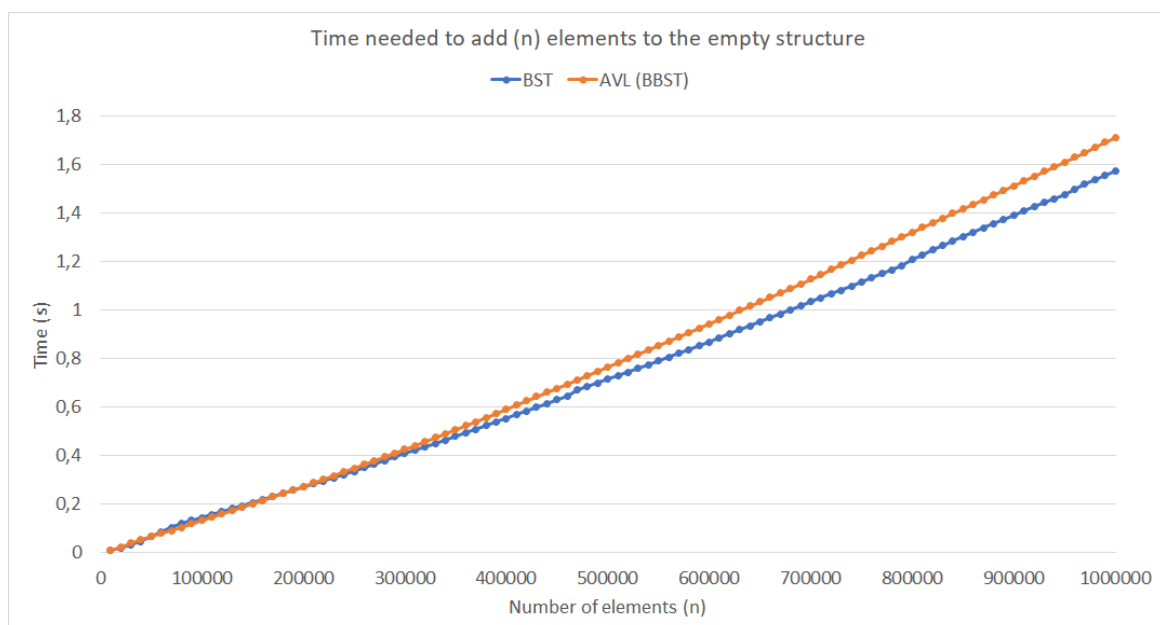
BBST is almost the same structure as the BST. The only difference is the fact that after every insertion or deletion algorithm checks if the balance of the tree is kept (in our implementation the difference between height of all leafs had to be non greater than 1. If the balance is disrupted, then the tree performs some rotations of branches to finally become balanced again. This operation takes some time, what can be noticed on the graph below.

Remark: On the plots above, it can be clearly seen that trees are incomparably faster than the linked list, therefore following graphs will contain only trees.

Adding / inserting



On the plot above, we can see that checking balance after every insertion, and balancing the tree makes AVL slower in terms of adding elements to the structure. As expected since adding element to tree structure has complexity equal to $O(\log_2(n))$, both graphs are lying about logarithmic function.

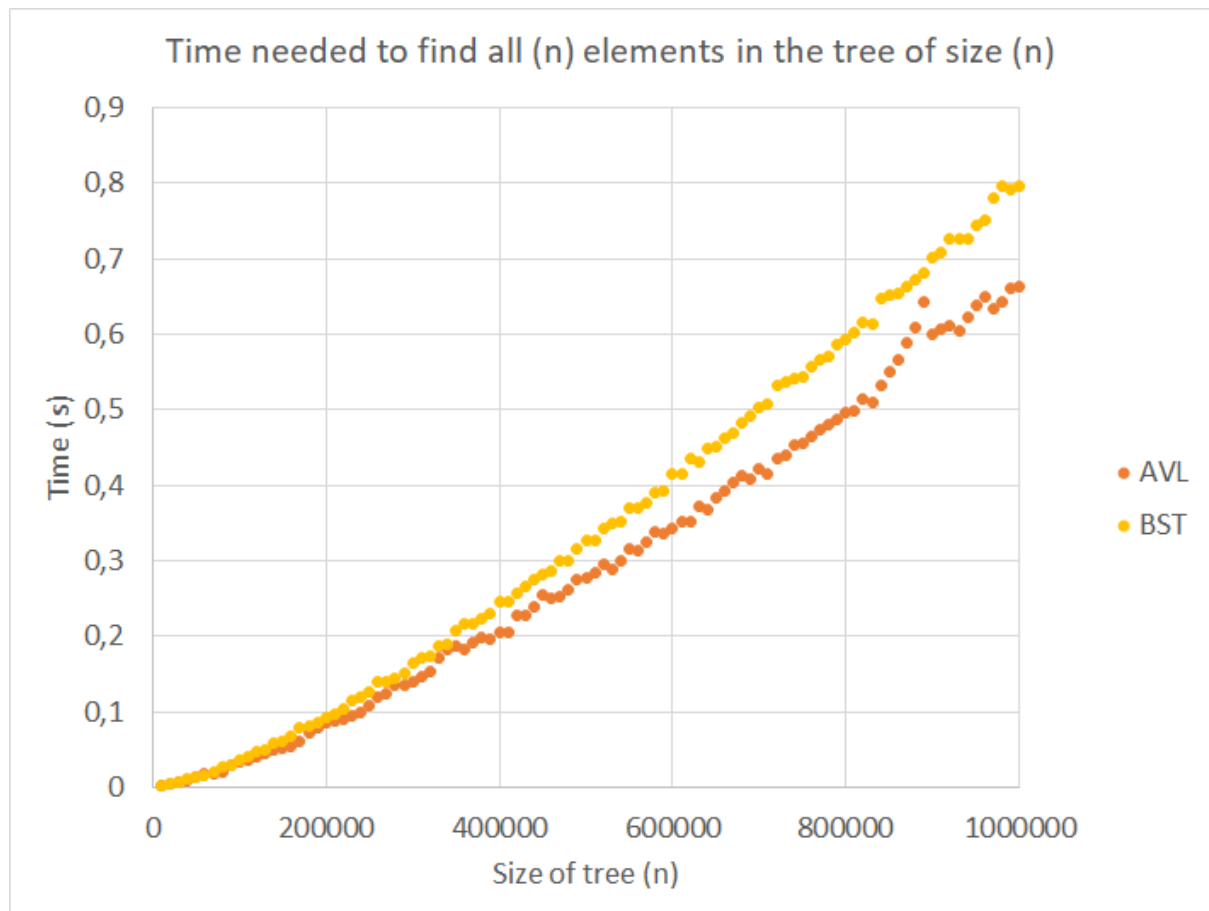


Same property (influence of balancing the tree) can be seen on the graph above. The bigger the structure is, the bigger is the time difference.

As expected, in the average case scenario standard BST is faster in terms of insertion of the new elements.

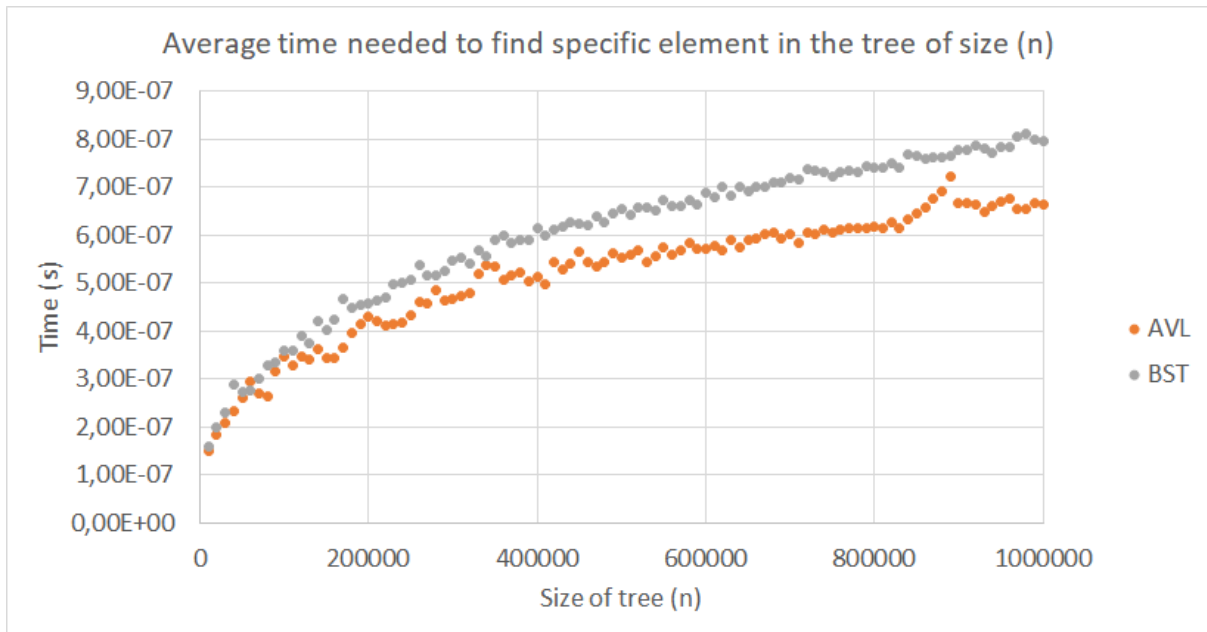
Searching:

Let us compare searching of the elements:



Now the balanced search tree takes an edge of advantage. Equal length of paths to the leaves makes AVL easier to search through, when in the BST lengths of branches are unknown and very likely unequal. Such an operation (searching all (n) elements in structure of size (n)) has a time complexity $O(n \cdot \log_2(n))$ which holds on our graph, since the shape of those functions are same as $y = n \cdot \log_2(n)$

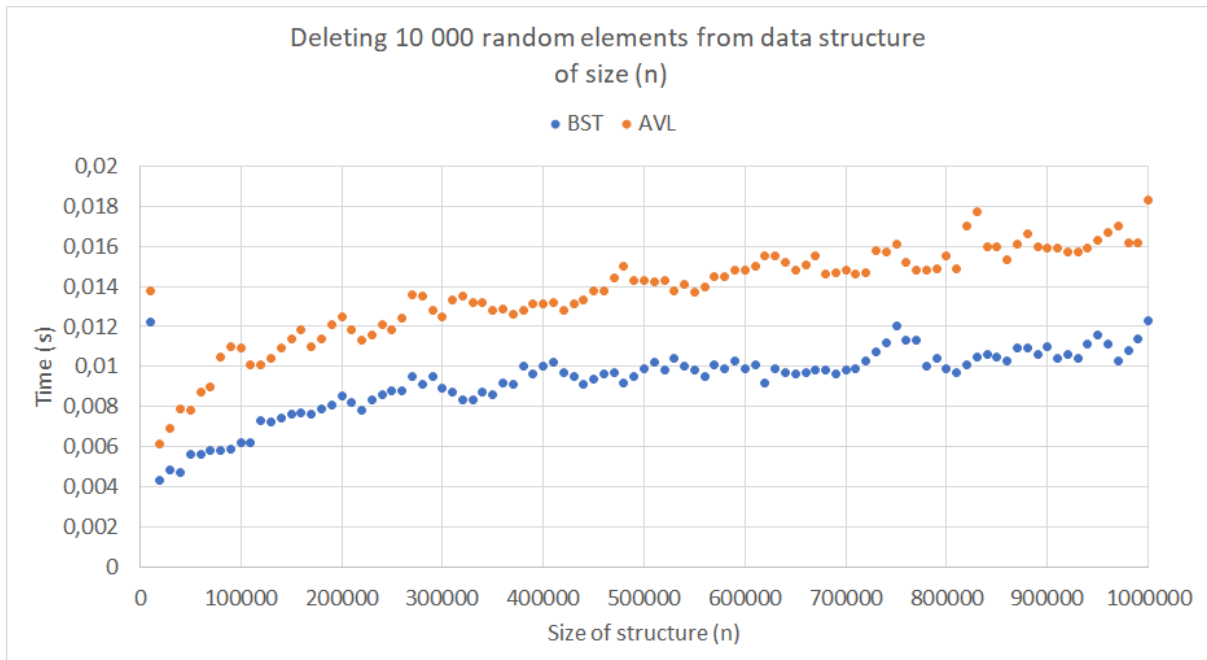
Graph of the mentioned function --> (<https://www.desmos.com/calculator/soejlimzky>).



Here, we can see that finding the specific element in structure has time complexity equal to $O(\log_2(n))$ as mentioned in the introduction. Also, a balanced tree is faster to search through.

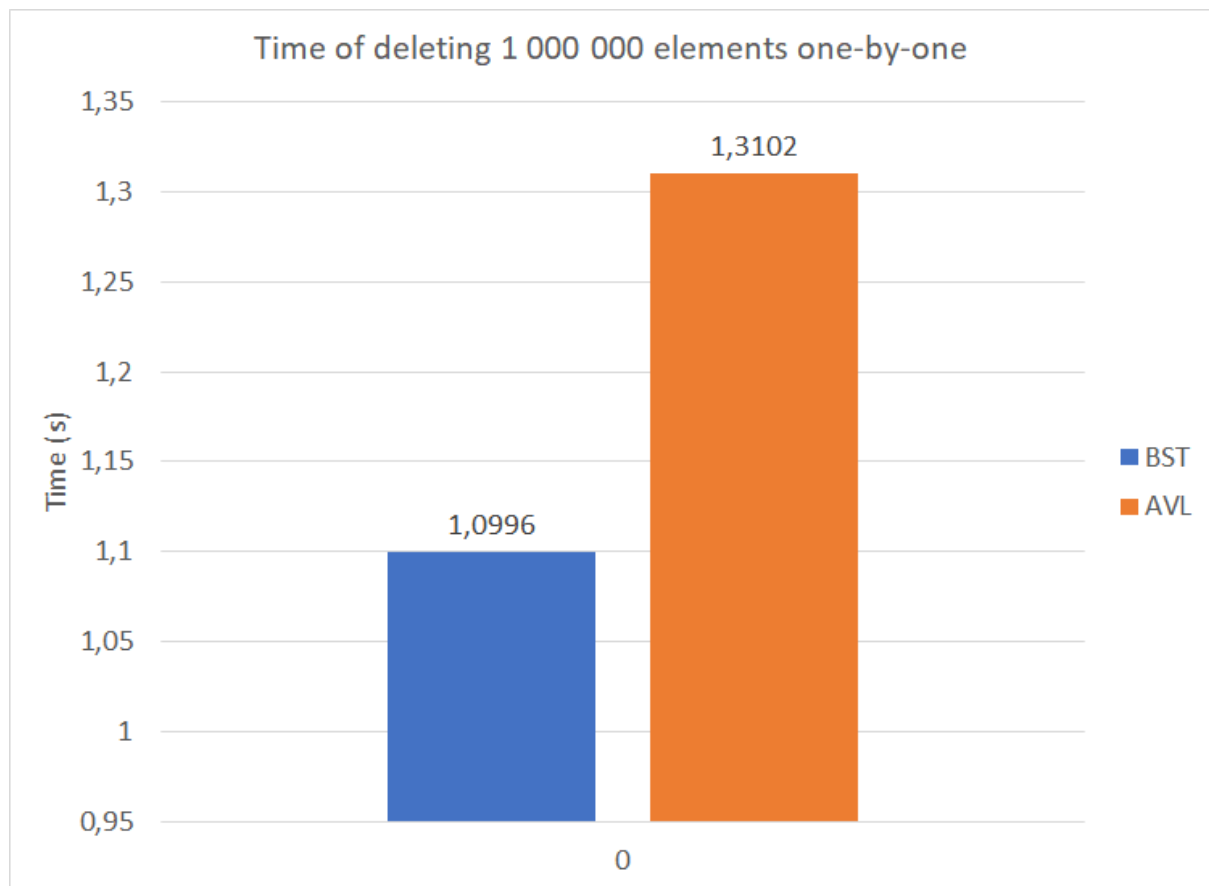
Deletion:

Deletion in AVL is more complex than in BST because of the need to balance the tree after operation, which can be clearly seen on the graph below.



Deletion of "k" elements from the structure with "n" elements has time complexity of $O(k \cdot \log_2(n))$.

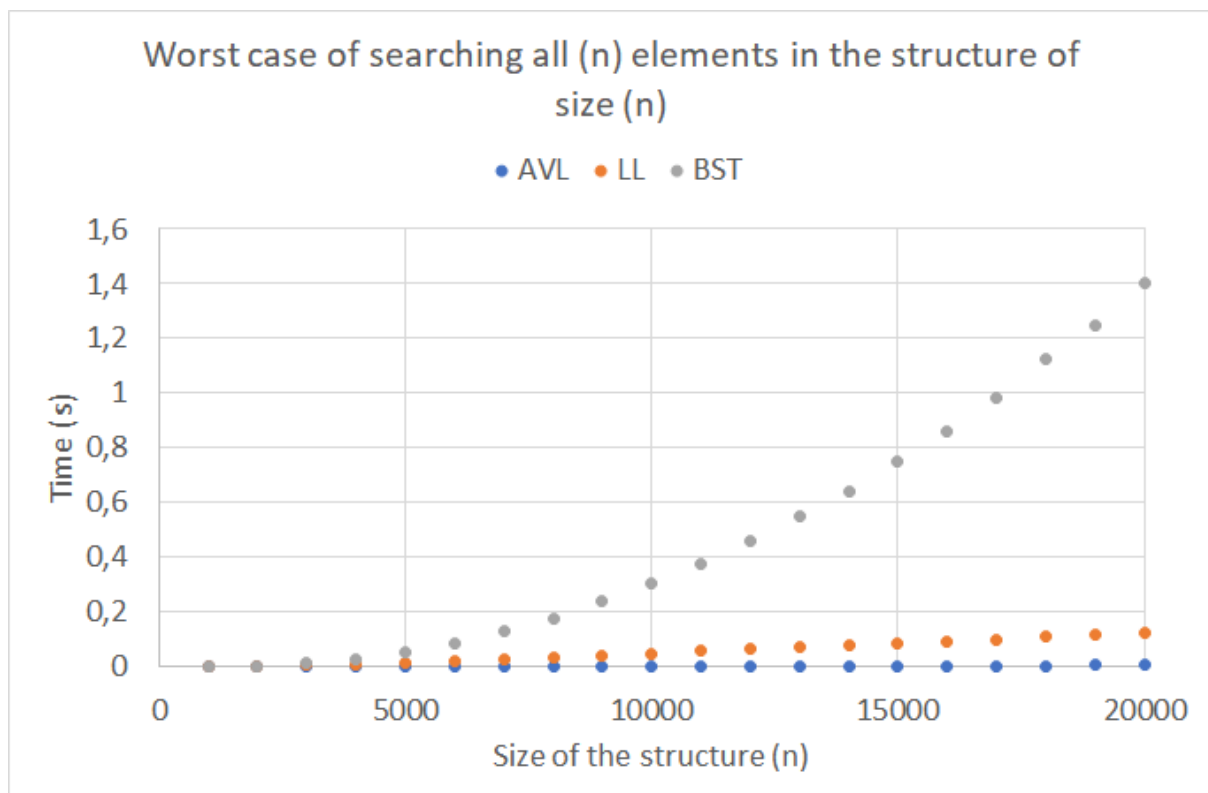
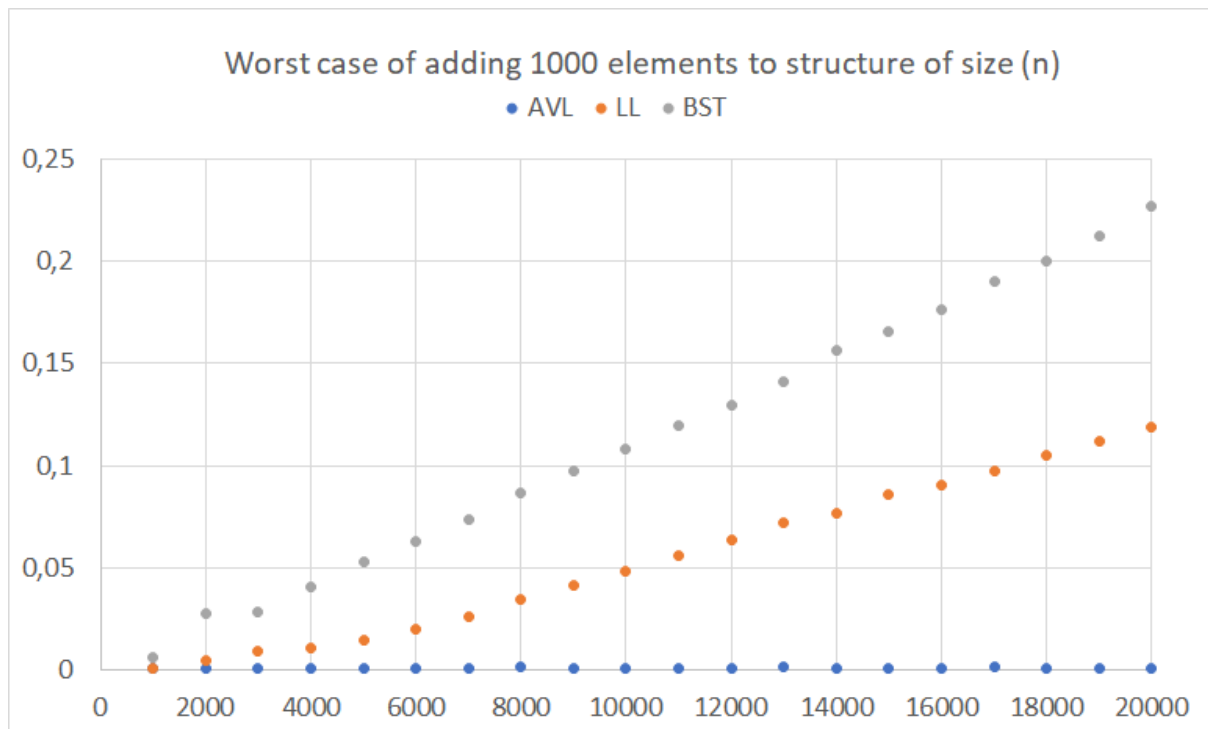
In our test, with 1 000 000 randomly inserted elements, deletion of all elements one by one in random order leasted:

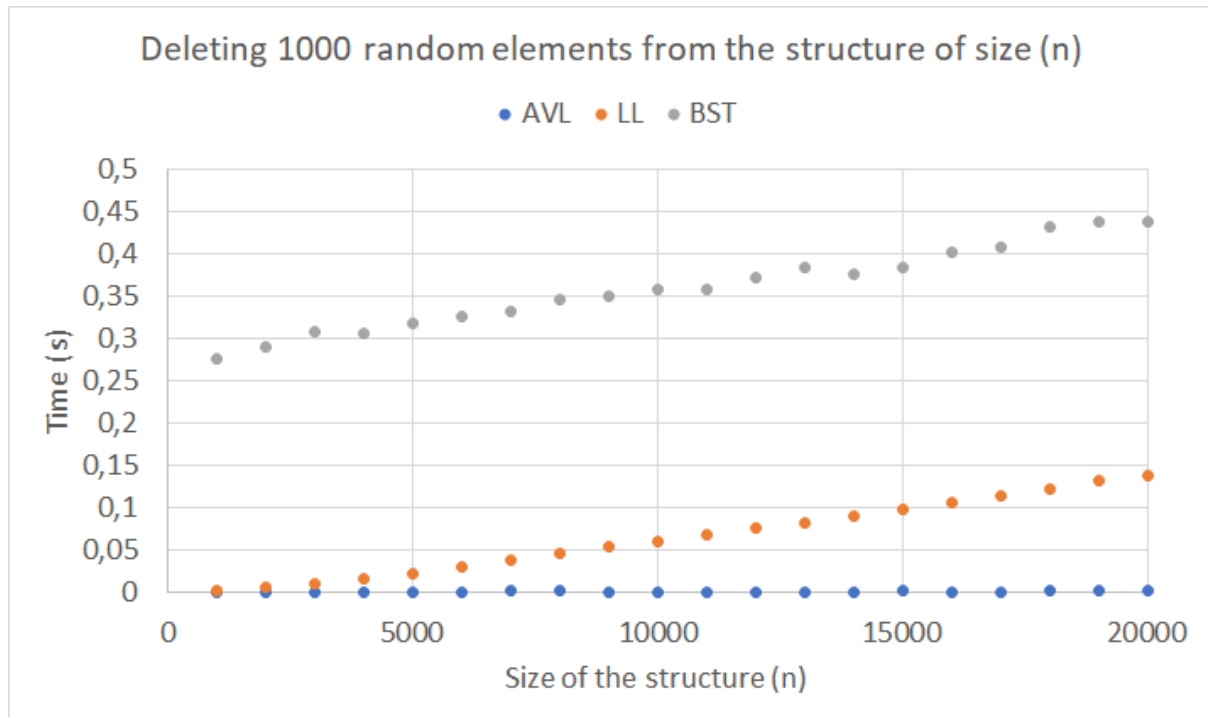


This graph also proves that operation of balancing the AVL tree takes some time, therefore BST is faster in this case. Nevertheless both of the trees are incomparably faster than the linked list in which deleting 100 000 elements in random order lasted 40,329 seconds (average of 10 runs).

Worst case scenario.

The biggest disadvantage of standard BST is the possibility of transforming into a Linked List. We ran a test in which we created a database with ordered indices. Here are some measurements from this test:





Expectations:

AVL should perform great and stay at its standard time complexity $O(\log_2(n))$. The Linked List should also stay at the same complexity, but operations should last longer thus every insertion of the element would have to iterate to the end of the current list. BST should become linked list i.e. change its complexity to $O(n)$.

Observations and conclusions:

What can be easily noticed, is the fact that BST started to behave dramatically worse. We expected it to be at the similar level as a linked list, but to our surprise graphs showed something different. After some investigation and checking our code, we came up with a conclusion. In our opinion the reason for such a behavior of BST is caused by a recursive nature of trees thus our functions (adding, searching and deleting) are recursive in contrast to the linked list, whose functions are implemented iteratively.

Also, as expected AVL outclassed BST and LL in such a case scenario thanks to the ability of balancing itself.

Final conclusions:

After testing and implementing all three data structures as expected, AVL turns out to be the safest data structure for storing data, but it's also the hardest to understand and implement. Nevertheless BST performs very similarly, and is noticeably easier to understand and implement. Worst case of inserting data is very unlikely and could be easily solved by simply shuffling the database. Last but not least is Ordered Linked List. This structure is definitely the easiest to understand and implement. It works acceptably well for small data instances, and can be found useful in many programs. (As a fun fact we can say that we've used this structure in our program for Low Level Programming laboratories.)

Below are presented average time complexities of tested structures, which were confirmed by our test.

	Addiction	Searching	Removing
Ordered linked list	n	n	n
Binary search tree	$\log n$	$\log n$	$\log n$
Balanced binary search tree	$\log n$	$\log n$	$\log n$

Below are all tables that we've created. Let's call it resources. In most cases time was measured every 10 000 elements i.e. after adding 10 000 elements timer was stopped, search started, after searching all elements added so far adding was resumed until adding next 10 000 elements, and so on.

1. Table with times related to addition of elements to structures.

LL iterator	Trees iterator	BST	BST prefix sum	Linked list	Linked list prefix	AVL	AVL prefix sum	AVL adding one element	BST adding one element	LL adding one element
1000	10000	0,0087	0,0087	0,0087	0,0009	0,0097	0,0097	0,0000097	0,0000087	0,0000009
2000	20000	0,0098	0,0185	0,0044	0,0053	0,0129	0,0226	0,00000129	0,00000098	0,0000044
3000	30000	0,0134	0,0319	0,0094	0,0147	0,0155	0,0381	0,00000155	0,00000134	0,0000094
4000	40000	0,0147	0,0466	0,0104	0,0251	0,0157	0,0538	0,00000157	0,00000147	0,0000104
5000	50000	0,0188	0,0654	0,0144	0,0395	0,0126	0,0664	0,00000126	0,00000188	0,0000144
6000	60000	0,0195	0,0849	0,0201	0,0596	0,0122	0,0786	0,00000122	0,00000195	0,0000201
7000	70000	0,019	0,1039	0,0261	0,0857	0,0126	0,0912	0,00000126	0,0000019	0,0000261
8000	80000	0,0159	0,1198	0,0348	0,1205	0,0131	0,1043	0,00000131	0,00000159	0,0000348
9000	90000	0,0117	0,1315	0,0417	0,1622	0,0137	0,118	0,00000137	0,00000117	0,0000417
10000	100000	0,0124	0,1439	0,0485	0,2107	0,0142	0,1322	0,00000142	0,00000124	0,0000485
11000	110000	0,0131	0,157	0,056	0,2667	0,0132	0,1454	0,00000132	0,00000131	0,000056
12000	120000	0,0112	0,1682	0,0633	0,33	0,0139	0,1593	0,00000139	0,00000112	0,0000633
13000	130000	0,0126	0,1808	0,0719	0,4019	0,014	0,1733	0,0000014	0,00000126	0,0000719
14000	140000	0,0121	0,1929	0,0768	0,4787	0,0141	0,1874	0,00000141	0,00000121	0,0000768
15000	150000	0,0122	0,2051	0,0858	0,5645	0,0137	0,2011	0,00000137	0,00000122	0,0000858
16000	160000	0,0125	0,2176	0,0901	0,6546	0,0138	0,2149	0,00000138	0,00000125	0,0000901
17000	170000	0,0131	0,2307	0,0973	0,7519	0,0143	0,2292	0,00000143	0,00000131	0,0000973
18000	180000	0,0129	0,2436	0,1049	0,8568	0,0142	0,2434	0,00000142	0,00000129	0,0001049
19000	190000	0,0126	0,2562	0,1118	0,9686	0,0143	0,2577	0,00000143	0,00000126	0,0001118
20000	200000	0,0128	0,269	0,1186	1,0872	0,0145	0,2722	0,00000145	0,00000128	0,0001186
21000	210000	0,0133	0,2823	0,1243	1,2115	0,015	0,2872	0,0000015	0,00000133	0,0001243
22000	220000	0,0128	0,2951	0,1334	1,3449	0,0147	0,3019	0,00000147	0,00000128	0,0001334
23000	230000	0,0131	0,3082	0,1384	1,4833	0,0153	0,3172	0,00000153	0,00000131	0,0001384
24000	240000	0,013	0,3212	0,148	1,6313	0,0157	0,3329	0,00000157	0,0000013	0,000148
25000	250000	0,0136	0,3348	0,1544	1,7857	0,0151	0,348	0,00000151	0,00000136	0,0001544
26000	260000	0,0147	0,3495	0,1611	1,9468	0,0152	0,3632	0,00000152	0,00000147	0,0001611
27000	270000	0,0143	0,3638	0,1671	2,1139	0,0151	0,3783	0,00000151	0,00000143	0,0001671
28000	280000	0,0153	0,3791	0,1746	2,2885	0,0154	0,3937	0,00000154	0,00000153	0,0001746
29000	290000	0,015	0,3941	0,1843	2,4728	0,0159	0,4096	0,00000159	0,0000015	0,0001843
30000	300000	0,0142	0,4083	0,1878	2,6606	0,0154	0,425	0,00000154	0,00000142	0,0001878
31000	310000	0,0133	0,4216	0,196	2,8566	0,0156	0,4406	0,00000156	0,00000133	0,000196
32000	320000	0,0134	0,435	0,2026	3,0592	0,0162	0,4568	0,00000162	0,00000134	0,0002026
33000	330000	0,0135	0,4485	0,2107	3,2699	0,0166	0,4734	0,00000166	0,00000135	0,0002107
34000	340000	0,0149	0,4634	0,2185	3,4884	0,0168	0,4902	0,00000168	0,00000149	0,0002185
35000	350000	0,0146	0,478	0,2242	3,7126	0,0164	0,5066	0,00000164	0,00000146	0,0002242
36000	360000	0,0155	0,4935	0,2313	3,9439	0,0163	0,5229	0,00000163	0,00000155	0,0002313
37000	370000	0,0143	0,5078	0,2419	4,1858	0,0162	0,5391	0,00000162	0,00000143	0,0002419
38000	380000	0,0146	0,5224	0,2487	4,4345	0,017	0,5561	0,0000017	0,00000146	0,0002487
39000	390000	0,0162	0,5386	0,2563	4,6908	0,0166	0,5727	0,00000166	0,00000162	0,0002563
40000	400000	0,0139	0,5525	0,2647	4,9555	0,018	0,5907	0,0000018	0,00000139	0,0002647
41000	410000	0,0163	0,5688	0,2731	5,2286	0,0171	0,6078	0,00000171	0,00000163	0,0002731
42000	420000	0,0144	0,5832	0,2735	5,5021	0,0174	0,6252	0,00000174	0,00000144	0,0002735
43000	430000	0,0147	0,5979	0,2853	5,7874	0,0172	0,6424	0,00000172	0,00000147	0,0002853
44000	440000	0,0149	0,6128	0,2881	6,0755	0,0175	0,6599	0,00000175	0,00000149	0,0002881
45000	450000	0,016	0,6288	0,2934	6,3689	0,0168	0,6767	0,00000168	0,0000016	0,0002934
46000	460000	0,0153	0,6441	0,3059	6,6748	0,0169	0,6936	0,00000169	0,00000153	0,0003059
47000	470000	0,0249	0,669	0,3217	6,9965	0,0179	0,7115	0,00000179	0,00000249	0,0003217
48000	480000	0,0151	0,6841	0,3208	7,3173	0,018	0,7295	0,0000018	0,00000151	0,0003208
49000	490000	0,0154	0,6995	0,3305	7,6478	0,0178	0,7473	0,00000178	0,00000154	0,0003305
50000	500000	0,0147	0,7142	0,3314	7,9792	0,0177	0,765	0,00000177	0,00000147	0,0003314
51000	510000	0,0148	0,729	0,3465	8,3257	0,0178	0,7828	0,00000178	0,00000148	0,0003465
52000	520000	0,0145	0,7435	0,3541	8,6798	0,0176	0,8004	0,00000176	0,00000145	0,0003541
53000	530000	0,0152	0,7587	0,3592	9,039	0,0175	0,8179	0,00000175	0,00000152	0,0003592
54000	540000	0,0147	0,7734	0,3569	9,3959	0,0176	0,8355	0,00000176	0,00000147	0,0003569
55000	550000	0,0156	0,789	0,3681	9,764	0,0183	0,8538	0,00000183	0,00000156	0,0003681
56000	560000	0,0165	0,8055	0,3793	10,1433	0,0175	0,8713	0,00000175	0,00000165	0,0003793
57000	570000	0,0151	0,8206	0,3871	10,5304	0,018	0,8893	0,0000018	0,00000151	0,0003871
58000	580000	0,0152	0,8358	0,3919	10,9223	0,0179	0,9072	0,00000179	0,00000152	0,0003919
59000	590000	0,0156	0,8514	0,4019	11,3242	0,018	0,9252	0,0000018	0,00000156	0,0004019
60000	600000	0,0163	0,8677	0,4071	11,7313	0,018	0,9432	0,0000018	0,00000163	0,0004071
61000	610000	0,0174	0,8851	0,4222	12,1535	0,018	0,9612	0,0000018	0,00000174	0,0004222
62000	620000	0,0169	0,902	0,4253	12,5788	0,018	0,9792	0,0000018	0,00000169	0,0004253
63000	630000	0,0158	0,9178	0,4298	13,0086	0,0182	0,9974	0,00000182	0,00000158	0,0004298
64000	640000	0,0167	0,9345	0,4381	13,4467	0,0181	1,0155	0,00000181	0,00000167	0,0004381
65000	650000	0,0168	0,9513	0,4509	13,8976	0,0182	1,0337	0,00000182	0,00000168	0,0004509
66000	660000	0,0159	0,9672	0,4536	14,3512	0,018	1,0517	0,0000018	0,00000159	0,0004536
67000	670000	0,0164	0,9836	0,465	14,8162	0,0183	1,07	0,00000183	0,00000164	0,000465
68000	680000	0,0158	0,9994	0,467	15,2832	0,0187	1,0887	0,00000187	0,00000158	0,000467
69000	690000	0,0162	1,0156	0,4796	15,7628	0,0184	1,1071	0,00000184	0,00000162	0,0004796
70000	700000	0,0178	1,0334	0,4947	16,2575	0,019	1,1261	0,0000019	0,00000178	0,0004947
71000	710000	0,0168	1,0502	0,4981	16,7556	0,0184	1,1445	0,00000184	0,00000168	0,0004981
72000	720000	0,0162	1,0664	0,5129	17,2685	0,0227	1,1672	0,00000227	0,00000162	0,0005129
73000	730000	0,0148	1,0812	0,5245	17,793	0,0187	1,1859	0,00000187	0,00000148	0,0005245
74000	740000	0,0161	1,0973	0,5411	18,3341	0,0198	1,2057	0,00000198	0,00000161	0,0005411
75000	750000	0,0168	1,1141	0,549	18,8831	0,0199	1,2256	0,00000199	0,00000168	0,000549
76000	760000	0,0181	1,1322	0,5602	19,4433	0,0188	1,2444	0,00000188	0,00000181	0,0005602
77000	770000	0,0165	1,1487	0,5593	20,0026	0,0188	1,2632	0,00000188	0,00000165	0,0005593
78000	780000	0,0163	1,165	0,5683	20,5709	0,0192	1,2824	0,00000192	0,00000163	0,0005683
79000	790000	0,0176	1,1826	0,6403	21,2112	0,019	1,3014	0,0000019	0,00000176	0,0006403
80000	800000	0,0246	1,2072	0,5908	21,802	0,0186	1,32	0,00000186	0,00000246	0,0005908
81000	810000	0,0174	1,2246	0,602	22,404	0,0203	1,3403	0,00000203	0,00000174	0,000602
82000	820000	0,0229	1,2475	0,6208	23,0248	0,0188	1,3591	0,00000188	0,00000229	0,0006208
83000	830000	0,0164	1,2639	0,6287	23,6535	0,0193	1,3784	0,00000193	0,00000164	0,0006287
84000	840000	0,0202	1,2841	0,6395	24,293	0,0193	1,3977	0,00000193	0,00000202	0,0006395
85000	850000	0,0177	1,3018	0,6529	24,9459	0,019	1,4167	0,0000019	0,00000177	0,0006529
86000	860000	0,0174	1,3192	0,6573	25,6032	0,0187	1,4354	0,00000187	0,00000174	0,0006573
87000	870000	0,0188	1,338	0,6662	26,2694	0,0191	1,4545	0,00000191	0,00000188	0,0006662
88000	880000	0,017	1,355	0,6865	26,9559	0,0193	1,4738	0,00000193	0,0000017	0,0006865
89000	890000	0,0163	1,3713	0,6949	27,6508	0,0194	1,4932	0,00000194	0,00000163	0,0006949
90000	900000	0,0173	1,3886	0,7223	28,3731	0,0191	1,5123	0,00000191	0,00000173	0,0007223
91000	910000	0,018								

2. Times related to searching of elements in structures:

Trees iterator	LL iterator	AVL	AVL time per one element	LL	LL time per one element	BST	BST time per one element
10000	1000	0,0015	1,50E-07	0,0026	0,00000026	0,0016	0,00000016
20000	2000	0,0037	1,85E-07	0,0062	0,00000031	0,004	0,0000002
30000	3000	0,0063	2,10E-07	0,0103	3,43333E-07	0,0069	0,00000023
40000	4000	0,0093	2,33E-07	0,0132	0,00000033	0,0116	0,00000029
50000	5000	0,0131	2,62E-07	0,0177	0,000000354	0,0137	0,000000274
60000	6000	0,0176	2,93E-07	0,0244	4,06667E-07	0,0165	0,000000275
70000	7000	0,019	2,71E-07	0,0308	0,00000044	0,021	0,0000003
80000	8000	0,021	2,63E-07	0,0402	5,025E-07	0,0263	3,2875E-07
90000	9000	0,0284	3,16E-07	0,0492	5,46667E-07	0,03	3,33333E-07
100000	10000	0,0348	3,48E-07	0,0566	0,000000566	0,0359	0,000000359
110000	11000	0,036	3,27E-07	0,0648	5,89091E-07	0,0397	3,60909E-07
120000	12000	0,0417	3,48E-07	0,0713	5,94167E-07	0,0469	3,90833E-07
130000	13000	0,0444	3,42E-07	0,0802	6,16923E-07	0,0489	3,76154E-07
140000	14000	0,0506	3,61E-07	0,0866	6,18571E-07	0,0588	0,00000042
150000	15000	0,0516	3,44E-07	0,0938	6,25333E-07	0,0602	4,01333E-07
160000	16000	0,0552	3,45E-07	0,1	0,000000625	0,0677	4,23125E-07
170000	17000	0,062	3,65E-07	0,1099	6,46471E-07	0,0793	4,66471E-07
180000	18000	0,0715	3,97E-07	0,1165	6,47222E-07	0,081	0,00000045
190000	19000	0,079	4,16E-07	0,1241	6,53158E-07	0,0867	4,56316E-07
200000	20000	0,0858	4,29E-07	0,1311	6,555E-07	0,0918	0,000000459
210000	21000	0,0883	4,20E-07	0,1372	6,53333E-07	0,0973	4,63333E-07
220000	22000	0,0909	4,13E-07	0,1461	6,64091E-07	0,1037	4,71364E-07
230000	23000	0,0953	4,14E-07	0,156	6,78261E-07	0,1146	4,98261E-07
240000	24000	0,1002	4,18E-07	0,1645	6,85417E-07	0,1199	4,99583E-07
250000	25000	0,1085	4,34E-07	0,1724	6,896E-07	0,127	0,000000508
260000	26000	0,1195	4,60E-07	0,1759	6,76538E-07	0,1395	5,36538E-07
270000	27000	0,1233	4,57E-07	0,1853	6,86296E-07	0,1397	5,17407E-07
280000	28000	0,1356	4,84E-07	0,2196	7,84286E-07	0,1446	5,16429E-07
290000	29000	0,1347	4,64E-07	0,2035	7,01724E-07	0,1521	5,24483E-07
300000	30000	0,1402	4,67E-07	0,2066	6,88667E-07	0,1641	0,000000547
310000	31000	0,1465	4,73E-07	0,2162	6,97419E-07	0,1718	5,54194E-07
320000	32000	0,1536	4,80E-07	0,2266	7,08125E-07	0,1735	5,42188E-07
330000	33000	0,1713	5,19E-07	0,2328	7,05455E-07	0,1878	5,69091E-07
340000	34000	0,1825	5,37E-07	0,2407	7,07941E-07	0,1887	0,000000555
350000	35000	0,1877	5,36E-07	0,2467	7,04857E-07	0,2066	5,90286E-07
360000	36000	0,1826	5,07E-07	0,2583	7,175E-07	0,2162	6,00556E-07
370000	37000	0,1909	5,16E-07	0,2716	7,34054E-07	0,2162	5,84324E-07
380000	38000	0,1982	5,22E-07	0,2746	7,22632E-07	0,2242	0,00000059
390000	39000	0,1963	5,03E-07	0,2812	7,21026E-07	0,2302	5,90256E-07
400000	40000	0,2052	5,13E-07	0,2879	7,1975E-07	0,2463	6,1575E-07
410000	41000	0,2046	4,99E-07	0,2975	7,2561E-07	0,2463	6,00732E-07
420000	42000	0,2279	5,43E-07	0,3	7,14286E-07	0,2572	6,12381E-07
430000	43000	0,2277	5,30E-07	0,3169	7,36977E-07	0,2662	6,1907E-07
440000	44000	0,3004	5,56E-07	0,4068	7,53333E-07	0,3523	6,52407E-07
450000	45000	0,3156	5,74E-07	0,4164	7,57091E-07	0,3707	0,000000674
460000	46000	0,3137	5,60E-07	0,4103	7,32679E-07	0,3708	6,62143E-07
470000	47000	0,3241	5,69E-07	0,4259	7,47193E-07	0,3766	6,60702E-07
480000	48000	0,3383	5,83E-07	0,4397	7,58103E-07	0,3911	6,7431E-07
490000	49000	0,3369	5,71E-07	0,4452	7,54576E-07	0,3925	6,65254E-07
500000	50000	0,3425	5,71E-07	0,4465	7,44167E-07	0,414	0,00000069
510000	51000	0,3521	5,77E-07	0,4563	7,48033E-07	0,4152	6,80656E-07
520000	52000	0,3529	5,69E-07	0,4643	7,48871E-07	0,4356	7,02581E-07
530000	53000	0,3713	5,89E-07	0,4979	7,90317E-07	0,4302	6,82857E-07
540000	54000	0,3683	5,75E-07	0,4949	7,73281E-07	0,4479	6,99844E-07
550000	55000	0,3843	5,91E-07	0,5035	7,74615E-07	0,4505	6,93077E-07
560000	56000	0,3914	5,93E-07	0,5772	8,74545E-07	0,4622	7,00303E-07
570000	57000	0,4046	6,04E-07	0,5157	7,69701E-07	0,4699	7,01343E-07
580000	58000	0,4124	6,06E-07	0,5268	7,74706E-07	0,482	7,08824E-07
590000	59000	0,4086	5,92E-07	0,5424	7,86087E-07	0,4907	7,11159E-07
600000	60000	0,4228	6,04E-07	0,5555	7,93571E-07	0,5029	7,18429E-07
610000	61000	0,4154	5,85E-07	0,5615	7,90845E-07	0,5077	7,1507E-07
620000	62000	0,4363	6,06E-07	0,5693	7,90694E-07	0,5312	7,37778E-07
630000	63000	0,4397	6,02E-07	0,5693	7,79863E-07	0,5372	7,3589E-07
640000	64000	0,4523	6,11E-07	0,5764	7,78919E-07	0,5409	7,30946E-07
650000	65000	0,4544	6,06E-07	0,6081	8,108E-07	0,5426	7,23467E-07
660000	66000	0,464	6,11E-07	0,6084	8,00526E-07	0,5572	7,33158E-07
670000	67000	0,4742	6,16E-07	0,619	8,03896E-07	0,5652	7,34026E-07
680000	68000	0,4807	6,16E-07	0,6275	8,04487E-07	0,5709	7,31923E-07
690000	69000	0,4869	6,16E-07	0,6295	7,96835E-07	0,5871	7,43165E-07
700000	70000	0,4953	6,19E-07	0,6547	8,18375E-07	0,5924	7,405E-07
710000	71000	0,4983	6,15E-07	0,6607	8,15679E-07	0,6014	7,42469E-07
720000	72000	0,5133	6,26E-07	0,6657	8,11829E-07	0,6151	7,50122E-07
730000	73000	0,51	6,14E-07	0,6761	8,14578E-07	0,6143	7,4012E-07
740000	74000	0,533	6,35E-07	0,6986	8,31667E-07	0,6466	7,69762E-07
750000	75000	0,5492	6,46E-07	0,6981	8,21294E-07	0,6512	7,66118E-07
760000	76000	0,5666	6,59E-07	0,7178	8,34651E-07	0,6543	7,60814E-07
770000	77000	0,5885	6,76E-07	0,7152	8,22069E-07	0,6625	7,61494E-07
780000	78000	0,6084	6,91E-07	0,7452	8,46818E-07	0,672	7,63636E-07
790000	79000	0,6604	6,67E-07	0,9255	9,34848E-07	0,7912	7,99192E-07
800000	80000	0,6634	6,63E-07	0,9443	9,443E-07	0,7965	7,965E-07

3. Table related to times of deleting elements from structures:

Iterator	Deleting 10 000 elements from LL	Deleting 10 000 elements from BST	Deleting 10 000 elements from AVL	LL / iterator	BST/iterator	AVL/iterator
10000	0,2849	0,0122	0,0138	0,00002849	1,22E-06	1,38E-06
20000	1,0236	0,0043	0,0061	0,00005118	2,15E-07	3,05E-07
30000	1,7748	0,0048	0,0069	0,00005916	1,60E-07	2,30E-07
40000	2,5664	0,0047	0,0079	0,00006416	1,18E-07	1,98E-07
50000	3,3661	0,0056	0,0078	0,000067322	1,12E-07	1,56E-07
60000	4,2348	0,0056	0,0087	0,00007058	9,33E-08	1,45E-07
70000	5,1551	0,0058	0,009	7,36443E-05	8,29E-08	1,29E-07
80000	6,1977	0,0058	0,0105	7,74713E-05	7,25E-08	1,31E-07
90000	7,2694	0,0059	0,011	8,07711E-05	6,56E-08	1,22E-07
100000	8,4562	0,0062	0,0109	0,000084562	6,20E-08	1,09E-07
110000		0,0062	0,0101		5,64E-08	9,18E-08
120000		0,0073	0,0101		6,08E-08	8,42E-08
130000		0,0072	0,0104		5,54E-08	8,00E-08
140000		0,0074	0,0109		5,29E-08	7,79E-08
150000		0,0076	0,0114		5,07E-08	7,60E-08
160000		0,0077	0,0118		4,81E-08	7,38E-08
170000		0,0076	0,011		4,47E-08	6,47E-08
180000		0,0079	0,0114		4,39E-08	6,33E-08
190000		0,0081	0,0121		4,26E-08	6,37E-08
200000		0,0085	0,0125		4,25E-08	6,25E-08
210000		0,0082	0,0118		3,90E-08	5,62E-08
220000		0,0078	0,0113		3,55E-08	5,14E-08
230000		0,0083	0,0116		3,61E-08	5,04E-08
240000		0,0086	0,0121		3,58E-08	5,04E-08
250000		0,0088	0,0118		3,52E-08	4,72E-08
260000		0,0088	0,0124		3,38E-08	4,77E-08
270000		0,0095	0,0136		3,52E-08	5,04E-08
280000		0,0091	0,0135		3,25E-08	4,82E-08
290000		0,0095	0,0128		3,28E-08	4,41E-08
300000		0,0089	0,0125		2,97E-08	4,17E-08
310000		0,0087	0,0133		2,81E-08	4,29E-08
320000		0,0083	0,0135		2,59E-08	4,22E-08
330000		0,0083	0,0132		2,52E-08	4,00E-08
340000		0,0087	0,0132		2,56E-08	3,88E-08
350000		0,0086	0,0128		2,46E-08	3,66E-08
360000		0,0092	0,0129		2,56E-08	3,58E-08
370000		0,0091	0,0126		2,46E-08	3,41E-08
380000		0,01	0,0128		2,63E-08	3,37E-08
390000		0,0096	0,0131		2,46E-08	3,36E-08
400000		0,01	0,0131		2,50E-08	3,28E-08
410000		0,0102	0,0132		2,49E-08	3,22E-08
420000		0,0097	0,0128		2,31E-08	3,05E-08
430000		0,0095	0,0131		2,21E-08	3,05E-08
440000		0,0091	0,0133		2,07E-08	3,02E-08
450000		0,0094	0,0138		2,09E-08	3,07E-08
460000		0,0096	0,0138		2,09E-08	3,00E-08
470000		0,0097	0,0144		2,06E-08	3,06E-08
480000		0,0092	0,015		1,92E-08	3,13E-08
490000		0,0095	0,0143		1,94E-08	2,92E-08
500000		0,0099	0,0143		1,98E-08	2,86E-08
510000		0,0102	0,0142		2,00E-08	2,78E-08
520000		0,0098	0,0143		1,88E-08	2,75E-08
530000		0,0104	0,0138		1,96E-08	2,60E-08
540000		0,01	0,0141		1,85E-08	2,61E-08
550000		0,0098	0,0137		1,78E-08	2,49E-08
560000		0,0095	0,014		1,70E-08	2,50E-08
570000		0,0101	0,0145		1,77E-08	2,54E-08
580000		0,0099	0,0145		1,71E-08	2,50E-08
590000		0,0103	0,0148		1,75E-08	2,51E-08
600000		0,0099	0,0148		1,65E-08	2,47E-08
610000		0,0101	0,015		1,66E-08	2,46E-08
620000		0,0092	0,0155		1,48E-08	2,50E-08
630000		0,0099	0,0155		1,57E-08	2,46E-08
640000		0,0097	0,0152		1,52E-08	2,38E-08
650000		0,0096	0,0148		1,48E-08	2,28E-08
660000		0,0097	0,0151		1,47E-08	2,29E-08
670000		0,0098	0,0155		1,46E-08	2,31E-08
680000		0,0098	0,0146		1,44E-08	2,15E-08
690000		0,0096	0,0147		1,39E-08	2,13E-08
700000		0,0098	0,0148		1,40E-08	2,11E-08
710000		0,0099	0,0146		1,39E-08	2,06E-08
720000		0,0103	0,0147		1,43E-08	2,04E-08
730000		0,0107	0,0158		1,47E-08	2,16E-08
740000		0,0112	0,0157		1,51E-08	2,12E-08
750000		0,012	0,0161		1,60E-08	2,15E-08
760000		0,0113	0,0152		1,49E-08	2,00E-08
770000		0,0113	0,0148		1,47E-08	1,92E-08
780000		0,01	0,0148		1,28E-08	1,90E-08
790000		0,0104	0,0149		1,32E-08	1,89E-08
800000		0,0099	0,0155		1,24E-08	1,94E-08
810000		0,0097	0,0149		1,20E-08	1,84E-08
820000		0,0101	0,017		1,23E-08	2,07E-08
830000		0,0105	0,0177		1,27E-08	2,13E-08
840000		0,0106	0,016		1,26E-08	1,90E-08
850000		0,0105	0,016		1,24E-08	1,88E-08
860000		0,0103	0,0153		1,20E-08	1,78E-08
870000		0,0109	0,0161		1,25E-08	1,85E-08
880000		0,0109	0,0166		1,24E-08	1,89E-08
890000		0,0106	0,016		1,19E-08	1,80E-08
900000		0,011	0,0159		1,22E-08	1,77E-08
910000		0,0104	0,0159		1,14E-08	1,75E-08
920000		0,0106	0,0157		1,15E-08	1,71E-08
930000		0,0104	0,0157		1,12E-08	1,69E-08
940000		0,0111	0,0159		1,18E-08	1,69E-08
950000		0,0116	0,0163		1,22E-08	1,72E-08
960000		0,0111	0,0167		1,16E-08	1,74E-08
970000		0,0103	0,017		1,06E-08	1,75E-08
980000		0,0108	0,0162		1,10E-08	1,65E-08
990000		0,0114	0,0162		1,15E-08	1,64E-08

4. Times related to measuring worst case scenario for BST:

WORST CASE	Adding			Searching			Deleting		
iterator	AVL	LL	BST	AVL	LL	BST	AVL	LL	BST
1000	0,0008	0,0009	0,006	0,0002	0,0026	0,0028	0,0008	0,002	0,276
2000	0,0008	0,0044	0,0278	0,0002	0,0062	0,0142	0,0002	0,0056	0,2898
3000	0,0006	0,0094	0,0282	0,0012	0,0103	0,0278	0,0006	0,01	0,308
4000	0,0006	0,0104	0,0404	0,0012	0,0132	0,0532	0,0006	0,0159	0,3064
5000	0,001	0,0144	0,0528	0,0008	0,0177	0,0878	0,0004	0,0227	0,319
6000	0,0006	0,0201	0,0628	0,0006	0,0244	0,1286	0,0008	0,03	0,326
7000	0,0008	0,0261	0,0738	0,0012	0,0308	0,1778	0,001	0,038	0,333
8000	0,0014	0,0348	0,0868	0,002	0,0402	0,2384	0,001	0,0458	0,3458
9000	0,0006	0,0417	0,0976	0,0012	0,0492	0,3018	0,0006	0,0541	0,3512
10000	0,0008	0,0485	0,1082	0,0016	0,0566	0,3772	0,0006	0,0608	0,3588
11000	0,0004	0,056	0,1192	0,0018	0,0648	0,4572	0,0008	0,0677	0,3588
12000	0,0008	0,0633	0,1298	0,0022	0,0713	0,5472	0,0006	0,0755	0,3728
13000	0,0012	0,0719	0,1414	0,003	0,0802	0,6424	0,0006	0,083	0,385
14000	0,001	0,0768	0,156	0,0036	0,0866	0,751	0,0006	0,09	0,377
15000	0,001	0,0858	0,1654	0,0028	0,0938	0,8608	0,001	0,0977	0,3854
16000	0,0008	0,0901	0,1762	0,003	0,1	0,9826	0,0004	0,1054	0,4018
17000	0,0014	0,0973	0,19	0,004	0,1099	1,1252	0,0008	0,1131	0,4082
18000	0,0008	0,1049	0,2004	0,0042	0,1165	1,2482	0,001	0,1226	0,4318
19000	0,0004	0,1118	0,2124	0,0044	0,1241	1,3998	0,002	0,1313	0,438
20000	0,001	0,1186	0,227	0,0042	0,1311	1,5568	0,0012	0,1373	0,4378