DISCRETE COSINE TRANSFORM
DISCRETE FOURIER TRANSFORM

Bonus Track:
- STFT
- Wavelet Transform

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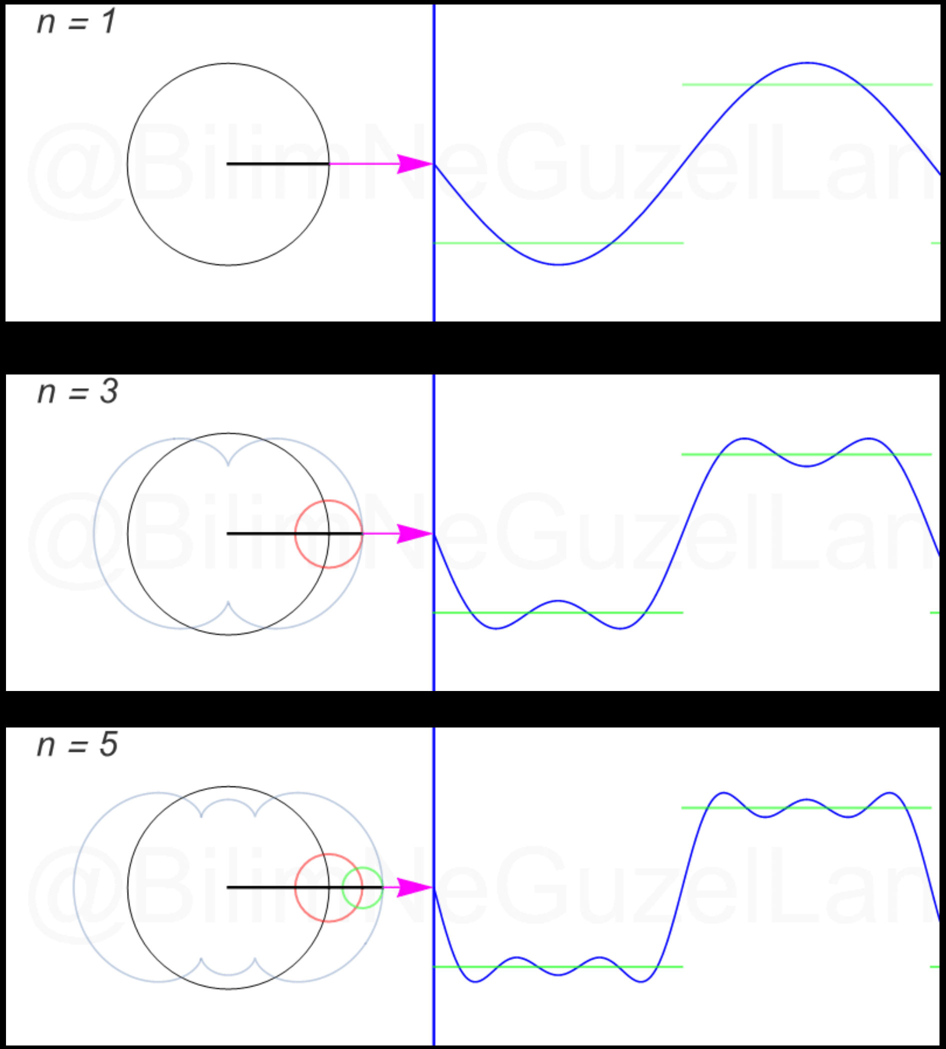
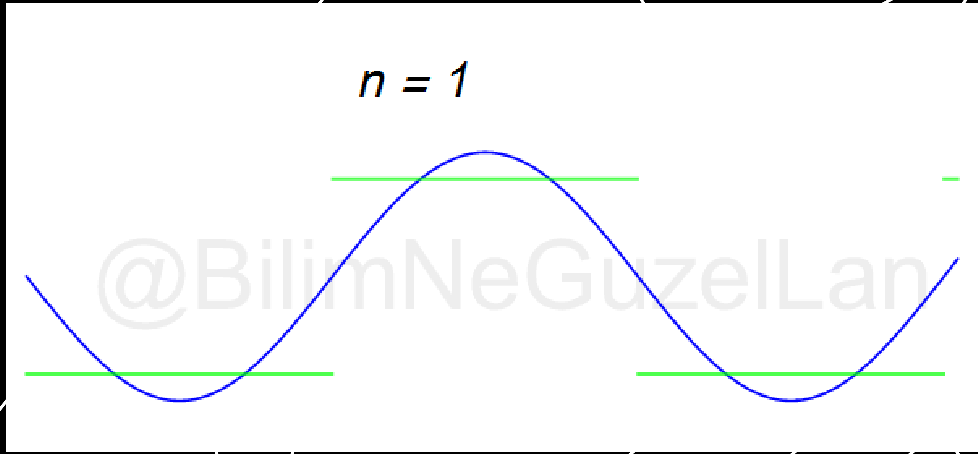
Special Topics: Brain-Body Music

SYNTHESIS





- Any wave can be constructed from a set of frequency components and their respective amplitudes
- The wave is the resultant summation of these sinusoidal frequency components
- Suppose a wave is the sum of cosines with a given set of frequencies, how would you find the amplitude for each frequency component?

DISCRETE COSINE TRANSFORM (DCT)

$$x_n = \frac{1}{N}\beta^* \sum_{k=0}^{N-1} X_k \cos\left[\left(\frac{\pi k}{2N}\right)(2n+1)\right]$$

$$X_k = \sum_{n=0}^{N-1} x_n \cos\left[\left(\frac{\pi k}{2N}\right)(2n+1)\right]$$

$$\begin{bmatrix} \vdots \\ X_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \cdots & \cdots & \cdots \\ \vdots & \cos\left[\left(\frac{\pi k}{2N}\right)(2n+1)\right] & \vdots \\ \cdots & \cdots & \cdots \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ x_n \\ \vdots \end{bmatrix}$$

$$k \times n$$

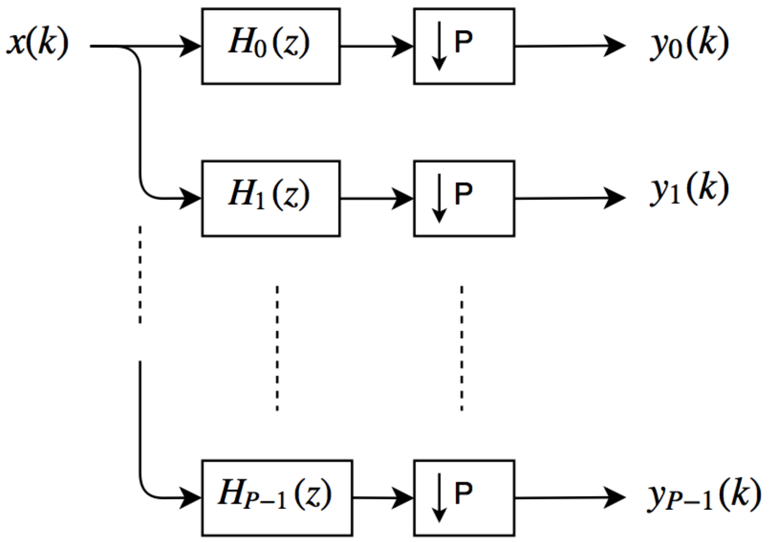
DISCRETE COSINE TRANSFORM (DCT)

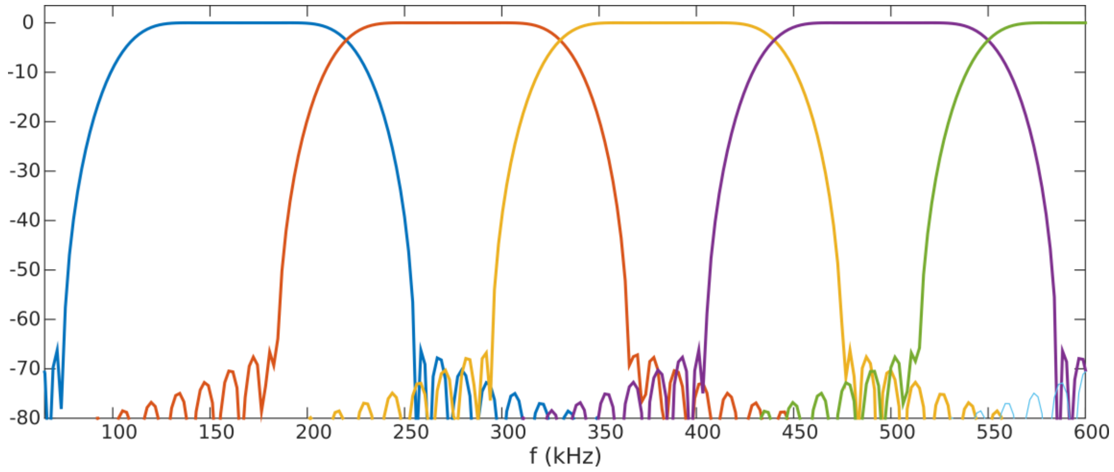
- DCT co-efficients are real valued (no imaginary parts)
- This makes it a preferred transformation for compression tasks
    - Data compression (MP3, MPEG, JPEG)
    - Audio Feedback cancellation
    - Biometric recognition (face, fingerprint, etc.)
    - Watermarking
- Many possible variation/implementations
    - DCT I
    - DCT II
    - DCT III
    - DCT IV
    - MDCT IV

  Thinkdsp implements DCT IV

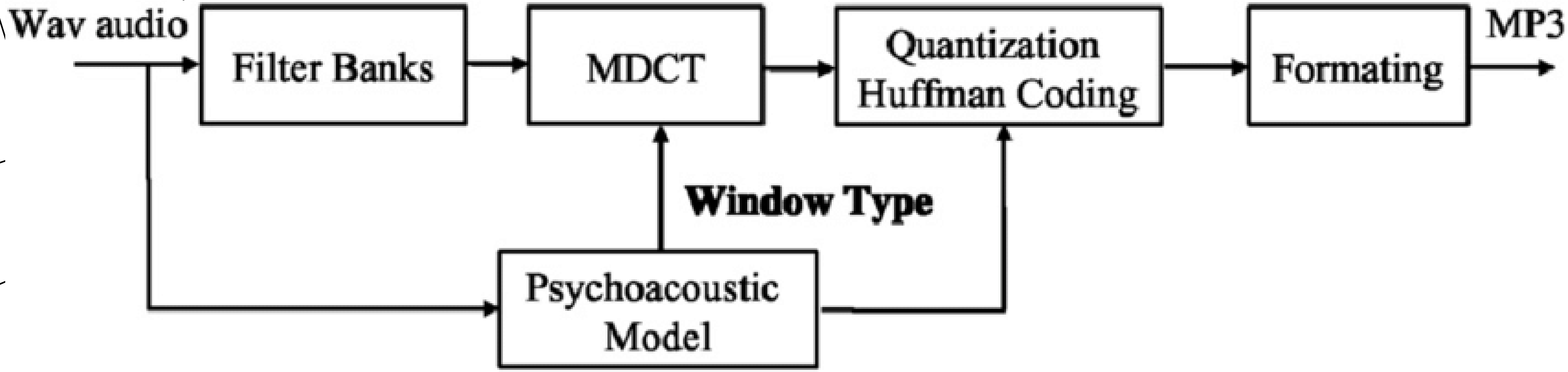
Example of Utilization

- Harmonic partials segregation





Example of Utilization



MP3 Block Diagram

ACTIVITY 1

COMPRESS HEARTBEAT SIGNALS USING DCT

DISCRETE FOURIER TRANSFORM (DFT)

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \left[\cos\left(\frac{2\pi kn}{N}\right) + i\sin\left(\frac{2\pi kn}{N}\right)\right]$$

$$X_k = \sum_{n=0}^{N-1} x_n \left[\cos\left(\frac{2\pi kn}{N}\right) - i\sin\left(\frac{2\pi kn}{N}\right)\right]$$

$$e^{i\frac{2\pi kn}{N}} = \cos\left(\frac{2\pi kn}{N}\right) + i\sin\left(\frac{2\pi kn}{N}\right)$$

Euler’s Formula

DISCRETE FOURIER TRANSFORM (DFT)

- We've been using the Discrete Fourier transform (DFT) since the first ThinkDSP exercise
- Complex Exponentials:

$$e^{i\phi} = \cos\phi + i\sin\phi$$

- This formula is a complex number with magnitude 1
- Think of it as a point in the complex plane, it is always on the unit circle.
- Just as we did with real sinusoids, we can create compound signals by adding up complex sinusoids with different frequencies.

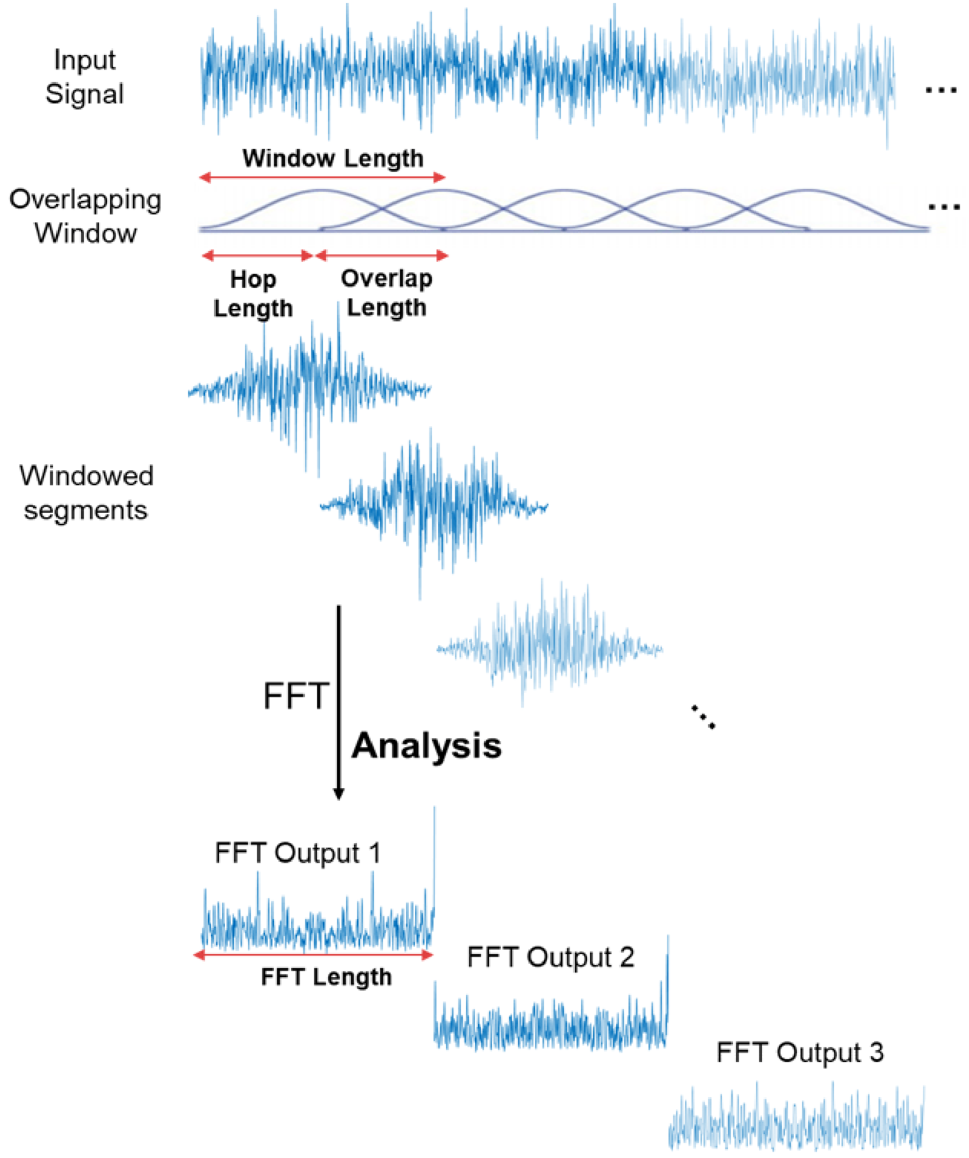
ACTIVITY 2

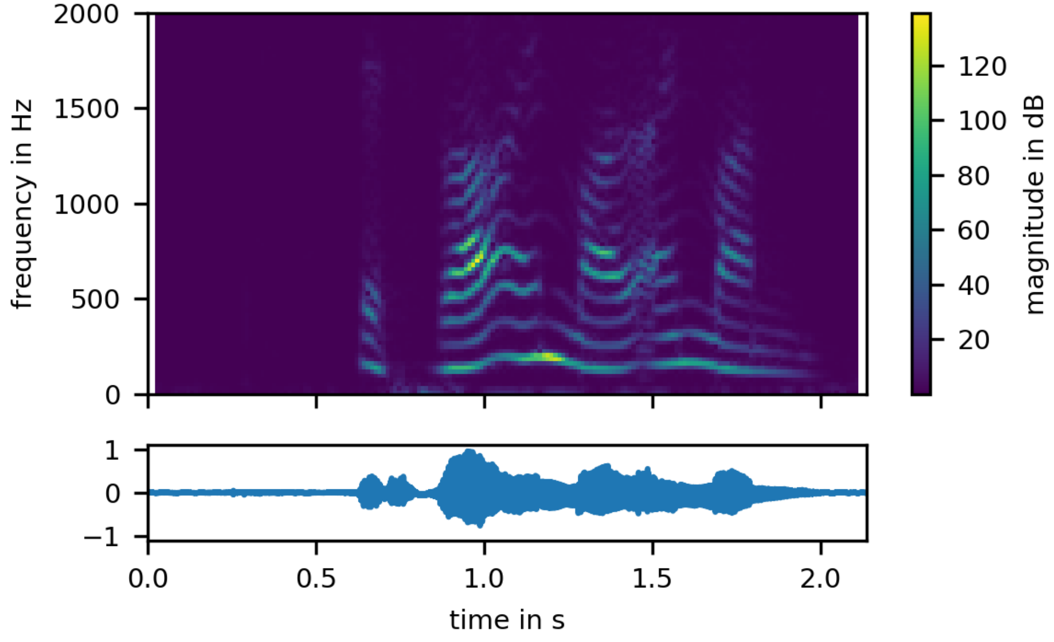
https://www.jezzamon.com/fourier/

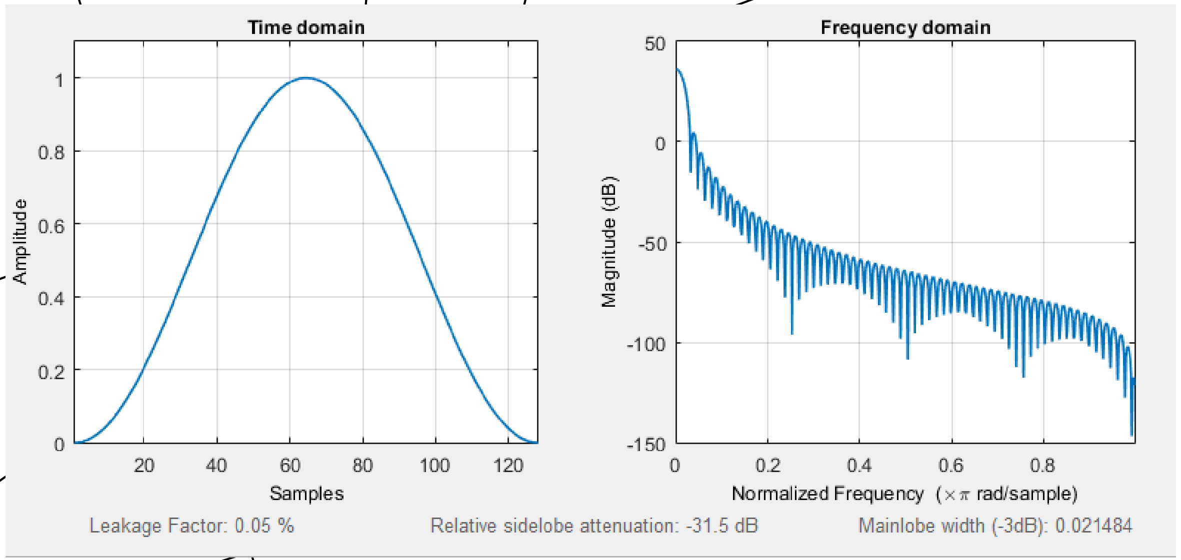
BONUS TRACK

- Short-time Fourier Transform
- Wavelet Transform

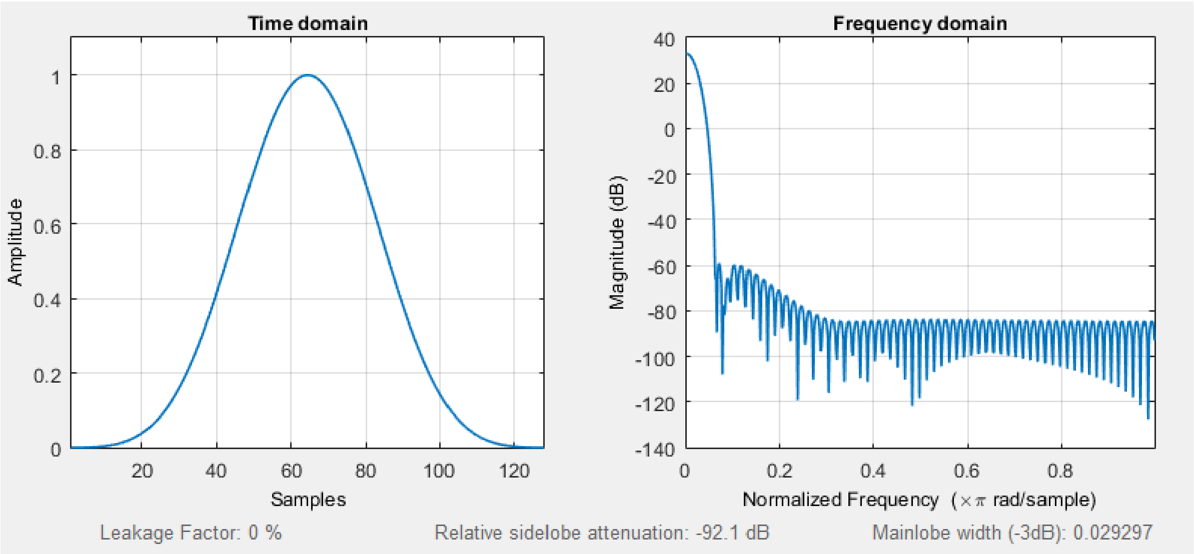
SHORT-TIME FOURIER TRANSFORM (STFT)



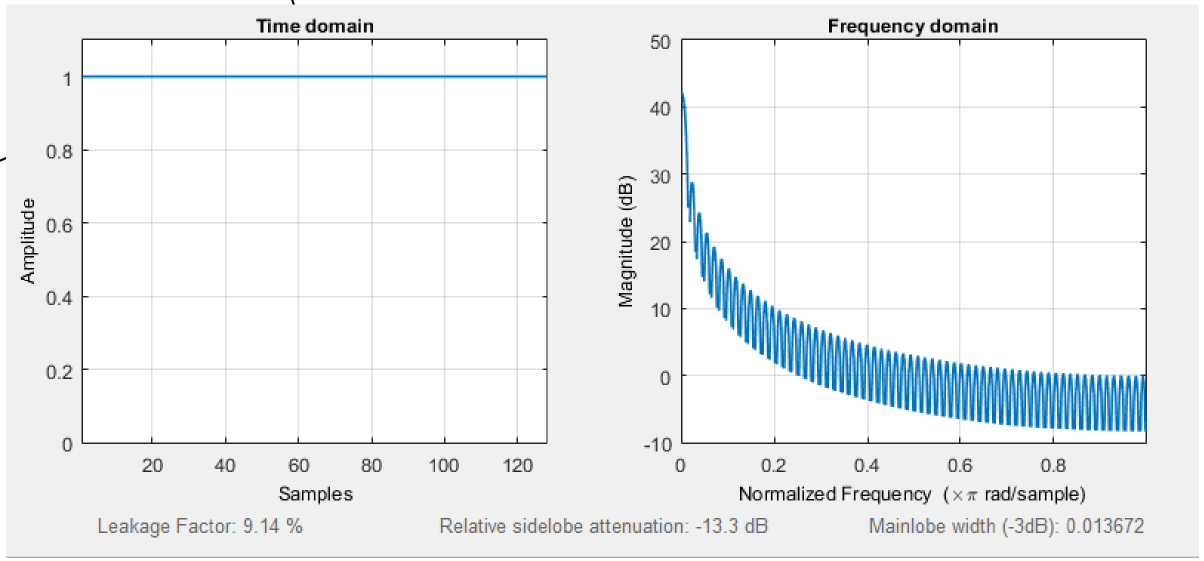




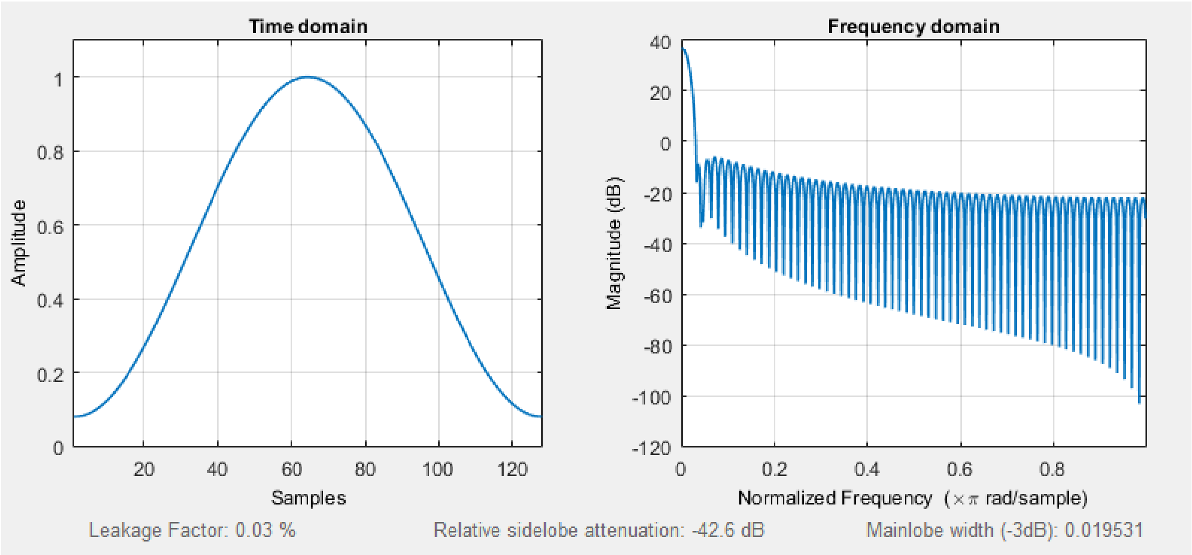
Tukey Window



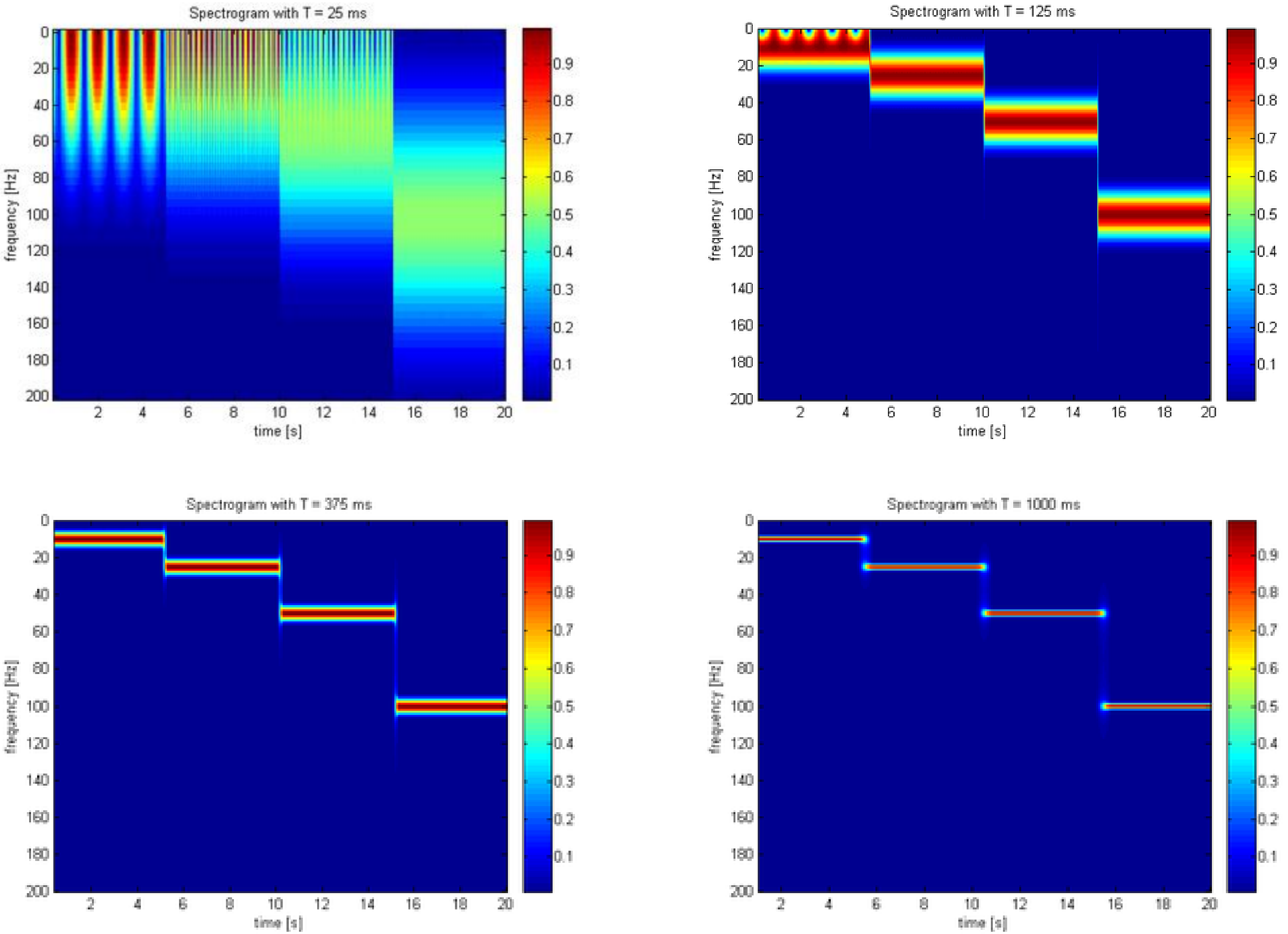
Blackman-Harris Window



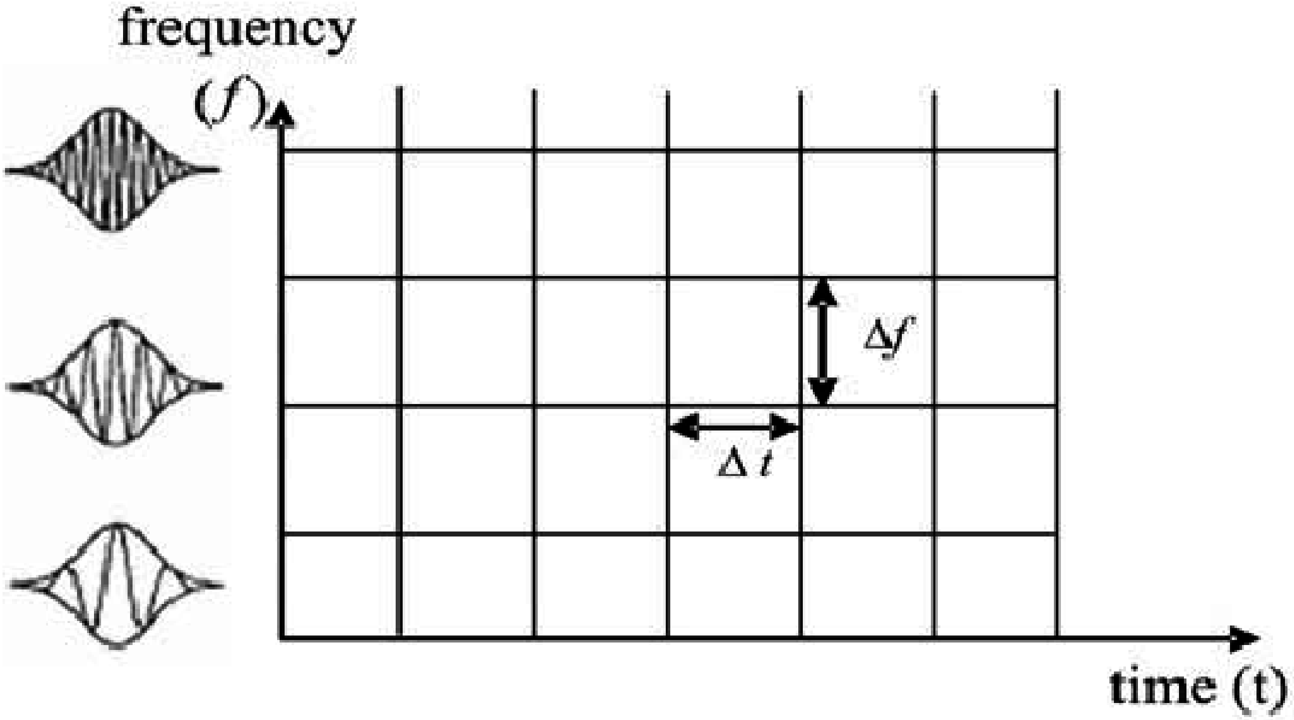
Rectangular Window



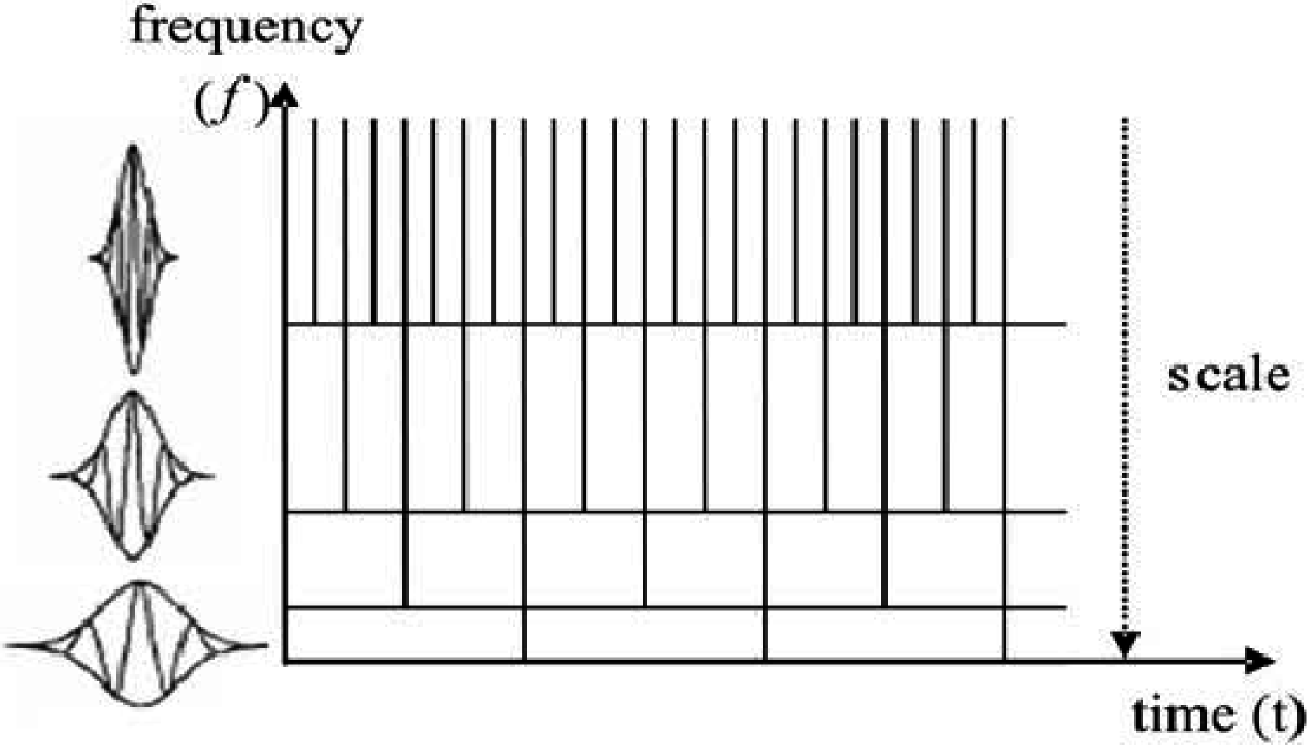
Hamming Window

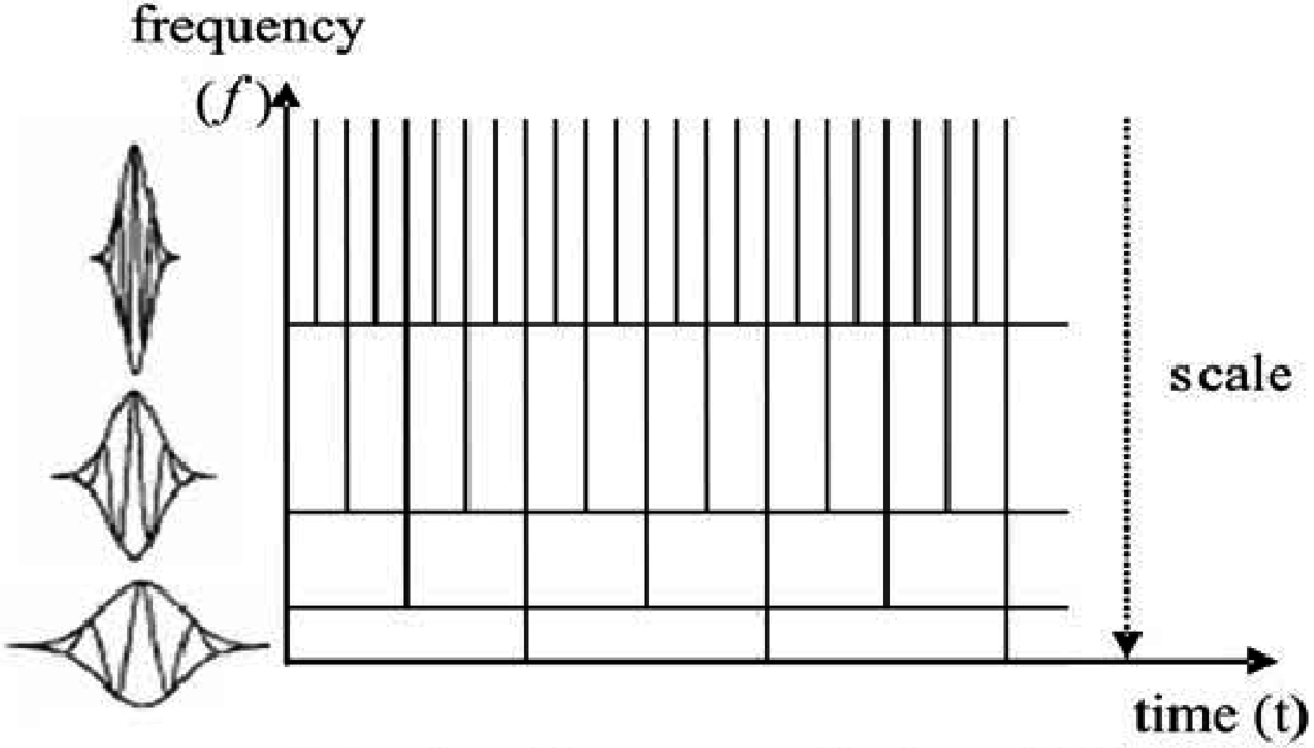


Spectrogram with T = 25 ms
Spectrogram with T = 125 ms
Spectrogram with T = 375 ms
Spectrogram with T = 1000 ms
frequency [Hz]
time [s]



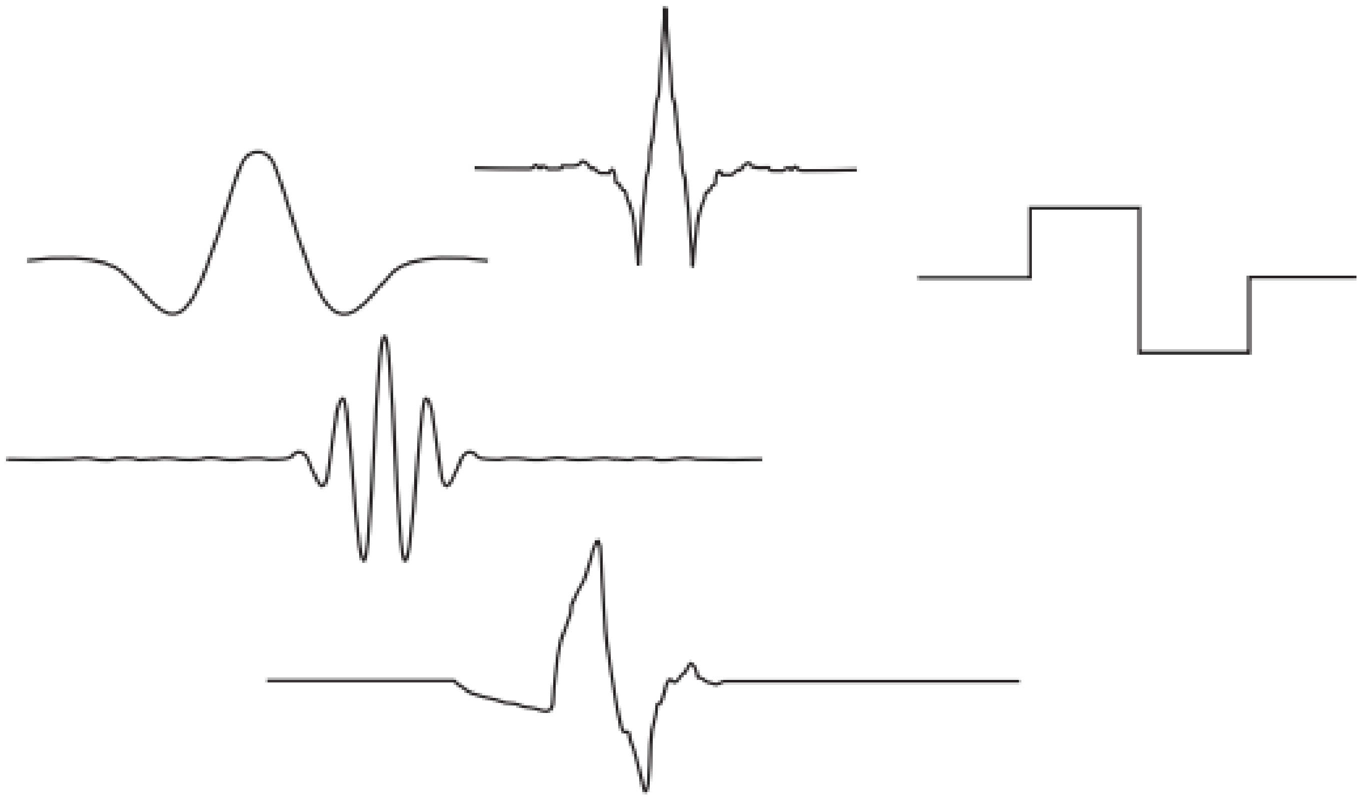
The time-frequency window of STFT


frequency
(f)
scale
time (t)



The time-frequency window of Wavelet

WAVELET TRANSFORM



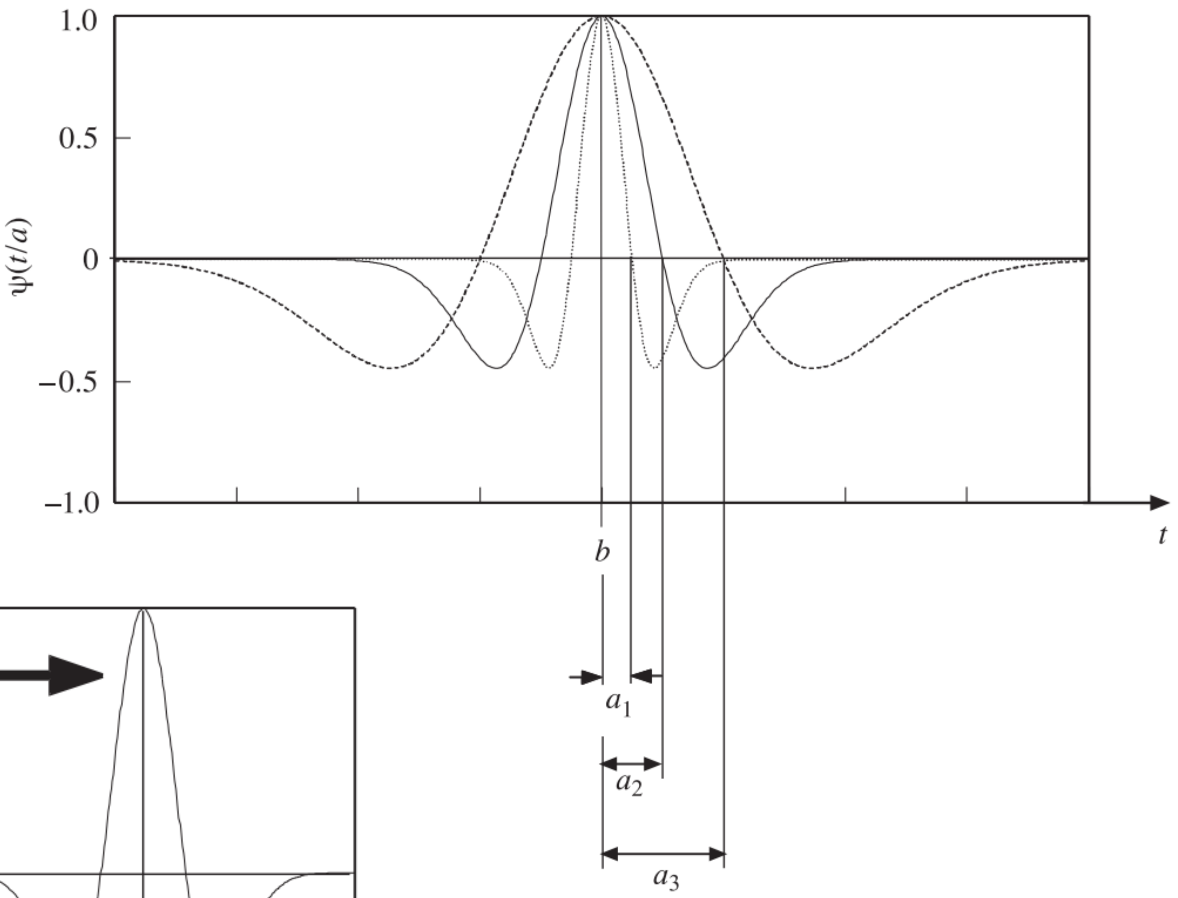
WAVELET TRANSFORM

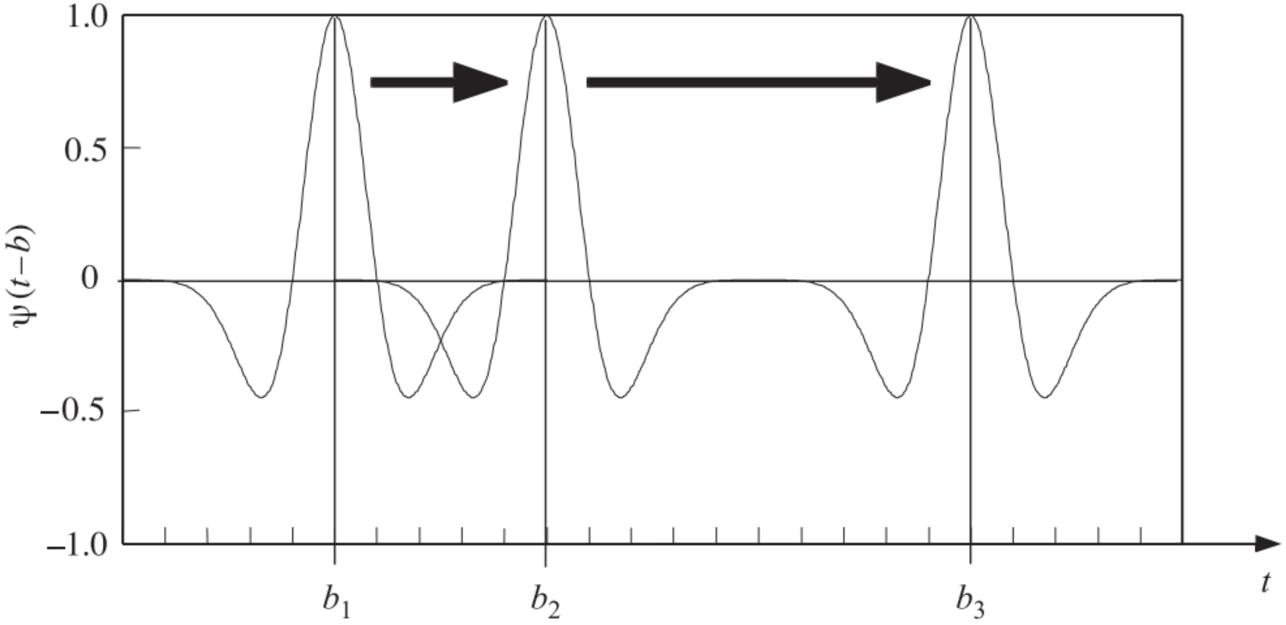
$$W_f(a,b) = < f, \psi_{a,b} > = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t)\, \psi^*(\frac{t-b}{a})$$

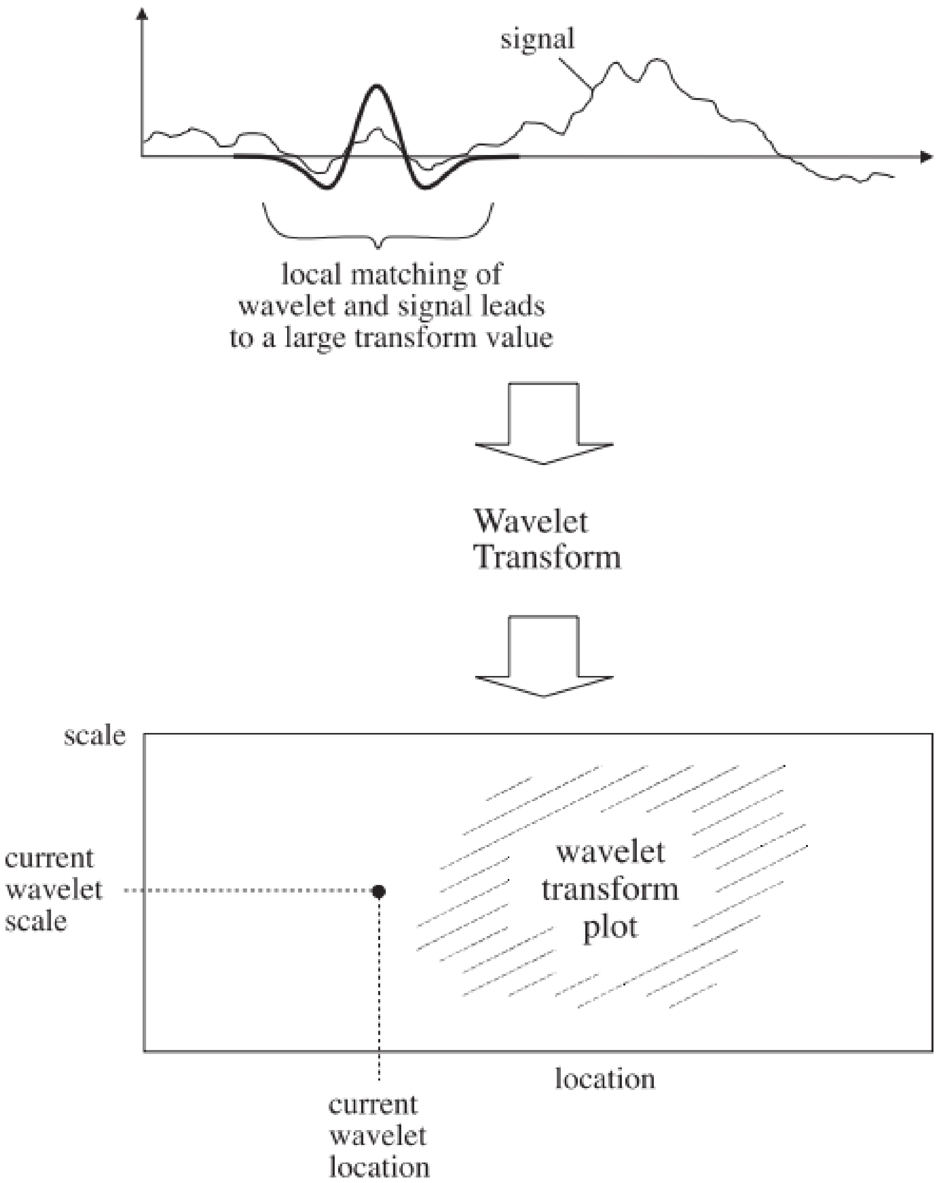
a → Scaling fator (dilatation)

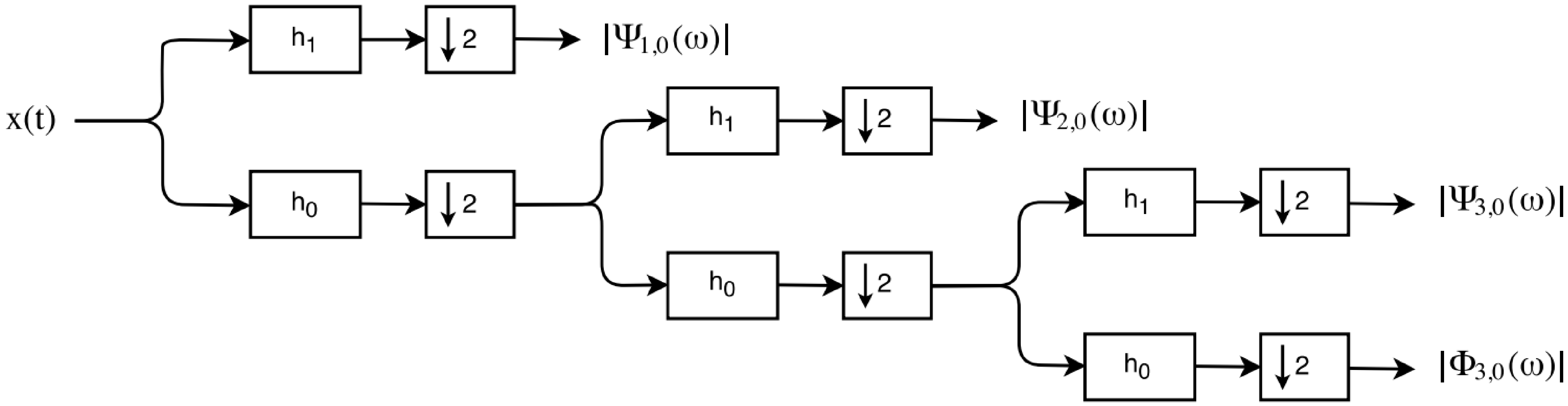
b → Translation factor

$$f(t) = \frac{1}{C} \iint_{-\infty}^{\infty} \frac{1}{|a|^2} W(a,b) \psi_{a,b}\, da\, db$$


1.0
0.5
0
−0.5
−1.0
ψ(t/a)
t
b
a1
a2
a3

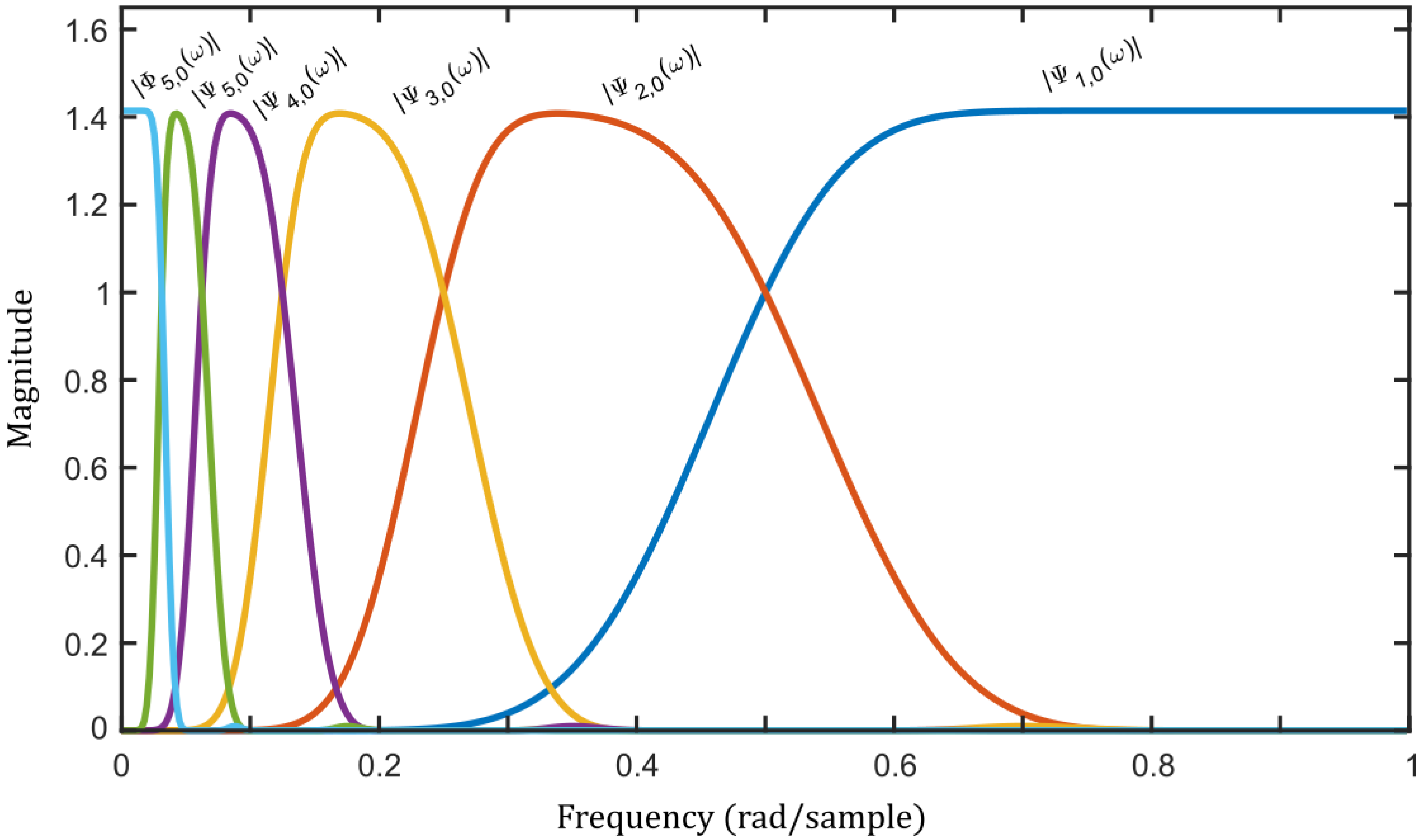

1.0
0.5
0
−0.5
−1.0
ψ(t−b)
t
b1
b2
b3


signal
local matching of
wavelet and signal leads
to a large transform value
Wavelet
Transform
scale
current
wavelet
scale
wavelet
transform
plot
location
current
wavelet
location

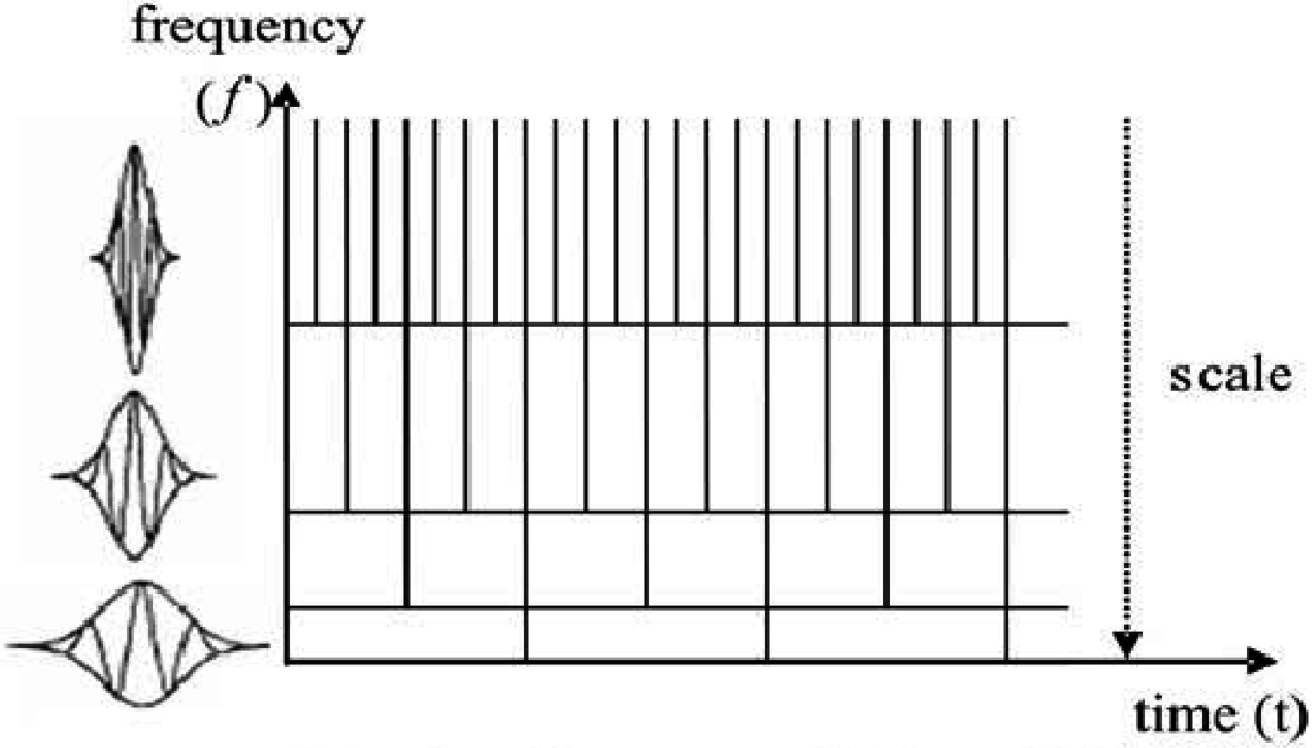


$h_1 \rightarrow highpass\ filter$

$h_0 \rightarrow lowpass\ filter$



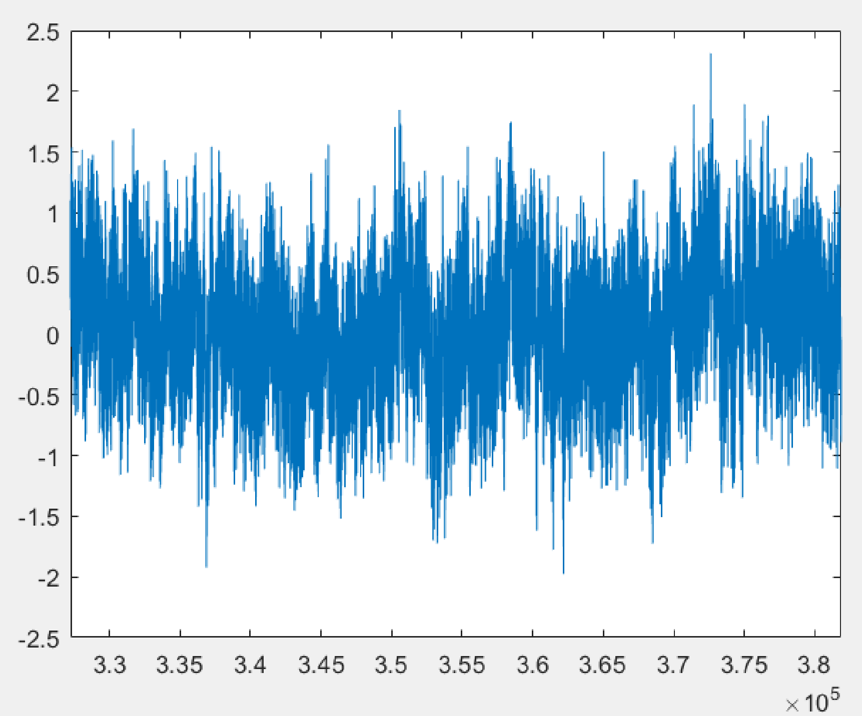
$|\Phi_{5,0}(\omega)|$
$|\Psi_{5,0}(\omega)|$
$|\Psi_{4,0}(\omega)|$
$|\Psi_{3,0}(\omega)|$
$|\Psi_{2,0}(\omega)|$
$|\Psi_{1,0}(\omega)|$
Magnitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Frequency (rad/sample)



The time-frequency window of Wavelet

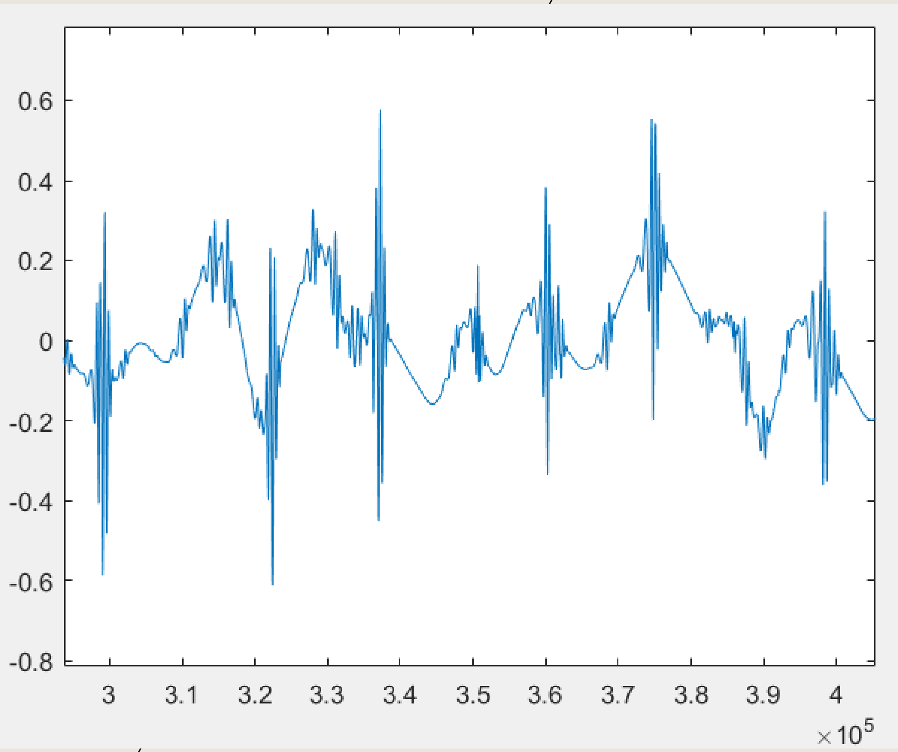
MOTIVATION FOR WAVELETS

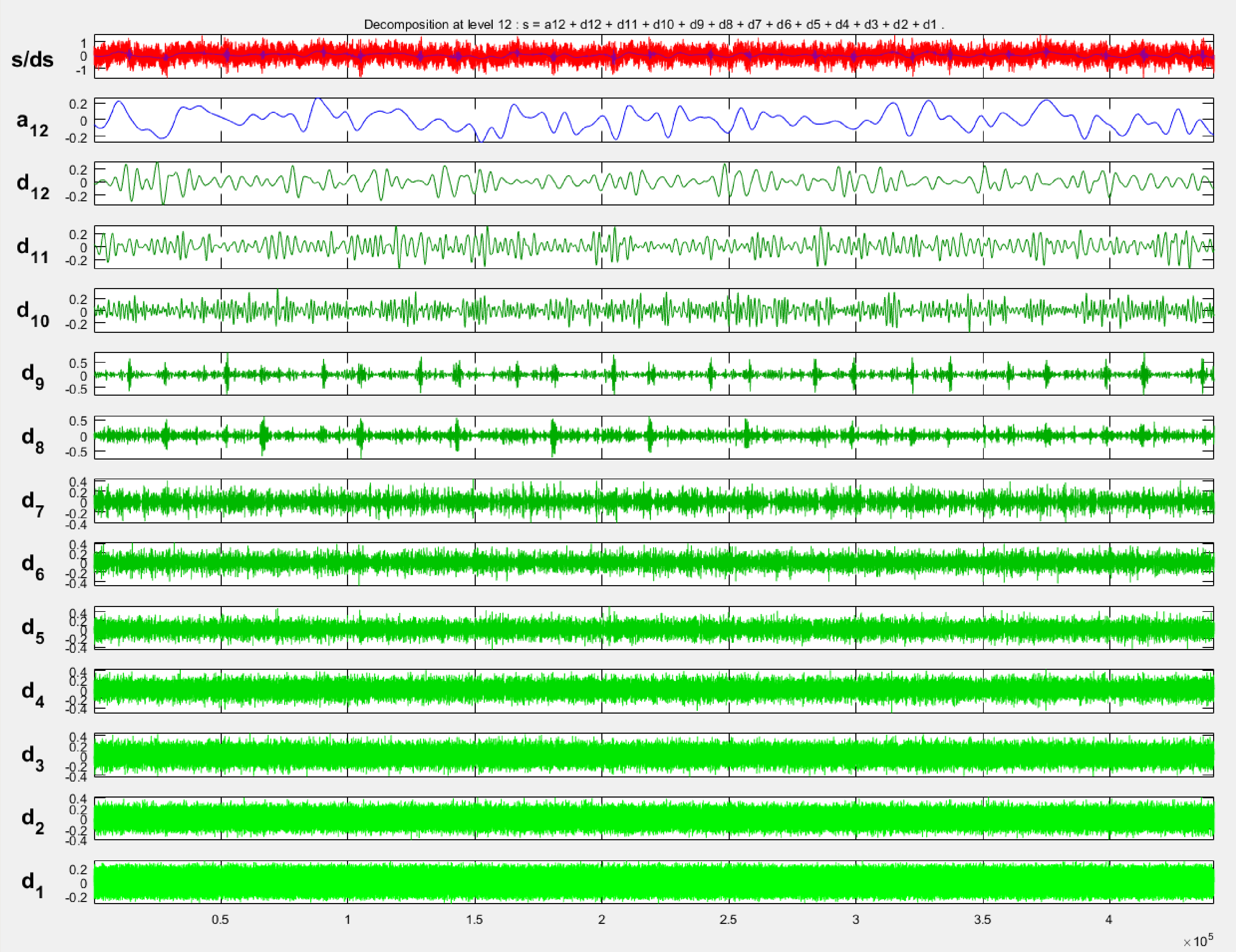
- Denoising biosignals



MOTIVATION FOR WAVELETS

- Denoised





Decomposition at level 12 : s = a12 + d12 + d11 + d10 + d9 + d8 + d7 + d6 + d5 + d4 + d3 + d2 + d1 .
s/ds
a_{12}
d_{12}
d_{11}
d_{10}
d_9
d_8
d_7
d_6
d_5
d_4
d_3
d_2
d_1
0.5
1
1.5
2
2.5
3
3.5
4
$\times 10^5$

THANK YOU!