V/F Control of Induction Machines

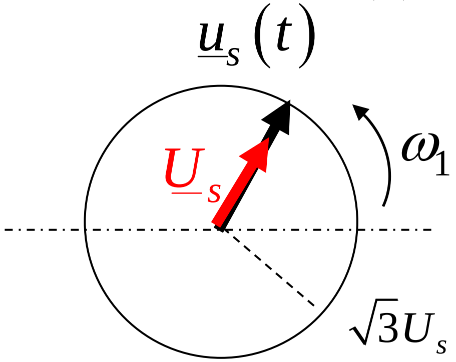
Operation of IM with Sinusoidal Symmetrical 3ϕ Supply

***** Steady State

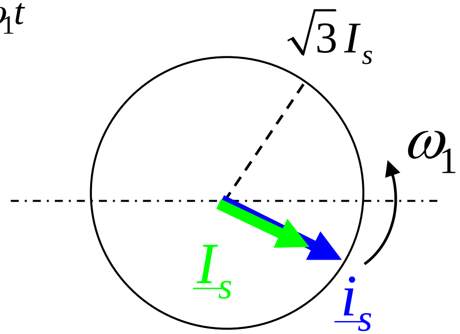
$$\begin{cases} u_{s1}(t) = \sqrt{2}U_s\cos(\omega_1 t+\tau_1) = \dfrac{\sqrt{2}}{2}\left[\underline{U}_s e^{j\omega_1 t} + \underline{U}_s^* e^{-j\omega_1 t}\right] \\ u_{s2}(t) = \sqrt{2}U_s\cos(\omega_1 t+\tau_1-\gamma) = \dfrac{\sqrt{2}}{2}\left[\underline{U}_s e^{j(\omega_1 t-\gamma)} + \underline{U}_s^* e^{-j(\omega_1 t-\gamma)}\right] \\ u_{s2}(t) = \sqrt{2}U_s\cos(\omega_1 t+\tau_1-2\gamma) = \dfrac{\sqrt{2}}{2}\left[\underline{U}_s e^{j(\omega_1 t-2\gamma)} + \underline{U}_s^* e^{-j(\omega_1 t-2\gamma)}\right] \end{cases}$$

$$\vec{u}_s(t) = u_{s1} + u_{s2}e^{j\gamma} + u_{s3}e^{j2\gamma} = \sqrt{3}\underline{U}_s e^{j\omega_1 t}$$

$\underline{U}_s$ แทน Phasor ของ $u_{s1}(t)$, $\underline{U}_s = U_s e^{j\tau_1}$ ในทำนองเดียวกัน



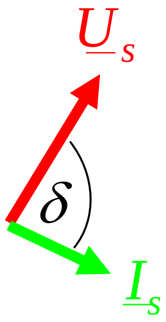
$$\vec{i}_s(t) = \sqrt{3}\,\underline{I}_s e^{j\omega_1 t}$$



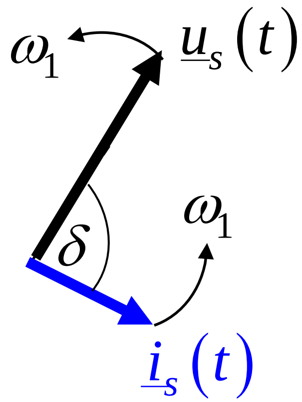
$\omega_1 = 50\ Hz$

$\omega = 45\ Hz$

$\omega_2 = 5\ Hz$



Phasor Diagram



Space Vector (diagram)

$\neq \omega_1$ (stator)

และ
$$\vec{i}_r{}' = \sqrt{3}\,\underline{I}_R e^{j\omega_2 t}$$
$$\vec{u}_r'(t) = \sqrt{3}\underline{U}_R e^{j\omega_2 t}$$

โดยที่ ω_2 : ความถี่ของแรงดัน, กระแสในขดลวดโรเตอร์

θ_r
$$\Rightarrow \quad \vec{i}_r(t) = \vec{i}_r'(t)\cdot e^{j\varepsilon} = \sqrt{3}\underline{I}_R e^{j\omega_2 t}e^{j\omega t}$$
$$= \sqrt{3}\underline{I}_R e^{j\omega_1 t} \quad ; \quad \boxed{\omega_1 = \omega_2 + \omega} \quad ; \quad u_r'(t) = \sqrt{3}\underline{U}_R\, e^{j\omega_1 t}$$

(คิดทางไฟฟ้าทั้งหมด)

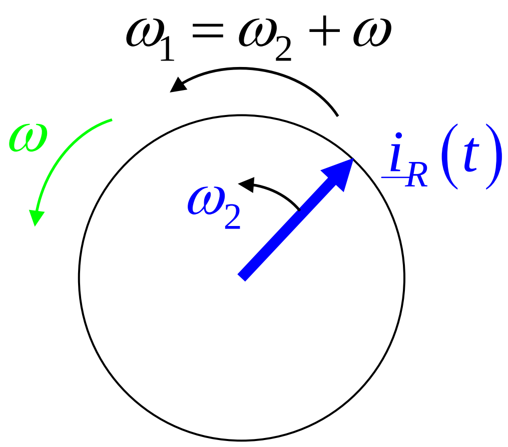
$\therefore$ เมื่อมองจาก Stator Reference Frame

$\vec{i}_s, \vec{u}_s, \vec{i}_r, \vec{u}_r$ จะหมุนด้วยความถี่ ω_1 เท่ากันหมด

$s\omega_1 = \omega_1 - \omega$

$\omega = (1-s)\omega_1$

$(\omega_1) = \omega_{mR} = \omega + \omega_s$



<u>**นิยาม**</u> $slip\ s = \dfrac{\omega_1 - \omega}{\omega_1}$; $Slip\ Frequency\ s\omega_1 = \omega_2$

$s = \dfrac{50-45}{50} = 10\%$

<u>**สมการไฟฟ้าในสภาวะอยู่ตัว**</u>

$$R_s\sqrt{3}\,\underline{I}_s e^{j\omega_1 t} + L_s\frac{d}{dt}\left(\sqrt{3}\,\underline{I}_s e^{j\omega_1 t}\right) + M\frac{d}{dt}\left(\sqrt{3}\,\underline{I}_R e^{j\omega_1 t}\right) = \sqrt{3}\underline{U}_s e^{j\omega_1 t}$$

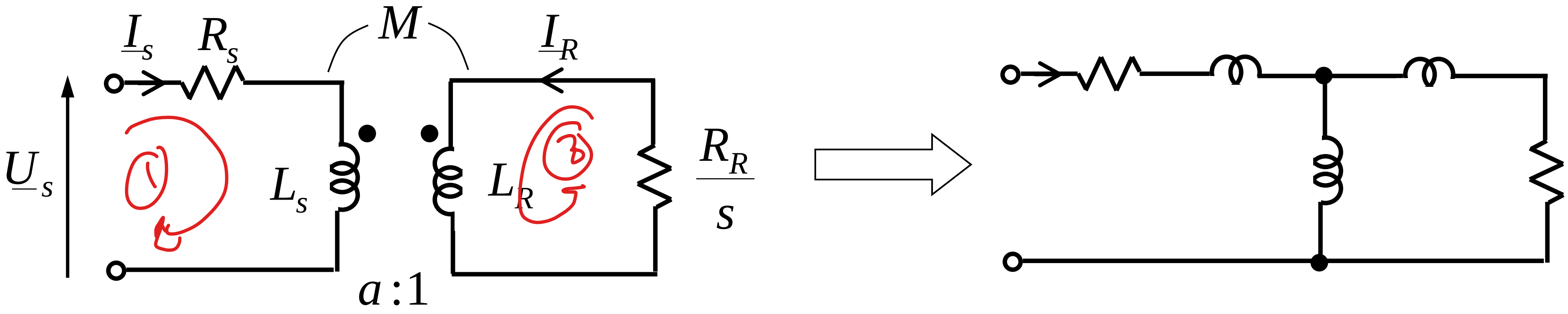
$$\boxed{R_s\,\underline{I}_s + j\omega_1 L_s \underline{I}_s + j\omega_1 M \underline{I}_R = \underline{U}_s \qquad (1)}$$

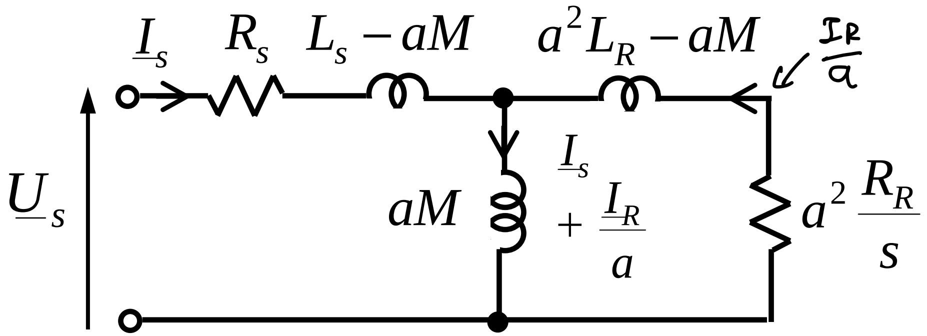
$$R_R\sqrt{3}\underline{I}_R e^{j\omega_1 t} + \left(\frac{d}{dt} - j\omega\right)\left[L_R\sqrt{3}\underline{I}_R e^{j\omega_1 t} + M\sqrt{3}\,\underline{I}_s e^{j\omega_1 t}\right] = \sqrt{3}\underline{U}_R e^{j\omega_1 t}$$

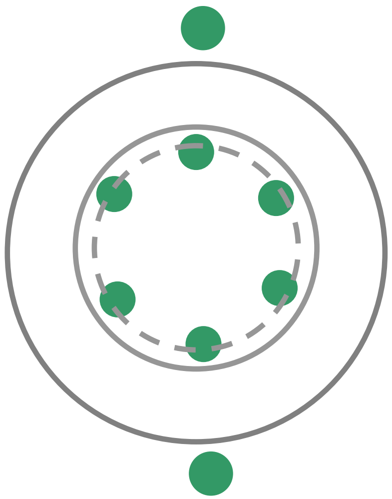
$$\boxed{R_R\,\underline{I}_R + j\underbrace{(\omega_1 - \omega)}_{\omega_2 = s\omega_1}(L_R\underline{I}_R + M\underline{I}_S) = \underline{U}_R \qquad (2)}$$

ในที่นี้เราจะพิจารณากรณีที่ $\underline{U}_R = 0$

$$\therefore \boxed{\frac{R_R}{s}\underline{I}_R + j\omega_1 L_R \underline{I}_R + j\omega_1 M \underline{I}_S = 0} \quad (3)$$

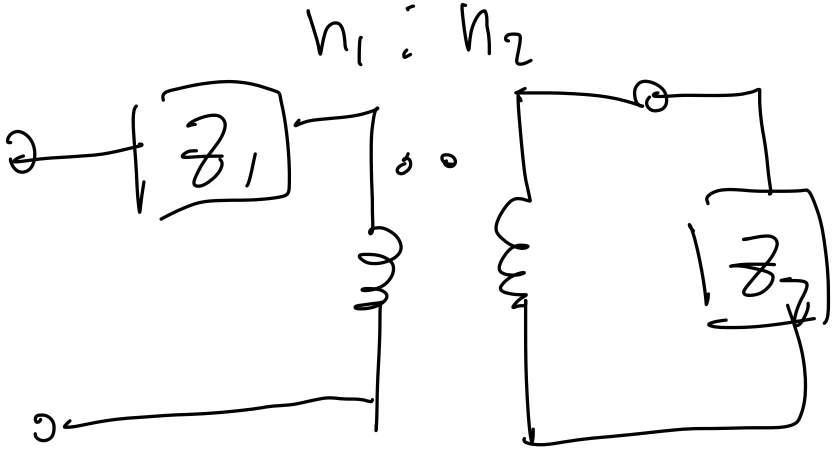






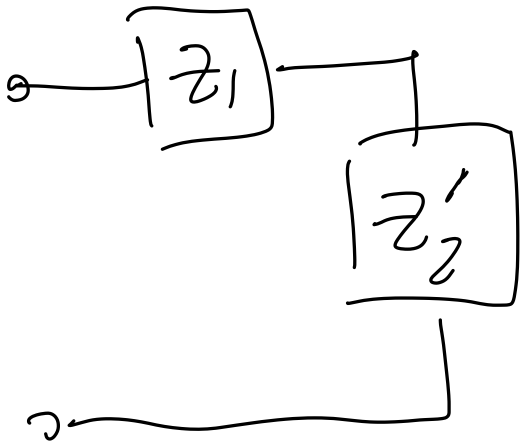
$\Rightarrow a$ เป็นอะไรก็ได้ $\because$ สมการไฟฟ้ายังคงเดิม

a : ไม่รู้ Turn Ratio จริงๆ



$a : 1$

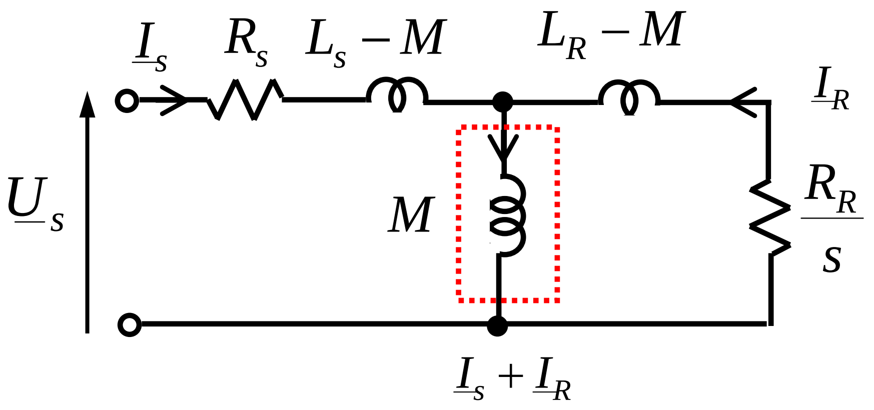
|||



$$= \left(\frac{n_1}{n_2}\right)^2 Z_2$$

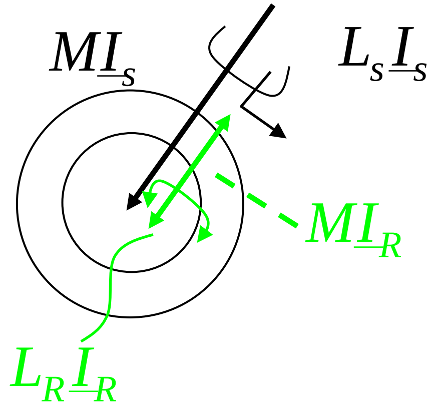
$$= a^2 Z_2$$

ก) $a = 1$; Airgap-Flux-Based Equivalent Circuit

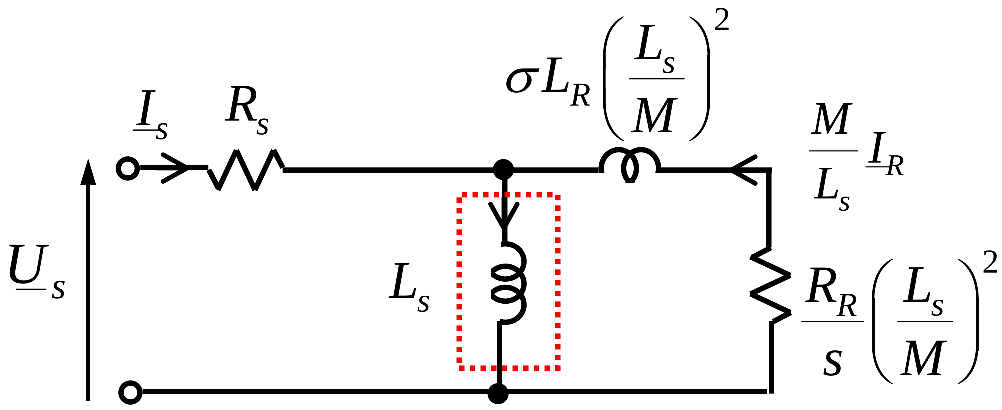


Flux $M\left(\underline{I}_s + \underline{I}_R\right)$ = Airgap Flux

$$L_s - M = \sigma_s M$$
$$L_R - M = \sigma_R M$$



ข) $a = L_s/M$; Stator-Flux-Based Equivalent Circuit



Flux $L_s\left(\underline{I}_s + \frac{M}{L_s}\underline{I}_R\right) = L_s\underline{I}_s + M\underline{I}_R$

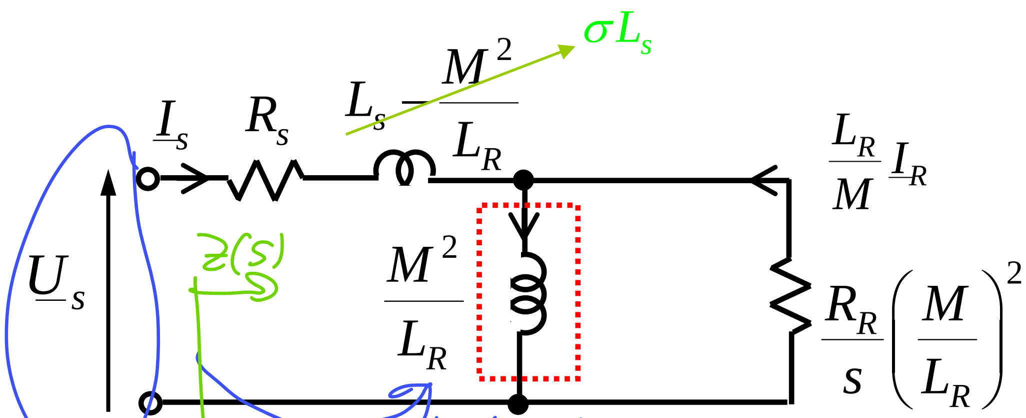
Stator Flux

$\sigma = 1 - \frac{M^2}{L_s L_R}$: Total Leakage Coefficient

$\neq \sigma_s, \sigma_R$

สัมประสิทธิ์ความรั่วไหลรวม

ค) $a = M / L_R$; Rotor-Flux-Based Equivalent Circuit



control Flux → keep constant.

Flux

$$\frac{M^2}{L_R}\left(\underline{I}_s + \frac{L_R}{M}\underline{I}_R\right)$$

$$= \frac{M}{L_R}\underline{\left(M\underline{I}_s + L_R\underline{I}_R\right)}$$

Rotor Flux

<u>Stator Impedance $Z(s)$</u> ⇒ ง) stator-flux based

$$R_R' = R_R\left(\frac{L_s}{M}\right)^2 ; \quad L_R' = L_R\left(\frac{L_s}{M}\right)^2$$

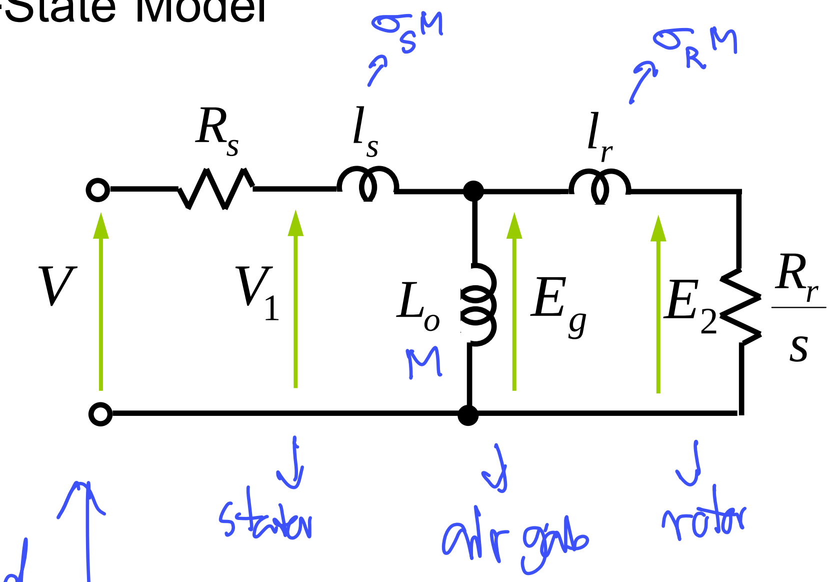
$$Z(s) = R_s + j\omega_1 L_s // \left(\frac{R_R'}{s} + j\omega_1\sigma L_R'\right)$$

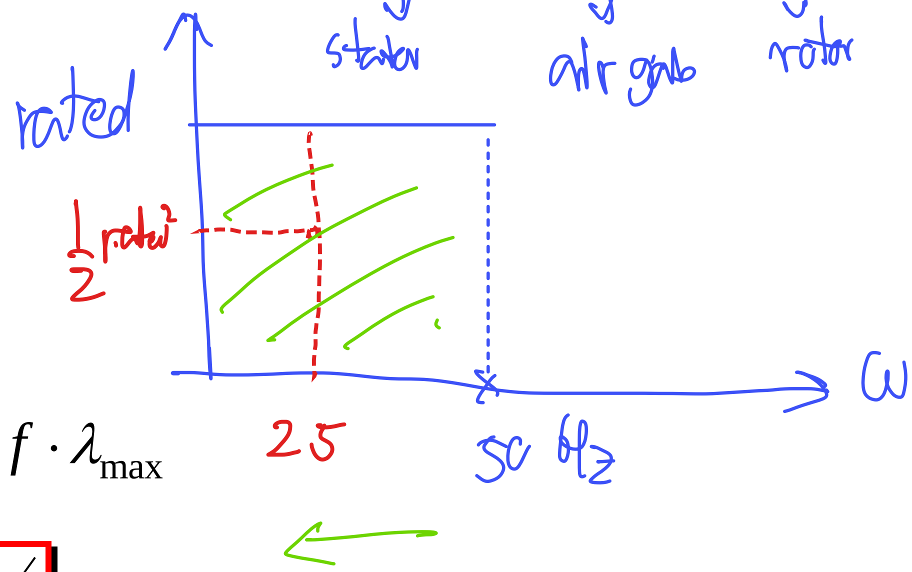
$$\frac{\underline{U}_s}{\underline{I}_s} = R_s + \frac{j\omega_1 L_s \cdot \left(R_R'/s + j\omega_1\sigma L_R'\right)}{R_R'/s + j\omega_1\left(L_s + \sigma L_R'\right)} = R_s + j\omega_1 L_s \cdot \frac{R_R'/s + j\omega_1\sigma L_R'}{R_R'/s + j\omega_1 L_R'}$$

(1) กรณีที่เป็น Voltage Source และใช้ Steady-State Model

<u>แนวคิด</u>

- รักษา flux ใน machine ให้มีค่าคงที่
  - มากเกินไป... เกิด Saturation
  - น้อยเกินไป... แรงบิดต่ำ ไม่ดี
- รักษา Torque-Speed Curve ให้ดีเหมือนเดิม
- ความเร็ว ~ frequency





Induced E.M.F $e = -d\lambda/dt \rightarrow E_{rms} = \sqrt{2}\pi f \cdot \lambda_{max}$

$$\boxed{\lambda_{max} \propto E_{rms}/f}$$

a) V/f มีค่าคงที่

$$Z(j\omega) = R_s + j\omega_1 l_s + j\omega_1 L_o // Z_2$$

$$Z_2(j\omega_1) = \frac{R_r}{s} + j\omega_1 l_r$$

$$j\omega_1 L_o // Z_2 = \frac{j\omega_1 L_o\left(\frac{R_r}{s} + j\omega_1 l_r\right)}{\frac{R_r}{s} + j\omega_1 (L_o + l_r)}$$

$$= \frac{j\omega_1 L_o\left(\frac{R_r}{s} + j\omega_1 l_r\right)\left(\frac{R_r}{s} - j\omega_1 L_R\right)}{\left(\frac{R_r}{s}\right)^2 + (\omega_1 L_R)^2}$$

$(\omega_s = s \cdot \omega_1)$

$$= \omega_1\left\{\frac{R_r}{\omega_s}\cdot\frac{\omega_s L_o}{R_r^2 + (\omega_s L_R)^2} + jL_o\frac{R_r^2 + \omega_s^2 l_r L_R}{R_r^2 + (\omega_s L_R)^2}\right\}$$

$$Z(j\omega_1) = R_s + \omega_1 \overbrace{\left\{\frac{R_r}{\omega_s}\cdot\frac{(\omega_s L_o)^2}{R_r^2 + (\omega_s L_R)^2}\right\}}^{A} + j\omega_1 \underbrace{\left\{l_s + L_o\cdot\frac{R_r^2 + \omega_s^2 l_r L_R}{R_r^2 + (\omega_s L_R)^2}\right\}}_{B}$$

$$= R_s + \omega_1 A + j\omega_1 B$$

$$\therefore \underline{U}_s \underline{I}_s^* = Z(j\omega_1)\cdot\underline{I}_s\cdot\underline{I}_s^*$$

$$= R_s \underline{I}_s \underline{I}_s^* + \omega_1 A \underline{I}_s \underline{I}_s^* + j\omega_1 B \underline{I}_s \underline{I}_s^*$$

copper loss $I_s^2 R_s$

Q Reactive Power

Airgap Power

$$P_{ag} = 3\times\omega_1 A I_s^2$$

$P_{ag} = T_e \cdot \omega_{syn}$

$= T_e\left(\frac{2}{P}\cdot\omega_m\right)$

$$T_e = m_M = \frac{3\times\omega_1 A I_s^2}{\left(\frac{2}{P}\right)\omega_1} = \left(\frac{P}{2}\right)\cdot 3AI_s^2$$

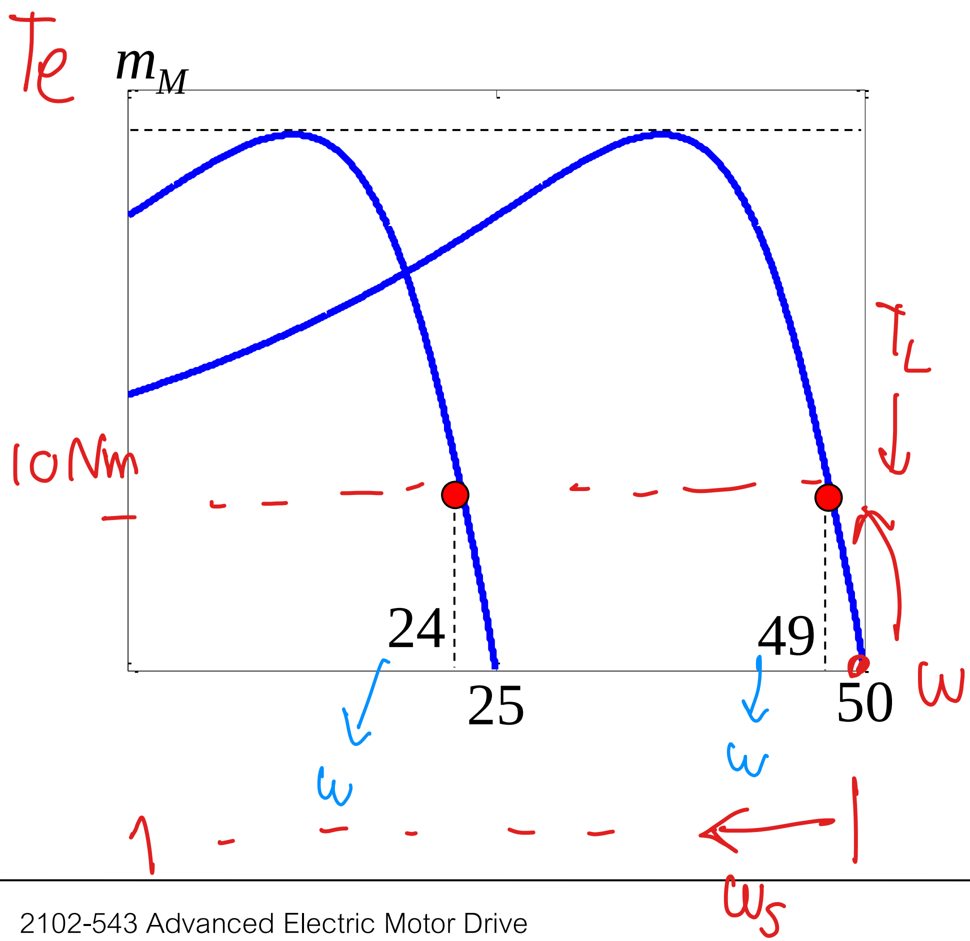
$P_{mech} = T_e \cdot \omega_m$

$= T_e\left(\frac{2}{P}\cdot\omega\right)$

$$I_s = \frac{U_s}{Z(j\omega)}$$

$$\boxed{m_M = \left(\frac{U_s}{\omega_1}\right)^2 \times \frac{3P}{2} \times \frac{A}{\left(\frac{R_s}{\omega_1} + A\right)^2 + B^2}}$$

$\frac{V}{f}$ คงที่ $\Rightarrow$ $\frac{U_s}{\omega_1}$: คงที่



$\frac{P}{2}$

ω หรือ ω_m

ถ้า $R_s = 0$

ω_s : Slip Frequency

$U_s \approx 220\,V$

$\omega_1 = 50\ Hz$ $\omega_s = 1\ Hz$

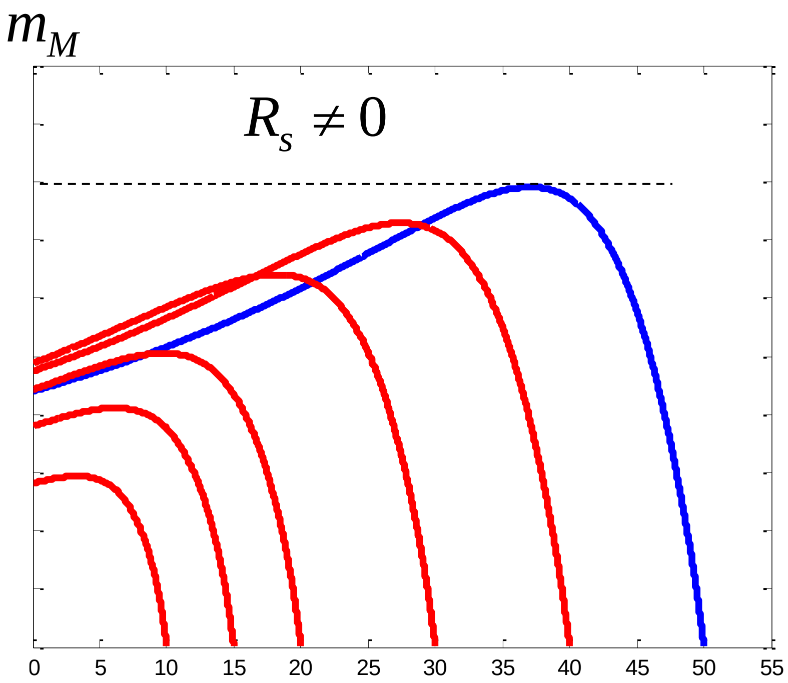
$\longrightarrow m_M = 10\ N \cdot m$

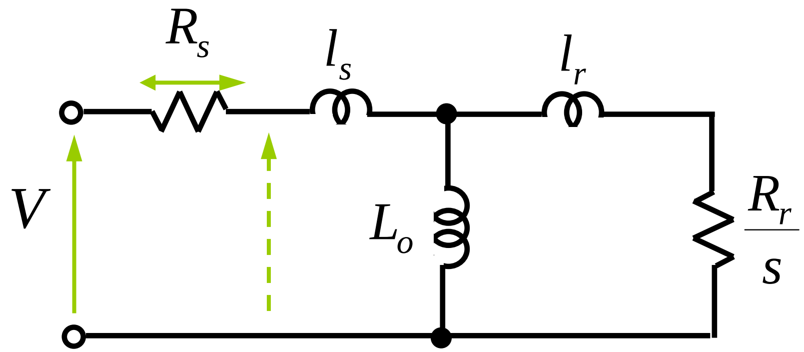
V/f คงที่; $R_s = 0$

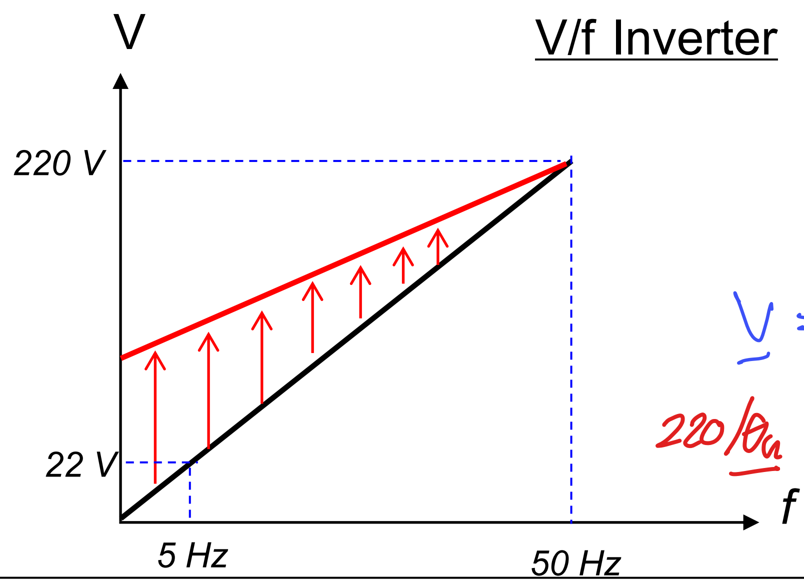
$U_s \approx 110\,V$

$\omega_1 = 25\ Hz$ $\omega_s = 1\ Hz$

$\longrightarrow m_M = 10\ N \cdot m$







Ex IM 380V/50Hz

$I_s = 5A;\ \ R_s = 1\Omega \qquad \therefore I_s R_s = 5V$

50 Hz → V= 220 V (per phase) → $V_1 \approx V$

25 Hz → V= 110 V → $V_1 \approx V$

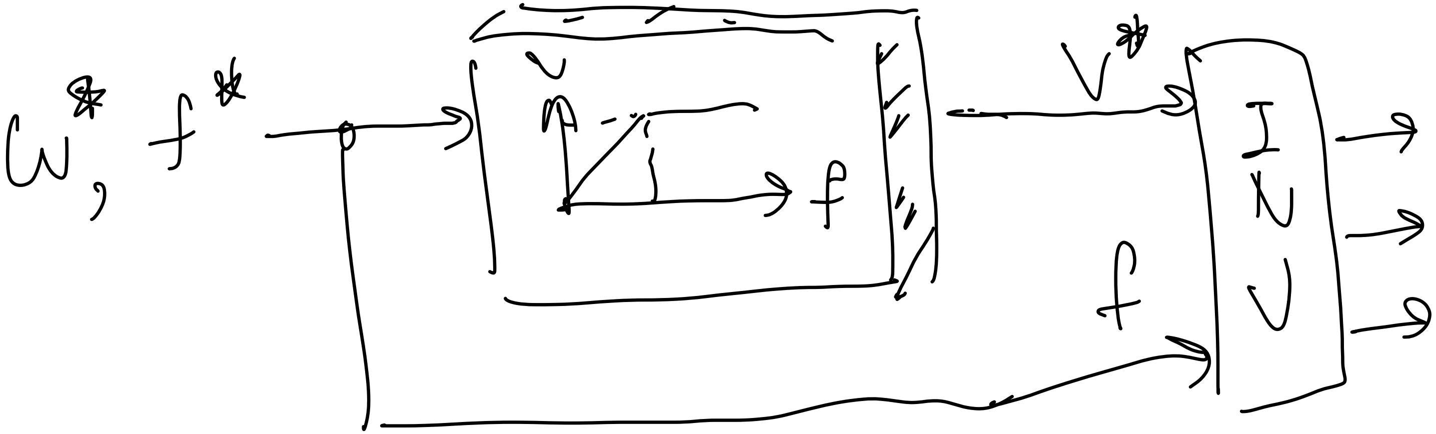
5 Hz → V= 22 V → ✗ ✗

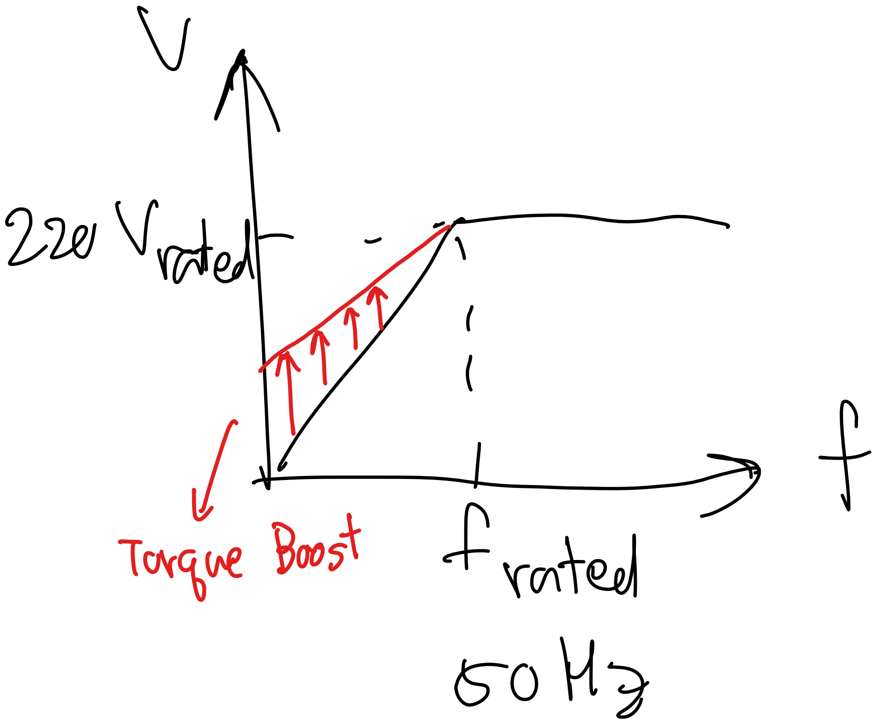
1 Hz → V= 4.4 V → ✗ ✗ ✗

$V = R_s I_s + V_1$

$220\angle\theta_u = 5\angle\theta_I + V_1$

V/f pattern





$$V_1(t) = \sqrt{2}\, V^* \cos(2\pi f^* t)$$

$$V_2(t) = \text{——————} - 120^\circ)$$

$$V_3(t) = \text{——————} - 240^\circ)$$

b) ควบคุม V_1/f คงที่

V_1 : สัมพันธ์กับ Stator flux λ_s

$$V_1 = \omega_1 \lambda_s$$

$$\frac{V_1}{f} = 2\pi\cdot\lambda_s$$

⇒ รักษาขนาดของ Stator flux ให้คงที่ (แม้ว่า ω_1 ↗)

$$\underline{V}_1 = \omega_1\left(A+jB\right)\underline{I}_s$$

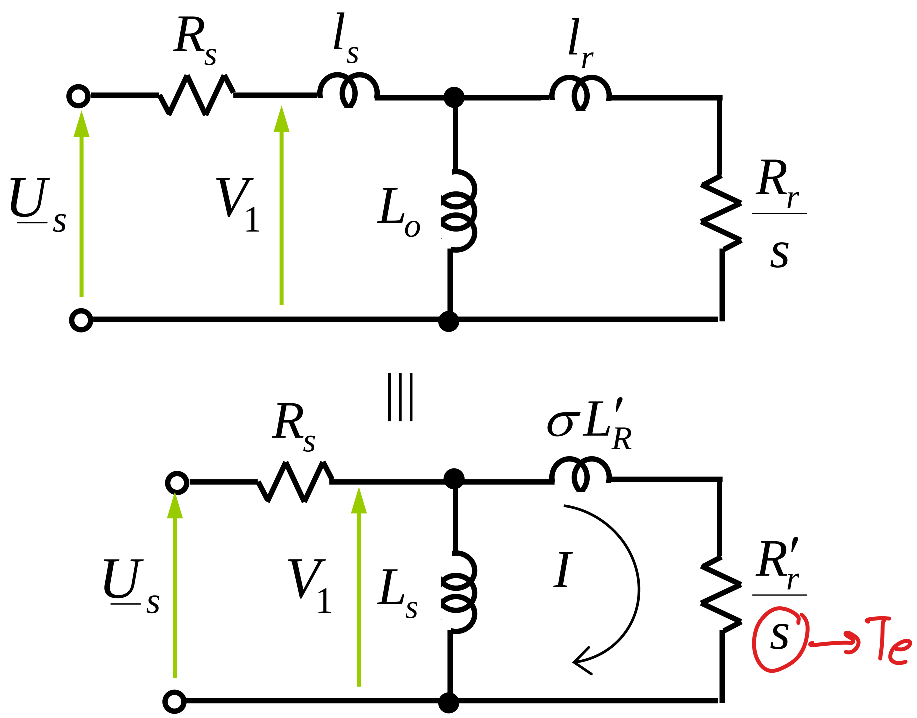
$\left(\underline{U}_s - R_s\underline{I}_s\right)$

$Z = R_s + \omega_1 A + j\omega B$

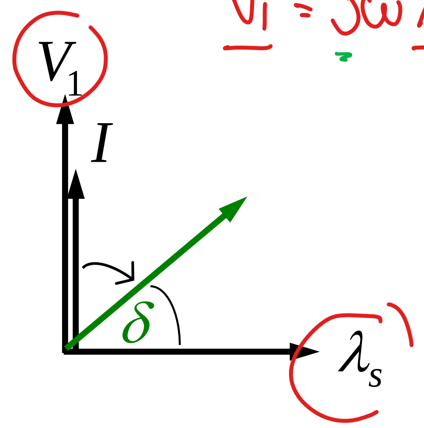
$$m_M = \frac{P_g}{\frac{2}{p}\omega_1} = \frac{3\times\omega_1 A I_s^2}{\frac{2}{p}\omega_1}$$

$$= \frac{3}{2}p\cdot A\cdot\frac{V_1^2}{\omega_1^2\left(A^2+B^2\right)}$$

no R_s !!

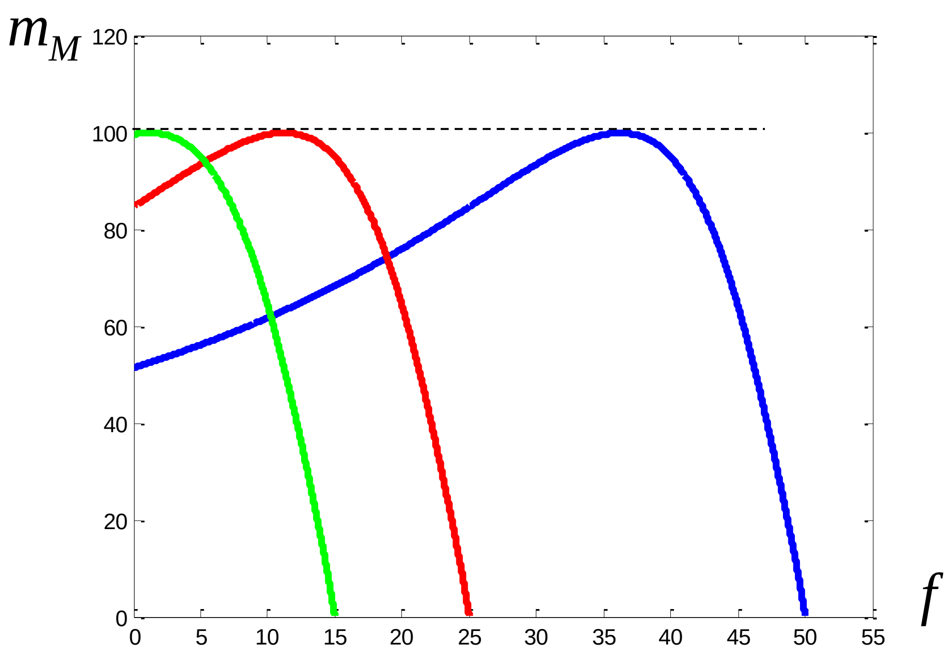


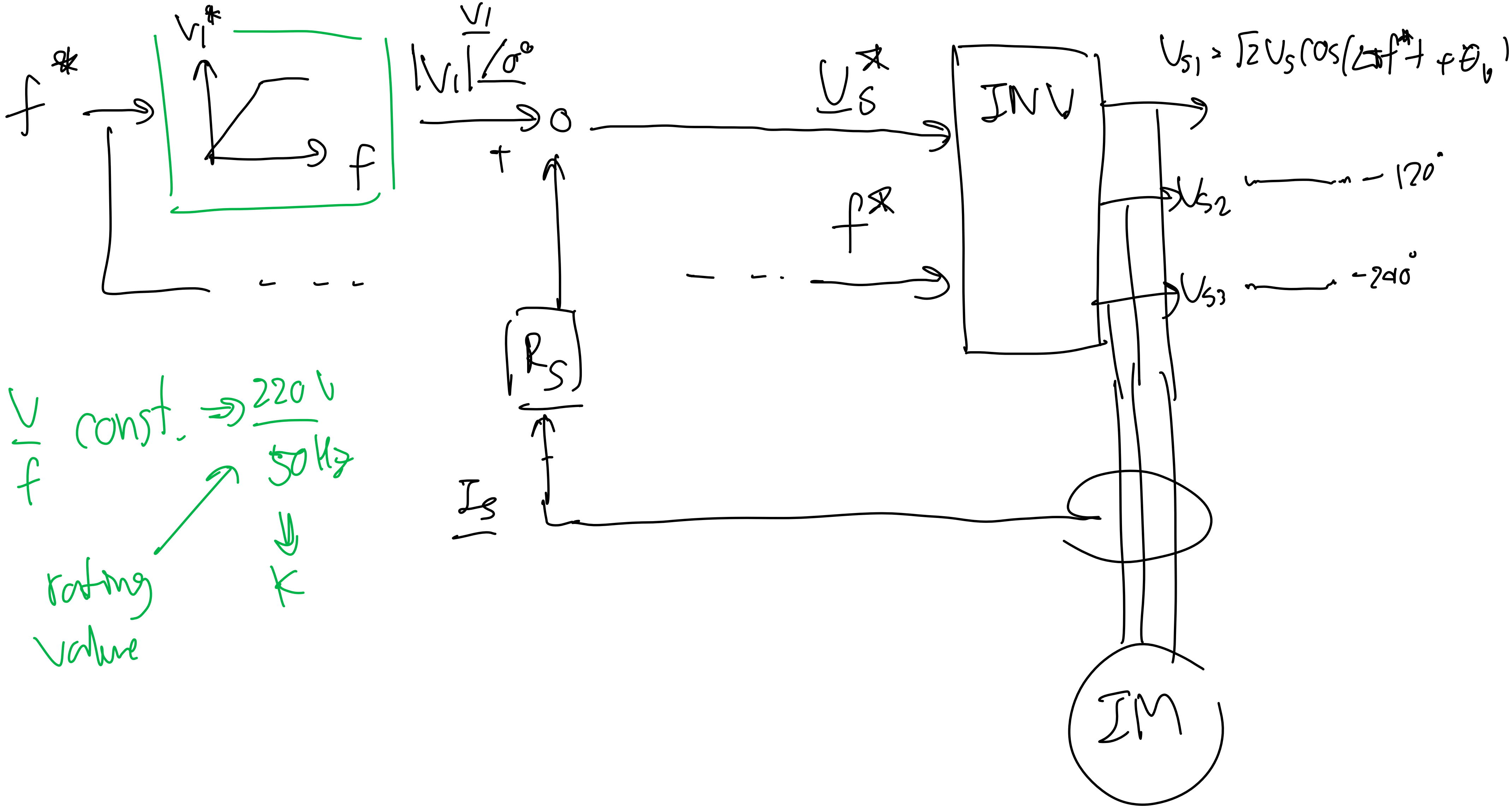
$\underline{V_1} = j\omega\underline{\lambda_s}$



no load
Te = 0 → s = 0
↓
I = 0

$$m_M = \frac{3}{2}p\cdot\underbrace{\frac{V_1^2}{\omega_1^2}}_{\text{คงที่}}\cdot\underbrace{\frac{A}{\left(A^2+B^2\right)}}_{\omega_s}$$

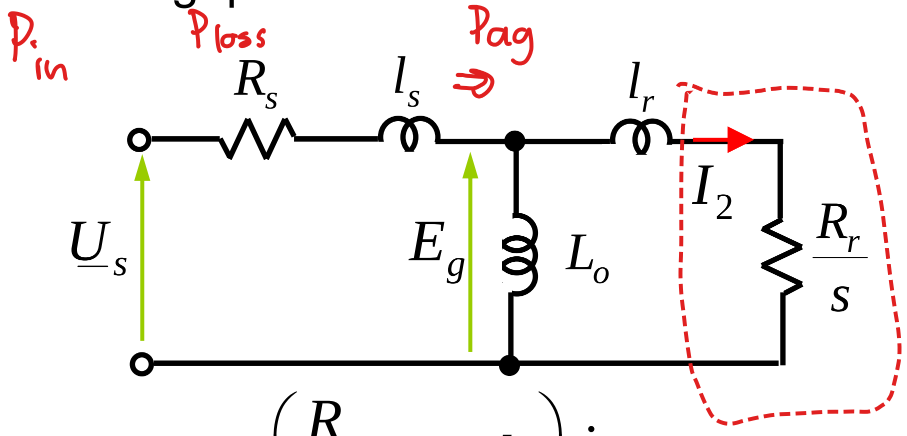




f*
V1*
f
|V1|∠0°
V1
+
+
Us*
f*
INV
Us1 = √2 Us cos(2πf* t + θ0)
Us2 — -120°
Us3 — -240°
Rs
Is
IM
V/f const ⇒ 220V/50Hz
⇓
K
rating value

c) ควบคุม E_g / f คงที่

Airgap flux จะมีขนาดคงที่



$$\underline{E}_g = \left(\frac{R_r}{s} + j\omega_1 l_r \right) \dot{\underline{I}}_2$$

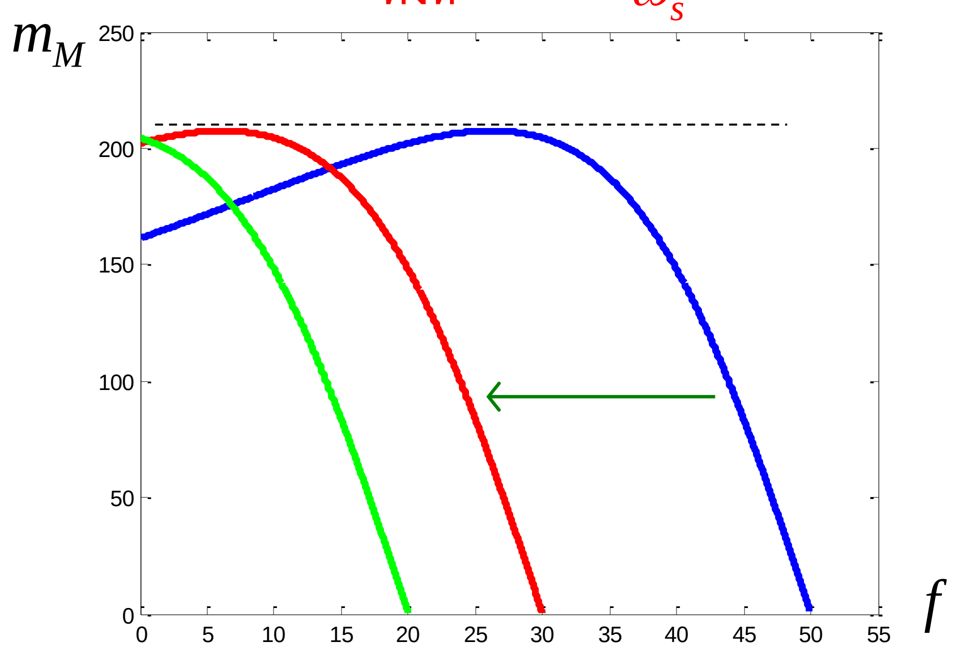
$U_s - R_s I_s - j\omega_1 l_s I_s$

$$m_M = \frac{P_g}{\frac{2}{p}\omega_1} = \frac{3 \times \boxed{\frac{R_r}{s} \times I_2^2}}{\frac{2}{p}\omega_1}$$

$$= \frac{3}{2} p \cdot \frac{R_r}{\omega_s} \cdot \frac{E_g^2}{\left(\frac{R_r}{s}\right)^2 + (\omega_1 l_r)^2}$$

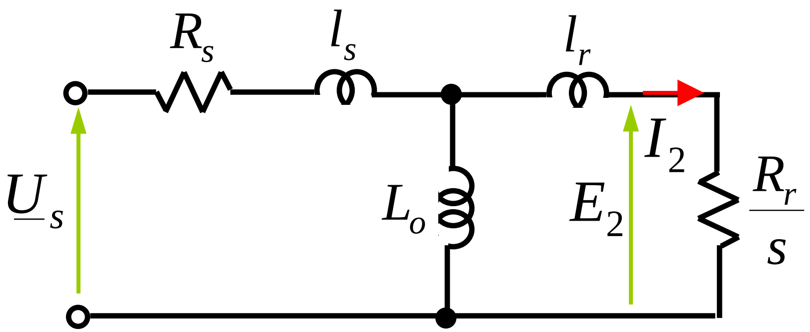
$$= \frac{3}{2} p \cdot \left(\frac{E_g}{\omega_1}\right)^2 \cdot \frac{R_r / \omega_s}{\left(\frac{R_r}{\omega_s}\right)^2 + l_r^2}$$

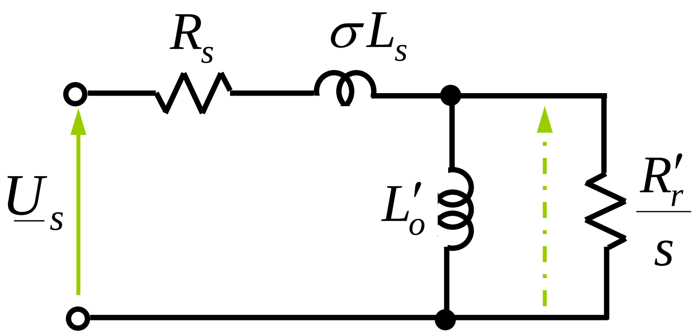
$$m_M = \frac{3}{2} p \cdot \underbrace{\left(\frac{E_g}{\omega_1}\right)^2}_{\text{คงที่}} \frac{\omega_s \cdot R_r}{\underbrace{R_r^2 + (\omega_s l_r)^2}_{\omega_s}}$$



d) ควบคุม E_2/f คงที่

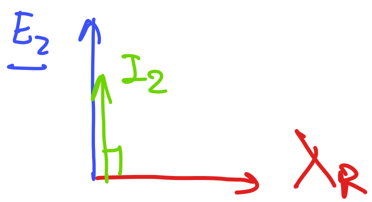
ขนาด Rotor flux คงที่





$$\underline{E}_2 = \frac{R_r}{s} \cdot \underline{I}_2$$

$$\left(\begin{array}{ll} \underline{E}_2 = j\omega_1 \underline{\lambda}_R \\ \therefore \quad \underline{I}_2 \perp \underline{\lambda}_R \end{array} \right)$$

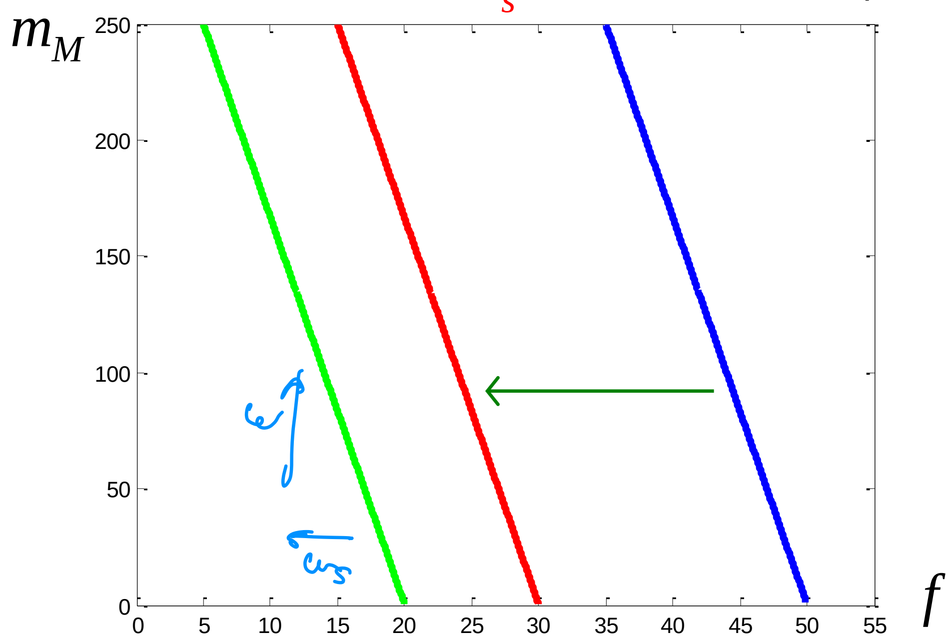


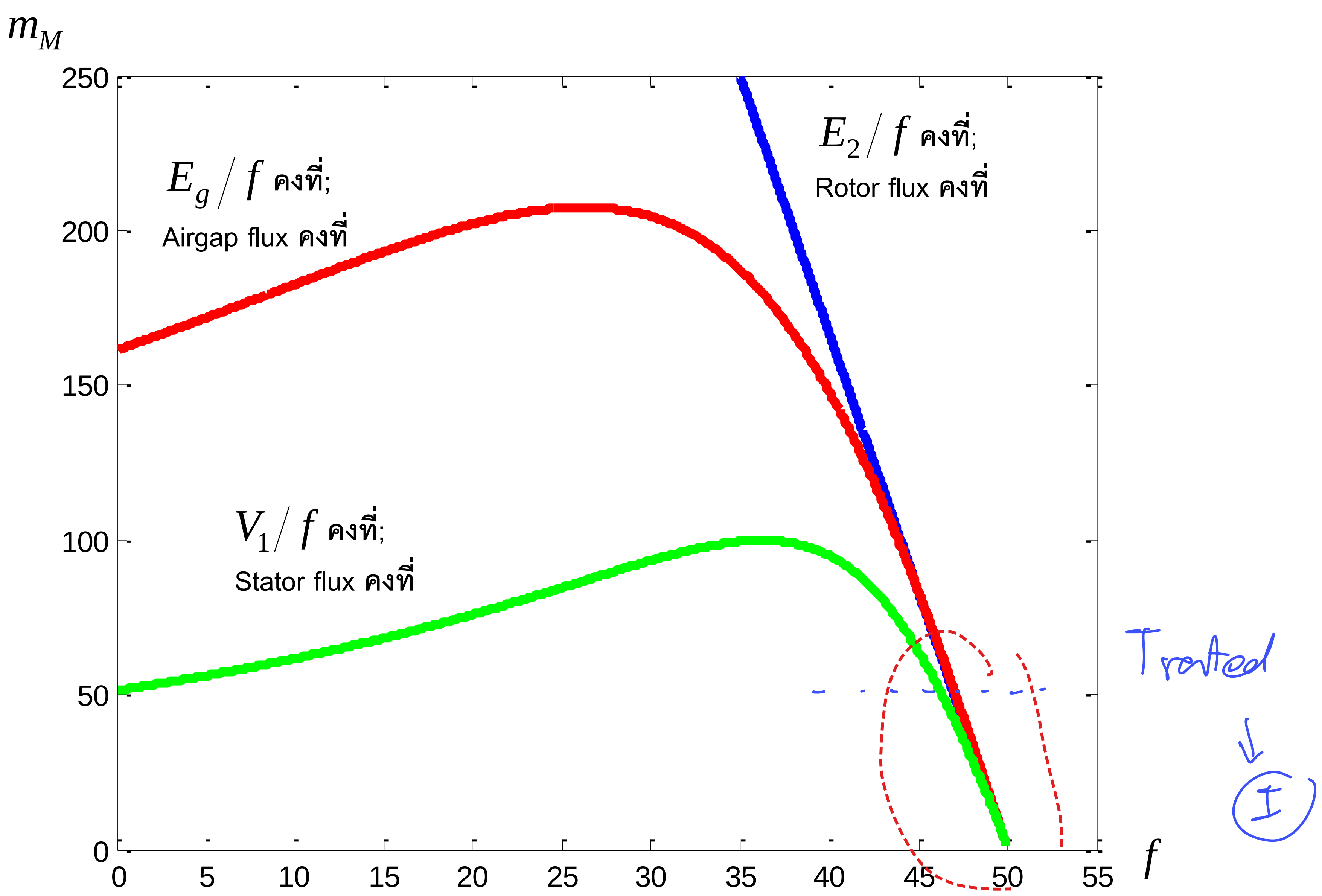
$$m_M = \frac{3 \times \frac{R_r}{s} \cdot I_2^2}{\frac{2}{p}\omega_1} = \frac{3}{2} p \cdot \frac{R_r}{s} \cdot \frac{E_2^2}{\omega_1 \left(\frac{R_r}{s} \right)^2}$$

$$= \frac{3}{2} p \cdot \underbrace{\left(\frac{E_2}{\omega_1} \right)^2}_{\text{คงที่}} \underbrace{\frac{\omega_s}{R_r}}_{\omega_s}$$

: linear !!!

- ไม่มีค่าจำกัดจาก Maximum Torque

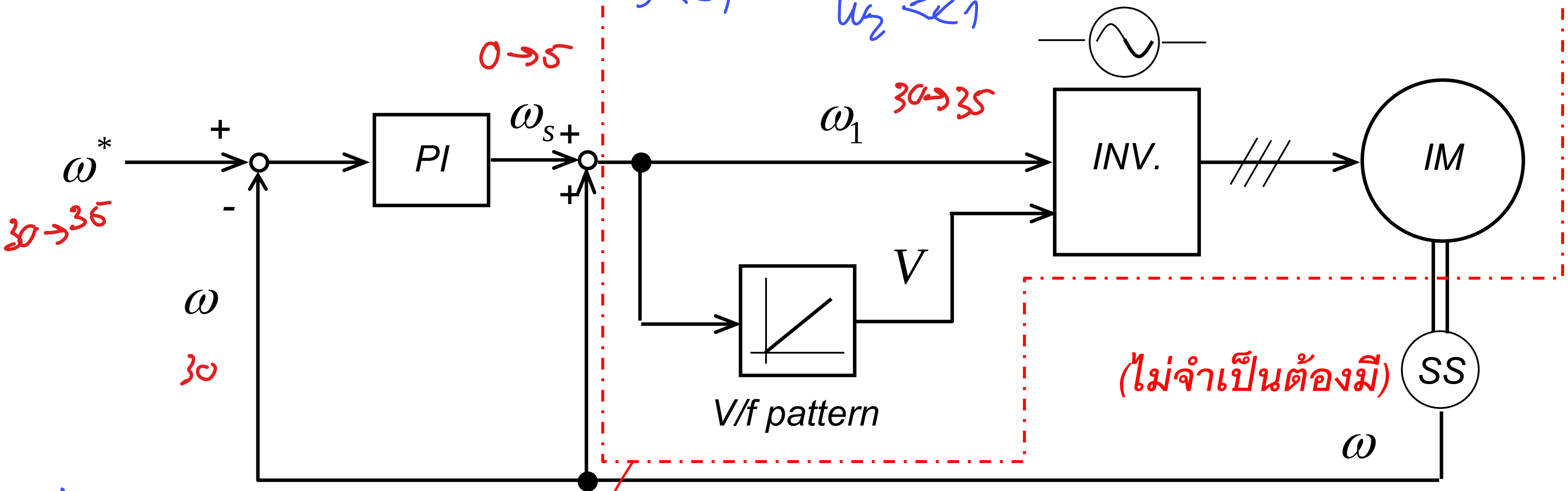



m_M
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40
45
50
55
f
E_g / f คงที่;
Airgap flux คงที่
E_2 / f คงที่;
Rotor flux คงที่
V_1 / f คงที่;
Stator flux คงที่
Trated
I

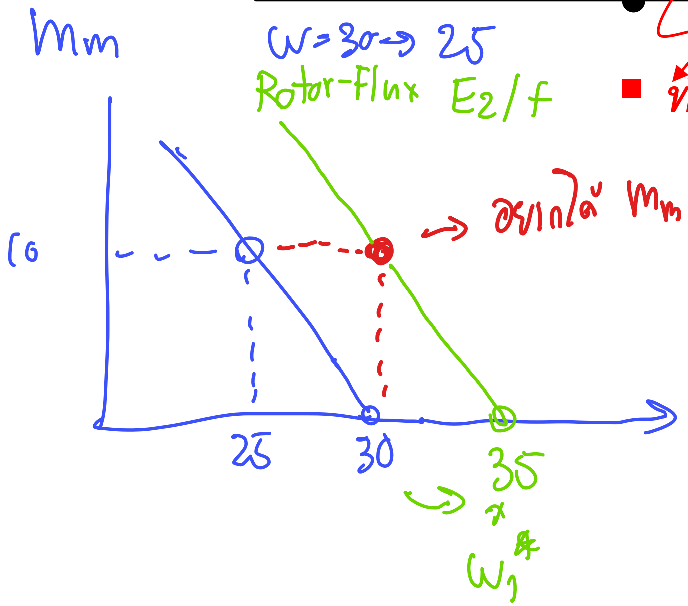
$M_m = 0 \Rightarrow \omega = \omega_1 \Rightarrow s = 0 \Rightarrow \omega_2 = 0$

$M_m \neq 0 \Rightarrow \omega \neq \omega_1 \Rightarrow s \neq 0$ $\quad \omega_2 \neq 0$

$s \ll 1$ $\quad \omega_2 \ll 1$



■ ทำงานเป็น open loop เฉพาะส่วนนี้ก็ได้โดยคิดว่า $\omega \approx \omega_1$



→ อยากได้ M_m = 10 Nm at ω = 30