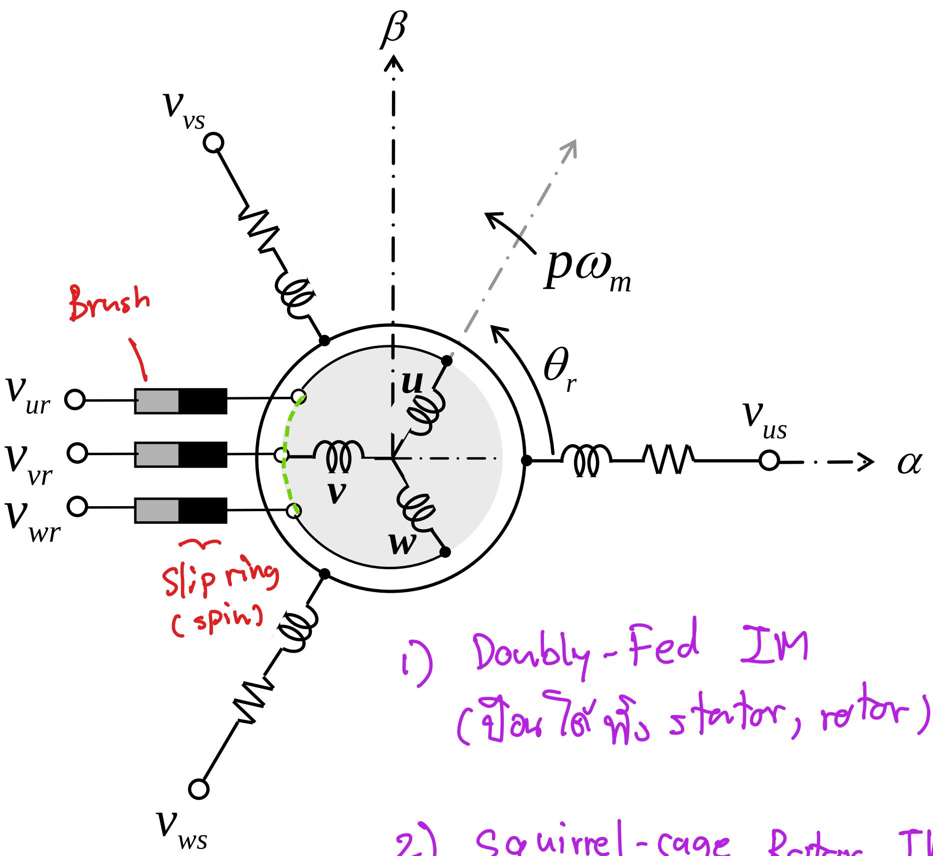
Chapter 6

Modeling and Control of Induction Machines; IMs

Modeling of Induction Motor



1) Doubly-Fed IM
(ป้อนไฟทั้ง stator, rotor)

2) Squirrel-cage Rotor IM
(short-circuit @ rotor)

Stator Equation on Stator Ref. Frame:

$$\begin{bmatrix} v_{us} \\ v_{vs} \\ v_{ws} \end{bmatrix} = R_s \begin{bmatrix} i_{us} \\ i_{vs} \\ i_{ws} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_{us} \\ \psi_{vs} \\ \psi_{ws} \end{bmatrix}$$

Rotor Equation on Rotor Ref. Frame:

$$\begin{bmatrix} v'_{ur} \\ v'_{vr} \\ v'_{wr} \end{bmatrix} = R_r \begin{bmatrix} i'_{ur} \\ i'_{vr} \\ i'_{wr} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi'_{ur} \\ \psi'_{vr} \\ \psi'_{wr} \end{bmatrix}$$

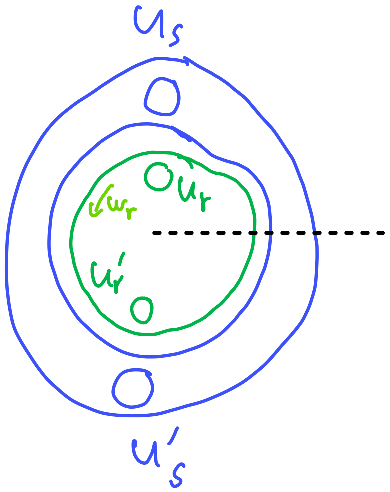
θ_r = elec angle

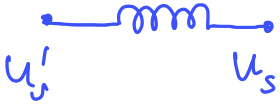
Stator Flux Linkage

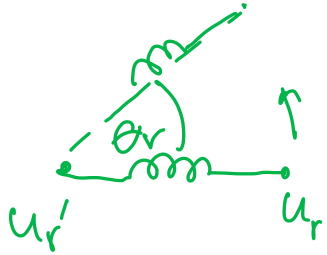
$$\begin{bmatrix}\psi_{us}\\ \psi_{vs}\\ \psi_{ws}\end{bmatrix}=\underbrace{\begin{bmatrix} l_s+M' & -\frac{1}{2}M' & -\frac{1}{2}M' \\ -\frac{1}{2}M' & l_s+M' & -\frac{1}{2}M' \\ -\frac{1}{2}M' & -\frac{1}{2}M' & l_s+M' \end{bmatrix}}_{\mathbb{L}_s'}\begin{bmatrix}i_{us}\\ i_{vs}\\ i_{ws}\end{bmatrix}+\underbrace{\begin{bmatrix} M'\cos\theta_r & M'\cos\left(\theta_r+\frac{2\pi}{3}\right) & M'\cos\left(\theta_r-\frac{2\pi}{3}\right) \\ M'\cos\left(\theta_r-\frac{2\pi}{3}\right) & M'\cos\theta_r & M'\cos\left(\theta_r+\frac{2\pi}{3}\right) \\ M'\cos\left(\theta_r+\frac{2\pi}{3}\right) & M'\cos\left(\theta_r-\frac{2\pi}{3}\right) & M'\cos\theta_r \end{bmatrix}}_{\mathbb{M}'}\begin{bmatrix}i'_{ur}\\ i'_{vr}\\ i'_{wr}\end{bmatrix}$$

Rotor Flux Linkage

$$\begin{bmatrix}\psi'_{ur}\\ \psi'_{vr}\\ \psi'_{wr}\end{bmatrix}=\underbrace{\begin{bmatrix} M'\cos\theta_r & M'\cos\left(\theta_r-\frac{2\pi}{3}\right) & M'\cos\left(\theta_r+\frac{2\pi}{3}\right) \\ M'\cos\left(\theta_r+\frac{2\pi}{3}\right) & M'\cos\theta_r & M'\cos\left(\theta_r-\frac{2\pi}{3}\right) \\ M'\cos\left(\theta_r-\frac{2\pi}{3}\right) & M'\cos\left(\theta_r+\frac{2\pi}{3}\right) & M'\cos\theta_r \end{bmatrix}}_{\mathbb{M}''}\begin{bmatrix}i_{us}\\ i_{vs}\\ i_{ws}\end{bmatrix}+\underbrace{\begin{bmatrix} l_r+M' & -\frac{1}{2}M' & -\frac{1}{2}M' \\ -\frac{1}{2}M' & l_r+M' & -\frac{1}{2}M' \\ -\frac{1}{2}M' & -\frac{1}{2}M' & l_r+M' \end{bmatrix}}_{\mathbb{L}_r'}\begin{bmatrix}i'_{ur}\\ i'_{vr}\\ i'_{wr}\end{bmatrix}$$







$\theta_r = 0 \rightarrow max$

Stator Equation on Stator Ref. Frame:

$$T\begin{bmatrix} v_{us} \\ v_{vs} \\ v_{ws} \end{bmatrix} = R_s T\begin{bmatrix} i_{us} \\ i_{vs} \\ i_{ws} \end{bmatrix} + \frac{d}{dt}\left\{ T\begin{bmatrix} & & \\ & \mathbb{L}'_s & \\ & & \end{bmatrix} T^{-1} T \begin{bmatrix} i_{us} \\ i_{vs} \\ i_{ws} \end{bmatrix} + T\begin{bmatrix} & & \\ & \mathbb{M}' & \\ & & \end{bmatrix} T^{-1} T \begin{bmatrix} i'_{ur} \\ i'_{vr} \\ i'_{wr} \end{bmatrix} \right\}$$

$$T\begin{bmatrix} & & \\ & \mathbb{L}'_s & \\ & & \end{bmatrix} T^{-1} = \begin{bmatrix} l_s + \frac{3}{2}M' & 0 \\ 0 & l_s + \frac{3}{2}M' \end{bmatrix} = \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix}$$

$$T\begin{bmatrix} & & \\ & \mathbb{M}' & \\ & & \end{bmatrix} T^{-1} = \frac{3}{2}M' \underbrace{\begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}}_{e^{J\theta_r}} = M \underbrace{\begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}}_{e^{J\theta_r}}$$

Stator Equation in Space Vector Representation:

$$\begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \end{bmatrix} = R_s \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + \frac{d}{dt}\left\{ L_s \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + M \overbrace{\begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}}^{e^{J\theta_r}} \underbrace{\begin{bmatrix} i'_{r\alpha} \\ i'_{r\beta} \end{bmatrix}} \right\}$$

Rotor Current on
Rotor Ref.
Frame

$$\begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \end{bmatrix} = R_s \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + \frac{d}{dt}\left\{ L_s \overbrace{\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}}^{\vec{i}_s} + M \overbrace{\underbrace{e^{J\theta_r} \begin{bmatrix} i'_{r\alpha} \\ i'_{r\beta} \end{bmatrix}}}^{\vec{i}_r} \right\}$$

$L_s \vec{i}_s + M \vec{i}_r$

Rotor Current on
Stator Ref.
Frame

$$\boxed{\vec{v}_s = R_s \vec{i}_s + L_s \frac{d\vec{i}_s}{dt} + M \frac{d\left(e^{J\theta_r}\, \vec{i}'_r\right)}{dt}}$$

(on Stator Ref. Frame)

Rotor Equation on Rotor Ref. Frame:

$$T\begin{bmatrix} v'_{ur} \\ v'_{vr} \\ v'_{wr} \end{bmatrix} = R_r\, T\begin{bmatrix} i'_{ur} \\ i'_{vr} \\ i'_{wr} \end{bmatrix} + \frac{d}{dt}\left\{ T\begin{bmatrix} & \mathbb{L}'_r & \end{bmatrix} T^{-1}\, T\begin{bmatrix} i'_{ur} \\ i'_{vr} \\ i'_{wr} \end{bmatrix} + T\begin{bmatrix} & \mathbb{M}'' & \end{bmatrix} T^{-1}\, T\begin{bmatrix} i_{us} \\ i_{vs} \\ i_{ws} \end{bmatrix} \right\}$$

$$T\begin{bmatrix} & \mathbb{L}'_r & \end{bmatrix} T^{-1} = \begin{bmatrix} l_r + \frac{3}{2}M' & 0 \\ 0 & l_r + \frac{3}{2}M' \end{bmatrix} = \begin{bmatrix} L_r & 0 \\ 0 & L_r \end{bmatrix}$$

$$T\begin{bmatrix} & \mathbb{M}' & \end{bmatrix} T^{-1} = \frac{3}{2}M' \underbrace{\begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix}}_{e^{-J\theta_r}} = M \underbrace{\begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix}}_{e^{-J\theta_r}}$$

Rotor Equation in Space Vector Representation:

$$\begin{bmatrix} v'_{r\alpha} \\ v'_{r\beta} \end{bmatrix} = R_r \begin{bmatrix} i'_{r\alpha} \\ i'_{r\beta} \end{bmatrix} + \frac{d}{dt}\left\{ L_r \begin{bmatrix} i'_{r\alpha} \\ i'_{r\beta} \end{bmatrix} + M \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \underbrace{\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}}_{\text{Stator Current on Stator Ref. Frame}} \right\}$$

$$\begin{bmatrix} v'_{r\alpha} \\ v'_{r\beta} \end{bmatrix} = R_s \begin{bmatrix} i'_{r\alpha} \\ i'_{r\beta} \end{bmatrix} + \frac{d}{dt}\left\{ L_r \begin{bmatrix} i'_{r\alpha} \\ i'_{r\beta} \end{bmatrix} + M \underbrace{e^{-J\theta_r} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}}_{\text{Stator Current on Rotor Ref. Frame}} \right\}$$

$$\boxed{\vec{v}_r{}' = R_r \vec{i}_r' + L_r \frac{d\,\vec{i}_r{}'}{dt} + M \frac{d\,\vec{i}_s{}'}{dt}}$$ (on Rotor Ref. Frame)

$$\vec{i}_s' = e^{-J\theta_r} \vec{i}_s$$

$$\boxed{\vec{v}_r{}' = R_r\,\vec{i}_r{}' + L_r\frac{d\,\vec{i}_r{}'}{dt} + M\,e^{-J\theta_r}\frac{d\,\vec{i}_s}{dt} - J\omega M\,e^{-J\theta_r}\,\vec{i}_s}$$ (on Rotor Ref. Frame)

Rotor Equation on Stator Ref. Frame:

$$e^{J\theta_r}\,\vec{v}_r{}' = e^{J\theta_r}\,R_r\,\vec{i}_r{}' + \underbrace{e^{J\theta_r}\,L_r\frac{d\,\vec{i}_r{}'}{dt}}_{} + e^{J\theta_r}\,M\,e^{-J\theta_r}\frac{d\,\vec{i}_s}{dt} - e^{J\theta_r}\,J\omega M\,e^{-J\theta_r}\,\vec{i}_s$$

$$\begin{bmatrix} v_{r\alpha}' \\ v_{r\beta}' \end{bmatrix}$$

$$L_r\frac{d\left(e^{J\theta_r}\,\vec{i}_r{}'\right)}{dt} - J\omega L_r\,e^{J\theta_r}\,\vec{i}_r{}'$$

$$\begin{bmatrix} v_{r\alpha} \\ v_{r\beta} \end{bmatrix} \quad \boxed{\vec{v}_r = R_r\,\vec{i}_r + \left(\frac{d}{dt}\left(-J\omega\right)\right)\left[L_r\vec{i}_r + M\vec{i}_s\right]}$$

$$\vec{i}_r = e^{J\theta_r}\,\vec{i}_r{}'$$

on stator Ref. Frame

สรุป

$$\vec{v}_s = R_s \vec{i}_s + L_s \frac{d\vec{i}_s}{dt} + M \frac{d\vec{i}_r}{dt}$$ ⇒ stator

$$\vec{v}_r = R_r \vec{i}_r + \left(\frac{d}{dt} - J\omega\right)\left[L_r \vec{i}_r + M\vec{i}_s\right]$$ ⇒ Rotor

การคำนวณหาแรงบิดจากสมการทางไฟฟ้า

$$\vec{i}_s \bullet \left\{ R_s \vec{i}_s + L_s \frac{d\vec{i}_s}{dt} + M \frac{d\vec{i}_r}{dt} = \vec{v}_s \right\}$$

$$\vec{i}_r \bullet \left\{ R_r \vec{i}_r + \left(\frac{d}{dt} - J\omega \right) \left[L_r \vec{i}_r + M \vec{i}_s \right] = \vec{v}_r \right\}$$

Note การคำนวณ Power ในรูป space vector notation

$$p(t) = \mathrm{Re}\left(\vec{i}^{\,*} \cdot \vec{v} \right) = i_{s1} v_{s1} + i_{s2} v_{s2} + i_{s3} v_{s3}$$

$$= \mathrm{Re}\left[\left(i_x - j i_y \right) \left(v_x + j v_y \right) \right]$$

$$= \left[i_x v_x + i_y v_y \right]$$

$$\begin{cases} i_x = \sqrt{\frac{2}{3}} \left(i_{s1} - \frac{1}{2} i_{s2} - \frac{1}{2} i_{s3} \right) = \sqrt{\frac{3}{2}} i_{s1} \\ i_y = \sqrt{\frac{2}{3}} \left(\frac{\sqrt{3}}{2} i_{s2} - \frac{\sqrt{3}}{2} i_{s3} \right) = \frac{1}{\sqrt{2}} \left(i_{s2} - i_{s3} \right) \end{cases}$$

$$\begin{cases} v_x = \sqrt{\frac{3}{2}} v_{s1} \\ v_y = \frac{1}{\sqrt{2}} \left(v_{s2} - v_{s3} \right) \end{cases}$$

$$p(t) = \left[\frac{3}{2} i_{s1} v_{s1} + \frac{1}{2} \left(i_{s2} v_{s2} + i_{s3} v_{s3} - i_{s3} v_{s2} - i_{s2} v_{s3} \right) \right]$$

แต่

$$-\frac{1}{2}i_{s3}v_{s2} = \frac{1}{2}(i_{s3}v_{s3}+i_{s3}v_{s1})$$
$$-\frac{1}{2}i_{s2}v_{s3} = \frac{1}{2}(i_{s2}v_{s2}+i_{s2}v_{s1})$$
$$\frac{3}{2}i_{s1}v_{s1} = \frac{1}{2}i_{s1}v_{s1}+i_{s1}v_{s1}$$
$$\therefore p(t)=i_{s1}v_{s1}+i_{s2}v_{s2}+i_{s3}v_{s3}$$ Q.E.D.

ในทาง Vector Notation

$$p(t) = \vec{i}\cdot\vec{v}$$ (dot product)

$$(\because = \begin{bmatrix} i_x & i_y \end{bmatrix}\begin{bmatrix} v_x \\ v_y \end{bmatrix}$$
$$= \begin{bmatrix} i_x v_x + i_y v_y \end{bmatrix})$$

จากสมการทางไฟฟ้า

$$R_s i_s^2 + \frac{d}{dt}\left(\frac{1}{2}L_s i_s^2\right) + \vec{i}_s \bullet M\frac{d\,\vec{i}_r}{dt} = \vec{i}_s \bullet \vec{u}_s$$

Power ทางด้านสเตเตอร์

$$R_r i_r^2 + \frac{d}{dt}\left(\frac{1}{2}L_r i_r^2\right) + \vec{i}_r \bullet M\frac{d\,\vec{i}_s}{dt} = \vec{i}_r \bullet \vec{u}_r$$
$$\underbrace{+\vec{i}_r \bullet \left(-J\omega L_R \vec{i}_r\right)}_{=0} + \vec{i}_r \bullet \left(-J\omega M\,\vec{i}_s\right)$$

Power ทางด้านโรเตอร์

Copper Losses

Magnetizing Energy

$$\boxed{\left(R_s i_s^2 + R_r i_r^2\right)} + \boxed{\frac{d}{dt}\left[\frac{1}{2}L_s i_s^2 + \frac{1}{2}L_r i_r^2 + M\vec{i}_s \bullet \vec{i}_r\right]}$$

$$\boxed{-\vec{i}_r \bullet J\omega M\,\vec{i}_s} = \boxed{\left(\vec{i}_s \bullet \vec{u}_s + \vec{i}_r \bullet \vec{u}_r\right)}$$

Mechanical Power

Input Power

$$\tau_e \cdot \left[\omega \times \frac{2}{p}\right]$$

$$\therefore \vec{\tau}_e = \left[-\vec{i}_r \bullet J\omega M\,\vec{i}_s\right].\vec{z} \times \frac{p}{2}\frac{1}{\omega}$$

$$\approx \quad i\,\vec{l} \times \vec{B}$$

$$= -M\,\vec{i}_s \times \vec{i}_r \times \frac{p}{2} = \frac{p}{2} \times \vec{i}_r \times M\,\vec{i}_s$$

<u>สรุป</u> Dynamic Model

$$\begin{cases} R_s\,\vec{i}_s + L_s \dfrac{d\,\vec{i}_s}{dt} + M\dfrac{d\,\vec{i}_r}{dt} = \vec{v}_s \\ R_r\,\vec{i}_r + \left(\dfrac{d}{dt} - J\omega_r\right)\left[L_r\vec{i}_r + M\vec{i}_s\right] = \vec{v}_r \\ \tau_e = \left(\dfrac{p}{2}\right)\mathrm{Im}\left(M\,\vec{i}_s\,\vec{i}_r^{\,*}\right) \end{cases}$$

โดยที่ ω : ความเร็วโรเตอร์ทางไฟฟ้า

[<u>ex</u>. Poles =4, ความเร็วโรเตอร์(ทางกล) = 750rpm

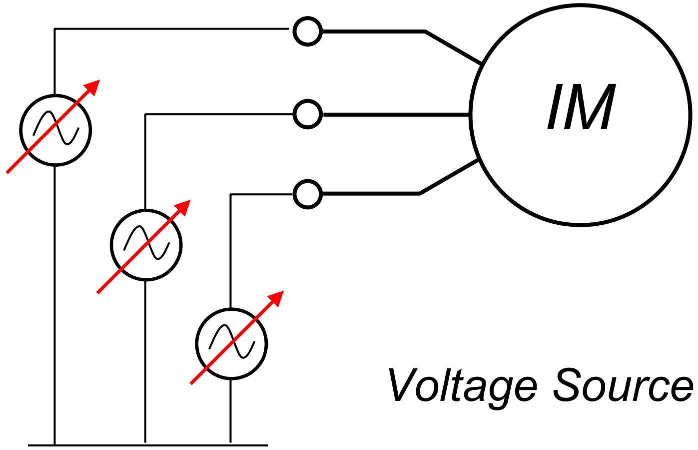
$$\omega = 50\pi\left[rad/s\right]\left(25\ Hz\right)$$]

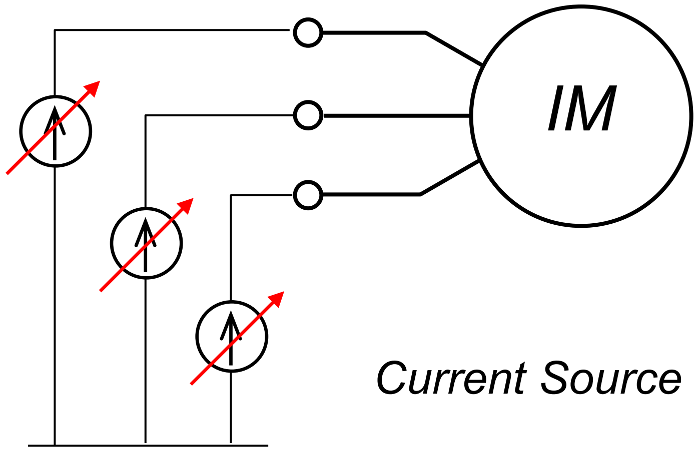
ε : มุมโรเตอร์ทางไฟฟ้า (เฟส R1)

$\parallel$

θ_r

Control of Induction Motor Drives





- แยกตามแหล่งจ่าย
  - แหล่งจ่ายแรงดัน $\rightarrow u_s$
  - แหล่งจ่ายกระแส $\rightarrow i_s$
- *- Steady-State Model (Equivalent Circuit Phasor)*

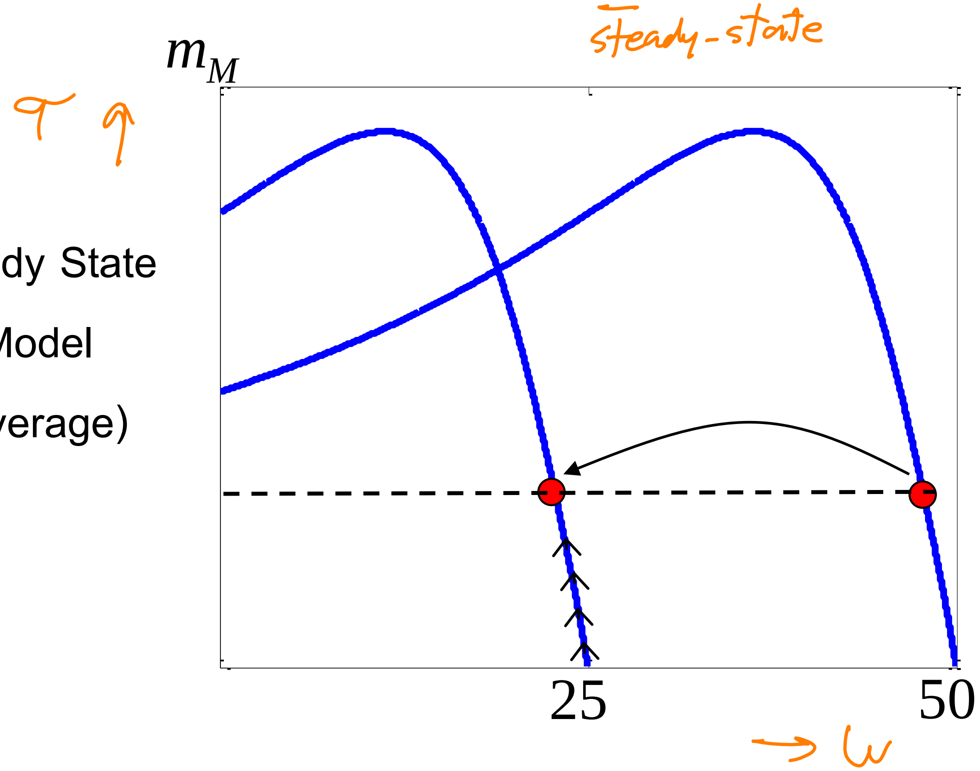
  - Dynamic Model (Space Vector)

Current Source + Dynamic Model

(Rotor Flux Oriented Control, Vector Control, Field Oriented Control)

- กรณีนี้จะง่ายกว่า Voltage Source เพราะว่า เราสามารถละเลย Stator Dynamic (สมการทางด้าน Stator) ได้
- ต้องการควบคุม IM ให้สร้าง Flux + Torque (ทุก ๆ ขณะ, Instantaneous) ตามที่ต้องการอย่างรวดเร็ว เมื่อเทียบกับกรณี V/f

Steady State Model (Average)

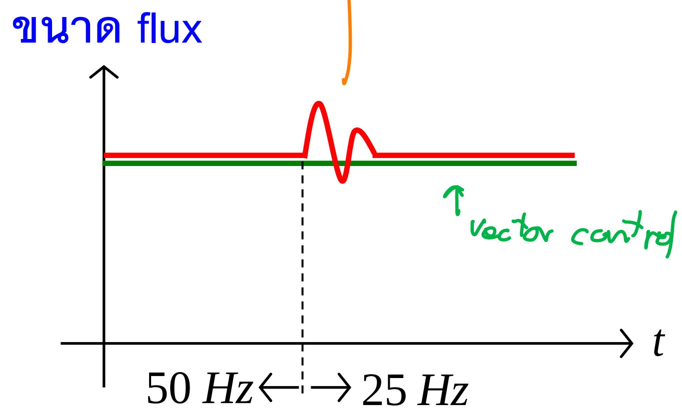


V/f Control:

Flux ✓ (average)

Torque ✗

Response ช้า



- High Performance Drives

Dynamic Model

- $$R_r \vec{i}_r' + L_r \frac{d\vec{i}_r'}{dt} + M \frac{d\vec{i}_s'}{dt} = 0$$ (สมการโรเตอร์)

$$\vec{i}_s' = e^{-J\theta_r}\,\vec{i}_s$$ (Rotor frame)

- $$\tau_e = \left(\frac{p}{2}\right)\mathrm{Im}\left(M\,\vec{i}_s\,\vec{i}_r^{\,*}\right) = \left(\frac{p}{2}\right)\mathrm{Im}\left(M\,\vec{i}_s\,e^{-J\theta_r}\vec{i}_r^{\,\prime *}\right)$$ (สมการแรงบิด)

$$\vec{\lambda}_r = L_r \vec{i}_r + M \vec{i}_s$$
$$= L_r e^{J\theta} \vec{i}_r' + M \vec{i}_s$$

<u>นิยาม</u> Magnetizing Current ของ Rotor Flux

$$\vec{i}_{mR}(t) = \vec{i}_s(t) + \frac{L_r}{M}\cdot e^{J\theta_r}\,\vec{i}_r'(t)$$
$$= i_{mR}(t)e^{j\rho(t)}$$

$$\triangleq \quad M\,\vec{i}_{mR}$$

$$\vec{i}_{mR} = i_{mR}(t)\,e^{J\rho(t)}$$

ρ

$$\tau_e(t) = PM\,\mathrm{Im}\left(\vec{i}_s\cdot\left(\vec{i}_{mR}-\vec{i}_s\right)^*\right)\cdot\frac{M}{L_r}$$

$$\left[\because\ e^{J\theta_r}\,\vec{i}_r' = \left(\vec{i}_{mR}-\vec{i}_s\right)\cdot\frac{M}{L_r}\right]$$

$\operatorname{Im}(\vec{i}_s \cdot \vec{i}_s^{\,*}) = 0$

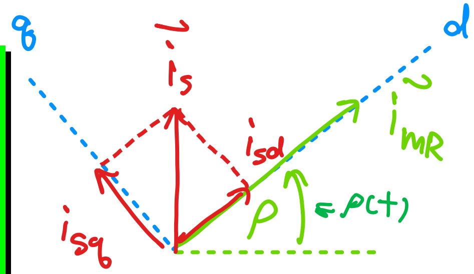
$$\tau_e(t) = P\frac{M^2}{L_r}\operatorname{Im}\left(\vec{i}_s \cdot \vec{i}_{mR}^{\,*}\right)$$

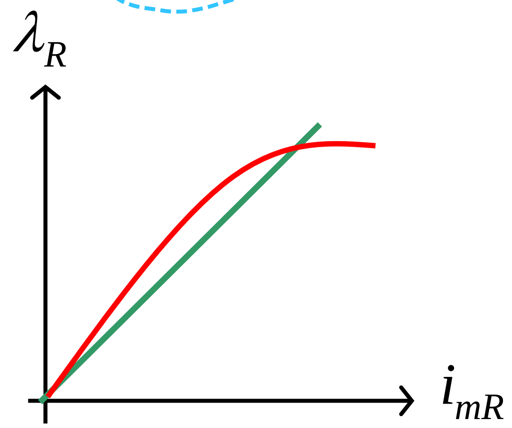
$$= P\frac{M^2}{L_r}\operatorname{Im}\left(\vec{i}_s \cdot e^{-j\rho}\begin{bmatrix} i_{mR} \\ 0 \end{bmatrix}\right) = P\frac{M^2}{L_r}\operatorname{Im}\left(\vec{i}_s \cdot i_{mR}\, e^{-j\rho}\right)$$

<u>นิยาม</u>

$$\vec{i}_s e^{-j\rho} = i_{sd} + j i_{sq}$$

<u>Note</u> i_{sq} : torque (producing) current

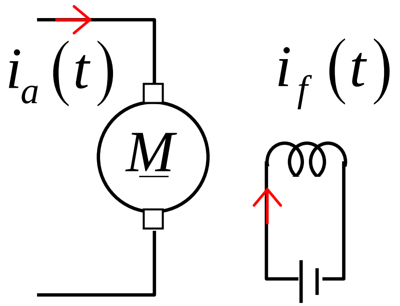




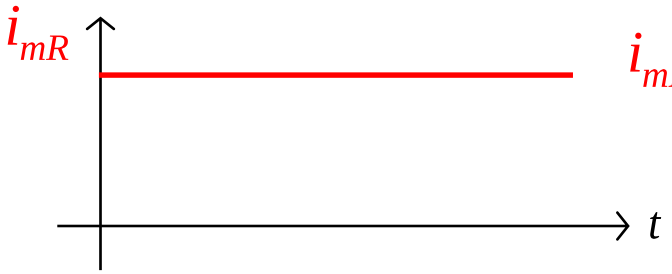
$$\boxed{\tau_e(t) = P\frac{M^2}{L_r} i_{mR} \cdot i_{sq}}$$

$\Updownarrow$

DC Motor : แรงบิด $\tau_e(t) = k\, i_f\, i_a$

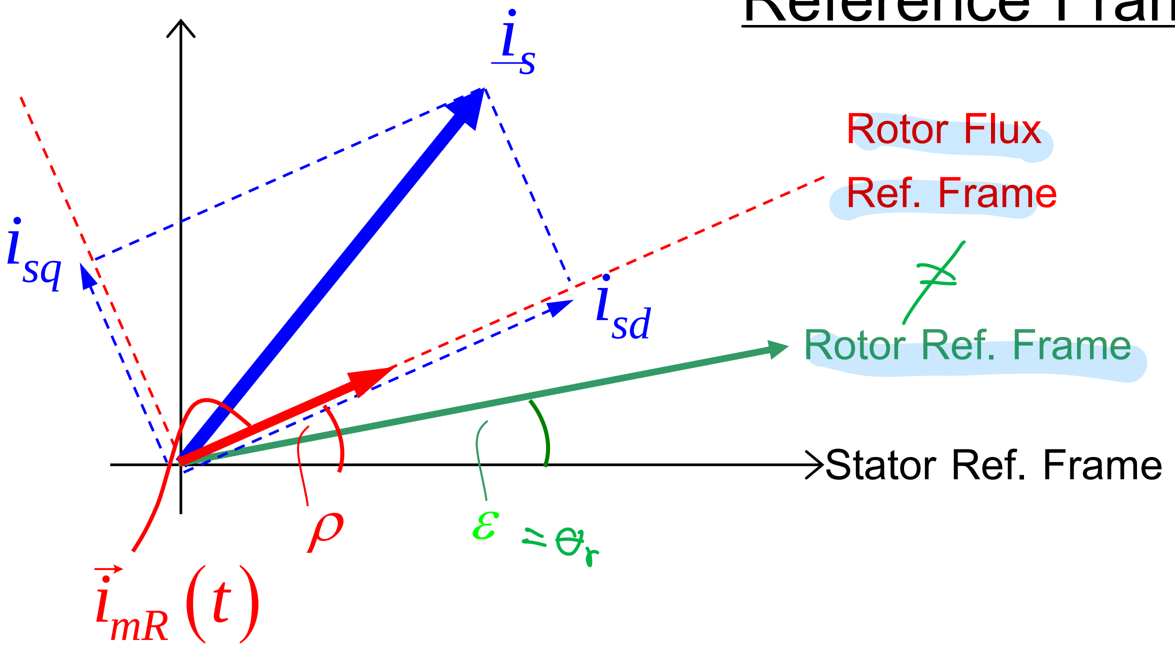


เพื่อให้บรรลุวัตถุประสงค์ $\longrightarrow$ ควบคุม $i_{mR}(t)$ และ $i_{sq}(t)$ $\Rightarrow$



i_{mR} คงที่

Reference Frame



$$\vec{i}_s e^{-j\rho(t)} = i_{sd} + j i_{sq}$$

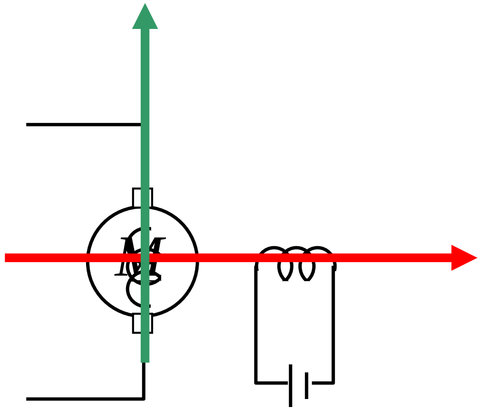
: space vector ของกระแส stator ที่อ้างอิง (มองจาก) Rotor Flux Ref. Frame

$$= i_{mR}(t) \cdot e^{j\rho(t)}$$

$\therefore \rho$ = มุมของ Mag. Current vector

= มุมของ Rotor Flux Vector

หาสมการ $\vec{i}_{mR}(t) \quad \leftrightarrow \quad \vec{i}_s$?



$$R_r \frac{M}{L_r}\left(\vec{i}_{mR} - \vec{i}_s\right)e^{-j\varepsilon} + \frac{d}{dt}\left(M\left[\vec{i}_{mR} - \vec{i}_s\right]e^{-j\varepsilon}\right)$$

$$+\frac{d}{dt}\left(M\vec{i}_s e^{-j\varepsilon}\right) \quad = \quad 0$$

$$\left(\frac{d\varepsilon}{dt} = \omega\right)$$

ความเร็วโรเตอร์

$$\therefore \quad \frac{R_r}{L_r} M\left(\vec{i}_{mR} - \vec{i}_s\right)e^{-j\varepsilon} + M\left[\frac{d\vec{i}_{mR}}{dt}\right]\cdot e^{-j\varepsilon} - j\omega M\vec{i}_{mR}\, e^{-j\varepsilon} \quad = \quad 0$$

$$\boxed{\frac{d\vec{i}_{mR}}{dt} \quad = \quad \left[-\frac{R_r}{L_r} + j\omega\right]\vec{i}_{mR} + \frac{R_r}{L_r}\vec{i}_s}$$

สมการ rotor flux

แต่ปริมาณที่เราสนใจ $i_{mR}(t), \rho(t)$

$$R_r \tilde{i}_r' + L_r \frac{d}{dt}\tilde{i}_r' + M \frac{d}{dt}\tilde{i}_s' = 0$$

$$\tilde{i}_s' = e^{-j\theta_r}\tilde{i}_s = e^{-j\varepsilon}\tilde{i}_s \Rightarrow \tilde{i}_s e^{-j\varepsilon}$$

$$\tilde{i}_{mR} = \tilde{i}_s + \frac{L_r}{M} e^{j\theta_r}\tilde{i}_r'$$

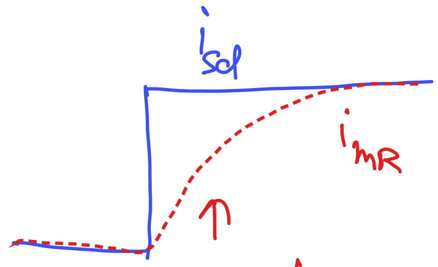
$$= \tilde{i}_s + \frac{L_r}{M} e^{j\varepsilon}\tilde{i}_r' = \tilde{i}_s + \frac{L_r}{M}\tilde{i}_r' e^{j\varepsilon}$$

$$\tilde{i}_r' = \frac{M}{L_r}\left(\tilde{i}_{mR} - \tilde{i}_s\right) e^{-j\varepsilon}$$

$$\frac{d}{dt}\left(i_{mR}\, e^{j\rho}\right) = \left[-\frac{R_r}{L_r} + j\omega\right] i_{mR} e^{j\rho} + \frac{R_r}{L_r}\vec{i}_s$$

$$\left[\frac{di_{mR}}{dt}\right]\cdot e^{j\rho} + j\omega_{mR}\, e^{j\rho} = \left[-\frac{R_r}{L_r} + j\omega\right] i_{mR} e^{j\rho} + \frac{R_r}{L_r}\vec{i}_s$$

(โดยที่ $\frac{d\rho}{dt} = \omega_{mR}$: rotor flux frequency)



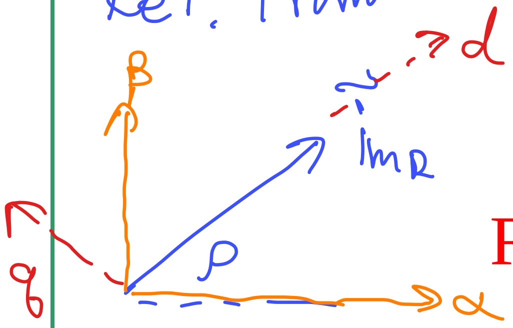
$\tau_R = \frac{L_r}{R_r}$

d-axis ↔ flux
q-axis ↔ torque

Rotor Flux Ref. Frame

Rotor ≠ Ref fram

$$\left[\frac{di_{mR}}{dt}\right] + j\omega_{mR} = \left[-\frac{R_r}{L_r} + j\omega\right] i_{mR} + \frac{R_r}{L_r}\underbrace{\vec{i}_s \cdot e^{-j\rho}}_{i_{sd} + j i_{sq}}$$



Re : $$\frac{di_{mR}}{dt} = -\frac{R_r}{L_r} i_{mR} + \frac{R_r}{L_r}\cdot i_{sd}$$ → ปกติ

Im : $$+ j\omega_{mR}\cdot i_{mR} = + j\omega \cdot i_{mR} + j\frac{R_r}{L_r}\cdot i_{sq}$$ → ตำแหน่ง ρ

$$\text{Re}: \quad \frac{di_{mR}}{dt} = -\frac{R_r}{L_r}\left(i_{mR} - i_{sd}\right)$$

$$\text{Im}: \quad \frac{d\rho}{dt} = \omega_{mR} = \omega + \frac{R_r}{L_r}\cdot\frac{i_{sq}}{i_{mR}}$$

$i_{sd} \rightarrow i_{mR}$

$i_{sq} \rightarrow \boxed{\times} \rightarrow T_e$

* * *

$\omega_{mR} = \omega + \omega_{slip}$

slip

$\langle$*Rotor Flux Dynamic Model*$\rangle$ $\quad 1\big/\frac{R_r}{L_r}$: *rotor time constant* $\sim 100ms$

<u>เปรียบเทียบ IM-DC</u>

		IM	DC
Flux control	:	i_{sd}	i_f
Torque control	:	i_{sq}	i_a
Flux position	:	rotating	fixed

$\therefore$ เราต้องรู้ว่า Rotor flux vector อยู่ที่ไหน (Magnetizing current vector $\vec{\lambda}_R, \vec{i}_{mR}$)

<u>ตัวอย่าง</u> ต้องการ $i_{mR} = 3A \rightarrow i_{sd}(t) = 3A$

$$\frac{di_{mR}}{dt} = -\frac{R_R}{L_R}\left(i_{mR} - i_{sd}\right)$$

ถ้าต้องการแรงบิด 10 Nm ; $M = L_r = 0.1\,H$

$$\tau_e(t) = \frac{M^2}{L_r}\cdot i_{mR}\cdot i_{sq}$$

$$10 = \frac{M^2}{L_r}\cdot 3\cdot i_{sq}$$

$$\therefore i_{sq}(t) = 10\times\left(\frac{L_r}{M^2}\right)\times\frac{1}{3} = 50\,A$$

แต่สิ่งที่เราจ่ายให้กับ IM จริงๆคือ กระแส 3 เฟส i_{s1}, i_{s2}, i_{s3}

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \begin{bmatrix} 3 \\ 50 \end{bmatrix} \xrightarrow{\boxed{\rho(t)}} \begin{bmatrix} i_{sx} \\ i_{sy} \end{bmatrix} \xrightarrow{\text{2/3 Trans.}} \begin{bmatrix} i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

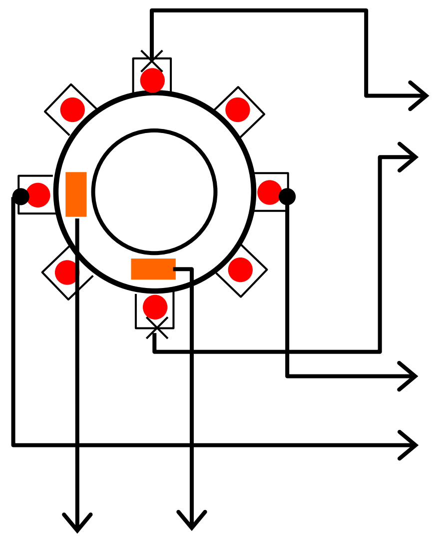
Rotor flux ref. Stator ref.

วิธีการหาตำแหน่งและขนาดของ flux (Flux Acquisition)

1) ใช้การติดตั้ง Sensors เพิ่มเติม ได้แก่ Search Coils พันในร่อง slots ของ stator/ Hall-Effect Sensors

→ - ทำได้ยากในทางปฏิบัติ

- มีผลของ noises จาก space harmonics



2) ใช้ Dynamic Model ของ IM มาคำนวณหา flux

→ ไม่มีปัญหาในการติดตั้ง Sensors (ทางปฏิบัติถือว่า O.K.)

ข้อจำกัด

[1] ต้องรู้ค่า Parameters ของ IM

[2] ค่า Parameters โดยทั่วไปจะเปลี่ยนแปลงตามสภาวะการทำงาน

[3] ต้องมี Speed Sensor/ Position Sensor (Encoder)

การแก้ไข

[1] → Off-Line Auto Tuning Technique

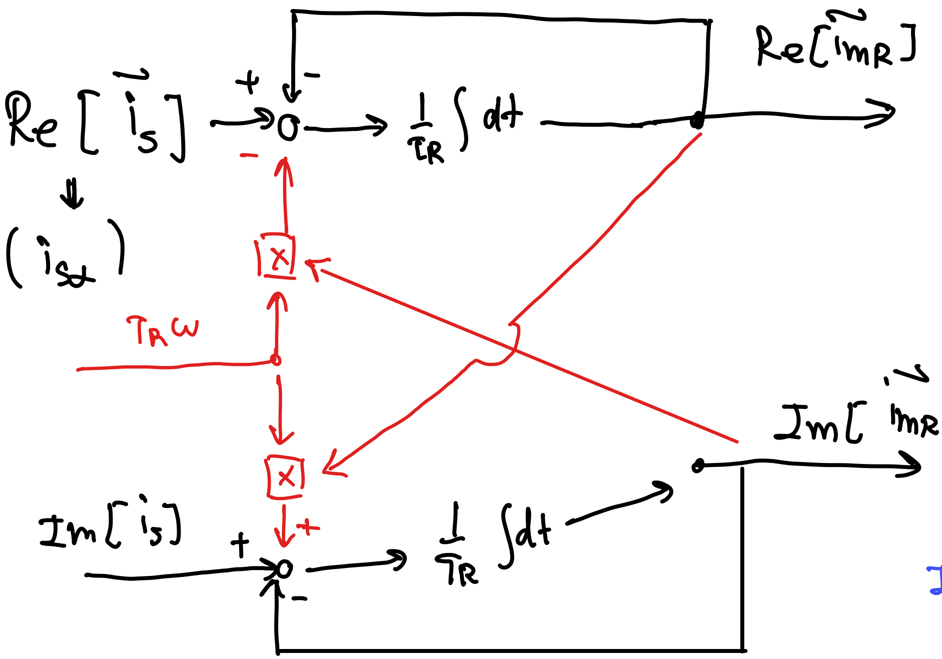
[2] → Adaptive/On-Line Tuning Technique

[3] → Speed Estimation Technique

[Speed Sensorless]

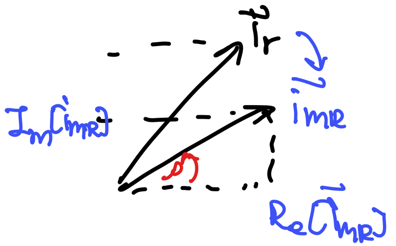
$$\frac{d}{dt}\tilde{i}_{mR} = \left[-\frac{R_r}{L_r}+j\omega\right]\tilde{i}_{mR} + \frac{R_r}{L_r}\tilde{i}_s \quad \text{(stator Ref.}$$

$$\frac{d}{dt}\begin{bmatrix} i_{mR.\alpha} \\ i_{mR.\beta} \end{bmatrix} = \left[-\frac{R_r}{L_r}+j\omega\right]\begin{bmatrix} i_{mR.\alpha} \\ i_{mR.\beta} \end{bmatrix} + \frac{R_r}{L_r}\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}$$



$$\rho = \tan^{-1}\left[\frac{Im(\tilde{i}_{mR})}{Re(\tilde{i}_{mR})}\right]$$

$$i_{mR} = \sqrt{Re^2[\,] + Im^2[\,]}$$

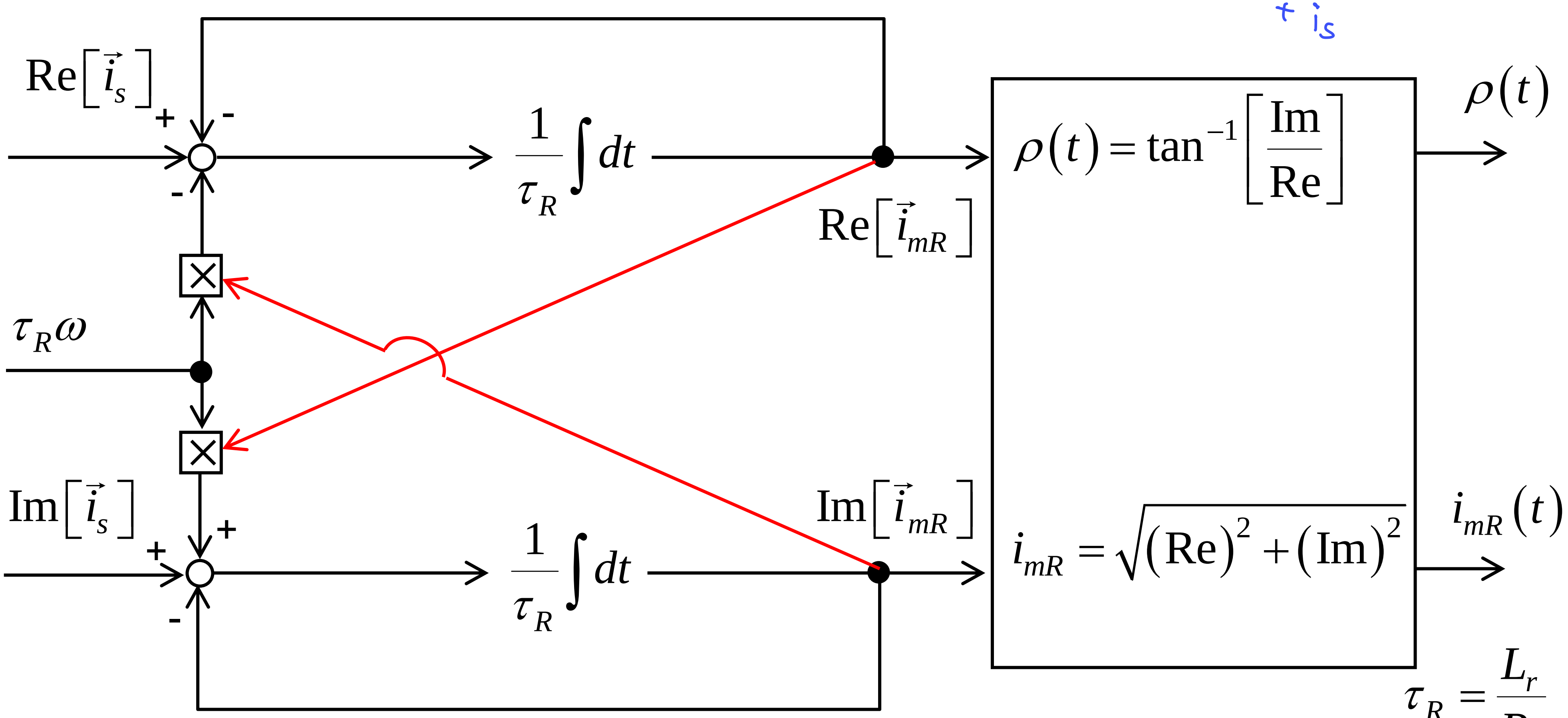


$\tau_R = \frac{L_r}{R_r}$

(a) คำนวณโดยอาศัยสมการบน Stator Ref. Frame

$$\frac{d\vec{i}_{mR}}{dt} = \left[-\frac{R_r}{L_r} + j\omega\right]\vec{i}_{mR} + \frac{R_r}{L_r}\vec{i}_s$$

$\Rightarrow \tau_R \frac{d}{dt}\tilde{i}_{mR} = \left[-1 + j\omega\tau_R\right]\tilde{i}_{mR} + \tilde{i}_s$

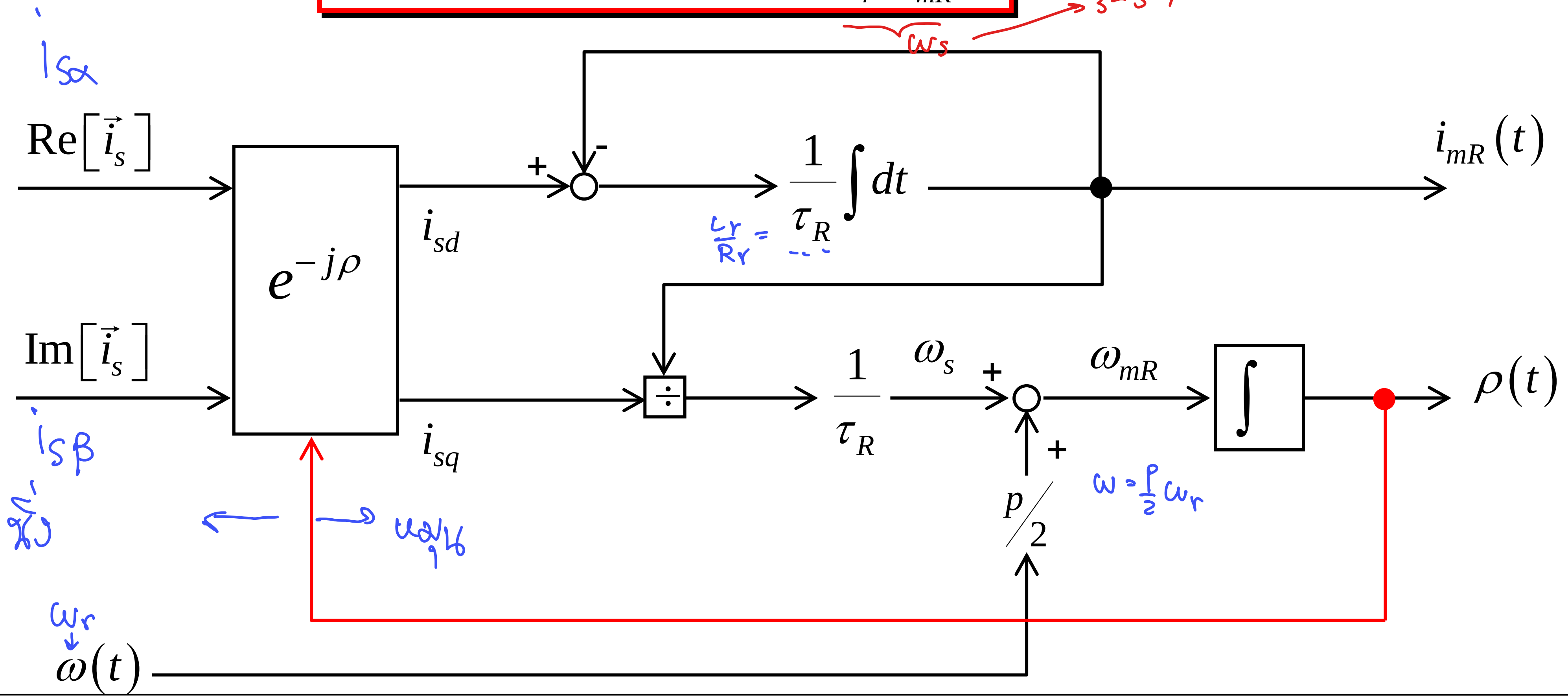


$$\tau_R = \frac{L_r}{R_r}$$

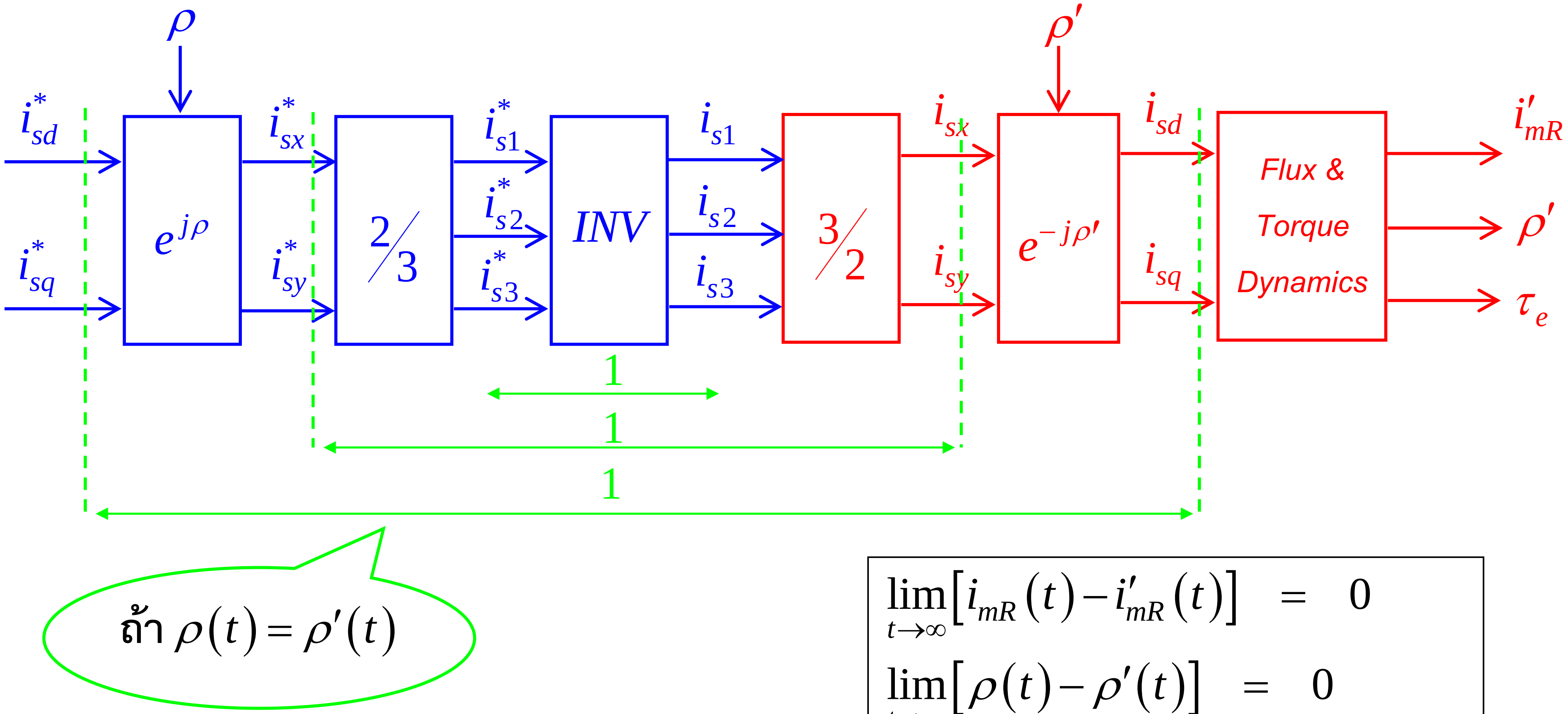
(b) คำนวณโดยอาศัยสมการบน Rotor Flux Ref. Frame

$$\text{Re:}\quad \frac{di_{mR}}{dt} = -\frac{R_r}{L_r}\left(i_{mR} - i_{sd}\right)$$

$$\text{Im:}\quad \frac{d\rho}{dt} = \omega_{mR} = \overset{\frac{p}{2}\omega_r}{\omega} + \underbrace{\frac{R_r}{L_r}\cdot\frac{i_{sq}}{i_{mR}}}_{\omega_s \to 3\text{-}5\%}$$



Block Diagram รวม IM



$$\lim_{t\to\infty}\left[i_{mR}(t) - i'_{mR}(t)\right] = 0$$

$$\lim_{t\to\infty}\left[\rho(t) - \rho'(t)\right] = 0$$

$$\Updownarrow$$

$$\lim_{t\to\infty}\left[\vec{i}_{mR}(t) - \vec{i}_{mR}{}'(t)\right] = 0$$

$(\ )'$: ค่าจริงในมอเตอร์

[Proof]

CAL. : $$\frac{d\vec{i}_{mR}}{dt} = \left[-\frac{R_r}{L_r} + j\omega\right]\vec{i}_{mR} + \frac{R_r}{L_r}\vec{i}_s^{\,*}$$

Motor : $$\frac{d\vec{i}_{mR}{}'}{dt} = \left[-\frac{R_r}{L_r} + j\omega\right]\vec{i}_{mR}{}' + \frac{R_r}{L_r}\vec{i}_s$$

ถ้า Inverter เป็น current source ในอุดมคติ $\vec{i}_s^{\,*} = \vec{i}_s$

$$\therefore \quad \frac{d\vec{e}}{dt} = \frac{d}{dt}\left(\vec{i}_{mR} - \vec{i}_{mR}{}'\right)$$

$$= \left[-\frac{R_r}{L_r} + j\omega\right]\vec{e}$$

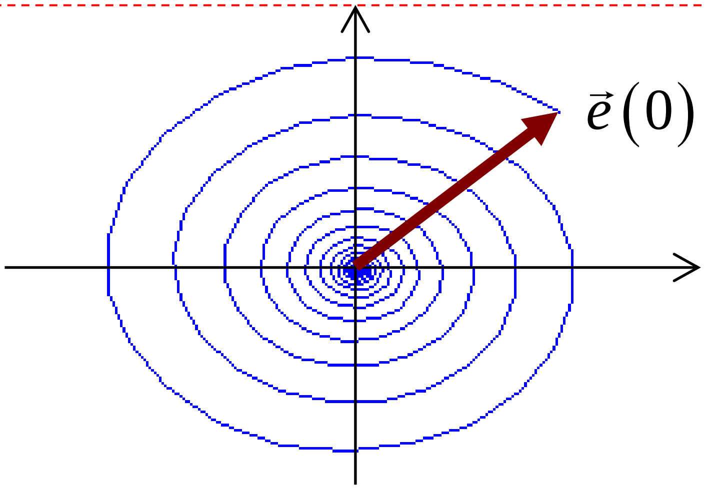
$$\vec{e}(t) = \vec{e}(0)\cdot\exp\left[\left(-\frac{R_r}{L_r} + j\omega\right)t\right]$$

$$= \underbrace{\vec{e}(0)\cdot e^{j\omega t}}\cdot\underbrace{e^{-\frac{R_r}{L_r}\cdot t}}$$

$\lim_{t\to\infty}\vec{e}(t) = 0$

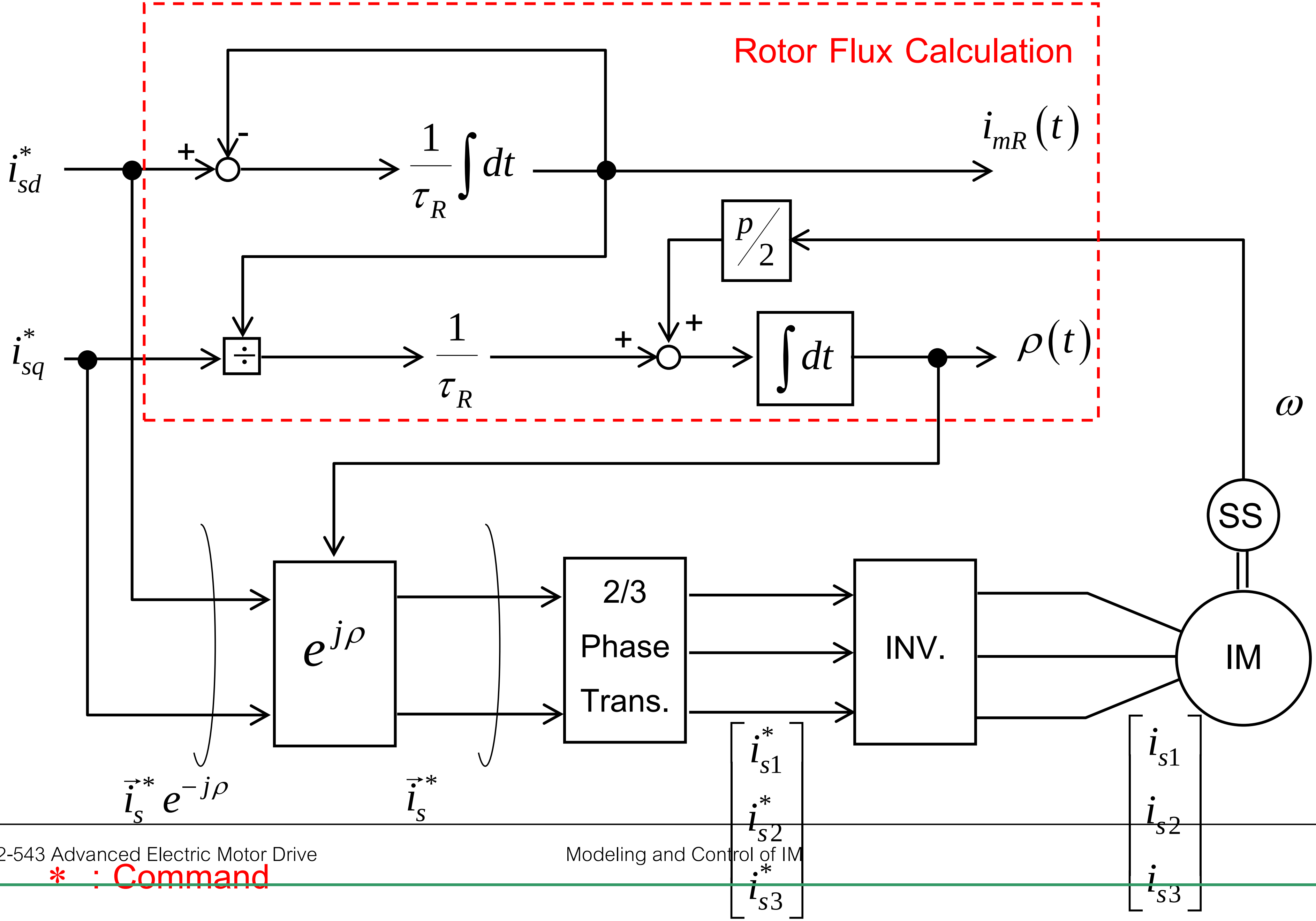
$\dot{x} = Ax$

$\lim_{t\to\infty} x = 0$ เมื่อไร



$\therefore$ Q.E.D

Block Diagram ของ Vector Control System (Field-Oriented Control)

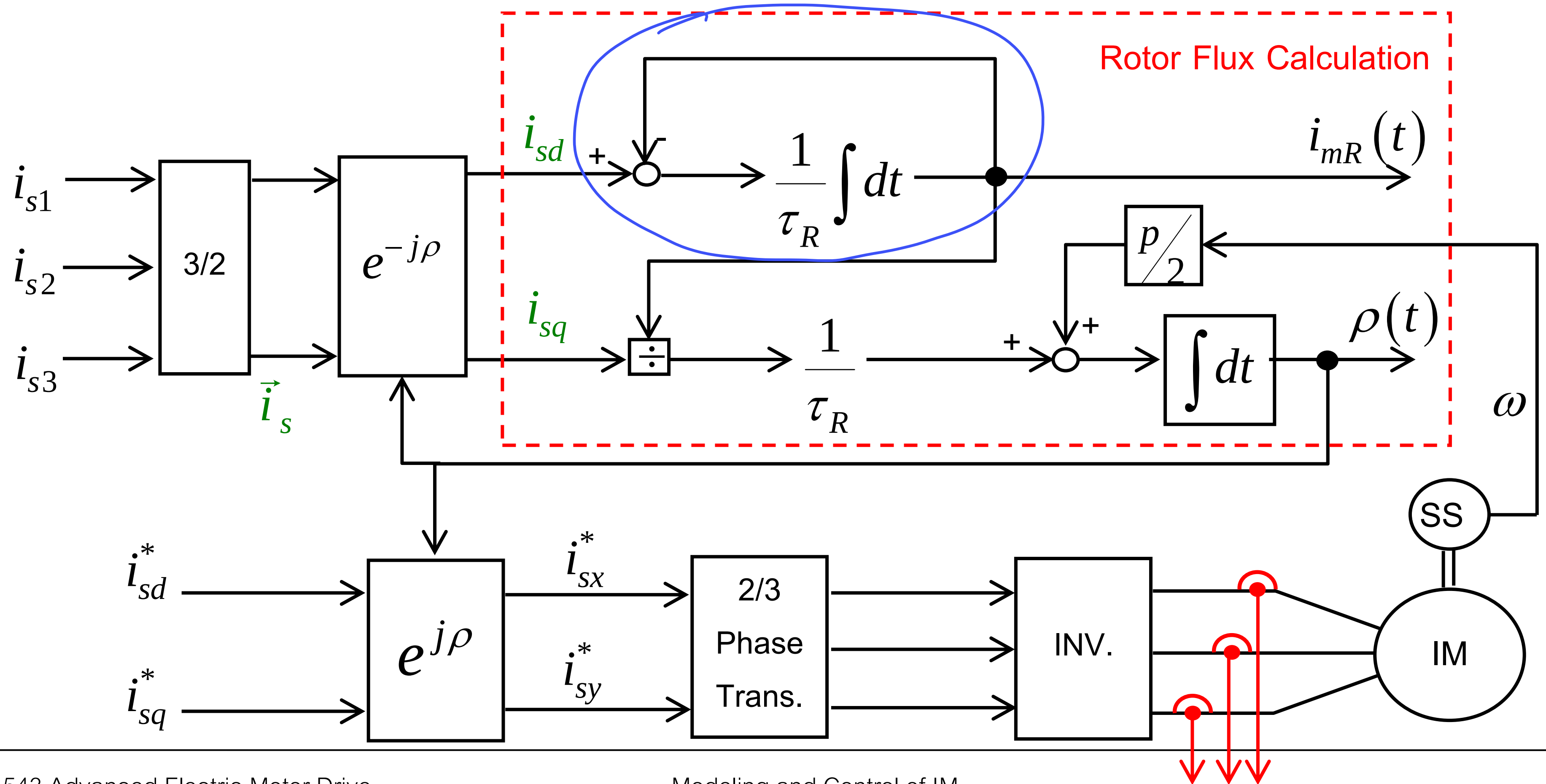


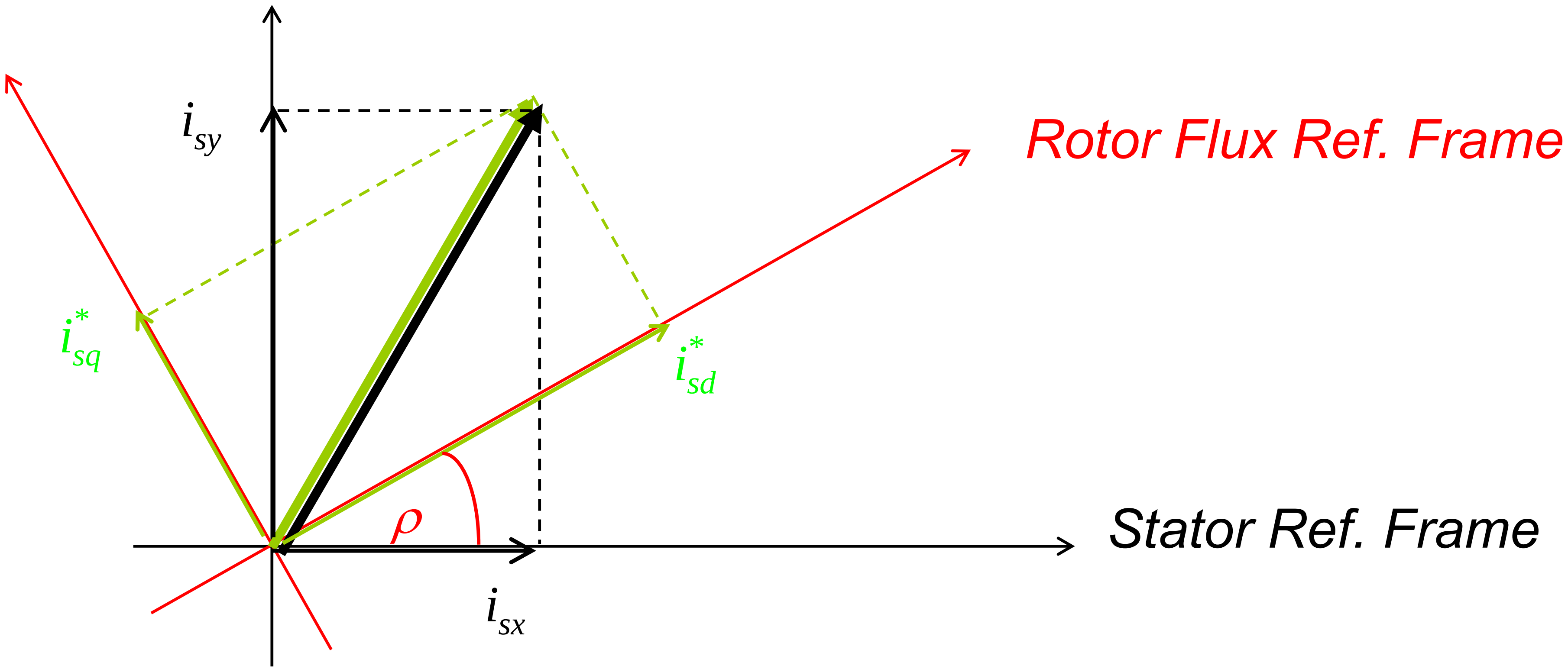
* : Command

ให้ INV. สามารถจ่ายกระแสได้เสมือน Current Source ในอุดมคติ

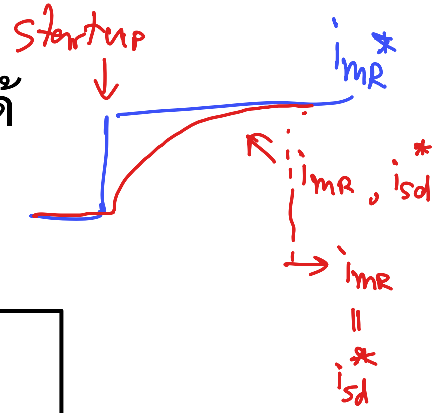
$$\rightarrow \begin{bmatrix} i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} = \begin{bmatrix} i_{s1}^* \\ i_{s2}^* \\ i_{s3}^* \end{bmatrix} \rightarrow \vec{i}_s \cdot e^{-j\rho} = \vec{i}_s^* \cdot e^{-j\rho} \rightarrow \quad \therefore \begin{matrix} i_{sd}^* = i_{sd} \\ i_{sq}^* = i_{sq} \end{matrix}$$

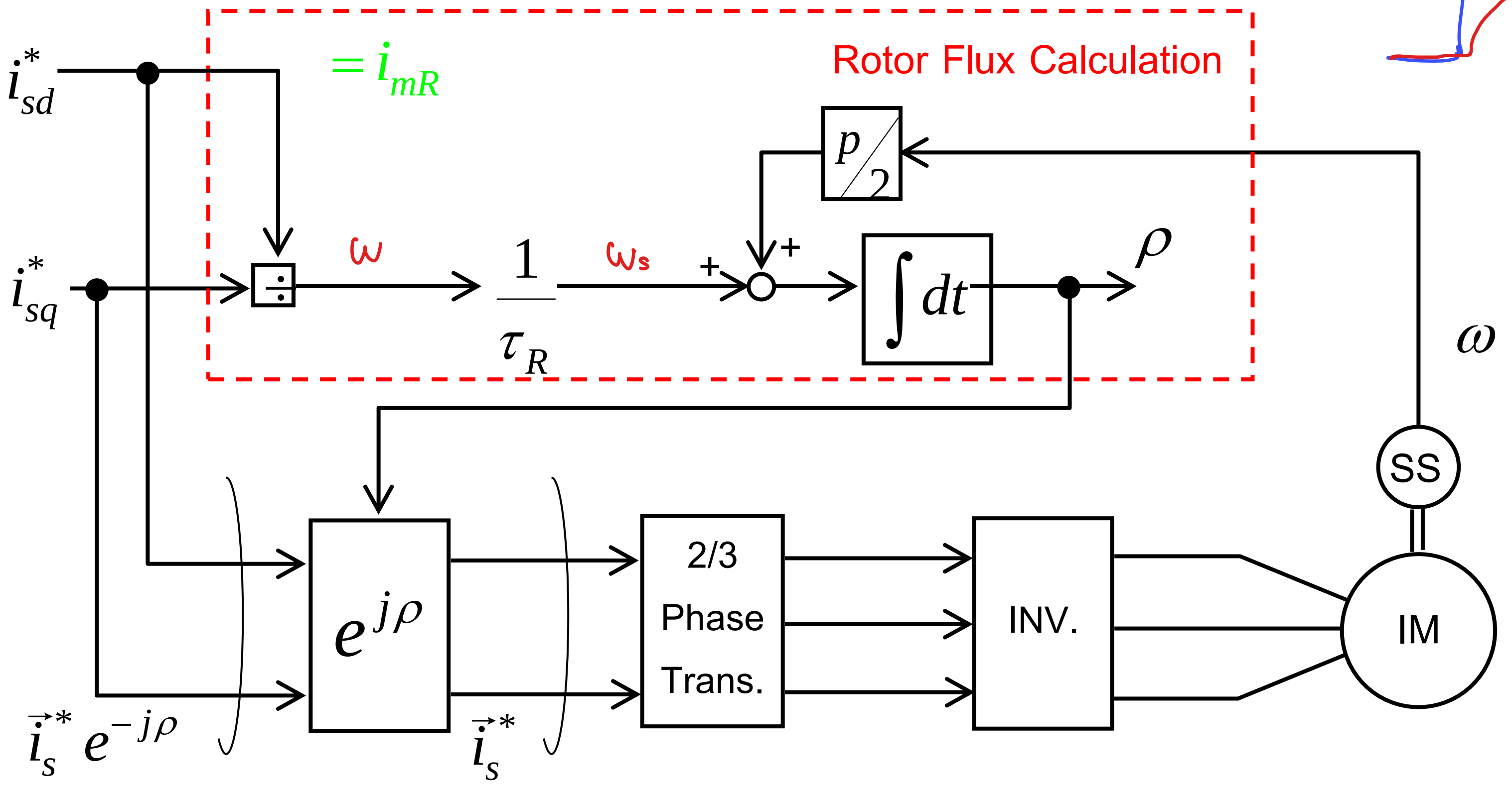
$i_{sd}^* = i_{sd} = i_{mR}$

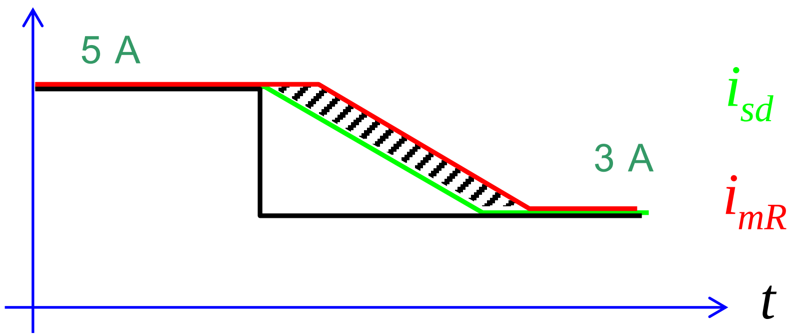



Rotor Flux Ref. Frame
i_{sy}
i_{sq}^*
i_{sd}^*
ρ
Stator Ref. Frame
i_{sx}

ถ้าเป็นระบบที่ flux คงที่ตลอดเวลา หรือ ละเลย Dynamic ของ $i_{mR}(t)$ ได้







- $i_{mR}(t)$, *flux เปลี่ยนตามความเร็วโรเตอร์* $\omega(t)$ *ซึ่งเปลี่ยนแปลงช้า (Mechanical Time Constant)*

4) Dynamic Model + Voltage Source

$\vec{i}_s \dashrightarrow \vec{i}_{mR}$

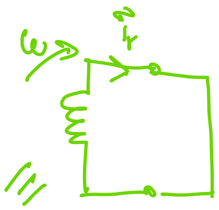
$$\begin{pmatrix} u_{s1} \\ u_{s2} \\ u_{s3} \end{pmatrix} \to \vec{u}_s \xrightarrow{\text{Stator eq}} \begin{matrix} i_{sd} \\ i_{sq} \end{matrix} \xrightarrow{\text{Rotor eq}} \begin{matrix} i_{mR} \\ m_M \end{matrix}$$

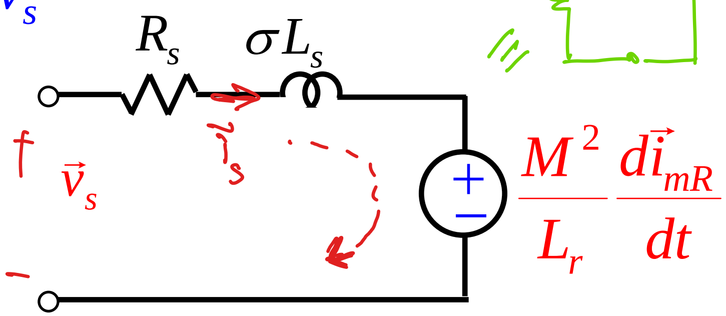
Stator dynamic

$$* \quad R_s \vec{i}_s + L_s \frac{d\vec{i}_s}{dt} + M \frac{d\vec{i}_r}{dt} = \vec{v}_s$$

แต่ $\vec{i}_r = \vec{i}_r{}' e^{j\theta_r} = \dfrac{M}{L_r}\left(\vec{i}_{mR} - \vec{i}_s\right)$

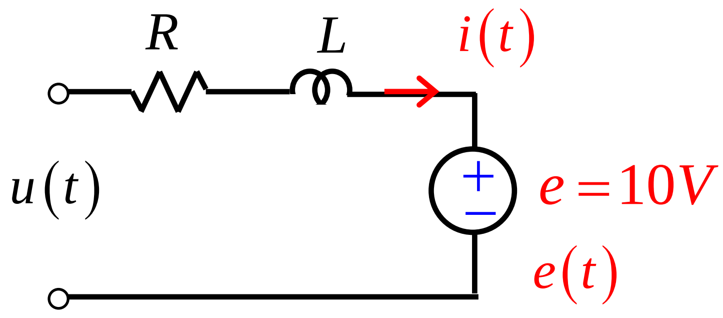
$$\therefore \quad R_s \vec{i}_s + \underbrace{\left(L_s - \frac{M^2}{L_r}\right)}_{\sigma L_s} \frac{d\vec{i}_s}{dt} + \underbrace{\frac{M^2}{L_r} \frac{d\vec{i}_{mR}}{dt}}_{\frac{M}{L_s}\frac{d}{dt}\left(\underbrace{M\vec{i}_{mR}}_{\vec{\lambda}_r}\right)} = \vec{v}_s$$





ex

$i(t) \rightarrow u(t)$



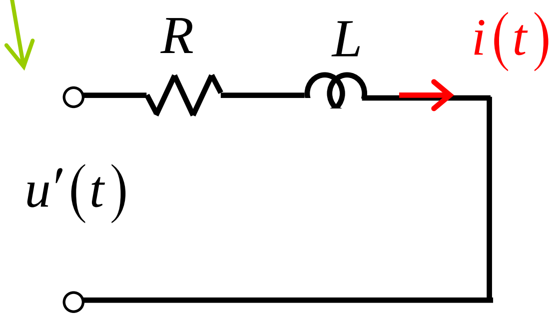
$$* \quad Ri(t)+L\frac{di(t)}{dt}+e(t) = u(t)$$

กำหนด

$$u(t)=\underbrace{10}_{e(t)}+u'(t)$$

จะได้

$$Ri(t)+L\frac{di(t)}{dt}+\underset{10}{\cancel{e(t)}} = u'(t)+\underset{e(t)}{\cancel{10}}$$



$$\boxed{Ri(t)+L\frac{di(t)}{dt} = u'(t)}$$ (ง่าย)

$$R\,i(t) + L\frac{di(t)}{dt} = i^{*}(t)\,R$$

สมมุติว่า $R = 1\Omega, \quad L = 1\,mH$

ต้องการให้ได้ $i(t) = 5A$

$$\frac{i(t)}{i^{*}(t)} = ?$$

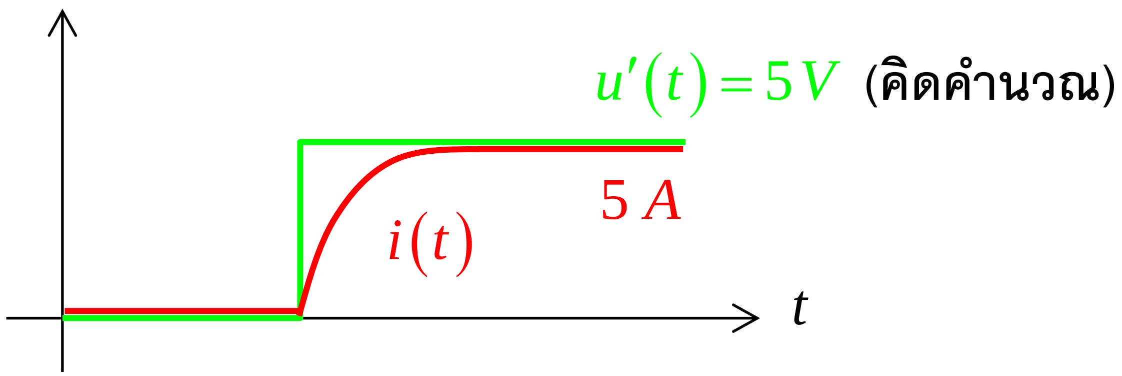
$$= i^{*}(t) \times R$$

$\Rightarrow$ **จ่ายแรงดัน** $u'(t) = 5 \times 1 = 5V$ **(คงที่)**

$$\therefore u(t) = 5 + e(t)$$

$$= 5 + 10 = \boxed{15V}$$

จ่ายจริง



1.11.๑๒

Rotor Dynamic

$$\frac{d\vec{i}_{mR}}{dt} = \left[-\frac{R_r}{L_r} + j\omega\right]\vec{i}_{mR} + \frac{R_r}{L_r}\vec{i}_s$$

$\frac{d\vec{i}_{mR}}{dt}$

Stator Dynamic บน Rotor Flux Ref. Frame

$$e^{-j\rho} \times \left\{ R_s\vec{i}_s + \sigma L_s\frac{d\vec{i}_s}{dt} + \frac{M^2}{L_r}\left[\left(-\frac{R_r}{L_r} + j\omega\right)\vec{i}_{mR} + \frac{R_r}{L_r}\vec{i}_s\right]\right\} = \vec{v}_s \cdot e^{-j\rho}$$

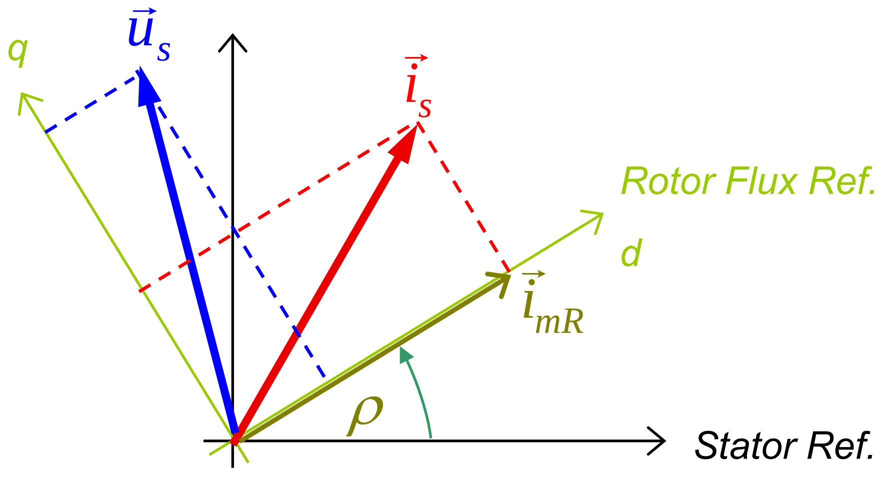
$\omega_{mR} = \frac{d\rho}{dt}$

$$R_s\vec{i}_s e^{-j\rho} + \sigma L_s\frac{d\vec{i}_s e^{-j\rho}}{dt} + j\omega_{mR}\sigma L_s\vec{i}_s e^{-j\rho}$$

$$+\frac{M^2}{L_r}\left(-\frac{R_r}{L_r} + j\omega\right)\vec{i}_{mR}\cdot e^{-j\rho} + \frac{M^2}{L_r^2}R_r\vec{i}_s \cdot e^{-j\rho} = \vec{v}_s \cdot e^{-j\rho}$$

นิยาม

$$\begin{cases} \vec{i}_{mR} e^{-j\rho} = i_{mR} \\ \vec{i}_s e^{-j\rho} = i_{sd} + j i_{sq} \\ \vec{u}_s e^{-j\rho} = u_{sd} + j u_{sq} \end{cases}$$



(d-axis)

Re :
$$R_s i_{sd} + \sigma L_s \frac{di_{sd}}{dt} - \underline{\underline{\omega_{mR} \sigma L_s i_{sq}}} - \frac{R_r M^2}{L_r^2}(i_{mR} - i_{sd}) = u_{sd}$$

(q-axis)

Im :
$$R_s i_{sq} + \sigma L_s \frac{di_{sq}}{dt} + \underline{\underline{\omega_{mR} \sigma L_s i_{sd}}} + \underline{\underline{\omega \frac{M^2}{L_r}}} i_{mR} + \frac{R_r M^2}{L_r^2} i_{sq} = u_{sq}$$

มีการเชื่อมโยงแรงเคลื่อนเหนี่ยวนำระหว่าง d-q axes !

กำหนดให้ Decoupling Control

control i_{sd}

ชดเชย e.m.f

$$
\begin{aligned}
u_{sd} &= u'_{sd} - \underline{\underline{\omega_{mR}\sigma L_s i_{sq}}} \\
u_{sq} &= u'_{sq} + \underline{\underline{\omega_{mR}\sigma L_s i_{sd}}} + \underline{\underline{\omega \frac{M^2}{L_r} i_{mR}}} + \underline{\underline{\frac{R_r M^2}{L_r^2} i_{sq}}}
\end{aligned}
$$

control i_{sq}

Decoupled Stator Dynamic

$$
\therefore \quad \begin{cases}
R_s i_{sd} + \sigma L_s \dfrac{di_{sd}}{dt} + \dfrac{R_r M^2}{L_r^2} \underbrace{\left(i_{sd} - i_{mR}\right)} = u'_{sd} \\
R_s i_{sq} + \sigma L_s \dfrac{di_{sq}}{dt} = u'_{sq}
\end{cases}
$$

term $i_{sd} - i_{mR}$: 1) steady state = 0

2) flux constant = 0

$$
\begin{array}{ccccc}
u'_{sd} & \longrightarrow & i_{sd} & \xrightarrow{\tau_R = L_r / R_r} & i_{mR} \\
u'_{sq} & \longrightarrow & i_{sq} & \longrightarrow & m_M
\end{array}
$$

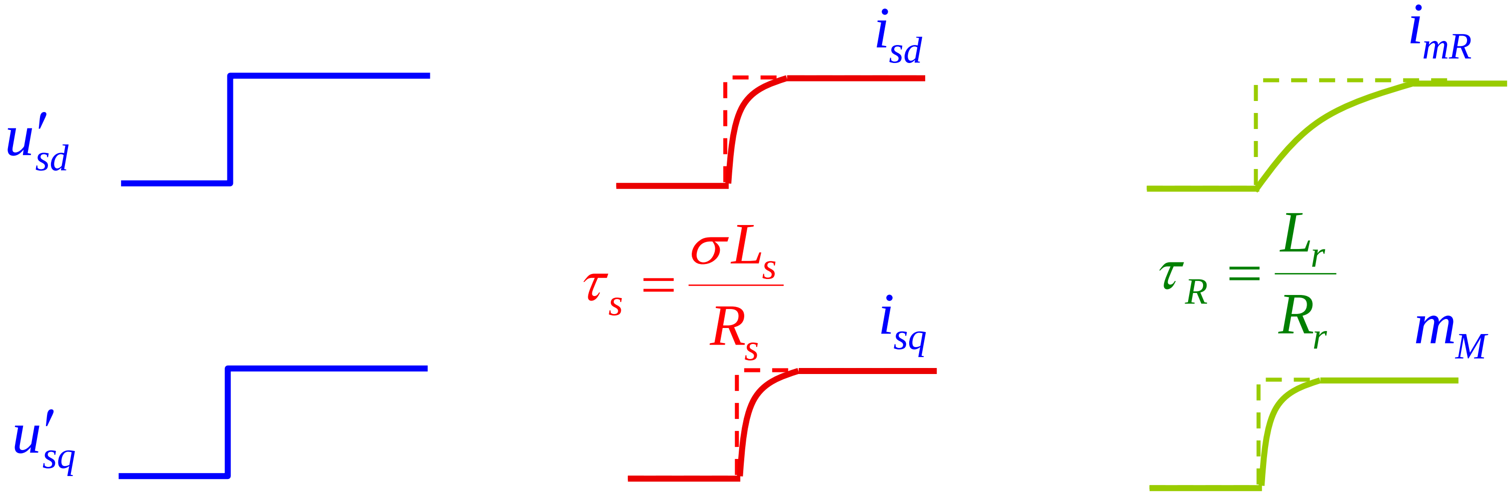
$$\tau_s = \frac{\sigma L_s}{R_s}$$

First-order lag

ไม่มี lag

10 ms

100 ms



Decoupling Control

$$u_{sd} = u'_{sd} - \omega_{mR}\sigma L_s i_{sq}$$

$$\left(\omega_s = \frac{R_r}{L_r}\frac{i_{sq}}{i_{mR}}\right)$$

$$u_{sq} = u'_{sq} + \omega_{mR}\sigma L_s i_{sd} + \omega\frac{M^2}{L_r}i_{mR} + \overbrace{\frac{M^2}{L_r}\omega_s i_{mR}}^{R_R\frac{M^2}{L_r^2}i_{sq}\ ;}$$

$$= u'_{sq} + \omega_{mR}\sigma L_s i_{sd} + \omega_{mR}\frac{M^2}{L_r}i_{mR}$$ ค่าจริง

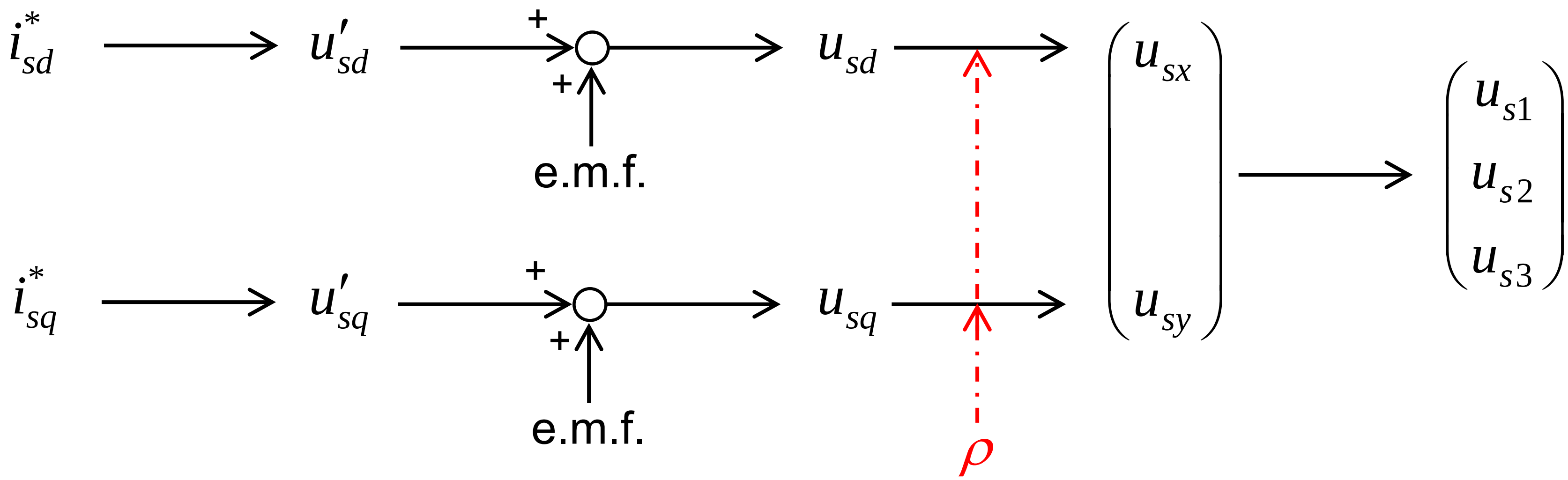
$$\sigma = 1 - \frac{M^2}{L_s L_r}$$

เนื่องจาก $i_{sd} \approx i_{mR}$

$$\therefore u_{sq} = u'_{sq} + \omega_{mR}\left[\sigma L_s + \frac{M^2}{L_r}\right]i_{sd}$$

$$\sigma L_s = L_s - \frac{M^2}{L_r}$$

$$= u'_{sq} + \omega_{mR}L_s i_{sd}$$ * ค่าโดยประมาณ



Decoupled Stator Dynamic ในเงื่อนไข $i_{mR} \cong i_{sd}$

$$\sigma L_s \frac{di_{sd}}{dt} + R_s i_{sd} = u'_{sd}$$
$$\sigma L_s \frac{di_{sq}}{dt} + R_s i_{sq} = u'_{sq}$$

เนื่องจากในสภาวะอยู่ตัว

$$u'_{sd} = R_s i_{sd}$$
$$u'_{sq} = R_s i_{sq}$$

$\therefore$ กำหนดให้

$$u'_{sd} = R_s i_{sd}^*$$
$$u'_{sq} = R_s i_{sq}^*$$

$\Rightarrow$ ในสภาวะอยู่ตัว $\begin{cases} i_{sd} = i_{sd}^* \\ i_{sq} = i_{sq}^* \end{cases}$

เนื่องจาก $\sigma L_s / R_s << 1$ (กระแสตอบสนองเร็ว)

เราประมาณได้ว่า $\begin{cases} i_{sd}(t) = i_{sd}^*(t) \\ i_{sq}(t) = i_{sq}^*(t) \end{cases}$

$\therefore$ Decoupling Control โดยประมาณจะเขียนได้เป็น

$$\boxed{\begin{aligned} u_{sd} &= R_s i_{sd}^* - \omega_{mR} \sigma L_s i_{sq}^* \\ u_{sq} &= R_s i_{sq}^* + \omega_{mR} L_s i_{sd}^* \end{aligned}}$$

Note เต็มรูปแบบ

$$u_{sd} = R_s i_{sd}^* - \omega_{mR} \sigma L_s i_{sq} + \left[\frac{M^2}{L_r^2} R_r \left[i_{sd} - i_{mR} \right] \right]$$

$$u_{sq} = R_s i_{sq}^* + \omega_{mR} \left[\sigma L_s i_{sd} + \frac{M^2}{L_r} i_{mR} \right]$$

ต้องคำนวณหา i_{sd}, i_{sq}, i_{mR}

Rotor Flux Dynamics

$$\frac{di_{mR}}{dt} = -\frac{R_r}{L_r}\left(i_{mR} - i_{sd} \right)$$

$$\frac{d\rho}{dt} = \omega_{mR} = \frac{p}{2}\omega + \frac{R_r}{L_r} \cdot \frac{i_{sq}}{i_{mR}}$$

<Decoupling Control>

ก) <u>เต็มรูปแบบ</u>

$$\begin{aligned} u_{sd} &= R_s i_{sd}^* - \omega_{mR}\sigma L_s i_{sq} + \left[\frac{M^2}{L_r^2} R_r \left[i_{sd} - i_{mR}\right]\right] \\ u_{sq} &= R_s i_{sq}^* + \omega_{mR}\left[\sigma L_s i_{sd} + \frac{M^2}{L_r} i_{mR}\right] \end{aligned}$$

$\Downarrow$

ประมาณว่า $i_{sd}(t) \cong i_{mR}(t)$

ข)

$$\begin{aligned} u_{sd} &= R_s i_{sd}^* - \omega_{mR}\sigma L_s i_{sq} \\ u_{sq} &= R_s i_{sq}^* + \omega_{mR} L_s i_{sd} \end{aligned}$$

$\Downarrow$

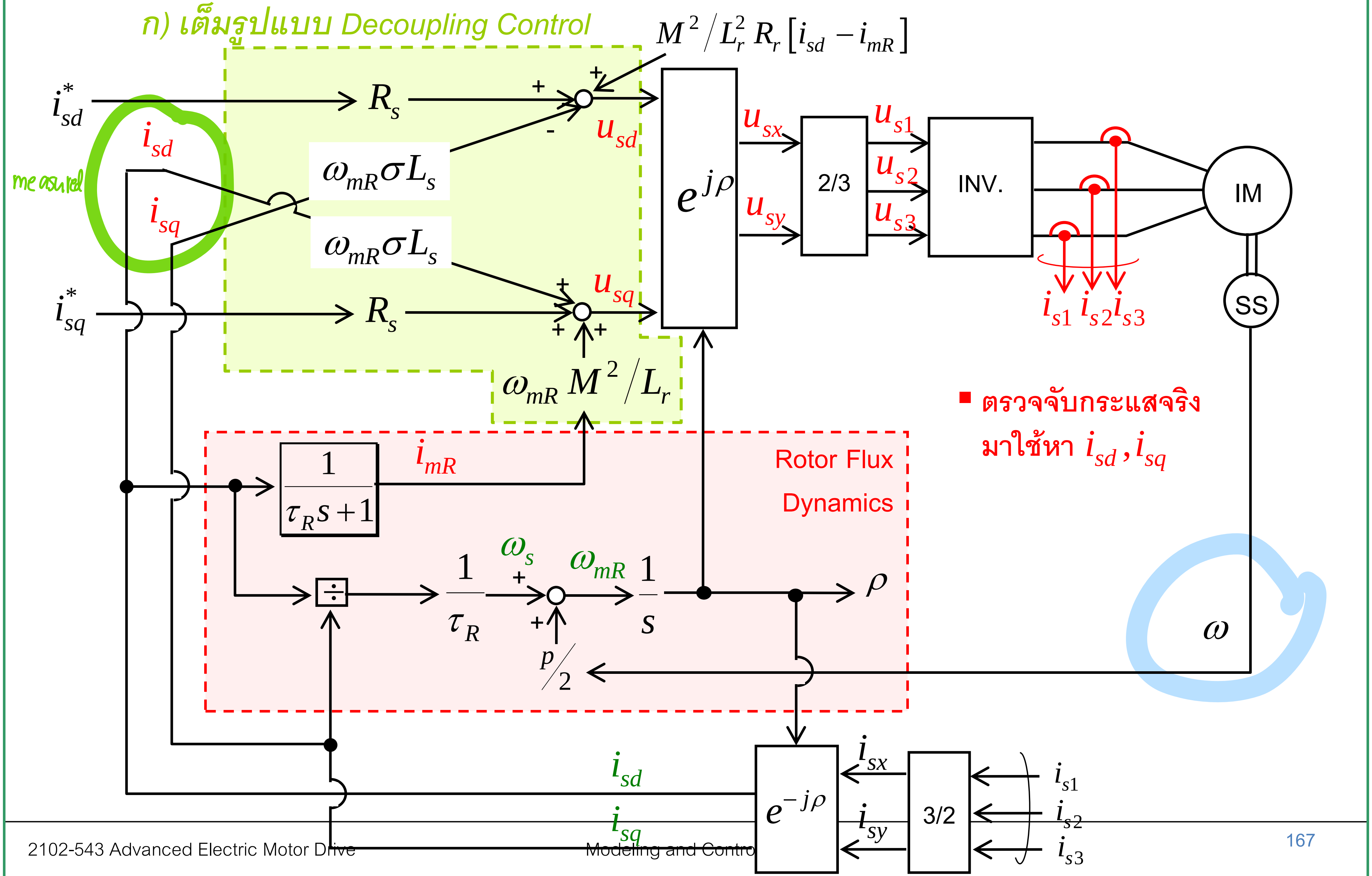
ประมาณว่า $i_{sd}(t) \cong i_{sd}^*(t)$

$i_{sq}(t) \cong i_{sq}^*(t)$

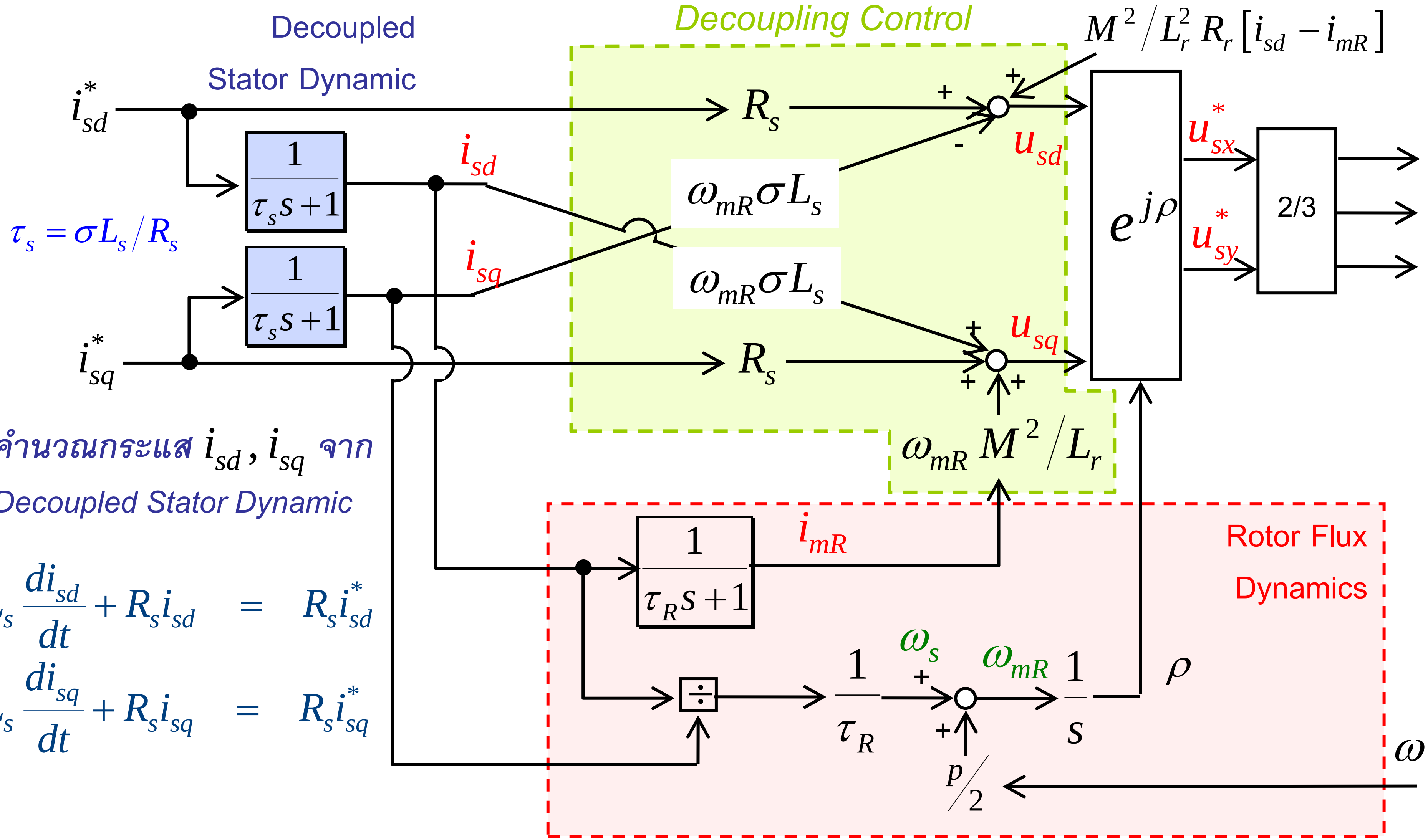
ค)

$$\begin{aligned} u_{sd} &= R_s i_{sd}^* - \omega_{mR}\sigma L_s i_{sq}^* \\ u_{sq} &= R_s i_{sq}^* + \omega_{mR} L_s i_{sd}^* \end{aligned}$$

ก) เต็มรูปแบบ Decoupling Control



- ตรวจจับกระแสจริง มาใช้หา i_{sd}, i_{sq}

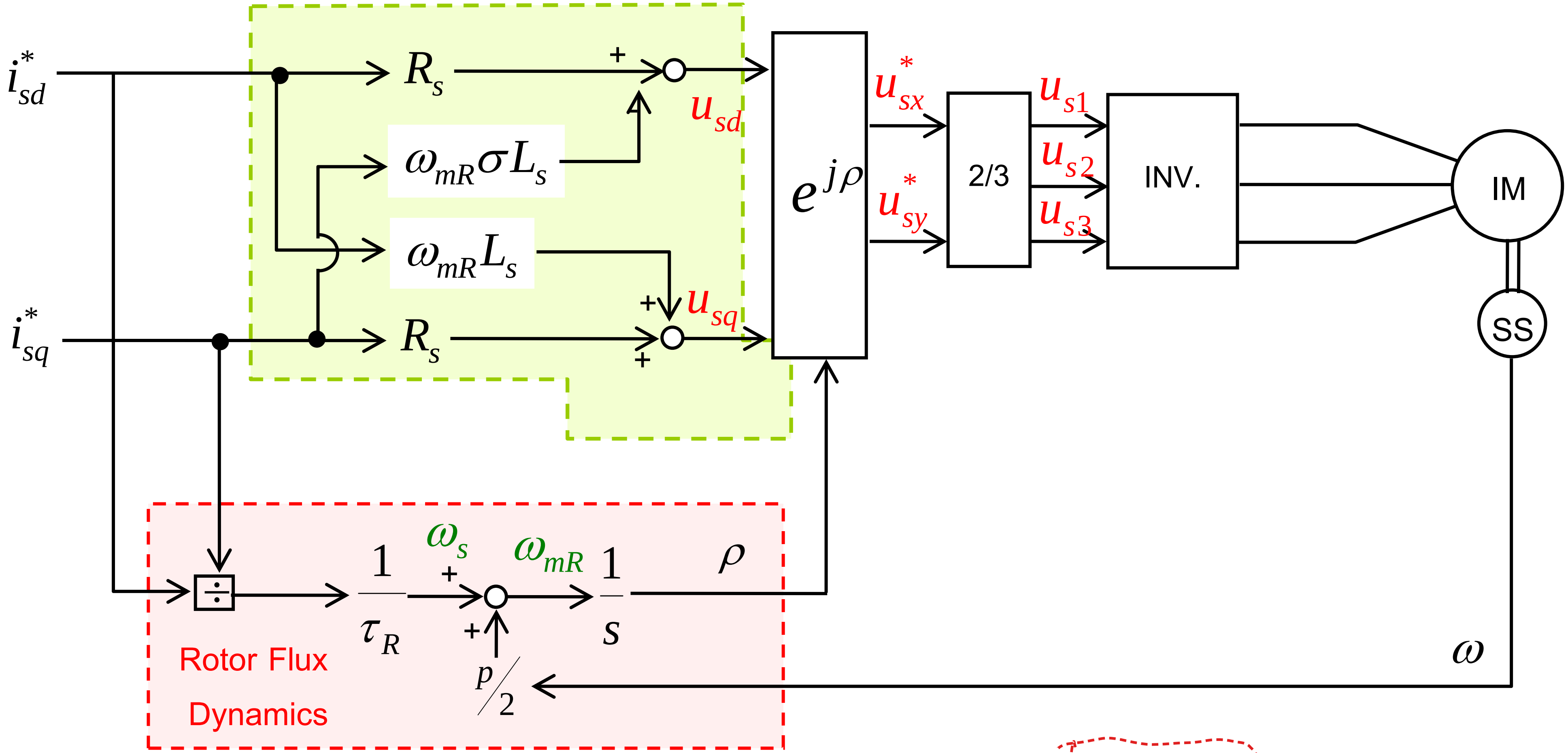


ข) คำนวณกระแส i_{sd}, i_{sq} จาก

Decoupled Stator Dynamic

$$
\begin{cases}
\sigma L_s \dfrac{di_{sd}}{dt} + R_s i_{sd} = R_s i_{sd}^* \\
\sigma L_s \dfrac{di_{sq}}{dt} + R_s i_{sq} = R_s i_{sq}^*
\end{cases}
$$

ค) Simplified Decoupling Control



Cond. $\begin{cases} i_{mR} \approx i_{sd} \approx i_{sd}^* \\ i_{sq} \approx i_{sq}^* \end{cases}$ (ประมาณ)

Decoupling Control with Current Feedback

Note: Decoupling Control เป็นการควบคุมแบบป้อนไปหน้า (feed forward)

$\rightarrow$ หากค่าพารามิเตอร์ไม่ถูกต้องหรือ Inverter สร้างแรงดันผิดเพี้ยนไปจากค่าคำสั่ง … $\rightarrow$ ทำให้ $\begin{cases} i_{sd} \neq i_{sd}^* \\ i_{sq} \neq i_{sq}^* \end{cases}$

$\Rightarrow$ เพิ่มการป้อนกลับกระแสสเตเตอร์จริง