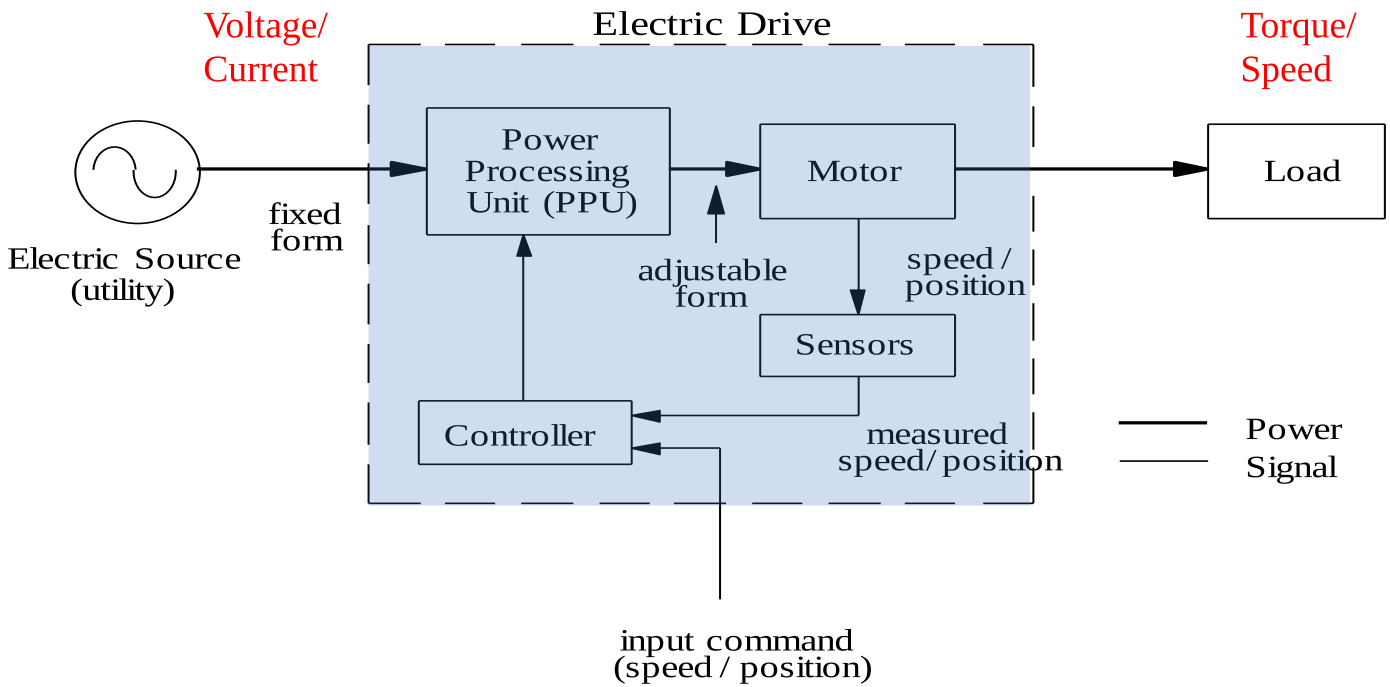
Understanding Mechanical System Requirements

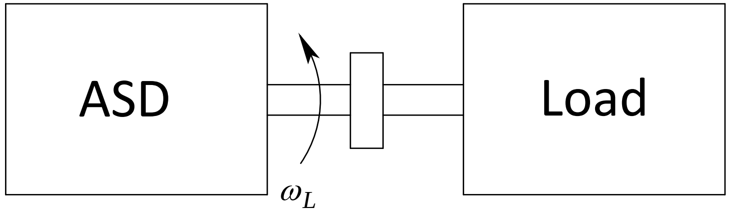
ASD

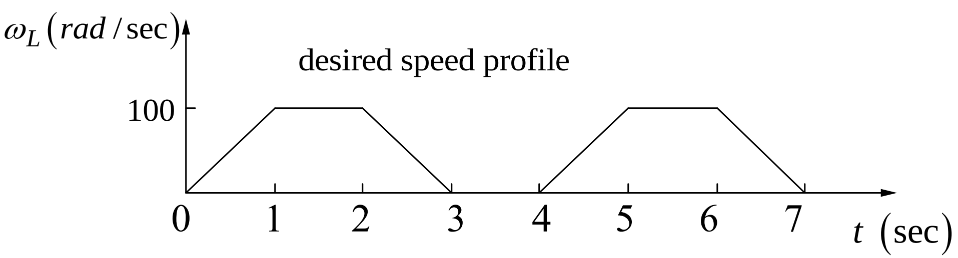
Block diagram of adjustable speed drives



- *Electric drives* are an interface between an *electrical system* and a *mechanical system*.

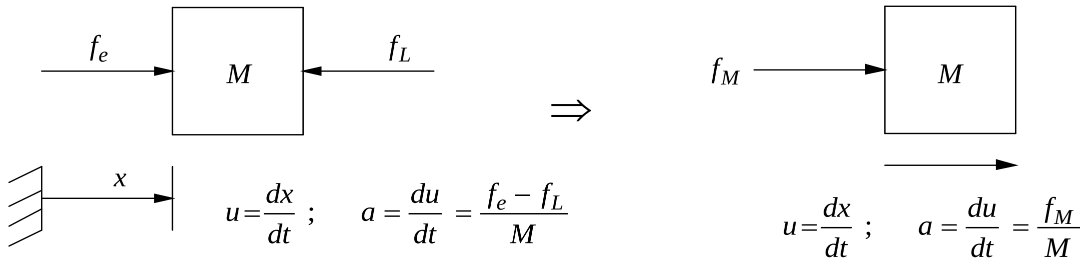
How can the ASD accelerate and decelerate the load to give desired speed profile?





- *Electric drive* must satisfy the requirements of *torque and speed* imposed by *mechanical loads* connected to them.
- The load in this figure may require a trapezoidal profile for the angular speed.
- Understanding *the requirements* from *mechanical system* is *necessary* for selecting an *appropriate electric drive* for a given application

Systems With Linear Motion



- Figure on left includes load force, f_L, that must be overcome
- Figure on right shows only the force, f_M, available to accelerate the mass, M

Accelaration

$$a = \frac{f_e - f_L}{M} = \frac{f_M}{M}$$

Power Input

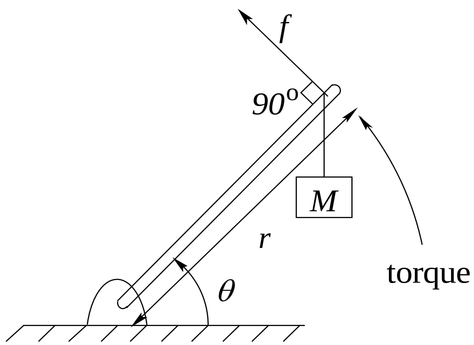
$$P_e(t) = f_e \cdot u = \underbrace{f_M \cdot u}_{\text{Power for accelerating the mass}} + \underbrace{f_L \cdot u}_{\text{Power for load (work or friction loss)}}$$

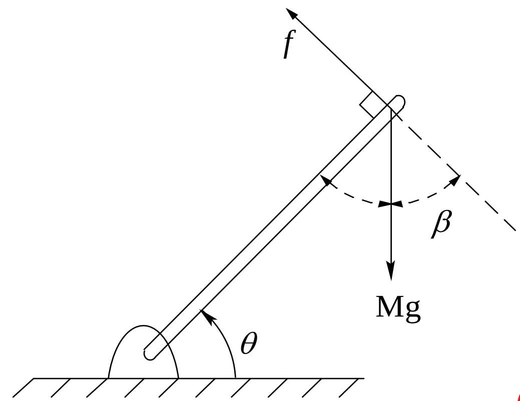
Kinetic energy

$$\underbrace{W_M = \frac{1}{2} M u^2}_{\text{Stored energy in the moving mass}}$$

Rotating Systems

- Most electric motors are of a *rotating* type.





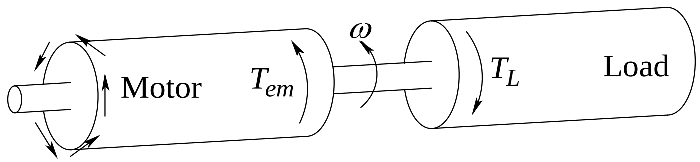
$f = Mg\cos\theta$

$\tau = Mgr\cos\theta$

- ❑ Torque = force radius
  $[Nm]$ $[N]$ $[m]$

- ❑ Example: what torque is needed to hold *M* motionless

Torque in an electric drive

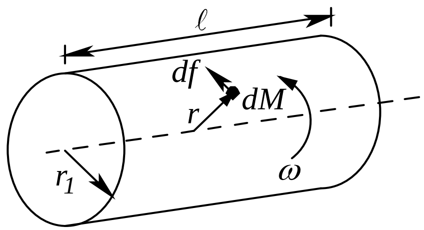


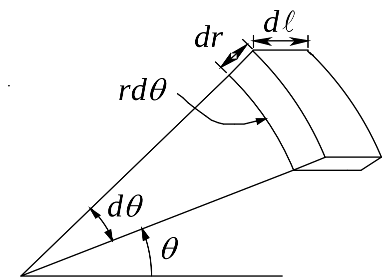
- T_{em} electromagnetic torque produced by motor
- T_{em} is opposed by load torque, T_L
- The difference, $T_{em} - T_L = T_J$, will accelerate the system
- $$\frac{d\omega}{dt} = \frac{T_{em} - T_L}{J} = \frac{T_J}{J}$$

  where J is the moment of inertia

- In a rotational system, the *angular acceleration*, due to the net torque acting on it, is determined by its *moment-of-inertia J*.

Calculation of Moment of Inertia J of a Uniform Cylinder





$$d\,f = dM\,\frac{d}{dt}v$$

$$\underset{arc}{dM = \rho\, rd\theta}\quad \underset{height}{dr}\quad \underset{length}{d\ell}$$

$$\Rightarrow dT = r^2\,dM\,\frac{d}{dt}\omega \;=\; \rho\,(\,r^3 dr\,d\theta\,d\ell\,)\frac{d}{dt}\omega$$

$$T = \rho\,(\int_0^{r_1} r^3 dr \int_0^{2\pi} d\theta \int_0^{\ell} d\ell\,)\frac{d}{dt}\omega \;=\; \underbrace{(\frac{\pi}{2}\rho\,\ell\,r_1^4\,)}_{J}\frac{d}{dt}\omega$$

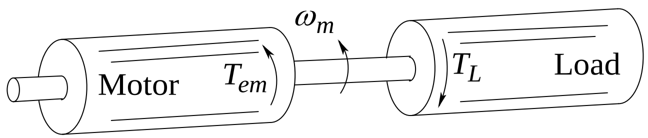
$$\boxed{J_{solid} = \frac{\pi}{2}\rho\,\ell\,r_1^4 = \frac{1}{2}M\,r_1^2}$$

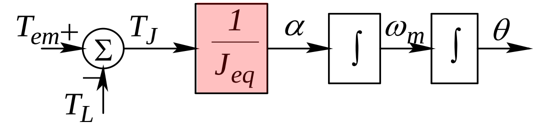
$$\because M = \rho\,(\,\pi r_1^2\,)\ell$$

$$\rho : material\;\; density\; in\; kg/m^3$$

$\rightarrow r \downarrow \Rightarrow J \downarrow$

Acceleration, Speed and Position, Power and Energy





Motor and load torque interaction with a rigid coupling

$$\boxed{\text{acceleration}, \quad \alpha = \frac{d\omega_m}{dt} = \frac{1}{(J_m + J_L)}(T_{em} - T_L) = \frac{T_J}{J_{eq}}}$$

$$\Rightarrow \text{speed}, \quad \omega_m(t) = \omega_m(0) + \int_0^t \alpha(\tau)\, d\tau$$

$$\Rightarrow \text{position}, \quad \theta(t) = \theta(0) + \int_0^t \omega(\tau)\, d\tau$$

Dynamic model of mechanical system

Torque is the fundamental variable for controlling speed and position.

Power Input

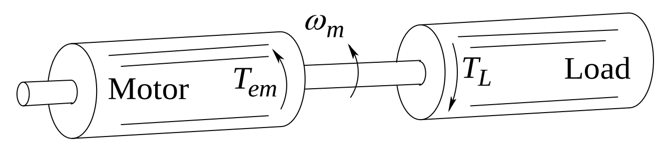
$$P_{em}(t) = T_{em} \cdot \omega = \underbrace{T_J \cdot \omega}_{\text{Power for accelerating the rotating mass}} + \underbrace{T_L \cdot \omega}_{\text{Power for load (work or friction loss)}}$$

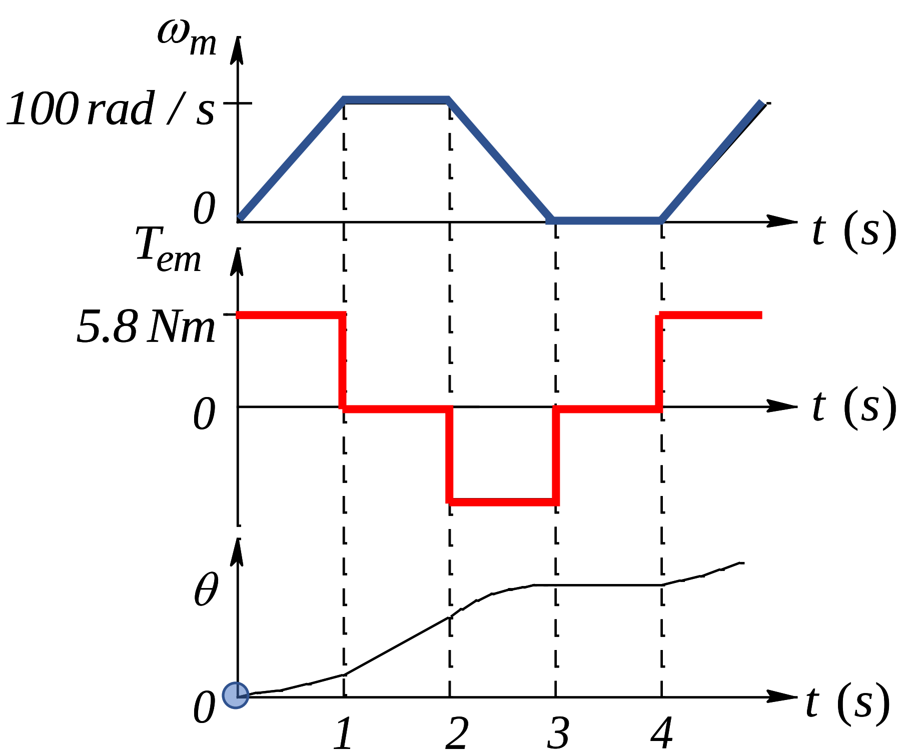
Kinetic energy

$$\underbrace{W = \frac{1}{2} J \omega^2}_{\text{Stored energy in the rotating mass of inertia } J}$$

Regenerative Braking.

Speed, torque, and angle variations with time





Consider that the above rotating system has the combined inertia *Jeq* = 0.058 kg.m^2 and is required to have a trapezoidal profile of angular speed profile shown on the left. The load torque is zero.

Calculate and plot the electromagnetic torque *Tem* required from the motor, and the change in position.

From the given waveform of angular speed, the magnitude of acceleration and the deceleration is 100 rad/s^2 .

During the intervals of acceleration and deceleration, since $T_L = 0$,

$$\boxed{T_{em} = T_J = J_{eq}\frac{d\omega_m}{dt} = \pm 5.8\ Nm}$$

During intervals with a constant speed, no torque is required

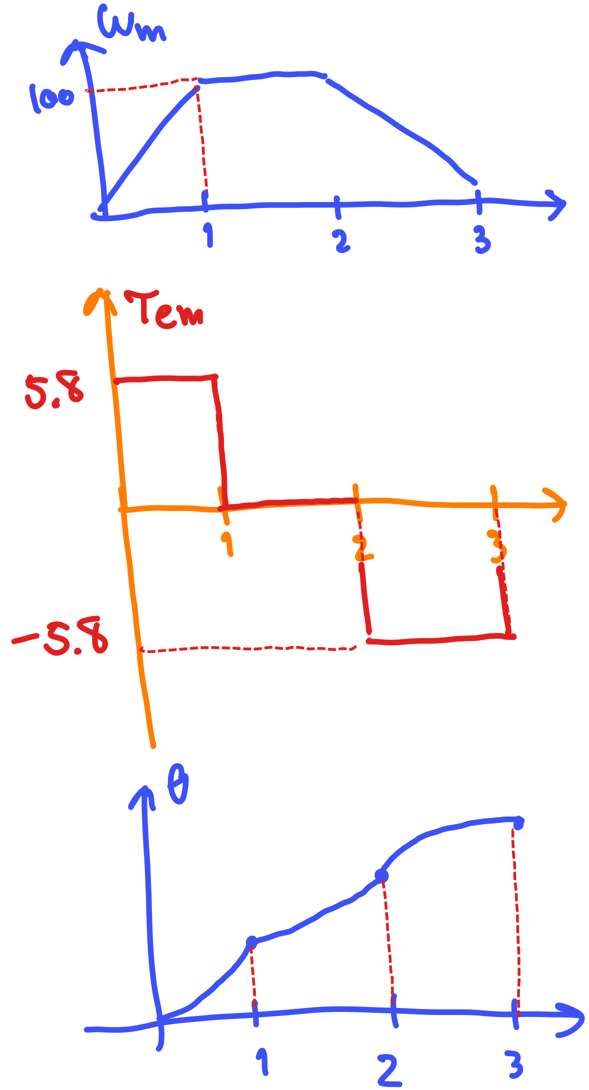
The position is the time-integral of speed :

$$\theta = \theta(0) + \int_0^t \omega_m d\tau \quad ; \quad assume\ \ \theta(0) = 0$$

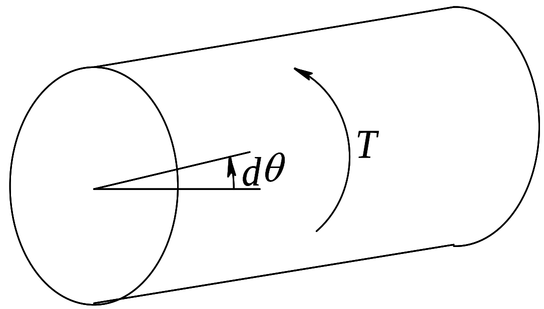
$$T_{em} - \overset{0}{\cancel{T_L}} = J_{eq} \frac{d\omega_m}{dt}$$

$$= 0.058 \times 100$$

$$T_{em} = 5.8 \text{ Nm}$$



Torque, work, and power



Rotational system

- If a net torque T causes the cylinder to rotate by a differential angle $d\theta$, the differential work done is

$$dW = T\,d\theta$$

- The power can be expressed as:

$$p = \frac{dW}{dt} = T\,\frac{d\theta}{dt} = T\,\omega_m$$

- Substitute for torque T :

$$p = \underbrace{J\,\frac{d\omega_m}{dt}}_{T}\,\omega_m$$

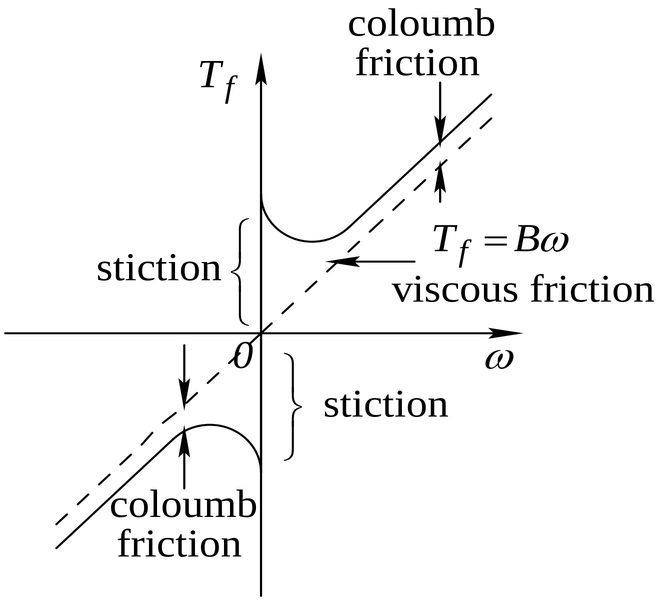
- The *kinetic energy* stored in the rotating mass of inertia J is :

$$W = \int_0^t p(\tau)\,d\tau = J\int_0^t \omega_m\,\frac{d\omega_m}{d\tau}\,d\tau = J\int_0^{\omega_m} \omega_m\,d\omega_m = \frac{1}{2}\,J\,\omega_m^2$$

- *This stored kinetic energy can be recovered by making the power p(t) reverse direction (p(t) <0) !!*

Friction in Machine
- Bearing
- Fan

Frictional Torque



- Stiction: static component
- Coulomb friction: dynamic component (constant magnitude)
- Viscous friction: speed dependent
- In general, friction is non-linear

- *Friction* within the motor and the load acts to *oppose rotation*.
- *Friction* occurs in the *bearings* that support rotating structure and moving objects in air encounter *windage or drag*.
- In *vehicles*, this *drag* is a major force that must be overcome.
- The *friction characteristic* may be linearized for an *approximate* analysis by means of the *dotted line*.
- With this approximation, the characteristic is similar to viscous friction in which

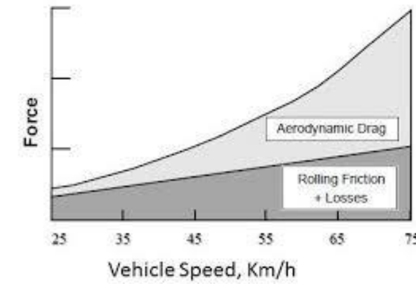
$$\boxed{T_f = B\omega_m}$$

Where B is the coefficient of viscous friction or viscous damping.

Example: Aerodynamic drag

- The aerodynamic *drag force* in automobiles is a kind of *resistance force*.
- It is dependent on the velocity u of the vehicles.

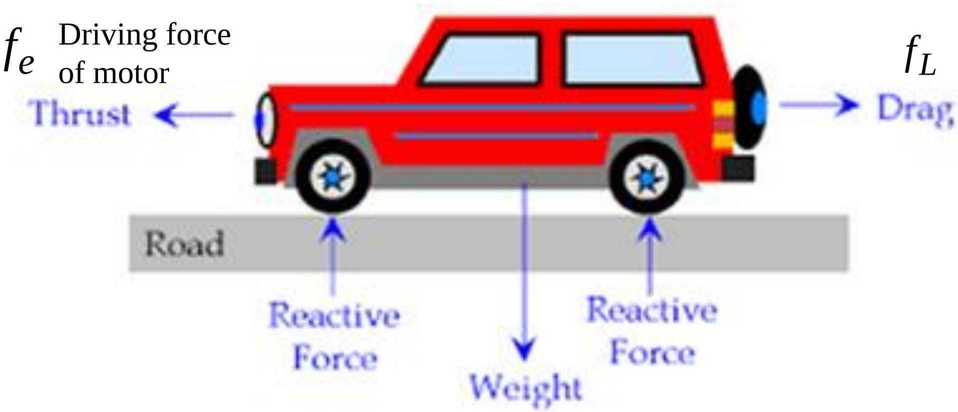
$$f_L = 0.046\, C_w\, A u^2$$



u (speed)

f_e Driving force of motor

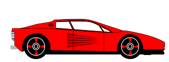
f_L



Drag power at different speeds

$$p = f_L \cdot u$$

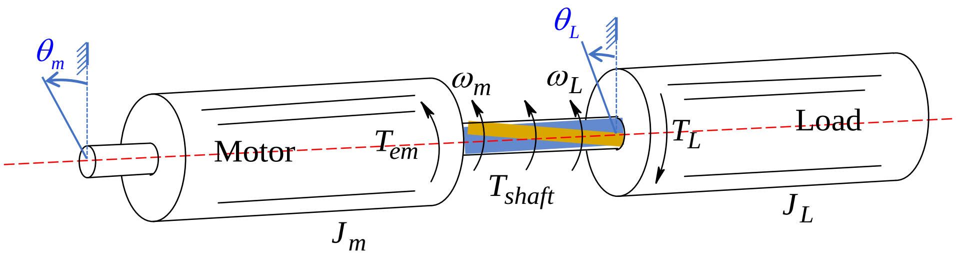
$\therefore$ power α speed^3

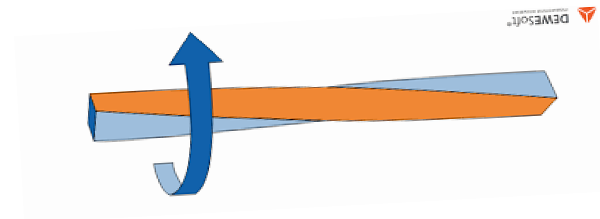
Speed (km/h)	Power (W)	
	 $C_w = 0.3$	 $C_w = 0.5$
50	0.86 kW	1.44 kW
100	6.9 kW	11.5 kW
150	23.3 kW	38.8 kW

C_w: drag coefficient (a unit-less quantity)

A: the vehicle cross-sectional area in m^2

Flexible Coupling /Torsional Resonances



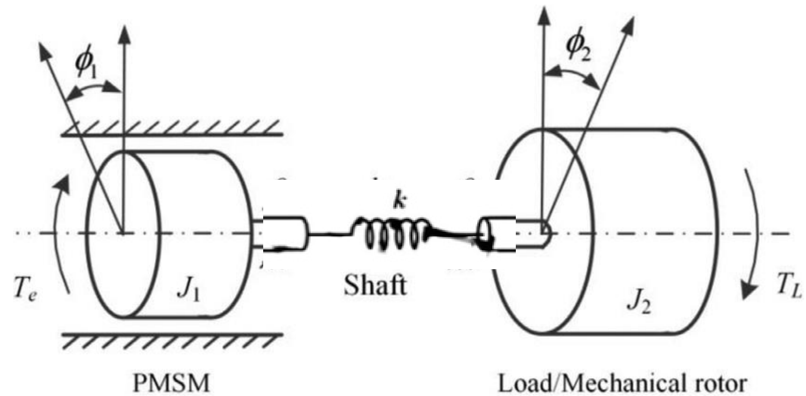


In reality, any shaft will twist (flex) as it transmits torque from one end to the other.

At motor end $T_{shaft} = T_{em} - J_m \dfrac{d\,\omega_m}{dt}$

At load end $T_{shaft} = T_L + J_L \dfrac{d\,\omega_L}{dt}$

$$\boxed{(\theta_m - \theta_L) = \frac{T_{shaft}}{K}}$$



K : shaft torsional or the compliance coefficient

θ_m and θ_L :angular rotation at the two ends of the shaft

- For rigid coupling: $K \to \infty,\ \theta_m = \theta_L,\ \omega_m = \omega_L$
  (J_M and J_L can be treated as one inertial mass: $J_{eq} = J_M + J_L$)
- Finite K may lead to resonances

- The shaft of finite compliance will act as a spring.
- This compliance in the presence of energy storage in the masses, and inertias of the system, can lead to resonance at certain frequencies.

Mechanical - Electrical Analogy

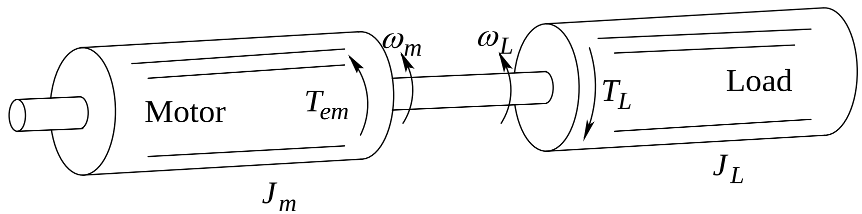
- Analogy with electrical circuits can be very useful when analyzing mechanical system.
- A commonly used analogy is to relate mechanical and electrical quantities, as show in Table 2-2.

TABLE 2-2 Torque–Current Analogy

Mechanical system	Electrical system
Torque (T)	Current (i)
Angular speed (ω_m)	Voltage (v)
Angular displacement (θ)	Flux linkage (ψ)
Moment of inertia (J)	Capacitance (C)
Spring constant (K)	1/Inductance ($1/L$)
Damping coefficient (B)	1/Resistance ($1/R$)
Coupling ratio (n_M/n_L)	Transformer ratio (n_L/n_M)

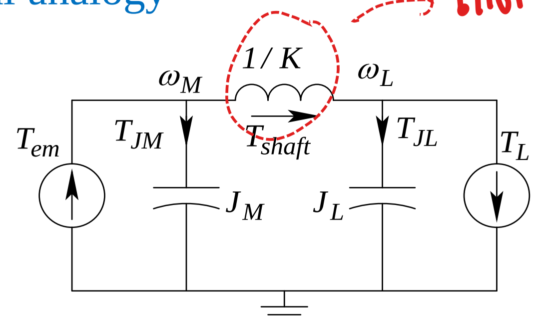
Electrical Analogy of Motor & Load

Mechanical system

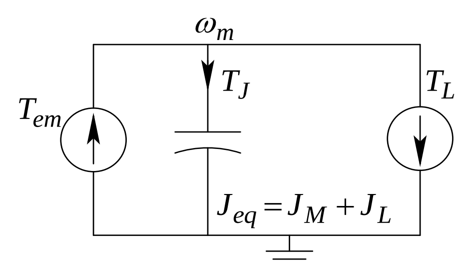


Electrical analogy

เกิด Resonance ได้



Finite shaft stiffness



Infinite shaft stiffness

- Each *inertia* is represented by a *capacitor* from its node to a reference (ground) node.
- In this circuit, the *similar differential equations* can be written.

- If the shaft is of infinite *stiffness* (K $\rightarrow \infty$), the *inductance* representing it becomes zero.
- In the resulting circuit, $\omega_m = \omega_L$,the two capacitors representing the two inertias can now be combined to result in a single equation.

Coupling Mechanisms

- It is preferable to couple the load directly to the motor, to avoid:
  - *Additional cost and power loss*
  - *Nonlinearity due to backlash*
  - *Wear and tear*

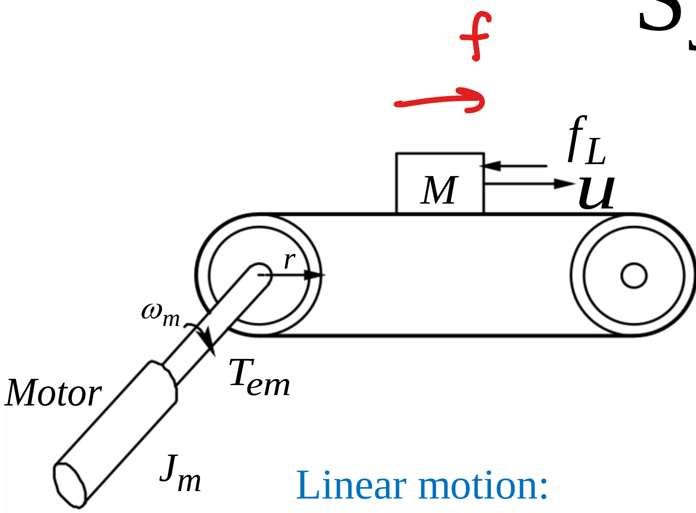
❑ Required when

- a (rotary) motor is driving a load which requires linear (translational) motion
- motors prefer higher rotational speed than that required by the load
- the axis of rotation needs to be changed

❑ Types

- Conveyor belts (belt and pulley)
- Rack and pinion or a lead-screw type of arrangement
- Gear mechanisms

Conversion between Linear and Rotary Systems



J_m = motor inertia

M = mass of load

r = pulley radius

Linear motion:

$$f = M\frac{du}{dt} + f_L$$

Angular and linear speeds are related by the radius r:

$$u = r\,\omega_m$$

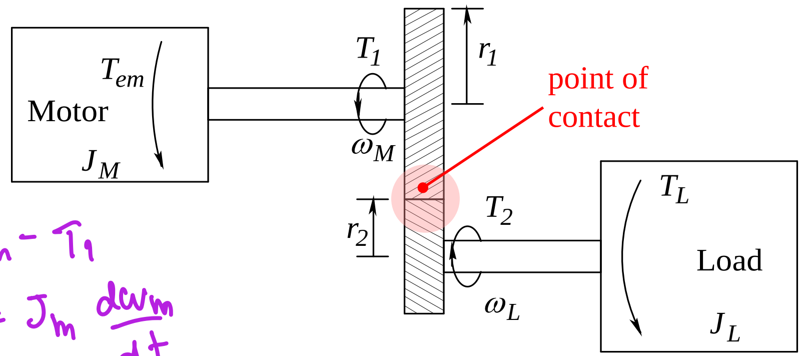
Force *f* is delivered by the motor in the form of a torque *T*:

$$T = r\,f = r^2 M\frac{d\omega_m}{dt} + r\,f_L$$

Electromagnetic torque required from the motor is:

$$T_{em} = \underbrace{J_m\frac{d\,\omega_m}{dt}}_{\text{required to accelerate motor}} + \underbrace{r^2 M\frac{d\,\omega}{dt} + r\,f_L}_{\text{due to load}}$$

Gears



$T_{em} - T_1 = J_m \frac{d\omega_m}{dt}$

$T_2 = T_L + J_L \frac{d\omega_L}{dt}$

Purpose:

- Gears are used for matching speeds.

Assumption:

- Shafts are assumed to be of infinite stiffness (rigid)
- Masses of gears are ignored.
- No power loss in the gears.

Principle:

- Both gears must have the same linear speed at the point of contact.

❑ Basic relationships: radius, speed, torque

Equal speeds at gear surfaces $\Rightarrow \boxed{r_1 \omega_M = r_2 \omega_L}$

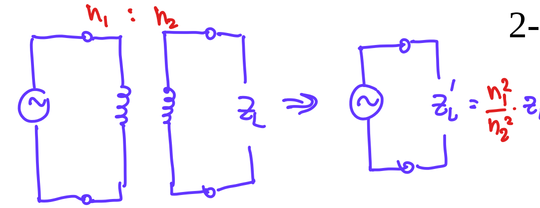
Power transferred across gears $\Rightarrow \omega_M T_1 = \omega_L T_2$, (… Assuming no power loss)

$$\Rightarrow \frac{r_1}{r_2} = \frac{\omega_L}{\omega_M} = \frac{T_1}{T_2} \qquad \& \quad \underbrace{\left(T_{em} - J_M \frac{d\omega_M}{dt}\right)}_{T_1} \frac{\omega_M}{\omega_L} = \underbrace{\left(T_L + J_L \frac{d\omega_L}{dt}\right)}_{T_2}$$

❑ Geared up: speed increased, torque decreased $\omega_L > \omega_M$; $T_2 < T_1$; $r_2 < r_1$

❑ Geared down: speed decreased, torque increased $\omega_L < \omega_M$; $T_2 > T_1$; $r_2 > r_1$

Gears (cont'd)



❑ Equivalent Inertia

$$T_{em} = \underbrace{\left[J_m + J_L \left(\frac{\omega_L}{\omega_m} \right)^2 \right]}_{J_{eq}} \frac{d\,\omega_m}{dt} + \left(\frac{\omega_L}{\omega_m} \right) T_L$$

$$\boxed{\Rightarrow J_{eq} = J_m + J_L \left(\frac{\omega_L}{\omega_m} \right)^2 = J_m + J_L \left(\frac{r_1}{r_2} \right)^2}$$

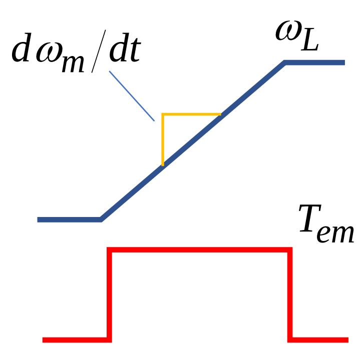
❑ Optimum gear ratio (to minimize T_{em})

$$T_{em} = \underbrace{\left[J_m + J_L \left(\frac{r_1}{r_2} \right)^2 \right]}_{J_{eq}} \frac{d\,\omega_m}{dt}$$

- The electromagnetic torque T_{em} required from motor to accelerate a motor-load combination depends on the gear ratio.

(For inertial load, TL can be assumed to be negligible.)

$$\left. \frac{d\,T_{em}}{d\left(r_1 / r_2 \right)} \right|_{d\omega_L/dt} = 0$$



- For a given load-acceleration $d\omega_m/dt$, T_{em} can be minimized by selecting an optimum gear ratio $\left(r_1/r_2 \right)_{opt}$
- The derivative of the optimum gear ratio shows that the load inertia "seen" by the motor should equal the motor inertia.

$$\boxed{J_m = \left(\frac{r_1}{r_2} \right)_{opt.}^2 \cdot J_L \quad \Rightarrow \quad \left(\frac{r_1}{r_2} \right)_{opt.} = \sqrt{\frac{J_m}{J_L}}}$$

and $\left(T_{em} \right)_{opt.} = 2\,J_m \dfrac{d\,\omega_m}{dt} = 2\,J_m \left(\dfrac{r_2}{r_1} \right)_{opt.} \dfrac{d\omega_L}{dt}$

Types of Loads

- Load torques normally act to oppose rotation.

- Loads can be classified into the following categories:

    1) Centrifugal (Squared) Torque

    2) Constant Torque

    3) Squared Power

    4) Constant Power

Centrifugal loads

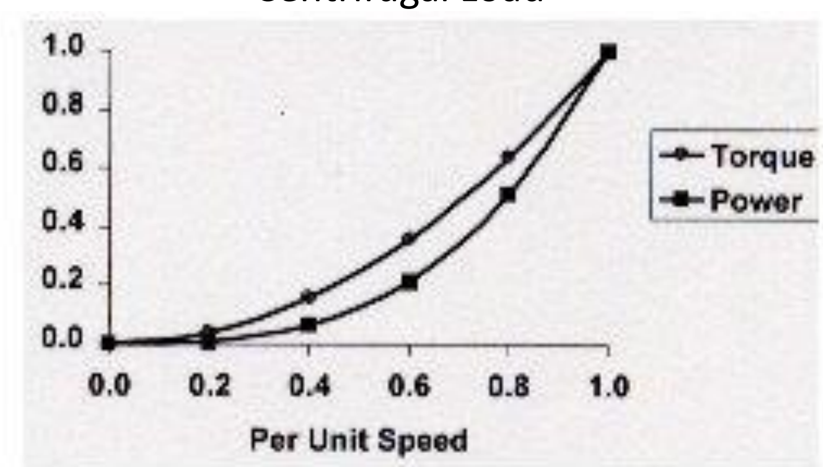
- Centrifugal loads, such as fans and blowers, require torque that varied with speed^2

$$\boxed{T_L = k\,\omega_L^2}$$

- And load power that varies with speed^3

$$P_L = T_L\,\omega_L = k\,\omega_L^3$$

Centrifugal Load



Blower

Constant-Torque Loads

- In constant-torque loads, such as conveyors, hoists, cranes and elevators, torque remains constant with speed

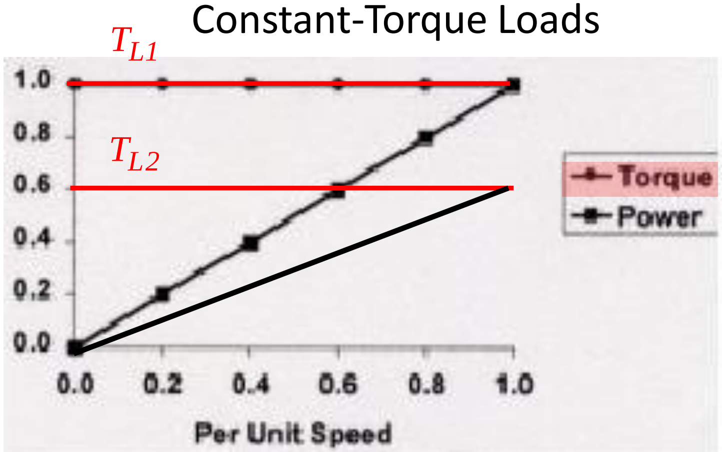
$$\boxed{T_L = const}$$

- And load power varies linearly with speed

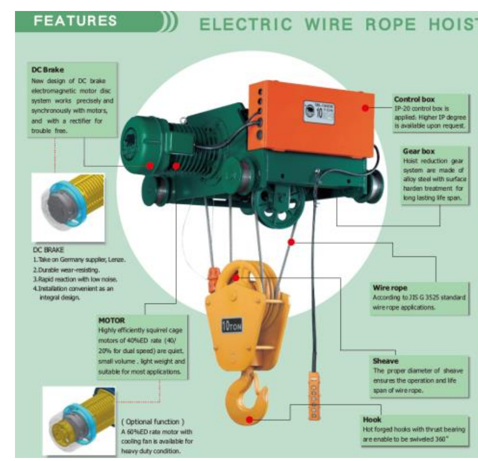
$$P_L = T_L \omega_L \propto \omega_L$$

Elevator

Constant-Torque Loads



Hoist



Squared-Power Loads

- In squared-power loads, such as compressors and rollers, torque vary linearly with speed

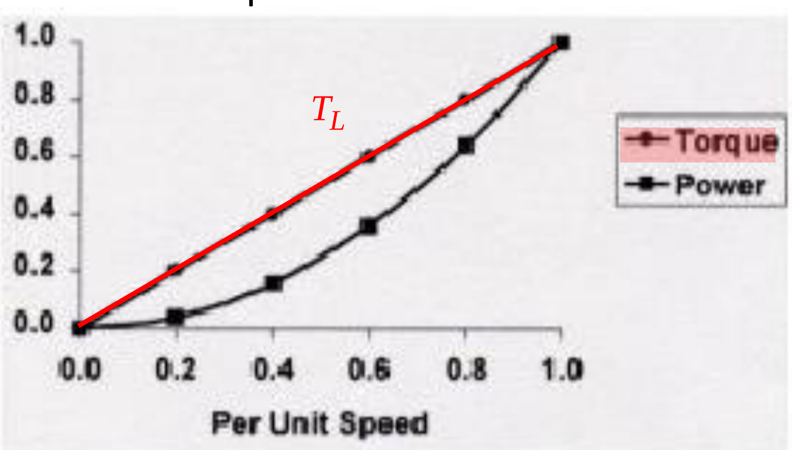
$$\boxed{T_L = k\,\omega_L}$$

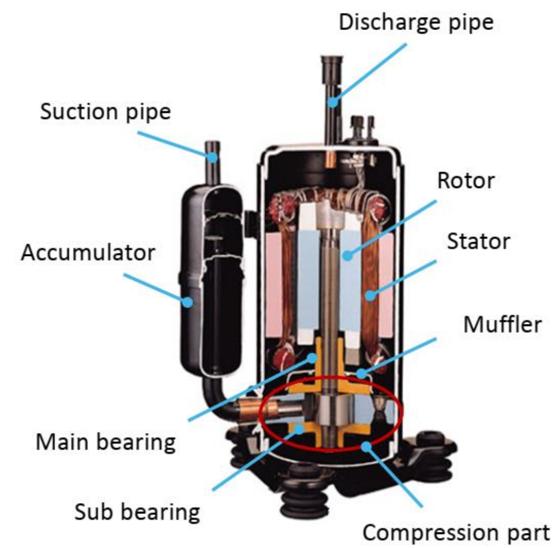
Compressor for Air Conditioner

- And load power varies with speed^2

$$P_L = T_L\,\omega_L \propto k\,\omega_L^2$$

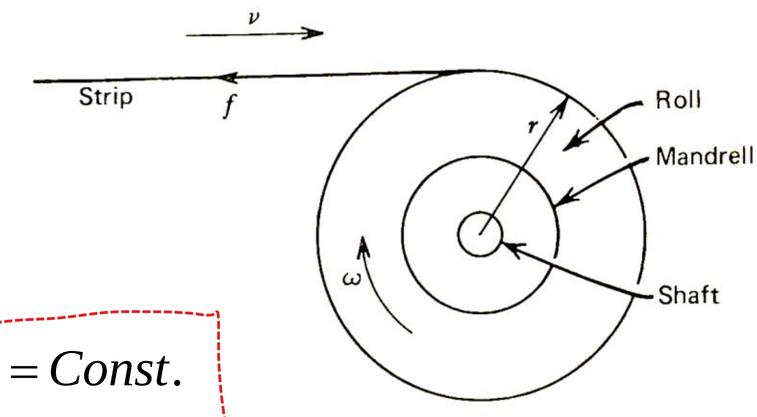
Squared-Power Load





Constant-Power Loads

- Typical applications of constant-power load over a range of speed are the take-up roll (or winder and unwinder) in a steel strip, plastic or paper mill.



- Basic operation of take-up roll:
  - If *a satisfactory roll is to be formed*, the strip tension or the force f must be constant :
  - The strip will emerge from the mill rolls at constant speed v:

$$f = Const.$$

$$v = Const.$$

- Torque and angular speed are:

$$T_L = f \cdot r \quad ; \quad \omega_L = \frac{v}{r}$$

$r \uparrow$

$\Rightarrow T_L \uparrow$

- The power is therefore constant :
- The relationship between torque and angular speed is hyperbola:
- The radius of the roll is varied; for the winder, the radius *r* will build up. On the other hand, for the unwinder, the radius *r* will fall off.

$$\boxed{P_L = f \cdot v = T_L \cdot \omega_L = Const.}$$

Constant-Power Loads (Cont.)

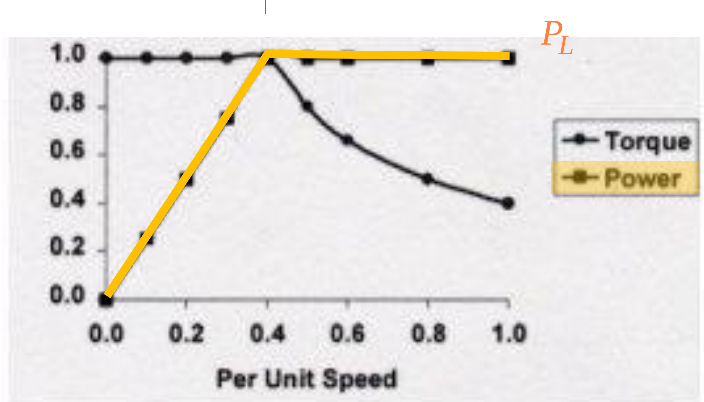
- In constant-power loads, such as winder and unwinder and rollers, the torque beyond a certain speed range varies inversely with speed

$$\boxed{T_L = \frac{k}{\omega_L}}$$

- And load power remains constant with speed

$$P_L = T_L\,\omega_L = k \quad (Const.)$$

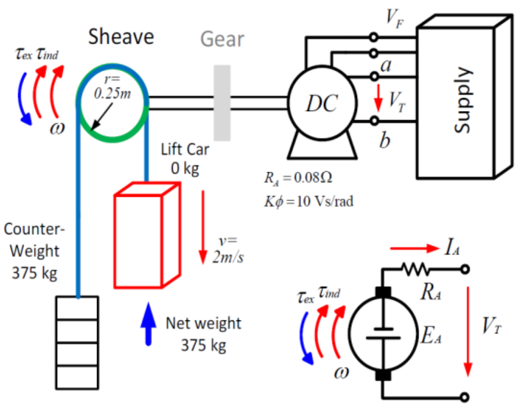
Constant power loads



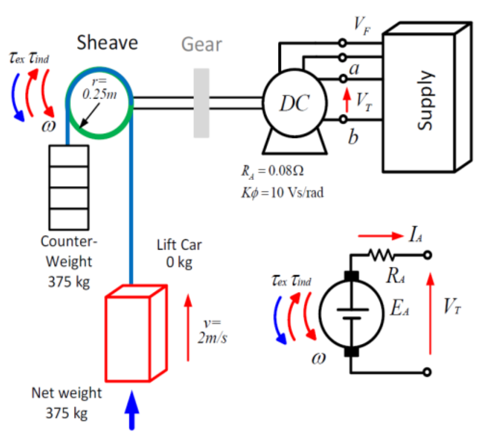
Winder and Unwinder

Four-Quadrant Operation

Elevator drive

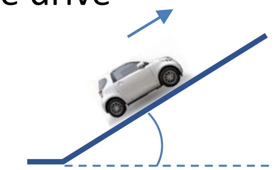


Operation in quadrant 3

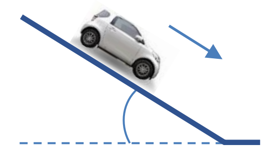


Operation in quadrant 4

Electric-Vehicle drive

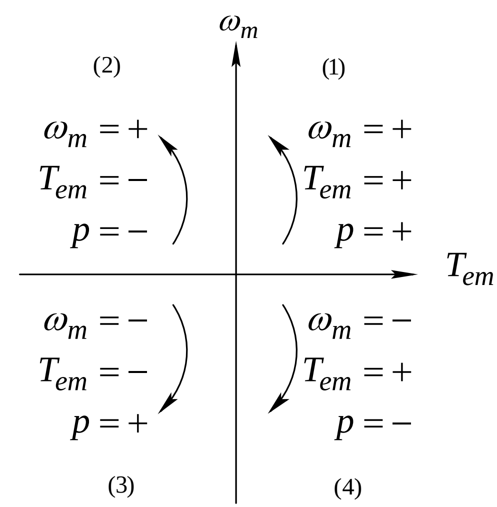


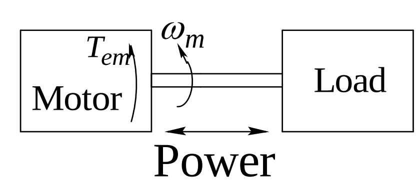
Operation in quadrant 1



Operation in quadrant 2

- In many high-performance systems, drives are required to operate in all four quadrants of the torque-speed plane.
- The motor drives the load in forward direction in quadrant 1, and in the reverse direction in quadrant 3. And the average power is positive (p = +) and flows from the motor to mechanical load.
- To control the load speed rapidly, it may be necessary to operate the system in the regenerative braking mode, where the power flows from the load into the motor.
- In quadrant 2, the speed is positive, but the torque produce by the motor is negative. In quadrant 4, the speed is negative, and the motor torque is positive.





Required Drive Characteristics

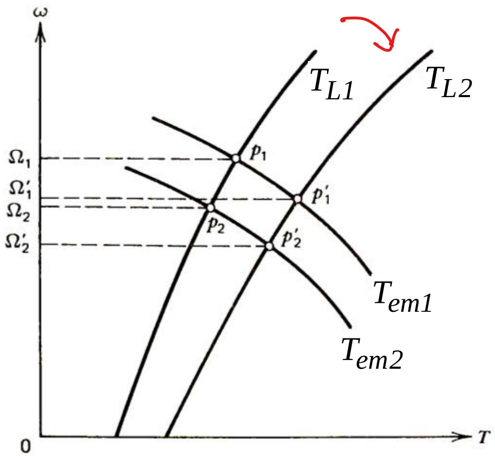
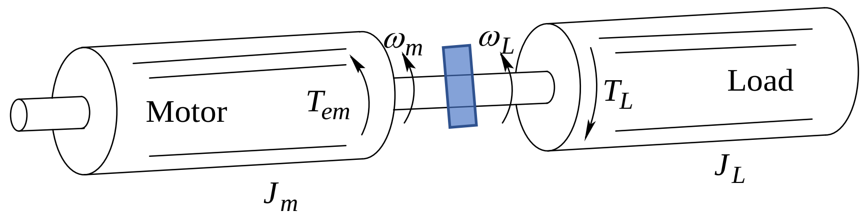
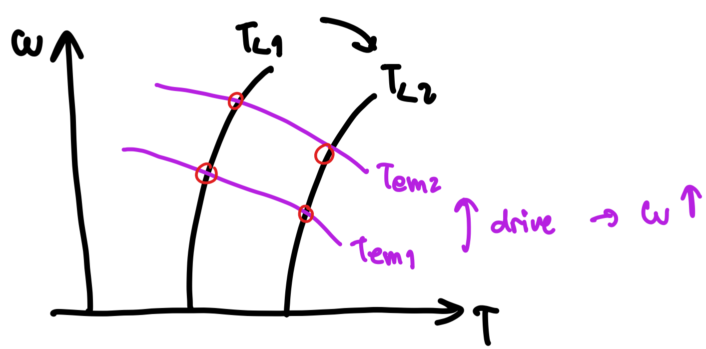


Fig. 1.18. Steady-state operating points.



Electromagnetic torque generated from electric machine	• Inertia (Jm, JL) • Shaft torsional (K)	Mechanical Coupling (for mech. Translation)	Friction Torque	Load Torque, 1) Squared Torque 2) Constant Torque 3) Squared Power 4) Constant Power
T_{em}	$J\frac{d\omega_m}{dt}$ $K(\theta_m-\theta_L)$	$\tilde{T}_L=\frac{r_1}{r_2}T_L$	T_f	T_L

- There are 2 torque-speed characteristics : one from electric machine (Tem-w), the other from load (TL-w).
- The crossing among these 2 characteristics indicates each steady-state operating point *p*.



Dynamic Operation

- How the operating point changes with time
- Important for High Performance Drives
- Speed change: rapid and without any oscillations
- Requires good controller design, as discussed in chapter 5

Summary

- What are the MKS units for force, torque, linear velocity, angular velocity, speed, and power?
- What is the relationship between force, torque, and power?
- Show that torque is the fundamental variable in controlling speed and position.
- What is the kinetic energy stored in a moving mass and a rotating inertia?

Summary

- What is the mechanism for torsional resonances?
- What are the various types of coupling mechanisms?
- What is the optimum gear ratio to minimize the torque required from the drive to accelerate a load?
- What are the torque-speed and the power-speed profiles for various types of loads?