

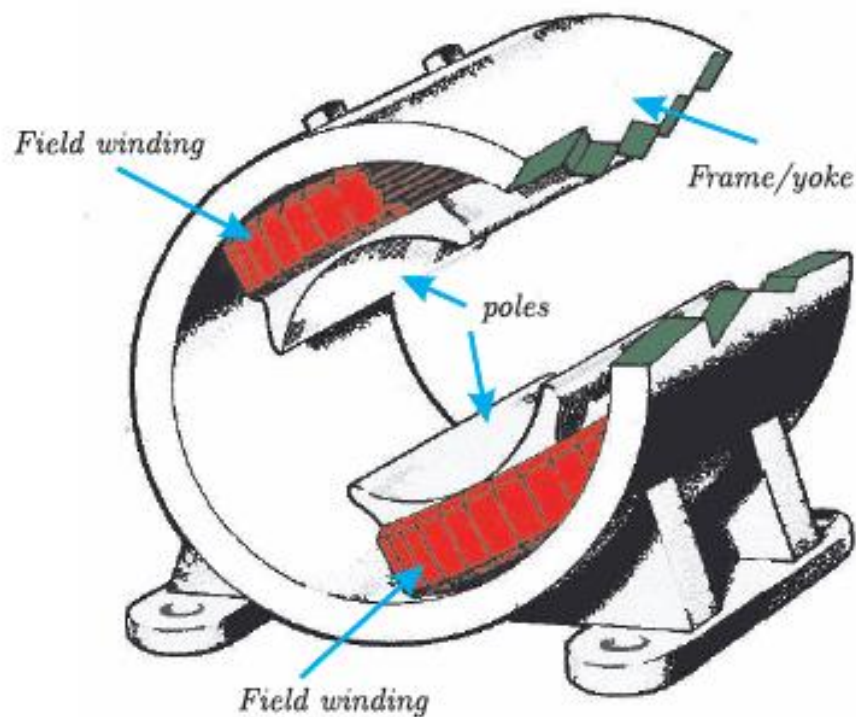
## Chapter 2

# Modeling and Control of DC Machines

$$\phi = M i_f, \quad T = k \phi i_a, \quad E_a = k \phi \omega, \quad V_t = R_a i_a + E_a$$

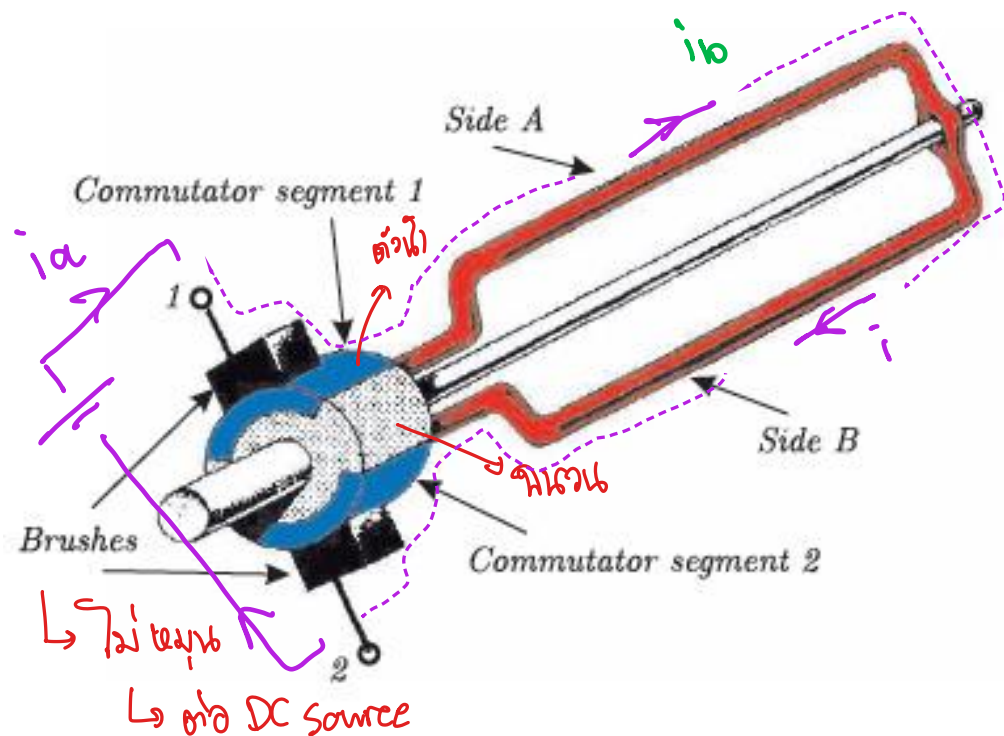
$$T = J \frac{d\omega}{dt} + T_{Load}$$

## 2.1 Machine Structure



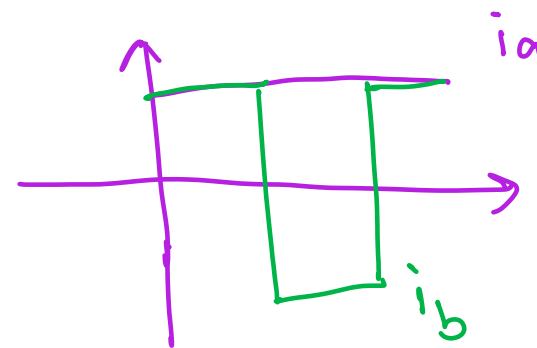
2-Pole DC Machine - Stator

- The field winding consists of  $N$  turn concentrated winding of which each half is wrapped around a pole.
- The frame is the yoke-part of the magnetic flux path.
- The use of a field winding gives us the ability to control the magnetic flux.
- Permanent magnets are often used to replace the field winding which leads to a more compact and efficient machine.
- However, we loose the control of flux magnitude and the potential of operating the machine on an AC source (universal machines)



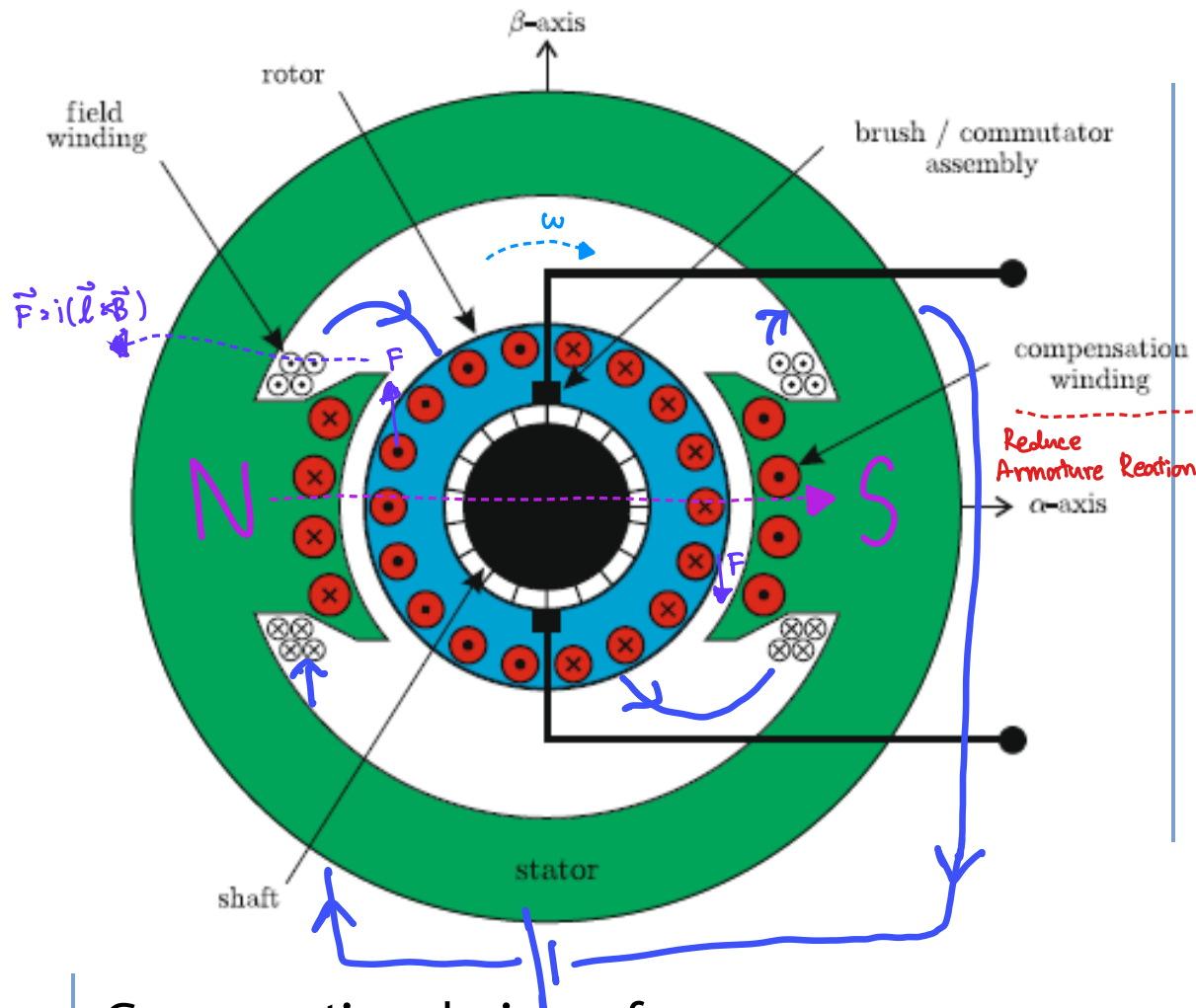
DC Machine - Rotor

- A very simple example of A DC rotor shows the same single turn winding introduced for the synchronous machine.
- The slip ring/brush combination is replaced by a so-called commutator.



# Separately Excited DC Machine

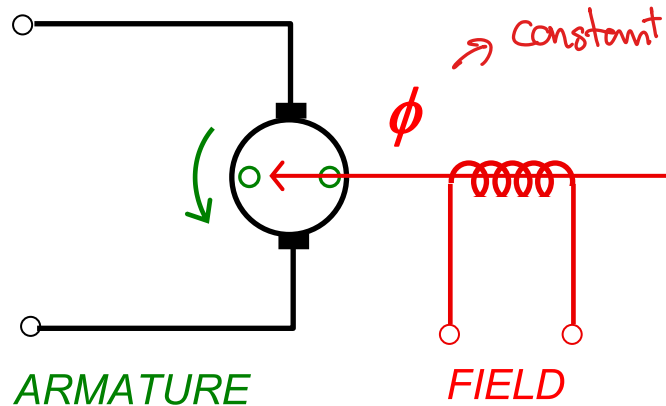
rotor → armature  
stator → field winding



- Field winding : stator based excitation winding.
- With regard to the compensation winding, the magnetizing flux will remain oriented along the  $\alpha$ - axis.
- In addition to the magnetizing flux, other component will appear due to the leakage inductance of the armature winding.

Cross-sectional view of a separately excited DC machine

## 2.2 Model of Separately-Excited DC Machine



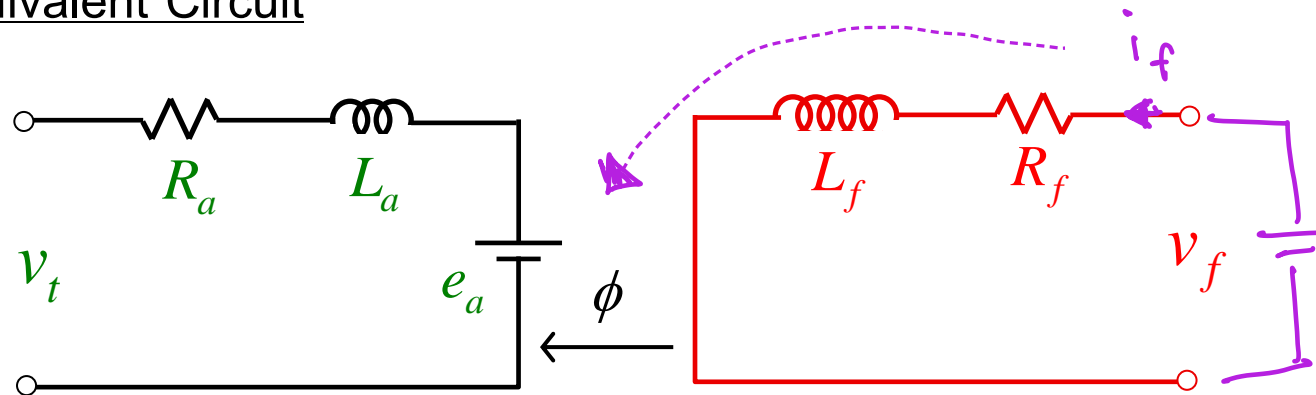
### Basic Equation

$$\underline{e_a} = k \underline{\phi} \underline{\omega}$$

$$\underline{T} = k \underline{\phi} \underline{i_a}$$

$k$  : Armature Constant

### Equivalent Circuit



Electrical Equation

field

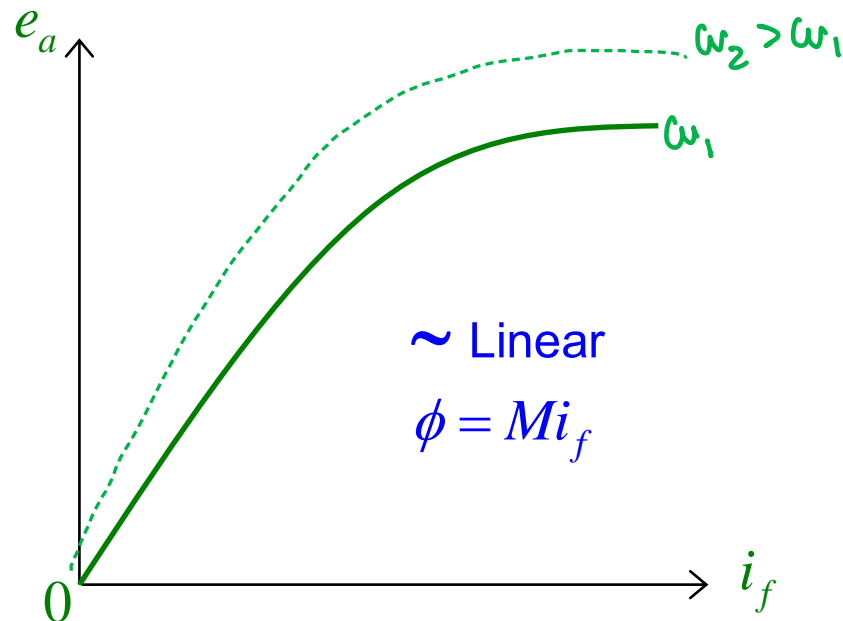
$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

armature

$$v_t = R_a i_a + L_a \frac{di_a}{dt} + \underbrace{k\phi\omega}_{e_a}$$

Mechanical Equation

$$T = k\phi i_a = J \frac{d\omega}{dt} + T_{Load}$$



MAGNETIZATION CURVE

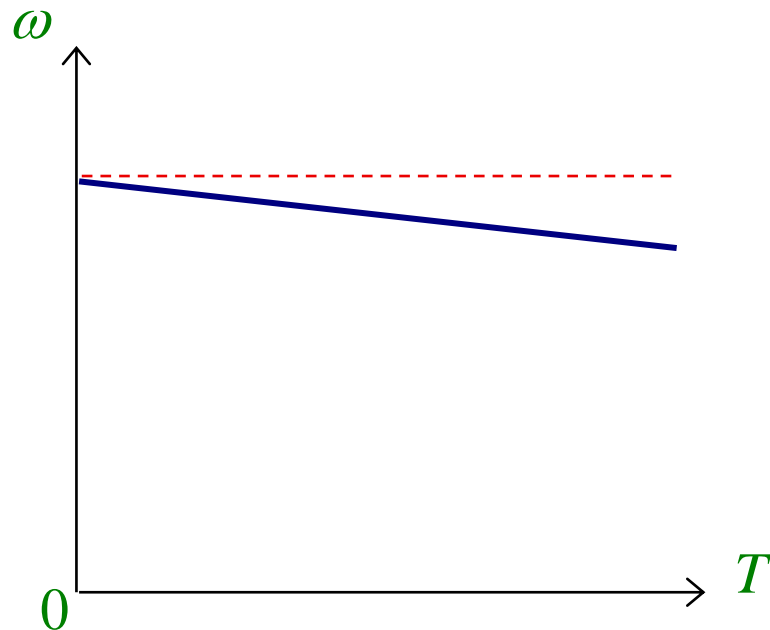
⚡ STEADY STATE

$$\begin{cases} v_f = R_f i_f \\ v_t = R_a i_a + k\phi\omega \\ T = T_{LOAD} = k\phi i_a \end{cases}$$

## 2.3 SPEED CONTROL

ที่ STEADY STATE

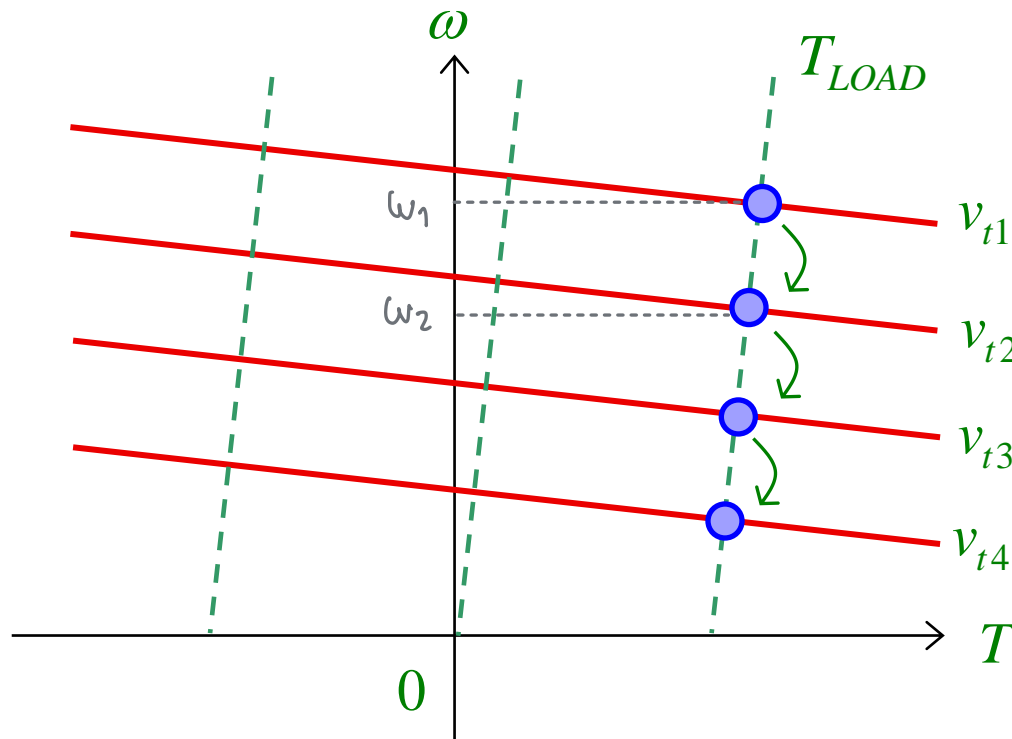
$$\omega = \frac{v_t}{k\phi} - \frac{R_a}{(k\phi)^2} \cdot T \quad (T = T_{LOAD})$$



จากสมการข้างต้น  $\omega$  สำหรับ  $T_{LOAD}$  ค่าหนึ่ง  
สามารถปรับเปลี่ยนได้ โดย

$v_t$ ,  $(k\phi)$ ,  $R_a$

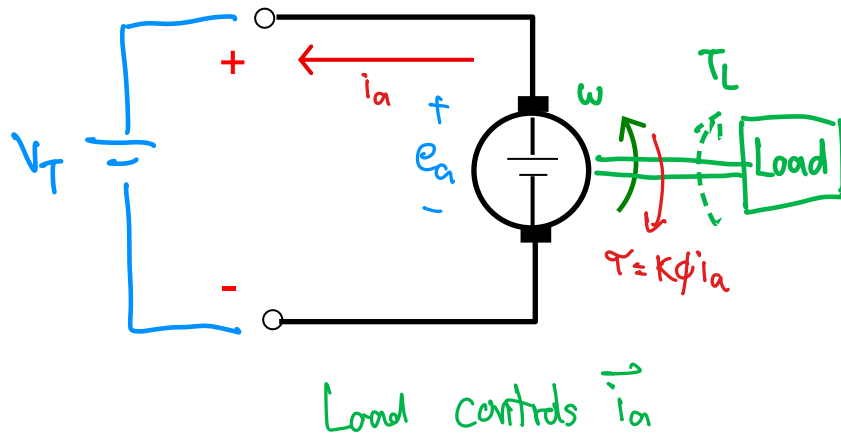
- SPEED CONTROL BY  $v_t$



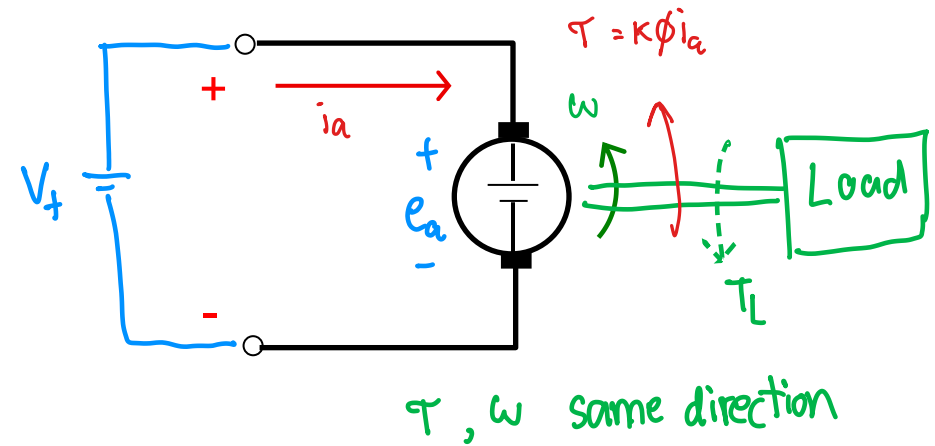
- Field ไม่เปลี่ยน
- $-v_{rate} < v_t < v_{rate}$
- $\omega < \omega_{base}$
- $i_a < i_{max}$  (พิกัด)



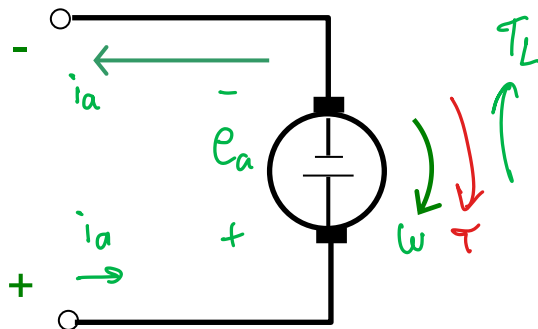
## REGENERATIVE BRAKING



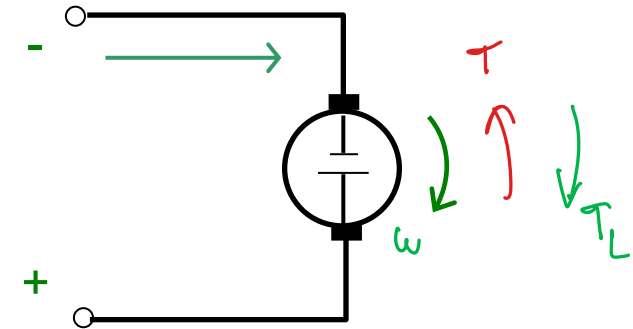
## FORWARD DRIVING



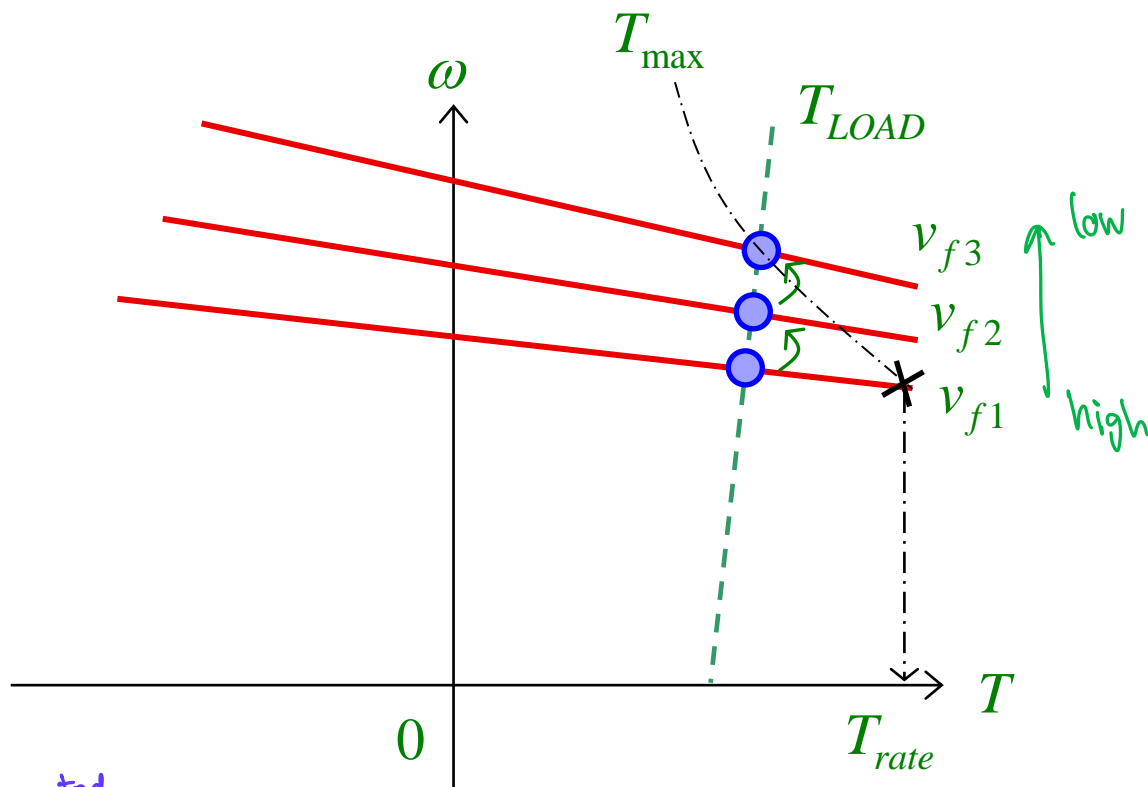
## REVERSE DRIVING



## REGENERATIVE BRAKING



## ● SPEED CONTROL BY FIELD WEAKENING



●  $v_t$  คงที่ ( $v_{rate}$ )

●  $v_{f1} > v_{f2} > v_{f3} \dots$

$\phi_1 > \phi_2 > \phi_3 \dots$

(rate)

$I_{f1} > I_{f2} > I_{f3} \dots$

●  $\omega > \omega_{base}$

●  $i_a < i_{max}$  (พิกัด)

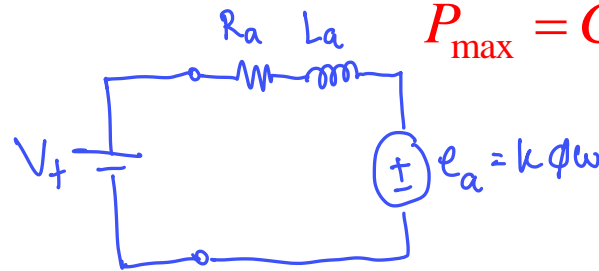
$T_{max} (= k\phi i_{max}) \propto \frac{1}{\omega}$

$P_{max} = Const.$

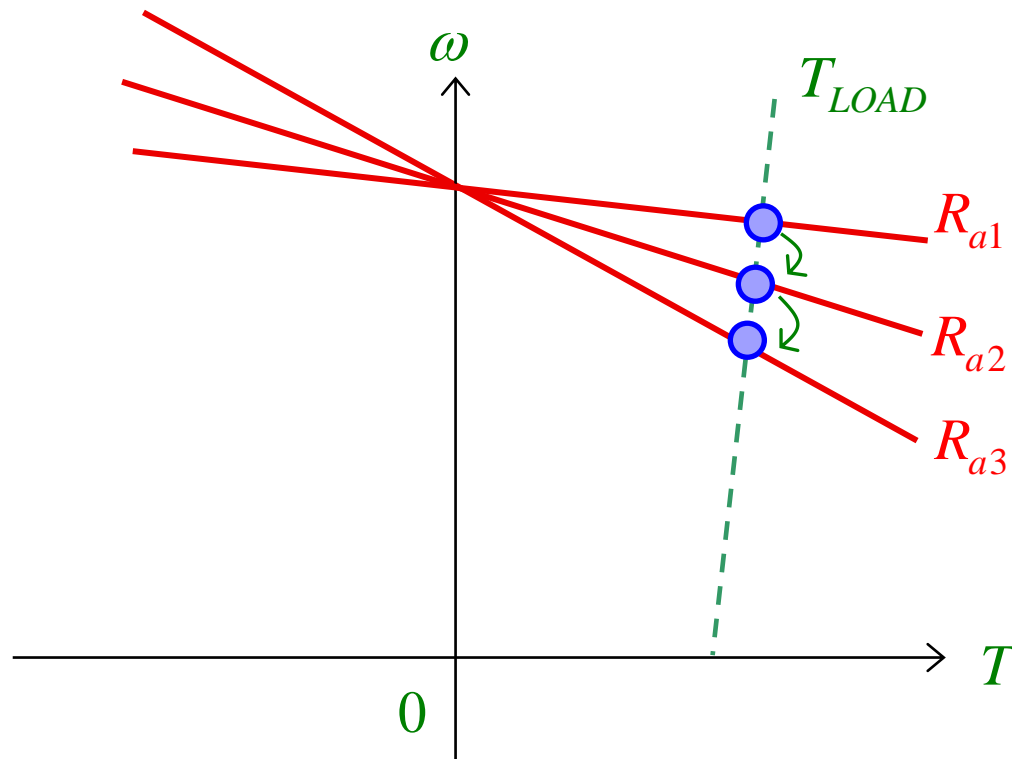
rated  
(น้อย)  
 $v_t = R_a i_a + k\phi\omega$   
 $\approx k\phi\omega$   
rated base

= คงที่

$\phi \downarrow \Rightarrow \omega \uparrow$



- SPEED CONTROL BY  $R_a$



- เพิ่ม Loss
- SPEED REGULATION เพิ่ม
- $\omega$  เปลี่ยนแปลงน้อยในช่วงโหลดต่ำ
- $i_a < i_{max}$  (พิกัด)

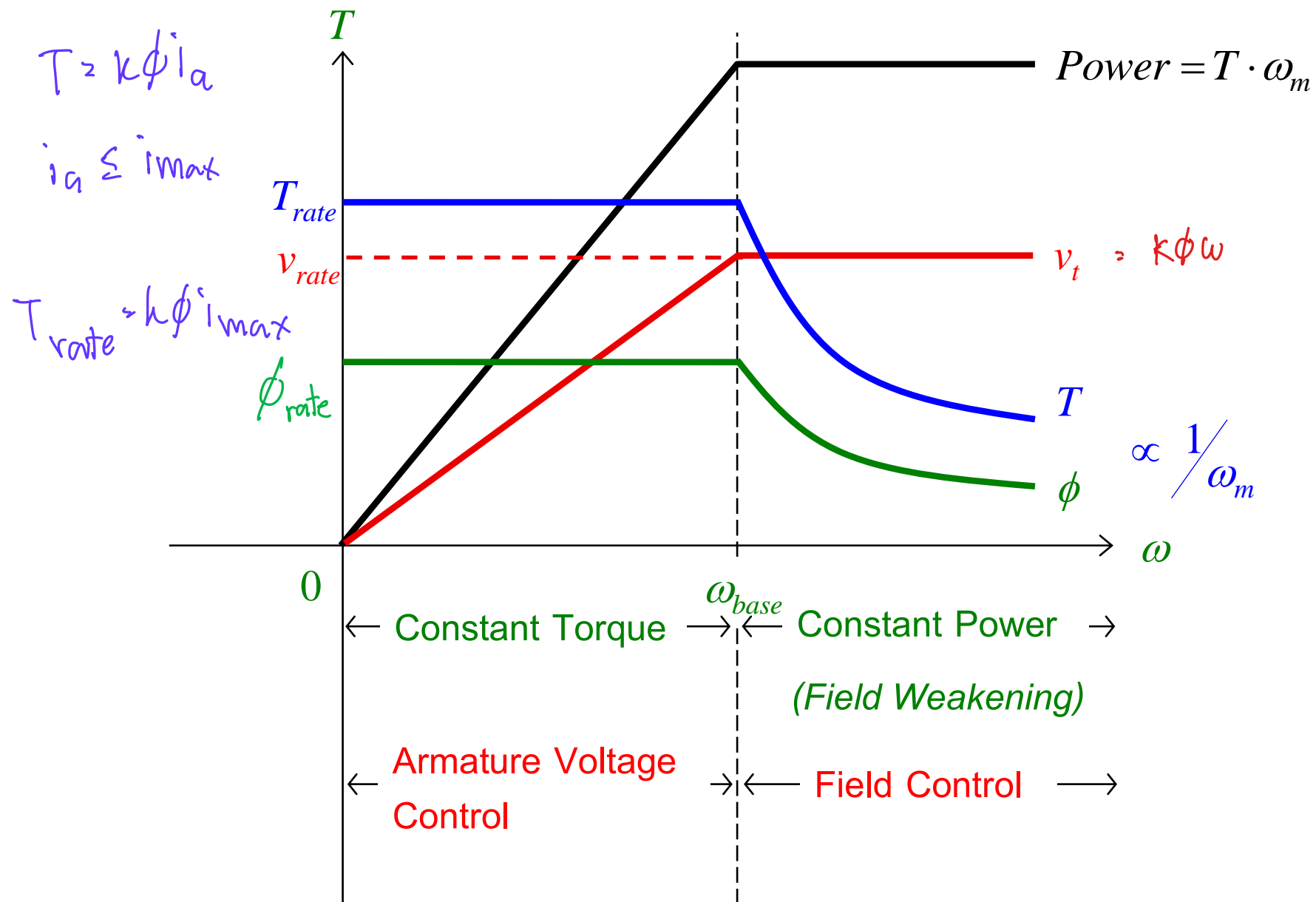
## LIMIT

$$|v_t| \leq v_{rate} = v_{\max}$$

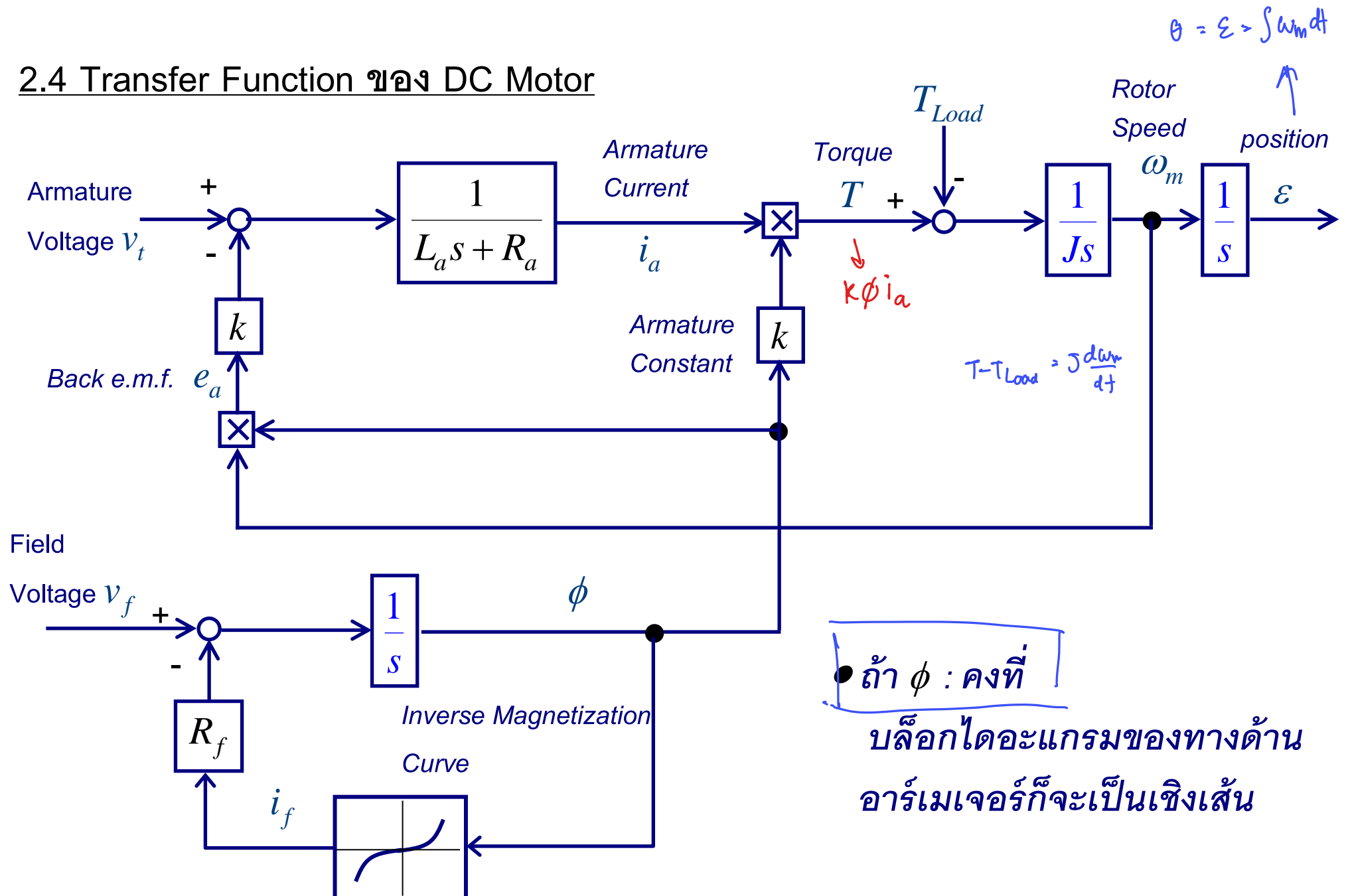
$$|i_a| \leq i_{rate} = i_{\max}$$

$$\omega \leq \omega_{\max} \quad (\text{อาจสูงกว่า } \omega_{base})$$

$$|i_f| \leq i_{frate}$$



## 2.4 Transfer Function ของ DC Motor



ถ้า  $\phi$  : คงที่  
 บล็อกไดอะแกรมของทางด้าน  
 อาร์เมเจอร์ก็จะเป็นเชิงเส้น

$$V_t = R_a i_a + L_a \frac{d}{dt} i_a + K \phi \omega$$

↓ Laplace transform

$$V_t = R_a i_a + L_a s i_a + K \phi \omega$$

$$V_t - K \phi \omega = (R_a + L_a s) i_a$$

$$\frac{i_a}{V_t - K \phi \omega} = \frac{1}{L_a s + R_a}$$

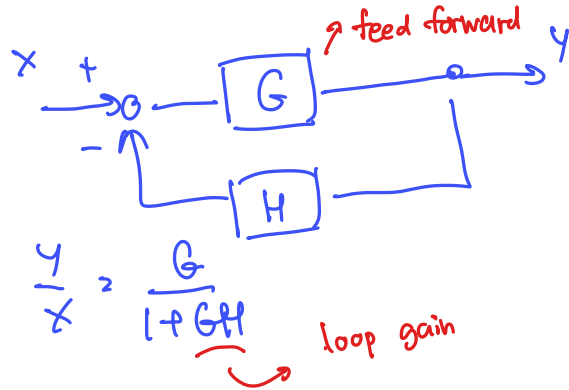
$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$= R_f i_f + \frac{d}{dt} (L_f i_f)$$

$\phi = L_f i_f = f(i_f)$   $\swarrow$  mag curve  $i_f$

$$V_f - R_f i_f = s \phi$$

$$\frac{\phi}{V_f - R_f i_f} = \frac{1}{s}$$



## Dynamic Behavior ของ DC Motor ที่ Const. Flux

⊗

$$\omega_m = F_1(s) \cdot v_t + F_2(s) \cdot T_{Load}$$

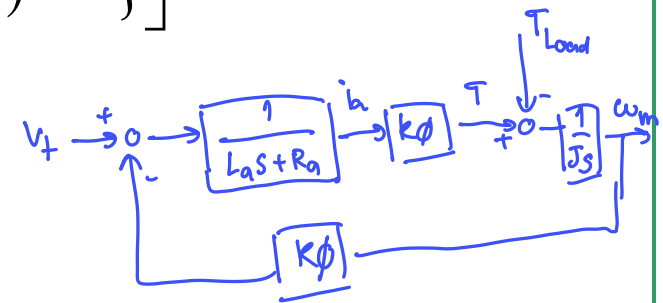
Superposition

$$F_1(s) = \frac{k\phi}{(L_a s + R_a) \cdot J s} \bigg/ \left[ 1 + \frac{(k\phi)^2}{\{(L_a s + R_a) \cdot J s\}} \right]$$

$$= \frac{k\phi}{J s (L_a s + R_a) + (k\phi)^2}$$

$$F_2(s) = -\frac{1}{J s} \bigg/ \left[ 1 + \frac{(k\phi)^2}{\{(L_a s + R_a) \cdot J s\}} \right]$$

$$= \frac{-(L_a s + R_a)}{J s (L_a s + R_a) + (k\phi)^2}$$





⊗

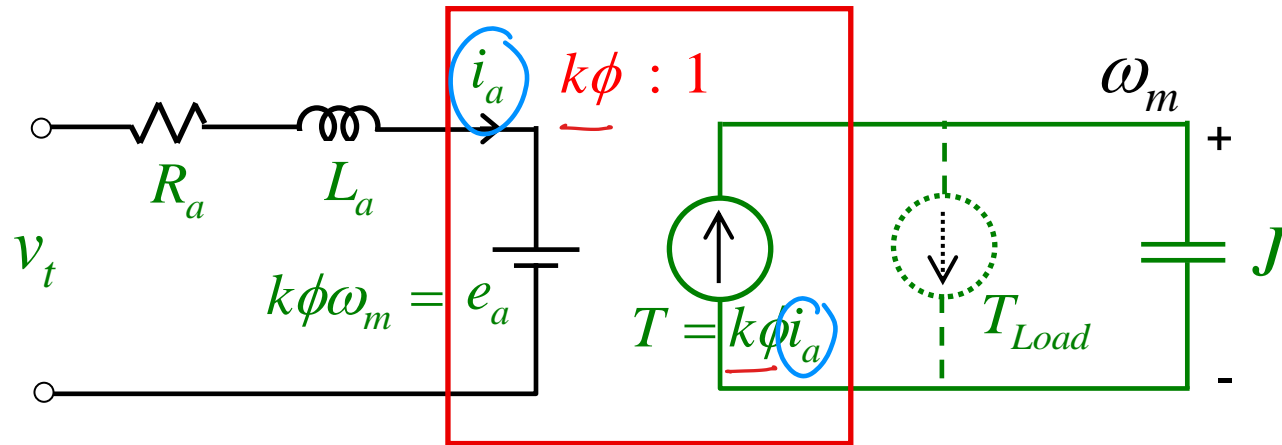
$$i_a = F_3(s) \cdot v_t + F_4(s) \cdot T_{Load}$$

$$F_3(s) = \frac{1}{(L_a s + R_a)} \bigg/ \left[ 1 + \frac{(k\phi)^2}{\{(L_a s + R_a) \cdot J s\}} \right]$$

$$= \frac{J s}{J s (L_a s + R_a) + (k\phi)^2} = \frac{\frac{J}{(k\phi)^2} \cdot s}{\frac{J}{(k\phi)^2} \cdot s (L_a s + R_a) + 1}$$

$$F_4(s) = \frac{k\phi}{J s (L_a s + R_a)} \bigg/ \left[ 1 + \frac{(k\phi)^2}{\{(L_a s + R_a) \cdot J s\}} \right]$$

$$= \frac{k\phi}{J s (L_a s + R_a) + (k\phi)^2} = F_1(s)$$

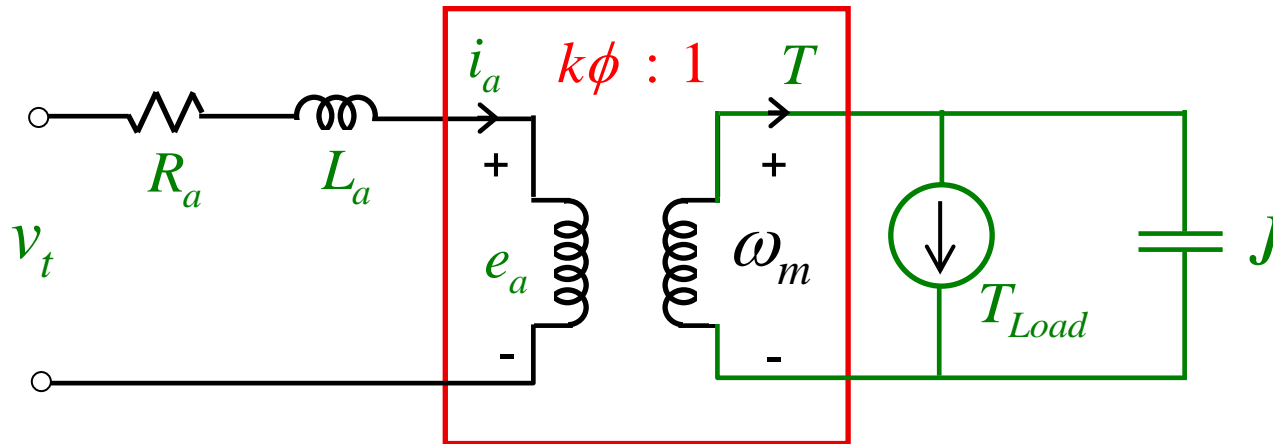


ทางด้านอาร์เมเจอร์

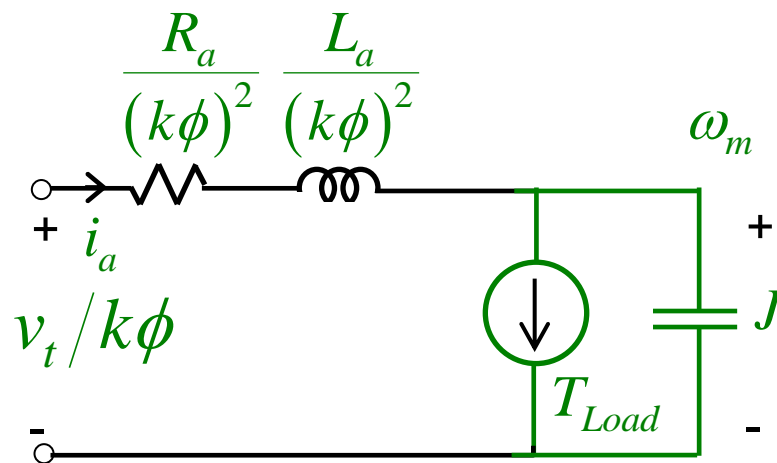
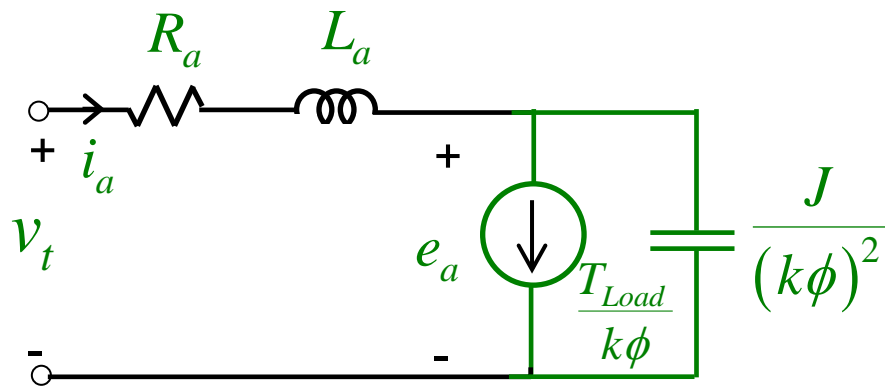
ทางด้านกล

$$J \frac{d\omega}{dt} = T - T_{Load}$$

$$c \frac{dv}{dt} = i$$

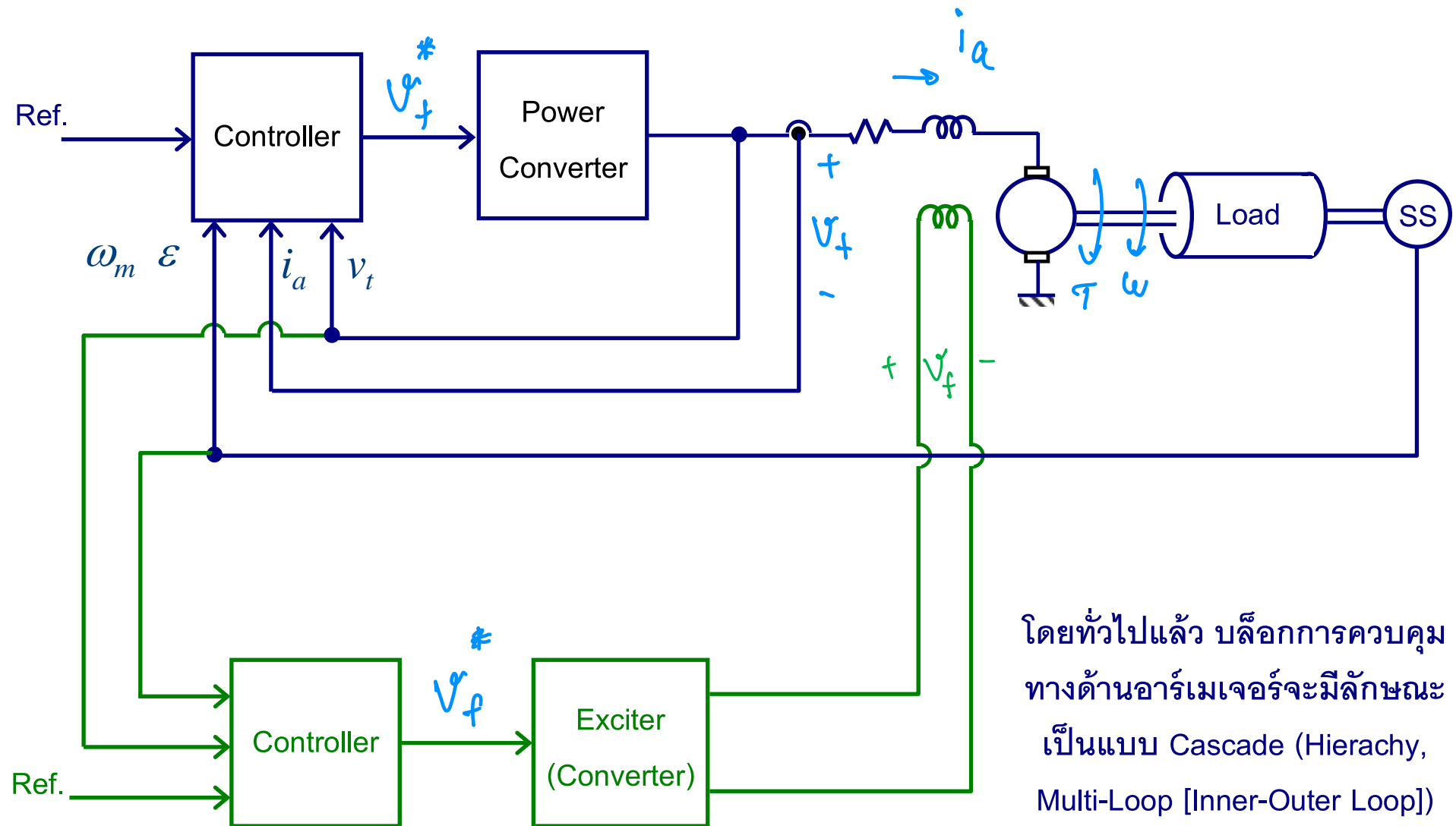


Ideal Transformer



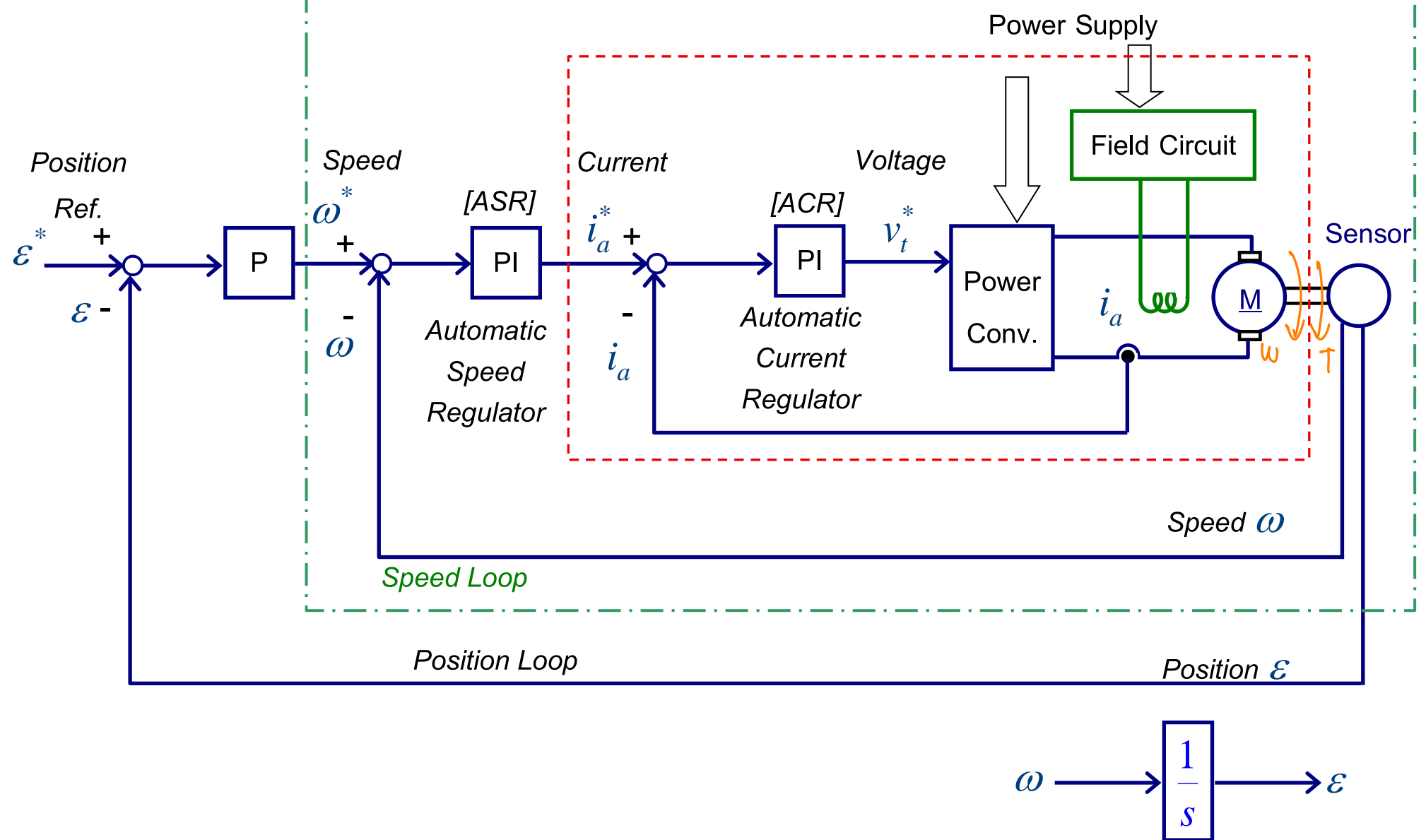
พลังงานที่สะสมใน  $J = \frac{1}{2} J \omega_m^2$

## Control of Separately Excited DC Motor



โดยทั่วไปแล้ว บล็อกการควบคุม  
ทางด้านอาร์เมเจอร์จะมีลักษณะ  
เป็นแบบ Cascade (Hierarchy,  
Multi-Loop [Inner-Outer Loop])

## Cascade Control

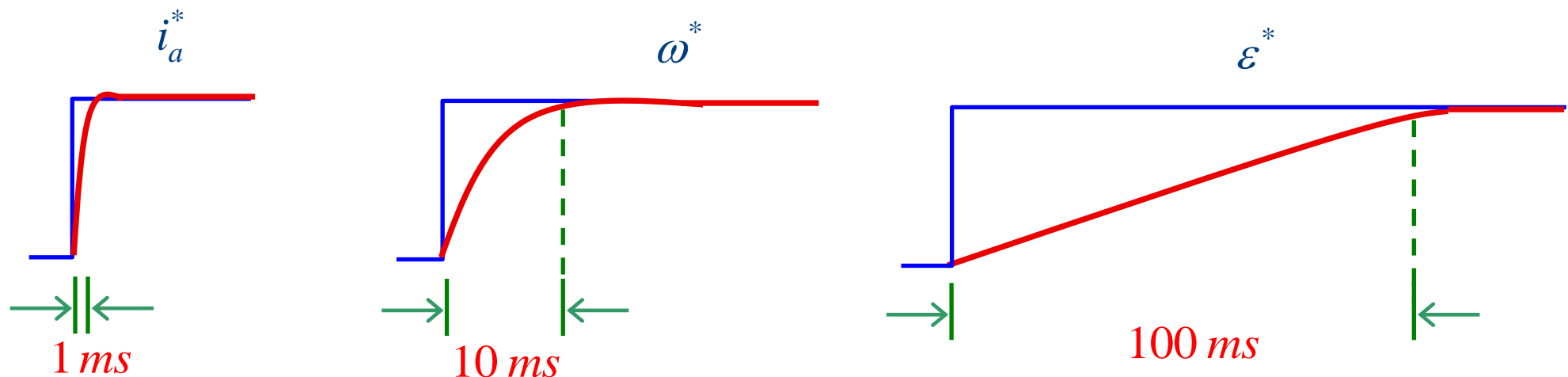


- การออกแบบระบบจะง่าย (อิสระต่อกัน) ถ้า

Response ของ Loop ใน >> Response ของ Loop นอก  
(Inner Loop) เร็วกว่า (Outer Loop)

Bandwidth เร็วกว่า 10 เท่า โดยทั่วไป

consider loop ใน โดยที่ loop นอกยังไม่เปลี่ยนไ้

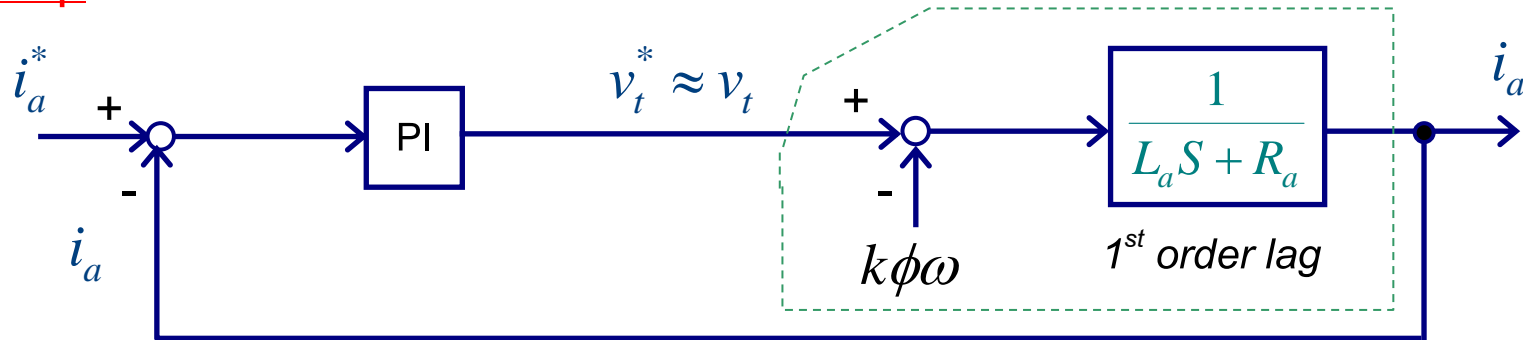


$$\tau = \frac{L_a}{R_a} \text{ (time const.)}$$

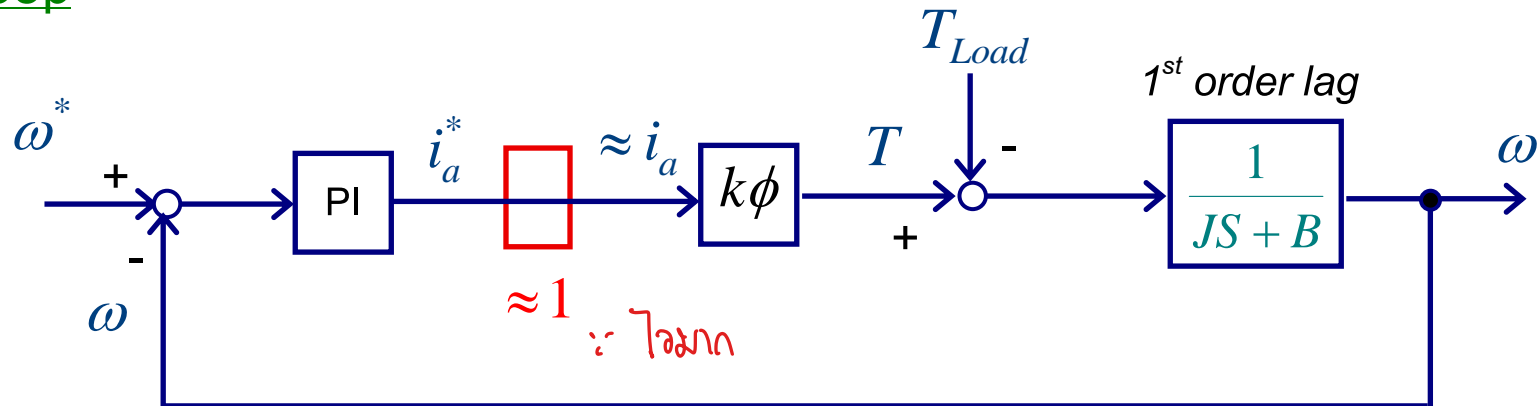
## Current Loop

Simple Control Problem !!

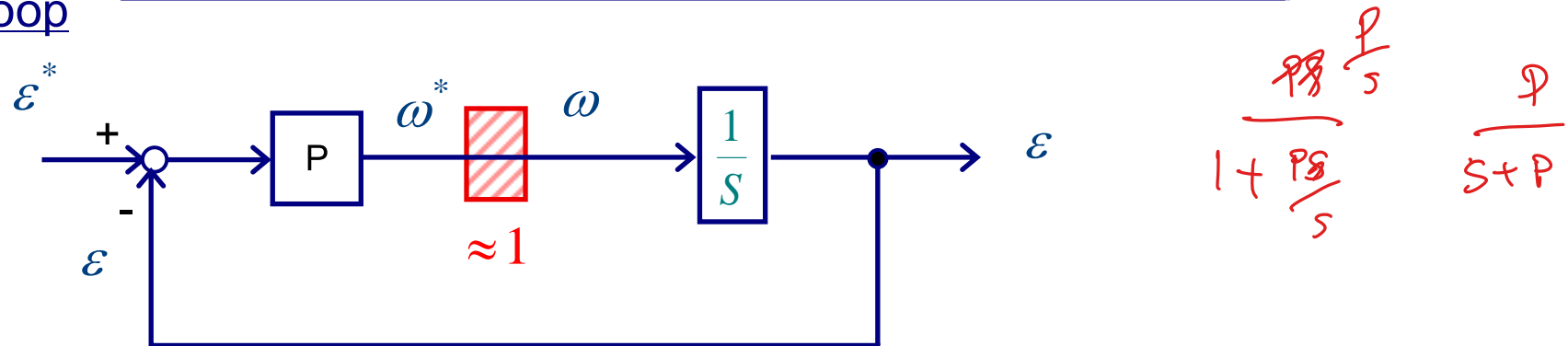
DC Motor



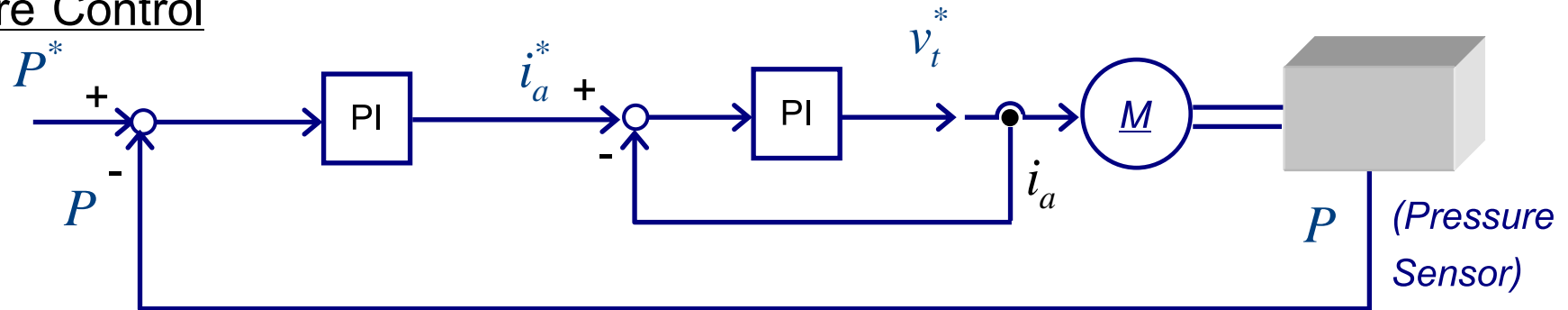
## Speed Loop



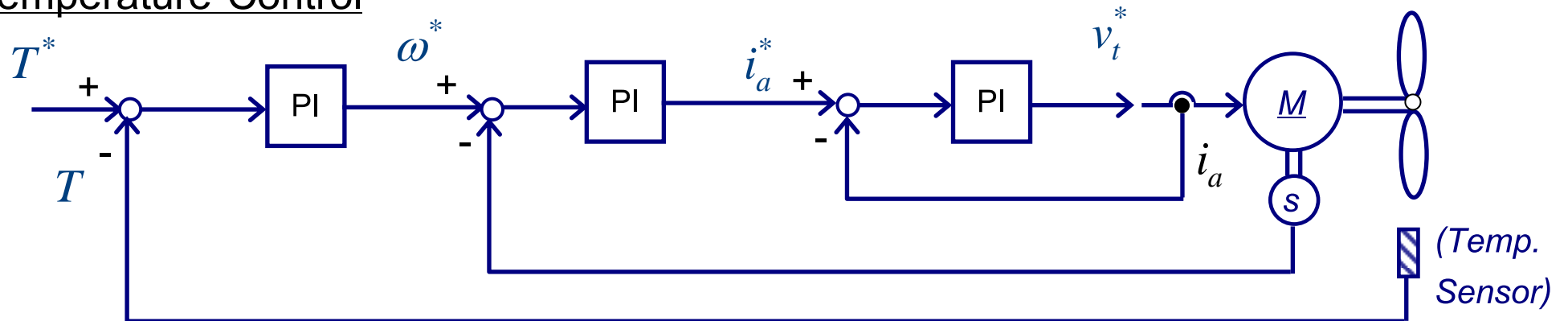
## Position Loop



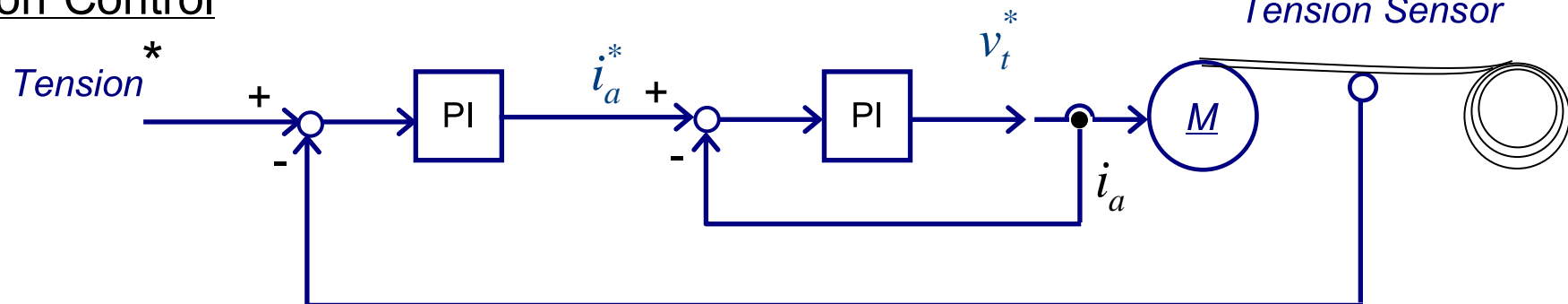
### Pressure Control



### Temperature Control



### Tension Control





## Basic Drive Circuit

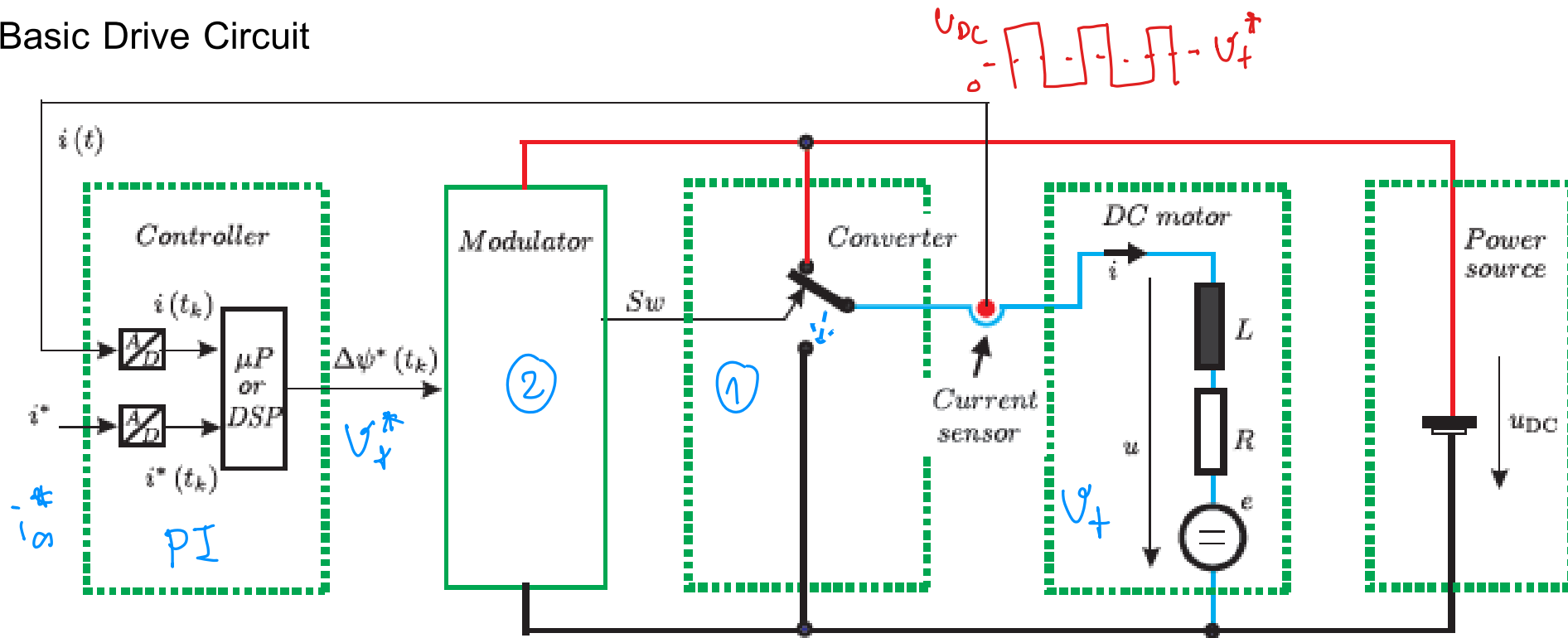
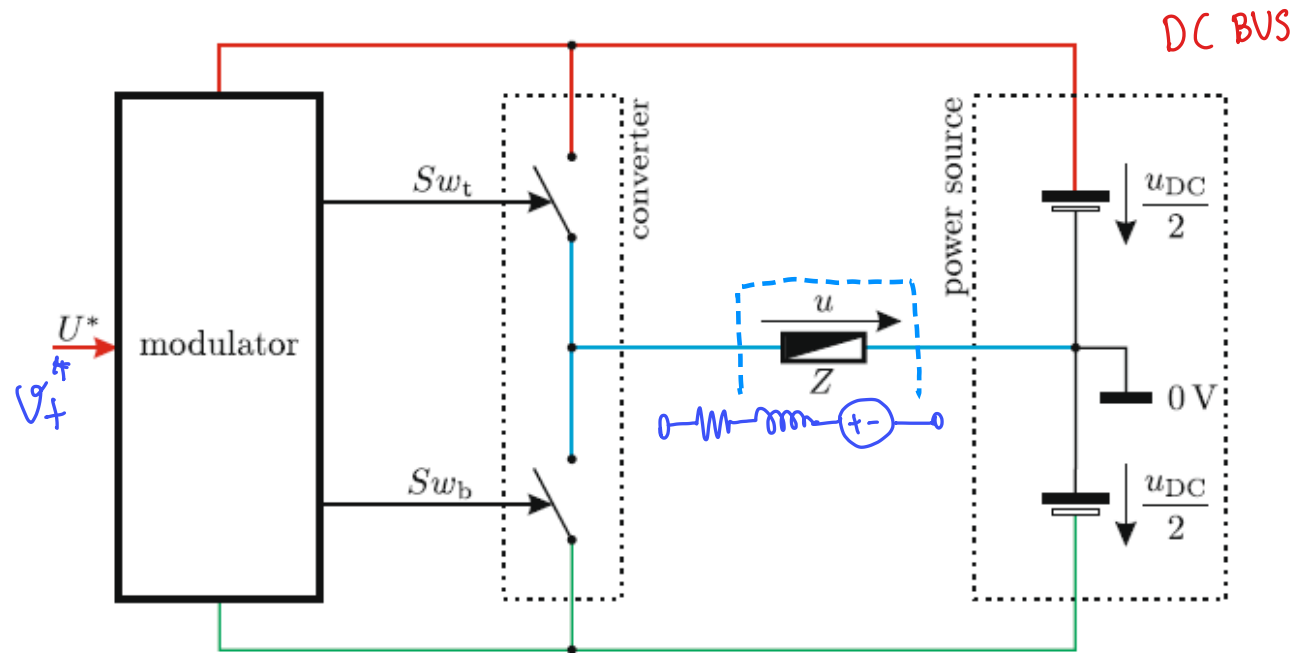


Figure 11.1. Basic electrical drive

The aim of this drive circuit is to control the current in the motor in such a way that the reference current value matches the actual load current under all circumstances, i.e. transient as well as in steady-state.

## Power Electronic Converters



Single-Phase Half-Bridge Converter:

- Consists of 2 switches.
- The average output voltage can be varied between  $+\frac{u_{DC}}{2}$  and  $-\frac{u_{DC}}{2}$ .
- Output waveform is bipolar.

Fig. 2.1 Two switch half-bridge converter with power source and mo

Table 2.1 Half-bridge switching states

$Sw_t$	$Sw_b$	Voltage $u$	Comment
0	0	—	Idle mode
0	1	$-\frac{u_{DC}}{2}$	Active mode
1	0	$\frac{u_{DC}}{2}$	Active mode
1	1	—	Shoot-through mode

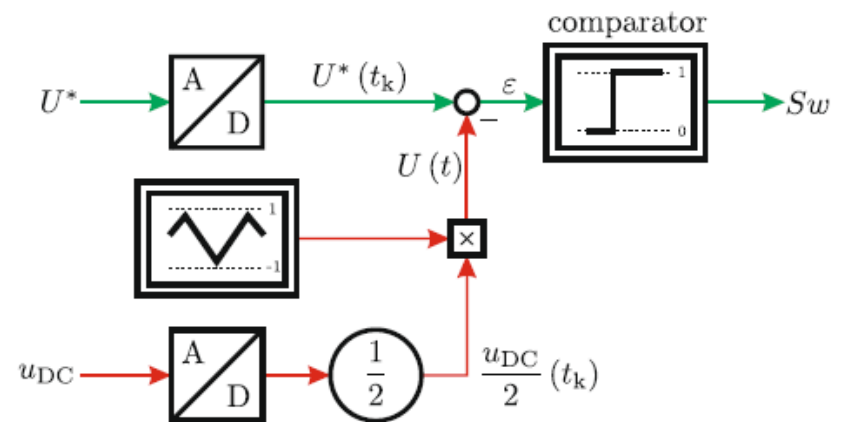
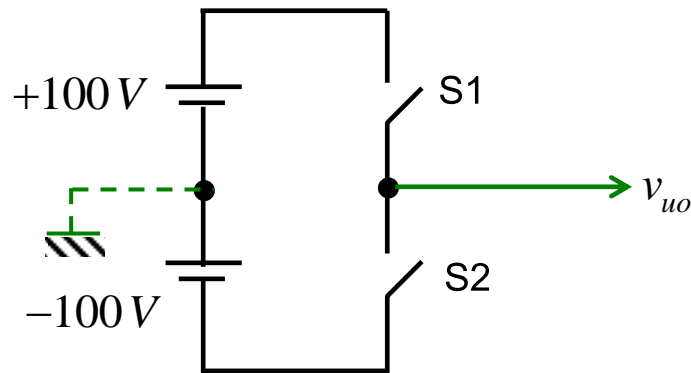


Fig. 2.4 Generic model of double edged PWM based half-bridge modulator

## Pulse Width-Modulation Converter

- ลด Harmonics ที่ความถี่ต่ำของ Six-Step Inverter
- ใช้การปรับ duty cycle (อัตราส่วน  $\frac{T_{on} \text{ ของ } S1}{T_{on}+T_{off} \text{ ของ } S1}$ )

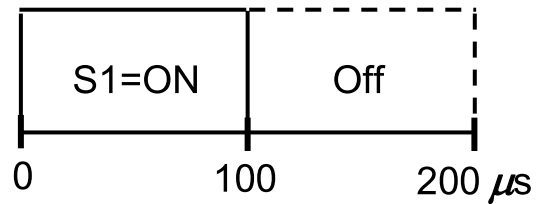


ให้ค่าเฉลี่ยแรงดันมีค่าตามต้องการ

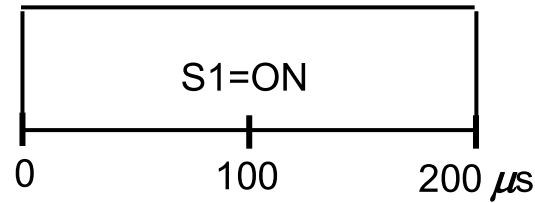
Ex  $E_d = 200 \text{ V}; T = 200 \mu\text{s}$

$$\therefore v_{uo} = \begin{cases} +100 \text{ V} \\ -100 \text{ V} \end{cases}$$

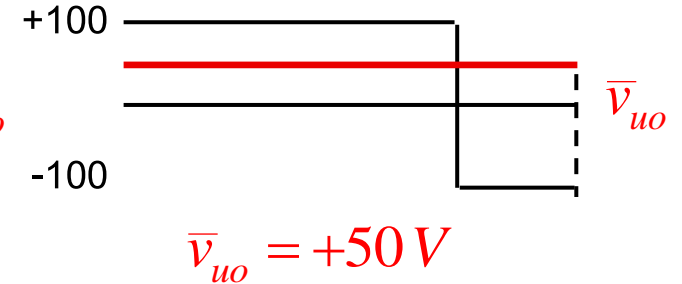
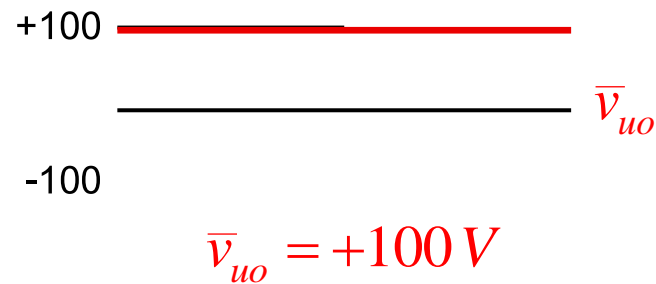
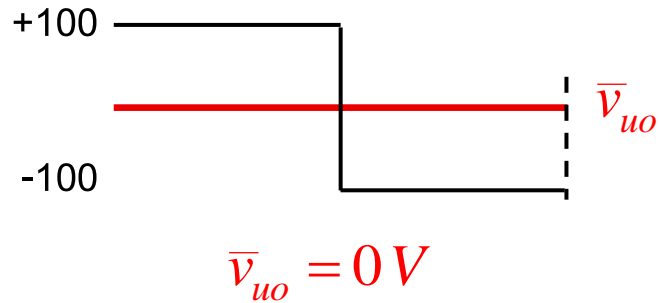
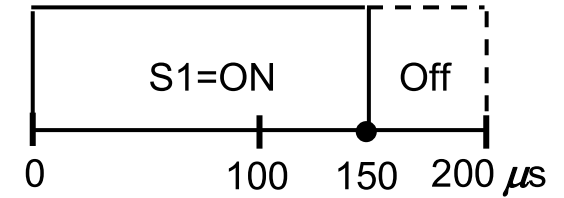
$$U^* = 0$$



$$U^* = 100 \text{ V}$$



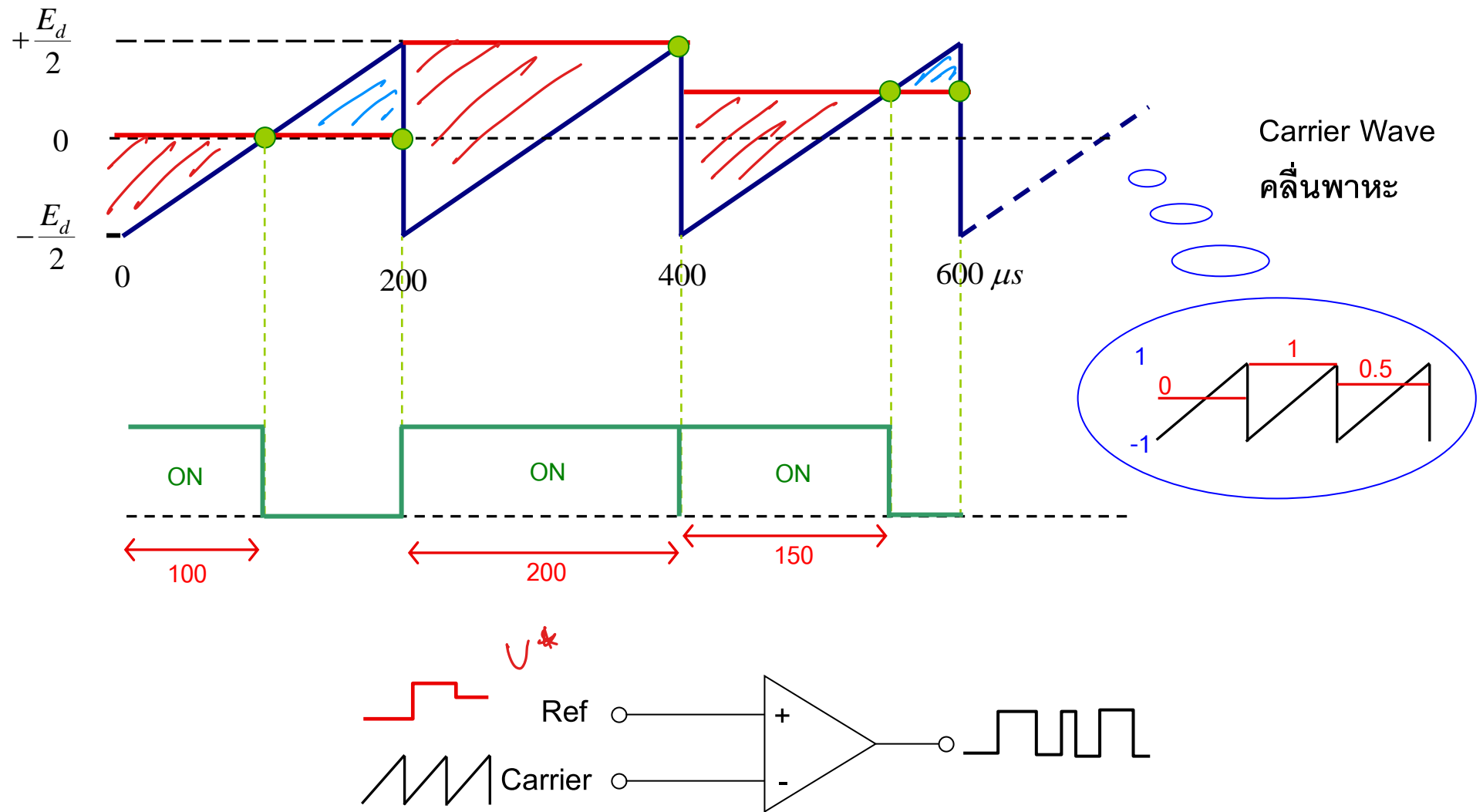
$$U^* = 50 \text{ V}$$



เราสามารถปรับค่าเฉลี่ยของแรงดันได้อย่างต่อเนื่องระหว่างค่า

$$+\frac{E_d}{2}, -\frac{E_d}{2} \quad (+100\text{V}, -100\text{V})$$

## วิธีการหาตำแหน่งเวลาการสวิตช์



# Single-Phase Full-Bridge Converter

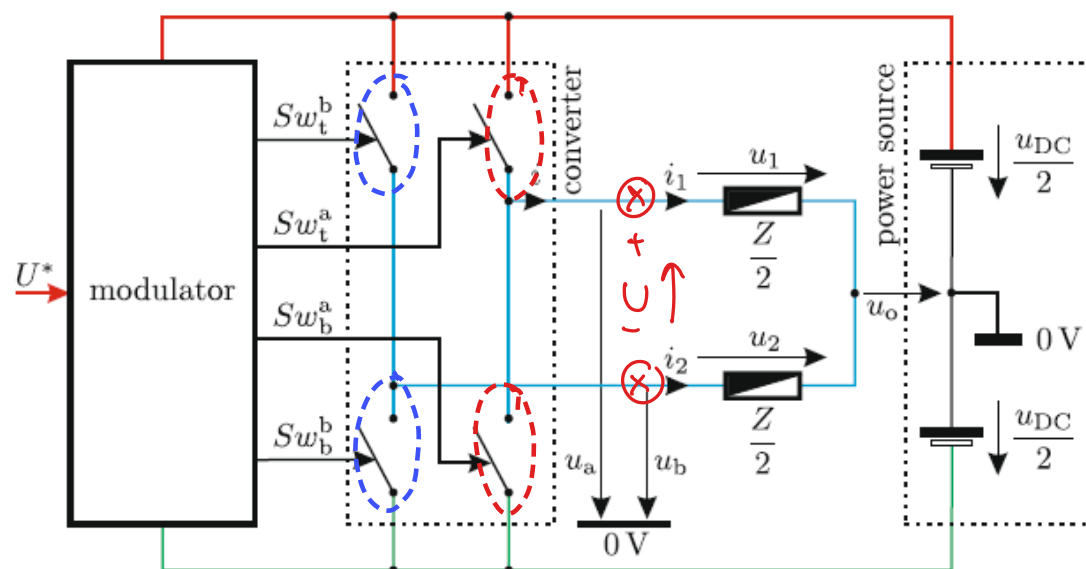
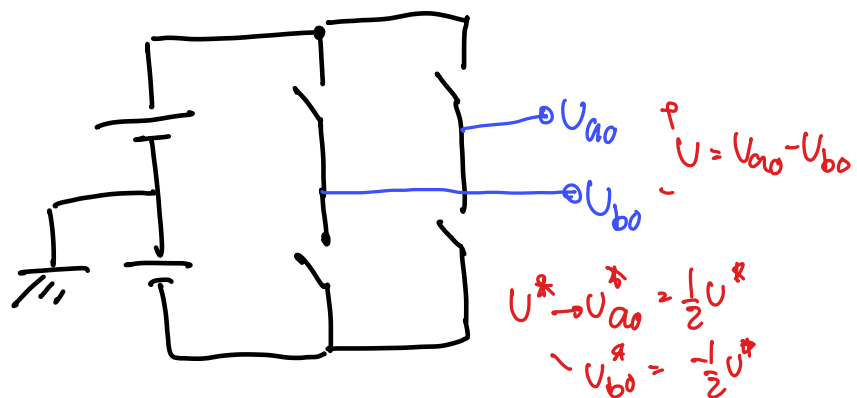


Fig. 2.5 H-bridge converter



Single-Phase Full-Bridge Converter:

- Consists of 4 switches.
- The average output voltage can be varied between  $+u_{DC}$  and  $-u_{DC}$ .
- Output waveform is unipolar.

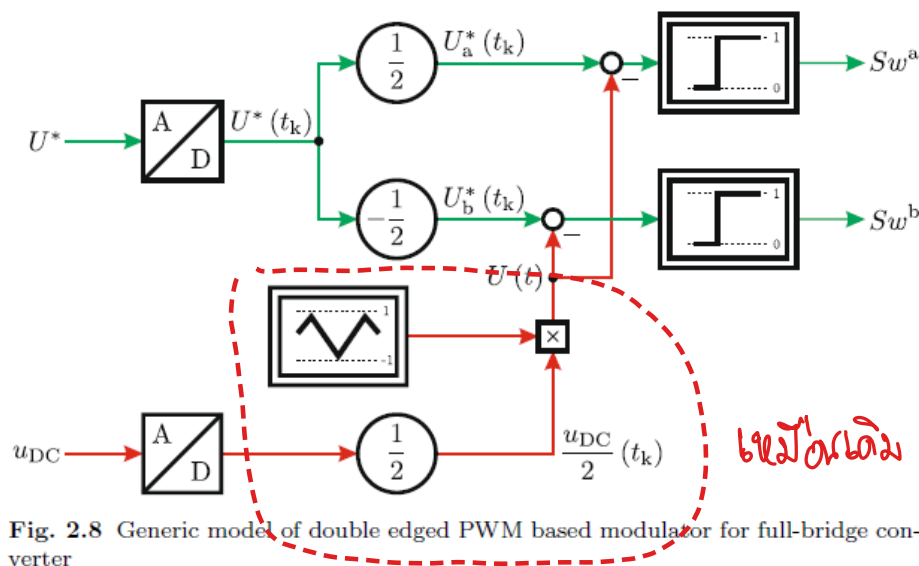
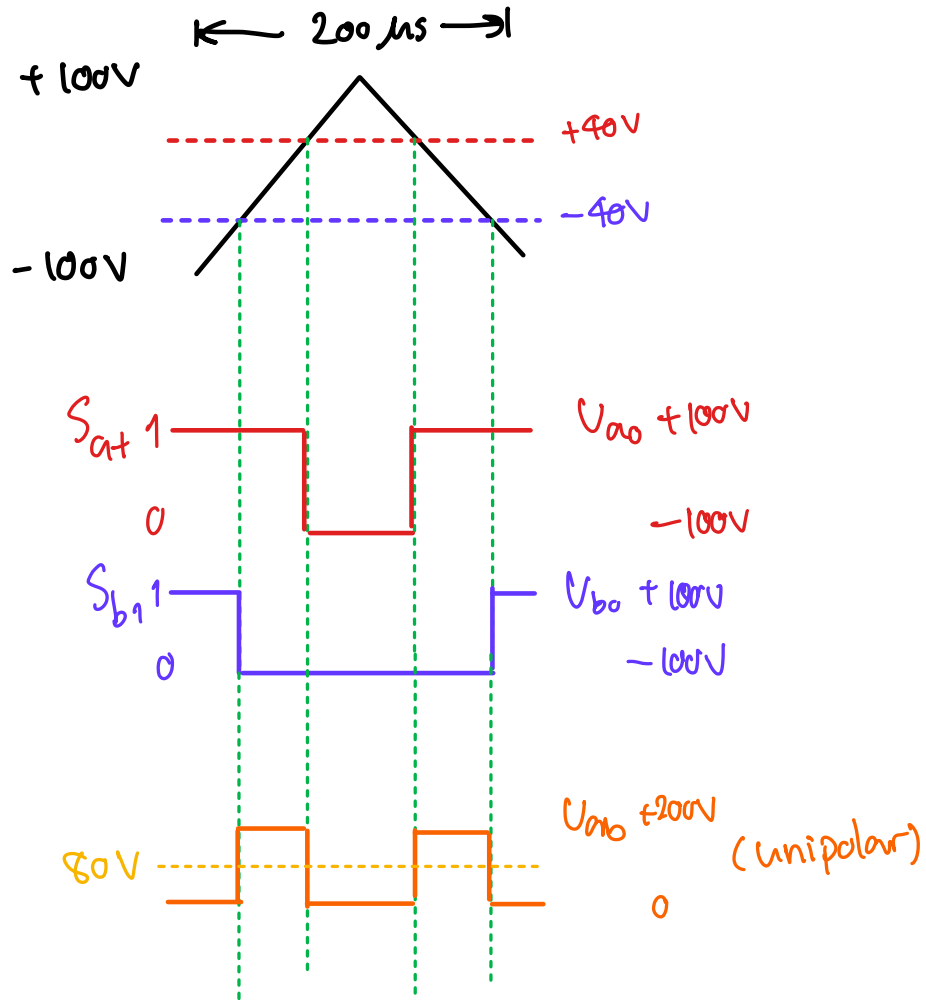


Fig. 2.8 Generic model of double edge PWM based modulator for full-bridge converter

$$U^* = +80V, T_s = 200\mu s, U_{DC} = 200V$$

$$\downarrow$$

$$U_{ao}^* = 40V, U_{bo}^* = -40V$$



## Current Control of Single-Phase Load

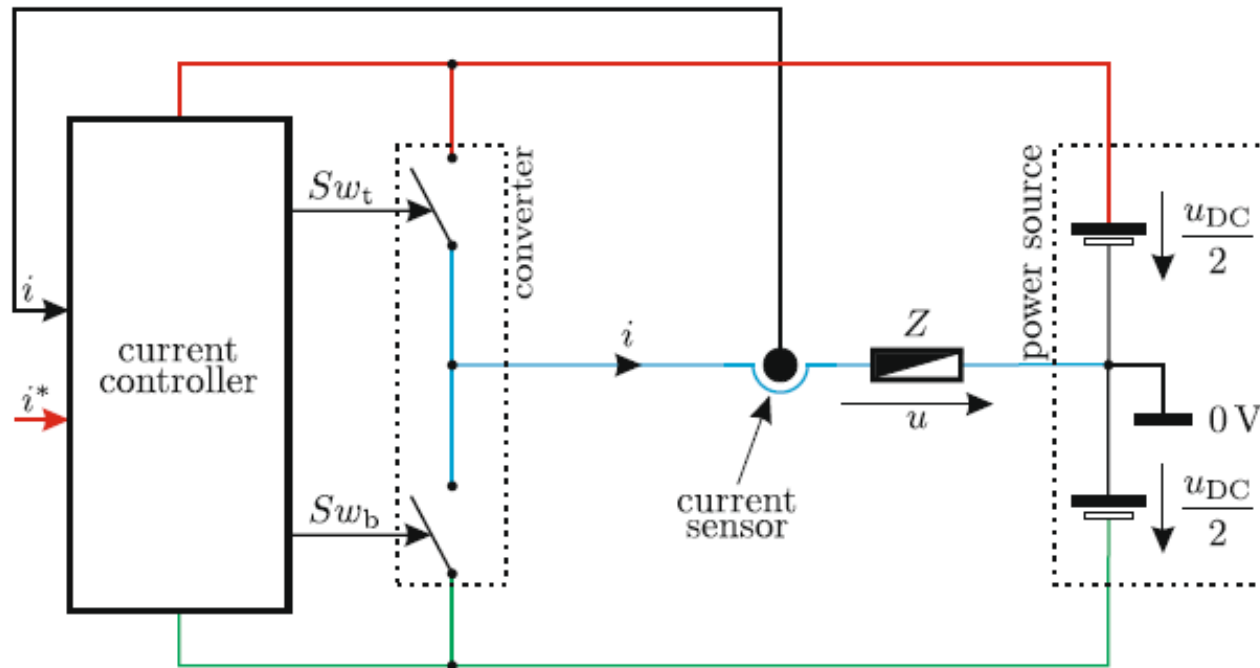


Fig. 3.2 Single-phase converter with hysteresis current controller

- 1) Hysteresis Current Control
- 2) PI (+ Feedforward) Current Control



## 1) Hysteresis Current Control

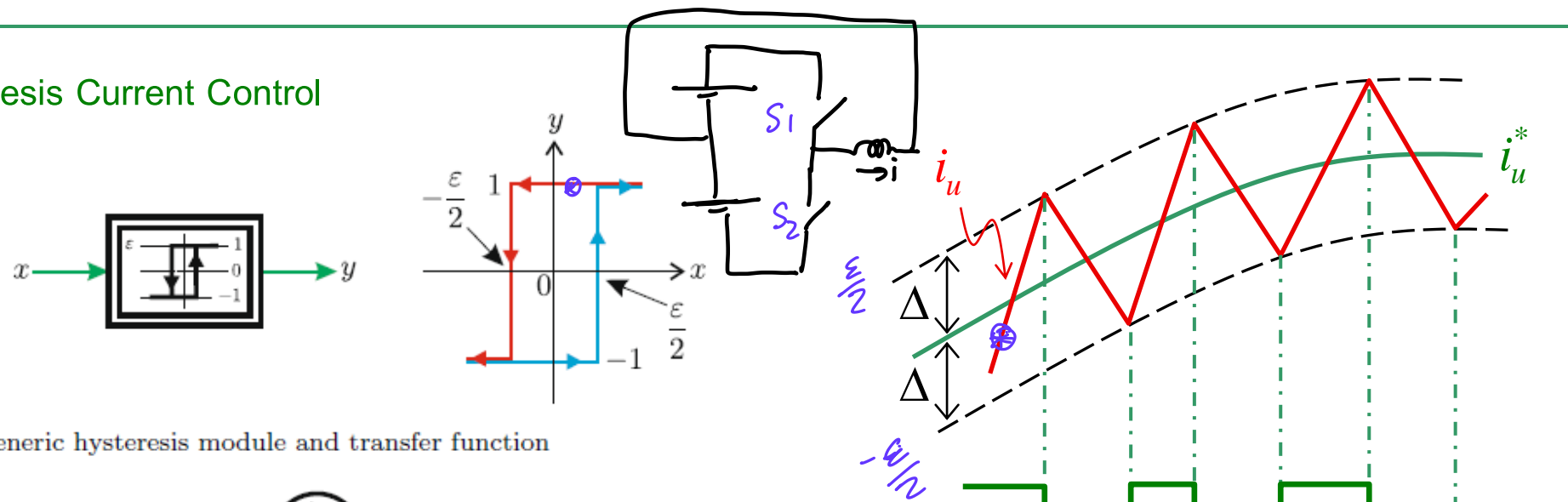


Fig. 3.1 Generic hysteresis module and transfer function

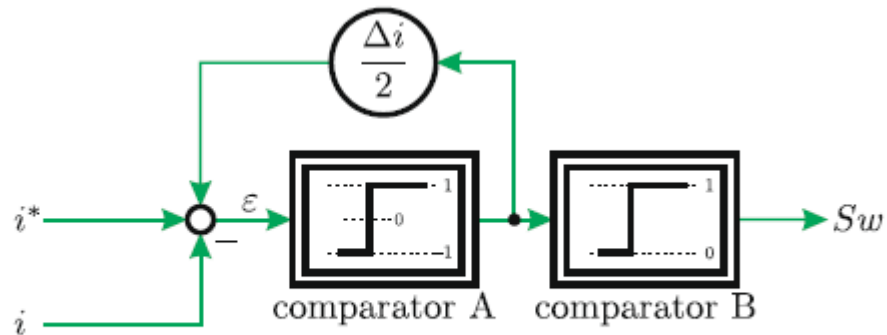
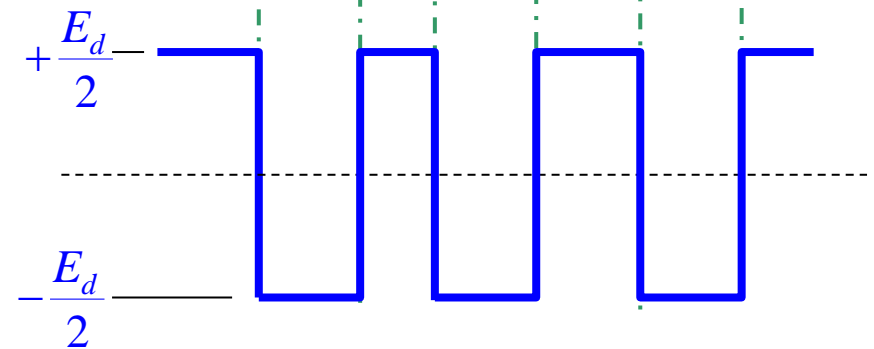


Fig. 3.3 Generic structure of hysteresis current controller

- ไม่มีการทำ PWM
- สัญญาณออกของ Hysteresis Comparator เป็น Switching Signal โดยตรง
- ไม่มี Carrier Frequency..... Switching Frequency ไม่คงที่
- เหมาะกับ Analog มากกว่า Digital
- Response เร็ว



## 2) PI (+ Feedforward) Current Control

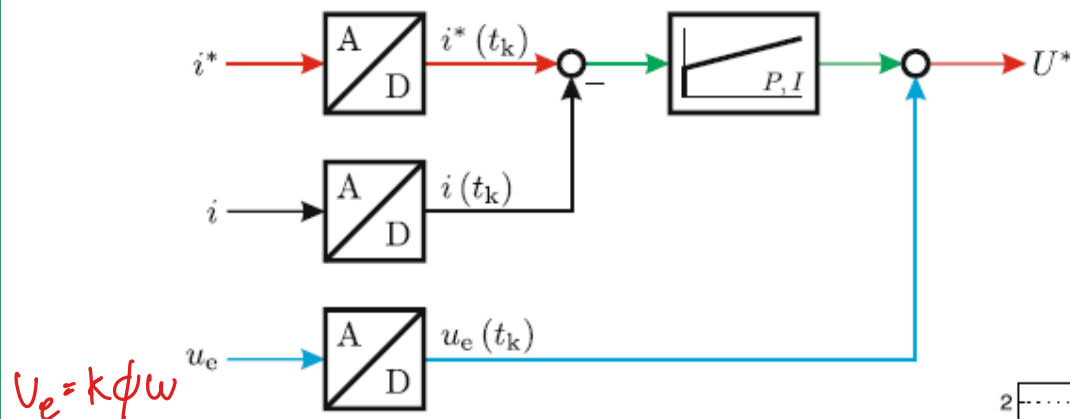


Fig. 3.7 Model based current controller structure

- มีการทำ PWM
- สัญญาณออกของตัวควบคุมเป็นคำสั่งแรงดัน
- มี Carrier Frequency..... มี Switching Frequency ตามที่กำหนด
- เหมาะทั้ง Analog และ Digital
- Feedforward Control ช่วยให้ Response เร็วขึ้น

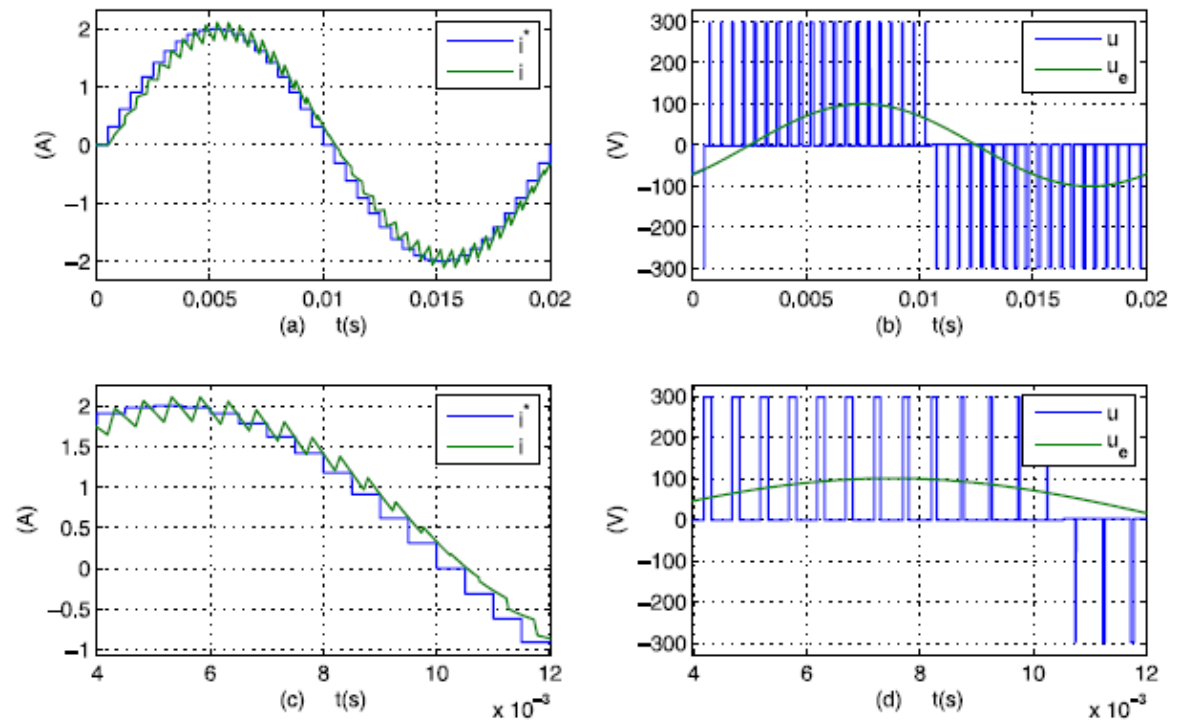


Fig. 3.8 Single-phase model based current control example

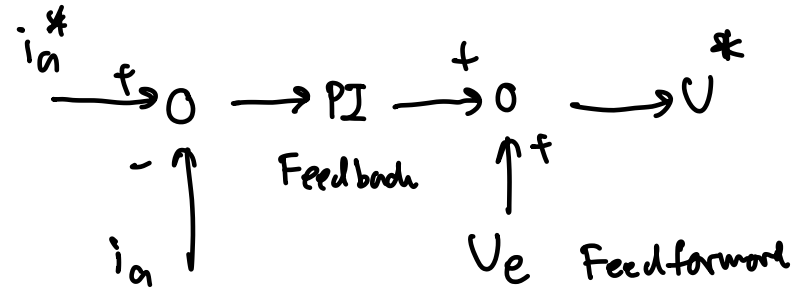
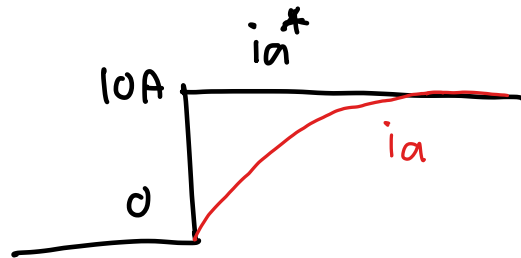
$$R_a = 1 \Omega, L_a = 10 \text{ mH}, i_a^* = 10 \text{ A}, U_e = 50 \text{ V}$$

$$U = \tilde{U} + 50 = R_a i_a + L_a \frac{di_a}{dt} + 50$$

$$\Downarrow$$

$$R_a i_a^* = R_a i_a + L_a \frac{di_a}{dt}$$

$$i_a^* = i_a + \frac{L_a}{R_a} \frac{di_a}{dt}$$



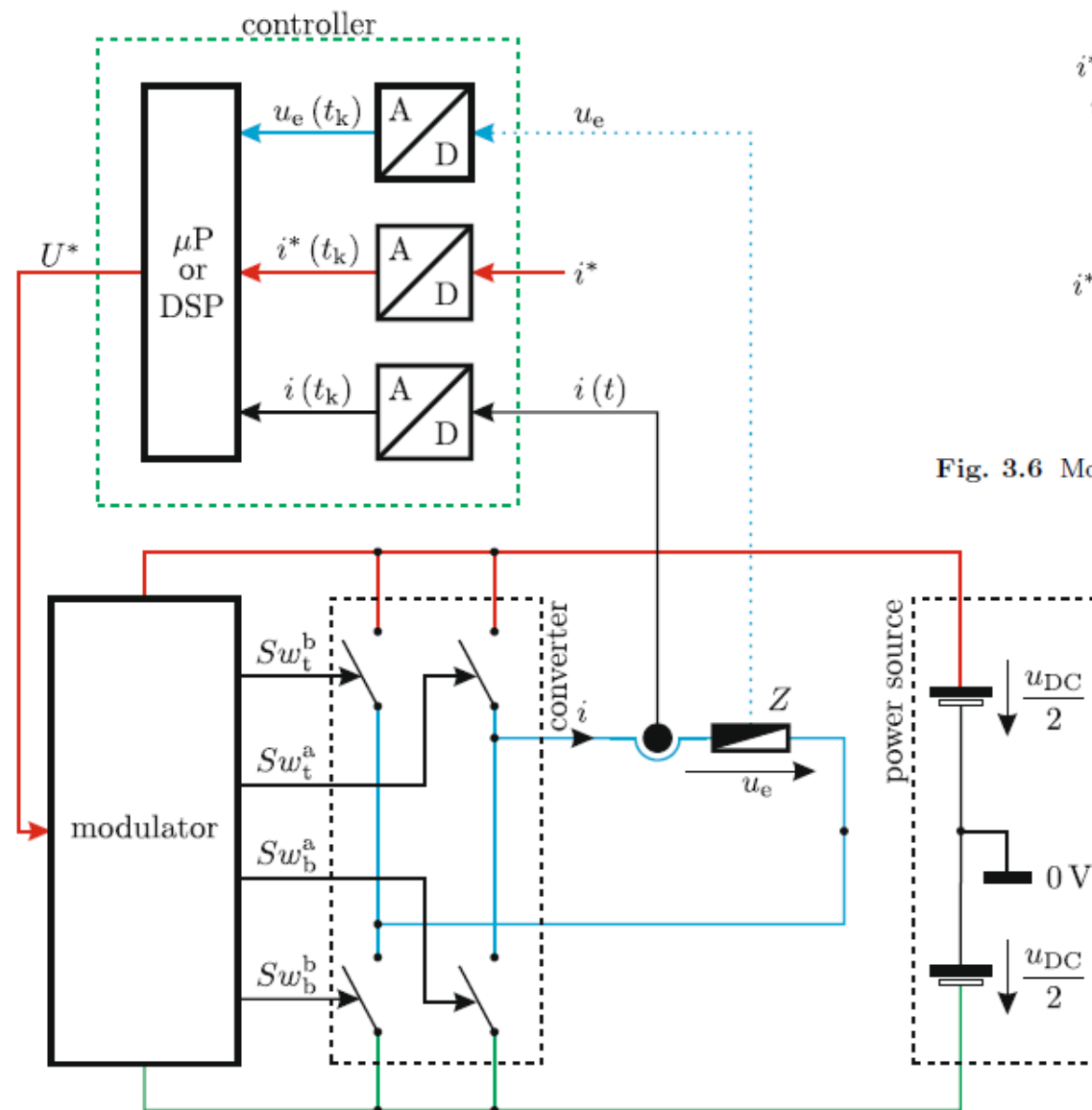


Fig. 3.5 Single-phase model based current control

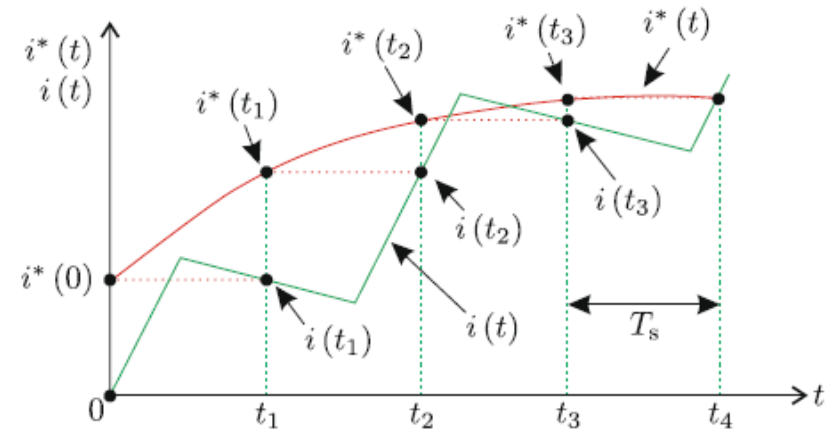


Fig. 3.6 Model based current control