

2.3 S-metaheuristics

Tabu Search (TS)

S-metaheuristics – Tabu Search Algorithm (TS)

- Main characteristics
 - Use of memory to store information related to the search process
 - Accept non improving solutions to escape from local optima

S-metaheuristics – Tabu Search Algorithm (TS)

Template of tabu search algorithm.

Save S as
best_solution

$s = s_0$; /* Initial solution */

Initialize the tabu list,

Repeat

Improve best_ s

Compare S with the
best_solution and
update if necessary

Find best admissible neighbor s' ; /* non tabu or aspiration criterion holds */

$s = s'$;

Update tabu list,

A Queue of K elements
with FIFO update

Until Stopping criteria satisfied

Output: Best solution found.

S-metaheuristics – Tabu Search Algorithm (TS)

- Best admissible solution (TSP prob): **Scenario 1**

Solution S:

1	2	3	4	5
4	3	2	5	1

 : Objective Fct = 33

2	3	4	5	1
---	---	---	---	---

Objective Fct : 50

4	2	3	5	1
---	---	---	---	---

Objective Fct : 39

Neighborhood of solution S

4	1	2	5	3
---	---	---	---	---

Objective Fct : 21



Tabu list empty

S-metaheuristics – Tabu Search Algorithm (TS)

- Best admissible solution (TSP prob): **Scenario 1**

Solution S:

1	2	3	4	5
4	3	2	5	1

 : Objective Fct = 33

2	3	4	5	1
---	---	---	---	---

Objective Fct : 50

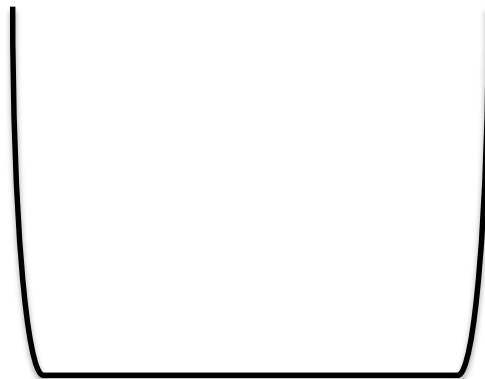
4	2	3	5	1
---	---	---	---	---

Objective Fct : 39

Neighborhood of solution S

4	1	2	5	3
---	---	---	---	---

Objective Fct : 21



Tabu list empty

Then best admissible solution is

4	1	2	5	3
---	---	---	---	---

Objective Fct : 21

S-metaheuristics – Tabu Search Algorithm (TS)

- Best admissible solution (TSP prob): **Scenario 2**

Solution S:

1	2	3	4	5
4	1	2	5	3

 : Objective Fct = 21

2	3	4	5	1
---	---	---	---	---

Objective Fct : 50

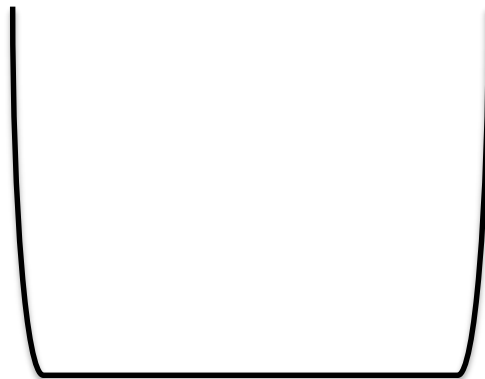
4	2	3	5	1
---	---	---	---	---

Objective Fct : 39

Neighborhood of solution S

4	3	2	5	1
---	---	---	---	---

Objective Fct : 33



Tabu list empty

Then best admissible solution is

4	3	2	5	1
---	---	---	---	---

Objective Fct : 33

S-metaheuristics – Tabu Search Algorithm (TS)

- Best admissible solution (TSP prob): **Scenario 3**

Solution S:

1	2	3	4	5
4	1	2	5	3

 : Objective Fct = 21

2	3	4	5	1
---	---	---	---	---

Objective Fct : 50

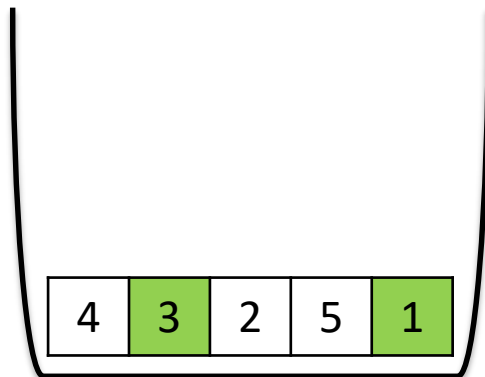
4	2	3	5	1
---	---	---	---	---

Objective Fct : 39

Neighborhood of solution S

4	3	2	5	1
---	---	---	---	---

Objective Fct : 33



Tabu list empty

Then best admissible solution is

4	2	3	5	1
---	---	---	---	---

Objective Fct : 39

S-metaheuristics – Tabu Search Algorithm (TS)

- Main characteristics
 - TS behaves like a steepest LS algorithm, but it accepts non improving solutions to escape from local optima when all neighbors are non improving solutions.
 - Usually, the whole neighborhood is explored in a deterministic manner
 - Dynamic neighborhood
 - Managing a short memory of visited neighbors called tabu list
 - Neighborhood of x may change according to the history of the search

S-metaheuristics – Tabu Search Algorithm (TS)

- Design issues
 - In addition to standard S-metaheuristics issues
 - Neighborhood definition
 - Initial solution generation
 - Tabu list definition
 - Which term for the memory
 - Short, medium, long (time consuming)
 - Aspiration criterion
 - Accept a tabu solution if it satisfies some “conditions”
➔ tabu solution better than the visited solutions

S-metaheuristics – Tabu Search Algorithm (TS)

- Additional search mechanisms
 - Intensification (medium-term memory)
 - Save best solutions (elite) during search then focus search around its neighborhood
 - Diversification (long-term memory)
 - Save visited solutions along search process and explore unvisited areas

S-metaheuristics – Tabu Search Algorithm (TS)

Template of tabu search algorithm.

Save S as
best_solution

$s = s_0$; /* Initial solution */

Initialize the tabu list, medium-term

Repeat

Improve best_ S

Compare S with the
best_solution and
update if necessary

Find best admissible neighbor s' ; /* non tabu or aspiration criterion holds */

$s = s'$;

Update tabu list, aspiration conditions, medium and long term memories ;

If intensification_criterion holds **Then** intensification ;

Until Stopping criteria satisfied

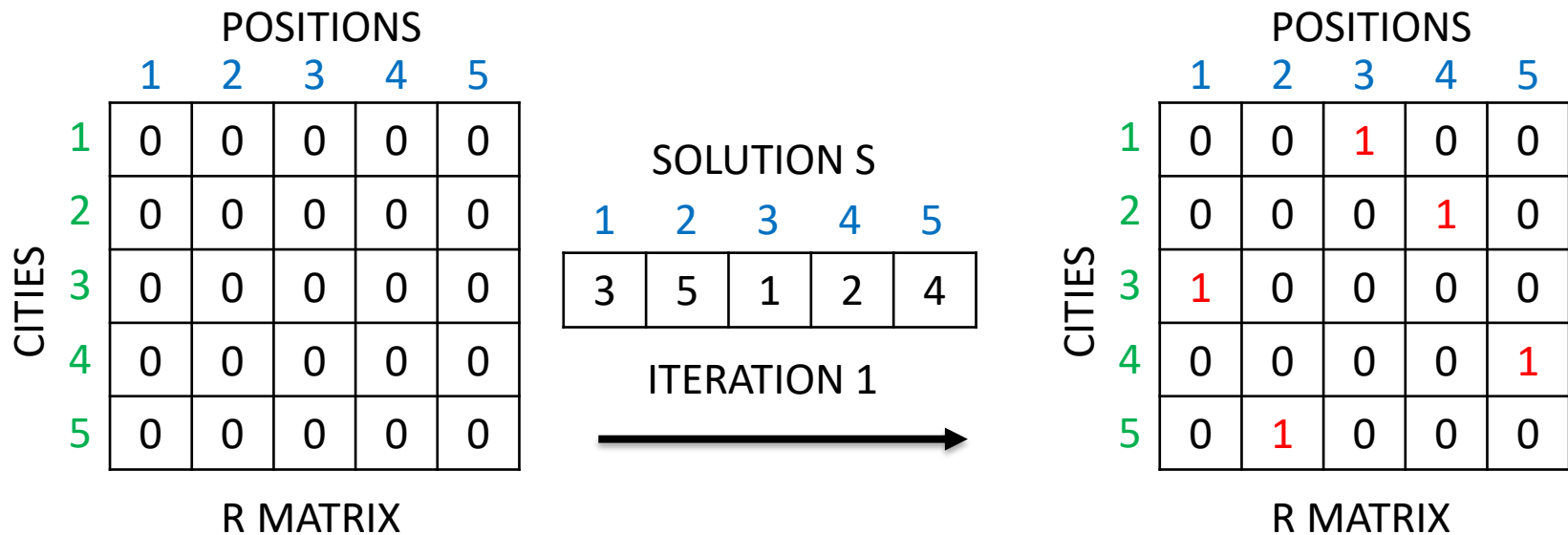
Output: Best solution found.

S-metaheuristics – Tabu Search Algorithm (TS)

- Intensification (medium term memory)
 - Exploit best solutions for guiding search in promising regions
 - Recency memory technic (problem dependent)
 - Start search from best found solution while keeping some of its components unchangeable
 - The ones that are present for a successive set of iterations
 - Example for TSP (Travelling Salesman problem)
 - Create a matrix R where $r(i,j)$ corresponding to number of successive iterations where city i is positioned at j
 - Intensification
 - Start search from best found solution S
 - Use largest values of R to freeze components of S
 - Focus search on other components

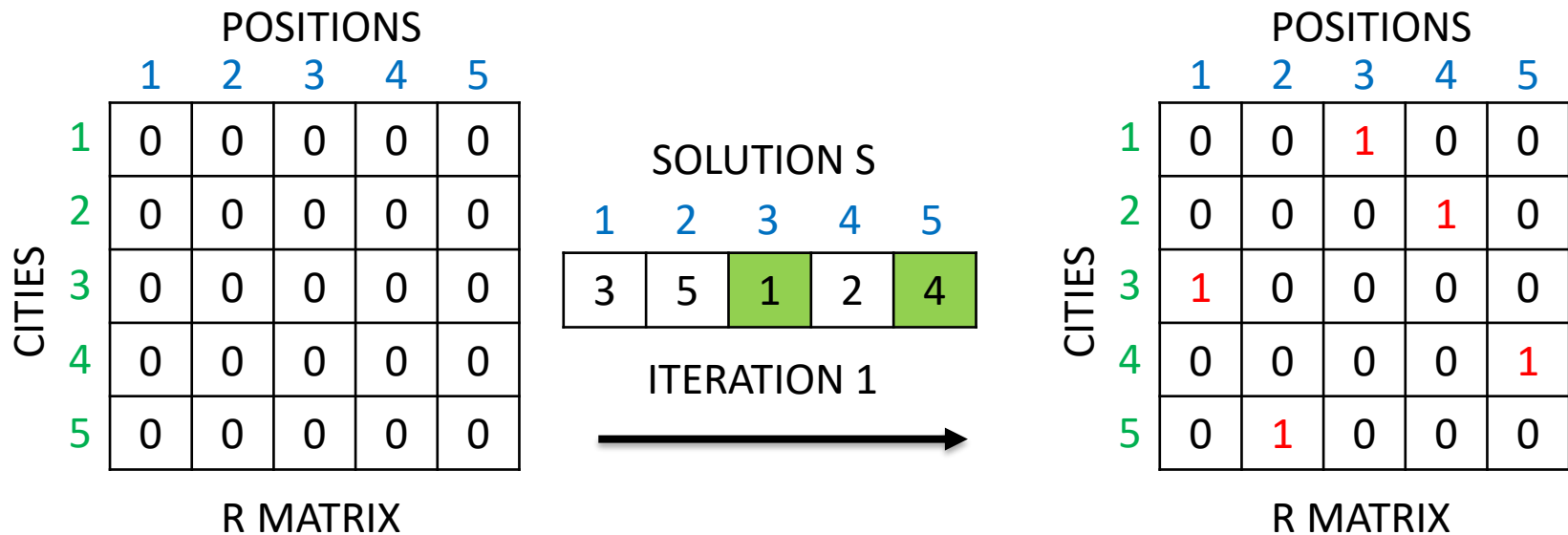
S-metaheuristics – Tabu Search Algorithm (TS)

- Intensification (Recency memory – TSP 5 cities)



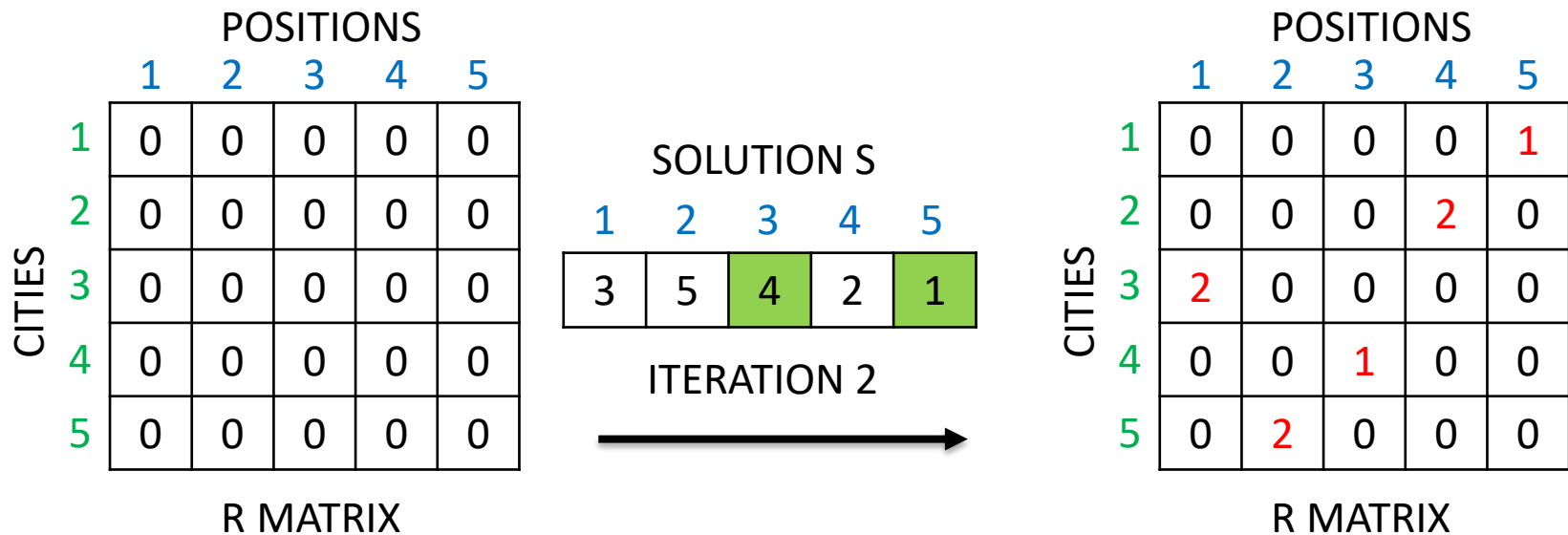
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- Intensification (Recency memory – TSP 5 cities)



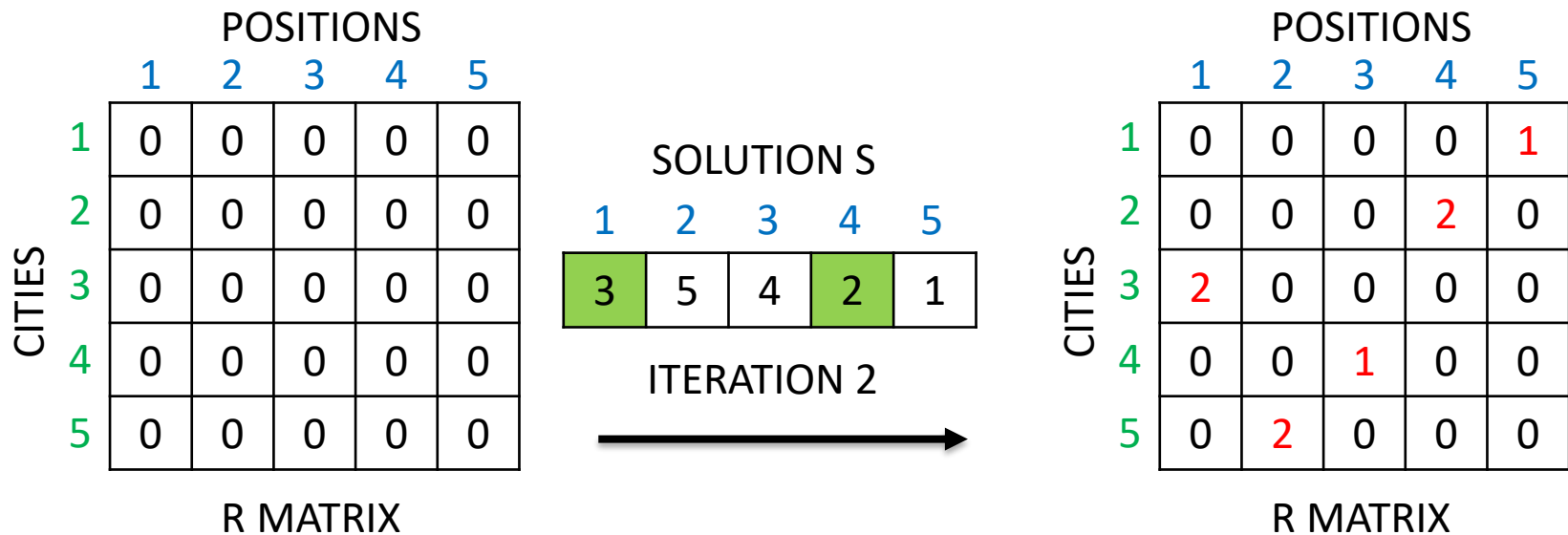
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- Intensification (Recency memory – TSP 5 cities)



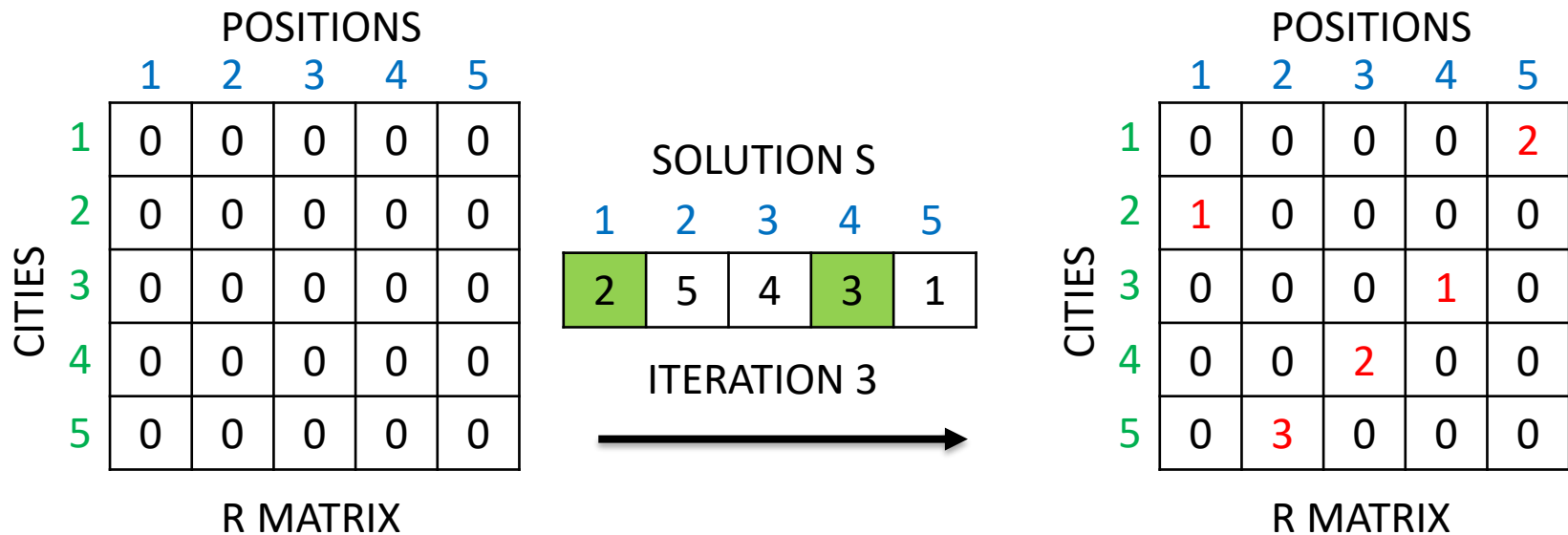
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- Intensification (Recency memory – TSP 5 cities)



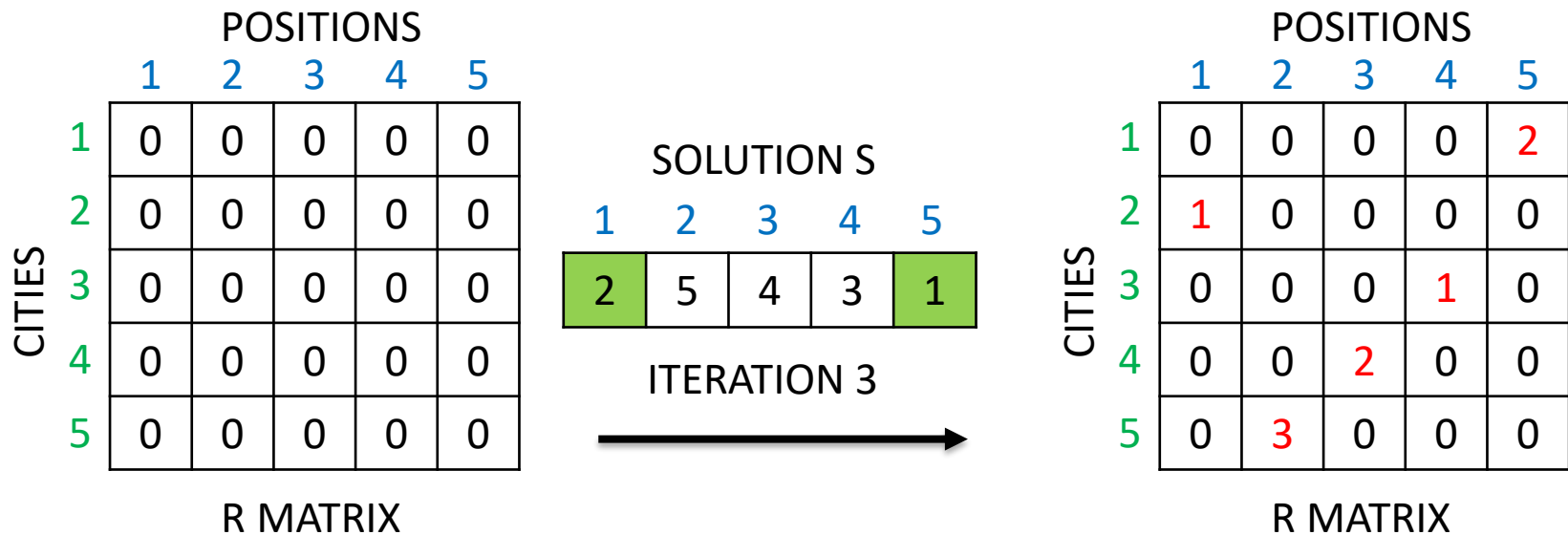
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- Intensification (Recency memory – TSP 5 cities)



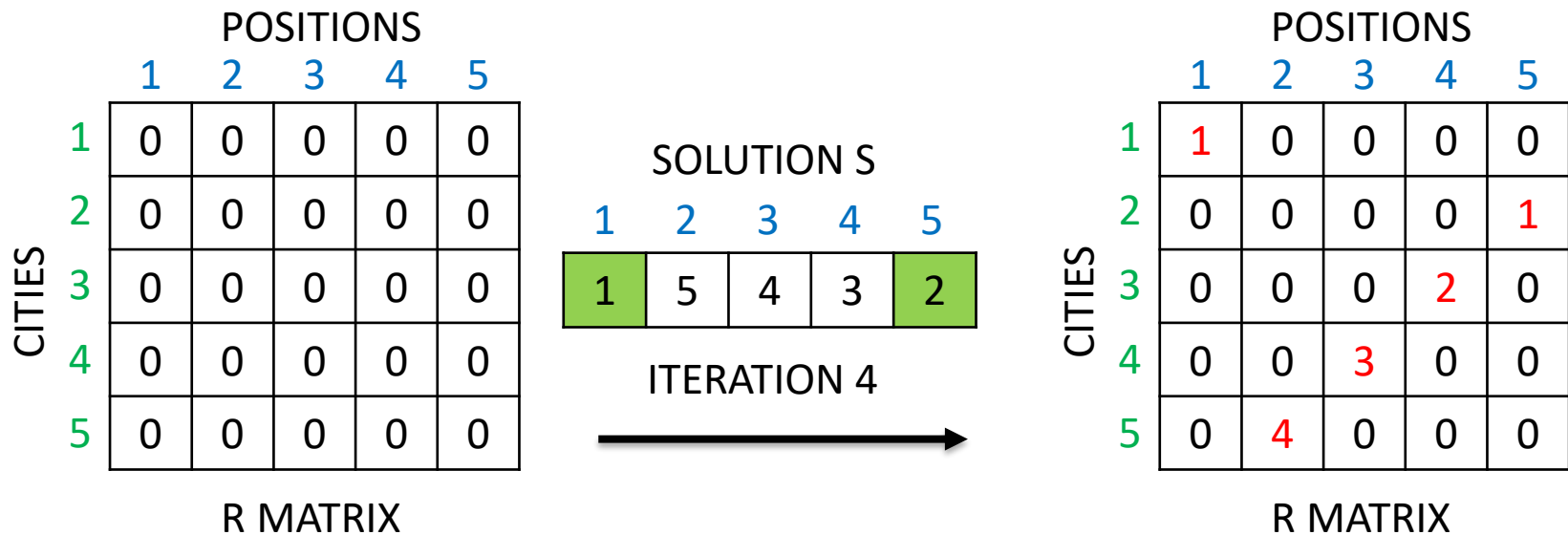
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- Intensification (Recency memory – TSP 5 cities)



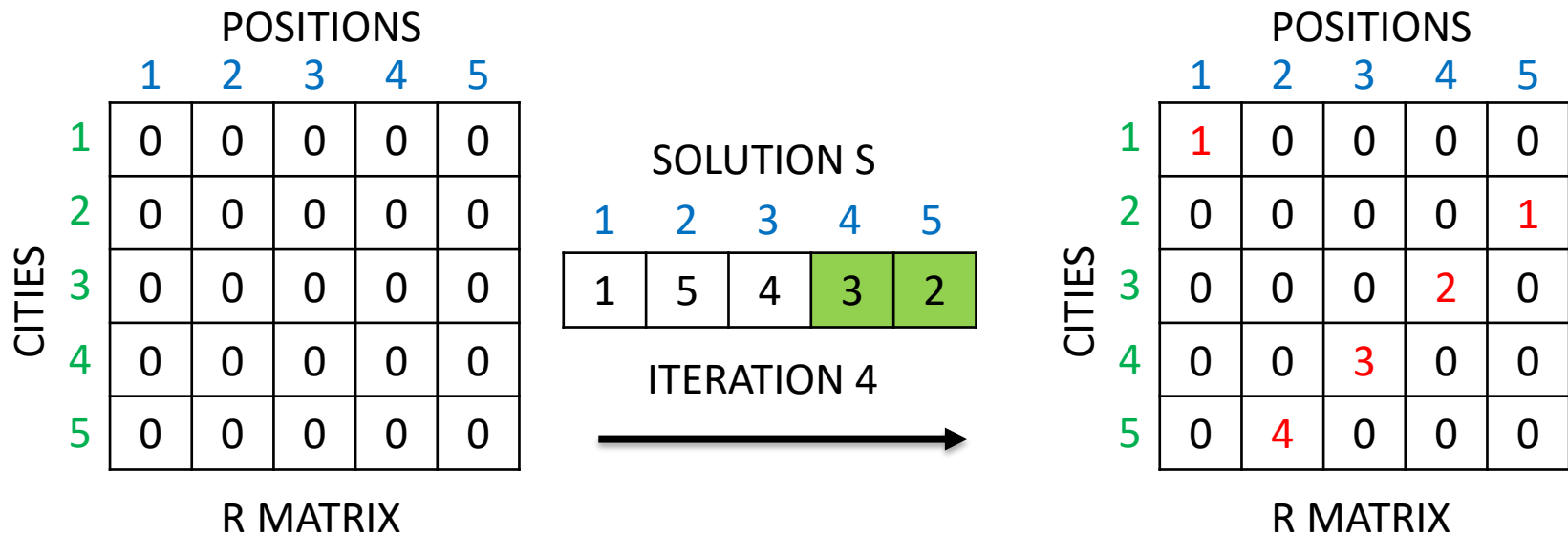
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- Intensification (Recency memory – TSP 5 cities)



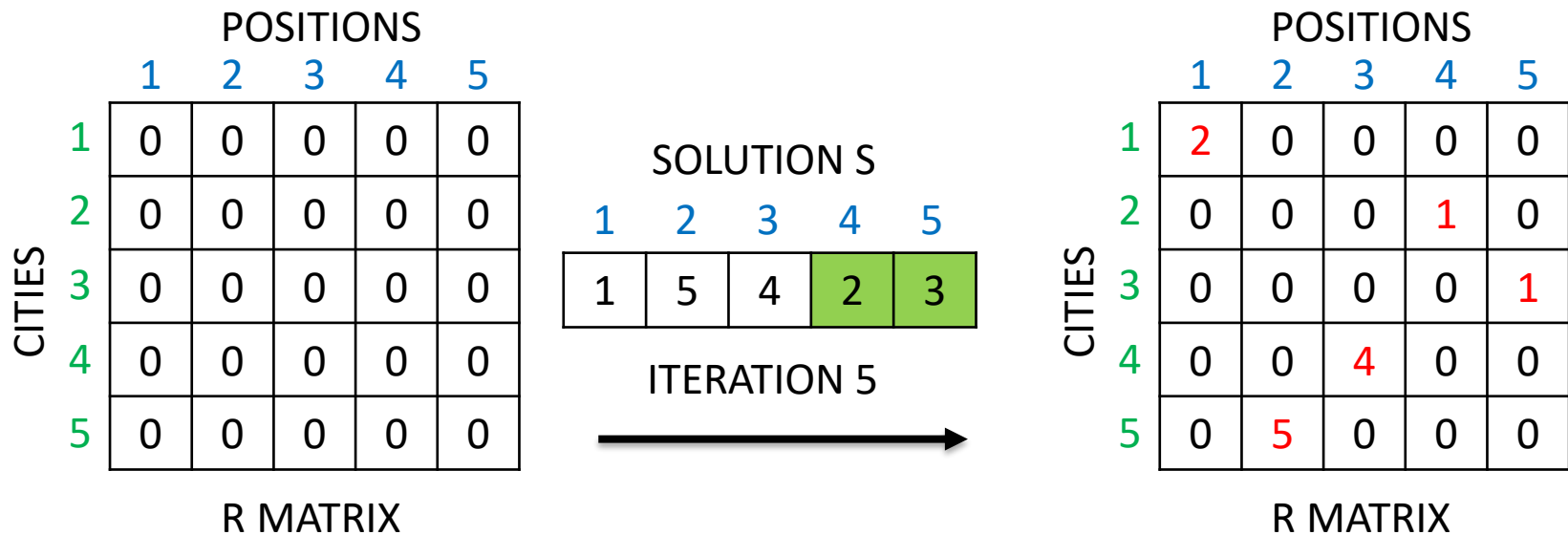
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- Intensification (Recency memory – TSP 5 cities)



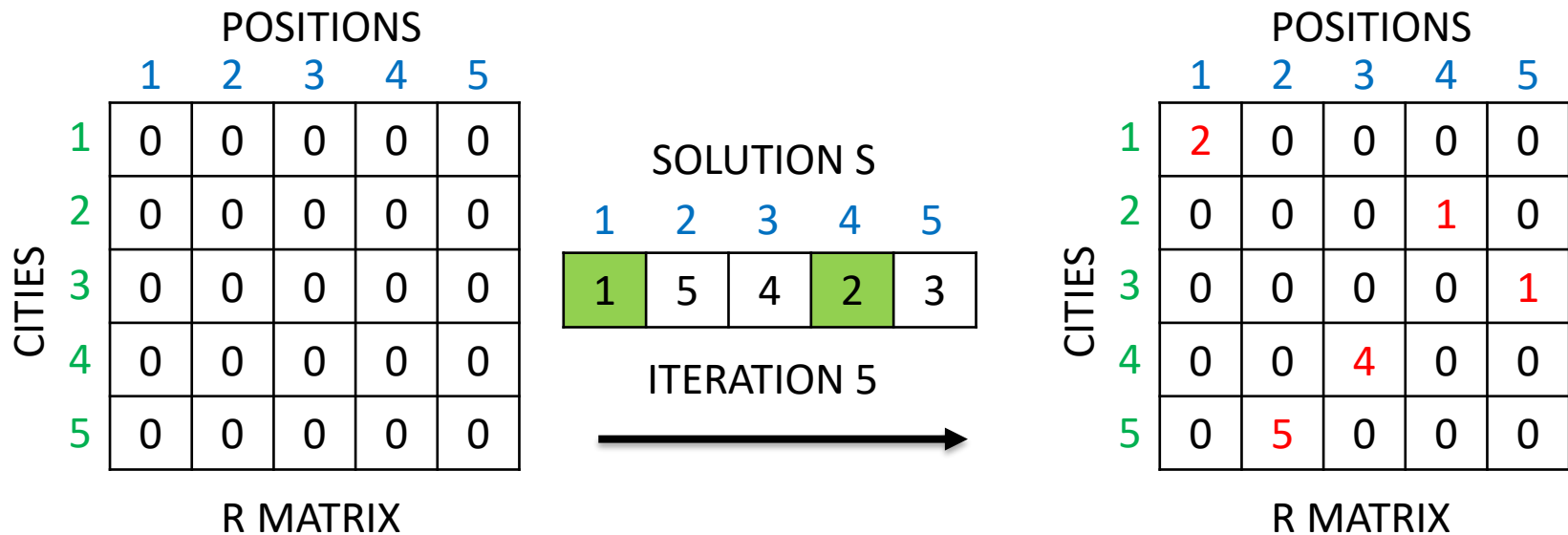
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- Intensification (Recency memory – TSP 5 cities)



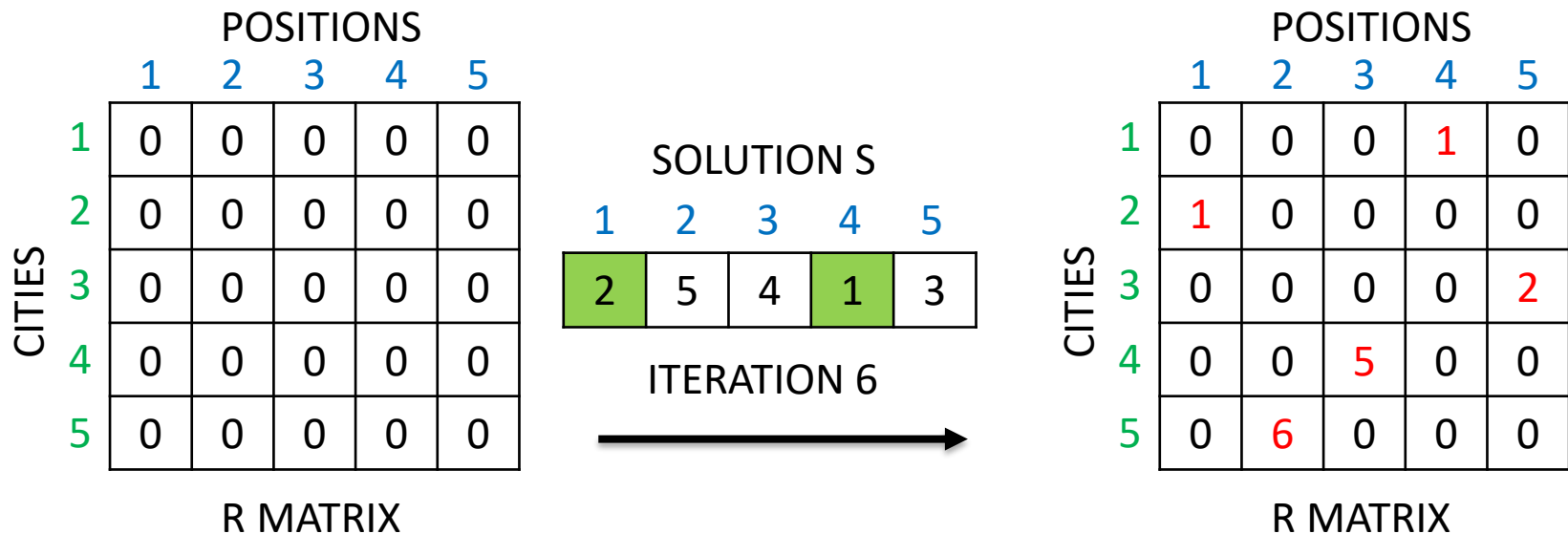
S-metaheuristics – Tabu Search Algorithm (TS)

- Intensification (Recency memory – TSP 5 cities)



S-metaheuristics – Tabu Search Algorithm (TS)

- Intensification (Recency memory – TSP 5 cities)



S-metaheuristics – Tabu Search Algorithm (TS)

• Intensification (Recency memory – TSP 5 cities)

- Intensification
 - Start search from best found solution S
 - Use largest values of R to freeze components of S
 - Focus search on other components

After a given number of iterations, start intensification

Best solution S (initial one)

1	2	3	4	5
3	5	1	2	4



	POSITIONS				
	1	2	3	4	5
1	0	0	0	1	0
2	1	0	0	0	0
3	0	0	0	0	2
4	0	0	5	0	0
5	0	6	0	0	0

R MATRIX



1	2	3	4	5
3	5	1	2	4

S-metaheuristics – Tabu Search Algorithm (TS)

• Intensification (Recency memory – TSP 5 cities)

- Intensification
 - Start search from best found solution S
 - Use largest values of R to freeze components of S
 - Focus search on other components

After a given number of iterations, start intensification

Best solution S (initial one)

1	2	3	4	5
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	POSITIONS				
	1	2	3	4	5
1	0	0	0	1	0
2	1	0	0	0	0
3	0	0	0	0	2
4	0	0	5	0	0
5	0	6	0	0	0

R MATRIX



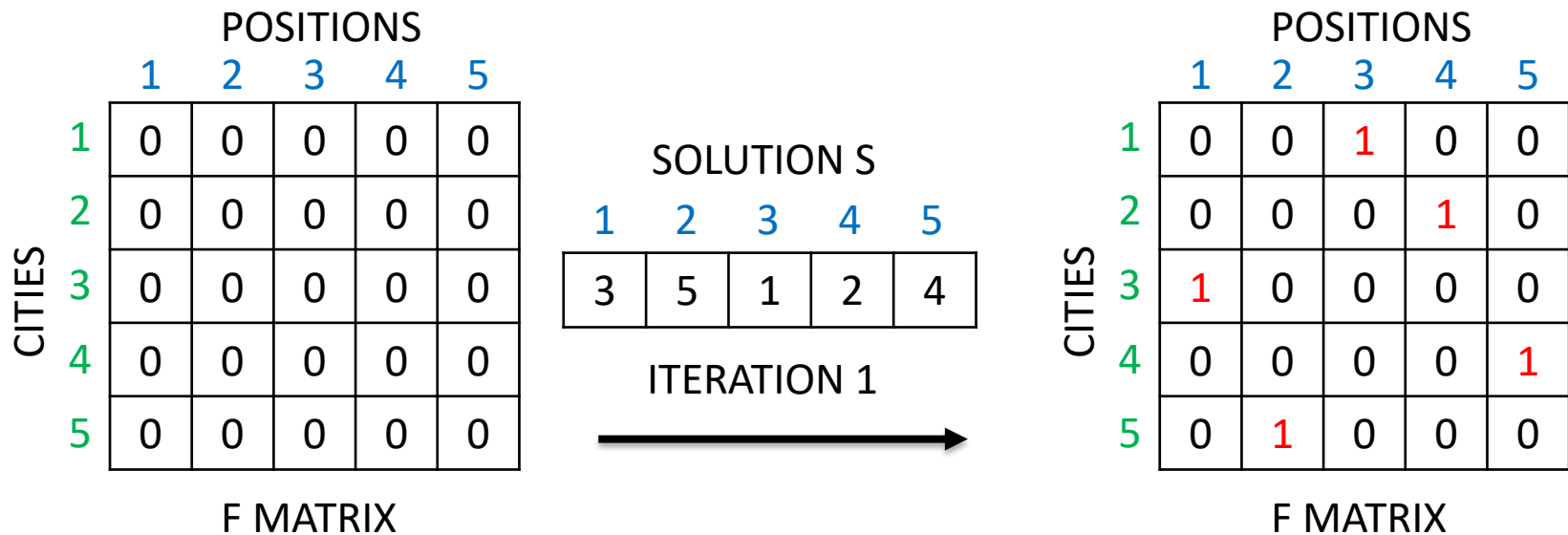
1	2	3	4	5
3	5	1	2	4

S-metaheuristics – Tabu Search Algorithm (TS)

- Diversification (long term memory)
 - Forcing search in unexplored regions
 - Frequency memory technic (problem dependent)
 - Focus search on less changed components in the search history
 - Example for TSP (Travelling Salesman problem)
 - Create a matrix F where $f(i,j)$ corresponding to number of iterations where city i is positioned at j from the starting of algorithm
 - Diversification
 - Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

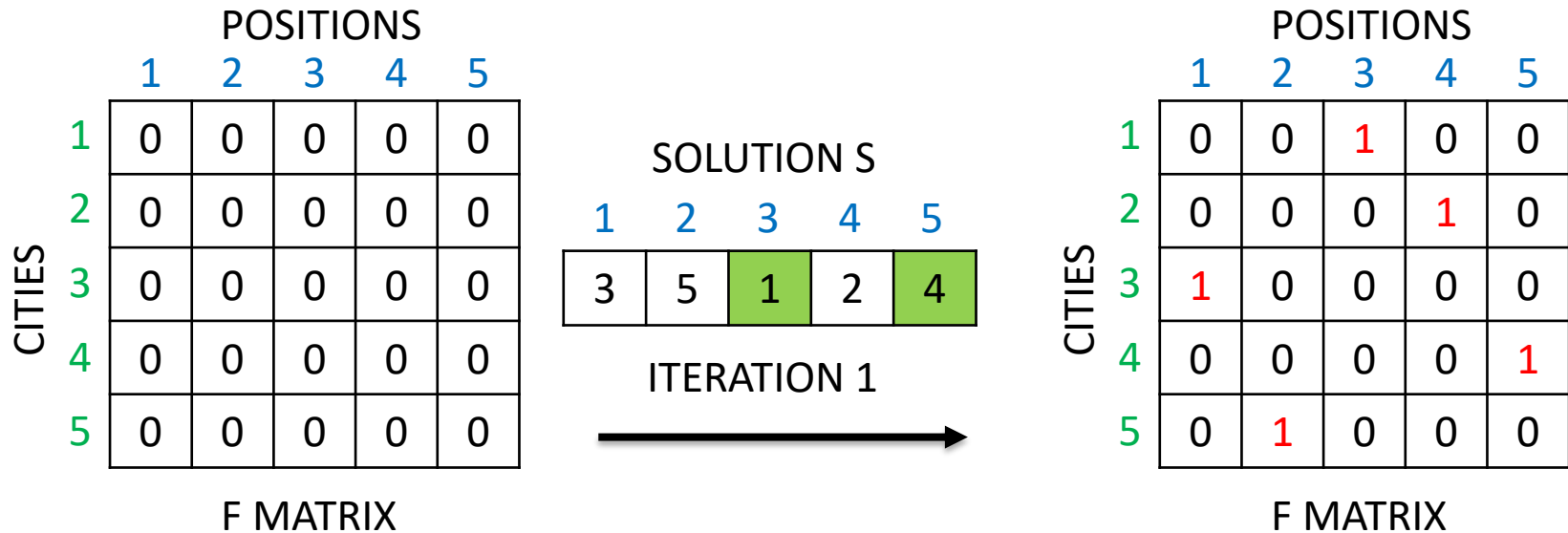
S-metaheuristics – Tabu Search Algorithm (TS)

- Diversification (Frequency memory – TSP 5 cities)



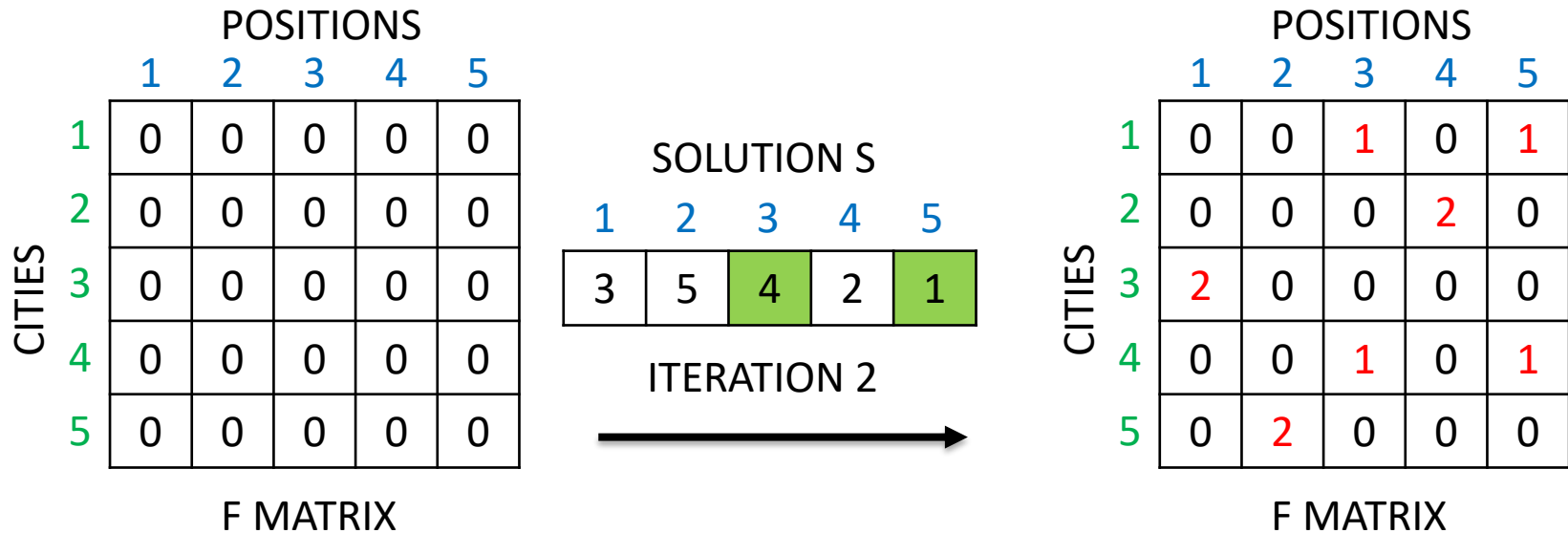
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- Diversification (Frequency memory – TSP 5 cities)



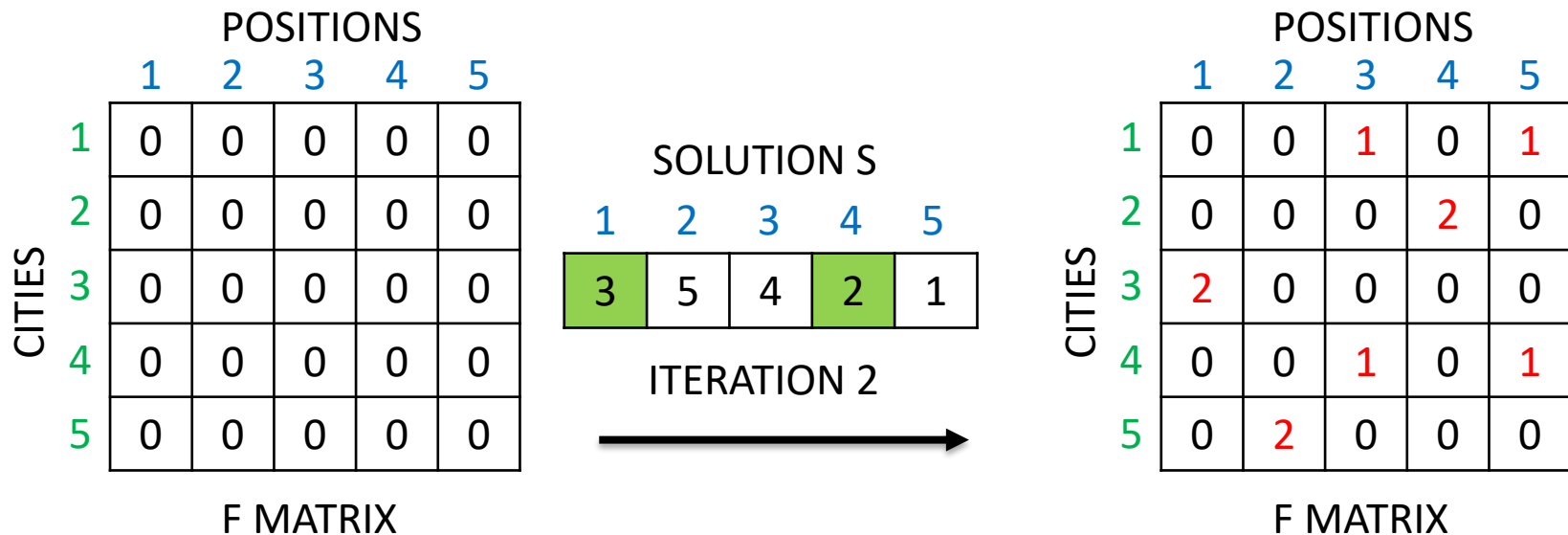
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- Diversification (Frequency memory – TSP 5 cities)



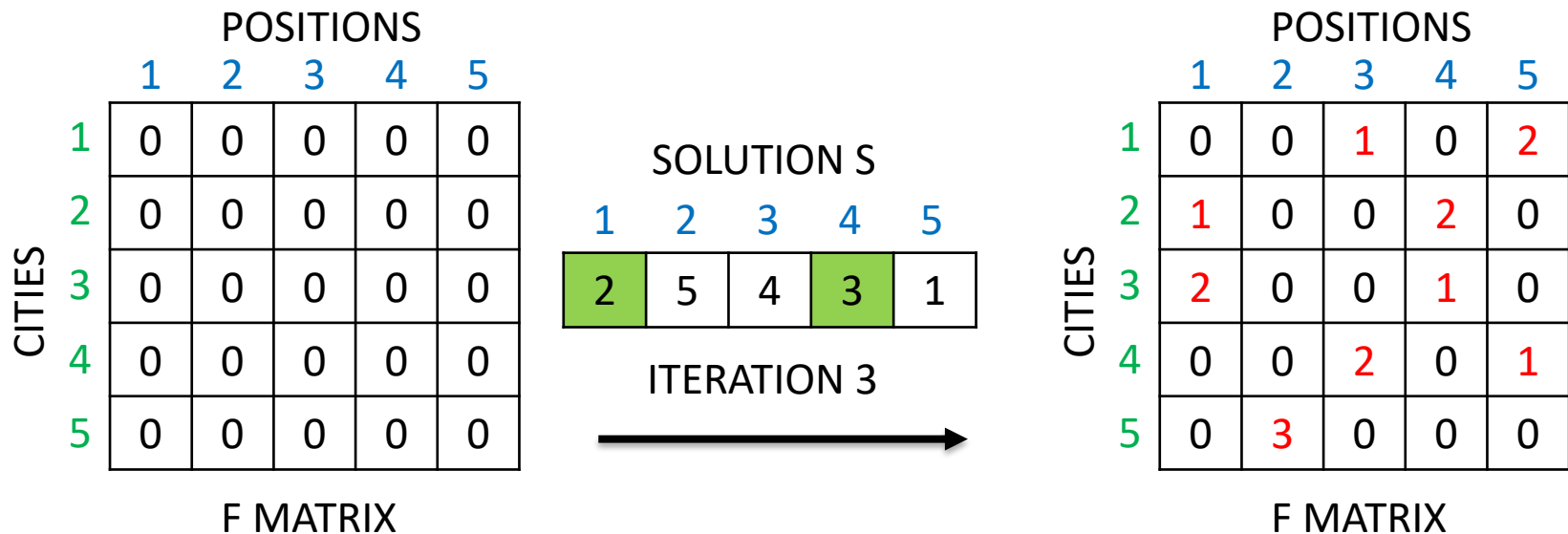
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- Diversification (Frequency memory – TSP 5 cities)



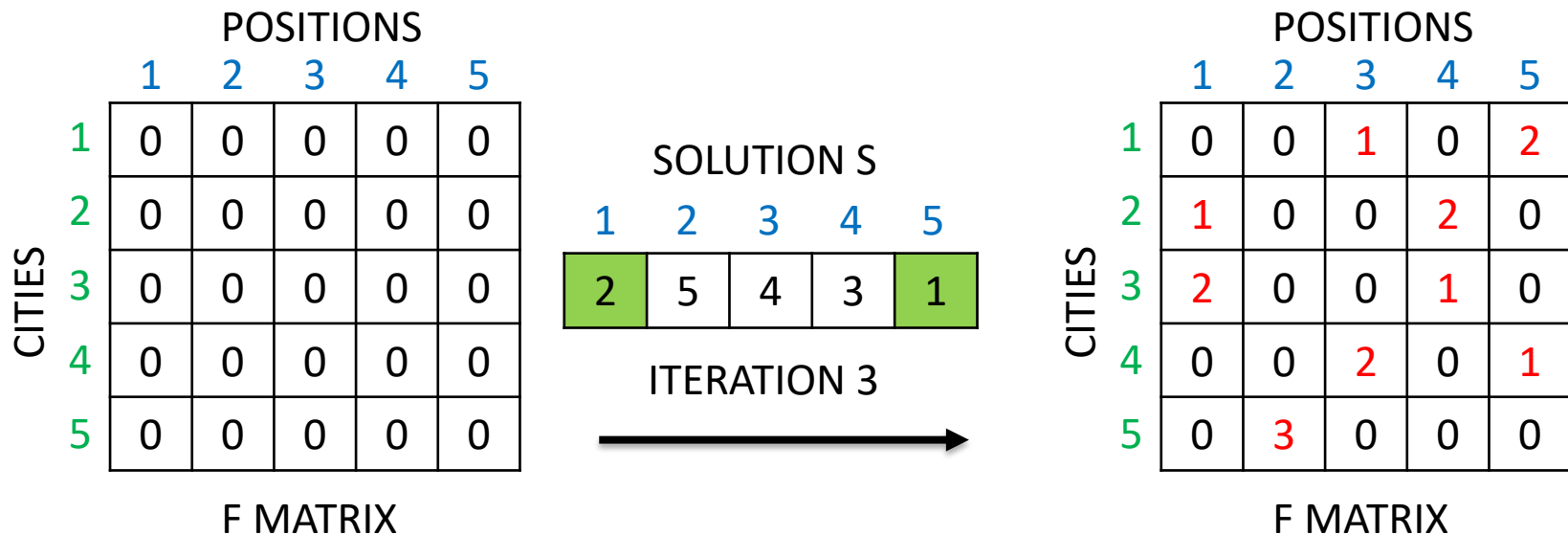
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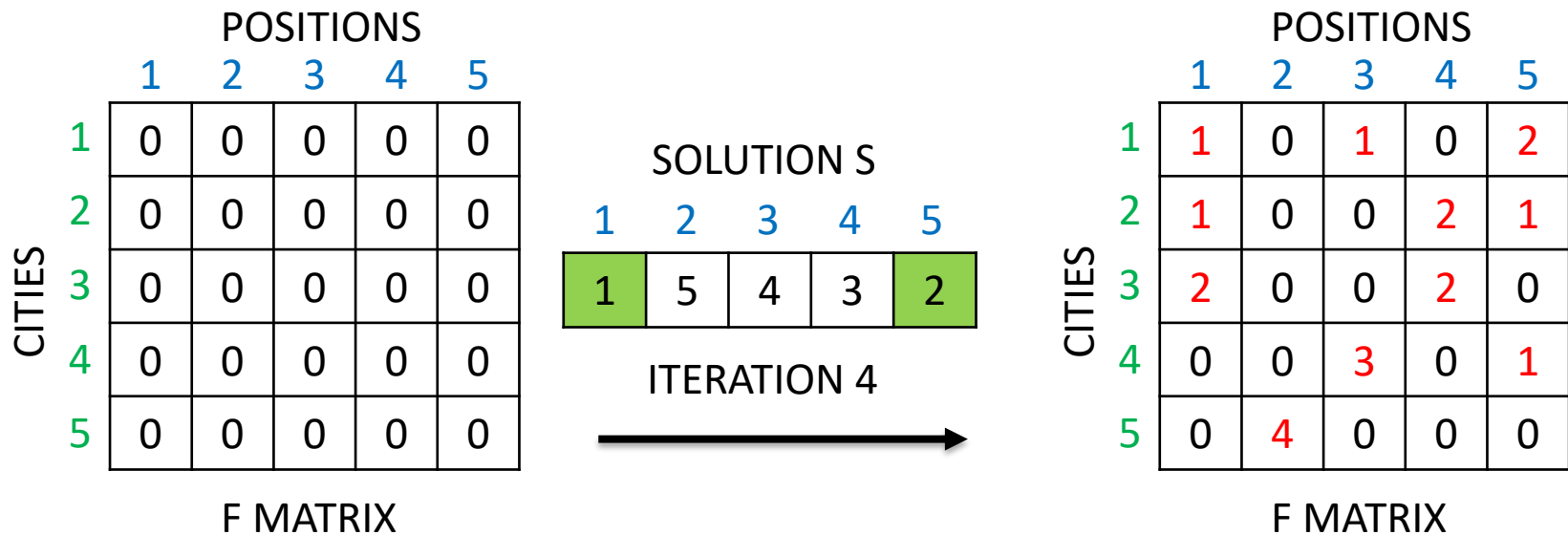
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- Diversification (Frequency memory – TSP 5 cities)



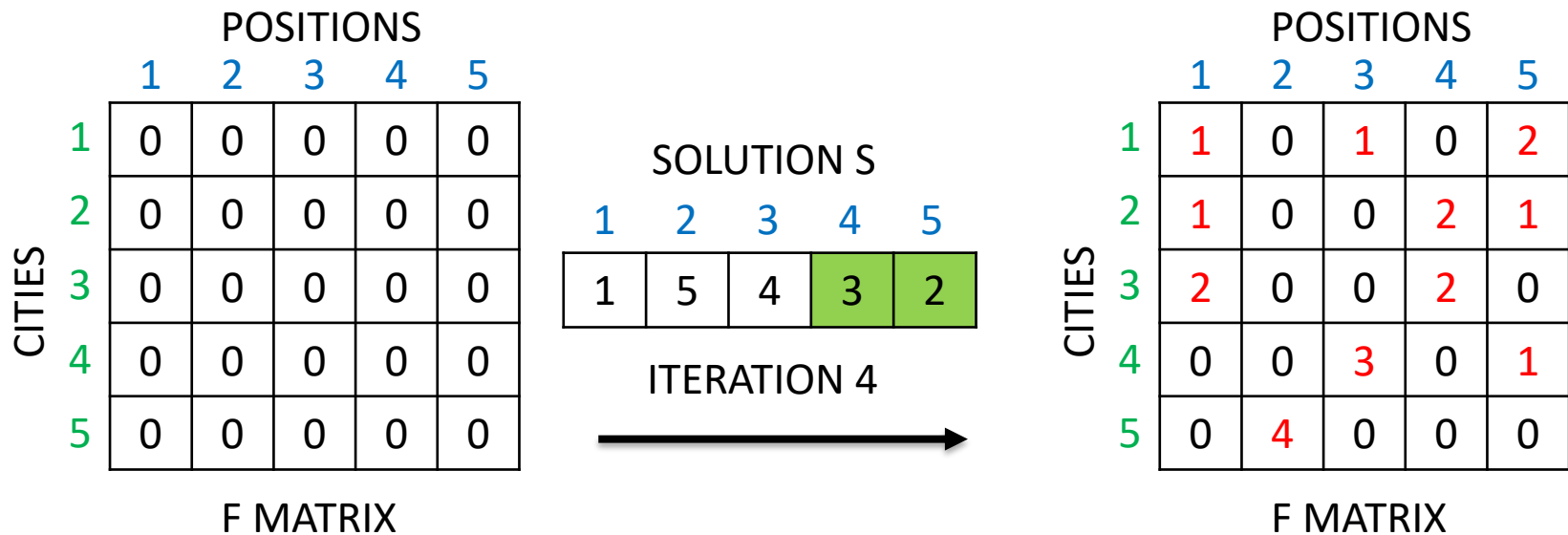
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- Diversification (Frequency memory – TSP 5 cities)



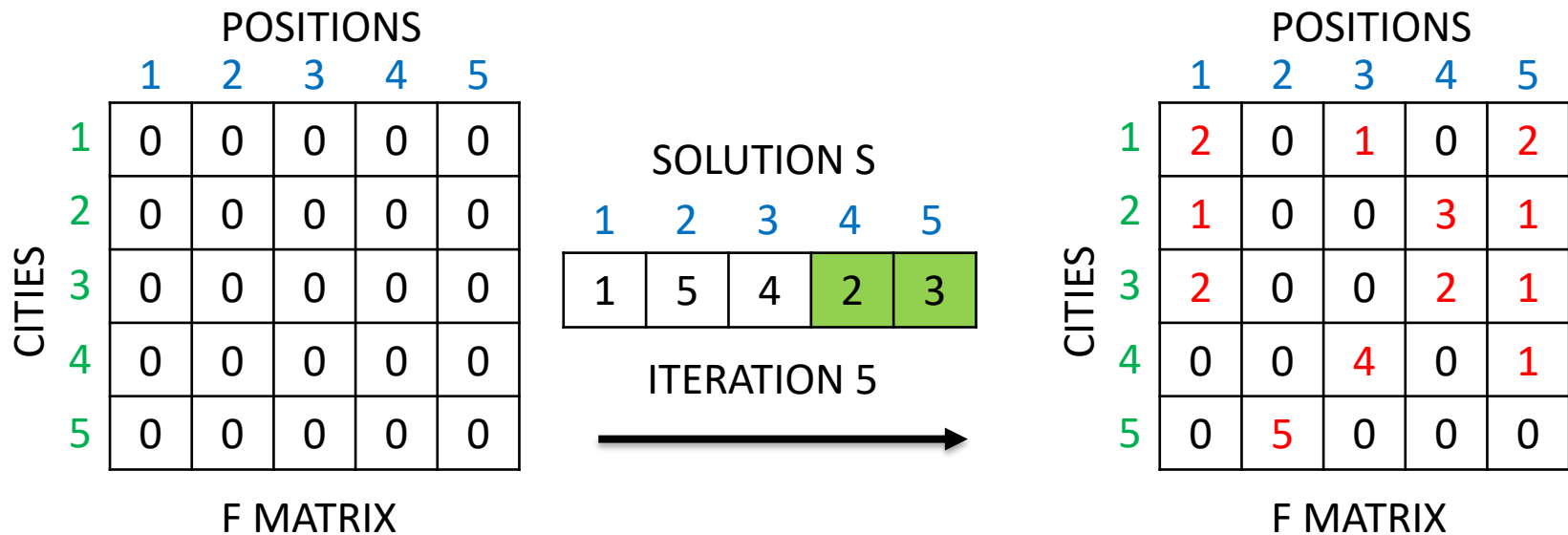
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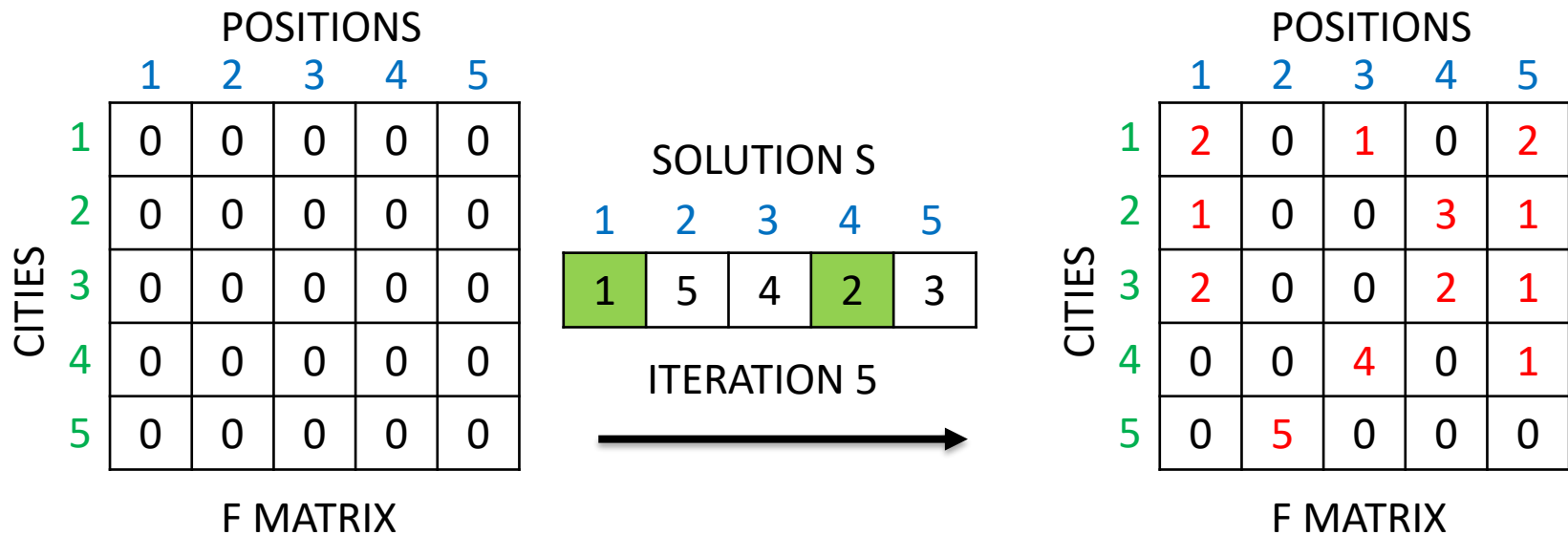
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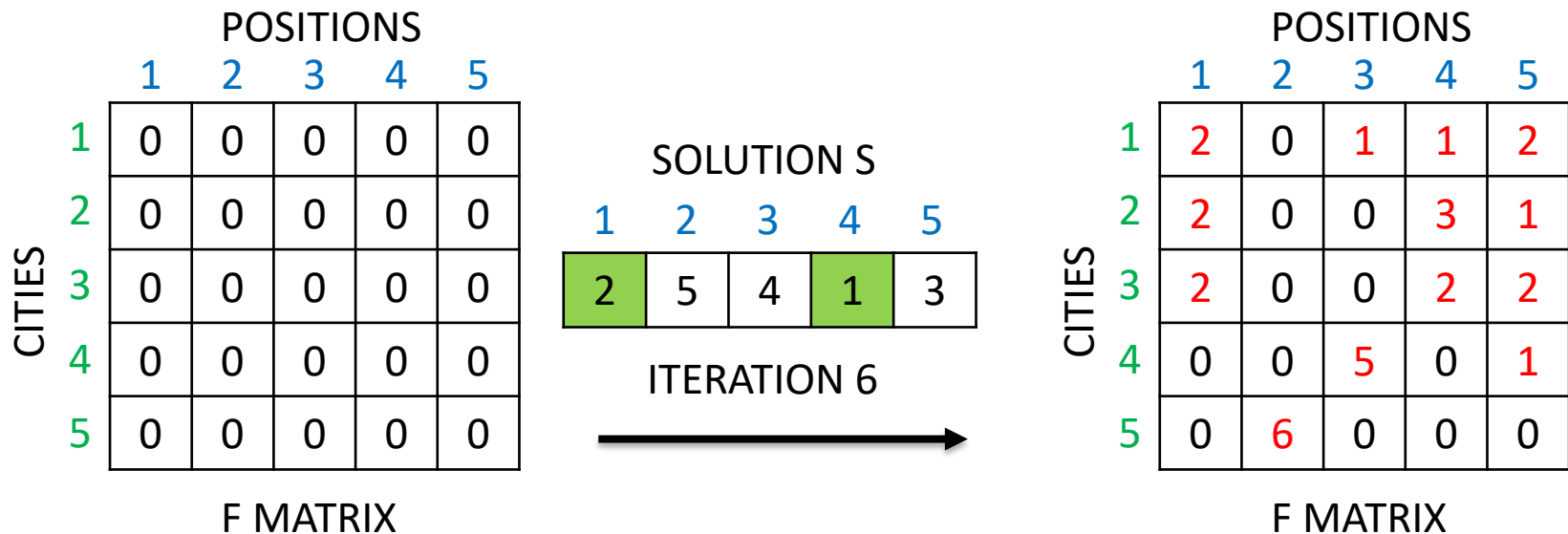
S-metaheuristics – Tabu Search Algorithm (TS)

- Diversification (Frequency memory – TSP 5 cities)



S-metaheuristics – Tabu Search Algorithm (TS)

- Diversification (Frequency memory – TSP 5 cities)



S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

After a given number of iterations, start diversification

New solution S

1	2	3	4	5



	POSITIONS				
	1	2	3	4	5
1	2	0	1	1	2
2	2	0	0	3	1
3	2	0	0	2	2
4	0	0	5	0	1
5	0	6	0	0	0

F MATRIX



S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

New solution S

1	2	3	4	5
5				



	POSITIONS				
	1	2	3	4	5
1	2	0	1	1	2
2	2	0	0	3	1
3	2	0	0	2	2
4	0	0	5	0	1
5	0	6	0	0	0

F MATRIX



After a given number of iterations, start diversification

S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

New solution S

1	2	3	4	5
5				



	POSITIONS				
	1	2	3	4	5
	1	0	1	1	2
	2	0	0	3	1
	3	0	0	2	2
	4	0	5	0	1
	5				

F MATRIX



After a given number of iterations, start diversification

S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

New solution S

1	2	3	4	5
5	1			



	POSITIONS				
	1	2	3	4	5
1		0	1	1	2
2		0	0	3	1
3		0	0	2	2
4		0	5	0	1
5					

F MATRIX



After a given number of iterations, start diversification

S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

After a given number of iterations, start diversification

New solution S

1	2	3	4	5
5	1			



POSITIONS

	1	2	3	4	5
1					
2					
3					
4					
5					

CITIES

	1	2	3
2	0	3	1
3	0	2	2
4	5	0	1

F MATRIX



S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

After a given number of iterations, start diversification

New solution S

1	2	3	4	5
5	1	3		



POSITIONS

	1	2	3	4	5
1					
2					
3					
4					
5					

CITIES	1	2	3
2	0	3	1
3	0	2	2
4	5	0	1

F MATRIX



S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

After a given number of iterations, start diversification

New solution S

1	2	3	4	5
5	1	3		



POSITIONS

	1	2	3	4	5
1					
2				3	1
3					
4				0	1
5					

CITIES

F MATRIX



S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

After a given number of iterations, start diversification

New solution S

1	2	3	4	5
5	1	3	4	



POSITIONS

	1	2	3	4	5
1					
2				3	1
3					
4				0	1
5					

CITIES

F MATRIX



S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

After a given number of iterations, start diversification

New solution S

1	2	3	4	5
5	1	3	4	



POSITIONS

	1	2	3	4	5
1					
2					1
3					
4					
5					

CITIES

F MATRIX



S-metaheuristics – Tabu Search Algorithm (TS)

• Diversification (Frequency memory – TSP 5 cities)

• Diversification

- Start search from a new initial solution S generated as follow
 - Use smallest values of F to replace components of S
 - Pursue search

After a given number of iterations, start diversification

New solution S

1	2	3	4	5
5	1	3	4	2



POSITIONS

	1	2	3	4	5
1					
2					
3					
4					
5					1

CITIES

F MATRIX



1	2	3	4	5
5	1	3	4	2

S-metaheuristics – Tabu Search Algorithm (TS)

Template of tabu search algorithm.

Save S as
best_solution

$s = s_0$; /* Initial solution */

Initialize the tabu list, medium-term and long-term memories ;

Improve S

Repeat

Find best admissible neighbor s' ; /* non tabu or aspiration criterion holds */

$s = s'$;

Update tabu list, aspiration conditions, medium and long term memories ;

If intensification_criterion holds **Then** intensification ;

If diversification_criterion holds **Then** diversification ;

Until Stopping criteria satisfied

Output: Best solution found.

Compare S with the
best_solution and
update if necessary

S-metaheuristics – Algorithms review

Local search

- Selection strategies of the best neighbor
 - Best improvement (steepest descent)
 - First improvement
 - Random improvement

High probability to fall into local optima

Simulated annealing

- Accepting the degradation of a solution under some conditions
 - High temperatures promote accepting bad solutions
 - Static temperatures prevent accepting very bad solutions

Tabu search

- Accepting the degradation of a solution if and only if
 - Non tabu solution
- Memory usage to optimize the search process
 - Short term → tabu list
 - Medium term → recency for intensification
 - Long term → frequency for diversification

Strategies to escape from local optima

S-metaheuristics – Tabu Search Algorithm (TS)



- Lab session – first part

Implement your third and last S-metaheuristic algorithm - the Tabu Search algorithm (TS) -

- Version 1, only Tabu list (recommended before next session)
- Version 2, add intensification process
- **Version 3, add diversification process**

Apply the 2 versions to

- The TSP problem – Data available on the campus
- Show the best solution and associated trajectory curve for each version