Metaheuristics

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Course session organization and evaluation

- 8 sessions of 3h45 [theory + lab]
- Written exam 2h
- Final score on 20

Material for this course

- Teams – Fichiers – supports de cours
  - This PPT
  - PDF document from [metaheuristics - from design to implementation - ResearchGate]

Course goals

- Design and develop multiple metaheuristic algorithms in Python.
- Apply these algorithms to solve real-world problems and formulate them as optimization problems.
- Analyze the performance of the algorithms and identify areas for improvement.

Chapters

- Introduction
  - Optimization problem definition
  - When to use metaheuristics
  - Metaheuristic algorithm design
- Single-solution based metaheuristics
  - Common concepts
  - Local search
  - Simulated Annealing
  - Tabu Search
- Population-based metaheuristics
  - Common concepts
  - Evolutionary algorithms (genetic Algorithms)
  - Swarm Intelligence
    - Ant colony
    - Bee colony (optional)

1. introduction

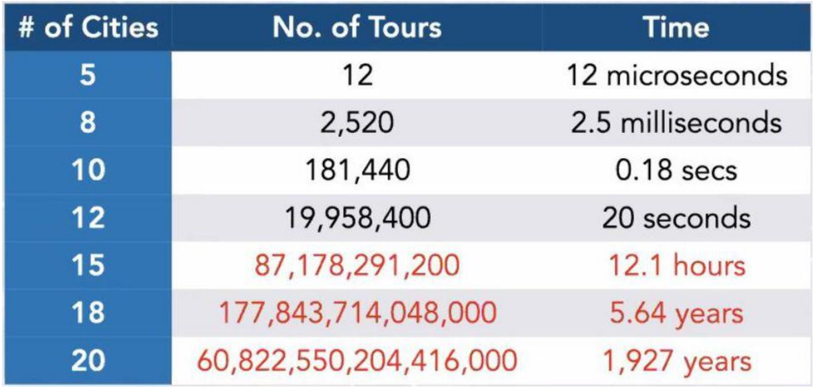
Combinatorial optimization problems / NP-hard problems

- Optimization problems -> computational problems in which the goal is to find the best of all possible solutions

- Many of them are NP-hard : not possible to solve in polynomial time

Combinatorial optimization problems / NP-hard problems

- Example
  - Traveling salesman problem – Visiting a number N of connected cities to each other, each one once and go back to starting point while minimizing the global distance



# of Cities	No. of Tours	Time
5	12	12 microseconds
8	2,520	2.5 milliseconds
10	181,440	0.18 secs
12	19,958,400	20 seconds
15	87,178,291,200	12.1 hours
18	177,843,714,048,000	5.64 years
20	60,822,550,204,416,000	1,927 years

$No.\ of\ Tours = n!$ without optimization

May be $\frac{1}{2}(n-1)!$ with optimization

Combinatorial optimization problems / NP-hard problems

- Example
  - Traveling salesman problem – Visiting a number N of connected cities to each other, each one once and go back to starting point while minimizing the global distance
  - Salesman applications
    - Logistic and transportations
    - Routing in networks
    - Manufacturing

Meataheuristic definition

- Metaheuristics definition - general methodology (template) used in designing the heuristic to solve optimization problems (F. Glover 1986)
- Metaheuristics provide a "good[1]" solution in a reasonable time for solving NP-hard problems
- Metaheuristics origins - Old Greek
  - Heuristics from *heuriskein* → Art of discovering new strategies (rules) to solve problems
  - Meta refers to upper methodology

[1] Not sure to be the best solution but close to the best one.

Meataheuristic algorithms

- A metaheuristic algorithm consists in an iterative process that attempt to find within the search space of solutions a good one. Each solution being associated with an objective function[1] value
- According to the nature of the problem, the algorithm will aim to maximize or minimize this objective function

[1] Called also fitness function which represent the quality of the solution.

Designing a metaheuristic algorithm

- Designing a metaheuristic algorithm
  - Choice of a Representation (Encoding) of solutions handled by the algorithm
  - Definition of the objective function that will guide the search

Designing a metaheuristic algorithm

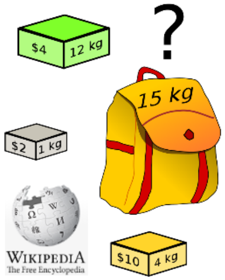
- Representation-Encoding
  - Completeness – all solutions associated to the problem must be represented
  - Connexity – any solution can be reached from another one in particular the global optimum
  - Efficiency – must be easy to manipulate by the search operators (time and space complexities reduced)

Designing a metaheuristic algorithm

- Representation-Encoding
  - Linear representation

1 0 0 0 1 1 0 1 1 1 0 1

Binary encoding



0/1 Knapsack problem

$$\forall i,\ s_i = \begin{cases} 1 & \text{if object } i \text{ is in the knapsack} \\ 0 & \text{otherwise} \end{cases}$$

5 7 6 6 4 3 8 4 2

Vector of discrete values

Generalized asignment problem

$$s[i] = j \quad \text{if the agent } j \text{ is assigned to task } i$$

1 4 8 9 3 6 5 2 7

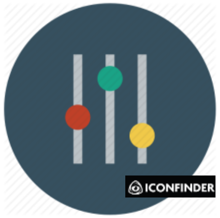
Permutation

Traveling salesman problem

$$\pi = (\pi_1, \pi_2, \ldots, \pi_n)$$

1.23 5.65 9.45 4.76 8.96

Vector of real values

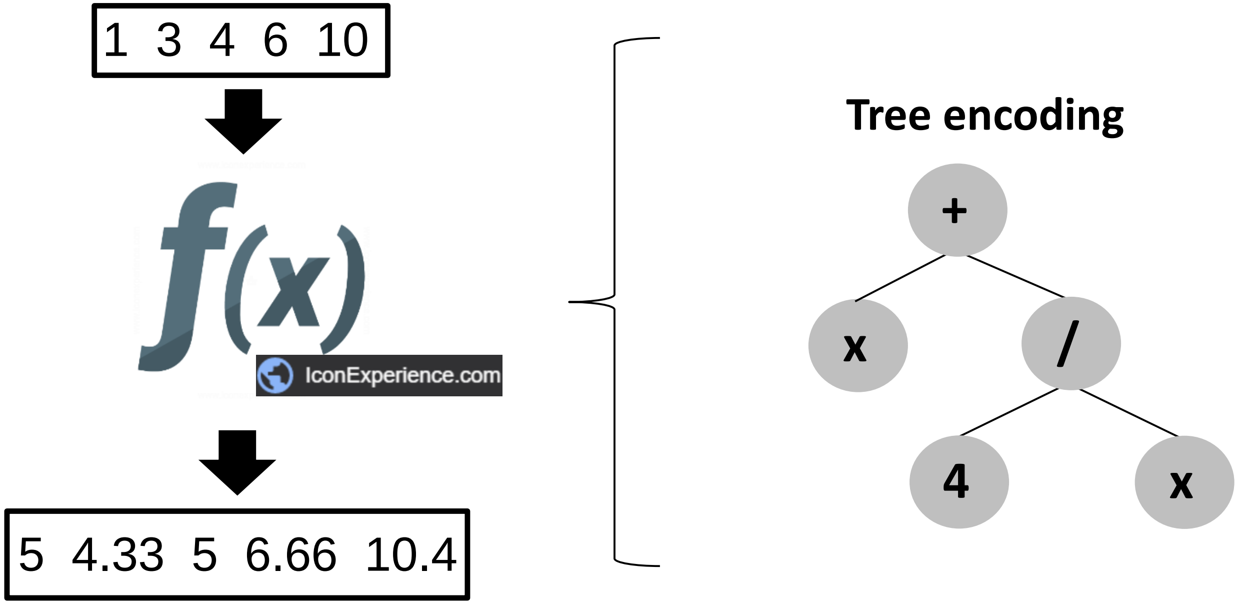


$$a_0 + 2x - 4y + 3yz + by = 0$$

Parameter identification

Designing a metaheuristic algorithm

- Representation-Encoding
  - Nonlinear representation



Regression problem

Designing a metaheuristic algorithm

- Objective function $f : S \to \mathbb{R}$
  - Real value for describing the quality
  - Solutions of search space sorted according to quality

1 4 8 9 3 6 5 2 7

Permutation

Traveling salesman problem

$\pi = (\pi_1, \pi_2, \ldots, \pi_n)$

Objective function →

$$f(s) = \sum_{i=1}^{n-1} d_{\pi(i),\pi(i+1)}$$

Metaheuristic algorithms

- Lab session

Using python or java and data (each file) provided in the Teams platform, propose an exact algorithm to solve the traveling salesman problem

For file ''p01.15.291.tsp''

Analyze, the associated running time and the obtained solutions (the best you reached) for the first n cities, n = 5,…,15 with optimized and non optimized version