Exercise 3.4

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Last updated Wednesday 28th February, 2024 at 15:35

If ν is a signed measure and λ, μ are positive measures such that $\nu = \lambda - \mu$, then $\lambda \geq \nu^+$ and $\mu \geq \nu^-$.

Solution. Let $X = P \cup N$ be a Hahn decomposition for X with respect to ν. Then for $E \in \mathcal{M}$

$$\nu^+(E) = \nu(E \cap P) = \lambda(E \cap P) - \mu(E \cap P) \leq \lambda(E \cap P) \leq \lambda(E) \tag{1}$$

since $E \cap P \subset E$ and λ is monotone as a positive measure. Similarly,

$$\nu^-(E) = -\nu(E \cap N) = -\lambda(E \cap N) + \mu(E \cap N) \leq \mu(E \cap N) \leq \mu(E) \tag{2}$$

since $E \cap N \subset E$ and μ is montone as positive measure.