Adding and Subtracting Fractions

Adding and subtracting fractions amounts to manipulating your expression in such a way as to make all denominators the same. When presented with two fractions which have the same denominators (bottom), we simply add (or subtract) the numerators (top) and create one fraction.

Examples 1.

$$\frac{1}{4}+\frac{3}{4}=\frac{4}{4} \qquad \frac{2}{3}+\frac{5}{3}=\frac{7}{3} \qquad \frac{3}{4}-\frac{9}{4}=-\frac{6}{4}$$

When faced with fractions which do not have the same denominators (bottoms), we will use this basic idea to manipulate the expression as desired: a fraction with the same numerator and denominator is equal to one.

Examples 2.

$$\frac{3}{3}=1 \qquad \frac{4}{4}=1 \qquad \frac{-4}{-4}=1 \qquad \frac{13423}{13423}=1 \qquad \frac{-78000}{-78000}=1$$

In fact, all of these fractions are ***equal***. They are *not* "different ones", they are all different ways to write the same number.

As we know, any number multiplied by 1 is that same number. So, for example,

$$\frac{7}{3}\cdot\frac{3}{3}=\frac{21}{9}=\frac{7\cdot 3}{3\cdot 3}=\frac{7}{3}$$

or

$$\frac{-13}{4}\cdot\frac{2}{2}=\frac{-26}{8}=\frac{(-13)\cdot 2}{4\cdot 2}=\frac{-13}{4}.$$

This is because, as explained, $\frac{3}{3}=\frac{2}{2}=1$. Knowing all this, let's add two fractions with different denominators. We will look for what factors each fraction is missing from the other fractions in the sum.

Examples 3.

1. Here, the $\frac{1}{2}$ is missing a 3 in the denominator, and the $\frac{2}{3}$ is missing a 2 in the denominator. So we perform the following manipulation:

$$\frac{1}{2}+\frac{2}{3}=\frac{1}{2}\cdot\frac{3}{3}+\frac{2}{3}\cdot\frac{2}{2}=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$$

2. Here, the $\frac{1}{3}$ is missing a 7 in the denominator and the $\frac{2}{7}$ is missing a 3 in the denominator. So we add them in:

$$\frac{1}{3}+\frac{2}{7}=\frac{1}{3}\cdot\frac{7}{7}+\frac{2}{7}\cdot\frac{3}{3}=\frac{7}{21}+\frac{6}{21}=\frac{13}{21}$$

3. It works with variables as well.

$$\frac{1}{x}+\frac{2}{y}=\frac{1}{x}\cdot\frac{y}{y}+\frac{2}{y}\cdot\frac{x}{x}=\frac{y}{xy}+\frac{2x}{xy}=\frac{y+2x}{xy}$$

$$\frac{a}{x}+\frac{4}{xy}=\frac{a}{x}\cdot\frac{y}{y}+\frac{4}{xy}=\frac{ay}{xy}+\frac{4}{xy}=\frac{ay+4}{xy}$$