Welcome back!

Nameplates please. And technology encouraged today!

All TF materials are available at github.com/nolankav/api-202.

If you want to follow along, download the dataset here:

In R: `df <- read.csv("http://tinyurl.com/api-202-tf-3")`

In Excel: http://tinyurl.com/api-202-tf-4

Multiple regression and omitted variables

API 202: TF Session 2

R

Nolan M. Kavanagh
February 6, 2026


I'm not like a regular TF
I'm a cool TF Right, Regina?
you in
dinner
CAPSULE

Goals for today

1. **Review core concepts in bivariate analysis.**
2. **Consider an example of omitted variable bias.**
3. **Learn how to run multiple regressions.**
4. **Practice interpreting multiple regressions.**

We'll treat this session like a workshop with an interactive example.

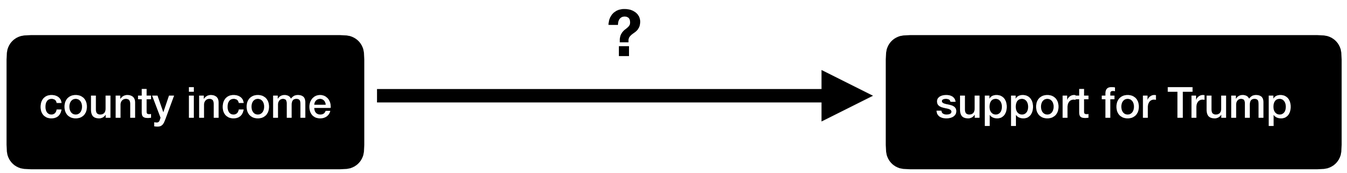
Overview of our sample data

Dataset of U.S. county-level characteristics in 2020

`state`	**State of county**	*Administrative*
`county_fips`	**County FIPS identifier**	*Administrative*
`pc_under_18`	**Percent of county under age 18**	*American Community Survey (2016–2020)*
`pc_over_65`	**Percent of county over age 65**	*American Community Survey (2016–2020)*
`pc_male`	**Percent of county that is male**	*American Community Survey (2016–2020)*
`pc_black`	**Percent of county that is Black**	*American Community Survey (2016–2020)*
`pc_latin`	**Percent of county that is Hispanic/Latino**	*American Community Survey (2016–2020)*
`pc_hs_grad`	**Percent of county that graduated high school**	*American Community Survey (2016–2020)*
`unemploy_rate`	**County unemployment rate (%)**	*American Community Survey (2016–2020)*
`med_income_000s`	**County median income ($1,000s)**	*American Community Survey (2016–2020)*
`pc_uninsured`	**Percent of county without health insurance**	*American Community Survey (2016–2020)*
`pc_trump`	**Percent of county votes for Trump in 2020**	*MIT Election Lab*

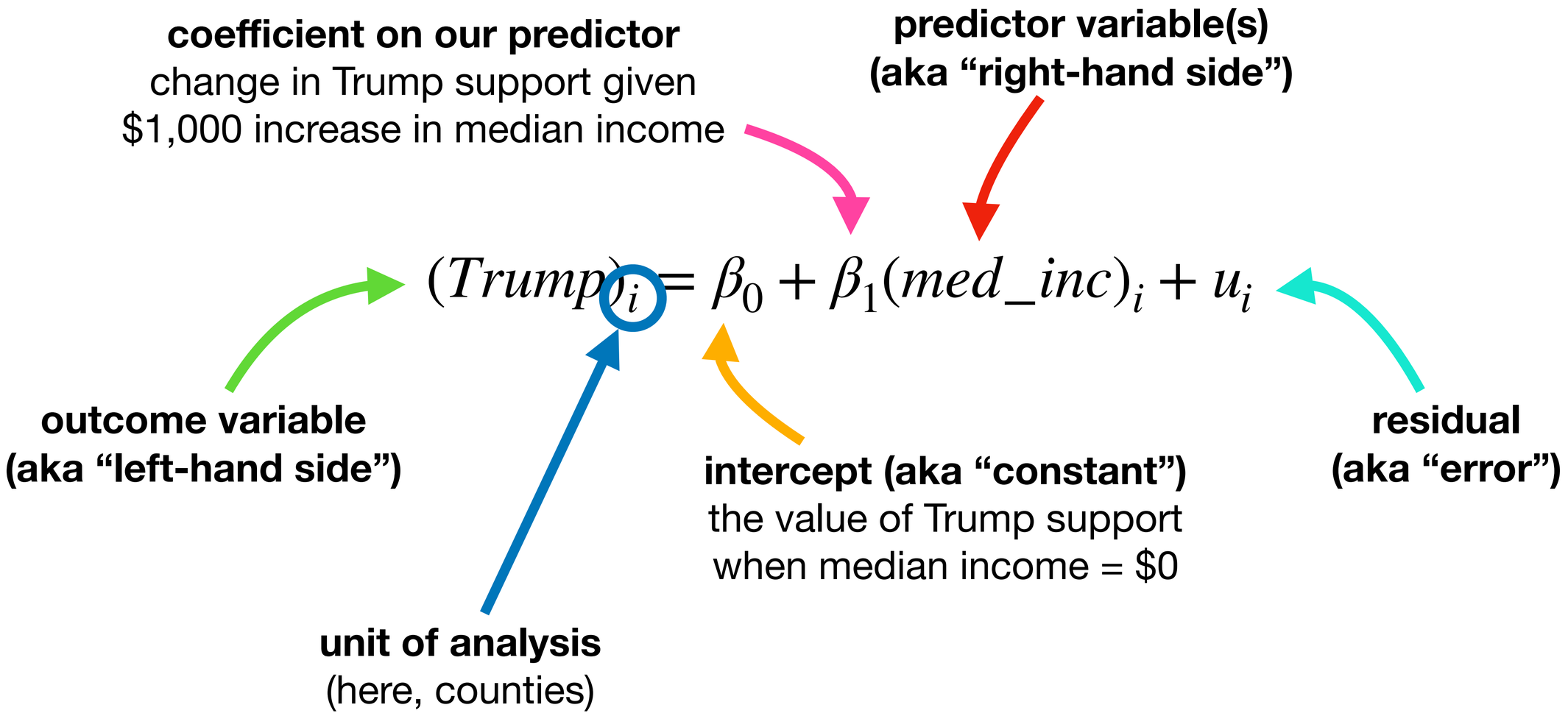
Hmm, I have an idea!

Was support for Trump about economic grievances?



This idea is (was?) very hot in political science and among pundits on MSNBC and Fox News.

Population regression function



Does the graph check out?

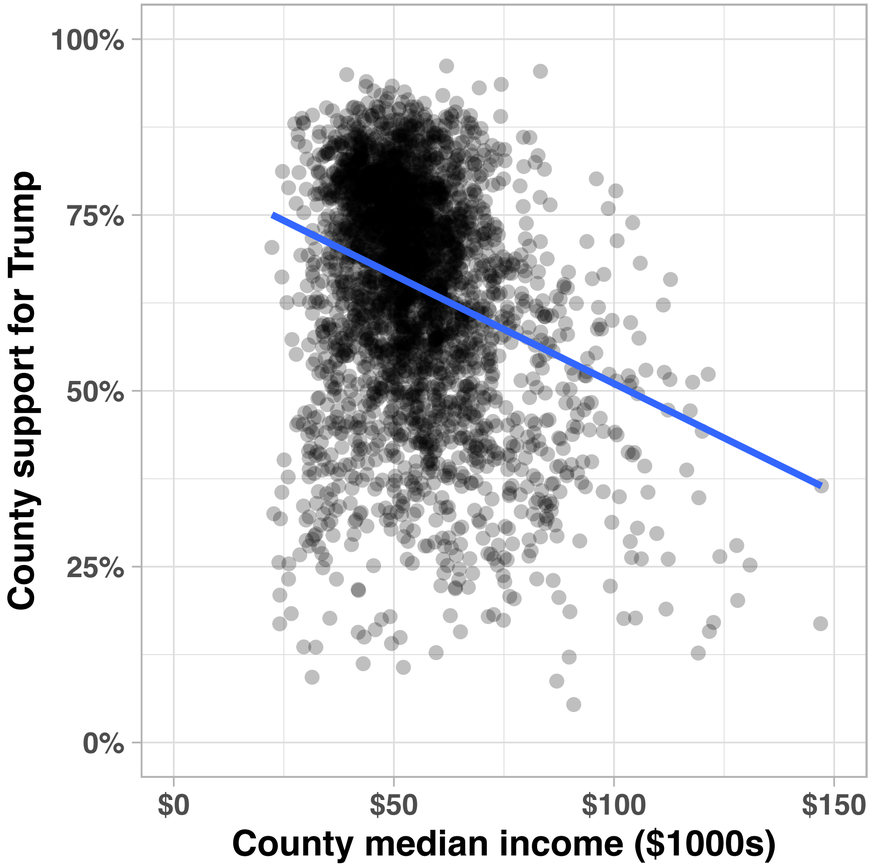
```
# Graph median income and Trump support
plot_1 <- ggplot(df, aes(x=med_inc_000s, y=pc_trump)) +

  # Add scatterplot points
  geom_point(alpha=0.25) +

  # Labels of axes
  xlab("County median income (000s)") +
  ylab("County support for Trump") +

  # Add best fit line
  geom_smooth(method="lm", se=F, formula = y~x) +

  # Cosmetic changes
  theme_light() + theme(text = element_text(face="bold")) +
  scale_y_continuous(limits=c(0,100),
                     labels = function(x) paste0(x,"%")) +
  scale_x_continuous(limits=c(0,150),
                     labels = scales::dollar_format())
```



Does the regression check out?

```
# Estimate regression
reg_1 <- lm(pc_trump ~ med_inc_000s, data=df)
summary(reg_1)

Call:
lm(formula = pc_trump ~ med_inc_000s, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-62.940  -8.985   3.256  11.042  39.239

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  81.93207    1.07913   75.92   <2e-16 ***
med_inc_000s -0.30905    0.01899  -16.28   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.5 on 3112 degrees of freedom
Multiple R-squared:  0.07845,      Adjusted R-squared:  0.07815
F-statistic: 264.9 on 1 and 3112 DF,  p-value: < 2.2e-16
```

Note: Trump support is measured from 0–100, so we don't have to multiply by 100 to interpret the coefficients.

Does the regression check out?

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reg_1 <- lm(pc_trump ~ med_inc_000s, data=df)
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```

Looks right to me!

Each $1,000 increase in county median income was associated with a statistically significant 0.31 percentage point (pp) decline in Trump support.

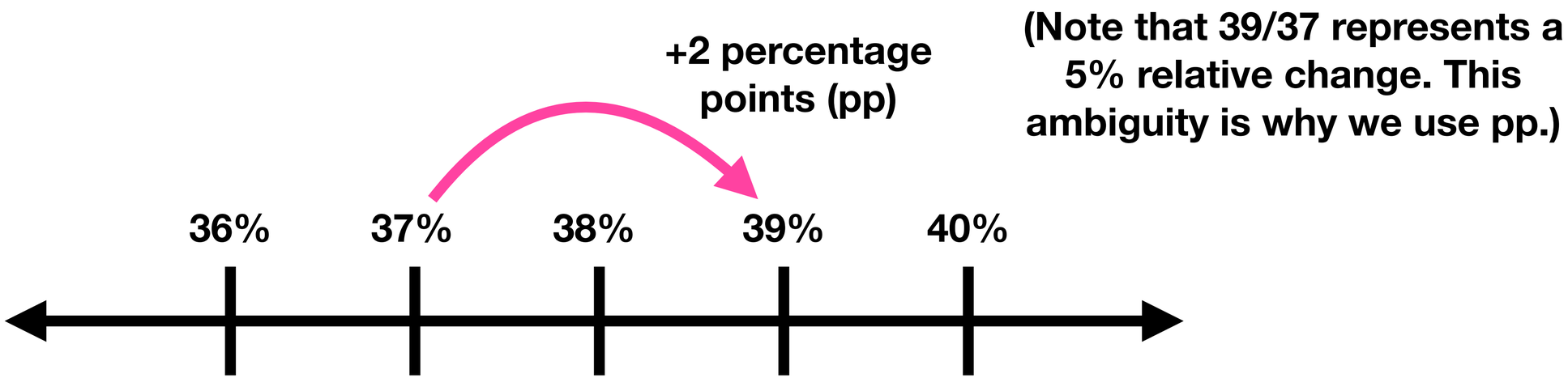
So Trump was all about economic grievances.

Case closed!

Note: Trump support is measured from 0–100, so we don't have to multiply by 100 to interpret the coefficients.

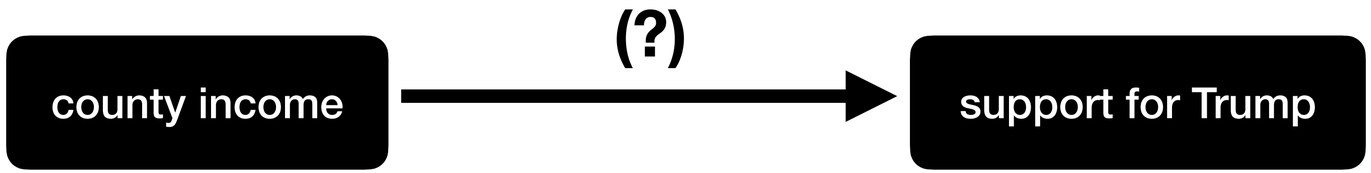
A quick detour on percentage points

- When our outcome is measured in percents (%), we describe any movement along the number line using percentage points.

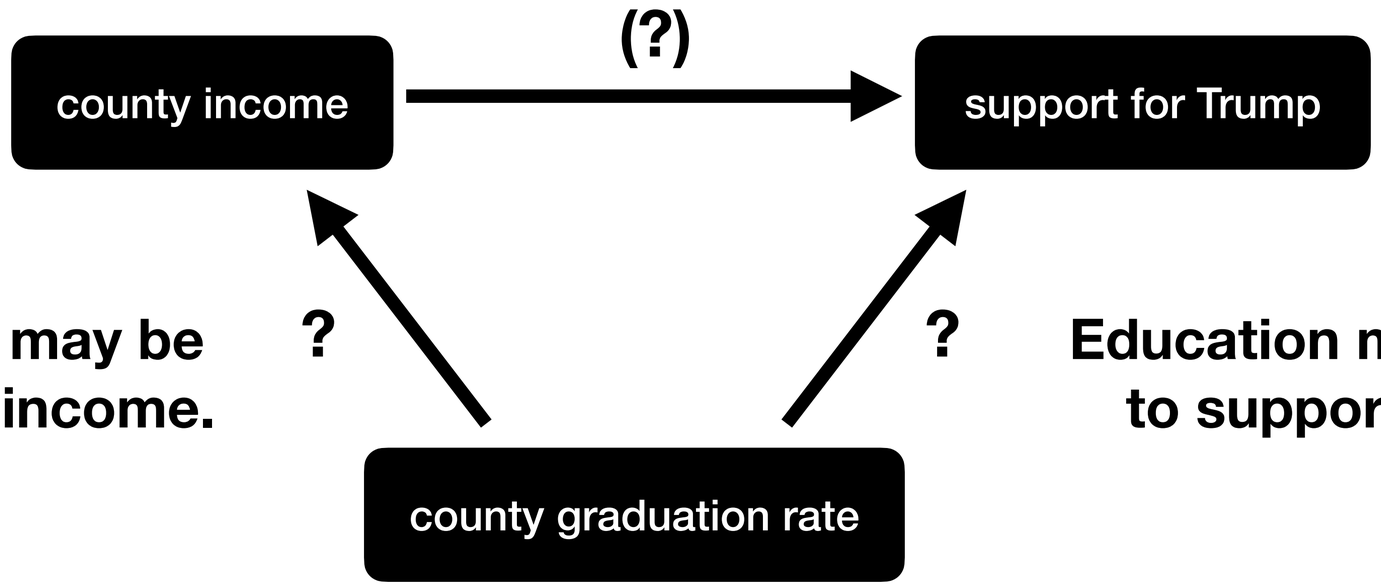


- If the outcome is measured from 0 to 100, you can interpret β_1 directly in pp. If measured 0 to 1, you must multiply by 100.

Or are we missing something?



Or are we missing something?



Graduation rates may be related to county income.

Education may be related to support for Trump.

The result? Bias in our regression.

Fine, let's add education to our analysis.

We use alpha vs. beta just to distinguish the different regressions.

Short regression $(Trump)_i = \hat{\alpha}_0 + \hat{\alpha}_1(med\_inc)_i + \hat{u}_i$

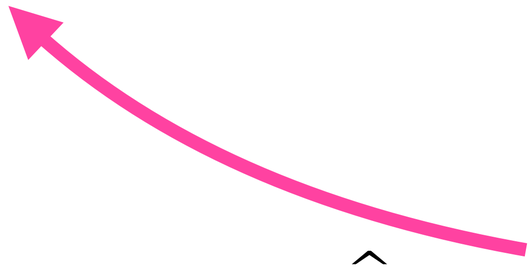
Long regression $(Trump)_i = \hat{\beta}_0 + \hat{\beta}_1(med\_inc)_i + \hat{\beta}_2(HS\_grad)_i + \hat{v}_i$

the omitted variable


You know
short and long regression functions?
Yeah, everyone in
can read
who has suffered API 202
short and long regression functions

Let's run the long regression.

```
# Estimate long regression
reg_2 <- lm(pc_trump ~ med_inc_000s + pc_hs_grad, data=df)
summary(reg_2)
```



$$(Trump)_i = \hat{\beta}_0 + \hat{\beta}_1(med\_inc)_i + \hat{\beta}_2(HS\_grad)_i + \hat{v}_i$$

To include multiple predictors in our regression, we just add them to the right-hand side with a "+".

Let’s run the long regression.

```
# Estimate long regression
reg_2 <- lm(pc_trump ~ med_inc_000s + pc_hs_grad, data=df)
summary(reg_2)

Call:
lm(formula = pc_trump ~ med_inc_000s + pc_hs_grad, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-57.641  -8.134   0.859   9.436  45.269

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  134.44740    2.30947  58.216   <2e-16 ***
med_inc_000s  -0.02966    0.02058  -1.442    0.149
pc_hs_grad    -1.02700    0.04086 -25.135   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 14.13 on 3111 degrees of freedom
Multiple R-squared:  0.234,           Adjusted R-squared:  0.2335
F-statistic: 475.2 on 2 and 3111 DF,  p-value: < 2.2e-16
```

Note: Trump support is measured from 0–100, so the coefficients are already in percentage points. No multiplication by 100 required here.

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              Estimate Std. Error t value Pr(>|t|)
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med_inc_000s  -0.02966    0.02058  -1.442    0.149
pc_hs_grad    -1.02700    0.04086 -25.135   <2e-16 ***
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Residual standard error: 14.13 on 3111 degrees of freedom
Multiple R-squared:  0.234,             Adjusted R-squared:  0.2335
F-statistic: 475.2 on 2 and 3111 DF,  p-value: < 2.2e-16
```

Well. **.**

Controlling for high school graduation rates, each $1,000 increase in county median income was associated with a 0.03 pp decline in Trump support.

And it's not statistically significant.

Note: Trump support is measured from 0–100, so the coefficients are already in percentage points. No multiplication by 100 required here.


She doesn't even
explain our outcome
after controls

Let's run the long regression.

```
# Estimate long regression
reg_2 <- lm(pc_trump ~ med_inc_000s + pc_hs_grad, data=df)
summary(reg_2)

Call:
lm(formula = pc_trump ~ med_inc_000s + pc_hs_grad, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-57.641  -8.134   0.859   9.436  45.269

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Note: Trump support is measured from 0–100, so the coefficients are already in percentage points. No multiplication by 100 required here.

Let's run the long regression.

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# Estimate long regression
reg_2 <- lm(pc_trump ~ med_inc_000s + pc_hs_grad, data=df)
summary(reg_2)

Call:
lm(formula = pc_trump ~ med_inc_000s + pc_hs_grad, data = df)

Residuals:
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

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F-statistic: 475.2 on 2 and 3111 DF,  p-value: < 2.2e-16
```

Meanwhile, each 1 pp increase in a county's high school graduation rate was associated with 1.0 pp less support for Trump, controlling for county median income.

This association is statistically significant at the 5% level.

Note: Trump support is measured from 0–100, so the coefficients are already in percentage points. No multiplication by 100 required here.

Let's run the long regression.

```
# Estimate long regression
reg_2 <- lm(pc_trump ~ med_inc_000s + pc_hs_grad, data=df)
summary(reg_2)

Call:
lm(formula = pc_trump ~ med_inc_000s + pc_hs_grad, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-57.641  -8.134   0.859   9.436  45.269

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
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F-statistic: 475.2 on 2 and 3111 DF,  p-value: < 2.2e-16
```

Note: Trump support is measured from 0–100, so the coefficients are already in percentage points. No multiplication by 100 required here.

Let's run the long regression.

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# Estimate long regression
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summary(reg_2)

Call:
lm(formula = pc_trump ~ med_inc_000s + pc_hs_grad, data = df)

Residuals:
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(Intercept)  134.44740    2.30947  58.216   <2e-16 ***
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pc_hs_grad    -1.02700    0.04086 -25.135   <2e-16 ***
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```

When county median income AND high school graduation rates are set to 0, the expected support for Trump is 134%.

(Obviously, this isn't a meaningful value.)

Note: Trump support is measured from 0–100, so the coefficients are already in percentage points. No multiplication by 100 required here.

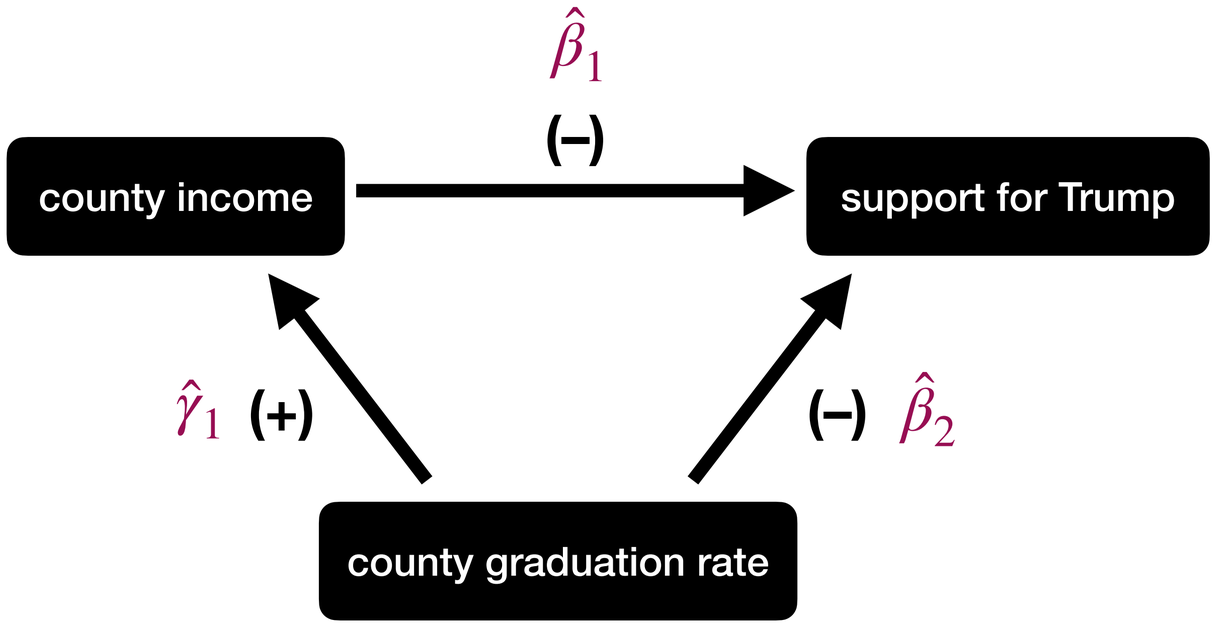
Womp.

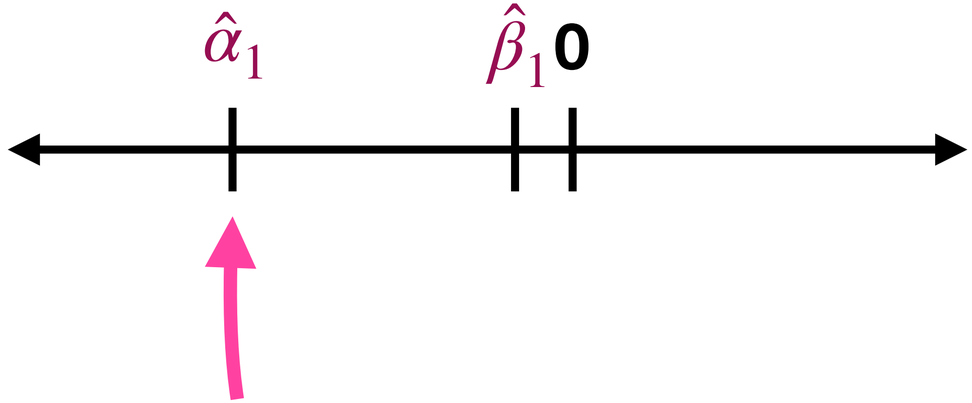
		Model 1	Model 2	
Intercept	$\hat{\alpha}_0$	**81.93** (-1.08) P<0.001	**134.45** (-2.31) P<0.001	$\hat{\beta}_0$
County median income ($1000s)	$\hat{\alpha}_1$	**-0.31** (0.02) P<0.001	**-0.03** (0.02) P=0.149	$\hat{\beta}_1$
County graduation rate			**-1.03** (0.04) P<0.001	$\hat{\beta}_2$
Num.Obs.		3114	3114	
R2		0.078	0.234	
R2 Adj.		0.078	0.234	

Short regression $(Trump)_i = \hat{\alpha}_0 + \hat{\alpha}_1(med\_inc)_i + \hat{u}_i$

Long regression $(Trump)_i = \hat{\beta}_0 + \hat{\beta}_1(med\_inc)_i + \hat{\beta}_2(HS\_grad)_i + \hat{v}_i$

Clearly, we were missing something.





Relative to the true β_1 (–), our estimate of α_1 was even more negative.

Bias formula $\alpha_1 - \beta_1 = \beta_2 * \gamma_1 = (-)(+) = (-)$


omitted variable
Nolan
'd be like, "Why didn't you
control for me
?"
And I'd be like, "Why are you so obsessed with me?"

Bias: sign or size?

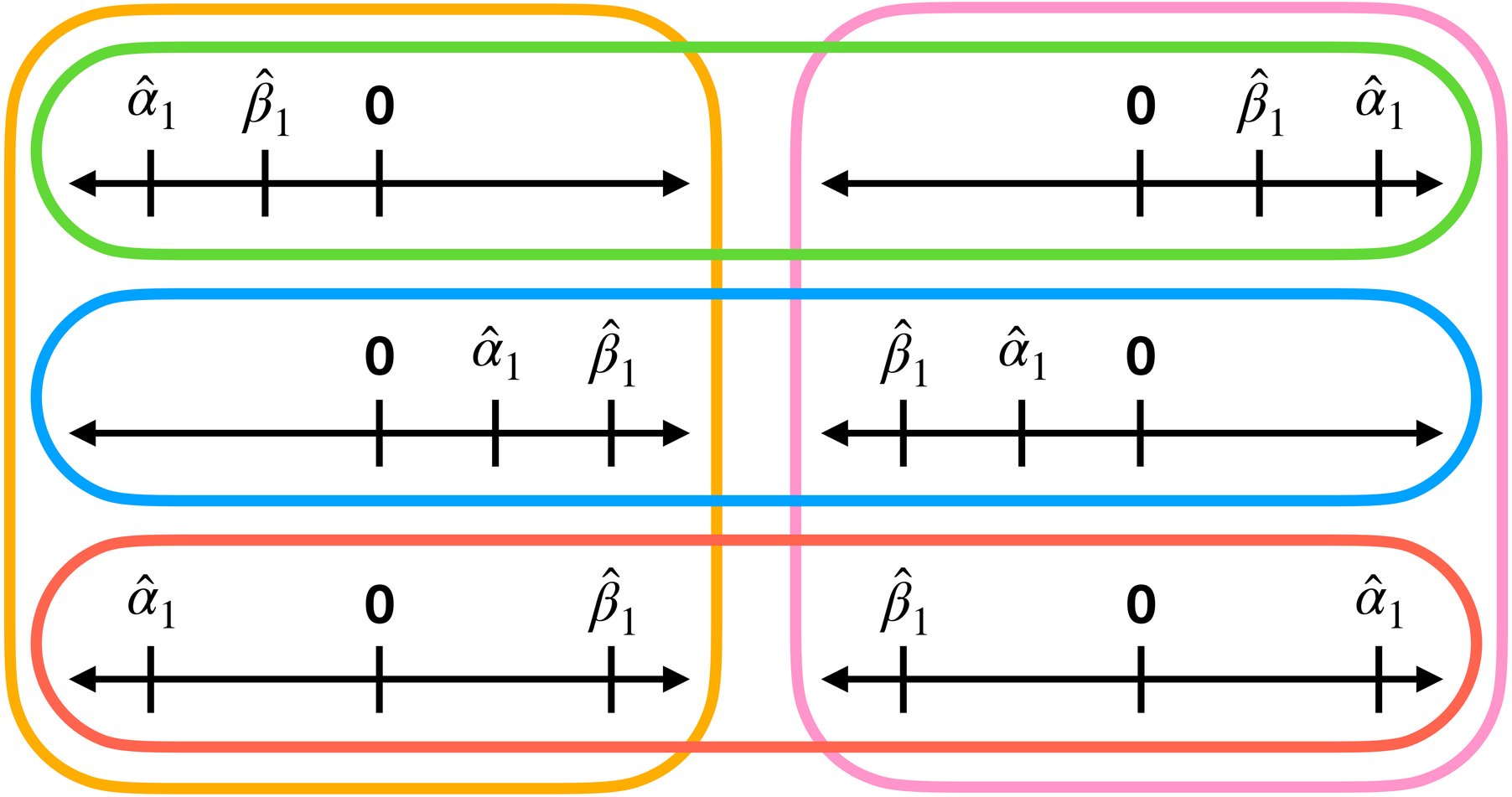
Overstatement

i.e. α_1 is farther from 0

Understatement

i.e. α_1 is closer to 0

Sign flip!



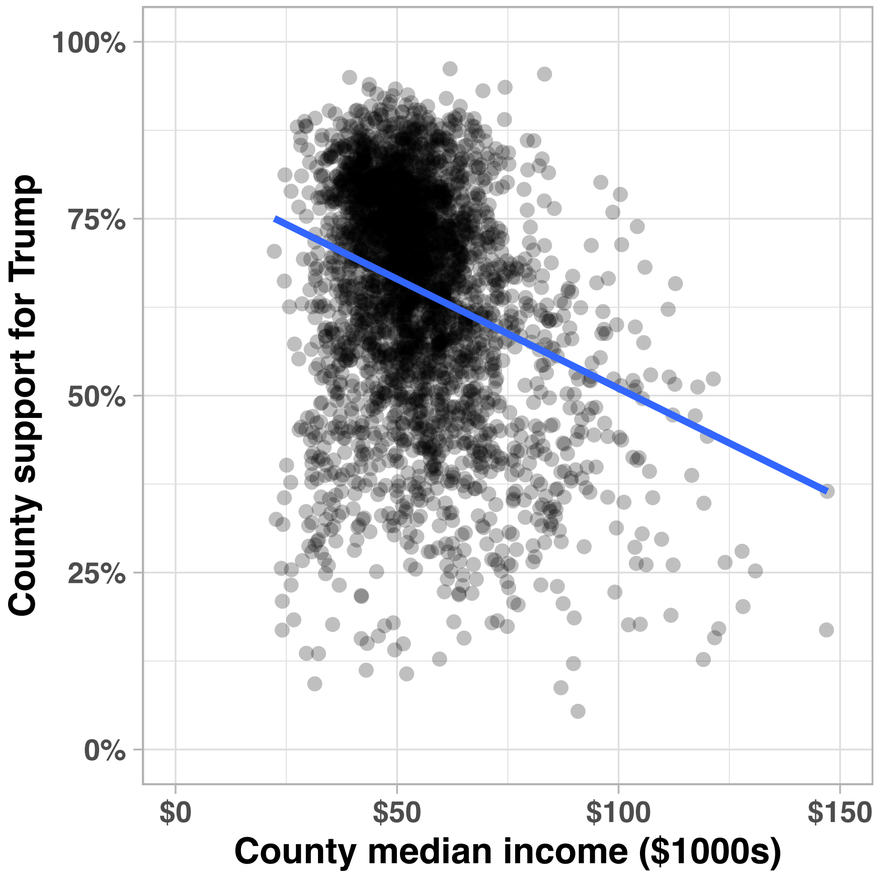
Negative bias (–)

i.e. α_1 is to the left of β_1

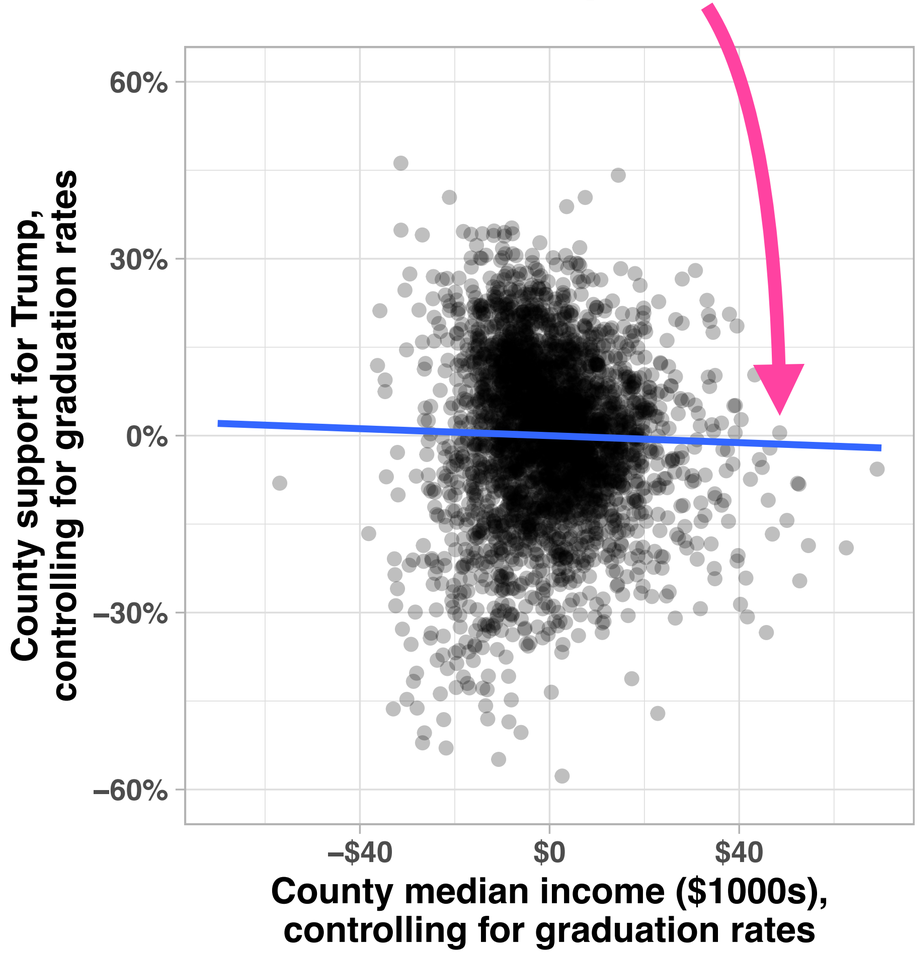
Positive bias (+)

i.e. α_1 is to the right of β_1

What happens to our graph when we control for education?



This is the same slope as Model 2.



P.S. The code to do this optional exercise is in the Github, but we won't be reviewing it in class.

OK, what did we learn?

Omitted variables can mess up our regressions.

Think carefully about what might be missing.

Is our new model causal? Or are we missing something *else*?

How many omitted variables can there be?