

# Welcome!

**Nameplates please. And technology encouraged today!**

**All TF materials are available at [github.com/nolankav/api-203](https://github.com/nolankav/api-203).**

**If you want to follow along, download the dataset here:**

**In R: `df <- read.csv("http://tinyurl.com/api-203-tf-1")`**



“Bring out yer omitted variables.”

# Fixed Effects and the Holy IV

API 203: TF Session 1

R

Nolan M. Kavanagh  
March 7, 2025

# Goals for today

- 1. Review the principles of fixed effects.**
- 2. Learn how to run fixed effects regressions in R.**
- 3. Practice interpreting fixed effects regressions.**
- 4. Review the principles of instrumental variables.**

(We'll learn how to run IVs in R next week.)

We'll treat this session like a workshop with an interactive example.

# Overview of our sample data

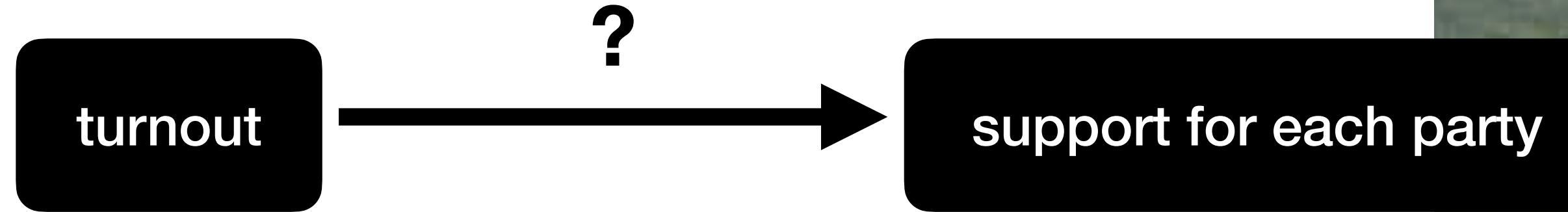
## Dataset of U.S. county characteristics in 2012, 2016, & 2020

county_fips	<b>County FIPS identifier</b>	<i>Administrative</i>
state	<b>State</b>	<i>Administrative</i>
year	<b>Year</b>	<i>Administrative</i>
pop_over_18	<b>County voting age population (i.e. over 18 years)</b>	<i>American Community Survey (5-year estimates)</i>
med_inc_000s	<b>County median income (in \$1,000s)</b>	<i>American Community Survey (5-year estimates)</i>
unemploy_rate	<b>County unemployment rate (0–100)</b>	<i>American Community Survey (5-year estimates)</i>
pc_uninsured	<b>Percent of county without health insurance (0–100)</b>	<i>American Community Survey (5-year estimates)</i>
all_votes	<b>Total number of votes cast in election</b>	<i>MIT Election Lab</i>
rep_votes	<b>Total number of votes for Republican candidate</b>	<i>MIT Election Lab</i>
turnout	<b>Percent of county VAP that voted in election (0–100)</b>	<i>MIT Election Lab/Constructed</i>
pc_rep	<b>Percent of county votes for Republican (0–100)</b>	<i>MIT Election Lab/Constructed</i>

# How do you win an election?

Some elections have higher turnout than others.

When more people turn out to vote, which party benefits: Democrats or Republicans?



# We could start with a pooled regression.

```
# Graph pooled turnout and percent Republican
plot_1 <- ggplot() +

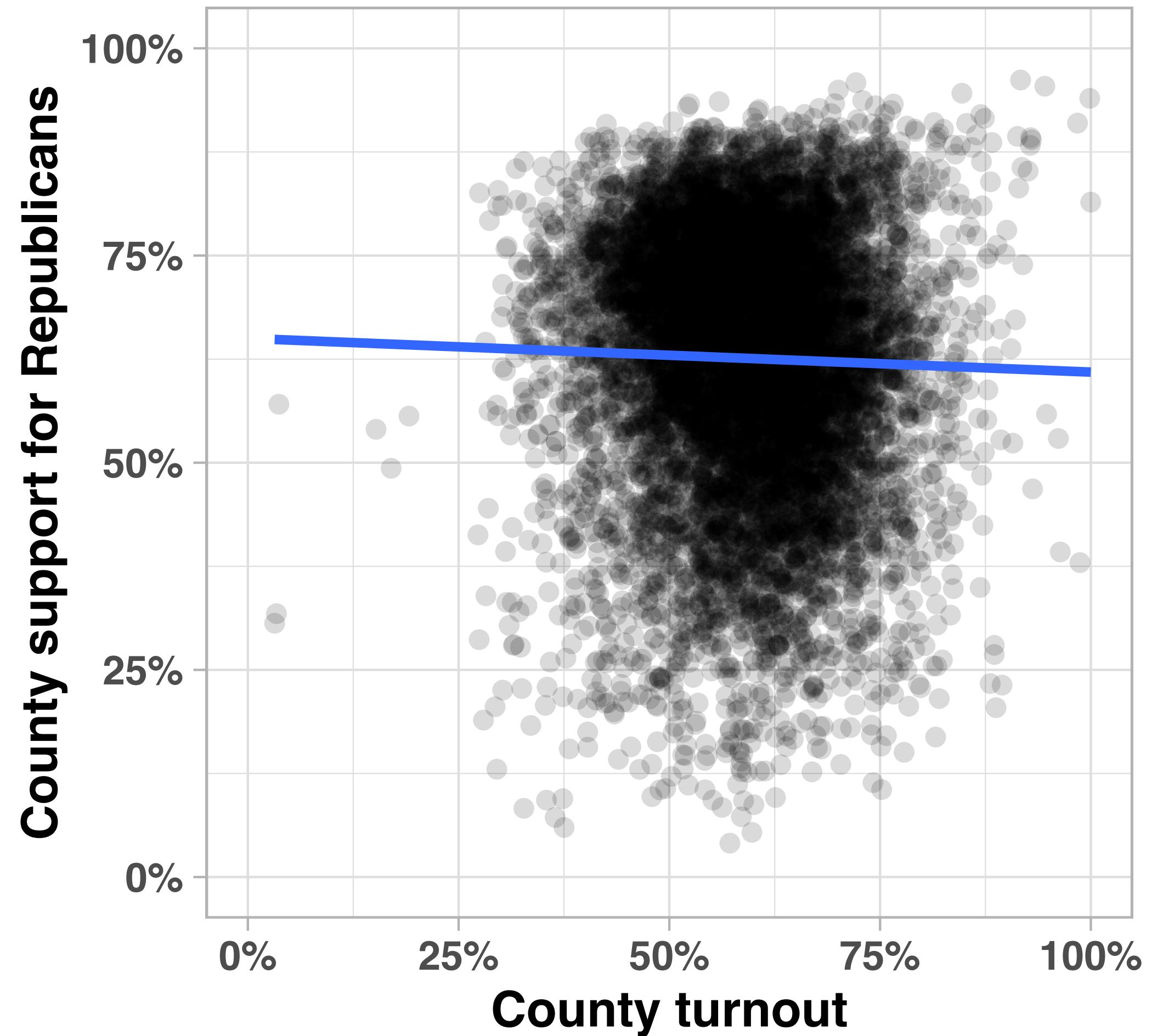
  # Add scatterplot points
  geom_point(data=df, aes(x=turnout, y=pc_rep), alpha=0.15) +

  # Labels of axes
  xlab("County turnout") +
  ylab("County support for Republicans") +

  # Cosmetic changes
  theme_light() +
  theme(text = element_text(face="bold")) +
  scale_y_continuous(limits=c(0,100),
                     labels = function(x) paste0(x,"%")) +
  scale_x_continuous(limits=c(0,100),
                     labels = function(x) paste0(x,"%")) +

  # Add line of best fit
  geom_smooth(data=df, aes(x=turnout, y=pc_rep),
              method="lm", se=F, formula = y~x)

# Print graph
print(plot_1)
```

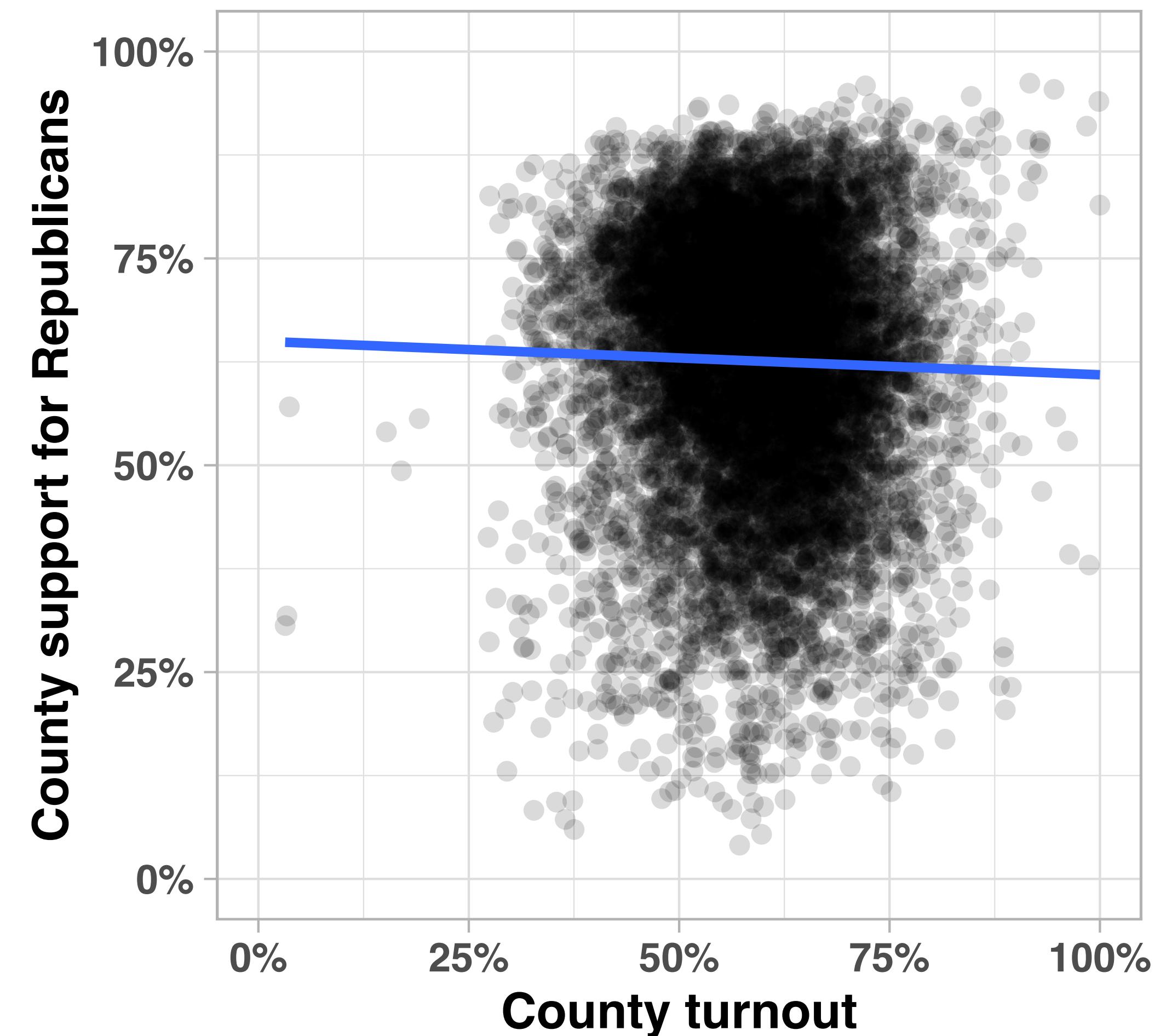


# We could start with a pooled regression.

Here, we have pooled all county-years from 2012, 2016, and 2020 together.

Each point is one county in one year, e.g. Philadelphia County in 2016.

It looks like higher turnout modestly hurts support for Republican presidential candidates.



# We could start with a pooled regression.

```
# Load package  
library(fixest) # Modeling tools  
  
# Estimate between-county regression  
model_1 <- feols(pc_rep ~ turnout | 0, df)  
summary(model_1)
```

```
OLS estimation, Dep. Var.: pc_rep  
Observations: 9,345  
Standard-errors: IID  
Estimate Std. Error t value Pr(>|t|)  
(Intercept) 65.084073 0.888754 73.23068 < 2.2e-16 ***  
turnout -0.042113 0.014729 -2.85919 0.0042566 **  
---  
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
RMSE: 15.7 Adj. R2: 7.673e-4
```

We will use “feols()” from the “fixest” package.

We set it up much like “lm()” except that fixed effects come after the bar. Here, “0” indicates that we don’t yet have any fixed effects.

Note: turnout and pc\_rep are coded 0–100.

# We could start with a pooled regression.

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# Load package
library(fixest) # Modeling tools

# Estimate between-county regression
model_1 <- feols(pc_rep ~ turnout | 0, df)
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---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 15.7  Adj. R2: 7.673e-4
```

Across all county-years from 2012 to 2020, a 1 pp increase in county turnout was associated with 0.09 pp less support for the Republican presidential candidate. It was statistically significant.

(Yes, it's a mouthful.)

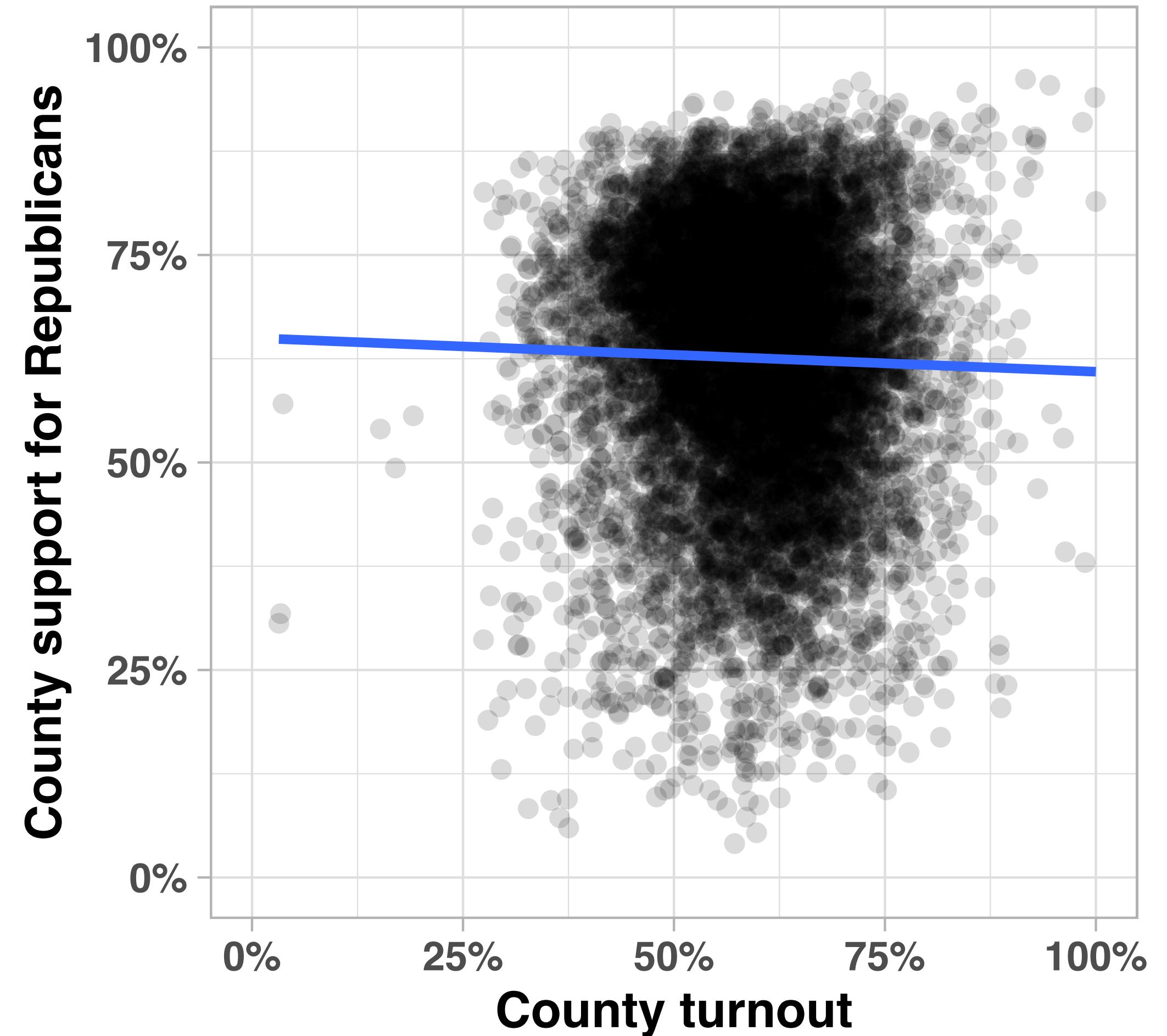
**Note:** turnout and pc\_rep are coded 0–100.

# We could add a few controls.

	Model 1	Model 2
<b>County turnout (0–100)</b>	<b>-0.04**</b> (0.01)	<b>-0.09***</b> (0.01)
<b>County median income (\$1,000s)</b>		<b>-0.40***</b> (0.01)
<b>County unemployment rate (0–100)</b>		<b>-2.17***</b> (0.05)
Num.Obs.	9345	9345
County fixed effects	No	No
R2	0.001	0.202
R2 Adj.	0.001	0.201

\*p<0.05, \*\*p<0.01, \*\*\*p<0.001

Note: pc\_rep is coded 0–100.



# **But what are we missing?**

# But what are we missing?

The potential omitted variables are endless!

- State election laws
- County election procedures
- County gender balance
- County age distribution
- County racial composition
- County major industries
- County insurance rate
- County partisan inclinations
- County geography
- And on and on

All of these are potential threats to inference!

What can we do?

One option is controlling for all of them, but we'd still end up missing omitted variables.

# Enter: Fixed effects.

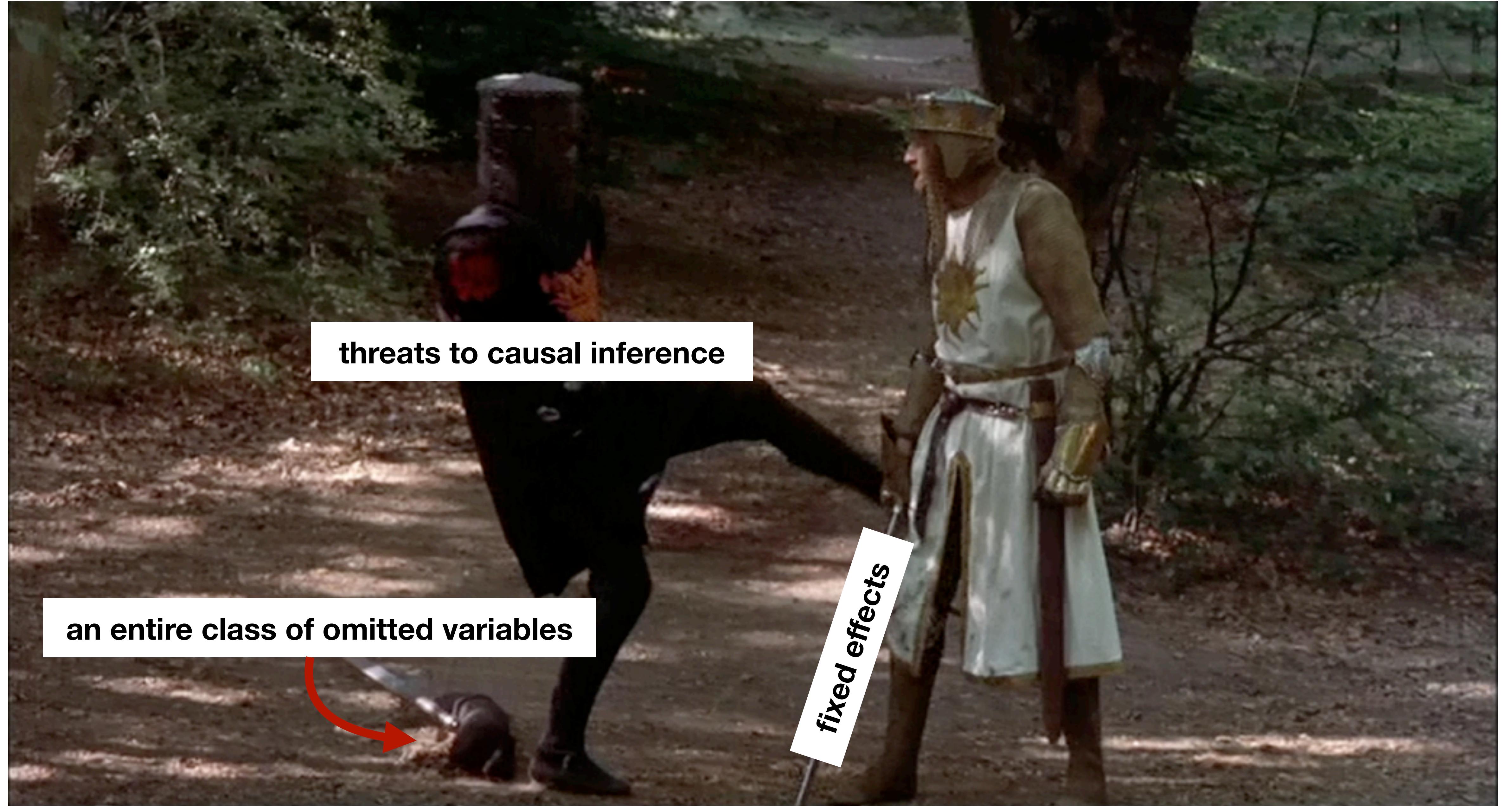
We want to know how changes in turnout *within a county* relate to votes.

Fixed effects let us control for entire classes of omitted variables.

A county fixed effect captures *all* time-invariant\* characteristics of a county, e.g. geography, demographics, political inclinations, etc.

\*meaning that they don't change meaningfully during our study period.

The result is a *within-county* estimate.



**an entire class of omitted variables**

**threats to causal inference**

**fixed effects**

# A brief note on notation.

## county fixed effects

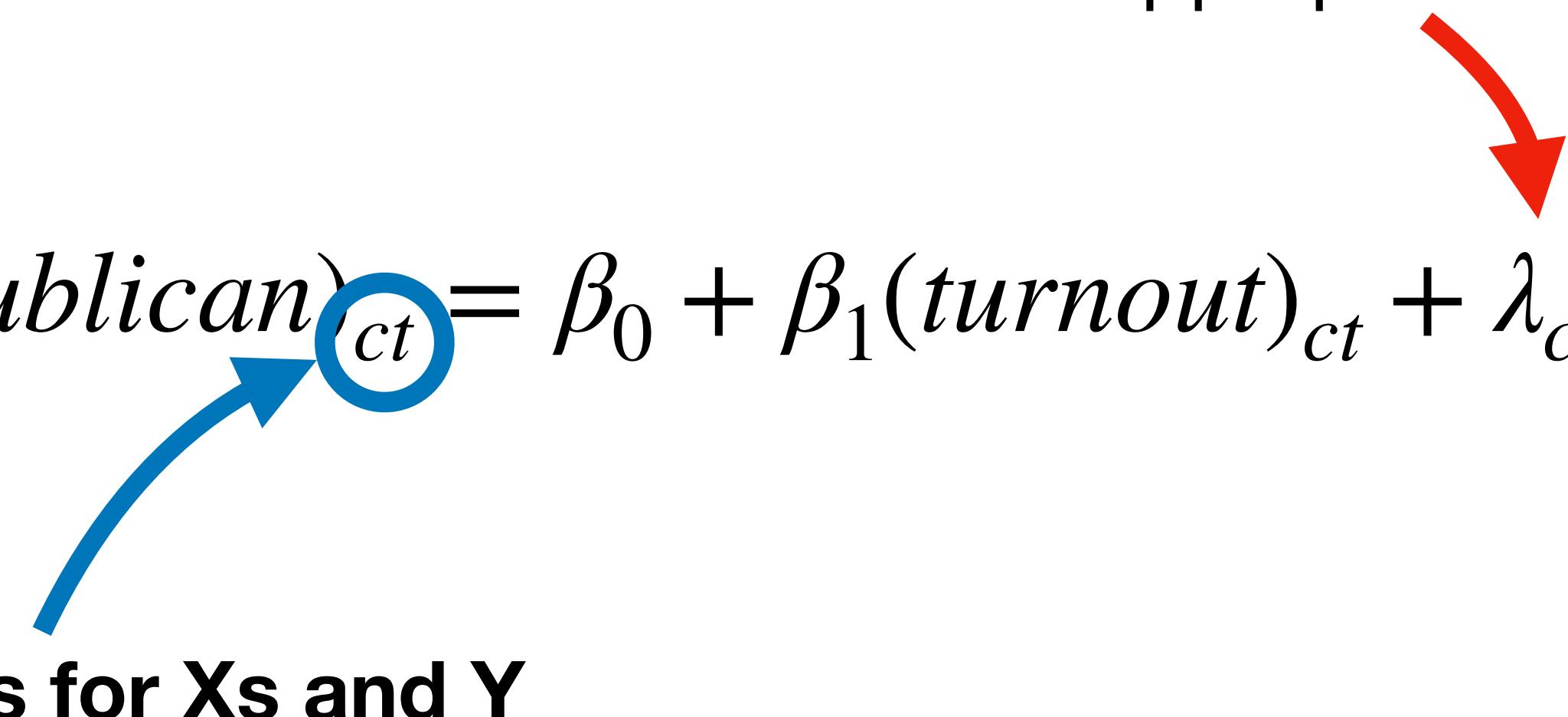
Fixed effects are usually just written with one Greek letter and the appropriate subscript.

$$(pc\_Republican)_{ct} = \beta_0 + \beta_1(turnout)_{ct} + \lambda_c + u_{ct}$$

## multiple indices for Xs and Y

Since we have the same county multiple times, we have to have enough indexes to identify each county-year.

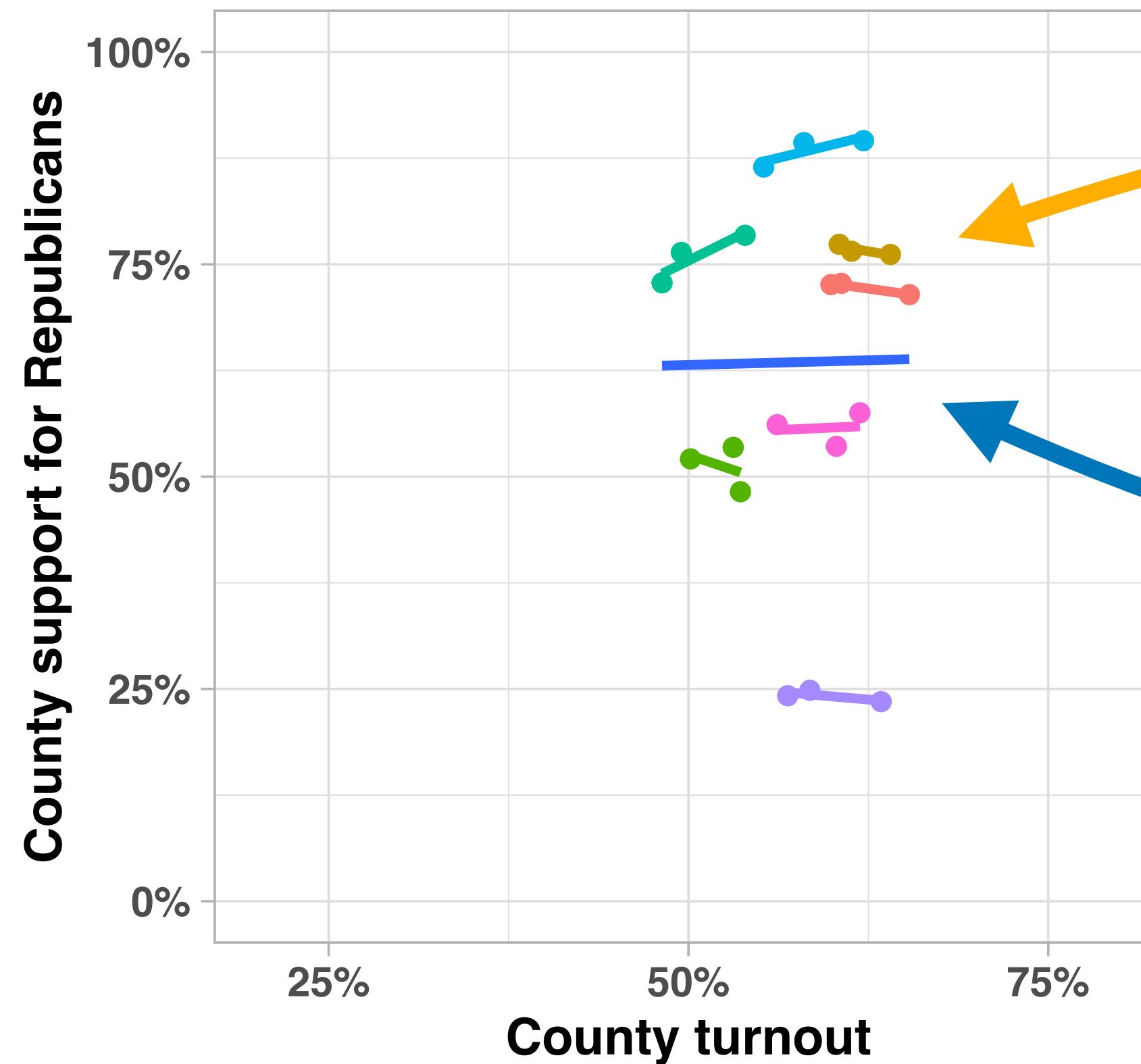
c = county; t = time



residual (a.k.a. “error”)  
Also needs both indices!

# Let's look under the hood.

Here, I've selected a few counties in Alabama, each a different color.



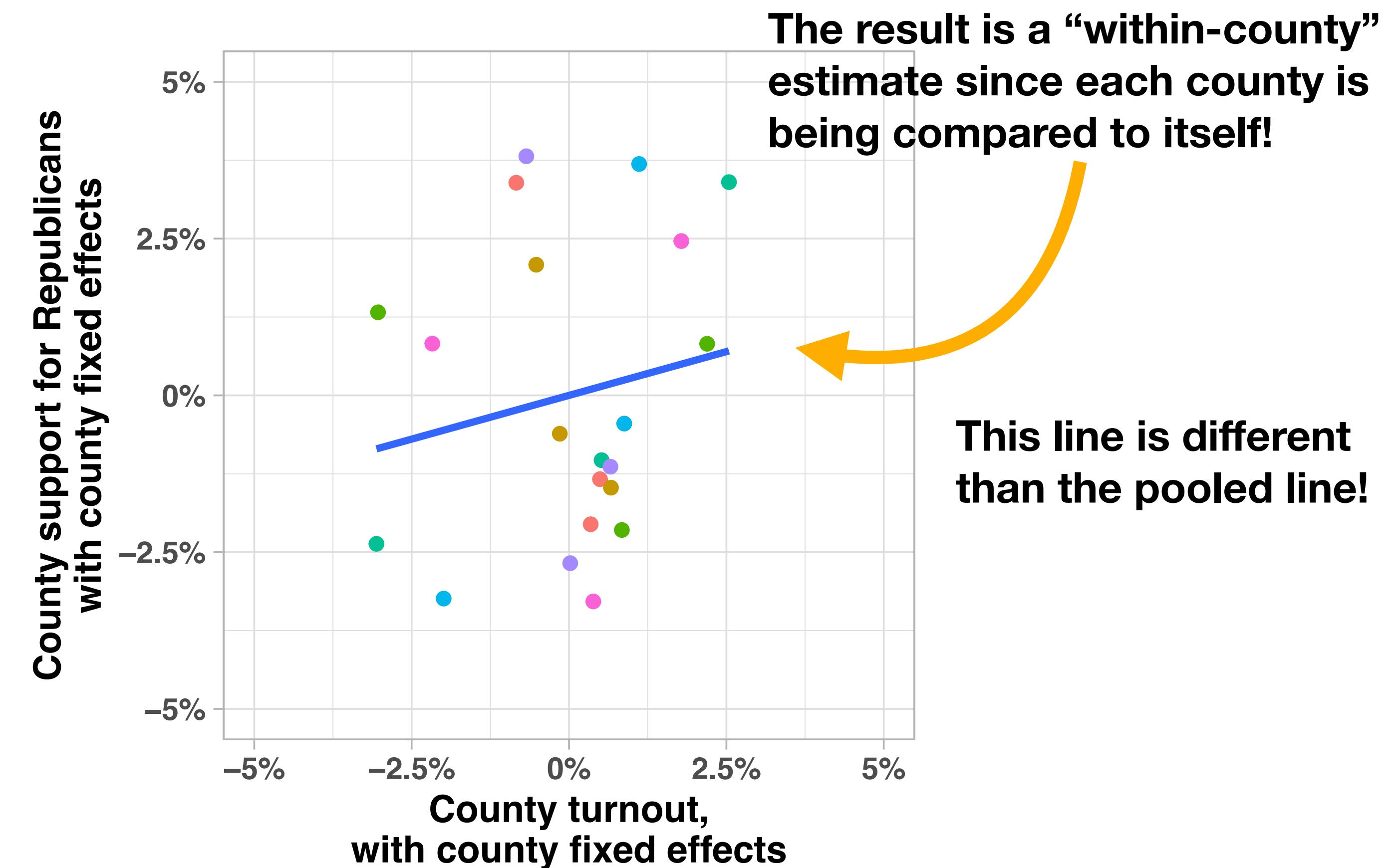
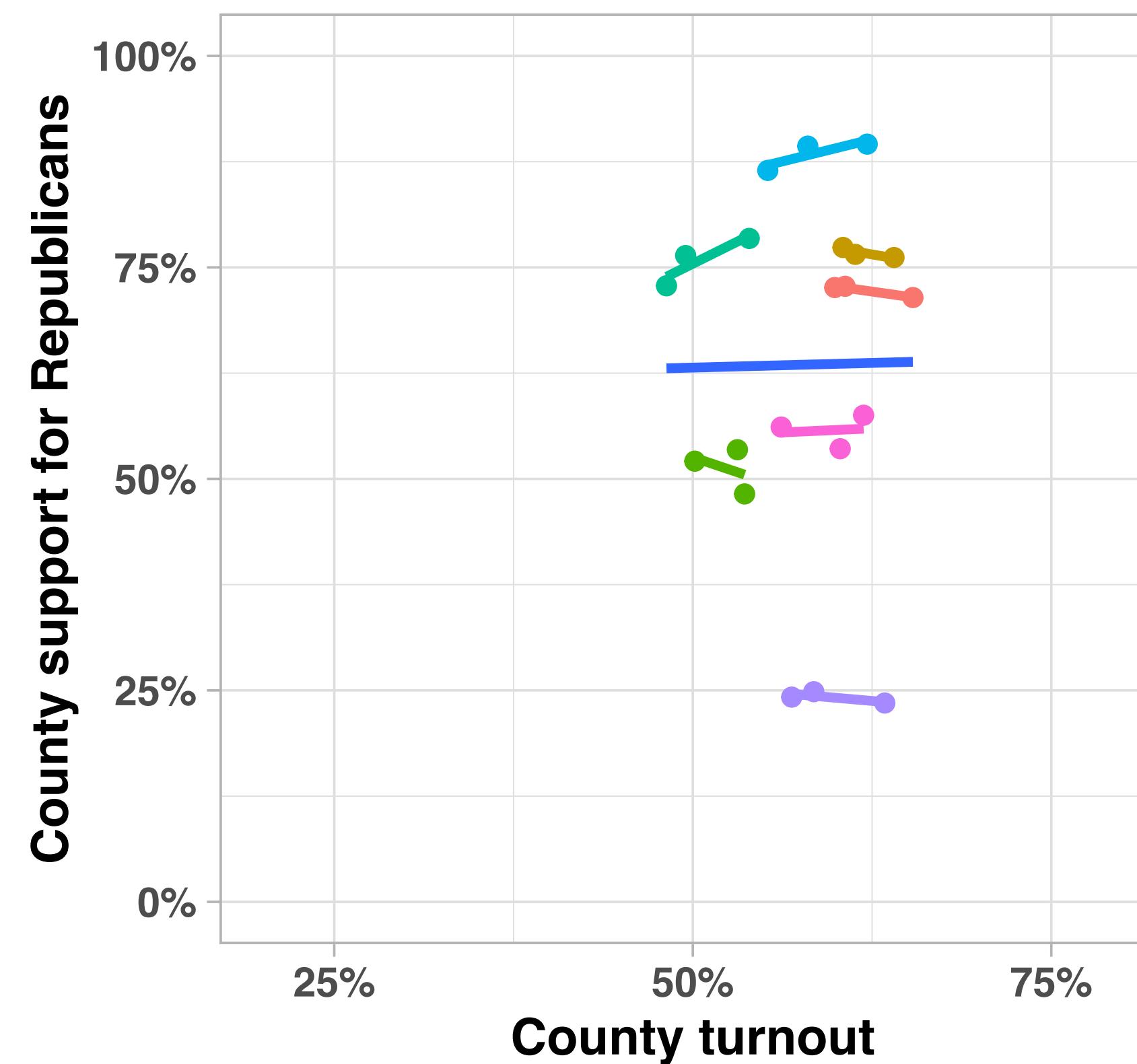
Here is one county and the best-fit line for that county.

Here is the best-fit line for the pooled sample of a few Alabama counties.

We can see that the association across counties is not necessarily the same as the association within counties!

# Let's “take out” the fixed effects.

Each county gets re-centered, and the best-fit line is refit.



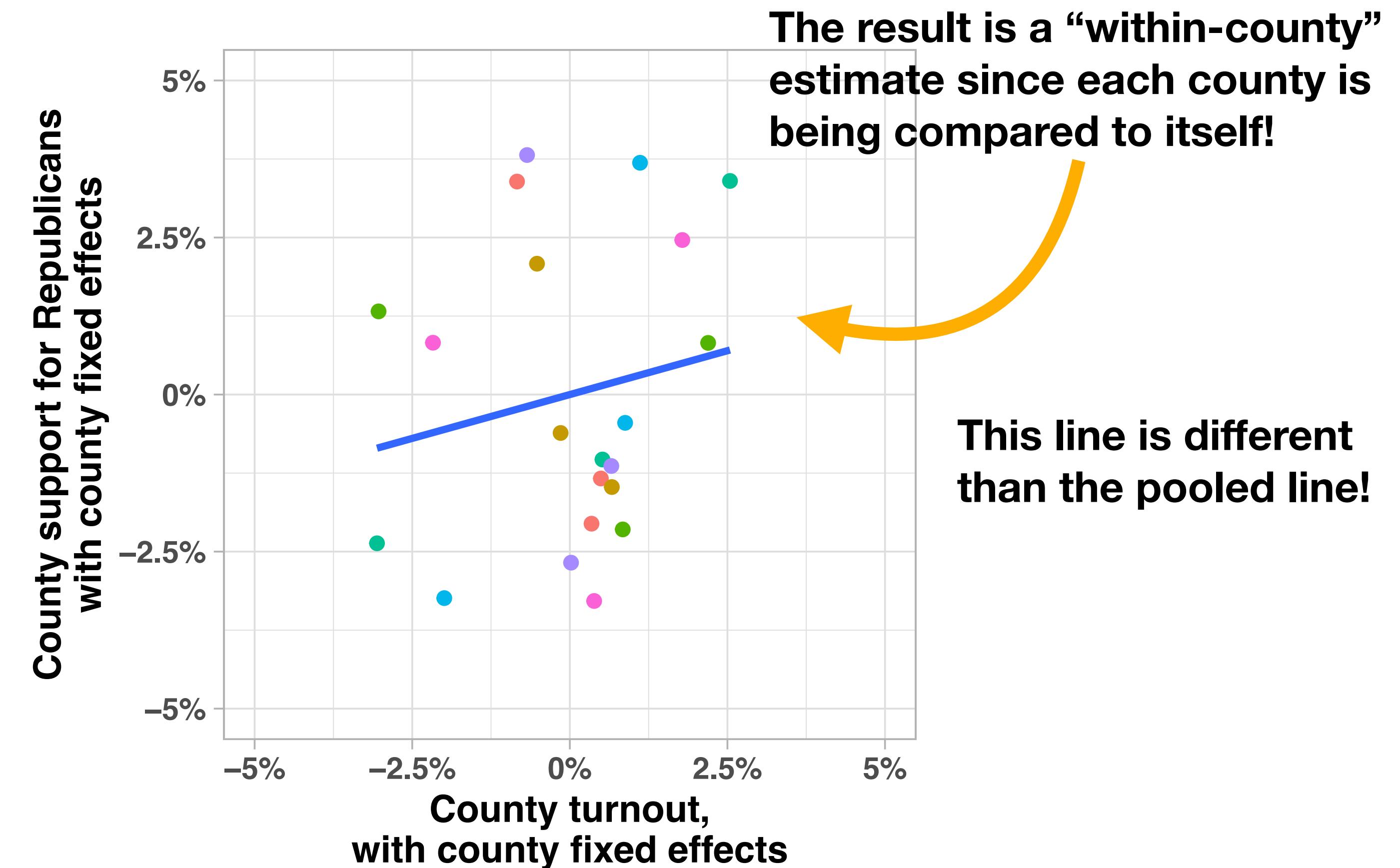
# Let's “take out” the fixed effects.

Each county gets re-centered, and the best-fit line is refit.

This process removes the differences between each county and leaves only the differences within counties over time!

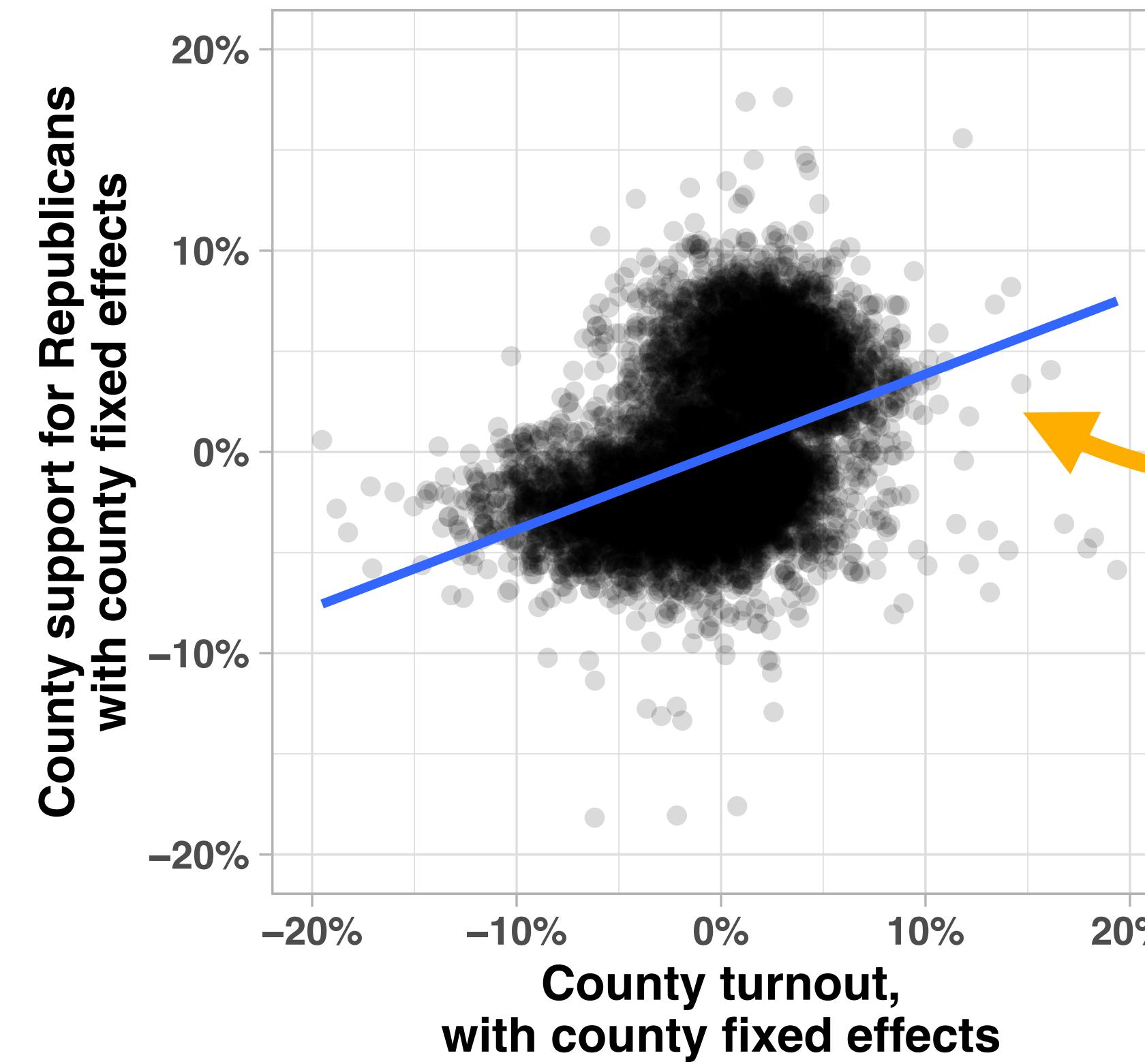
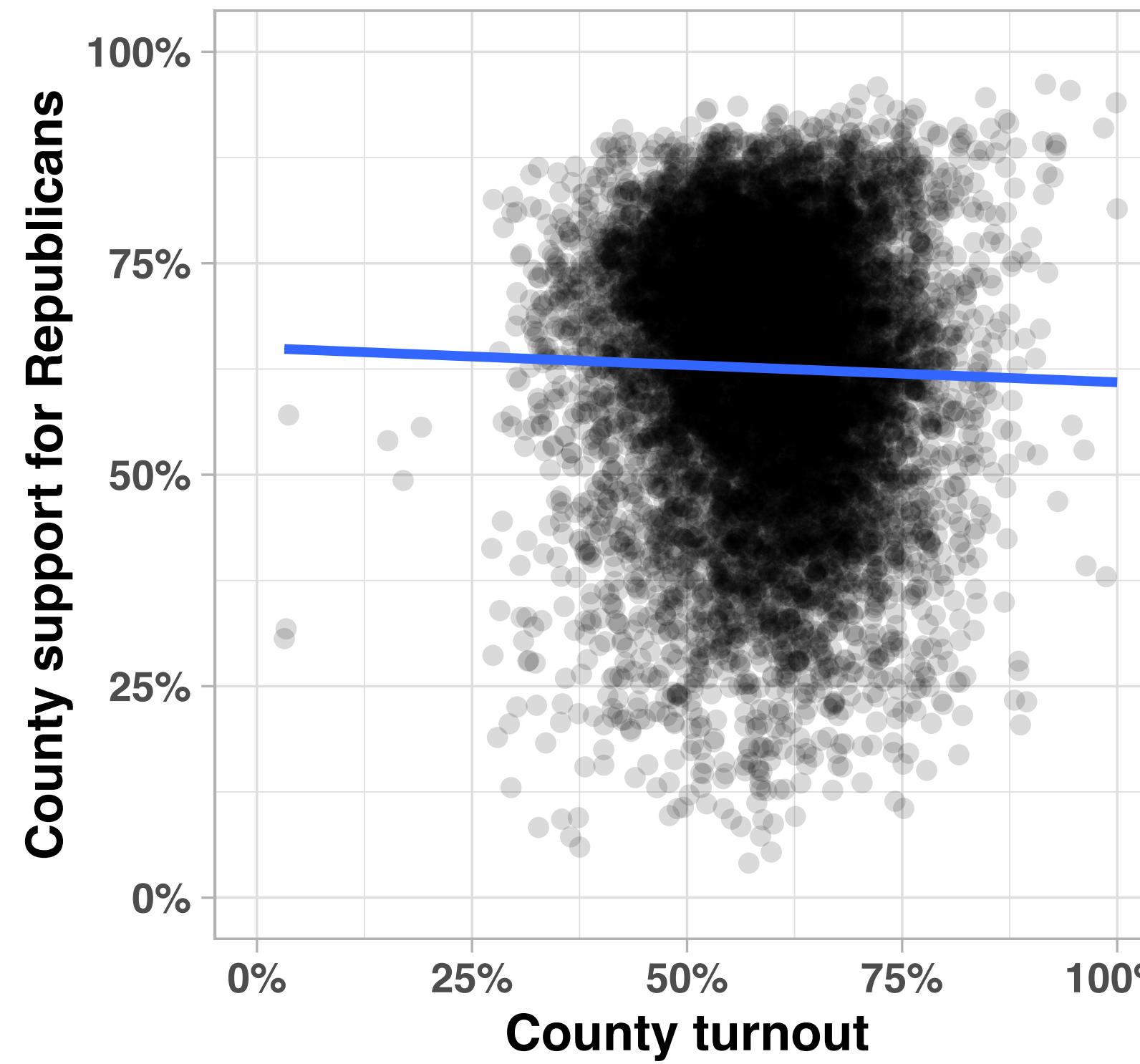
Note: The fixed effects get “taken out” of both the X and Y variables.

So it’s a re-centering on both dimensions, not just one. Hence, it’s not as visually obvious as expected.



# Let's “take out” the fixed effects.

Each county gets re-centered, and the best-fit line is refit.



When we look within all counties, Republicans appear to benefit big from higher turnout!

# Let's “take out” the fixed effects.

```
# Estimate within-county regressions
model_3 <- feols(pc_rep ~ turnout | county_fips, df)
summary(model_3)

OLS estimation, Dep. Var.: pc_rep
Observations: 9,345
Fixed-effects: county_fips: 3,117
Standard-errors: Clustered (county_fips)
    Estimate Std. Error t value Pr(>|t|)
turnout 0.370447  0.013371 27.7063 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 3.48412      Adj. R2: 0.926479
                           Within R2: 0.144134
```

Note: turnout and pc\_rep are coded 0–100.

To add our fixed effect, we place it after the bar.



Note that our output does not show the estimates for the individual fixed effects or intercept.

The fixed effects get estimated by a clever de-meaning procedure behind the scenes, and the intercept gets wrapped up with them.

# Let's “take out” the fixed effects.

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OLS estimation, Dep. Var.: pc_rep
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RMSE: 3.48412 Adj. R2: 0.926479
Within R2: 0.144134
```

Note: turnout and pc\_rep are coded 0–100.

A 1 pp increase in a given county's turnout was associated with a statistically significant 0.37 pp increase in support for the Republican presidential candidate from 2012 to 2020.

# Let's “take out” the fixed effects.

	Model 1	Model 2	Model 3	Model 4
<b>County turnout (0–100)</b>	<b>-0.04**</b> (0.01)	<b>-0.09***</b> (0.01)	<b>0.37***</b> (0.01)	<b>0.15***</b> (0.02)
<b>County median income (\$1,000s)</b>		<b>-0.40***</b> (0.01)		<b>0.09***</b> (0.02)
<b>County unemployment rate (0–100)</b>		<b>-2.17***</b> (0.05)		<b>-0.50***</b> (0.03)
Num.Obs.	9345	9345	9345	9345
County fixed effects	No	No	Yes	Yes
R2	0.001	0.202	0.951	0.955
R2 Adj.	0.001	0.201	0.926	0.933

\*p<0.05, \*\*p<0.01, \*\*\*p<0.001

**Collectively, the time-invariant differences between counties added enough omitted variable bias to flip the sign of the relationship!**

**Note:** pc\_rep is coded 0–100.

**You don't frighten me, statistical nonsense thing! Go  
and boil your standard errors, son of a silly model. I  
blow my residuals on you, so-called Fixed-effect,  
you and your silly statistical C...ontrols!**



**(OK, I'll admit this meme is a little sloppy.)**

# Consider what the fixed effect captures.

For example, what about state-level omitted variables?

What happens when we include state fixed effects?

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For example, what about state-level omitted variables?

What happens when we include state fixed effects?

No worries!

Each state fixed effect is just the sum of all the county fixed effects in that state, so the state FEs get dropped from the regression.

# **Consider what the fixed effect captures.**

**But what about time-variant omitted variables?**

# **Consider what the fixed effect captures.**

**But what about time-variant omitted variables?**

**These are not covered by our county fixed effects.**

**We might consider adding year fixed effects, which would address national trends that affect all counties in our sample.**

**We could add time-county-varying controls, like unemployment rate.**

# **Consider what the fixed effect captures.**

**Can we add county-year fixed effects?**

# Consider what the fixed effect captures.

Can we add county-year fixed effects?

No, because that's the source of all our variation!

```
# Estimate regression with county-year FEs
feols(pc_rep ~ turnout + med_inc_000s + unemploy_rate | county_fips*year, df)

Error: in feols(pc_rep ~ turnout + med_inc_000s + unemploy_....:
All variables 'turnout', 'med_inc_000s' and 'unemploy_rate' are collinear with
the fixed effects. In such circumstances, the estimation is void.
```

# Is it causal?

Causality is a spectrum.

Fixed effects eliminate entire classes of potential omitted variables.

We consider a study “causal” if the most worrisome classes of omitted variables (and other biases) have been adequately addressed.

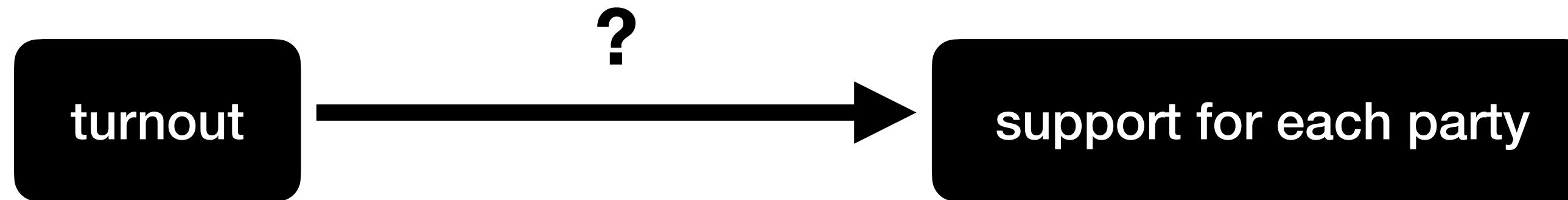
In this case, probably no. But in other fixed effects regressions, maybe yes.



AND NOW  
FOR SOMETHING  
COMPLETELY  
DIFFERENT

# What's up with instrumental variables?

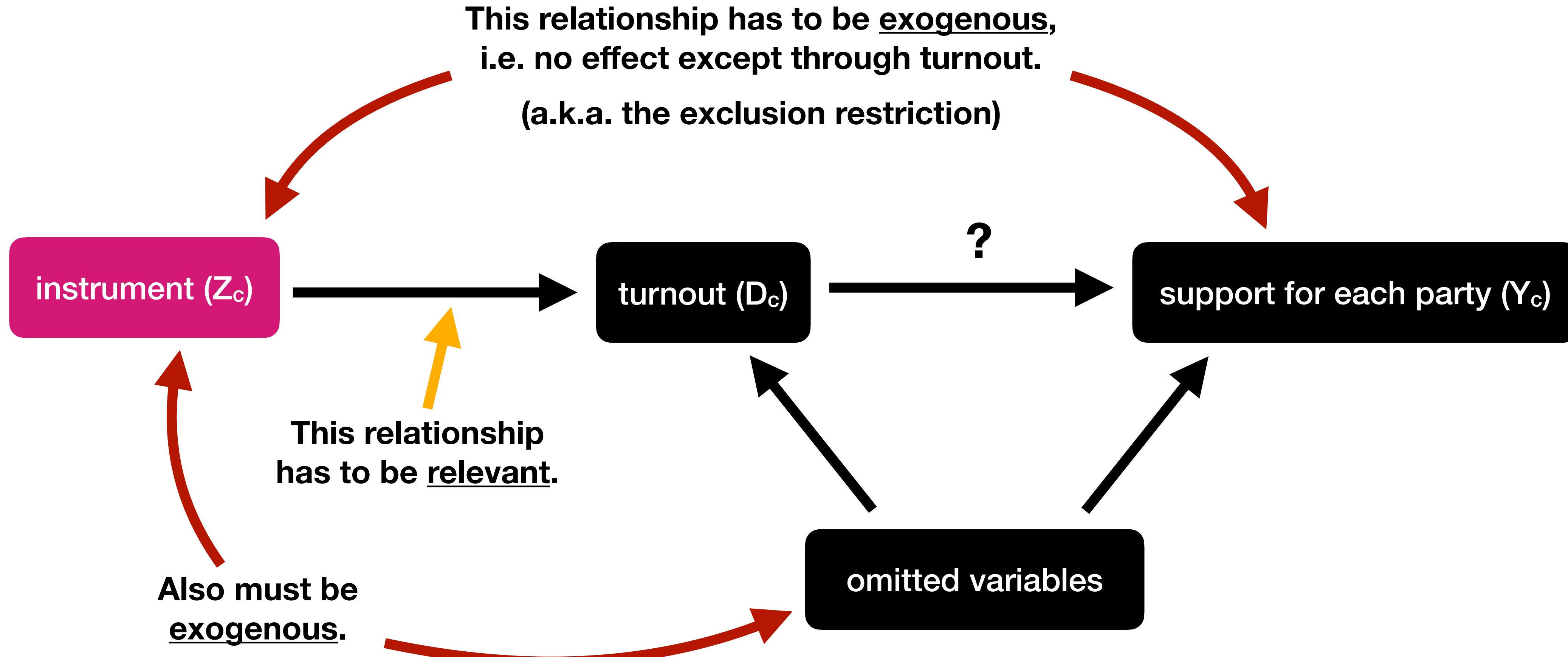
We still want to know about turnout and Republican support.



Some of this relationship is causal; some is due to omitted variables.

IVs basically split the relationship into causal and non-causal parts by taking advantage of random variation in a third variable.

# What's up with instrumental variables?



# Let's say our instrument is rain.

The idea is that random variation in rain = random variation in turnout.

**Stage 1:** Regress D on Z and take the predicted values of D.

This captures the part of our predictor that is “random.”

$$(turnout)_c = \beta_0 + \beta_{FS}(rain)_c + \beta_2(average\_rain)_c + u_c$$

I've included a control  
for the average rainy-  
ness of each county.



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I've included a control  
for the average rainy-  
ness of each county.

**Stage 2:** Then, regress Y on the predicted values of D.

This estimates the change in our outcome due to random variation in our predictor.

$$(pc\_Republican)_c = \beta_0 + \beta_{IV}(\widehat{turnout})_c + \beta_2(average\_rain)_c + u_c$$

This process is called “two-stage least squares.”



**Phillip G. Wright** when people rediscovered instrumental variables after he casually invented them in 1928 while writing about vegetable oil tariffs, only for the method to be forgotten for several decades.

Of course it's a good idea

# What can go wrong?

As it turns out, potentially a lot:

1. Weak IV: The first stage just isn't very strong ("irrelevant").
2. Violations of exclusion restriction: Does rain *only* affect turnout?
3. Not "independent": The instrument might not actually be random.

Unfortunately, we can only prove 1 and maybe 3.

# Where do we go from here?

Next week, we will dive deeper into IVs.

We will also learn how to estimate them in R.

For now, focus on your understanding.

IVs fundamentally hinge on whether we “buy” that their assumptions are met.

