ME 163 Homework 0

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1 Problem 1

Problem Statement

Given $m\ddot{x} + kx = Asin(\omega t)$, where m = k = ω = A = 1, use Matlab to numerically integrate the ODE over the time interval $0 \leq t \leq 15$. Use the initial conditions $x(0) = 1, \dot{x}(0) = 0$. Use the Runga-Kutta numerical scheme (ode45 in Matlab). Plot

- $x(t)$ vs. t
- $\dot{x}(t)$ vs. t

Solution

Using the code shown below the ode was solved and plotted:

```
funn = @(t,y)([y(2);sin(t) - y(1)])
[t,y] = ode45(funn,[0:0.01:15],[1;0]);
plot(t,y(:,1))
xlim([0 10])
ylim([-5 5])
hold off
plot(t,y(:,1))
xlim([0 10])
ylim([-5 5])
```

Below are the plots generated:

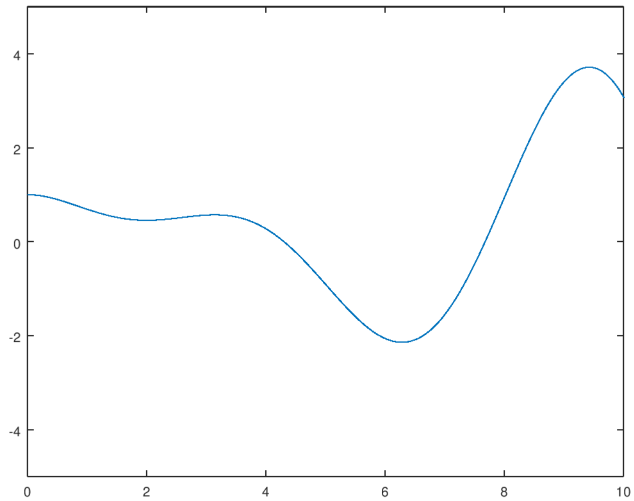

Figure 1: $x(t)$ vs. t

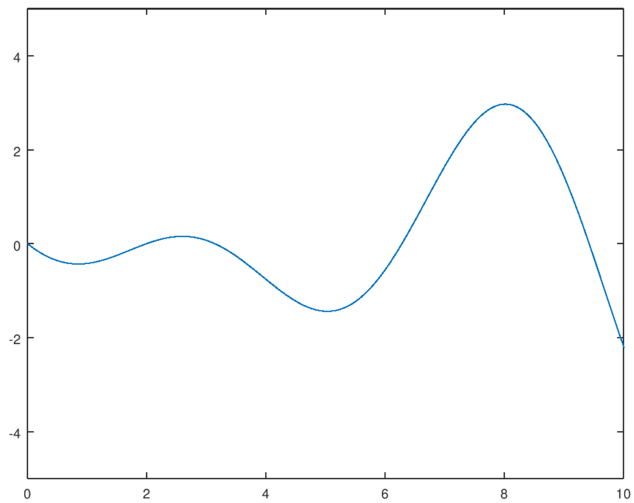

Figure 2: $\dot{x}(t)$ vs. t

2 Problem 2

Problem Statement

Find the fourier series for a square wave with amplitude 0.5 and period 1.

Soution

We can set up the solution for the fourier series as follows:

$$b_n = \frac{1}{L}(\int_0^L \sin\left(\frac{\pi n t}{L}\right) dt + \int_{-L}^0 \left(-\sin\left(\frac{\pi n t}{L}\right)\right) dt) = -\frac{2\cos(\pi n)}{\pi n} + \frac{2}{\pi n}$$

Therefore if L = 1/2:

$$b_n = \begin{cases} \frac{4}{\pi n}, & n = 1, 3, 5... \\ 0, & n = 2, 4, 6... \end{cases}$$

Since the function is odd we know that:

$$a_n = 0$$

Since the average value of the function is 0 we know that:

$$a_0 = 0$$

3 Problem 3

Solving for eigenvalues of the matrix A

$$A = \begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}$$

$$det \begin{bmatrix} 0-\lambda & 1 \\ -3 & 2-\lambda \end{bmatrix} = (0-\lambda)(2-\lambda) - (1)(-3) = -\lambda * (-\lambda + 2) + 3 = 0$$

The eigenvalues are the values of λ such that the determinant equals zero, therefore:

$$\lambda = \begin{bmatrix} 1-\sqrt{2}i, & 1+\sqrt{2}i \end{bmatrix}$$

These answers were then confirmed with matlabs eig function. The output is shown below:

```
>> A = [0 1;-3 2]
A =

   0   1
  -3   2
```

```
>> eig(A)
ans =

   1.0000 + 1.4142i
   1.0000 - 1.4142i
```

4 Problem 4

To solve for the pendulum system we will find the torque on the pendulum in polar coordinates. The torque from gravity is written as:

$$\tau_g = -glm\sin(\theta)$$

The torque from the spring is written as:

$$\tau_s = -\frac{4kl^2\sin(\theta)\cos(\theta)}{9}$$

The torque from friction is written as:

$$\tau_f = -b\dot{\theta}$$

For small angles we can make the approximations that:

$$sin(\theta) = \theta$$

$$cos(\theta) = 1$$

And linearize the equation as such:

$$ml^2\ddot{\theta} = -b\dot{\theta} - \theta glm - \frac{4\theta kl^2}{9}$$

Assuming the solution takes the form:

$$\theta(t) = e^{rt}$$

We can solve for r:

$$br + glm + \frac{4kl^2}{9} + l^2mr^2 = 0$$

$$r = \left[\frac{-3b + \sqrt{9b^2 - 36gl^3m^2 - 16kl^4m}}{6l^2m}, \quad -\frac{3b + \sqrt{9b^2 - 36gl^3m^2 - 16kl^4m}}{6l^2m}\right]$$