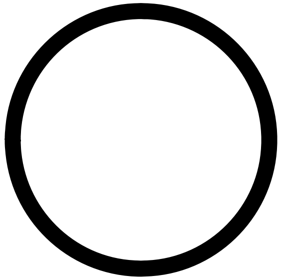
Lect. 12: Ring Resonator

Consider a ring waveguide



Assuming there is light circulating inside the ring

Resonance condition:

$$e^{-j\beta(2\pi r)} = 1$$

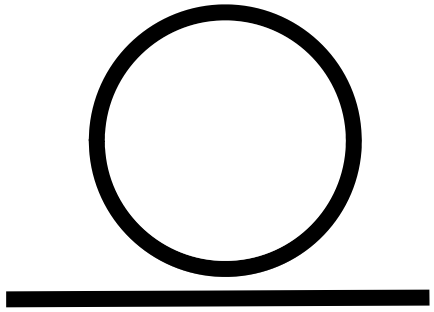
$$n_{eff} \frac{2\pi}{\lambda} 2\pi r = 2m\pi \qquad 2\pi r = m \frac{\lambda}{n_{eff}}$$

What can we do with this?

How to couple light in and out?

Lect. 12: Ring Resonator

How to couple light in and out?



Use directional coupler

Coupling coefficient: $-j\kappa$

Through coefficient: γ

$$|\kappa|^2 + |\gamma|^2 = 1$$

Consider loss in the ring waveguide

Loss coefficient: α

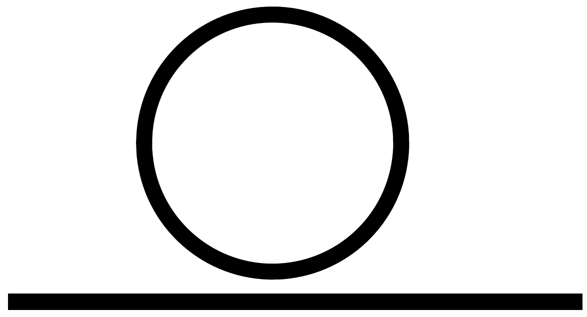
E (after one roundtrip) = α xE(before the round trip)

$$E_{out} = \gamma E_{in} - (j\kappa) E_{in} \alpha e^{-j\beta L}(-j\kappa) + (-j\kappa) E_{in} \alpha e^{-j\beta L} \gamma \alpha e^{-j\beta L}(-j\kappa) + \ldots$$

$$\frac{E_{out}}{E_{in}} = \gamma - \kappa^2 \alpha e^{-j\beta L}[1 + \gamma \alpha e^{-j\beta L} + (\gamma \alpha e^{-j\beta L})^2 + \ldots]$$

$$= \gamma - \frac{\kappa^2 \alpha e^{-j\beta L}}{1 - \gamma \alpha e^{-j\beta L}} = \frac{\gamma - \gamma^2 \alpha e^{-j\beta L} - \kappa^2 \alpha e^{-j\beta L}}{1 - \gamma \alpha e^{-j\beta L}} = \frac{\gamma - \alpha e^{-j\beta L}}{1 - \gamma \alpha e^{-j\beta L}}$$

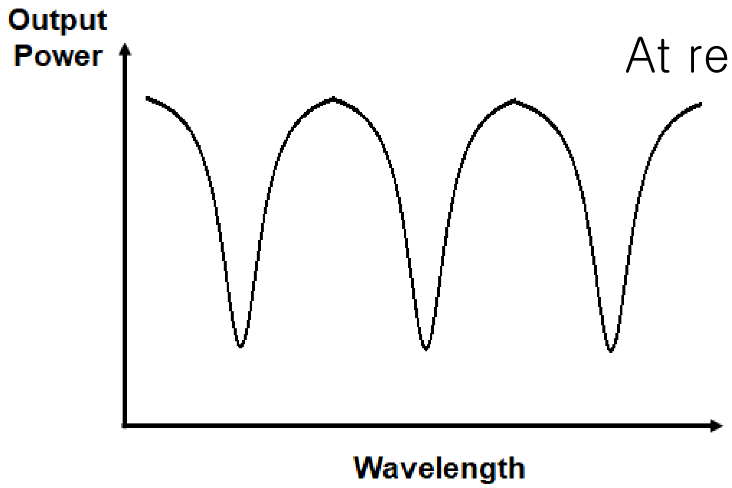
Lect. 12: Ring Resonator



$$\frac{E_{out}}{E_{in}} = \frac{\gamma - \alpha e^{-j\beta L}}{1 - \gamma\alpha e^{-j\beta L}}$$

$$T = \left|\frac{E_{out}}{E_{in}}\right|^2 = \frac{\gamma^2 + \alpha^2 - 2\gamma\alpha\cos\beta L}{1 + \gamma^2\alpha^2 - 2\gamma\alpha\cos\beta L}$$

T minimum when $\beta L = 2m\pi$ (resonance) $\quad L = m\dfrac{\lambda}{n_{eff}}$



Wavelength filter

At resonance,

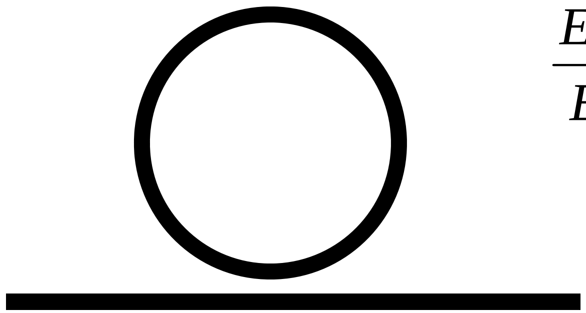
$$T = \frac{\gamma^2 + \alpha^2 - 2\gamma\alpha}{1 + \gamma^2\alpha^2 - 2\gamma\alpha} = \frac{(\gamma - \alpha)^2}{(1 - \gamma\alpha)^2}$$

T=0 if $\gamma = \alpha$; critical coupling

$\gamma < \alpha$; over coupling

$\gamma > \alpha$; under coupling

Lect. 12: Ring Resonator



$$\frac{E_{out}}{E_{in}} = \frac{\gamma - \alpha e^{-j\beta L}}{1-\gamma\alpha e^{-j\beta L}} \qquad T = \frac{\gamma^2 + \alpha^2 - 2\gamma\alpha\cos\beta L}{1+\gamma^2\alpha^2 - 2\gamma\alpha\cos\beta L}$$

If α=1, $$T = \frac{1+\gamma^2 - 2\gamma\cos\beta L}{1+\gamma^2 - 2\gamma\cos\beta L} = 1$$

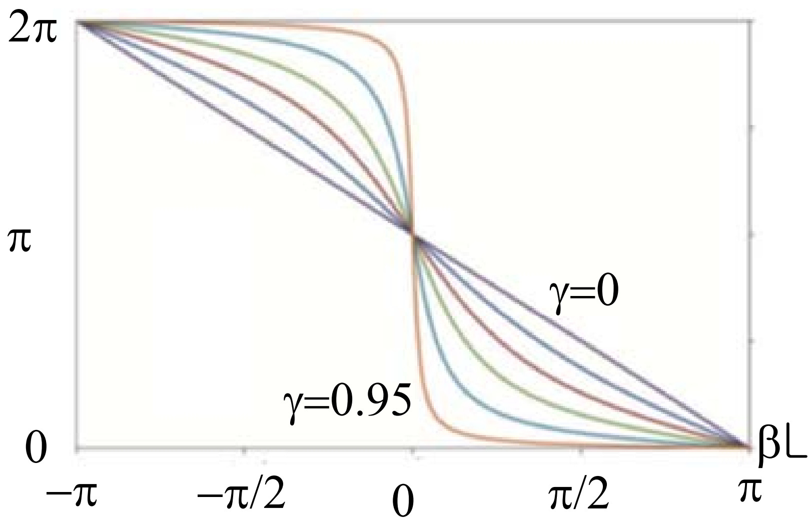
What good is this?

$$\frac{E_{out}}{E_{in}} = \frac{\gamma - e^{-j\beta L}}{1-\gamma e^{-j\beta L}}$$

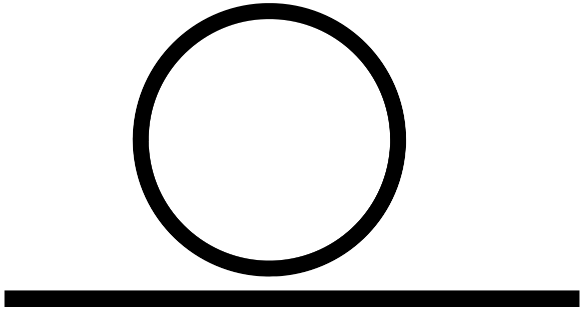
Phase-domain filter

➔ Optical All-Pass Filter

Optical Delay Line

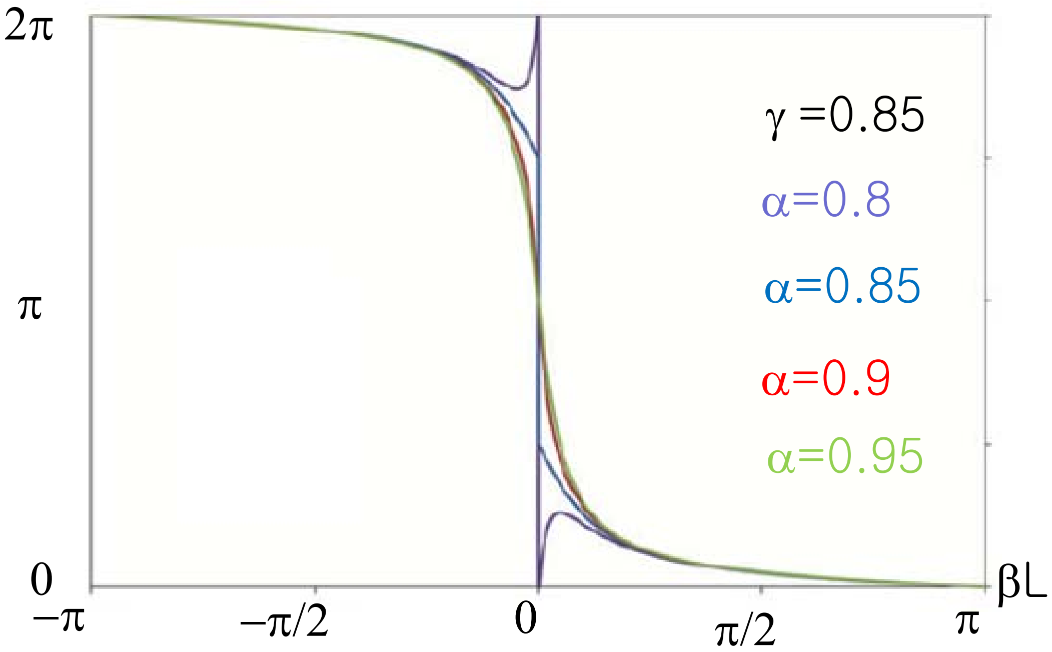


Lect. 12: Ring Resonator



$$\frac{E_{out}}{E_{in}} = \frac{\gamma - \alpha e^{-j\beta L}}{1 - \gamma\alpha e^{-j\beta L}} \qquad T = \frac{\gamma^2 + \alpha^2 - 2\gamma\alpha \cos\beta L}{1 + \gamma^2\alpha^2 - 2\gamma\alpha\cos\beta L}$$

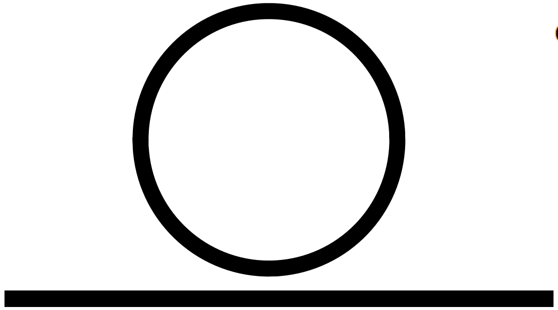
Phase response for $\alpha<1$

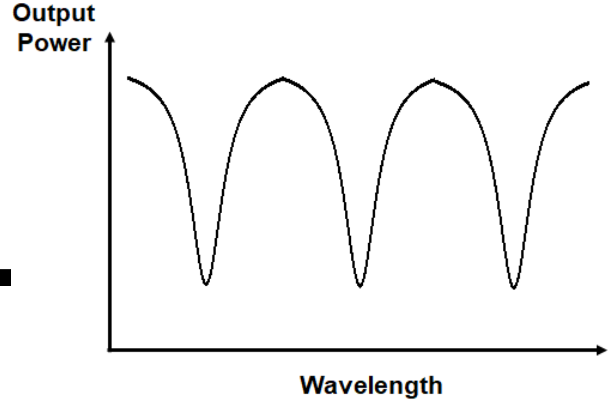


Sing change at resonance

Sharp transition from 2π to 0 around resonance

Lect. 12: Ring Resonator





$$T = \frac{\gamma^2 + \alpha^2 - 2\gamma\alpha\cos\beta L}{1 + \gamma^2\alpha^2 - 2\gamma\alpha\cos\beta L}$$

FSR

$$\Delta\beta L = 2\pi \quad => \Delta\beta = \frac{2\pi}{L}$$

$$\Delta\lambda = ? \qquad |\Delta\lambda| \sim \left|\frac{d\lambda}{d\beta}\right|\Delta\beta = n_{eff}\frac{2\pi}{\beta^2}\Delta\beta \ = \frac{\lambda^2}{n_{eff}L} \quad ==> \frac{\lambda^2}{n_g L}$$

$$\lambda = n_{eff}\frac{2\pi}{\beta}$$

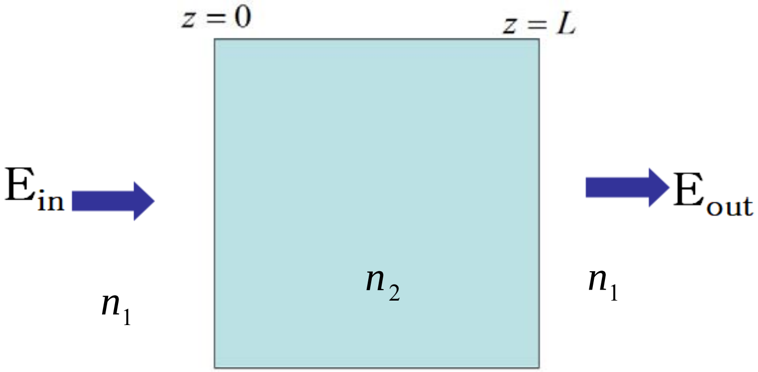
Group effective index (Lecture 8)

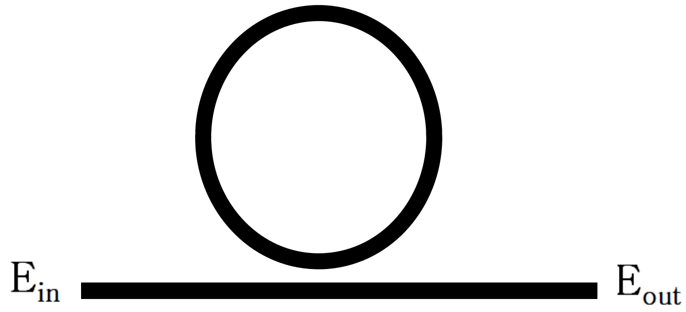
$$n_g = n_{eff}(\lambda) - \lambda\frac{\partial n_{eff}(\lambda)}{\partial\lambda}$$

Above results are effectively same as those for F-P interferometer

Lect. 12: Ring Resonator

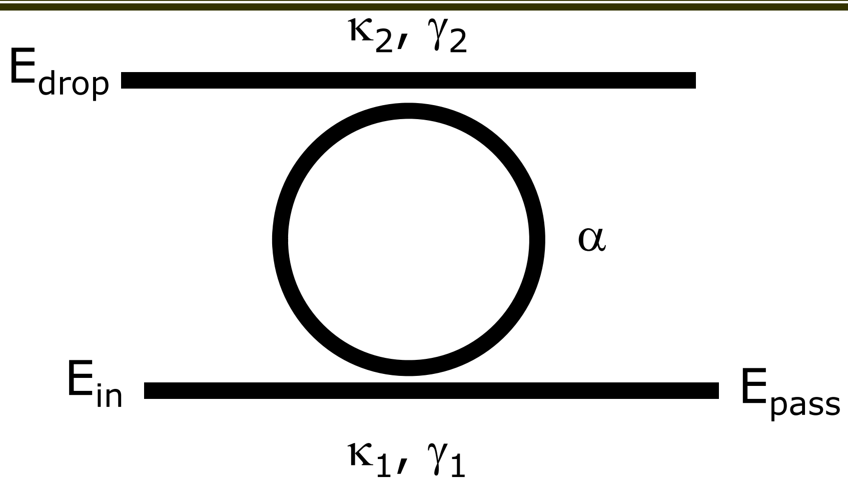
(Lecture 8)

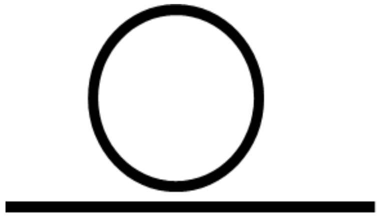




Slab (Lecture 8)		Ring resonator	
FSR:	$\dfrac{\lambda^2}{2n_2L}$	FSR:	$\dfrac{\lambda^2}{n_gL}$
FWHM:	$\dfrac{\lambda^2}{2n_2L}\dfrac{(1-R)}{\pi\sqrt{R}}$	FWHM:	$\dfrac{\lambda^2(1-\alpha\gamma)}{n_gL\pi\sqrt{\alpha\gamma}}$
Finesse:	$\dfrac{\pi\sqrt{R}}{(1-R)}$	Finesse:	$\dfrac{\pi\sqrt{\alpha\gamma}}{(1-\alpha\gamma)}$
Q-factor:	$\dfrac{2n_2L\pi\sqrt{R}}{\lambda_{res}(1-R)}$	Q-factor:	$\dfrac{n_gL\pi\sqrt{\alpha\gamma}}{\lambda_{res}(1-\alpha\gamma)}$

Lect. 12: Ring Resonator





$$\frac{E_{out}}{E_{in}} = \frac{\gamma - \alpha e^{-j\beta L}}{1 - \gamma\alpha e^{-j\beta L}}$$

$$T = \frac{\gamma^2 + \alpha^2 - 2\gamma\alpha\cos\beta L}{1 + \gamma^2\alpha^2 - 2\gamma\alpha\cos\beta L}$$

$$\gamma ==> \gamma_1 \qquad \alpha ==> \alpha\gamma_2$$

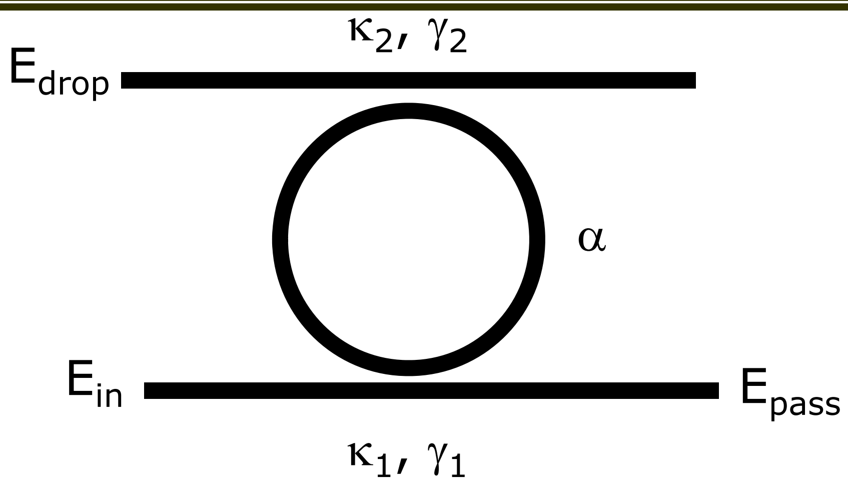
$$\frac{E_{pass}}{E_{in}} = \frac{\gamma_1 - \alpha\gamma_2 e^{-j\beta L}}{1 - \alpha\gamma_1\gamma_2 e^{-j\beta L}} \quad T_{pass} = \frac{\gamma_1^2 + \alpha^2\gamma_2^2 - 2\alpha\gamma_1\gamma_2\cos\beta L}{1 + \alpha^2\gamma_1^2\gamma_2^2 - 2\alpha\gamma_1\gamma_2\cos\beta L}$$

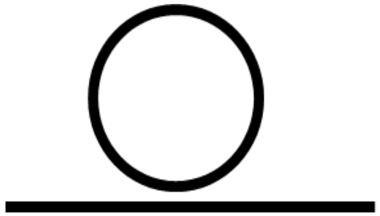
Resonance $\beta L=2m\pi$ T=0 if $\gamma_1=\alpha\gamma_2$; critical coupling

$\gamma_1<\alpha\gamma_2$; over coupling

$\gamma_1>\alpha\,\gamma_2$; under coupling

Lect. 12: Ring Resonator





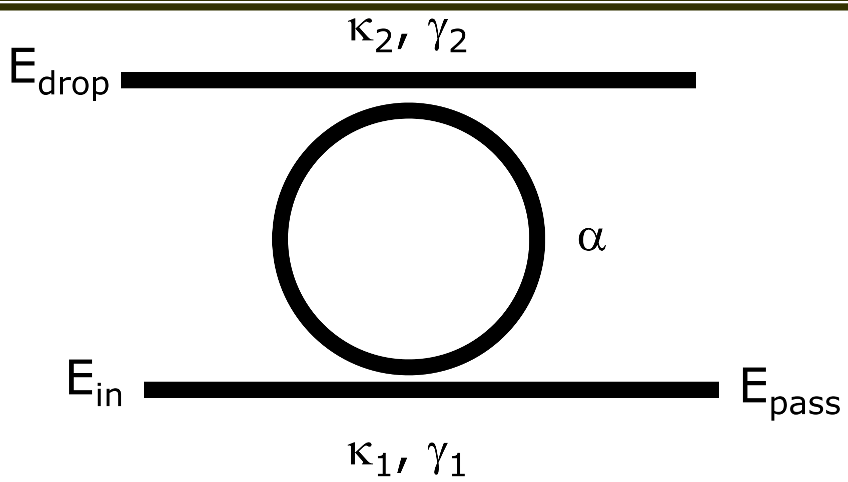
$$\frac{E_{out}}{E_{in}} = \frac{\gamma - \alpha e^{-j\beta L}}{1 - \gamma\alpha e^{-j\beta L}}$$

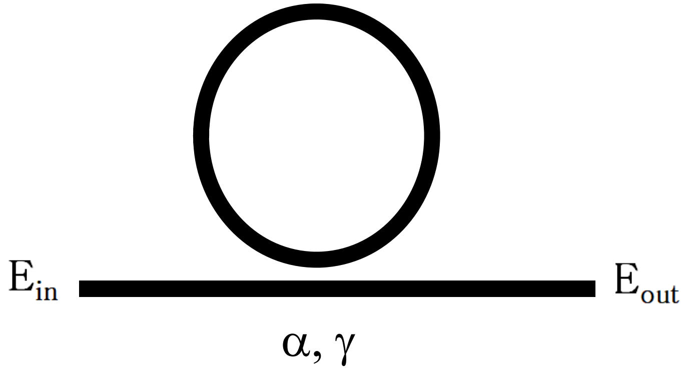
$$T = \frac{\gamma^2 + \alpha^2 - 2\gamma\alpha\cos\beta L}{1 + \gamma^2\alpha^2 - 2\gamma\alpha\cos\beta L}$$

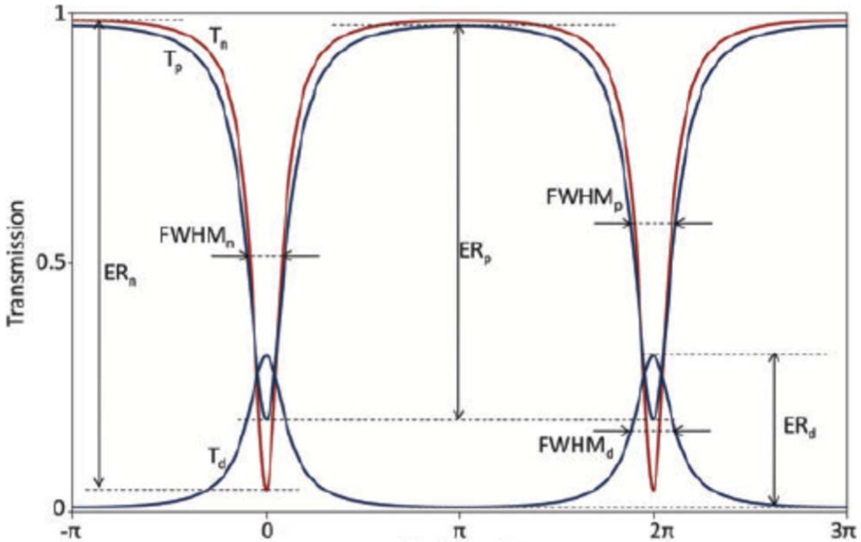
$$\frac{E_{drop}}{E_{in}} = -j\kappa_1\sqrt{a}\,e^{-j\frac{\beta L}{2}}(-j\kappa_2)[1 + a\gamma_1\gamma_2 e^{-j\beta L} + \ldots] = -\frac{\kappa_1\kappa_2\sqrt{a}\,e^{-j\frac{\beta L}{2}}}{1 - a\gamma_1\gamma_2 e^{-j\beta L}}$$

$$T_{pass} = \frac{\kappa_1^{\,2}\kappa_2^{\,2}\alpha^2}{1 + \alpha^2\gamma_1^{\,2}\gamma_2^{\,2} - 2\alpha\gamma_1\gamma_2\cos\beta L} = \frac{(1-\gamma_1^{\,2})(1-\gamma_2^{\,2})\alpha^2}{1 + \alpha^2\gamma_1^{\,2}\gamma_2^{\,2} - 2\alpha\gamma_1\gamma_2\cos\beta L}$$

Lect. 12: Ring Resonator







$$\alpha = 0.85$$
$$\gamma = \gamma_1 = \gamma_2 = 0.9$$

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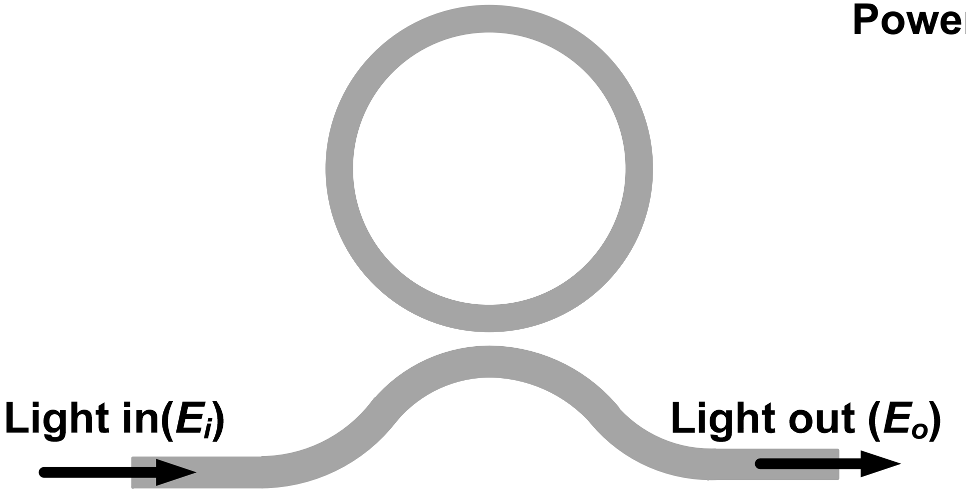
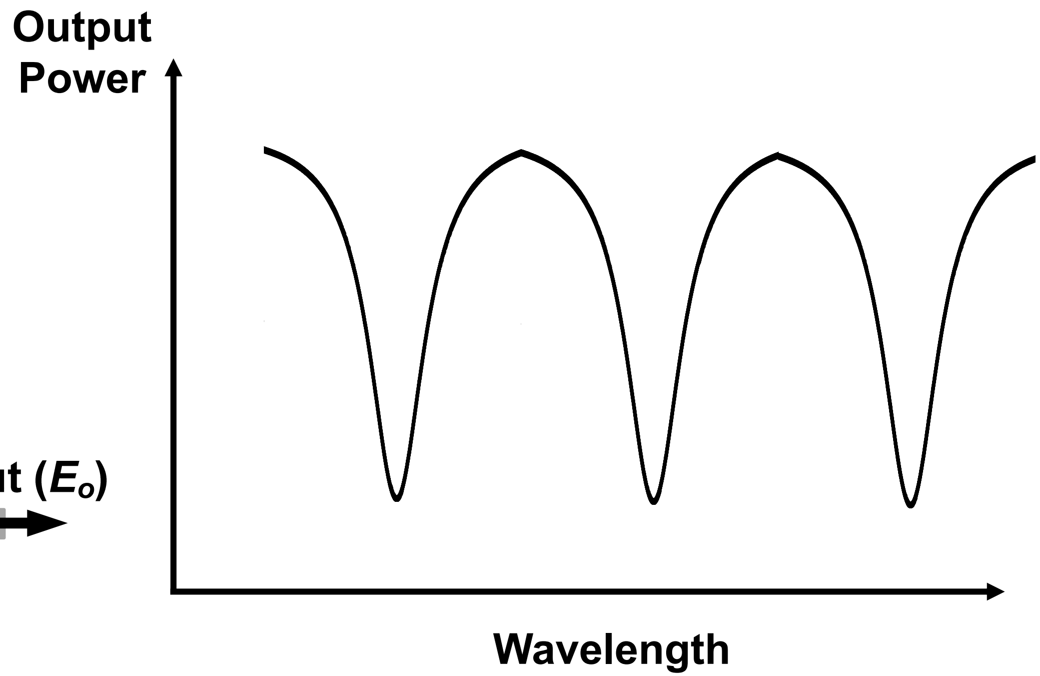
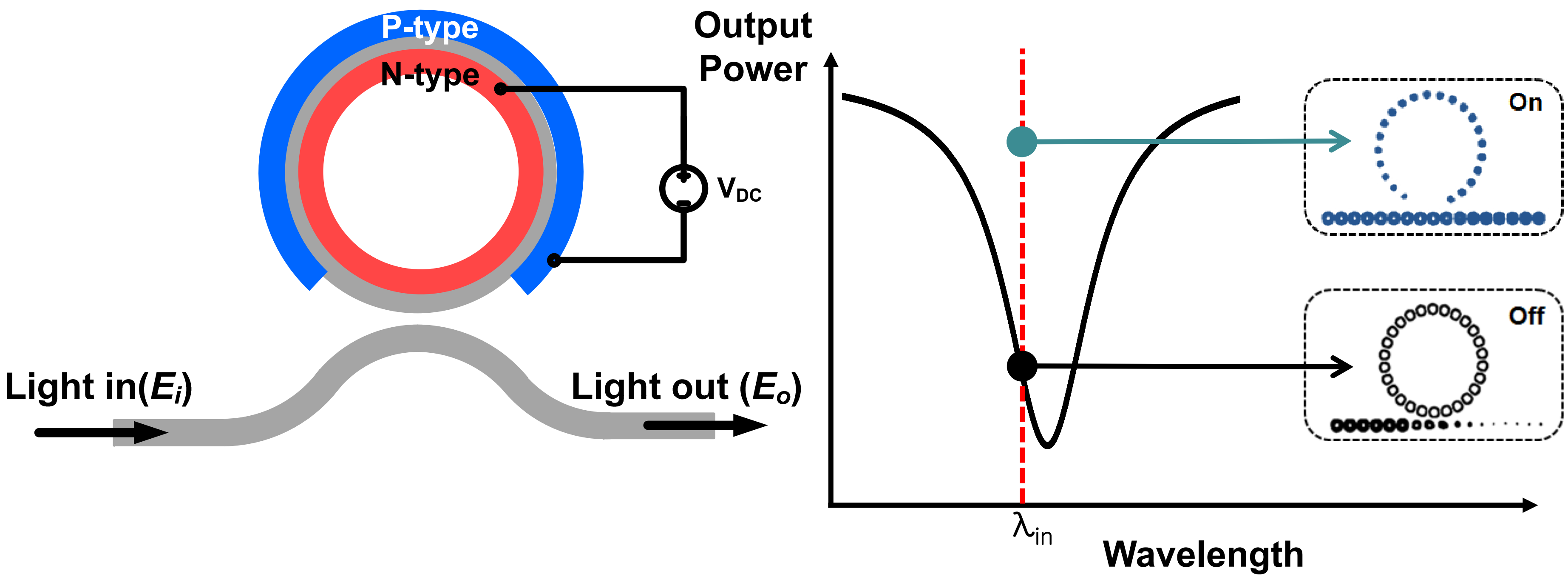




Si intensity modulator based on ring resonator?

Lect. 12: Ring Resonator



Lect. 12: Ring Resonator

Design Exercise 8 (12/4)

Simulate ring resonator characteristics
(Field intensity profile, spectrum …)

Compare with equations derived in the class