

UNIT 1

Kinematics describes the motion of objects without considering the forces acting on the objects.

SCALARS AND VECTORS

Scalar	Describes magnitude	Mass, distance, <u>speed</u>
Vector	Describes magnitude and direction	Force, <u>velocity</u> , displacement

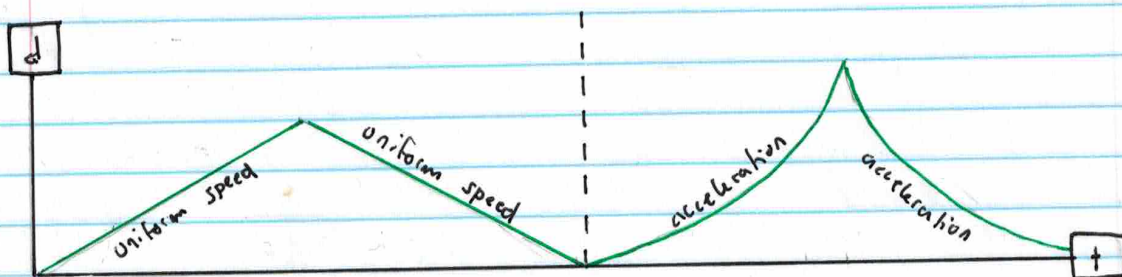
SPEED AND VELOCITY

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

Speed describes how fast an object is moving
Velocity describes the speed and direction of an object

$$a = \frac{\Delta v}{\Delta t} = \frac{V_f - V_i}{t}$$

Acceleration describes the change in speed over a certain amount of time



KINEMATIC FORMULAE

$$d = V_i t + \frac{1}{2} a t^2$$

$$d = V_f t - \frac{1}{2} a t^2$$

$$d = \frac{V_f + V_i}{2} \times t$$

$$V_f^2 = V_i^2 + 2ad$$

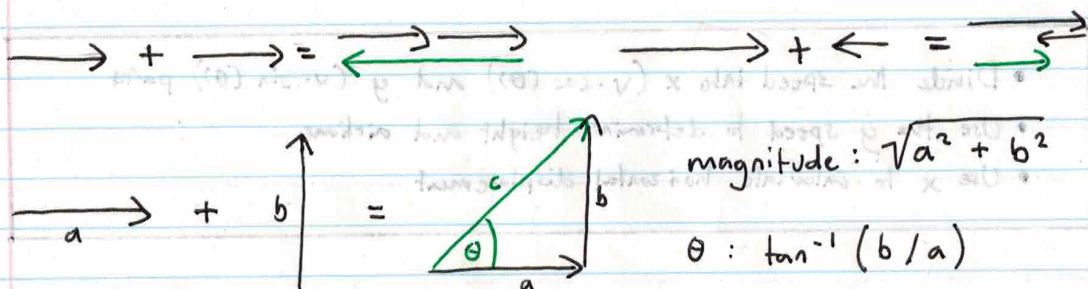
★ Many problems involve gravity. For those, use $a = -9.81 \text{ m/s}^2$

UNIT 2

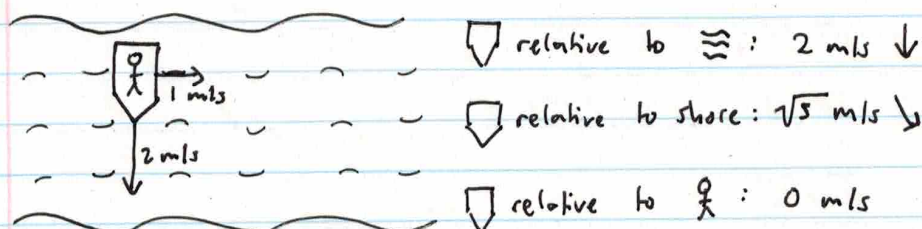
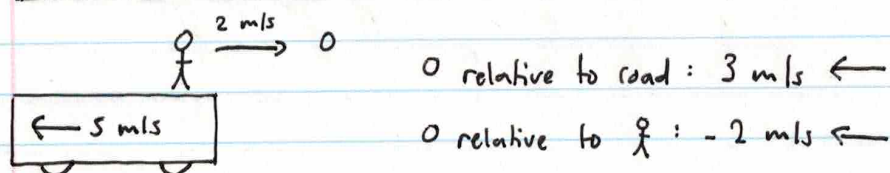
Vectors have a direction and magnitude

ADDING VECTORS

Vectors must be added tip to tail

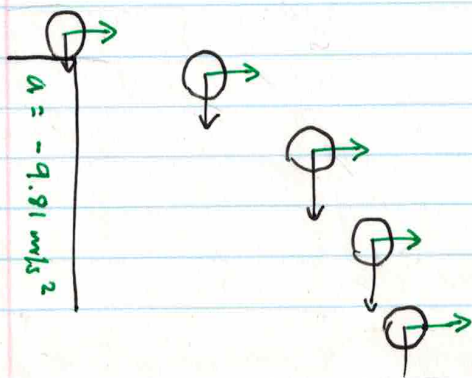


RELATIVE MOTION



HORIZONTAL PROJECTILES

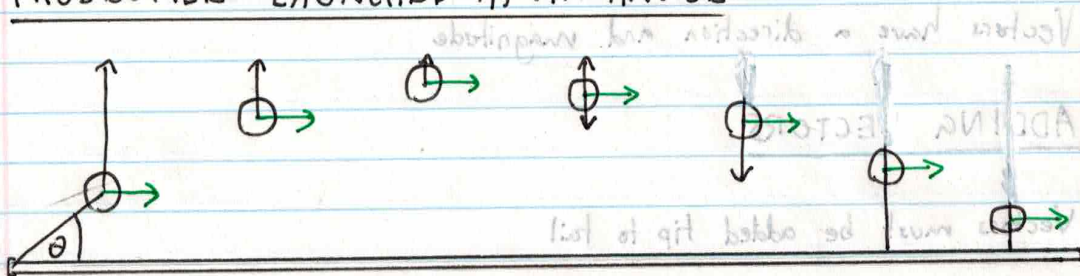
Projectiles launched horizontally will move horizontally at a constant rate



$$\bullet t = t_{\text{air}} = v_x$$

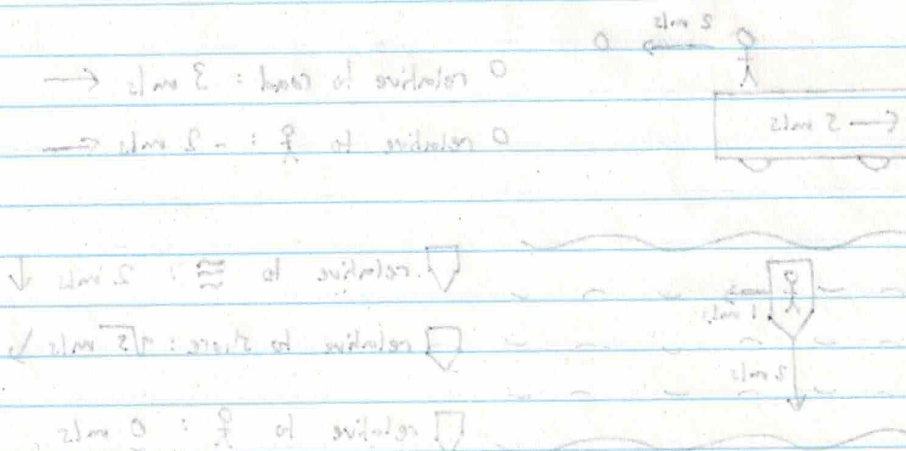
- The rest of the problem can be solved using kinematic formulae

PROJECTILES LAUNCHED AT AN ANGLE



- Divide the speed into x ($v \cdot \cos(\theta)$) and y ($v \cdot \sin(\theta)$) parts
- Use the y speed to determine height and airtime
- Use x to calculate horizontal displacement

RELATIVE MOTION

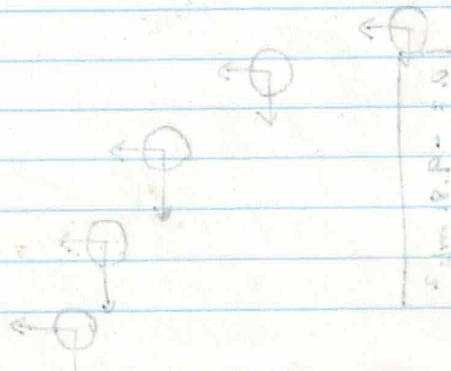


HORIZONTAL PROJECTILES

Projectiles launched horizontally will move horizontally at a constant rate

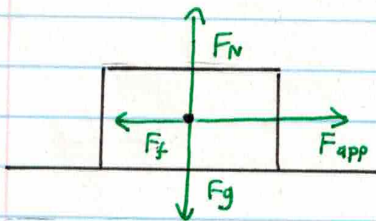
$$v_x = v \cdot \cos(\theta)$$

The rest of the problem can be solved using kinematic formulas



UNIT 3

Dynamics studies the forces acting on objects



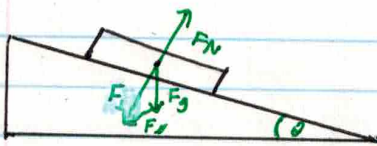
F_g : Force of gravity ($= mg$)

F_N : Normal force (perpendicular to surface) $= mg \cdot \cos$

F_{app} : Applied force

F_f : Force of friction

F_{NET} : Total force acting on the object



F_{\perp} : Perpendicular component of F_g on an incline $= mg \cdot \cos$

F_{\parallel} : Parallel component of F_g , usually F_{NET} $= mg \cdot \sin$

If an object isn't moving, $F_{NET} = 0$

NEWTON'S LAWS

I. An object will only accelerate if an external force acts on it
If $F_{NET} = 0$, $\Delta v = 0$

II. If a force acts on an object, it will accelerate
 $F = ma$

III. Every action has an equal and opposite reaction
 $F_1 = F_2$, $m_1 a_1 = m_2 a_2$

FRICTION



Friction is a force that opposes movement

Static friction	Applies when an object isn't moving ($F_{NET} = 0$)	Static friction > Dynamic friction
Dynamic friction	Applies when an object is moving ($F_{NET} > 0$)	

$F_f = \mu |F_N|$, where μ is the coefficient friction of the surfaces

UNIT 4

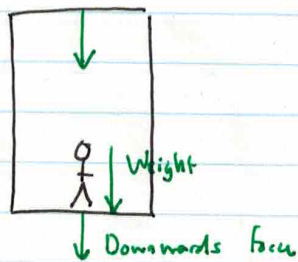
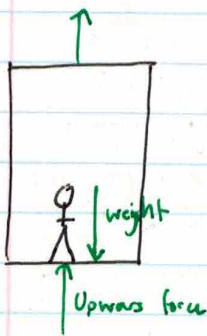
GRAVITATION

Weight (in N) = $m \cdot a_g$ ($a = 9.81 \text{ m/s}^2$ on earth)

Mass creates a gravitational field defined by $F_g = \frac{Gm_1m_2}{r^2}$ (force exerted)

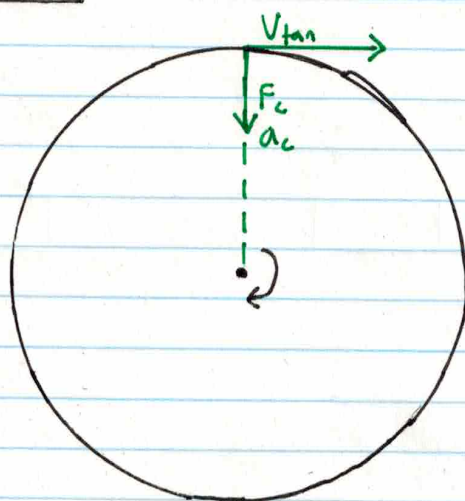
The force of a field on an object is $F_g = \frac{Gm}{r^2}$ — mass of object producing the field

ELEVATOR PROBLEMS



Subtract the movement force from the weight to get the apparent weight

UNIT 5



Circular motion is a type of periodic motion

$$T = \frac{1}{f} \quad \begin{array}{l} T = \text{period (s)} \\ f = \text{frequency (Hz)} \end{array}$$

$$V_{tan} = \frac{2\pi r}{T} \quad V_{tan} = 2\pi r f$$

$$a_c = \frac{v^2}{r} \rightarrow F_c = \frac{mv^2}{r} \quad \text{Using the formula for centrifugal acceleration and Newton's second law}$$

CIRCLE PROBLEMS

Horizontal	$F_c = F_x$ (wire), $F_c = F_t$ (equilibrium)
Vertical	$F_g = F_g + F_N$ (roller coaster), $F_g = F_g + F_T$ (wire)

SATELLITES AND KEPLER'S LAWS

I. Orbits are elliptical, and the object being orbited is at one of the foci

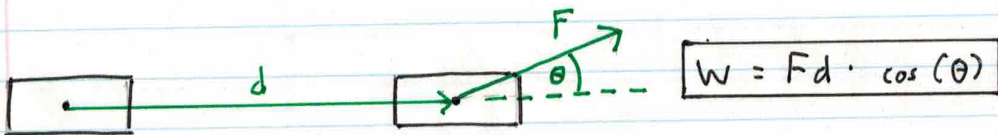
II. The satellite sweeps out equal areas (gets faster near the focus)

III. $k = \frac{T^2}{r^3}$, $\frac{T^2}{r^3} = \frac{T^2}{r^3}$, $\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$ (r = average radius in m)

Satellites satisfy the equation $F_g = F_c$, and $v = \sqrt{\frac{GM}{r}}$

UNIT 6

Work is defined as the transfer of energy ($W = \Delta E$)



The area under a force - displacement graph is the work done

POTENTIAL AND KINETIC ENERGY

$$E_p = mgh$$

$$E_k = \frac{1}{2}mv^2$$

$$W = \Delta E_p + \Delta E_k$$

HOOKE'S LAW AND ELASTIC POTENTIAL ENERGY

$$F_s = -kx$$

Where F_s is the restoring force in N, k is the spring constant and x is the distance in m

$$E_p = \frac{1}{2}kx^2$$

Elastic potential energy in J

MECHANICAL ENERGY

$$E_m = E_k + E_p$$

Only applies in an isolated system (no mass or energy exchanged)

POWER AND EFFICIENCY

$$P = \frac{W}{t}$$

$$P = \frac{\Delta E}{t}$$

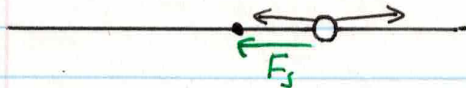
$$P = Fv$$

Power is measured in W (watts)

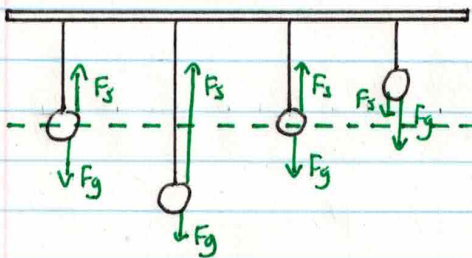
$$\text{Efficiency} = \frac{\text{Energy Out}}{\text{Energy In}}$$

UNIT 7

SIMPLE HARMONIC MOTION



An object moving in a fixed pattern with a restoring force bringing it to equilibrium



Equilibrium: $F_s = 0$

$$F_s = -kx$$

Equilibrium w/ gravity: $F_{NET} = F_s + F_g = 0$



SHM can describe the motion of pendulums as long as they don't exceed 15°

EQUATIONS FOR SHM

$$a = \frac{-kx}{m}$$

$$V_{max} = \sqrt{\frac{kA^2}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = 2\pi \sqrt{\frac{l}{g}} < 15^\circ$$

RESONANCE

The resonant frequency of a SHM object is the same as its period. If a force is applied at the same frequency, and $F_{app} > F_f$, A grows quickly

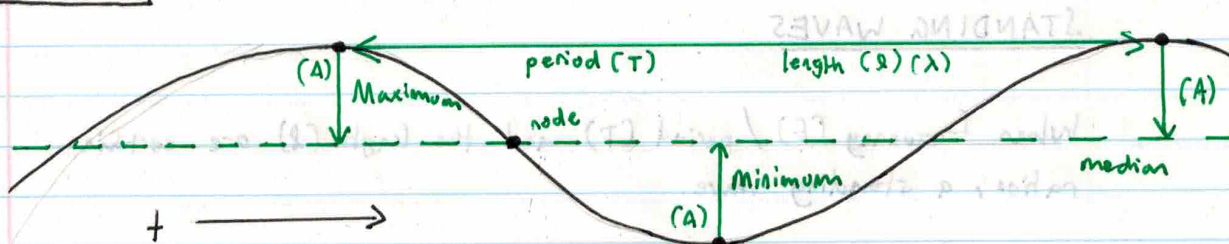
Every object has a resonant frequency

EQUATIONS

$$x(t) = A \cdot \sin(2\pi f \cdot t) \quad \text{If the motion starts at "0"}$$

$$x(t) = A \cdot \cos(2\pi f \cdot t) \quad \text{If the motion starts at a max or min}$$

UNIT 8



WAVE PROPERTIES

Transverse		ex. sine and cosine function
Longitudinal		ex. sound and light

$$v = \frac{d}{t} = \frac{\lambda}{t}$$

$$v = f\lambda$$

v and λ change when density changes, but f stays constant

DENSITY CHANGES

L \rightarrow H	Most of the wave is reflected w/ inversion	$v_{\text{tran}} \downarrow$
H \rightarrow L	Most of the wave is reflected w/o inversion	$v_{\text{tran}} \uparrow$
Small difference	Most of the wave is transmitted w/o inv	v_{tran}

SUPERPOSITIONING

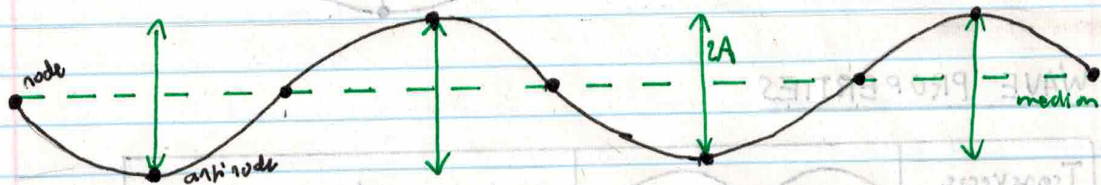
When waves intersect, their amplitudes get added together

Constructive	
Destructive	

Interference between two points can create a central maximum, central minimums. These are classified into orders

STANDING WAVES

When frequency (f) / period (T) and the length (l) are certain ratios, a standing wave.



Nodes are constant, and antinodes have a constant x -value.

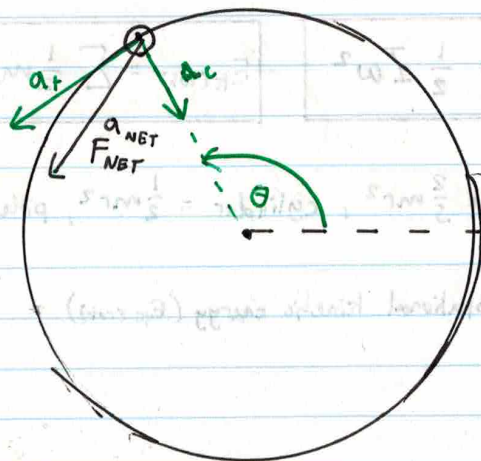
HARMONICS

Various frequencies (f) can create standing waves in air columns (open or closed) of a given length (L)

Harmonic	Open air	Equation	Closed air	Equation
Fundamental		$L = \frac{1}{2}\lambda \quad f = \frac{v}{2L}$		$L = \frac{1}{4}\lambda \quad f = \frac{v}{4L}$
2nd harmonic		$L = \frac{2}{2}\lambda \quad f = \frac{2v}{2L}$	X	X
3rd harmonic		$L = \frac{3}{2}\lambda \quad f = \frac{3v}{2L}$		$L = \frac{3}{4}\lambda \quad f = \frac{3v}{4L}$
4th harmonic		$L = \frac{4}{2}\lambda \quad f = \frac{4v}{2L}$	X	X

Closed strings are similar to open air, but there needs to be a node at each end instead of an opening

AP: ROTATIONAL MOTION



$$\text{Angular distance } (\theta) = \Delta\theta \quad \text{rad}$$

$$\text{Angular speed } (\omega) = \Delta\theta / t \quad \text{rad/s}$$

$$\text{Angular acceleration } (\alpha) = \Delta\omega / t \quad \text{rad/s}^2$$

$$\text{Distance} = \Delta\theta \cdot R$$

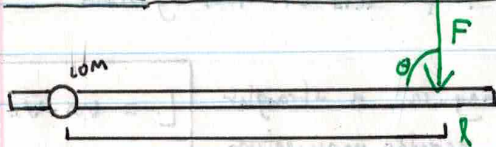
$$\text{Speed} = \omega \cdot R$$

$$\text{Tangential Acceleration } (a_{\text{tan}}) = \alpha \cdot R$$

Tangential and centrifugal acceleration can be added as vectors.

Kinematic formulae apply ($d \rightarrow \Delta\theta$, $v \rightarrow \omega$, $a \rightarrow \alpha$)

TORQUE AND ROTATIONAL INERTIA



Torque applies when a force acts on an object away from its center of mass

Torque's unit is the newton-meter (N·m)

$$\tau = F \cdot l \cdot \sin(\theta)$$

Torque is analogous to force (rotational motion)

Clockwise = negative torque

Counter-clockwise = positive torque

If an object isn't rotating, $\tau_{\text{net}} = 0$

$$\tau = mr^2 \alpha = I \alpha$$

$$\alpha = \frac{\tau}{mr^2} = \frac{\tau}{I}$$

$$I = mr^2$$

$$I = \sum mr^2$$

I is rotational inertia, which describes how hard it is to rotate something
Rotational inertia is analogous to mass ($m \rightarrow I$)

ANGULAR KINETIC ENERGY

$$E_{K(trans)} = \frac{1}{2}mv^2$$

$$E_{K(rot)} = \frac{1}{2}I\omega^2$$

$$E_{K(rot)} = \sum \frac{1}{2}mv^2$$

I is different for every shape; sphere = $\frac{2}{5}mr^2$, cylinder = $\frac{1}{2}mr^2$, pole = $\frac{1}{3}mr^2$ (end)

Translational kinetic energy ($E_{K(trans)}$) + Rotational kinetic energy ($E_{K(rot)}$) =
Total kinetic energy (E_K)

ANGULAR MOMENTUM

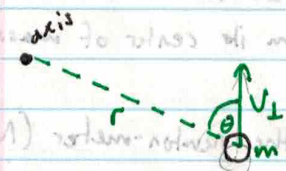
$$p = mv = F \cdot \Delta t$$

$$L = m \cdot v_{\perp} \cdot r$$

$$L = \omega \cdot mr^2 = \omega I$$

$$L = T \cdot \Delta t$$

Angular momentum is conserved if no net torque acts on the system



A mass moving in a straight line can have angular momentum in relation to an axis

$$L = m \cdot v_{\perp} \cdot r \cdot \sin(\theta)$$

ANGULAR MOMENTUM PROBLEMS (AND OTHER TYPES)

Rolling without slipping is defined by $v_{(trans)} = r\omega$ (v of COM)

$$\text{Rolling: } E_m = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$$

$$\text{Falling on a string: } mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\text{Ball hits rod: } L_i = L_f \quad (\text{expand for parameters needed})$$

GRAVITATIONAL POTENTIAL ENERGY

$$U_g = -G \frac{m_1 m_2}{d}$$

Notice that d isn't squared like in the force formula