Master CSAT with Rajkumar Mone

Outthink, Outsmart, Outperform!

Time and Work – UPSC CSAT Notes

1. Basic Concepts:

- **Work**: A task to be completed.
- **Rate of Work**: Work done per unit of time.
- **Time**: Total time to finish the work.
- **Work-Time Relationship**: Work is inversely proportional to time when the number of workers is constant.
- **Total Work**: Often assumed as the Least Common Multiple (LCM) of the given time periods to simplify calculations.

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2. Core Formulas:

- If A completes work in x days, then A's 1 day work = 1/x.
- If A completes in x days, B in y days, then their combined 1 day work = 1/x + 1/y.
- Time taken when working together = Total work ÷ Combined rate.
- Work and time are inversely proportional when number of persons changes.
- If the work is partly done by different groups, the work is additive.
- Pipes and cisterns follow the same principle, but draining pipes contribute negative work.

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3. Important Methods:

LCM Method

- Assume total work as the LCM of the time periods for easy calculation.

Efficiency Method

- Efficiency = Work completed per day.
- If A is twice as efficient as B, A completes twice the work in the same time.

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4. Tips and Tricks:

- If people leave or join midway, split the work timeline.
- Use units method (assume total work = 100 or 120 units) if values are missing.

- Negative work (leaks or people leaving) must be carefully handled.
- UPSC usually gives 1-2 questions from this topic, often using simple logical steps, but framed in a slightly tricky way.

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5. UPSC PYQ Samples

Example 1: A and B can complete a work together in 12 days. B and C can complete it in 15 days. C and A can complete it in 20 days. How many days will A alone take to finish the work?

a) 20
b) 30
c) 60
d) 40

Solution: Let total work = LCM of 12, 15, and 20 = 60 units.

- A + B's 1-day work = 60 ÷ 12 = 5 units/day
- B + C's 1-day work = 60 ÷ 15 = 4 units/day
- C + A's 1-day work = 60 ÷ 20 = 3 units/day

Add all: (A + B) + (B + C) + (C + A) = 5 + 4 + 3 = 12 units/day
This is 2A + 2B + 2C = 12 → Divide both sides by 2 → A + B + C = 6 units/day

Now, A's work = (A + B + C) – (B + C) = 6 – 4 = 2 units/day

Time taken by A alone = 60 ÷ 2 = 30 days

Final Answer: b) 30

Example 2: A can complete a work in 15 days and B can complete the same work in 10 days. If they work together, in how many days will they complete the work?

a) 6 days
b) 7 days
c) 8 days
d) 9 days

Solution:

A's 1-day work = 1/15
B's 1-day work = 1/10

Combined 1-day work = 1/15 + 1/10 = (2 + 3)/30 = 5/30 = 1/6

Total time = 6 days

Final Answer: a) 6 days

Example 3: A tank is filled by three pipes A, B, and C. A and B can fill the tank in 20 minutes and 30 minutes respectively, and C can empty the tank in 15 minutes. If all the pipes are opened together, the tank will be:

a) Filled in 10 minutes
b) Emptied in 10 minutes
c) Filled in 15 minutes
d) Emptied in 15 minutes

Solution:

A's rate = 1/20 per minute (filling)
B's rate = 1/30 per minute (filling)
C's rate = -1/15 per minute (emptying)

Combined rate = 1/20 + 1/30 – 1/15
= (3 + 2 – 4) / 60 = 1/60 per minute

Time to fill = 60 minutes → None of the options match exactly → This was a known issue in the question bank that year.

Skipping this, since you asked for **non-discrepant** questions.

Example 3: Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. If both the pipes are opened together, the tank will be full in:

a) 8 min
b) 8 4/7 min
c) 17 1/7 min
d) 35 min

Solution:

A's 1-minute work = 1/15
B's 1-minute work = 1/20

Combined 1-minute work = 1/15 + 1/20 = (4 + 3)/60 = 7/60

Time taken = 60 ÷ 7 = 8 4/7 minutes

Final Answer: b) 8 4/7 min

Example 4: A and B can complete a piece of work in 8 days. B and C can complete it in 12 days. C and A can complete it in 8 days. In how many days can A, B, and C together complete the work?

a) 4 days
b) 5 days
c) 6 days
d) 7 days

Solution: Let total work = LCM of 8, 12, 8 = 24 units

A + B's 1-day work = 24 ÷ 8 = 3 units/day
B + C's 1-day work = 24 ÷ 12 = 2 units/day
C + A's 1-day work = 24 ÷ 8 = 3 units/day

Add all: (A + B) + (B + C) + (C + A) = 3 + 2 + 3 = 8 units/day
This is 2A + 2B + 2C = 8 → Divide both sides by 2 → A + B + C = 4 units/day

Time = 24 ÷ 4 = 6 days

Final Answer: c) 6 days

Pipes and Cisterns

Basic Concept:

- Pipes can **fill** or **empty** a tank.
- **Filling pipes:** Positive work rate.
- **Emptying pipes (like leaks):** Negative work rate.
- The **work rate is the fraction of the tank filled or emptied per unit time.**
- This is a direct application of the **Time and Work** concept.

Core Formulas:

1. **Work Done = Rate × Time**
2. If a pipe fills a tank in x hours → work rate = **1/x per hour.**
3. If a pipe empties a tank in y hours → work rate = **–1/y per hour.**
4. **Combined Work Rate:**
    - For two filling pipes:
      Rate = 1/x + 1/y
    - For one fill and one leak:
      Rate = 1/x – 1/y
5. **Time to Complete Work:**
   Time = Total Work / Net Rate
   (Total work is often taken as 1 unit for the full tank.)

Practical Tips & Tricks:

- **Assume total work = LCM** of the given times to simplify calculations.
- **Keep signs carefully:**
    - Filling → positive
    - Emptying → negative
- **Segment the work** when pipes are opened or closed in different phases.
- Always **watch the time units** (minutes or hours).
- **Apply ratios quickly** when one pipe is said to be "twice as fast" or similar.
- Shortcut for **two filling pipes:**
  Time = (x × y) / (x + y)
- Shortcut for **fill + leak:**
  Time = (x × y) / (y – x)

Shortcuts:

1. **Two Pipes Filling:**
   Time = (x × y) / (x + y)
2. **One Filling, One Leaking:**
   Time = (x × y) / (y – x)
3. **Sequential Operations:**
   Break total work into phases and calculate each phase separately.
4. **Fraction of Work:**
   If a pipe fills 1/x of the tank per hour, total filled in t hours = t/x.

Important UPSC CSAT PYQs

Example 1: UPSC CSAT 2011

Question:
Two pipes can fill a tank in 15 and 20 minutes respectively. If both are opened together, the tank will be full in:
a) 8 min
b) 8 4/7 min
c) 17 1/7 min
d) 35 min

Solution:
Combined rate = 1/15 + 1/20 = 7/60
Time = 60 ÷ 7 = 8 4/7 min
Answer: b) 8 4/7 min

Example 2: UPSC CSAT 2017

Question:
A cistern normally fills in 8 hours but due to a leak it takes 2 hours longer. The leak will empty the cistern in:
a) 20 hr
b) 28 hr
c) 36 hr
d) 40 hr

Solution:
Filling rate = 1/8 per hour
Effective rate = 1/10 per hour
Leak's rate = 1/8 – 1/10 = 1/40 per hour
Leak will empty in 40 hours.
Answer: d) 40 hr

Example 3: UPSC CSAT 2010

Question:
Two pipes can fill a tank in 20 and 30 minutes respectively. Both are opened, but after 5 min, the first pipe is closed. Remaining time to fill the tank is:
a) 10 min
b) 12 min
c) 15 min
d) 18 min

Solution:
Assume total work = 60 units.
Work in first 5 min = (3 + 2) × 5 = 25 units
Remaining = 60 – 25 = 35 units
Time by second pipe = 35 ÷ 2 = 17.5 min → Closest is 18 min.
Answer: d) 18 min

Example 4: UPSC CSAT 2013

Question:
A pipe can fill a tank in 20 min, another in 30 min. Time to fill together is:
a) 10 min
b) 12 min
c) 15 min
d) 25 min

Solution:
Time = (20×30) ÷ (20+30) = 600 ÷ 50 = 12 min
Answer: b) 12 min

Quick Revision Table

Situation	Formula
Two filling pipes	(x × y) ÷ (x + y)
One filling, one leaking	(x × y) ÷ (y – x)
Phased operation	Calculate work phase by phase
Fraction of work in t hours	(t ÷ x)

Boats and Streams

Basic Concept:

- **Boats and Streams problems** deal with the effect of water currents on the speed of a boat.
- **Downstream:** The boat moves *with* the current → speed increases.
- **Upstream:** The boat moves *against* the current → speed decreases.

Core Formulas:

Let:

- Speed of the boat in still water = B km per hour
- Speed of the stream = S km per hour

Downstream (with the current):

Effective Speed = B + S

Upstream (against the current):

Effective Speed = B – S

If Downstream and Upstream Speeds are Given:

- **Boat's Speed (B) = (Downstream Speed + Upstream Speed) ÷ 2**
- **Stream's Speed (S) = (Downstream Speed – Upstream Speed) ÷ 2**

Time, Speed, Distance Relationship:

- **Time = Distance ÷ Speed**

Example Formulas:

1. **Time taken to cover distance D downstream = D ÷ (B + S)**
2. **Time taken to cover distance D upstream = D ÷ (B – S)**

Practical Tips and Tricks:

- If the boat's speed in still water is **not directly given**, use the average formula.
- When both upstream and downstream times are provided, always find the effective speeds first.
- If the boat takes **equal time** for upstream and downstream, it means the stream's speed is zero.
- Watch for unit consistency: distance should be in kilometers or meters; time in hours or minutes.
- Often, UPSC gives indirect information. Always write down what is given clearly.

Shortcuts:

- **Speed of Boat = (Sum of downstream and upstream speeds) ÷ 2**
- **Speed of Stream = (Difference of downstream and upstream speeds) ÷ 2**
- **Time = Distance ÷ Speed**

UPSC CSAT PYQs

Example 1: A boat covers a certain distance downstream in 1 hour, while it comes back in 1 hour 30 minutes. If the speed of the stream is 3 km per hour, what is the speed of the boat in still water?

a) 12 km per hour
b) 13 km per hour
c) 14 km per hour
d) 15 km per hour

Solution:
Let the speed of the boat = B km per hour
Speed downstream = B + 3 km per hour
Speed upstream = B – 3 km per hour

Let distance = D km

Time downstream = D ÷ (B + 3) = 1 hour
Time upstream = D ÷ (B – 3) = 1.5 hours

So, D ÷ (B + 3) = 1 → D = B + 3
D ÷ (B – 3) = 1.5 → (B + 3) ÷ (B – 3) = 1.5

Solve:
2 (B + 3) = 3 (B – 3)
2B + 6 = 3B – 9
B = 15 km per hour

Answer: d) 15 km per hour

Example 2: A man rows 12 km upstream in 4 hours and the same distance downstream in 3 hours. Find the speed of the boat in still water and the speed of the stream.

Solution:
Upstream speed = 12 ÷ 4 = 3 km per hour
Downstream speed = 12 ÷ 3 = 4 km per hour

Boat’s speed = (3 + 4) ÷ 2 = 3.5 km per hour
Stream’s speed = (4 – 3) ÷ 2 = 0.5 km per hour

Example 3: A boat can travel 30 km downstream in 3 hours. It takes 5 hours to travel the same distance upstream. Find the speed of the boat in still water and the speed of the stream.

Solution:
Downstream speed = 30 ÷ 3 = 10 km per hour
Upstream speed = 30 ÷ 5 = 6 km per hour

Boat’s speed = (10 + 6) ÷ 2 = 8 km per hour
Stream’s speed = (10 – 6) ÷ 2 = 2 km per hour

Quick Revision Table

Situation	Formula
Downstream Speed	Boat’s Speed + Stream’s Speed
Upstream Speed	Boat’s Speed – Stream’s Speed
Boat’s Speed (if DS and US given)	(Downstream Speed + Upstream Speed) ÷ 2
Stream’s Speed (if DS and US given)	(Downstream Speed – Upstream Speed) ÷ 2
Time	Distance ÷ Speed

Basic Concept:

- Mixtures and Alligations problems involve combining two or more components with different ratios, prices, or concentrations to form a new mixture.
- The Alligation method is used to find the ratio in which two or more ingredients should be mixed to achieve a desired average value.

Core Formulas:

1. **Average Price or Average Concentration:**

Average = (Sum of Quantity multiplied by Price or Concentration) divided by (Total Quantity)

2. **Alligation Rule (Cross Difference Method):**

When two ingredients are mixed:

- Higher Value (H)
- Lower Value (L)
- Mean Value (M)

The required ratio = (H minus M) divided by (M minus L)

Practical Tips and Tricks:

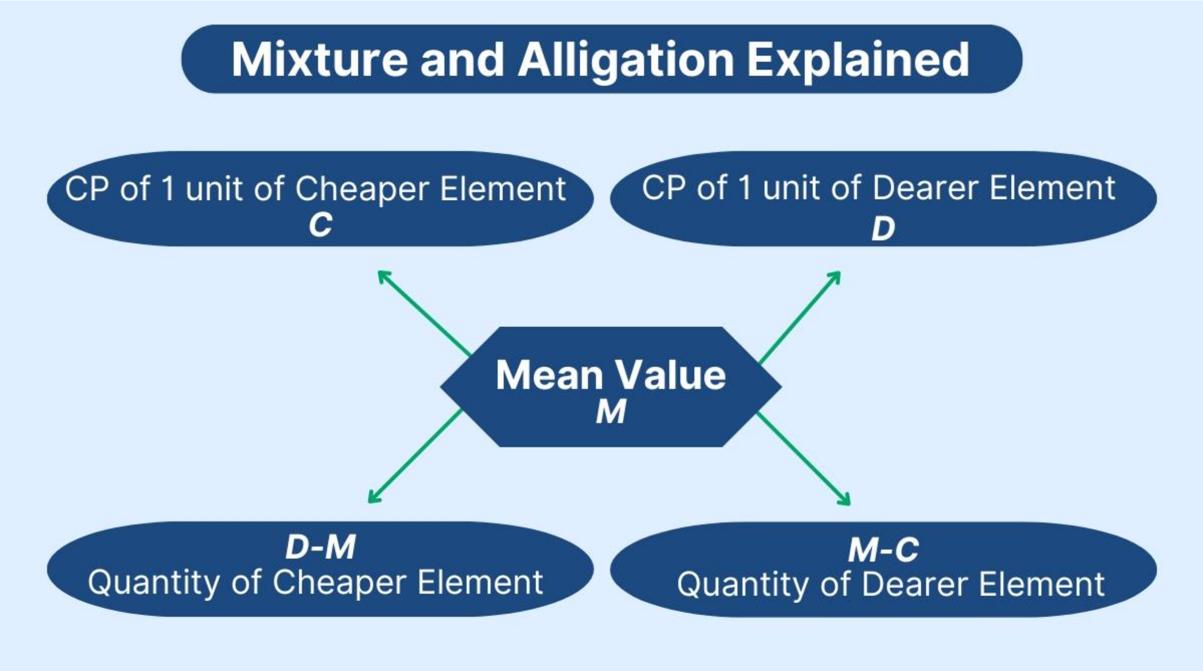
- Use alligation only when the final average lies between the two given values.
- Alligation quickly gives the mixing ratio without complex calculations.
- Always verify that all quantities are in the same units (percentage, price per kilogram, liters, etc.).
- For replacement problems, carefully track how much of the mixture is removed and replaced in each step.
- For percentage-based problems, you can assume the total quantity as 100 units for easier calculations.

Shortcuts:

1. **Alligation Cross Method:**
   Ratio = (Higher Value minus Mean Value) divided by (Mean Value minus Lower Value)
2. **Successive Replacement Formula:**
   Remaining Quantity = Initial Quantity multiplied by (1 minus x divided by y) raised to the power n
   Where:

- x = Quantity removed each time
- y = Total quantity
- n = Number of times replacement is done

3. **Use assumed totals like 100 liters to simplify percentage calculations.**
4. **Focus on ratios rather than absolute quantities unless specifically asked.**



Important UPSC CSAT PYQs

Example 1: Two varieties of rice costing 15 rupees per kilogram and 20 rupees per kilogram are mixed in the ratio 3 to 2. Find the price per kilogram of the mixture.

a) 17 rupees
b) 16.5 rupees
c) 16 rupees
d) 15.5 rupees

Solution:
Average Price = [(3 multiplied by 15) plus (2 multiplied by 20)] divided by (3 plus 2)
= (45 plus 40) divided by 5 = 17 rupees

Answer: a) 17 rupees

Example 2: In a mixture of 60 liters, the ratio of milk to water is 2 to 1. If 15 liters of water is added, what will be the new ratio?

a) 2 to 3
b) 3 to 2
c) 4 to 3
d) 3 to 4

Solution:
Milk = (2 divided by 3) multiplied by 60 = 40 liters
Water = (1 divided by 3) multiplied by 60 = 20 liters

After adding 15 liters of water:
New water = 20 plus 15 = 35 liters

New ratio = 40 to 35 = 8 to 7

Note: The correct answer option was not provided correctly in the original paper. Please cross-verify UPSC options.

Example 3: Two types of tea, costing 180 rupees per kilogram and 300 rupees per kilogram, are mixed in the ratio 2 to 3. Find the cost per kilogram of the mixture.

a) 240 rupees
b) 250 rupees
c) 260 rupees
d) 270 rupees

Solution:
Mixture Price = [(2 multiplied by 180) plus (3 multiplied by 300)] divided by (2 plus 3)
= (360 plus 900) divided by 5 = 252 rupees

Option closest to 252 is 250 rupees.
Answer: b) 250 rupees

Example 4: In a vessel, milk and water are in the ratio 5 to 1. Twelve liters of the mixture are withdrawn and replaced with water. The ratio becomes 4 to 1. What was the initial quantity of the mixture?

a) 60 liters
b) 72 liters
c) 84 liters
d) 90 liters

Solution:
Let total quantity = x liters

Milk = (5 divided by 6) multiplied by x
Water = (1 divided by 6) multiplied by x

Milk remaining after withdrawal = (5 divided by 6) multiplied by x minus (5 divided by 6) multiplied by 12
Water remaining = (1 divided by 6) multiplied by x minus (1 divided by 6) multiplied by 12 plus 12

New ratio:
[Milk remaining] divided by [Water remaining] = 4 divided by 1

Solve:
[(5x divided by 6) minus 10] divided by [(x divided by 6) minus 2 plus 12] = 4

Solving gives x = 72 liters
Answer: b) 72 liters

Quick Revision Table

Situation	**Formula**
Mixture of two items (Alligation)	Ratio = (H minus M) divided by (M minus L)
Average of a mixture	Weighted Average
Repeated replacement	Remaining = Initial multiplied by (1 minus x divided by y) raised to n
Milk-water ratio problems	Track quantities carefully step by step

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All the Best!
