VAJIRAO IAS ACADEMY Pvt. Ltd.

A Success Story...

CSAT

Quantitative Aptitude

VOLUME - III

For UPSC and State Civil Services Examinations

Contents

Basic Numeracy

01 Numbers

Numbers are a collection of certain symbols or figures called digits. The most common number system in use is Decimal System which has ten symbols, each representing a digit. These are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. A combination of these figures representing a number is called a numeral.

We also have Binary Number system. It uses only 0 and 1. There are other Number systems too.

Every digit in a number has a **face value** and a **place value.** The face value of a digit equals the value of the digit itself, irrespective of its place in the numeral.

For determining the place value of a digit in a given number we begin from the extreme right as Unit's place, Ten's place, Hundred's place, Thousand's place and so on. It has been illustrated with an example given below :

The number 2465971385 may be represented as :

Billion	Ten	Crore's	Ten Lac's	Lac's	Ten	Thousand's	Hundred's	Ten's	Unit's
	Crore's		(million)			Thousand's			
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
2	4	6	5	9	7	1	3	85	

We may also write this number as

$2465971385 = 2 \times 10^9 + 4 \times 10^8 + 6 \times 10^7 + 5 \times 10^6 + 9 \times 10^5 + 7 \times 10^4 + 1 \times 10^3 + 3 \times 10^2 + 8 \times 10^1 + 5 \times 10^0$

Here, the place value of various digits are

Digit	***Place Value***
5	$5 \times 10^0 = 5$
8	$8 \times 10^1 = 80$
3	$3 \times 10^2 = 300$
1	$1 \times 10^3 = 1{,}000$
7	$7 \times 10^4 = 70{,}000$
9	$9 \times 10^5 = 9{,}00{,}000$
5	$5 \times 10^6 = 5{,}000{,}000$
6	$6 \times 10^7 = 60{,}000{,}000$
4	$4 \times 10^8 = 400{,}000{,}000$
2	$2 \times 10^9 = 2{,}000{,}000{,}000$

Thus, we see that the place value of a digit depends on its location in the number. For instance, take the digit 5 in the above example. Its face value remains five at both the places while its place values at two places are 5 and 5,000,000 respectively.

Classification of Numbers

Natural Number : These are also called counting numbers as these are the numbers we use for counting purposes. It is represented by

N : {1, 2, 3, 4... .}.

Whole Number : It includes all natural numbers plus zero. So, we can denote it by W : {0, 1, 2, 3 ....}.

I : {........–5, –4, –3, –2, –1, 0, 1, 2, 3,.....}

Thus, we see that whole numbers consist of positive integers and zero. Similarly, natural numbers consist of positive integers.

Even Number : A number which is completely divisible by 2 is called an even number. In other words, such numbers have 2 as a factor when they are written as product of different numbers. For instance;

$$30 = \mathbf{2} \times 3 \times 5, \quad 42 = \mathbf{2} \times 3 \times 7$$

Other examples of even numbers are 2, 4, 6, 8, 10..... (A number is said to be a factor or sub-multiple of another when it divides the other exactly.

For example, 5 and 3 are factors of 15.

Odd Number : These numbers are not completely divisible by 2. In other words, a number which is not even is an odd number.

For example, 1, 3, 5, 7, 9 ....

It may be noted that zero is an exception to this even-odd classification.

Cyclic Number : It is such an integer of *n* digits which on being multiplied by any number from 1 through *n*, gives a product containing the same *n* digits as in the original number and these digits are in the same cyclic order in the product.

For example, 142857

2 × 142857 = 285714;

3 × 142857 = 428571

4 × 142857 = 571428;

5 × 142857 = 714285

Fractions : A fraction is a number which represents a ratio or division of two integers. It is expressed in the form $\frac{p}{q}$ where '*p*' and '*q*' are integers. '*p*' is called the numerator and '*q*' is called the denominator.

A fraction cannot have zero ($q \neq 0$) as its denominator since division by zero is not defined.

Zero divided by any integer is always zero.

A fraction with 1 as the denominator is the same as the whole number which is its numerator.

For example, $\frac{8}{1}$ is 8; $\frac{0}{5}$ is 0.

Equivalent Fractions : Two fractions are said to be equivalent if they represent the same ratio or number. So, if we multiply or divide the numerator and denominator of a fraction by the same non-zero integer, the result obtained will be equivalent to the original fraction.

For example, ; $\frac{2}{7} = \frac{3\times 2}{3\times 7} = \frac{6}{21}$ $\frac{15}{25} = \frac{\frac{15}{5}}{\frac{25}{5}} = \frac{3}{5}$

In a multiple-choice test, the answer to a problem that one obtains need not necessarily be the same as given in the choices, however, one of the choices may be equivalent. In such cases one needs to express the answer as equivalent fraction.

To find a fraction with a known denominator equal to a given fraction :

(*i*) We divide the desired denominator by the denominator of the given fraction.

(*ii*) Then we multiply the result of (*i*) by the numerator of the given fraction which gives the numerator of the required fraction.

Example : Find the fraction with denominator 20 which is equal to $\frac{3}{5}$.

Sol. (*i*) 20 ÷ 5 = 4 (*ii*) 4 × 3 = 12

Answer is . $\mathbf{\frac{12}{20}}$

To reduce a fraction to its lowest terms, we cancel all the factors common to the numerator and denominator.

Real Number : Any measurement carried out in the physical world gives some meaningful figure or value or number. This number is called **Real Number**. It consists of two groups :

(*i*) Rational Number : A rational number can always be represented by a fraction of the form $\frac{p}{q}$, where *p* and *q* are integers and *q* is not equal to zero ($q \neq 0$). All integers and fractions are rational numbers.

For example, $2\left(=\frac{2}{1}\right), \frac{2}{3}, \frac{4}{5}, \frac{6}{13}$ etc.

(*ii*) Irrational Number : An irrational number can't be expressed in the form of , $\frac{p}{q}$ where $q \neq 0$.

For example, $\sqrt{3}, \sqrt{2}$. It gives only an approximate answer in the form of a fraction or decimal number. The digits after the decimal point are non-ending. The same holds true for π = 3.14....... which again is irrational. Alternatively, we can say that an infinite non-recurring decimal number is an irrational number.

Prime Number : A prime number is a number which has no factor besides itself and unity, i.e; it is divisible only by itself and 1 but not by any other number.

For example : 2, 3, 5, 7, 11, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107.

Note : **(*i*)** '2' is the only even number which is prime.

(*ii*) All prime numbers other than 2 are odd numbers but all odd numbers are not prime numbers. For example, 9 is an odd number but it is not a prime number as it is divisible by 3.

Composite Number : A composite number is one which has other factors besides itself and unity. Thus, it is a non-prime number. For example, 4, 6, 9, 14, 15 etc.

Note : **(*i*)** '1' is neither prime nor composite.

(*ii*) A composite number may be even or odd.

..Notes

The number of ways in which a number N can be expressed as product of two factors which are relatively prime to each other is $\mathbf{2}^{m-1}$, where m is the number of different prime factors of N.

For example, $540 = 2^2 \times 3^3 \times 5$ ($= 4 \times 27 \times 5$)

$\therefore$ $m = 3$(i.e., 2, 3, 5)

$\therefore$ No. of ways $= 2^{3-1} = 4$

i.e., 20×27, 4×135, 108×5, 540×1.

The largest prime number known so far is $2^{2281}-1$ which is of about 700 digits.

Consecutive Integers : These are series of numbers differing by 1 in ascending or descending order.

For example, 12, 13, 14, 15.....

Similarly, examples of consecutive even numbers are

4, 6, 8, 10 .... 22, 24, 26, 28 ..... and so on.

Examples of consecutive prime numbers are

7, 11, 13, 17 ....

Test for Prime Numbers

For numbers less than 100, it is not very difficult to determine whether it is a prime number or not. We can also check it up from the list given under prime numbers.

For testing any number greater than 100 whether it is a prime number or not :

I. We take the nearest integer larger than the approximate square root of that number. Suppose it is x.

II. We test the divisibility of the given number by every prime number less than x.

III. If the number is not divisible by any of them, then it is a prime number, otherwise it is a composite number.

Let us test the three numbers

(*a*) 331, (*b*) 481 and (*c*) 881.

(*a*) For 331, $18 \times 18 = 324$, $19 \times 19 = 361$, $\therefore$ $x = 19$.

The prime numbers less than 19 are 2, 3, 5, 7, 11, 13 and 17. Now try to divide 331 by these prime numbers. We see that 331 is not completely divisible by any of these prime numbers. Therefore, 331 is a prime number.

(*b*) For 481, $x = 22$.

The prime numbers less than 22 are 2, 3, 5, 7, 11, 13, 17, 19. We find that 481 is divisible by 13.

Therefore, 481 is not a prime number.

(*c*) For 881, $x = 30$

The prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

We see that 881 is not divisible by any of the above prime numbers. Therefore, 881 is a prime number.

Any whole number can be written as a product of factors which are prime numbers. To write a number as a product of prime factors :

I. Divide the number by 2 if possible; continue to divide by 2 until the factor you get is not divisible by 2.

II. Divide the result from I by 3 if possible; continue to divide by 3 until the factor you get is not divisible by 3.

III. Divide the result from II by 5 if possible; continue to divide by 5 until the factor you get is not divisible by 5.

IV. Continue the procedure with 7, 11 and so on, until all the factors are primes.

For example, $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$

Perfect Number : If the sum of the divisors of a number N excluding N itself is equal to N, then N is called a perfect number. For example, 6, 28, 496, 8128

For 6, divisors are 1, 2, and 3

6 : $1 + 2 + 3 = 6$

28 : $1 + 2 + 4 + 7 + 14 = 28$

496 : $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$

8128 : $1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 + 1016 + 2032 + 4064 = 8128$

Note : The sum of the reciprocals of the divisors of a perfect number including that of its own is always equals 2.

For example, for 28 the factors are 1, 2, 4, 7 and 14.

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}$$

$$=\frac{28+14+7+4+2+1}{28}=\frac{56}{28}=2$$

Decimal Number : A collection of digits (0, 1, 2, 3, ...9) after a period (called the decimal point) is called a decimal fraction.

For example, 0.629, 0.53, 0.023 etc.

0.53 is read as decimal five three and not as decimal fifty three.

12.142 is read as twelve decimal one four two.

Every decimal fraction represents a fraction. These fractions have denominators with powers of 10.

For example, $0.5=\frac{5}{10}$

$$0.42=\frac{4}{10}+\frac{2}{100}=\frac{42}{100}$$

$$0.429=\frac{4}{10}+\frac{2}{100}+\frac{9}{1000}=\frac{429}{1000}$$

A number containing a decimal point is called a decimal number.

$$35.467=(3\times10^1)+(5\times10^0)+\frac{4}{10^1}+\frac{6}{10^2}+\frac{7}{10^3}$$

$$=(3\times10)+(5\times1)+\frac{4}{10}+\frac{6}{100}+\frac{7}{1000}$$

$$=30+5+\frac{4}{10}+\frac{6}{100}+\frac{7}{1000}$$

$$=35+\frac{400+60+7}{1000}=35+\frac{467}{100}=\frac{35467}{1000}$$

Mixed Number : A mixed number consists of whole number and a fraction.

For example, $3\frac{4}{5}$ is a mixed number. This is equivalent to and hence can be written as : $3\frac{4}{5}=3+\frac{4}{5}$.

Here 3 is the whole number and $\frac{4}{5}$ is the fraction.

Changing a Mixed Number into an Equivalent Fraction :

A mixed number can be changed into a fraction as follows :

I. Multiply the whole number by the denominator of the fraction.

II. Add the numerator of the fraction to the resultant product.

III. The Equivalent Fraction has numerator given by II above and denominator same as that of the fractional part of the given mixed number.

In the above example : $3\frac{4}{5}=\frac{(3\times5)+4}{5}=\frac{19}{5}$

We can also do the reverse of it, i.e, when the numerator of a given fraction is greater than the denominator, we can change it into a mixed number using the following procedure :

I. Divide the numerator by the denominator.

II. The quotient, gives the whole number and the remainder gives the numerator of the fractional part. The denominator of the fractional part will be the same as the denominator of the given fraction.

For example, $\frac{21}{4}=5\frac{1}{4}$

Signed Number : A number which has either a plus or a minus sign attached as prefix is called a Signed Number. Such numbers are also called Directed Numbers.

For example, + 3, – 6, –7 $+\frac{5}{2}$, $-\frac{6}{7}$, $+3\frac{1}{2}$ $-4\frac{1}{3}$, etc.

If no sign is given with a number, a plus sign is assumed and vice versa.

Singed numbers are often used to distinguish different concepts. For example, a profit of Rs. 25 may be indicated by + Rs. 25 and a loss of Rs. 25 by – Rs. 25. Thus, plus and minus signs have opposite meanings. A height of 2500 metres above the sea level may be denoted by +2500 metres while a depth of 2500 metres below the sea level is denoted by – 2500 metres. Similarly, we can write for temperature above and below 0°C. So, we see that in all cases we must have a datum or reference with respect to which the given quantity is measured and expressed. In the above examples, zero, cost price, sea level, 0°C are the reference points. The quantity on the positive side of the datum is given plus sign and its absolute value is obtained by adding the quantity to the datum/reference value. On the other hand, the other (negative) side of the datum is given minus sign and its absolute value is given by subtracting the given quantity from the datum which, in other words, is equivalent to adding the quantity with minus sign to the datum.

For example, 0 + 25 = + 25 = 25

0 – 25 = 0 + (–25) = –25

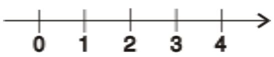
Thus, we see that **adding a negative quantity to any number is equivalent to subtracting its absolute (positive) value from the given number.**

You can imagine numbers to be arranged on a line called a number line.

Draw a line which extends indefinitely (i.e., upto infinity) in both directions. Mark a reference point on this line and call it zero. The portion on the right of zero is called positive side and that on the left of zero as negative side. (Instead of a horizontal line we can alternatively draw a vertical line in which case the portion above zero is positive and that below zero is negative).

Mark another point on the line to the right of zero and call it '1'.

The point to the right of 1 which is exactly as far away from 1 as 1 is from 0 is marked 2. The point to the right of 2 just as far from 2 as 1 is from 0 (or 2 is from 1) is called 3. Similarly, we have 4, 5, 6 and so on.

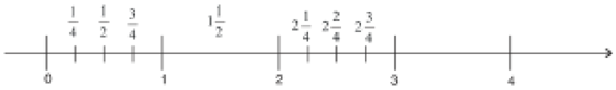


The point mid-way or half-way between 0 and 1 is called $\frac{1}{2}$, the point midway between 1 and 2 is called $1\frac{1}{2}$ or $\frac{3}{2}$ and so on. The point halfway between 0 and $\frac{1}{2}$ is called $\frac{1}{4}$ and the point midway between $\frac{1}{2}$ and 1 is called $\frac{3}{4}$. Thus any whole number or fraction can be identified.

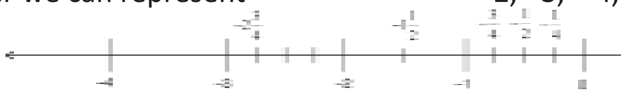
For example, $2\frac{3}{4}$.

It lies between 2 and 3.

Divide the distance between 2 and 3 into 4 equal parts. The point at the third ark between 2 and 3 represents $2\frac{3}{4}$.



If we go to the left of zero, the same distances as we did from 0 to 1, the point is called –1. In a similar manner we can represent – 2, –3, – 4, $-\frac{1}{4}$, $-\frac{1}{2}$, $-\frac{3}{4}$, $-1\frac{1}{2}$, $-2\frac{3}{4}$



and so on. Thus,

Zero is neither positive nor negative and therefore we can write, 0 = + 0 = – 0

Any non-zero number is either positive or negative but can't be both.

It is not always necessary to take zero as datum/reference point. Datum or Reference Point is chosen as per convenience and depends on the problem to be solved. The same holds true for positive and negative too. But once chosen, their uniformity must be maintained throughout the solution. It is usual practice to assign zero to the reference point/datum, whatever its value might be.

For instance, we take ground level of a particular place instead of sea level as datum for expressing the height of a building. Similarly, the reference for local time is different in different countries.

Modulus or Absolute Value : This Absolute Value must be differentiated from the absolute value discussed above. The Modulus or Absolute Value of any signed number (be it positive or negative) is always taken as positive and is equal to the distance of the number from the zero datum. It is denoted by writing the number between two vertical lines. In other words, it denotes only the magnitude of the number. If A is any signed number, its Modulus or Absolute Value is written as |A|.

If A is positive, |A| = A and if A is negative, |A| = –A

So, what we do effectively is simply drop the sign attached to the number, when we are taking its Modulus or Absolute Value.

For example,

|+3| = + 3 = 3
|–8| = – (–8) = + 8 = 8
|+8| = + 8 = 8
|–12| = –(–12) = + 12 = 12

Absolute value of zero is always zero.

Mathematical Operations on Signs :

$$\left.\begin{array}{l} +(+20)=+20 \\ +(-20)=-20 \\ -(+20)=-20 \\ -(-20)=+20 \end{array}\right\}$$

$$\left.\begin{array}{l} (+)\times(+)=+ \\ (+)\times(-)=- \\ (-)\times(+)=- \\ (-)\times(-)=+ \\ (+)\div(+)=+ \\ (+)\div(-)=- \\ (-)\div(+)=- \\ (-)\div(-)=+ \end{array}\right\}$$

..Notes

Solved Examples

Ex.1. *What will be the digit in unit's place in the product of* $7^{35} \times 3^{71} \times 11^{55}$ *?*

Sol. 7 × 7 = 49, 7 × 7 × 7 = 343 and 7 × 7 × 7 × 7 = 2401

∴ Digit in unit's place in 7^4 is 1.

Now, $7^{35} = (7^4)^8 \times 7^3$

Therefore, digit in unit's place in $7^{35} = 1^8.3 = 3$

Similarly $3^3 = 3 \times 3 \times 3 = 27$ and $3^4 = 3 \times 3 \times 3 \times 3 = 81$

$3^{71} = (1)^{17}.7 = 1 \times 7 = 7$

∴ Digit in unit's place in

and in the product of 11 × 11 × 11 . . . . 55 times, the digit in unit's place is 1.

∴ Digit in unit's place in the product of $7^{35} \times 7^{71} \times 11^{55}$ *i.e.* 3 × 7 × 1 = 21 is 1.

Ex.2. *Find the sum of even numbers between 1 and 31 is*

Sol. Required number n e 2, 4, 6 . . . . , 30. This is an A.P containing 15 terms

∴ Required sum = last term)

= $\frac{15}{2}(2+30) = 240$.

Ex.3. *What least value must be assigned to * so that the number* $197*5462$ *is divisible by 9 ?*

Sol. Let the missing digit be *x*.

∴ Sum of digits = (1 + 9 + 7 + *x* + 5 + 4 + 6 + 2)

= (34 + *x*)

For (34 + *x*) to be divisible by 9, *x* must be replaced by 2

Hence the digit in place of * must be 2.

Ex.4. *How many terms are there in 2, 4, 8, 16. . . . 1024 ?*

Sol. Since, 2, 4, 8, 1 . . . 1024 form a G.P. with

a = 2 and *r* = $\frac{4}{2}$ = 2

Let the number of terms be *n*. Then

$$2 \times 2^{n-1} = 1024 \Rightarrow 2^{n-1} = 512 = 2^9$$

∴ $n - 1 = 9$ or $n = 10$

Ex.5. The first odd number is 1, the second odd number is 3, the third odd number is 5 and so on. The 200th odd number is

Sol. First odd number = 1

Second odd number = 3

Third odd number = 5

∴ n th odd number = 1 + (n – 1) 2 = 2n – 1

∴ 200th odd number = 2 × 200 – 1= 400 – 1 = 399

Ex.6. The number of digits in the square root of 625686734489 is

The number of digits in 625686734489 is 12.

$\therefore$ Number of digits in its square root = 6

i.e., $\sqrt{625686734489}$ = 791003.625

Practice Exercise

1. In a three digit number the digit in the unit's place is twice the digit in the ten's place and 1.5 times the digit in the hundred's place. If the sum of all the three digits of the number is 13, what is the number ?

(A) 364 (B) 436 (C) 238 (D) 634

2. Two numbers are less than the third number by 50% and 54% respectively. By how much per cent is the second number less than the first number ?

(A) 13 (B) 10 (C) 12 (D) 8

3. If $(28)^3$ is subtracted from the square of a number, the answer so obtained is 1457. What is the number ?

(A) 127 (B) 136 (C) 142 (D) 153

4. The difference between a number and $\frac{2}{5}$th of the number is 30. The number is

(A) 50 (B) 75 (C) 57 (D) None of these

5. What is the greater of two numbers whose product is 1092 and the sum of the two numbers exceeds their difference by 42?

(A) 48 (B) 44 (C) 52 (D) 54

6. The difference between 58% of a number and 39% of the same number is 247. What is 62% of that number ?

(A) 1,300 (B) 806 (C) 754 (D) 1,170

7. If $(12)^3$ is subtracted from the square of a number, the answer so obtained is 976. What is the number ?

(A) 58 (B) 56 (C) 54 (D) 52

8. If the sum of four consecutive even numbers is 228, which is the smallest of the numbers ?

(A) 52 (B) 54 (C) 56 (D) 4 8

9. The product of two successive positive integers is 462. Which is the smaller integer?

(A) 20 (B) 22 (C) 21 (D) 23

10. The total number of integers between 200 and 400, each of which either begins with 3 or ends with 3 or both, is

(A) 10 (B) 100 (C) 110 (D) 120

11. If *a* and *b* are two distinct natural numbers, which one of the following is true ?

(A) $\sqrt{a+b} > \sqrt{a} + \sqrt{b}$ (B) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

(C) $\sqrt{a+b} < \sqrt{a} + \sqrt{b}$ (D) $ab = 1$

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12. The number obtained by interchanging the digits of a two digit number is less than the original number by 18. If sum of the digits is 6, what is the original two digit number?

(A) 51 (B) 24 (C) 42 (D) 15

13. How many numbers less than 1000 are multiples of both 10 and 13 ?

(A) 9 (B) 8 (C) 6 (D) 7

14. The difference between a two digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number ?

(A) 4 (B) 9 (C) 3 (D) Cannot be determined

15. By how much is two-fifth of 200 greater than three-fifth of 125?

(A) 15 (B) 3 (C) 5 (D) 30

16. The average of four consecutive even numbers is one-fourth of the sum of these numbers. What is the difference between the first and the last number?

(A) 4 (B) 6 (C) 2 (D) Cannot be determined

17. In a party there are 120 members. Two-third of them are men and rest are women. All the members are married except 12 women members. How many married women are there in the club ?

(A) 28 (B) 30 (C) 32 (D) 40

18. If the digits of a two digit number are interchanged, the newly formed number is more than the original number by 18, and sum of the digits is 8; then what is the original number ?

(A) 53 (B) 26 (C) 35 (D) Cannot be determined

19. Difference between a two digit number and the number obtained by interchanging the digit is 27. If the sum of the digit is 7, what is the original number ?

(A) 52 (B) 25 (C) 43 (D) Cannot be determined

20. $\frac{1}{2}$ of $\frac{3}{4}$ of a number is $2\frac{1}{2}$ of 10. What is the number ?

(A) 50 (B) 60 (C) $66\frac{2}{3}$ (D) 56

21. The digit in the unit's place of a number is equal to the digit in the ten's place of half that number and the digit in the ten's place of that number is less than the digit in the unit's place of half of the number by 1. If the sum of the digits of the number is seven, then what is the number ?

(A) 52 (B) 162 (C) 34 (D) Data inadequate

22. A classroom has equal number of boys and girls. Eight girls left to play kho-kho, leaving twice as many boys as girls in the classroom. What is the total number of girls and boys present initially ?

(A) Cannot be determined (B) 16 (C) 24

(D) 32

23. The digits of a two-digit number are in the ratio 2 : 3 and the number obtained by interchanging the digits is bigger than the original number by 27. What was the original number ?

(A) 64 (B) 46 (C) 96 (D) 69

24. The number 0.121212.... in the form $\frac{p}{q}$ is equal to

(A) $\frac{4}{11}$ (B) $\frac{2}{11}$ (C) $\frac{4}{33}$ (D) $\frac{2}{33}$

25. Two natural numbers are in the ratio 3 : 5 and their product is 2160. The smaller of the numbers is

(A) 36 (B) 24 (C) 18 (D) 12

Answer Key

1. (B)	**2. (D)**	**3. (D)**	**4. (A)**	**5. (C)**
6. (B)	**7. (D)**	**8. (B)**	**9. (C)**	**10. (C)**
11. (C)	**12. (C)**	**13. (D)**	**14. (A)**	**15. (C)**
16. (B)	**17. (A)**	**18. (C)**	**19. (A)**	**20. (C)**
21. (A)	**22. (D)**	**23. (D)**	**24. (C)**	**25. (A)**

..Notes

02 LCM and HCF

A number is said to be multiple of another when it is exactly divisible by the other. Similarly, we can define common multiple as the number which is exactly divisible by numbers for whom it is common. For example, 10 is a common multiple for 2 as well as 5, 20, 30, 40 etc. are other common multiples of 2 and 5.

For 2 and 3, common multiples are 6, 12, 18 and so on.

For 4 and 6, common multiples are 12, 24, 36 and so on.

LEAST COMMON MULTIPLE (LCM)

LCM of two or more numbers is the smallest number which is common multiple of the given numbers. In other words, LCM of given numbers is the smallest number which is exactly divisible by each of them.

In the above examples, the LCM for

2 and 5 is 10

2 and 3 is 6

4 and 6 is 12

We can find LCM by two methods.

Method A :

I. Write each of the given numbers as product of prime factors.

II. Find the product of the highest powers of the prime factors which will be the LCM.

Note : Don't repeat any factor while writing the product in Step II.

Ex. 1. Find the LCM of 27 and 63.

Sol. $27 = 3 \times 3 \times 3 = 3^3$

$63 = 3 \times 3 \times 7 = 3^2 \times 7$

(3^3 includes 3^2 and therefore, we will not write 3^2 while finding the product for LCM).

LCM $= 3^3 \times 7 = 189$

Ex. 2. Find the LCM of 54 and 21.

Sol. $54 = 2 \times 3^3$

$21 = 3 \times 7$

LCM $= 2 \times 3^3 \times 7 = 378$

Ex. 3. Find the LCM of 36, 56, 105 and 108.

Sol. $36 = 2^2 \times 3^2$

$56 = 2^3 \times 7$

$105 = 3 \times 5 \times 7$

$108 = 2^2 \times 3^3$

The LCM must contain every prime factors of each of the numbers. Also, it must include the highest power of each prime factor which appears in any of them.

So, it must contain 2^3 or it would not be a multiple of 56, it must contain 3^3 or it would not be a multiple of 108, it must contain 5 or it would not be a multiple of 105, and it must contain 7 or it would not be a multiple of 56 or of 105.

Therefore, the LCM $= 2^3 \times 3^3 \times 5 \times 7 = 7560$

Note : 36 is itself a factor of 108 and hence any multiple of 108 must be a multiple of 36 too. Therefore, we could have struck out 36.

Method B :

This is a quicker method to find the prime factors and hence LCM. In this method there can be more than one arrangements for the same numbers.

I. Write the given numbers in a row and strike out those numbers which are factors of any other number in the Set.

II. Write the factor on the left hand side which can divide maximum of the numbers.

III. Write in the next row the quotients obtained and also those numbers (as they are) which are not divisible by that factor. You can strike out from any row 1, if it appears.

IV. Repeat Steps II and III until we get a row where no two numbers have a common factor or divisor, i.e., all the numbers in the row are prime to each other, though individually they may not be prime numbers.

V. Multiply all the factors/divisors and the numbers left in the last row. The product gives the LCM of the given numbers.

Let us now see how it works and how simple it is.

Ex. 4. Find the LCM of 36, 56, 105 and 108.

Sol.

2	36	56,	105,	108
2		28,	105,	54
3		14,	105,	27
7		14,	35,	9
		2,	5,	9

36 is a factor/submultiple of 108 (36 × 3 = 108) and hence we strike it off. 56 and 108 are divisible by 2. So we write 2 on the left side and perform Step III.

Next factors are 2, 3, and 7.

Thereafter, we find that 2, 5 and 9 left in the last row have no common divisor i.e., 2, 5, 9 are prime to each other, though 9 itself is not a prime number.

So, we find the product of 2, 2, 3, 7, 2, 5 and 9 to get the required LCM.

LCM = 2 × 2 × 3 × 7 × 2 × 5 × 9 = 7560

We may have another arrangement as given below which will save more time. Also, in order to save time we can find the product from the table itself instead of writing multiplication separately as all the numbers to be multiplied lie on the periphery of the table as indicated by arrow below. The basic logic remains the same as explained earlier.

4	36,	56,	105,	108
3		14,	105,	27
7		14,	35,	9
		2,	5,	9

LCM = 4 × 3 × 7 × 2 × 5 × 9 = 7560.

The basic principle followed in both the methods of finding LCM is to ensure that the factors which are common to two or more numbers do not get repeated in the final product.

Note **:** The LCM of a set of numbers which are prime relative to each other is equal to their product. This is so because such a set of numbers do not have any common factor. The numbers individually may not be prime numbers.

Ex. 5. Find the LCM of 3, 4, and 7.

Sol. Here, 4 itself is not a prime number but the three given numbers 3, 4 and 7 are prime to each other as none of them have any factor common to the other.

So, LCM = 3 × 4 × 7 = 84

Ex. 6. Find the LCM of 4, 6 and 7.

Sol. Here, 4 and 6 are not prime to each other as they have 2 as common factor and this should not get repeated in the product for finding LCM. So, we can use either of the two methods to find LCM which ensures non-repetition of such common factors in the product.

Method A : $4 = 2^2$

$6 = 2 \times 3$

$7 = 7$

$\therefore$ LCM = $2^2 \times 3 \times 7 = 84$

Method B :

2	4,	6,	7
	2,	3,	7

$\therefore$ LCM = 2 × 2 × 3 × 7 = 84

LCM OF MORE THAN TWO NUMBERS

LCM of three numbers is equal to the LCM of (LCM of any two numbers the remaining third number).

For four numbers this can be extended to LCM of (LCM of any three numbers and fourth number) and so on.

We can also break four numbers in two subgroups of two numbers each and take their LCMs separately and thereafter, we take the LCM of these two LCMs.

Ex. 7. Find the LCM of 36, 56, 105 and 108.

Sol. Method A :

$36 = 2^2 \times 3^2$

$56 = 2^3 \times 7$

$105 = 3 \times 5 \times 7$

$108 = 2^2 \times 3^3$

Let us take 36 and 56 first.

LCM of 36 and 56 :

$LCM_2 = 2^3 \times 3^2 \times 7 = 504$

Now LCM of LCM_2 and 105 :

$LCM_3 = 2^3 \times 3^2 \times 5 \times 7 = 2520$

LCM of LCM_3 and 108 :

$LCM_4 = 2^3 \times 3^3 \times 5 \times 7 = 7560$

So, LCM of 36, 56, 105 and 108 = 7560

Let us now try to find the LCM of above numbers by breaking them into two subgroups.

Subgroup-1	**Subgroup-2**
36, 56	105, 108
$LCM_1 = 2^3 \times 3^2 \times 7$	$LCM_2 = 2^2 \times 3^3 \times 5 \times 7$

LCM of all four numbers = LCM of (LCM_1 and LCM_2)

$= 2^3 \times 3^3 \times 5 \times 7$

LCM = 7560

Method B :

$$\begin{array}{r|ll} 4 & 36, & 56 \\ \hline & 9, & 14 \end{array}$$

$LCM_2 = 4 \times 9 \times 14 = 504$

LCM_3 = LCM of 504 and 105

$$\begin{array}{r|ll} 3 & 504, & 105 \\ \hline 7 & 168, & 35 \\ \hline & 24, & 5 \end{array}$$

$LCM_3 = 3 \times 7 \times 24 \times 5 = 2520$

LCM_4 = LCM of 2520 and 108

$$\begin{array}{r|ll} 4 & 2520, & 108 \\ \hline 3 & 630, & 27 \\ \hline 3 & 210, & 9 \\ \hline & 70, & 3 \end{array}$$

LCM = $4 \times 3 \times 3 \times 70 \times 3$

LCM = 7560

Now, let us break the four numbers again into two subgroups.

Subgroup-1

$$\begin{array}{r|ll} 4 & 36, & 56 \\ \hline & 9, & 14 \end{array}$$

$LCM_1 = 4 \times 9 \times 14 = 504$

Subgroup -2

$$\begin{array}{r|ll} 3 & 105, & 108 \\ \hline & 35, & 36 \end{array}$$

$LCM_2 = 3 \times 35 \times 36 = 3780$

LCM of 504 and 3780

4	504,	3780
9	126,	945
7	14,	105
	2,	15

LCM = 4 × 9 × 7 × 2 × 15 = 7560

HIGHEST COMMON FACTOR (HCF)

HCF of any given set of numbers is the greatest (highest) factor common to them. It is also known as Greatest Common Factor (GCF) or Greatest Common Measure (GCM) or Highest Common Divisor (HCD).

HCF too can be found by two methods.

Method A : By Factorisation

I. Write each number as product of its prime factors.

II. Multiply the factors with lowest powers which are common to all numbers (Only such factors can divide all the given numbers).

HCF is equal to the product of such factors.

Ex. 8. Find the HCF of 126, 396 and 5400.

Sol. I. $126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$

$396 = 2 \times 2 \times 3 \times 3 \times 11 = 2^2 \times 3^2 \times 11$

$5400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 = 2^3 \times 3^3 \times 5^2$

II. The factors common to all the given numbers are 2 and 3 and their lowest powers are 1 (2^1) and 2 (3^2) respectively. Only the product of 2 and 3^2 can divide all the given numbers.

HCF = $2 \times 3^2 = 18$

2 × 3 = 6 is also a factor common to all the numbers i.e., all given numbers are divisible by 6 but it is less than 18 and hence is not the Highest Common Factor (HCF).

Note : It is not always necessary to factorise all the numbers into prime factors. Factorise only the smallest number and check which of its factors can divide all the other numbers. HCF will be equal to the product of such common divisors.

Ex. 9. Find the HCF of 360, 440, 680, 5000 and 720.

Sol. 360 = 10 × 4 × 9

Now, we see that all the other numbers are divisible by 10. They are also divisible by 4. But except 720 other numbers i.e., 440, 680 and 5000 are not divisible by 9.

So, we have only 10 and 4 as common divisors to all the given numbers.

∴ HCF = 10 × 4 = 40

You can check up that the quotients obtained on dividing the numbers by HCF, are prime to each other.

Once again we would like to remind that the numbers prime to each other do not imply that such numbers are prime numbers individually. It only means that they do not have any common factor.

Method B : By Division

This method for finding HCF of given numbers is based on the following two facts :

(*i*) If a given number is divisible by another number (divisor), then the multiples of the given number will also be divisible by that divisor. In other words, if a given number is divisible by the other (divisor), then the divisor is a factor/submultiple of the given number and any multiple of the given number shall always contain such a divisor as a factor. It means that the multiple of the given number is also divisible by that divisor.

For example, 6 is divisible by 2. Then all multiples of 6 such as 12,18, 24 .... will be also divisible by 2.

(*ii*) Any number which divides each of the two numbers also divides their sum, their difference and also the sum and difference of their multiples. In other words, a factor common to two numbers is also common to their sum, their difference and also to the sum and difference of their multiples. This follows basically from the distributive law which we will discuss in detail in later chapter. Here, we will mention it only briefly. Let *'A'* be the factor common to the numbers

B and C and the multiplication factors be p and q. So, $(p \times B)$and $(q \times C)$ will be multiples of B and C respectively.

Distributive Laws :

$A \times (B + C) = (A \times B) + (A \times C)$

$A \times (B - C) = (A \times B) - (A \times C)$

$A \times (pB + qC) = A \times (pB) + A \times (qC)$

$A \times (pB - qC) = A \times (pB) - A \times (qC)$

You can take any example and check it.

To find the HCF of two numbers proceed as follows:

I. Divide the greater number by the smaller number.

II. Next, divide the preceding divisor by the remainder. Continue until you get zero as remainder. The last divisor is the required HCF.

Ex. 10. Find the HCF of 45 and 108.

Sol.

```
45 ) 108 (2
     90
     18) 45 ( 2
         36
          9) 18 (2
             18
              ×
```

∴ Required HCF = 9

Ex. 11. Find the HCF of 568 and 2569.

Sol.

```
568 ) 2569 (4
      2272
       297) 568 (1
            297
             271) 297 (1
                  271
                   26) 271(10
                       260
                        11) 26 (2
                            22
                             4) 11 (2
                                 8
                                 3) 4 (1
                                    3
                                    1) 3 (3
                                       3
                                       ×
```

∴ Required HCF = 1

Note : HCF 1 implies that the given numbers have no common factor except 1.

The given numbers are prime to each other.

Ex. 12. Find the HCF of 110448 and 244024.

Sol.

```
110448 ) 244024 (2
         220896
          23128) 110448 (4
                  92512
                  17936) 23128 (1
                         17936
                          5192) 17936 (3
                                15576
                                 2360) 5192 (2
                                       4720
                                        472) 2360 (5
                                             2360
                                               ×
```

∴ Required HCF = 472

Alternatively, 110448 = 16 × 9 × 767

244024 = 8 × 11 × 2773

In the above example, 9 and 11 are clearly prime to each other and hence can be struck out.

HCF of 16 and 8 is 8 ...(*A*)

Now, we find the HCF of 767 and 2773.

767) 2773 (3
2301
472) 767 (1
472
295) 472 (1
295
177) 295 (1
177
118) 177 (1
118
59) 118 (2
118
×

HCF of 767 and 2773 is 59. ...(B)

The HCF of 110448 and 244024 will be given by
(A) × (B). HCF = 8 × 59 = 472

HCF OF MORE THAN TWO NUMBERS

When we have more than two such numbers which cannot be easily written as product of factors, we proceed to first find the HCF of any two of the given numbers and then the HCF of the result and a third number and so on. The final HCF is the required HCF for the given numbers.

Ex. 13. Find the HCF of 48, 168, 324 and 1400.

Sol.

48) 168 (3
144
24) 48 (2
48
×

$(HCF)_1$ = 24

Next, we find the HCF of 24 and 324.

24) 324 (13
24
84
72
12) 24 (2
24
×

$(HCF)_2$ = 12

Now, we find the HCF of 12 and 1400.

12) 1400 (116
12
20
12
80
72
8) 12 (1
8
4) 8 (2
8
×

Thus, the HCF of 48, 168, 324 and 1400 = 4

RULES FOR SIMPLIFICATION FOR FINDING HCF

(*i*) Any easily recognisable common factors may be taken out which may be multiplied, in the end, with the HCF of the quotients.

(*ii*) All prime factors which are **not** common to all can be rejected altogether.

Let us take an example just to illustrate the above rules.

Ex. 14. Find the HCF of 980, 1240 and 5304.

Sol. 980 = 2 × 7 × 70

1240 = 2 × 10 × 62

5304 = 2 × 3 × 884

(*i*) We can take **2** as the common factor.

(*ii*) We can reject 7, 10 and 3 as they are not common to all the numbers.

Now, we have to find the HCF of 70, 62 and 884.

```
62 ) 70 (1
     62
     ---
      8 ) 62 (7
          56
          ---
           6 ) 8 (1
               6
               ---
               2 ) 6 (3
                   6
                   ---
                   0
```

∴ Required HCF of 62 and 70 is 2

Now, HCF of 2 and 884 is obviously 2.

So, the final HCF will be the product of the last HCF and the common factor of step (*i*).

∴ Required HCF = 2 × 2 = 4

You can check it up by finding directly the HCF of 980, 1240 and 5304.

HCF AND LCM OF DECIMAL NUMBERS

If the given set of numbers has decimal fractions, then first of all make the same numbers of places of decimals by suffixing zeros, if necessary. Thereafter, treat these numbers without decimal point and find HCF/LCM as required. In the result put the decimal point leaving as many digits on its right as there are in each of the numbers.

Ex. 15. Find the HCF of 12.05 and 13.5.

Sol. 12.05 has two digits after the decimal point whereas 13.5 has only one digit. Therefore, we suffix a zero on 13.5 to make two places of decimals i.e., 13.50.

Now, we ignore the decimal point and find the HCF of 1205 and 1350.

```
1205 ) 1350 (1
       1205
       ----
        145 ) 1205 (8
              1160
              ----
                45 ) 145 (3
                     135
                     ---
                      10 ) 45 (4
                           40
                           --
                            5 ) 10 (2
                                10
                                --
                                 0
```

We get 5 as HCF of 1205 and 1350.

Since we started with two decimal places, we will convert 5 also into the same form i.e., 0.05

∴ Required HCF = 0.05

Ex. 16. Find the LCM of 14.25 and 5.775

Sol. 5.775 has three decimal places. Therefore, we will write 14.25 as 14.250. Now, ignoring the decimal point we find the LCM of 5775 and 14250.

25	5775,	14250
3	231,	570
	77,	190

25 × 3 × 77 × 190 = 1097250

∴ Required LCM = 1097.250

LCM AND HCF OF FRACTIONS

First of all we must reduce fractions to their lowest terms and also convert mixed numbers into fractions. Then we can use the following relations.

LCM of Fractions $= \dfrac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$

and HCF of Fractions $= \dfrac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$

These relations are easy to remember but don't get confused. In the numerator we write (LCM/HCF) that we have to find out and in the denominator the other one.

Note : LCM of fractions may be a fraction or an integer but the HCF of fractions is always a fraction. This is explained as follows.

We need to do two things before we apply the above relations.

I. The fractions must be reduced to their lowest terms. For example, $\frac{10}{14}$ must be reduced $\frac{5}{7}$ which is its lowest term i.e., the factors common to the numerator and the denominator have to be cancelled out.

II. Next, we convert mixed number into fraction.

For example, $3\frac{1}{4} = \frac{13}{4}$, $4\frac{1}{2} = \frac{9}{2}$

Ex. 17. Find the LCM of $3\frac{1}{4}, \frac{3}{4}, 9\frac{3}{5}$ and $4\frac{6}{9}$.

Sol. $4\frac{6}{9} = 4\frac{2}{3} = \frac{14}{3}$; $9\frac{3}{5} = \frac{48}{5}$; $3\frac{1}{4} = \frac{13}{4}$ and $\frac{3}{4}$

LCM of Numerators 14, 48, 13 and 3 :

$$\begin{array}{r|rrrr} 2 & 14, & 48, & 13, & 3 \\ \hline 3 & 7, & 24, & 13, & 3 \\ \hline & 7, & 8, & 13, & 1 \end{array}$$

∴ LCM= 2 × 3 × 7 × 8 × 13 = 4368

HCF of Denominators 3, 5, 4, 4 :

We can clearly see that there is no factor which is common to all. Hence, HCF will be 1.

LCM $= \dfrac{\text{LCM of Numerators}}{\text{HCF of Denominators}} = \dfrac{4368}{1}$

∴ Required LCM = 4368

Ex. 18. Find the LCM of $\frac{7}{4}, \frac{3}{2}$ and $\frac{1}{6}$.

Sol. LCM of Numerators 7, 3 and 1 will be 7 × 3 × 1 = 21

We have simply multiplied the numerators here because all of them are prime numbers and hence can't have any common factor which needs to be taken care of.

HCF of Denominators 4, 2 and 6 will be 2.This is clear as there is no other factor common to all.

∴ Required LCM $= \frac{21}{2} = 10\frac{1}{2}$

Ex. 19. Find the HCF of $3\frac{24}{46}$ and $4\frac{18}{92}$.

Sol. $3\frac{24}{46} = 3\frac{12}{23} = \frac{81}{23}$; $4\frac{18}{92} = 4\frac{9}{46} = \frac{193}{46}$

HCF of numerators 81 and 193 is 1.

LCM of denominators 23 and 46 is 46.

$\therefore$ Required HCF $= \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}} = \frac{1}{46}$

Ex. 20. Find the HCF of $\frac{124}{136}$ and $\frac{186}{36}$.

Sol. $\frac{124}{136} = \frac{31}{34}$; $\frac{186}{36} = \frac{93}{18}$

HCF of Numerators 31 and 93 is obviously 31 as 93 is a multiple of 31.

LCM of Denominators 34 and 18

$34 = 2 \times 17$ and $18 = 2 \times 9$

LCM $= 2 \times 17 \times 9 = 306$

$\therefore$ Required HCF $= \frac{31}{306}$

In the above examples LCM of fractions can be a fraction as well as an integer but HCF of fractions is always a fraction. This is so because LCM of two numbers is always greater than 1 unless both the numbers are 1; in this case the numbers can't be called fractions. And HCF of two fractions has in its denominator the LCM of denominators of the fraction.

Contrary to this, the HCF of two numbers may or may not be 1 and hence the LCM of fractions may or may not be integers.

Note : If the given set of numbers includes fractions as well as whole numbers, treat whole numbers too as fractions with 1 in their denominators.

Ex. 21. Find the HCF of $\frac{2}{5}$, 12 and $\frac{32}{49}$.

Sol. Take 12 as

HCF $= \frac{\text{HCF of } 2, 12 \text{ and } 32}{\text{LCM of } 5, 1 \text{ and } 49} = \frac{2}{5 \times 49} = \frac{2}{245}$

Solved Examples

Ex. 1. Find the LCM of $\frac{2}{5}$, 8 and $\frac{3}{25}$.

Sol. LCM $= \frac{\text{LCM of } 2, 8 \text{ and } 3}{\text{HCF of } 5 \text{ and } 25} = \frac{8 \times 3}{5}$

$\therefore$ Required LCM $= \frac{24}{5}$

Now let us move from numbers to quantities with their units given in the problem. How do we find out HCF, LCM of quantities expressed in different units?

Note : Before we do any mathematical operation, whether it is LCM, HCF, or addition, subtraction we must ensure that all quantities are expressed in the same unit. Otherwise, performing these operations will be meaningless.

For example, 1 m 40 cms, 21m 60 cms must be first converted to 140 cms and 2160 cms or 1.40 m and 21.60 m respectively.

Similarly, kg and gms or hour, minute and second and so on should be converted into the same unit.

Further, two quantities of different measures can't be combined. For instance, we can't perform above operations where one quantity is in cms and another in gms.

The most important property of LCM and HCF of two numbers is that their product is equal to the product of the two numbers.

Product of any two numbers = (LCM × HCF) of the two numbers.

Ex. 2. For 6 and 8 find LCM and HCF and check whether the above relation holds good.

Sol. LCM of 6 and 8 is 24.

HCF of 6 and 8 is 2.

LCM × HCF = 24 × 2 = 48

Product of 6 and 8 = 6 × 8 = 48

Hence, the above relation holds good.

Ex. 3. LCM of 852 and 1491 is 5964. Find the HCF.

Sol. Since the LCM of the two numbers is already given, we need not calculate the HCF by fundamental method. Instead, we will use the relation.

HCF × LCM = Product of the two numbers

$$\text{HCF} = \frac{\text{Product of the two numbers}}{\text{LCM}}$$

$$= \frac{852 \times 1491}{5964}$$

∴ Required HCF = 213

Ex. 4. HCF of 2873 and 5083 is 221. Find their LCM.

Sol. LCM $= \frac{\text{Product of the numbers}}{\text{HCF}} = \frac{2873 \times 5083}{221}$

LCM = 66079

Ex. 5. The HCF of two numbers is 4 and their LCM is 576. If one of the numbers is 64, find the other number.

Sol. Let the unknown number be x.

We know the relation,

Product of two numbers

= (HCF × LCM) of the two numbers

or, $64 \times x = 4 \times 576$

Divide both sides by 64,

or, $\frac{64 \times x}{64} = \frac{4 \times 576}{64}$

or, $x = 36$

∴ The other number is 36.

Ex. 6. Reduce the following fractions to their lowest terms.

(*i*) $\frac{42}{72}$ (*ii*) $\frac{18}{63}$ (*iii*) $\frac{205}{495}$ (*iv*) $\frac{147}{259}$

(*v*) $\frac{117}{243}$

Sol. (*i*) $\frac{42}{72} = \frac{2 \times 3 \times 7}{2 \times 3 \times 12} = \frac{7}{12}$ (*ii*) $\frac{18}{63} = \frac{9 \times 2}{9 \times 7} = \frac{2}{7}$

(*iii*) $\frac{205}{495} = \frac{5 \times 41}{5 \times 99} = \frac{41}{99}$ (*iv*) $\frac{147}{259} = \frac{7 \times 21}{7 \times 37} = \frac{21}{37}$

(*v*) $\frac{117}{243} = \frac{9 \times 13}{9 \times 27} = \frac{13}{27}$

Ex. 7. Pick out the correct answer from amongst the given alternatives.

(*i*) LCM of fractions is
(A) an integer (B) a fraction
(C) either (A) or (B) (D) Data inadequate

(*ii*) HCF of fraction is
(A) an integer (B) a fraction
(C) either (A) or (B) (D) Data inadequate

(*iii*) LCM of fractions is an integer when
(A) LCM of numerators is unity
(B) LCM of denominators is unity
(C) HCF of numerators is unity
(D) HCF of denominators is unity.

(*iv*) LCM of fractions is a fraction when
(A) LCM of numerators is greater than 1.
(B) LCM of denominators is greater than 1.
(C) HCF of numerators is greater than 1.
(D) HCF of denominators is greater than 1.

(*v*) HCF of fractions is a fraction because
(A) LCM of numerators is always greater than 1.
(B) LCM of denominators is always greater than 1.
(C) HCF of numerators is always greater than 1.
(D) HCF of denominators is always greater than 1.

Answers : (*i*) C (*ii*) B (*iii*) D
(*iv*) D (*v*) B

Ex. 8. Arrange the following fractions in ascending order :

$$\frac{2}{7}, \frac{5}{9}, \frac{1}{4}, \frac{2}{3}, \frac{5}{6}$$

Sol. First take the LCM of the denominators.

$$\begin{array}{c|ccccc} 2 & 7, & 9, & 4, & 3, & 6 \\ \hline 3 & 7, & 9, & 2, & 3, & 3 \\ \hline & 7, & 3, & 2, & 1, & 1 \end{array}$$

LCM = 3 × 4 × 7 × 3 = 252

Now, express all the fractions with common denominator equal to the LCM.

$$\frac{(2\times36),(5\times28),(1\times63),(2\times84),(5\times42)}{252}$$

$$= \frac{72, 140, 63, 168, 210}{252}$$

Now, arrange the numerators in the ascending order which will is also true for the fractions as all of them now have equal denominator.

63, 72, 140, 168, 210 :

∴ **Answer :** $\frac{1}{4} < \frac{2}{7} < \frac{5}{9} < \frac{2}{3} < \frac{5}{6}$

Ex. 9. Express $\frac{7}{15}$ and $\frac{5}{18}$ with common denominator 90 and find which of the two fractions is smaller.

Sol. $\frac{7}{15} = \frac{7\times6}{15\times6} = \frac{42}{90}$; $\frac{5}{18} = \frac{5\times5}{18\times5} = \frac{25}{90}$

Since, the two fractions now have common denominator (i.e., 90), the one with smaller numerator will be the smaller fraction.

25 < 42

$\therefore$ **Answer** = $\frac{5}{18}$

Ex. 10. Arrange the following fractions in descending order.

$$\frac{1}{6}, \frac{2}{15}, \frac{3}{10}, \frac{5}{21}, \frac{9}{14}.$$

Sol. LCM of the denominators

2	6,	15,	10,	21,	14
3	3,	15,	5,	21,	7
5	1,	5,	5,	7,	7
7		1,	1,	7,	7
				1,	1

LCM = 2 × 3 × 5 × 7 = 210

Rewrite the fractions with common denominator equal to the LCM 210.

$$\frac{(1\times35),(2\times14),(3\times21),(5\times10),(9\times15)}{210} = \frac{35, 28, 63, 50, 135}{210}$$

Rearrange the numerators in the descending order which will also be the order for the given fractions as they now have a common denominator.

$$\frac{135, 63, 50, 35, 28}{210}$$

Answer : $\frac{9}{14} > \frac{3}{10} > \frac{5}{21} > \frac{1}{6} > \frac{2}{15}$.

Note : Instead of writing LCM as 210, you should prefer to write it as 2 × 3 × 5 × 7. This will help you in writing the numerators easily.

$$\frac{(5\times7),(2\times2\times7),(3\times3\times7),(5\times2\times5),(9\times3\times5)}{(2\times3\times5\times7)}$$

Ex. 11. Find the smallest number which when increased by 5 is divisible by 9, 21, 25 and 30.

Sol. LCM of the given numbers.

3	9,	21,	25,	30
5	3,	7,	25,	10
	3,	7,	5,	2

LCM = 3 × 5 × 3 × 7 × 5 × 2 = 3150

By definition, LCM of a given set of numbers is their smallest common multiple. In other words, it is the smallest numbers which is divisible by all the given numbers.

So, to find the answer we will simply deduct 5 from the LCM.

3150 – 5 = 3145

Ex. 12. Find the smallest number which when reduced by 3 is divisible by 21, 25, 27 and 35.

Sol.

3	21,	25,	27,	35
5	7,	25,	9,	35
7	7,	5,	9,	7
	1,	5,	9,	1

LCM = 3 × 5 × 7 × 5 × 9 × 1 = 4725

..Notes

Answer : 4725 + 3 = 4728

Ex. 13. Find the smallest number which when added to 172 leaves 3 as remainder in each case when divided by 4, 6, 9 and 15 respectively.

Sol. First find the LCM of the divisors

$$\begin{array}{c|cccc} 2 & 4, & 6, & 9, & 15 \\ \hline 3 & 2, & 3, & 9, & 15 \\ \hline & 2, & 1, & 3, & 5 \end{array}$$

LCM = 2 × 3 × 2 × 3 × 5 = 180

Now, 180 will be exactly divisible by 4, 6, 9 and 15. To get 3 as remainder the dividend must be (180 + 3) = 183.

Therefore, we must add (183 – 172) = 11 to 172.

∴ **Answer** = 11.

Practice Exercise

1. The sum of two numbers is 45. Their difference is $\frac{1}{9}$ of their sum. Their L.C.M. is

(A) 200 (B) 250 (C) 100 (D) 150

2. The largest number of five digits which, when divided by 16, 24, 30, or 36 leaves the same remainder 10 in each case, is :

(A) 99279 (B) 99370 (C) 99269 (D) 99350

3. Two numbers are in the ratio 4 : 5 and their L.C.M. is 180. The smaller number is:

(A) 9 (B) 15 (C) 36 (D) 45

4. The H.C.F. of two numbers, each having three digits , is 17 and their L.C.M. is 714. The sum of the numbers will be :

(A) 289 (B) 391 (C) 221 (D) 731

5. The smallest positive integer *n*, for which 864*n* is a perfect cube, is :

(A) 1 (B) 2 (C) 3 (D) 4

6. The number nearest to 43582 divisible by each of 25, 50 and 75 is :

(A) 43500 (B) 43650 (C) 43600 (D) 43550

17. The H.C.F and L.C.M of two numbers are 12 and 336 respectively. If one of the numbers is 84, the other is

(A) 36 (B) 48 (C) 72 (D) 96

8. If the HCF of two numbers (each greater than 13) be 13 and LCM 273, then the sum of the numbers will be

(A) 288 (B) 290 (C) 130 (D) 286

9. The sum of the H.C.F. and L.C.M of two number is 680 and the L.C.M. is 84 times the H.C.F. If one of the numbers is 56, the other is :

(A) 84 (B) 12 (C) 8 (D) 96

10. The LCM of two numbers is 1820 and their HCF is 26. If one number is 130, then the other number is

(A) 70 (B) 1690 (C) 364 (D) 1264

11. Find the greatest number of five digits which when divided by 3, 5, 8, 12 have 2 as remainder.

(A) 99999 (B) 99958 (C) 99960 (D) 99962

12. What is the LCM of $(x + 5)^2$ $(x + 1)^3$ $(x - 3)$ and $(x + 1)$ $(x - 3)^2$ $(x + 5)$?

(A) $(x + 5)(x + 1)(x - 3)$ (B) $(x + 5)^2 (x + 1)(x - 3)$

(C) $(x + 5)(x - 3)$ (D) $(x + 5)^2 (x + 1)^3 (x - 3)^2$

13. The GCD of 1.08, 0.36 and 0.9 is

(A) 0.03 (B) 0.9 (C) 0.18 (D) 0.108

14. 4 bells ring at intervals of 30 minutes, 1 hour, $1\frac{1}{2}$ hours and 1 hour 45 minutes respectively. All the bells ring simultaneously at 12 noon. They will again ring simultaneously at

(A) 12 mid night (B) 3 a.m. (C) 6 a.m. (D) 9 a.m.

15. The HCF of 1.75, 5.6 and 7 is

(A) 0.07 (B) 0.7 (C) 3.5 (D) 0.35

16. A gardener wants to plant trees in a garden. If the number of trees in each row is the same and there are 35 or 14 or 21 rows, then no tree is left. Find the least number of trees the planter has.

(A) 280 (B) 350 (C) 140 (D) 210

17. The GCD of $\frac{3}{16}, \frac{5}{12}, \frac{7}{18}$ is

(A) $\frac{105}{48}$ (B) $\frac{1}{144}$ (C) $\frac{1}{48}$ (D) $\frac{105}{4}$

18. The LCM and the HCF of the numbers 28 and 42 are in the ratio

(A) 6 : 1 (B) 2 : 3 (C) 3 : 2 (D) 7 : 2

19. HCF of 3240, 3600 and a third number is 36 and their LCM is $2^4 \times 3^5 \times 5^2 \times 7^2$. The third number is

(A) $2^2 \times 5^3 \times 7^2$ (B) $2^2 \times 3^5 \times 7^2$ (C) $2^3 \times 3^5 \times 7^2$ (D) $2^5 \times 5^2 \times 7^2$

20. The sum of two numbers is 216 and their HCF is 27. How many pairs of such numbers are there?

(A) 1 (B) 2 (C) 3 (D) 0

21. The product of the LCM and the HCF of two numbers is 24. If the difference of the numbers is 2, then the greater of the number is

(A) 3 (B) 4 (C) 6 (D) 8

22. Two numbers are in the ratio 3 : 4. If their LCM is 240, the smaller of the two number is

(A) 100 (B) 80 (C) 60 (D) 50

23. The HCF and product of two numbers are 15 and 6300 respectively. The number of possible pairs of the numbers is

(A) 4 (B) 3 (C) 2 (D) 1

24. The product of two numbers is 4107. If the HCF of the numbers is 37, the greater number is

(A) 185 (B) 111 (C) 107 (D) 101

25. The LCM of two numbers is 12 times their HCF. The sum of the HCF and the LCM is 403. If one of the numbers is 93, then the other number is

(A) 124 (B) 128 (C) 134 (D) 138

Answers Key

1. (C)	2. (B)	3. (C)	4. (C)	5. (B)
6. (B)	7. (B)	8. (C)	9. (D)	10. (C)
11. (D)	12. (D)	13. (C)	14. (D)	15. (A)
16. (D)	17. (B)	18. (A)	19. (B)	20. (B)
21. (C)	22. (C)	23. (C)	24. (B)	25. (A)

03 Probability

Basic Terms

Experiment : An operation which results in some well defined outcomes is called an experiment.

Random Experiment : An experiment whose outcome cannot be predicted with certainty is called a random experiment. In other words, if an experiment is performed many times under similar conditions and the outcome each time is not the same this experiment is called a random experiment.

Sample Space : The set of all possible outcomes of a random experiment is called the sample space for that experiment; it is usually denoted by S.

Sample Point or Event Point : Each element of the sample space is called a sample point or an event point.

Discrete Sample Space : A sample space S is called a discrete sample space if S is a finite set.

Events : A subset of the sample space S is called an event.

Simple Events or Elementary Events : An event is called a simple event if it is a singleton subset of the sample space S.

Mixed Events or Compound Events : A subset of the simple space S which contains more than one element is called mixed events.

Trial : When an experiment is repeated under similar condition and it does not give the same result each time but may result in any one of the several possible outcomes the experiment is called a trial and the outcomes are called Cases. The number of times the experiment is repeated is called the number of trials.

Mutually Exclusive Events : Two or more events are said to be mutually exclusive if one of them occurs, other cannot occur.

Events $A_1, A_2, \ldots, A_n$ are mutually exclusive if and only if $A_i \cap A_j = \phi$ for $i \neq j$.

Independent Events : Two or more events are said to be independent or mutually independent if the probability of occurrence or non-occurrence of any of them does not change by occurrence or non-occurrence of other events.

Key Points

1. If E_1 and E_2 are any two events associated with an experiment, then
   $P(E_1 \cup E_2) \leq (E_1) + P(E_2)$
2. If $E_1, E_2, E_3, \ldots, E_n$ be n independent events (associated with an experiment) with respective probabilities $P_1, P_2, \ldots, P_n$, then P(at least one of $E_1, E_2, \ldots, E_n$ occurs)
   $= P(E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n)$
   $= 1 - P(E_1')\, P(E_2') \ldots P(E_n')$
   $= 1 - (1 - P_1)(1 - P_2) \ldots (1 - P_n)$

3. If E_1, E_2 and E_3 be any three events associated with an experiment, then
$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) -$
$P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)$

4. If E_1 and E_2 are independent events, then
(A) $P(E_1$ and E_2 occur$) = P(E_1)\,P(E_2)$
(B) $P(E_1$ or E_2 occurs$)$
$= P($at least one of E_1 and E_2 occurs$)$
$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$= P(E_1) + P(E_2) - P(E_1) \cap P(E_2)$
(C) $P($neither E_1 nor E_2 occurs$)$
$= P((E_1 \cup E_2)') = P(E_1' \cap E_2') = P(E_1')\,P(E_2')$
$= (1 - P(E_1))\,(1 - P(E_2))$

5. **Probability of occurence of an event :** Let S be the sample space and E be an event. Then the probability (or chance) of occurrence of the event E is denoted by P(E) and is defined as

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{\text{Number of cases favourable to event E}}{\text{Total number of cases}}$$

clearly, $0 \le P(E) \le 1$
$P(E) = 0 \quad \Leftrightarrow \quad E = \phi$ and $P(E) = 1$, $E = S$

6. **Odd in favour and odds of an event E**

(A) Odds in favour of an event $E = \dfrac{n(E)}{n(E')}$

$$= \frac{\text{number of cases favourable to event E}}{\text{number of cases against E}}$$

(B) Odds against an event

$$E = \frac{n(E')}{n(E)} = \frac{\text{number of cases against E}}{\text{number of cases in favour of E}}$$

(C) Odds in favour of $E = \dfrac{P(E)}{P(E')}$

(D) Odds against $E = \dfrac{P(E')}{P(E)}$

[**Note :** If any one of P(E), odds in favour of E and odds against E, is given other can be determined.]

7. **Some symbols :** Let A and B be any two events, Then
(A) $A \cup B$ denotes the events of occurrence of at least one of the events A and B.
(B) $A \cap B$ denotes the event of occurrence of both the events A and B.
(C) P(A/B) denotes the probability of occurrence of event A when B has already occurred.

$P(A/B) = \dfrac{P(A \cap B)}{P(B)}$, when $B \ne \phi$

Theorem 1: The probability of happening of an event in one trial being known, to find the probability of its happening in exactly 1,2,3,.......*r* times in *n* trials = ${}^nC_r p^r q^{n-r}$

Theorem 2: The probability that an event would happen at least *r* times in *n* trials is
$p^n + {}^nC_1 p^{n-1} q + {}^nC_2 p^{n-2} q^2 + {}^nC_3 p^{n-3} q^3 + \ldots {}^nC_r p^r q^{n-r}$

Theorem 3: The probability of simultaneous occurrence of two events A and B is equal to the probability of A multiplied by the conditional probablity of B given that A has occurred.

P(AB) = P(A) P (B/A) = P(B) P(A/B)

Theorem 4: If $A_1 \subset A_2$, then

$P(A_1) \le P(A_2)$ and $P(A_2 - A_1) = P(A_2) - P(A_1)$

Theorem 5: If A, B be mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

Theorem 6: For any two events A and B,

$P(A–B) = P(A) - P(A \cap B)$

Baye's Theorem 7: If an event A is known to occur together with one of the events A_1, A_2,..........., A_n forming a mutually exclusive and collectively exhaustive events (*i.e.,* A_1, A_2,............... A_n) form a partition of the sample space, then

$$P(A_k/A)=\frac{P(A_k)P(A/A_k)}{P(A_1)P(A/A_1)+P(A_2).P(A/A_2)+......}$$

$+P(A_n).\ P(A/A_n)$

Bayer's theorm can be used only when it is known that an event A has occured and we have to find due probability of occurrence of another event A_k such that events A_1, A_2,............. A_k..............A_n are mutually exclusive and exhaustive they are mutually exclusive and exhaustive they are mutually exclusive and cover all possible cases of an experiment and A is knwon to occur with one of A.

Solved Examples

Example 1. A random variable X has Poisson distribution with mean 2. Then P(X > 1.5) equals

(A) $1-\frac{3}{e^2}$ (B) $\frac{3}{e^2}$ (C) $\frac{2}{e^2}$ (D) 0

Explanation: (A): $P(r)=\frac{\lambda^r e^{-\lambda}}{\lfloor r}$

$$P(x>1.5)=1-\{P(x=0)+P(x=1)\} \quad =1-\left\{e^{-2}+\frac{2e^{-2}}{\lfloor 1}\right\}=1-\left(\frac{1}{e^2}+\frac{2}{e^2}\right)=1-\frac{3}{e^2}$$

Example 2. An arrangement of the letters of the word "ARRANGE" is formed at random, the probability that neither two R's nor two A's occur together is

(A) $\frac{900}{1260}$ (B) $\frac{660}{1260}$ (C) $\frac{540}{1260}$ (D) None of these

Explanation: (B): Total number of arrangements $=\frac{\lfloor 7}{2!\ 2!}=1260$

Number of arrangements when neither two R's nor two A's occur together

$$=1260-\frac{\lfloor 6}{\lfloor 2}-\frac{\lfloor 6}{\lfloor 2}+\lfloor 5=660$$

Example 3. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow the Poisson distribution with an average of 5 phone calls during 10-minute time interval. The probability that there is at the most one phone call during a 10-minute time period is

(A) $\frac{6}{e^5}$ (B) $\frac{6}{5^e}$ (C) $\frac{5}{6}$ (D) $\frac{6}{55}$

Explanation: (A): Required probability

= P(X = 0) + P(X = 1)

..Notes

$= \frac{e^{-5}}{0!}.5^0 + \frac{e^{-5}}{1!}.5^1 = 6.e^{-5} = \frac{6}{e^5}$

Example 4. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is equal to

(A) 1.4 (B) 0.8 (C) 1.2 (D) 1.4

Explanation: (B): Here P(A ∪ B) = 0.6, P(A ∩ B) = 0.2

Using the fact that

P(A ∪ B) = P(A) + P(B) − P(A ∩ B), we get,

$0.6 = \left[1 - P(\overline{A})\right] + \left[1 - P(\overline{B})\right] - 0.2$

or $P(\overline{A}) + P(\overline{B}) = 1.2$

Practice Exercise

1. The chance that a person aged *m* years dies in a year is P. The probability that out of *n* persons $P_1, P_2, P_3, \ldots, P_n$ each aged *m* years, P_1 will die and be the first to die is

(A) $1 - (1 - P)^n$ (B) $\frac{1}{n^2}\{1-(1-P)^n\}$

(C) $\frac{1}{n}\{1-(1-P)^n\}$ (D) None of these

2. There are four letters and four addressed envelopes. The chance that all letters are not despatched in the right envelope is

(A) $\frac{19}{24}$ (B) $\frac{21}{23}$ (C) $\frac{23}{24}$ (D) None of these

3. If in two events A and B, P (A ∪B) = 0.6, P $(A \cup \overline{B})$ = 0.8, then P (A) is equal to

(A) 0.2 (B) 0.4 (C) 0.5 (D) 0.6

4. The probability that an event will fail to happen is 0.05. The probability that the event will take place on 4 consecutive occasions is

(A) 0.00000625 (B) 0.18543125 (C) 0.00001875 (D) 0.81450625

5. A pair of dice is rolled again and again till a total of 5 or 7 is obtained. The chance that a total of 5 comes before a total of 7 is

(A) $\frac{2}{5}$ (B) $\frac{3}{7}$ (C) $\frac{3}{13}$ (D) None of these

6. The records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die is

(A) 1458×10^{-5} (B) 1458×10^{-6} (C) 41×10^{-6} (D) $8748 \times 10^{-}$

7. An urn contains (2*n* + 1) coins of which *n* coins have a head on both sides and the remaining *n* + 1 coins are fair. A coin is taken out of the bag and tossed if the probability of obtaining a head be 37/50, then *n* is equal to

(A) 10 (B) 11 (C) 12 (D) 13

8. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

(A) 37/256 (B) 219/256 (C) 128/256 (D) 28/256

9. Ram and Shyam stand in a queue at random along with 10 other persons. The probability that there are exactly three persons between them, is

(A) $\frac{4}{33}$ (B) $\frac{2}{33}$ (C) $\frac{2}{99}$ (D) None of these.

10. A fair coin is tossed 99 times, if x is the number of times head occurs P(x = r) is maximum when r is

(A) 49 (B) 50 (C) 51 (D) None of these

11. The probability of India winning a test match against England is 1/2. Assuming independence of the result of various matches, the chance that in a 5 match series. India's second win occurs at 3rd test is

(A) $\frac{2}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{2}$

12. A random variable X has the probability distribution :

X :	1	2	3	4	5	6	7	8
P(X)	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is prime number} and

F = {X < 4}, the probability P(E ∪ F) is

(A) 0.87 (B) 0.77 (C) 0.35 (D) 0.50

13. If $\frac{1-3P}{2}, \frac{1+4P}{3}, \frac{1+P}{6}$ are the probabilities of three matually exclusive and exhausitive events, then the set of all of P is

(A) (0,1) (B) [−1/4, 1/3] (C) (0, 1/3) (D) (0, ∞)

14. There are 8 coloured balls and 8 coloured boxes (colours on the boxes are same as those on the balls). One ball is put into each box at random. The chance that exactly 5 of the balls are put into their respective (coloured) boxes is

(A) $\frac{1}{120}$ (B) $\frac{5}{8}$ (C) $\frac{1}{360}$ (D) None of these

15. Fifteen coupons are numbered 1, 2,....,15 respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is

(A) $\left(\frac{9}{10}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$ (C) $\left(\frac{3}{5}\right)^7$ (D) None of these

16. One hundred identical coins, each with probability p, showing up heads are tossed. If 0 < p < 1 and the probability of heads showing on 50 coins is equal to that of the heads showing 51 coins, then the value of p is

(A) $\frac{1}{2}$ (B) $\frac{49}{101}$ (C) $\frac{50}{101}$ (D) $\frac{51}{101}$

17. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiment are formed. The probability that the event A happens at least once is

(A) 0.936 (B) 0.784 (C) 0.904 (D) None of these

..Notes

18. A pack of cards contains 4 aces, 4 kings, 4 queens and 4 jacks. Two cards are drawn at random. The probability that the one ace is drawn least number of times is

(A) $\frac{1}{5}$ (B) $\frac{3}{16}$ (C) $\frac{1}{6}$ (D) $\frac{1}{9}$

19. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or in tests I and III. The probabilities of the student passing in tests I, II and III are *p*, *q* and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(A) $p = q = 1$ (B) $p = q = \frac{1}{2}$ (C) $p = 1, q = 0$ (D) $p = 1, q = \frac{1}{2}$

20. In a box, there are 10 black and 5 red balls. If one ball is picked up randomly, what is the probability that it is a black ball ?

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{5}$

21. One-third of 12 mangoes got rotten. If 4 mangoes are taken out randomly, what is the probability that no mango is rotten ?

(A) $\frac{14}{99}$ (B) $\frac{2}{3}$ (C) $\frac{16}{99}$ (D) $\frac{85}{99}$

22. In a box, there are 3 red and 7 black caps. One cap is picked up randomly. What is the probability that it is not a red cap ?

(A) $\frac{3}{10}$ (B) $\frac{2}{5}$ (C) $\frac{9}{10}$ (D) $\frac{7}{10}$

23. A bag contains 2 red, 3 green and 2 blue balls. 2 balls are to be drawn randomly. What is the probability that the balls drawn contain no blue ball ?

(A) $\frac{5}{7}$ (B) $\frac{10}{21}$ (C) $\frac{2}{7}$ (D) $\frac{11}{21}$

24. In a box there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green ?

(A) $\frac{7}{19}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{9}{21}$

25. Two cards are drawn at random from a pack of 52 cards. The probability that the drawn cards are both aces, is

(A) $\frac{2}{445}$ (B) $\frac{1}{218}$ (C) $\frac{4}{1569}$ (D) $\frac{1}{221}$

Answers Key

1.(C)	2.(C)	3.(B)	4.(D)	5.(A)
6.(A)	7.(C)	8.(D)	9.(A)	10.(A),(B)
11.(B)	12.(B)	13.(B)	14.(D)	15.(C)
16.(D)	17.(B)	18.(A)	19.(C)	20.(C)
21.(A)	22.(D)	23.(B)	24.(C)	25.(D)

04 Permutations and Combinations

FUNDAMENTAL PRINCIPLE OF COUNTING

Multiplication Principle : If an operation can be performed in *m* different ways; following which a second operation can be performed in *n* different ways, then the two operations in succession can be performed in $m \times n$ different ways.

Addition Principle : If an operation can be performed in *m* different ways and another operation, which is independent of the first operation, can be performed in *n* different ways. Then either of the two operations can be performed in $(m + n)$ ways.

Note : The above two principles can be extended for any finite number of operations.

Permutation : Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a *permutation*.

Note : Permutation of things means arrangement of things. The word arrangement is used if order of things is taken into account. Thus, if order of different things changes, then their arrangement also changes.

Notations : Let *r* and *n* be positive integers such that $1 \le r \le n$. Then, the number of permutations of *n* different things, taken *r* at a time, is denoted by the symbol ${}^nP_r$ or P(*n*, *r*).

Some Results of Permutations

1. ${}^nP_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots\ldots(n-r+1)\, 0 \le r \le n.$
2. The number of permutations of *n* different things taken all at a time = ${}^nP_n = n!$
3. The number of permutations of *n* things taken all at a time, out of which *p* are alike and are of one type, *q* are alike and are of second type and rest are all different $= \frac{n!}{p!q!}$.
4. The number of permutations of *n* different things taken *r* at a time when each thing may be repeated any number of times is n^r.
5. **Circular Permutations :**
   - (*i*) Number of circular arrangements (permutations) of *n* different things = $(n - 1)!$
   - (*ii*) Number of circular arrangements (permutations) of *n* different things when clockwise and anti-clockwise arrangements are not different, *i.e.*, when observation can be made from both sides $= \frac{1}{2}(n-1)!$
   - (*iii*) Number of circular permutations of *n* different things, taken *r* at a time, when clockwise, and anti-clockwise orders are taken as different, is $\frac{{}^nP_r}{r}$.
   - (*iv*) Number of circular permutations of *n* different things, taken *r* at a time, when clockwise and anti-clockwise orders are not different, is $= \frac{{}^nP_r}{2r}$.

Combination : Each of the different groups or selections which can be made by taking some or all of a number of things (irrespective of order) is called a combination.

Note : Combination of things means selection of things obviously. In selection of things order of things has no importance. Thus, with the change of order of things selection of things does not change.

Notations : The number of combinations of n different things taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$.

$$\text{Thus, } {}^nC_r = \frac{n!}{r!(n-r)!} \quad (0 \le r \le n) \qquad = \frac{{}^nP_r}{r!} = \frac{n(n-1)(n-2)....(n-r+1)}{r(r-1)(r-2)...3.2.1}$$

If $r > n$, then ${}^nC_r = 0$.

Properties of ${}^nC_r$

(*i*) ${}^nC_r = {}^nC_{n-r}$

(*ii*) ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = n$

(*iii*) If ${}^nC_x = {}^nC_y$, then either $x = y$ or $x = n - y$ or $x + y = n$

(*iv*) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(*v*) $n \,.\, {}^{n-1}C_{r-1} = (n - r + 1)\, {}^nC_{r-1}$

(*vi*) $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$

(*vii*) If n is even, then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.

(*viii*) If n is odd, then the greatest value of ${}^nC_r$ is

$${}^nC_{\frac{n+1}{2}} \text{ or } {}^nC_{\frac{n-1}{2}}$$

(*ix*) ${}^nC_r = \dfrac{r \text{ decreasing numbers starting with } n}{r \text{ increasing numbers starting with } 1}$

i.e., ${}^nC_r = \dfrac{n(n-1)(n-2).....(n-r+1)}{1.2.3......r}$

(*x*) ${}^nP_r = r$ decreasing numbers starting with n

i.e., ${}^nP_r = n(n - 1)\,(n - 2).....\,(n - r + 1)$

(*xi*) ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

(*xii*) ${}^nC_0 + {}^nC_2 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}$.

Some Results on Combinations

1. Number of selections of r things out of n different things

   (*i*) When p particular things are always included = ${}^{n-p}C_{r-p}$

   (*ii*) When p particular things are excluded = ${}^{n-p}C_r$

   (*iii*) When p particular things are not together in any selection = ${}^{n-p}C_{r-p}$

2. (*i*) Number of selections of r consecutive things out of n things in a row = $n - r + 1$

   (*ii*) Number of selections of r consecutive things out of n things along a circle

   $$= \begin{bmatrix} n & \text{when} & r < n \\ 1 & \text{when} & r = n \end{bmatrix}$$

   (*iii*) Number of selections of zero or more things out of n identical things = $n + 1$.

   (*iv*) Number of selections of one or more things out of n identical things = n.

3. (*i*) Number of ways of dividing $m + n$ different things in two groups containing m and n things respectively

$$(m \neq n) = \frac{(m+n)!}{m!\,n!}.$$

(*ii*) Number of ways of dividing $m + n + p$ different things in three groups containing m, n and p things respectively

$$(m \neq n \neq p) = \frac{(m+n+p)!}{m!\,n!\,p!}.$$

(*iii*) Number of ways of dividing $2n$ different things in two groups, each containing n things and the order of the groups is not important is $\frac{(2n)!}{2!(n!)^2}$.

(*iv*) Number of ways of dividing 3 m different things in three groups each containing m things and the order of the groups is important is $\frac{(3\quad)!}{(\quad)}$

4. (*i*) Number of ways of dividing n identical things into r groups, if blank groups are allowed is ${}^{n+r-1}C_{r-1}$.

(*ii*) Number of ways of dividing n identical things into r groups, if blank groups are not allowed is ${}^{n-1}C_{r-1}$.

(*iii*) Number of ways of dividing n identical things into r groups such that no group contains less than m things and more than $k(m < k)$ things is coefficient of x^n in the expansion of $(x^m + x^{m+1} + ... + x^k)^r$.

5. The number of ways in which r identical things can be distributed among n persons when each person can get zero or more things

= Coefficient of x^r in $(1 + x + x^2 + ... + x^r)^n$

= Coefficient of x^r in $(1 - x)^{-n} = {}^{n+r-1}C_r$ 6

6. The number of non-negative integral solutions of the equation $x_1 + x_2 + ... + x_r = n$ is ${}^{n+r-1}C_r$

7. The number of terms of the expansion of

$(a_1 + a_2 + ... + a_n)^r$ is ${}^{n+r-1}C_r$

Some Useful Results

1. If n distinct points are given in the plane such that no three of which are collinear, then the number of line segments formed = ${}^nC_2$.

If m of these points are collinear ($m \geq 3$), then the number of line segments is $({}^nC_2 - {}^mC_2) + 1$

2. The number of diagonals in an n-sided closed polygon = ${}^nC_2 - n$.

3. If n distinct points are given on the circumference of a circle, then

(A) Number of straight lines = ${}^nC_2$

(B) Number of triangles = ${}^nC_3$

(C) Number of quadrilaterals = ${}^nC_4$ and so on.

4. The sum of the digits in the unit place of all numbers formed with the help of a_1, a_2, ..., a_n taken all at a time is

= $(n - 1)!\ (a_1 + a_2 + ... + a_n)$

(Repetition of digits not allowed)

5. The sum of all n digit numbers that can be formed using the digits $a_1, a_2, ..., a_n$ is

$$= (n-1)!\,(a_1 + a_2 + ... + a_n)\,\frac{(10^n - 1)}{9}.$$

Solved Examples

Example 1. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the nubmer of ways in which he can vote, is

(A) 1110 (B) 5040 (C) 6210 (D) 385

Explanation. (D) : Total number of ways of voting ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$

Example 2. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(A) 603 (B) 602 (C) 601 (D) 600

Explanation. (C) :

Letters starting with A = $\lfloor 5$

Letters starting with C = $\lfloor 5$

Letters starting with H = $\lfloor 5$

Letters starting with I = $\lfloor 5$

Letters starting with N = $\lfloor 5$

SACHIN is the Ist starting with S

Hence, its rank = $\lfloor 5 + \lfloor 5 + \lfloor 5 + \lfloor 5 + \lfloor 5 + 1$

= 601

Example 3. How many words can be formed from the letters of the word COMMITTEE ?

(A) $\dfrac{\lfloor 9}{(\lfloor 2)^2}$ (B) $\dfrac{\lfloor 9}{(\lfloor 2)^3}$

(C) $\dfrac{\lfloor 9}{\lfloor 2}$ (D) $\lfloor 9$.

Explanation. *Ans.* (B) *:* Here, C = 1, O = 1,

M = 2, I = 1, T = 2, E= 2

$$\therefore \quad \textbf{Required number} = \frac{9!}{2!\times 2!\times 2!} = \frac{9!}{(2!)^3}$$

Practice Exercise

1. In how many ways can 5 red and 4 white balls be drawn from a bag containing 10 red and 8 white balls ?

(A) ${}^8C_5 \times {}^{10}C_4$ (B) ${}^{10}C_5 \times {}^8C_4$ (C) ${}^{18}C_9$ (D) None of these

2. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

(A) 69, 760 (B) 30, 240 (C) 99, 784 (D) None of these

3. A letter lock consists of three rings each marked with 10 different letters. In how many ways it is possible to make an unsuccessful attempt to open the lock ?

(A) 27 (B) 1000 (C) 720 (D) 999

4. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(A) 140 (B) 196 (C) 280 (D) 346

5. If ${}^{9}P_{5} + 5.\,{}^{9}P_{4} = {}^{10}P_{r}$, then r =

(A) 5 (B) 4 (C) 9 (D) 10

6. If ${}^{k+5}P_{k+1} = \frac{11(k-1)}{2}\,{}^{k+2}P_{k}$, then the values of k are

(A) 2 and 6 (B) 2 and 11 (C) 6 and 7 (D) 7 and 11

7. In a football championship, there were played 153 matches. Every team played one match with each other. The number of teams participating in the championship is

(A) 9 (B) 18 (C) 11 (D) None of these

8. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is

(A) 12 (B) 18 (C) 10 (D) 9

9. The number of permutations that can be formed by arranging all the letters of the word 'NINETEEN' in which no two E's occur together is

(A) $\frac{8\,!}{3\,!\,3\,!}$ (B) $\frac{5\,!}{3\,!} \times {}^{6}C_{3}$ (C) $\frac{5\,!}{3\,! \times {}^{6}C_{2}}$ (D) $\frac{8\,!}{5\,!} \times {}^{6}C_{3}$

10. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty, is

(A) 5 (B) 21 (C) 3^8 (D) ${}^{8}C_{3}$

11. In a college examination, a candidate is required to attempt 6 out of 10 questions which are divided into two sections each containing 5 questions. Further the candidate is not permitted to attempt more than 4 questions from either of the section. The number of ways in which he can make up a choice of 6 questions is

(A) 15 (B) 60 (C) 100 (D) 200

12. On a railway there are 15 stations. The number of tickets required in order that it may be possible to book a passenger from every station to every other is

(A) $\frac{15!}{2!}$ (B) 15 ! (C) $\frac{15!}{13!}$ (D) $\frac{15!}{13!\,2!}$

13. The number of words that can be made by writing down the letters of the word CALCULATE such that each word starts and ends with a consonant is

(A) $\frac{5(7!)}{2}$ (B) $\frac{3(7!)}{2}$ (C) 2(7!) (D) None of these

14. The number of triangles that can be formed with 10 points as vertices, *n* of them being collinear is 110, then *n* is

(A) 3 (B) 4 (C) 5 (D) 6

15. A bag contains 3 black, 4 white and 2 red balls, all the balls being different, the number of selections of at most 6 balls (containing balls of all the colours) is

(A) 42 (4!) (B) $2^6 \times 4!$ (C) $(2^6 - 1)(4!)$ (D) None of these

16. The total number of selections of at most *n* things from (2*n* + 1) different things is 63, then the value of *n* is

(A) 3 (B) 2 (C) 4 (D) None of these

17. The number of different pairs of words ($\boxed{}\boxed{}\boxed{}\boxed{}$, $\boxed{}\boxed{}\boxed{}$) that can be made with letters of the words STATICS is

(A) 828 (B) 1260 (C) 396 (D) None of these

18. From 7 gentlemen and 4 ladies, a committee of 5 is to be formed. The number of ways in which this can be done so as to include at least one lady is

(A) 805 (B) 403 (C) 754 (D) None of these

19. From a class of 20 students, 2 are chosen for a competition. In how many ways can this be done?

(A) 190 (B) 180 (C) 240 (D) 380

20. Three gentlemen and three ladies are candidates for two vacancies. A voter has to vote for two candidates. In how many ways can one cast his vote?

(A) 9 (B) 30 (C) 36 (D) 15

21. If there are 12 persons in a party and if each two of them shake hands with each other, how many hand shakes happen in the party?

(A) 24 (B) 55 (C) 66 (D) 48

22. A question paper has two parts, Part A and Part B, each containing 10 questions. If the student has to choose 8 from Part A and 5 from Part B, in how many ways can he choose the questions?

(A) 11340 (B) 12750 (C) 40 (D) 320

23. Find the number of ways in which 5 identical balls can be distributed among 10 identical boxes, if not more than one ball can go into a box

(A) 256 (B) 252 (C) 50 (D) 25

24. Find the number of triangles which can be formed by joining the angular points of a polygon of 8 sides as vertices.

(A) 56 (B) 16 (C) 24 (D) 8

25. On a new year day every student of a class sends a card to every other student. The post man delivers 600 cards. How many students are there in the class?

(A) 25 (B) 52 (C) 300 (D) 200

Answers key

1.(B)	2.(A)	3.(D)	4.(B)	5.(A)
6.(C)	7.(B)	8.(B)	9.(B)	10.(B)
11.(D)	12.(D)	13.(A)	14.(C)	15.(A)
16.(A)	17.(B)	18.(D)	19.(A)	20. (D)
21.(C)	22.(A)	23.(B)	24.(A)	25.(A)

05 Alligation or Mixutre

FACTS AND FORMULAE

I. Alligation

Alligation deals with the calculation of value or proporties of a mixture. Alligation is the rule that enables us

(*i*) To find the proportion in which the two or more ingredients at the given prices must be mixed to yield a mixture at the given price. This is termed as *Alligation Alternate.*

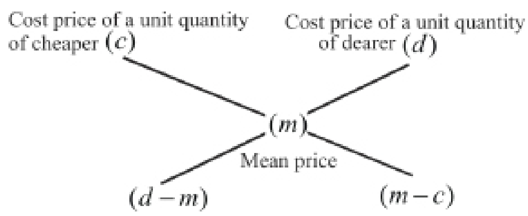
(*ii*) To calculate the average or mean value of a mixture when the prices of two or more ingredients which are to be mixed together and proposition in which they are to be mixed are given. This is termed as *Alligation of Medial*.

II. Rule of Alligation : If two ingredients are mixed, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{Cost price of dearer} - \text{Mean price}}{\text{Mean price} - \text{Cost price of cheaper}}$$

Here cost price of unit quantity of the mixture is called the *mean price*.

We present as under



∴ (Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$

III. Short-cut Methods

(*i*) An ingredient of a mixture in its pure form has percentage of value 100% and fractional value 1.

(*ii*) If a group of 2-legged and 4-legged animals give a head-count as H and their total number of legs is L, then

the number of 4-legged animals = $\frac{L - 2H}{2}$, and

the number of 2-legged animals = $\frac{4H - L}{2}$

(*iii*) Let a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid will be $\left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

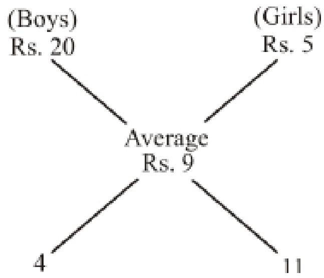
(*iv*) m gm of sugar solution has x % sugar in it. To increase the sugar content in the solution to y %, then

the quantity of sugar need to be added = $\frac{m(y - x)}{(100 - y)}$

Solved Examples

Ex.1. *Rs. 675 was divided among 75 boys and girls. Each boy gets Rs. 20 whereas a girl gets Rs. 55. Find the number of boys and girls.*

Sol. Average money per head (boy or girl) $= \frac{675}{75}$ = Rs. 9

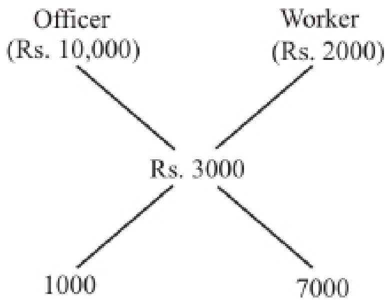


$\therefore$ Number of boys $= \frac{4}{4+1} \times 75 = 60$

$\therefore$ Number of girls $= \frac{1}{4+1} \times 75 = 5$.

Ex.2. *The average monthly salary of employees consisting of officers and workers of an organisation is Rs. 3000. The average salary of an officer is Rs. 10,000 while that of a worker is Rs. 2000 per month. If there are total 400 employers in the organisation. Find the number of officers and workers separately.*

Sol.



$$\frac{\text{Number of officers}}{\text{Number of workers}} = \frac{1000}{7000} = \frac{1}{7}$$

Number of officers $= \frac{1}{1+7} \times 400 = 50$
Number of workers = 400 – 50 = 350

Ex.3. *A person has Rs. 5000. He invests a part of it at 3% per annum and the remainder at 8% per annum simple interest. His total income in 3 years is Rs. 750. Find the sum invested at different rates of interest.*

Sol. Average rate of interest $= \frac{100 \times 750}{5000 \times 3}$
= 5% per annum

$$\begin{array}{ccc} 3 & & 8 \\ & 5 & \\ 3 & & 2 \end{array}$$

Investment at 3% per annum = $\frac{3}{3+2} \times 5000$

= Rs. 3000

Investment at 8% per annum = $\frac{2}{3+2} \times 5000$

= Rs. 2000.

Practice Exercise

Directions. Each of the questions given below is followed by four or five alternatives of which one is correct. Find out the correct answer.

1. **Three glasses of capacity 2 litres, 5 litres and 9 litres contain mixture of milk and water with milk concentrations 90%, 80% and 70% respectively. The contents of three glasses are emptied into a large vessel. Find the milk concentration and ratio of milk to water in the resultant mixture.**

   (A) 121 : 39 (B) 7 : 12 (C) 55 : 80 (D) 80 : 81

2. **A vessel contains wine with 30% spirit. A part of it is stolen and replaced by same quantity of wine with 10% spirit. The resultant mixture has only 25% spirit. How much of wine was stolen ?**

   (A) $\frac{1}{2}$th (B) $\frac{1}{3}$th (C) $\frac{1}{4}$th (D) $\frac{4}{5}$th

3. **Milk and water are in the ratio 3 : 2 in a mixture of 80 litres. How much water should be added so that the ratio of the milk and water becomes 2 : 3 ?**

   (A) 25 litres (B) 40 litres (C) 35 litres (D) 20 litres

4. **Find the proportion in which 4 types of tea priced @ Rs. 40, Rs. 50, Rs. 80 and Rs. 100 be mixed so as to obtain a mixture worth Rs. 75 per kg.**

   (A) 1 : 5 : 7 : 5 (B) 1 : 6 : 8 : 9 (C) 2 : 5 : 6 : 8 (D) 3 : 12 : 7 : 1.

5. **Prabhu purchased 30 kg of rice at the rate Rs. 17.50 per kg and another 30 kg rice at a certain rate. He mixed the two and sold the entire quantity at the rate of Rs. 18.60 per kg and made 20 per cent overall profit. At what price per kg did he purchase the lot of another 30 kg rice ?**

   (A) Rs. 14.50 (B) Rs. 12.50 (C) Rs. 15.50 (D) Rs. 13.50

6. **From a cask of wine containing 25 litres, 5 litres are withdrawn and the cask is refilled with water. The process is repeated a second and then a third wine. Find the quantity of wine left in the cask and also the ratio of wine of water in the resulting mixture.**

   (A) 65.41 (B) 64.61 (C) 31.38 (D) 40.54

7. **19 litres are drawn from a vessel full of wine and it is filled with water. Again 19 litres of the mixture are drawn and the vessel is again filled with water. The ratio of the wine to water new present in the vessel is 81 : 19. What is the overall capacity of the vessel ?**

   (A) 1280 litres (B) 190 litres (C) 1542 litres (D) 1000 litres

8. **A vessel contains mixture of liquids A and B in the ratio 3 : 2 when 20 litres of the mixture is taken out and replaced by 20 litres of liquid B, the ratio changes to 1 : 4. How many litres of liquid A was there initially present in the vessel ?**

   (A) 7 litres (B) 18 litres (C) 5 litres (D) 2 litres

9. **A piece of an alloy of two metals (A and B) weighs 15 gm and costs Rs. 150. If the weights of the two metals be interchanged, the new alloy would be worth Rs. 120. If the price of metal A is Rs. 6 per gm, find the weight of the other metal in the original piece of alloy.**

   (A) 5 gm (B) 10 gm (C) 7 gm (D) 15 gm

10. **If 4 kg of an alloy made of $\frac{1}{4}$th iron and rest is mixed with 6 kg of another alloy made of $\frac{2}{3}$rd iron and rest tin. Find the ratio of iron to tin in the resultant mixture.**

   (A) 5 kg (B) 15 kg (C) 28 kg (D) 30 kg.

11. **A herd of 2-legged and 4-legged animals give a head count as H. When legs are counted, it comes to L numbers. Find the number of 2-legged and 4-legged animals in terms of H and L.**

   (A) $\frac{3H-T}{5}$ (B) $\frac{4H-L}{2}$ (C) $\frac{3H-A}{5}$ (D) $\frac{4T-H}{5}$

12. **Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, then price of the third variety per kg will be**

   (A) Rs. 175.50 (B) Rs. 171.50 (C) Rs. 172.50 (D) Rs. 178.50

13. **8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask hold originally ?**

   (A) 10 (B) 12.4 (C) 24 (D) 21

14. **The cost of type 1 rice is Rs. 15 per kg and type 2 rice is Rs. 20 per kg. If both type 1 and type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is**

   (A) 18 per kg (B) 10 per kg (C) 40 per kg (D) 250 per kg

15. **In a 45 litres mixture of milk and water, the ratio of the milk to water is 2 : 1. When some quantity of water is added to the mixture, this ratio becomes 1 : 2. The quantity of water added is**

   (A) 10 litres (B) 21 litres (C) 35 litres (D) 45 litres

16. The ratio in which tea costing Rs. 192 per kg is to be mixed with tea costing Rs. 150 per kg so that the mixed tea, when sold for Rs. 194.40 per kg, gives a profit of 20%, is

(A) 2 : 5 (B) 3 : 5 (C) 5 : 3 (D) 5 : 2

17. An alloy contains zinc, copper and tin in the ratio 2 : 3 : 1 and another contains copper, tin and lead in the ratio 5 : 4 : 3. If equal weights of both alloys are melted together to form a third alloy, then the weight of lead per kg in the new alloy will be

(A) $\frac{1}{2}$ kg (B) $\frac{1}{8}$ kg (C) $\frac{3}{14}$ kg (D) $\frac{7}{9}$ kg

18. In a 729 litres mixture of milk and water, the ratio of milk to water is 7 : 2. To get a new mixture containing milk and water in the ratio 7 : 3, the amount of water to be added is

(A) 81 litres (B) 71 litres (C) 56 litres (D) 50 litres

19. In 40 litres mixture of milk and water, the ratio of milk to water is 7 : 1. In order to make the ratio of milk and water 3 : 1, the quantity of water (in litres) that should be added to the mixture will be

(A) 6 (B) $6\frac{1}{2}$ (C) $6\frac{2}{3}$ (D) $6\frac{3}{4}$

20. In what ratio must a mixture of 30% alcohol strength be mixed with that of 50% alcohol strength so as to get a mixture of 45% alcohol strength ?

(A) 1 : 2 (B) 1 : 3 (C) 2 : 1 (D) 3 : 1

21. A shopkeeper bought 80 kg of sugar at the rate of Rs. 13.50 per kg. He mixed it with 120 kg of sugar costing Rs. 16 per kg. In order to make a profit of 20%, he must sell the mixture at

(A) Rs. 18 per kg (B) Rs. 17 per kg (C) Rs. 16.40 per kg (D) Rs. 15 per kg

22. 1 litre of water is added to 5 litres of alcohol-water soution containing 40% alcohol strength. The strength of alcohol in the new solution will be

(A) 30% (B) 33% (C) $33\frac{2}{3}$% (D) $33\frac{1}{3}$%

23. A liquid 'P' is $1\frac{3}{7}$ times as heavy as water and water is $1\frac{2}{5}$ times as heavy as another liquid 'Q'. The amount of liquid 'P' that must be added to 7 litres of the liquid 'Q' so that the mixture may weigh as much as an equal volume of water, will be

(A) 7 litres (B) $5\frac{1}{6}$ litres (C) 5 litres (D) $4\frac{2}{3}$ litres

24. Zinc and copper are in the ratio of 5 : 3 in 200 gm of an alloy. How much grams of copper be added to make the ratio as 3 : 5 ?

(A) $133\frac{1}{3}$ (B) $\frac{1}{200}$ (C) 72 (D) 66

25. 200 litres of a mixture contains milk and water in the ratio 17 : 3. After the addition of some more milk to it, the ratio of milk to water in the resulting mixture becomes 7 : 1. The quantity of milk added to it was

(A) 20 litres (B) 40 litres (C) 60 litres (D) 80 litres

Answers Key

1.(A)	**2.(C)**	**3.(B)**	**4.(A)**	**5.(D)**
6.(B)	**7.(B)**	**8.(B)**	**9.(B)**	**10.(A)**
11.(B)	**12.(A)**	**13.(C)**	**14.(A)**	**15.(D)**
16.(A)	**17.(B)**	**18.(A)**	**19.(C)**	**20.(B)**
21.(A)	**22.(D)**	**23.(D)**	**24.(A)**	**25.(B)**

06 Percentage

Percentage : It is a fraction whose denominator is 100 and the numerator of such a fraction is termed as rate per cent. The following example will illustrate the per cent.

A student gets 50 per cent in Mathematics means that he obtained 50 marks out of every hundred of full marks.

If the full marks be 200, he gets 100 marks in Mathematics. i.e.

50% of 200 = $\frac{50}{100} \times 200 = 100$

Percentage as a Conversion Tool : It provides an alternative method to express fractions of any quantity or a decimal number into an equivalent simpler form which is easier to comprehend.

(i) Decimal to Percentage Conversion : This conversion implies multiplication by 100. For example,

2.75 = 275 %

0.258 = 25.8%

(ii) Percentage to Decimal Conversion : This is reverse of the previous operation (i) . For example,

125 % = 1.25

1.867% = 0.01867

(iii) Fraction to Percentage Conversion : This conversion implies multiplication of the given fraction or number by 100. The result obtained is its equivalent percentage. For example,

1. What is the ratio 5 : 4 equal to when expressed as a per cent ?

Solution. Required ratio = $\frac{5}{4} \times 100 = 125\%$

(iv) Percentage to Fraction Conversion : This conversion implies division by 100. For example,

$$65\% = \frac{65}{100} = \frac{13}{20}$$

As we know our key operators in this chapter are prime fractions of per cent value. Let's collect some of the frequently used prime fractions. These will help in making calculations quicker in objective type exams.

$5\% = \frac{1}{20}$ $\qquad$ $20\% = \frac{1}{5}$

$6\frac{1}{4}\% = \frac{1}{16}$ $\qquad$ $25\% = \frac{1}{4}$

$8\% = \frac{2}{25}$ $\qquad$ $33\frac{1}{3}\% = \frac{1}{3}$

$8\frac{1}{3}\% = \frac{1}{12}$ $\qquad$ $37\frac{1}{2}\% = \frac{3}{8}$

$10\% = \frac{1}{10}$ $\quad$ $40\% = \frac{2}{5}$

ü $= \frac{3}{25}$ $\quad$ $60\% = \frac{3}{5}$

$12\frac{1}{2}\% = \frac{1}{8}$ $\quad$ $66\frac{2}{3}\% = \frac{2}{3}$

$14\frac{2}{7}\% = \frac{1}{7}$ $\quad$ $75\% = \frac{3}{4}$

$15\% = \frac{3}{20}$ $\quad$ $87\frac{1}{2}\% = \frac{7}{8}$

$16\% = \frac{4}{25}$ $\quad$ $16\frac{2}{3}\% = \frac{1}{6}$

Some Results on Percentage:

(a) If two values are respectively *x* % and *y* % more than a third value, then the first is the $\frac{100+x}{100+y}\times 100\%$ of the second.

(b) If A is *x* % of C and B is *y* % of C, then A is $\frac{x}{y}\times 100\%$ of B.

(c) Let the population of a town be P now and suppose it increases at the rate of R % per annum, then:

(i) Population after n years = $P\left(1+\frac{R}{100}\right)^n$

(ii) Population n years ago = $\frac{P}{\left(1+\frac{R}{100}\right)^n}$

(d) If the price of a commodity increases by R%, then reduction in consumption, not to increase the expenditure, is :

$$\left\{\frac{R}{(100+R)}\times 100\right\}\%$$

(e) If the price of a commodity decreases by R%, then the increase in consumption, not to decrease the expenditure is:

$$\left\{\frac{R}{(100-R)}\times 100\right\}\%$$

(f) Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then,

(i) Value of the machine after *n* years = $P\left(1-\frac{R}{100}\right)^n$

(ii) Value of the machine n years ago = $\dfrac{P}{\left(1-\dfrac{R}{100}\right)^n}$

(g) If first value is r% more than the second value, then the second is $\left[\dfrac{r}{100+r}\times 100\right]\%$ less than the first value.

(h) If the first value is r % less than the second value then, the second value is $\left[\dfrac{r}{100-r}\times 100\right]$ % more than the first value.

(i) If the value of a number is first increased by x % and then decreased by y%, the net change is always a decrease that is equal to x % of x or $\dfrac{x^2}{100}$

(j) If the value is first increased by x % and then decreased by y %, the net change is $\left[x-y-\dfrac{xy}{100}\right]$ % increase or decrease, according to the positive or negative sign respectively.

If the value is increased successively by x% and y%, then final increase is given by $\left[x+y+\dfrac{xy}{100}\right]\%$

Solved Examples

Ex.1. *A person who spend $66\frac{2}{3}\%$ of his income is able to save Rs. 1200 per month. What is his monthly expenses?*

Sol. Let the monthly income be Rs. x. Then,

$\left(100\% - 6\ \frac{2}{3}\%\right)$ of x = 1200

$\Rightarrow\ 33\frac{1}{3}\%$ of x = 1200 $\Rightarrow \dfrac{100}{3\times 100}\times x = 1200$ $\Rightarrow$ x = 3600

$\therefore$ Monthly expenses = 3600 – 1200 = Rs. 2400

Ex.2. *If the price of sugar be raised by 25%, then find by how much percent consumption of sugar should be reduced so that the expenditure on it may not increase?*

Sol. Short-cut Method :

Percentage increase = 25%, *i.e.* x = 25

To retain the previous expenditure level, consumption of sugar should be reduced by

$$\left(\frac{100\times x}{100+x}\right)\% = \left(\frac{100\times 25}{100+25}\right)\% = \left(\frac{100\times 25}{125}\right)\% = 20\%$$

Ex.3. *Two varieties of tea were prepared by adding 40 gm sugar with 270 gm tea and 30 gm sugar with 190 gm tea. Which of the two varieties of tea will be more sweetened? Also find out what will be the percentage of tea in the mixture formed by blending these two varieties.*

Sol. Per cent amount of sugar in the first variety of tea $=\left(\dfrac{40}{40+270}\right)\times 100\% = 12\dfrac{28}{31}\%$

and per cent amount of sugar in the second variety of tea $=\left(\frac{30}{30+190}\right)\times 100\% = 13\frac{7}{11}\%$

Therefore, it is clear that the second variety of tea will be more sweetened on blending the two varieties. Amount of tea in the mixture = 270 + 190 = 460 gm and amount of sugar in the mixture = 40 + 30 = 70 gm.

$\therefore$ % of tea in the mixture $=\left(\frac{460}{460+70}\right)\times 100\%$

$$=\frac{460}{530}\times 100\% = 86\frac{42}{53}\%$$

Ex.4. *The present population of a town is 12,000. After an increase of 6% in the male population and 8% in the female population, its population becomes 12800. What is the number of males in the present population of the town.*

Sol. Let the number of males in the town be *x*. Then,

member of females in the town = (12000 – *x*)

According to the question, *x* + 6% of *x* + (12000 – *x*) + 8% of (12000 – *x*) = 12800

$\Rightarrow x+\frac{6x}{100}+12000-x+960-\frac{8x}{100}=12800$

$\Rightarrow$ 6*x* + 1200000 + 96000 – 8*x* = 1280000

$\Rightarrow$ 8*x* – 6*x* = 1280000 – 1200000 – 96000

$\Rightarrow$ 2*x* = 16000 or $x=\frac{16000}{2}=8,000$

Therefore, the number of males in the town is 8,000.

Ex.5. *The numerator of a fraction is increased by 20% and the denominator is increased by 150%. If the resultant fraction is* $\frac{9}{50}$, *then what was the original fraction?*

Sol. Let the original fraction be $\frac{x}{y}$

According to the question,

$$\frac{x+20\% \text{ of } x}{y+150\% \text{ of } y}=\frac{9}{50} \text{ or } \frac{x+\frac{x}{5}}{y+\frac{3y}{2}}=\frac{9}{50}$$

$$\Rightarrow \frac{\frac{6x}{5}}{\frac{5y}{2}}=\frac{9}{50} \text{ or } \frac{ü\ x}{25y}=\frac{}{50} \text{ or } \frac{x}{y}=\frac{9\times 25}{12\times 50}=\frac{3}{8}$$

Practice Exercise

Directions. Each of the questions given below is followed by four or five alternatives of which one is correct. Mark (√) against correct answer.

1. If 15% of (A + B) = 25% of (A – B), then what percent of B is equal to A ?

(A) 10% (B) 60% (C) 200% (D) 400%

✍..Notes

2. **If *x* is 90% of *y*, then what percent of *x* is *y*?**

(A) 111.1 (B) 90 (C) 190 (D) 101.1

3. **What approximate value should come in the place of question mark (?) in the following equation?**

$$161\sqrt{5} + ?\% \text{ of } 960 = 1052 - 124$$

(A) 35 (B) 45 (C) 60 (D) 25

4. **If a number is increased by 10% and thereafter decreased by 10%, then by how many percent the number has been increased or decreased?**

(A) 1% increase (B) 1% decrease (C) 2% increase (D) None of these

5. **When the price of radio set was increased by 15%, then the number of sets sold reduce by 20%. What was the effect on gross receipts of the shop?**

(A) increase by 8% (B) increase by 10%
(C) decrease by 8% (D) decrease by 10%.

6. **In 80 litres mixture of milk and water, the amount of water is 20%. How much water should be added to this mixture to make the quantity of water 30%?**

(A) 5 litres (B) $11\frac{3}{7}$ litres (C) $12\frac{3}{7}$ litres (D) None of these

7. **List price of an article is Rs. 400. After allowing two successive discounts of 20% and 15%, its selling price will be**

(A) Rs. 272 (B) Rs. 282 (C) Rs. 262 (D) Rs. 252

8. **In an examination 60% of the students pass in English, 70% pass in Hindi and 40% pass in both. What percent of students fails in both English and Hindi ?**

(A) 10 (B) 20 (C) 25 (D) 30

9. **If food price go up by 10 percent, by how much should a man reduce his consumption to not to increase his expenditure ?**

(A) $9\frac{1}{1}\%$ (B) 10% (C) $11\frac{1}{9}\%$ (D) Data insufficient

10. **20 litres milk contains 2 per cent water. What quantity of pure milk should be added so that water content comes down to one per cent,**

(A) 10 litres (B) 20 litres
(C) Cannot be determined (D) None of these

11. **A shopkeeper increases the price of an article by 25%. Later he further increases the new price by 20%. What is the total percentage increase on the original price of the article?**

(A) 50% (B) 40% (C) 55% (D) 38%

12. **8% of the voters in an election did not cast their votes. In the election, there were only two candidates and the winner by obtaining 48% of the total votes, defeated his contestant by 1100 votes. The total number of voters in the election was**

(A) 21000 (B) 23500 (C) 22000 (D) 27500

13. A number is increased by 10% and then decreased by 10%. Finally the number

(A) does not change (B) decreases by 1%

(C) increases by 1% (D) increases by 0.1%

14. A man spend $2\frac{1}{2}$ per cent of his of his salary on items of daily use and 30 per cent of the remainder on house rent. Therefore that he is left with Rs. 2940. How much is his salary?

(A) Rs. 4800 (B) Rs. 5200 (C) Rs. 4500 (D) Rs. 4000

15. In a group of students, 70 per cent can speak English and 65 per cent can speak Hindi. If 27 per cent of the students can speak none of the two languages, then what per cent of the group can speak both the languages?

(A) 38 percent (B) 62 percent (C) 28 percent (D) 23 percent

16. In a factory, 80% of the employees are males. Among them 20% are matriculates and the remaining are graduates. Among the females, 25% are matriculates and the remaining are graduates. If the total number of female employees in the factory is 600, how many graduates are there in the factory ?

(A) 2370 (B) 3000 (C) 1940 (D) None of these

17. In an examination 56% students fail in English and 48% students fail in Science. If 18% fail in English and Science both, then find the percentage of students who have been successful in both the subjects.

(A) 16% (B) 14% (C) 20% (D) 22%

18. A man ordered 4 pairs of black socks and some pairs of brown socks. The price of a black pair is double that of a brown pair. While preparing the bill, the clerk interchanged the number of black and brown pairs by mistake which increased the bill by 50%. The ratio of the numbers of black and brown pairs of socks in the original order was

(A) 4 : 1 (B) 2 : 1 (C) 1 : 4 (D) 1 : 2

19. In a survey of political preference, 78% of those asked were in favour at least one of the proposals, I, II and III. 50% of those asked favoured proposal I, 30% favoured proposal II, and 20% favoured proposal III. If 5% of those asked favoured all three proposals, what percentage of those asked favoured more than one of the three proposals?

(A) 12 (B) 17 (C) 22 (D) 10

20. From the salary of an officer, 10% is deducted as house rent, 20% of the rest, he spends on conveyance, 20% of the rest he pays as income tax and 10% of the balance, he spends on clothes. Then, he is left with Rs. 15552.. Find his total salary (in rupees).

(A) 25000 (B) 30000 (C) 35000 (D) 40000

21. A chemist has 10 litres of a solution that has 10 per cent nitric acid by volume. He wants to dilute the solution to 4 per cent strength by adding water. How many liter of water must he add ?

(A) 15 (B) 20 (C) 18 (D) 25

..Notes

22. A company received two shipments of ball bearings. In the first shipment, 1 percent of the ball bearings were defective, in the second shipment which was twice as large as the first, 4.5 per cent of the ball bearings were defective. If the company received a total of 100 defective ball bearings, how many ball bearings were there in the first shipment ?

(A) 990 (B) 2000 (C) 1000 (D) 3000

23. A company management was taken over by a multinational and wages of all the employees were increased by 20%. It has also decided to pay an employer's contribution to ESI at 4%. The multinational decided to withdraw and handed over the management of the company back to the old owner who again reduced the wages by 20% but continued the contribution to ESI. What was the net effect of these changes as compared to the original payout ?

(A) 4.0% increase in payout (B) 1.16% increase in payout

(C) 1.16% decrease in payout (D) None of these

24. There are 3 containers A, B and C which contain water, milk and acid respectively in equal quantities. 10% of the content of A is taken out and poured into B. Then the same amount from B is transferred to C, from which again the same amount is transferred to A. What is the proportion of milk in container A at the end of the process?

(A) $\frac{9}{10}$ (B) $\frac{1}{11}$ (C) $\frac{1}{121}$ (D) $\frac{2}{13}$

25. In a school, 60% of the students of class X were boys. 75% of boys passed the class X exam. 40% of the passed boys got first rank, 80% of the total students passed the exam and 50% of the passed students got first division. Which of the following conclusions is not correct?

(A) 75% of the failed sudents are boys

(B) 55% of the first-divisioners are girls

(C) Number of passed girls is more than that of boys

(D) If *x* students failed, 2*x* got first division

Answers Key

1.(D)	**2.(A)**	**3.(C)**	**4.(B)**	**5.(C)**
6.(B)	**7.(A)**	**8.(A)**	**9.(A)**	**10.(B)**
11.(A)	**12.(D)**	**13.(B)**	**14.(A)**	**15.(B)**
16.(A)	**17.(B)**	**18.(C)**	**19.(C)**	**20.(B)**
21.(A)	**22.(C)**	**23.(D)**	**24.(C)**	**25.(C)**

07 Simple Interest

Key Points

1. 'Interest' is the money paid by a borrower to the lender for the use of money borrowed.

2. The money borrowed or lent out for a certain period is called the 'Principal' or 'the sum'.

3. The sum of Interest and Principal is called the Amount. Thus

   Amount = Principal + Interest

4. The duration of period for which the money is borrowed is called the Time.

5. The interest is charged according to some pre-condition which is expressed in general as a rate per cent of the principal for each year and is called 'rate percent per annum'. Thus if the rate of interest is 5% per annum, it means that the interest on principal of Rs. 100 for one year is Rs. 5.

   Here 'per annum' means 'for a year'.

6. If throughout the loan period, the interest is charged on the original sum (Principal) borrowed, it is called Simple Interest.

7. Some abbreviations to be used frequently :

   P = Principal R = Rate of interest

   T = Time S.I. = Simple interest

   A = Amount

8. **Formulae :**

If P is the Principal, R is the rate per cent per annum, T is the time in years and S.I. is the simple interest, then

(i) S.I. $= \dfrac{P \times R \times T}{100}$ (ii) P $= \dfrac{S.I. \times 100}{R \times T}$

(iii) R $= \dfrac{S.I. \times 100}{P \times T}$ (iv) $T = \dfrac{S.I. \times 100}{P \times R}$

(v) A = P + I = P + $\dfrac{PRT}{100}$

$\Rightarrow$ 100A = 100 × P + PRT $\Rightarrow$ 100 × A = P (100 + RT)

$\therefore\ P = \dfrac{100 \times A}{100 + RT}$

(vi) The annual payment that will discharge a debt of Rs. x due in T years at the rate of interest R% per annum is $\dfrac{100x}{100T + \dfrac{RT(T-1)}{2}}$

(vii) If a sum of money becomes x times in T years at SI, the rate of interest is given by $\dfrac{100(x-1)}{T}\%$

(viii) A sum of Rs. x is lent out in parts in such a way that the interest on 1st part at R_1% for T_1 years, the interest on second part at R_2% for T_2 years and so on, are equal, the ratio in which the sum was divided in parts is given by $\frac{1}{R_1T_1} : \frac{1}{R_2T_2} : ..... \frac{1}{R_nT_n}$.

(ix) When different sums mature to the same amount at simple rate of interest, the ratio of the sums invested are in the inverse ratio of (100 + time × rate), i.e., the ratio in which the sums are invested is $\frac{1}{100+R_1T_1} : \frac{1}{100+R_2R_2}$: ....

9. 1 Ordinary year = 365 days

10. 1 Leap year = 366 days

11. The number of days in February of leap year is 29

12. If the rate per cent is given half-yearly or quarterly then to find the rate per cent per annum, multiply it by 2 or 4 respectively.

13. In counting the number of days between the two given dates, the either day (first or last) is excluded. You have to keep in mind that interest is not charged for the day on which money is borrowed but it is charged for the day it is returned.

Solved Examples

Ex.1. *Out of a sum of Rs. 1600 Tarun lent some money at 4% per annum simple interest and remaining sum of money at 3% p.a. simple interest. If after 3 years, he was paid Rs. 162 as interest, how much money had he lent at 3% p.a. ?*

Sol. Let Tarun be lent Rs. x at the rate of 4%

∵ He lent Rs. (1600 – x) at the rate of 3%

∵ S.I. Rs. x for 3 years at the rate of 4% p.a.

$$= \frac{x \times 4 \times 3}{100} = \text{Rs. } \frac{12x}{100}$$

Also, S.I. on Rs. (1600 – x) for 3 years at the rate of 3% p.a. = Rs. $\frac{(1600-x)\times 3\times 3}{100}$ = Rs. $\frac{9(1600-x)}{100}$

According to the question,

S.I. on the total sum of money = Rs. 162

∴ $\frac{9}{100}(1600-x)+\frac{12x}{100}=162$ or x = 600

∴ Money lent at 3% p.a. = Rs. 1600 – Rs. 600 = Rs. 1000

Ex.2. *The simple interest on a certain sum of money is $\frac{1}{10}$ th of the sum for four years. Find the rate.*

Sol. Let the sum of money (P) = Rs. x

Simple Interest = $\frac{1}{10}$ th of Rs. x = Rs. $\frac{x}{10}$

Time (T) = 4 years

Rate percent = $\frac{x/10 \times 100}{x \times 4} = \frac{100}{10 \times 4}$ = 2.5 % or $2\frac{1}{2}\%$.

Hence, the rate of interest p.a. will be 2.5%.

Ex.3. *In how many years will a sum of money double itself at the rate of 4% p.a. simple interest ?*

Sol. Let P = Rs. *x*, A = Rs. 2*x*

∵ Simple Interest (S.I) = 2*x* – *x* = Rs. *x*, and rate (R) = 4%

∴ Time = $\frac{\text{S.I} \times 100}{\text{P} \times \text{R}} = \frac{x \times 100}{x \times 4}$ = 25 years

Therefore, the sum will double itself at the rate of 4% p.a. in 25 years.

Ex.4. *A sum of Rs. 2444 is divided among three parts in such a manner that simple interest on the three divided parts at 6% p.a. after 3, 4 and 5 years respectively remains equal. Find such three parts.*

Sol. Short-cut Method

Ratio among such three parts of Rs. 2444

$= \frac{1}{r_1t_1} : \frac{1}{r_2t_2} : \frac{1}{r_3t_3} = \frac{1}{6 \times 3} : \frac{1}{6 \times 4} : \frac{1}{6 \times 5}$

$= \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$ = 20 : 15 : 12

Sum of proportionals = 20 + 15 + 12 = 47

∴ Ist part = $\frac{20}{47} \times 2444$ = Rs. 1040

IInd part = $\frac{15}{47} \times 2444$ = Rs. 780

IIIrd part = $\frac{12}{47} \times 2444$ = Rs. 624

Ex.5. *Amount accrued by the investment of Rs. 1500 for $2\frac{1}{2}$ years at the rate of 8% p.a. is equal to the amount accrued by investment of a certain sum of money for 2 years at the rate of 5% p.a. Find the sum of investment in the second case.*

Sol. Short-cut Method

Let the sum of money invested in the second case

be Rs. *x*.

Since amount in each case remains equal.

∴ Two parts will be in the ratio of $\frac{1}{100 + r_1t_1} : \frac{1}{100 + r_2t_2}$

⇒ $1500 : x :: \frac{1}{100 + 8 \times 2\frac{1}{2}} : \frac{1}{100 + 5 \times 2}$

$\Rightarrow$ $1500 : x :: \frac{1}{120} : \frac{1}{110}$ or $\frac{1500}{x} = \frac{1/120}{1/110} = \frac{110}{120}$

$\Rightarrow$ $x = \frac{1500 \times 120}{110} = 1636\frac{4}{11}$

$\therefore$ Required sum of money is Rs. $1636\frac{4}{1}$

Ex.6. *In how many years, Rs. 150 will produce the same interest @ 8% as Rs. 800 produce in 3 years @ $4\frac{1}{2}\%$?*

Sol. Given, P = Rs. 800, R = $4\frac{1}{2}\%$ = $\frac{9}{2}\%$, and T = 3 years

$\therefore$ S.I. = Rs.$\left(800 \times \frac{9}{2} \times \frac{3}{100}\right)$ = Rs. 108

Since, P = Rs. 150, S.I. = Rs. 108, R = 8%

$\because$ Time = $\left(\frac{100 \times 108}{150 \times 8}\right)$ years = 9 years

Ex.7. *A lent Rs. 5000 to B for 2 years and Rs. 3000 to C for 4 years on simple interest at the same rate of interest and received Rs. 2200 in all from both of them as interest, the rate of interest per annum is*

Sol. Let the rate be R% p.a.. Then,

$$\left(\frac{5000 \times R \times 2}{100}\right) + \left(\frac{3000 \times R \times 4}{100}\right) = 2200$$

$\Rightarrow$ 100 R + 120 R = 2200 or R = $\left(\frac{2200}{220}\right)$ = 10

$\therefore$ Rate = 10%.

Ex.8. *David invested certain amount in three different schemes A, B and C with the rate of interest 10% p.a 12% p.a. and 15% p.a. respectively. If the total interest accured in one year was Rs. 3200 and the amount invested in scheme C was 150% of the amount invested in scheme A and 240% of the amount invested in scheme B, what was the amount invested in the scheme B ?*

Sol. Let x, y, and z be the amounts invested in the schemes A, B and C respectively. Then,

$$\left(\frac{x \times 10 \times 1}{100}\right) + \left(\frac{y \times 12 \times 1}{100}\right) + \left(\frac{z \times 15 \times 1}{100}\right) = 3200$$

$\Rightarrow$ $10x + 12y + 15z = 320000$

Now, z = 240% of y = $\frac{12}{5}$ y and z = 150% of x = $\frac{3x}{2}$

$\Rightarrow$ $x = \frac{2}{3}z = \left(\frac{2}{3} \times \frac{12}{5}y\right) = \frac{8y}{5}$ $\therefore$ 16y + 12y + 36y = 320000

or 64y = 320000 or y = 5000.

$\therefore$ Sum invested in Scheme B = Rs. 5000.

Ex.9. *A sum of money becomes* $\frac{7}{4}$ *of itself in 6 years at a certain rate of interest. What is the rate of interest ?*

Sol. Let the sum (Principal) be Rs. 100.

$\therefore$ Amount (A) = $\frac{7}{4}$ of 100 = Rs. 175

$\therefore$ I = 175 – 100 = Rs. 75.

Time (T) = 6 years

$$\therefore \; R = \frac{I \times 100}{P \times T} = \frac{75 \times 100}{100 \times 6} = \frac{75}{6} = \frac{25}{2} = 12.5\%$$

Practice Exercise

Directions. Each of the questions given below is followed by four or five alternatives of which one is correct. Mark (✓) against correct answer.

1. A sum of Rs. 1550 was lent partly at 5% and partly at 8% simple interest. The total interest received after 3 years is Rs. 300. The ratio of money lent at 5% to that at 8% is

(A) 5 : 8 (B) 8 : 5 (C) 31 : 6 (D) 16 : 15

2. Simple interest on a certain sum at a certain annual rate of interest is $\frac{16}{25}$ of the sum. If the numbers representing rate percent and time in years be equal, then the rate of interest is

(A) $11\frac{1}{2}$ percent (B) 8 percent (C) $12\frac{1}{2}$ percent (D) $12\frac{1}{4}$ percent

3. Ravi gave Rs. 1200 on loan. Some amount he gave on 4% per annum S.I. and remaining at 5% per annum S.I. After two years he got Rs. 110 as interest, then the amounts given on 4% and 5% per annum simple interest are respectively.

(A) Rs. 500, Rs. 700 (B) Rs. 400, Rs. 800

(C) Rs. 800, Rs. 300 (D) Rs. 1110, Rs. 1100

4. If the simple interest for 6 years be equal to 30% of the principal, it will be equal to the principal after

(A) 20 years (B) 30 years (C) 10 years (D) 22 years

5. Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively. If the difference in interests for two periods was Rs. 412.50, then each sum is

(A) Rs. 3250 (B) Rs. 3500 (C) Rs. 3750 (D) Rs. 4250

6. A borrowed 830 Rs. money from B at 12% p.a. S.I. for 3 years. He then added some more money to the borrowed sum and lent it to C for the same periods at 14% p.a. rate of interest. If A gains Rs. 93.90 in the whole transaction, how much money did he add from his side ?

(A) Rs. 35 (B) Rs. 55 (C) Rs. 80 (D) Rs. 105.

7. A person borrowed Rs. 500 @ 3% per annum S.I. and Rs. 600 @ $4\frac{1}{2}\%$ per annum on the agreement that the whole sum will be returned only when the total interest becomes Rs. 126. The number of years, after which the borrowed sum is to be returned is
(A) 2 (B) 3 (C) 4 (D) 5

8. The rates of simple interest in two banks A and B are in the ratio 5 : 4. A person wants to deposit his total savings into banks in such a way that he receives equal half yearly interests from both. He should deposit the savings in banks A and B in the ratio
(A) 2 : 5 (B) 4 : 5 (C) 5 : 2 (D) 5 : 4

9. What annual payment will discharge a debt of Rs. 6450 due in 4 years at 5% simple interest ?
(A) Rs. 1500 (B) Rs. 1600 (C) Rs. 1525 (D) Rs. 1450

10. What is the simple interest on Rs. 3000 at $6\frac{1}{4}\%$ per annum for the periods from 4th Feb, 2005 to 18 April, 2005 ?
(A) Rs. 100.2 (B) Rs. 37.50 (C) Rs. 42.2 (D) Rs. 1.33

11. A sum of Rs. 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, then it would amount to how much ?
(A) Rs. 992 (B) Rs. 882 (C) Rs. 100 (D) Rs. 113

12. A sum of Rs. 1550 is lent out into two parts. One at 8% and the other at 6%. If the total interest is Rs. 106, find the money lent at each rate.
(A) 200 (B) 900 (C) 100 (D) None of these

13. What annual payment will discharge a debt of Rs. 6,450 due in 4 years at 5% per annum simple interest ?
(A) Rs. 1,400 (B) Rs. 1,500 (C) Rs. 1,550 (D) Rs. 1,600

14. If a sum of money at simple interest doubles in 12 years, the rate of interest per annum is
(A) $16\frac{2}{3}\%$ (B) 7.5% (C) $8\frac{1}{3}\%$ (D) 10%

15. Out of Rs. 50,000, that a man has, he lends Rs. 8000 at $5\frac{1}{2}\%$ per annum simple interest and Rs. 24,000 at 6 % per annum simple interest. He lends the remaining money at a certain rate of interest so that he gets total annual interest of Rs. 3680. The rate of interest per annum, at which the remaining money is lent, is
(A) 5% (B) 7% (C) 10% (D) 12%

16. Kamal took Rs. 6800 as a loan which along with interest is to be repaid in two equal annual instalments. If the rate of interest is $12\frac{1}{2}\%$, compounded annually, then the value of each instalment is
(A) Rs. 8100 (B) Rs. 4150 (C) Rs. 4050 (D) Rs. 4000

17. A man lent Rs. 60,000, partly at 5% and the rest at 4% simple interest. If the total annual interest is Rs. 2560, the money lent at 4% was
(A) Rs. 40000 (B) Rs. 44000 (C) Rs. 30000 (D) Rs. 45000

18. If Rs 12,000 is divided into two parts such that the simple interest on the first part for 3 years at 12% per annum is equal to the simple interest on the second part for $4\frac{1}{2}$ years at 16% per annum, the greater part is

(A) Rs. 8,000 (B) Rs. 6,000 (C) Rs. 7,000 (D) Rs. 7,500

19. At what rate of simple interest per annum will a sum become $\frac{7}{4}$ of itself in 4 years ?

(A) 18% (B) $18\frac{1}{4}\%$ (C) $18\frac{3}{4}\%$ (D) $18\frac{1}{2}\%$

20. A sum of money at a certain rate per annum of simple interest doubles in the 5 years and at a different rate becomes three times in 12 years. The lower rate of interest per annum is

(A) 15% (B) 20% (C) $15\frac{3}{4}\%$ (D) $16\frac{2}{3}\%$

21. Rs. 6,000 becomes Rs. 7,200 in 4 years at a certain rate of simple interest. If the rate becomes 1.5 times of itself, the amount of the same principal in 5 years will be

(A) Rs. 8,000 (B) Rs. 8,250 (C) Rs. 9,250 (D) Rs. 9,000

22. Simple interest on Rs. 500 for 4 years at 6.25% per annum is equal to the simple interest on Rs. 400 at 5% per annum for a certain period of time. The period of time is

(A) 4 years (B) 5 years (C) $6\frac{1}{4}$ years (D) $8\frac{2}{3}$ years

23. With a given rate of simple inerest, the ratio of principal and amount for a certain period of time is 4 : 5. After 3 years, with the same rate of interest, the ratio of the principal and amount becomes 5 : 7. The rate of interest is

(A) 4% (B) 6% (C) 5% (D) 7%

24. Rs. 1,000 is invested at 5% per annum simple interest. If the interest is added to the principal after every 10 years, the amount will become Rs. 2,000 after

(A) 15 years (B) 18 years (C) 20 years (D) $16\frac{2}{3}$ years

25. A sum of money amounts to Rs. 5,200 in 5 years and to Rs. 5,680 in 7 years at simple interest. The rate of interest per annum is

(A) 3% (B) 4% (C) 5% (D) 6%

Answers Key

1.(D)	2.(B)	3.(A)	4.(A)	5.(C)
6.(D)	7.(B)	8.(B)	9.(A)	10.(B)
11.(A)	12.(B)	13.(B)	14.(C)	15.(C)
16.(C)	17.(B)	18.(A)	19.(C)	20.(D)
21.(B)	22.(C)	23.(C)	24.(D)	25.(D)

..Notes

8 Compound Interest

Key Points:

1. We have learnt that if Principal = Rs. P, Rate = R% per annum and Time = T years, then the simple interest is given by the formula

   S.I. $= \frac{P \times R \times T}{100}$

   Clearly, when money is borrowed on simple interest, the interest is calculated uniformly on the original principal throughout the loan period.

   However, in post offices, banks, insurance corporations and the other money lending and deposit taking companies, the method of calculating interest is quite different.

   Under this method, the borrower and the lender agree to fix up a certain unit of time, say yearly or half-yearly or quarterly, to settle the previous account.

   In such cases, the interest accrued during the first unit of time is **added** to the original principal and the amount so obtained is taken as the Principal for the second unit of time. The Amount of this Principal at the end of second unit of time becomes the Principal for the third unit of time and so on.

   After a certain specified period, the difference between the Amount and the money borrowed is called the Compound Interest (C.I.) for that period.
2. The fixed unit of time is known as the conversion period.
3. Let us take an example to explain the process. Let us suppose Rs. 1000 is lent for 2 years at 10%.

   Obviously,

   S.I. = $\frac{1000 \times 2 \times 10}{100} = \text{Rs. } 200.$

   Hence, amount after 2 years under S.I.

   = Rs. 1000 + Rs. 200 = Rs. 1200.

   Now, interest on Rs. 1000 at 10% after 1 year.

   $= \frac{1000 \times 10 \times 1}{100} = \text{Rs. } 100$

   Under C.I. this interest will be added to the original principal i.e. Rs. 1000 so that the amount after 1 year = Rs. 1000 + Rs. 100. i.e. Rs. 1100. This amount becomes the principal for the 2nd year. Now, interest on Rs. 1100 at 10% for 1 year

   $= \frac{1100 \times 10 \times 1}{100} = \text{Rs. } 110$

   Hence, amount after 2 years

   = Rs. 1100 + Rs. 110 = Rs. 1210

   It is the final amount.

   ∴ C.I. = Final amount – Original amount

   = Rs. (1210 – 1000) = Rs. 210
4. **Formulae**

   (i) If P is the original Principal, R is the rate of interest per annum, T is the number of years, for which the money is lent and A is the final amount, then

   $$A = P\left(1 + \frac{R}{100}\right)^T$$

   C.I. = A – P

   (ii) If the interest is payable half-yearly, then time is multiplied by 2 and the rate is halved.

   i.e., Amount (A) = $P\left[1 + \frac{(R/2)}{100}\right]^{2T}$

(iii) If the interest is payable quarterly, then time is multiplied by 4 and the rate is divided by 4.

i.e., Amount (A) $= P\left[1+\frac{(R/4)}{100}\right]^{4T}$

(iv) When interest is compounded annually but time is in fraction, say $5\frac{2}{3}$ years, then

Amount (A) $= P\left[1+\frac{R}{100}\right]^{5}\left(1+\frac{\frac{2}{3}R}{100}\right)$

(v) When rates are different for different years, say R_1%, R_2%, R_3% for 1st, 2nd and 3rd year respectively.

Then, Amount $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

5. If a sum becomes x times in y years at CI then it will be $(x)^n$ times in (ny) years.

6. If a certain sum becomes 'm' times in 't' years, the rate of compound interest R is equal to $100\left[(\mathrm{m})^{1/t}-1\right]$

7. If the difference between SI and CI on a certain sum of money for 2 years at R% per annum is D. Then the sum (Principal) $P = \left(\frac{100}{R}\right)^2 \times D$

8. If the difference between SI and CI for a certain sum of money for 3 years at R% per annum is D. Then the sum (Principal) $P = \frac{D \times 100^3}{R^2(R+300)}$

9. If a sum 'A' becomes 'B' in T_1 years at compound rate of interest, then after T_2 years the sum becomes

$\frac{(B)^{T_2/T_1}}{A^{T_2/T_1-1}}$ rupees.

10. The compound interest is calculated annually in general unless some other period is clearly otherwise stated.

11. The compound interest is always greater than the simple interest for the same period and at the same rate. However, the C.I. for one year is equal to the S.I. for one year if calculated annually.

An Important Table

Time→	1 year	2 years	3 years	4 years
Rate↓	$\left[1+\frac{r}{100}\right]$	$\left[1+\frac{r}{100}\right]^2$	$\left[1+\frac{r}{100}\right]^3$	$\left[1+\frac{r}{100}\right]^4$
4%	$\frac{26}{25}$	$\frac{676}{625}$	$\frac{17576}{15625}$	$\frac{456976}{390625}$
5%	$\frac{21}{20}$	$\frac{441}{400}$	$\frac{9261}{8000}$	$\frac{194481}{160000}$
10%	$\frac{11}{10}$	$\frac{121}{100}$	$\frac{1331}{1000}$	$\frac{14641}{10000}$
20%	$\frac{6}{5}$	$\frac{36}{25}$	$\frac{216}{125}$	$\frac{1296}{625}$

..Notes

Solved Examples

Ex.1. *If the compound interest on a sum of money for 2 years at 4% per annum is Rs. 10 more than the simple interest on the same sum for the same period and at the same rate. The find the sum.*

Sol. Let the sum be Rs. *x*.

$\therefore$ C.I. = $x\left[\left(1+\frac{4}{100}\right)^2-1\right]=x\left[\left(\frac{26}{25}\right)^2-1\right]$

$= x\times\frac{676}{625}-x=\frac{676x-625x}{625}$ = Rs. $\frac{51x}{625}$

$\therefore$ Simple interest = $\frac{x\times4\times2}{100}$ = Rs. $\frac{2x}{25}$

According to the question,
C.I. – S.I. = Rs. 10

$\therefore$ $\frac{51x}{625}-\frac{2x}{25}=10$ or $\frac{51x-50x}{625}=10$

$\Rightarrow$ x = 625 × 10 = Rs. 6250

Hence, the sum of money = Rs. 6250

Short-cut Method

Since the difference of interest in the two cases is Rs. 10.

$\therefore$ For two years, x = 10, R = 4%

Sum = $x\times\left(\frac{100}{R}\right)^2=10\times\left(\frac{100}{4}\right)^2$

= 10 × 25 × 25 = Rs. 6250

Ex.2. *The difference between compound and simple interest on a certain sum of money for 3 years at 10% per annum is Rs. 15.50. Find the sum.*

Sol. Let the sum be Rs. *x*.

$\therefore$ C.I. = $x\left[\left(1+\frac{10}{100}\right)^3-1\right]=x\left[\left(\frac{11}{10}\right)^3-1\right]$

= $x\left[\frac{1331-1000}{1000}\right]$ = Rs. $\frac{331}{1000}x$

and S.I. = $\frac{x\times3\times10}{100}$ = Rs. $\frac{3x}{10}$

According to the question,

$\Rightarrow$ C.I. – S.I. = Rs. 15.50

$\frac{331}{1000}x-\frac{3x}{10}$ = 15.50 $\Rightarrow$ $\frac{331x-300x}{1000}$ = 15.50

$\Rightarrow$ $31x$ = 15500 or x = 500

Hence the sum = Rs. 500

Short-cut Method

Since the difference in interest for 3 years. *i.e.,*

x = Rs. 15.15 and R = 10%.

$$\therefore \quad \text{Sum} = \frac{x \times (100)^3}{R^2(300+R)} = \frac{15.50 \times (100)^3}{10^2(300+10)}$$

$$= \frac{15.50 \times 100 \times 100}{310} = \text{Rs. } 500$$

Ex.3. *A and B borrowed a total sum of Rs. 8280 from a money lender at 7% per annum compound interest in such a proportion that to settle his loan after 3 years, A paid as much amount as was paid by B after 4 years from the date of borrowing. Find the part of money borrowed by B.*

Sol. Let A's part = Rs. x, B's part = Rs. $(8280 - x)$

$$\therefore \quad \text{For A's amount} = x\left(1+\frac{7}{100}\right)^3 = x \times \left(\frac{107}{100}\right)^3$$

and for B's amount $= (8280 - x) \times \left(\frac{107}{100}\right)^4$

Since amount in these two cases remains equal, therefore

$$\therefore \quad x \times \left(\frac{107}{100}\right)^3 = (8280 - x) \times \left(\frac{107}{100}\right)^4$$

$$\Rightarrow \quad x = (8280 - x)\left(\frac{107}{100}\right) \quad \text{or } 100x = 8280 \times 107 - 107x$$

$$\Rightarrow \quad 100x + 107x = 8280 \times 107$$

$$\Rightarrow \quad 207x = 8280 \times 107 \quad \text{or } x = 4280$$

$\therefore$ B borrowed Rs. (8280 – 4280) = Rs. 4000

Short-cut Method

$\because$ For A, Time = 3 years, Rate = 7%

For B, Time = 4 years, Rate = 7%

Since amount in both the cases remains equal, therefore

$\therefore$ Ratio A's part to B's part

$$= \frac{1}{\left(1+\frac{R_1}{100}\right)^{t_1}} : \frac{1}{\left(1+\frac{R_2}{100}\right)^{t_2}} = \frac{1}{\left(1+\frac{7}{100}\right)^3} : \frac{1}{\left(1+\frac{7}{100}\right)^4}$$

$$\Rightarrow \quad \frac{1}{\left(\frac{107}{100}\right)^3} : \frac{1}{\left(\frac{107}{100}\right)^4}$$

or $\frac{107}{100}:1$ or 107 : 100

∴ Sum of proportionals = 107 + 100 = 207

∴ Part of B out of total sum borrowed *i.e.* 8280

= $\frac{\text{ü}}{\text{ü}}\times 8280$ = Rs. 4000

Ex.4. *A sum of money placed at compound interest doubles itself in 10 years. In how many years will it amount to four times itself ?*

Sol. Short-cut Method

t_1 = 10 years, $n_1 = 2$, $t_2 = ?$, $n_2 = 4$

$$(n_1)^{1/t_1} = (n_2)^{1/t_2} \text{ or } (4)^{\frac{1}{t_1}} = (4)^{\frac{1}{t_2}}$$

$$\Rightarrow \quad (2)^{\frac{1}{10}} = (2^2)^{\frac{1}{t_2}} \quad \Rightarrow \quad \frac{1}{10} = \frac{2}{t_2}$$

Hence, the sum will become four-fold of itself in 20 years.

Ex.5. *Find the compound interest on Rs. 16000 at 20% per annum for 9 months, compounded quarterly.*

Sol. Given, principal = Rs. 16000; Time = 9 months = 3 quarters; Rate = 20% per annum = 5% per quarter.

∴ Amount = Rs. $\left[16000\times\left(1+\frac{5}{100}\right)^3\right]$

$$= \left(16000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right) = \text{Rs.}18522$$

∴ C.I. = Rs. (18522 – 16000) = Rs. 2522

Ex.6. *Divide Rs.* 1301 *between* A *and* B, *so that the amount of* A *after* 7 *years is equal to the amount of* B *after* 9 *years, the interest being compounded at* 4% *per annum.*

Sol. Let the two parts be Rs. x and Rs. $(1301 - x)$.

$$\therefore \quad x\left(1+\frac{4}{100}\right)^7 = (1301-x)\left(1+\frac{4}{100}\right)^9$$

$$\Rightarrow \quad \frac{x}{(1301-x)} = \left(1+\frac{4}{100}\right)^2 = \left(\frac{26}{25}\times\frac{26}{25}\right)$$

⇔ $625x = 676\ (1301 - x)$ or $1301x = 676 \times 1301$

⇔ $x = 676$

∴ Two parts are Rs. 676 and Rs. (1301 – 676) *i.e.*, Rs. 676 and Rs. 625.

Practice Exercise

Directions: Each of the questions given below is followed by four or five alternatives of which one is correct. Mark **(√)** against the correct answer.

1. **The compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly is**
   (A) Rs. 636.80 (B) Rs. 824.32 (C) Rs. 912.86 (D) Rs. 828.82

2. **The difference between the compound interest (compounded annually) and the simple interest on a sum of Rs. 1000 at a certain rate of interest for 2 years is Rs. 10. The rate of interest per annum is**
   (A) 5% (B) 6% (C) 10% (D) 12%

3. **What would be compound interest on a sum of Rs. 6000 at the end of three years at the rate 5 per cent per annum ?**
   (A) Rs. 615.00 (B) Rs. 945.75 (C) Rs. 1293.04 (D) Rs. 900.00

4. **The compound interest on Rs. 16000 for 9 months at 20 per cent per annum, interest being compounded quarterly, is**
   (A) Rs. 2520 (B) Rs. 2524 (C) Rs. 2522 (D) Rs. 2518

5. **The compound interest on a certain sum at 5% annum for 2 years is Rs. 328. The simple interest for that sum at the same rate and for the same period will be**
   (A) Rs. 320 (B) Rs. 312 (C) Rs. 305 (D) Rs. 300

6. **The difference between the compound interest and simple interest on a certain amount of money at 5% per annum for 2 years is Rs. 15. Find the principal sum.**
   (A) Rs. 4500 (B) Rs. 7500 (C) Rs. 5000 (D) Rs. 6000.

7. **The sum of money at compound interest doubles itself in 5 years. It will amount to 8 times itself at the same rate of interest in**
   (A) 10 years (B) 15 years (C) 7 years (D) 20 years

8. **If the compound interest on a certain sum for 2 years at 10% per annum is Rs. 2100, then simple interest on it at the same rate for 2 years would be**
   (A) Rs. 1700 (B) Rs. 1800 (C) Rs. 1900 (D) Rs. 2000

9. **The simple interest on a sum of money Rs. 1800 for 2 years is Rs. 108. What would be the compound interest on the same sum of money for the same period, if the rate of interest is further increased by 4% ?**
   (A) 260.82 (B) 245.80 (C) 270.72 (D) 250.82

10. **Present ages of Tarun and Gulshan are 12 years and 10 years respectively. They want to deposit in post office a total sum of Rs. 5204 in such a proportion that at the age of 15, each of them may get an equal amount if the post office pays 4% p.a. compound interest on the total sum of deposit. Find the part of money deposited by each of them.**
    (A) Rs. 2500, Rs. 2704 (B) Rs. 2704, Rs. 2500
    (C) Rs. 2804, Rs. 2400 (D) Rs. 2400, Rs. 3000

11. **The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly is**

(A) 6.06% (B) 6.07% (C) 6.08% (D) 6.09%

12. **Mr. Das invested money in two schemes A and B offering compound interest @ 8 p.c.p.a. and 9 p.c.p.a. respectively. If the total amount of interest accrued through two schemes together in two years was Rs. 4818.30 and the total amount invested was Rs. 27000, then what was the amount invested in Scheme A?**

(A) Rs. 12,000 (B) Rs. 13,500
(C) Rs. 15,000 (D) Cannot be determined

13. **If a sum on compound interest becomes three times in 4 years, then with the same interest rate, the sum will become 27 times in**

(A) 8 years (B) 12 years (C) 24 years (D) 36 years

14. **A sum of money is borrowed and paid back in two annual instalment of Rs. 882 each allowing 5% compound interest. The sum borrowed was**

(A) Rs. 1620 (B) Rs. 1640 (C) Rs 1680 (D) Rs. 1700

15. **A sum of money at compound interest doubles itself in 15 years. It will become eight times of itself in**

(A) 45 years (B) 48 years (C) 54 years (D) 60 years

16. **At what rate per cent per annum will a sum of Rs. 1,000 amount to Rs. 1,102.50 in 2 years at compound interest ?**

(A) 5 (B) 5.5 (C) 6 (D) 6.5

17. **In what time will Rs. 10,000 amount to Rs. 13310 at 20% per annum compounded half yearly?**

(A) $1\frac{1}{2}\%$ years (B) 2 years (C) $2\frac{1}{2}$ years (D) 3 years

18. **At a certain rate per annum, the simple interest on a sum of money for one year is Rs. 260 and the compound interest on the same sum for two years is Rs. 540.80. The rate of interest per annum is**

(A) 4% (B) 6% (C) 8% (D) 10%

19. **A certain sum of money yields Rs. 1261 as compound interest for 3 years at 5% per annum. The sum is**

(A) Rs. 9000 (B) Rs. 8400 (C) Rs. 7500 (D) Rs. 8000

20. **The simple interest on a sum of money at 4% per annum for 2 years is Rs. 80. The compound interest in the same sum for the same period is**

(A) Rs. 82.60 (B) Rs. 82.20 (C) Rs. 81.80 (D) Rs. 81.60

21. **A sum of Rs 13,360 was borrowed at $8\frac{3}{4}$% per annum compound interest and paid back in two years in two equal annual instalments. What was the amount of each instalment ?**

(A) Rs. 5,769 (B) Rs. 7,569 (C) Rs. 7,009 (D) Rs. 7,500

22. A certain sum, invested at 4% per annum compound interest, compounded half - yearly, amounts to Rs 7,803 at the end of one year. The sum is

(A) Rs 7,000 (B) Rs 7,200 (C) Rs 7,500 (D) RS 7,700

23. The difference between compound and simple interests on a certain sum for 3 years at 5% per annum is Rs. 122. The sum is

(A) Rs 16,000 (B) Rs 15,000 (C) Rs 12,000 (D) Rs 10,000

24. A certain sum amount to Rs. 5,832 in 2 years at 8% per annum compound interest, the sum is

(A) Rs. 5,000 (B) Rs. 5,200 (C) Rs. 5,280 (D) Rs. 5,400

25. The compound interest on a certain sum of money at 5% per annum for 2 years is Rs. 246. The simple interest on the same sum for 3 years at 6% per annum is

(A) Rs 435 (B) Rs 450 (C) Rs 430 (D) Rs 432

Answers key

1.(B)	2.(C)	3.(B)	4.(C)	5.(A)
6.(D)	7.(B)	8.(D)	9.(A)	10.(B)
11.(D)	12.(A)	13.(B)	14.(B)	15.(A)
16.(A)	17.(A)	18.(C)	19.(D)	20.(D)
21.(B)	22.(C)	23.(A)	24.(A)	25.(D)

9 Square Roots & Cube Roots

FACTS AND FORMULAE

I. Square Roots

The square root of a number is the number which when multiplied by itself produces the number in question i.e., if $x^2 = y$, then the square root of y is x or $\sqrt{y} = x$.

Square root of a given number can be obtained by two methods.

- Prime Factorisation Method
- Division Method

II. Cube Roots : The cube root of a number is the number, the third power of which gives the number in question. The cube root of a given number x is the number whose cube is x and cube root of x is denoted by $\sqrt[3]{x}$. We can find the cube root of a number by Prime Factorisation Method only.

III. Conceptual Facts

(*i*) $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

(*ii*) $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$

Solved Examples

Ex.1. *Find the square root of 9, 25, 144 and 32 +* $\sqrt{5+\sqrt{121}}$ *by Prime Factorisation Method.*

Sol. Square root of 9 = $\sqrt[2]{9} = \sqrt{3 \times 3} = 3$

Similarly, square root of 25 = $\sqrt[2]{25} = \sqrt{5 \times 5} = 5$

Square root of 144 = $\sqrt[2]{144} = \sqrt{12 \times 12} = 12$, and

Square root of $32 + \sqrt{5+\sqrt{121}}$ = $\sqrt[2]{32+\sqrt{5+\sqrt{121}}}$

= $\sqrt[2]{32+\sqrt{5+11}}$ = $\sqrt{32+\sqrt{16}}$

= $\sqrt[2]{32+4}$ = $\sqrt[2]{36}\ \Re\sqrt[2]{6\ \ 6}\ \ \ 6$

Ex.2. *If* $\sqrt{3} = 1.732$, *then find the value of* $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ *correct to 3 places of decimal.*

Sol. Given, $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$

= $\sqrt{64 \times 3} - \frac{1}{2}\sqrt{16 \times 3} - \sqrt{25 \times 3}$

= $8\sqrt{3} - \frac{1}{2} \times 4\sqrt{3} - 5\sqrt{3}$

= $3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$ = 1.732

Ex.3. *Find the least perfect square, which is divisible by each of 21,36 and 66.*

Sol. LCM of 21, 36, 66 = 2772, and

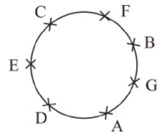
2772 = 2 × 2 × 3 × 3 × 7 × 11

To make it a perfect square, it must be multiplied by 7 × 11.

∴ Required number = $2^2 \times 3^2 \times 7^2 \times 11^2$ = 213444

Ex.4. *The smallest number added to 680621 to make the sum a perfect square ?*

Sol.



∴ Number to be added = $(825)^2 - 680621$

= 680625 – 680621 = 4

Practice Exercise

Directions. Each of the questions given below is followed by four or five alternatives of which one is correct. Mark (✓) against the correct answer.

1. **If** $x = \left(\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)$ and $y = \left(\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$, **then find the value of** (x^2+y^2).

(A) 31 (B) 56 (C) 62 (D) 96

2. **If** $\sqrt{1+\dfrac{x}{144}} = \dfrac{13}{12}$, **then *x* equals to**

(A) 1 (B) 13 (C) 27 (D) None of these

3. **One-third of the square root of which number is 0.001?**

(A) 0.0009 (B) 0.000001 (C) 0.000009 (D) None of these

4. **The value of** $\sqrt{\dfrac{(0.1)^2+(0.01)^2+(0.009)^2}{(0.01)^2+(0.001)^2+(0.0009)^2}}$ **is**

(A) 10^2 (B) 10 (C) 0.1 (D) 0.01

5. **What is the least number which should be subtracted from 0.000326 to have perfect square?**

(A) 0.000004 (B) 0.000002 (C) 0.04 (D) 0.02.

6. **The value of** $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$ **is**

(A) 1 (B) 2 (C) 3 (D) 8

7. **If** $\sqrt{3} = 1.7321$, **then value of** $\sqrt{192} - \dfrac{1}{2}\sqrt{48} - \sqrt{75}$ **correct to 3 places of decimal is**

(A) 8.661 (B) 4.331 (C) 1.732 (D) – 1.732

8. **What is the value of** $\sqrt[3]{\sqrt{0.000064}}$ **?**

(A) 0.02 (B) 0.2 (C) 2.0 (D) 20.0

9. If $3a = 4b = 6c$ and $a + b + c = 27\sqrt{29}$, then the value of $\sqrt{a^2+b^2+c^2}$ is

(A) $3\sqrt{29}$ (B) 81 (C) 87 (D) 91

10. $\sqrt{0.0169\times(?)}=1.3$

(A) 10 (B) 100 (C) 1000 (D) None of these

11. If $\sqrt{4096}=64$, then the value of

$\sqrt{40.96}+\sqrt{0.4096}+\sqrt{0.004096}+\sqrt{0.00004096}$ is

(A) 7.09 (B) 7.10114 (C) 7.1104 (D) 7.12

12. The value of x in the equation

$\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=2\frac{1}{6}$ is

(A) $\frac{5}{13}$ (B) $\frac{7}{13}$ (C) $\frac{9}{13}$ (D) None of these

13. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of on variety only. Minimum number of rows in which the trees may be planted is

(A) 20 (B) 17 (C) 15 (D) 3

14. If the ratio between the roots of the equation $lx^2+nx+n=0$ is $p : q$, then the value of $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}$ is

(A) 4 (B) 3 (C) 0 (D) -1

15. If $x^{1/3}+y^{1/3}+z^{1/3}=0$, then

(A) $x+y+z=0$ (B) $(x+y+z)^3=27xyz$

(C) $x+y+z=3xyz$ (D) $x^3+y^3+z^3=0$

16. The square root of

$\frac{(0.75)^3}{1-0.75}+\left[0.75+(0.75)^2+1\right]$ is

(A) 1 (B) 2 (C) 3 (D) 4

17. The greatest number among $\sqrt{5},\sqrt[3]{4},\sqrt[5]{2},\sqrt[7]{3}$ is

(A) $\sqrt[3]{4}$ (B) $\sqrt[7]{3}$ (C) $\sqrt{5}$ (D) $\sqrt[5]{2}$

18. $\sqrt[3]{(13.608)^2-(13.392)^2}$ is equal to

(A) 0.6 (B) 0.06 (C) 1.8 (D) 2.6

19. $\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$ **is equal to**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

20. The square root of

$\frac{9.5\times0.0085\times18.9}{0.0017\times1.9\times2.1}$ **is**

(A) 15 (B) 45 (C) 75 (D) 225

21. $\frac{(0.87)^3+(0.13)^3}{(0.87)^2+(0.13)^2-(0.87)\times(0.13)}$ **is equal to**

(A) $\frac{1}{2}$ (B) 2 (C) 1 (D) $2\frac{1}{2}$

22. The largest among the numbers

$\sqrt{7}-\sqrt{5}, \sqrt{5}-\sqrt{3}, \sqrt{9}-\sqrt{7}, \sqrt{11}-\sqrt{9}$ **is**

(A) $\sqrt{7}-\sqrt{5}$ (B) $\sqrt{5}-\sqrt{3}$ (C) $\sqrt{9}-\sqrt{7}$ (D) $\sqrt{\text{ü}}-\sqrt{\ }$

23. If $x^{1/3}+y^{1/3}=z^{1/3}$ **,then**

$(x+y-z)^3+27\,xyz$ is equal to

(A) 0 (B) 1 (C) −1 (D) 27

24. If $\sqrt{7\sqrt{7\sqrt{7\sqrt{7....}}}}=(343)^{y-1}$ **, then *y* is equal to**

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) $\frac{3}{4}$

25. $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1}$ **is equal to**

(A) $\sqrt[3]{2}+1$ (B) $\sqrt[3]{2}-1$ (C) $-\sqrt[3]{2}-1$ (D) $1-\sqrt[3]{2}$

Answers Key

1.(C)	2.(D)	3.(C)	4.(B)	5.(B)
6.(B)	7.(C)	8.(B)	9.(C)	10.(B)
11.(C)	12.(C)	13.(B)	14.(C)	15.(B)
16.(B)	17.(C)	18.(C)	19.(A)	20.(A)
21.(C)	22.(B)	23.(A)	24.(C)	25.(B)

..Notes

10 Profit and Loss

A consumer goes to the market and buys certain goods. The buyer is called a customer and the shopkeeper who sells the goods to him is called a retailer. The retailer in turn purchases goods in bulk from a wholesaler who keeps a large stock of goods. In the retailer-wholesaler deal, the retailer becomes the customer.

Cost Price (C.P.) : The price at which a person buys an article is called the *Cost Price* (C.P.) of the article.

Selling Price (S.P.) : Thte price at which an article is sold is called the *Selling Price* (S.P.) of the article. Whenever there is a transaction between the two parties i.e., the buyer and the seller, the price at which the article is sold by the seller is same as the price at which it is purchased by the buyer. Normally, the Cost Price and the Selling Price in a problem are commonly used in a slightly modified sense. This is clarified by the following illustration :

A sells an article to *B* at price Rs. *x*, *B* in turn sells it to *C* at price Rs. *y*. Here, Rs. *x* is the Selling Price for *A* as well as the Cost Price for *B*. But the Selling Price for *B* is Rs. *y* which is also the Cost Price for *C*.

Profit or Gain : Whenever a person sells an article at price greater than the Cost Price, he is said to have made a profit or gain.

Profit or Gain = S.P. – C.P.

In the above example *B*'s profit = Rs. y – Rs. x

Loss : When S.P. is less than the C.P., one makes a loss.

Loss = C.P. – S.P.

In the above example *B*'s loss = Rs. x – Rs. y.

Note : S.P. and C.P. must be for the same article and for the same quantity and also must be referred to the same person while calculating profit or loss.

For example, we can't take the difference of the Price of table and chair; or difference of Cost Price of 10 kgs of Sugar and Selling Price of 2 kgs of Sugar for calculating profit or loss. Further, if we are calculating *B*'s profit or loss, then *B*'s C.P. and S.P. should be used and not *A*'s or *C*'s.

Note : Profit or Loss are always reckoned on Cost Price.

Gain per cent or % gain is gain on the reference value & Rs. 100.

Loss per cent or % loss is loss on the reference value & Rs. 100.

Some Basic Formulae :

Gain = S.P. – C.P.

Loss = C.P. – S.P.

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\%$$

$$\% \text{ loss} = \frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100\%$$

From these we can write direct expressions for S.P. and C.P.

$$\text{S.P.} = \left(\frac{100 + \%\text{ Profit}}{100}\right) \times \text{C.P. in case of profit}$$

$$\text{and S.P.} = \left(\frac{100 - \%\text{ Loss}}{100}\right) \times \text{C.P. in case of loss}$$

These can be rewritten as

$$\text{C.P.} = \left(\frac{100}{100 + \%\text{Profit}}\right) \times \text{S.P. in case of profit}$$

$$\text{and C.P.} = \left(\frac{100}{100 - \%\text{ Loss}}\right) \times \text{S.P. in case of loss}$$

Overheads : The expenses incurred on transportation, rent, personnel salary, maintenance, packaging, advertisements and the like are included under the general heading of Overheads. These overheads and the profit when added to the cost price determine the Selling Price. If the overheads are not separately mentioned in the problem, we assume it to be zero or else they have been included in the Cost Price itself.

Discount : It is an offer made by the seller to the buyer for reduction in price to be paid. There are several cases where discounts are allowed. For instance, to dispose off old goods, to increase its market share, when the customer is ready to pay the whole amount in cash instead of payments in instalments and so on. It is subtracted from the original price and is usually expressed as per cent or a fraction of the market price. The price obtained after deducting the discount from the original price is the selling price which the customer has to pay.

Solved Examples

Ex.1. *A shopkeeper allows 10% discount on the marked price of an article and still gains 25%. Find the cost price of the article which has been marked as Rs. 150.*

Sol. S.P. of the article after allowing 10% discount on marked price = Rs. 150 – 10% of Rs. 150

= Rs. 135 and profit % = 25%

$\therefore$ C.P. of the article $= \text{S.P.} \times \frac{100}{125} = 135 \times \frac{100}{125}$ = Rs. 108

Hence, C.P. of the article = Rs. 108

Ex.2. *By selling a radio set for Rs. 1300, Sanjay suffered some loss and his CP. had he sold the radio set for Rs. 1480, he would have gained 20% of the loss suffered in the first case. Find the cost price of the radio ?*

Sol. Let Sanjay sold the radio set for Rs. 1300 at a loss of Rs. x. Then,

C.P. of the radio set = Rs. $(1300 + x)$

According to question

S.P. = Rs. 1480, Profit = 20% of x = Rs. $\frac{20x}{100}$ = Rs. $\frac{x}{5}$

C.P. = S.P. – Profit = Rs. 1480 – Rs. $\frac{x}{5}$ . . . . (*ii*)

By hypothesis,

$1300 + x = 1480 - \frac{x}{5} \Rightarrow \frac{6x}{5} = 180$ or $x = 150$

C.P. of the ratio = Rs. 1300 + Rs. 150 = Rs. 1450

Ex.3. *A man bought 2 cows for Rs. 1600. He sold one cow at a gain of 25% and the other at a loss of 25%. If he sold the two cows for the same price, find the cost price of each the cows.*

Sol. Let the selling price of each of the cows = Rs. x. Then,

For the Ist cow

S.P. = Rs. x, Profit = 25%

C.P. $= \text{S.P.} \times \frac{100}{125} = x \times \frac{4}{5}$ = Rs. $\frac{4x}{5}$

For the IInd cow

S.P. = Rs. x and loss % = 25%

C.P. = S.P. × $\frac{100}{75}$ $= x \times \frac{4}{3}$ = Rs. $\frac{4x}{3}$

Since C.P. of the two cows is Rs. 1600, therefore

$$\frac{4x}{5} + \frac{4x}{3} = 1600 \Rightarrow \frac{12x + 20x}{15} = 1600$$

or $32x = 24000$ or $x = 750$

∴ C.P. of the first cows = Rs. $\frac{4x}{5} = \frac{4 \times 750}{5}$ = Rs. 600

and C.P. of the second cow

$$= \frac{4x}{3} = \text{Rs.}\frac{4 \times 750}{5}$$ = Rs. 1000

Ex.4. *An article displayed for sale in a shop is marked at Rs. 60. If this marked price is inclusive of 20% profit of the shopkeeper, then find the cost price of this article of the shopkeeper.*

Sol. Marked price = Rs.60, Profit = 20%

∴ C.P. of the article for the shopkeeper

= Rs. 60 × $\frac{100}{120}$ = Rs.50

Ex.6. *Suresh bought an article at 3/4 of its marked price and sold it at 20% more than its marked price. Find his per cent profit.*

Sol. Let marked price of the article = Rs. *x*.

∴ C.P. of the article for Suresh

= $\frac{3}{4}$ of Rs. x = Rs. $\frac{3x}{4}$

He sold the article at 20% more than its marked price

∴ S.P. of the article = Rs. x + 20% of Rs. x

= Rs. $\frac{6x}{5}$

Profit = S.P. – C.P. $= \frac{6x}{5} - \frac{3x}{4} = \frac{24x - 15x}{20} = \text{Rs.}\frac{9x}{20}$

Profit % $= \frac{\text{Profit} \times 100}{\text{C.P.}}$ $= \frac{\frac{9x}{20} \times 100}{3x/4}$ = 60%

Practice Exercise

Directions. Each of the questions given below is followed by four alternatives of which one is correct. Mark (✓) the correct answer.

1. A bought a horse for Rs. 9000 and sold it to B at a loss of 10%. B sold the same horse again to A at a gain of 10%. As a result of this transaction,

(A) B made a profit of Rs. 810 (B) A suffered a loss of Rs. 900

(C) A made a profit of Rs. 800 (D) B made a profit of Rs. 900

2. A man after getting two successive discounts of 10% and 10% on the list price of an article bought it for Rs. 445.50. The article was listed at

(A) Rs. 650 (B) Rs. 560 (C) Rs. 550 (D) Rs. 540

3. Saransh purchased 120 rims of paper at Rs. 80 per rim. He spent Rs. 280 on transportation, paid octroi at the rate of 40 paise per rim and paid Rs. 72 to the coolie. If he wants to have a gain of 8%, then what must be the selling price per rim ?

(A) Rs. 86 (B) Rs. 87.40 (C) Rs. 89 (D) Rs. 90

4. A sum of Rs. 1550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is Rs. 106, the money (in rupees) lent at 8% is

(A) 650 (B) 720 (C) 840 (D) 900

5. I purchased 120 exercise books at the rate of Rs. 3 each and sold $\frac{1}{3}$ of them at the rate of Rs. 4 each, $\frac{1}{2}$ of them at the rate of Rs. 5 each and the rest at the cost price. My profit per cent was

(A) 44% (B) $44\frac{4}{9}\%$ (C) $44\frac{2}{3}\%$ (D) 45%

6. A horse and a cow were sold for Rs. 12000 each. The horse was sold at a loss 20% and the cow at a gain 20%. The entire transaction resulted in

(A) no loss or gain (B) loss of Rs. 1000

(C) gain of Rs. 1000 (D) gain of Rs. 2000

7. A dealer offers a discount of 10% on the marked price of an article and still makes a profit of 20% if its marked price is Rs. 800, then the cost price of the article is

(A) Rs. 900 (B) Rs. 800 (C) Rs. 700 (D) Rs. 600

8. A dishonest dealer sells his goods at the cost price, but still earns a profit of 25% by under-weighing. What weight does he use for a kg?

(A) 750 gm (B) 800 gm (C) 825 gm (D) 725 gm

9. By giving a discount of 10% on the marked price of Rs. 1100 of a cycle, a deal gains 10%. The cost price of the cycle is

(A) Rs. 1100 (B) Rs. 900 (B) Rs. 1089 (D) Rs. 891

10. If the cost price of the 12 pens is equal to the selling price of 8 pens, the gain per cent is

(A) $33\frac{1}{3}\%$ (B) $66\frac{2}{3}\%$ (C) 25% (D) 50%

11. By selling an article for Rs. 69 there is a loss of 8 per cent. When the article is sold for Rs. 78, the gain or loss per cent is

(A) neither loss nor gain (B) 4% gain
(C) 4% loss (D) 40% gain

12. Nikita bought 30 kg of wheat at the rate Rs. 9.50 per kg and 40 kg of wheat at the rate of Rs. 8.50 per kg. Her total profit or loss in the transaction was

(A) Rs. 2 loss (B) Rs. 2 profit (C) Rs. 7 loss (D) Rs. 7 profit

13. A man bought a horse and a carriage for Rs. 3000. He sold the horse at a of 20% gain and the carriage at a loss of 10%, thereby gainig 2% on the whole. Find the cost of the horse.

(A) Rs. 500 (B) Rs. 1200 (C) Rs. 1500 (D) Rs. 2000

14. At what percentage above the C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?

(A) 20% (B) 40% (C) 60% (D) 65%

15. A person brought some articles at the rate of 5 per rupee and the same number at the rate of 4 per rupee. He mixed both the types and sold at the rate of 9 for 2 rupees. In this business, he suffered a loss of Rs. 3. The total number of articles bought by him was

(A) 1090 (B) 1080 (C) 540 (D) 545

16. A company blends two varieties of tea from two different tea gardens, one variety costing Rs. 20 per kg and the other Rs. 25 per kg in the ratio 5 : 4. He sells the blended tea a Rs. 23 per kg. Find his profit or loss per cent.

(A) 5% profit (B) 5% loss (C) 3.5% profit (D) No profit, no loss

17. A video magazine distributor made 3500 copies of the March issue of the magazine at a cost of Rs. 350000. He gave 500 cassettes free to some key video libraries. He also allowed a 25% discount on the market price of the cassette and gave one extra cassette free with every 29 cassettes baught at a time. In this manner, he was able to sell all the 3500 cassettes that were produced, if the market price of a cassette was Rs. 150, then what is his gain or loss per cent for the March issue of video magazine ?

(A) 25% loss (B) 10% gain (C) 40% gain (D) 6.8% loss

18. Company C sells a line 25 products with an average retail price of Rs. 1200. If none of these products sells for less than Rs. 420 and exactly 10 of the products sells for less than Rs. 1000, what is the greatest possible sellings price of the most expensive products ?

(A) Rs. 2,600 (B) Rs. 7,800 (C) Rs. 3,900 (D) Rs. 11,800

19. In a society, there are 100 members. Each of them has been allotted membership number, from 1 to 100. They started a business in which nth member contributed Rs $(10' 2^n - 5)$ after one year 4th member gets 62 as his share. Find the total profit in the business after one year ?

(A) Rs. 8 [2^{100} – 26] (B) Rs. 4 [2^{99} – 26]
(C) Rs. 2 [2^{100} – 26] (D) None of these

20. When a producer allows 36% commission on the retail price of his products, he earns a profit of 8.8%. What would be his profit per cent if the commission is reduced by 24%?

(A) 37.8% (B) 49.6% (C) 57.3% (D) 6.3%

21. Deepa bought a calculator with 30% discount on the listed price. Had she not got the discount, she would have Rs. 82.50 extra. At what price did she buy the calculator ?

(A) Rs. 192.50 (B) Rs. 275
(C) Rs. 117.85 (D) Cannot be determined

22. Some toffees were bought at the rate of 11 for Rs. 10 and the same number at the rate of 9 for Rs. 10. If the whole lot was sold at one rupee per toffee, then the gain or loss in the whole transaction was

(A) loss of 1% (B) gain of 1%
(C) neither gain nor loss (D) gain of 1.5%

23. A merchant finds his profit as 20% of the selling price. His actual profit is

(A) 20% (B) 22% (C) 25% (D) 30%

24. If a man were to sell his chair for Rs. 720, he would lose 25%. To gain 25% he should sell it for

(A) Rs. 1,200 (B) Rs. 1,000 (C) Rs. 960 (D) Rs. 900

25. A fruit seller buys lemons at 2 for a rupee and sells them at 5 for three rupees. His profit per cent is

(A) 10 (B) 15 (C) 20 (D) 25

Answers KEY

1.(A)	2.(C)	3.(D)	4.(A)	5.(B)
6.(B)	7.(D)	8.(B)	9.(B)	10.(D)
11.(B)	12.(A)	13.(B)	14.(B)	15.(B)
16.(C)	17.(D)	18.(D)	19.(A)	20.(B)
21.(A)	22.(A)	23.(C)	24.(A)	25.(C)

✍..Notes

11 Average

Average : The average of a number of quantities of observations of the same kind is their sum divided by their number. The average is also called average value or mean value or more accurately an arithmetic mean.

Thus,

Average $=\dfrac{\text{Sum of quantities or observations}}{\text{Number of quantities or observations}}$

It is self evident that

(a) Number of observations/quantities

$=\dfrac{\text{Sum of quantities}}{\text{Average}}$

(b) Sum of quantities = Average × Number of quantities

For example,

Ex. 1. The marks of a student in English, Mathematics, Physics and Chemistry are respectively 59, 82, 76 and 43. Find his average marks.

Sol. : Average marks

$=\dfrac{59+82+76+43}{4}=\dfrac{260}{4}=65$

Here the number of quantities is 4. Hence, to find average, we have divided their sum (260) by the number of quantities.

Ex. 2. In a family of five members, the age of father is 42 years, mother is 38 years and sum of three children's ages is 80 years. Find the average age of the family.

Sol. : Average age $=\dfrac{42+38+80}{5}$

(Number of total members = Father + Mother + 3 Sons = 5)

$=\dfrac{160}{5}=32$ years

Various Types of Questions Asked in Exam

Type-I: Questions Related to Temperature

Ex.1. The average temperature of Monday, Tuesday and Wednesday was 60°. The average temperature of Tuesday, Wednesday and Thursday was 60°. The temperature of Thursday was 60°. Find the temperature of Monday.

Sol. : For quick calculation, we can write M for Monday, T for Tuesday and so on.

Hence,

Total of M + T + W = 3 × 60 = 180

and T + W + TH = 3 × 65 = 195

– – –

M – TH = – 15

or, M = – 15 + TH = – 15 + 60 = 45°

∴ Temperature of Monday = 45°

Ex. 2. The average daily temperature from 7th January to 14th January (both days inclusive) was 40°C and from 8th to 15th (both days inclusive) was 42°C. The temperature on the 7th was 41°C. What was the temperature on the 15th January?

Sol. : By mental action, we have the difference of temperature between 15th day and 7th day

= 8 × 42 – 8 × 40 = 8 (42 – 40) = 8 × 2 = 16°C

∴ Temperature on 15th day

= 16°C + 7th day's temperature

= 16 + 41 = 57°C

Ex. 3. The average temperature of Monday, Tuesday, Wednesday and Friday is 58°. The average temperature of Monday, Tuesday, Thursday and Friday is 62°. The ratio of temperature of Wednesday and Thursday is 15 : 19. Find the temperatures of Wednesday and Thursday.

Sol. : By mental action,

Temp. of Thursday – Temp. of Wednesday

= 4 × 62 – 4 × 58 = 4 (62 – 58) = 4 × 4 = 16°

Now, because the temperatures of Wednesday and Thursday are in the ratio of 15 : 19, hence suppose the temperature of Wednesday = 15x

and that of Thursday = 19x

∴ 19x – 15x = 16°

∴ x = 4°

∴ Temperature of Wednesday

= 15 × 4 = 60°

and temperature of Thursday

= 19 × 4 = 76°

Type-II: Problems Related to Numbers

A. Average related to natural numbers

Ex. 1. The average of the first n natural numbers $=\frac{n+1}{2}$

Sol. : As we know,

Sum of the first n natural numbers $=\frac{n(n+1)}{2}$

Number of quantities = n

∴ Average $=\frac{\frac{n(n+1)}{2}}{n}$

$=\frac{n(n+1)}{2\times n}=\frac{n+1}{2}$

Ex. 2. Find the average of 1, 2, 3, 4, 5, 6, 7, 8

Sol. : $=\frac{8+1}{2}=\frac{9}{2}=4.5$ (Here, n = 8)

Note : Obviously, the average of first n whole numbers $=\frac{n}{2}$, because the set of whole numbers starts with 0.

Ex. 3. The average of the squares of the first n natural numbers $=\frac{(n+1)(2n+1)}{6}$

Sol. : As we know

Sum = $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

No. of quantities = n

$\therefore$ Average

$$=\frac{n(n+1)(2n+1)}{6\times n}=\frac{(n+1)(2n+1)}{6}$$

3. Average of the cubes of the first n natural numbers $=\frac{n(n+1)^2}{4}$

Sol. : As we know

Sum = $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3$

$$=\left[\frac{n(n+1)}{2}\right]^2=\frac{n^2(n+1)^2}{4}$$

No. of quantities = n

$\therefore$ Average $=\frac{n^2(n+1)^2}{4\times n}=\frac{n(n+1)^2}{4}$

Example : (Case-b)

Average of $1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

$$=\frac{(6+1)(2\times6+1)}{6}=\frac{7\times13}{6}=\frac{91}{6}=15.16$$

Example : Average of $1^3, 2^3, 3^3, 4^3 = \frac{4(4+1)^2}{4}=25$

B. Average related to even numbers

1. Average of consecutive even numbers or n first even numbers = n + 1

Sol. : Let the first n even numbers be 2, 4, 6 ...... 2n.

$\therefore$ Sum = 2 + 4 + 6 + ... + 2n

= 2 (1 + 2 + 3 + ... n) $=2\frac{n(n+1)}{2}=n(n+1)$

Number of quantities = n

$\therefore$ Average $=\frac{n(n+1)}{n}=n+1$

Example : Average of 2, 4, 6, 8 and 10.

= 5 + 1 = 6 (Here, n = no. of even number' = 5)

2. Average of consecutive even numbers to n $=\frac{n+2}{2}$ where n is the last even number

Example : Average of consecutive even numbers to 10 $=\frac{ü\ +}{2}=6$

Average of 2, 4, 6, 8, 10 $=\frac{ü\ +}{2}=6$

3. Average of squares of n consecutive even numbers

$$=\frac{2(n+1)(2n+1)}{3}$$

Example : Average of 2^2, 4^2, 6^2 and 8^2

$$= \frac{2(4+1)(2\times4+1)}{3} = \frac{\text{ü}\times\ \times}{3} = 30$$

(Here, n = no. of even numbers = 4)

4. Average of squares of consecutive even numbers to n

$$= \frac{(n+1)(n+2)}{3}$$

Where n is the last even number.

Example : Average of squares of consecutive even numbers to 8 i.e., 2^2, 4^2, 6^2 and 8^2.

$$= \frac{(8+1)(8+2)}{3} = 30$$

C. Average related to odd numbers

1. Average of n first consecutive odd numbers = n

Explanation : Sum of n consecutive odd numbers = n^2

No. of quantities = n

$\therefore$ Average $= \frac{n^2}{n} = n$

More clearly, it can be illustrated as below :

Let the odd numbers to n terms be 1, 3, 5 ....

S (let) = 1 + 3 + 5 + ... + to n terms.

Again, let the nth term be t_n

$\therefore$ S = 1 + 3 + 5 + ... + t_n

Now, we find t_n

$t_n = a + (n - 1) d$

where, a = first term = 1

d = common difference = 2 = 1 + (n – 1) 2 = 2n – 1

$\therefore S = \frac{n}{2}$ (First term + Last term)

$$= \frac{\text{ü}}{2}(1+2n-1) = \frac{2\ \times}{2} = n^2$$

$\therefore$ Average $= \frac{n^2}{n} = n$

2. The average of consecutive odd numbers to $n = \frac{n+1}{2}$

Where n is the last odd number.

Example : Average of 1, 3, 5, 7, 9 and 11.

$= \frac{11+1}{2} = 6$ [Here n = 11 (odd)]

3. Average of squares of consecutive odd numbers to n

$$= \frac{n(n+2)}{3}$$

(Where n = odd number)

Example : Average of squares of consecutive odd numbers to 9.

= Average of 1^2, 3^2, 5^2, 7^2 and 9^2

$$=\frac{9(9+2)}{3}=33$$

4. Average of an arithmetic sequence with first and last number (term) known

$$=\frac{\text{First number}+\text{Last number}}{2}$$

Example : Average of 2, 5, 8, 11, 14 and 17.

$$=\frac{2+17}{2}=\frac{19}{2}=9.5$$

Here, the given sequence has common difference 3, hence it is arithmetic sequence and first term (number) = 2 and last term (number) = 17

Type-III: Problems Related to Speed

1. Average speed $=\frac{\text{Total distance covered}}{\text{Total time taken}}$

Example : A man walks 2000 metres in 30 minutes, 1500 metres in 40 minutes and 500 metres in 10 minutes. Then what is the average speed for whole journey?

(A) 50 metres/minute

(B) 55 metres/minute

(C) 60.5 metres/minute

(D) 50.5 metres/ minute

Sol. : Average

$$=\frac{(2000+1500+500)\text{ metres}}{(30+40+10)\text{ minutes}}$$

$$=\frac{4000\text{ metres}}{80\text{ minutes}}$$

2. If an object travels a distance at a speed of x km/hr and the same distance at a speed of y km/hr, then the average speed during the whole journey is given by $\frac{2xy}{x+y}$ km/hr

Here, it should be noted that the distance travelled is same but the speeds are unequal.

Above mentioned result can be summarized in the following way too :

If half of the journey is travelled at a speed of x km/hr and the rest half at a speed of y km/hr, then average speed during the whole journey is given by

$\frac{2xy}{x+y}$ km/hr.

Sol. : Let the distance covered be A km

Now, time taken to cover the distance at the speed of x km/hr $=\frac{A}{x}$ hrs. and time taken to cover the same distance at the speed of y km/hr = $\frac{A}{y}$ hrs.

Total time $=\left(\frac{A}{x}+\frac{A}{y}\right)$ hrs.

This total time is taken to cover a distance of (A + A) = 2A kms.

$\therefore$ Average speed $=\frac{2A}{\frac{A}{x}+\frac{A}{y}}$

$$= \frac{2A}{A\left(\frac{y+x}{xy}\right)} = \frac{2xy}{x+y} \text{ km/hr.}$$

Note : It is clear that if equal distances are covered by unequal speeds, then the average speed is given by Harmonic Mean of the relevant speeds.

3. If an object travels three equal distances at a speed of x km/hr, y km/hr and z km/hr respectively, then the average speed during the whole journey is given by

$$\frac{3\ddot{u}}{xy+yz+xz} \text{km/hr}$$

Ex. 1. A vehicle travels from A to B at the speed of 40 km/hr but from B to A at the speed of 60 km/hr. What is its average speed during the whole journey?

(A) 48 km/hr (B) 48.5 km/hr

(C) 46 km/hr (D) 50 km/hr

Sol. (A) By the formula,

Average speed $= \frac{2\times40\times60}{40+60} = \frac{4800}{100}$ = 48 km/hr.

Therefore, option (A) is true.

Ex. 2. A person divides his total distance of journey into three equal parts and decides to travel the three parts with speeds of 40, 30 and 15 km/hr. respectively. Find his average speed.

(A) 24 km/hr. (B) 25 km/hr.

(C) 28.3 km/hr. (D) 24.2 km/hr.

Sol. (A) Average speed

$$= \frac{3\times40\times30\times15}{40\times30+30\times15+40\times15}$$

$$= \frac{3\times40\times30\times15}{1200+450+600} = \frac{3\times40\times30\times15}{2250}$$

$$= 24 \text{ km/hr.}$$

Hence, option (A) is true.

Ex. 3. One-third of a certain journey is covered at the rate 20 km/hr, one-fourth at the rate of 30 km/hr and the rest at the rate of 50 km/hr. Find the average speed for the whole journey.

(A) $54\frac{6}{11}$ km/hr. (B) $33\frac{1}{3}$ km/hr.

(C) 30 km/hr. (D) 35 km/hr.

Sol. (C) Here, three parts of the journey are unequal. Hence we should find the average by dividing total distance by total time taken.

Let the distance of total journey be x km.

Then journey of $\frac{x}{3}$ km is covered at the speed of 20 km/hr.

Journey of $\frac{x}{4}$ km is covered at the speed of 30 km/hr.

and the rest distance $\left(x-\frac{x}{3}-\frac{x}{4}\right)=\frac{5}{12}x$ km is covered at the speed of 50 km/hr.

∴ Total time taken

$$=\left(\frac{x}{3\times20}+\frac{x}{4\times30}+\frac{5x}{12\times50}\right)\text{hrs}$$

$$\Re\left(\frac{ü}{60}\quad\frac{}{120}\quad\frac{}{120}\right)ü\quad=\frac{4x}{120}=\frac{x}{30}\text{ hrs.}$$

∴ Average speed $=\dfrac{x}{\dfrac{x}{30}}=30$ km/hr

Hence, option (C) is true.

Type–IV: Questions Related to Height, Weight and Age

Average Age

Tricks subject to certain conditions :

A. When a person leaves a group and another person replaces him, then two cases arise :

Case I : When the average age increases age of new comer

= Age of person who left + no. of persons in the group × increase in average age.

Case II : When the average age decreases, age of the new comer = Age of the person who left – No. of persons in the group × decrease in average age value.

1. The average age of 8 men is increased by 4 years when one of them of age 30 years is replaced by a new man. What is the age of new man?

(A) 55 years (B) 62 years
(C) 42 years (D) 69 years

Sol. (B) The average value of age increases after a new man replaces a man of the group. Hence, we apply the formula of **Case I**

The age of the new man = 30 + 8 × 4 = 62 years

Hence option (B) is true.

Ex. 2. The average age of 45 persons is decreased by $\frac{1}{9}$ years when one of them of age 60 years is replaced by a new comer. What is the age of new comer?

(A) 40 years (B) 62 years
(C) 55 years (D) 59 years

Sol. (C) Here, we see decrease in average age. Hence case II is applicable.

∴ Age of new comer $\Re 60-45\quad\frac{1}{9}\quad 55$ years

Hence, option (C) is true.

B. When a person joins a group without any replacement, then two cases arise.

Case I : When the average age increases, age of the new comer

= Previous average (when the new comer is not available) of ages + No. of all persons (including the new comer) × increase in average value.

Case II : When the average age decreases then, age of the new comer = Previous average age – No. of all persons (including new comer × decrease in average value.)

Ex. 1. The average age of 6 men is 32 years which is increased by 1 year when a new man joins the group. Then what is the age of new man?

(A) 42 years (B) 35 years

(C) 45 years (D) 39 years

Sol. (D) Age of new man

= 32 + (6 + 1) × 1 = 32 + 7 = 39 years

Hence, option (D) is correct.

Note : After a new man joins the group, the number of men increases to 7.

Ex. 2. The average age of 20 teachers is 45 years which is decreased by $\frac{6}{7}$ years when a student joins the group. Then what is the age of that student?

(A) 15 years (B) 27 years

(C) 18 years (D) 25 years

Sol. (B) Age of the students $= 45-(20+1)\times\frac{6}{7}$ = 27 years

Hence, option (B) is true.

C. When a person leaves the group without any replacement, again two cases arise.

Case I : When the average age increases, the age of the outgoing person = Previous average age of the group – No. of present persons (excluding the person who left) × increase in the average age value.

Case II : When the average age decreases

Age of man who left = Past average value + No. of present persons × decrease in average value

Ex. 1. The average age of 10 girls in a hostel is 19 years. But one girl left the hostel and the average age is increased by $\frac{1}{2}$ year. Then find the age of the girl who left the hostel.

(A) $14\frac{1}{2}$ years (B) 15 years

(C) $15\frac{1}{2}$ years (D) 18 years

Sol. (A) Age of the girl who left

$\Re 19-(10-1)\ \frac{1}{2}\quad 14\frac{1}{2}$ years

Hence, option (A) is true.

Ex. 2. The average age of 26 labourers is 30 years. It is decreased by $\frac{1}{5}$ year when a labourer went home. Then the value of age of that labour is.

(A) 30 years (B) 32 years

(C) 24 years (D) 35 years

Sol. (D) Age of the labourer

$= 30+(26-1)\times\frac{1}{5}$ = 35 years

Hence, option (D) is correct.

Ex. 3. In a boat there are 8 men whose average weight is increased by 1 kg. When a man of 60 kg is replaced by a new man. What is the weight of new comer?

(A) 68 kg (B) 64 kg
(C) 62.5 kg (D) 63 kg

Sol. (A) Weight of new comer = Wt. of outgoing man + increase of wt. × No. of men = 60 + 1 × 8 = 68 kg

∴ Option (A) is correct.

Ex. 4. In a class there are 30 boys whose average weight is decreased by 200 gms. When one boy whose weight was 25 kilos leaves the class and new one is admitted, find the weight of new comer.

(A) 19 kg (B) 20 kg
(C) 20.5 kg (D) 20.45 kg

Sol. (A) Wt. of New Comer = Wt. of outgoing boy – decrease in average wt. × No. of boys.

$= 25 - \frac{ü}{1000} \times 30$ = 25 – 6 = 19 kg

Hence, option (A) is true.

Type-V: Problems on Cricket Score

Ex. 1. Score of a batsman in his five innings is 68, 72, 3, 42* and 26. Find his average of these innings (* means not out).

(A) 52.75 (B) 53
(C) 53.5 (D) 54

Sol. (A) [It is to be noted that in cricket scrore in not out innings is not counted in total innings while its score is calculated in total score]

Now, total score of five innings

= 68 + 72 + 3 + 42 + 26 = 211

But he has remained not out in one innings, therefore total innings counted = 4

∴ Average $= \frac{ü}{4} = 52.75$

∴ Hence, option (A) is true.

Ex. 2. A batsman makes a score of 87 runs in the 17th innings and thus increases his average by 3. Find his averge after 17th innings.

(A) 39 (B) 35
(C) 40 (D) 39.5

Sol. (A) Let the average after 17th innings = x

Then, average after 16th innings = x – 3

∴ 17x – 16 (x – 3) = 87

or, x = 87 – 48 = 39

Ex. 3. In a cricket match 6 players had a certain average of their runs. Seventh player makes a scores of 112 runs, thereby increaseing the average of their runs by 10. Find the average of first 6 players.

(A) 42 (B) 43
(C) 43.5 (D) 44

Sol. (A) Suppose the average of first six players' runs = x

∴ 7 (x + 10) – 6x = 112

(By mental action)

or, x = 42

Therefore, option (A) is true.

Ex. 4. A cricketer has a certain average for 9 innings. In the tenth innings, he scores 100 runs, thereby increasing his average by 8 runs. His new average is

(A) 20 runs (B) 23 runs

(C) 28 runs (D) 30 runs

Sol. (C) Let average for 9 innings be x. Then

9x + 100 =10x + 80

or, x = 20

∴ New average = x + 8 = 28 runs

Therefore, option (C) is true.

Solved Examples

Ex. 1. The average monthly salary of 660 workers in a factory is Rs. 380. The average monthly salary of officers is Rs. 2,100 and the average monthly salary of the other workers is Rs. 340. The number of other workers is

Sol. Total salary of 660 workers

= 660 × 380 = Rs. 250800

If the number of other workers be x

Then the number of officers = 660 – x

∴ (660 – x) × 2100 + 340x = 250800

or, 138600 – 2100x + 340x = 250800

or, $x = \frac{1135200}{1760} = \text{ü}$

Ex. 2. The average of 25 results is 18. The average of first 12 of those is 14 and the average of last 12 is 17. What is the 13th result?

Sol. Total of 25 results = 25 × 18 = 450

Total of first 12 results = 12 × 14 = 168

Total of last 12 results = 12 × 17 = 204

∴ 13th result = 450 – (168 + 204) = 450 – 372 = 78

By short cut method 13th result

= 25 × 18 – (12 × 14 + 12 × 17) = 450 – 372 = 78

(By mental action)

Ex. 3. Of the three numbers the first is twice the second and the second is twice the third. The average of three numbers is 35. What are the numbers ?

Sol. Let the third numbers be *x*

∴ Average of the number

$= \frac{\text{ü}+2 \quad +4}{3} = 35$

or, 7*x* = 105

or, *x* = 15

∴ Required numbers are 15, 30, 60.

Ex. 4. Visitors to a show were charged Rs. 15 each on the first day, Rs. 7.5 on the second, Rs. 2.5 on the third day and the total attendance on three days were in the ratio 2 : 5 : 13 respectively. Find the average charge per person for the whole show.

Sol. Total attendance on three days

= 2*x* + 5*x* + 13*x* = 20x

Total money collected on three days

$$= \text{Rs.}\left(15\times 2\ddot{u} + \frac{\ddot{u}}{2}\times 5 \quad + \frac{}{2}\times 13\right)$$

= Rs. (30*x* + 37.50*x* + 32.50*x*) = Rs. 100 *x*

$\therefore$ The average charge per person for the whole show $= \frac{\ddot{u} \quad x}{20x} = \text{Rs. } 5$

Ex. 5. The mean of $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$ is

Sol. Mean or average

$$= \frac{(7+1)(2\times 7+1)}{6} = \frac{\ddot{u}\times}{6} = 20$$

Practice Exercise

1. **The average of 30 numbers is 12. The average of the first 20 of them is 11 and that of the next 9 is 10. The last number is**

   (A) 60 (B) 45 (C) 40 (D) 50

2. **The average of eight numbers is 20. If the sum of first two numbers is 31, the average of the next three numbers is and the seventh and eighth numbers exceed the sixth number by 4 and 7 respectively, then the eighth number is**

   (A) 20 (B) 25 (C) 21.6 (D) 25.3

3. **The bowling average of a cricketer was 12.4. He improves his bowling average by 0.2 points when he takes 5 wickets for 26 runs in his last match. The number of wickets taken by him before the last match was**

   (A) 125 (B) 150 (C) 175 (D) 200

4. **In a certain year, the average monthly income of a person was Rs. 3,400. For the first eight months of the year, his average monthly income was Rs. 3,160 and for the last five months, it was Rs. 4,120. His income in the eighth month of the year was**

   (A) Rs. 3,160 (B) Rs. 5,080 (C) Rs. 15,520 (D) Rs. 5,520

5. **The average age of 40 students of a class is 18 years. When 20 new students are admitted to the same class, the average age of the students of the class is increased by 6 months. The average age of newly admitted students is**

   (A) 19 years (B) 19 years 6 months (C) 20 years (D) 20 years 6 months

6. **Of the three numbers, the second is twice the first and thrice the third. If the average of the three numbers is 44, the largest number is**

   (A) 24 (B) 72 (C) 36 (D) 108

7. **A cricketer had a certain average of runs for his 64 innings. In his 65th innings, he is bowled out for no score on his part. This brings down his average by 2 runs. His new average of runs is**

   (A) 130 (B) 128 (C) 70 (D) 68

8. Out of seven given numbers, the average of the first four numbers is 4 and that of the last four numbers is also 4. If the average of all the seven numbers is 3, the fourth number is

(A) 3 (B) 4 (C) 7 (D) 11

9. The average of 100 numbers is 44. The average of these 100 numbers and 4 other new numbers is 50. The average of the four new numbers will be

(A) 800 (B) 200 (C) 176 (D) 24

10. The average of 6 observations is 45.5. If one new observation is added to the previous observations, then the new average becomes 47. The new observation is

(A) 58 (B) 56 (C) 50 (D) 46

11. Of the three numbers, the second is twice the first and is also thrice the third. If the average of these three numbers is 44, the largest number is

(A) 24 (B) 36 (C) 72 (D) 108

12. The average of 30 numbers is 15. The average of the first 18 numbers is 10 and that of the next 11 numbers is 20. The last number is

(A) 56 (B) 52 (C) 60 (D) 50

13. The average age of 40 students of a class is 15 years. When 10 new students are admitted, the average age is increased by 0.2 years. The average age of new students is

(A) 15.2 years (B) 16 years (C) 16.2 years (D) 16.4 years

14. A man travels a distance of 24 km at 6 km/hr another distance of 24 km at 8 km/hr and a third distance of 24 km at 12 km/hr . His average speed for the whole journey (in km/hr) is

(A) (B) 8 (C) (D) 9

15. A cricketer has a certain average of runs for his 8 innings. In the ninth innings, he scores 100 runs, thereby increases his average by 9 runs. His new average of runs is

(A) 20 (B) 24 (C) 28 (D) 32

16. A man buys a certain number of oranges at 20 for Rs. 60 and an equal number at 30 for Rs. 60. He mixes them and sells them at 25 for Rs. 60. What is gain or loss per cent ?

(A) Gain of 4% (B) Loss of 4%

(C) Neither gain nor loss (D) Loss of 5%

17. A man goes from A to B at a uniform speed of 12 km/hr and returns with a uniform speed of 4 km/hr His average speed (in km/hr) for the whole journey is :

(A) 8 (B) 7.5 (C) 6 (D) 4.5

18. The average expenditure of a man for the first five months of a year is Rs. 5,000 and for the next seven months it is Rs. 5,400. He saves Rs. 2,300 during the year. His average monthly income is :

(A) Rs. 5,425 (B) Rs. 5,500 (C) Rs. 5,446 (D) Rs. 5,600

19. A batsman, in his 12th innings, makes a score of 63 runs and thereby increases his average score by 2. The average of his score after 12th innings is
    (A) 41 (B) 42 (C) 34 (D) 35

20. The average of two numbers A and B is 20, that of B and C is 19 and of C and A it is 21. What is the value of A ?
    (A) 24 (B) 22 (C) 20 (D) 18

21. A cricket player has an average score of 30 runs for 42 innings played by him. In an innings, his highest score exceeds his lowest score by 100 runs. If these two innings are excluded, his average of the remaining 40 innings is 28 runs. His highest score in an innings is
    (A) 125 (B) 120 (C) 110 (D) 100

22. The average of four consecutive even numbers is 27. The largest of these numbers is
    (A) 28 (B) 26 (C) 32 (D) 30

23. The average of the marks of 28 students in Mathematics was 50. When 8 students left the school, then the average increased by 5. What is the average of the marks obtained by the students who left the school ?
    (A) 50.5 (B) 37.5 (C) 42.5 (D) 45

24. The average monthly salary of the workers in a workshop is Rs. 8,500. If the average monthly salary of 7 technicians is Rs. 10,000 and average monthly salary of the rest is Rs. 7,800, the total number of workers in the workshop is
    (A) 18 (B) 20 (C) 22 (D) 24

25. The average of the first 100 positive integers is
    (A) 100 (B) 51 (C) 50.5 (D) 49.5

Answers Key

1. (D)	2. (B)	3. (C)	4. (B)	5. (B)
6. (B)	7. (B)	8. (D)	9. (B)	10. (B)
11. (C)	12. (D)	13. (B)	14. (B)	15. (C)
16. (B)	17. (C)	18. (A)	19. (A)	20. (B)
21. (B)	22. (D)	23. (B)	24. (C)	25. (C)

12 Ratio and Proportion

FACTS AND FORMULAE

I. Ratio

The ratio of two quantities in the same unit is a fraction that one quantity is of the other. There cannot be a ratio between quanitites of different kinds.

Ratio of two quantities a and b of the same kind is written as $a : b$ or $\frac{a}{b}$, where a and b are called *terms of ratio*.

The first term of the ratio is called *antecedent*, while the second term is known as *consequent*.

Thus, the ratio 5 : 9 represents $\frac{5}{9}$ with antecedent 5 and consequent 9.

(A) **Compound ratio :** When the numerators and the denominators of two or more ratios are multiplied to obtain the new numerator and denominator, a new ratio is formed called the *compound ratio*.

For example : The compound ratio of 2 : 3, 3 : 4, 5 : 6 is (2 × 3 × 5) : (3 × 4 × 6) = 30 : 72 = 5 : 12

(B) **Duplicate ratio :** When a ratio is compounded with itself, and thus the resulting ratio is called the *duplicate ratio* of the given ratio. Thus $a^2 : b^2$ is the duplicate ratio of $a : b$.

Similarly, the ratio compounded of three equal ratios is called the *triplicate ratio* of the given ratio. Thus $a^3 : b^3$ is the triplicate ratio of $a : b$.

For example : Divide 1455 into two parts such that one may be to the other of 2 : 3.

Sol. Ist part = $2\times\frac{1455}{2+3}=\frac{2}{5}\times1455=582$

2nd part = $3\times\frac{1455}{5}=873$

For example : Two numbers are in the ratio 8 : 11. If 6 is subtracted from each, the resulting numbers are 7 : 10. Find the two numbers.

Sol. Let the given number be $8x$ and $11x$.

By hypothesis,

$$\frac{8x-6}{11x-6}=\frac{7}{10} \text{ or } 80x - 60 = 77x - 42$$

$3x = 18$ or $x = 6$ Hence, numbers are 48 and 66.

II. Proportion

Four quantities are said to be in proportion when the ratio of the first two quantities is same as the ratio of the last two quantities. Since 3 : 4 = 15 : 20, so 3, 4, 15 and 20 are called in *proportion*.

It can be remembered that, "If four quantities be in proportion, the product of the extreme is equal to the product of the means."

If $a : b = c : d$ or $a \times d = b \times c$; the proportion may be written as

3 : 2 : : 135 : 90

This is read as 3 is to 2 as 135 to 90.

The numbers 3, 2, 135, 90 are called the terms. 3 is the *first term*, 2 is the *second term*, 135 is the *third term* and 90 is the *fourth term*.

The first and fourth terms are called *extremes* and the second and third terms are called *means*.

(A) **Continued proportion :** Three quantities of same kind are said to be in continued proportion, if $a : b = b : c$, then *abc* are called *continued proportion*. The second term is also called the *mean proportion*. The second term is also called the mean proportion between the other two and the third term is called the *third proportion* of the first two.

(B) **Direct Proportion :** In direct proportion, the ratio of the first two terms is equal to the ratio of last two terms, and the four quantities are said to be in *direct proportion*.

(C) **Indirect proportion :** In indirect proportion a greater number require a smaller number and vice versa.

(*i*) Fourth proportional

$$= \frac{\text{Second term} \times \text{Third term}}{\text{First term}}$$

(*ii*) Mean proportion = $\sqrt{ab}$

(*iii*) Third proportion = $\frac{b^2}{a}$

where *a* and *b* are first and second terms respectively.

III. Proportion Rules

(*i*) Denote the quantity to be found by the letter *x*, and set it down as the 4th term.

(*ii*) Of the three given quantities, set that down for the third term, which is of the same kind as the quantity to be found.

(*iii*) Now consider carefully whether the quantity to be found will be greater or less than the third term; if greater, make the greater of the two remaining quantities as the 2nd term, and the 1st term, but if less, make the less quantity as the 2nd term and the greater the 1st term.

(*iv*) Having thus arranged the terms, divide the 2nd term by the first and multiply the quotient by the 3rd term, the product will be the required quantity.

Solved Examples

Ex.1. *If 40 per cent of a number is added in the other, then it becomes 125 per cent of itself. What will be the ratio of first and second number ?*

Sol. Suppose the two numbers are *x* and *y*.

By hypothesis,

40% of $x + y$ = 125% of x

or $\frac{40}{100}\ddot{u}+ = \frac{125}{100}$ or $\frac{85}{100}x = y$

or $\frac{x}{y} = \frac{100}{85} = \frac{20}{17}$

Ex.2. *A sum of Rs. 9000 is to be distributed among A, B and C in the ratio 4 : 5 : 6. What will be the difference between A's and C's shares?*

Sol. A's share = $9000 \times \frac{4}{4+5+6} = 9000 \times \frac{4}{15}$ = Rs. 2400

C's share = $9000 \times \frac{6}{4+5+6} = 9000 \times \frac{6}{15}$ = Rs. 3600

∴ Difference between share's of A and C

= 3600 – 2400 = Rs. 1200

Ex.3. *The ratio of the school ages of Neeta and Samir is 5 : 6. If the ratio of the one-third of Neeta's age and one-half of Samir's age be 5 : 9, then what is the school age of Samir ?*

Sol. Here the ratio of their respective ages is given, but the sum of their ages is not given. Hence, the given data are inadequate.

Practice Exercise

1. A and B are two alloys of gold and copper prepared by mixing metals in the ratio 7 : 2 and 7 : 11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the ratio of gold and copper in C will be

(A) 5 : 7 (B) 5 : 9 (C) 7 : 5 (D) 9 : 5

2. Zinc and copper are melted together in the ratio 9 : 11. What is the weight of melted mixture, if 28.8 kg of zinc has been consumed in it ?

(A) 58 kg (B) 60 kg (C) 64 kg (D) 70 kg

3. A sum of Rs. 1300 is divided amongst P, Q, R and S such that

$\frac{\text{P's share}}{\text{Q's share}} \Re \frac{\text{Q's share}}{\text{R's share}} \; \frac{\text{R's share}}{\text{S's share}} \; \frac{2}{3}$. Then, P's share is

(A) Rs. 140 (B) Rs. 160 (C) Rs. 240 (D) Rs. 320

4. Three containers have their volumes in the ratio 3 : 4 : 5. They are full of mixture of milk and water. The mixture contain milk and water in the ratio of (4 : 1), (3 : 1) and (5 : 2) resepctively. The contents of all these three containers are poured into a fourth container. The ratio of milk and water in the fourth container is

(A) 4 : 1 (B) 151 : 48 (C) 157 : 53 (D) 5 : 2

5. Ratio of the earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What are A's earnings ?

(A) Rs. 21,000 (B) Rs. 26,000

(C) Rs. 28,000 (D) Data inadequate

6. If Rs. 510 be divided among A, B, C in such a way that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets, then shares are respectively

(A) Rs. 120, Rs. 240, Rs. 150 (B) Rs. 60, Rs. 90, Rs. 360

(C) Rs. 150, Rs. 300, Rs. 60 (D) None of these.

..Notes

7. **The ratio between the length and the perimeter of a rectangular plot is 1 : 3 and the ratio between the breadth and perimeter of that plot is 1 : 6. What is the ratio between the length and area of that plot?**

   (A) 2 : 1 (B) 1 : 6
   (C) 1 : 8 (D) Data inadequate.

8. **Hariprasad and Madhusudan started a business investing amounts in the ratio of 2 : 3 respectively. If Hariprasad had invested an additional amount of Rs. 10,000, the ratio of Hariprasad's investment to Madhusudan's investment would have been 3 : 2. What was the amount invested by Hariprasad ?**

   (A) Rs. 8,000 (B) Rs. 12,000
   (C) Rs. 9,000 (D) Data inadequate

9. **Two numbers are in the ratio 3 : 5. If each number is increased by 10, the ratio becomes 5 : 7. The numbers are**

   (A) 3, 5 (B) 7, 9 (C) 13, 22 (D) 15, 25

10. **If $a : b : c = 3 : 4 : 7$, then the ratio $(a + b + c) : c$ is equal to**

    (A) 2 : 1 (B) 14 : 3 (C) 7 : 2 (D) 1 : 2

11. **The number of students in 3 classes are in the ratio 2 : 3 : 4. If 12 students are increased in each class, this ratio changes to 8 : 11 :14. The total number of students in the three classes in the beginning was**

    (A) 162 (B) 108 (C) 96 (D) 54

12. **A box has 210 coins of denominations one-rupee and fifty paise only. The ratio of their respective values is 13 : 11. The number of one-rupee coins is**

    (A) 65 (B) 66 (C) 77 (D) 78

13. **If $\frac{2}{3}$ of A = 75% of B = 0.6 of C, then A : B : C is**

    (A) 2 : 3 : 3 (B) 3 : 4 : 5 (C) 4 : 5 : 6 (D) 9 : 8 : 10

14. **If A and B are in the ratio 3 : 4, and B and C in the ratio 12 : 13, then A and C will be in the ratio**

    (A) 3 :13 (B) 9 : 13 (C) 36 :13 (D) 13 : 9

15. **The salaries of A, B and C are in the ratio 1 : 3 : 4. If the salaries are increased by 5%, 10% and 15% respectively, then the increased salaries will be in the ratio**

    (A) 20 : 66 : 95 (B) 21 : 66 : 95 (C) 21 : 66 : 92 (D) 19 : 66 : 92

16. **The total marks obtained by Arun in English and Mathematics are 170. If the difference between his marks in these two subjects is 10, then the ratio of his marks in these subjects is**

    (A) 7 : 8 (B) 8 : 7 (C) 9 : 8 (D) 9 : 7

17. **Ther ratio of incomes of A and B is 5 : 6. If A gets Rs. 1,100 less than B, their total income (in rupees) is**

    (A) 9,900 (B) 12,100 (C) 14,400 (D) 10,000

18. In an innings of a cricket match, three players A, B and C scored a total of 361 runs. If the ratio of the number of runs scored by A to that scored by B and also number of runs scored by B to that scored by be 3 : 2, the number of runs scored by A was

(A) 171 (B) 181 (C) 185 (D) 161

19. In a school having roll strength 286, the ratio of boys and girls is 8 : 5. If 22 more girls get admitted into the school, the ratio of boys and girls becomes

(A) 12 : 7 (B) 10 : 7 (C) 8 : 7 (D) 4 : 3

20. If 20% of A = 30% of B = $\frac{1}{6}$ of C, then A : B : C is

(A) 2 : 3 : 16 (B) 3 : 2 : 16 (C) 10 : 15 : 18 (D) 15 : 10 : 18

21. A box contains 1-rupee, 50-paise and 25-paise coins in the ratio 8 : 5 : 3. If the total amount of money in the box is Rs. 112.50, the number of 50-paise coins is

(A) 80 (B) 50 (C) 30 (D) 42

22. Rs. 33,630 are divided among A, B and C in such a manner that the ratio of the amount of A to that of B is 3 : 7 and the ratio of the amount of B to that of C is 6 : 5. The amount of money received by B is

(A) Rs.14,868 (B) Rs.16,257 (C) Rs.13,290 (D)Rs.12,390

23. The bus fare and train fare of a place from Kolkata were Rs. 20 and Rs. 30 respectively. Train fare has been increased by 20% and the bus fare has been increased by 10%. The ratio of new train fare to new bus fare is

(A) 11 : 18 (B) 18 : 11 (C) 5 : 3 (D) 3 : 5

24. If A : B = 2 : 3 and B : C = 4 : 5, then A : B : C is

(A) 2 : 3 : 5 (B) 5 : 4 : 6 (C) 6 : 4 : 5 (D) 8 : 12 : 15

25. If two times of A is equal to three times of B and also equal to four times of C, then A : B : C is

(A) 2 : 3 : 4 (B) 3 : 4 : 2 (C) 4 : 6 : 3 (D) 6 : 4 : 3

26. In a cricket match to total number of runs scored by Sachin, Vinod and Sourav is 285. The ratio of the number of runs scored by Sachin and Sourav is 3 : 2 and that of the runs scored by Sourav and Vinod is also 3 : 2. The number of runs scored by Sachin in that match is

(A) 135 (B) 90 (C) 60 (D) 140

27. In a bag, there are three types of coins — 1-rupee, 50 paise and 25-paise in the ratio of 3 : 8 : 20. Their total value is Rs. 372. The total number of coins is

(A) 1200 (B) 961 (C) 744 (D) 612

28. If A : B = 3 : 2 and B : C = 3 : 4, then A: C is equal to

(A) 1 : 2 (B) 2 : 1 (C) 8 : 9 (D) 9 : 8

29. The ratio of two numbers is 4:5 when the first is increased by 20% and the second is decreased by 20%, the ratio of the resulting numbers is

(A) 4 : 5 (B) 5 : 4 (C) 5 : 6 (D) 6 : 5

✍..Notes

30. The radii of the bases of two cylinders are in the ratio 3 : 5 and their heights in the ratio 2 : 3. The ratio of their curved surfaces will be :

(A) 2 : 5 (B) 2 : 3 (C) 3 : 5 (D) 5 : 3

31. When a particular number is subtracted from each of 7, 9, 11 and 15, the resulting numbers are in proportion. The number to be subtracted is :

(A) 1 (B) 2 (C) 3 (D) 5

32. Two numbers are in the ratio 4 : 5 and their L.C.M. is 180. The smaller number is

(A) 9 (B) 15 (C) 36 (D) 45

33. If A : B = 2 : 3, B : C = 4 : 5 and C : D = 5 : 9, then A : D is equal to

(A) 11 : 17 (B) 8 : 27 (C) 5 : 9 (D) 2 : 9

34. If $a + b + c = 1$ and $ab + bc + ca = \frac{1}{3}$ then $a : b : c$ is

(A) 1 : 2 : 2 (B) 2 : 1 : 2 (C) 1 : 1 : 1 (D) 1 : 2 : 1

35. In a class, the number of girls is 20% more than that of the boys. The strength of the class is 66. If 4 more girls are admitted to the class, the ratio of the number of boys to that of the girls is

(A) 1 : 2 (B) 3 : 4 (C) 1 : 4 (D) 3 : 5

36. If A : B = 3 : 4 and B : C = 8 : 9, then the ratio A : B : C is

(A) 3 : 4 : 5 (B) 1 : 2 : 3 (C) 7 : 12 : 17 (D) 6 : 8 : 9

37. If $a : b = b : c$, then the ratio $a^4 : b^4$ is equal to

(A) $ac : b^2$ (B) $a^2 : c^2$ (C) $c^2 : a^2$ (D) $b^2 : ac$

38. A bag contains Rs. 145 in the form of one-rupee, 50-paise and 25-paise coins in the ratio 3 : 5 : 7. The number of one-rupee coins is

(A) 60 (B) 50 (C) 48 (D) 45

39. Ram and Gopal have money in the ratio 7 : 17 and Gopal and Krishna also have money in the same ratio 7 : 17. If Ram has Rs. 490, Krishna has

(A) Rs. 2,330 (B) Rs. 2,680 (C) Rs. 2,890 (D) Rs. 3,000

40. What must be added to each term of the ratio 7 : 11, so as to make it equal to 3 : 4 ?

(A) 8 (B) 7.5 (C) 6.5 (D) 5

41. If 60% of A = $\frac{3}{4}$ of B, then A : B is

(A) 9 : 20 (B) 20 : 9 (C) 4 : 5 (D) 5 : 4

42. Two numbers are in the ratio 7 : 11. If 7 is added to each of the numbers, the ratio becomes 2 : 3. The smaller number is

(A) 39 (B) 49 (C) 66 (D) 77

43. If $W_1 : W_2 = 2 : 3$ and $W_1 : W_3 = 1 : 2$ then $W_2 : W_3$ is

(A) 3 : 4 (B) 4 : 3 (C) 2 : 3 (D) 4 : 5

Answers Key

1.(C)	2.(C)	3.(B)	4.(C)	5.(D)
6.(B)	7.(D)	8.(A)	9.(D)	10.(A)
11.(A)	12.(D)	13.(D)	14.(B)	15.(C)
16.(C)	17.(B)	18.(A)	19.(D)	20.(D)
21.(B)	22.(A)	23.(B)	24.(D)	25.(D)
26.(A)	27.(B)	28.(D)	29.(D)	30.(A)
31.(C)	32.(C)	33.(B)	34.(C)	35.(B)
36.(D)	37.(B)	38.(A)	39.(C)	40.(D)
41.(D)	42.(B)	43.(A)		

13 Time, Work and Wages

FACTS AND FORMULAE

I. If a person can complete a piece of work in *n* days, then he will complete $\frac{1}{n}$ th of the work in one day. Conversely, if he does $\frac{1}{n}$ th of the work in one day, then he will complete the whole work in *n* days.

II. When two or more men jointly complete a job and get some money in lieu of that work done, each man and share in the money will be proportional to his their work done.

III. If the number of men employed for a job be increased in a certain ratio, the time required to do the job is decreased in the same ratio and vice versa.

IV. Short-cut Methods

(*i*) If A completes a work in *x* days, B in *y* days and C in *z* days, then

(A) A and B will complete the work in $\frac{xy}{x+y}$ days.

(B) A, B and C will complete the work in $\frac{xyz}{xy+yz+xz}$ days.

(*ii*) If A + B completes a work in *x* days, B + C in *y* days and A + C completes it in *z* days, then

(A) A, B and C will complete the work in $\frac{2xyz}{xy+yz+xz}$ days.

(B) A will complete the work in $\frac{2xyz}{xy+yz-xz}$ days.

(*iii*) $\frac{\text{A's work}}{\text{B's work}} = \frac{\text{B's time}}{\text{A's time}}$

(*iv*) If a takes *n* times to completes B and C's work, then A will complete the work in (*n* + 1) × time taken by A, B and C to complete the work.

(*v*) If A completes a work in *x* days, B in *y* days and C in *z* days, then

(A) A and B start a work together, but A joins the work *n* days after, then work will be completed in $\frac{xy}{x+y}\left(1+\frac{n}{x}\right)=\frac{(x+n)y}{x+y}$ days

(B) If A, B and C start a work together, but A leaves the work *n* days after, then work will be completed in

$$\frac{xyz}{xy+yz+xz}\left(1+\frac{n}{x}\right)=\frac{(x+n)yz}{xy+yz+xz} \text{ days}$$

(C) If A, B and C start a work together but A leaves working *n* days before completion of work and B leaves *m* days before too, then the work be completed by C in

$$\frac{xyz}{xy+yz+xz}\left(1+\frac{n}{x}+\frac{m}{y}\right) \text{ days.}$$

(*vi*) A completes a work in *x* days, B in *y* days and C in *z* days. If all of three start work together, but

(A) A leaves the work after *n* days, then it would be completed in

$$\frac{yz}{y+z}\left(1-\frac{n}{x}\right) \text{ days.}$$

(B) A leaves the work after *n* days, B leaves after *m* days, then the work would be completed in

$$z\left(1-\frac{n}{x}-\frac{m}{y}\right) \text{ days.}$$

Solved Examples

Ex.1. *A, B and C can complete a piece of work in 10, 15 and 18 days respectively. In how many days would all of them complete the same work working together?*

Sol. Work done by A in 1 day $=\frac{1}{10}$

Work done by B in 1 day $=\frac{1}{15}$

Work done by C in 1 day $=\frac{1}{18}$

∵ Work done by (A + B + C) in 1 day

$$=\frac{1}{10}+\frac{1}{15}+\frac{1}{18}=\frac{9+6+5}{90} \quad =\frac{2}{9} \text{ th work}$$

∵ (A + B + C) do $\frac{2}{9}$ th work in 1day

∴ (A + B + C) will do 1 work in $\frac{1\times 9}{2} \quad =4\frac{1}{2}$ days.

Short-cut Method:

Given, x = 10, y = 15 and z = 18

∴ Working together A, B and C will complete the work in $\left(\frac{xyz}{xy+yz+zx}\right)$

$$=\left(\frac{10\times 15\times 18}{10\times 15+15\times 18+18\times 10}\right)$$

$$=\left(\frac{2700}{150+270+180}\right)=\frac{2700}{600}=4\frac{1}{2} \text{ days.}$$

Ex.2. *A farmer appointed three persons A, B and C on harvesting a field for Rs. 3200. A, B and C can harvest the field in 16,24 and 16 days respectively. If they harvest the field working together. What will be the share of A?*

A, B, C, can harvest the field together in 6 days

Sol. Work done by A in 1 day $=\frac{1}{16}$

∴ Work done by A in 6 days $=\frac{1}{16}\times 6=\frac{3}{8}$

Wages of A = Rs. $\frac{3}{8}\times 3200$ = Rs. 1200

Similarly, work done by B in 1 day $=\frac{1}{24}$

∴ Work done by B in 6 days $=\frac{1}{24}\times 6=\frac{1}{4}$

∴ Wages of B = Rs. $\frac{1}{4}\times 3200=\text{Rs. }800$

∴ Wages of C for 6 days = Rs. 3200 – Rs. (1200 + 800)

= Rs. 3200 – Rs. 2000 = Rs. 1200

∴ Wages of A, B and C and C will be Rs. 1200, Rs. 800 and Rs. 1200 respectively.

Short-cut Method:

∵ Wages of A for 6 days = work done by A in 6 days

Rs. 3200

$=\left(\frac{6}{16}\right)\times 3200$ = Rs. 1200

∴ Wages of B for 6 days

= Work done by B in 6 days × Rs 3200

$=\left(\frac{6}{24}\right)\times 3200=\text{Rs. }800$

∴ Wages of C for 6 days

= Rs. [3200 – (1200 + 800)] = Rs. (3200 – 2000)
= Rs. 1200

Ex.3. *A can complete a piece of work in 10 days. B is 25% more efficient than A and C is 60% more efficient than B. Working together how long would they take to finish the job?*

Sol. Work done by A in 1 day $=\frac{1}{10}$

∴ B is 25% better worker than A

∵ Work done by B in 1 day $=\frac{1}{10}\times 125\%$

$=\frac{1}{10}\times\frac{125}{100}=\frac{1}{8}$

Since, C is 60% better worker than B, therefore work done by C in 1 day $=\frac{1}{8}\times 160\% \quad = \frac{1}{8}\times\frac{160}{100}=\frac{1}{5}$

Now work done by (A + B + C) in 1 day

$$=\frac{1}{10}+\frac{1}{8}+\frac{1}{3}=\frac{4+5+8}{40}=\frac{17}{40}$$

Since (A + B + C) do $\frac{17}{40}$ th work in 1 day

∴ (A + B + C) will do whole work in $\frac{40}{17}$ days

i.e., $2\frac{6}{7}$ days

Short-cut Method:

∵ A does the piece of work in 10 days.

∴ B will complete the work in $10\times\frac{100}{125}=8$ days,

and C will complete the same work in

$$8\times\frac{100}{160}=5 \text{ days}$$

Given, x = 10, y = 8 and z = 5, therefore working together A, B and C will complete the work in

$$\left(\frac{xyz}{xy+yz+zx}\right)=\left(\frac{10\times 8\times 5}{10\times 8+8\times 5+5\times 10}\right)$$

$$=\left(\frac{400}{80+40+50}\right)=\frac{400}{170}=\frac{40}{17}=2\frac{6}{7} \text{ days.}$$

Ex.4. *A can do a piece of work in 6 days. B can do the same work in 3 days. How long would both of them take to do the same work working together*

Sol. Work done by A in 1 day $=\frac{1}{6}$ and work done by B in 1 day $=\frac{1}{3}$

∴ Work done by (A + B) in 1 day $=\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$

∴ (A + B) do $\frac{1}{2}$ work in 1 day.

i.e., (A + B) will do 1 work in 2 days.

Short-cut Method:

Given, x = 6 and y = 3

∴ A and B working together will complete the work in $\left(\frac{xy}{x+y}\right)$ days $=\left(\frac{6\times 3}{6+3}\right)=\frac{18}{9}$ = 2 days.

Ex.5. *A and B can complete a piece of work in 10 and 15 days respectively. If after working alone for 4 days, A leaves the work and goes home, then how long would B take to finish the remaining work ?*

Sol. Work done by A in 1 day $=\frac{1}{10}$

Work done by A in 4 days $=\frac{1}{10}\times 4=\frac{2}{5}$

..Notes

∴ Remaining work

Since B can do 1 work in 15 days

∴ B can do remaining $\frac{3}{5}$ work in $\frac{15\times3}{5}=9$ days

Short-cut Method:

Suppose B would take x days in completing the remaining work.

∴ Work done by A in 4 days and work done by B in x days = Whole work

∴ $\frac{4}{10}+\frac{x}{15}=1$ or $\frac{x}{15}=1-\frac{4}{10}$

⇒ $\frac{x}{15}=\frac{10-4}{10}$ or $\frac{x}{15}=\frac{6}{10}$

or $x=\frac{6\times15}{10}$ = 9 days

Ex.6. *A, B and C working together can complete a piece work in 24 days. C alone can complete this work in 48 days, if A works twice as fast as B. How many days would A take to finish the work working alone ?*

Sol. Work done by (A + B + C) in 1 day $=\frac{1}{24}$ and work done by C in 1 day $=\frac{1}{48}$

∴ Work done by (A + B) in 1 day $=\frac{1}{24}-\frac{1}{48}=\frac{1}{48}$

According to the question,

∴ A works twice as fast as B

∴ In 1 day, work done by B is equal to half of the work done by A *i.e.,* B's work $=\frac{1}{2}$ of A's work.

∴ Work done by $\left(A+\frac{A}{2}\right)$ or $\frac{3A}{2}$ in 1 day $=\frac{1}{48}$

∴ Work done by A in 1 day $=\frac{1}{48}\times\frac{2}{3A}\times A=\frac{1}{72}$

∵ A does $\frac{1}{2}$ work in 1 day

∴ A will do 1 work in $\frac{72\times1}{1}=72$ days

Short-cut Method:

Suppose A working alone does this work in D days.

∴ B working alone will do this work in 28 days

Here x = D, y = 2D and C = 48

and A, B and C working together to this work in 24 days.

$\therefore \quad \dfrac{xyz}{xy+yz+zx}=24$ or $\dfrac{D+2D+48}{D\times 2D+2D\times 48+48D}=24$

$\Rightarrow \quad \dfrac{48D^2}{D^2+48D+24D}=24$

$\Rightarrow \quad 48D^2 = 24(D^2 + 48D + 24\ D)$

$2D^2 = D^2 + 72D$ or $D = 72$

$\therefore$ A working alone will complete this work in 72 days.

Ex.7. *A and B can do a piece of work in 45 days and 40 days respectively. They started the work together but after working a few days together A dropped out. If after that B finished the remaining work in 23 days. How long did A work together job ?*

Sol. Work done by A in 1 day $=\dfrac{1}{45}$ and work done by B in 1 day $=\dfrac{1}{40}$

$\therefore$ Work done by (A + B) in 1 day $=\dfrac{1}{45}+\dfrac{1}{40}=\dfrac{17}{360}$

Work done by B in 23 days $=\dfrac{23}{40}$

$\therefore$ Work done by (A + B) $=1-\dfrac{23}{40}=\dfrac{17}{40}$

$\because$ (A + B) do $\dfrac{17}{360}$ th work in 1 day

$\therefore$ (A + B) did $\dfrac{17}{40}$ work in $\dfrac{360}{17}\times\dfrac{17}{40}$ = 9 days

Therefore, A dropped out after working 9 days together.

Short-cut Method:

Suppose A dropped out after x days.

$\therefore$ A worked for x days, B worked for $(x + 23)$ days

Work done by A in x days + work done by B in $(x + 23)$ days = 1 work.

$\therefore \quad \dfrac{x}{45}+\dfrac{x+23}{40}=1 \quad \Rightarrow \dfrac{8x+9x+207}{360}=1$

$\Rightarrow \quad 17x + 207 = 360$

$\Rightarrow \quad 17x = 153$ or $x = 9$

$\therefore$ A dropped out after 9 days.

Ex. 8. *A, B and C together earn Rs. 900 in 10 days. A and B together earn Rs. 350 in 5 days. B and C together earn Rs. 400 in 8 days. Find the daily earning of each.*

Sol. $(A + B + C)$'s daily earning $=\dfrac{\text{Rs.}900}{10}=\text{Rs. }90$

$(A + B)$'s daily earning $=\dfrac{\text{Rs. }350}{5}=\text{Rs. }70$

✍..Notes

$(B + C)$'s daily earning $= \dfrac{\text{Rs. } 400}{8} = \text{Rs. } 50$

$\therefore$ A's daily earning = Rs.90 – Rs. 50 = Rs. 40

B's daily earning = Rs. 70. – Rs. 40 = Rs. 30

C's daily earning = Rs. 50. – Rs. 30 = Rs. 20

Practice Exercise

Directions. Each of the questions given below is followed by four or five alternatives of which one is correct. Mark **(√)** against the correct answer.

1. A can cultivate $\frac{2}{5}$th of a land in 6 days and B can cultivate $\frac{1}{3}$rd of the same land in 10 days, working together A and B can cultivate $\frac{4}{5}$th of the land in

(A) 4 days (B) 5 days (C) 8 days (D) 10 days

2. If 3 men or 4 women can plough a field in 43 days. How long will 7 men and 5 women take to plough it ?

(A) 10 days (B) 11 days (C) 9 days (D) 12 days

3. A is three times more efficient worker than B and is, therefore, able to complete a work in 60 days earlier. The number of days, that A and B together will take to complete the work is

(A) $22\frac{1}{2}$ (B) 25 (C) $27\frac{1}{2}$ (D) 30

4. Babu and Asha can do a job together in 7 days. Asha is $1\frac{3}{4}$ times as efficient as Babu. The same job can be done by Asha alone in

(A) $\frac{49}{4}$ days (B) $\frac{49}{3}$ days (C) 11 days (D) $\frac{28}{3}$ days

5. A and B can do a piece of work in 20 days and 12 days respectively. A started the work alone and then after 4 days B joined him till the completion of the work. How long did the work last ?

(A) 10 days (B) 20 days (C) 15 days (D) 6 days

6. A contractor undertook to do a certain piece of work in 9 days. He employed certain number of men, but 6 of them being absent from the very first day, the rest could finish the work in 15 days. The number of men originally employed were

(A) 12 (B) 15 (C) 18 (D) 24

7. A completes $\frac{7}{10}$ of a work in 15 days, then he completes the remaining work with the help of B in 4 days, the time required for A and B together to complete the entire work is

(A) $10\frac{1}{3}$ days (B) $12\frac{2}{3}$ days (C) $13\frac{1}{3}$ days (D) $8\frac{1}{4}$ days

8. A group of workers accepted to do a piece of work in 25 days. If 6 of them did not turn up for the work and the remaining workers did the work in 40 days, then the original number of workers was

(A) 22 (B) 18 (C) 20 (D) 16

9. A and B weave a carpet in 10 days and 15 days respectively. They begin to work together but B leaves after 2 days. In what time will A complete the remaining work ?

(A) $\frac{1}{3}$ days (B) $6\frac{2}{3}$ days (C) 7 days (D) 8 days

10. 12 men complete a work in 18 days. After 6 days, they had started working, four men joined them. How many days will all of them take to complete the remaining work ?

(A) 10 days (B) 12 days (C) 15 days (D) 9 days

11. A child can do a piece of work 15 hours slower than a women. The child work for 18 hours on the job and then the women takes charges for 6 hours. In this manner $\frac{3}{5}$ of the work can be completed. To complete the job now, how much time will the women take ?

(A) 24 hours (B) 18 hours (C) 12 hours (D) 30 hours

12. Tap 'A' can fill a water tank in 25 minutes, tap 'B' can fill the same tank in 40 minutes and tap 'C' can empty that tank in 30 minutes. If all the three taps are opened together, in how many minutes will the tank be completely filled up or emptied ?

(A) $16\frac{2}{13}$ (B) $15\frac{5}{13}$ (C) $3\frac{2}{13}$ (D) $31\frac{11}{19}$

13. The quantity of work done by a woman in 8 hours is equivalent to the quantity of work done by a man in 6 hours and also to the quantity of work done by a child in 12 hours. If by working for 6 hours per day 9 men can complete the work in 6 days. In how many days, will 12 men, 12 women and 12 children together complete the work by working 8 hours per day ?

(A) $1\frac{1}{2}$ (B) 3 (C) $3\frac{2}{3}$ (D) $1\frac{1}{3}$

14. Samiara, Mahira and Kiara rented a set of DVDs at a rent of Rs. 578. If they used it for 8 hours, 12 hours and 14 hours respectively, what is Kiara's share of rent to be paid ?

(A) Rs. 238 (B) Rs. 204 (C) Rs. 192 (D) Rs. 215

15. A daily-wage labourer was engaged for a certain number of days for Rs 5,750; but being absent on some of those days he was paid only Rs 5,000. What was his maximum possible daily wage?

(A) Rs 125 (B) Rs 250 (C) Rs 375 (D) Rs 500

16. A man and a boy received Rs. 800 as wages for 5 days for the work they did together. The man's efficiency in the work was three times that of the boy. What are the daily wages of the boy ?

(A) Rs. 76 (B) Rs. 56 (C) Rs. 44 (D) Rs. 40

17. Ram can do a work in 20 days. Ram and Shyam together do the same work in 15 days. If they are paid Rs. 400 for that work, what is the share of each?

(A) 50 (B) 100 (C) 150 (D) 200

..Notes

18. *A* and *B* contract to do a work together for Rs. 300. *A* alone can do it in 8 days and *B* alone in 12 days. But with the help of *C* they finish it in 4 days. How much is the money to be divided among them?

(A) 50 (B) 60 (C) 70 (D) 80

19. *A*, *B* and *C* together earn Rs. 900 in 10 days. *A* and *B* together earn Rs. 350 in 5 days. *B* and *C* together earn Rs. 400 in 8 days. Find the daily earning of each.

(A) 5 (B) 10 (C) 15 (D) 20

20. Wages of 20 boys for 15 days is Rs. 9000. If the daily wage of a man is one and half times that of a boy, how many men must work for 30 days to earn Rs. 13500?

(A) 10 (B) 15 (C) 20 (D) 25

21. Ram and Harsh together can complete a work in 10 days. Ram alone can complete the same work in 15 days. In how many days will Harsh alone complete the work?

(A) 25 (B) 5
(C) 30 (D) Cannot be determined

22. If 5 men or 8 women can do a piece of work in 12 days, how many days will be taken by 2 men and 4 women to do the same work?

(A) 15 days (B) $13\frac{1}{2}$ days (C) $13\frac{1}{3}$ days (D) 10 days

23. Suresh can complete a job in 15 hours. Ashutosh alone can complete the same job in 10 hours. If Suresh works alone for 9 hours and then stops, in how many hours Ashutosh will complete the job alone?

(A) 4 (B) 5 (C) 6 (D) 12

24. 12 men complete a work in 9 days. After they have worked for 6 days, 6 more men join them. How many days will they take to complete the remaining work?

(A) 2 days (B) 3 days (C) 4 days (D) 5 days

25. *A* and *B* can do piece of work in 10 days, *B* and *C* in 15 days and *C* and *A* in 20 days, *C* alone can do the work in

(A) 60 days (B) 120 days (C) 80 days (D) 30 days

Answers key

1.(C)	2.(D)	3.(A)	4.(C)	5.(A)
6.(B)	7.(C)	8.(D)	9.(B)	10.(D)
11.(C)	12.(D)	13.(A)	14.(A)	15.(B)
16.(D)	17. (B)	18. (A)	19. (D)	20.(A)
21.(C)	22.(C)	23.(A)	24.(A)	25.(B)

14 Time and Distance

The **Speed** of a body is the rate at which it is moving that is distance travelled in unit time. It is a measure by the distance, a moving body would cover in a given time. Thus, we see that the distance covered by a moving body depends on the speed of the body or person and the time taken.

Basic Rules :

(*i*) More distance, more time; at the same speed.

(*ii*) More speed; less time; for the same distance.

(*iii*) More speed; more distance in the same time.

(*iv*) If the speed of a body is changed in the ratio *a* : *b*, the ratio of time taken to cover a given distance changes in the ratio *b* : *a*.

Formulae : (*i*) Distance = Speed × Time

(*ii*) Speed = $\frac{\text{Distance}}{\text{Time}}$

and, (*iii*) Time = $\frac{\text{Distance}}{\text{Speed}}$

Conversion of Units :

x km/hr $= x\frac{1000}{3600}$ m/s $\left(\begin{array}{l}\because 1\text{ km} = 1000\text{ m} \\ \quad 1\text{ hr} = 3600\text{ seconds}\end{array}\right)$

x km/hr $= \frac{5}{18}x$ m/s

and, x m/s $= \frac{18}{5}x$ km/hr

[Here,
km = Kilometre; m = Metre; hr = Hour; S = Second]

Average Speed : If a certain distance is covered in parts at different speeds, the average speed is given by,

Average Speed $= \frac{\text{Total distance covered}}{\text{Total time taken}}$

Velocity : The speed of a moving body is called its velocity if the direction of motion is also taken into consideration. Though, speed and velocity are interchangeably used in daily life, the two are different quantities. It is given by

Velocity $= \frac{\text{Net displacement of the body}}{\text{Time taken}}$

Relative Speed :

(*a*) ***Bodies moving in the same direction :*** (*i*) If two bodies move in the same **direction,** the relative speed of one with respect to the other is the difference of their speeds. For example, if the two cars *A* and *B* move in the same direction at speeds of 40 km/hr and

30 km/hr respectively, the relative speed of *A* with respect to *B* is (40 – 30) = 10 km/hr.

(*ii*) When two bodies move in the same direction, the distance between them increases/ decreases at the rate of difference in their speeds. In other words, increase (or decrease) in distance between them after time *t* is equal to the product of difference in their speeds and time *t*.

(*b*) ***Bodies moving in the opposite directions :*** (*i*) Relative speed of one with respect to the other is sum of their speeds.

(*ii*) Increase or decrease in distance between them equals product of their relative speed and time.

(*iii*) The distance between two bodies moving towards each other will get reduced at the rate of their relative speed (i.e., sum of their speeds). The time of their meeting (or crossing) is given by,

$$\text{Meeting time} = \frac{\text{Initial distance between the two bodies}}{\text{Sum of their speeds}}$$

SOLVED EXAMPLES

Ex.1. *Starting from a point at a speed of 4 km/hr, a man reaches at a certain place and returns back to the same point from where he had started journey on bicycle at the speed of 16 km/hr. Find his average speed during the entire journey.*

Sol. Given, x = 4 km/hr, y = 16 km/hr.

Since distances covered by the man in the two cases are equal,

∴ Average speed during the entire journey

$$= \frac{2xy}{x+y} = \frac{2\times4\times16}{4+16} = \frac{8\square16}{20}\ \square 6.4\,\text{km/hr}$$

Ex.2. *Two points A and B are 150 km apart. A man completes his onward journey from A to B in 3 hours 20 minutes and return from B to A in 4 hours 10 minutes. Find his average speed reduced during the entire journey in comparison to his average speed during the journey from A to B.*

Sol. *During onward journey* from A to B,

Distance covered = 150 km

Time taken = 3 hours 20 minutes = $3\frac{1}{3} = \frac{10}{3}$ hours

∴ Average speed = $\frac{150}{10/3} = \frac{150\times3}{10} = 45$ km/hr

During return journey from B to A,

Distance covered = 150 km

Time taken = 4 hours 10 minutes

$= 4\frac{1}{6} = \frac{25}{6}$ hours

∴ Average speed = $\frac{150}{25/6} = \frac{150\times6}{25} = 36$ km/hr.

Since distances covered during onward and return journies are equal.

∴ Average speed during the entire journey

$= \frac{2\times45\times36}{45+36} = \frac{90\times36}{81}$ = 40 km/hr

Hence, his average speed during the entire journey is (45 – 40) = 5 km/hr less than his average speed during the journey from A to B.

Ex.3. *Kanchan walks from her home at 4 km/hr and reaches her school 5 minutes late. If she walks from her home at 5 km/hr, then she reaches the school $2\frac{1}{2}$ minutes earlier. How far is the school from her home?*

Sol. Let the distance between her house to the school be x km. Then,
time spent in covering x km at the rate of 4 km/hr

$= \frac{x}{4}$ hours

But Kanchan reaches her school late by 5 minutes.

∴ Usual time of reaching the school

$= \frac{x}{4}\text{hour} - 5\text{ minutes} = \left(\frac{x}{4} - \frac{1}{12}\right)\text{hours}$ .... (*i*)

Again, time spent in covering x km/hr $= \frac{x}{5}$ hours.

But this time Kanchan reaches her school $2\frac{1}{2}$ minutes earlier.

∴ Usual time of reaching the school

$= \frac{x}{5}\text{hours} + 2\frac{1}{2}\text{minutes} = \left(\frac{x}{5} + \frac{1}{24}\right)$ hours .... (*ii*)

Comparing equations (*i*) and (*ii*), we get

$$\frac{x}{4} - \frac{1}{12} = \frac{x}{5} + \frac{1}{24}$$

$$\Rightarrow x = \frac{1}{8} \times 20 = 2.5\,\text{km}$$

Therefore, her school is at a distance of 2.5 km from her home.

Ex.4. *A monkey wants to climb up a glazed pole. It climbs 12 metre in 1 minute and then slips back 3 metre in the next minute. If the pole is 63 metres high, how long does it take to climb at the top of the pole?*

Sol. The monkey climbs in first 2 minutes

= 12 – 3 = 9 metres

∴ The monkey climbs in first 12 minutes

$= 9 \times 6 = 54$ metres

Remaining height of the pole to be covered

= 63 – 54 = 9 metres

∴ The monkey will climb the height of 9 metres in the 13th minutes.

∵ The monkey climbs 12 metres in 1 minute.

∴ The monkey climbs 9 metre in

$\frac{1}{12} \times 9 = \frac{3}{4}$ minutes

∴ Time spent in climbing at the top of the pole

$= \left(12 + \frac{3}{4}\right)$ minutes $= 12\frac{3}{4}$ minutes.

Ex.5. *Two men A and B start walking from certain place in the same direction at 3 km/hr and $3\frac{1}{2}$ km/hr. respectively. What will be the distance between them after $2\frac{1}{2}$ hours ?*

Sol. Since, they start walking in the same direction,

∴ Their relative speed per hour $= 3\frac{1}{2} - 3 = \frac{1}{2}$ km/hr.

∴ Distance between them after $2\frac{1}{2}$ hours

= Relative speed × Time = $\frac{1}{2} \times 2\frac{1}{2} = 1.25$ km

∴ They will be 1.25 km apart after $2\frac{1}{2}$ hours.

Ex.6. *A motorist covers a certain distance at 80 km/hr and during the return journey, he covers the same distance at 60 km/hr. Find the average speed of the motorist.*

Sol. Let distance between the two destinations of the motorist be *x* km. Then,

Time spent in covering *x* km at 80 km/hr = $\frac{x}{80}$ hours

And time spent in covering *x* km at 60 km/hr = $\frac{x}{60}$ hr

∴ Total time spent during his entire journey

$$= \frac{x}{80} + \frac{x}{60} = \frac{7x}{240} \text{ hours}$$

and total distance covered = $x + x = 2x$ kms.

∴ Average speed of the motorist = $\frac{2x}{7x/240}$

$$= \frac{2 \times 240}{7} = 68\frac{4}{7} \text{ km/hr.}$$

Practice Exercise

Directions. Each of the questions given below is followed by four or five alternatives, of which one is correct. Mark (✓) against the correct answer.

1. A and B start at the same time with speeds of 40 km/hr and 50 km/hr respectively. If A takes 15 minutes more than B to complete the journey, then total distance is

(A) 46 km (B) 48 km (C) 50 km (D) 52 km

2. A, B and C start at the same time in the same direction to run around a curicular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds; all starting at the same point. After what time will they meet at the starting point again ?

(A) 40 minutes 12 seconds (B) 45 minutes
(C) 42 minutes 36 seconds (D) 26 minutes 18 seconds

3. The distance between two towns is 800 kms. A car starts from the first town at 30 km/hr. At the same time another car starts from the other town. If the distance of the point where these two cars meet is 500 km from the first town, then at what speed did the second car travel ?

(A) 40 km/hr (B) 30 km/hr
(C) 50 km/hr (D) None of these

4. Vipin travelled 1200 km by air which is 2/5 of his trip. One third of the whole trip, he travelled by car and the rest of the journey by train. The distance travelled by train was

(A) 480 km (B) 800 km (C) 160 km (D) 1800 km

5. Points A and B are 70 km apart on a highway. One car starts from A and another one from B at the same time. If they travel in the same direction, they meet in 7 hours. But if they travel towards each other, they meet in one hour. The speed of the two cars are

(A) 45 and 25 km/hr (B) 70 and 10 km/hr
(C) 40 and 30 km/hr (D) 60 and 40 km/hr

6. An aeroplane flew with a speed of 440 kmph and covered a certain distance. It still had to cover a distance 770 km less than what it had already covered but it flew with a speed of 660 kmph. The average speed for the entire flight was 500 kmph. Find the total distance covered.

(A) 3250 km (B) 2750 km (C) 4400 km (D) 1375 km

7. An aeroplane flies along the four sides of a square field at speeds of 200, 400, 600 and 800 km/hr. The average speed of the plane in the flight around the field in km/hr is

(A) 384 (B) 400 (C) 500 (D) 284

8. A man covers a certain distance on scooter. Had he moved 3 km/hr faster, he would have taken 40 minutes less. If he had moved 2 km/hr slower, then he would have taken 40 minutes more. The distance (in km) is

(A) 20 (B) 36 (C) 37.5 (D) 40

9. The ratio between the rates of walking of A and B is 2 : 3 and therefore A takes 10 minutes more than the time taken by B to reach the destination. If A had walked at double the speed, he would have covered the distance in

(A) 15 minutes (B) 20 minutes (C) 25 minutes (D) 30 minutes

10. Two cars A and B start from the same point by the same route to reach the same destination 120 km away from the starting point. Car B starts exactly one hour later than car A and but both the cars reach the destination at the same time *i.e.,* three hours after A started the journey. What is the speed of car B in km/hr ?

(A) 80 (B) 40
(C) 60 (D) Cannot be determined

11. A train A leaves Meerut at 5 a.m. and reaches Delhi at 9 a.m. Another train B leaves Delhi at 7 a.m. and reaches Meerut at 10:30 a.m. At what time do the two train cross each other ?

(A) 7.36 a.m. (B) 7.56 a.m. (C) 8 a.m. (D) 8.26 a.m.

12. Two cars P and Q start at the same time from A and B which are 120 km apart. When the two cars travel in opposite directions, they meet after one hour and if they travel in same direction (from A towards B), then P meets Q after 6 hours. What is the speed of car P ?

(A) 60 kmph (B) 40 kmph
(C) 120 kmph (D) Data inadequate

13. Buses start from a bus terminal with a speed of 20 km/hr at intervals of 10 minutes. What is the speed of a man coming from the opposite direction towards the bus terminal if he meets the buses at intervals of 8 minutes ?

(A) 3 km/hr (B) 4 km/hr (C) 5 km/hr (D) 7 km/hr

14. By walking at $\frac{3}{4}$ of his usual speed, a man reaches his office 20 minutes later than his usual time. The usual time taken by him to reach his office is

(A) 75 minutes (B) 60 minutes (C) 40 minutes (D) 30 minutes

15. A and B started at the same time from the same place for a certain destination. B walking at $\frac{5}{6}$ of A's speed reached the destination 1 hour 15 minutes after A. B reached the destination in

(A) 6 hours 45 minutes (B) 7 hours 15 minutes
(C) 7 hours 30 minutes (D) 8 hours 15 minutes

16. Two men start together from the some place in the same direction to go round a circular path. If one takes 10 minutes and the other takes 15 minutes to make one complete round they will meet after

(A) 30 minutes (B) 33 minutes (C) 40 minutes (D) 45 minutes

17. A man takes 6 hours 15 minutes in walking a distance and riding back to the starting place. He could walk both ways in 7 hours 45 minutes. The time taken by him to ride both ways, is

(A) 4 hours (B) 4 hours 30 minutes
(C) 4 hours 45 minutes (D) 5 hours

18. A man completed a certain journey by a car. If he covered 30% of the distance at the speed of 20km/hr, 60% of the distance at 40km/hr and the remaining distance at 10km/hr; his average speed for the whole journey was

(A) 25 km/hr (B) 28 km/hr (C) 30 km/hr (D) 33 km/hr

19. A boy has a few coins of denominations 50 paise, 25 paise and 10 paise in the ratio 1 : 2 : 3. If the total amount of the coins is Rs 6.50, the number of 10 paise coins is

(A) 5 (B) 10 (C) 15 (D) 20

20. If p men working p hours per day for p days produce p units of work, then the units of work produced by n men working n hours a day for n days is

(A) $\frac{p^2}{n^2}$ (B) $\frac{p^3}{n^2}$ (C) $\frac{n^2}{p^2}$ (D) $\frac{n^3}{p^2}$

21. A, B, and C start together from the same place to walk round a circular path of length 12km. A walks at the rate of 4 km/hr., B 3 km./hr and C $\frac{3}{2}$ km/hr. They will meet together at the starting place at the end of

(A) 10 hours (B) 12 hours (C) 15 hours (D) 24 hours

22. Ravi and Ajay start simultaneously from a place A towards B, 60 km apart. Ravi's speed is 4km/hr less than that of Ajay. Ajay, after reaching B, turns back and meets Ravi at a places 12 km away from B. Ravi's speed is

(A) 12 km/hr (B) 10 km/hr (C) 8 km/hr (D) 6 km/hr

23. The speeds of A and B are in the ratio 3 : 4. A takes 20 minutes more than B to reach a destination. In what time does A reach the destination ?

(A) $1\frac{1}{3}$ hours (B) 2 hours (C) $2\frac{2}{3}$ hours (D) $1\frac{2}{3}$ hours

24. A man covers half of his journey at 6km/hr and the remaining half at 3km/hr. His average speed is

(A) 9 km/hr (B) 4.5 km/hr (C) 4 km/hr (D) 3 km/hr

25. A student walks from his house at a speed of $2\frac{1}{2}$ km per hour and reaches his school 6 minutes late. The next day he increases his speed by 1 km per hour and reaches 6 minutes before school time. How far is the school from his house ?

(A) $\frac{5}{4}$ km (B) $\frac{7}{4}$ km (C) $\frac{9}{4}$ km (D) $\frac{11}{4}$ km

Answers Key

1.(C)	2.(A)	3.(D)	4.(B)	5.(C)
6.(B)	7.(A)	8.(D)	9.(A)	10.(C)
11.(B)	12.(B)	13.(C)	14.(B)	15.(C)
16.(A)	17.(C)	18.(A)	19.(C)	20.(D)
21.(D)	22.(C)	23.(A)	24.(C)	25.(B)

15 Area

FACTS AND FORMULAE

It is study of measurement, especially of the dimension of geometric figure in order to calculate their areas.

I. Square

AB = BC = CD = DA = side

(*i*) Area of a square = $(\text{side})^2$

(*ii*) Perimeter of a square = 4 × side

(*iii*) Diagonal of a square $=\sqrt{2}$ × side

II. Rectangle

(*i*) Area of a rectangle = length × breadth

(*ii*) Perimeter of a rectangle

= 2 (length + breadth)

(*iii*) Diagonal of a rectangle

$=\sqrt{(\text{length})^2+(\text{breadth})^2}$ [∵ AB = CD, AD = CB]

III. Circle

(*i*) Area of a circle

= π × $(\text{radius})^2$

(*ii*) Radius $=\dfrac{\text{Diameter}}{2}$

or Diameter = 2 × radius

(*iii*) Circumference of a circle = $2\pi\times(\text{radius})$

IV. Triangle

(*i*) Area of a triangle

$=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\dfrac{a+b+c}{2}$

(*ii*) Area of a right angled triangle

$=\dfrac{1}{2}\times\text{base}\times\text{height}$

(*iii*) Area of an equilateral triangle $=\dfrac{\sqrt{3}}{4}\times(\text{side})^2$

(*iv*) Radius of incircle of an equilateral triangle of side (A) $=\dfrac{a}{2\sqrt{3}}$

(*v*) Radius of circumcircle of an equilateral triangle of side (a) $=\dfrac{a}{\sqrt{3}}$

(*vi*) Radius of incircle of a triangle of area (Δ) and semi-perimeter $(s)=\dfrac{\Delta}{r}$

V. Quadrilateral

(*i*) Area of a quadrilateral

$=\dfrac{1}{2}\times$ diagonal × sum of the offsets.

where AC = diagonal, BF and DE are offsets.

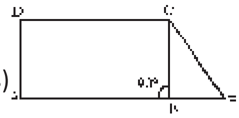
(*ii*) Area of a parallelogram
= length of a side ×
corresponding altitude

where AB = Side of the parallelogram ABCD and
h = corresponding altitude

(*iii*) Area of a rhombus $= \frac{1}{2} \times$ product of the diagonals where AC, BD = diagonals and AB = BC = CD = DA

(*iv*) Area of a trapezium

$= \frac{1}{2}$ × (sum of the parallel sides)



× distance between them.
where AB, CD = parallel sides,
CE = distance between the parallel sides.

VI. Fourwalls (or room)

(*i*) Area of the four walls (A) = 2 × h (l + b)

(*ii*) Height of the room (h) $= \dfrac{A}{2(l+b)}$

where h = height of the room, b = breadth of the room
l = length of the room, A = area of four walls

Short-cut Methods

(*i*) If the length and the breadth of a rectangle are increased by x% and y% respectively, then the area of the rectangle will increase by $\left(x+y+\dfrac{xy}{100}\right)\%$

(*ii*) If the side of a square is increased by x%, then its area will increase by $\left(2x+\dfrac{x^2}{100}\right)\%$.

(*iii*) If the side of a square is decreased by x%, then its area will decrease by $\left(2x-\dfrac{x^2}{100}\right)\%$.

(*iv*) If area of two squares are in the ratio of $a : b$, then their perimeters will be in the ratio of $\sqrt{a} : \sqrt{b}$.

(*v*) Area of the largest circle inscribed in a square of side x cm will be $\left(\pi \times \dfrac{x^2}{4}\right)$ sq.cm.

(*vi*) Area of the square inscribed in a circle of radius x cm will be $2x^2$ sq.cm.

SOLVED EXAMPLES

Ex.1. *By how much percent area of a square will increase if its side is increased by 10% ?*

Sol. Let side of the square be x metre.

In the first case :

Area of the square = x^2 sq.m.

In the second case :

On effecting 10% increase in the side of the square side of the new square = x + 10% of x
= 1.1x metre

✍..Notes

∴ Area of the new square = $1.1x \times 1.1x = 1.21x^2$ sq.m.

Increase in area = $1.21x^2 - x^2 = 0.21x^2$ sq.m.

∴ Percentage increase $= \frac{0.21x^2}{x^2} \times 100 = 21\%$

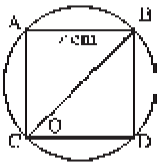
Short-Cut Method

Percentage increase in the side of the square = 10%

∴ Percentage increase in the area of the square $= \left(2x + \frac{x^2}{100}\right)\% = \left(2 \times 10 + \frac{10 \times 10}{100}\right)\%$ = 21%

Ex.2. *Area of a square inscribed in a circle is 50 sq.cm. Find the radius of this circle.*

Sol.



Area of the square ABCD inscribed in the circle

= 50 sq.cm

∴ AB = BD = CD = AC = x cm

∴ Area of the square = $x^2 = 50$

$x = \sqrt{50} = 5\sqrt{2}$ cm

∵ Diagonal BC of the square = Diameter of the circle

$= \sqrt{x^2 + x^2} = x\sqrt{2} = 5\sqrt{2} \times \sqrt{2} = 10$ cm

∴ Radius of the circle $= \frac{10}{2}$ = 5 cm

Short-cut Method

Area of the square inscribed in the circle = 50 sq. cm

∴ $2x^2 = 50$ or $x^2 = 25$ or $x = 5$

∴ Radius of the circle = 5 cm

Ex.3. *If a circle of largest area is cut from the square of 5 cm. What will be the area of the remaining portion of the square?*

Sol. In the given square ABCD,

AB = BC = CD = DA = 5 cm

∴ Area of the square

= 5 × 5 = 25 sq. cm.

Diameter of the circle inscribed in the square will be equal to the side of the square, *i.e.*, diameter of the circle = 5 cm.

∴ Radius of the circle $= \frac{5}{2} = 2.5$ cm

∴ Area of the circle = $\pi r^2 = \pi \times (2.5)^2$

$= 6.25\pi$ sq cm

∴ Area of the remaining portion of the square

= $(25 - 6.25\pi)$ sq cm.

Short-cut Method

∴ Side of the square = 5 cm.

∴ Area of the square = 25 sq. m.

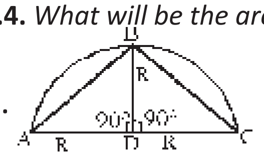
∴ Area of the largest circle inscribed in the square

$= \pi\frac{(5)^2}{4}$ = 6.25 sq cm

∴ Area of remaining portion of the square

= $(25 - 6.25\pi)$ sq. cm

Ex.4. *What will be the area of the largest triangle inscribed in a semi circle of radius R ?*

Sol. 

Radius of the semicircle = AD = DC = DB = R

∴ Area of the Δ ABC

= Area of Δ ADB + Area of Δ DBC

$$= \frac{1 \times R \times R}{2} + \frac{1 \times R \times R}{2} = R^2$$

∴ Area of the largest triangle inscribed in semicircle of radius R will be R^2.

Ex.5. *A rectangular tank is 25 m long, 12 m wide and 6 m high. Find the cost of plastering its walls and battom at 75 paise per square metre.*

Sol. Length = 25 m, breadth = 12 m and height = 6 m

∴ Area of the four walls of the tank

= 2 × height (length + breadth)

= 2 × 6 × (25 + 12) = 12 × 37 = 444 sq.m.

And area of the bottom of the tank

= length × breadth = 25 × 12 = 300 sq. m.

∴ Total area = 444 + 300 = 744 sq. m.

∴ Cost of plastering $= 744 \times \frac{75}{100}$ = Rs. 558.

Ex.6. If area of an equilateral triangle is $\sqrt{3}$ sq. cm, then find the length of its sides.

Sol. Given, area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

∴ $\frac{\sqrt{3}}{4} \times (\text{side})^2 = \sqrt{3}$ or or $(\text{side})^2 = \frac{\sqrt{3} \times 4}{\sqrt{3}} = 4$

or side $= \sqrt{4} = 2$

Hence, side of the triangle = 2 cm.

Practice Exercise

Directions: Each of the questions given below is followed four or five alternatives. Find the one which is correct. Mark (√) against the correct answer.

1. If the length of a rectangle is increased by 20% and its breadth is decreased by 20%, then its area

(A) increases by 20% (B) decreases by 4%
(C) decreases by 1% (D) remains unchanged

2. In the given figure, a circle of radius 2 cm has been set in to a rectangle of dimensions 7 cm × 11 cm, find the area of the shaded portion.

(A) 71.5 cm^2 (B) 64.43 cm^2 (C) 76.2 cm^2 (D) 56.5 cm^2

3. A sector of 120°, cut out from a circle, has an area of $9\frac{3}{7}$ sq. cm. Find the radius of the circle.

(A) 3 cm (B) 6 cm (C) 7 cm (D) 12 cm

4. The perimeters of a square and a circular field are the same, if the area of the circular field is 3,850 sq metre. What is the area (in m^2) of the square?

(A) 4225 (B) 3025 (C) 2500 (D) 2025

5. A circular wire of radius 42 cm is cut and bent into the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle (in cm) is

(A) 30 (B) 60 (C) 72 (D) 132

6. The length of a rectangular garden is 12 metres and its breadth is 5 metre. Find the length of the diagonal of a square garden having the same area as that of the rectangular garden.

(A) $2\sqrt{30}$ m (B) $\sqrt{13}$ m (C) 13 m (D) $8\sqrt{15}$ m

7. A took 15 sec to cross a rectangular field diagonally walking at the rate of 52 m/min and B took the same to cross the same field along its sides walking at the rate of 68 m/min. The area of the field is

(A) 30 m^2 (B) 40 m^2 (C) 50 m^2 (D) 60 m^2

8. The area of the biggest circle which can be drawn inside a square with a side of 21 cm is

(A) 344.5 sq m (B) 364.5 sq m (C) 366.5 sq m (D) 346.5 sq m

9. The perimetres of five squares are 24 cm, 32 cm, 40 cm, 76 cm and 80 cm respectively. The perimeter of another square equal in area to sum of the areas of these squares is

(A) 31 cm (B) 62 cm (C) 124 cm (D) 961 cm

10. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of a side of the square by 5 cm and a breadth is less than the length of the side of the square by 3 cm. The perimeter of the rectangle is

(A) 17 cm (B) 26 cm (C) 30 cm (D) 34 cm

11. If the number of square inches in the area of a circle is half the number of inches of its circumference, the diameter of the circle is

(A) 2 inches (B) 1 inch (C) 4 inches (D) 3 inches

12. The length of a rectangle is decreased by *r*%, and the breadth is increased by (*r* + 5)%. find *r*, if the area of the rectangle is unaltered.

(A) 5 (B) 8 (C) 15 (D) 20

13. A piece of paper is in the shape of a right angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was a 35 per cent reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?

(A) 16.665 (B) 16.565 (C) 15.465 (D) 14.365

14. The area of the square ABCD given below is

(A) 60 cm^2 (B) 90 cm^2 (C) 169 cm^2 (D) 144 cm^2

15. ABCD is a square. F is midpoint of AB and E is a point on BC such that BE is one third of BC. If area of ΔFBE = 108 m^2, then length of AC is

(A) 63 m (B) $36\sqrt{2}$ m (C) $63\sqrt{2}$ m (D) $72\sqrt{2}$ m

16. The area of four walls of a room is 120m. The length is twice the breadth. If the height of the room is 4 m, then find the area of the floor

(A) 48 m^2 (B) 49 m^2 (C) 50 m^2 (D) 52 m^2

17. A wire bent in the form of a square, encloses an area of 484 cm^2. If the same wire is bent so as to form a circle, then the area enclosed will be

(A) 484 cm^2 (B) $538\frac{2}{7}$ cm^2 (C) 616 cm^2 (D) 644 cm^2

18. A circular disc of area 0.49 *x* square metres rolls down a length of 1.76 km. The number of revolutions it makes is

(A) 300 (B) 400 (C) 600 (D) 4000

19. The area of a rectangle gets reduced by 9m^2. If its length is reduced by 5 m and breadth is increased by 3m. If we increase the length by 3m and breadth by 2 m, then area is increased by 67 m^2. The length of the rectangle is

(A) 9 cm (B) 15.6 cm (C) 17 m (D) 18.5 m

20. Find the area of the sector of a circle of radius 14 cm and angle of sector is 45°.

(A) 144 cm^2 (B) 77 cm^2 (C) 225 cm^2 (D) None of these

21. Capacity of a cylindrical vessel is 25872 litres. If the height of the cylinder is three times the radius of its base. What is the area of the base in square cm?

(A) 336 (B) 1232
(C) 616 (D) Can't be determined

22. What is the area of the shaded portion? ABCD is a square with each side of 10 cm.

(A) 80 cm^2 (B) 60 cm^2
(C) 75 cm^2 (D) Cannot be determined

..Notes

23. The area of a circle is 154 cm^2. Its radius is

$\left[\text{Use } \pi = \frac{22}{7}\right]$

(A) 149 cm (B) π cm (C) 7 cm (D) 122 cm

24. The ratio of breadth and length of a rectangular field is 2 : 3. If the cost of the wall is Rs. 260 per metre, and its area is 1350 sq. metres, then find the cost to build the wall?

(A) Rs. 31,200 (B) Rs. 39,000 (C) Rs. 37,700 (D) Data inadequate

25. The area of a rectangle is 110 cm^2. If each of its sides is decreased by 4 cm, then the area is 42 cm^2. What is the width of the rectangle ?

(A) 6 cm (B) 11 cm

(C) 10 cm (D) Can't be determined

Answers Key

1.(B)	2.(B)	3.(A)	4.(B)	5.(B)
6.(A)	7.(D)	8.(D)	9.(C)	10.(D)
11.(A)	12.(D)	13.(D)	14.(C)	15.(B)
16.(C)	17.(C)	18.(B)	19.(C)	20.(B)
21.(C)	22.(D)	23.(C)	24.(B)	25.(C)

16 Volume and Surface Area

FACTS AND FORMULAE

I. Solids : Any thing which resists changes of shape is full of matter *i.e,* which is not hollow, occupies space and has three dimensions (*i.e.,* length, breadth and height) is called *solid*.

II. Volume : The space enclosed within the bounding faces of a body is called its *volume*. Volume is the measurement of cubical content of a solid body.

III. Cuboid (Rectagular Parallelopiped) : A solid body having six rectangular faces, is called *cuboid*.

or

A parallelopiped whose faces are rectangles is called rectangular parallelopiped or cuboid.

(*i*) Volume of cuboid = Length × breadth × height

= $l \times b \times h$

(*ii*) Surface area of a cuboid = 2 (length × breadth + breadth × height + height × length)

= 2 ($lb + bh + hl$)

(*iii*) Longest diagonal of a cuboid

= $\sqrt{(\text{length})^2 + (\text{breadth})^2 + (\text{height})^2}$ = $\sqrt{l^2 + b^2 + h^2}$

IV. Cube : A solid body having six equal square faces, is called *cube*.

(*i*) Volume of a cube = $(\text{side})^3$ [∵ Side = l = b = h]

(*ii*) Surface area of cube = 6 × $(\text{side})^2$

(*iii*) Longest diagonal of a cube

= $\sqrt{3} \times \text{side}$

V. Cylinder (i.e., Right Circular Cylinder)

(*i*) Volume of the cylinder = Area of the base × height

= $\pi r^2 \times h = \pi r^2 h$ cubic units

(*ii*) Area of the curved surface

= Circumference of the base × height

= $2\pi r \times h = 2\pi r h$ sq. units

(*iii*) Area of the total surface

= Area of the curved surface

+ Area of the two circular ends

= $2\pi r h + 2\pi r^2 = 2\pi r(h + r)$ sq. units

VI. Cone

(*i*) Volume of a cone = $\frac{1}{3}$ × area of base × height

$= \frac{1}{3}\pi r^2 h$

(*ii*) Slant height of a cone (*l*)

$= \sqrt{r^2 + h^2}$

(*iii*) Area of the curved surface of a cone

$= \frac{1}{2}$ × perimeter of base × slant height

$= \pi r \times \text{slant height} = \pi r\sqrt{r^2 + h^2} = \pi rl$

VII. Sphere

(*i*) Volume of a sphere $= \frac{4}{3}\pi r^3$ cubic unit where *r* is a radius of the sphere.

(*ii*) Surface area or area of the curved surface of a sphere $= 4\pi r^2$ sq. units

(*iii*) Volume of a hollow sphere = $\frac{4}{3}\pi(R^3 - r^3)$ cubic units where R is outer radius and *r* is inner radius.

VIII. Hemisphere

(*i*) Volume of a hemisphere = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.

(*ii*) Curved surface area = $(2\pi r^2)$ sq. units.

(*iii*) Total surface area = $(3\pi r^2)$ sq. units.

IX. Hollow Cylinder

A solid bounded by two co-axial cylinders of the same height, is called a *hollow cylinder*.

(*i*) Volume of the hollow cylinder

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

whose R and *r* are the external and internal radii of the hollow cylinder and *h* is its height.

(*ii*) Curved surface area of hollow cylinder

$= 2\pi Rh + 2\pi rh = 2\pi(R + r)h$

X. Frustum of a Cone

If a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and base is called the *frustum of the cone*.

(*i*) Lateral surface area of frustum of a right circular cone

$= \pi l(R + r)$ sq. units

where R, *r* be the radii of base and top of the frustum of a cone, *h* is the height of the frustum, and

$$l^2 = h^2 + (R - r)^2$$

(*ii*) Volume of a frustum of a cone

$= \frac{\pi h}{3}(R^2 + r^2 + Rr)$ cubic units

(*iii*) Total surface area of frustum of right circular cone

= Area of base + Area of top + Lateral surface area

$= \pi R^2 + \pi r^2 + \pi l(R + r)$

$= \pi[R^2 + r^2 + l(R + r)]$ sq. units

XI. Prism

A prism is a solid whose side faces are parallelograms and whose base are equal and parallel rectilinear figures.

A prism is called a *right prism,* if the axis is perpendicular to the base.

(*i*) Volume of right prism
= (Area of the base × height) cubic units

(*ii*) Lateral surface area of a right prism
= (Perimeter of the base × height) sq. units

(*iii*) Total surface area of a right prism
= Lateral area + 2 (area of one base) sq. units.

XII. Pyramid

A polyhedron whose one face is a polygon and the other faces are triangles having a common vertex.

(*i*) Volume of a right pyramid

= $\frac{1}{3}$(Area of the base) × height cubic units

(*ii*) Surface area of a right pyramid = $\frac{1}{2}$ (Perimeter of the base) × slant height sq. units

(*iii*) Total surface area of a right pyramid = Surface Area + Area of the base sq. units.

XIII. Short-cut Methods

(A) If three edges or dimensions of a sphere, cuboid, cube, cylinder or cone are increased or decreased by *x*%, *y*% and *z*% respectively, then volume of the figure will increase or decrease by

$$\left(x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}\right)\%$$

(B) If the two edges or dimensions which are included in the surface area of a sphere, cuboid, cube, cylinder or cone are increased or decreased by *x*% and *y*%, then the surface area of the figure will increase or decrease by

$$\left(x + y + \frac{xy}{100}\right)\%$$

We can also say that in case of percentage increase, values of *x*, *y* and *z* are positive and in case of percentage decrease, values of *x*, *y* and *z* are negative.

Solved Examples

Ex.1. *If the height and the base radius of a cone are each increased by 100%, find the ratio of the volumes of the original and the new cones.*

Sol. Let the height and base radius of the cone be *h* and *r* respectively

∴ Volume of the cone = $\frac{1}{3}\pi r^2 h$

In the second case

Height of the cone = *h* + 100% of *h* = 2*h*

Radius of the cone = *r* + 100% of *r* = 2*r*

∴ Volume of the new cone

$$= \frac{1}{3}\pi(2r)^2 2h = \frac{1}{3}\pi(8r^2)h = \left(\frac{1}{3}\pi r^2 h\right)\times 8$$

Hence, ratio of the volumes of the original and the new cones = $\frac{1}{3}\pi r^2 h : \frac{1}{3}\pi r^2 h \times 8 = 1:8$

Short-cut Method

Volume of the cone comprises two radii and one height which together constitute three edges of cone.

100% increase in base radius of cone implies that two edges (radii) are each increased by 100%. (*i.e*, *x*% = *y*% = 100%) and height is increased by 100% *i.e*, *z*% = 100% in case of percentage increase in *x*, *y* and *z*, their values will be positive.

∴ Percentage increase in the volume of the cone

$$= \left(3\times100+\frac{3\times(100)^2}{100}+\frac{(100)^3}{100^2}\right)\%$$

= (300 + 300 + 100)% = 700%

∴ If previous volume is 100 cubic cm, then the volume of the new cone

= 100 + 700% at 100 = 800 cubic cm

Hence, the ratio of the volumes of the original cone and new cone will be 100 : 800 = 1 : 8.

Ex.2. *If radius of a cylinder is increased by 10% and height is decreased by 10%, then by how much percent will the area of its curved surface be increased or decreased ?*

Sol. Suppose the base radius and the height of the cylinder are *r* cm and *h* cm respectively.

∴ Surface area of the cylinder = $2\pi h$ sq. cm

In the second case

Radius of the new cylinder

= *r* + 10% of *r* = 1.1*r* cm

And height of the new cylinder

= *h* – 10% of *h* = 0.9 *h* cm

Surface area of the new cylinder

= $2\pi\times1.1r\times0.9h$ = $2\pi\times0.99\ rh$ sq. cm

Decrease in surface area

= $2\pi rh-2\pi1(0.99)rh$ = 2p ´ $0.01rh$ sq. cm

∴ Percentage decrease = $\frac{2\pi\times.01rh}{2\pi rh}$ × 100 = 1 %

Therefore, surface area of the cylinder will be decreased by 1%.

Short-cut Method

Surface area of a cylinder comprises one edge (*i.e.,* base radius of the cylinder) and another edge (*i.e.,* height of the cylinder).

∴ One edge (radius) of the cylinder is increased by 10% while one edge (height) is decreased by 10%. Hence *x*% = 10%, *y* = 10% in case of percentage increase, the value of *x* will be positive (+) and in case of percentage decrease, value of *y* will be negative (–).

∴ Change in surface area of the cylinder

$$= \left[x+(-y)+\frac{(x)\times(-y)}{100}\right]\%$$

$$= \left[10-10+\frac{10\times(-10)}{100}\right]\% = \left[\frac{-100}{100}\right]\% = -1\%$$

Since the negative sign represents decrease, therefore surface area of the cylinder will be decreased by 1%.

Ex.3. *How many spherical bullets can be made out of a lead cylinder 28 cm high and with base radius 6 cm, each bullet being 1.5 cm in diameter?*

Sol. Volume of cylinder = (π × 6 × 6 × 28) cm^3

= (36 × 28) π cm^3

$\therefore$ Volume of each bullet $= \left(\frac{4}{3}\pi\times\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\right)$ cm^3

$= \frac{9\pi}{16}$ cm^3

$\therefore$ Number of bullets $= \frac{\text{Volume of cylinder}}{\text{Volume of each bullet}}$

$= \left[(36\times 28)\pi\times\frac{16}{9\pi}\right] = 1792$

Ex.4. *A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone 6 cm and its height 4 cm. Calculate the surface area of the toy.*

Sol. Given, Radius of the base of cone, r = 3 cm

Radius of the base of hemisphere, R = 3 cm

Surface area of hemispherical base

$= 2\pi R^2 = 2\pi(3)^2 = 18\pi$

Similarly, surface area of cone $= \pi rl$

$= \pi r\sqrt{h^2+r^2} = \pi.3.\sqrt{(3)^2+(4)^2}$

$= \pi.3\sqrt{9+16} \Rightarrow \pi.3\sqrt{25} = 15$

Hence, surface area of toy

= Surface area of hemispherical base + Surface area of cone

$= 18\pi + 15\pi = 33\pi$ = 33 × 3.14 = 103.62 sq. cm

Ex.5. *The radii of the ends of a bucket of height 24 cm are 15 cm and 5 cm. Find its capacity.*

Sol. The bucket is in the form of a frustum of a cone. Here

$r_2 = 5\,\text{cm},\ r_1 = 15\,\text{cm},\ h = 24\,\text{cm}$

Capacity of bucket (*i.e.*, volume)

$= \frac{\pi h}{3}[r_1^2 + r_2^2 + r_1.r_2]$

$= \frac{22}{7\times 3}\times 24[(15)^2 + (5)^2 + 15\times 5]$ cm^2

$= \frac{22\times 8}{7}[225+25+75]$ cm^3

$= \frac{22\times 8\times 325}{7}$ cm^3 = 8171.43 cm^3.

Ex.6. *The interior of a building is in the form of a cylinder of base radius 12 m and height 3.5 m, surmounted by a cone of equal base and slant height 12.5 m. Find the internal curved surface area and the capacity of the building.*

Sol. For cylinder height, h = 3.5 m

Radius = 12 m

A_1 = curved surface area $= 2\pi rh$

= 2 × 3.14 × 12 × 3.5 sq.m

= 263.76 sq. m.

V_1 = Volume of cylinder $= \pi r^2 h$

= 3.14 × 144 × 3.5 cube m.

..Notes

For cone, slant height l = 12.5 m

∴ Curved surface area = πrl

= 3.14 × 12 × 12.5

or A_2 = 471 sq. m.

Let V_2 be the volume of cone,

= $\frac{1}{3}\pi r^2 h$ = $\frac{1}{3} \times 3.14 \times 144 \times h$

V_2 = $\frac{1}{3} \times (3.14) \times 144 \times 3.5$

$$\left[h = \sqrt{(12.5)^2 - 12^2} = 3.5\right]$$

∴ Total area of surface = $A_1 + A_2$

= 263.76 + 471 = 734.76 sq. m.

and total volume of the building = $V_1 + V_2$

= 3.14 × 144 (3.5 + 3.5) = 3168 cube m.

Practice Exercise

Directions. Each of the questions given below is followed by four or five alternatives. Find out the which is correct. Mark (?) against the correct answer.

1. **How much water will flow of a pipe of 1.5 cm radius in one hour, if the speed of water is 3 km/hr ?**

   (A) $\frac{20}{3}\pi m^3$ (B) $\frac{27}{40}\pi m^3$ (C) $\frac{3}{40}\pi m^3$ (D) $\frac{40}{3}\pi m^3$

2. **If length of a swimming pool is 12 metres, width is 9 metres while depth is 4 metres, its volume will be**

   (A) 432 cubic metres (B) 360 cubic metres
   (C) 270 cubic metres (D) 208 cubic metres

3. **A wooden pillar is 7 metres high and its diameter is 20 cm. Find its weight, if the weight of wood is 225 kg/cubic metre.**

   (A) 155.4 kg (B) 56 kg (C) 49.5 kg (D) 16.5 kg

4. **Find the volume of a cone when its slant height is 17 cm and radius of base is 8 cm.**

   (A) 3017.1 cm^3 (B) 2015.2 cm^3 (C) 951.4 cm^3 (D) 1005.7 cm^3

5. **A hemisphere and a cone have equal bases. If their heights are also equal, then ratio of their curved surfaces will be**

   (A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$ (C) 1 : 2 (D) 2 : 1

6. **The areas of three adjacent faces of a cuboid are *p*, *q* and *r*, its volume will be**

   (A) $\sqrt{pqr}$ (B) pqr (C) $p^2q^2r^2$ (D) $\sqrt{p^2+q^2+r^2}$

7. **A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm, if the thickness of the pipe is 1 cm and iron weight $8 g/cm^3$, then the weight of the pipe (Take $\pi = \frac{2}{7}$) is**

(A) 3.696 kg (B) 3.6 kg (C) 36 kg (D) 36.9 kg

8. A hollow spherical metallic ball has an external diameter 6 cm and is $\frac{1}{2}$ cm thick. Then volume of the ball (in cm^3) is

(A) $41\frac{2}{3}$ (B) $37\frac{2}{3}$ (C) $47\frac{2}{3}$ (D) $40\frac{2}{3}$

9. If 12 spheres of the same size are made by melting a solid cylinder of 16 cm diameter and 2 cm height, then diameter of each sphere is

(A) 2 cm (B) 4 cm (C) 3 cm (D) $\sqrt{3}$ cm

10. A right cylindrical pack is full of ice-cream. How many ice-cream cones having same diameter and height as that of the cylinder can be filled with the ice-cream?

(A) 2 (B) 3 (C) 4 (D) None of these

11. If the height curved surface area and volume of a cone are h, c and v respectively, then $3\pi v h^3 - c^2h^2 + 9v^2$ will be equal to

(A) 0 (B) 1 (C) chv (D) v^2h

12. If V be the volume and S be the surface area of a cuboid of dimensions a, b, c, then, $\frac{1}{V}$ is equal to

(A) $\frac{s}{2}(a+b+c)$ (B) $\frac{2}{s}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(C) $\frac{2s}{a+b+c}$ (D) $2s(a+b+c)$

13. The sum of length, breadth and height of a room is 19 m. The length of the diagonal is 11 m. The cost of painting the total surface area of the room at the rate of Rs. 10 per m^2 is

(A) Rs. 240 (B) Rs. 2400 (C) Rs. 420 (D) Rs.4200

14. A rectangular tank measuring 5 m × 4.5 m × 2.1 m is dug in the centre of the field measuring 13.5 m × 2.5 m. The earth dug out is spread evenly over the remaining portion of the field. How much is the level of the field raised?

(A) 4.0 m (B) 4.1 m (C) 4.2 m (D) 4.3 m

15. A rectangular tank is 225 m by 162 m at the base. With what speed must water flow into it through an aperture 60 cm by 45 cm that the level may be raised 20 cm in 5 hours?

(A) 5000 m/hr (B) 5400 m/hr (C) 5200 m/hr (D) 5600 m/hr

16. If a solid piece of iron of dimensions 49 cm × 33 cm × 24 cm is moulded into a sphere, then radius of the sphere is

(A) 21 cm (B) 28 cm (C) 35 cm (D) None of these

17. Volume of a cylindrical tank is 3080 cubic metres. If the height of the tank is increased by 5 metres keeping the base same, then volume increases to 3850 cubic metres. What is the radius of the base?

(A) 14 metres (B) 21 metres

(C) 7 metres (D) can't be determined

..Notes

18. The length, breadth and height of a cuboid are in the ratio 1 : 2 : 3, the length, breadth and height of the cuboid are increased by 100%, 200% and 200% respectively, then the increase in the volume of the cuboid is

(A) 5 times (B) 6 times (C) 12 times (D) 17 times

19. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, then the volume of the beverage in the cylindrical vessel is

(A) $66\frac{2}{3}\%$ (B) $78\frac{1}{2}\%$ (C) 100 % (D) More than 100%

20. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical ball each of half the radius of the cylinder. The number of spherical ball is

(A) 24 (B) 30 (C) 48 (D) 64

21. How many spherical bullets can be made out of a lead cylinder 15 cm high and with base radius 3 cm, each bullet being 5 mm in diameter?

(A) 6480 (B) 6820 (C) 7180 (D) 7800

22. If the areas of three adjacent faces of a rectangular block are in the ratio of 2 : 3 : 4 and its volume is 9000 cuuic cm, then the length of the shortest side is

(A) 5 cm (B) 10 cm (C) 15 cm (D) 20 cm

23. A hemispherical tank of radius $1\frac{3}{4}$m is full of water. It is connected with a pipe which empties it at the rate 7 litres per second. How much time will it take to empty the tank completely?

(A) 11 min (B) 26.7 min (C) 30.3 min (D) 47.7 min

24. A cone is cut transversely to obtain cone of half of its height and a frustum. Find the ratio of the volumes of the two parts of the cone.

(A) 7 : 1 (B) 1 : 7 (C) 16 : 1 (D) 1 : 16.

25. A circus tent has cylindrial shape surmounted by a conical roof. The radius of the cylindrical base is 20 m. The heights of the cylindrical and conical portions are 4.2 m and 2.1 m respectively. Find the volume of the tent.

(A) 5740 m^3 (B) 6068 m^3 (C) 6160 m^3 (D) 7940 m^3

Answers Key

1.(D)	2.(A)	3.(C)	4.(D)	5.(B)
6.(B)	7.(A)	8.(C)	9.(B)	10.(B)
11.(A)	12.(B)	13.(B)	14.(C)	15.(B)
16.(A)	17.(C)	18.(D)	19.(C)	20.(C)
21.(A)	22.(C)	23.(B)	24.(B)	25.(C)

17 Problem on Trains

FACTS AND FORMULAE

I. x km/hr = $\left(x \times \frac{5}{18}\right)$ m/s

or x m/s = $\left(x \times \frac{18}{5}\right)$ km/hr

II. When a train has to cross a stationary object, it has to cover at the given speed and in the given time a distance equal to the length of the train, *i.e.*,

Length of the train = Speed of the train $\times$ Time

taken to cross the object.

III. When a train has to cross a stationary object which has some length (*viz*., a railway platform or a bridge, it has to cover the sum of its own length and the length of the stationary object, *i.e,* Time taken in crossing the stationary object _

$$= \frac{\text{Length of the train} + \text{Length of the journey object}}{\text{Speed of the train}}$$

IV. When two trains running either in the same direction or in opposite direction pass each other, they cover the distance equal to sum of their lengths. Therefore, the distance covered

= Length of the first train + Length of the second train.

V. If two trains start moving at the same time in the same direction from a certain place at x km/hr and y km/hr respectively, their relative speed per hour will be the difference of speeds of the two trains, *i,e.,* Relative speed per hour

= Speed of the first train – Speed of the second train.

= $(x - y)$ km/hr $(\because x > y)$

If lengths of the trains are u km and v km respectively, then time taken by the faster train to cross the slower train (*i.e.*, in same direction)

= $\left(\frac{u+v}{x-y}\right)$ hrs.

VI. If two trains start moving at the same time in opposite directions from a certain place at

x km/hr and y km/hr respectively, then their relative speed per hour will be the difference of speeds of the two trains, *i.e.*, Relative speed per hour

= Speed of the first train (x) + Speed of the second train (y) = $(x + y)$ km/hr

If lengths of the trains are u km and v km respectively, then time taken to cross each other (*i.e.*, in opposite direction)

$= \left(\frac{u+v}{x+y}\right)$hrs

VII. If two trains start at the same time from two points P and Q towards each other and after crossing, they take t_1 and t_2 hours in reaching Q and P respectively

i.e., P'*s* speed : Q'*s* speed = $\sqrt{t_2}$: $\sqrt{t_1}$

Solved Examples

Ex.1. *Two trains running in the same direction at 40 km/hr and 22 km/hr completely pass one another in 1 minute. If the length of the first train is 125 m, then find length of the second train.*

Sol. Relative speed = 40 – 22 = 18 km/hr

$= 18 \times \frac{5}{18}$ m/sec = 5 m/sec

Let the length of second train be x metres. Then,

$\frac{125+x}{5} = 60 \Rightarrow 125 + x = 300$ or $x = 175$

$\therefore$ Length of the second train = 175 m

Ex.2. *A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of 120 metres long train running at 45 kmph in the same direction. In how much time will the train pass the jogger ?*

Sol. Speed of train relative to jogger

= (45 – 9) km/hr = 36 km/hr.

$= \left(36 \times \frac{5}{18}\right)$ m/sec = 10 m/sec.

Distance to be covered = (240 + 120) m = 360 m.

$\therefore$ Time taken = $\left(\frac{360}{10}\right)$ sec = 36 sec.

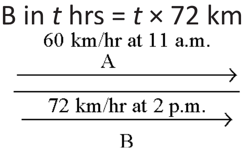
Ex.3. *Train A leaves Mumbai Central station for Lucknow at 11 a.m. running at a speed for 60 kmph. Train B leaves Mumbai Central station for Lucknow by the same route at 2 p.m. on the same day running at the speed of 72 kmph . At what time will the two trains meet each other ?*

Sol. Let two trains meet after t hours from 2 p.m.

Distance covered by train A in $(t + 3) \times 60$ km.

Distance covered by train B in t hrs = $t \times 72$ km

$\therefore$ $(t + 3) \times 60 = t \times 72$



$\Rightarrow$ $5t + 15 = 6t$ or $t = 15$ hours

Hence, they will meet after 15 hours from 2 p.m. i.e. at 2 p.m. + 10 hrs + 5 hrs = 5 a.m. next day.

OBJECTIVE TYPE QUESTIONS

Directions. Each of the questions given below is followed by four or five alternatives of which one is correct. Mark (√) against the correct answer.

1. **Two trains one 160 m and the other 140 m long are running in opposite directions on parallel rails. The first at 77 km an hour and the other at 67 km an hour. How long will they take to cross each other ?**

   (A) 7 seconds (B) $7\frac{1}{2}$ seconds (C) 6 seconds (D) 10 seconds

2. **A passenger train takes two hours less for a journey of 300 km. If its speed is increased by 5 km/hr from its normal speed, then normal speed is**

   (A) 35 km/hr (B) 50 km/hr (C) 25 km/hr (D) 30 km/hr

3. **A train 110 m in length travels at 60 km/hr. How much time does the train take in passing a man walking at 6 km/hr against the train ?**

   (A) 6 sec (B) 12 sec (C) 16 sec (D) 18 sec

4. **A train passes two persons walking in the same direction at a speed of 3 km/hr and 5 km/hr respectively in 10 seconds and 11 seconds. The speed of the train is**

   (A) 28 km/hr (B) 27 km/hr (C) 25 km/hr (D) 24 km/hr

5. **A passenger train running at the speed of 80 km/hr leaves the railway station 6 hours after a goods train leaves and overtakes it in 4 hours. The speed of the goods train is**

   (A) 32 km/hr (B) 40 km/hr (C) 50 km/hr (D) 60 km/hr

6. **Two trains 100 metres and 120 metres long are running in the same direction with speed of 72 km/hr and 54 km/hr. In how much time will the first train cross the second ?**

   (A) 50 sec (B) 8 sec (C) 44 sec (D) 12 sec.

7. **A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds of pass a telegraph post, then length of the train is**

   (A) 90 m (B) 100 m (C) 120 m (D) 140 m

8. **A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, then what is the length of the platform ?**

   (A) 120 m (B) 240 m. (C) 300 m (D) None of these

9. **Two stations P and Q are 110 km apart on a straight line. One train starts from P at 7 a.m. and travels towards Q at 20 km/hr. Another train starts from Q at 8 a.m. and travels towards P at a speed at 25 km/hr. At what time will they meet ?**

   (A) 9 a.m. (B) 10 a.m. (C) 10.30 a.m. (D) 11 a.m.

10. **A train 110 m long passes a man, running at 6 km/hr in the direction opposite to that of the train, in 6 seconds. The speed of the train is**

    (A) 54 km/hr (B) 60 km/hr (C) 66 km/hr (D) 72 km/hr.

11. Two trains of equal length use running of parallel lines in the same direction at 46 km/hr and 36 km/hr. The faster train passes the slower train in 36 seconds. The length of each train is

(A) 50 m (B) 72 m (C) 80 m (D) 82 m

12. Two trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is

(A) 2 : 3 (B) 3 : 4 (C) 4 : 3 (D) 9 : 16

13. A train, 300m long, passed a man, walking along the line in the same direction at the rate of 3 km/hr in 33 seconds. The speed of the train is

(A) 30 km/h (B) 32 km/h (C) $32\frac{8}{11}$ km/h (D) $35\frac{8}{11}$ km/h

14. Two trains started at the same time, one from A to B and the other from B to A. If they arrived at B and A respectively 4 hours and 9 hours after they passed each other, the ratio of the speeds of the two trains was

(A) 2 : 1 (B) 3 : 2 (C) 4 : 3 (D) 5 : 4

15. Two trains of equal length, running in opposite directions, pass a pole in 18 and 12 seconds. The trains will cross each other in

(A) 14.4 seconds (B) 15.5 seconds
(C) 18.8 seconds (D) 20.2 seconds

16. A moving train crosses a man standing on a platform and a bridge 300 metres long in 10 seconds and 25 seconds respectively. What will be the time taken by the train to cross a platform 200 metres long ?

(A) $16\frac{2}{3}$ seconds (B) 18 seconds
(C) 20 seconds (D) 22 seconds

17. A train covers a distance of 3584 km in 2 days 8 hours. If it covers 1440 km on the first day and 1608 km on the second day, by how much does the average speed of the train for the remaining part of the journey differ from that for the entire journey ?

(A) 3 km/hour more (B) 3 km/hour less
(C) 4 km/hour more (D) 5 km/hour less

18. A train, 150m long, passes a pole in 15 seconds and another train of the same length travelling in the opposite direction in 12 seconds. The speed of the second train is

(A) 45 km./hr (B) 48 km./hr (C) 52 km./hr (D) 54 km./hr

19. A train travelling at 48 km/hr crosses another train, having half its length and travelling in opposite direction at 42 km/hr, in 12 seconds. It also passes a railway platform in 45 seconds. The length of the railway platform is

(A) 200 m (B) 300 m (C) 350 m (D) 400 m

20. Two trains 105 metres and 90 metres long, run at the speeds of 45 km/hr and 72 km/hr respectively, in opposite directions on parallel tracks. The time which they take to cross each other, is

(A) 8 seconds (B) 6 seconds (C) 7 seconds (D) 5 seconds

21. A train travelling at a speed of 30 m/sec crosses a platform, 600 metres long, in 30 seconds. The length (in metres) of train is

(A) 120 (B) 150 (C) 200 (D) 300

22. A train passes a platform 110 m long in 40 seconds and a boy standing on the platform in 30 seconds . The length of the train is

(A) 100 m (B) 110 m (C) 220 m (D) 330 m

23. A train with a uniform speed passes a platform, 122 metres long, in 17 seconds and a bridge, 210 metres long, in 25 seconds. The speed of the train is

(A) 46.5 km/hour (B) 37.5 km/hour

(C) 37.6 km/hour (D) 39.6 km/hour

24. A train passes a platform 90 metres long in 30 seconds and a man standing on the platform in 15 seconds. The speed of the train is :

(A) 12.4 kmph (B) 14.6 kmph (C) 18.4 kmph (D) 21.6 kmph

25. A train passes two persons walking in the same direction at a speed of 3 km/hour and 5km/hour respectively in 10 seconds and 11 seconds respectively. The speed of the train is

(A) 28 km/hour (B) 27 km/hour (C) 25 km/hour (D) 24 km/hour

Answers key

1. (B)	**2.(C)**	**3.(A)**	**4.(C)**	**5.(A)**
6. (C)	**7.(C)**	**8.(B)**	**9.(A)**	**10.(B)**
11.(A)	**12.(C)**	**13.(D)**	**14.(B)**	**15.(A)**
16.(C)	**17.(A)**	**18.(D)**	**19.(D)**	**20.(B)**
21.(D)	**22.(D)**	**23.(D)**	**24.(D)**	**25. (C)**

..Notes

18 Calendar

If someone asks you what day it was on 10th May 1575 or what day it would be on 12th September 2340, you may call him crazy for asking such silly questions. If you don't know the rule how to find it, it may look like a Herculean task for you. But, truly speaking, it is not so difficult.

In this chapter we will concentrate our discussion on finding its answer i.e; on what day of the week a particular date falls. The clue to the process of finding it lies in calculating the number of odd days, which is quite different from the odd numbers. The number of days more than the complete number of weeks in a given period are called *odd days.* In other words it is the remainder left when the given number of days is converted into weeks on dividing it by 7.

In ancient times many civilizations used calendars based on movement of the moon. These lunar calendars were not accurate and corrections had to be made frequently. Later, based on the fact that in the solar system all planets including Earth revolved around the Sun, solar calendars were developed. These solar calendars proved to be more accurate.

A solar year consists of 365 days, 5 hours, 48 minutes and 48 seconds. In Julian calendar, arranged in 47 BC by Julius Caesar, the year was taken as $365\frac{1}{4}$ days. In order to make up for the odd quarter of a day, an extra or intercalary day was added once in every fourth year and this was called a *Leap year*. Thus, an ordinary year consists of 365 days and a leap year has 366 days. In a leap year, February has 29 days instead of 28 days for ordinary year. The calendar based on this system is known as the Old Style Calendar. But, it can be noticed on comparison, since the solar year is 11 minutes 12 seconds less than a quarter of a day, in due course of several years, Julian Calendar too became inaccurate by several days. It again called for a further correction to be made. To rectify this discrepancy Pope Gregory XIII devised another calendar known as Gregorian Calendar. According to it, not all century years are leap years, although all of them are divisible by 4. He made centurial years leap years only once in 4 centuries. Accordingly, only those century years which are divisible by 400 are leap years, while other century years are ordinary years. For example, 1300, 1400 and 1500 are ordinary years but 1600 is a leap year. With this modification, the Gregorian Calendar came in close exactitude with the solar year and the difference between the two is only 26 seconds which amounts to a day in 3323 years. These calendars are called as the New Style Calendars.

In India, Vikrami and many other calendars were used earlier. Now the Government of India has adopted the National Calendar based on Saka era with Chaitra as its first month. The days of this national calendar have a direct permanent correspondence with the days of Gregorian — **Chaitra 1 falls on March 22 in an ordinary year and on March 21 in a leap year.**

Key Points

1. An ordinary year contains 365 days i.e., 52 weeks and 1 odd day.

2. A leap year contains 366 days i.e., 52 weeks and 2 odd days.

Note : For an year to be a *leap year,* both the following conditions should be satisfied:

(*i*) An year divisible by 4 is a leap year.

For example, 1984, 1988, 1992, 1996, 2000 etc.

(*ii*) In case of century years, only those divisible by 400 are leap years, while other century years are not leap years.

For example, 400, 800, 1200, 1600, 2000 etc. are leap years.

500, 600,700, 900, 1000 etc. are not leap years.

3. 100 years contains 24 leap years and 76 ordinary years.

∴ 100 years

= [(24 × 52) weeks + (24 × 2) add days] +

[(76 × 52) weeks + (76 × 1) odd days]

= (24 + 76) × 52 weeks + (48 + 76) odd days.

= 5200 weeks + 124 odd days

= 5200 weeks + 17 weeks + 5 odd days

= 5217 weeks + 5 odd days

i.e., **100 years contains 5 odd days.**

200 years contains (5 × 2) = 10 =1 week + 3 odd days, i.e., **3 odd days**.

300 years contains (5 × 3) = 15 = 2 weeks + 1 odd day, i.e, **1 odd day**.

400 years is a leap year and hence it will contain

(5 × 4) + 1 = 21 days which equals 3 weeks and and hence

no odd day.

Similarly, 800, 1200, 1600, 2000 years each contain no odd day.

4. To find the day of the week on a particular date when no reference day is given :

(*i*) Count the net number of odd days on the given date.

(*ii*) Write Sunday for 0 odd day

Monday for 1 odd day

Tuesday for 2 odd days

⋮

Saturday for 6 odd days

Sunday for 7 odd days which is same as 0 odd day.

Assumption : First January 1 A.D. was Monday.

Practice Exercise

Directions: Each of the given questions are followed by four or five alternatives which one is correct. Mark (✓) against the correct answer.

1. What was the day of the week on 16th July, 1776?

(A) Tuesday (B) Monday (C) Wendesday (D) Friday

2. On what dates of July 2004 did Monday fall ?

(A) 5th, 12th, 19th, 26th (B) 4th, 11th, 18th, 25th

(C) 1th, 8th, 9th, 16th (D) 2nd, 9th, 16th, 1st

3. The first Republic day of India was celebrated on 26th Januray 1950. It was

(A) Tuesday (B) Wendesday (C) Thursday (D) Friday.

4. **On what dates of April 2001 did Sunday fall?**
   (A) 1st, 8th, 15th, 22th, 29th (B) 2nd, 9th, 16th, 23th, 30th
   (C) 4th, 11th, 18th, 25th (D) 6th, 13th, 20th, 27th

5. **What was the day of the week on 16th April, 2000?**
   (A) Tuesday (B) Saturday (C) Sunday (D) Monday

6. **January 1, 2004 was a Thursday. What day of the week lies on Jan. 1, 2005 ?**
   (A) Thursday (B) Friday (C) Saturday (D) Sunday

7. **Any date in march of a years is the same day of the week as the corresponding date is _______ that year.**
   (A) 11th Sept. (B) 2nd Nov. (C) 1st Nov. (D) 11th May

8. **What was the day of the week on 15th August 1947 ?**
   (A) Saturday (B) Tuesday (C) Monday (D) Wednesday

9. **Today is Friday. After 62 days, it will be**
   (A) Saturday (B) Monday (C) Thursday (D) Tuesday

10. **The calendar for the year 2005 is the same as for the years**
    (A) 2010 (B) 2011 (C) 2012 (D) 2013

11. **Dec. 9, 2001 is Sunday. What was the day on Dec. 9, 1971 ?**
    (A) Thursday (B) Wednesday (C) Saturday (D) Sunday

12. **If the day before yesterday was Sunday, what day will it be three days after the day after tomorrow ?**
    (A) Sunday (B) Monday (C) Wednesday (D) Saturday

13. **Raju and Nirmala celebrated their first wedding anniversary on Sunday, the 5th of December 1993. What would be the day of their wedding anniversary in 1997?**
    (A) Wednesday (B) Thursday (C) Friday (D) Tuesday

14. **If the day before yesterday was Friday, what day will two days after the day after tomorrow be?**
    (A) Saturday (B) Thursday (B) Friday (D) Sunday

15. **If the day before yesterday was Thursday, when will Sunday be?**
    (A) Today (B) Two days after today
    (C) Tomorrow (D) Day after Tomorrow

16. **If day before yesterday was Tuesday, the day after tomorrow will be**
    (A) Monday (B) Wednesday (C) Friday (D) Saturday

17. **If three days after today will be Tuesday, what day was four days before yesterday?**
    (A) Tuesday (B) Sunday (C) Monday (D) Wednesday

18. **If 15th June falls 3 days after tomorrow, that is Friday, on what day will the last of the month fall?**
    (A) Monday (B) Tuesday (C) Wednesday (D) Thursday

19. If the day after tomorrow is Friday, what day will third day after the tomorrow be?

(A) Saturday (B) Monday (C) Sunday (D) Friday

20. If the day after tomorrow is Tuesday, what day will two days after the day after tomorrow be?

(A) Monday (B) Wednesday (C) Saturday (D) Thursday

21. Rajesh correctly remembers that his friend Sanjay started working after April but before September. Vinod correctly remembers that Sanjay did not have a job before May. Madan correctly remembers that the month Sanjay started working had 30 days. In which month of the year did Sanjay definitely start working?

(A) July (B) August (C) September (D) June

22. Prerna correctly remembers that last year the exams were held before June but after February. Simran correctly remembers that last year the exams were before September but after April. In which month of the year were the exams definitely held ?

(A) October (B) January (C) July (D) May

23. Pooja correctly remembers that she had bought an article from a shop before Saturday and after Tuesday. The shopkeeper correctly remembers that she had bought the article after Thursday but before Sunday. On which of the following days did she definitely buy the article ?

(A) Monday (B) Wednesday (C) Friday (D) Thursday

24. Snehal correctly remembers that Kiran's birthday was after Tuesday but before Friday. Rajan correctly remembers that Kiran's birthday was after Wednesday but before Sunday, on which day of the week does Kiran's birthday definitely fall ?

(A) Monday (B) Thursday
(C) Saturday (D) Cannot be determined

25. If the first day of a month is Saturday, which of the following will be the third day from 17th of that month?

(A) Monday (B) Tuesday
(C) Thursday (D) Wednesday

Answers Key

1.(A)	2.(A)	3.(C)	4.(A)	5.(C)
6.(C)	7.(C)	8.(A)	9.(C)	10.(C)
11.(A)	12.(A)	13.(C)	14.(B)	15.(C)
16.(D)	17.(C)	18.(B)	19.(C)	20.(D)
21.(D)	22.(D)	23.(C)	24.(B)	25.(C)

19 Clocks

FACTS AND FORMULAE

I. Spaces

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute divisions (*i.e.*, spaces).

As we know, a clock has two hands, the smaller one is called the *hour hand* (*i.e.*, short hand), while the the larger one is called the *minute hand* (*i.e.*, long hand)

II. In 60 minutes, the minute hand gains 55 minutes on the hour hand.

III. The hands are in the same straight line when they are coincident or opposite to each other.

IV. In every hour, both the hands coincide once.

V. When the two hands are at right angles, they are 15 minute divisions apart.

VI. When the two hands are in opposite directions, they are 30 minute divisions apart.

VII. Angle traced by hour hand in 12 hrs = 360°.

VIII. Angle traced by minutes hand in 60 min = 360°

Too Fast and Too Slow : If a watch or a clock indicates 10.15, whereas the correct time is 10, it is said to be 15 minutes too fast.

On the other hand, if it indicates 9.45, whereas the correct time is 10, it is said to be 15 minutes too slow.

Solved Examples

Ex.1. *How many times are the hands of a clock at right angle in a day ?*

Sol. In 12 hours, they are at right angles 22 times.

∴ In 24 hours, they are at right angles 44 times.

Ex.2. *An accurate clock shows* **8 O'** *clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 O'clock in the afternoon ?*

Sol. Angle traced by the hour hand in 6 hours

$$= \left(\frac{360}{12} \times 6\right)^{\circ} = 180^{\circ}$$

Ex.3. *A watch which gains 5 seconds in 3 minutes was set right at 7 am. In the afternoon of the same day, when the watch indicated quarter past 4 O'clock, the true time is*

Sol. Time from 7 am to 4.15 pm = 9 hrs 15 min. = $\frac{37}{4}$ hrs.

3 min. 5 sec. of this clock = 3 min of the correct clock

$= \frac{37}{720}$ hrs. of this clock = $\frac{1}{20}$ hrs. of the correct clock.

$= \frac{37}{4}$ hrs. of this clock $= \left(\frac{1}{20} \times \frac{720}{37} \times \frac{37}{4}\right)$ hrs. of the correct clock.

= 9 hrs. of the correct clock.

$\therefore$ The correct time is 9 hrs. after 7 am, *i.e.,* 4 pm.

Practice Exercise

Directions: Each of the questions given below is followed by four or five alternatives of which one is correct. Mark (✓) against the correct answer.

1. Find at what time between 8 and 9 o'clock will the hands of a clock be in the same straight line but not together.

(A) $10\frac{10}{11}$ min. past 8 (B) $10\frac{10}{11}$ min. past 7

(C) $10\frac{10}{11}$ min. past 9 (D) $10\frac{11}{10}$ min. past 8.

2. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or loss ?

(A) $10\frac{44}{10}$ min. (B) $10\frac{10}{43}$ min. in 24 hours

(C) $10\frac{1}{2}$ in 2 hours (D) 6 min. in 8 hours.

3. A clock is set right at 5 am. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 pm on 4th day ?

(A) 1 am (B) 2 am (C) 12 pm (D) 11 pm

4. What is the angle between the hour and the minute hand of a clock when the time is 3.25.

(A) $47\frac{1}{2}^{\circ}$ (B) $48\frac{1}{2}^{\circ}$ (C) $47\frac{2}{1}^{\circ}$ (D) $47\frac{2}{3}^{\circ}$

5. At what time between 2 and 3 O'clock will the hands of a clock be together ?

(A) $10\frac{10}{11}$ min. past 2 (B) $10\frac{10}{12}$ past 2

(C) $10\frac{10}{11}$ past 3 (D) $10\frac{10}{11}$ past 1

6. A clock is set right at 8 am. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 1 pm. on the following day ?

(A) 48 past 11 (B) 48 min. past 4

(C) 48 min. past 12 (D) 48 min. past 2

✍..Notes

7. At what time between 5.30 and 6 will the hands of a clock be at right angles ?

(A) $43\frac{5}{11}$ min. past 5 (B) $43\frac{7}{11}$ min. past 5

(C) 40 min. past 5 (D) 45 min. past 5

8. At what time between 9 and 10 O'clock will the hands of a watch be together ?

(A) 45 min. past 9 (B) 50 min. past 9

(C) $49\frac{1}{11}$ min. past 9 (D) $48\frac{2}{11}$ min. past 9

9. The angle between the minute hand and the hour hand of a clock, when the time is 8.30, is

(A) 80° (B) 75° (C) 60° (D) 105°

10. A clock is started at noon. By 10 min, past 5 the hour hand has turned through

(A) 145° (B) 150° (C) 155° (D) 160°

11. How many times do the hands of a clock coincide in a day ?

(A) 20 (B) 21 (C) 22 (D) 24

12. At what angle the hands of a clock are inclined at 15 min. past 5 ?

(A) $58\frac{1}{2}^{\circ}$ (B) 64° (C) $67\frac{1}{2}^{\circ}$ (D) $72\frac{1}{2}^{\circ}$

13. How much does a watch lose per day, if its hands coincide every 64 min. ?

(A) $32\frac{8}{11}$ min. (B) $36\frac{5}{11}$ min. (C) 90 min. (D) 96 min.

14. At what time, in minutes between 3 O'clock and 4 O'clock both the needles will coincide with each other?

(A) $5\frac{1}{11}''$ (B) $12\frac{4}{11}''$ (C) $13\frac{4}{11}''$ (D) $16\frac{4}{11}''$

15. The reflex angle between the hands of a clock at 10.25 is

(A) 180° (B) $192\frac{1}{2}^{\circ}$ (C) 195° (D) $197\frac{1}{2}^{\circ}$

16. At what time between 4 and 5O'clock will the hands of a watch point in opposite directions ?

(A) 45 min. past 4 (B) 40 min. past 4

(C) $50\frac{4}{11}$ min past 4 (D) $54\frac{6}{11}$ min past 4

17. At what time between 7 and 8 O'clock will the hands of a clock be in the same straight line but not together ?

(A) 5 min. past 7 (B) $5\frac{2}{11}$ min. past 7

(C) $5\frac{3}{11}$ min. past 7 (D) $5\frac{5}{11}$ min. past 7

18. At what time between 5 and 6 O'clock are the hands of a clock 3 minutes apart?

(A) $31\frac{5}{11}$ min. past 8 (B) $31\frac{5}{11}$ min. past 5

(C) $31\frac{6}{11}$ min. past 5 (D) $31\frac{4}{11}$ min. past 5

19. A watch which gains uniformly is 5 min. show at 8 O'clock in the morning on sunday and it is 5 min. 48 sec. fast at 8 pm on the following Sunday. When was it correct?

(A) 20 min. past 7 on Tuesday (B) 18 min. past 8 on Friday

(C) 20 min. past 7 pm on Sunday (D) 20 min. 7 pm on Wednesday

20. How many times in a day, the hands of a clock are straight?

(A) 22 (B) 24 (C) 44 (D) 48

21. By what time, in minutes between 3 O'clock both the needles coincide each other?

(A) $12\frac{4}{11}$' ' (B) $5\frac{1}{11}$' ' (C) $13\frac{4}{11}$" (D) $16\frac{4}{11}$"

22. The first period of a class starts at 10 : 30 hours and fourth ends at 13 : 45 hours. If periods are of equal duration and after each period a break of 5 minutes is given to the students, the exact duration of each period is:

(A) 35 minutes (B) 42 minutes (C) 45 minutes (D) 40 minutes

23. The minute hand of a clock overtakes the hour hand at intervals of 64 minutes of correct time. How much a day does the clock gain or lose?

(A) $32\frac{8}{11}$ minutes (B) $30\frac{8}{11}$ minutes

(C) $34\frac{8}{14}$ minutes (D) $34\frac{8}{11}$ minutes

24. Find the angle between the two hands of a clock at 15 minutes past 4 O'clock.

(A) 32.5° (B) 34.5° (C) 37.5° (D) 39.5°

25. At what time between 2 and 3 O'clock are the two hands of the clock together?

(A) $8\frac{10}{11}$ minutes (B) minutes (C) $14\frac{10}{11}$ minutes (D) $12\frac{10}{11}$ minutes

Answers Key

1.(A)	2.(B)	3.(D)	4.(A)	5.(A)
6.(C)	7.(B)	8.(C)	9.(B)	10.(C)
11.(C)	12.(C)	13.(A)	14.(D)	15.(D)
16.(D)	17.(D)	18.(B)	19.(D)	20.(C)
21.(D)	22.(C)	23.(A)	24.(C)	25.(B)

..Notes

20 Discount

FACTS AND FORMULAE

I. Discount : The amount which is charged less on S.P. is called *the rebate or discount*. When discount is offered in cash, it is either given as lumpsum or as certain percentage of the selling price.

Discount is given on the list price or catalogue price or marked price. There may be more than one discount allowed on the goods. When there is more than one discount, it is called *discount series* or *successive discounts*. The first discount may be on behalf of the company, second discount may be off season and third one may be if you pay the money in cash to the merchant.

II. Marked Price : When discount is given, a certain price is attached to the article which the *shopkeeper* professes to be the cost of the article for the customers. This price is called the *marked price* or *list price* of the article. The discount is always offered on marked price.

$\therefore$ Selling Price = Marked Price – Discount.

III. True Discount : Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. Thus, the payment of Rs. 100 will clear off the debt of Rs. 156 due 4 years hence.

$\therefore$ Sum due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

$\therefore$ True Discount (T.D.) = Rs. (156 – 100) = Rs. 56

= (Sum due) – (P.W.).

T.D. = Interest on P.W.

$\Rightarrow$ **Amount = (P.W.) + (T.D.).**

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IV. Let rate = R% per annum and Time = T years. Then,

(*i*) $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (R \times T)} = \dfrac{100 \times \text{T.D.}}{R \times T}$

(*ii*) $\text{T.D.} = \dfrac{(\text{P.W.}) \times R \times T}{100} = \dfrac{\text{Amount} \times R \times T}{100 + (R \times T)}$

(*iii*) $\text{Sum} = \dfrac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$

(*iv*) (S.I.) – (T.D.) = S.I. on T.D.

(*v*) When the sum is put at compound interest, then P.W. $= \dfrac{\text{Amount}}{\left(1 + \dfrac{R}{100}\right)^T}$

V. Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.

The date exactly after 5 months is called *nominally* due date. Three days (*i.e.*, grace days) are added to it to get a date, known as *legally due date*.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (Rs. 10,000 in this case) for the period from the date on which the bill was discounted (paid by the banker) and the legally due date. This amount is known as *Banker's Discount (B.D.)*.

Thus, B.D. is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.

Banker's Gain (B.G.) = (B.D.) – (T.D.) for the unexpired time.

Note : When the date of the bill is not given, grace days are not to be added.

VI. Short-cut Methods

(*i*) B.D. = S.I. on bill for unexpired time.

(*ii*) B.G. = (B.D.) – (T.D.) = S.I. on T.D. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$

(*iii*) T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$

(*iv*) B.D. = $\left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}\right)$

(*v*) Amount = $\left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$

(*vi*) T.D. = $\left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}\right)$

Solved Examples

Ex.1. *Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.*

Sol. P.W. = $\frac{100 \times \text{Amount}}{100 + (R \times T)}$ = Rs. $\left[\frac{100 \times 930}{100 + (8 \times 3)}\right]$

= Rs. $\left(\frac{100 \times 930}{124}\right)$ = Rs. 750.

T.D. = (Amount) – (P.W.) = Rs. (930 – 750) = Rs. 180.

Ex.2. *The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 resspectively. Find the sum and the rate percent.*

Sol. Sum = $\left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$

= Rs. $\left(\frac{120 \times 110}{120 - 110}\right)$ = Rs. 1320

✍..Notes

Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.

$\therefore$ Rate $= \left(\dfrac{100 \times 120}{1320 \times \frac{2}{3}}\right)\% = 13\dfrac{7}{11}\%$

Ex.3. *The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.*

Sol. $\text{T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}}$

$\therefore$ $\text{B.G.} = \dfrac{(\text{T.D.})^2}{\text{P.W.}} = \text{Rs.}\left(\dfrac{110 \times 110}{1100}\right)$ = Rs. 11

$\therefore$ B.D. = (T.D. + B.G.) = Rs. (110 + 11) = Rs. 121

Ex.4. *The true discount on a bill due 9 months hence at 12% per annum is Rs. 540. Find the amount of the bill and its present worth.*

Sol. Let amount be Rs. *x*. Then,

$$\frac{x \times R \times T}{100 + (R \times T)} = \text{T.D.}$$

$\Rightarrow$ $\dfrac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)}$ = 540

$\Rightarrow$ $x = \left(\dfrac{540 \times 109}{9}\right)$ = Rs. 6540.

$\therefore$ Amount = Rs. 6540.

P.W. = Rs. (6540 – 540) = Rs. 6000.

Ex.5. *The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.*

Sol. $\text{Sum} = \dfrac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \dfrac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}}$

$\therefore$ $\dfrac{\text{T.D.}}{\text{B.G.}} = \dfrac{\text{Sum}}{\text{B.D.}} = \dfrac{1650}{165} = \dfrac{10}{1}$

If B.D. is Rs. 11, T.D. = Rs. 10.

If B.D. is Rs. 165, T.D.

= Rs. $\left(\dfrac{10}{11} \times 165\right)$ = Rs. 150

And, B.G. = Rs. (165 – 150) = Rs. 15

Practice Exercise

Directions: Each of the questions given below is followed by four or five alternatives of which one is correct. Mark (✓) against the correct answer.

1. Goods were bought for Rs. 600 and sold the same day for Rs. 688.50 at a credit of 9 months and thus gaining 2%. The rate of interest per annum is

(A) $16\frac{2}{3}\%$ (B) $14\frac{1}{2}\%$ (C) $13\frac{1}{3}\%$ (D) 15%

2. Rs. 20 is the true discount on Rs. 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same ?

(A) Rs. 10 (B) Rs. 10.40 (C) Rs. 15.20 (D) Rs. 13

3. The present worth of Rs. 1404 due in two equal half-yearly instalments at 8% per annum, simple interest is

(A) Rs. 1325 (B) Rs. 1300 (C) Rs. 1350 (D) Rs. 1500

4. A has to pay Rs. 220 to B after 1 year. B asks A to pay Rs. 110 in cash and defer the payment of Rs. 110 for 2 years. A agrees to it. If the rate of interest be 10% per annum, in this mode of payment, then

(A) There is no gain or loss to any one
(B) A gains Rs. 7.34
(C) A loses Rs. 7.34
(D) A gains Rs. 11

5. A trader owes a merchant Rs. 10,028 due 1 year hence. The trader wants to settle the account after 3 months. If the rate of interest is 12% per annum, how much cash should he pay ?

(A) Rs. 9025.20 (B) Rs. 9200 (C) Rs. 9600 (D) Rs. 9560

6. A man buys a watch for Rs. 1950 in cash and sells it for Rs. 2200 at a credit of 1 year. If the rate of interest is 10% per annum, then man

(A) gains Rs. 55 (B) gains Rs. 50 (C) loses Rs. 30 (D) gains Rs. 30

7. A owes B, Rs. 1573 payable $1\frac{1}{2}$ years hence. Also B owes A, Rs. 1444.50 payable 6 months hence. If they want to settle the account forthwith, keeping 14% as the rate of interest, then who should pay and how much ?

(A) A, Rs. 28.50 (B) B, Rs. 37.50 (C) A, Rs. 50 (D) B, Rs. 50

8. A man purchased a cow for Rs. 3000 and sold it the same day for Rs. 3600, allowing the buyer a credit of 2 years. If the rate of interest be 10% per annum, then the man has a gain of

(A) 0% (B) 5% (C) 7.5% (D) 10%

9. The banker's discount on Rs. 1600 at 15% per annum is the same as true discount on Rs. 1680 for the same time and at the same rate. The time is

(A) 3 months (B) 4 months (C) 6 months (D) 8 months

10. The true discount on a bill of Rs. 540 is Rs. 90. The banker's discount is

(A) Rs. 60 (B) Rs. 108 (C) Rs. 110 (D) Rs. 112

..Notes

11. **The present worth of a certain sum due sometime hence is Rs. 1600 and the true discount is Rs. 160. The banker's gain is**
    (A) Rs. 20 (B) Rs. 24 (C) Rs. 16 (D) Rs. 12

12. **The banker's discount on a bill due 4 months hence at 15% is Rs. 420. The true discount is**
    (A) Rs. 400 (B) Rs. 360 (C) Rs. 480 (D) Rs. 320

13. **The present worth of a sum due sometime hence is Rs. 576 and the banker's gain is Rs. 16. The true discount is**
    (A) Rs. 36 (B) Rs. 72 (C) Rs. 48 (D) Rs. 96

14. **The banker's gain on a gain on a sum due 3 years hence at 12% per anum is Rs. 270. The banker's discount is**
    (A) Rs. 400 (B) Rs. 840 (C) Rs. 1020 (D) Rs. 760

15. **The present worth of a certain bill due sometimes hence is Rs. 800 and the true discount is Rs. 36. The banker's discount is**
    (A) Rs. 37 (B) Rs. 37.62 (C) Rs. 34.38 (D) Rs. 38.98

16. **The banker's gain of a certain sum due 2 years hence at 10% per annum is Rs. 24. The present worth is**
    (A) Rs. 480 (B) Rs. 520 (C) Rs. 600 (D) Rs. 960

17. **A fan listed at Rs. 1500 and a discount of 20% is offered on the list price. What additional discount must be offered to the customer to bring the net price to Rs. 1104 ?**
    (A) 8% (B) 10% (C) 12% (D) 15%

18. **A person first increases the price of a commodity by 10% and then he announces a discount of 15%. The actual discount on the original price is**
    (A) 5% (B) 6.5% (C) 12.5% (D) 7.5%

19. **A shopkeeper sold a TV set for Rs. 16560 after giving 10% discount on the labelled price and earned 15% profit. What would have been the per cent profit if no discount was given ?**
    (A) $27\frac{2}{9}$ (B) $27\frac{7}{9}$ (C) $28\frac{7}{9}$ (D) Data inadequate

20. **At what percentage above the cost price must an article be marked so as to gain 33% after allowing a customer a discount of 5% ?**
    (A) 48% (B) 43% (C) 40% (D) 38%

Answers Key

1.(A)	2.(B)	3.(A)	4.(B)	5.(B)
6.(B)	7.(D)	8.(A)	9.(B)	10.(B)
11.(C)	12.(A)	13.(D)	14.(C)	15.(B)
16.(C)	17.(A)	18.(B)	19.(B)	20.(C)