1.1 OVERVIEW OF ELECTRICAL ENGINEERING

Electrical engineers design systems that have two main objectives:

1. To gather, store, process, transport, and present *information*.
2. To distribute, store, and convert *energy* between various forms.

In many electrical systems, the manipulation of energy and the manipulation of information are interdependent.

For example, numerous aspects of electrical engineering relating to information are applied in weather prediction. Data about cloud cover, precipitation, wind speed, and so on are gathered electronically by weather satellites, by land-based radar stations, and by sensors at numerous weather stations. (Sensors are devices that convert physical measurements to electrical signals.) This information is transported by electronic communication systems and processed by computers to yield forecasts that are disseminated and displayed electronically.

In electrical power plants, energy is converted from various sources to electrical form. Electrical distribution systems transport the energy to virtually every factory, home, and business in the world, where it is converted to a multitude of useful forms, such as mechanical energy, heat, and light.

No doubt you can list scores of electrical engineering applications in your daily life. Increasingly, electrical and electronic features are integrated into new products. Automobiles and trucks provide just one example of this trend. The electronic content of the average automobile is growing rapidly in value. Auto designers realize that electronic technology is a good way to provide increased functionality at lower cost. Table 1.1 shows some of the applications of electrical engineering in automobiles.

You may find it interesting to search the web for sites related to "mechatronics."

As another example, we note that many common household appliances contain keypads for operator control, sensors, electronic displays, and computer chips, as well as more conventional switches, heating elements, and motors. Electronics have become so intimately integrated with mechanical systems that the name **mechatronics** is used for the combination.

Subdivisions of Electrical Engineering

Next, we give you an overall picture of electrical engineering by listing and briefly discussing eight of its major areas.

1. Communication systems transport information in electrical form. Cellular phone, radio, satellite television, and the Internet are examples of communication systems. It is possible for virtually any two people (or computers) on the globe to communicate almost instantaneously. A climber on a mountaintop in Nepal can call or send e-mail to friends whether they are hiking in Alaska or sitting in a New York City office. This kind of connectivity affects the way we live, the way we conduct business, and the design of everything we use. For example, communication systems will change the design of highways because traffic and road-condition information collected by roadside sensors can be transmitted to central locations and used to route traffic. When an accident occurs, an electrical signal can be emitted automatically when the airbags deploy, giving the exact location of the vehicle, summoning help, and notifying traffic-control computers.

Computers that are part of products such as appliances and automobiles are called *embedded computers*.

2. Computer systems process and store information in digital form. No doubt you have already encountered computer applications in your own field. Besides the computers of which you are aware, there are many in unobvious places, such as household appliances and automobiles. A typical modern automobile contains several

Table 1.1. **Current and Emerging Electronic/Electrical Applications in Automobiles and Trucks**

Safety
Antiskid brakes
Inflatable restraints
Collision warning and avoidance
Blind-zone vehicle detection (especially for large trucks)
Infrared night vision systems
Heads-up displays
Automatic accident notification
Rear-view cameras
Communications and entertainment
AM/FM radio
Digital audio broadcasting
CD/DVD player
Cellular phone
Computer/e-mail
Satellite radio
Convenience
Electronic GPS navigation
Personalized seat/mirror/radio settings
Electronic door locks
Emissions, performance, and fuel economy
Vehicle instrumentation
Electronic ignition
Tire inflation sensors
Computerized performance evaluation and maintenance scheduling
Adaptable suspension systems
Alternative propulsion systems
Electric vehicles
Advanced batteries
Hybrid vehicles

dozen special-purpose computers. Chemical processes and railroad switching yards are routinely controlled through computers.

3. Control systems gather information with sensors and use electrical energy to control a physical process. A relatively simple control system is the heating/cooling system in a residence. A sensor (thermostat) compares the temperature with the desired value. Control circuits operate the furnace or air conditioner to achieve the desired temperature. In rolling sheet steel, an electrical control system is used to obtain the desired sheet thickness. If the sheet is too thick (or thin), more (or less) force is applied to the rollers. The temperatures and flow rates in chemical processes are controlled in a similar manner. Control systems have even been installed in tall buildings to reduce their movement due to wind.

4. Electromagnetics is the study and application of electric and magnetic fields. The device (known as a magnetron) used to produce microwave energy in an oven is one application. Similar devices, but with much higher power levels, are employed in manufacturing sheets of plywood. Electromagnetic fields heat the glue between layers of wood so that it will set quickly. Cellular phone and television antennas are also examples of electromagnetic devices.

5. Electronics is the study and application of materials, devices, and circuits used in amplifying and switching electrical signals. The most important electronic devices are transistors of various kinds. They are used in nearly all places where electrical information or energy is employed. For example, the cardiac pacemaker is an electronic circuit that senses heart beats, and if a beat does not occur when it should, applies a minute electrical stimulus to the heart, forcing a beat. Electronic instrumentation and electrical sensors are found in every field of science and engineering. Many of the aspects of electronic amplifiers studied later in this book have direct application to the instrumentation used in your field of engineering.

Electronic devices are based on controlling electrons. Photonic devices perform similar functions by controlling photons.

6. Photonics is an exciting new field of science and engineering that promises to replace conventional computing, signal-processing, sensing, and communication devices based on manipulating electrons with greatly improved products based on manipulating photons. Photonics includes light generation by lasers and light-emitting diodes, transmission of light through optical components, as well as switching, modulation, amplification, detection, and steering light by electrical, acoustical, and photon-based devices. Current applications include readers for DVD disks, holograms, optical signal processors, and fiber-optic communication systems. Future applications include optical computers, holographic memories, and medical devices. Photonics offers tremendous opportunities for nearly all scientists and engineers.

7. Power systems convert energy to and from electrical form and transmit energy over long distances. These systems are composed of generators, transformers, distribution lines, motors, and other elements. Mechanical engineers often utilize electrical motors to empower their designs. The selection of a motor having the proper torque–speed characteristic for a given mechanical application is another example of how you can apply the information in this book.

8. Signal processing is concerned with information-bearing electrical signals. Often, the objective is to extract useful information from electrical signals derived from sensors. An application is machine vision for robots in manufacturing. Another application of signal processing is in controlling ignition systems of internal combustion engines. The timing of the ignition spark is critical in achieving good performance and low levels of pollutants. The optimum ignition point relative to crankshaft rotation depends on fuel quality, air temperature, throttle setting, engine speed, and other factors.

If the ignition point is advanced slightly beyond the point of best performance, *engine knock* occurs. Knock can be heard as a sharp metallic noise that is caused by rapid pressure fluctuations during the spontaneous release of chemical energy in the combustion chamber. A combustion-chamber pressure pulse displaying knock is shown in Figure 1.1. At high levels, knock will destroy an engine in a very short time. Prior to the advent of practical signal-processing electronics for this application, engine timing needed to be adjusted for distinctly suboptimum performance to avoid knock under varying combinations of operating conditions.

By connecting a sensor through a tube to the combustion chamber, an electrical signal proportional to pressure is obtained. Electronic circuits process this signal to determine whether the rapid pressure fluctuations characteristic of knock are present. Then electronic circuits continuously adjust ignition timing for optimum performance while avoiding knock.

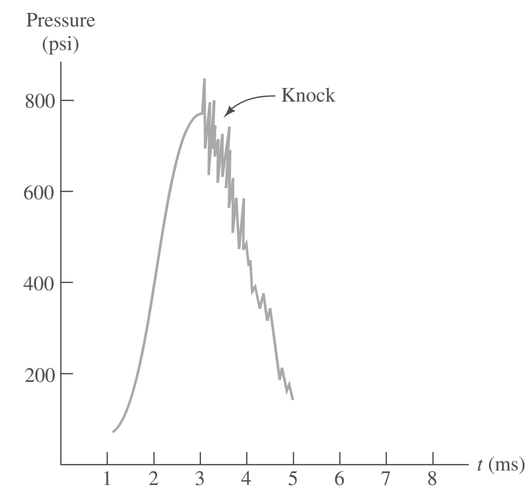


Figure 1.1 Pressure versus time for an internal combustion engine experiencing knock. Sensors convert pressure to an electrical signal that is processed to adjust ignition timing for minimum pollution and good performance.

Why You Need to Study Electrical Engineering

As a reader of this book, you may be majoring in another field of engineering or science and taking a required course in electrical engineering. Your immediate objective is probably to meet the course requirements for a degree in your chosen field. However, there are several other good reasons to learn and retain some basic knowledge of electrical engineering:

1. *To pass the Fundamentals of Engineering (FE) Examination as a first step in becoming a Registered Professional Engineer.* In the United States, before performing engineering services for the public, you will need to become registered as a Professional Engineer (PE). This book gives you the knowledge to answer questions relating to electrical engineering on the registration examinations. Save this book and course notes to review for the FE examination. (See Appendix C for more on the FE exam.)

Save this book and course notes to review for the FE exam.

2. *To have a broad enough knowledge base so that you can lead design projects in your own field.* Increasingly, electrical engineering is interwoven with nearly all scientific experiments and design projects in other fields of engineering. Industry has repeatedly called for engineers who can see the big picture and work effectively in teams. Engineers or scientists who narrow their focus strictly to their own field are destined to be directed by others. (Electrical engineers are somewhat fortunate in this respect because the basics of structures, mechanisms, and chemical processes are familiar from everyday life. On the other hand, electrical engineering concepts are somewhat more abstract and hidden from the casual observer.)

3. *To be able to operate and maintain electrical systems, such as those found in control systems for manufacturing processes.* The vast majority of electrical-circuit malfunctions can be readily solved by the application of basic electrical-engineering principles. You will be a much more versatile and valuable engineer or scientist if you can apply electrical-engineering principles in practical situations.

4. *To be able to communicate with electrical-engineering consultants.* Very likely, you will often need to work closely with electrical engineers in your career. This book will give you the basic knowledge needed to communicate effectively.

Content of This Book

Circuit theory is the electrical engineer's fundamental tool.

Electrical engineering is too vast to cover in one or two courses. Our objective is to introduce the underlying concepts that you are most likely to need. Circuit theory is the electrical engineer's fundamental tool. That is why the first six chapters of this book are devoted to circuits.

Embedded computers, sensors, and electronic circuits will be an increasingly important part of the products you design and the instrumentation you use as an engineer or scientist. Chapters 7, 8, and 9 treat digital systems with emphasis on embedded computers and instrumentation. Chapters 10 through 14 deal with electronic devices and circuits.

As a mechanical, chemical, civil, industrial, or other engineer, you will very likely need to employ energy-conversion devices. The last three chapters relate to electrical energy systems treating transformers, generators, and motors.

Because this book covers many basic concepts, it is also sometimes used in introductory courses for electrical engineers. Just as it is important for other engineers and scientists to see how electrical engineering can be applied to their fields, it is equally important for electrical engineers to be familiar with these applications.

1.2 CIRCUITS, CURRENTS, AND VOLTAGES

Overview of an Electrical Circuit

Before we carefully define the terminology of electrical circuits, let us gain some basic understanding by considering a simple example: the headlight circuit of an automobile. This circuit consists of a battery, a switch, the headlamps, and wires connecting them in a closed path, as illustrated in Figure 1.2.

The battery voltage is a measure of the energy gained by a unit of charge as it moves through the battery.

Chemical forces in the battery cause electrical charge (electrons) to flow through the circuit. The charge gains energy from the chemicals in the battery and delivers energy to the headlamps. The battery voltage (nominally, 12 volts) is a measure of the energy gained by a unit of charge as it moves through the battery.

Electrons readily move through copper but not through plastic insulation.

The wires are made of an excellent electrical conductor (copper) and are insulated from one another (and from the metal auto body) by electrical insulation (plastic) coating the wires. Electrons readily move through copper but not through the plastic insulation. Thus, the charge flow (electrical current) is confined to the wires until it reaches the headlamps. Air is also an insulator.

The switch is used to control the flow of current. When the conducting metallic parts of the switch make contact, we say that the switch is **closed** and current flows through the circuit. On the other hand, when the conducting parts of the switch do not make contact, we say that the switch is **open** and current does not flow.

Electrons experience collisions with the atoms of the tungsten wires, resulting in heating of the tungsten.

Energy is transferred by the chemical action in the battery to the electrons and then to the tungsten.

The headlamps contain special tungsten wires that can withstand high temperatures. Tungsten is not as good an electrical conductor as copper, and the electrons experience collisions with the atoms of the tungsten wires, resulting in heating of the tungsten. We say that the tungsten wires have electrical resistance. Thus, energy is transferred by the chemical action in the battery to the electrons and then to the tungsten, where it appears as heat. The tungsten becomes hot enough so that copious light is emitted. We will see that the power transferred is equal to the product of current (rate of flow of charge) and the voltage (also called electrical potential) applied by the battery.

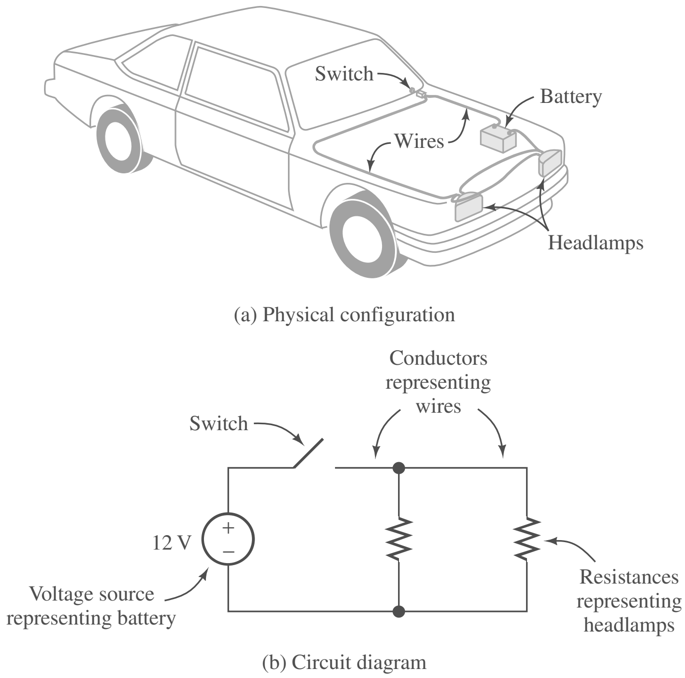


Figure 1.2 The headlight circuit. (a) The actual physical layout of the circuit. (b) The circuit diagram.

(Actually, the simple description of the headlight circuit we have given is most appropriate for older cars. In more modern automobiles, sensors provide information to an embedded computer about the ambient light level, whether or not the ignition is energized, and whether the transmission is in park or drive. The dashboard switch merely inputs a logic level to the computer, indicating the intention of the operator with regard to the headlights. Depending on these inputs, the computer controls the state of an electronic switch in the headlight circuit. When the ignition is turned off and if it is dark, the computer keeps the lights on for a few minutes so the passengers can see to exit and then turns them off to conserve energy in the battery. This is typical of the trend to use highly sophisticated electronic and computer technology to enhance the capabilities of new designs in all fields of engineering.)

Fluid-Flow Analogy

Electrical circuits are analogous to fluid-flow systems. The battery is analogous to a pump, and charge is analogous to the fluid. Conductors (usually copper wires) correspond to frictionless pipes through which the fluid flows. Electrical current is the counterpart of the flow rate of the fluid. Voltage corresponds to the pressure differential between points in the fluid circuit. Switches are analogous to valves. Finally, the electrical resistance of a tungsten headlamp is analogous to a constriction in a fluid system that results in turbulence and conversion of energy to heat. Notice that current is a measure of the flow of charge *through* the cross section of a circuit element, whereas voltage is measured *across* the ends of a circuit element or *between* any other two points in a circuit.

The fluid-flow analogy can be very helpful initially in understanding electrical circuits.

Now that we have gained a basic understanding of a simple electrical circuit, we will define the concepts and terminology more carefully.

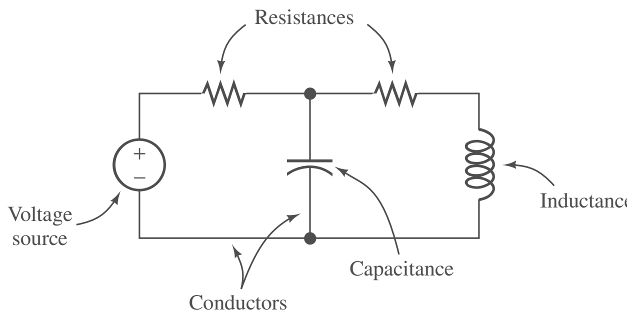


Figure 1.3 An electrical circuit consists of circuit elements, such as voltage sources, resistances, inductances, and capacitances, connected in closed paths by conductors.

Electrical Circuits

An electrical circuit consists of various types of circuit elements connected in closed paths by conductors.

An **electrical circuit** consists of various types of circuit elements connected in closed paths by conductors. An example is illustrated in Figure 1.3. The circuit elements can be resistances, inductances, capacitances, and voltage sources, among others. The symbols for some of these elements are illustrated in the figure. Eventually, we will carefully discuss the characteristics of each type of element.

Charge flows easily through conductors.

Charge flows easily through conductors, which are represented by lines connecting circuit elements. Conductors correspond to connecting wires in physical circuits. Voltage sources create forces that cause charge to flow through the conductors and other circuit elements. As a result, energy is transferred between the circuit elements, resulting in a useful function.

Electrical Current

Current is the time rate of flow of electrical charge. Its units are amperes (A), which are equivalent to coulombs per second (C/s).

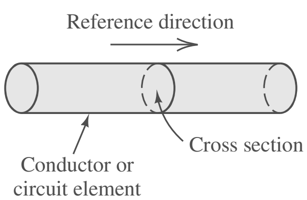


Figure 1.4 Current is the time rate of charge flow through a cross section of a conductor or circuit element.

Electrical current is the time rate of flow of electrical charge through a conductor or circuit element. The units are amperes (A), which are equivalent to coulombs per second (C/s). (The charge on an electron is -1.602×10^{-19} C.)

Conceptually, to find the current for a given circuit element, we first select a cross section of the circuit element roughly perpendicular to the flow of current. Then, we select a **reference direction** along the direction of flow. Thus, the reference direction points from one side of the cross section to the other. This is illustrated in Figure 1.4.

Next, suppose that we keep a record of the net charge flow through the cross section. Positive charge crossing in the reference direction is counted as a positive contribution to net charge. Positive charge crossing opposite to the reference is counted as a negative contribution. Furthermore, negative charge crossing in the reference direction is counted as a negative contribution, and negative charge against the reference direction is a positive contribution to charge.

Thus, in concept, we obtain a record of the net charge in coulombs as a function of time in seconds denoted as $q(t)$. The electrical current flowing through the element in the reference direction is given by

Colored shading is used to indicate key equations throughout this book.

$$i(t) = \frac{dq(t)}{dt} \tag{1.1}$$

A constant current of one ampere means that one coulomb of charge passes through the cross section each second.

To find charge given current, we must integrate. Thus, we have

$$q(t) = \int_{t_0}^{t} i(t)\,dt + q(t_0) \qquad (1.2)$$

in which t_0 is some initial time at which the charge is known. (Throughout this book, we assume that time t is in seconds unless stated otherwise.)

Current flow is the same for all cross sections of a circuit element. (We reexamine this statement when we introduce the capacitor in Chapter 3.) The current that enters one end flows through the element and exits through the other end.

Example 1.1 Determining Current Given Charge

Suppose that charge versus time for a given circuit element is given by

$$q(t) = 0 \qquad \text{for } t < 0$$

and

$$q(t) = 2 - 2e^{-100t}\,\text{C} \qquad \text{for } t > 0$$

Sketch $q(t)$ and $i(t)$ to scale versus time.

Solution First we use Equation 1.1 to find an expression for the current:

$$\begin{aligned} i(t) &= \frac{dq(t)}{dt} \\ &= 0 \qquad \text{for } t < 0 \\ &= 200e^{-100t}\,\text{A} \qquad \text{for } t > 0 \end{aligned}$$

Plots of $q(t)$ and $i(t)$ are shown in Figure 1.5. ■

Reference Directions

In analyzing electrical circuits, we may not initially know the *actual direction* of current flow in a particular circuit element. Therefore, we start by assigning current

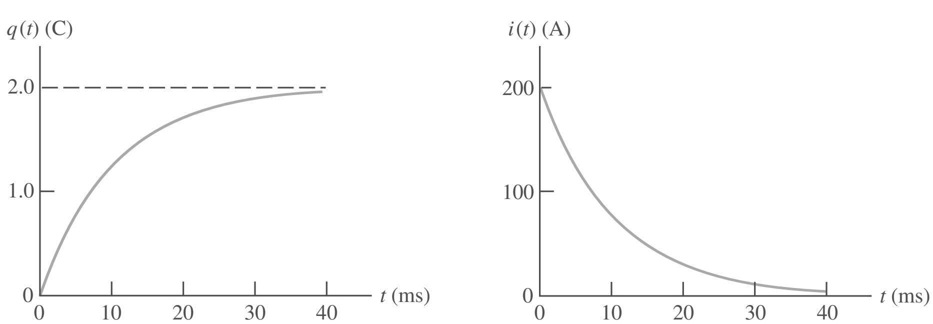


Figure 1.5 Plots of charge and current versus time for Example 1.1. *Note*: The time scale is in milliseconds (ms). One millisecond is equivalent to 10^{-3} seconds.

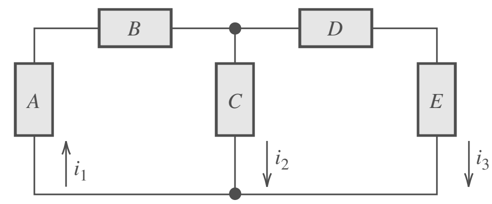


Figure 1.6 In analyzing circuits, we frequently start by assigning current variables i_1, i_2, i_3, and so forth.

variables and arbitrarily selecting a *reference direction* for each current of interest. It is customary to use the letter i for currents and subscripts to distinguish different currents. This is illustrated by the example in Figure 1.6, in which the boxes labeled A, B, and so on represent circuit elements. After we solve for the current values, we may find that some currents have negative values. For example, suppose that $i_1 = -2$ A in the circuit of Figure 1.6. Because i_1 has a negative value, we know that current actually flows in the direction opposite to the reference initially selected for i_1. Thus, the actual current is 2 A flowing downward through element A.

Direct Current and Alternating Current

Dc currents are constant with respect to time, whereas ac currents vary with time.

When a current is constant with time, we say that we have **direct current**, abbreviated as dc. On the other hand, a current that varies with time, reversing direction periodically, is called **alternating current**, abbreviated as ac. Figure 1.7 shows the values of a dc current and a sinusoidal ac current versus time. When $i_b(t)$ takes a negative value, the actual current direction is opposite to the reference direction for $i_b(t)$. The designation ac is used for other types of time-varying currents, such as the triangular and square waveforms shown in Figure 1.8.

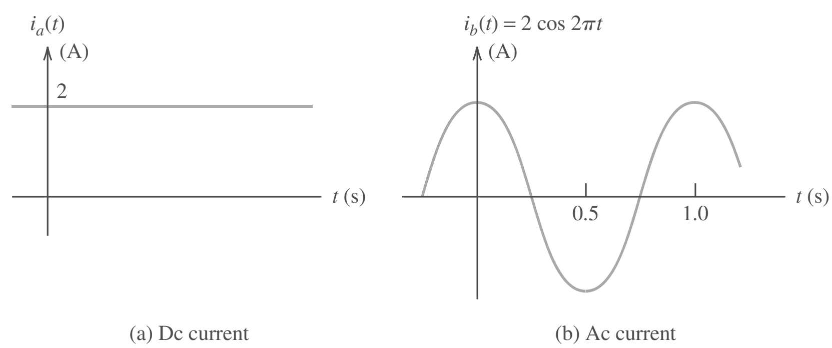


Figure 1.7 Examples of dc and ac currents versus time.

Double-Subscript Notation for Currents

So far we have used arrows alongside circuit elements or conductors to indicate reference directions for currents. Another way to indicate the current and reference direction for a circuit element is to label the ends of the element and use double subscripts to define the reference direction for the current. For example, consider the resistance of Figure 1.9. The current denoted by i_{ab} is the current through the element with its reference direction pointing from a to b. Similarly, i_{ba} is the current with its reference directed from b to a. Of course, i_{ab} and i_{ba} are the same in magnitude and

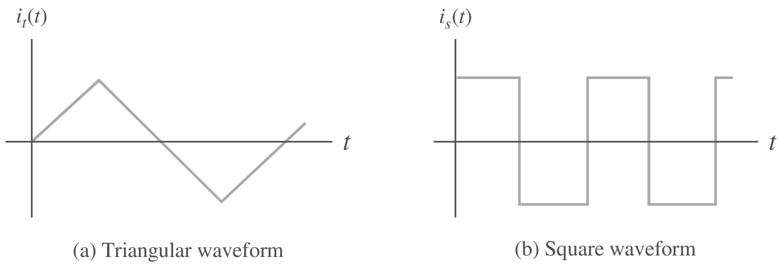


Figure 1.8 Ac currents can have various waveforms.

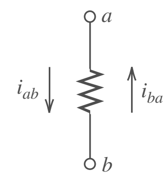


Figure 1.9 Reference directions can be indicated by labeling the ends of circuit elements and using double subscripts on current variables. The reference direction for i_{ab} points from a to b. On the other hand, the reference direction for i_{ba} points from b to a.

opposite in sign, because they denote the same current but with opposite reference directions. Thus, we have

$$i_{ab} = -i_{ba}$$

Exercise 1.1 A constant current of 2 A flows through a circuit element. In 10 seconds (s), how much net charge passes through the element?
Answer 20 C. □

Exercise 1.2 The charge that passes through a circuit element is given by $q(t) = 0.01\sin(200t)$ C, in which the angle is in radians. Find the current as a function of time.
Answer $i(t) = 2\cos(200t)$ A. □

Exercise 1.3 In Figure 1.6, suppose that $i_2 = 1$ A and $i_3 = -3$ A. Assuming that the current consists of positive charge, in which direction (upward or downward) is charge moving in element C? In element E?
Answer Downward in element C and upward in element E. □

Voltages

When charge moves through circuit elements, energy can be transferred. In the case of automobile headlights, stored chemical energy is supplied by the battery and absorbed by the headlights where it appears as heat and light. The **voltage** associated with a circuit element is the energy transferred per unit of charge that flows through the element. The units of voltage are volts (V), which are equivalent to joules per coulomb (J/C).

Voltage is a measure of the energy transferred per unit of charge when charge moves from one point in an electrical circuit to a second point.

For example, consider the storage battery in an automobile. The voltage across its terminals is (nominally) 12 V. This means that 12 J are transferred to or from the battery for each coulomb that flows through it. When charge flows in one direction, energy is supplied by the battery, appearing elsewhere in the circuit as heat or light or perhaps as mechanical energy at the starter motor. If charge moves through the battery in the opposite direction, energy is absorbed by the battery, where it appears as stored chemical energy.

Notice that voltage is measured across the ends of a circuit element, whereas current is a measure of charge flow through the element.

Voltages are assigned polarities that indicate the direction of energy flow. If positive charge moves from the positive polarity through the element toward the negative polarity, the element absorbs energy that appears as heat, mechanical energy, stored chemical energy, or as some other form. On the other hand, if positive charge moves from the negative polarity toward the positive polarity, the element supplies energy. This is illustrated in Figure 1.10. For negative charge, the direction of energy transfer is reversed.

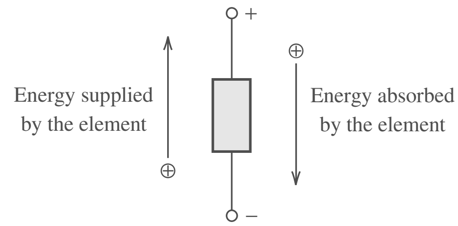


Figure 1.10 Energy is transferred when charge flows through an element having a voltage across it.

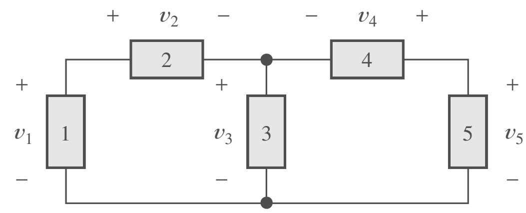


Figure 1.11 If we do not know the voltage values and polarities in a circuit, we can start by assigning voltage variables choosing the reference polarities arbitrarily. (The boxes represent unspecified circuit elements.)

Reference Polarities

When we begin to analyze a circuit, we often do not know the actual polarities of some of the voltages of interest in the circuit. Then, we simply assign voltage variables choosing *reference* polarities arbitrarily. (Of course, the *actual* polarities are not arbitrary.) This is illustrated in Figure 1.11. Next, we apply circuit principles (discussed later), obtaining equations that are solved for the voltages. If a given voltage has an actual polarity opposite to our arbitrary choice for the reference polarity, we obtain a negative value for the voltage. For example, if we find that $v_3 = -5$ V in Figure 1.11, we know that the voltage across element 3 is 5 V in magnitude and its actual polarity is opposite to that shown in the figure (i.e., the actual polarity is positive at the bottom end of element 3 and negative at the top).

In circuit analysis, we frequently assign reference polarities for voltages arbitrarily. If we find at the end of the analysis that the value of a voltage is negative, then we know that the true polarity is opposite of the polarity selected initially.

We usually do not put much effort into trying to assign "correct" references for current directions or voltage polarities. If we have doubt about them, we make arbitrary choices and use circuit analysis to determine true directions and polarities (as well as the magnitudes of the currents and voltages).

Voltages can be constant with time or they can vary. Constant voltages are called **dc voltages**. On the other hand, voltages that change in magnitude and alternate in polarity with time are said to be **ac voltages**. For example,

$$v_1(t) = 10 \text{ V}$$

is a dc voltage. It has the same magnitude and polarity for all time. On the other hand,

$$v_2(t) = 10 \cos(200\pi t)\text{V}$$

is an ac voltage that varies in magnitude and polarity. When $v_2(t)$ assumes a negative value, the actual polarity is opposite the reference polarity. (We study sinusoidal ac currents and voltages in Chapter 5.)

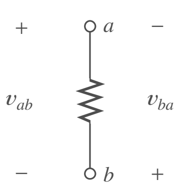


Figure 1.12 The voltage v_{ab} has a reference polarity that is positive at point a and negative at point b.

Double-Subscript Notation for Voltages

Another way to indicate the reference polarity of a voltage is to use double subscripts on the voltage variable. We use letters or numbers to label the terminals between which the voltage appears, as illustrated in Figure 1.12. For the resistance shown in the figure, v_{ab} represents the voltage between points a and b with the positive reference at point a. The two subscripts identify the points between which the voltage appears, and the first subscript is the positive reference. Similarly, v_{ba} is the voltage between a and b with the positive reference at point b. Thus, we can write

$$v_{ab} = -v_{ba} \tag{1.3}$$

because v_{ba} has the same magnitude as v_{ab} but has opposite polarity.

Still another way to indicate a voltage and its reference polarity is to use an arrow, as shown in Figure 1.13. The positive reference corresponds to the head of the arrow.

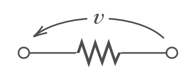


Figure 1.13 The positive reference for v is at the head of the arrow.

Switches

Switches control the currents in circuits. When an ideal switch is open, the current through it is zero and the voltage across it is determined by the remainder of the circuit. When an ideal switch is closed, the voltage across it is zero and the current through it is determined by the remainder of the circuit.

Exercise 1.4 The voltage across a given circuit element is $v_{ab} = 20$ V. A positive charge of 2 C moves through the circuit element from terminal b to terminal a. How much energy is transferred? Is the energy supplied by the circuit element or absorbed by it?

Answer 40 J are supplied by the circuit element. □

1.3 POWER AND ENERGY

Consider the circuit element shown in Figure 1.14. Because the current i is the rate of flow of charge and the voltage v is a measure of the energy transferred per unit of charge, the product of the current and the voltage is the rate of energy transfer. In other words, the product of current and voltage is power:

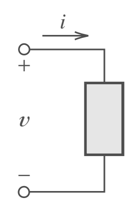


Figure 1.14 When current flows through an element and voltage appears across the element, energy is transferred. The rate of energy transfer is $p = vi$.

$$p = vi \tag{1.4}$$

The physical units of the quantities on the right-hand side of this equation are

$$\begin{aligned} \text{volts} \times \text{amperes} &= \\ (\text{joules/coulomb}) \times (\text{coulombs/second}) &= \\ \text{joules/second} &= \\ \text{watts}& \end{aligned}$$

Passive Reference Configuration

Now we may ask whether the power calculated by Equation 1.4 represents energy supplied by or absorbed by the element. Refer to Figure 1.14 and notice that the current reference enters the positive polarity of the voltage. We call this arrangement the **passive reference configuration**. Provided that the references are picked in this manner, a positive result for the power calculation implies that energy is being absorbed by the element. On the other hand, a negative result means that the element is supplying energy to other parts of the circuit.

If the current reference enters the negative end of the reference polarity, we compute the power as

$$p = -vi \tag{1.5}$$

Then, as before, a positive value for p indicates that energy is absorbed by the element, and a negative value shows that energy is supplied by the element.

If the circuit element happens to be an electrochemical battery, positive power means that the battery is being charged. In other words, the energy absorbed by the battery is being stored as chemical energy. On the other hand, negative power indicates that the battery is being discharged. Then the energy supplied by the battery is delivered to some other element in the circuit.

Sometimes currents, voltages, and powers are functions of time. To emphasize this fact, we can write Equation 1.4 as

$$p(t) = v(t)i(t) \tag{1.6}$$

Example 1.2 Power Calculations

Consider the circuit elements shown in Figure 1.15. Calculate the power for each element. If each element is a battery, is it being charged or discharged?

Solution In element A, the current reference enters the positive reference polarity. This is the passive reference configuration. Thus, power is computed as

$$p_a = v_a i_a = 12 \text{ V} \times 2 \text{ A} = 24 \text{ W}$$

Because the power is positive, energy is absorbed by the device. If it is a battery, it is being charged.

In element B, the current reference enters the negative reference polarity. (Recall that the current that enters one end of a circuit element must exit from the other end, and vice versa.) This is opposite to the passive reference configuration. Hence, power is computed as

$$p_b = -v_b i_b = -(12 \text{ V}) \times 1 \text{ A} = -12 \text{ W}$$

Since the power is negative, energy is supplied by the device. If it is a battery, it is being discharged.

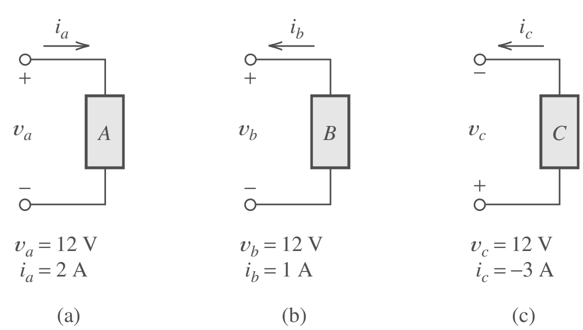


Figure 1.15 Circuit elements for Example 1.2.

In element C, the current reference enters the positive reference polarity. This is the passive reference configuration. Thus, we compute power as

$$p_c = v_c i_c = 12\text{ V} \times (-3\text{ A}) = -36\text{ W}$$

Since the result is negative, energy is supplied by the element. If it is a battery, it is being discharged. (Notice that since i_c takes a negative value, current actually flows downward through element C.) ■

Energy Calculations

To calculate the energy w delivered to a circuit element between time instants t_1 and t_2, we integrate power:

$$w = \int_{t_1}^{t_2} p(t)\, dt \tag{1.7}$$

Here we have explicitly indicated that power can be a function of time by using the notation $p(t)$.

Example 1.3 **Energy Calculation**

Find an expression for the power for the voltage source shown in Figure 1.16. Compute the energy for the interval from $t_1 = 0$ to $t_2 = \infty$.

Solution The current reference enters the positive reference polarity. Thus, we compute power as

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= 12 \times 2e^{-t} \\ &= 24e^{-t}\text{ W} \end{aligned}$$

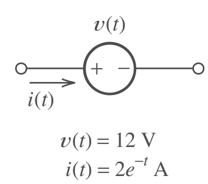


Figure 1.16 Circuit element for Example 1.3.

Subsequently, the energy transferred is given by

$$\begin{aligned} w &= \int_0^\infty p(t)\, dt \\ &= \int_0^\infty 24e^{-t}\, dt \\ &= \left[-24e^{-t}\right]_0^\infty = -24e^{-\infty} - (-24e^0) = 24\text{ J} \end{aligned}$$

Because the energy is positive, it is absorbed by the source. ■

Prefixes

In electrical engineering, we encounter a tremendous range of values for currents, voltages, powers, and other quantities. We use the prefixes shown in Table 1.2 when working with very large or small quantities. For example, 1 milliampere (1 mA) is equivalent to 10^{-3} A, 1 kilovolt (1 kV) is equivalent to 1000 V, and so on.

Table 1.2. Prefixes Used for Large or Small Physical Quantities

Prefix	Abbreviation	Scale Factor
giga-	G	10^9
meg- or mega-	M	10^6
kilo-	k	10^3
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}

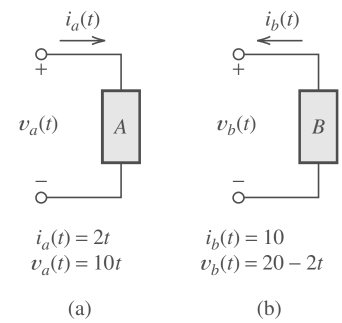


Figure 1.17 See Exercise 1.6.

Exercise 1.5 The ends of a circuit element are labeled a and b, respectively. Are the references for i_{ab} and v_{ab} related by the passive reference configuration? Explain.

Answer The reference direction for i_{ab} enters terminal a, which is also the positive reference for v_{ab}. Therefore, the current reference direction enters the positive reference polarity, so we have the passive reference configuration. □

Exercise 1.6 Compute the power as a function of time for each of the elements shown in Figure 1.17. Find the energy transferred between $t_1 = 0$ and $t_2 = 10$ s. In each case is energy supplied or absorbed by the element?

Answer **a.** $p_a(t) = 20t^2$ W, $w_a = 6667$ J; since w_a is positive, energy is absorbed by element A. **b.** $p_b(t) = 20t - 200$ W, $w_b = -1000$ J; since w_b is negative, energy is supplied by element B. □

~~1.4 KIRCHHOFF'S CURRENT LAW~~

Kirchhoff's current law states that the net current entering a node is zero.

~~A **node** in an electrical circuit is a point at which two or more circuit elements are joined together. Examples of nodes are shown in Figure 1.18.~~

~~An important principle of electrical circuits is **Kirchhoff's current law**: *The net current entering a node is zero.* To compute the *net* current entering a node, we add the currents entering and subtract the currents leaving. For illustration, consider the nodes of Figure 1.18. Then, we can write:~~

$$\text{Node } a: \quad i_1 + i_2 - i_3 = 0$$

$$\text{Node } b: \quad i_3 - i_4 = 0$$

$$\text{Node } c: \quad i_5 + i_6 + i_7 = 0$$

1.6 INTRODUCTION TO CIRCUIT ELEMENTS

In this section, we carefully define several types of ideal circuit elements:

Conductors
Voltage sources
Current sources
Resistors

Later in the book, we will encounter additional elements, including inductors and capacitors. Eventually, we will be able to use these idealized circuit elements to describe (model) complex real-world electrical devices.

Conductors

We have already encountered conductors. Ideal conductors are represented in circuit diagrams by unbroken lines between the ends of other circuit elements. We define ideal circuit elements in terms of the relationship between the voltage across the element and the current through it.

The voltage between the ends of an ideal conductor is zero regardless of the current flowing through the conductor.

All points in a circuit that are connected by ideal conductors can be considered as a single node.

The voltage between the ends of an ideal conductor is zero regardless of the current flowing through the conductor. When two points in a circuit are connected together by an ideal conductor, we say that the points are **shorted** together. Another term for an ideal conductor is **short circuit**. All points in a circuit that are connected by ideal conductors can be considered as a single node.

If no conductors or other circuit elements are connected between two parts of a circuit, we say that an **open circuit** exists between the two parts of the circuit. No current can flow through an ideal open circuit.

Independent Voltage Sources

An ideal independent voltage source maintains a specified voltage across its terminals.

An **ideal independent voltage source** maintains a specified voltage across its terminals. The voltage across the source is independent of other elements that are connected to it and of the current flowing through it. We use a circle enclosing the reference polarity marks to represent independent voltage sources. The value of the voltage is indicated alongside the symbol. The voltage can be constant or it can be a function of time. Several voltage sources are shown in Figure 1.30.

In Figure 1.30(a), the voltage across the source is constant. Thus, we have a dc voltage source. On the other hand, the source shown in Figure 1.30(b) is an ac voltage source having a sinusoidal variation with time. We say that these are *independent* sources because the voltages across their terminals are independent of all other voltages and currents in the circuit.

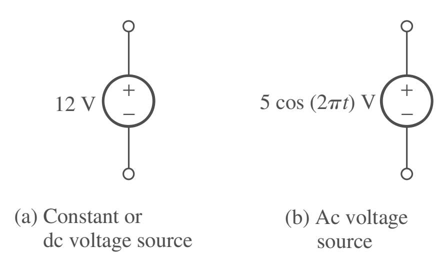


Figure 1.30 Independent voltage sources.

Ideal Circuit Elements versus Reality

Here we are giving definitions of *ideal* circuit elements. It is possible to draw ideal circuits in which the definitions of various circuit elements conflict. For example, Figure 1.31 shows a 12-V voltage source with a conductor connected across its terminals. In this case, the definition of the voltage source requires that $v_x = 12$ V. On the other hand, the definition of an ideal conductor requires that $v_x = 0$. In our study of ideal circuits, we avoid such conflicts.

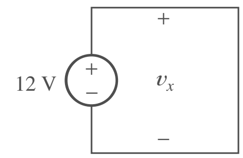


Figure 1.31 We avoid self-contradictory circuit diagrams such as this one.

In the real world, an automobile battery is nearly an ideal 12-V voltage source, and a short piece of heavy-gauge copper wire is nearly an ideal conductor. If we place the wire across the terminals of the battery, a very large current flows through the wire, stored chemical energy is converted to heat in the wire at a very high rate, and the wire will probably melt or the battery be destroyed.

When we encounter a contradictory idealized circuit model, we often have an undesirable situation (such as a fire or destroyed components) in the real-world counterpart to the model. In any case, a contradictory circuit model implies that we have not been sufficiently careful in choosing circuit models for the real circuit elements. For example, an automobile battery is not exactly modeled as an ideal voltage source. We will see that a better model (particularly if the currents are very large) is an ideal voltage source in series with a resistance. (We will discuss resistance very soon.) A short piece of copper wire is not modeled well as an ideal conductor, in this case. Instead, we will see that it is modeled better as a small resistance. If we have done a good job at picking circuit models for real-world circuits, we will not encounter contradictory circuits, and the results we calculate using the model will match reality very well.

Dependent Voltage Sources

A **dependent** or **controlled voltage source** is similar to an independent source except that the voltage across the source terminals is a function of other voltages or currents in the circuit. Instead of a circle, it is customary to use a diamond to represent controlled sources in circuit diagrams. Two examples of dependent sources are shown in Figure 1.32.

A **voltage-controlled voltage source** is a voltage source having a voltage equal to a constant times the voltage across a pair of terminals elsewhere in the network.

A voltage-controlled voltage source maintains a voltage across its terminals equal to a constant times a voltage elsewhere in the circuit.

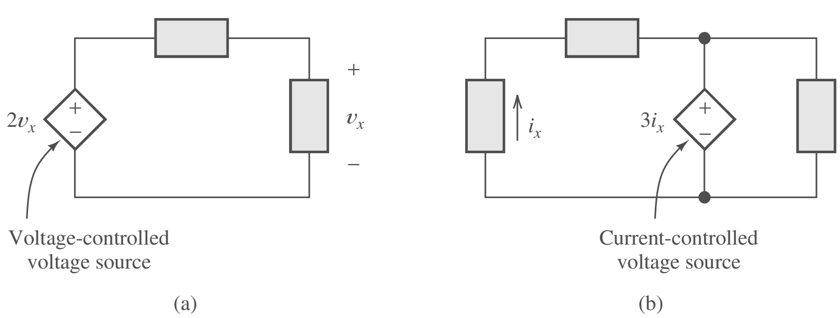


Figure 1.32 Dependent voltage sources (also known as controlled voltage sources) are represented by diamond-shaped symbols. The voltage across a controlled voltage source depends on a current or voltage that appears elsewhere in the circuit.

An example is shown in Figure 1.32(a). The dependent voltage source is the diamond symbol. The reference polarity of the source is indicated by the marks inside the diamond. The voltage v_x determines the value of the voltage produced by the source. For example, if it should turn out that $v_x = 3$ V, the source voltage is $2v_x = 6$ V. If v_x should equal -7 V, the source produces $2v_x = -14$ V (in which case, the actual positive polarity of the source is at the bottom end).

A current-controlled voltage source maintains a voltage across its terminals equal to a constant times a current flowing through some other element in the circuit.

A **current-controlled voltage source** is a voltage source having a voltage equal to a constant times the current through some other element in the circuit. An example is shown in Figure 1.32(b). In this case, the source voltage is three times the value of the current i_x. The factor multiplying the current is called the **gain parameter**. We assume that the voltage has units of volts and the current is in amperes. Thus, the gain parameter [which is 3 in Figure 1.32(b)] has units of volts per ampere (V/A). (Shortly, we will see that the units V/A are the units of resistance and are called ohms.)

Returning our attention to the voltage-controlled voltage source in Figure 1.32(a), we note that the gain parameter is 2 and is unitless (or we could say that the units are V/V).

Later in the book, we will see that controlled sources are very useful in modeling transistors, amplifiers, and electrical generators, among other things.

Independent Current Sources

An ideal independent current source forces a specified current to flow through itself.

An ideal **independent current source** forces a specified current to flow through itself. The symbol for an independent current source is a circle enclosing an arrow that gives the reference direction for the current. The current through an independent current source is independent of the elements connected to it and of the voltage across it. Figure 1.33 shows the symbols for a dc current source and for an ac current source.

If an open circuit exists across the terminals of a current source, we have a contradictory circuit. For example, consider the 2-A dc current source shown in Figure 1.33(a). This current source is shown with an open circuit across its terminals. By definition, the current flowing into the top node of the source is 2 A. Also by definition, no current can flow through the open circuit. Thus, KCL is not satisfied at this node. In good models for actual circuits, this situation does not occur. Thus, we will avoid current sources with open-circuited terminals in our discussion of ideal networks.

A battery is a good example of a voltage source, but an equally familiar example does not exist for a current source. However, current sources are useful in constructing theoretical models. Later, we will see that a good approximation to an ideal current source can be achieved with electronic amplifiers.

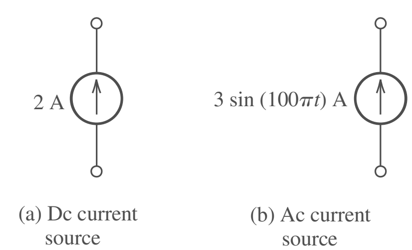


Figure 1.33 Independent current sources.

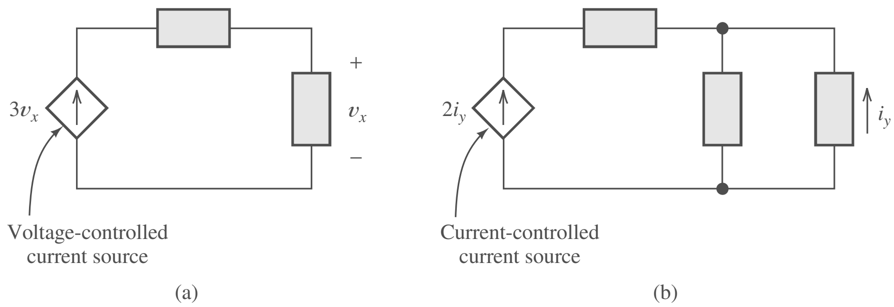


Figure 1.34 Dependent current sources. The current through a dependent current source depends on a current or voltage that appears elsewhere in the circuit.

Dependent Current Sources

The current flowing through a dependent current source is determined by a current or voltage elsewhere in the circuit.

The current flowing through a **dependent current source** is determined by a current or voltage elsewhere in the circuit. The symbol is a diamond enclosing an arrow that indicates the reference direction. Two types of controlled current sources are shown in Figure 1.34.

In Figure 1.34(a), we have a **voltage-controlled current source**. The current through the source is three times the voltage v_x. The gain parameter of the source (3 in this case) has units of A/V (which we will soon see are equivalent to siemens or inverse ohms). If it turns out that v_x has a value of 5 V, the current through the controlled current source is $3v_x = 15$ A.

Figure 1.34(b) illustrates a **current-controlled current source**. In this case, the current through the source is twice the value of i_y. The gain parameter, which has a value of 2 in this case, has units of A/A (i.e., it is unitless).

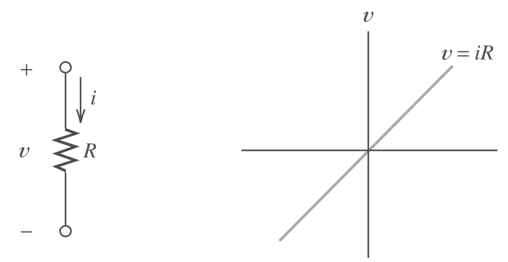
Like controlled voltage sources, controlled current sources are useful in constructing circuit models for many types of real-world devices, such as electronic amplifiers, transistors, transformers, and electrical machines. If a controlled source is needed for some application, it can be implemented by using electronic amplifiers. In sum, these are the four kinds of controlled sources:

1. Voltage-controlled voltage sources
2. Current-controlled voltage sources
3. Voltage-controlled current sources
4. Current-controlled current sources

Resistors and Ohm's Law

The voltage v across an ideal **resistor** is proportional to the current i through the resistor. The constant of proportionality is the resistance R. The symbol used for a resistor is shown in Figure 1.35(a). Notice that the current reference and voltage polarity reference conform to the passive reference configuration. In other words, the reference direction for the current is into the positive polarity mark and out of the negative polarity mark. In equation form, the voltage and current are related by **Ohm's law**:

$$v = iR$$



(a) Resistance symbol (b) Ohm's law

Figure 1.35 Voltage is proportional to current in an ideal resistor. Notice that the references for v and i conform to the passive reference configuration.

The units of resistance are V/A, which are called ohms. The uppercase Greek letter omega (Ω) represents ohms. In practical circuits, we encounter resistances ranging from milliohms (mΩ) to megohms (MΩ).

Except for rather unusual situations, the resistance R assumes positive values. (In certain types of electronic circuits, we can encounter negative resistance, but for now we assume that R is positive.) In situations for which the current reference direction enters the *negative* reference of the voltage, Ohm's law becomes

$$v = -iR$$

This is illustrated in Figure 1.36.

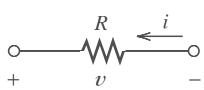


Figure 1.36 If the references for v and i are opposite to the passive configuration, we have $v = -Ri$.

The relationship between current direction and voltage polarity can be neatly included in the equation for Ohm's law if double-subscript notation is used. (Recall that to use double subscripts, we label the ends of the element under consideration, which is a resistance in this case.) If the order of the subscripts is the same for the current as for the voltage (i_{ab} and v_{ab}, for example), the current reference direction enters the first terminal and the positive voltage reference is at the first terminal. Thus, we can write

$$v_{ab} = i_{ab}R$$

On the other hand, if the order of the subscripts is not the same, we have

$$v_{ab} = -i_{ba}R$$

Conductance

Solving Ohm's law for current, we have

$$i = \frac{1}{R}v$$

We call the quantity $1/R$ a **conductance**. It is customary to denote conductances with the letter G:

$$G = \frac{1}{R} \tag{1.8}$$

Conductances have the units of inverse ohms (Ω^{-1}), which are called siemens (abbreviated S). Thus, we can write Ohm's law as

$$i = Gv \tag{1.9}$$

Resistors

It turns out that we can construct nearly ideal resistors by attaching terminals to many types of conductive materials. This is illustrated in Figure 1.37. Conductive materials that can be used to construct resistors include most metals, their alloys, and carbon.

On a microscopic level, current in metals consists of electrons moving through the material. (On the other hand, in solutions of ionic compounds, current is carried partly by positive ions.) The applied voltage creates an electric field that accelerates the electrons. The electrons repeatedly collide with the atoms of the material and lose their forward momentum. Then they are accelerated again. The net effect is a constant average velocity for the electrons. At the macroscopic level, we observe a current that is proportional to the applied voltage.

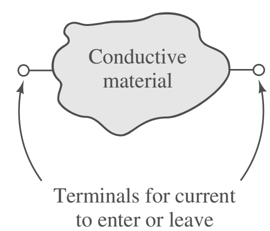


Figure 1.37 We construct resistors by attaching terminals to a piece of conductive material.

Resistance Related to Physical Parameters

The dimensions and geometry of the resistor as well as the particular material used to construct a resistor influence its resistance. We consider only resistors that take the form of a long cylinder or bar with terminals attached at the ends, as illustrated in Figure 1.38. The cross-sectional area A is constant along the length of the cylinder or bar. If the length L of the resistor is much greater than the dimensions of its cross section, the resistance is approximately given by

$$R = \frac{\rho L}{A} \tag{1.10}$$

in which ρ is the *resistivity* of the material used to construct the resistor. The units of resistivity are ohm meters (Ωm).

Materials can be classified as conductors, semiconductors, or insulators, depending on their resistivity. **Conductors** have the lowest resistivity and easily conduct electrical current. **Insulators** have very high resistivity and conduct very little current (at least for moderate voltages). **Semiconductors** fall between conductors and insulators. We will see in Chapters 10, 12, and 13 that certain semiconductors are very useful in constructing electronic devices. Table 1.3 gives approximate values of resistivity for several materials.

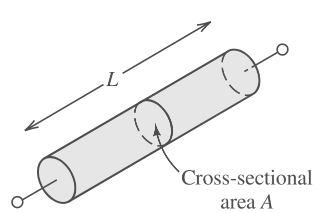


Figure 1.38 Resistors often take the form of a long cylinder (or bar) in which current enters one end and flows along the length.

Table 1.3. Resistivity Values (Ωm) for Selected Materials at 300 K

Conductors	
Aluminum	2.73×10^{-8}
Carbon (amorphous)	3.5×10^{-5}
Copper	1.72×10^{-8}
Gold	2.27×10^{-8}
Nichrome	1.12×10^{-6}
Silver	1.63×10^{-8}
Tungsten	5.44×10^{-8}
Semiconductors	
Silicon (device grade) depends on impurity concentration	10^{-5} to 1
Insulators	
Fused quartz	$> 10^{21}$
Glass (typical)	1×10^{12}
Teflon	1×10^{19}

Example 1.4 Resistance Calculation

Compute the resistance of a copper wire having a diameter of 2.05 mm and a length of 10 m.

Solution First, we compute the cross-sectional area of the wire:

$$A = \frac{\pi d^2}{4} = \frac{\pi(2.05 \times 10^{-3})^2}{4} = 3.3 \times 10^{-6}\ \mathrm{m}^2$$

Then, the resistance is given by

$$R = \frac{\rho L}{A} = \frac{1.72 \times 10^{-8} \times 10}{3.3 \times 10^{-6}} = 0.052\ \Omega$$

These are the approximate dimensions of a piece of 12-gauge copper wire that we might find connecting an electrical outlet to the distribution box in a residence. Of course, two wires are needed for a complete circuit. ■

Power Calculations for Resistances

Recall that we compute power for a circuit element as the product of the current and voltage:

$$p = vi \tag{1.11}$$

If v and i have the passive reference configuration, a positive sign for power means that energy is being absorbed by the device. Furthermore, a negative sign means that energy is being supplied by the device.

If we use Ohm's law to substitute for v in Equation 1.11, we obtain

$$p = Ri^2 \tag{1.12}$$

On the other hand, if we solve Ohm's law for i and substitute into Equation 1.11, we obtain

$$p = \frac{v^2}{R} \tag{1.13}$$

Notice that power for a resistance is positive regardless of the sign of v or i (assuming that R is positive, which is ordinarily the case). Thus, power is absorbed by resistances. If the resistance results from collisions of electrons with the atoms of the material composing a resistor, this power shows up as heat.

Some applications for conversion of electrical power into heat are heating elements for ovens, water heaters, cooktops, and space heaters. In a typical space heater, the heating element consists of a nichrome wire that becomes red hot in operation. (Nichrome is an alloy of nickel, chromium, and iron.) To fit the required length of wire in a small space, it is coiled rather like a spring.

PRACTICAL APPLICATION 1.1

Using Resistance to Measure Strain

Civil and mechanical engineers routinely employ the dependence of resistance on physical dimensions of a conductor to measure strain. These measurements are important in experimental stress–strain analysis of mechanisms and structures. (Strain is defined as fractional change in length, given by $\epsilon = \Delta L/L$.)

A typical resistive strain gauge consists of nickel–copper alloy foil that is photoetched to obtain multiple conductors aligned with the direction of the strain to be measured. This is illustrated in Figure PA1.1. Typically, the conductors are bonded to a thin polyimide (a tough flexible plastic) backing, which in turn is attached to the structure under test by a suitable adhesive, such as cyanoacrylate cement.

The resistance of a conductor is given by

$$R = \frac{\rho L}{A}$$

As strain is applied, the length and area change, resulting in changes in resistance. The strain and the change in resistance are related by the gauge factor:

$$G = \frac{\Delta R/R_0}{\epsilon}$$

in which R_0 is the resistance of the gauge before strain. A typical gauge has $R_0 = 350\ \Omega$ and $G = 2.0$. Thus, for a strain of 1%, the change in resistance is $\Delta R = 7\ \Omega$. Usually, a Wheatstone bridge (discussed in Chapter 2) is used to measure the small changes in resistance associated with accurate strain determination.

Sensors for force, torque, and pressure are constructed by using resistive strain gauges.

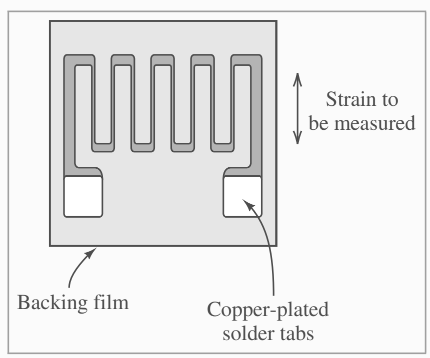


Figure PA1.1

Resistors versus Resistances

As an aside, we mention that resistance is often useful in modeling devices in which electrical power is converted into forms other than heat. For example, a loudspeaker appears to have a resistance of 8 Ω. Part of the power delivered to the loudspeaker is converted to acoustic power. Another example is a transmitting antenna having a resistance of 50 Ω. The power delivered to an antenna is radiated, traveling away as an electromagnetic wave.

There is a slight distinction between the terms *resistor* and *resistance*. A resistor is a two-terminal device composed of a conductive material. Resistance is a circuit property for which voltage is proportional to current. Thus, resistors have the property of resistance. However, resistance is also useful in modeling antennas and loudspeakers, which are quite different from resistors. Often, we are not careful about this distinction in using these terms.

Example 1.5 **Determining Resistance for Given Power and Voltage Ratings**

A certain electrical heater is rated for 1500 W when operated from 120 V. Find the resistance of the heater element and the operating current. (Resistance depends on temperature, and we will find the resistance at the operating temperature of the heater.)

Solution Solving Equation 1.13 for resistance, we obtain

$$R = \frac{v^2}{p} = \frac{120^2}{1500} = 9.6\ \Omega$$

Then, we use Ohm's law to find the current:

$$i = \frac{v}{R} = \frac{120}{9.6} = 12.5\ \text{A}$$

■

Exercise 1.11 The 9.6-Ω resistance of Example 1.5 is in the form of a nichrome wire having a diameter of 1.6 mm. Find the length of the wire. (*Hint:* The resistivity of nichrome is given in Table 1.3.)

Answer $L = 17.2$ m. □

Exercise 1.12 Suppose we have a typical incandescent electric light bulb that is rated for 100 W and 120 V. Find its resistance (at operating temperature) and operating current.

Answer $R = 144\ \Omega, i = 0.833$ A. □

Exercise 1.13 A 1-kΩ resistor used in a television receiver is rated for a maximum power of 1/4 W. Find the current and voltage when the resistor is operated at maximum power.

Answer $v_{\max} = 15.8$ V, $i_{\max} = 15.8$ mA. □

1.7 INTRODUCTION TO CIRCUITS

In this chapter, we have defined electrical current and voltage, discussed Kirchhoff's laws, and introduced several ideal circuit elements: voltage sources, current sources, and resistances. Now we illustrate these concepts by considering a few relatively simple circuits. In the next chapter, we consider more complex circuits and analysis techniques.

Consider the circuit shown in Figure 1.39(a). Suppose that we want to know the current, voltage, and power for each element. To obtain these results, we apply

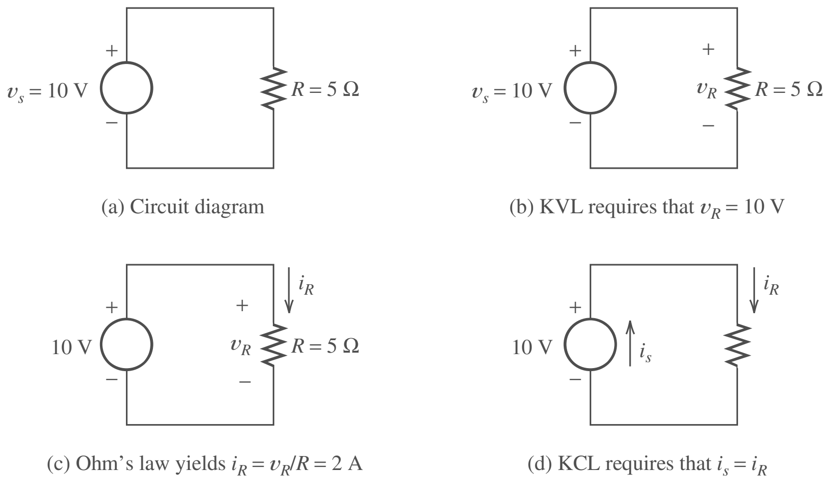


Figure 1.39 A circuit consisting of a voltage source and a resistance.

the basic principles introduced in this chapter. At first, we proceed in small, methodical steps. Furthermore, for ease of understanding, we initially select reference polarities and directions that agree with the actual polarities and current directions.

KVL requires that the sum of the voltages around the circuit shown in Figure 1.39 must equal zero. Thus, traveling around the circuit clockwise, we have $v_R - v_s = 0$. Consequently, $v_R = v_s$, and the voltage across the resistor v_R must have an actual polarity that is positive at the top end and a magnitude of 10 V.

An alternative way of looking at the voltages in this circuit is to notice that the voltage source and the resistance are in parallel. (The top ends of the voltage source and the resistance are connected, and the bottom ends are also connected.) Recall that when elements are in parallel, the voltage magnitude and polarity are the same for all elements.

Now consider Ohm's law. Because 10 V appears across the 5-Ω resistance, the current is $i_R = 10/5 = 2$ A. This current flows through the resistance from the positive polarity to the negative polarity. Thus, $i_R = 2$ A flows downward through the resistance, as shown in Figure 1.39(c).

According to KCL, the sum of the currents entering a given node must equal the sum of the currents leaving. There are two nodes for the circuit of Figure 1.39: one at the top and one at the bottom. The current i_R leaves the top node through the resistance. Thus, an equal current must enter the top node through the voltage source. The actual direction of current flow is upward through the voltage source, as shown in Figure 1.39(d).

Another way to see that the currents i_s and i_R are equal is to notice that the voltage source and the resistance are in series. In a series circuit, the current that flows in one element must continue through the other element. (Notice that for this circuit the voltage source and the resistance are in parallel and they are also in series. A two-element circuit is the only case for which this occurs. If more than two elements are interconnected, a pair of elements that are in parallel cannot also be in series, and vice versa.)

Notice that in Figure 1.39, the current in the voltage source flows from the negative polarity toward the positive polarity. It is only for resistances that the current is required to flow from plus to minus. For a voltage source, the current can flow in either direction, depending on the circuit to which the source is connected.

It is only for resistances that the current is required to flow from plus to minus. Current may flow in either direction for a voltage source depending on the other elements in the circuit.

Now let us calculate the power for each element. For the resistance, we have several ways to compute power:

$$p_R = v_R i_R = 10 \times 2 = 20 \text{ W}$$
$$p_R = i_R^2 R = 2^2 \times 5 = 20 \text{ W}$$
$$p_R = \frac{v_R^2}{R} = \frac{10^2}{5} = 20 \text{ W}$$

Of course, all the calculations yield the same result. Energy is delivered to the resistance at the rate of 20 J/s.

To find the power for the voltage source, we have

$$p_s = -v_s i_s$$

where the minus sign is used because the reference direction for the current enters the negative voltage reference (opposite to the passive reference configuration). Substituting values, we obtain

$$p_s = -v_s i_s = -10 \times 2 = -20 \text{ W}$$

Because p_s is negative, we understand that energy is being delivered by the voltage source.

As a check, if we add the powers for all the elements in the circuit, the result should be zero, because energy is neither created nor destroyed in an electrical circuit. Instead, it is transported and changed in form. Thus, we can write

$$p_s + p_R = -20 + 20 = 0$$

Using Arbitrary References

In the previous discussion, we selected references that agree with actual polarities and current directions. This is not always possible at the start of the analysis of more complex circuits. Fortunately, it is not necessary. We can pick the references in an arbitrary manner. Application of circuit laws will tell us not only the magnitudes of the currents and voltages but the true polarities and current directions as well.

Example 1.6 **Circuit Analysis Using Arbitrary References**

Analyze the circuit of Figure 1.39 using the current and voltage references shown in Figure 1.40. Verify that the results are in agreement with those found earlier.

Solution Traveling clockwise and applying KVL, we have

$$-v_s - v_x = 0$$

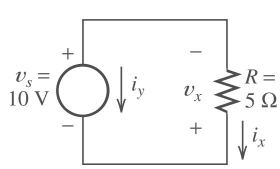


Figure 1.40 Circuit for Example 1.6.

This yields $v_x = -v_s = -10$ V. Since v_x assumes a negative value, the actual polarity is opposite to the reference. Thus, as before, we conclude that the voltage across the resistance is actually positive at the top end.

According to Ohm's law,

$$i_x = -\frac{v_x}{R}$$

where the minus sign appears because v_x and i_x have references opposite to the passive reference configuration. Substituting values, we get

$$i_x = -\frac{-10}{5} = 2 \text{ A}$$

Since i_x assumes a positive value, the actual current direction is downward through the resistance.

Next, applying KCL at the bottom node of the circuit, we have

$$\text{total current entering} = \text{total current leaving}$$

$$i_y + i_x = 0$$

Thus, $i_y = -i_x = -2$ A, and we conclude that a current of 2 A actually flows upward through the voltage source.

The power for the voltage source is

$$p_s = v_s i_y = 10 \times (-2) = -20 \text{ W}$$

Finally, the power for the resistance is given by

$$p_R = -v_x i_x$$

where the minus sign appears because the references for v_x and i_x are opposite to the passive reference configuration. Substituting, we find that $p_R = -(-10) \times (2) = 20$ W. Because p_R has a positive value, we conclude that energy is delivered to the resistance. ■

Sometimes circuits can be solved by repeated application of Kirchhoff's laws and Ohm's law. We illustrate with an example.

Example 1.7 Using KVL, KCL, and Ohm's Law to Solve a Circuit

Solve for the source voltage in the circuit of Figure 1.41 in which we have a current-controlled current source and we are given that the voltage across the 5-Ω resistance is 15 V.

Solution First, we use Ohm's Law to determine the value of i_y:

$$i_y = \frac{15 \text{ V}}{5 \text{ }\Omega} = 3 \text{ A}$$

Next, we apply KCL at the top end of the controlled source:

$$i_x + 0.5 i_x = i_y$$

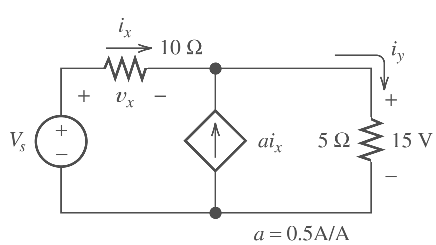


Figure 1.41 Circuit for Example 1.7.

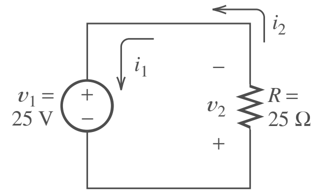


Figure 1.42 Circuit for Exercise 1.14.

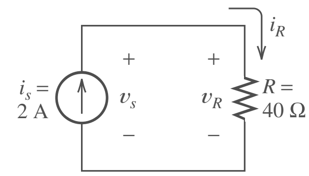


Figure 1.43 Circuit for Exercise 1.15.

Substituting the value found for i_y and solving, we determine that $i_x = 2$ A. Then Ohm's law yields $v_x = 10i_x = 20$ V. Applying KCL around the periphery of the circuit gives

$$V_s = v_x + 15$$

Finally, substituting the value found for v_x yields $V_s = 35$ V. ■

Exercise 1.14 Analyze the circuit shown in Figure 1.42 to find the values of i_1, i_2, and v_2. Use the values found to compute the power for each element.

Answer $i_1 = i_2 = -1$ A, $v_2 = -25$ V, $p_R = 25$ W, $p_s = -25$ W. □

Exercise 1.15 Figure 1.43 shows an independent current source connected across a resistance. Analyze to find the values of i_R, v_R, v_s, and the power for each element.

Answer $i_R = 2$ A, $v_s = v_R = 80$ V, $p_s = -160$ W, $p_R = 160$ W. □

Summary

1. Electrical and electronic features are increasingly integrated into the products and systems designed by engineers in other fields. Furthermore, instrumentation in all fields of engineering and science is based on the use of electrical sensors, electronics, and computers.
2. Some of the main areas of electrical engineering are communication systems, computer systems, control systems, electromagnetics, photonics, electronics, power systems, and signal processing.
3. Some important reasons to learn basic electrical engineering principles are to pass the Fundamentals of Engineering Examination, to have a broad enough knowledge base to lead design projects in your own field, to be able to identify and correct simple malfunctions in electrical systems, and to be able to communicate efficiently with electrical engineering consultants.
4. Current is the time rate of flow of electrical charge. Its units are amperes (A), which are equivalent to coulombs per second (C/s).
5. The voltage associated with a circuit element is the energy transferred per unit of charge that flows through the element. The units of voltages are volts (V), which are equivalent to joules per coulomb (J/C). If positive charge moves from the positive reference to the negative reference, energy is absorbed by the circuit element. If the charge moves in the opposite direction, energy is delivered by the element.
6. In the passive reference configuration, the current reference direction enters the positive reference polarity.
7. If the references have the passive configuration, power for a circuit element is computed as the product of the current through the element and the voltage across it:

$$p = vi$$

If the references are opposite to the passive configuration, we have

$$p = -vi$$

In either case, if p is positive, energy is being absorbed by the element.
8. A node in an electrical circuit is a point at which two or more circuit elements are joined together. All points joined by ideal conductors are electrically equivalent and constitute a single node.
9. Kirchhoff's current law (KCL) states that the sum of the currents entering a node equals the sum of the currents leaving.
10. Elements connected end to end are said to be in series. For two elements to be in series, no other current path can be connected to their common node. The current is identical for all elements in a series connection.

11. A loop in an electrical circuit is a closed path starting at a node and proceeding through circuit elements eventually returning to the starting point.
12. Kirchhoff's voltage law (KVL) states that the algebraic sum of the voltages in a loop must equal zero. If the positive polarity of a voltage is encountered first in going around the loop, the voltage carries a plus sign in the sum. On the other hand, if the negative polarity is encountered first, the voltage carries a minus sign.
13. Two elements are in parallel if both ends of one element are directly connected to corresponding ends of the other element. The voltages of parallel elements are identical.
14. The voltage between the ends of an ideal conductor is zero regardless of the current flowing through the conductor. All points in a circuit that are connected by ideal conductors can be considered as a single point.
15. An ideal independent voltage source maintains a specified voltage across its terminals independent of other elements that are connected to it and of the current flowing through it.
16. For a controlled voltage source, the voltage across the source terminals depends on other voltages or currents in the circuit. A voltage-controlled voltage source is a voltage source having a voltage equal to a constant times the voltage across a pair of terminals elsewhere in the network. A current-controlled voltage source is a voltage source having a voltage equal to a constant times the current through some other element in the circuit.
17. An ideal independent current source forces a specified current to flow through itself, independent of other elements that are connected to it and of the voltage across it.
18. For a controlled current source, the current depends on other voltages or currents in the circuit. A voltage-controlled current source produces a current equal to a constant times the voltage across a pair of terminals elsewhere in the network. A current-controlled current source produces a current equal to a constant times the current through some other element in the circuit.
19. For constant resistances, voltage is proportional to current. If the current and voltage references have the passive configuration, Ohm's law states that $v = Ri$. For references opposite to the passive configuration, $v = -Ri$.

Problems

Section 1.1: Overview of Electrical Engineering

P1.1. Broadly speaking, what are the two main objectives of electrical systems?

P1.2. Name eight subdivisions of electrical engineering.

P1.3. Briefly describe four important reasons that other engineering students need to learn the fundamentals of electrical engineering.

P1.4. Write a few paragraphs describing an interesting application of electrical engineering in your field. Consult engineering journals and trade magazines such as the *IEEE Spectrum, Automotive Engineering, Chemical Engineering*, or *Civil Engineering* for ideas.

Section 1.2: Circuits, Currents, and Voltages

P1.5. Carefully define or explain each of the following terms in your own words giving units where appropriate: **a.** Electrical current; **b.** Voltage; **c.** An open switch; **d.** A closed switch; **e.** Direct current; **f.** Alternating current.

P1.6. In the fluid-flow analogy for electrical circuits, what is analogous to: **a.** a conductor; **b.** an open switch; **c.** a resistance; **d.** a battery?

***P1.7.** The ends of a length of wire are labeled a and b. If the current in the wire is $i_{ab} = -3$ A, are electrons moving toward a or b? How much charge passes through a cross section of the wire in 3 seconds?

* Denotes that answers are contained in the Student Solutions files. See Appendix E for more information about accessing the Student Solutions.

***P1.8.** The net charge through a cross section of a circuit element is given by $q(t) = 2t + t^2$ C. As usual, t is in seconds. Find the current through the element in amperes.

***P1.9.** The current through a given circuit element is given by $i(t) = 2e^{-t}$ A. As usual, time t is in seconds. Find the net charge that passes through the element in the interval from $t = 0$ to $t = \infty$. (*Hint:* Current is the rate of flow of charge. Thus, to find charge, we must integrate current with respect to time.)

***P1.10.** A certain lead-acid storage battery has a mass of 30 kg. Starting from a fully charged state, it can supply 5 A for 24 hours with a terminal voltage of 12 V before it is totally discharged. **a.** If the energy stored in the fully charged battery is used to lift the battery with 100-percent efficiency, what height is attained? Assume that the acceleration due to gravity is 9.8 m/s^2 and is constant with height. **b.** If the stored energy is used to accelerate the battery with 100 percent efficiency, what velocity is attained? **c.** Gasoline contains about 4.5×10^7 J/kg. Compare this with the energy content per unit mass for the fully charged battery.

***P1.11.** A typical "deep-cycle" battery (used for electric trolling motors for fishing boats) is capable of delivering 12.6 V and 10 A for a period of 10 hours. How much charge flows through the battery in this interval? How much energy does the battery deliver?

P1.12. An ac current given by $i(t) = 5\sin(200\pi t)$ A, in which t is in seconds and the angle is in radians, flows through an element of an electrical circuit. **a.** Sketch $i(t)$ to scale versus time for t ranging from 0 to 15 ms. **b.** Determine the net charge that passes through the element between $t = 0$ and $t = 10$ ms. **c.** Repeat for the interval from $t = 0$ to $t = 15$ ms.

P1.13. Consider the headlight circuit of Figure 1.2 on page 7. For current to flow through the headlight, should the switch be open or closed? In the fluid-flow analogy for the circuit, would the valve corresponding to the switch be open or closed? What state for a valve, open or closed, is analogous to an open switch?

P1.14. What is the net number of electrons per second that pass through the cross-section of a wire carrying 5 A of dc current? The current flow is due to electrons, and the magnitude of the charge of each electron is 1.60×10^{-19} C.

P1.15. The circuit element shown in Figure P1.15 has $v = 15$ V and $i = -3$ A. What are the values of v_{ba} and i_{ba}? Be sure to give the correct algebraic signs. Is energy being delivered to the element or taken from it?

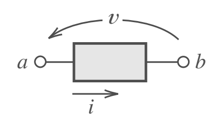


Figure P1.15

P1.16. Suppose that the net charge passing through the cross section of a certain circuit element is given by $q(t) = 2t + 3t^2$ C, with time t in seconds. Determine the current through the element as a function of time.

P1.17. In typical residential wiring, the copper wire has a diameter of 2.05 mm and carries a maximum current of $15\sqrt{2}$ A due solely to electrons, each of which has a charge of 1.60×10^{-19} C. Given that the free electron (these are the electrons capable of moving through the copper) concentration in copper is 10^{29} electrons/m^3, find the average velocity of the electrons in the wire when the maximum current is flowing.

P1.18. The charge carried by an electron is -1.60×10^{-19} C. Suppose that an electron moves through a voltage of 120 V from the negative polarity to the positive polarity. How much energy is transferred? Does the electron gain or lose energy?

P1.19. Consider a circuit element, with terminals a and b, that has $v_{ab} = -12$ V and $i_{ab} = 3$ A. Over a period of 2 seconds, how much charge moves through the element? If electrons carry the charge, which terminal do they enter? How much energy is transferred? Is it delivered to the element or taken from it?

Section 1.3: Power and Energy

P1.20. What does the term *passive reference configuration* imply? When do we have this configuration if we are using double subscript notation for an element having terminals a and b?

***P1.21.** Compute the power for each element shown in Figure P1.21. For each element, state whether energy is being absorbed by the element or supplied by it.

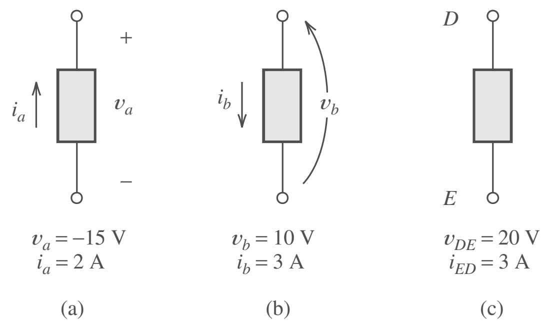


Figure P1.21

***P1.22.** A certain battery has terminals labeled a and b. The battery voltage is $v_{ab} = 12$ V. To increase the chemical energy stored in the battery by 600 J, how much charge must move through the battery? Should electrons move from a to b or from b to a?

P1.23. The terminals of an electrical device are labeled a and b. If $v_{ab} = 25$ V, how much energy is exchanged when a positive charge of 4 C moves through the device from a to b? Is the energy delivered to the device or taken from it?

P1.24. Consider the element shown in Figure P1.24 which has $v(t) = -15$ V and $i(t) = 3e^{-2t}$ A. Compute the power for the circuit element. Find the energy transferred between $t = 0$ and $t = \infty$. Is this energy absorbed by the element or supplied by it?

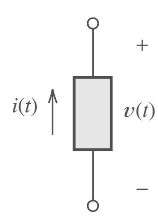


Figure P1.24

P1.25. Suppose that the cost of electrical energy is \$0.12 per kilowatt hour and that your electrical bill for 30 days is \$60. Assume that the power delivered is constant over the entire 30 days. What is the power in watts? If a voltage of 120 V supplies this power, what current flows? Part of your electrical load is a 60-W light that is on continuously. By what percentage can your energy consumption be reduced by turning this light off?

P1.26. An electrical device has $i_{ab}(t) = 2$ A and $v_{ab}(t) = 10\sin(100\pi t)$ V, in which the angle is in radians. **a.** Find the power delivered to the device and sketch it to scale versus time for t ranging from 0 to 30 ms. **b.** Determine the energy delivered to the device for the interval from $t = 0$ to $t = 10$ ms. **c.** Repeat for the interval from $t = 0$ to $t = 20$ ms.

P1.27. A fully charged deep-cycle lead–acid storage battery is rated for 12.6 V and 100 ampere hours. (The ampere-hour rating of the battery is the operating time to discharge the battery multiplied by the current.) This battery is used aboard a sailboat to power the electronics which consume 30 W. Assume that the battery voltage is constant during the discharge. For how many hours can the electronics be operated from the battery without recharging? How much energy in kilowatt hours is initially stored in the battery? If the battery costs \$95 and has a life of 250 charge–discharge cycles, what is the cost of the energy in dollars per kilowatt hour? Neglect the cost of recharging the battery.

***P1.28.** Figure P1.28 shows an ammeter (AM) and voltmeter (VM) connected to measure the current and voltage, respectively, for circuit element A. When current actually enters the + terminal of the ammeter the reading is positive, and when current leaves the + terminal the reading is negative. If the actual voltage polarity is positive at the + terminal of the VM, the reading is positive; otherwise, it is negative. (Actually, for the connection shown, the ammeter reads the sum of the current in element A and the very small current taken by the voltmeter. For purposes of this problem, assume that the current taken by the voltmeter is negligible.) Find the power for element A and state whether energy is being delivered to element A or taken from it if: **a.** the ammeter reading is +2 A and the voltmeter reading is −25 V; **b.** the ammeter reading is −2 A and the voltmeter reading is +25 V; **c.** the ammeter reading is −2 A and the voltmeter reading is −25 V.

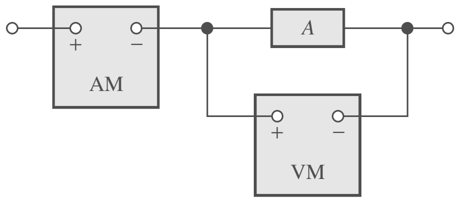


Figure P1.28

P1.29. Repeat Problem P1.28 with the meters connected as shown in Figure P1.29.

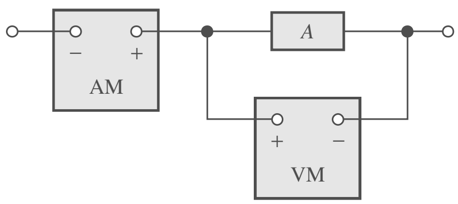


Figure P1.29

P1.30. Determine the cost per kilowatt hour for the energy delivered by a typical alkaline 9-V "transistor" battery that costs \$1.95 and is capable of delivering a current of 50 mA for a period of 10 hours. (For comparison, the approximate cost of energy purchased from electric utilities in the United States is \$0.12 per kilowatt hour.)

Section 1.4: Kirchhoff's Current Law

P1.31. Define the term *node* as it applies in electrical circuits. Identify the nodes in the circuit of Figure P1.31. Keep in mind that all points connected by ideal conductors are considered to be a single node in electrical circuits.

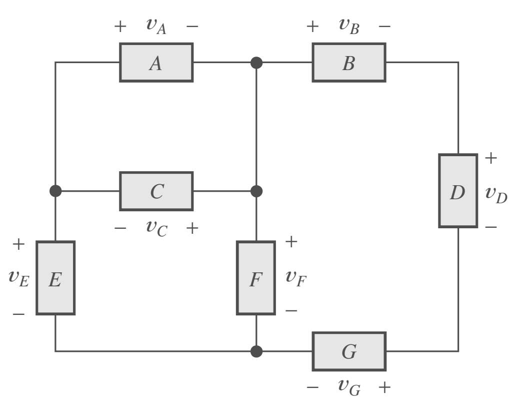


Figure P1.31

***P1.32.** Use KCL to find the values of i_a, i_c, and i_d for the circuit of Figure P1.32. Which elements are connected in series in this circuit?

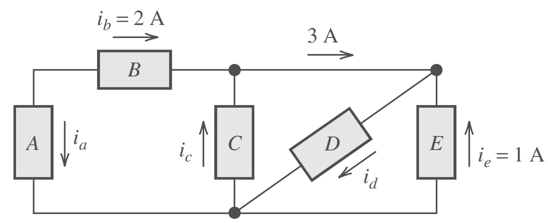


Figure P1.32

P1.33. What can you say about the currents through series-connected elements in an electrical circuit?

P1.34. State Kirchhoff's current law in your own words. Why is it true?

***P1.35.** Identify elements that are in series in the circuit of Figure P1.31.

P1.36. In the fluid-flow analogy for an electrical circuit, the analog of electrical current is volumetric flow rate with units of cm^3/s. For a proper analogy to electrical circuits, must the fluid be compressible or incompressible? Must the walls of the pipes be elastic or inelastic? Explain your answers.

***P1.37.** Given that $i_a = 2$ A, $i_b = 3$ A, $i_d = -5$ A, and $i_h = 4$ A, determine the values of the other currents in Figure P1.37.

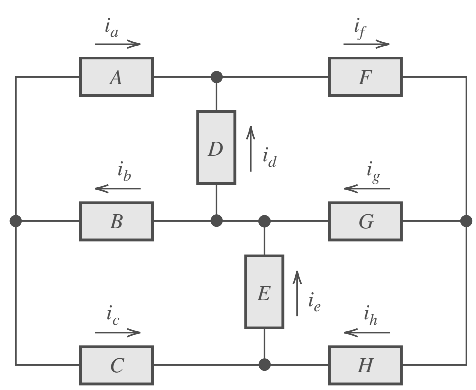


Figure P1.37

P1.38. Determine the values of the other currents in Figure P1.37, given that $i_a = 2$ A, $i_c = -3$ A, $i_g = 6$ A, and $i_h = 1$ A.

P1.39. **a.** Which elements are in series in Figure P1.39? **b.** What is the relationship between i_d and i_c? **c.** Given that $i_a = 6$ A and $i_c = -2$ A, determine the values of i_b and i_d .

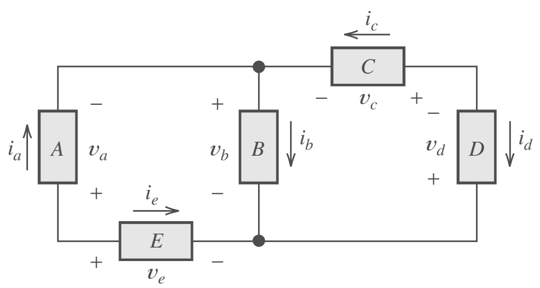


Figure P1.39

Section 1.5: Kirchhoff's Voltage Law

P1.40. Explain Kirchhoff's voltage law in your own words. Why is it true?

***P1.41.** Use KVL and KCL to solve for the labeled currents and voltages in Figure P1.41. Compute the power for each element and show that power is conserved (i.e., the algebraic sum of the powers is zero).

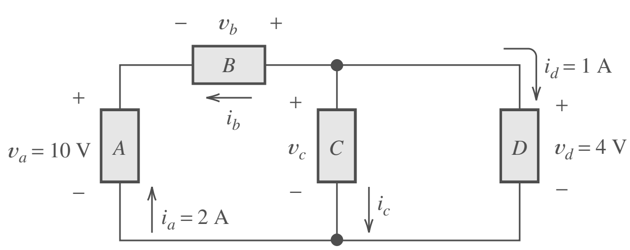


Figure P1.41

***P1.42.** Use KVL to solve for the voltages v_a, v_b, and v_c in Figure P1.42.

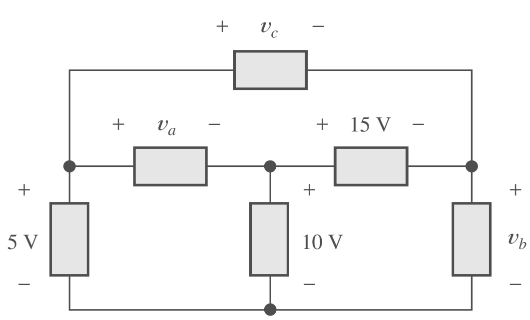


Figure P1.42

P1.43. We know that $v_a = 10$ V, $v_b = -3$ V, $v_f = 12$ V, and $v_h = 5$ V, solve for the other voltages shown in Figure P1.43.

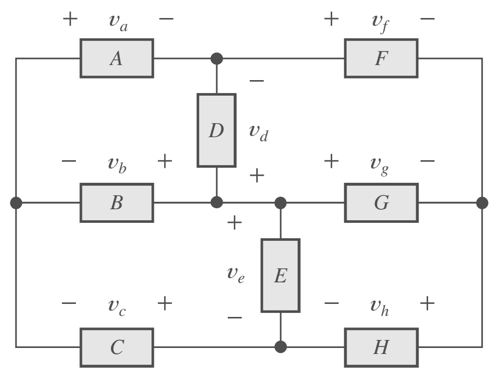


Figure P1.43

P1.44. Consider the circuit shown in Figure P1.31. **a.** Which elements are in parallel? **b.** What is the relationship between v_A and v_C? **c.** Given that $v_A = 8$ V and $v_E = 4$ V, determine the values of v_C and v_F.

P1.45. Identify the elements that are in parallel: **a.** in Figure P1.32, **b.** in Figure P1.43.

P1.46. Identify the nodes in Figure P1.46. Which elements are in series? Which are in parallel?

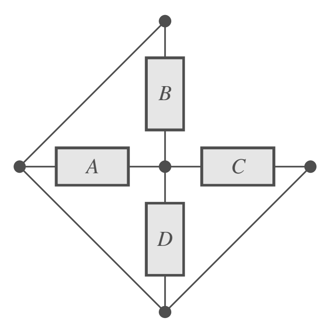


Figure P1.46

P1.47. Consider a circuit containing four nodes labeled *a*, *b*, *c*, and *d*. Also, we know that $v_{ab} = 12$ V, $v_{cb} = -4$ V, and $v_{da} = 8$ V. Determine the values of v_{ac} and v_{cd}. (*Hint:* Draw a picture showing the nodes and the known voltages.)

P1.48. A typical golf cart uses a number of 6-V batteries (which, for the purposes of this problem, can be modeled as ideal 6-V voltage sources). If the motor requires 36 V, what is the minimum number of batteries needed? How should they be connected? Sketch a diagram for the battery connections showing the polarity of each battery.

Section 1.6: Introduction to Circuit Elements

P1.49. Explain Ohm's law in your own words, including references.

P1.50. Define these terms in your own words: **a.** an ideal conductor; **b.** an ideal voltage source; **c.** an ideal current source; **d.** short circuit; **e.** open circuit.

P1.51. Name four types of dependent sources and give the units for the gain parameter of each type.

P1.52. **a.** Show that for wires of identical dimensions but made of different materials, such as copper and aluminum, the resistance is proportional to the resistivity of the material. (See Table 1.3 on page 28 for resistivity values of some materials used in electrical wiring.) **b.** We know that the resistance of a certain copper wire is 1.5 Ω. Determine the resistance of an aluminum wire having the same dimensions as the copper wire.

***P1.53.** Draw a circuit that contains a 5-Ω resistance, a 10-V independent voltage source, and a 2-A independent current source. Connect all three elements in series. (Because the polarity of the voltage source and reference direction for the current source are not specified, several correct answers are possible.)

P1.54. Repeat Problem P1.53, placing all three elements in parallel.

P1.55. Suppose that a certain wire has a resistance of 10 Ω. Find the new resistance: **a.** if the length of the wire is doubled; **b.** if the diameter of the wire is doubled.

P1.56. Suppose we have a copper wire with a resistance of 2 Ω. We want to replace the copper wire with an aluminum wire of the same length and resistance. By what factor must the diameter of the wire be increased?

***P1.57.** A power of 100 W is delivered to a certain resistor when the applied voltage is 100 V. Find the resistance. Suppose that the voltage is reduced by 10 percent (to 90 V). By what percentage is the power reduced? Assume that the resistance remains constant.

P1.58. Sketch the diagram of a circuit that contains a 3-Ω resistor, a 10-V voltage source, and a current-controlled current source having a gain constant of 0.4 A/A. Assume that the current through the resistor is the control current for the controlled source. Place all three elements in parallel. Several answers are possible, depending on the polarities and reference directions chosen.

P1.59. Sketch the diagram of a circuit that contains a 3-Ω resistor, a 2-A current source, and a voltage-controlled current source having a gain constant of 2 S. Assume that the voltage across the resistor is the control voltage for the controlled source. Place all three elements in parallel. Several answers are possible, depending on the polarities and reference directions chosen.

P1.60. The current through a 10-Ω resistor is given by $i(t) = 10\exp(-2t)$ A. Determine the energy delivered to the resistor between $t = 0$ and $t = \infty$.

P1.61. The voltage across a 12-Ω resistor is given by $v(t) = 24\cos(2\pi t)$ V. Calculate the energy delivered to the resistor between $t = 0$ and $t = 2$ s. The argument of the cosine function, $2\pi t$, is in radians.

Section 1.7: Introduction to Circuits

***P1.62.** Which of the following are self-contradictory combinations of circuit elements? **a.** A 12-V voltage source in parallel with a 2-A current source. **b.** A 2-A current source in series with a 3-A current source. **c.** A 2-A current source in parallel with a short circuit. **d.** A 2-A current source in series with an open circuit. **e.** A 5-V voltage source in parallel with a short circuit.

***P1.63.** Consider the circuit shown in Figure P1.63. Use repeated applications of Ohm's law, KVL, and KCL to eventually find V_x.

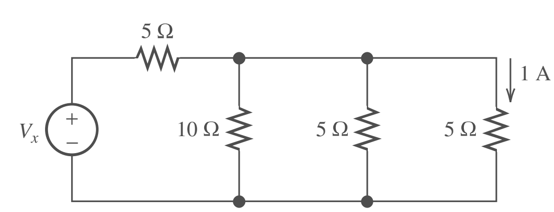


Figure P1.63

***P1.64.** Consider the circuit shown in Figure P1.64. Find the current i_R flowing through the resistor. Find the power for each element in the circuit. Which elements are absorbing power?

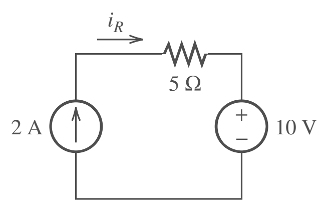


Figure P1.64

P1.65. Plot, to scale, i_{ab} versus v_{ab} for each of the parts of Figure P1.65.

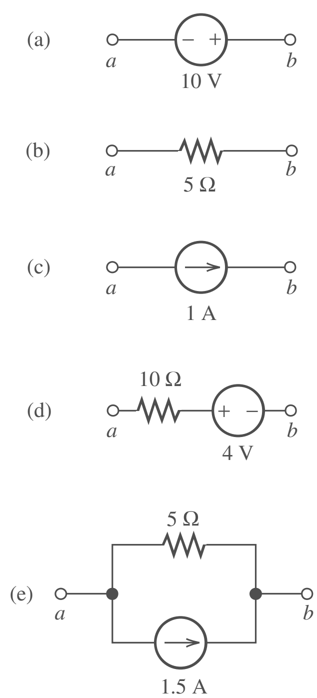


Figure P1.65

P1.66. Consider the circuit shown in Figure P1.66. **a.** Which elements are in series? **b.** Which elements are in parallel? **c.** Apply Ohm's and Kirchhoff's laws to solve for v_x.

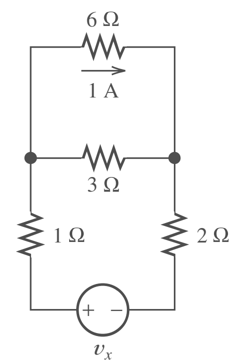


Figure P1.66

P1.67. Given the circuit shown in Figure P1.67, find the power for each source. Which source is absorbing power? Which is delivering power?

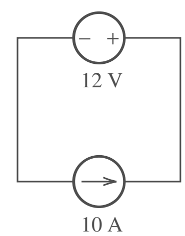


Figure P1.67

P1.68. Consider the circuit shown in Figure P1.68. Find the current i_R flowing through the resistor. Find the power for each element in the circuit. Which elements are absorbing energy?

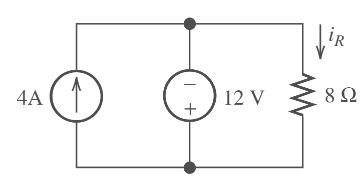


Figure P1.68

P1.69. Use repeated applications of Ohm's law, KVL, and KCL to eventually find the value of I_x in the circuit of Figure P1.69.

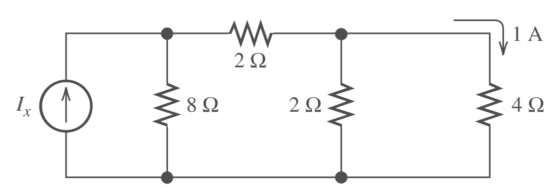


Figure P1.69

***P1.70.** The circuit shown in Figure P1.70 contains a voltage-controlled voltage source. **a.** Use KVL to write an equation relating the voltages and solve for v_x. **b.** Use Ohm's law to find the current i_x. **c.** Find the power for each element in the circuit and verify that power is conserved.

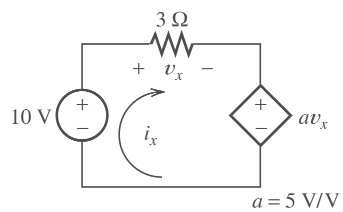


Figure P1.70

***P1.71.** What type of controlled source is shown in the circuit of Figure P1.71? Solve for v_s.

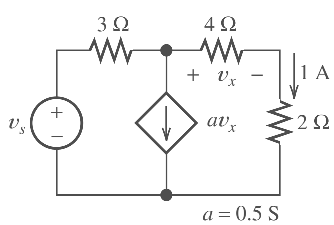


Figure P1.71

P1.72. Consider the circuit shown in Figure P1.72. **a.** Which elements are in series? **b.** Which elements are in parallel? **c.** Apply Ohm's and Kirchhoff's laws to solve for R_x.

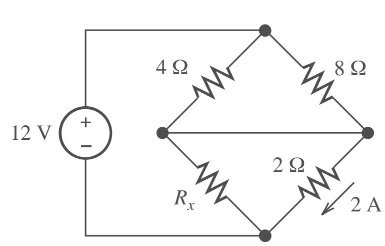


Figure P1.72

P1.73. Solve for the currents shown in Figure P1.73.

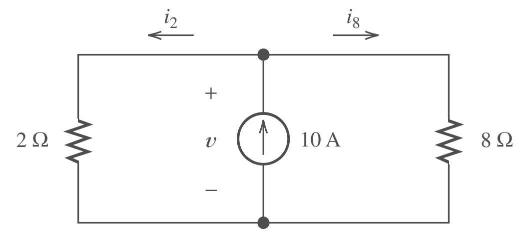


Figure P1.73

P1.74. Figure P1.74 is the electrical model for an electronic megaphone, in which the 16-Ω resistance models a loudspeaker, the source I_x and the 5-Ω resistance represent a microphone, and the remaining elements model an amplifier. What is the name of the type of controlled source shown? Given that the power delivered to the 16-Ω resistance is 16 W, determine the current flowing in the controlled source. Also, determine the value of the microphone current I_x.

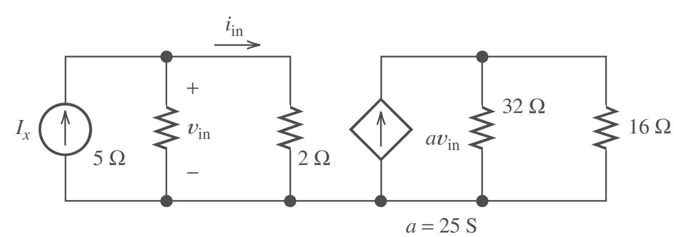


Figure P1.74

P1.75. What type of controlled source appears in the circuit of Figure P1.75? Determine the values of v_x and i_y.

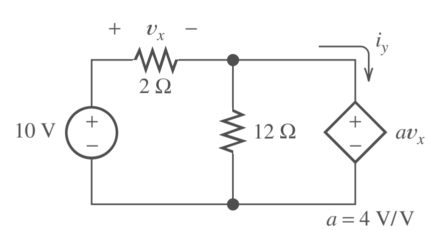


Figure P1.75

P1.76. A 10-A independent current source is connected in series with a 2-Ω resistance between terminals a and b. What can you say about

i_{ab} and v_{ab}? What single circuit element is equivalent to this series combination?

P1.77. A 10-V independent voltage source is connected in parallel with a 2-Ω resistance between terminals *a* and *b*. What can you say about i_{ab} and v_{ab}? What single circuit element is equivalent to this parallel combination?

P1.78. Consider the circuit shown in Figure P1.78, given $R_1 = 15\ \Omega$, $R_2 = 10\ \Omega$, and $I_s = 4$ A. **a.** Use KCL to write an equation relating the currents. **b.** Use Ohm's law to write equations relating i_1 and i_2 to the voltage v. **c.** Substitute the equations from part (b) into the equation from part (a) and solve for v. **d.** Find the power for each element in the circuit and verify that power is conserved.

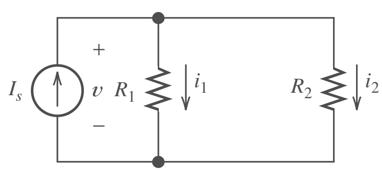


Figure P1.78

P1.79. Consider the circuit shown in Figure P1.79, given $R_1 = 3\ \Omega$, $R_2 = 4\ \Omega$, and $I_s = 2$ A. **a.** Use KVL to write an equation relating the voltages. **b.** Use Ohm's law to write equations relating v_1 and v_2 to the current I_s. **c.** Substitute the equations from part (b) into the equation from part (a) and solve for v. **d.** Find the power for each element in the circuit and verify that power is conserved.

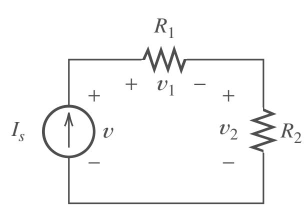


Figure P1.79

P1.80. What types of sources are present in the circuit shown in Figure P1.80? Solve for I_s and the voltage v.

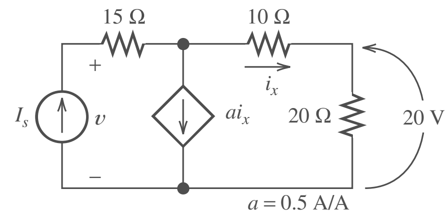


Figure P1.80

P1.81. KVL is a consequence of KCL and conservation of power in electrical circuits. This problem illustrates this fact for the circuit of Figure P1.81. **a.** Write the KCL equation for the circuit. **b.** Apply conservation of power to the circuit, summing the powers for the elements (observing the proper algebraic signs) and setting the sum to zero. **c.** Use the KCL equation to substitute for i_3 in the equation of part (b) and put the result into the form $Ai_1 + Bi_2 = 0$, in which A and B are algebraic sums of the voltages. **d.** Now, suppose that we have control of the current sources. For any values we select for i_1 and i_2, the equation of part (c) must be satisfied. This can only happen if we have $A = 0$ and $B = 0$. Show that this gives the KVL equations for the circuit.

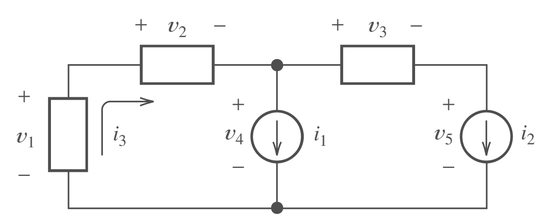


Figure P1.81

P1.82. What types of sources are present in the circuit of Figure P1.82? Solve for the current i_x.

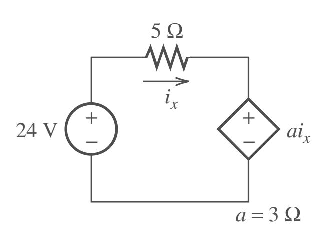


Figure P1.82

Practice Test

Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in Appendix D and complete solutions are included in the Student Solutions files. See Appendix E for more information about the Student Solutions.

T1.1. Match each entry in Table T1.1(a) with the best choice from the list given in

Table T1.1

Item	Best Match
(a)	
a. Node	
b. Loop	
c. KVL	
d. KCL	
e. Ohm's law	
f. Passive reference configuration	
g. Ideal conductor	
h. Open circuit	
i. Current source	
j. Parallel connected elements	
k. Controlled source	
l. Units for voltage	
m. Units for current	
n. Units for resistance	
o. Series connected elements	
(b)	
1. $v_{ab} = Ri_{ab}$	
2. The current reference for an element enters the positive voltage reference	
3. A path through which no current can flow	
4. Points connected by ideal conductors	
5. An element that carries a specified current	
6. An element whose current or voltage depends on a current or voltage elsewhere in the circuit	
7. A path starting at a node and proceeding from node to node back to the starting node	
8. An element for which the voltage is zero	
9. A/V	
10. V/A	
11. J/C	
12. C/V	
13. C/s	
14. Elements connected so their currents must be equal	
15. Elements connected so their voltages must be equal	
16. The algebraic sum of voltages for a closed loop is zero	
17. The algebraic sum of the voltages for elements connected to a node is zero	
18. The sum of the currents entering a node equals the sum of those leaving	

Table T1.1(b). [Items in Table T1.1(b) may be used more than once or not at all.]

T1.2. Consider the circuit of Figure T1.2 with $I_s = 3\,\text{A}$, $R = 2\ \Omega$, and $V_s = 10\,\text{V}$. **a.** Determine the value of v_R. **b.** Determine the magnitude of the power for the voltage source and state whether the voltage source is absorbing energy or delivering it. **c.** How many nodes does this circuit have? **d.** Determine the magnitude of the power for the current source and state whether the current source is absorbing energy or delivering it.

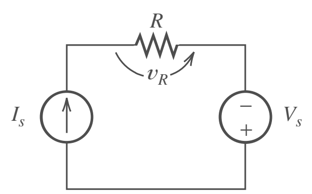


Figure T1.2

T1.3. The circuit of Figure T1.3 has $I_1 = 3\,\text{A}, I_2 = 1\,\text{A}, R_1 = 12\,\Omega$, and $R_2 = 6\,\Omega$. **a.** Determine the value of v_{ab}. **b.** Determine the power for each current source and state whether it is absorbing energy or delivering it. **c.** Compute the power absorbed by R_1 and by R_2.

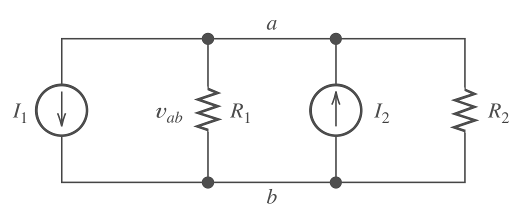


Figure T1.3

T1.4. The circuit shown in Figure T1.4 has $V_s = 12\,\text{V}$, $v_2 = 4\,\text{V}$, and $R_1 = 4\ \Omega$. **a.** Find the values of: **a.** v_1; **b.** i; **c.** R_2.

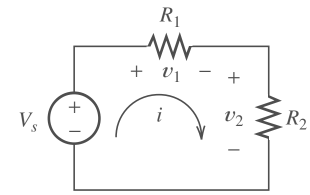


Figure T1.4

T1.5. We are given $V_s = 15\,\text{V}$, $R = 10\ \Omega$, and $a = 0.3\,\text{S}$ for the circuit of Figure T1.5. Find the value of the current i_{sc} flowing through the short circuit.

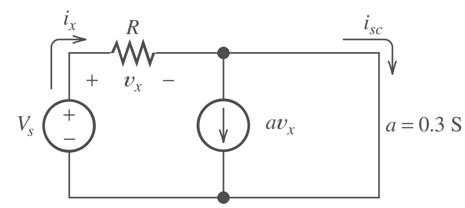


Figure T1.5

2.1 RESISTANCES IN SERIES AND PARALLEL

In this section, we show how to replace series or parallel combinations of resistances by equivalent resistances. Then, we demonstrate how to use this knowledge in solving circuits.

Series Resistances

Consider the series combination of three resistances shown in Figure 2.1(a). Recall that in a series circuit the elements are connected end to end and that the same current flows through all of the elements. By Ohm's law, we can write

$$v_1 = R_1 i \tag{2.1}$$

$$v_2 = R_2 i \tag{2.2}$$

and

$$v_3 = R_3 i \tag{2.3}$$

Using KVL, we can write

$$v = v_1 + v_2 + v_3 \tag{2.4}$$

Substituting Equations 2.1, 2.2, and 2.3 into Equation 2.4, we obtain

$$v = R_1 i + R_2 i + R_3 i \tag{2.5}$$

Factoring out the current i, we have

$$v = (R_1 + R_2 + R_3)i \tag{2.6}$$

Now, we define the equivalent resistance R_{eq} to be the sum of the resistances in series:

$$\boxed{R_{eq} = R_1 + R_2 + R_3} \tag{2.7}$$

Using this to substitute into Equation 2.6, we have

$$v = R_{eq} i \tag{2.8}$$

Thus, we conclude that the three resistances in series can be replaced by the equivalent resistance R_{eq} shown in Figure 2.1(b) with no change in the relationship between

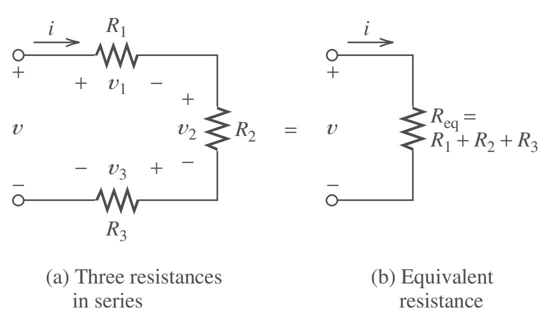


Figure 2.1 Series resistances can be combined into an equivalent resistance.

the voltage v and current i. If the three resistances are part of a larger circuit, replacing them by a single equivalent resistance would make no changes in the currents or voltages in other parts of the circuit.

A series combination of resistances has an equivalent resistance equal to the sum of the original resistances.

This analysis can be applied to any number of resistances. For example, two resistances in series can be replaced by a single resistance equal to the sum of the original two. To summarize, *a series combination of resistances has an equivalent resistance equal to the sum of the original resistances.*

Parallel Resistances

Figure 2.2(a) shows three resistances in parallel. In a parallel circuit, the voltage across each element is the same. Applying Ohm's law in Figure 2.2(a), we can write

$$i_1 = \frac{v}{R_1} \tag{2.9}$$

$$i_2 = \frac{v}{R_2} \tag{2.10}$$

$$i_3 = \frac{v}{R_3} \tag{2.11}$$

The top ends of the resistors in Figure 2.2(a) are connected to a single node. (Recall that all points in a circuit that are connected by conductors constitute a node.) Thus, we can apply KCL to the top node of the circuit and obtain

$$i = i_1 + i_2 + i_3 \tag{2.12}$$

Now using Equations 2.9, 2.10, and 2.11 to substitute into Equation 2.12, we have

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} \tag{2.13}$$

Factoring out the voltage, we obtain

$$i = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v \tag{2.14}$$

Now, we define the equivalent resistance as

$$\boxed{R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}} \tag{2.15}$$

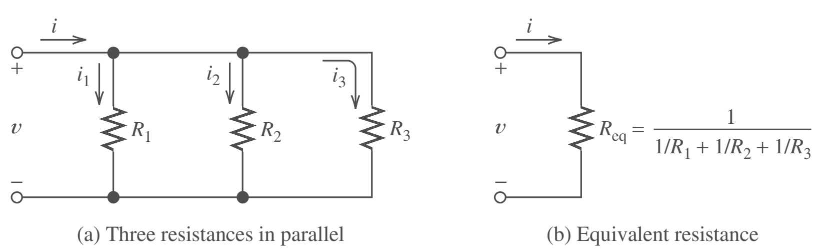


Figure 2.2 Parallel resistances can be combined into an equivalent resistance.

In terms of the equivalent resistance, Equation 2.14 becomes

$$i = \frac{1}{R_{eq}} v \tag{2.16}$$

Comparing Equations 2.14 and 2.16, we see that i and v are related in the same way by both equations provided that R_{eq} is given by Equation 2.15. Therefore, a parallel combination of resistances can be replaced by its equivalent resistance without changing the currents and voltages in other parts of the circuit. The equivalence is illustrated in Figure 2.2(b).

A parallel combination of resistances can be replaced by its equivalent resistance without changing the currents and voltages in other parts of the circuit.

This analysis can be applied to any number of resistances in parallel. For example, if four resistances are in parallel, the equivalent resistance is

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4} \tag{2.17}$$

Similarly, for two resistances, we have

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} \tag{2.18}$$

This can be put into the form

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \tag{2.19}$$

(Notice that Equation 2.19 applies only for two resistances. The product over the sum does not apply for more than two resistances.)

The product over the sum does not apply for more than two resistances.

Sometimes, resistive circuits can be reduced to a single equivalent resistance by repeatedly combining resistances that are in series or parallel.

Example 2.1 Combining Resistances in Series and Parallel

Find a single equivalent resistance for the network shown in Figure 2.3(a).

Solution First, we look for a combination of resistances that is in series or in parallel. In Figure 2.3(a), R_3 and R_4 are in series. (In fact, as it stands, no other two resistances in this network are either in series or in parallel.) Thus, our first step is to combine R_3 and R_4, replacing them by their equivalent resistance. Recall that for a series combination, the equivalent resistance is the sum of the resistances in series:

1. Find a series or parallel combination of resistances.
2. Combine them.
3. Repeat until the network is reduced to a single resistance (if possible).

$$R_{eq1} = R_3 + R_4 = 5 + 15 = 20\ \Omega$$

Figure 2.3(b) shows the network after replacing R_3 and R_4 by their equivalent resistance. Now we see that R_2 and R_{eq1} are in parallel. The equivalent resistance for this combination is

$$R_{eq2} = \frac{1}{1/R_{eq1} + 1/R_2} = \frac{1}{1/20 + 1/20} = 10\ \Omega$$

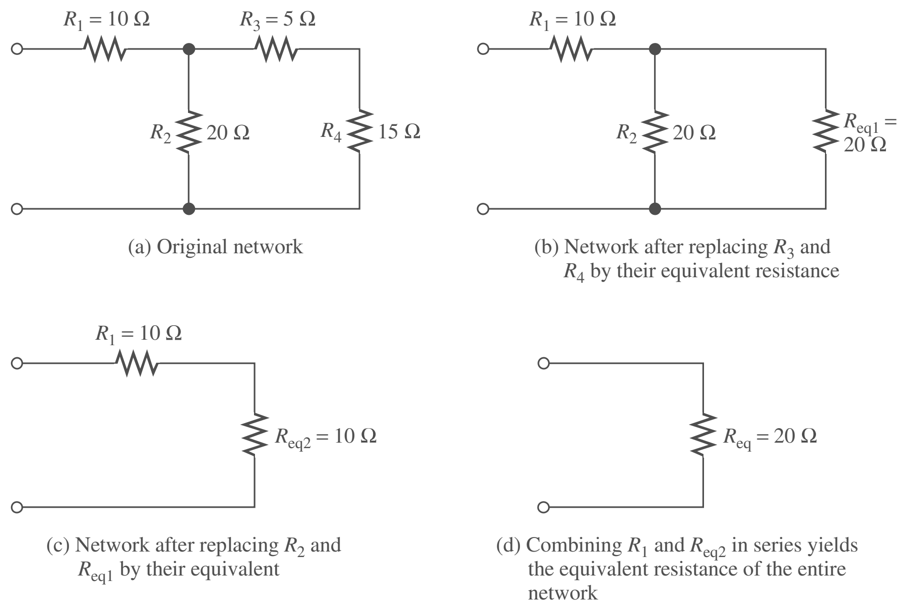


Figure 2.3 Resistive network for Example 2.1.

Making this replacement gives the equivalent network shown in Figure 2.3(c).

Finally, we see that R_1 and R_{eq2} are in series. Thus, the equivalent resistance for the entire network is

$$R_{eq} = R_1 + R_{eq2} = 10 + 10 = 20\ \Omega$$ ■

Exercise 2.1 Find the equivalent resistance for each of the networks shown in Figure 2.4. [*Hint for part (b):* R_3 and R_4 are in parallel.]
Answer **a.** 3 Ω; **b.** 5 Ω; **c.** 52.1 Ω; **d.** 1.5 kΩ. □

Conductances in Series and Parallel

Recall that conductance is the reciprocal of resistance. Using this fact to change resistances to conductances for a series combination of n elements, we readily obtain:

Combine conductances in series as you would resistances in parallel. Combine conductances in parallel as you would resistances in series.

$$G_{eq} = \frac{1}{1/G_1 + 1/G_2 + \cdots + 1/G_n} \tag{2.20}$$

Thus, we see that conductances in series combine as do resistances in parallel. For two conductances in series, we have:

$$G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$

For n conductances in parallel, we can show that

$$G_{eq} = G_1 + G_2 + \cdots + G_n \tag{2.21}$$

Conductances in parallel combine as do resistances in series.

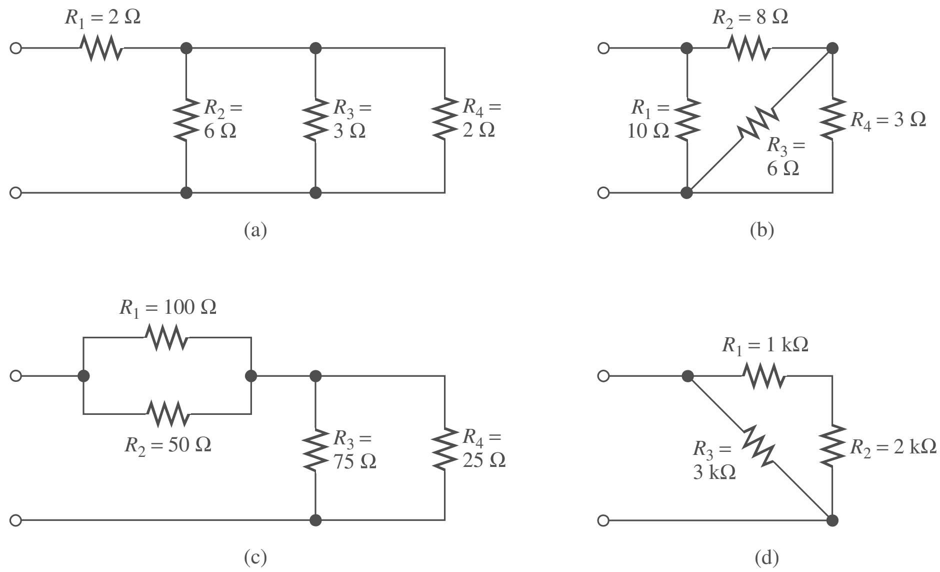


Figure 2.4 Resistive networks for Exercise 2.1.

Series versus Parallel Circuits

An element such as a toaster or light bulb that absorbs power is called a **load**. When we want to distribute power from a single voltage source to various loads, we usually place the loads in parallel. A switch in series with each load can break the flow of current to that load without affecting the voltage supplied to the other loads.

When we want to distribute power from a single voltage source to various loads, we usually place the loads in parallel.

Sometimes, to save wire, strings of Christmas lights consist of bulbs connected in series. The bulbs tend to fail or "burn out" by becoming open circuits. Then the entire string is dark and the defective bulb can be found only by trying each in turn. If several bulbs are burned out, it can be very tedious to locate the failed units. In a parallel connection, only the failed bulbs are dark.

2.2 NETWORK ANALYSIS BY USING SERIES AND PARALLEL EQUIVALENTS

An electrical **network** (or electrical circuit) consists of circuit elements, such as resistances, voltage sources, and current sources, connected together to form closed paths. **Network analysis** is the process of determining the current, voltage, and power for each element, given the circuit diagram and the element values. In this and the sections that follow, we study several useful techniques for network analysis.

An electrical network consists of circuit elements such as resistances, voltage sources, and current sources, connected together to form closed paths.

Sometimes, we can determine the currents and voltages for each element in a resistive circuit by repeatedly replacing series and parallel combinations of resistances by their equivalent resistances. Eventually, this may reduce the circuit sufficiently that the equivalent circuit can be solved easily. The information gained from the simplified circuit is transferred to the previous steps in the chain of equivalent circuits. In the end, we gain enough information about the original circuit to determine all the currents and voltages.

Circuit Analysis Using Series/Parallel Equivalents

Here are the steps in solving circuits using series/parallel equivalents:

1. **1.** Begin by locating a combination of resistances that are in series or parallel. Often the place to start is farthest from the source.
2. **2.** Redraw the circuit with the equivalent resistance for the combination found in step 1.
3. **3.** Repeat steps 1 and 2 until the circuit is reduced as far as possible. Often (but not always) we end up with a single source and a single resistance.
4. **4.** Solve for the currents and voltages in the final equivalent circuit. Then, transfer results back one step and solve for additional unknown currents and voltages. Again transfer the results back one step and solve. Repeat until all of the currents and voltages are known in the original circuit.
5. **5.** Check your results to make sure that KCL is satisfied at each node, KVL is satisfied for each loop, and the powers add to zero.

Some good advice for beginners: Don't try to combine steps. Be very methodical and do one step at a time. Take the time to redraw each equivalent carefully and label unknown currents and voltages consistently in the various circuits. The slow methodical approach will be faster and more accurate when you are learning. Walk now—later you will be able to run.

Example 2.2 Circuit Analysis Using Series/Parallel Equivalents

Find the current, voltage, and power for each element of the circuit shown in Figure 2.5(a).

Steps 1, 2, and 3.

Solution First, we combine resistances in series and parallel. For example, in the original circuit, R_2 and R_3 are in parallel. Replacing R_2 and R_3 by their parallel equivalent, we obtain the circuit shown in Figure 2.5(b). Next, we see that R_1 and R_{eq1} are in series. Replacing these resistances by their sum, we obtain the circuit shown in Figure 2.5(c).

After we have reduced a network to an equivalent resistance connected across the source, we solve the simplified network. Then, we transfer results back through the chain of equivalent circuits. We illustrate this process in Figure 2.6. (Figure 2.6 is identical to Figure 2.5, except for the currents and voltages shown in Figure 2.6.

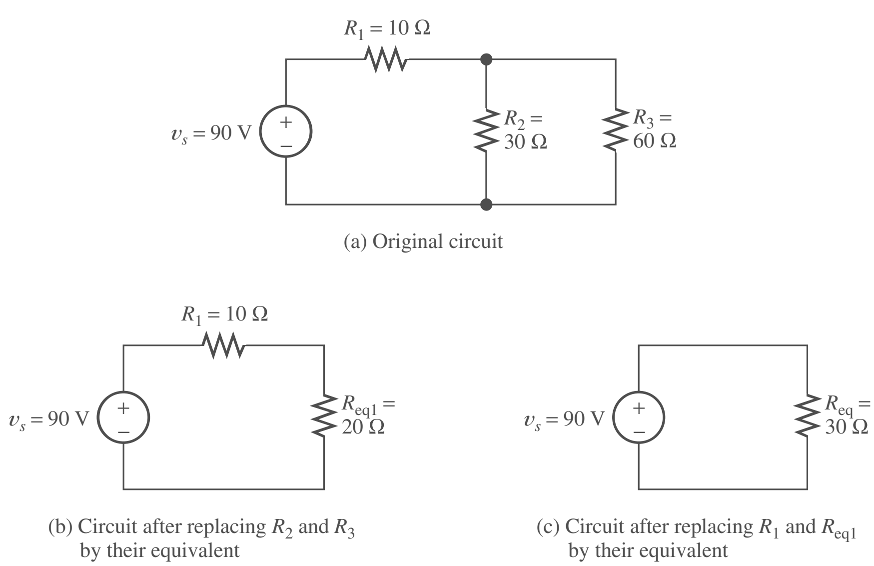


Figure 2.5 A circuit and its simplified versions. See Example 2.2.

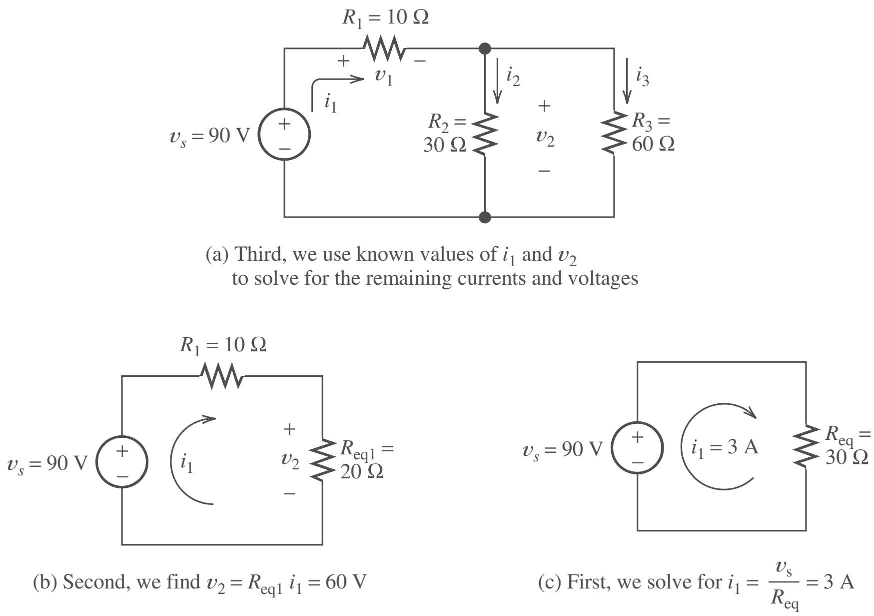


Figure 2.6 After reducing the circuit to a source and an equivalent resistance, we solve the simplified circuit. Then, we transfer results back to the original circuit. Notice that the logical flow in solving for currents and voltages starts from the simplified circuit in (c).

Usually, in solving a network by this technique, we first draw the chain of equivalent networks and then write results on the same drawings. However, this might be confusing in our first example.)

First, we solve the simplified network shown in Figure 2.6(c). Because R_{eq} is in parallel with the 90-V voltage source, the voltage across R_{eq} must be 90 V, with its positive polarity at the top end. Thus, the current flowing through R_{eq} is given by

Step 4.

$$i_1 = \frac{v_s}{R_{eq}} = \frac{90\ \text{V}}{30\ \Omega} = 3\ \text{A}$$

We know that this current flows downward (from plus to minus) through R_{eq}. Since v_s and R_{eq} are in series in Figure 2.6(c), the current must also flow upward through v_s. Thus, $i_1 = 3$ A flows clockwise around the circuit, as shown in Figure 2.6(c).

Because R_{eq} is the equivalent resistance seen by the source in all three parts of Figure 2.6, the current through v_s must be $i_1 = 3$ A, flowing upward in all three equivalent circuits. In Figure 2.6(b), we see that i_1 flows clockwise through v_s, R_1, and R_{eq1}. The voltage across R_{eq1} is given by

$$v_2 = R_{eq1} i_1 = 20\ \Omega \times 3\ \text{A} = 60\ \text{V}$$

Because R_{eq1} is the equivalent resistance for the parallel combination of R_2 and R_3, the voltage v_2 also appears across R_2 and R_3 in the original network.

At this point, we have found that the current through v_s and R_1 is $i_1 = 3$ A. Furthermore, the voltage across R_2 and R_3 is 60 V. This information is shown in

Figure 2.6(a). Now, we can compute the remaining values desired:

$$i_2 = \frac{v_2}{R_2} = \frac{60\ \text{V}}{30\ \Omega} = 2\ \text{A}$$

$$i_3 = \frac{v_2}{R_3} = \frac{60\ \text{V}}{60\ \Omega} = 1\ \text{A}$$

(As a check, we can use KCL to verify that $i_1 = i_2 + i_3$.)

Next, we can use Ohm's law to compute the value of v_1:

$$v_1 = R_1 i_1 = 10\ \Omega \times 3\ \text{A} = 30\ \text{V}$$

Step 5.

(As a check, we use KVL to verify that $v_s = v_1 + v_2$.)

Now, we compute the power for each element. For the voltage source, we have

$$p_s = -v_s i_1$$

We have included the minus sign because the references for v_s and i_1 are opposite to the passive configuration. Substituting values, we have

$$p_s = -(90\ \text{V}) \times 3\ \text{A} = -270\ \text{W}$$

Because the power for the source is negative, we know that the source is supplying energy to the other elements in the circuit.

The powers for the resistances are

$$p_1 = R_1 i_1^2 = 10\ \Omega \times (3\ \text{A})^2 = 90\ \text{W}$$

$$p_2 = \frac{v_2^2}{R_2} = \frac{(60\ \text{V})^2}{30\ \Omega} = 120\ \text{W}$$

$$p_3 = \frac{v_2^2}{R_3} = \frac{(60\ \text{V})^2}{60\ \Omega} = 60\ \text{W}$$

(As a check, we verify that $p_s + p_1 + p_2 + p_3 = 0$, showing that power is conserved.) ■

Power Control by Using Heating Elements in Series or Parallel

Resistances are commonly used as heating elements for the reaction chamber of chemical processes. For example, the catalytic converter of an automobile is not effective until its operating temperature is achieved. Thus, during engine warm-up, large amounts of pollutants are emitted. Automotive engineers have proposed and studied the use of electrical heating elements to heat the converter more quickly, thereby reducing pollution. By using several heating elements that can be operated individually, in series, or in parallel, several power levels can be achieved. This is useful in controlling the temperature of a chemical process.

Exercise 2.2 Find the currents labeled in Figure 2.7 by combining resistances in series and parallel.

Answer **a.** $i_1 = 1.04$ A, $i_2 = 0.480$ A, $i_3 = 0.320$ A, $i_4 = 0.240$ A; **b.** $i_1 = 1$ A, $i_2 = 1$ A; **c.** $i_1 = 1$ A, $i_2 = 0.5$ A, $i_3 = 0.5$ A. □

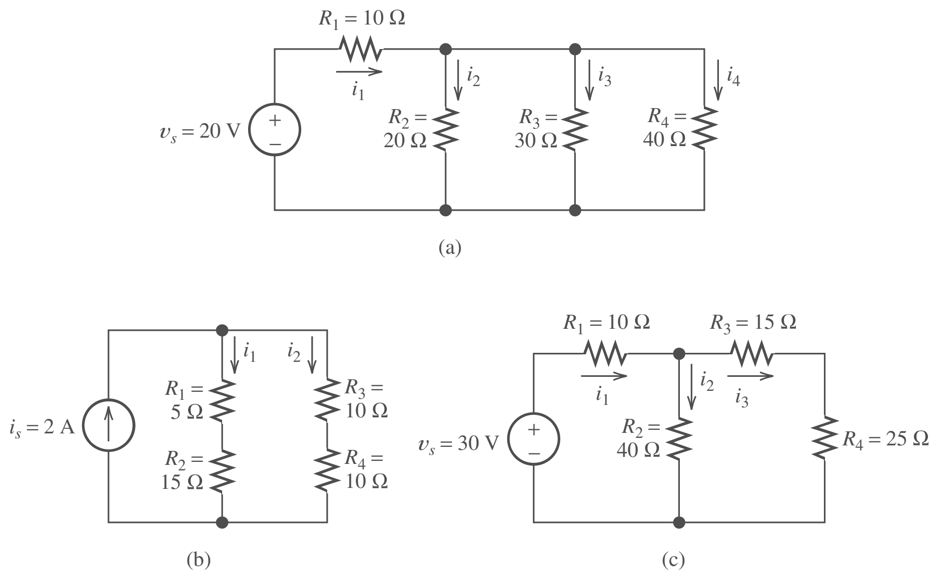


Figure 2.7 Circuits for Exercise 2.2.

2.3 VOLTAGE-DIVIDER AND CURRENT-DIVIDER CIRCUITS

Voltage Division

When a voltage is applied to a series combination of resistances, a fraction of the voltage appears across each of the resistances. Consider the circuit shown in Figure 2.8. The equivalent resistance seen by the voltage source is

$$R_{\text{eq}} = R_1 + R_2 + R_3 \tag{2.22}$$

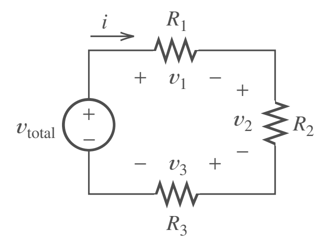


Figure 2.8 Circuit used to derive the voltage-division principle.

The current is the total voltage divided by the equivalent resistance:

$$i = \frac{v_{\text{total}}}{R_{\text{eq}}} = \frac{v_{\text{total}}}{R_1 + R_2 + R_3} \tag{2.23}$$

Furthermore, the voltage across R_1 is

$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}} \tag{2.24}$$

Similarly, we have

$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}} \tag{2.25}$$

and

$$v_3 = R_3 i = \frac{R_3}{R_1 + R_2 + R_3} v_{\text{total}} \tag{2.26}$$

Of the total voltage, the fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.

We can summarize these results by the statement: *Of the total voltage, the fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.* This is known as the **voltage-division principle**.

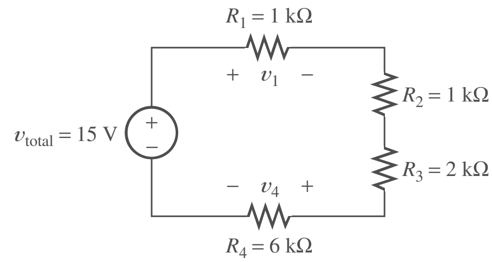


Figure 2.9 Circuit for Example 2.3.

We have derived the voltage-division principle for three resistances in series, but it applies for any number of resistances as long as they are connected in series.

Example 2.3 Application of the Voltage-Division Principle

Find the voltages v_1 and v_4 in Figure 2.9.

Solution Using the voltage-division principle, we find that v_1 is the total voltage times the ratio of R_1 to the total resistance:

$$\begin{aligned} v_1 &= \frac{R_1}{R_1 + R_2 + R_3 + R_4} v_{\text{total}} \\ &= \frac{1000}{1000 + 1000 + 2000 + 6000} \times 15 = 1.5 \text{ V} \end{aligned}$$

Similarly,

$$\begin{aligned} v_4 &= \frac{R_4}{R_1 + R_2 + R_3 + R_4} v_{\text{total}} \\ &= \frac{6000}{1000 + 1000 + 2000 + 6000} \times 15 = 9 \text{ V} \end{aligned}$$

Notice that the largest voltage appears across the largest resistance in a series circuit. ■

Current Division

The total current flowing into a parallel combination of resistances divides, and a fraction of the total current flows through each resistance. Consider the circuit shown in Figure 2.10. The equivalent resistance is given by

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} \tag{2.27}$$

The voltage across the resistances is given by

$$v = R_{\text{eq}} i_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} i_{\text{total}} \tag{2.28}$$

Now, we can find the current in each resistance:

$$\boxed{i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{\text{total}}} \tag{2.29}$$

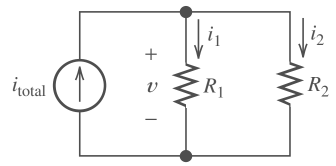


Figure 2.10 Circuit used to derive the current-division principle.

and

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{\text{total}} \tag{2.30}$$

We can summarize these results by stating the **current-division principle**: *For two resistances in parallel, the fraction of the total current flowing in a resistance is the ratio of the other resistance to the sum of the two resistances.* Notice that this principle applies only for two resistances. If we have more than two resistances in parallel, we should combine resistances so we only have two before applying the current-division principle.

For two resistances in parallel, the fraction of the total current flowing in a resistance is the ratio of the other resistance to the sum of the two resistances.

An alternative approach is to work with conductances. For n conductances in parallel, it can be shown that

Current division using conductances uses a formula with the same form as the formula for voltage division using resistances.

$$i_1 = \frac{G_1}{G_1 + G_2 + \cdots + G_n} i_{\text{total}}$$

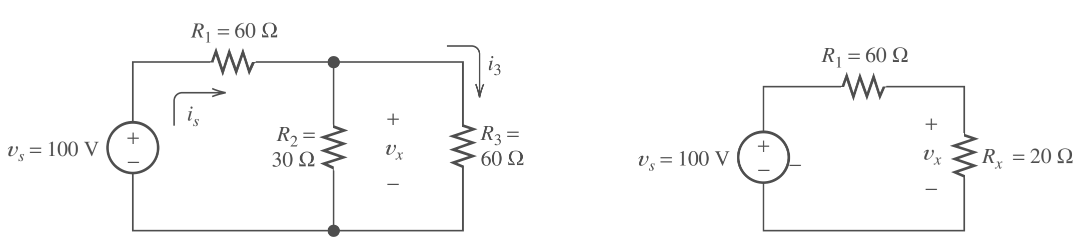
$$i_2 = \frac{G_2}{G_1 + G_2 + \cdots + G_n} i_{\text{total}}$$

and so forth. In other words, current division using conductances uses a formula with the same form as the formula for voltage division using resistances.

Example 2.4 Applying the Current- and Voltage-Division Principles

Use the voltage-division principle to find the voltage v_x in Figure 2.11(a). Then find the source current i_s and use the current-division principle to compute the current i_3.

Solution The voltage-division principle applies only for resistances in series. Therefore, we first must combine R_2 and R_3. The equivalent resistance for the parallel



(a) Original circuit

(b) Equivalent circuit obtained by combining R_2 and R_3

Figure 2.11 Circuit for Example 2.4.

combination of R_2 and R_3 is

$$R_x = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\ \Omega$$

The equivalent network is shown in Figure 2.11(b).

Now, we can apply the voltage-division principle to find v_x. The voltage v_x is equal to the total voltage times R_x divided by the total series resistance:

$$v_x = \frac{R_x}{R_1 + R_x} v_s = \frac{20}{60 + 20} \times 100 = 25\ \text{V}$$

The source current i_s is given by

$$i_s = \frac{v_s}{R_1 + R_x} = \frac{100}{60 + 20} = 1.25\ \text{A}$$

Now, we can use the current-division principle to find i_3. The fraction of the source current i_s that flows through R_3 is $R_2/(R_2 + R_3)$. Thus, we have

$$i_3 = \frac{R_2}{R_2 + R_3} i_s = \frac{30}{30 + 60} \times 1.25 = 0.417\ \text{A}$$

As a check, we can also compute i_3 another way:

$$i_3 = \frac{v_x}{R_3} = \frac{25}{60} = 0.417\ \text{A}$$

■

Example 2.5 Application of the Current-Division Principle

The current-division principle applies for *two* resistances in parallel. Therefore, our first step is to combine R_2 and R_3.

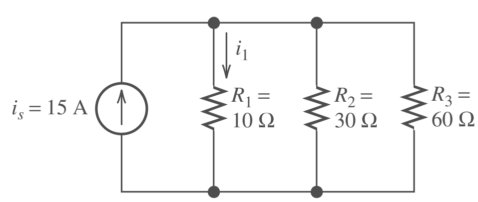
Use the current-division principle to find the current i_1 in Figure 2.12(a).

Solution The current-division principle applies for two resistances in parallel. Therefore, our first step is to combine R_2 and R_3:

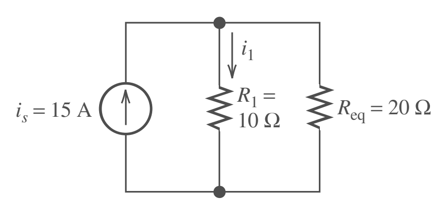
$$R_{\text{eq}} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\ \Omega$$

The resulting equivalent circuit is shown in Figure 2.12(b). Applying the current-division principle, we have

$$i_1 = \frac{R_{\text{eq}}}{R_1 + R_{\text{eq}}} i_s = \frac{20}{10 + 20} 15 = 10\ \text{A}$$



(a) Original circuit



(b) Circuit after combining R_2 and R_3

Figure 2.12 Circuit for Example 2.5.

Reworking the calculations using conductances, we have

$$G_1 = \frac{1}{R_1} = 100\,\text{mS}, \quad G_2 = \frac{1}{R_2} = 33.33\,\text{mS}, \quad \text{and} \quad G_3 = \frac{1}{R_3} = 16.67\,\text{mS}$$

Then, we compute the current

$$i_1 = \frac{G_1}{G_1 + G_2 + G_3} i_s = \frac{100}{100 + 33.33 + 16.67} 15 = 10\,\text{A}$$

which is the same value that we obtained working with resistances. ■

Position Transducers Based on the Voltage-Division Principle

Transducers are used to produce a voltage (or sometimes a current) that is proportional to a physical quantity of interest, such as distance, pressure, or temperature. For example, Figure 2.13 shows how a voltage that is proportional to the rudder angle of a boat or aircraft can be obtained. As the rudder turns, a sliding contact moves along a resistance such that R_2 is proportional to the rudder angle θ. The total resistance $R_1 + R_2$ is fixed. Thus, the output voltage is

$$v_o = v_s \frac{R_2}{R_1 + R_2} = K\theta$$

where K is a constant of proportionality that depends on the source voltage v_s and the construction details of the transducer. Many examples of transducers such as this are employed in all areas of science and engineering.

Exercise 2.3 Use the voltage-division principle to find the voltages labeled in Figure 2.14.

Answer **a.** $v_1 = 10$ V, $v_2 = 20$ V, $v_3 = 30$ V, $v_4 = 60$ V; **b.** $v_1 = 6.05$ V, $v_2 = 5.88$ V, $v_4 = 8.07$ V. □

Exercise 2.4 Use the current-division principle to find the currents labeled in Figure 2.15.

Answer **a.** $i_1 = 1$ A, $i_3 = 2$ A; **b.** $i_1 = i_2 = i_3 = 1$ A. □

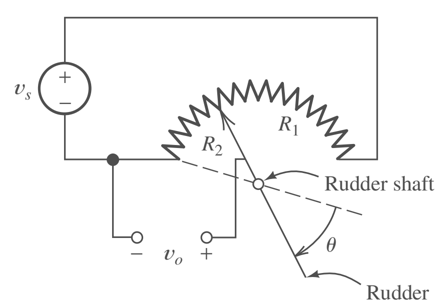


Figure 2.13 The voltage-division principle forms the basis for some position sensors. This figure shows a transducer that produces an output voltage v_o proportional to the rudder angle θ.

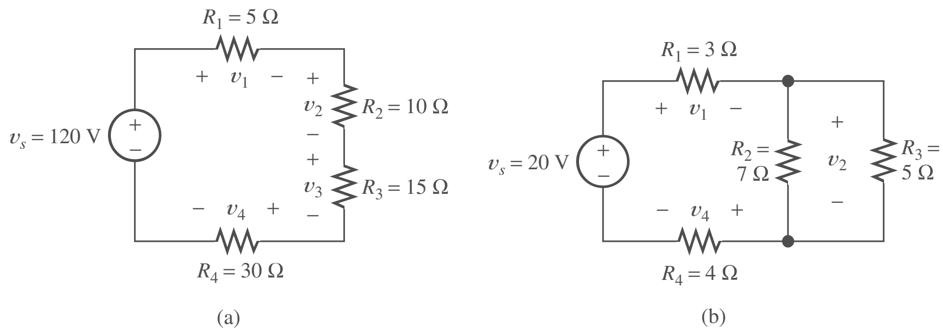


Figure 2.14 Circuits for Exercise 2.3.

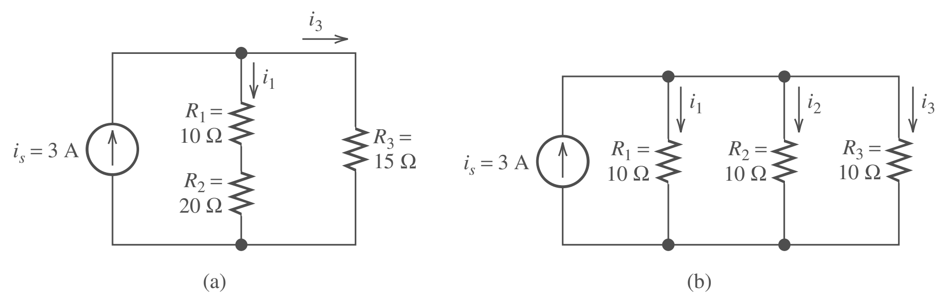


Figure 2.15 Circuits for Exercise 2.4.

~~2.4 NODE-VOLTAGE ANALYSIS~~

Although they are very important concepts, series/parallel equivalents and the current/voltage division principles are not sufficient to solve all circuits.

~~The network analysis methods that we have studied so far are useful, but they do not apply to all networks. For example, consider the circuit shown in Figure 2.16. We cannot solve this circuit by combining resistances in series and parallel because no series or parallel combination of resistances exists in the circuit. Furthermore, the voltage division and current division principles cannot be applied to this circuit. In this section, we learn **node-voltage analysis**, which is a general technique that can be applied to any circuit.~~

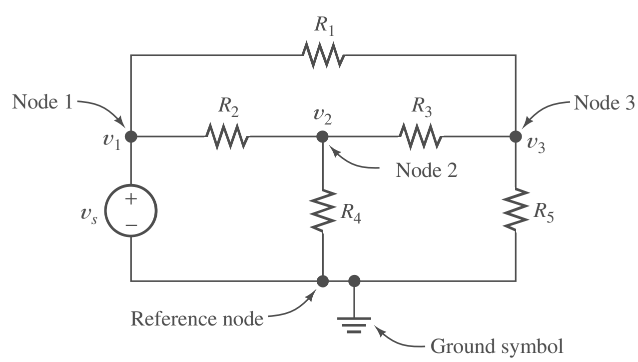


Figure 2.16 The first step in node analysis is to select a reference node and label the voltages at each of the other nodes.

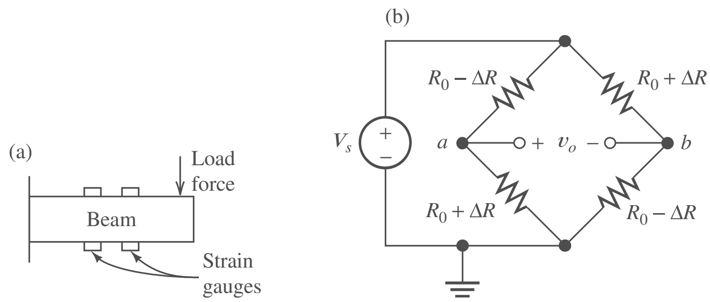


Figure 2.65 Strain measurements using the Wheatstone bridge.

~~The resistances labeled $R_0 + \Delta R$ are the gauges on the top of the beam and are being stretched, and those labeled $R_0 - \Delta R$ are those on the bottom and are being compressed. Before the load is applied, all four resistances have a value of R_0, the Wheatstone bridge is balanced, and the output voltage v_o is zero.~~

~~It can be shown that the output voltage v_o from the bridge is given by~~

$$\cancel{v_o = V_s \frac{\Delta R}{R_0} = V_s G \frac{\Delta L}{L}} \qquad \cancel{(2.93)}$$

~~Thus, the output voltage is proportional to the strain of the beam.~~

~~In principle, the resistance of one of the gauges could be measured and the strain determined from the resistance measurements. However, the changes in resistance are very small, and the measurements would need to be very precise. Furthermore, gauge resistance changes slightly with temperature. In the bridge arrangement with the gauges attached to the beam, the temperature changes tend to track very closely and have very little effect on v_o.~~

~~Usually, v_o is amplified by an instrumentation-quality differential amplifier such as that discussed in Section 14.8 which starts on page 685. The amplified voltage can be converted to digital form and input to a computer or relayed wirelessly to a remote location for monitoring.~~

Summary

1. Series resistances have an equivalent resistance equal to their sum. For n resistances in series, we have

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

2. Parallel resistances have an equivalent resistance equal to the reciprocal of the sum of their reciprocals. For n resistances in parallel, we get

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_n}$$

3. Some resistive networks can be solved by repeatedly combining resistances in series or parallel. The simplified network is solved, and results are transferred back through the chain of equivalent circuits. Eventually, the currents and voltages of interest in the original circuit are found.

4. The voltage-division principle applies when a voltage is applied to several resistances in series. A fraction of the total voltage appears across each resistance. The fraction that appears across a given resistance is the ratio of the given resistance to the total series resistance.

5. The current-division principle applies when current flows through two resistances in parallel. A fraction of the total current flows through each resistance. The fraction of the total current flowing through R_1 is equal to $R_2/(R_1 + R_2)$.
6. The node-voltage method can be used to solve for the voltages in any resistive network. A step-by-step summary of the method is given starting on page 76.
7. A step-by-step procedure to write the node-voltage equations directly in matrix form for circuits consisting of resistances and independent current sources appears on page 66.
8. The mesh-current method can be used to solve for the currents in any planar resistive network. A step-by-step summary of the method is given on page 88.
9. A step-by-step procedure to write the mesh-current equations directly in matrix form for circuits consisting of resistances and independent voltage sources appears on page 84. For this method to apply, all of the mesh currents must flow in the clockwise direction.
10. A two-terminal network of resistances and sources has a Thévenin equivalent that consists of a voltage source in series with a resistance. The Thévenin voltage is equal to the open-circuit voltage of the original network. The Thévenin resistance is the open-circuit voltage divided by the short-circuit current of the original network. Sometimes, the Thévenin resistance can be found by zeroing the independent sources in the original network and combining resistances in series and parallel. When independent voltage sources are zeroed, they are replaced by short circuits. Independent current sources are replaced by open circuits. Dependent sources must not be zeroed.
11. A two-terminal network of resistances and sources has a Norton equivalent that consists of a current source in parallel with a resistance. The Norton current is equal to the short-circuit current of the original network. The Norton resistance is the same as the Thévenin resistance. A step-by-step procedure for determining Thévenin and Norton equivalent circuits is given on page 95.
12. Sometimes source transformations (i.e., replacing a Thévenin equivalent with a Norton equivalent or vice versa) are useful in solving networks.
13. For maximum power from a two-terminal network, the load resistance should equal the Thévenin resistance.
14. The superposition principle states that the total response in a resistive circuit is the sum of the responses to each of the independent sources acting individually. The superposition principle does not apply to any circuit that has element(s) described by nonlinear equation(s).
15. The Wheatstone bridge is a circuit used to measure unknown resistances. The circuit consists of a voltage source, a detector, three precision calibrated resistors, of which two are adjustable, and the unknown resistance. The resistors are adjusted until the bridge is balanced, and then the unknown resistance is given in terms of the three known resistances.

Here's the answer to the trick question on page 97: Suppose that we open circuit the terminals. Then, no current flows through the Thévenin equivalent, but a current I_n circulates in the Norton equivalent. Thus, the box containing the Norton equivalent will become warm because of power dissipation in the resistance. The point of this question is that the circuits are equivalent in terms of their terminal voltage and current, not in terms of their internal behavior.

***Note:* You can check the answers to many of the problems in this chapter by using a computer-aided circuit-analysis program such as Multisim from National Instruments or OrCAD Capture from Cadence Inc.**

Problems

Section 2.1: Resistances in Series and Parallel

***P2.1.** Reduce each of the networks shown in Figure P2.1 to a single equivalent resistance by combining resistances in series and parallel.

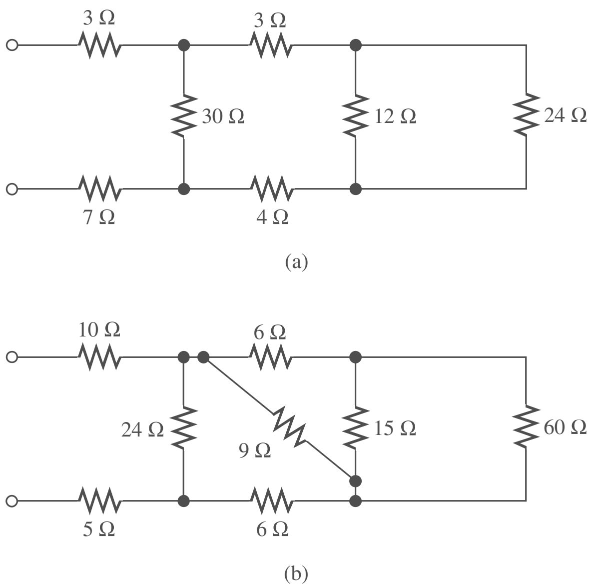


Figure P2.1

***P2.2.** A 4-Ω resistance is in series with the parallel combination of a 20-Ω resistance and an unknown resistance R_x. The equivalent resistance for the network is 8 Ω. Determine the value of R_x.

***P2.3.** Find the equivalent resistance between terminals *a* and *b* in Figure P2.3.

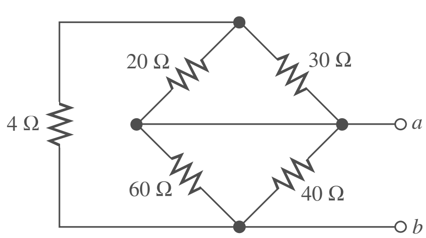


Figure P2.3

***P2.4.** Find the equivalent resistance looking into terminals *a* and *b* in Figure P2.4.

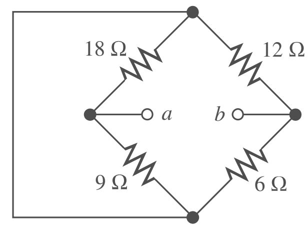


Figure P2.4

***P2.5.** Suppose that we need a resistance of 1.5 kΩ and you have a box of 1-kΩ resistors. Devise a network of 1-kΩ resistors so the equivalent resistance is 1.5 kΩ. Repeat for an equivalent resistance of 2.2 kΩ.

P2.6. **a.** Determine the resistance between terminals *c* and *d* for the network shown in Figure P2.6. **b.** Repeat after removing the short circuit between terminals *a* and *b*.

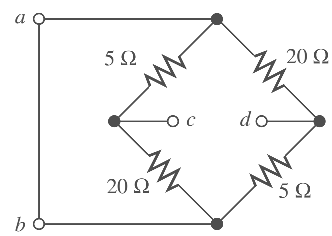


Figure P2.6

P2.7. Two resistances R_1 and R_2 are connected in series. We know that $R_1 = 60\ \Omega$ and that the voltage across R_2 is three times the value of the voltage across R_1. Determine the value of R_2.

P2.8. Find the equivalent resistance between terminals *a* and *b* for each of the networks shown in Figure P2.8.

P2.9. What resistance in parallel with 70 Ω results in an equivalent resistance of 20 Ω?

P2.10. Two resistances having values of $2R$ and $3R$ are in parallel. R and the equivalent resistance are both integers. What are the possible values for R?

P2.11. A network connected between terminals *a* and *b* consists of two parallel combinations that are in series. The first parallel

* Denotes that answers are contained in the Student Solutions files. See Appendix E for more information about accessing the Student Solutions.

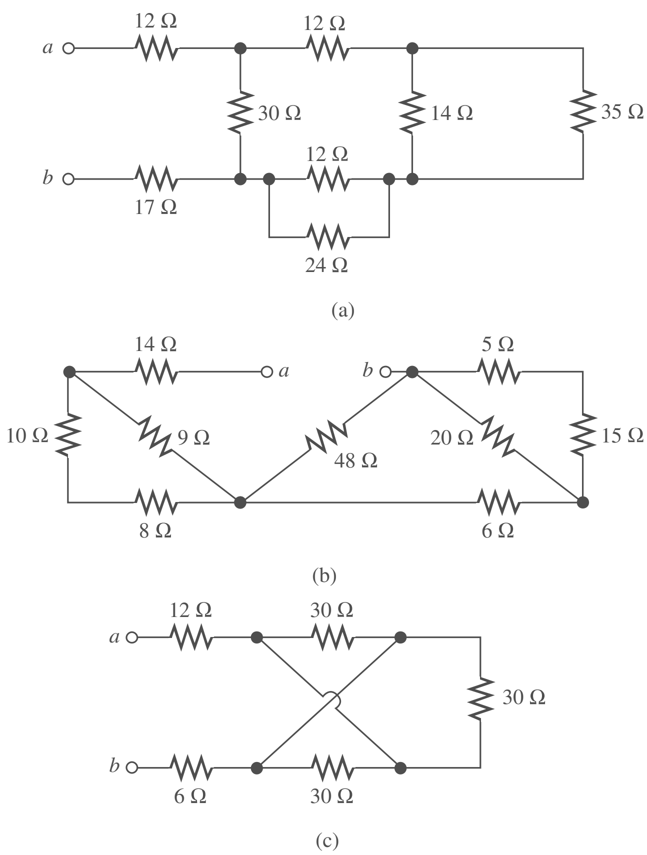


Figure P2.8

combination is composed of a 10-Ω resistor and a 15-Ω resistor. The second parallel combination is composed of a 14-Ω resistor and a 35-Ω resistor. Draw the network and determine its equivalent resistance.

P2.12. The heating element of an electric cook top has two resistive elements, $R_1 = 40\ \Omega$ and $R_2 = 100\ \Omega$, which can be operated separately, in series, or in parallel from voltages of either 120 V or 240 V. For the lowest power, R_1 is in series with R_2, and the combination is operated from 120 V. What is the lowest power? For the highest power, how should the elements be operated? What power results? List three more modes of operation and the resulting power for each.

P2.13. Find the equivalent resistance for the infinite network shown in Figure P2.13(a). Because of its form, this network is called a semi-infinite ladder. (*Hint:* If another section is added to the ladder as shown in Figure P2.13(b), the equivalent resistance is the same. Thus, working from Figure P2.13(b), we can write an expression for R_{eq} in terms of R_{eq}. Then, we can solve for R_{eq}.)

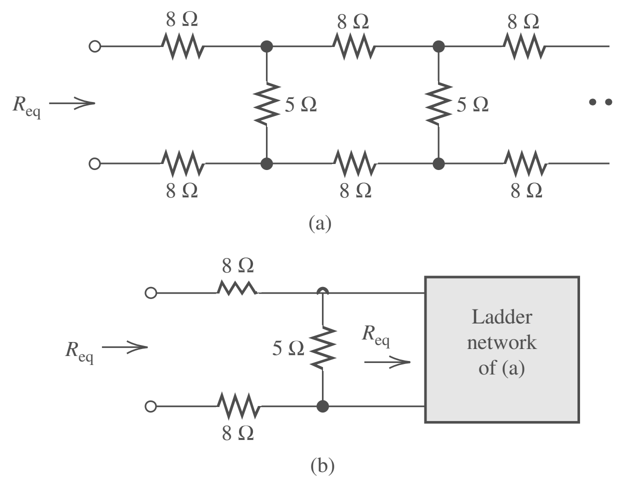


Figure P2.13

P2.14. If we connect n 1000-Ω resistances in parallel, what value is the equivalent resistance?

P2.15. We are designing an electric space heater to operate from 120 V. Two heating elements with resistances R_1 and R_2 are to be used that can be operated in parallel, separately, or in series. The highest power is to be 960 watts, and the lowest power is to be 180 watts. What values are needed for R_1 and R_2? What intermediate power settings are available?

P2.16. The equivalent resistance between terminals a and b in Figure P2.16 is $R_{ab} = 40\ \Omega$. Determine the value of R.

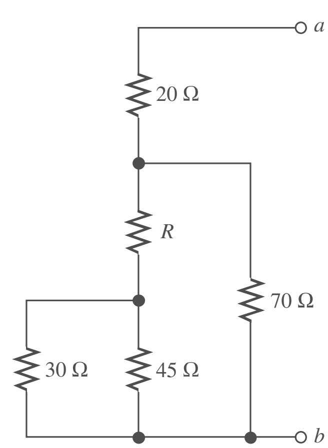


Figure P2.16

P2.17. Sometimes, we can use symmetry considerations to find the resistance of a circuit that cannot be reduced by series or parallel combinations. A classic problem of this type is illustrated in Figure P2.17. Twelve 1-Ω resistors are arranged on the edges of a cube, and terminals a and b are connected to diagonally opposite corners of the cube. The problem is to find the resistance between the terminals. Approach the problem this way: Assume that 1 A of current enters terminal a and exits through terminal b. Then, the voltage between terminals a and b is equal to the unknown resistance. By symmetry considerations, we can find the current in each resistor. Then, using KVL, we can find the voltage between a and b.

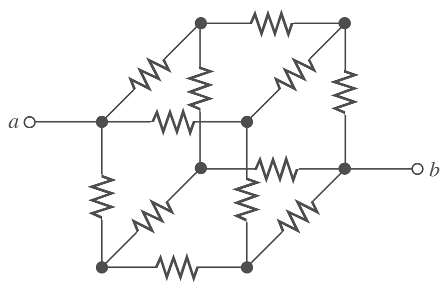


Figure P2.17

P2.18. **a.** Three conductances G_1, G_2, and G_3 are in series. Write an expression for the equivalent conductance $G_{eq} = 1/R_{eq}$ in terms of G_1, G_2, and G_3. **b.** Repeat part (a) with the conductances in parallel.

P2.19. Most sources of electrical power behave as (approximately) ideal voltage sources. In this case, if we have several loads that we want to operate independently, we place the loads in parallel with a switch in series with each load. Thereupon, we can switch each load on or off without affecting the power delivered to the other loads.

How would we connect the loads and switches if the source were an ideal independent current source? Draw the diagram of the current source and three loads with on–off switches such that each load can be switched on or off without affecting the power supplied to the other loads. To turn a load off, should the corresponding switch be opened or closed? Explain.

P2.20. Often, we encounter delta-connected loads, such as that illustrated in Figure P2.20, in three-phase power distribution systems (which are treated in Section 5.7). If we only have access to the three terminals, a method for determining the resistances is to repeatedly short two terminals together and measure the resistance between the shorted terminals and the third terminal. Then, the resistances can be calculated from the three measurements. Suppose that the measurements are $R_{as} = 24\ \Omega$, $R_{bs} = 30\ \Omega$, and $R_{cs} = 40\ \Omega$, where R_{as} is the resistance between terminal a and the short between b and c, etc. Determine the values of R_a, R_b, and R_c. (*Hint*: You may find the equations easier to deal with if you work in terms of conductances rather than resistances. Once the conductances are known, you can easily invert their values to find the resistances.)

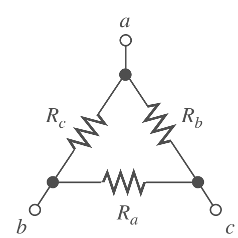


Figure P2.20

P2.21. The resistance between terminals a and b with c open circuited for the network shown in Figure P2.21 is $R_{ab} = 30\ \Omega$. Similarly, the resistance between terminals b and c with a open is $R_{bc} = 50\ \Omega$, and between c and a with b open, the resistance is $R_{ca} = 40\ \Omega$. Now, suppose that a short circuit is connected from terminal b to terminal c, and determine the resistance between terminal a and the shorted terminals b–c.

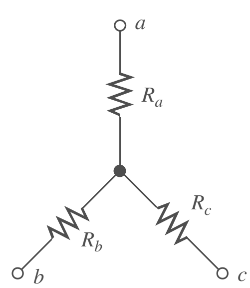


Figure P2.21

Section 2.2: Network Analysis by Using Series and Parallel Equivalents

P2.22. From memory, list the steps in solving a circuit by network reduction (series/parallel combinations). Does this method always provide the solution? Explain.

***P2.23.** Find the values of i_1 and i_2 in Figure P2.23.

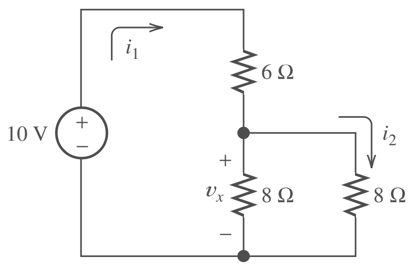


Figure P2.23

***P2.24.** Find the values of i_1 and i_2 in Figure P2.24. Find the power for each element in the circuit, and state whether each is absorbing or delivering energy. Verify that the total power absorbed equals the total power delivered.

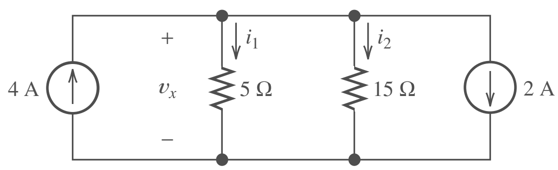


Figure P2.24

***P2.25.** Find the values of v and i in Figure P2.25.

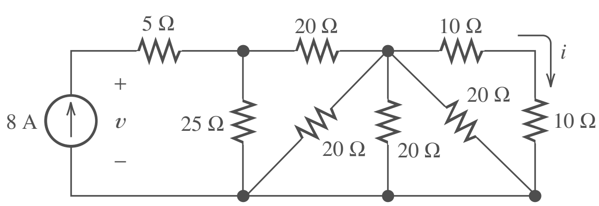


Figure P2.25

***P2.26.** Find the voltages v_1 and v_2 for the circuit shown in Figure P2.26 by combining resistances in series and parallel.

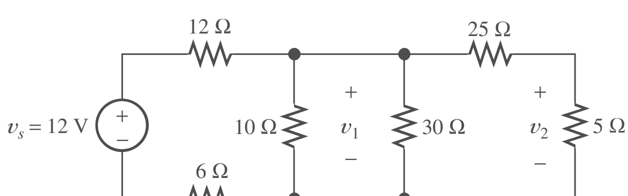


Figure P2.26

P2.27. Consider the circuit shown in Figure P2.26. Suppose that the value of v_s is adjusted until $v_1 = 10$ V. Determine the new values for v_2 and v_s. (*Hint*: Start at the location of v_2 and compute currents and voltages, moving to the right and left.)

P2.28. Find the values of v_s, v_1, and i_2 in Figure P2.28.

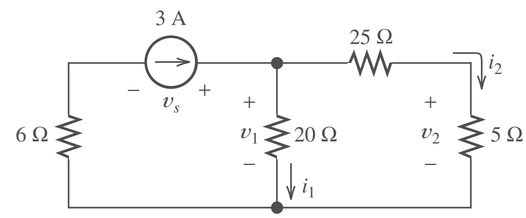


Figure P2.28

P2.29. Determine the values of i_1 and i_2 in Figure P2.29.

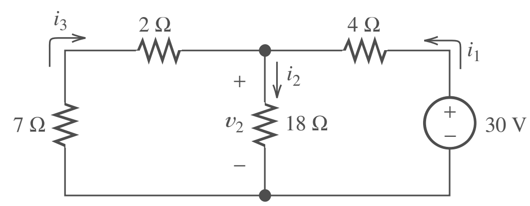


Figure P2.29

P2.30. Find the voltage v and the currents i_1 and i_2 for the circuit shown in Figure P2.30.

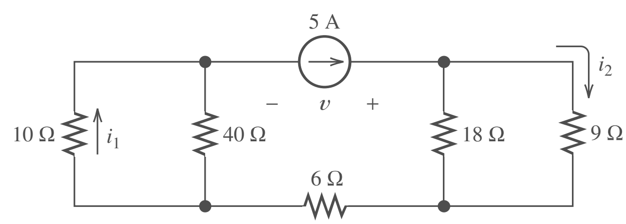


Figure P2.30

P2.31. Solve for the values of i_1, i_2, and the powers for the sources in Figure P2.31. Is the current source absorbing energy or delivering it? Is the voltage source absorbing energy or delivering it? Check to see that power is conserved in the circuit.

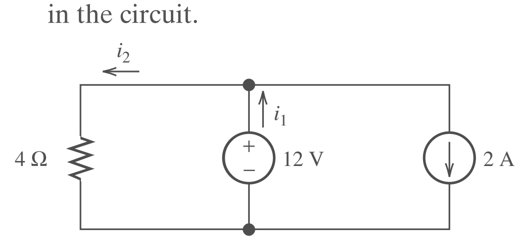


Figure P2.31

P2.32. Consider the circuit shown in Figure P2.32. With the switch open, we have $v_2 = 10$ V. On the other hand, with the switch closed, we have $v_2 = 8$ V. Determine the values of R_2 and R_L.

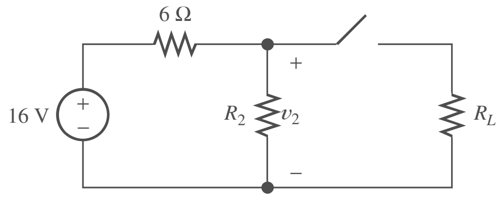


Figure P2.32

P2.33. Consider the circuit shown in Figure P2.33. Find the values of v_1, v_2, v_{ab}, v_{bc}, and v_{ca}.

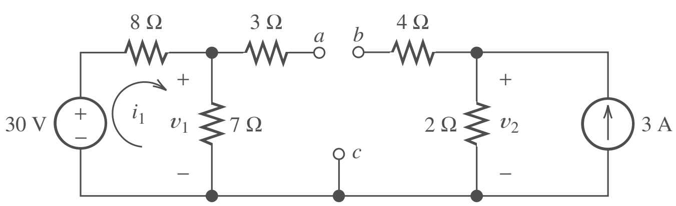


Figure P2.33

P2.34. We know that the 10-V source in Figure P2.34 is delivering 4 W of power. All four resistors have the same value R. Find the value of R.

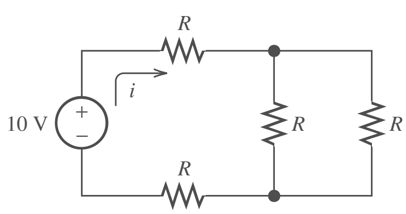


Figure P2.34

***P2.35.** Find the values of i_1 and i_2 in Figure P2.35.

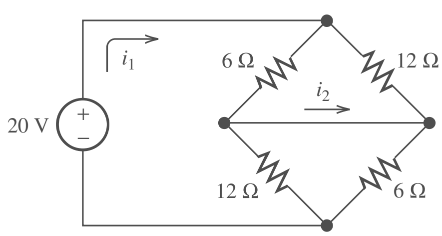


Figure P2.35

Section 2.3: Voltage-Divider and Current-Divider Circuits

***P2.36.** Use the voltage-division principle to calculate v_1, v_2, and v_3 in Figure P2.36.

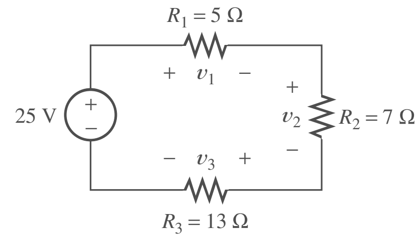


Figure P2.36

***P2.37.** Use the current-division principle to calculate i_1 and i_2 in Figure P2.37.

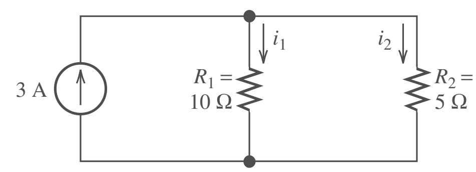


Figure P2.37

***P2.38.** Use the voltage-division principle to calculate v in Figure P2.38.

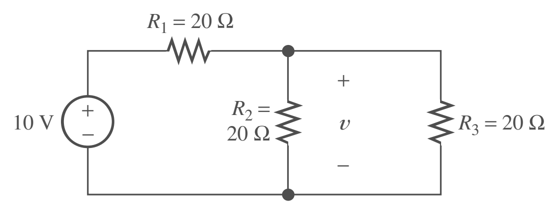


Figure P2.38

P2.39. Use the current-division principle to calculate the value of i_3 in Figure P2.39.

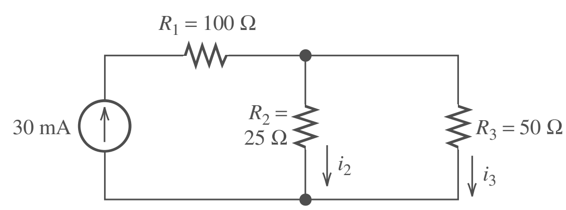


Figure P2.39

P2.40. We want to design a voltage-divider circuit to provide an output voltage $v_o = 5$ V from a 9-V battery as shown in Figure P2.40. The current taken from the 9-V source with no load connected is to be 10 mA. **a.** Find the values of $R_1$and R_2. **b.** Now suppose that a load resistance of 1 kΩ is connected across the output terminals (i.e., in parallel with R_2). Find the loaded value of v_o. **c.** How could we change the design so the voltage remains closer to 5 V when the load is connected? How would this affect the life of the battery?

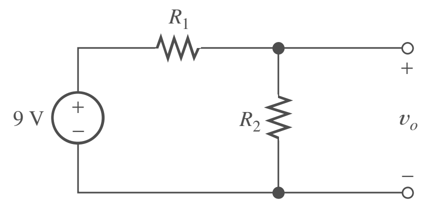


Figure P2.40

P2.41. A series-connected circuit has a 240-V voltage source, a 10-Ω resistance, a 5-Ω resistance, and an unknown resistance R_x. The voltage across the 5-Ω resistance is 30 V. Determine the value of the unknown resistance.

P2.42. A parallel circuit (i.e., all elements are in parallel with one another) has a 60-Ω resistance, a 20-Ω resistance, an unknown resistance R_x, and 30 mA current source. The current through the unknown resistance is 10 mA. Determine the value of R_x.

P2.43. The circuit of Figure P2.43 is similar to networks used in some digital-to-analog converters. For this problem, assume that the circuit continues indefinitely to the right. Find the values of i_1, i_2, i_3, and i_4. How is i_{n+2} related to i_n? What is the value of i_{18}? (*Hint:* See Problem P2.13 for hints on how to handle semi-infinite networks.)

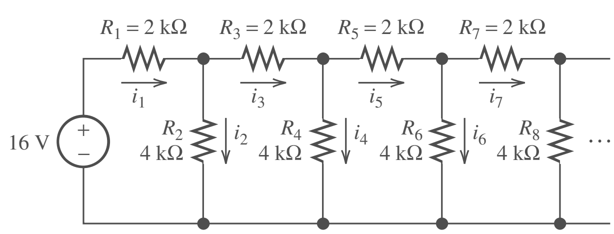


Figure P2.43

***P2.44.** A worker is standing on a wet concrete floor, holding an electric drill having a metallic case. The metallic case is connected through the ground wire of a three-terminal power outlet to power-system ground. The resistance of the ground wire is R_g. The resistance of the worker's body is $R_w = 500\ \Omega$. Due to faulty insulation in the drill, a current of 2 A flows into its metallic case. The circuit diagram for this situation is shown in Figure P2.44. Find the maximum value of R_g so that the current through the worker does not exceed 0.1 mA.

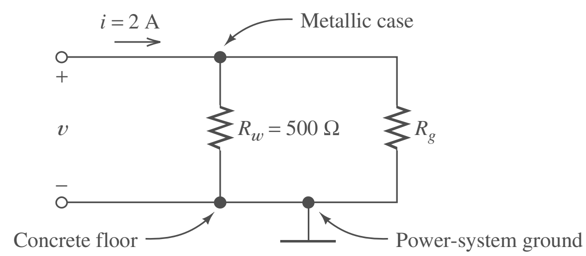


Figure P2.44

P2.45. We have a 15-V source and a load that absorbs power and requires a current varying between 0 and 100 mA. The voltage across the load must remain between 4.7 and 5.0 V for all values of load current. Design a voltage-divider network to supply the load. You may assume that resistors of any value desired are available. Also, give the minimum power rating for each resistor.

P2.46. A load resistance of 150 Ω needs to be supplied with 5 V. A 12.6-V voltage source and resistors of any value needed are available. Draw a suitable circuit consisting of the voltage source, the load, and one additional resistor. Specify the value of the resistor.

P2.47. Suppose that we wish to supply 500 mW to a 200-Ω load resistance R_L. A 100-mA current source and resistors of any value needed are available. Draw a suitable circuit consisting of the current source, the load, and one additional resistor. Specify the value of the resistor.

Section 2.4: Node-Voltage Analysis

P2.48. On your own, using analytical thinking and memory, list the steps to follow in analyzing a general circuit with the node-voltage technique.

***P2.49.** Write equations and solve for the node voltages shown in Figure P2.49. Then, find the value of i_1.

P2.50. Solve for the node voltages shown in Figure P2.50. What are the new values of the node voltages after the direction of the current source is reversed? How are the values related?

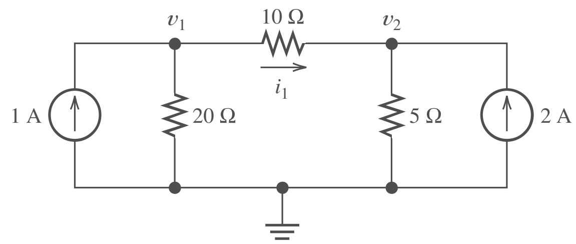


Figure P2.49

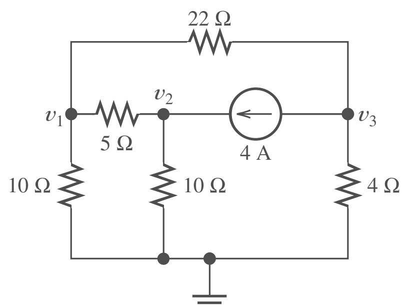


Figure P2.50

P2.51. Given $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 8\ \Omega$, $R_4 = 6\ \Omega$, $R_5 = 8\ \Omega$, and $I_s = 4$ A, solve for the node voltages shown in Figure P2.51.

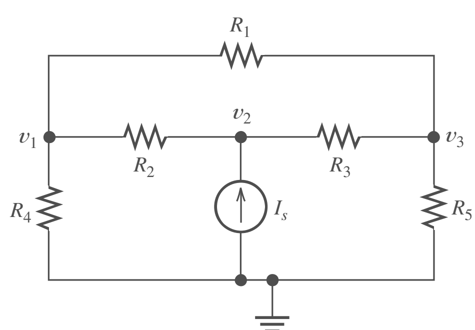


Figure P2.51

P2.52. Given $R_1 = 15\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 10\ \Omega$, $R_5 = 8\ \Omega$, $R_6 = 4\ \Omega$, and $I_s = 2$ A, solve for the node voltages shown in Figure P2.52.

***P2.53.** Solve for the node voltages shown in Figure P2.53. Then, find the value of i_s.

P2.54. Determine the value of i_1 in Figure P2.54 using node voltages to solve the circuit. Select the location of the reference node to minimize the number of unknown node voltages. What effect does the 17-Ω resistance have on the answer? Explain.

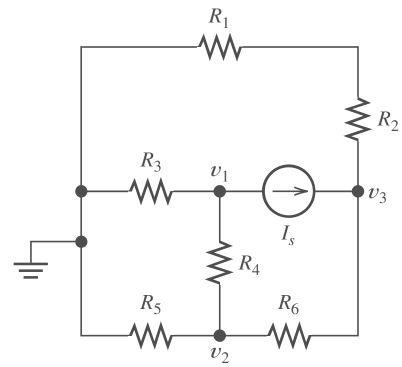


Figure P2.52

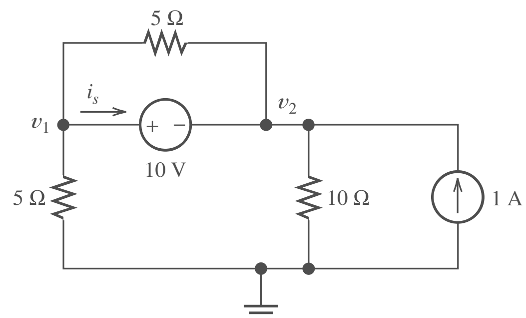


Figure P2.53

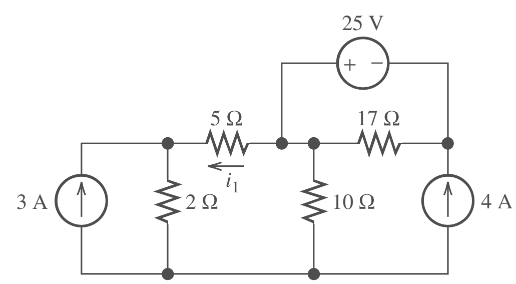


Figure P2.54

P2.55. In solving a network, what rule must you observe when writing KCL equations? Why?

P2.56. Use the symbolic features of MATLAB to find an expression for the equivalent resistance for the network shown in Figure P2.56. (*Hint:* First, connect a 1-A current source across terminals *a* and *b*. Then, solve the network by the node-voltage technique. The voltage across the current source is equal in value to the equivalent resistance.) Finally, use the subs command to evaluate for $R_1 = 15\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 15\ \Omega$, $R_4 = 10\ \Omega$, and $R_5 = 10\ \Omega$.

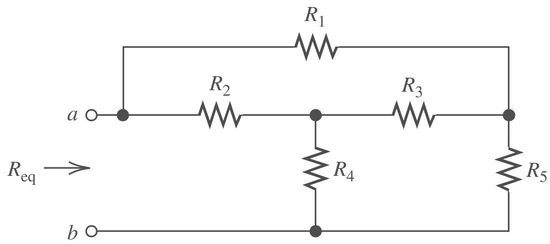


Figure P2.56

***P2.57.** Solve for the values of the node voltages shown in Figure P2.57. Then, find the value of i_x.

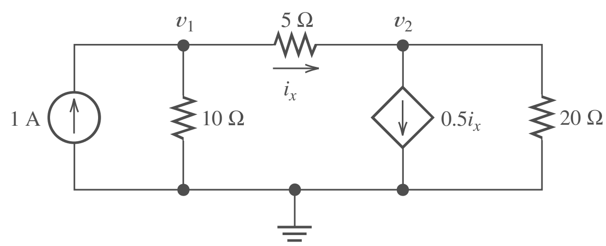


Figure P2.57

***P2.58.** Solve for the node voltages shown in Figure P2.58.

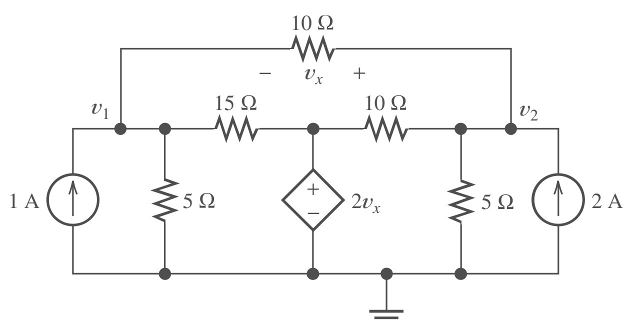


Figure P2.58

P2.59. Solve for the node voltages shown in Figure P2.59.

P2.60. Solve for the power delivered to the 8-Ω resistance and solve for the node voltages shown in Figure P2.60.

P2.61. Find the equivalent resistance looking into terminals a–b for the network shown in Figure P2.61. (*Hint:* First, connect a 1-A current source across terminals a and b. Then, solve the network by the node-voltage technique. The voltage across the current source is equal in value to the equivalent resistance.)

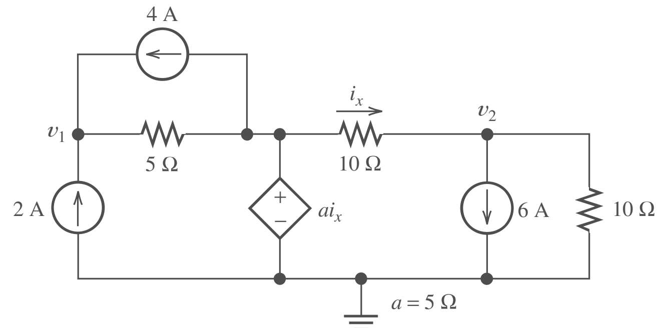


Figure P2.59

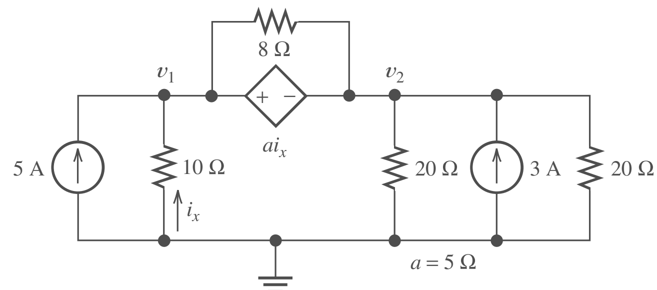


Figure P2.60

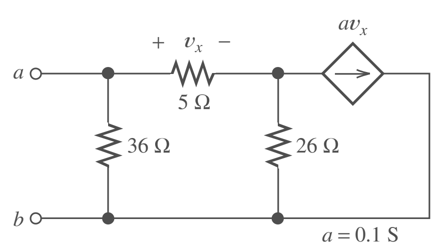


Figure P2.61

P2.62. Find the equivalent resistance looking into terminals a–b for the network shown in Figure P2.62. (*Hint:* First, connect a 1-A current source across terminals a and b. Then, solve the network by the node-voltage technique. The voltage across the current source is equal in value to the equivalent resistance.)

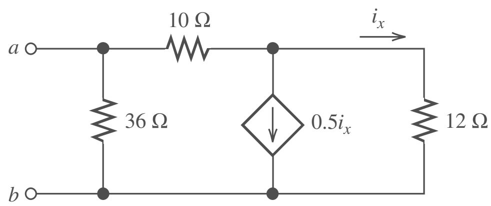


Figure P2.62

P2.63. We have a cube with 1-Ω resistances along each edge as illustrated in Figure P2.63, in which we are looking into the front face which has corners at nodes 1, 2, 7, and the reference node. Nodes 3, 4, 5, and 6 are the corners on the rear face of the cube. (Alternatively, you can consider it to be a planar network.) We want to find the resistance between adjacent nodes, such as node 1 and the reference node. We do this by connecting a 1-A current source as shown and solving for v_1, which by symmetry is equal in value to the resistance between any two adjacent nodes. **a.** Use MATLAB to solve the matrix equation $\mathbf{GV} = \mathbf{I}$ for the node voltages and determine the resistance. **b.** Modify your work to determine the resistance between nodes at the ends of a diagonal across a face, such as node 2 and the reference node. **c.** Finally, find the resistance between opposite corners of the cube. (*Comment*: Part (c) is the same as Problem 2.17 in which we suggested using symmetry to solve for the resistance. Parts (a) and (b) can also be solved by use of symmetry and the fact that nodes having the same value of voltage can be connected by short circuits without changing the currents and voltages. With the shorts in place, the resistances can be combined in series and parallel to obtain the answers. Of course, if the resistors have arbitrary values, the MATLAB approach will still work, but considerations of symmetry will not.)

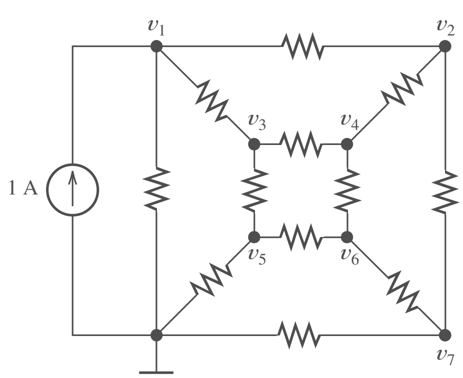


Figure P2.63 Each resistance is 1 Ω.

P2.64. Figure P2.64 shows an unusual voltage-divider circuit. Use node-voltage analysis and the symbolic math commands in MATLAB to solve for the voltage-division ratio V_{out}/V_{in} in terms of the resistances. Notice that the node voltage variables are V_1, V_2, and V_{out}.

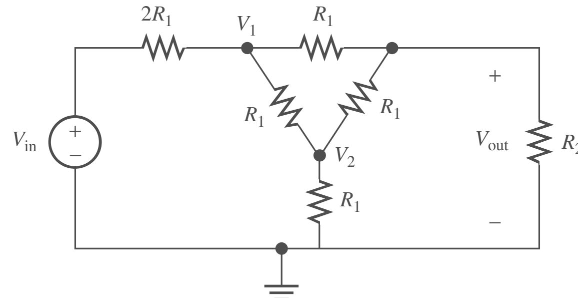


Figure P2.64

P2.65. Solve for the node voltages in the circuit of Figure P2.65. (Disregard the mesh currents, i_1, i_2, i_3, and i_4 when working with the node voltages.)

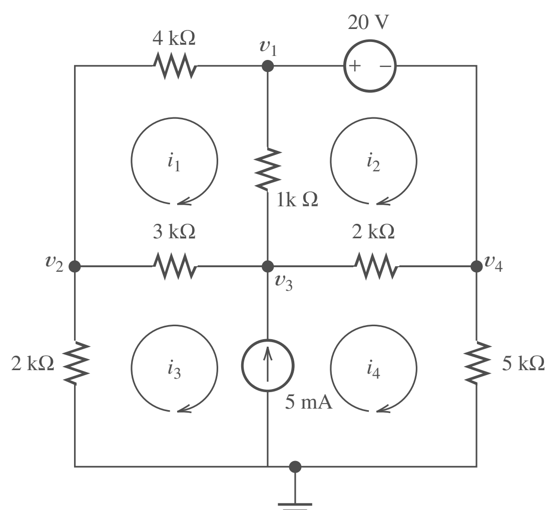


Figure P2.65

Section 2.5: Mesh-Current Analysis

P2.66. List the steps in analyzing a planar network using the mesh-current method. Attempt this as a "closed-book" exam problem.

***P2.67.** Solve for the power delivered to the 15-Ω resistor and for the mesh currents shown in Figure P2.67.

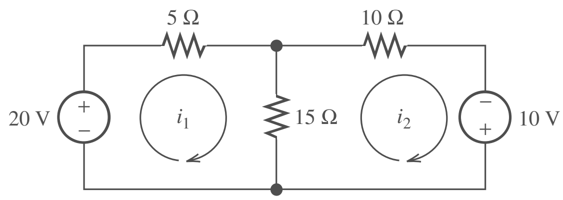


Figure P2.67

***P2.68.** Determine the value of v_2 and the power delivered by the source in the circuit of Figure P2.26 by using mesh-current analysis.

***P2.69.** Use mesh-current analysis to find the value of i_1 in the circuit of Figure P2.49.

P2.70. Solve for the power delivered by the voltage source in Figure P2.70, using the mesh-current method.

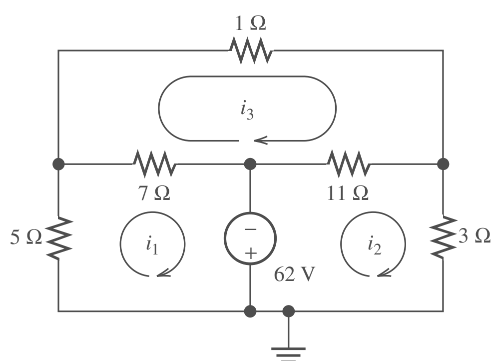


Figure P2.70

P2.71. Use mesh-current analysis to find the value of v in the circuit of Figure P2.38.

P2.72. Use mesh-current analysis to find the value of i_3 in the circuit of Figure P2.39. (Choose your mesh-current variables as i_A and i_B to avoid confusion with the current labels on the circuit diagram.)

P2.73. Use mesh-current analysis to find the values of i_1 and i_2 in Figure P2.30. Select i_1 clockwise around the left-hand mesh, i_2 clockwise around the right-hand mesh, and i_3 clockwise around the center mesh.

P2.74. Find the power delivered by the source and the values of i_1 and i_2 in the circuit of Figure P2.23, using mesh-current analysis.

P2.75. Use mesh-current analysis to find the values of i_1 and i_2 in Figure P2.29. First, select i_A clockwise around the left-hand mesh and i_B clockwise around the right-hand mesh. After solving for the mesh currents, i_A and i_B, determine the values of i_1 and i_2.

P2.76. Use mesh-current analysis to find the values of i_1 and i_2 in Figure P2.28. First, select i_A clockwise around the left-hand mesh and i_B clockwise around the right-hand mesh. After solving for the mesh currents, i_A and i_B, determine the values of i_1 and i_2.

P2.77. The circuit shown in Figure P2.77 is the dc equivalent of a simple residential power distribution system. Each of the resistances labeled R_1 and R_2 represents various parallel-connected loads, such as lights or devices plugged into outlets that nominally operate at 120 V, while R_3 represents a load, such as the heating element in an oven that nominally operates at 240 V. The resistances labeled R_w represent the resistances of wires. R_n represents the "neutral" wire. **a.** Use mesh-current analysis to determine the voltage magnitude for each load and the current in the neutral wire. **b.** Now, suppose that due to a fault in the wiring at the distribution panel, the neutral wire becomes an open circuit. Again, compute the voltages across the loads and comment on the probable outcome for a sensitive device such as a computer or plasma television that is part of the 20-Ω load.

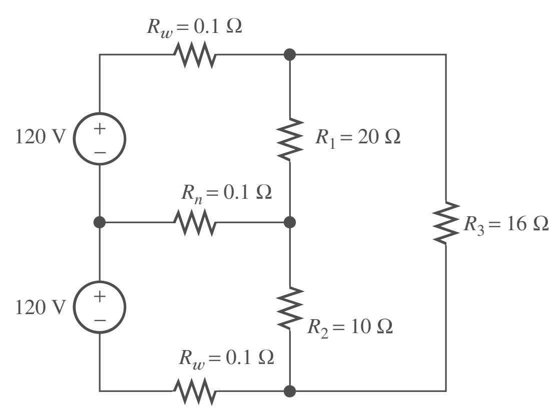


Figure P2.77

P2.78. Use MATLAB and mesh-current analysis to determine the value of v_2 in the circuit of Figure P2.51. The component values are $R_1 = 4\ \Omega, R_2 = 5\ \Omega, R_3 = 8\ \Omega, R_4 = 6\ \Omega, R_5 = 8\ \Omega$, and $I_s = 4$ A.

P2.79. Connect a 1-V voltage source across terminals a and b of the network shown in Figure P2.56. Then, solve the network by the mesh-current technique to find the current through the source. Finally, divide the source voltage by the current to determine the equivalent resistance looking into terminals a and b. The resistance values are $R_1 = 15\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 15\ \Omega$, $R_4 = 10\ \Omega$, and $R_5 = 10\ \Omega$.

P2.80. Connect a 1-V voltage source across the terminals of the network shown in Figure P2.1(a). Then, solve the network by the mesh-current technique to find the current through the source. Finally, divide the source voltage by the current to determine the equivalent resistance looking into the terminals. Check your answer by combining resistances in series and parallel.

P2.81. Use MATLAB to solve for the mesh currents in Figure P2.65.

Section 2.6: Thévenin and Norton Equivalent Circuits

P2.82. List the steps in determining the Thévenin and Norton equivalent circuits for a general two-terminal circuit. Try this as if it were a "closed-book" exam question.

***P2.83.** Find the Thévenin and Norton equivalent circuits for the two-terminal circuit shown in Figure P2.83.

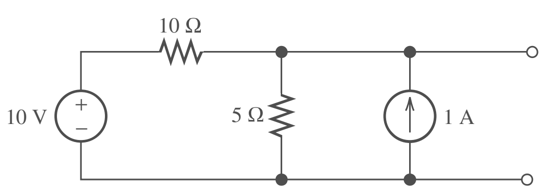


Figure P2.83

***P2.84.** We can model a certain battery as a voltage source in series with a resistance. The open-circuit voltage of the battery is 9 V. When a 100-Ω resistor is placed across the terminals of the battery, the voltage drops to 6 V. Determine the internal resistance (Thévenin resistance) of the battery.

P2.85. Find the Thévenin and Norton equivalent circuits for the circuit shown in Figure P2.85. Take care that you orient the polarity of the voltage source and the direction of the current source correctly relative to terminals a and b. What effect does the 9-Ω resistor have on the equivalent circuits? Explain your answer.

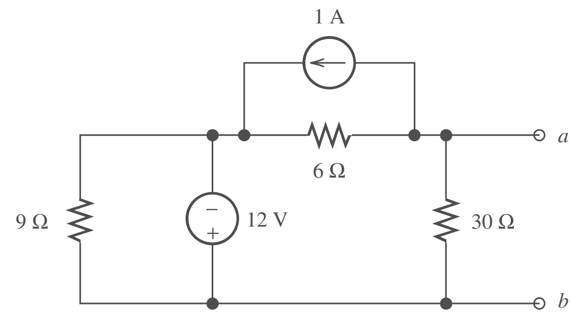


Figure P2.85

P2.86. Find the Thévenin and Norton equivalent circuits for the circuit shown in Figure P2.86.

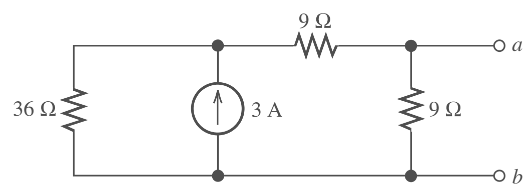


Figure P2.86

P2.87. Find the Thévenin and Norton equivalent circuits for the two-terminal circuit shown in Figure P2.87.

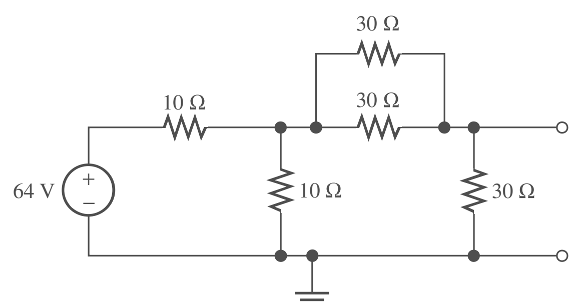


Figure P2.87

P2.88. A somewhat discharged automotive battery has an open-circuit voltage of 12.5 V and supplies 50 A when a 0.1-Ω resistance is connected across the battery terminals. Draw the Thévenin and Norton equivalent circuits, including values for the circuit parameters.

What current can this battery deliver to a short circuit? Considering that the energy stored in the battery remains constant under open-circuit conditions, which of these equivalent circuits seems more realistic? Explain.

P2.89. A certain two-terminal circuit has an open-circuit voltage of 9 V. When a 200-Ω load is attached, the voltage across the load is 7 V. Determine the Thévenin resistance for the circuit.

P2.90. If we measure the voltage at the terminals of a two-terminal network with two known (and different) resistive loads attached, we can determine the Thévenin and Norton equivalent circuits. When a 1-kΩ load is attached to a two-terminal circuit, the load voltage is 8 V. When the load is increased to 2 kΩ, the load voltage becomes 10 V. Find the Thévenin voltage and resistance for the circuit.

P2.91. Find the Thévenin and Norton equivalent circuits for the circuit shown in Figure P2.91.

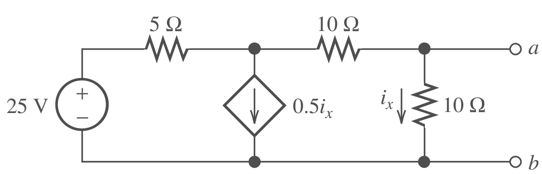


Figure P2.91

P2.92. Find the maximum power that can be delivered to a resistive load by the circuit shown in Figure P2.83. For what value of load resistance is the power maximum?

P2.93. Find the maximum power that can be delivered to a resistive load by the circuit shown in Figure P2.86. For what value of load resistance is the power maximum?

P2.94. A battery can be modeled by a voltage source V_t in series with a resistance R_t. Assuming that the load resistance is selected to maximize the power delivered, what percentage of the power taken from the voltage source V_t is actually delivered to the load? Suppose that $R_L = 9R_t$; what percentage of the power taken from V_t is delivered to the load? Usually, we want to design battery-operated systems so that nearly all of the energy stored in the battery is delivered to the load. Should we design for maximum power transfer?

***P2.95.** Figure P2.95 shows a resistive load R_L connected to a Thévenin equivalent circuit. For what value of Thévenin resistance is the power delivered to the load maximized? Find the maximum power delivered to the load. (*Hint:* Be careful; this is a tricky question if you don't stop to think about it.)

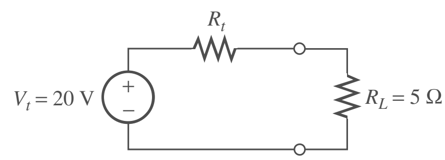


Figure P2.95

P2.96. Starting from the Norton equivalent circuit with a resistive load R_L attached, find an expression for the power delivered to the load in terms of I_n, R_t, and R_L. Assuming that I_n and R_t are fixed values and that R_L is a variable, show that maximum power is delivered for $R_L = R_t$. Find an expression for maximum power delivered to the load in terms of I_n and R_t.

Section 2.7: Superposition Principle

***P2.97.** Use superposition to find the current i in Figure P2.97. First, zero the current source and find the value i_v caused by the voltage source alone. Then, zero the voltage source and find the value i_c caused by the current source alone. Finally, add the results algebraically.

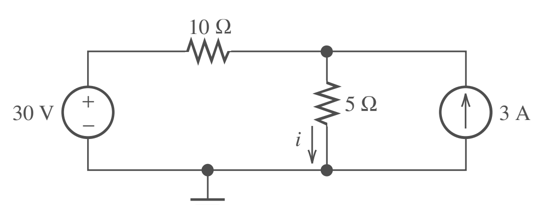


Figure P2.97

***P2.98.** Solve for i_s in Figure P2.53 by using superposition.

P2.99. Solve the circuit shown in Figure P2.49 by using superposition. First, zero the 1-A source and find the value of i_1 with only the 2-A source activated. Then, zero the 2-A source and find the value of i_1 with only the 1-A source activated. Finally, find the total value of i_1, with both sources activated, by algebraically adding the previous results.

P2.100. Solve for i_1 in Figure P2.24 by using superposition.

P2.101. Another method of solving the circuit of Figure P2.26 is to start by assuming that $v_2 = 1$ V. Accordingly, we work backward toward the source, using Ohm's law, KCL, and KVL to find the value of v_s. Since we know that v_2 is proportional to the value of v_s, and since we have found the value of v_s that produces $v_2 = 1$ V, we can calculate the value of v_2 that results when $v_s = 12$ V. Solve for v_2 by using this method.

P2.102. Use the method of Problem P2.101 for the circuit of Figure P2.23, starting with the assumption that $i_2 = 1$ A.

P2.103. Solve for the actual value of i_6 for the circuit of Figure P2.103 with $V_s = 10$ V, starting with the assumption that $i_6 = 1$ A. Work back through the circuit to find the value of V_s that results in $i_6 = 1$ A. Then, use proportionality to determine the value of i_6 that results for $V_s = 10$ V.

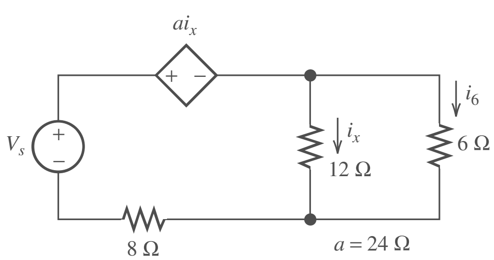


Figure P2.103

P2.104. Device A shown in Figure P2.104 has $v = 2i^3$. **a.** Solve for v with the 2-A source active and the 1-A source zeroed. **b.** Solve for v with the 1-A source active and the 2-A source zeroed. **c.** Solve for v with both sources active. Why doesn't superposition apply?

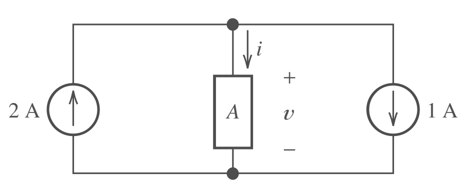


Figure P2.104

Section 2.8: Wheatstone Bridge

P2.105. **a.** The Wheatstone bridge shown in Figure 2.64 on page 105 is balanced with $R_1 = 1$ kΩ, $R_3 = 3419$ Ω, and $R_2 = 1$ kΩ. Find R_x. **b.** Repeat if R_2 is 100 kΩ and the other values are unchanged.

***P2.106.** The Wheatstone bridge shown in Figure 2.64 on page 105 has $v_s = 10$ V, $R_1 = 10$ kΩ, $R_2 = 10$ kΩ, and $R_x = 5932$ Ω. The detector can be modeled as a 5-kΩ resistance. **a.** What value of R_3 is required to balance the bridge? **b.** Suppose that R_3 is 1 Ω higher than the value found in part (a). Find the current through the detector. (*Hint:* Find the Thévenin equivalent for the circuit with the detector removed. Then, place the detector across the Thévenin equivalent and solve for the current.) Comment.

P2.107. In theory, any values can be used for R_1 and R_3 in the Wheatstone bridge of Figure 2.64 on page 105. For the bridge to balance, it is only the *ratio* R_3/R_1 that is important. What practical problems might occur if the values are very small? What practical problems might occur if the values are very large?

P2.108. Derive expressions for the Thévenin voltage and resistance "seen" by the detector in the Wheatstone bridge in Figure 2.64 on page 105. (In other words, remove the detector from the circuit and determine the Thévenin resistance for the remaining two-terminal circuit.) What is the value of the Thévenin voltage when the bridge is balanced?

P2.109. Derive Equation 2.93 for the bridge circuit of Figure 2.65 on page 107.

P2.110. Consider a strain gauge in the form of a long thin wire having a length L and a cross-sectional area A before strain is applied. After the strain is applied, the length increases slightly to $L + \Delta L$ and the area is reduced so the volume occupied by the wire is constant. Assume that $\Delta L/L << 1$ and that the resistivity ρ of the wire material is constant. Determine the gauge factor

$$G = \frac{\Delta R/R_0}{\Delta L/L}$$

(*Hint:* Make use of Equation 1.10.)

P2.111. Explain what would happen if, in wiring the bridge circuit of Figure 2.65 on page 107, the gauges in tension (i.e., those labeled $R + \Delta R$) were both placed on the top of the bridge circuit diagram, shown in part (b) of the figure, and those in compression were both placed at the bottom of the bridge circuit diagram.

Practice Test

Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in Appendix D and complete solutions are included in the Student Solutions files. See Appendix E for more information about the Student Solutions.

T2.1. Match each entry in Table T2.1(a) with the best choice from the list given in Table T2.1(b)

Table T2.1

Item	Best Match
(a)	
a. The equivalent resistance of parallel-connected resistances…	
b. Resistances in parallel combine as do…	
c. Loads in power distribution systems are most often connected…	
d. Solving a circuit by series/parallel combinations applies to…	
e. The voltage-division principle applies to…	
f. The current-division principle applies to…	
g. The superposition principle applies to…	
h. Node-voltage analysis can be applied to…	
i. In this book, mesh-current analysis is applied to…	
j. The Thévenin resistance of a two-terminal circuit equals…	
k. The Norton current source value of a two-terminal circuit equals…	
l. A voltage source in parallel with a resistance is equivalent to…	
(b)	
1. conductances in parallel	
2. in parallel	
3. all circuits	
4. resistances or conductances in parallel	
5. is obtained by summing the resistances	
6. is the reciprocal of the sum of the reciprocals of the resistances	
7. some circuits	
8. planar circuits	
9. a current source in series with a resistance	
10. conductances in series	
11. circuits composed of linear elements	
12. in series	
13. resistances or conductances in series	
14. a voltage source	
15. the open-circuit voltage divided by the short-circuit current	
16. a current source	
17. the short-circuit current	

for circuits composed of sources and resistances. [Items in Table T2.1(b) may be used more than once or not at all.]

T2.2. Consider the circuit of Figure T2.2 with $v_s = 96\,\text{V}$, $R_1 = 6\,\Omega$, $R_2 = 48\,\Omega$, $R_3 = 16\,\Omega$, and $R_4 = 60\,\Omega$. Determine the values of i_s and i_4.

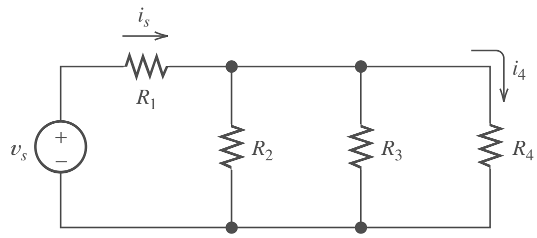


Figure T2.2

T2.3. Write MATLAB code to solve for the node voltages for the circuit of Figure T2.3.

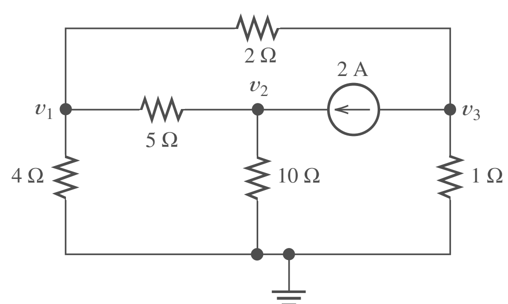


Figure T2.3

T2.4. Write a set of equations that can be used to solve for the mesh currents of Figure T2.4. Be sure to indicate which of the equations you write form the set.

T2.5. Determine the Thévenin and Norton equivalent circuits for the circuit of Figure T2.5. Draw the equivalent circuits labeling the terminals to correspond with the original circuit.

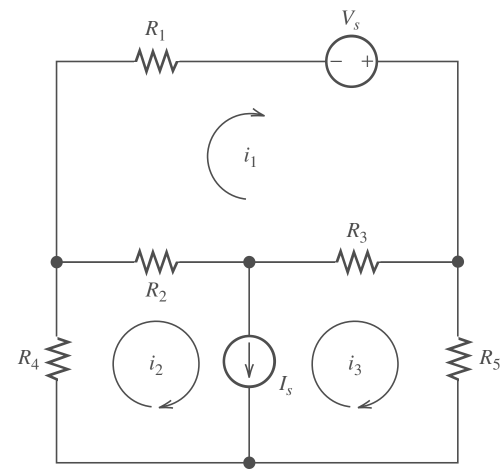


Figure T2.4

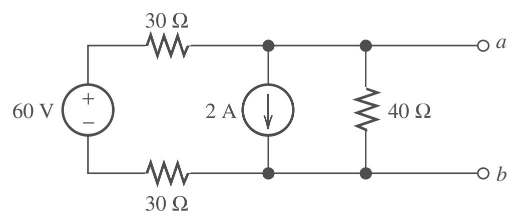


Figure T2.5

T2.6. According to the superposition principle, what percentage of the total current flowing through the 5-Ω resistance in the circuit of Figure T2.6 results from the 5-V source? What percentage of the power supplied to the 5-Ω resistance is supplied by the 5-V source? Assume that both sources are active when answering both questions.

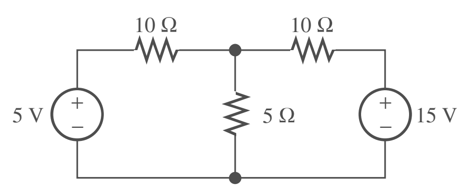


Figure T2.6

In Chapter 4, we saw that the response of a network has two parts: the forced response and the natural response. In most circuits, the natural response decays rapidly to zero. The forced response for sinusoidal sources persists indefinitely and, therefore, is called the steady-state response. Because the natural response quickly decays, the steady-state response is often of highest interest. In this chapter, we learn efficient methods for finding the steady-state responses for sinusoidal sources.

We also study three-phase circuits, which are used in electric power-distribution systems. Most engineers who work in industrial settings need to understand three-phase power distribution.

5.1 SINUSOIDAL CURRENTS AND VOLTAGES

A sinusoidal voltage is shown in Figure 5.1 and is given by

$$v(t) = V_m \cos(\omega t + \theta) \tag{5.1}$$

where V_m is the **peak value** of the voltage, ω is the **angular frequency** in radians per second, and θ is the **phase angle**.

Sinusoidal signals are periodic, repeating the same pattern of values in each **period** T. Because the cosine (or sine) function completes one cycle when the angle increases by 2π radians, we get

$$\omega T = 2\pi \tag{5.2}$$

The **frequency** of a periodic signal is the number of cycles completed in one second. Thus, we obtain

$$f = \frac{1}{T} \tag{5.3}$$

We refer to ω as angular frequency with units of radians per second and f simply as frequency with units of hertz (Hz).

The units of frequency are hertz (Hz). (Actually, the physical units of hertz are equivalent to inverse seconds.) Solving Equation 5.2 for the angular frequency, we have

$$\omega = \frac{2\pi}{T} \tag{5.4}$$

Using Equation 5.3 to substitute for T, we find that

$$\omega = 2\pi f \tag{5.5}$$

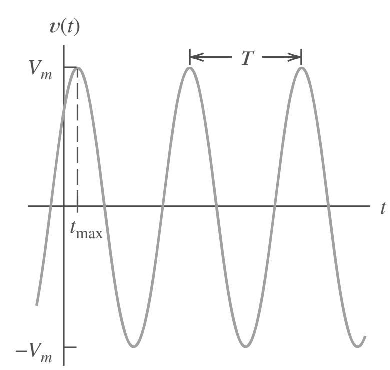


Figure 5.1 A sinusoidal voltage waveform given by $v(t) = V_m \cos(\omega t + \theta)$. *Note:* Assuming that θ is in degrees, we have $t_{max} = \frac{-\theta}{360} \times T$. For the waveform shown, θ is $-45°$.

Throughout our discussion, the argument of the cosine (or sine) function is of the form

$$\omega t + \theta$$

We assume that the angular frequency ω has units of radians per second (rad/s). However, we sometimes give the phase angle θ in degrees. Then, the argument has mixed units. If we wanted to evaluate $\cos(\omega t + \theta)$ for a particular value of time, we would have to convert θ to radians before adding the terms in the argument. Usually, we find it easier to visualize an angle expressed in degrees, and mixed units are not a problem.

Electrical engineers often write the argument of a sinusoid in mixed units: ωt is in radians and the phase angle θ is in degrees.

For uniformity, we express sinusoidal functions by using the cosine function rather than the sine function. The functions are related by the identity

$$\sin(z) = \cos(z - 90^\circ) \tag{5.6}$$

For example, when we want to find the phase angle of

$$v_x(t) = 10\sin(200t + 30^\circ)$$

we first write it as

$$\begin{aligned} v_x(t) &= 10\cos(200t + 30^\circ - 90^\circ) \\ &= 10\cos(200t - 60^\circ) \end{aligned}$$

Thus, we state that the phase angle of $v_x(t)$ is -60°.

Root-Mean-Square Values

Consider applying a periodic voltage $v(t)$ with period T to a resistance R. The power delivered to the resistance is given by

$$p(t) = \frac{v^2(t)}{R} \tag{5.7}$$

Furthermore, the energy delivered in one period is given by

$$E_T = \int_0^T p(t)\,dt \tag{5.8}$$

The average power P_{avg} delivered to the resistance is the energy delivered in one cycle divided by the period. Thus,

$$P_{\text{avg}} = \frac{E_T}{T} = \frac{1}{T}\int_0^T p(t)\,dt \tag{5.9}$$

Using Equation 5.7 to substitute into Equation 5.9, we obtain

$$P_{\text{avg}} = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\,dt \tag{5.10}$$

This can be rearranged as

$$P_{\text{avg}} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}\right]^2}{R} \tag{5.11}$$

Now, we define the **root-mean-square** (rms) value of the periodic voltage $v(t)$ as

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \tag{5.12}$$

Using this equation to substitute into Equation 5.11, we get

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} \tag{5.13}$$

Power calculations are facilitated by using rms values for voltage or current.

Thus, if the rms value of a periodic voltage is known, it is relatively easy to compute the average power that the voltage can deliver to a resistance. The rms value is also called the **effective value**.

Similarly for a periodic current $i(t)$, we define the rms value as

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} \tag{5.14}$$

and the average power delivered if $i(t)$ flows through a resistance is given by

$$P_{\text{avg}} = I_{\text{rms}}^2 R \tag{5.15}$$

RMS Value of a Sinusoid

Consider a sinusoidal voltage given by

$$v(t) = V_m \cos(\omega t + \theta) \tag{5.16}$$

To find the rms value, we substitute into Equation 5.12, which yields

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta)\,dt} \tag{5.17}$$

Next, we use the trigonometric identity

$$\cos^2(z) = \frac{1}{2} + \frac{1}{2}\cos(2z) \tag{5.18}$$

to write Equation 5.17 as

$$V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\int_0^T [1 + \cos(2\omega t + 2\theta)]\,dt} \tag{5.19}$$

Integrating, we get

$$V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\left[t + \frac{1}{2\omega}\sin(2\omega t + 2\theta)\right]_0^T} \tag{5.20}$$

Evaluating, we have

$$V_{\text{rms}} = \sqrt{\frac{V_m^2}{2T}\left[T + \frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2\omega}\sin(2\theta)\right]} \tag{5.21}$$

Referring to Equation 5.2, we see that $\omega T = 2\pi$. Thus, we obtain

$$\begin{aligned}\frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2\omega}\sin(2\theta) &= \frac{1}{2\omega}\sin(4\pi + 2\theta) - \frac{1}{2\omega}\sin(2\theta)\\ &= \frac{1}{2\omega}\sin(2\theta) - \frac{1}{2\omega}\sin(2\theta)\\ &= 0\end{aligned}$$

Therefore, Equation 5.21 reduces to

$$\boxed{V_{\text{rms}} = \frac{V_m}{\sqrt{2}}} \tag{5.22}$$

This is a useful result that we will use many times in dealing with sinusoids.

Usually in discussing sinusoids, the rms or effective value is given rather than the peak value. For example, ac power in residential wiring is distributed as a 60-Hz 115-V rms sinusoid (in the United States). Most people are aware of this, but probably few know that 115 V is the rms value and that the peak value is $V_m = V_{\text{rms}} \times \sqrt{2} = 115 \times \sqrt{2} \cong 163$ V. (Actually, 115 V is the nominal residential distribution voltage. It can vary from approximately 105 to 130 V.)

Keep in mind that $V_{\text{rms}} = V_m/\sqrt{2}$ applies to sinusoids. To find the rms value of other periodic waveforms, we would need to employ the definition given by Equation 5.12.

The rms value for a sinusoid is the peak value divided by the square root of two. This is not true for other periodic waveforms such as square waves or triangular waves.

Example 5.1 Power Delivered to a Resistance by a Sinusoidal Source

Suppose that a voltage given by $v(t) = 100\cos(100\pi t)$ V is applied to a 50-Ω resistance. Sketch $v(t)$ to scale versus time. Find the rms value of the voltage and the average power delivered to the resistance. Find the power as a function of time and sketch to scale.

Solution By comparison of the expression given for $v(t)$ with Equation 5.1, we see that $\omega = 100\pi$. Using Equation 5.5, we find that the frequency is $f = \omega/2\pi = 50$ Hz. Then, the period is $T = 1/f = 20$ ms. A plot of $v(t)$ versus time is shown in Figure 5.2(a).

The peak value of the voltage is $V_m = 100$ V. Thus, the rms value is $V_{\text{rms}} = V_m/\sqrt{2} = 70.71$ V. Then, the average power is

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} = \frac{(70.71)^2}{50} = 100 \text{ W}$$

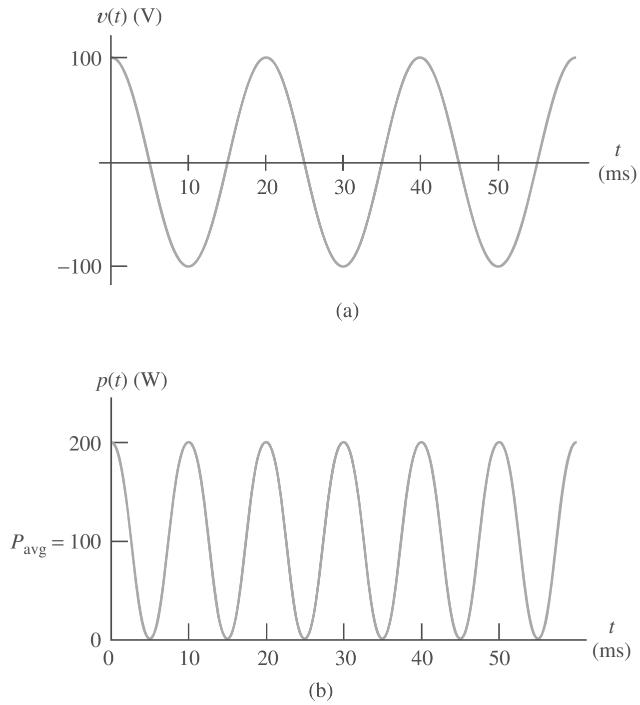


Figure 5.2 Voltage and power versus time for Example 5.1.

The power as a function of time is given by

$$p(t) = \frac{v^2(t)}{R} = \frac{100^2 \cos^2(100\pi t)}{50} = 200 \cos^2(100\pi t)\ \mathrm{W}$$

A plot of $p(t)$ versus time is shown in Figure 5.2(b). Notice that the power fluctuates from 0 to 200 W. However, the average power is 100 W, as we found by using the rms value. ■

For a sinusoidal current flowing in a resistance, power fluctuates periodically from zero to twice the average value.

RMS Values of Nonsinusoidal Voltages or Currents

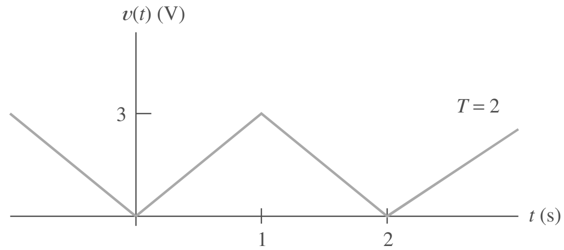
Sometimes we need to determine the rms values of periodic currents or voltages that are not sinusoidal. We can accomplish this by applying the definition given by Equation 5.12 or 5.14 directly.

Example 5.2 RMS Value of a Triangular Voltage

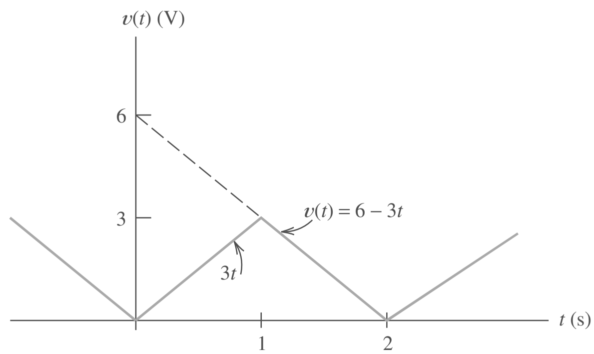
The voltage shown in Figure 5.3(a) is known as a triangular waveform. Determine its rms value.

Solution First, we need to determine the equations describing the waveform between $t = 0$ and $t = T = 2$ s. As illustrated in Figure 5.3(b), the equations for the first period of the triangular wave are

$$v(t) = \begin{cases} 3t & \textit{for} \quad 0 \le t \le 1 \\ 6 - 3t & \textit{for} \quad 1 \le t \le 2 \end{cases}$$



(a) Triangular voltage waveform



(b) Equations for the first period

Figure 5.3 Triangular voltage waveform of Example 5.2.

Equation 5.12 gives the rms value of the voltage.

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

Dividing the interval into two parts and substituting for $v(t)$, we have

$$V_{\text{rms}} = \sqrt{\frac{1}{2}\left[\int_0^1 9t^2 dt + \int_1^2 (6-3t)^2 dt\right]}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{2}\left[3t^3\big|_{t=0}^{t=1} + (36t - 18t^2 + 3t^3)\big|_{t=1}^{t=2}\right]}$$

Evaluating, we find

$$V_{\text{rms}} = \sqrt{\frac{1}{2}\left[3 + (72 - 36 - 72 + 18 + 24 - 3)\right]} = \sqrt{3}\text{ V}$$

■

The integrals in this example are easy to carry out manually. However, when the integrals are more difficult, we can sometimes obtain answers using the MATLAB Symbolic Toolbox. Here are the MATLAB commands needed to perform the integrals in this example:

```
>> syms Vrms t
>> Vrms = sqrt((1/2)*(int(9*t^2,t,0,1) + int((6-3*t)^2,t,1,2)))
    Vrms =
    3^(1/2)
```

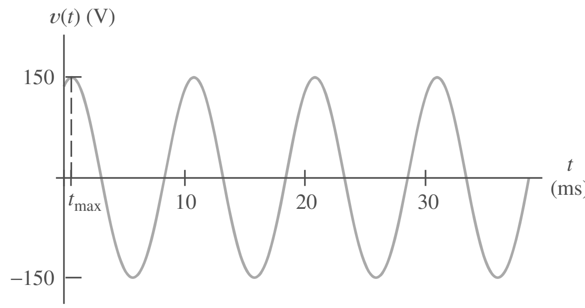


Figure 5.4 Answer for Exercise 5.1(c).

Exercise 5.1 Suppose that a sinusoidal voltage is given by

$$v(t) = 150\cos(200\pi t - 30°) \text{ V}$$

a. Find the angular frequency, the frequency in hertz, the period, the peak value, and the rms value. Also, find the first value of time $t_{\max}$ after $t = 0$ such that $v(t)$ attains its positive peak. **b.** If this voltage is applied to a 50-Ω resistance, compute the average power delivered. **c.** Sketch $v(t)$ to scale versus time.

Answer **a.** $\omega = 200\pi$, $f = 100$ Hz, $T = 10$ ms, $V_m = 150$ V, $V_{\text{rms}} = 106.1$ V, $t_{\max} = \frac{30°}{360°} \times T = 0.833$ ms; **b.** $P_{\text{avg}} = 225$ W; **c.** a plot of $v(t)$ versus time is shown in Figure 5.4. □

Exercise 5.2 Express $v(t) = 100\sin(300\pi t + 60°)$ V as a cosine function.

Answer $v(t) = 100\cos(300\pi t - 30°)$ V. □

Exercise 5.3 Suppose that the ac line voltage powering a computer has an rms value of 110 V and a frequency of 60 Hz, and the peak voltage is attained at $t = 5$ ms. Write an expression for this ac voltage as a function of time.

Answer $v(t) = 155.6 \cos(377t - 108°)$ V. □

~~5.2 PHASORS~~

~~In the next several sections, we will see that sinusoidal steady-state analysis is greatly facilitated if the currents and voltages are represented as vectors (called **phasors**) in the complex-number plane. In preparation for this material, you may wish to study the review of complex-number arithmetic in Appendix A.~~

~~We start with a study of convenient methods for adding (or subtracting) sinusoidal waveforms. We often need to do this in applying Kirchhoff's voltage law (KVL) or Kirchhoff's current law (KCL) to ac circuits. For example, in applying KVL to a network with sinusoidal voltages, we might obtain the expression~~

$$\cancel{v(t) = 10\cos(\omega t) + 5\sin(\omega t + 60°) + 5\cos(\omega t + 90°)} \tag{5.23}$$

~~To obtain the peak value of $v(t)$ and its phase angle, we need to put Equation 5.23 into the form~~

$$\cancel{v(t) = V_m\cos(\omega t + \theta)} \tag{5.24}$$

~~This could be accomplished by repeated substitution, using standard trigonometric identities. However, that method is too tedious for routine work. Instead, we will see~~

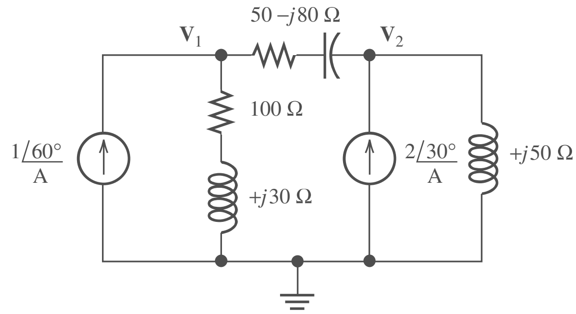


Figure 5.48 Circuit for Exercise 5.18.

Exercise 5.18 Use MATLAB to solve for the phasor node voltages in polar form for the circuit of Figure 5.48.

Answer The MATLAB commands are:

```
clear
Y = [(1/(100+i*30)+1/(50-i*80)) (-1/(50-i*80));...
      (-1/(50-i*80)) (1/(i*50)+1/(50-i*80))];
I = [pin(1,60); pin(2,30)];
V = inv(Y)*I;
pout(V(1))
pout(V(2))
```

and the results are $\mathbf{V}_1 = 79.98\ \underline{/106.21^\circ}$ and $\mathbf{V}_2 = 124.13\ \underline{/116.30^\circ}$. □

Summary

1. A sinusoidal voltage is given by $v(t) = V_m \cos(\omega t + \theta)$, where V_m is the peak value of the voltage, ω is the angular frequency in radians per second, and θ is the phase angle. The frequency in hertz is $f = 1/T$, where T is the period. Furthermore, $\omega = 2\pi f$.
2. For uniformity, we express sinusoidal voltages in terms of the cosine function. A sine function can be converted to a cosine function by use of the identity $\sin(z) = \cos(z - 90^\circ)$.
3. The root-mean-square (rms) value (or effective value) of a periodic voltage $v(t)$ is

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

The average power delivered to a resistance by $v(t)$ is

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R}$$

Similarly, for a current $i(t)$, we have

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

and the average power delivered if $i(t)$ flows through a resistance is

$$P_{\text{avg}} = I_{\text{rms}}^2 R$$

For a sinusoid, the rms value is the peak value divided by $\sqrt{2}$.

4. We can represent sinusoids with phasors. The magnitude of the phasor is the peak value of the sinusoid. The phase angle of the phasor is the phase angle of the sinusoid (assuming that we have written the sinusoid in terms of a cosine function).
5. We can add (or subtract) sinusoids by adding (or subtracting) their phasors.
6. The phasor voltage for a passive circuit is the phasor current times the complex impedance of the circuit. For a resistance, $\mathbf{V}_R = R\mathbf{I}_R$, and the voltage is in phase with the current. For an

inductance, $\mathbf{V}_L = j\omega L\mathbf{I}_L$, and the voltage leads the current by 90°. For a capacitance, $\mathbf{V}_C = -j(1/\omega C)\mathbf{I}_C$, and the voltage lags the current by 90°.

7. Many techniques learned in Chapter 2 for resistive circuits can be applied directly to sinusoidal circuits if the currents and voltages are replaced by phasors and the passive circuit elements are replaced by their complex impedances. For example, complex impedances can be combined in series or parallel in the same way as resistances (except that complex arithmetic must be used). Node voltages, the current-division principle, and the voltage-division principle also apply to ac circuits.

8. When a sinusoidal current flows through a sinusoidal voltage, the average power delivered is $P = V_{\text{rms}} I_{\text{rms}} \cos(\theta)$, where θ is the power angle, which is found by subtracting the phase angle of the current from the phase angle of the voltage (i.e., $\theta = \theta_v - \theta_i$). The power factor is $\cos(\theta)$.

9. Reactive power is the flow of energy back and forth between the source and energy-storage elements (L and C). We define reactive power to be positive for an inductance and negative for a capacitance. The net energy transferred per cycle by reactive power flow is zero. Reactive power is important because a power distribution system must have higher current ratings if reactive power flows than would be required for zero reactive power.

10. Apparent power is the product of rms voltage and rms current. Many useful relationships between power, reactive power, apparent power, and the power angle can be obtained from the power triangles shown in Figure 5.23 on page 236.

11. In steady state, a network composed of resistances, inductances, capacitances, and sinusoidal sources (all of the same frequency) has a Thévenin equivalent consisting of a phasor voltage source in series with a complex impedance. The Norton equivalent consists of a phasor current source in parallel with the Thévenin impedance.

12. For maximum-power transfer from a two-terminal ac circuit to a load, the load impedance is selected to be the complex conjugate of the Thévenin impedance. If the load is constrained to be a pure resistance, the value for maximum power transfer is equal to the magnitude of the Thévenin impedance.

13. Because of savings in wiring, three-phase power distribution is more economical than single phase. The power flow in balanced three-phase systems is smooth, whereas power pulsates in single-phase systems. Thus, three-phase motors generally have the advantage of producing less vibration than single-phase motors.

Problems

Section 5.1: Sinusoidal Currents and Voltages

P5.1. Give the units for angular frequency, ω, and frequency, f. What is the relationship between f and ω?

P5.2. In terms of physical units, such as m, kg, C, and s, what are the units of radians? What are the *physical* units for angular frequency?

P5.3. Consider the plot of the sinusoidal voltage $v(t) = V_m \cos(\omega t + \theta)$ shown in Figure 5.1 on page 210. Which of the numbered statements below best describes **a.** Increasing the peak amplitude V_m? **b.** Increasing the frequency f? **c.** Increasing θ? **d.** Decreasing the angular frequency ω? **e.** Decreasing the period?

1. Stretches the sinusoidal curve horizontally.
2. Compresses the sinusoidal curve horizontally.
3. Translates the sinusoidal curve to the right.
4. Translates the sinusoidal curve to the left.
5. Stretches the sinusoidal curve vertically.
6. Compresses the sinusoidal curve vertically.

* Denotes that answers are contained in the Student Solutions files. See Appendix E for more information about accessing the Student Solutions.

***P5.4.** A voltage is given by $v(t) = 10\sin(1000\pi t + 30°)$ V. First, use a cosine function to express $v(t)$. Then, find the angular frequency, the frequency in hertz, the phase angle, the period, and the rms value. Find the power that this voltage delivers to a 50-Ω resistance. Find the first value of time after $t = 0$ that $v(t)$ reaches its peak value. Sketch $v(t)$ to scale versus time.

P5.5. Repeat Problem P5.4 for $v(t) = 5\sin(500\pi t + 120°)$.

***P5.6.** A sinusoidal voltage $v(t)$ has an rms value of 20 V, a period of 100 μs, and reaches a positive peak at $t = 20$ μs. Write an expression for $v(t)$.

P5.7. We have a sinusoidal current $i(t)$ that has an rms value of 20 A, a period of 1 ms, and reaches a positive peak at $t = 0.3$ ms. Write an expression for $i(t)$.

P5.8. The voltage $v(t) = 10\sin(250\pi t)$ V appears across a 20-Ω resistance. Sketch $v(t)$ and $p(t)$ to scale versus time. Find the average power delivered to the resistance.

P5.9. A sinusoidal voltage has a peak value of 15 V, a frequency of 500 Hz, and crosses zero with positive slope at $t = 0.1$ ms. Write an expression for the voltage.

P5.10. A current $i(t) = 2\cos(1000\pi t)$ A flows through a 5-Ω resistance. Sketch $i(t)$ and $p(t)$ to scale versus time. Find the average power delivered to the resistance.

P5.11. Is the rms value of a periodic waveform always equal to the peak value divided by the square root of two? When is it?

***P5.12.** Find the rms value of the current waveform shown in Figure P5.12.

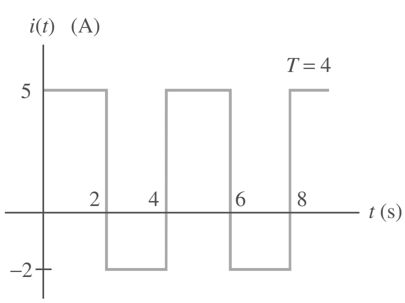


Figure P5.12

P5.13. Find the rms value of the voltage waveform shown in Figure P5.13.

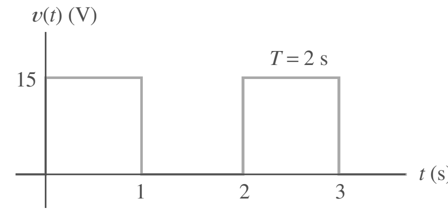


Figure P5.13

P5.14. Compute the rms value of the periodic waveform shown in Figure P5.14.

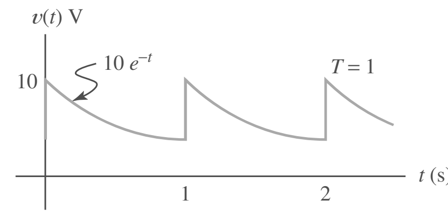


Figure P5.14

P5.15. Determine the rms value of $v(t) = A\cos(2\pi t) + 2B\sin(2\pi t)$.

P5.16. Calculate the rms value of the full-wave rectified sinusoidal wave shown in Figure P5.16, which is given by $v(t) = 4|\cos(20\pi t)|$.

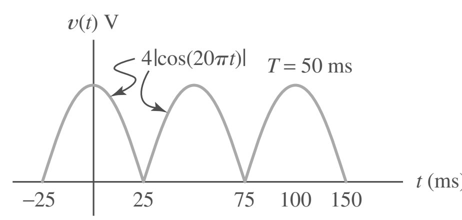


Figure P5.16

P5.17. Find the rms value of the voltage waveform shown in Figure P5.17.

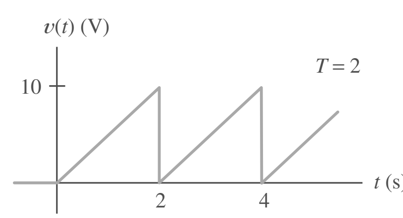


Figure P5.17

P5.18. Determine the rms value of $v(t) = 3 + 4\sqrt{2}\cos(20\pi t)$ V.

Section 5.2: Phasors

P5.19. What are the steps we follow in adding sinusoidal currents or voltages? What must be true of the sinusoids?

P5.20. **a.** Explain how the phase relationship between two sinusoids of the same frequency can be determined working from the phasor diagram. **b.** Explain how to determine the phase relationship between two sinusoids of the same frequency working from plots of the sinusoids versus time.

***P5.21.** Reduce $5\cos(\omega t + 75°) - 3\cos(\omega t - 75°) + 4\sin(\omega t)$ to the form $V_m\cos(\omega t + \theta)$.

***P5.22.** Suppose that $v_1(t) = 100\cos(\omega t)$ and $v_2(t) = 100\sin(\omega t)$. Use phasors to reduce the sum $v_s(t) = v_1(t) + v_2(t)$ to a single term of the form $V_m\cos(\omega t + \theta)$. Draw a phasor diagram, showing $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_s$. State the phase relationships between each pair of these phasors.

***P5.23.** Consider the phasors shown in Figure P5.23. The frequency of each signal is $f = 200$ Hz. Write a time-domain expression for each voltage in the form $V_m\cos(\omega t + \theta)$. State the phase relationships between pairs of these phasors.

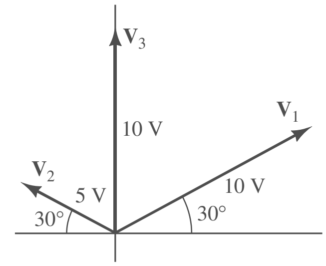


Figure P5.23

P5.24. Suppose we have two sinusoidal voltages of the same frequency with rms values of 12 V and 7 V, respectively. The phase angles are unknown. What is the smallest rms value that the sum of these voltages could have? The largest? Justify your answers.

P5.25. Find an expression for the sinusoid shown in Figure P5.25 of the form $v(t) = V_m\cos(\omega t + \theta)$, giving the numerical values of V_m, ω, and θ. Also, determine the phasor and the rms value of $v(t)$.

P5.26. A sinusoidal current $i_1(t)$ has a phase angle of $-30°$. Furthermore, $i_1(t)$ attains its positive peak 0.25 ms later than current $i_2(t)$ does. Both the currents have a frequency of 1000 Hz. Determine the phase angle of $i_2(t)$.

P5.27. Suppose that $v_1(t) = 90\cos(\omega t - 15°)$ and $v_2(t) = 50\sin(\omega t - 60°)$. Use phasors to

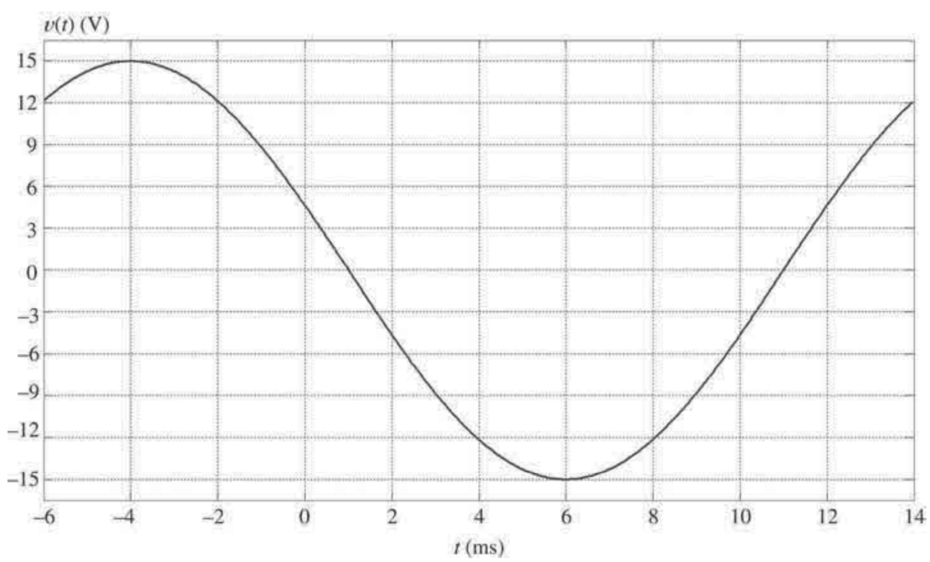


Figure P5.25

reduce the difference $v_s(t) = v_1(t) - v_2(t)$ to a single term of the form $V_m \cos(\omega t + \theta)$. State the phase relationships between each pair of these phasors.

P5.28. Reduce the expression

$$5\sin(\omega t+45^\circ)+15\cos(\omega t-30^\circ)+10\cos(\omega t-120^\circ)$$

to the form $V_m \cos(\omega t + \theta)$.

P5.29. Suppose we have a circuit in which the voltage is $v_1(t) = 15\cos(\omega t + 30^\circ)$ V. Furthermore, the current $i_1(t)$ has an rms value of 5 A and lags $v_1(t)$ by 40°. (The current and the voltage have the same frequency.) Draw a phasor diagram and write an expression for $i_1(t)$ of the form $I_m \cos(\omega t + \theta)$.

Section 5.3: Complex Impedances

P5.30. What is the phase relationship between current and voltage for a pure resistance? For an inductance? For a capacitance?

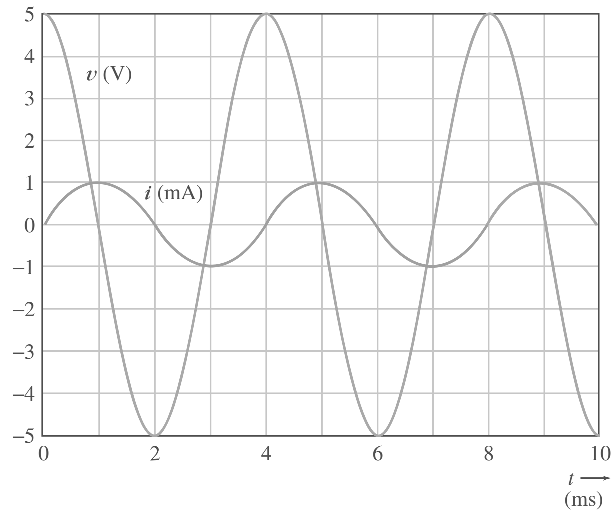
P5.31. Write the relationship between the phasor voltage and phasor current for an inductance. Repeat for capacitance. Repeat for a resistance.

***P5.32.** A voltage $v_L(t) = 10\cos(2000\pi t)$ is applied to a 100-mH inductance. Find the complex impedance of the inductance. Find the phasor voltage and current, and construct a phasor diagram. Write the current as a function of time. Sketch the voltage and current to scale versus time. State the phase relationship between the current and voltage.

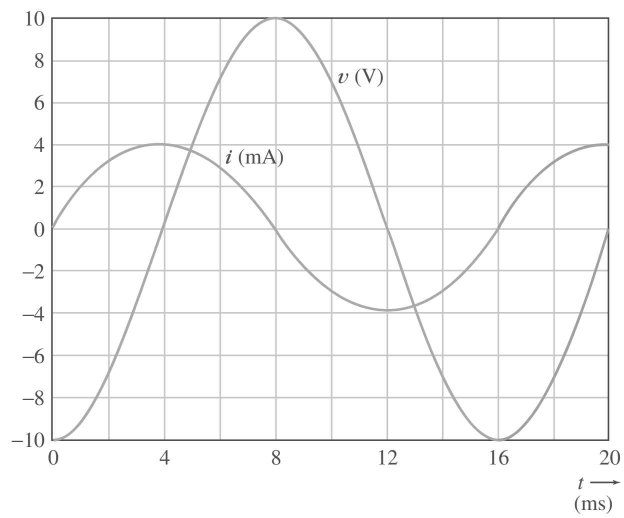
***P5.33.** A voltage $v_C(t) = 10\cos(2000\pi t)$ is applied to a 10-μF capacitance. Find the complex impedance of the capacitance. Find the phasor voltage and current, and construct a phasor diagram. Write the current as a function of time. Sketch the voltage and current to scale versus time. State the phase relationship between the current and voltage.

P5.34. A certain circuit element is known to be a pure resistance, a pure inductance, or a pure capacitance. Determine the type and value (in ohms, henrys, or farads) of the element if the voltage and current for the element are given by: **a.** $v(t) = 100\cos(200t + 30^\circ)$ V, $i(t) = 2.5\sin(200t + 30^\circ)$ A; **b.** $v(t) = 100\sin(200t + 30^\circ)$ V, $i(t) = 4\cos(200t + 30^\circ)$ A; **c.** $v(t) = 100\cos(100t + 30^\circ)$ V, $i(t) = 5\cos(100t + 30^\circ)$ A.

P5.35. **a.** The current and voltage for a certain circuit element are shown in Figure P5.35(a). Determine the nature and value of the element. **b.** Repeat for Figure P5.35(b).



(a)



(b)

Figure P5.35

P5.36. **a.** A certain element has a phasor voltage of $\mathbf{V} = 100\angle 30^\circ$ V and current of $\mathbf{I} = 5\angle 120^\circ$ A. The angular frequency is 1000 rad/s. Determine the nature and value of the element. **b.** Repeat for $\mathbf{V} = 20\angle -45^\circ$ V and current of $\mathbf{I} = 2\angle -135^\circ$ A. **c.** Repeat for $\mathbf{V} = 50\angle 45^\circ$ V and current of $\mathbf{I} = 10\angle 45^\circ$ A.

Section 5.4: Circuit Analysis with Phasors and Complex Impedances

P5.37. Explain the step-by-step procedure for steady-state analysis of circuits with sinusoidal sources. What condition must be true of the sources?

***P5.38.** Find the phasors for the current and for the voltages of the circuit shown in Figure P5.38. Construct a phasor diagram showing $\mathbf{V}_s$, $\mathbf{I}$, $\mathbf{V}_R$, and $\mathbf{V}_L$. What is the phase relationship between $\mathbf{V}_s$ and $\mathbf{I}$?

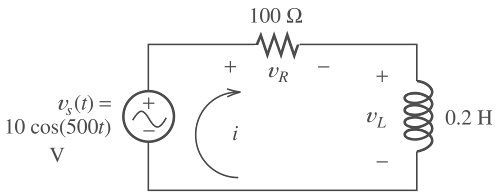


Figure P5.38

P5.39. Change the inductance to 0.4 H, and repeat Problem P5.38.

***P5.40.** Find the phasors for the current and the voltages for the circuit shown in Figure P5.40. Construct a phasor diagram showing $\mathbf{V}_s$, $\mathbf{I}$, $\mathbf{V}_R$, and $\mathbf{V}_C$. What is the phase relationship between $\mathbf{V}_s$ and $\mathbf{I}$?

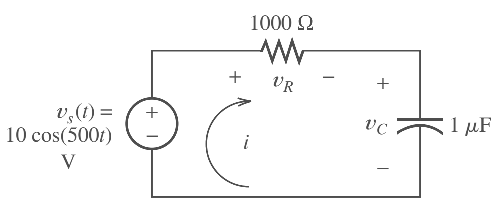


Figure P5.40

P5.41. Repeat Problem P5.40, changing the capacitance value to 2 μF.

***P5.42.** Find the complex impedance in polar form of the network shown in Figure P5.42 for $\omega = 500$. Repeat for $\omega = 1000$ and $\omega = 2000$.

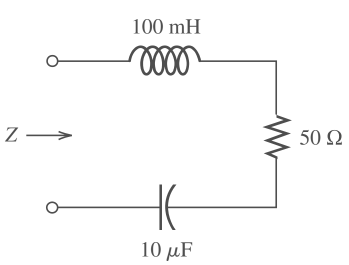


Figure P5.42

P5.43. Compute the complex impedance of the network shown in Figure P5.43 for $\omega = 500$. Repeat for $\omega = 1000$ and $\omega = 2000$. Give the answers in both polar and rectangular forms.

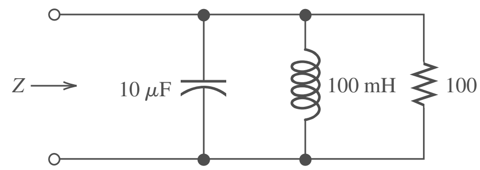


Figure P5.43

P5.44. A 100-μF capacitance is connected in parallel with the series combination of a 10-mH inductance and a 5-Ω resistance. Calculate the impedance of the combination in rectangular form and in polar form for angular frequencies of 500, 1000, and 2000 radians per second.

***P5.45.** Consider the circuit shown in Figure P5.45. Find the phasors $\mathbf{I}_s$, $\mathbf{V}$, $\mathbf{I}_R$, $\mathbf{I}_L$, and $\mathbf{I}_C$. Compare the peak value of $i_L(t)$ with the peak value of $i_s(t)$. Do you find the answer surprising? Explain.

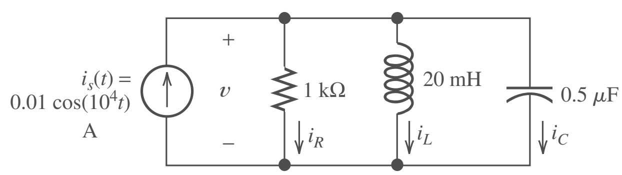


Figure P5.45

P5.46. Find the phasors for the voltage and the currents of the circuit shown in Figure P5.46. Construct a phasor diagram showing $\mathbf{I}_s$, $\mathbf{V}$, $\mathbf{I}_R$, and $\mathbf{I}_L$. What is the phase relationship between $\mathbf{V}$ and $\mathbf{I}_s$?

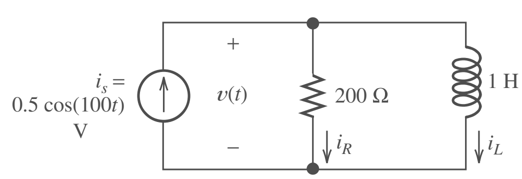


Figure P5.46

P5.47. Consider the circuit shown in Figure P5.47. Find the phasors $\mathbf{V}_s$, $\mathbf{I}$, $\mathbf{V}_L$, $\mathbf{V}_R$, and $\mathbf{V}_C$ in polar form. Compare the peak value of $v_L(t)$ with the peak value of $v_s(t)$. Do you find the answer surprising? Explain.

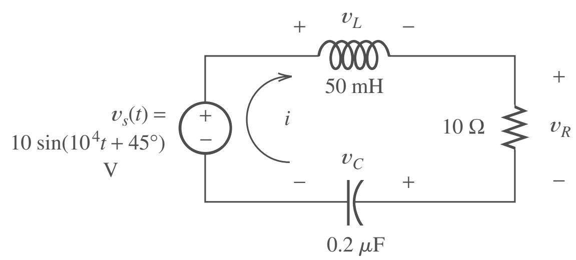


Figure P5.47

***P5.48.** Find the phasors for the voltage and the currents for the circuit shown in Figure P5.48. Construct a phasor diagram showing $\mathbf{I}_s$, $\mathbf{V}$, $\mathbf{I}_R$, and $\mathbf{I}_C$. What is the phase relationship between $\mathbf{V}$ and $\mathbf{I}_s$?

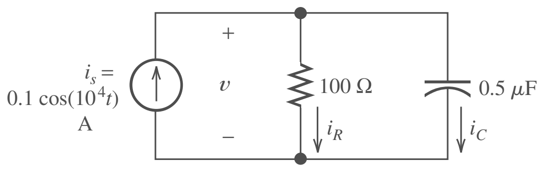


Figure P5.48

P5.49. Consider the circuit shown in Figure P5.49. Find the phasors $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_R, \mathbf{V}_L$, and $\mathbf{I}$. Draw the phasor diagram to scale. What is the phase relationship between $\mathbf{I}$ and $\mathbf{V}_1$? Between $\mathbf{I}$ and $\mathbf{V}_L$?

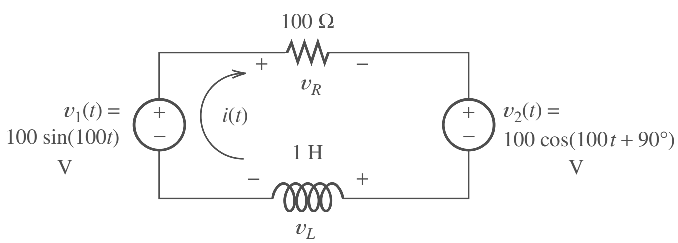


Figure P5.49

P5.50. Solve for the node voltage shown in Figure P5.50.

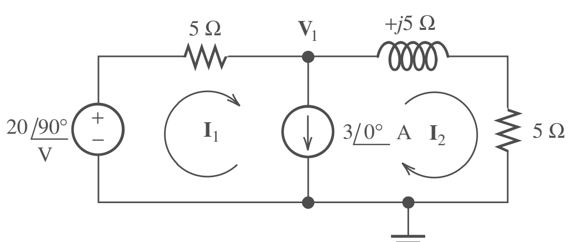


Figure P5.50

P5.51. Find the phasors $\mathbf{I}, \mathbf{I}_R$, and $\mathbf{I}_C$ for the circuit shown in Figure P5.51. Also, draw the phasor diagram for the currents.

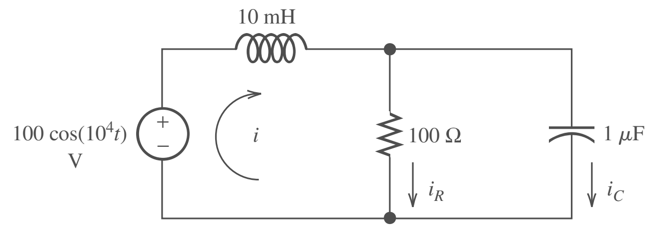


Figure P5.51

P5.52. **a.** At what frequency or frequencies is the series combination of elements shown in Figure P5.52 equivalent to an open circuit? A short circuit? **b.** Repeat with the elements in parallel.

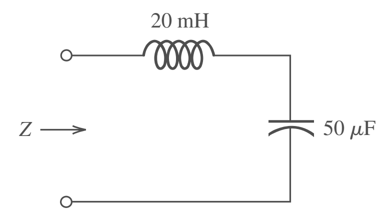


Figure P5.52

P5.53. Solve for the node voltage shown in Figure P5.53.

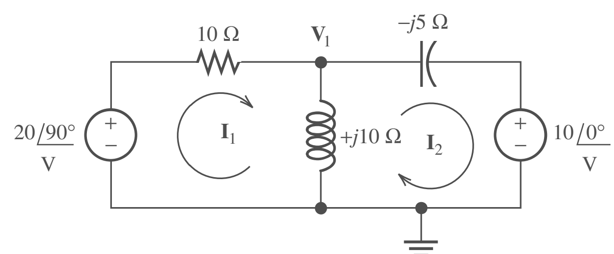


Figure P5.53

P5.54. **a.** At what frequency or frequencies is the series combination of elements shown in Figure P5.54 equivalent to an open circuit? A short circuit? **b.** Repeat with the elements in parallel.

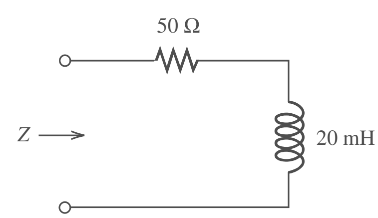


Figure P5.54

Section 5.5: Power in AC Circuits

P5.55. What are the customary units for real power? For reactive power? For apparent power?

P5.56. How do we compute the complex power delivered to a circuit component? How are average power and reactive power related to complex power?

P5.57. Explain how power factor and power angle are related. How is the power angle determined if we know the phasors for the current through and voltage across a load?

P5.58. A load is said to have a leading power factor. Is it capacitive or inductive? Is the reactive power positive or negative? Repeat for a load with lagging power factor.

P5.59. Assuming that a nonzero ac source is applied, state whether the power and reactive power are positive, negative, or zero for: **a.** a pure resistance; **b.** a pure inductance; **c.** a pure capacitance.

P5.60. Define what we mean by "power-factor correction." For power-factor correction of an inductive load, what type of element should we place in parallel with the load?

P5.61. **a.** Sketch a power triangle for an inductive load, label the sides, and show the power angle. **b.** Repeat for a capacitive load.

P5.62. Discuss why power plant and distribution system engineers are concerned with: **a.** the real power absorbed by a load; **b**. with the reactive power.

***P5.63.** Consider the circuit shown in Figure P5.63. Find the phasor current **I**. Find the power, reactive power, and apparent power delivered by the source. Find the power factor and state whether it is lagging or leading.

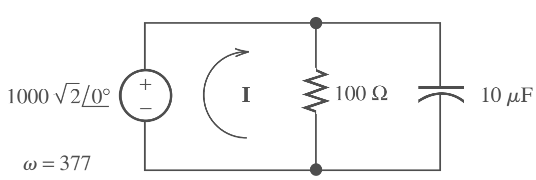


Figure P5.63

P5.64. Repeat Problem P5.63, replacing the capacitance by a 1-H inductance.

***P5.65.** Consider a load that has an impedance given by $Z = 100 - j50\ \Omega$. The current flowing through this load is $\mathbf{I} = 15\sqrt{2}\ \angle 30^\circ$ A. Is the load inductive or capacitive? Determine the power factor, power, reactive power, and apparent power delivered to the load.

P5.66. Determine the complex power, power factor, power, reactive power, and apparent power delivered to the load shown in Figure P5.66, given that $\mathbf{I} = 15\sqrt{2}\angle 75^\circ$ A. Also, determine the load impedance. Is the power factor leading or lagging?

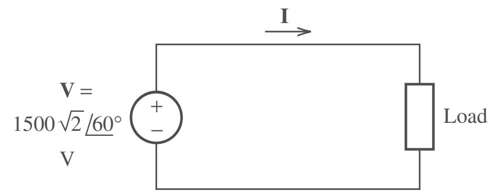


Figure P5.66

P5.67. Determine the power for each element, including the sources, shown in Figure P5.67. Also, state whether each element is delivering or absorbing energy.

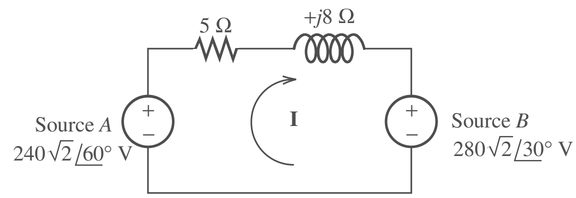


Figure P5.67

P5.68. Suppose that the load impedance shown in Figure P5.66 is given by $Z = 40 - j30\Omega$. (This is a different value from that of Problem P5.66.) Is the load inductive or capacitive? Determine the power factor, complex power, real power, reactive power, and apparent power delivered to the load.

P5.69. The voltage across a load is $v(t) = 10^4\sqrt{2}\cos(\omega t + 75^\circ)$ V, and the current through the load is $i(t) = 25\sqrt{2}\cos(\omega t + 30^\circ)$ A. The reference direction for the current points into the positive reference for the voltage. Determine the complex power, the power factor, the real power, the reactive power, and the apparent power of the load. Is this load inductive or capacitive?

P5.70. Given that a nonzero ac voltage source is applied, state whether the power and reactive power are positive, negative, or zero for: **a.** a pure capacitance; **b.** a resistance in series with an inductance; **c.** a resistance in series with a capacitance; **d.** a pure resistance. (Assume that the resistances, inductance, and capacitance are nonzero and finite in value.)

P5.71. Given that a nonzero ac voltage source is applied, what can you say about whether the power and reactive power are positive, negative, or zero for a pure capacitance in series with a pure inductance? Consider cases in which the impedance magnitude of the capacitance is greater than, equal to, or less than the impedance magnitude of the inductance.

P5.72. Repeat Problem P5.71 for the inductance and capacitance in parallel.

P5.73. A 60-Hz 240-V-rms source supplies power to a load consisting of a resistance in series with an inductance. The real power is 1500 W, and the apparent power is 2500 VA. Determine the value of the resistance and the value of the inductance.

P5.74. Determine the power for each element, including the sources, shown in Figure P5.74. Also, state whether each element is delivering or absorbing energy.

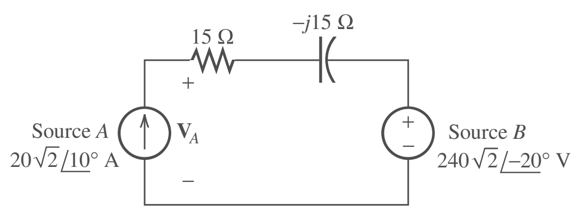


Figure P5.74

***P5.75.** Two loads—A and B—are connected in parallel across a 1-kV rms 60-Hz line, as shown in Figure P5.75. Load A consumes 10 kW with a 90-percent-lagging power factor. Load B has an apparent power of 15 kVA with an 80-percent-lagging power factor. Find the power, reactive power, and apparent power delivered by the source. What is the power factor seen by the source?

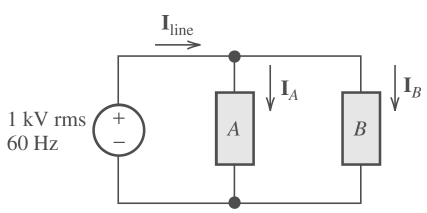


Figure P5.75

P5.76. Repeat Problem P5.75 given that load A consumes 50 kW with a 60-percent-lagging power factor and load B consumes 75 kW with an 80-percent-leading power factor.

P5.77. Find the power, reactive power, and apparent power delivered by the source in Figure P5.77. Find the power factor seen by the source and state whether it is leading or lagging.

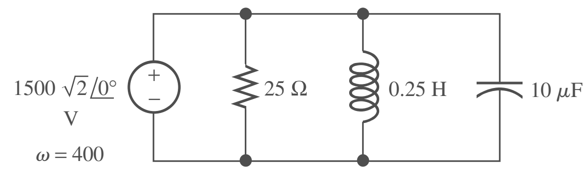


Figure P5.77

P5.78. Repeat Problem P5.77 with the resistance, inductance, and capacitance connected in series rather than in parallel.

***P5.79.** Consider the situation shown in Figure P5.79. A 1000-V rms source delivers power to a load. The load consumes 100 kW with a power factor of 25 percent lagging. **a.** Find the phasor **I**, assuming that the capacitor is not connected to the circuit. **b.** Find the value of the capacitance that must be connected in parallel with the load to achieve a power factor of 100 percent. Usually, power-systems engineers rate capacitances used for power-factor correction in terms of their reactive power rating. What is the rating of this capacitance in kVAR? Assuming that this capacitance is connected, find the new value for the phasor **I**. **c.** Suppose that the source is connected to the load by a long distance. What are the potential advantages and disadvantages of connecting the capacitance across the load?

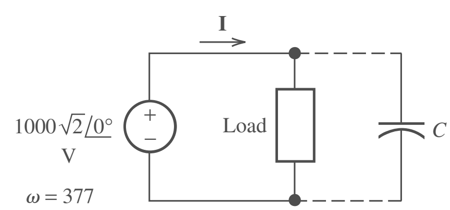


Figure P5.79

Section 5.6: Thévenin and Norton Equivalent Circuits

P5.80. Of what does an ac steady-state Thévenin equivalent circuit consist? A Norton equivalent circuit? How are the values of the parameters of these circuits determined?

P5.81. For an ac circuit consisting of a load connected to a Thévenin circuit, is it possible for the load voltage to exceed the Thévenin voltage in magnitude? If not, why not? If so, under what conditions is it possible? Explain.

P5.82. To attain maximum power delivered to a load, what value of load impedance is required if: **a.** the load can have any complex value; **b.** the load must be pure resistance?

***P5.83.** **a.** Find the Thévenin and Norton equivalent circuits for the circuit shown in Figure P5.83. **b.** Find the maximum power that this circuit can deliver to a load if the load can have any complex impedance; **c.** if the load is purely resistive.

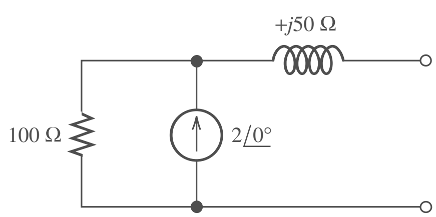


Figure P5.83

P5.84. Draw the Thévenin and Norton equivalent circuits for Figure P5.84, labeling the elements and terminals.

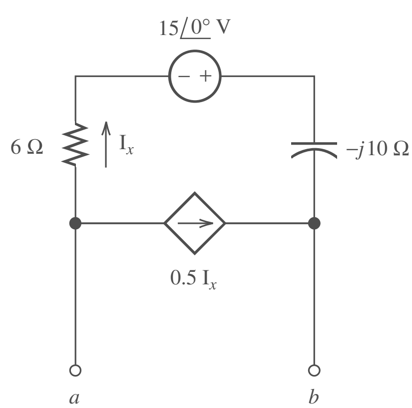


Figure P5.84

P5.85. Find the Thévenin voltage, Thévenin impedance, and Norton current for the two-terminal circuit shown in Figure P5.85.

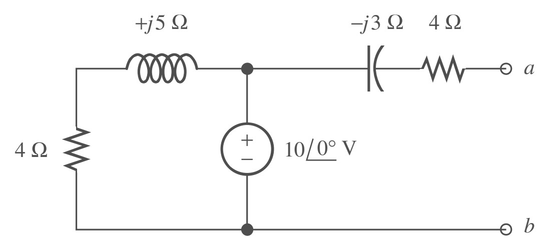


Figure P5.85

P5.86. Find the Thévenin and Norton equivalent circuits for the circuit shown in Figure P5.86. Find the maximum power that this circuit can deliver to a load if the load can have any complex impedance. Repeat if the load must be purely resistive.

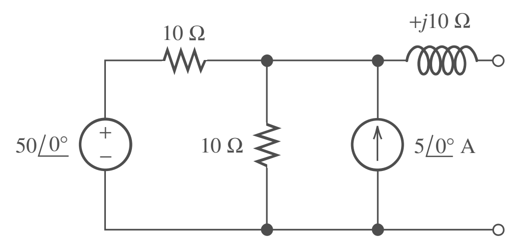


Figure P5.86

***P5.87.** The Thévenin equivalent of a two-terminal network is shown in Figure P5.87. The frequency is $f = 60$ Hz. We wish to connect a load across terminals a–b that consists of a resistance and a capacitance in parallel such that the power delivered to the resistance is maximized. Find the value of the resistance and the value of the capacitance.

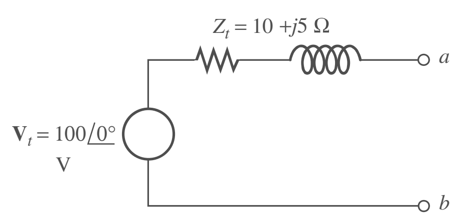


Figure P5.87

P5.88. Repeat Problem P5.87 with the load required to consist of a resistance and a capacitance in series.

Section 5.7: Balanced Three-Phase Circuits

P5.89. A positive-sequence three-phase source has $v_{an}(t) = 200\cos(\omega t + 120°)$ V. Find time-domain expressions for $v_{bn}(t)$, $v_{cn}(t)$, $v_{ab}(t)$, $v_{bc}(t)$, and $v_{ca}(t)$ and sketch their phasor diagram.

P5.90. We have a balanced positive-sequence three-phase source for which

$$v_{an}(t) = 150\cos(400\pi t + 15^\circ)\ \text{V}$$

a. Find the frequency of this source in Hz. **b.** Give expressions for $v_{bn}(t)$ and $v_{cn}(t)$. **c.** Repeat part (b) for a negative-sequence source.

***P5.91.** Each phase of a wye-connected load consists of a 50-Ω resistance in parallel with a 100-μF capacitance. Find the impedance of each phase of an equivalent delta-connected load. The frequency of operation is 60 Hz.

***P5.92.** A balanced wye-connected three-phase source has line-to-neutral voltages of 440 V rms. Find the rms line-to-line voltage magnitude. If this source is applied to a wye-connected load composed of three 30-Ω resistances, find the rms line-current magnitude and the total power delivered.

P5.93. What can you say about the flow of power as a function of time between a balanced three-phase source and a balanced load? Is this true of a single-phase source and a load? How is this a potential advantage for the three-phase system? What is another advantage of three-phase power distribution compared with single phase?

P5.94. A delta-connected source delivers power to a delta-connected load, as shown in Figure P5.94. The rms line-to-line voltage at the source is $V_{ab\text{rms}} = 440$ V. The load impedance is $Z_\Delta = 12 + j3$. Find $\mathbf{I}_{aA}, \mathbf{V}_{AB}, \mathbf{I}_{AB}$, the total power delivered to the load, and the power lost in the line.

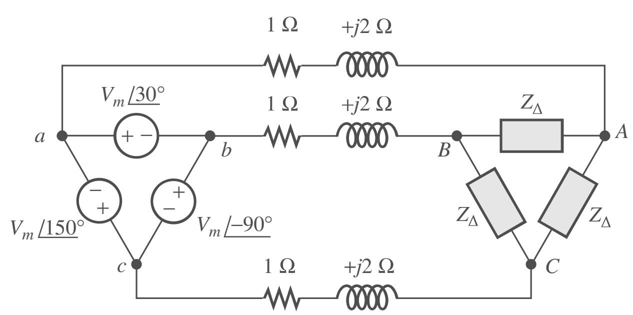


Figure P5.94

***P5.95.** Repeat Problem P5.94, with $Z_\Delta = 15 - j6$.

P5.96. A balanced wye-connected three-phase source has line-to-neutral voltages of 277 V rms. Find the rms line-to-line voltage. This source is applied to a delta-connected load, each arm of which consists of a 15-Ω resistance in parallel with a $+j30$-Ω reactance. Determine the rms line current magnitude, the power factor, and the total power delivered.

P5.97. A negative-sequence wye-connected source has line-to-neutral voltages $V_{an} = V_Y \angle 180^\circ$, $V_{bn} = V_Y \angle -60^\circ$, and $V_{cn} = V_Y \angle 60^\circ$. Find the line-to-line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$. Construct a phasor diagram showing both sets of voltages and compare with Figure 5.41 on page 255.

P5.98. A balanced positive-sequence wye-connected 60-Hz three-phase source has line-to-line voltages of $V_L = 208$ V rms. This source is connected to a balanced wye-connected load. Each phase of the load consists of an impedance of $30 + j40$ Ω. Find the line-to-neutral voltage phasors, the line-to-line voltage phasors, the line-current phasors, the power, and the reactive power delivered to the load. Assume that the phase of $\mathbf{V}_{an}$ is zero.

P5.99. In this chapter, we have considered balanced loads only. However, it is possible to determine an equivalent wye for an unbalanced delta, and vice versa. Consider the equivalent circuits shown in Figure P5.99. Derive formulas for the impedances of the wye in terms of the impedances of the delta. (*Hint*: Equate the impedances between corresponding pairs of terminals of the two circuits with the third terminal open. Then, solve the equations for Z_a, Z_b, and Z_c in terms of Z_A, Z_B, and Z_C. Take care in distinguishing between upper- and lowercase subscripts.)

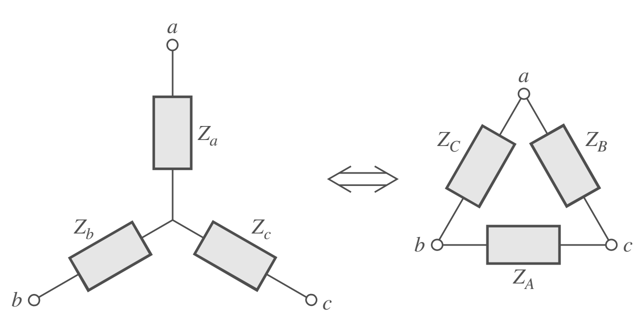


Figure P5.99

P5.100. Repeat Problem P5.99, but solve for the impedances of the delta in terms of those of the wye. [*Hint*: Start by working in terms of the admittances of the delta (Y_A, Y_B, and Y_C) and the impedances of the wye (Z_a, Z_b, and Z_c). Short terminals b and c for each circuit. Then, equate the admittances between terminal a and the shorted terminals for the two circuits. Repeat this twice more with shorts between the remaining two pairs of terminals. Solve the equations to determine Y_A, Y_B, and Y_C in terms of Z_a, Z_b, and Z_c. Finally, invert the equations for Y_A, Y_B, and Y_C to obtain equations relating the impedances. Take care in distinguishing between upper- and lowercase subscripts.]

Section 5.8: AC Analysis using MATLAB

***P5.101.** Use MATLAB to solve for the node voltages shown in Figure P5.101.

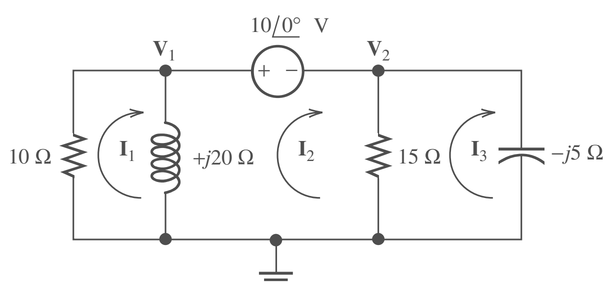


Figure P5.101

P5.102. Use MATLAB to solve for the mesh currents shown in Figure P5.101.

***P5.103.** Use MATLAB to solve for the mesh currents shown in Figure P5.53.

P5.104. Use MATLAB to solve for the mesh currents shown in Figure P5.50.

P5.105. Use MATLAB to solve for the node voltages shown in Figure P5.105.

P5.106. A **Lissajous figure** results if one sinusoid is plotted versus another. Consider $x(t) = \cos(\omega_x t)$ and $y(t) = \cos(\omega_y t + \theta)$. Use MATLAB to generate values of x and y for 20 seconds at 100 points per second and obtain a plot of y versus x for: **a.** $\omega_x = \omega_y = 2\pi$ and $\theta = 0°$; **b.** $\omega_x = \omega_y = 2\pi$ and $\theta = 45°$; **c.** $\omega_x = \omega_y = 2\pi$ and $\theta = 90°$; **d.** $\omega_x = 6\pi$, $\omega_y = 2\pi$, and $\theta = 90°$.

P5.107. Use the MATLAB Symbolic Toolbox to determine the rms value of $v(t)$ which has a period of 1 s and is given by $v(t) = 10\exp(-5t)\sin(20\pi t)$ V for $0 \le t \le 1$ s.

P5.108. Use MATLAB to obtain a plot of $v(t) = \cos(19\pi t) + \cos(21\pi t)$ for t ranging from 0 to 2 seconds. Explain why the terms in this expression cannot be combined by using phasors. Then, considering that the two terms can be represented as the real projection of the sum of two vectors rotating at different speeds in the complex plane, comment on the plot.

P5.109. Use MATLAB or manually produce plots of the magnitudes of the impedances of a 20-mH inductance, a 10-μF capacitance, and a 40-Ω resistance to scale versus frequency for the range from zero to 1000 Hz.

P5.110. **a.** Use MATLAB to produce a plot of the impedance magnitude versus angular frequency for the circuit of Figure P5.52. Allow ω to range from zero to 2000 rad/s and the vertical axis to range from 0 to 100 Ω. **b.** Repeat with the inductance and capacitance in parallel.

P5.111. **a.** Use MATLAB to produce a plot of the impedance magnitude versus angular frequency for the circuit shown in Figure P5.54. Allow ω to range from zero to 5000 rad/s. **b.** Repeat with the inductance and resistance in parallel.

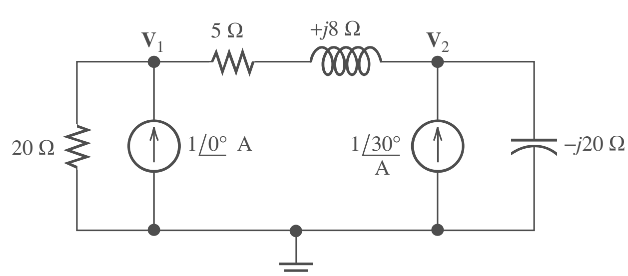


Figure P5.105

Practice Test

Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in Appendix D and complete solutions are included in the Student Solutions files. See Appendix E for more information about the Student Solutions.

T5.1. Determine the rms value of the current shown in Figure T5.1 and the average power delivered to the 50-Ω resistance.

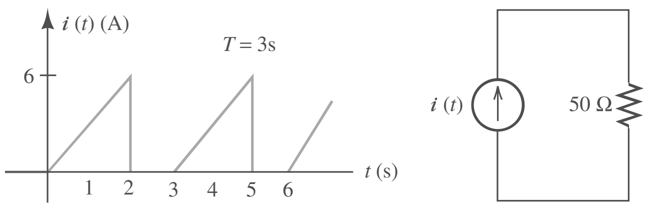


Figure T5.1

T5.2. Reduce the expression

$$v(t) = 5\sin(\omega t + 45°) + 5\cos(\omega t - 30°)$$

to the form $V_m \cos(\omega t + \theta)$.

T5.3. We have two voltages $v_1(t) = 15\sin(400\pi t + 45°)$ V and $v_2(t) = 5\cos(400\pi t - 30°)$ V. Determine (including units): **a.** the rms value of $v_1(t)$; **b.** the frequency of the voltages; **c.** the angular frequency of the voltages; **d.** the period of the voltages; **e.** the phase relationship between $v_1(t)$ and $v_2(t)$.

T5.4. Find the phasor values of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in polar form for the circuit of Figure T5.4.

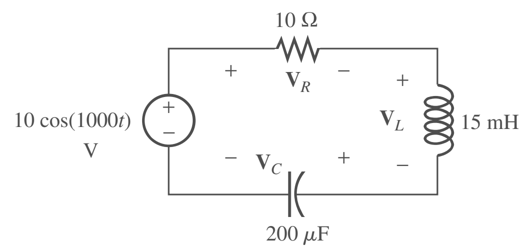


Figure T5.4

T5.5. Determine the complex power, power, reactive power, and apparent power absorbed by the load in Figure T5.5. Also, determine the power factor for the load.

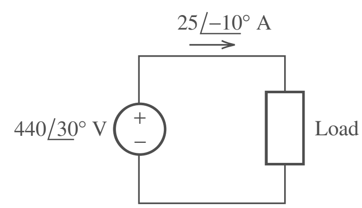


Figure T5.5

T5.6. Determine the line current $\mathbf{I}_{aA}$ in polar form for the circuit of Figure T5.6. This is a positive-sequence, balanced, three-phase system with $\mathbf{V}_{an} = 208\,\angle 30°$ V and $Z_\Delta = 6 + j8\ \Omega$.

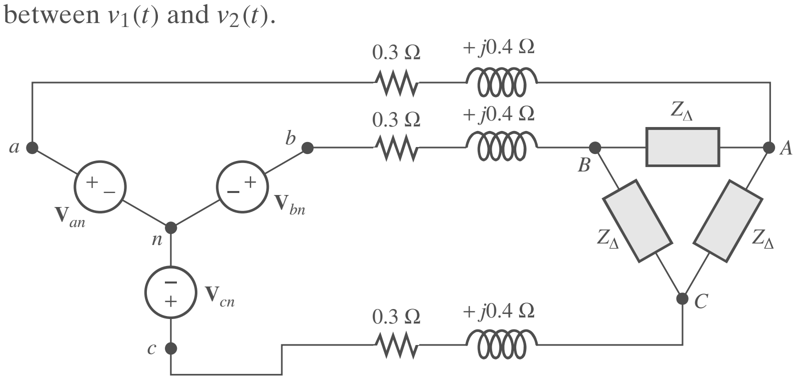


Figure T5.6

T5.7. Write the MATLAB commands to obtain the values of the mesh currents of Figure T5.7 in polar form. You may use the pin and pout functions defined in this chapter if you wish.

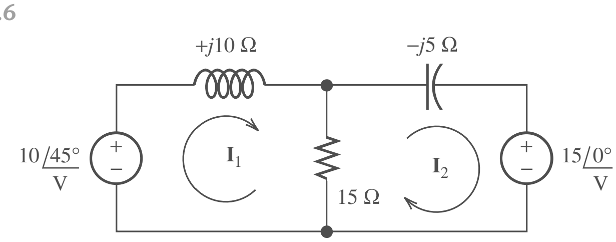


Figure T5.7

10.1 BASIC DIODE CONCEPTS

The diode is a basic but very important device that has two terminals, the **anode** and the **cathode**. The circuit symbol for a diode is shown in Figure 10.1(a), and a typical volt–ampere characteristic is shown in Figure 10.1(b). As shown in Figure 10.1(a), the voltage v_D across the diode is referenced positive at the anode and negative at the cathode. Similarly, the diode current i_D is referenced positive from anode to cathode.

Diodes readily conduct current from anode to cathode (in the direction of the arrow), but do not readily allow current to flow in the opposite direction.

Notice in the characteristic that if the voltage v_D applied to the diode is positive, relatively large amounts of current flow for small voltages. This condition is called **forward bias**. Thus, current flows easily through the diode in the direction of the arrowhead of the circuit symbol.

On the other hand, for moderate negative values of v_D, the current i_D is very small in magnitude. This is called the **reverse-bias region**, as shown on the diode characteristic. In many applications, the ability of the diode to conduct current easily in one direction, but not in the reverse direction, is very useful. For example, in an automobile, diodes allow current from the alternator to charge the battery when the engine is running. However, when the engine stops, the diodes prevent the battery from discharging through the alternator. In these applications, the diode is analogous to a one-way valve in a fluid-flow system, as illustrated in Figure 10.1(d).

If a sufficiently large reverse-bias voltage is applied to the diode, operation enters the reverse-breakdown region of the characteristic, and currents of large magnitude flow.

If a sufficiently large reverse-bias voltage is applied to the diode, operation enters the **reverse-breakdown region** of the characteristic, and currents of large magnitude flow. Provided that the power dissipated in the diode does not raise its temperature too high, operation in reverse breakdown is not destructive to the device. In fact, we

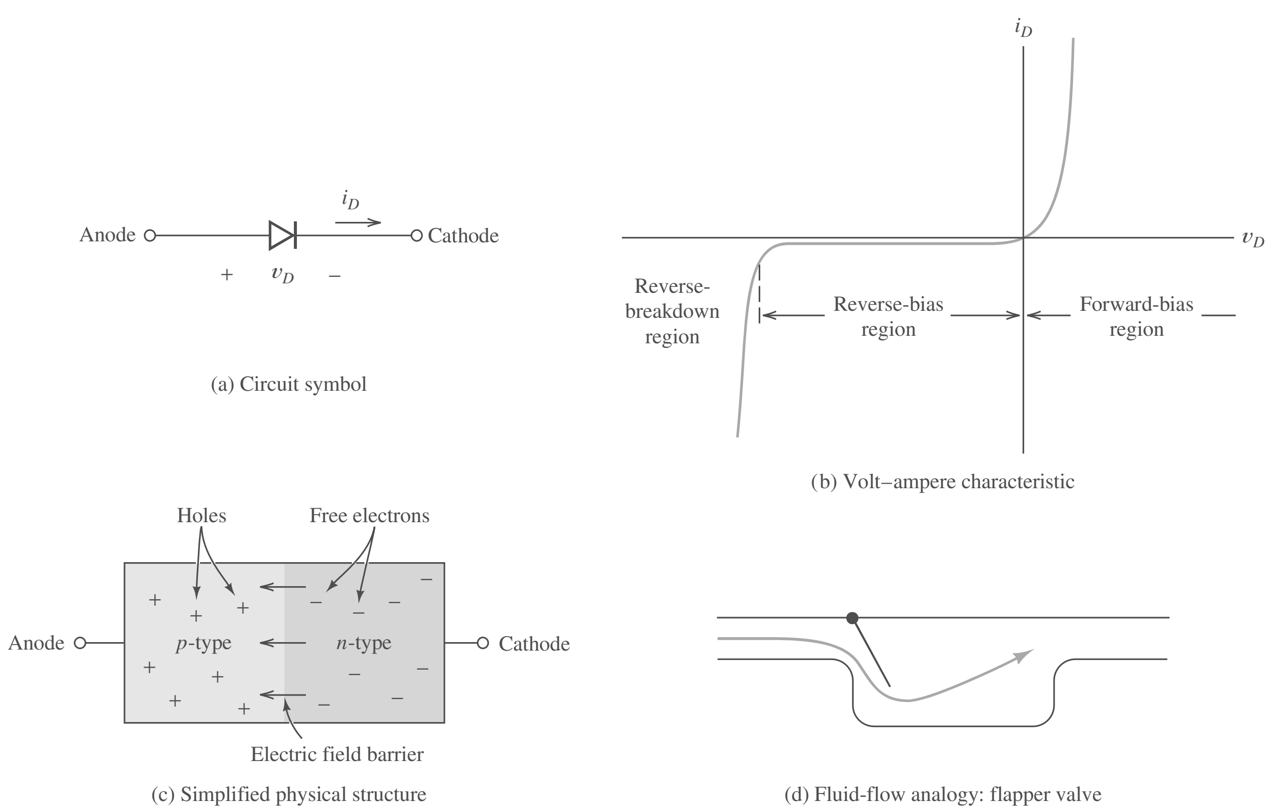


Figure 10.1 Semiconductor diode.

will see that diodes are sometimes deliberately operated in the reverse-breakdown region.

Brief Sketch of Diode Physics

We concentrate our discussion on the external behavior of diodes and some of their circuit applications. However, at this point, we give a thumbnail sketch of the internal physics of the diode.

The diodes that we consider consist of a junction between two types of semiconducting material (usually, silicon with carefully selected impurities). On one side of the junction, the impurities create ***n*-type material**, in which large numbers of electrons move freely. On the other side of the junction, different impurities are employed to create (in effect) positively charged particles known as **holes**. Semiconductor material in which holes predominate is called ***p*-type material**. Most diodes consist of a junction between *n*-type material and *p*-type material, as shown in Figure 10.1(c).

Even with no external applied voltage, an electric-field **barrier** appears naturally at the *pn* junction. This barrier holds the free electrons on the *n*-side and the holes on the *p*-side of the junction. If an external voltage is applied with positive polarity on the *n*-side, the barrier is enhanced and the charge carriers cannot cross the junction. Thus, virtually no current flows. On the other hand, if a voltage is applied with positive polarity on the *p*-side, the barrier is reduced and large currents cross the junction. Thus, the diode conducts very little current for one polarity and large current for the other polarity of applied voltage. The anode corresponds to the *p*-type material and the cathode is the *n*-side.

Small-Signal Diodes

Various materials and structures are used to fabricate diodes. For now, we confine our discussion to small-signal silicon diodes, which are the most common type found in low- and medium-power electronic circuits.

The characteristic curve of a typical small-signal silicon diode operated at a temperature of 300 K is shown in Figure 10.2. Notice that the voltage and current scales

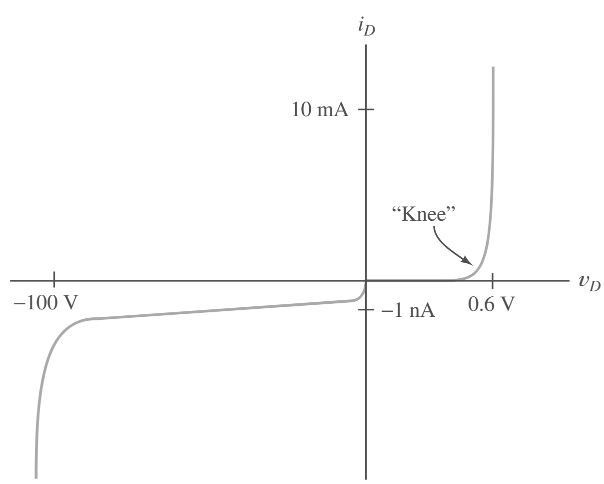


Figure 10.2 Volt–ampere characteristic for a typical small-signal silicon diode at a temperature of 300 K. Notice the change of scale for negative current and voltage.

for the forward-bias region are different than for the reverse-bias region. This is necessary for displaying details of the characteristic, because the current magnitudes are much smaller in the reverse-bias region than in the forward-bias region. Furthermore, the forward-bias voltage magnitudes are much less than typical breakdown voltages.

In the forward-bias region, small-signal silicon diodes conduct very little current (much less than 1 mA) until a forward voltage of about 0.6 V is applied (assuming that the diode is at a temperature of about 300 K). Then, current increases very rapidly as the voltage is increased. We say that the forward-bias characteristic displays a *knee* in the forward bias characteristic at about 0.6 V. (The exact value of the knee voltage depends on the device, its temperature, and the current magnitude. Typical values are 0.6 or 0.7 V.) As temperature is increased, the knee voltage decreases by about 2 mV/K. (Because of the linear change in voltage with temperature, diodes are useful as temperature sensors. The diode is operated at a fixed current, and the voltage across the diode depends on its temperature. Electronic thermometers used by physicians contain a diode sensor, amplifiers, and other electronic circuits that drive the liquid-crystal temperature display.)

In the reverse-bias region, a typical current is about 1 nA for small-signal silicon diodes at room temperature. As temperature increases, reverse current increases in magnitude. A rule of thumb is that the reverse current doubles for each 10-K increase in temperature.

When reverse breakdown is reached, current increases in magnitude very rapidly. The voltage for which this occurs is called the **breakdown voltage**. For example, the breakdown voltage of the diode characteristic shown in Figure 10.2 is approximately −100 V. Breakdown-voltage magnitudes range from several volts to several hundred volts. Some applications call for diodes that operate in the forward-bias and nonconducting reverse-bias regions without entering the breakdown region. Diodes intended for these applications have a specification for the minimum magnitude of the breakdown voltage.

Shockley Equation

Under certain simplifying assumptions, theoretical considerations result in the following relationship between current and voltage for a junction diode:

$$i_D = I_s\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right] \tag{10.1}$$

This is known as the **Shockley equation**. The **saturation current**, I_s, has a value on the order of 10^{-14} A for small-signal junction diodes at 300 K. (I_s depends on temperature, doubling for each 5-K increase in temperature for silicon devices.) The parameter n, known as the **emission coefficient**, takes values between 1 and 2, depending on details of the device structure. The voltage V_T is given by

$$V_T = \frac{kT}{q} \tag{10.2}$$

and is called the **thermal voltage**. The temperature of the junction in kelvin is represented by T. Furthermore, $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, and

$q = 1.60 \times 10^{-19}$ C is the magnitude of the electrical charge of an electron. At a temperature of 300 K, we have $V_T \cong 0.026$ V.

If we solve the Shockley equation for the diode voltage, we find that

$$v_D = nV_T \ln[\left(\frac{i_D}{I_s}\right) + 1] \tag{10.3}$$

For small-signal junction diodes operated at forward currents between 0.01 μA and 10 mA, the Shockley equation with n taken as unity is usually very accurate. Because the derivation of the Shockley equation ignores several phenomena, the equation is not accurate for smaller or larger currents. For example, under reverse bias, the Shockley equation predicts $i_D \cong -I_s$, but we usually find that the reverse current is much larger in magnitude than I_s (although still small). Furthermore, the Shockley equation does not account for reverse breakdown.

With forward bias of at least several tenths of a volt, the exponential in the Shockley equation is much larger than unity; with good accuracy, we have

$$i_D \cong I_s \exp\left(\frac{v_D}{nV_T}\right) \tag{10.4}$$

This approximate form of the equation is often easier to use.

Occasionally, we are able to derive useful analytical results for electronic circuits by use of the Shockley equation, but much simpler models for diodes are usually more useful.

Zener Diodes

Diodes that are intended to operate in the breakdown region are called **Zener diodes**. Zener diodes are useful in applications for which a constant voltage in breakdown is desirable. Therefore, manufacturers try to optimize Zener diodes for a nearly vertical characteristic in the breakdown region. The modified diode symbol shown in Figure 10.3 is used for Zener diodes. Zener diodes are available with breakdown voltages that are specified to a tolerance of ±5%.

Figure 10.3 Zener-diode symbol.

Exercise 10.1 At a temperature of 300 K, a certain junction diode has $i_D = 0.1$ mA for $v_D = 0.6$ V. Assume that n is unity and use $V_T = 0.026$ V. Find the value of the saturation current I_s. Then, compute the diode current at $v_D = 0.65$ V and at 0.70 V.

Answer $I_s = 9.50 \times 10^{-15}$ A, $i_D = 0.684$ mA, $i_D = 4.68$ mA. □

Exercise 10.2 Consider a diode under forward bias so that the approximate form of the Shockley equation (Equation 10.4) applies. Assume that $V_T = 0.026$ V and $n = 1$. **a.** By what increment must v_D increase to double the current? **b.** To increase the current by a factor of 10?

Answer **a.** $\Delta v_D = 18$ mV; **b.** $\Delta v_D = 59.9$ mV. □

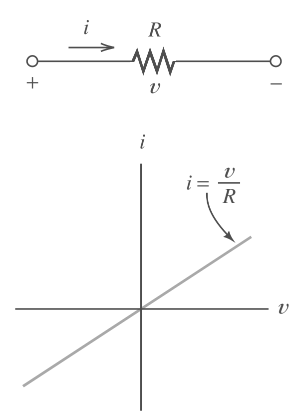


Figure 10.4 In contrast to diodes, resistors have linear volt–ampere characteristics.

~~10.2 LOAD-LINE ANALYSIS OF DIODE CIRCUITS~~

~~In Section 10.1, we learned that the volt–ampere characteristics of diodes are nonlinear. We will see shortly that other electronic devices are also nonlinear. On the other hand, resistors have linear volt–ampere characteristics, as shown in Figure 10.4.~~

Because of this nonlinearity, many of the techniques that we have studied for linear circuits in Chapters 1 through 6 do not apply to circuits involving diodes. In fact, much of the study of electronics is concerned with techniques for analysis of circuits containing nonlinear elements.

Graphical methods provide one approach to analysis of nonlinear circuits. For example, consider the circuit shown in Figure 10.5. By application of Kirchhoff's voltage law, we can write the equation

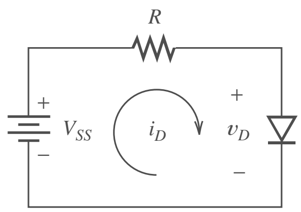


Figure 10.5 Circuit for load-line analysis.

$$V_{SS} = Ri_D + v_D \tag{10.5}$$

We assume that the values of V_{SS} and R are known and that we wish to find i_D and v_D. Thus, Equation 10.5 has two unknowns, and another equation (or its equivalent) is needed before a solution can be found. This is available in graphical form in Figure 10.6, which shows the volt–ampere characteristic of the diode.

We can obtain a solution by plotting Equation 10.5 on the same set of axes used for the diode characteristic. Since Equation 10.5 is linear, it plots as a straight line, which can be drawn if two points satisfying the equation are located. A simple method is to assume that $i_D = 0$, and then Equation 10.5 yields $v_D = V_{SS}$. This pair of values is shown as point A in Figure 10.6. A second point results if we assume that $v_D = 0$, for which the equation yields $i_D = V_{SS}/R$. The pair of values is shown as point B in Figure 10.6. Then, connecting points A and B results in a plot called the **load line**. The **operating point** is the intersection of the load line and the diode characteristic. This point represents the simultaneous solution of Equation 10.5 and the diode characteristic.

Example 10.1 Load Line Analysis

If the circuit of Figure 10.5 has $V_{SS} = 2$ V, $R = 1$ kΩ, and a diode with the characteristic shown in Figure 10.7, find the diode voltage and current at the operating point.

Solution First, we locate the ends of the load line. Substituting $v_D = 0$ and the values given for V_{SS} and R into Equation 10.5 yields $i_D = 2$ mA. These values plot as point B in Figure 10.7. Substitution of $i_D = 0$ and circuit values results in

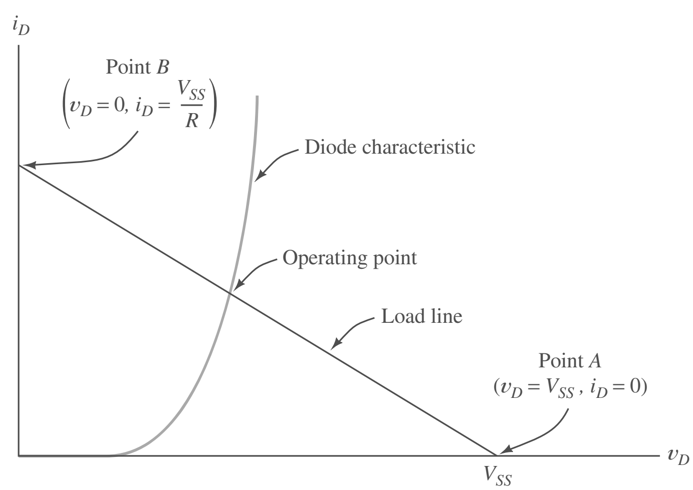


Figure 10.6 Load-line analysis of the circuit of Figure 10.5.

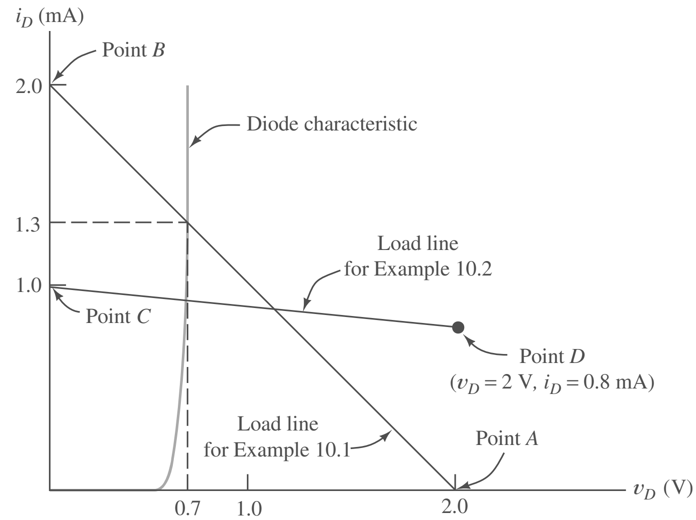


Figure 10.7 Load-line analysis for Examples 10.1 and 10.2.

~~$v_D = 2$ V. These values plot as point A in the figure. Constructing the load line results in an operating point of $V_{DQ} \cong 0.7$ V and $I_{DQ} \cong 1.3$ mA, as shown in the figure.~~ ■

~~Example 10.2 Load-Line Analysis~~

~~Repeat Example 10.1 if $V_{SS} = 10$ V and $R = 10$ kΩ.~~

~~**Solution** If we let $v_D = 0$ and substitute values into Equation 10.5, we find that $i_D = 1$ mA. This is plotted as point C in Figure 10.7.~~

~~If we proceed as before by assuming that $i_D = 0$, we find that $v_D = 10$ V. This is a perfectly valid point on the load line, but it plots at a point far off the page. Of course, we can use any other point satisfying Equation 10.5 to locate the load line. Since we already have point C on the i_D axis, a good point to use would be~~ on

When an intercept of the load line falls off the page, we select a point at the edge of the page.

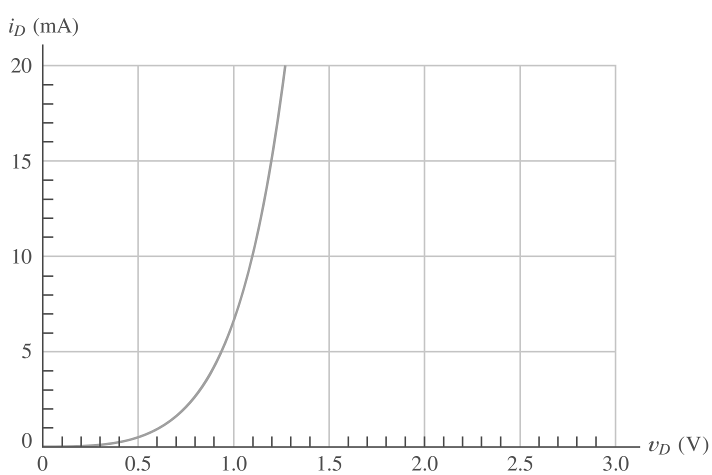


Figure 10.8 Diode characteristic for Exercise 10.3.

~~the right-hand edge of Figure 10.7. Thus, we assume that $v_D = 2$ V and substitute values into Equation 10.5, resulting in $i_D = 0.8$ mA. These values plot as point D. Then, we can draw the load line and find that the operating point values are $V_{DQ} \cong 0.68$ V and $I_{DQ} \cong 0.93$ mA.~~ ■

~~**Exercise 10.3** Find the operating point for the circuit of Figure 10.5 if the diode characteristic is shown in Figure 10.8 and: **a.** $V_{SS} = 2$ V and $R = 100\ \Omega$; **b.** $V_{SS} = 15$ V and $R = 1$ kΩ; **c.** $V_{SS} = 1.0$ V and $R = 20\ \Omega$.~~

~~**Answer** **a.** $V_{DQ} \cong 1.1$ V, $I_{DQ} \cong 9.0$ mA; **b.** $V_{DQ} \cong 1.2$ V, $I_{DQ} \cong 13.8$ mA; **c.** $V_{DQ} \cong 0.91$ V, $I_{DQ} \cong 4.5$ mA.~~ □

10.3 ZENER-DIODE VOLTAGE-REGULATOR CIRCUITS

A voltage regulator circuit provides a nearly constant voltage to a load from a variable source.

Sometimes, a circuit that produces constant output voltage while operating from a variable supply voltage is needed. Such circuits are called **voltage regulators**. For example, if we wanted to operate computer circuits from the battery in an automobile, a voltage regulator would be needed. Automobile battery voltage typically varies between about 10 and 14 V (depending on the state of the battery and whether or not the engine is running). Many computer circuits require a nearly constant voltage of 5 V. Thus, a regulator is needed that operates from the 10 to 14 V supply and produces a nearly constant 5-V output.

In this section, we use the load-line technique that we introduced in Section 10.2 to analyze a simple regulator circuit. The regulator circuit is shown in Figure 10.9. (For proper operation, it is necessary for the minimum value of the variable source voltage to be somewhat larger than the desired output voltage.) The Zener diode has a breakdown voltage equal to the desired output voltage. The resistor R limits the diode current to a safe value so that the Zener diode does not overheat.

Assuming that the characteristic for the diode is available, we can construct a load line to analyze the operation of the circuit. As before, we use Kirchhoff's voltage law to write an equation relating v_D and i_D. (In this circuit, the diode operates in the breakdown region with negative values for v_D and i_D.) For the circuit of Figure 10.9, we obtain

$$V_{SS} + Ri_D + v_D = 0 \tag{10.6}$$

Once again, this is the equation of a straight line, so location of any two points is sufficient to construct the load line. The intersection of the load line with the diode characteristic yields the operating point.

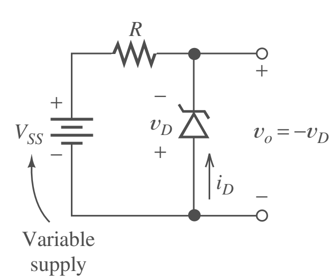


Figure 10.9 A simple regulator circuit that provides a nearly constant output voltage v_o from a variable supply voltage.

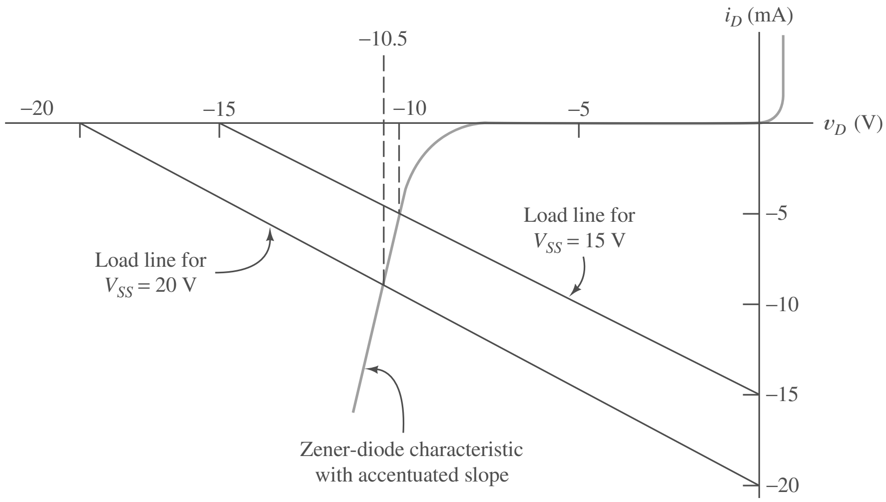


Figure 10.10 See Example 10.3.

Example 10.3 Load-Line Analysis of a Zener-Diode Voltage Regulator

The voltage-regulator circuit of Figure 10.9 has $R = 1\ \text{k}\Omega$ and uses a Zener diode having the characteristic shown in Figure 10.10. Find the output voltage for $V_{SS} =$ 15 V. Repeat for $V_{SS} = 20$ V.

Solution The load lines for both values of V_{SS} are shown in Figure 10.10. The output voltages are determined from the points where the load lines intersect the diode characteristic. The output voltages are found to be $v_o = 10.0$ V for $V_{SS} =$ 15 V and $v_o = 10.5$ V for $V_{SS} = 20$ V. Thus, a 5-V change in the supply voltage results in only a 0.5-V change in the regulated output voltage.

Actual Zener diodes are capable of much better performance than this. The slope of the characteristic has been accentuated in Figure 10.10 for clarity—actual Zener diodes have a more nearly vertical slope in breakdown. ■

Slope of the Load Line

Notice that the two load lines shown in Figure 10.10 are parallel. Inspection of Equation 10.5 or Equation 10.6 shows that the slope of the load line is $-1/R$. Thus, a change of the supply voltage changes the position, but not the slope of the load line.

Load lines for different source voltages (but the same resistance) are parallel.

Load-Line Analysis of Complex Circuits

Any circuit that contains resistors, voltage sources, current sources, and a single two-terminal nonlinear element can be analyzed by the load-line technique. First, the Thévenin equivalent is found for the linear portion of the circuit as illustrated in Figure 10.11. Then, a load line is constructed to find the operating point on the characteristic of the nonlinear device. Once the operating point of the nonlinear element is known, voltages and currents can be determined in the original circuit.

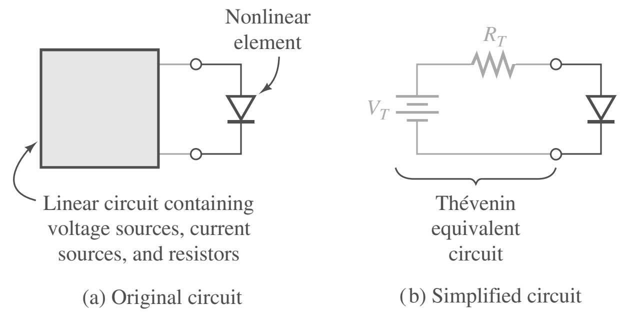


Figure 10.11 Analysis of a circuit containing a single nonlinear element can be accomplished by load-line analysis of a simplified circuit.

Example 10.4 Analysis of a Zener-Diode Regulator with a Load

Consider the Zener-diode regulator circuit shown in Figure 10.12(a). The diode characteristic is shown in Figure 10.13. Find the load voltage v_L and source current I_S if $V_{SS} = 24$ V, $R = 1.2$ kΩ, and $R_L = 6$ kΩ.

Solution First, consider the circuit as redrawn in Figure 10.12(b), in which we have grouped the linear elements together on the left-hand side of the diode. Next, we find the Thévenin equivalent for the linear portion of the circuit. The Thévenin voltage is the open-circuit voltage (i.e., the voltage across R_L with the diode replaced by an open circuit), which is given by

$$V_T = V_{SS} \frac{R_L}{R + R_L} = 20 \text{ V}$$

The Thévenin resistance can be found by zeroing the voltage source and looking back into the circuit from the diode terminals. This is accomplished by reducing V_{SS}

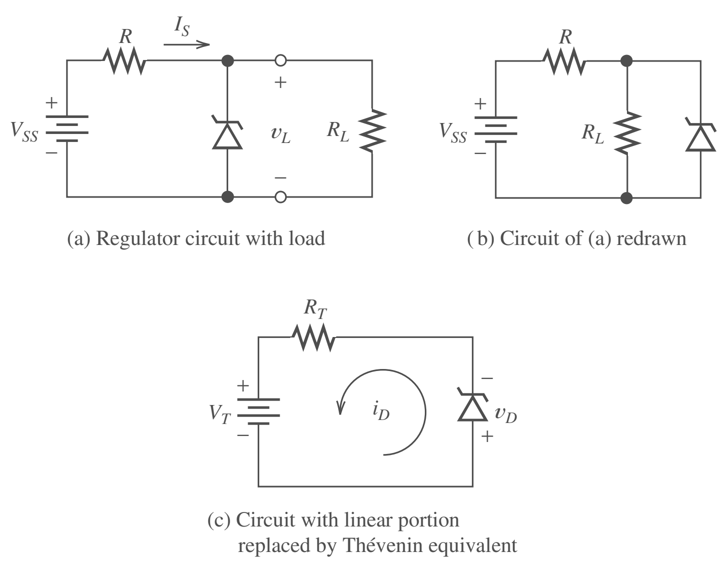


Figure 10.12 See Example 10.4.

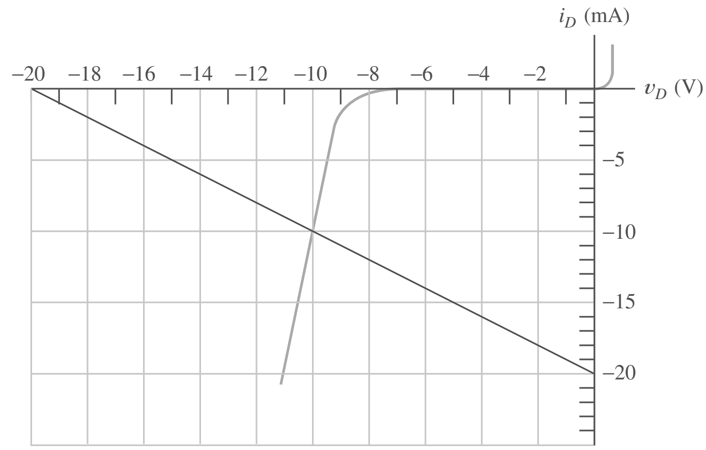


Figure 10.13 Zener-diode characteristic for Example 10.4 and Exercise 10.4.

to zero so that the voltage source becomes a short circuit. Then, we have R and R_L in parallel, so the Thévenin resistance is

$$R_T = \frac{RR_L}{R + R_L} = 1\ \text{k}\Omega$$

The resulting equivalent circuit is shown in Figure 10.12(c).

Now, we can use Kirchhoff's voltage law to write the load-line equation from the equivalent circuit as

$$V_T + R_T i_D + v_D = 0$$

Using the values found for V_T and R_T, we can construct the load line shown in Figure 10.13 and locate the operating point. This yields $v_L = -v_D = 10.0$ V.

Once v_L is known, we can find the voltages and currents in the original circuit. For example, using the output voltage value of 10.0 V in the original circuit of Figure 10.12(a), we find that $I_S = (V_{SS} - v_L)/R = 11.67$ mA. ■

Exercise 10.4 Find the voltage across the load in Example 10.4 if: **a.** $R_L = 1.2\ \text{k}\Omega$; **b.** $R_L = 400\ \Omega$.

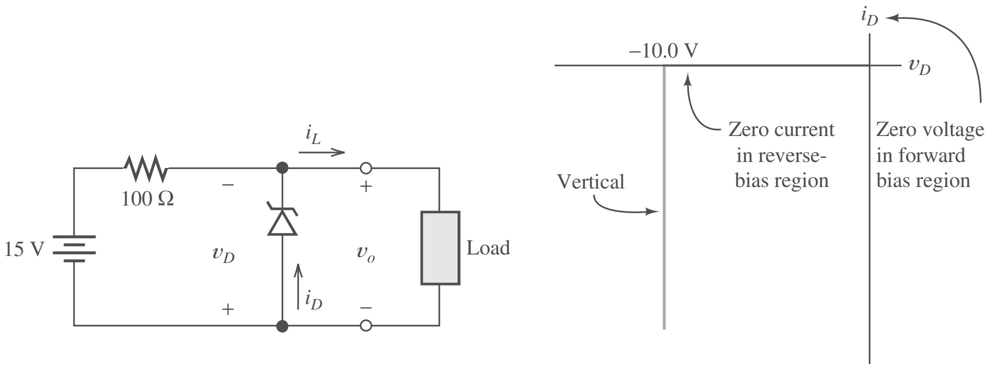
Answer **a.** $v_L \cong 9.4$ V; **b.** $v_L \cong 6.0$ V. (Notice that this regulator is not perfect because the load voltage varies as the load current changes.) □

Exercise 10.5 Consider the circuit of Figure 10.14(a). Assume that the breakdown characteristic is vertical, as shown in Figure 10.14(b). Find the output voltage v_o for: **a.** $i_L = 0$; **b.** $i_L = 20$ mA; **c.** $i_L = 100$ mA. [*Hint:* Applying Kirchhoff's voltage law to the circuit, we have

$$15 = 100(i_L - i_D) - v_D$$

Construct a different load line for each value of i_L.]

Answer **a.** $v_o = 10.0$ V; **b.** $v_o = 10.0$ V; **c.** $v_o = 5.0$ V. (Notice that the regulator is not effective for large load currents.) □



(a) Circuit diagram (b) Ideal Zener-diode characteristic

Figure 10.14 See Exercise 10.5.

~~10.4 IDEAL-DIODE MODEL~~

~~Graphical load line analysis is useful for some circuits, such as the voltage regulator studied in Section 10.3. However, it is too cumbersome for more complex circuits. Instead, we often use simpler models to approximate diode behavior.~~

The ideal diode acts as a short circuit for forward currents and as an open circuit with reverse voltage applied.

~~One model for a diode is the **ideal diode**, which is a perfect conductor with zero voltage drop in the forward direction. In the reverse direction, the ideal diode is an open circuit. We use the ideal-diode assumption if our judgment tells us that the forward diode voltage drop and reverse current are negligible, or if we want a basic understanding of a circuit rather than an exact analysis.~~

~~The volt–ampere characteristic for the ideal diode is shown in Figure 10.15. If i_D is positive, v_D is zero, and we say that the diode is in the *on* state. On the other hand, if v_D is negative, i_D is zero, and we say that the diode is in the *off* state.~~

~~Assumed States for Analysis of Ideal Diode Circuits~~

~~In analysis of a circuit containing ideal diodes, we may not know in advance which diodes are on and which are off. Thus, we are forced to make a considered guess. Then, we analyze the circuit to find the currents in the diodes assumed to be on and the voltages across the diodes assumed to be off. If i_D is positive for the diodes assumed to be on and if v_D is negative for the diodes assumed to be off, our assumptions are correct, and we have solved the circuit. (We are assuming that i_D is referenced positive in the forward direction and that v_D is referenced positive at the anode.)~~

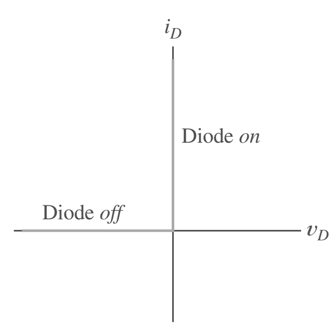


Figure 10.15 Ideal-diode volt–ampere characteristic.

Summary

1. A *pn*-junction diode is a two-terminal device that conducts current easily in one direction (from anode to cathode), but not in the opposite direction. The volt–ampere characteristic has three regions: forward bias, reverse bias, and reverse breakdown.
2. The Shockley equation relates current and voltage in a *pn*-junction diode.
3. Nonlinear circuits, such as those containing a diode, can be analyzed by using the load-line technique.
4. Zener diodes are intended to be operated in the reverse-breakdown region as constant-voltage references.
5. Voltage regulators are circuits that produce a nearly constant output voltage while operating from a variable source.
6. The ideal-diode model is a short circuit (on) if current flows in the forward direction and an open circuit (off) if voltage is applied in the reverse direction.
7. In the method of assumed states, we assume a state for each diode (on or off), analyze the circuit, and check to see if the assumed states are consistent with the current directions and voltage polarities. This process is repeated until a valid set of states is found.
8. In a piecewise-linear model for a nonlinear device, the volt–ampere characteristic is approximated by straight-line segments. On each segment, the device is modeled as a voltage source in series with a resistance.
9. Rectifier circuits can be used to charge batteries and to convert ac voltages into constant dc voltages. Half-wave rectifiers conduct current only for one polarity of the ac input, whereas full-wave circuits conduct for both polarities.
10. Wave-shaping circuits change the waveform of an input signal and deliver the modified waveform to the output terminals. Clipper circuits remove that portion of the input waveform above (or below) a given level. Clamp circuits add or subtract a dc voltage, so that the positive (or negative) peaks have a specified voltage.
11. The small-signal (incremental) equivalent circuit of a diode consists of a resistance. The value of the resistance depends on the operating point (Q point).
12. Dc sources and coupling capacitors are replaced by short circuits in small-signal ac equivalent circuits. Diodes are replaced with their dynamic resistances.

Problems

Section 10.1: Basic Diode Concepts

P10.1. Describe a fluid-flow analogy for a diode.

P10.2. Draw the circuit symbol for a diode, labeling the anode and cathode.

P10.3. Sketch the volt–ampere characteristic of a typical diode and label the various regions.

P10.4. Write the Shockley equation and define all of the terms.

P10.5. Determine the values of V_T for temperatures of 20°C and 175°C.

P10.6. Suppose that we have a junction diode that has $i_D = 0.5$ mA for $v_D = 0.6$ V. Assume that $n = 2$ and $V_T = 0.026$ V. Use the Shockley equation to compute the diode current at $v_D = 0.65$ V and at $v_D = 0.70$ V.

***P10.7.** A diode operates in forward bias, in which it is described by Equation 10.4, with $V_T = 0.026$ V. For $v_{D1} = 0.600$ V, the current is $i_{D1} = 1$ mA. For $v_{D2} = 0.680$ V, the current is $i_{D2} = 10$ mA. Determine the values of I_s and n.

* Denotes that answers are contained in the Student Solutions files. See Appendix E for more information about accessing the Student Solutions.

***P10.8.** Sketch i versus v to scale for the circuits shown in Figure P10.8. The reverse-breakdown voltages of the Zener diodes are shown. Assume voltages of approximately 0.6 V for all diodes including the Zener diodes when current flows in the forward direction.

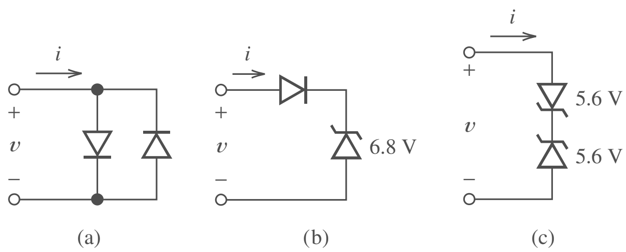


Figure P10.8

P10.9. Repeat Problem P10.8 for the circuits shown in Figure P10.9.

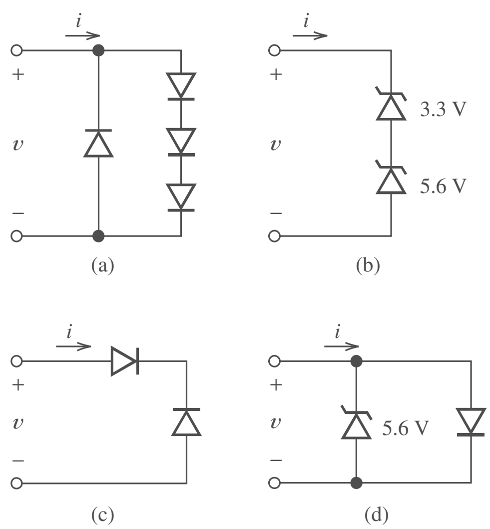


Figure P10.9

P10.10. As a rule of thumb, with constant current flowing in the forward direction in a small-signal silicon diode, the voltage across the diode decreases with temperature by about 2 mV/K. Such a diode has a voltage of 0.650 V, with a current of 1 mA at a temperature of 25°C. Find the diode voltage at 1 mA and a temperature of 150°C.

P10.11. We have a silicon diode described by the Shockley equation. The diode has $n = 2$ and operates at 100°C with a current of 1 mA and voltage of 0.5 V. Determine the current after the voltage is increased to 0.6 V.

P10.12. Consider a junction diode operating at a constant temperature of 300 K. With a forward current of 1 mA, the voltage is 600 mV. Furthermore, with a current of 10 mA, the voltage is 700 mV. Find the value of n for this diode.

P10.13. Assume a diode with $n = 2$, $I_s = 20$ nA, and $V_T = 26$ mV. **a.** Using a computer program of your choice, obtain a plot of i_D versus v_D for i_D ranging from 10 μA to 10 mA. Choose a logarithmic scale for i_D and a linear scale for v_D. What type of curve results? **b.** Place a 100-Ω resistance in series with the diode, and plot current versus voltage across the series combination on the same axes used for part (a). Compare the two curves. When is the added series resistance significant?

***P10.14.** The diodes shown in Figure P10.14 are identical and have $n = 1$. The temperature of the diodes is constant at 300 K. Before the switch is closed, the voltage v is 600 mV. Find v after the switch is closed. Repeat for $n = 2$.

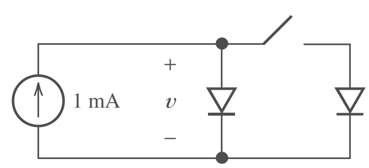


Figure P10.14

***P10.15.** **Current hogging**. The diodes shown in Figure P10.15 are identical and have $n = 1$. For each diode, a forward current of 100 mA results in a voltage of 700 mV at a temperature of 300 K. **a.** If both diodes are at 300 K, what are the values of I_A and I_B? **b.** If diode A is at 300 K and diode B is at 305 K, again find I_A and I_B, given that I_s doubles in value for a 5-K increase in temperature. [*Hint:* Answer part (a) by use of symmetry. For part (b), a transcendental equation for the voltage across the diodes can be found. Solve by trial and error. An important observation to be made from this problem is that, starting at the same temperature, the diodes should theoretically each conduct half of the total current. However, if one diode conducts slightly more, it becomes warmer, resulting in even more current. Eventually, one of the diodes “hogs” most of the current. This is particularly noticeable for devices that are thermally isolated from one another with large currents, for which significant heating occurs.]

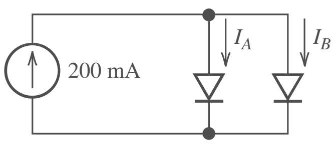


Figure P10.15

Section 10.2: Load-Line Analysis of Diode Circuits

***P10.16.** The nonlinear circuit element shown in Figure P10.16 has $i_x = [\exp(v_x) - 1]/10$, in which the units of v_x and i_x are V and A, respectively. Also, we have $V_s = 3$ V and $R_s = 1\ \Omega$. Use graphical load-line techniques to solve for i_x and v_x. (You may prefer to use a computer program to plot the characteristic and the load line.)

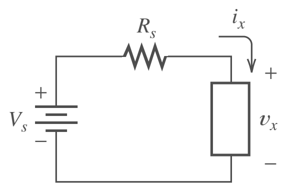


Figure P10.16

P10.17. Repeat Problem P10.16 for $V_s = 4$ V, $R_s = 1\ \Omega$, and $i_x = v_x + 2v_x^2$, in which the units of v_x and i_x are V and A, respectively.

P10.18. Repeat Problem P10.16 for $V_s = 15$ V, $R_s = 3\ \mathrm{k}\Omega$, and $i_x = 0.01/(1 - v_x/4)^3$, in which the units of v_x and i_x are V and mA, respectively.

P10.19. Repeat Problem P10.16 for $V_s = 8$ V, $R_s = 2\ \Omega$, and $i_x = v_x^3/4$, in which the units of v_x and i_x are V and A, respectively.

P10.20. Several types of special-purpose diodes exist. One is the constant-current diode for which the current is constant over a wide range of voltage. The circuit symbol and volt–ampere characteristic for a particular constant-current diode are shown in Figure P10.20(a). Another special type is the LED for which the circuit symbol and a typical volt–ampere characteristic are shown in Figure P10.20(b). Sometimes, the series combination of these two devices is used to provide constant current to the LED from a variable voltage source. **a.** Sketch the overall volt–ampere characteristic to scale for the series combination shown in Figure P10.20(c). **b.** Sketch the overall volt–ampere characteristic to scale for the parallel combination shown in Figure P10.20(d).

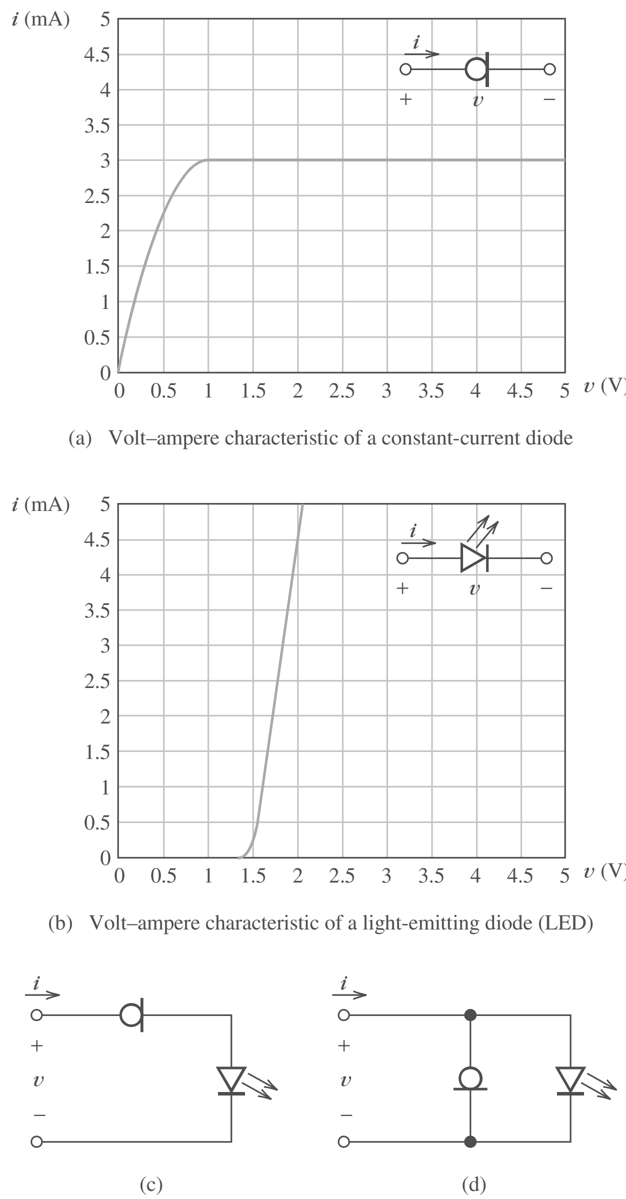


Figure P10.20

P10.21. Determine the values for i and v for the circuit of Figure P10.21. The diode is the LED having the characteristic shown in Figure P10.20(b).

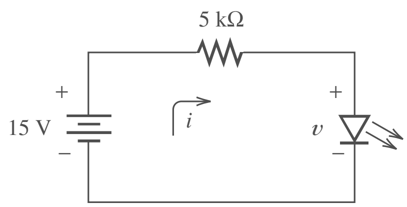


Figure P10.21

Section 10.3: Zener-Diode Voltage-Regulator Circuits

P10.22. What is a Zener diode? For what is it typically used? Draw the volt–ampere characteristic of an ideal 5.8-V Zener diode.

***P10.23.** Draw the circuit diagram of a simple voltage regulator.

P10.24. Consider the Zener-diode regulator shown in Figure 10.14 on page 478. What is the smallest load resistance for which v_o is 10 V?

P10.25. A simple voltage regulator is shown in Figure P10.25. The source voltage V_s varies from 8 to 12 V, and the load current i_L varies from 50 to 150 mA. Assume that the Zener diode is ideal. Determine the largest value allowed for the resistance R_s so that the load voltage v_L remains constant with variations in load current and source voltage. Determine the maximum power dissipation in R_s.

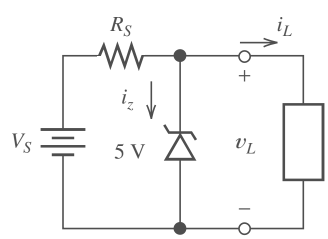


Figure P10.25

P10.26. You are required to design a simple voltage-regulator circuit that provides a constant voltage of 5 V to a load from a variable supply voltage. The load current varies from 0 to 100 mA, and the source voltage varies from 8 to 10 V. You may assume that ideal Zener diodes are available. Resistors of any value may be specified. Draw the circuit diagram of your regulator, and specify the value of each component. Also, find the worst case (maximum) power dissipated in each component in your regulator. Try to use good judgment in your design.

P10.27. Repeat Problem P10.26 given that the supply voltage ranges from 6 to 10 V.

P10.28. Repeat Problem P10.26 given that the load current varies from 0 to 1 A.

P10.29. Explain in general terms the method for solving a circuit that contains a single nonlinear element plus resistors, dc voltage sources, and dc current sources, given the volt–ampere characteristic of the nonlinear device.

***P10.30.** A certain linear two-terminal circuit has terminals a and b. Under open-circuit conditions, we have $v_{ab} = 10$ V. A short circuit is connected across the terminals, and a current of 2 A flows from a to b through the short circuit. Determine the value of v_{ab} when a nonlinear element that has $i_{ab} = \sqrt[3]{v_{ab}}$ is connected across the terminals.

P10.31. Determine the values for i_1 and i_2 for the circuit of Figure P10.31. The device is the constant-current diode having the characteristic shown in Figure P10.20(a).

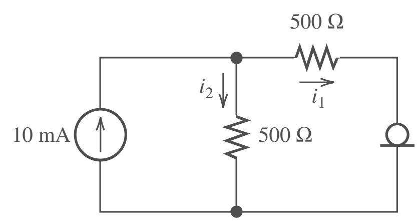


Figure P10.31

P10.32. Determine the values for i and v for the circuit of Figure P10.32. The diode is the LED having the characteristic shown in Figure P10.20(b).

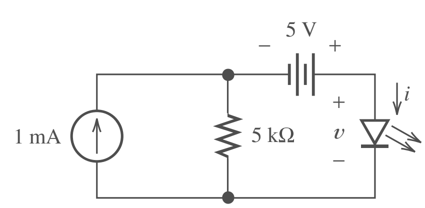


Figure P10.32

P10.33. Repeat Problem P10.32 for the circuit of Figure P10.33. The diode is the LED having the characteristic shown in Figure P10.20(b).

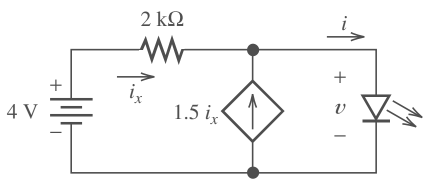


Figure P10.33

Section 10.4: Ideal-Diode Model

P10.34. What is an ideal diode? Draw its volt–ampere characteristic. After solving a circuit with ideal diodes, what check is necessary for diodes initially assumed to be on? Off?

P10.35. Consider two ideal diodes in series, pointing in opposite directions. What is the equivalent circuit for the combination? What is the equivalent circuit if the diodes are in parallel and pointing in opposite directions?

***P10.36.** Find the values of I and V for the circuits of Figure P10.36, assuming that the diodes are ideal.

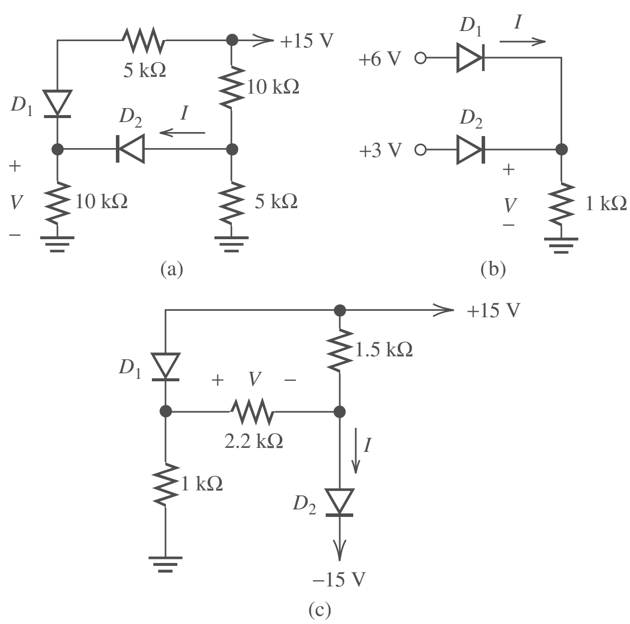


Figure P10.36

P10.37. Find the values of I and V for the circuits of Figure P10.37, assuming that the diodes are ideal.

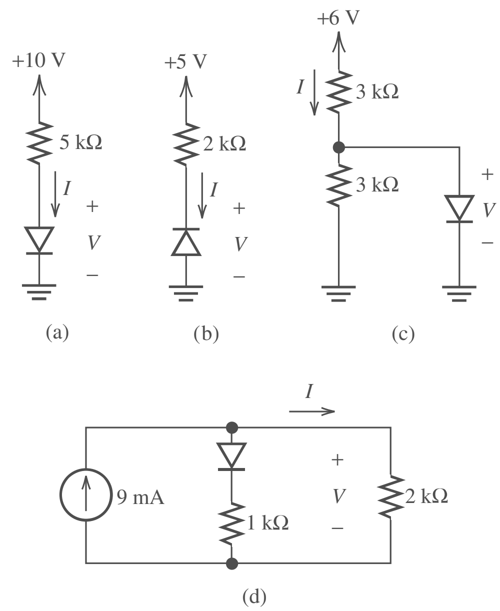


Figure P10.37

P10.38. Find the values of I and V for the circuits of Figure P10.38, assuming that the diodes are ideal. For part (b), consider $V_{in} = 0, 2, 6,$ and 10 V. Also, for part (b) of the figure, plot V versus V_{in} for V_{in} ranging from −10 V to 10 V.

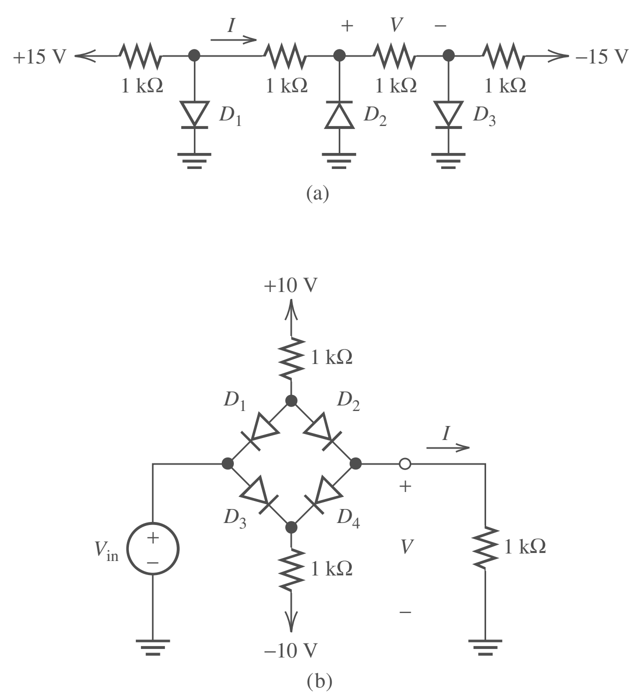
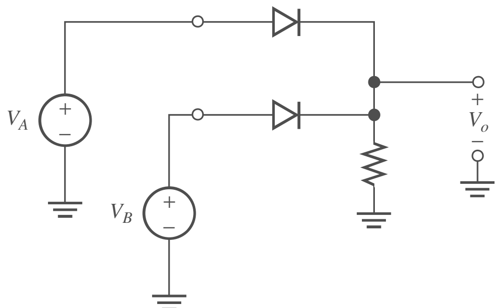
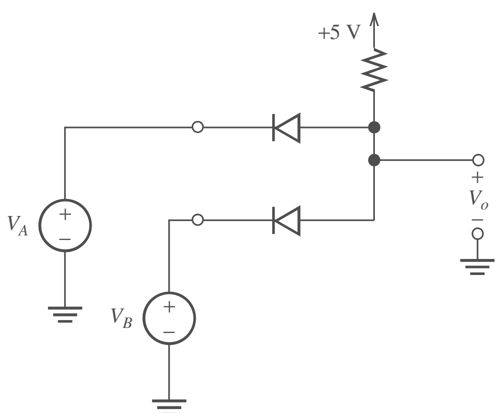


Figure P10.38

P10.39. **a.** Figure P10.39(a) shows a type of logic gate. Assume that the diodes are ideal. The voltages V_A and V_B independently have values of either 0 V (for logic 0, or low) or 5 V (for logic 1, or high). For which of the four combinations of input voltages is the output high (i.e., $V_o = 5$ V)? What type of logic gate is this? **b.** Repeat for Figure P10.39(b).



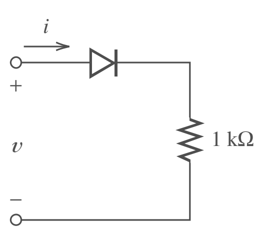
(a)



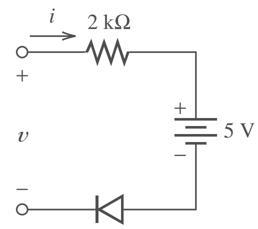
(b)

Figure P10.39

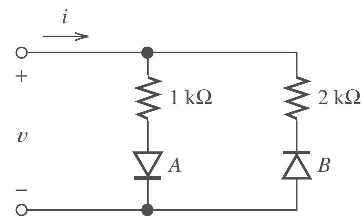
P10.40. Sketch a plot of i versus v to scale for each of the circuits shown in Figure P10.40. Assume that the diodes are ideal and allow v to range from −10 V to +10 V.



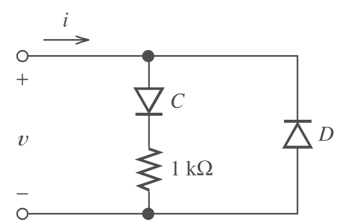
(a)



(b)



(c)



(d)

Figure P10.40

P10.41. Sketch $v_o(t)$ to scale versus time for the circuit shown in Figure P10.41. Assume that the diodes are ideal.

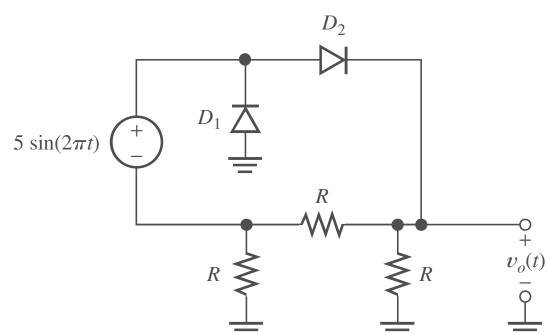


Figure P10.41

Section 10.5: Piecewise-Linear Diode Models

P10.42. If a nonlinear two-terminal device is modeled by the piecewise-linear approach, what is the equivalent circuit of the device for each linear segment?

P10.43. A resistor R_a is in series with a voltage source V_a. Draw the circuit. Label the voltage across the combination as v and the current as i. Draw and label the volt–ampere characteristic (i versus v).

P10.44. The volt–ampere characteristic of a certain two-terminal device is a straight line that passes through the points (2 V, 5 mA) and (3 V, 15 mA). The current reference points into the positive reference for the voltage. Determine the equivalent circuit for this device.

***P10.45.** Assume that we have approximated a nonlinear volt–ampere characteristic by the straight-line segments shown in Figure P10.45(c). Find the equivalent circuit for each segment. Use these equivalent circuits to find v in the circuits shown in Figure P10.45(a) and (b).

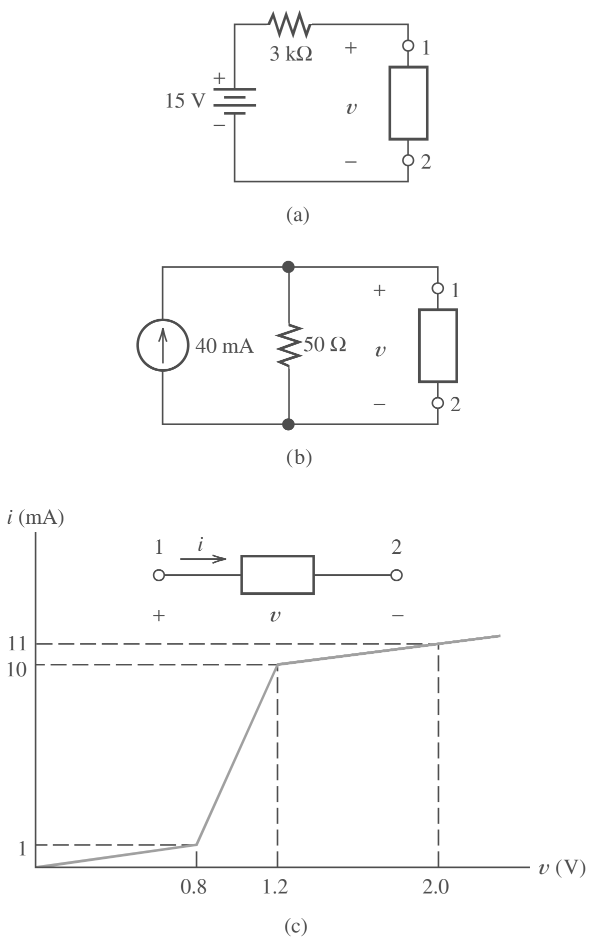


Figure P10.45

P10.46. Consider the volt–ampere characteristic of an ideal 10-V Zener diode shown in Figure 10.14 on page 478. Determine the piecewise-linear equivalent circuit for each segment of the characteristic.

***P10.47.** The Zener diode shown in Figure P10.47 has a piecewise-linear model shown in Figure 10.19 on page 481. Plot load voltage v_L versus load current i_L for i_L ranging from 0 to 100 mA.

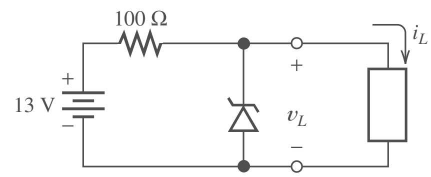


Figure P10.47

P10.48. In this problem, we will assume that the diode shown in Figure P10.48 can be represented by the model of Figure 10.23 on page 483, with $V_f = 0.7$ V. **a.** Assume that the diode operates as an open-circuit and solve for the node voltages v_1 and v_2. Are the results consistent with the model? Why or why not? **b.** Repeat part (a), assuming that the diode operates as a 0.7-V voltage source.

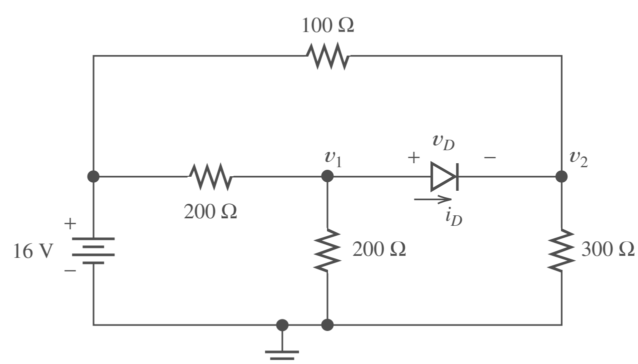


Figure P10.48

Section 10.6: Rectifier Circuits

P10.49. Draw the circuit diagram of a half-wave rectifier for producing a nearly steady dc voltage from an ac source. Draw two different full-wave circuits (one of which requires two ac sources).

P10.50. We have a 15-V rms 60-Hz ac source is in series with an ideal diode and a 50-Ω resistance. Determine the peak current and peak inverse voltage (PIV) for the diode.

P10.51. This problem relates to the half-wave rectifier shown in Figure 10.26 on page 485. The ac source has an rms value of 15 V and a frequency of 60 Hz. The diodes are ideal, and the capacitance is very large, so the ripple voltage V_r is very small. The load is a 50-Ω resistance. Determine the peak inverse voltage across the diode and the charge that passes through the diode per cycle. What can you say about the peak current through the diode?

P10.52. This problem relates to the battery-charging circuit shown in Figure 10.25 on page 484. The ac source voltage is $v_s(t) = V_m \sin(\omega t) = 24 \sin(120\pi t)$. The resistance is 0.5 Ω, the diode is ideal, and $V_B = 12$ V. Determine the average current (i.e., the value of the charge that passes through the battery in 1 second). Suppose that the battery starts from a totally discharged state and has a capacity of 100 ampere hours. How long does it take to fully charge the battery?

P10.53. Dc voltmeters produce readings equal to the average values of the voltages measured. The average value of a periodic waveform is

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)dt$$

in which T is the period of the voltage $v(t)$ applied to the meter.

a. What does a dc voltmeter read if the applied voltage is $v(t) = V_m \sin(\omega t)$?

b. What does the meter read if the applied voltage is a half-wave rectified version of the sinewave?

c. What does the meter read if the applied voltage is a full-wave rectified version of the sinewave?

***P10.54.** A half-wave rectifier is needed to supply 15-V dc to a load that draws an average current of 250 mA. The peak-to-peak ripple is required to be 0.2 V or less. What is the minimum value allowed for the smoothing capacitance? If a full-wave rectifier is needed?

***P10.55.** Design a half-wave rectifier power supply to deliver an average voltage of 9 V with a peak-to-peak ripple of 2 V to a load. The average load current is 100 mA. Assume that ideal diodes and 60-Hz ac voltage sources of any amplitudes needed are available. Draw the circuit diagram for your design. Specify the values of all components used.

P10.56. Repeat Problem P10.55 with a full-wave bridge rectifier.

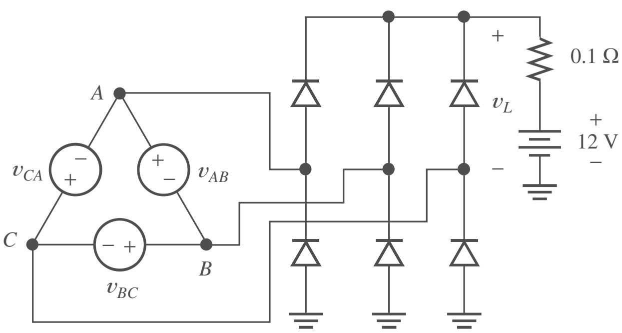
P10.57. Repeat Problem P10.55 with two diodes and out-of-phase voltage sources to form a full-wave rectifier.

P10.58. Repeat Problem P10.55, assuming that the diodes have forward drops of 0.8 V.

P10.59. Consider the battery-charging circuit shown in Figure 10.25 on page 484, in which $v_s(t) = 20 \sin(200\pi t)$, $R = 80$ Ω, $V_B = 12$ V, and the diode is ideal. **a.** Sketch the current $i(t)$ to scale versus time. **b.** Determine the average charging current for the battery. (*Hint*: The average current is the charge that flows through the battery in one cycle, divided by the period.)

P10.60. **a.** Consider the full-wave rectifier shown in Figure 10.27 on page 486, with a large smoothing capacitance placed in parallel with the load R_L and $V_m = 12$ V. Assuming that the diodes are ideal, what is the approximate value of the load voltage? What peak inverse voltage (PIV) appears across the diodes? **b.** Repeat for the full-wave bridge shown in Figure 10.28 on page 487.

P10.61. Figure P10.61 shows the equivalent circuit for a typical automotive battery charging system. The three-phase delta-connected source represents the stator coils of the alternator. (Three-phase ac sources are discussed in Section 5.7. Actually, the alternator stator is usually wye connected, but the terminal voltages are the same as for the equivalent delta.) Not shown in the figure is a voltage regulator that controls the current applied to the rotor coil of the alternator and, consequently, V_m and the charging current to the battery. **a.** Sketch the load voltage $v_L(t)$ to scale versus time. Assume ideal diodes and that V_m is large enough that current flows into the battery at all times. (*Hint*: Each source and four of the diodes form a full-wave bridge rectifier.) **b.** Determine the peak-to-peak ripple and the average load voltage in terms of V_m. **c.** Determine the value of V_m needed to provide an average charging current of 30 A. **d.** What additional factors would need to be considered in a realistic computation of V_m?



$v_{AB} = V_m \cos(\omega t)$

$v_{BC} = V_m \cos(\omega t - 120°)$

$v_{CA} = V_m \cos(\omega t + 120°)$

Figure P10.61 Idealized model of an automotive battery-charging system.

Section 10.7: Wave-Shaping Circuits

P10.62. What is a clipper circuit? Draw an example circuit diagram, including component values, an input waveform, and the corresponding output waveform.

P10.63. Sketch the transfer characteristic (v_o versus v_{in}) to scale for the circuit shown in Figure P10.63. Allow v_{in} to range from −10 V to +10 V and assume that the diodes are ideal.

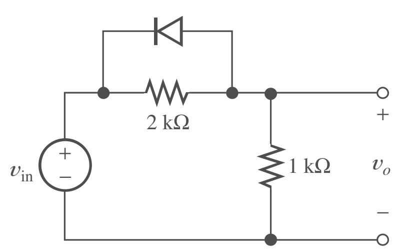


Figure P10.63

P10.64. Sketch to scale the output waveform for the circuit shown in Figure P10.64. Assume that the diodes are ideal.

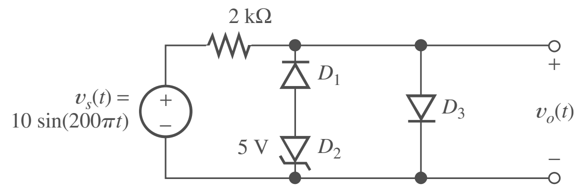


Figure P10.64

P10.65. Sketch the transfer characteristic (v_o versus v_{in}) to scale for the circuit shown in Figure P10.65. Allow v_{in} to range from −10 V to +10 V and assume that the diodes are ideal.

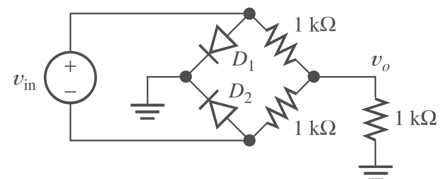


Figure P10.65

P10.66. Sketch the transfer characteristic (v_o versus v_{in}) to scale for the circuit shown in Figure P10.66. Allow v_{in} to range from −10 V to +10 V and assume that the diodes are ideal.

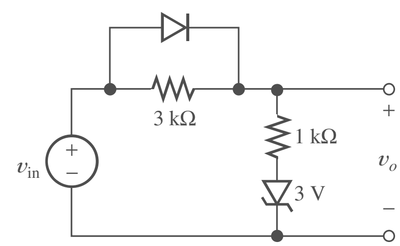


Figure P10.66

P10.67. Sketch the transfer characteristic (v_o versus v_{in}) for the circuit shown in Figure P10.67,

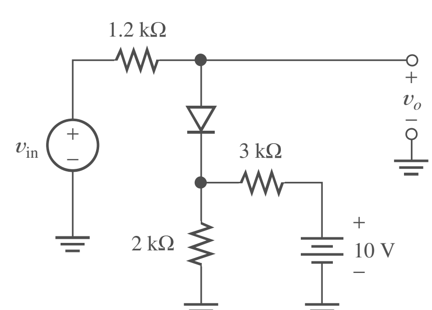


Figure P10.67

carefully labeling the breakpoint and slopes. Allow v_{in} to range from -10 V to $+10$ V and assume that the diodes are ideal.

P10.68. What is a clamp circuit? Draw an example circuit diagram, including component values, an input waveform, and the corresponding output waveform.

***P10.69.** Sketch to scale the steady-state output waveform for the circuit shown in Figure P10.69. Assume that RC is much larger than the period of the input voltage and that the diodes are ideal.

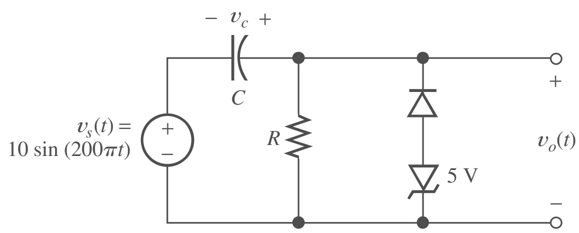


Figure P10.69

***P10.70.** Design a clipper circuit to clip off the portions of an input voltage that fall above 3 V or below -5 V. Assume that diodes having a constant forward drop of 0.7 V are available. Ideal Zener diodes of any breakdown voltage required are available. Dc voltage sources of any value needed are available.

P10.71. Repeat Problem P10.70, with clipping levels of $+5$ V and $+10$ V (i.e., every part of the input waveform below $+5$ or above $+10$ is clipped off).

P10.72. Consider the circuit shown in Figure P10.72, in which the RC time constant is very long compared with the period of the input and in which the diode is ideal. Sketch several cycles of $v_o(t)$ to scale versus time.

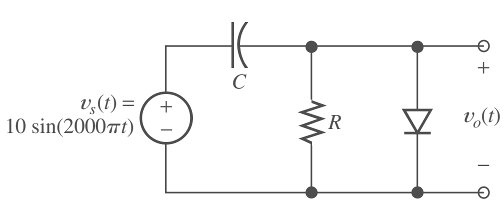


Figure P10.72

P10.73. Voltage-doubler circuit. Consider the circuit of Figure P10.73. The capacitors are very large, so they discharge only a very small amount per cycle. (Thus, no ac voltage appears across the capacitors, and the ac input plus the dc voltage of C_1 must appear at point A.) Sketch the voltage at point A versus time. Find the voltage across the load. Why is this called a voltage doubler? What is the peak inverse voltage across each diode?

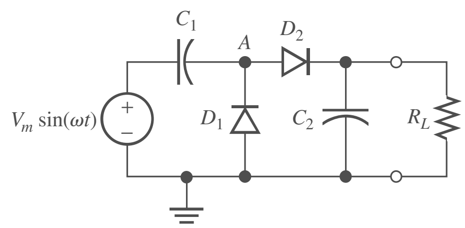
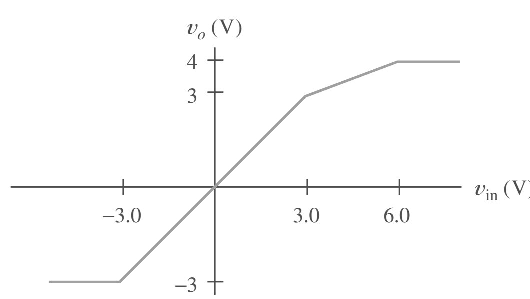


Figure P10.73 Voltage doubler

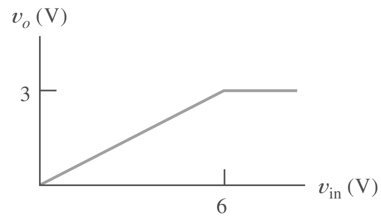
***P10.74.** Design a clamp circuit to clamp the negative extreme of a periodic input waveform to -5 V. Use diodes, Zener diodes, and resistors of any values required. Assume a 0.6-V forward drop for all diodes and that the Zener diodes have an ideal characteristic in the breakdown region. Power-supply voltages of ± 15 V are available.

P10.75. Repeat Problem P10.74 for a clamp voltage of $+5$ V.

P10.76. Design circuits that have the transfer characteristics shown in Figure P10.76. Assume that v_{in} ranges from -10 to $+10$ V. Use diodes, Zener diodes, and resistors of any values needed. Assume a 0.6-V forward drop for all diodes and that the Zener diodes have an ideal characteristic in the breakdown region. Power-supply voltages of ± 15 V are available.



(a)



(b)

Figure P10.76

Section 10.8: Linear Small-Signal Equivalent Circuits

P10.77. A certain diode has $I_{DQ} = 4$ mA and $i_d(t) = 0.5\cos(200\pi t)$ mA. Find an expression for $i_D(t)$, and sketch several cycles to scale versus time.

P10.78. Of what does the small-signal equivalent circuit of a diode consist? How is the dynamic resistance of a nonlinear circuit element determined at a given operating point?

P10.79. With what are dc voltage sources replaced in a small-signal ac equivalent circuit? Why?

P10.80. With what should we replace a dc current source in a small-signal ac equivalent circuit? Justify your answer.

***P10.81.** A certain nonlinear device has $i_D = v_D^3/8$. Sketch i_D versus v_D to scale for v_D ranging from −2 V to +2 V. Is this device a diode? Determine the dynamic resistance of the device and sketch it versus v_D to scale for v_D ranging from −2 V to +2 V.

P10.82. A breakdown diode has

$$i_D = \frac{-10^{-6}}{(1 + v_D/7)^2} \quad \text{for } -7 \text{ V} < v_D < 0$$

where i_D is in amperes. Plot i_D versus v_D for the ranges $-10 \text{ mA} \le i_D \le 0$ and $-7 \text{ V} \le v_D \le 0$. Find the dynamic resistance of this diode at $I_{DQ} = -0.5$ mA and at $I_{DQ} = -10$ mA.

P10.83. A diode is operating with an applied voltage given by

$$v_D(t) = 4 + 0.02\cos(\omega t)\text{V}$$

The current is given by

$$i_D(t) = 7 + 0.2\cos(\omega t) \text{ mA}$$

Determine the dynamic resistance and Q point of the diode under the conditions given.

P10.84. The voltage of an ideal Zener diode is constant in the breakdown region. What does this imply about the dynamic resistance in the breakdown region for an ideal Zener diode?

***P10.85.** Consider the voltage-regulator circuit shown in Figure P10.85. The ac ripple voltage is 1 V peak to peak. The dc (average) load voltage is 5 V. What is the Q-point current in the Zener diode? What is the maximum dynamic resistance allowed for the Zener diode if the output ripple is to be less than 10 mV peak to peak?

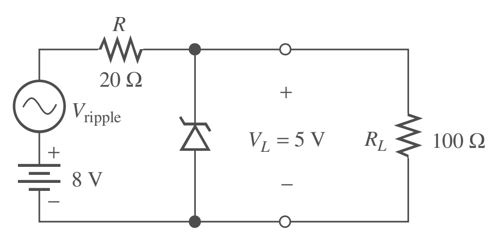


Figure P10.85

Practice Test

Here is a practice test you can use to check your comprehension of the most important concepts in this chapter. Answers can be found in Appendix D and complete solutions are included in the Student Solutions files. See Appendix E for more information about the Student Solutions.

T10.1. Determine the value of i_D for each of the circuits shown in Figure T10.1. The characteristic for the diode is shown in Figure 10.8 on page 473.

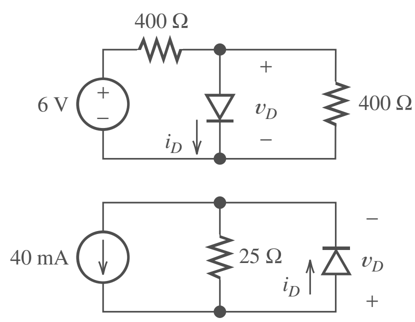


Figure T10.1

T10.2. The diode shown in Figure T10.2 is ideal. Determine the state of the diode and the values of v_x and i_x.

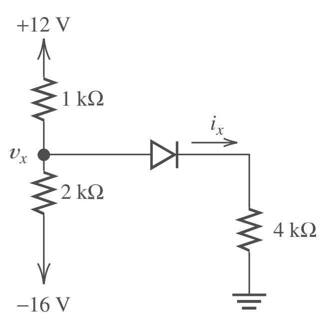


Figure T10.2

T10.3. The current versus voltage characteristic of a certain two-terminal device passes through the points (5 V, 2 mA) and (10 V, 7 mA). The reference for the current points into the positive reference for the voltage. Determine the values for the resistance and voltage source for the piecewise linear equivalent circuit for this device between the two points given.

T10.4. Draw the circuit diagram for a full-wave bridge rectifier with a resistance as the load.

T10.5. Suppose we have a 10-V-peak sinusoidal voltage source. Draw the diagram of a circuit that clips off the part of the sinusoid above 5 V and below −4 V. The circuit should be composed of ideal diodes, dc voltage sources, and other components as needed. Be sure to label the terminals across which the clipped output waveform $v_o(t)$ appears.

T10.6. Suppose we have a 10-Hz sinusoidal voltage source, $v_{\text{in}}(t)$. Draw the diagram of a circuit that clamps the positive peaks to −4 V. The circuit should be composed of ideal diodes, dc voltage sources, and other components as needed. List any constraints that should be observed in selecting component values. Be sure to label the terminals across which the clamped output waveform $v_o(t)$ appears.

T10.7. Suppose we have a silicon diode operating with a bias current of 5 mA at a temperature of 300 K. The diode current is given by the Shockley equation with $n = 2$. Draw the small-signal equivalent circuit for the diode including numerical values for the components.