DATA605 - Assignment 13

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Assignment 13

1

Use integration by substitution to solve the integral below.

Answer

$$\int 4e^{-7x}dx$$

$$u = -7x$$

$$du = -7dx \Rightarrow dx = \frac{du}{-7}$$

by substitution

$$\int 4e^{u}\frac{du}{-7}$$

$$-\frac{4}{7}\int e^{u}du$$

$$-\frac{4}{7}e^{u} + C$$

$$-\frac{4}{7}e^{-7x} + C$$

2

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

Answer

We know the rate of change is the derivative of the function $N(t)$ so to find $N(t)$ we take the integral

$$\frac{dN}{dt} = N'(t) = \frac{3150}{t^4} - 220$$

$$N(t) = \int \frac{3150}{t^4} - 220dt$$

$$= \frac{1050}{t^3} - 220t + C$$

We are given $N(1) = 6530$ so we can plug that in to solve for C

$$6530 = \frac{1050}{1^3} - 220 * 1 + C$$
$$6530 = 830 + C$$
$$C = 5700$$

This gives us

$$N(t) = \frac{1050}{t^3} - 220t + 5700$$

3

Find the total area of the red rectangles in the figure below, where the equation of the line is f (x) = 2x - 9.

Answer

By looking at the graph the limits are observed as 4.5 as the lower bound and 8.5 as the upper bound. Using R to perform the integration

```
f <- function(x){2*x - 9}
integrate(f, 4.5, 8.5)
```

```
## 16 with absolute error < 1.8e-13
```

16 is the area of the rectangle

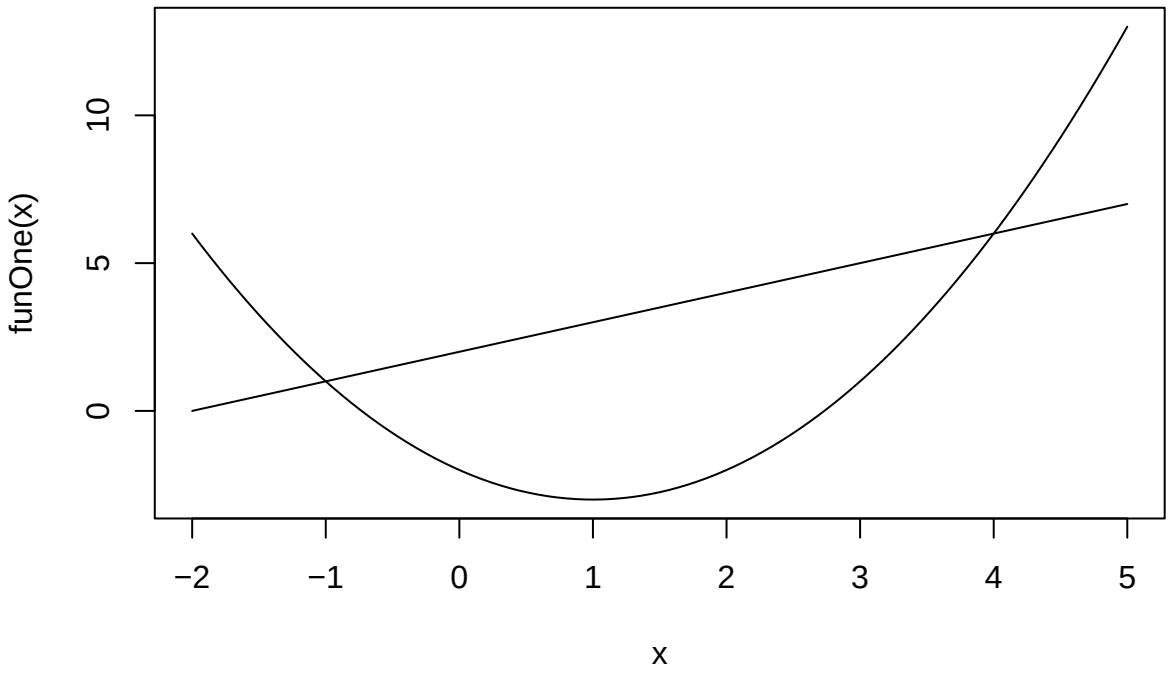
4

Find the area of the region bounded by the graphs of the given equations. $y = x^2 - 2x - 2, y = x + 2$

Answer

First plot the curves to find the intersection

```
funOne <- function(x){x^2 - 2*x - 2}
funTwo <- function(x){x+2}
curve(funOne, -2,5)
curve(funTwo,-2,5,add=T)
```



From the plot of the curves the intersection occurs at x=-1 and x=4 which are our lower and upper bounds. To find the area we take the integral of the upper function minus the lower function

$$\int_{-1}^{4} (x^2 - 2x - 2) - (x + 2)dx$$

$$\int_{-1}^{4} x^2 - 3x - 4dx$$

```
funDiff <- function(x){x^2 - 3*x - 4}
integrate(funDiff, -1,4)
```

```
## -20.83333 with absolute error < 2.3e-13
```

Area is ≈ -20.833

5

A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store one flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

Answer

To minimize inventory costs we need to create a function $C(x)$ using what was given then take its derivative.

- Costs 3.75 to store one iron per year
- Each order costs 8.25
- Expect to sell 110 per next year

$C(x) = 8.25x + \frac{3.75}{2x}$

```
D(expr = expression(8.25 * (110/x) + 3.75* (x/2)), 'x')
```

```
## 3.75 * (1/2) - 8.25 * (110/x^2)
```

$C'(x) = 3.75 * (1/2) - 8.25 * (110/x^2)$

Setting $C'(x) = 0$ you get $x = 22$, which means by plugging 22 in for the number of unknown orders 110/22 = 5 orders will minimize costs

6

Use integration by parts to solve the integral below.

$$\int ln(9x)x^6 dx$$

Answer

Integration by parts

$$\int udv = uv - \int vdu$$

$$u = ln(9x), du = \frac{1}{x}, v = \frac{1}{7}x^7, dv = x^6$$

$$ln(9x)\frac{1}{7}x^7 - \int \frac{1}{7}x^7 \frac{1}{x} dx$$

$$ln(9x)\frac{1}{7}x^7 - \frac{1}{7}\int x^6 dx$$

$$ln(9x)\frac{1}{7}x^7 - \frac{1}{7} \times \frac{1}{7}x^7 + C$$

$$ln(9x)\frac{1}{7}x^7 - \frac{x^7}{49} + C$$

7

Determine whether f (x) is a probability density function on the interval $\left[_{1,e^6}\right]$. If not, determine the value of the definite integral $f(x) = \frac{1}{6x}$

Answer

In order for f(x) to be a PDF the integral must be = 1

```
fX <- function(x){1/(6*x)}
integrate(fX, 1,exp(1)^6)
```

```
## 1 with absolute error < 9.3e-05
```