DATA605 - Assignment 15

Nick Oliver

Assignment 15

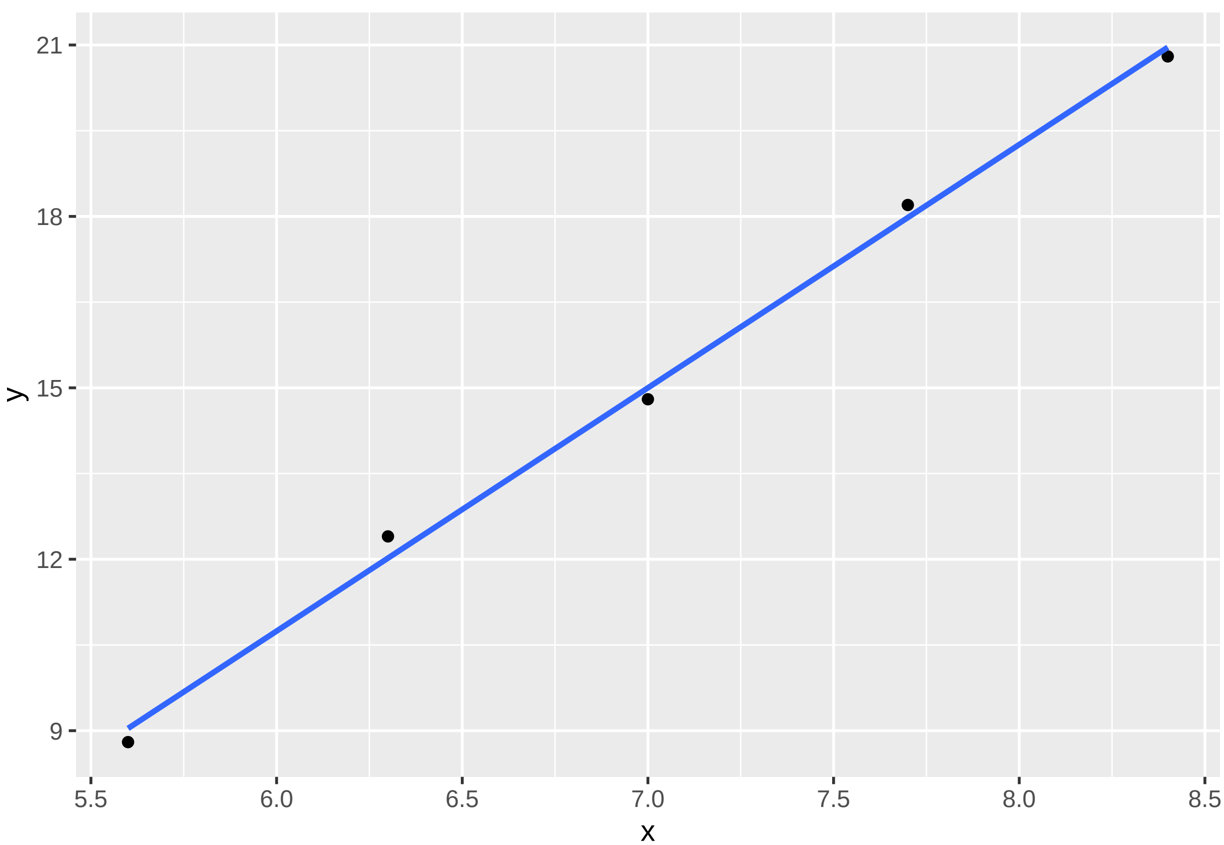
1.

Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary. (5.6,8.8), (6.3,12.4), (7,14.8), (7.7,18.2), (8.4,20.8)

Answer: $y = -14.8 + 4.26 \times x$

```
x<-c(5.6, 6.3, 7, 7.7, 8.4)
y<-c(8.8, 12.4,14.8,18.2,20.8)
df <- data.frame(x,y)
df %>% ggplot(aes(x=x,y)) +
  geom_point() +
stat_smooth(method = "lm", se = FALSE)
```

```
## `geom_smooth()` using formula 'y ~ x'
```



```r
summary(lm(y~x))
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##     1     2     3     4     5
## -0.24  0.38 -0.20  0.22 -0.16
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -14.8000     1.0365  -14.28 0.000744 ***
## x             4.2571     0.1466   29.04 8.97e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3246 on 3 degrees of freedom
## Multiple R-squared:  0.9965, Adjusted R-squared:  0.9953
## F-statistic: 843.1 on 1 and 3 DF,  p-value: 8.971e-05
```

2.

Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma. $f(x, y) = 24x - 6xy^2 - 8y^3$

Find derivatives with respect to x and y

```r
f <- expression(24*x - 6 * x * y^2 - 8*y^3)
D(f,'x')
```

```
## 24 - 6 * y^2
```

```r
D(f,'y')
```

```
## -(6 * x * (2 * y) + 8 * (3 * y^2))
```

$\frac{\partial}{\partial x} = 24 - 6y^2$ $\frac{\partial}{\partial y} = -12xy - 24y^2$

Set derivatives equal to 0 to find critical points

$0 = 24 - 6y^2 \rightarrow y = \pm 2$

Solve for x when $y = 2$

$0 = -12x * 2 - 24 * 2^2 \rightarrow x = -4$

Solve for x when $y = -2$

$0 = (-12x * -2) - 24 * (-2)^2 \rightarrow x = 4$

Obtain z values by plugging in values for x,y

```r
x<--4
y<-2
print(paste0('z=',24*x - 6 * x * y^2 - 8*y^3))
```

```
## [1] "z=-64"
```

```r
x<-4
y<--2
print(paste0('z=',24*x - 6 * x * y^2 - 8*y^3))
```

```
## [1] "z=64"
```

Critical points (-4,2,-64),(4,-2,64)

To find if local extrema you can use the second derivative test

```r
f <- expression(24*x - 6 * x * y^2 - 8*y^3)
D(D(f,'x'),'x')
```

```
## [1] 0
```

```r
D(D(f,'y'),'y')
```

```
## -(6 * x * 2 + 8 * (3 * (2 * y)))
```

```r
D(D(f,'x'),'y')
```

```
## -(6 * (2 * y))
```

$f_{xx} = 0$

$f_{yy} = -12x - 48y$

$f_{xy} = -12y$

$D = f_{xx} * f_{yy} - f_{xy}^2 = 0 * (-12x - 48y) - (12y)^2 = -144y^2$

$D = -144 * 2^2 = -576$

$D = -144 * -2 ^ 2 = -576 $

Because D is < 0 we conclude they are both saddle points

3.

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1. Find the revenue function $R(x, y)$.

$R(x, y)$ is equal to the "house" brand revenue function times x plus the "name" brand revenue function times y

$R(x, y) = (81 - 21x + 17)x + (40 + 11x - 23y)y = -21x^2 + 28xy + 81x - 23y^2 + 40y$

Step 2. What is the revenue if she sells the "house" brand for $2.30 and the "name" brand for $4.10?

```r
x <- 2.3
y <- 4.1
-21*x^2 + 28*x*y + 81*x - 23*y^2 + 40*y
```

```
## [1] 116.62
```

Answer: Revenue is $116.62

4.

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

We know that 96 units total will be produced from two different plants, LA & Denver. Setting LA to x and Denver to y we get $96 = x + y$

$x = 96 - y$ so we can rewrite the cost function as $C(96 - y, y) = \frac{1}{6}(96 - y)^2 + \frac{1}{6}y^2 + 7(96 - y) + 25y + 700$ which reduces to $\frac{1}{3}(y^2 - 42y + 8724)$

Take the derivative

```
D(expression(1/3*(y^2 - 42*y + 8724)),'y')
```

```
## 1/3 * (2 * y - 42)
```

You get $1/3 * (2 * y - 42)$ set that equal to 0 and solve for y you get $y = 21$

To find x we simply enter 21 into previous equation $x = 96 - 21 \to x = 75$

Answer: LA produces 75 units and Denver produces 21 units

5.

Evaluate the double integral on the given region.

$$\iint_R (e^{8x+3y})dA; R : 2 \leq x \leq 4and2 \leq y \leq 4$$

Write your answer in exact form without decimals.

```
library(pracma)
```

```
##
## Attaching package: 'pracma'

## The following object is masked from 'package:purrr':
##
##     cross
```

```
fun <- function(x,y) {exp(8*x + 3 *y)}
xmin <- 2
xmax <- 4
ymin <- 2
ymax <- 4

integral2(fun, xmin, xmax, ymin, ymax)
```

```
## $Q
## [1] 5.341559e+17
##
## $error
## [1] 15214781912
```