DATA605 - Assignment 9

Nick Oliver

Assignment 9

1. 11 page 363

11 The price of one share of stock in the Pilsdorff Beer Company (see Exercise 8.2.12) is given by Y_n on the nth day of the year. Finn observes that the differences $X_n = Y_{n+1} - Y_n$ appear to be independent random variables with a common distribution having mean $\mu = 0$ and variance $\sigma^2 = 1/4$. If $Y_1 = 100$, estimate the probability that Y_{365} is

```
n <- 364
y1 <- 100
variance  <- 1/4
mu <- 0
sd <- sqrt(n * variance)
```

(a) $\geq$ 100.

$Pr(Y_{365} \geq 100) = 0.5$

```
pnorm(100 - y1, mu, sd, lower.tail = F)
```

```
## [1] 0.5
```

(b) $\geq$ 110.

$Pr(Y_{365} \geq 110) = 0.1472537$

```
pnorm(110 - y1, mu, sd, lower.tail = F)
```

```
## [1] 0.1472537
```

(c) $\geq$ 120.

$Pr(Y_{365} \geq 120) = 0.01801584$

```
pnorm(120 - y1, mu, sd, lower.tail = F)
```

```
## [1] 0.01801584
```

2. Calculate the expected value and variance of the binomial distribution using the moment generating function.

From Wikipedia the $M_x(t)$ for a binomial distribution is

$$(1 - p + pe^t)^n$$

The mean is equal to the first derivative of the moment generating function at t=0 and the variance is equal to the second derivative of the moment generating function at t=0 minus the mean squared

Mean

$$\begin{aligned}\mu = M'(0) &= n(1-p+pe^0)^{n-1}(pe)^0 \\ &= n(1-p+p)^{n-1}p(1) \\ &= np\end{aligned}$$

Variance

$$\begin{aligned}M''(0) &= n(1-p+pe^0)^{n-1}(pe^0) + (pe^0)n(n-1)(1-p+pe^0)^{n-2}pe^0 \\ &= n(1)^{n-1}p + pn(n-1)(1)^{n-2}p(1) \\ &= np + n^2p^2 - np^2\end{aligned}$$

$$\begin{aligned}\sigma^2 = M''(0) - M'(0)^2 &= np + n^2p^2 - np^2 - np^2 \\ &= np + n^2p^2 - np^2 - n^2p^2 \\ &= np - np^2\end{aligned}$$

3. Calculate the expected value and variance of the exponential distribution using the moment generating function.

From Wikipedia the $M_x(t)$ for a exponential distribution is

$$\left(1 - t\lambda^{-1}\right)^{-1},\ t < \lambda = \frac{\lambda}{\lambda - t}$$

The mean is equal to the first derivative of the moment generating function at t=0 and the variance is equal to the second derivative of the moment generating function at t=0 minus the mean squared

Mean

$$\begin{aligned}\mu = M'(0) &= \frac{\lambda}{(\lambda-0)^2} \\ &= \frac{\lambda}{\lambda^2} \\ &= \frac{1}{\lambda}\end{aligned}$$

Variance

$$\begin{aligned}M''(0) &= \frac{2\lambda}{(\lambda-0)^3} \\ &= \frac{2}{\lambda^2}\end{aligned}$$

$$\begin{aligned}\sigma^2 = M''(0) - M'(0)^2 &= \frac{2}{\lambda^2} - (\frac{1}{\lambda})^2 \\ &= \frac{1}{\lambda^2}\end{aligned}$$