DATA605 - Assignment 9

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Assignment 9

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11 The price of one share of stock in the Pilsdorff Beer Company (see Exercise 8.2.12) is given by Y_n on the nth day of the year. Finn observes that the differences $X_n = Y_{n+1} - Y_n$ appear to be independent random variables with a common distribution having mean $\mu = 0$ and variance $\sigma^2 = 1/4$. If $Y_1 = 100$, estimate the probability that Y_{365} is

```
n <- 364
y1 <- 100
variance  <- 1/4
mu <- 0
sd <- sqrt(n * variance)
```

(a) $\geq$ 100.

$Pr(Y_{365} \geq 100) = 0.5$

```
pnorm(100 - y1, mu, sd, lower.tail = F)
```

```
## [1] 0.5
```

(b) $\geq$ 110.

$Pr(Y_{365} \geq 110) = 0.1472537$

```
pnorm(110 - y1, mu, sd, lower.tail = F)
```

```
## [1] 0.1472537
```

(c) $\geq$ 120.

$Pr(Y_{365} \geq 120) = 0.01801584$

```
pnorm(120 - y1, mu, sd, lower.tail = F)
```

```
## [1] 0.01801584
```

2. Calculate the expected value and variance of the binomial distribution using the moment generating function.

From Wikipedia the $M_x(t)$ for a binomial distribution is

$$(1 - p + pe^t)^n$$

The mean is equal to the first derivative of the moment generating function at t=0 and the variance is equal to the second derivative of the moment generating function at t=0

$$
\begin{aligned}
\mu = M'(0) &= n(1-p+pe^0)^{n-1}(pe)^0 \\
&= n(1-p+p)^{n-1}p(1) \\
&= np
\end{aligned}
$$

3. Calculate the expected value and variance of the exponential distribution using the moment generating function.

From Wikipedia the $M_x(t)$ for a exponential distribution is

$$\left(1 - t\lambda^{-1}\right)^{-1},\ t < \lambda$$

`The mean is equal to the first derivative of the moment generating function at t=0 and the variance is`