DATA605 - Assignment 9

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Assignment 10

Smith is in jail and has 1 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollars, he wins A dollars with probability .4 and loses A dollars with probability .6. Find the probability that he wins 8 dollars before losing all of his money if

(a) he bets 1 dollar each time (timid strategy).

A fit's the form of the Gambler's Ruin problem from the text "A gambler starts with a "stake" of size s. She plays until her capital reaches the value M or the value 0."

s = 1 M = 8

$$q_z = \frac{\frac{q}{p}^z - 1}{\frac{q}{p}^M - 1}$$

```
z <- 1
M <- 8
q <- .6
p <- .4
qz <- ((q/p)^z -1) / ((q/p)^M -1)
qz
```

```
## [1] 0.02030135
```

Answer: There is a 0.0203 probability that Smith wins the game without running out of money

(b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy).

Using the bold strategy there are now only transition states

Starting with one dollar:

bets 1 dollar then .4 wins 1 dollar → 2 dollars || .6 loses 1 dollar → 0 dollars

bets 2 dollars then .4 wins 2 dollars → 4 dollars || .6 loses 2 dollars → 0 dollars

bets 4 dollars then .4 wins 4 dollars → 8 dollars || .6 loses 4 dollars → 0 dollars

You can model this with an absorbing state markov chain

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0.4 & 0 & 0 \\ 0.6 & 0 & 0 & 0.4 & 0 \\ 0.6 & 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

You can use Theorem 11.2 from the book $u^{(n)} = uP^n$ where n = 3

```r
#using library expm
P <- matrix(c(
  c(1,0,0,0,0),
  c(0.6,0,0.4,0,0),
  c(0.6,0,0,0.4,0),
  c(0.6,0,0,0,0.4),
  c(0,0,0,0,1)),nrow = 5, byrow=TRUE)
P
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  1.0    0  0.0  0.0  0.0
## [2,]  0.6    0  0.4  0.0  0.0
## [3,]  0.6    0  0.0  0.4  0.0
## [4,]  0.6    0  0.0  0.0  0.4
## [5,]  0.0    0  0.0  0.0  1.0
```

```r
n <- 3
u1 <- c(0,1,0,0,0)
Pn <- P %^% n
u1 %*% Pn
```

```
##       [,1] [,2] [,3] [,4]  [,5]
## [1,] 0.936    0    0    0 0.064
```

Answer: There is a 0.064 probability that Smith ends up with 8 dollars before losing all his money

(c) Which strategy gives Smith the better chance of getting out of jail?

The bold strategy has a higher probability