DATA605 - Assignment 2

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C24† Find the eigenvalues, eigenspaces, algebraic and geometric multiplicities for

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\lambda I_3 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\lambda I_3 - A = \begin{bmatrix} \lambda - 1 & 1 & -1 \\ 1 & \lambda - 1 & 1 \\ -1 & 1 & \lambda - 1 \end{bmatrix}$$

Apply rule of Sarrus

$$\lambda I_3 - A = \begin{bmatrix} \lambda - 1 & 1 & -1 \\ 1 & \lambda - 1 & 1 \\ -1 & 1 & \lambda - 1 \end{bmatrix} \begin{pmatrix} \lambda - 1 & 1 \\ 1 & \lambda - 1 \\ -1 & 1 \end{pmatrix}$$

$$(\lambda-1)(\lambda-1)(\lambda-1)+(1)(1)(-1)+(-1)(1)(1)-(1)(1)(\lambda-1)-(\lambda-1)(1)(1)-(-1)(\lambda-1)(-1)$$

$$1 + (-1 + \lambda)^3 - 3\lambda = 01 + (-1 + 3)^3 - 3 * 3 = 0\lambda = 31 + (-1 + 0)^3 - 3 * 0 = 0\lambda = 0$$

```
A = matrix(c(1,-1,1,-1,1,-1,1), nrow = 3, byrow=TRUE)
A
```

```
## [,1] [,2] [,3]
## [1,] 1 -1 1
## [2,] -1 1 -1
## [3,] 1 -1 1
```

eigen(A)

```
## eigen() decomposition
## $values
## [1] 3.000000e+00 8.881784e-16 -2.372134e-17
##
## $vectors
## [,1] [,2] [,3]
```

```
## [1,] 0.5773503 0.8164966 0.0000000
## [2,] -0.5773503 0.4082483 0.7071068
## [3,] 0.5773503 -0.4082483 0.7071068
```