DATA605 - Assignment 7

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Assignment 7

1. Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y .

I know the solution is supposed to be For 1 j k, $m(j) = \frac{(k-j+1)n-(k-j)}{k^n}$ from this website https://math.dartmouth.edu/archive/m20f10/public_html/HW5Solutions.pdf/ but I will fully admit I do not understand why.

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

Probability

$$Pr(X = x) = (1 - p)^k p$$

```
geomProb <- (1 - (.1))^(8) * .1
geomProb
```

```
## [1] 0.04304672
```

```
dgeom(8,.1)
```

```
## [1] 0.04304672
```

Answer: **0.04304672**

Expected Value

$$E[X] = \frac{1}{p}$$

```
1/.1
```

```
## [1] 10
```

Answer: **10**

Standard Deviation

$$StdDev = \frac{\sqrt{1-p}}{p}$$

```r
sqrt(1 - .1) / .1
```

```
## [1] 9.486833
```

Answer:**9.486833**

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

Probability

$$Pr(X = k) = \lambda e^{-\lambda x}$$

```r
prob <- .1
prob * exp(-prob * 8)
```

```
## [1] 0.0449329
```

```r
dexp(8,.1)
```

```
## [1] 0.0449329
```

Answer: **0.0449329**

Expected Value

$$E[x] = \frac{1}{\lambda}$$

```r
1 / .1
```

```
## [1] 10
```

Answer: **10**

Standard Deviation

$$StdDev = \sqrt{\frac{1}{\lambda^2}}$$

```r
sqrt(1/(.1^2))
```

```
## [1] 10
```

Answer: **10**

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

Probability

$$Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

```r
k <- 0
prob <- 1/10
n <- 8
nCk <- choose(n,k)
 (nCk * (prob ^ k)) * (1 - prob) ^ (n-k)
```

```
## [1] 0.4304672
```

```r
dbinom(k,n,prob)
```

```
## [1] 0.4304672
```

Answer: **0.4304672**

Expected Value

$$E[x] = np$$

Answer: **0.8**

Standard Deviation

$$StdDev = \sqrt{np(1-p)}$$

```r
prob <- 1/10
n <- 8
sqrt(n * prob * ( 1 - prob))
```

```
## [1] 0.8485281
```

Answer: **0.8485281**

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson

Probability

$$Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

```r
k <- 0
prob <- 1/10

lambda <- 8 * prob / 1

((lambda ^ k) * exp(-lambda))/factorial(k)
```

```
## [1] 0.449329
```

```r
dpois(0,.8)
```

```
## [1] 0.449329
```

Answer: **0.449329**

Expected Value

$$E[X] = \lambda$$

Answer: **0.8**

Standard Deviation

$$StdDev = \sqrt{\lambda}$$

```r
sqrt(lambda)
```

```
## [1] 0.8944272
```

Answer: **0.8944272**