

\times is used to denote vector cross product, \bullet is used to denote vector dot product, and \cdot is used to denote vector-scalar multiplication.

The course notes give us the derivative of an edge e_x of a rigid body x having an angular velocity of ω_x :

$$\frac{d}{dt}e_x = \omega_x \times e_x \quad (1)$$

Using this, we can find the derivative of $e_a \times e_b$ using the product rule (https://en.wikipedia.org/wiki/Cross_product#Differentiation):

$$\frac{d}{dt}(e_a \times e_b) = \frac{de_a}{dt} \times e_b + e_a \times \frac{de_b}{dt} = (\omega_a \times e_a) \times e_b + e_a \times (\omega_b \times e_b) \quad (2)$$

The derivative of the norm of a vector is given by (<https://math.stackexchange.com/questions/291318/derivative-of-the-2-norm-of-a-multivariate-function>):

$$\frac{d}{dt}\|n\| = \frac{1}{\|n\|}(n \bullet \frac{d}{dt}n) \quad (3)$$

With which we find the derivative of $\|e_a \times e_b\|$:

$$\frac{d}{dt}\|e_a \times e_b\| = \frac{1}{\|e_a \times e_b\|}(e_a \times e_b \bullet \frac{d}{dt}(e_a \times e_b)) \quad (4)$$

With the quotient rule and Equations 2 and 4, we can derive \hat{n} :

$$\begin{aligned} \frac{d\hat{n}}{dt} &= \frac{d}{dt} \frac{e_a \times e_b}{\|e_a \times e_b\|} = \frac{\|e_a \times e_b\| \cdot \frac{d}{dt}(e_a \times e_b) - (e_a \times e_b) \cdot \frac{d}{dt}\|e_a \times e_b\|}{\|e_a \times e_b\|^2} \\ &= \frac{\|e_a \times e_b\| \cdot \frac{d}{dt}(e_a \times e_b) - (e_a \times e_b) \cdot \left(\frac{1}{\|e_a \times e_b\|}(e_a \times e_b \bullet \frac{d}{dt}(e_a \times e_b))\right)}{\|e_a \times e_b\|^2} \\ &= \frac{\frac{d}{dt}(e_a \times e_b) - \frac{e_a \times e_b}{\|e_a \times e_b\|} \cdot \left(\frac{e_a \times e_b}{\|e_a \times e_b\|} \bullet \frac{d}{dt}(e_a \times e_b)\right)}{\|e_a \times e_b\|} \end{aligned} \quad (5)$$

Finally, we can now state the correct code listing. The $*$ symbols in the **n1** and **z** calculations should have been \wedge indicating cross products. The **n** variable on the last line should have been **n1**, the normalized cross product.

```
triple computeNdot(Contact *c)
{
    if (c->vf)
    {
        return c->b->omega ^ c->n;
    }
    else
    {
        triple eadot = c->a->omega ^ ea,
               ebdot = c->b->omega ^ eb,
               n1 = ea ^ eb,
               z = eadot ^ eb + ea ^ ebdot;
        double l = length(n1);
        n1 = n1 / l;
        return (z - ((z * n1) * n1)) / l;
    }
}
```