Introduction to many-body perturbation theory, GW band gap calculations and RPA energy calculations

Lucia Reining
Palaiseau Theoretical Spectroscopy Group

Introduction to many-body perturbation theory, GW band gap calculations and RPA energy calculations

→ Why do we need Green's functions?

→ From Green's functions to observables

→ A new auxiliary world

→ The GW approximation: (dynamical) screening!

→ Flavours of the GWA

→ What is wrong, and outlook

T=0, N fixed

$$O = \langle \hat{O} \rangle$$

$$O = \int ... \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N) \sum_{i,j,...} O(x_i, x_j, ...) \Psi(x_1, ...x_N)$$

$$O = \langle \hat{O} \rangle$$

$$O = \int ... \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N) \sum_{i,j,...} O(x_i, x_j, ...) \Psi(x_1, ...x_N)$$

$$O = O[\Psi]$$

$$\Psi \longrightarrow O$$

Density Functional Theory: not only $O = O[\Psi]$

$$O_1 = \int dx_1 \, O_1(x_1) n(x_1)$$

$\downarrow$

$\tilde{O}[v_{\text{ext}}]$

$\downarrow$

$\mathcal{O}[n]$

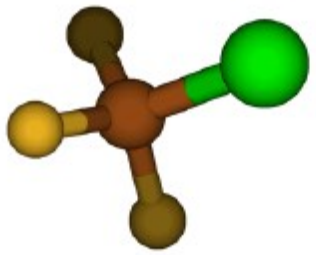
…….but for all ground state O:

P. Hohenberg and W. Kohn, Phys. Rev. 136 B864 (1964)

The descriptor:

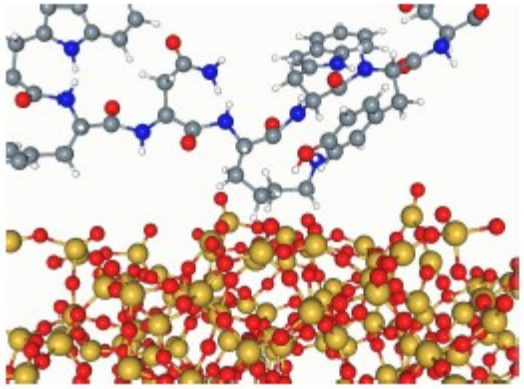
$\Psi(x_1, x_2, ..., x_N; t)$

CI, QMC



$n(\mathbf{r}; t)$

Density Functionals



Density functional theory

$$\mathcal{O}[n]$$

And where do we get the density from ?

Density functional theory

$$\mathcal{O}[n]$$

And where do we get the density from ?

$$n(r) = \langle \Psi | \hat{\psi}^{\dagger}(r) \hat{\psi}(r) | \Psi \rangle$$

Density functional theory

$$\mathcal{O}[n]$$

And where do we get the density from ?

$$n(r) = \langle \Psi | \hat{\psi}^{\dagger}(r) \hat{\psi}(r) | \Psi \rangle$$

But we do not want to calculate $|\Psi\rangle$

Density functional theory

$$\mathcal{O}[n]$$

And where do we get the density from ?

Next great idea in DFT:

→ Build an auxiliary non-interacting system that yields the correct density

The Kohn-Sham simulation chamber

Image from pixabay:
www.noft-traders.com/establish-zero-gravity-zones-with-supply-and-demand/

Density Functional Theory and the Kohn-Sham auxiliary system

$$\left(-\frac{1}{2}\boldsymbol{\nabla}^2 + v_{\text{eff}}(\mathbf{r})\right)\psi_i(\mathbf{r}) = \varepsilon_i\psi_i(\mathbf{r})$$

$$v_{\text{eff}}(\mathbf{r}) = \quad v_{\text{ext}}(\mathbf{r}) + v_{\text{H}}([n],\mathbf{r}) + v_{\text{xc}}([n],\mathbf{r})$$

W. Kohn and L. J. Sham, Phys. Rev. 140:A1133–1138, 1965

Density Functional Theory and the Kohn-Sham auxiliary system

$$\left(-\frac{1}{2}\boldsymbol{\nabla}^2 + v_{\text{eff}}(\mathbf{r})\right)\psi_i(\mathbf{r}) = \varepsilon_i\psi_i(\mathbf{r})$$

$$v_{\text{eff}}(\mathbf{r}) = v_{\text{ext}}(\mathbf{r}) + v_{\text{H}}([n],\mathbf{r}) + v_{\text{xc}}([n],\mathbf{r})$$

W. Kohn and L. J. Sham, Phys. Rev. 140:A1133–1138, 1965

$$v_{\text{xc}}(\mathbf{r},[n]) = \frac{\delta E_{\text{xc}}[n]}{\delta n(\mathbf{r})}$$

We do not know $E_{\text{xc}}[n]$, hence the potential → Approximate

The Kohn-Sham simulation chamber

Image from pixabay:
www.noft-traders.com/establish-zero-gravity-zones-with-supply-and-demand/

The Kohn-Sham simulation chamber

Image from pixabay:
www.noft-traders.com/establish-zero-gravity-zones-with-supply-and-demand/

Density Functional Theory and the Kohn-Sham auxiliary syste

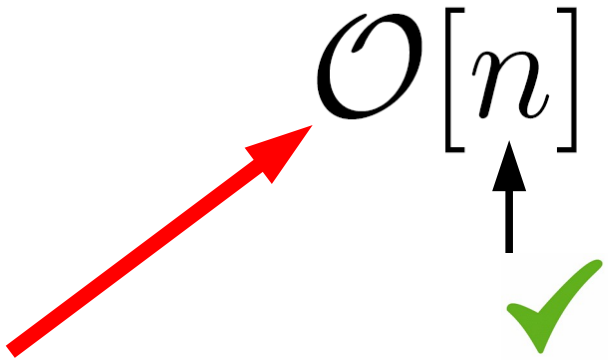
$$\left(-\frac{1}{2}\boldsymbol{\nabla}^2 + v_{\text{eff}}(\mathbf{r})\right)\psi_i(\mathbf{r}) = \varepsilon_i\psi_i(\mathbf{r})$$

$$v_{\text{eff}}(\mathbf{r}) = \quad v_{\text{ext}}(\mathbf{r}) + v_{\text{H}}([n],\mathbf{r}) + v_{\text{xc}}([n],\mathbf{r})$$

W. Kohn and L. J. Sham, Phys. Rev. 140:A1133–1138, 1965

This system is designed to yield the correct density. Nothing else.

Density functional theory

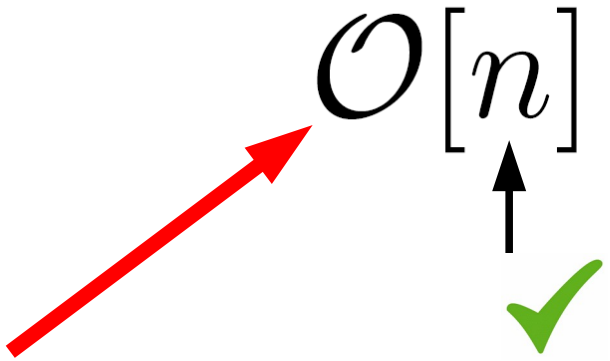


And where do we get this functional from ?

Very often: we do not know

→ we calculate observables in the KS system

Density functional theory

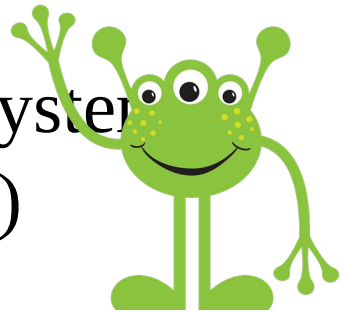


And where do we get this functional from ?

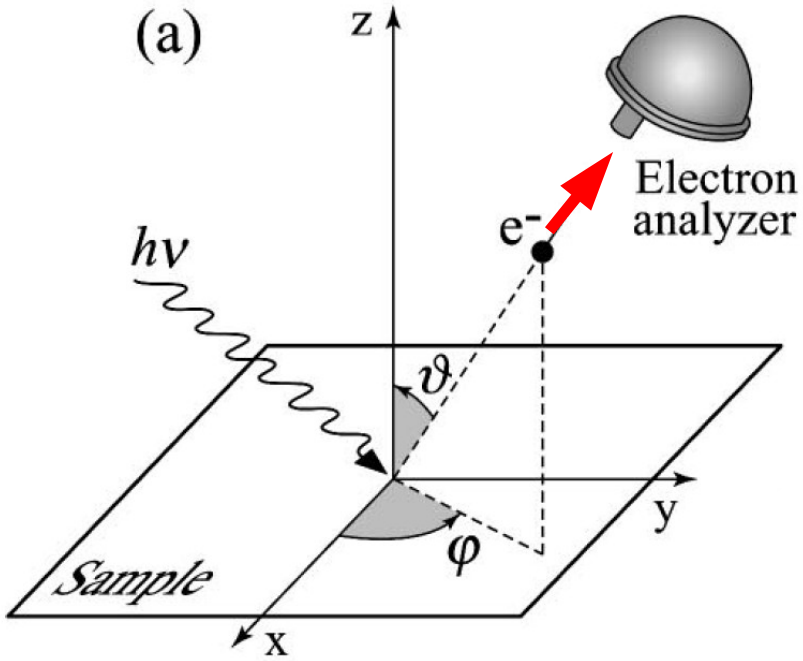
Very often: we do not know

→ we calculate observables in the KS syste

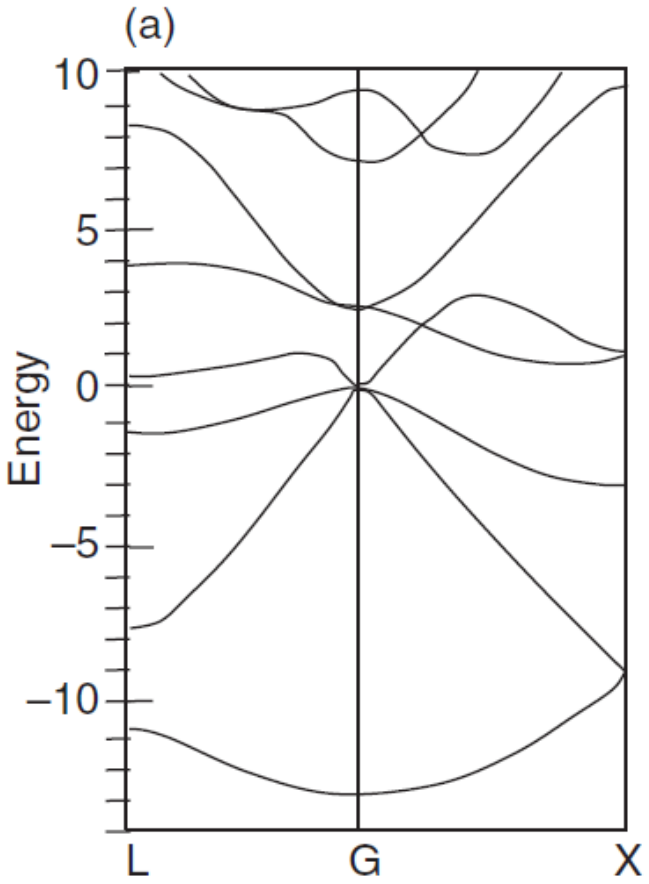
(This can be seen as approximate functional)



Can we use this approximation for photoemission spectra?



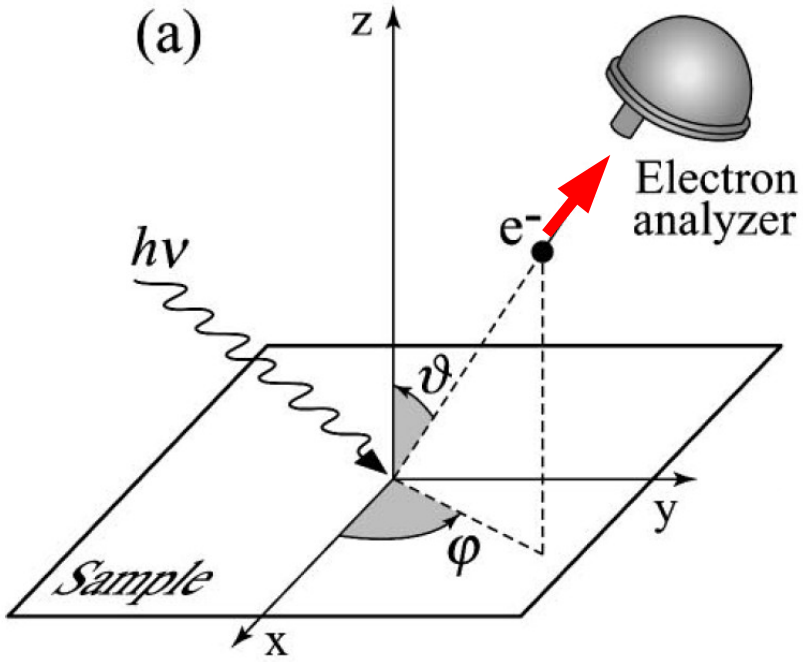
Photoemission geometry



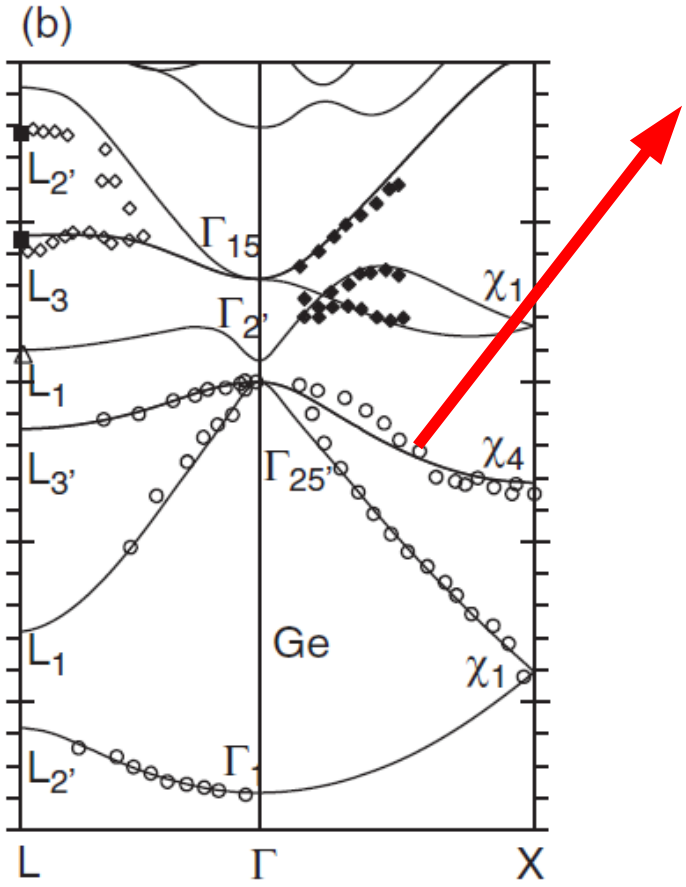
From Damascelli et al., RMP 75, 473 (2003)

+.....

Can we use this approximation for photoemission spectra?



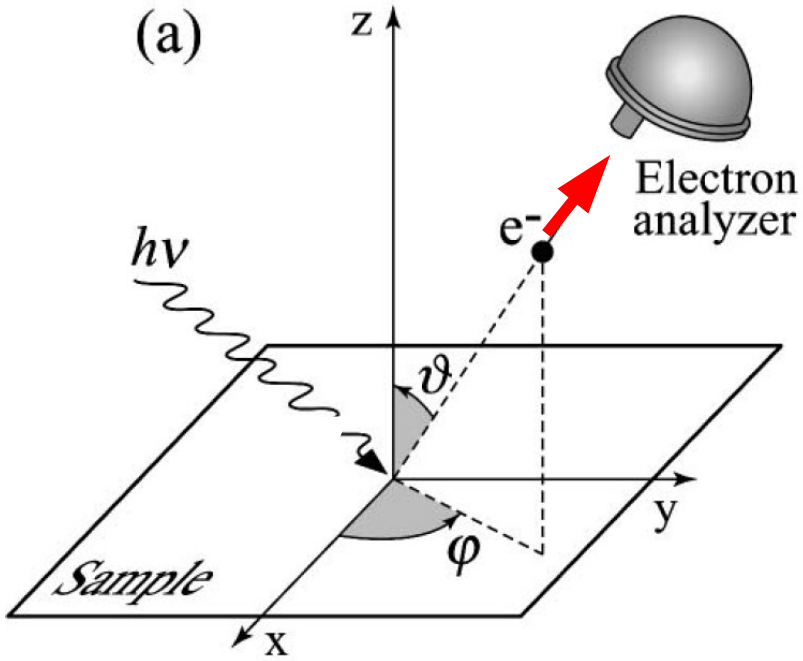
Photoemission geometry



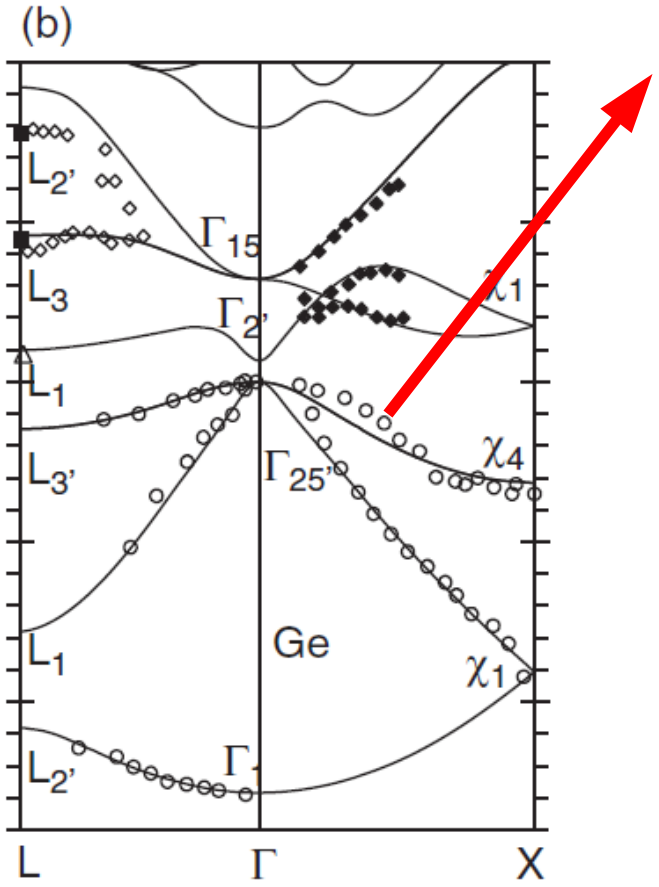
From Damascelli et al., RMP 75, 473 (2003)

+.....

Can we use this approximation for photoemission spectra?



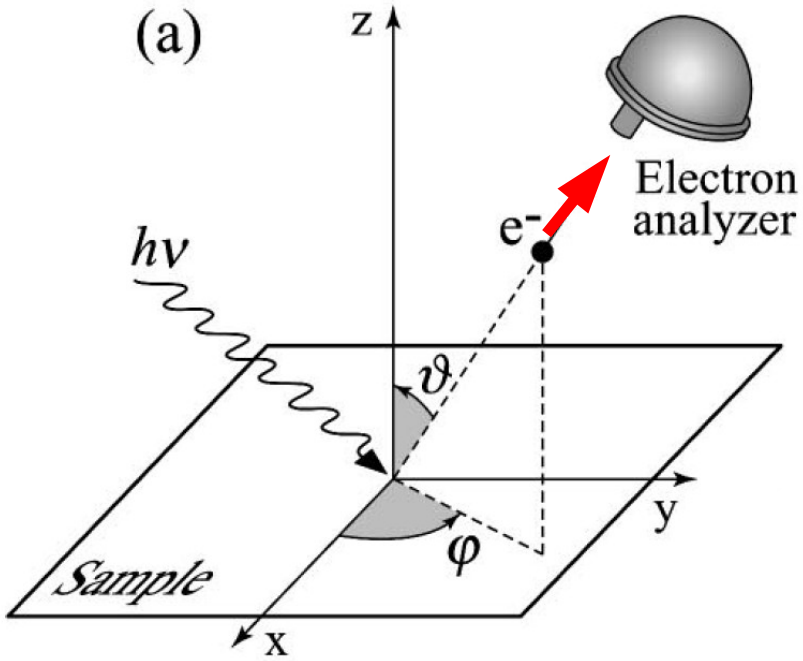
Photoemission geometry



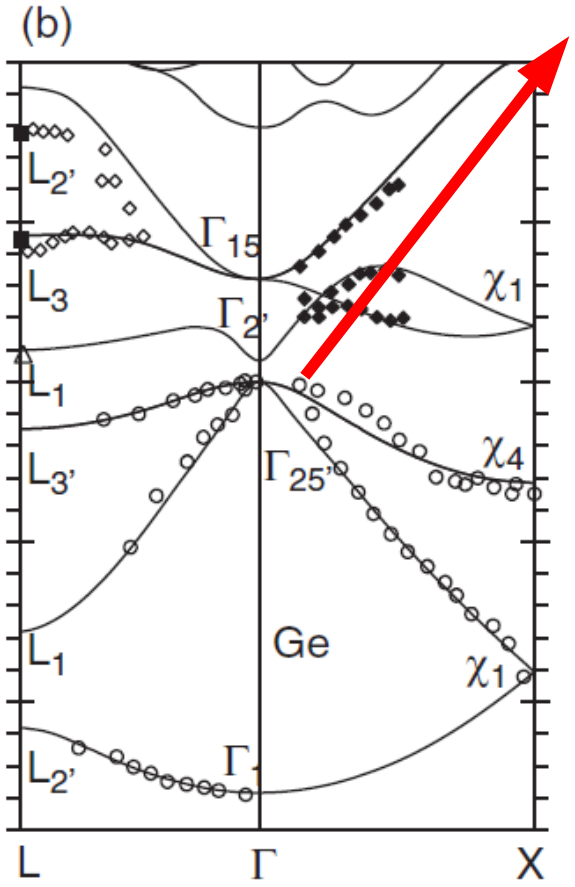
From Damascelli et al., RMP 75, 473 (2003)

+.....

Can we use this approximation for photoemission spectra?



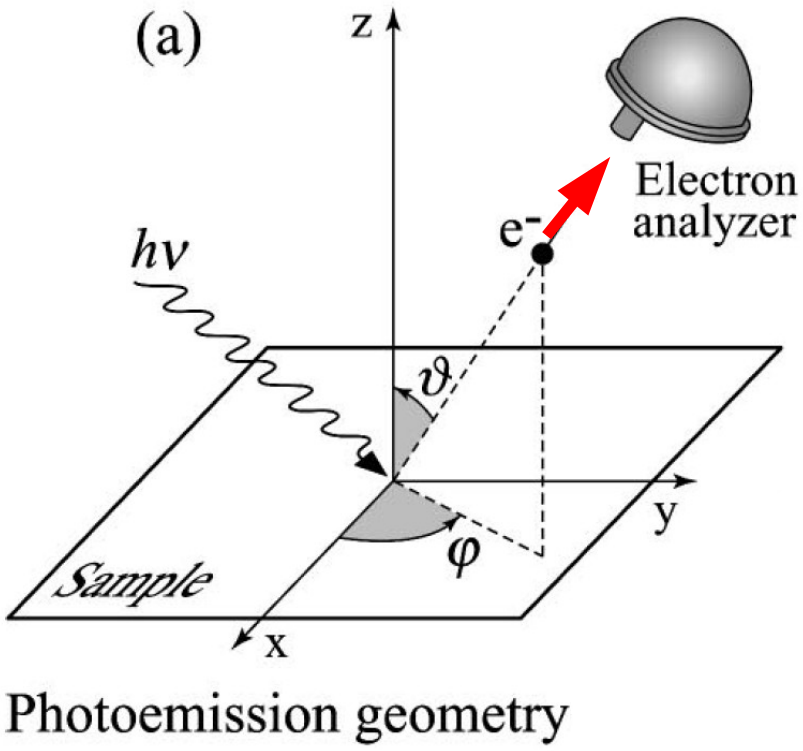
Photoemission geometry



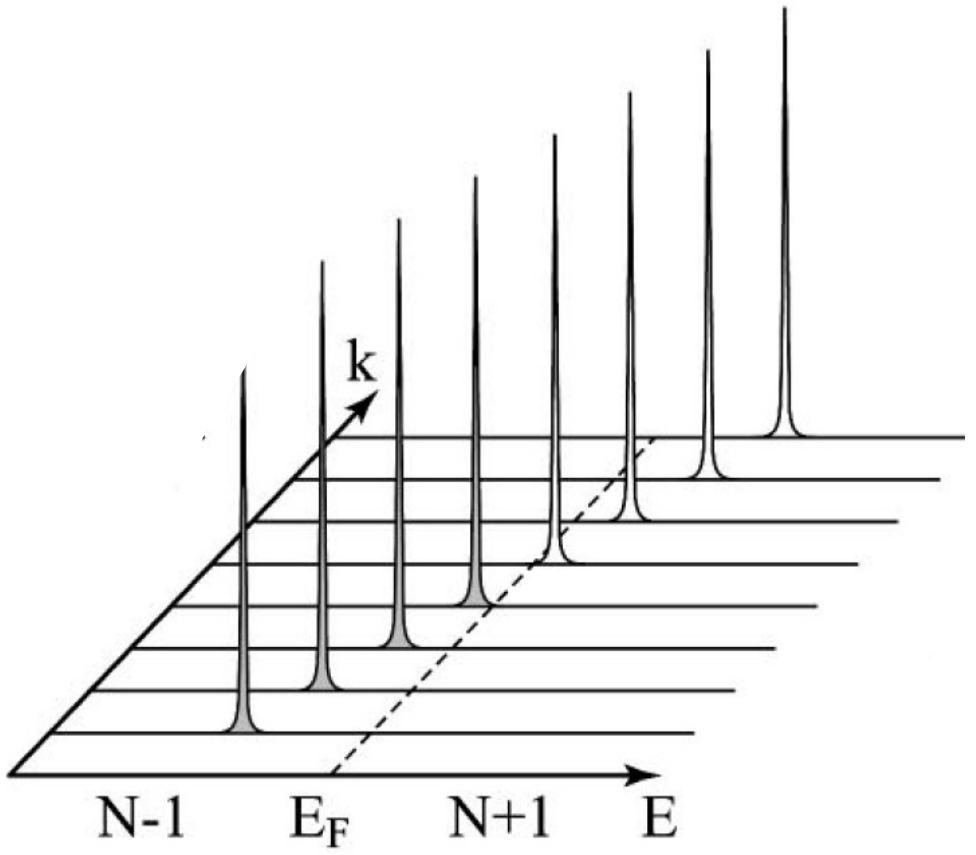
From Damascelli et al., RMP 75, 473 (2003)

+.....

Can we use this approximation for photoemission spectra?



Photoemission geometry

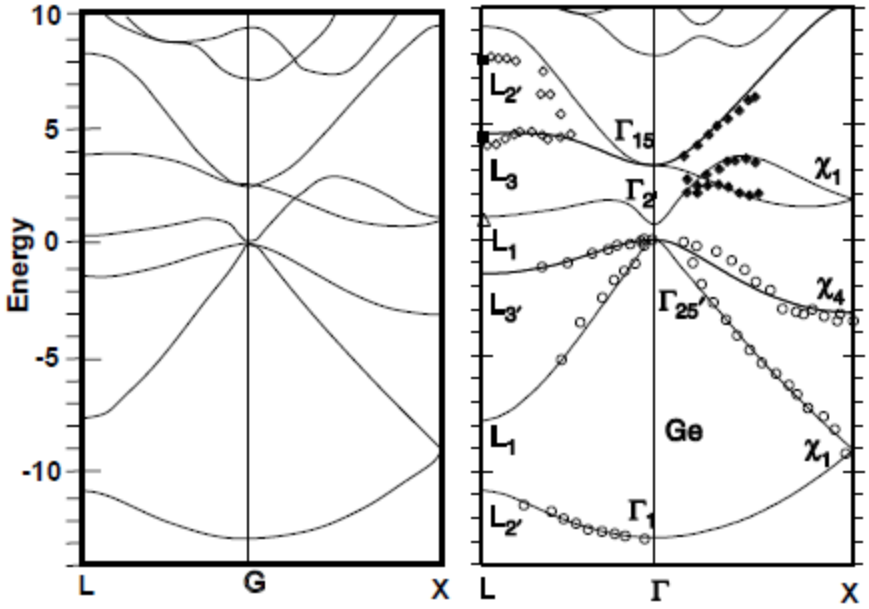


From Damascelli et al., RMP 75, 473 (2003)

+.....

Can we use this approximation for photoemission spectra?

, plus LDA



Band structure of bulk germanium

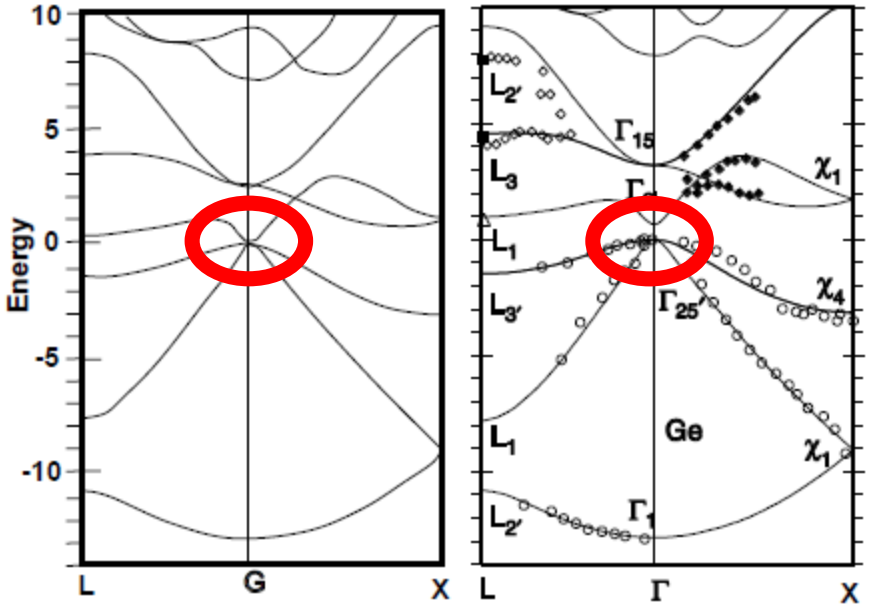
Rohlfing et al., PRB 48, 17791 (1993)

GW calculations, Rohlfing et al., PRB 48, 17791 (1993)

Bandstructure of germanium, theory versus experiment

“The Kohn-Sham band gap problem”

, plus LDA



Band structure of bulk germanium

Rohlfing et al., PRB 48, 17791 (1993)

GW calculations, Rohlfing et al., PRB 48, 17791 (1993)

Bandstructure of germanium, theory versus experiment

“Exact” KS band gaps (in eV)
using xc potential reconstructed from AFQMC density:

	Si		NaCl
	indirect	direct at Γ	(direct at Γ)
AFQMC	0.69	2.72	5.25
PBE	0.66	2.60	5.08
LDA	0.49	2.55	4.59
Exp:	1.17	>3	8.5

A. Aouina, M. Gatti, S. Chen, S. Zhang, L. Reining Phys. Rev. B 107, 195123

Confirming previous work:

R. W. Godby, M. Schlüter, and L. J. Sham, Phys. Rev. Lett. 56, 2415 (1986); Phys. Rev. B 37, 101 (1988)

Y. M. Niquet and X. Gonze, Phys. Rev. B 70, 245115 (2004)

M. Grüning, A. Marini, and A. Rubio, Phys. Rev. B 74,161103 (2006); J.Chem.Phys. 124, 154108 (2006).

T. Kotani, J. Phys.: Condens. Matter 10, 9241 (1998).

J. Klimeš and G. Kresse, The Journal of Chemical Physics 140, 054516 (2014).

S. Riemelmoser, M. Kaltak, and G. Kresse, J. Chem. Phys. 154, 154103 (2021).

“Exact” KS band gaps (in eV) using xc potential reconstructed from AFQMC density:

	Si		NaCl
	indirect	direct at Γ	(direct at Γ)
Exp:	1.17	>3	8.5

The exact KS gap is definitely smaller than the photoemission gap

The “band gap problem” comes from the approx. use of the KS system

A. Aouina, M. Gatti, S. Chen, S. Zhang, L. Reining Phys. Rev. B 107, 195123 (2023)

Confirming previous work:

R. W. Godby, M. Schlüter, and L. J. Sham, Phys. Rev. Lett. 56, 2415 (1986); Phys. Rev. B 37, 10159 (1988)
Y. M. Niquet and X. Gonze, Phys. Rev. B 70, 245115 (2004)
M. Grüning, A. Marini, and A. Rubio, Phys. Rev. B 74,161103 (2006); J.Chem.Phys. 124, 154108 (2006).
T. Kotani, J. Phys.: Condens. Matter 10, 9241 (1998).
J. Klimeš and G. Kresse, The Journal of Chemical Physics 140, 054516 (2014).
S. Riemelmoser, M. Kaltak, and G. Kresse, J. Chem. Phys. 154, 154103 (2021).

"Exact" KS band gaps (in eV)
using xc potential reconstructed from AFQMC density:

	Si		NaCl
	indirect	direct at Γ	(direct at Γ)

The exact KS gap is definitely smaller than the photoemission gap

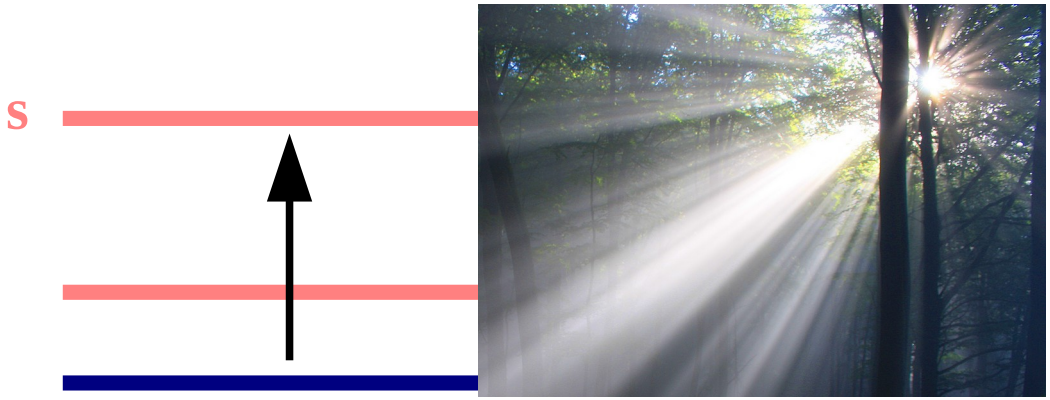
The "band gap problem" comes from the approx. use of the KS system

BUT: we have no clue about $E_{gap}[n]$!!!!!

Confirming previous work:
R. W. Godby, M. Schlüter, and L. J. Sham, Phys. Rev. Lett. 56, 2415 (1986); Phys. Rev. B 37, 10159 (1988)
Y. M. Niquet and X. Gonze, Phys. Rev. B 70, 245115 (2004)
M. Grüning, A. Marini, and A. Rubio, Phys. Rev. B 74,161103 (2006); J.Chem.Phys. 124, 154108 (2006).
T. Kotani, J. Phys.: Condens. Matter 10, 9241 (1998).
J. Klimeš and G. Kresse, The Journal of Chemical Physics 140, 054516 (2014).
S. Riemelmoser, M. Kaltak, and G. Kresse, J. Chem. Phys. 154, 154103 (2021).

Spectroscopy: have to describe transitions

$$f_{ds}(t) = N \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N; t) d(x_1) e^{i\omega t} \Psi_s(x_1, ...x_N; t)$$



Spectroscopy: have to describe transitions

$$f_{ds}(t) = N \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N; t) d(x_1) e^{i\omega t} \Psi_s(x_1, ...x_N; t)$$

$$= e^{i(E_s - E_0 + \omega)t} N \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N) d(x_1) \Psi_s(x_1, ...x_N)$$

Phase factor: excitation energy

Spectroscopy: have to describe transitions

$$f_{ds}(t) = N \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N; t) d(x_1) e^{i\omega t} \Psi_s(x_1, ...x_N; t)$$

$$= e^{i(E_s - E_0 + \omega)t} N \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N) d(x_1) \Psi_s(x_1, ...x_N)$$

Phase factor: excitation energy

For the transition amplitude

Spectroscopy: have to describe transitions

$$f_{ds}(t) = N \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N; t) d(x_1) e^{i\omega t} \Psi_s(x_1, ...x_N; t)$$

$$= e^{i(E_s - E_0 + \omega)t} N \int dx_1 ... dx_N \, \Psi^*(x_1, ...x_N) d(x_1) \Psi_s(x_1, ...x_N)$$

Phase factor: excitation energy

For the transition amplitude

Need amplitudes to excited states
and phase factors/energies

For photoemission: we need transition energies and amplitudes N → N-1

$$E_s \equiv E_0 - E_{N-1,s}$$

$$f_s(x_1,t) \equiv e^{iE_s t} N \int dx_2, \ldots\ldots dx_N \, \Psi^*(x_1, \ldots x_N)\Psi_{N-1,s}(x_2, \ldots . x_N)$$

$$f_s(x_1,t) = e^{iE_s t} f_s(x_1)$$

We build an object that contains the desired information (and also N → N+1)

$$G(x_1, x_1', t, t') = -ie^{-iE_s(t-t')}\Big[\Theta(t-t')\Theta(E_s-\mu) - \Theta(t'-t)\Theta(\mu-E_s)\Big] f_s(x_1) f_s^*(x_1')$$

One-body Green's function,

describing propagation of electrons and holes

Written using second quantization:

$$G(x_1, x_1', t, t') = -i\langle N|T[\hat{\Psi}(x_1,t)\hat{\Psi}^\dagger(x_1',t')]|N\rangle$$

Introduction to many-body perturbation theory, GW band gap calculations and RPA energy calculations

→ Why do we need Green's functions?

→ From Green's functions to observables

→ A new auxiliary world

→ The GW approximation: (dynamical) screening!

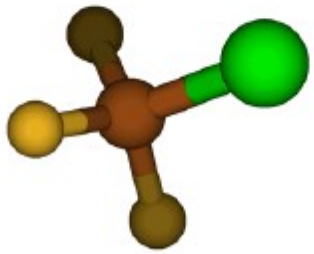
→ Flavours of the GWA

→ What is wrong, and outlook

The descriptor:

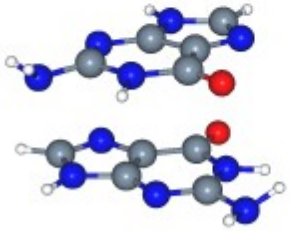
$\Psi(x_1, x_2, ..., x_N; t)$

CI, QMC



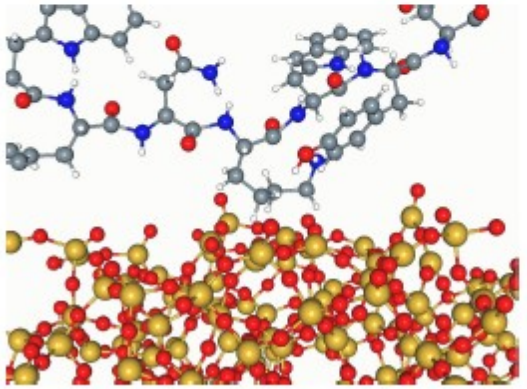
$G(x_1, x_2; t_1, t_2)$

Green's Functions



$n(\mathbf{r}; t)$

Density Functionals



The spectral function is a simple functional of the Green's function

Spectral function: peaks at transition energies, probabilities → intensity

$$\frac{1}{\pi}\left|\mathrm{Im}\, G_{\mathbf{k}}(\omega)\right| = \sum_{\lambda} \left|\mathrm{f}_{\lambda}(\mathbf{k})\right|^2 \delta(\omega - \varepsilon_{\lambda})$$

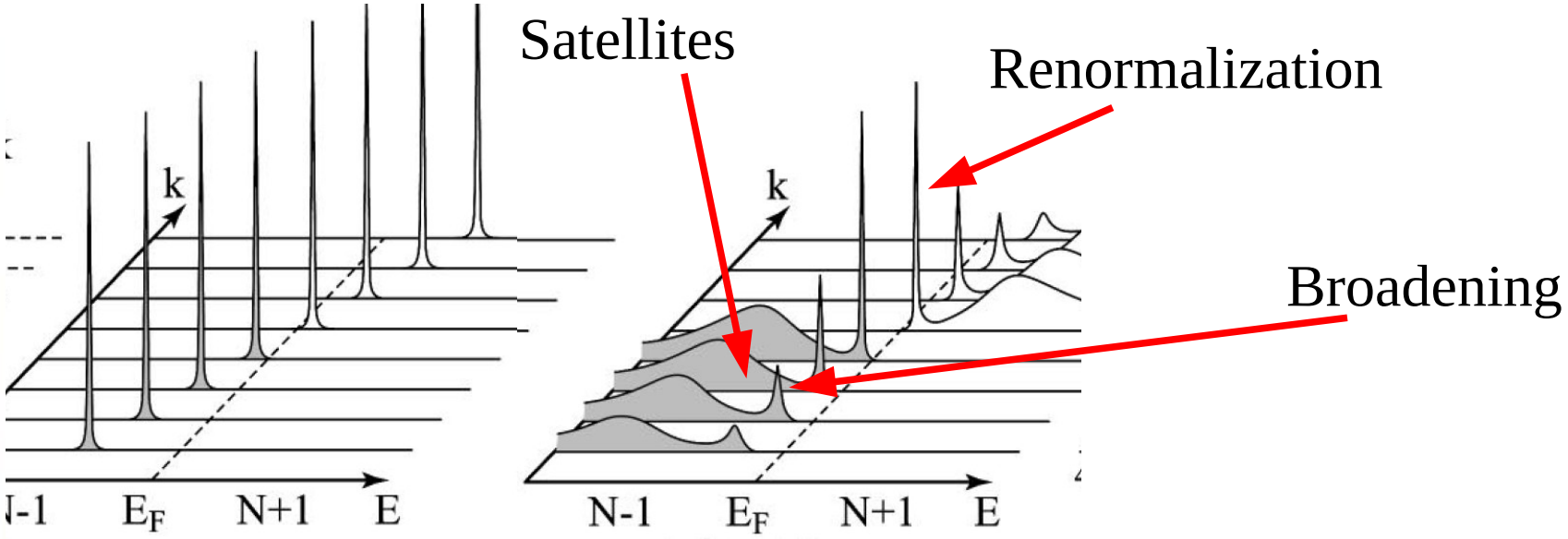
$$G^{T}(x_1, x_2, \omega) = \lim_{\eta \to 0^+} \sum_{\lambda} \frac{\mathrm{f}_{\lambda}(x_1)\mathrm{f}^{*}_{\lambda}(x_2)}{\omega - \varepsilon_{\lambda} + i\eta\, \mathrm{sgn}\,(\varepsilon_{\lambda} - \mu)}$$

$$\varepsilon_{\lambda} = E(N+1, \lambda) - E_0 > \mu \qquad \varepsilon_{\lambda} = E_0 - E(N-1, \lambda) < \mu$$

One-body GF

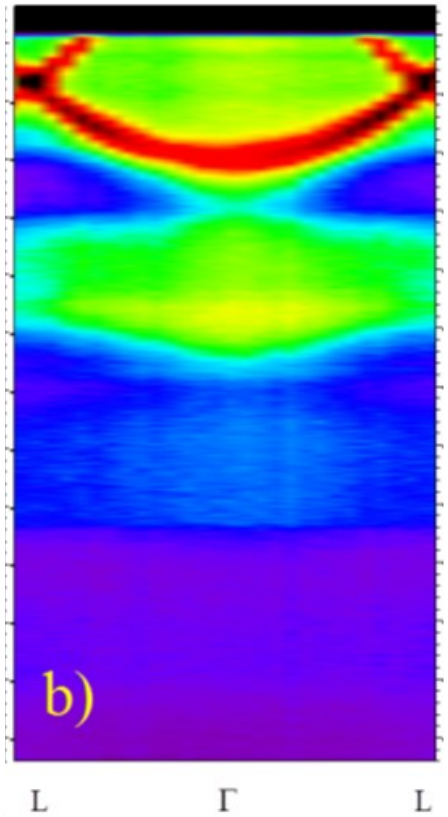
Spectral function: peaks at transition energies, probabilities → intensity

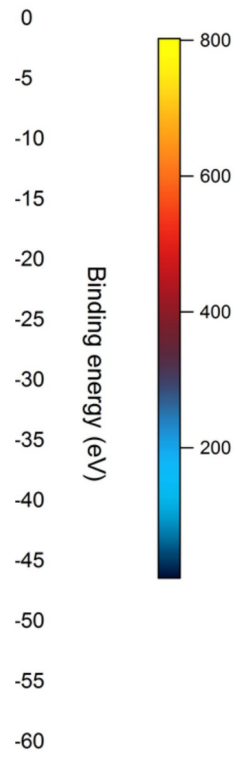
$$A_{\mathbf{k}}(\omega) = \frac{1}{\pi} |\mathrm{Im}\, G_{\mathbf{k}}(\omega)|$$



Photoemission of bulk aluminum

Experiment





Zhou, Reining, Nicolaou, Bendounan, Ruotsalainen, Vanzini, Kas, Rehr, Muntwiler, Strocov, Sirotti, Gatti, PNAS 117 (46), 28596 (2020)

$$\mathcal{O}[n] \rightarrow \mathrm{O}\ ???$$

$$O[G] \rightarrow \mathrm{O}$$ ✓

$$A_{\mathbf{k}}(\omega) = \frac{1}{\pi} |\mathrm{Im}\, G_{\mathbf{k}}(\omega)|$$

$$A(\omega) = \sum_{\mathbf{k}} A_{\mathbf{k}}(\omega)$$

Also: $\rho(\mathbf{r}, \mathbf{r}') = -iG(\mathbf{r}, \mathbf{r}', t, t^+)$

$$n(\mathbf{r}) = -iG(\mathbf{r}, \mathbf{r}, t, t^+)$$

$$\mathcal{O}[n] \rightarrow \mathrm{O}\ ???$$

$$O[G] \rightarrow \mathrm{O}$$ ✓

$$A_{\mathbf{k}}(\omega) = \frac{1}{\pi} |\mathrm{Im}\, G_{\mathbf{k}}(\omega)|$$

But where do we get G from?

$$A(\omega) = \sum_{\mathbf{k}} A_{\mathbf{k}}(\omega)$$

$$G(x_1, x_1', t, t') = -i\langle N|T[\hat{\Psi}(x_1, t)\hat{\Psi}^\dagger(x_1', t')]|N\rangle$$

We know how to calculate G in PRINCIPLE……

……but this is exactly what we do NOT want to do in PRACTICE!!!

$$G_u(1,1') = G^0(1,1') + G^0(1,\bar{2})\left\{[u(\bar{2}) + v_H(\bar{2})]G_u(\bar{2},1') + \right.$$
$$\left. + iv_c(\bar{2},\bar{3})\frac{\delta G_u(\bar{2},1')}{\delta u(\bar{3}^+)}\right\}$$

L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics*

Introduction to many-body perturbation theory, GW band gap calculations and RPA energy calculations

→ Why do we need Green's functions?

→ From Green's functions to observables

→ A new auxiliary world

→ The GW approximation: (dynamical) screening!

→ Flavours of the GWA

→ What is wrong, and outlook

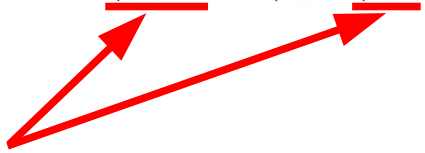
$$\Big(-\frac{\nabla^2}{2} + v_{\mathrm{ext}}(\mathbf{r}) + v_H(\mathbf{r}) + v_{\mathrm{xc}}(\mathbf{r}) \Big) \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r})$$

More advanced simulation chamber

Image from pixabay:
www.noft-traders.com/establish-zero-gravity-zones-with-supply-and-demand/

$$\left(-\frac{\nabla^2}{2} + v_{\rm ext}(\mathbf{r}) + v_H(\mathbf{r}) + v_{\rm xc}(\mathbf{r}) \right) \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r})$$

$$\left(-\frac{\nabla^2}{2} + v_{\rm ext}(\mathbf{r}) + v_H(\mathbf{r}) \right) \varphi_i(\mathbf{r};\omega) + \int d\mathbf{r}'\, \Sigma_{\rm xc}(\mathbf{r},\mathbf{r}';\omega) \varphi_i(\mathbf{r}';\omega) = \varepsilon_i(\omega) \varphi_i(\mathbf{r};\omega)$$



New features: → non-locality in space

$$\left(-\frac{\nabla^2}{2} + v_{\rm ext}(\mathbf{r}) + v_H(\mathbf{r}) + v_{\rm xc}(\mathbf{r}) \right) \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r})$$

$$\left(-\frac{\nabla^2}{2} + v_{\rm ext}(\mathbf{r}) + v_H(\mathbf{r}) \right) \varphi_i(\mathbf{r};\omega) + \int d\mathbf{r}'\, \Sigma_{\rm xc}(\mathbf{r},\mathbf{r}';\omega) \varphi_i(\mathbf{r}';\omega) = \varepsilon_i(\omega) \varphi_i(\mathbf{r};\omega)$$

New features: → non-locality in space

→ non-locality in time, hence, frequency-dependence

Descriptor	Auxiliary "potential"	
$n(\mathbf{r})$	$v_{\mathrm{xc}}(\mathbf{r})$	
$G(\mathbf{r}_1, \mathbf{r}_2, \omega)$	$\Sigma(\mathbf{r}_1, \mathbf{r}_2, \omega)$	
$G(\mathbf{r}, \mathbf{r}, \omega)$	$v_{SF}(\mathbf{r}, \omega)$	

Gatti, Olevano, Reining, Tokatly, PRL 99, 057401 (2007)

$G_{\ell\ell}(\omega)$	$\Sigma_{\ell}^{loc}(\omega)$	DMFT

A. Georges et al., Rev. Mod. Phys. 68, 13 (1996)

S. Y. Savrasov and G. Kotliar, Phys. Rev. B 69, 245101 (2004)

$$\Big(-\frac{\nabla^2}{2} + v_{\rm ext}(\mathbf{r}) + v_H(\mathbf{r}) + v_{\rm xc}(\mathbf{r}) \Big) \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r})$$

$$\Big(-\frac{\nabla^2}{2} + v_{\rm ext}(\mathbf{r}) + v_H(\mathbf{r}) \Big) \varphi_i(\mathbf{r};\omega) + \int d\mathbf{r}'\, \Sigma_{\rm xc}(\mathbf{r},\mathbf{r}';\omega) \varphi_i(\mathbf{r}';\omega) = \varepsilon_i(\omega) \varphi_i(\mathbf{r};\omega)$$

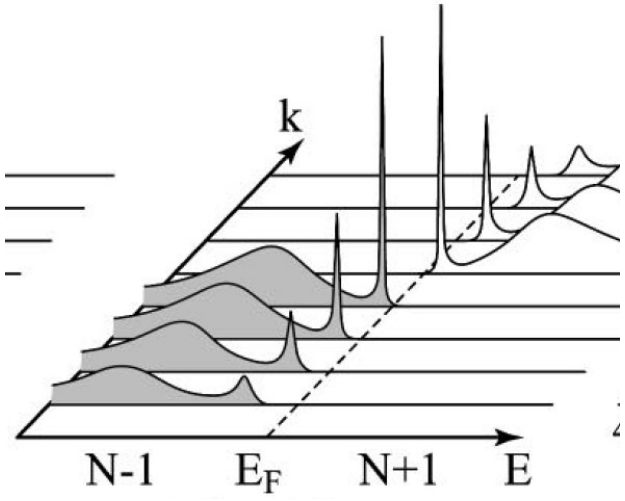
Usually formulated as Dyson equation:

$$G = G_0 + G_0 (v_H + \Sigma_{\rm xc}) G$$

Dyson equation: $G = G_0 + G_0 \Sigma G$

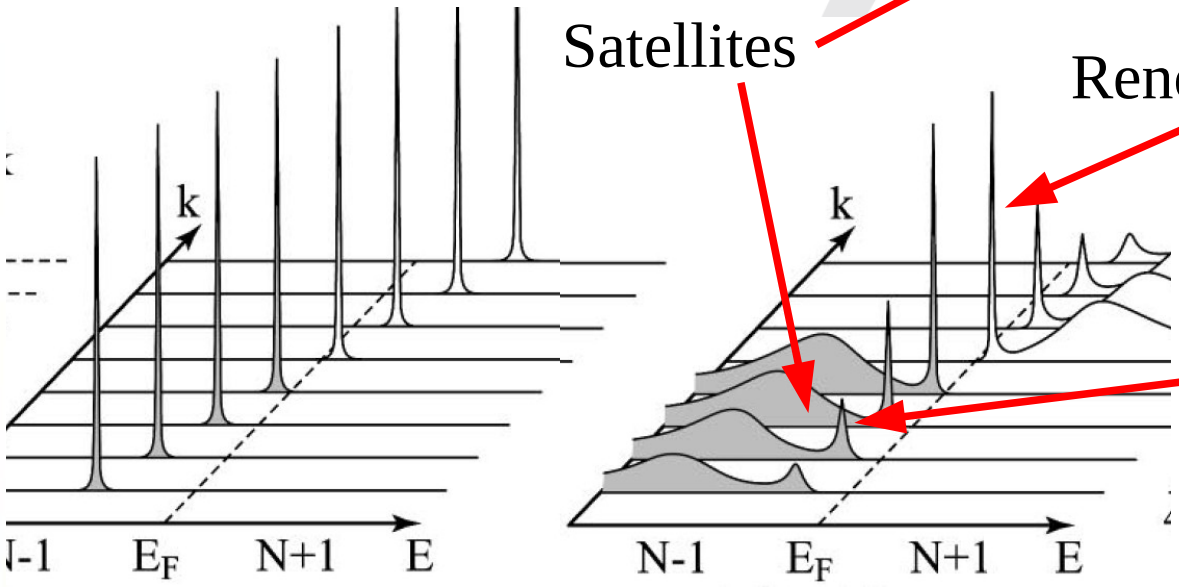
$$A_{\ell\ell}(\omega) = \frac{1}{\pi} |\mathrm{Im}\, G_{\ell\ell}(\omega)|$$

Solving the Dyson equation would give a full spectral function



→ The spectral function
Quasi-particles and beyond

$$A_{\mathbf{k}}(\omega) = \frac{1}{\pi}|\mathrm{Im}\, G_{\mathbf{k}}(\omega)| = \frac{1}{\pi}\frac{|\mathrm{Im}\, \Sigma_{\mathbf{k}}(\omega)|}{[\omega - \varepsilon^0_{\mathbf{k}} - \mathrm{Re}\, \Sigma_{\mathbf{k}}(\omega)]^2 + [\mathrm{Im}\, \Sigma_{\mathbf{k}}(\omega)]^2}$$



Satellites

Renormalization

Broadening

QP calculations in practice

Supposing wavefunctions do not change

Supposing imaginary part is small

$$\varepsilon_{\mathbf{k}} = \varepsilon^0_{\mathbf{k}} + \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega = \varepsilon_{\mathbf{k}})$$

$$\mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega) \approx \mathrm{Re}\,\Sigma_{\mathbf{k}}(\varepsilon^0_{\mathbf{k}}) + (\omega - \varepsilon^0_{\mathbf{k}}) \frac{\partial \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega')}{\partial \omega'}_{|\omega'=\varepsilon^0_{\mathbf{k}}}$$

$$Z_{\mathbf{k}} = \frac{1}{1 - \frac{\partial \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega')}{\partial \omega'}_{|\omega'=\varepsilon^0_{\mathbf{k}}}}$$

$$\varepsilon_{\mathbf{k}} \approx \varepsilon^0_{\mathbf{k}} + Z_{\mathbf{k}} \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega = \varepsilon^0_{\mathbf{k}})$$

Introduction to many-body perturbation theory, GW band gap calculations and RPA energy calculations

→ Why do we need Green's functions?

→ From Green's functions to observables

→ A new auxiliary world

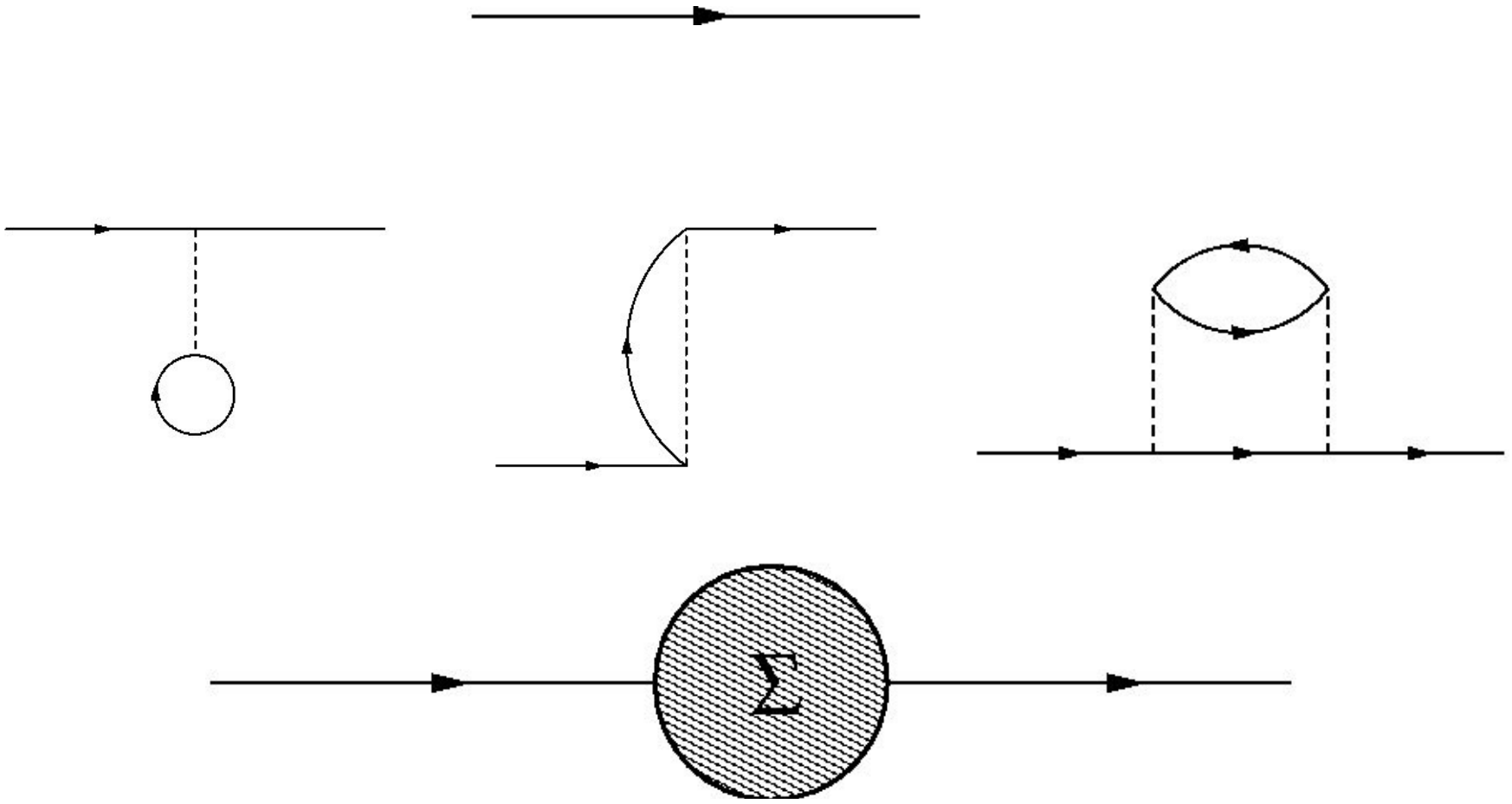
→ The GW approximation: (dynamical) screening!

→ Flavours of the GWA

→ What is wrong, and outlook

$$G(x_1, x_1', t, t') = -i\langle N|T[\hat{\Psi}(x_1, t)\hat{\Psi}^\dagger(x_1', t')]|N\rangle$$

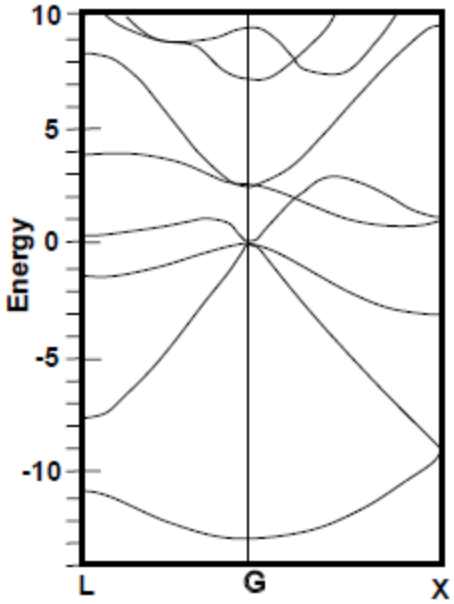
Many things can happen to a particle that propagates in the middle of others.....

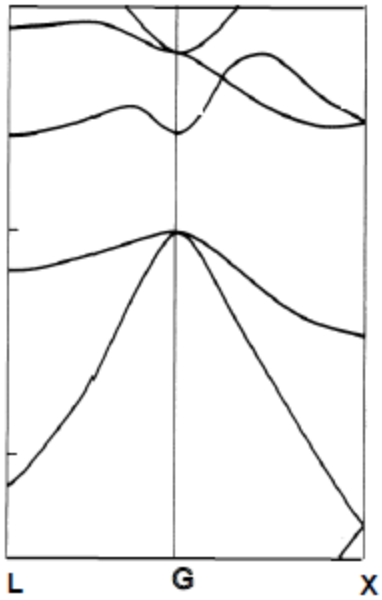


Dyson equation: $G = G_0 + G_0 \Sigma G$

LDA

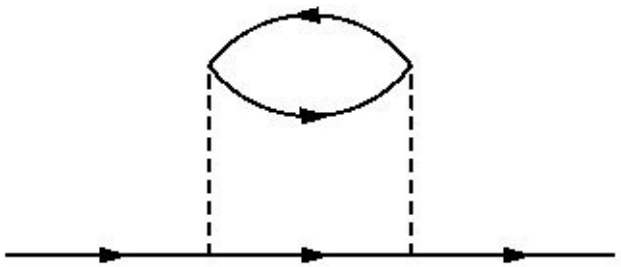
HF





GW calculations, Rohlfing et al., PRB 48, 17791 (1993)

Bandstructure of germanium, theory versus experiment



+ ……..

→ $\Sigma \sim i\,\mathcal{W}\mathcal{G}$ “GW”

L. Hedin, Phys. Rev. 139:A796–823, 1965

$$W = \varepsilon^{-1}(\omega)\, v$$

GW

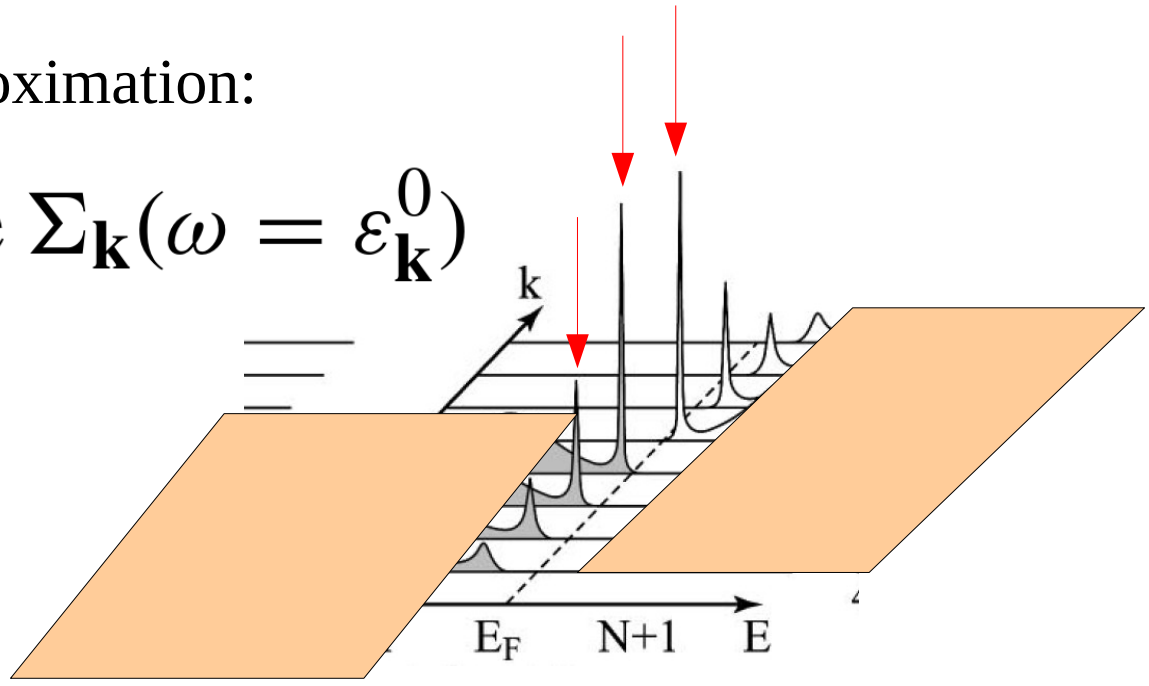
Hartree-Fock

Dyson equation: $G = G_0 + G_0 \Sigma G$

$$A_{\ell\ell}(\omega) = \frac{1}{\pi} |\mathrm{Im}\, G_{\ell\ell}(\omega)|$$

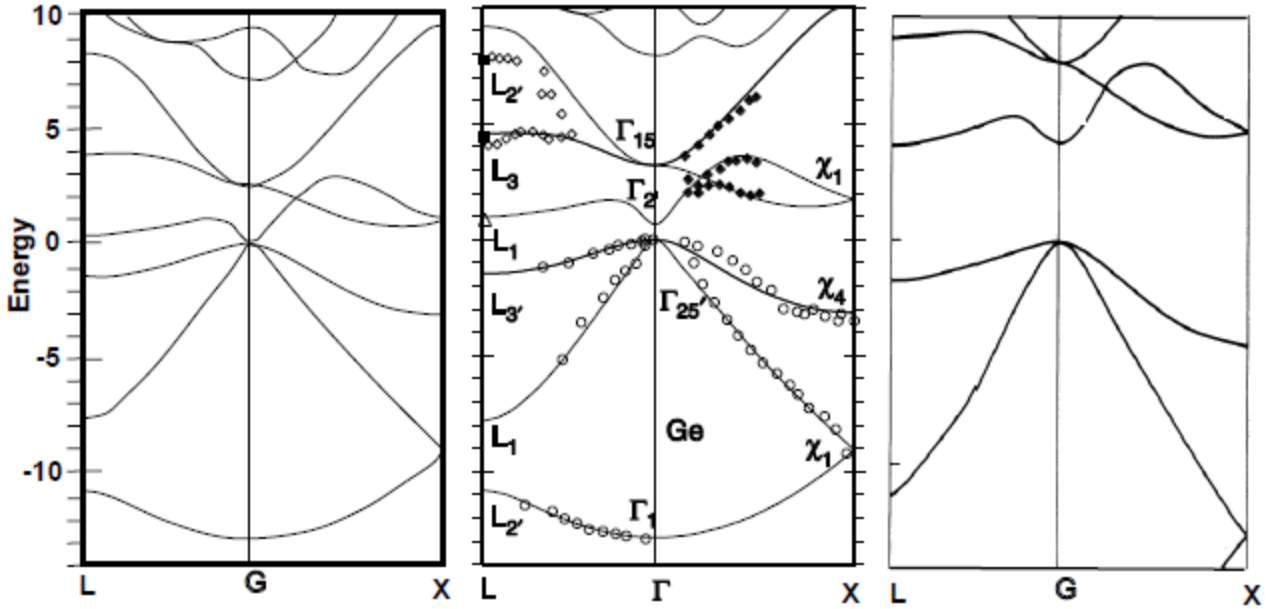
Quasiparticle approximation:

$$\varepsilon_{\mathbf{k}} = \varepsilon^0_{\mathbf{k}} + Z_{\mathbf{k}} \mathrm{Re}\, \Sigma_{\mathbf{k}}(\omega = \varepsilon^0_{\mathbf{k}})$$



Usually good gaps and band structures in GW

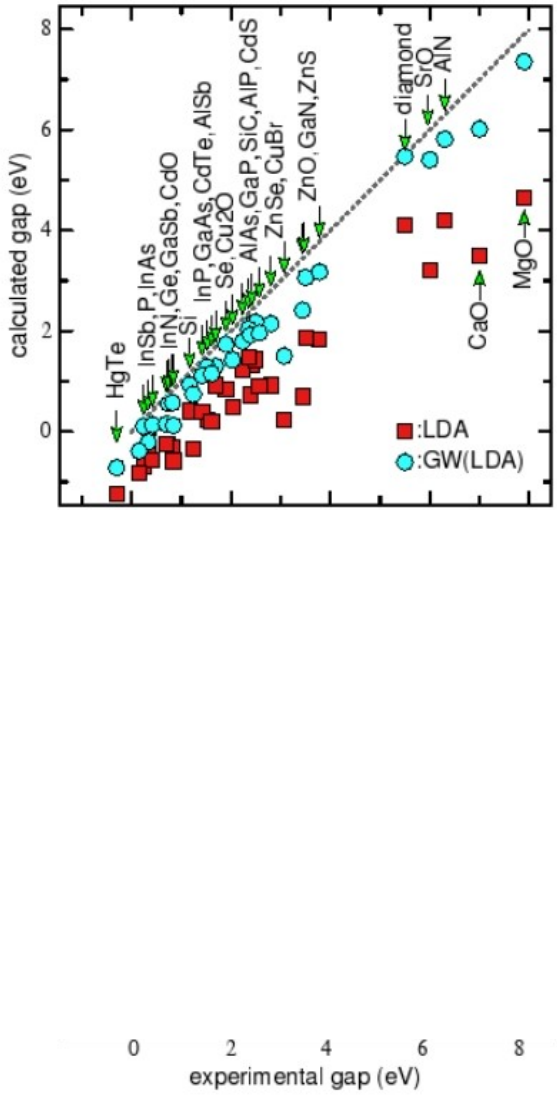
LDA GW HF



GW calculations, Rohlfing et al., PRB 48, 17791 (1993)

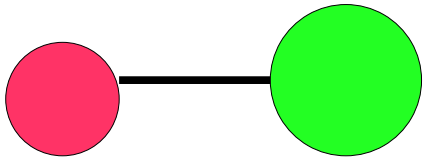
Bandstructure of germanium, theory versus experiment

Usually good gaps and band structures in GW

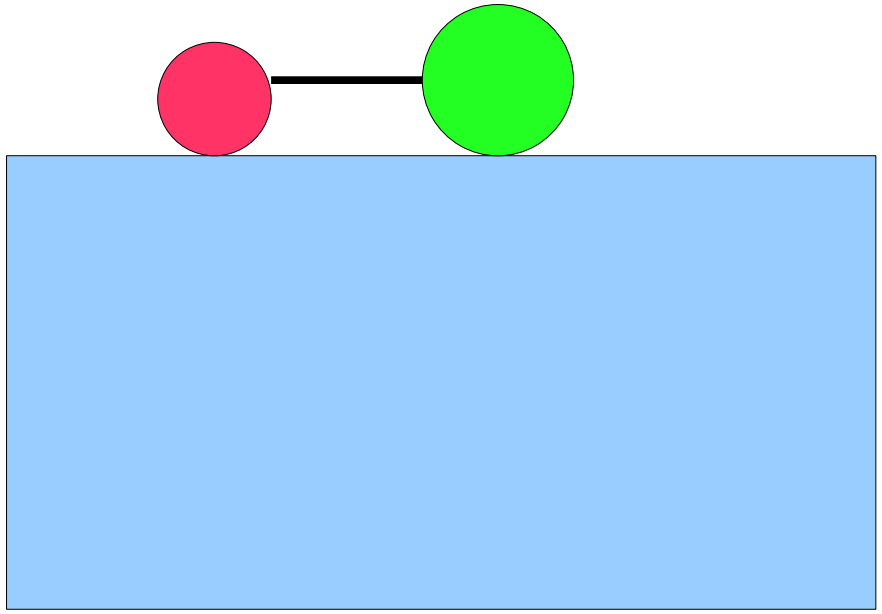


van Schilfgaarde, Kotani, Faleev,
Phys. Rev. Lett. 96, 226402 (2006)

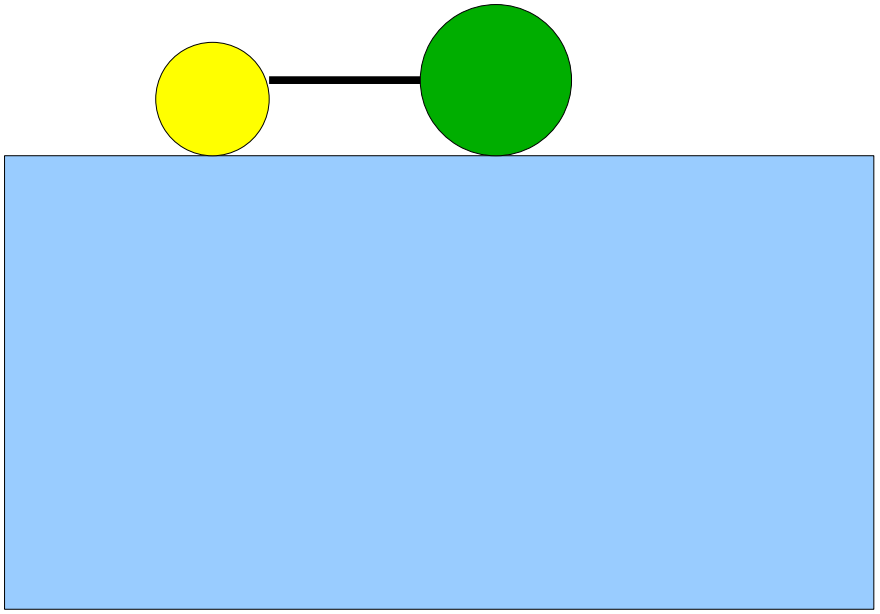
Molecules on surfaces



Molecules on surfaces

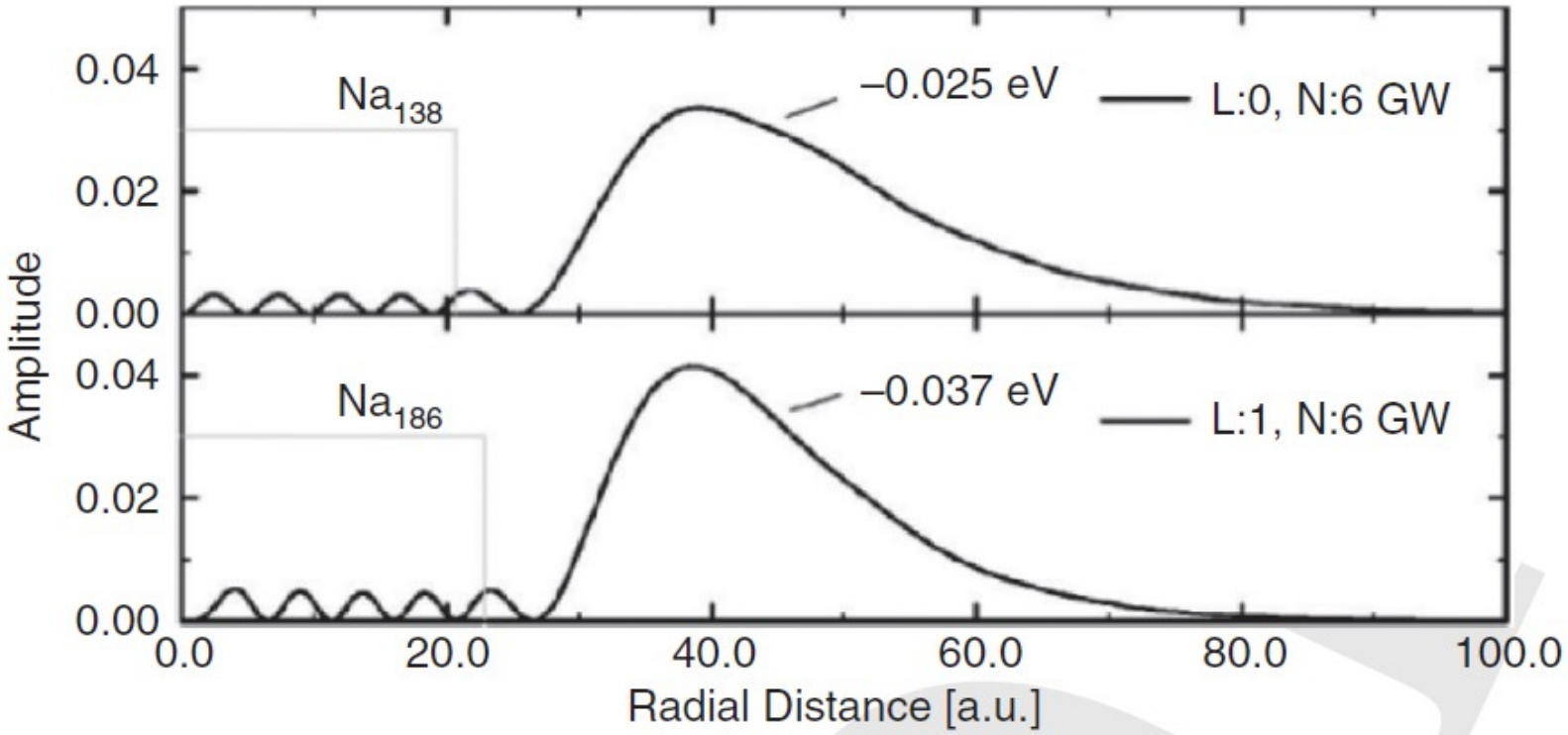


Molecules on surfaces



C. Freysoldt, et al., Phys. Rev. Lett. 103:056803, 2009.
J. M. Garcia-Lastra, et al, Phys. Rev. B 80:245427, 2009.

Image states



P. Rinke, et al., Phys. Rev. A 70:063201, 2004.

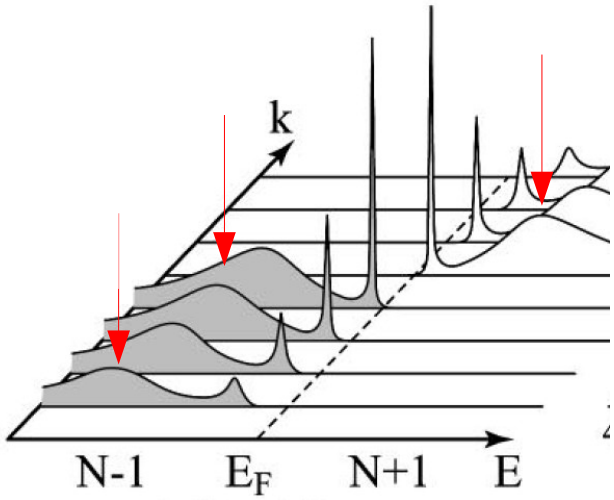
Also: vdW García-González & Godby, Phys. Rev. Lett. 88, 056406 (2002)

Dyson equation: $G = G_0 + G_0 \Sigma G$

$$A_{\ell\ell}(\omega) = \frac{1}{\pi} |\mathrm{Im}\, G_{\ell\ell}(\omega)|$$

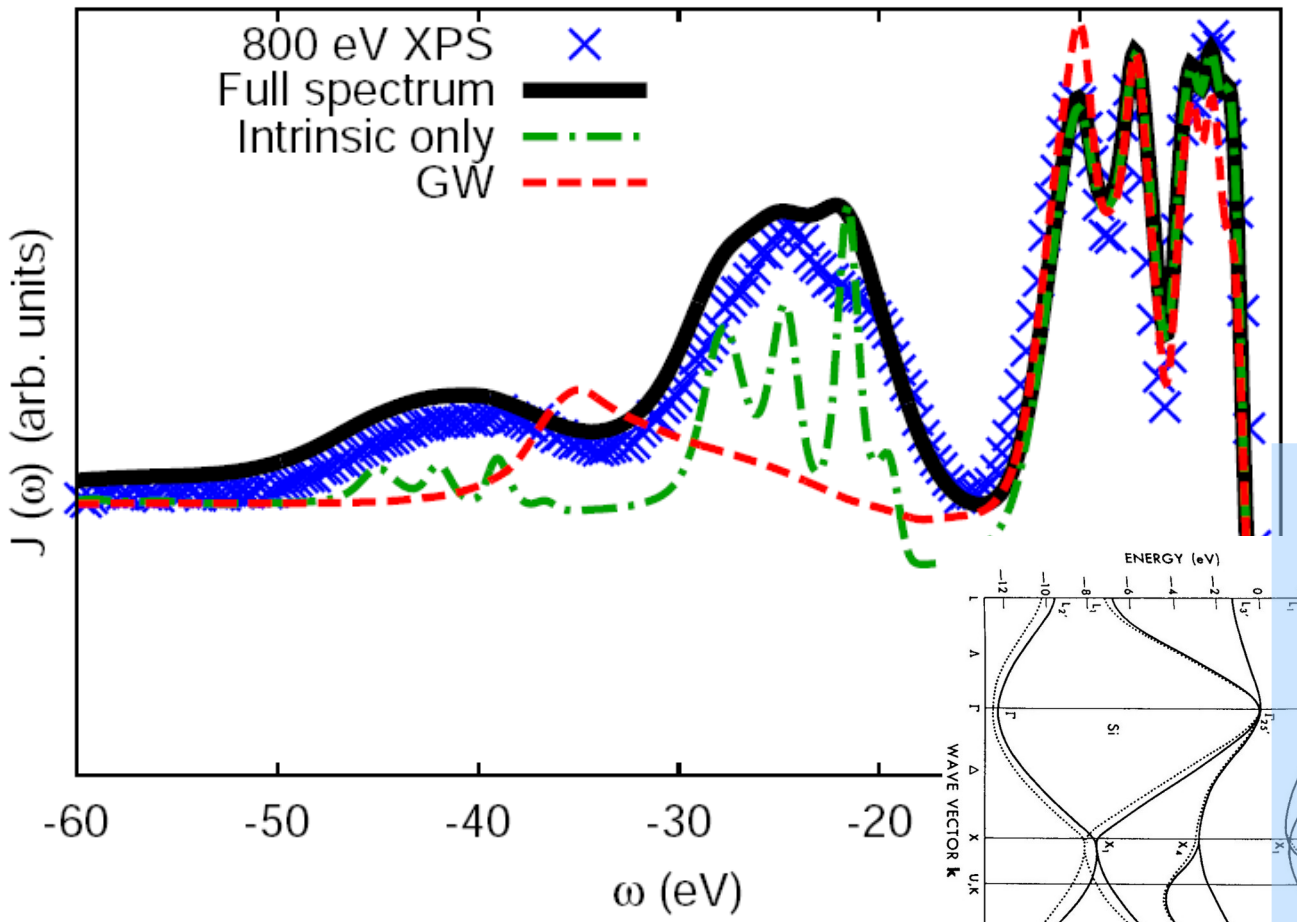
Quasiparticle approximation **and beyond:**

Additional excitations contained in $W(\omega)$



→ ARPES of simple bulk silicon:

Obviously far from an i.p. picture!



Cohen and Chelikowsky: "Electronic Structure and Optical Properties of Semiconductors" Solid-State Sciences 75, Springer-Verlag 1988)

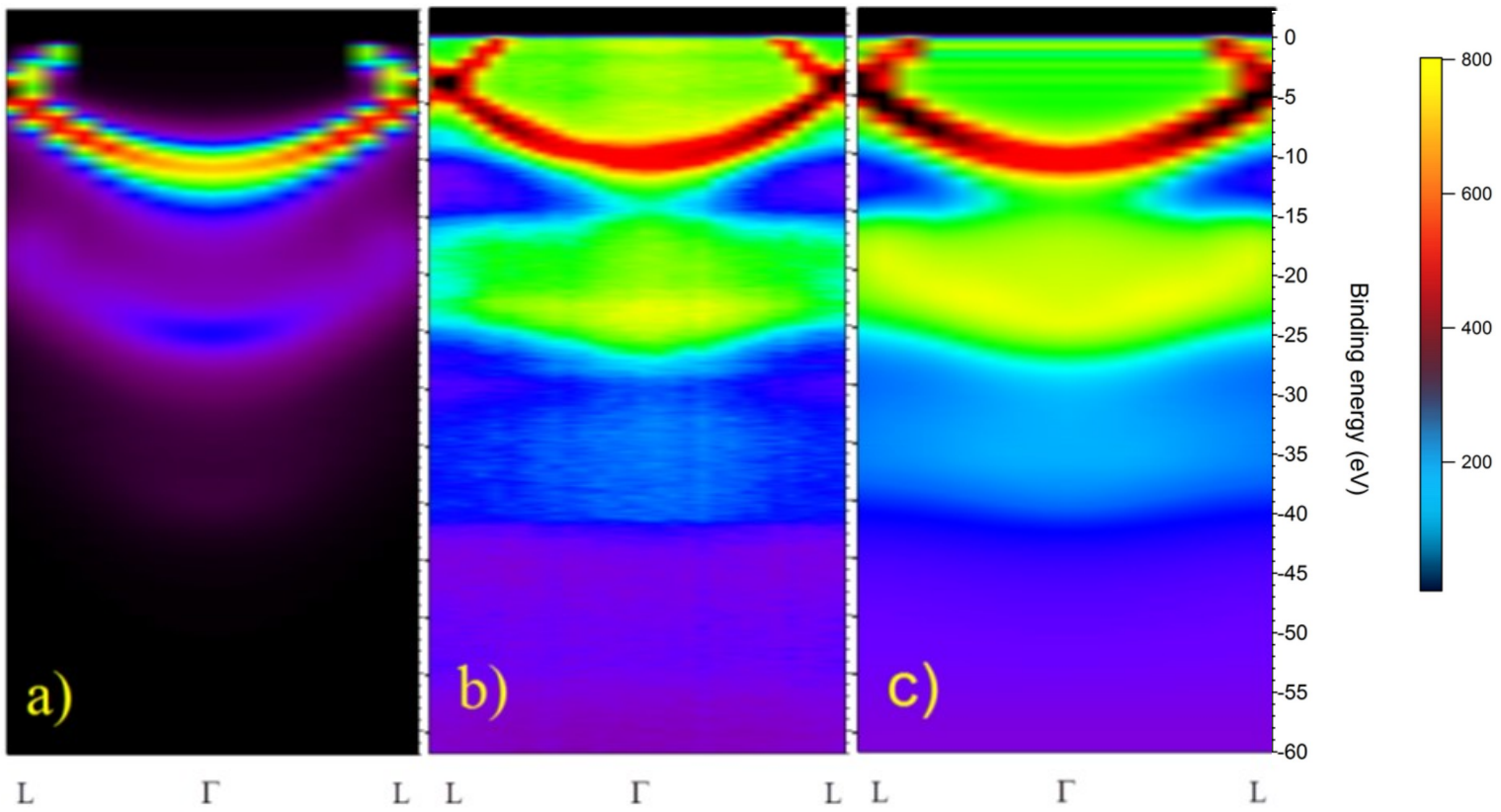
Exp.: F. Sirotti et al., TEMPO beamline SOLEIL

Photoemission of bulk aluminum

GW+C spectrum

Experiment

GW+C++



Zhou, Reining, Nicolaou, Bendounan, Ruotsalainen,Vanzini, Kas, Rehr, Muntwiler, Strocov, Sirotti, Gatti, PNAS 117 (46), 28596 (2020)

Introduction to many-body perturbation theory, GW band gap calculations and RPA energy calculations

→ Why do we need Green's functions?

→ From Green's functions to observables

→ A new auxiliary world

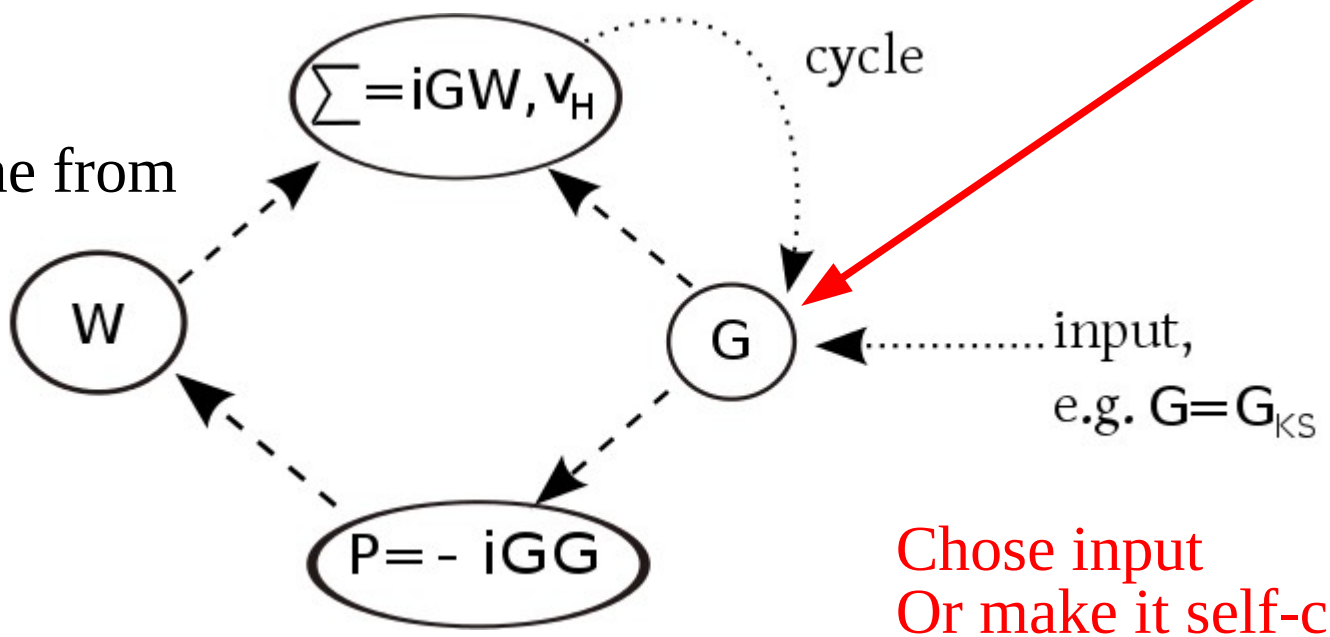
→ The GW approximation: (dynamical) screening!

→ Flavours of the GWA

→ What is wrong, and outlook

Output G or QP energies

G in input and output...



Can come from TDDFT

Chose input
Or make it self-consistent

RPA
Or beyond

QP calculations in practice

Supposing wavefunctions do not change

Supposing imaginary part is small

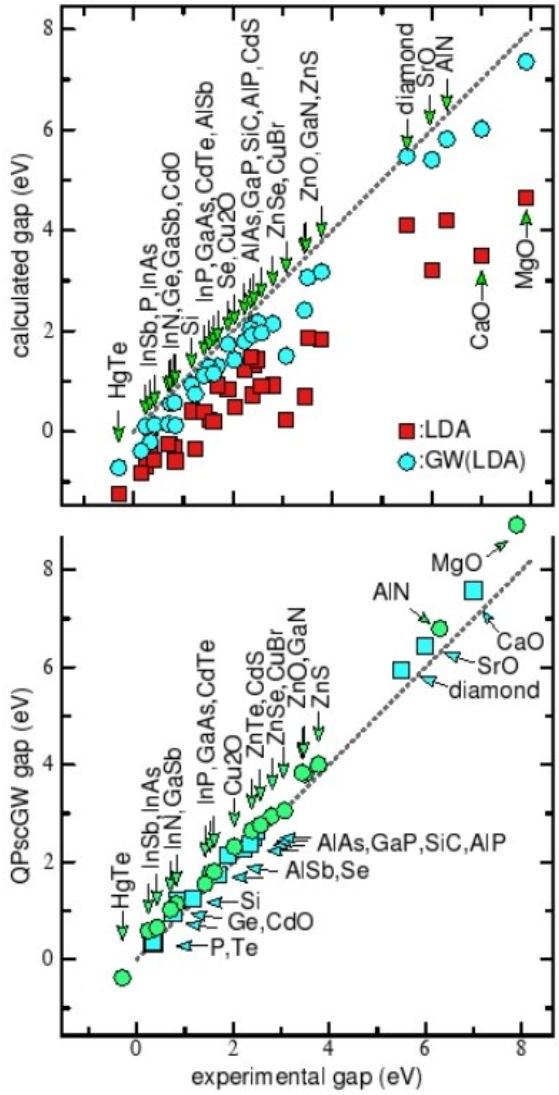
$$\varepsilon_{\mathbf{k}} = \varepsilon^0_{\mathbf{k}} + \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega = \varepsilon_{\mathbf{k}})$$

$$\mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega) \approx \mathrm{Re}\,\Sigma_{\mathbf{k}}(\varepsilon^0_{\mathbf{k}}) + (\omega - \varepsilon^0_{\mathbf{k}}) \frac{\partial \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega')}{\partial \omega'}_{|\omega'=\varepsilon^0_{\mathbf{k}}}$$

$$Z_{\mathbf{k}} = \frac{1}{1 - \frac{\partial \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega')}{\partial \omega'}_{|\omega'=\varepsilon^0_{\mathbf{k}}}}$$

$$\varepsilon_{\mathbf{k}} \approx \varepsilon^0_{\mathbf{k}} + Z_{\mathbf{k}} \mathrm{Re}\,\Sigma_{\mathbf{k}}(\omega = \varepsilon^0_{\mathbf{k}})$$

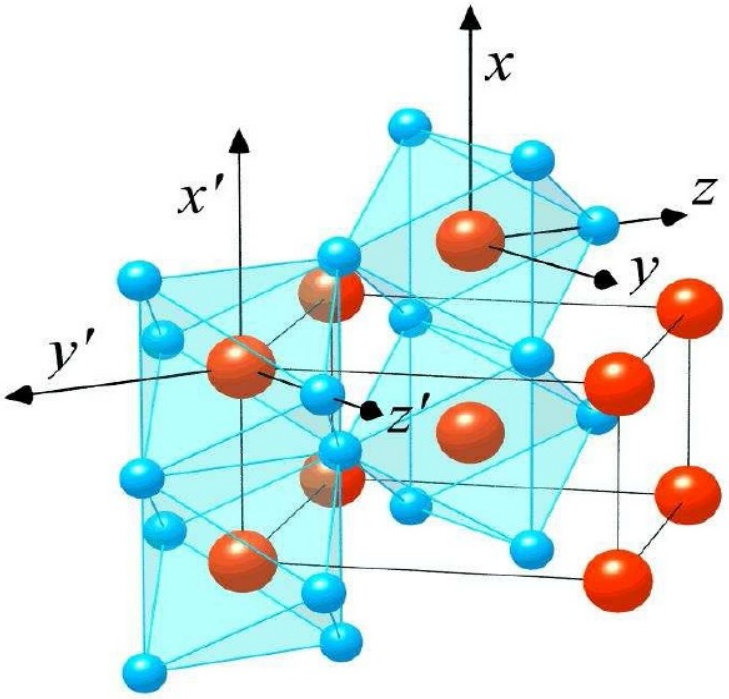
Usually good gaps and band structures in GW



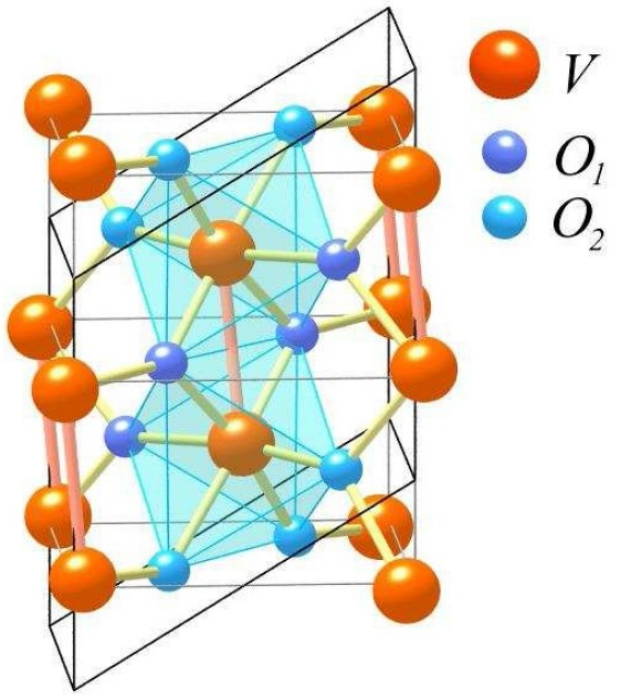
van Schilfgaarde, Kotani, Faleev, Phys. Rev. Lett. 96, 226402 (2006)

Gaps...even for TMO's

VO_2

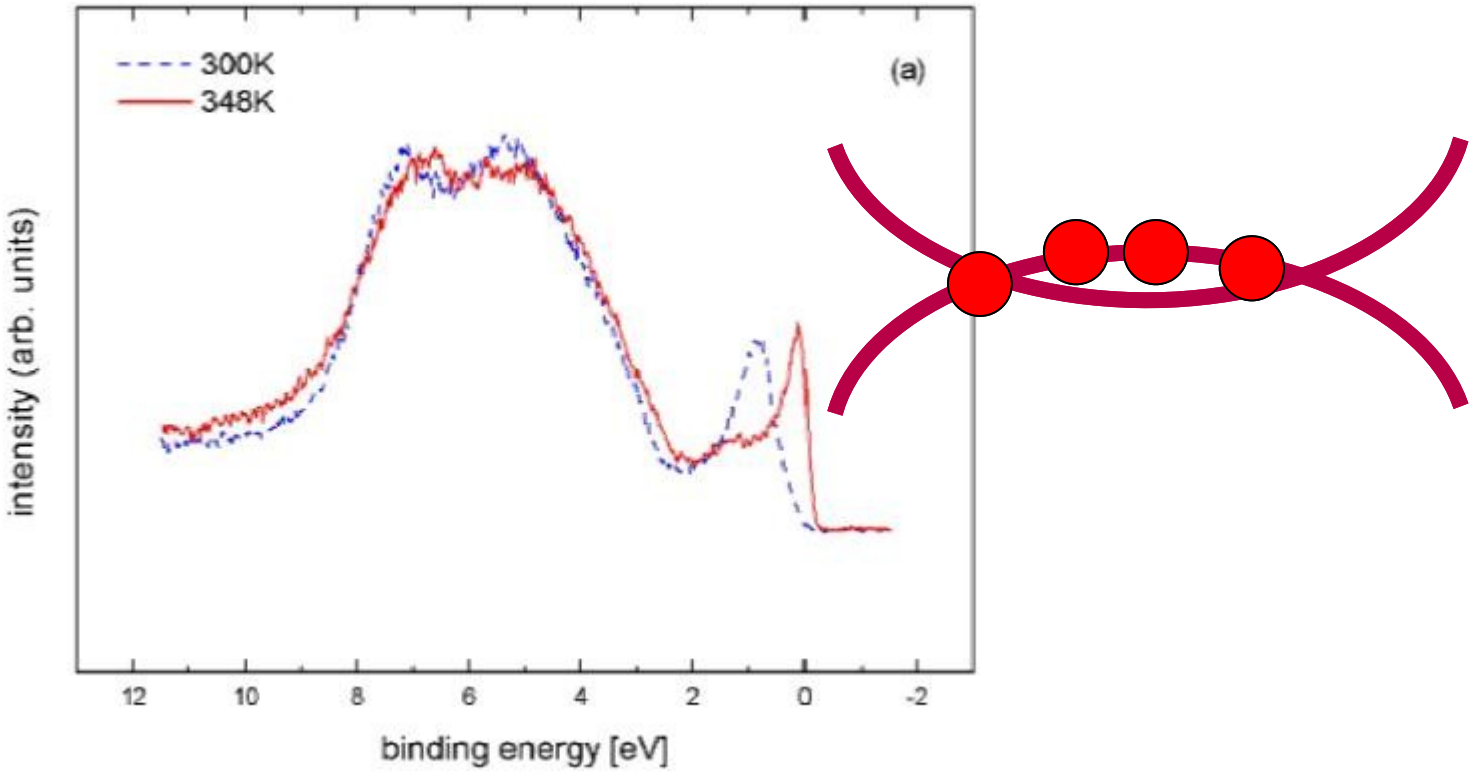


rutile



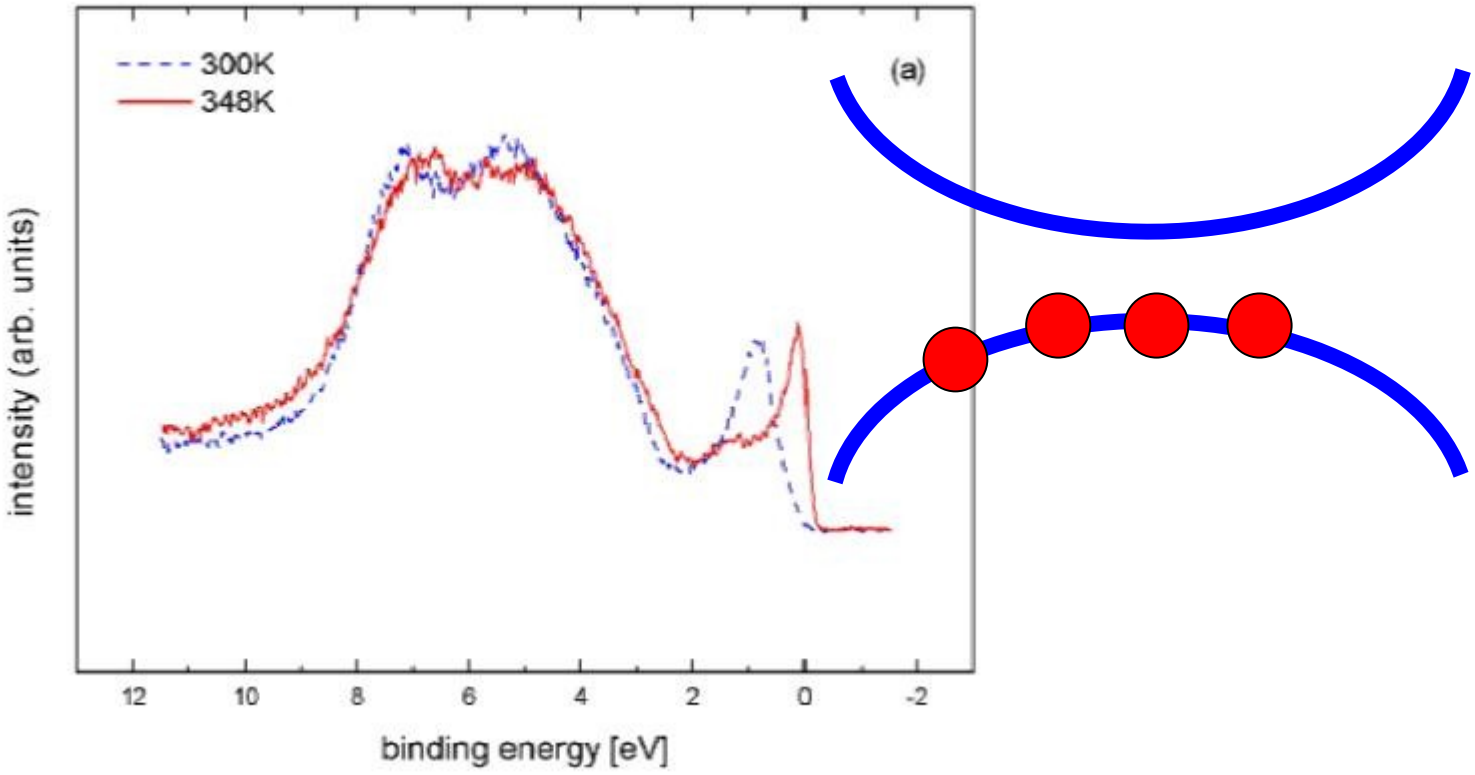
monoclinic

Matteo Gatti et al.



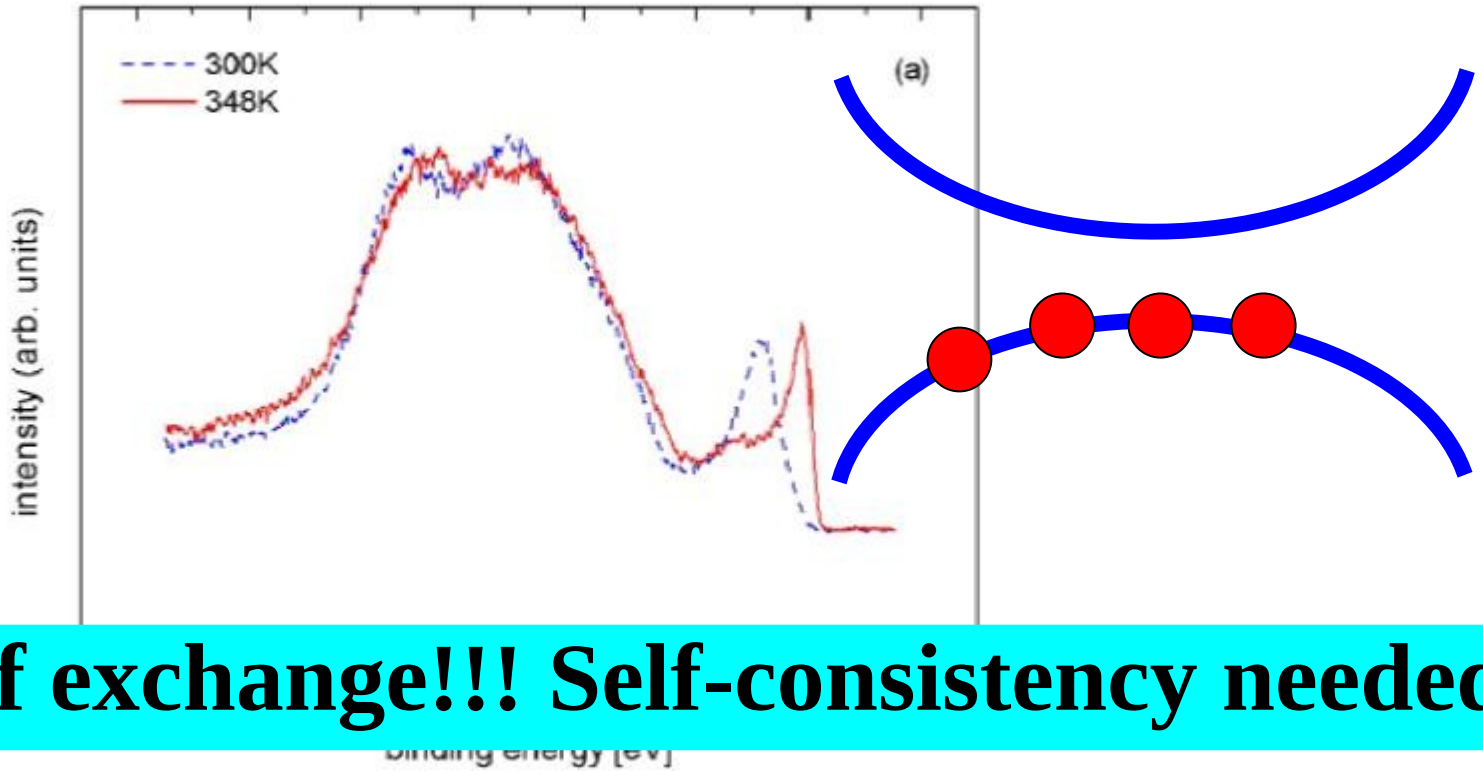
T. C. Koethe et al., Phys. Rev. Lett. 97, 116402 (2006).

T. C. Koethe et al., Phys. Rev. Lett. 97, 116402 (2006).



In GW: M. Gatti, F. Bruneval, V. Olevano and L. Reining, Phys. Rev. Lett. **99**, 266402 (2007)

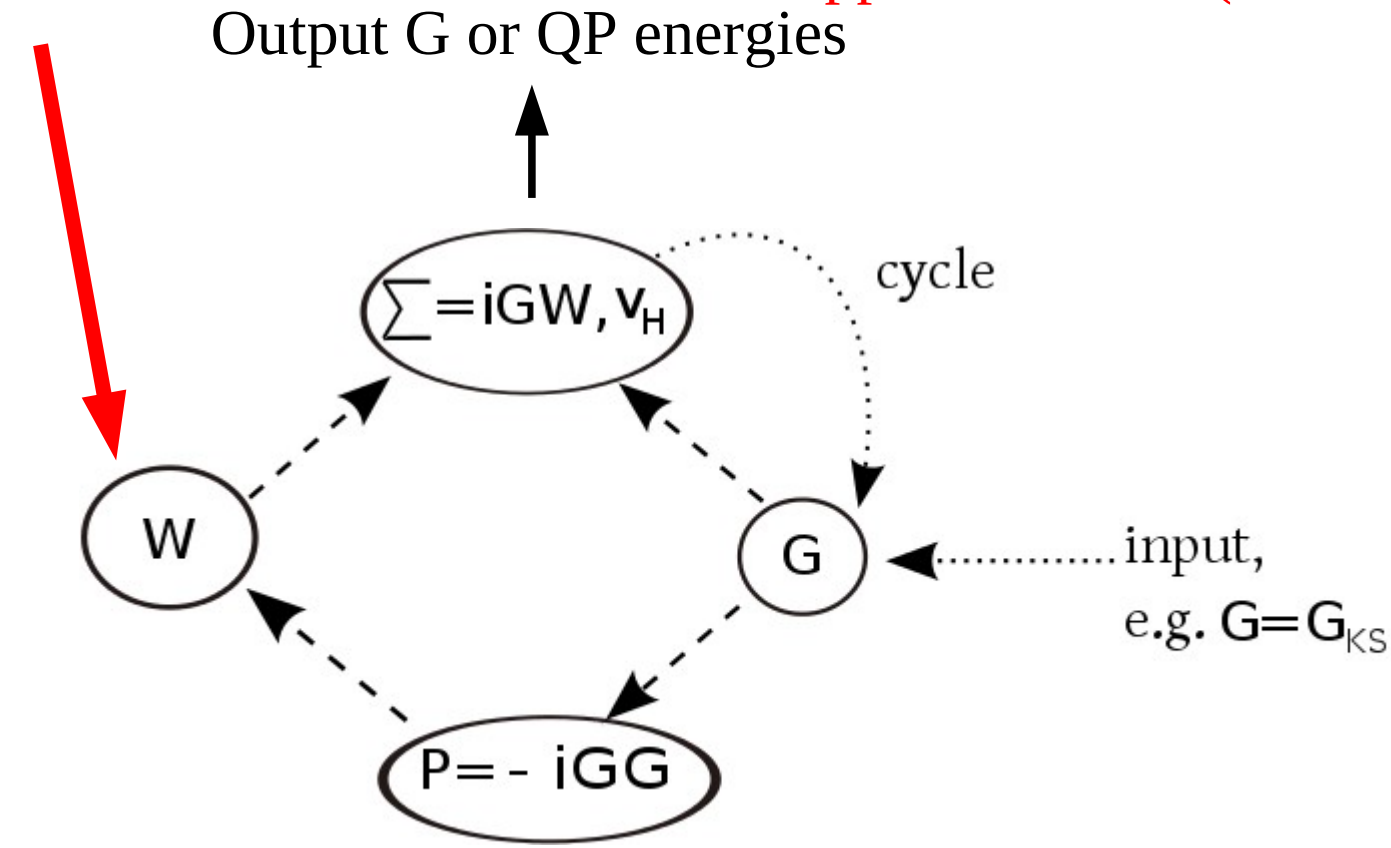
T. C. Koethe et al., Phys. Rev. Lett. 97, 116402 (2006).



Effect of exchange!!! Self-consistency needed

In GW: M. Gatti, F. Bruneval, V. Olevano and L. Reining, Phys. Rev. Lett. **99**, 266402 (2007)

Can be calculated with different approximations (see TDDFT)



Introduction to many-body perturbation theory, GW band gap calculations and RPA energy calculations

→ Why do we need Green's functions?

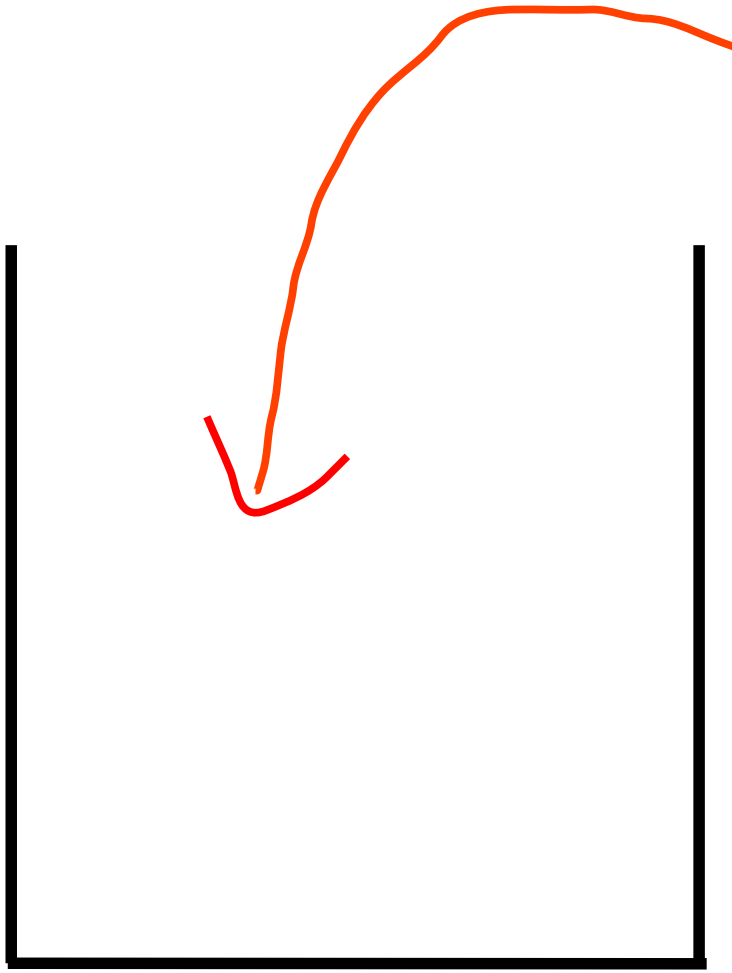
→ From Green's functions to observables

→ A new auxiliary world

→ The GW approximation: (dynamical) screening!

→ Flavours of the GWA

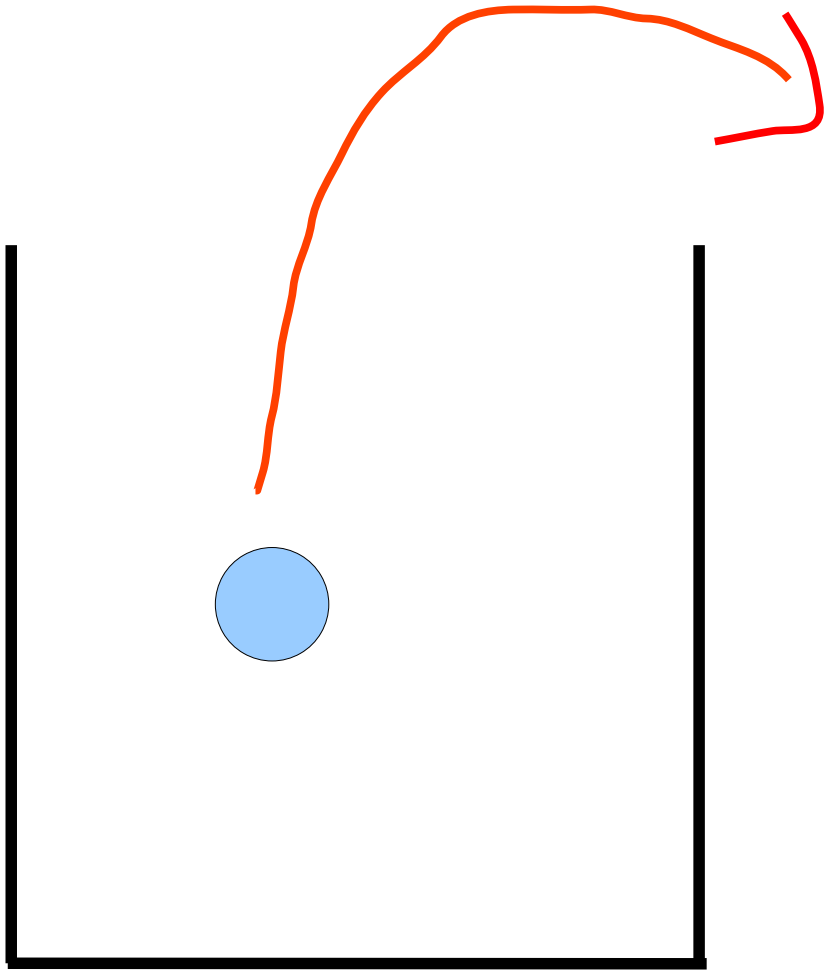
→ What is wrong, and outlook



$$W = v_c$$

HF = exact

Romaniello P., Guyot S., and Reining L., J. Chem. Phys. 131, 154111 (2009)

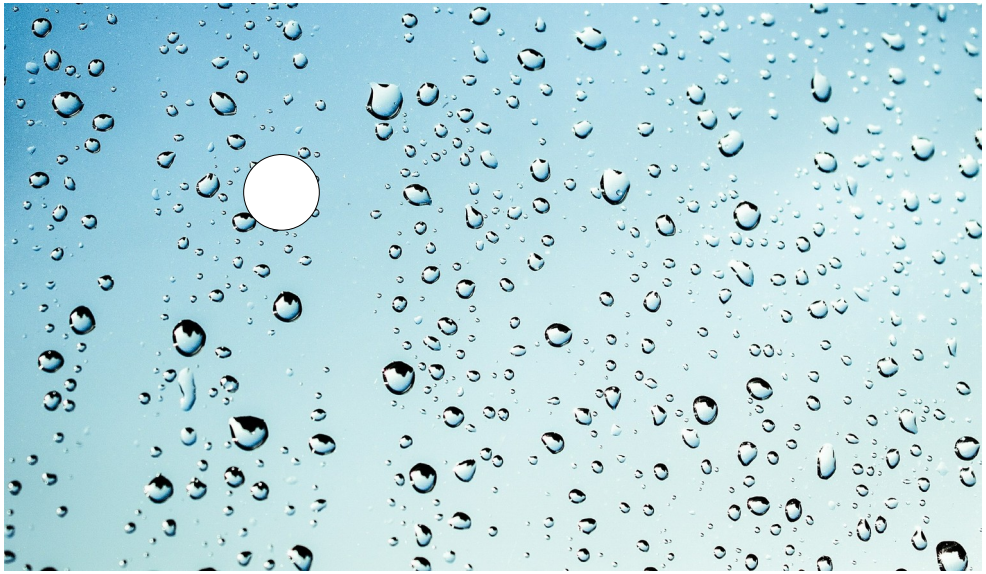


$$W \neq v_c$$

self-screening

Romaniello P., Guyot S., and Reining L., J. Chem. Phys. 131, 154111 (2009)

GW is good for QP energies of electrons in “normal” densities



GW fails when explicit correlation with individual electrons is neded

Corrections beyond GW: “vertex corrections”

$$\Sigma_{\mathrm{xc}} = iGW\Gamma \approx iG^{\mathrm{app}}W^{\mathrm{app}}\Gamma^{\mathrm{app}} \approx iG^{\mathrm{app}}W^{\mathrm{app}}$$

To do a GW calculation, we need W:

Next topic!

Suggested Reading

L. Hedin, “On correlation effects in electron spectroscopies and the GW approximation,” J. Phys. C 11:R489–528, 1999. *Short review, very good for photoemission!*

F. Aryasetiawan and O. Gunnarsson, “The GW method,” Rep. Prog. Phys. 61:237–312, 1998; and:

W. G. Aulbur, L. Jonsson, and J. W. Wilkins, “Quasiparticle calculations in solids,” Solid State Phys. 54:1–218, 2000;
Two nice and quite complete reviews on GW

Strinati, G., “Application of the Green’s function method to the study of the optical-properties of semiconductors,” Rivista del Nuovo Cimento 11, 1, 1988. *Pedagogical review of the theoretical framework underlying today’s Bethe–Salpeter calculations. Derivation of the main equations and link to spectroscopy.*

Onida, G., Reining, L., and Rubio, A., “Electronic excitations: density-functional versus many-body Greens-function approaches,” Rev. Mod. Phys. 74, 601, 2002.
Review of ab initio calculations of electronic excitations with accent on optical properties and a comparison between Bethe–Salpeter and TDDFT

R.M. Martin, L. Reining, D.M. Ceperley, “Interacting Electrons: Theory and Computational Approaches, Cambridge May 2016
Recent book containing many-body perturbation theory, DMFT and QMC