Homework 1

Due: 2023 Sep. 25

1. Go over Sec. 1.1 to 1.4 of the textbook.

2. Do Exercise 1 on page 7.

3. Consider the function $h(x)$ given by

$$h(x) = \begin{cases} \frac{1}{2}x^2 & 0 \le x < 0.5 \\ \frac{1}{4} - \frac{1}{2}(x-1)^2 & 0.5 \le x \le 1 \end{cases}$$

(a) Is $h(x)$ in the space C^1?
(b) Is $h(x)$ in the space C^2?
(c) Is $h(x)$ in the space H^2?

4. Consider a boundary-value problem (i.e. strong-form problem) with the Dirichlet boundary conditions imposed on both ends, that is,

$$\begin{cases} u_{,xx} + f = 0, \\ u(0) = g_0, \\ u(1) = g_1. \end{cases}$$

(a) State its corresponding weak-form problem.
(b) Prove the equivalence between the strong- and weak-form problems.