NERS521

Cross Sections Cheat Sheet

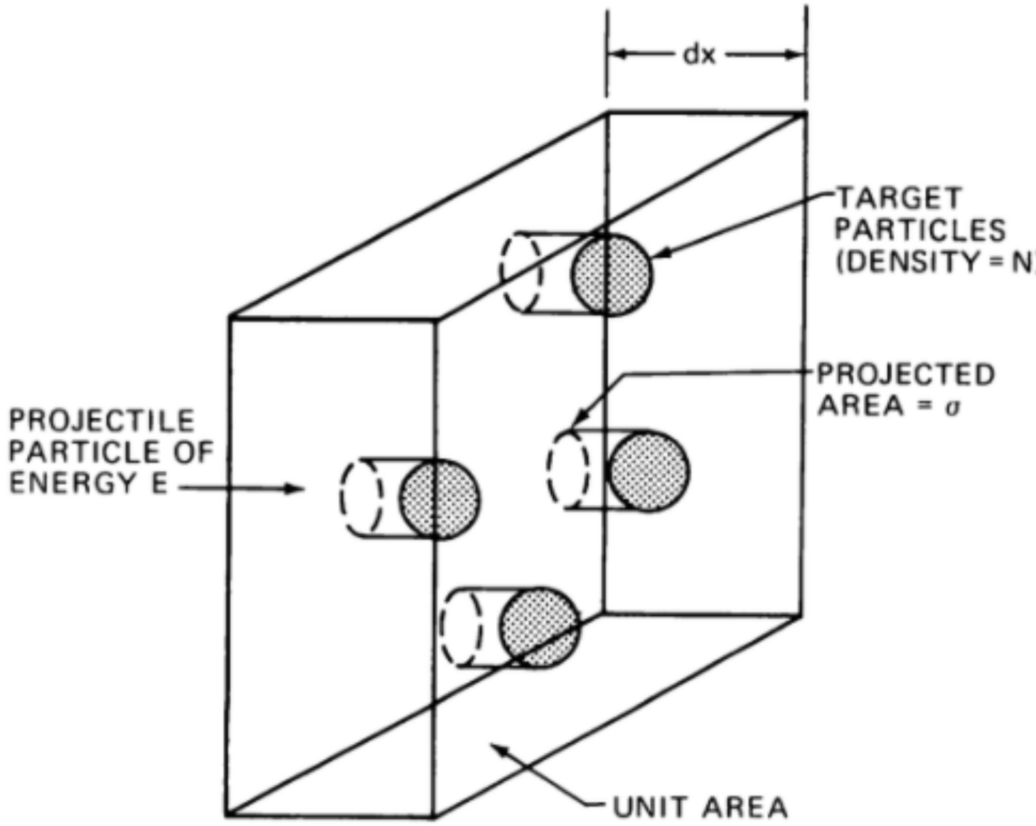
This document provides a brief description and reference sheet for cross sections as they pretain to radiation damage, and specifically in the calculation of displacement per atom using a Kinchin-Pease approach. A good overview is provided in [Fundamentals Aspects of Nuclear Reactor Fuel Elements] from which this content is derived.

Total scattering cross section

A simple schematic to demonstrate the concept of a collision cross section is shown in the Figure below. If you have a single energetic particle that interacts with a target volume made of atoms (or scattering centers) then the value, N, is the number of atoms per unit volume. For simple cross sections, we assume random distribution of atoms (or scattering centers) and do not take into account crystallinity. The probability. P then of a collision with an atom in the target is proportional to dx and N, or simply

$$P = N\sigma_s(E)dx$$

where $sigma$ is the projected area (commonly given in units of barn = $\times 10^{-24}\ cm^2$) , N is the particles/atoms per unit volume, and dx is the path length traveled by the incident particle.



Total scattering cross section, $\sigma_s(E)$ in a target of number density N:

- $N\sigma_s(E)dx$ = Probability of the collision on an incident particle with a target dx

Similarly, cross sections of varying types of interactions can also be derived. Simplified ones for energy transfer and angular cross section are produced here for reference.

Differential energy transfer cross section

Differential energy transfer cross section, $\sigma_s(E_i, T)$:

- $N\sigma_s(E_i, T)dTdx$ = Probability of the collision in distance dx which transfers energy in the range (T, dT) to the target particle

Differential angular cross section

Differential angular cross section, $\sigma_s(E_i, \phi)$:

- $N\sigma_s(E_i, \phi)d\Omega dx$ = Probability of the collision in distance dx which scatters the incident particle into a center-of-mass angle in the rang $(\phi, d\Omega)$

Conversions for cross sections:

$$\sigma_s(E_i, T)\ dT = \sigma_s(E_i, \phi)\ d\Omega = 2\pi\ \sigma_s(E_i, \phi)\ sin\phi\ d\phi$$

$$\sigma_s(E) = \int_{\check{T}}^{\hat{T}} \sigma_s(E_i, T)\ dT$$

$$\sigma_s(E) = \int_{0}^{\Omega} \sigma_s(E_i, \phi)\ d\Omega$$

Equations for energy transfer

$$\bar{T} = \frac{\int_{\check{T}}^{\hat{T}} T\sigma_s(E_i, T)\ dT}{\int_{\check{T}}^{\hat{T}} \sigma_s(E_i, T)\ dT}$$

$$T_{hardsphere} = \frac{4M_1M_2}{(M_1 + M_2)^2} E_i(1 - cos\phi)$$

$$\bar{T}_{hardsphere} = \frac{1}{2}\gamma E_i$$

$$\hat{T} = T_{max} = \gamma E_i$$