

# Voids II

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# Nucleation vs. Growth

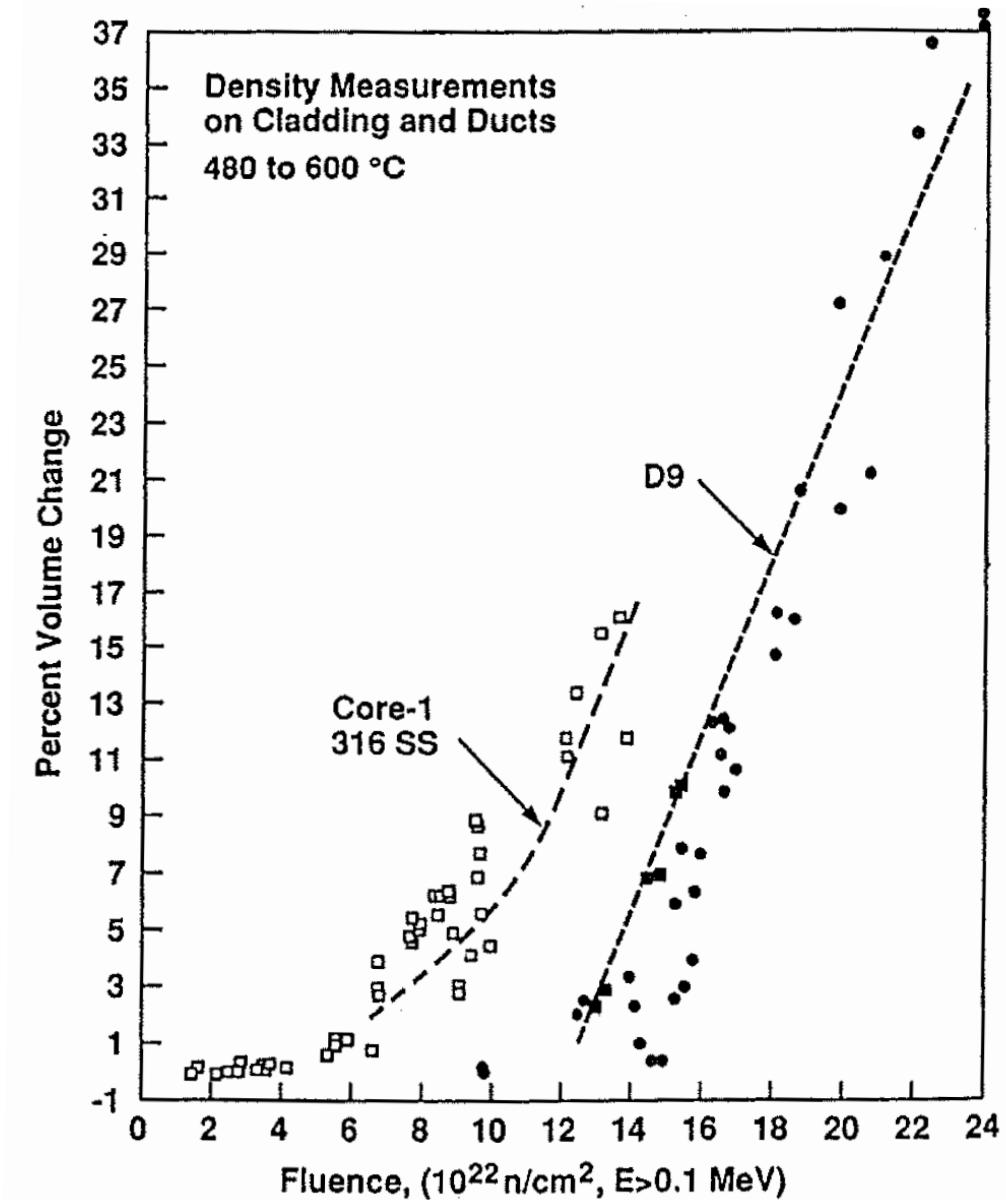
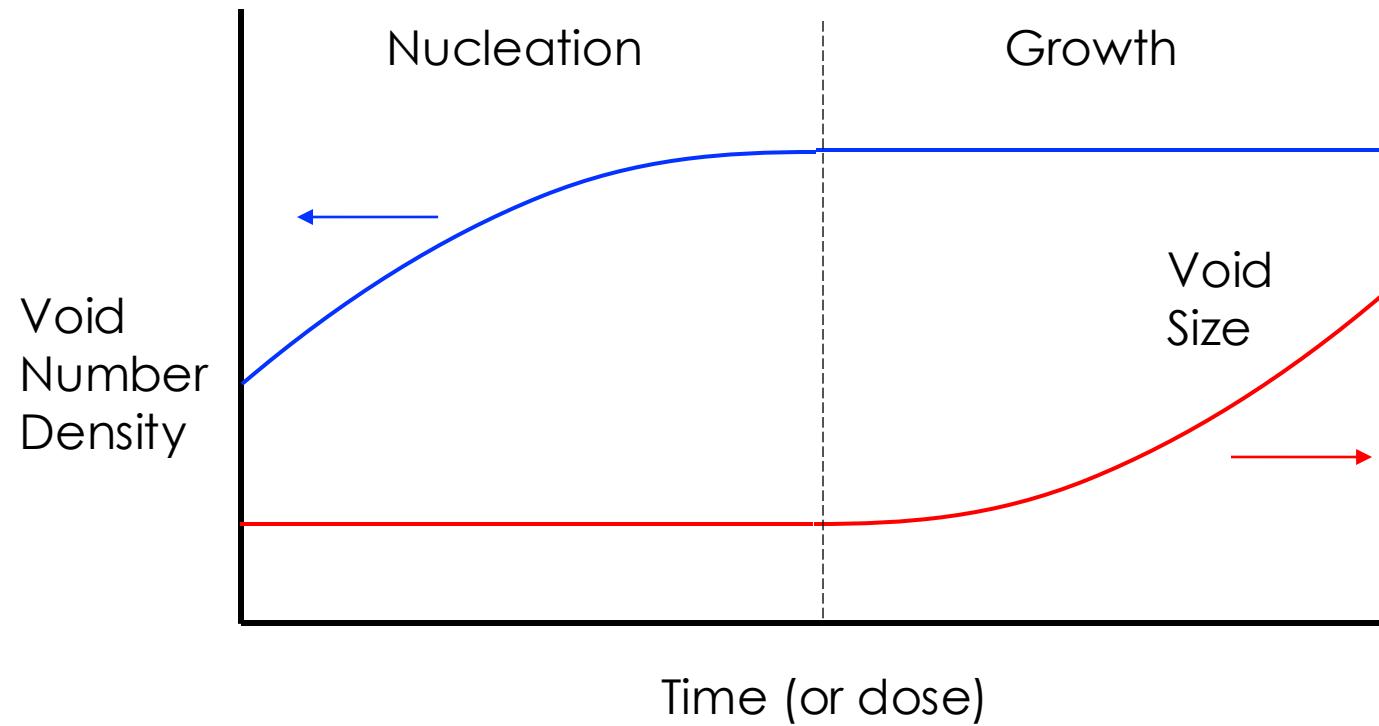
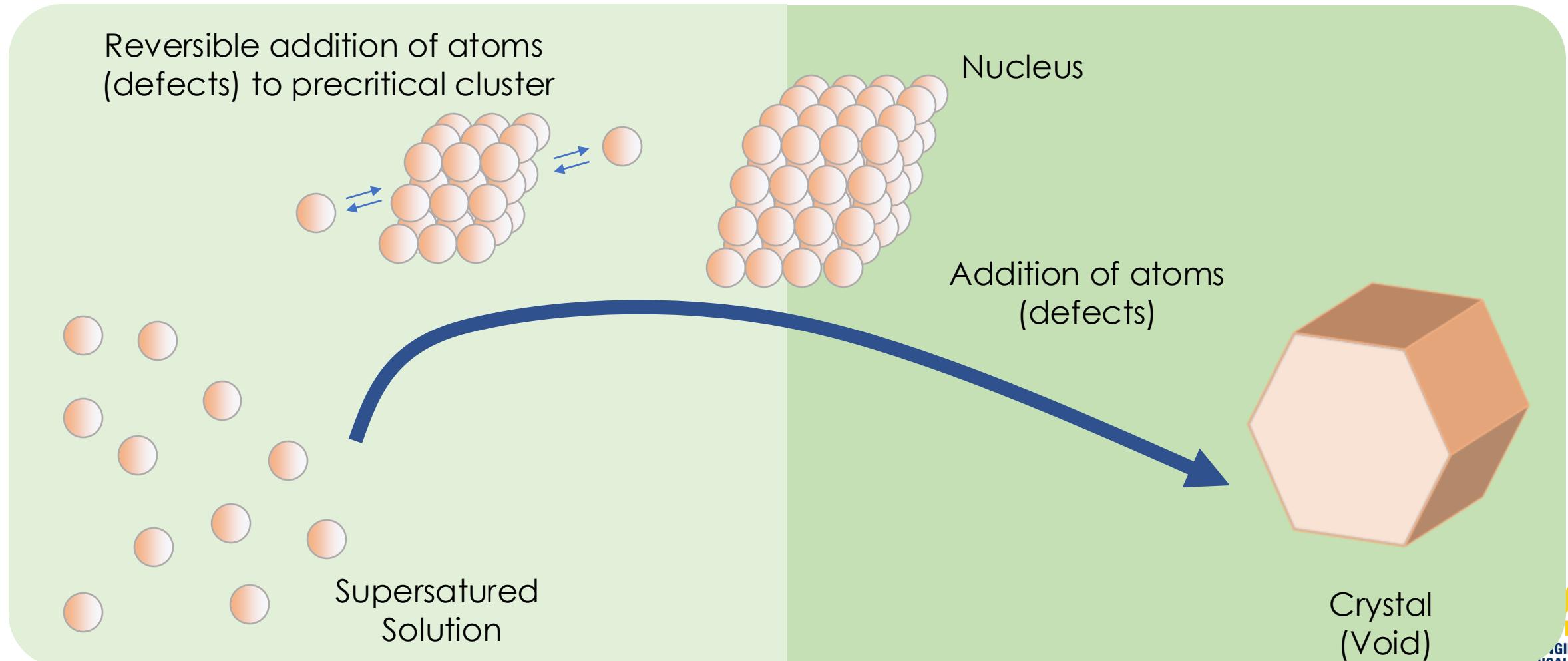


Fig. 3. Swelling observed in two cold-worked austenitic alloys after serving as fuel cladding in the open core of FFTF [23].

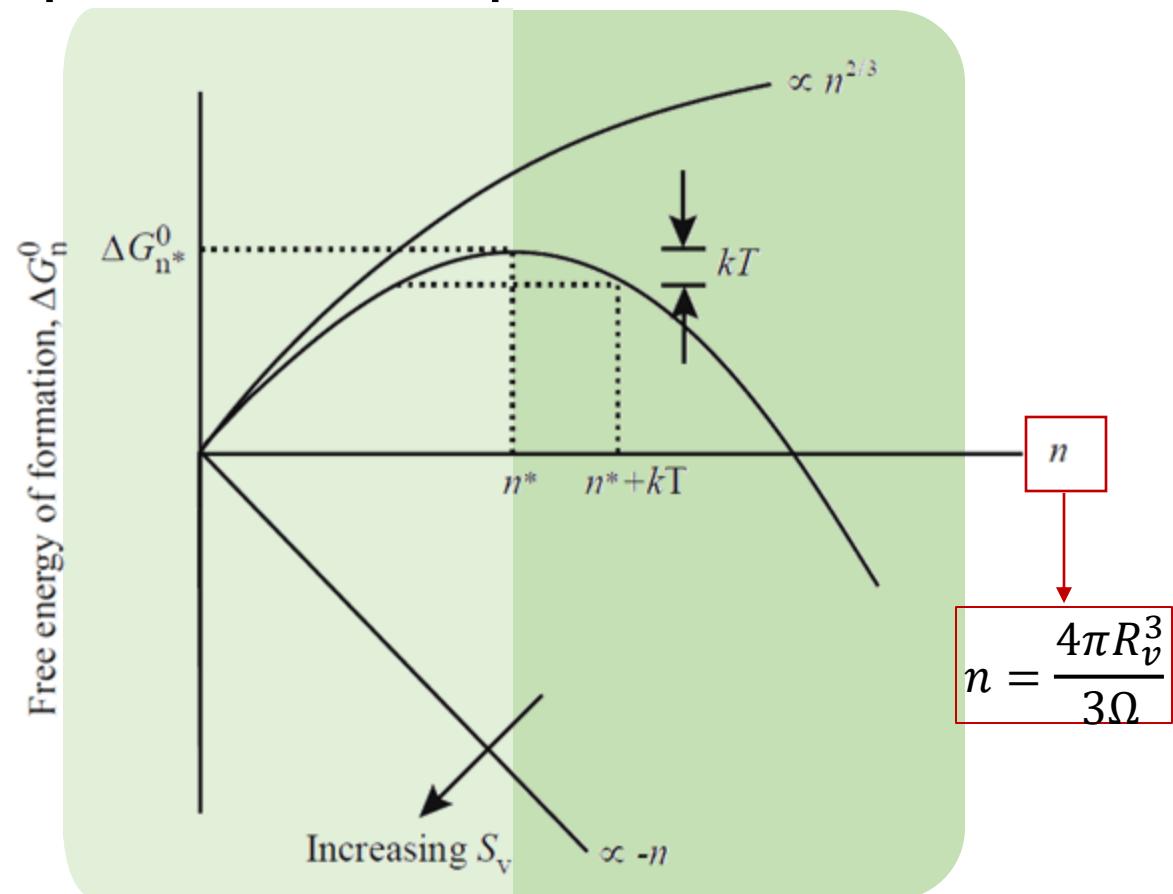
# Nucleation

The nucleation theory used in nuclear materials is commonly the classical pathway description



# Void Nucleation Theory: Graphical depiction

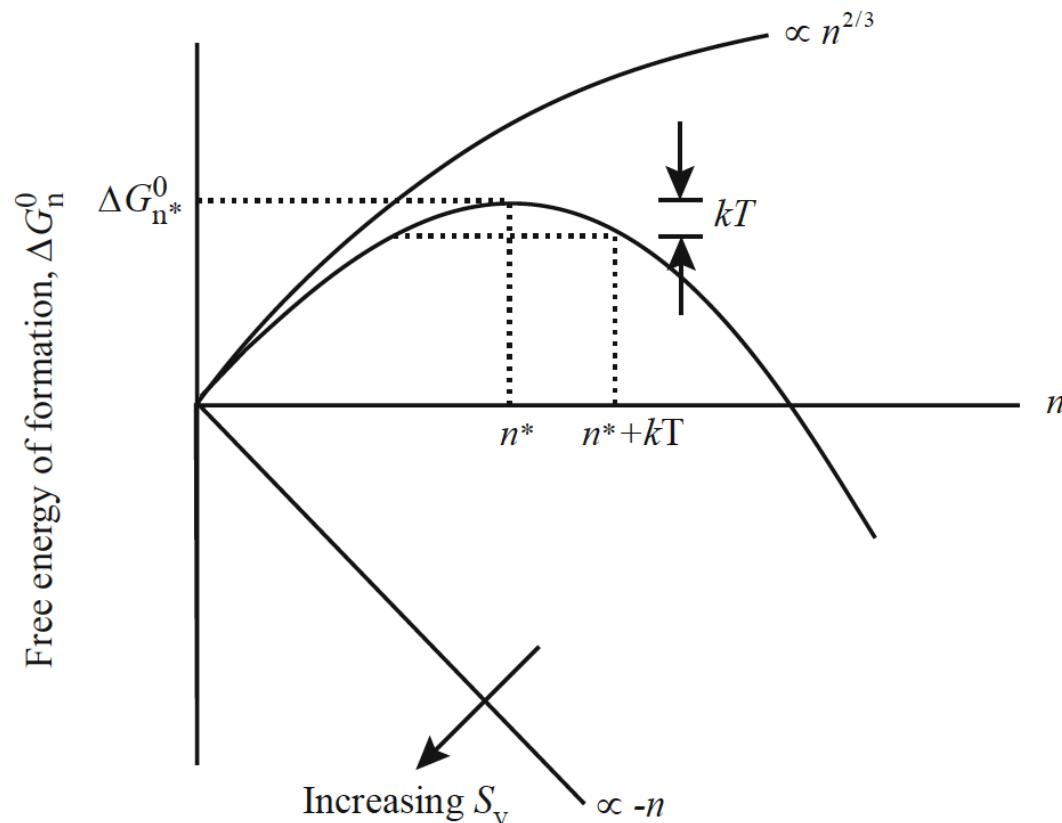
$$\Delta G_n^0 = -nkT \cdot \ln(S_v) + (36\pi\Omega^2)^{1/3} \gamma n^{2/3}$$



Full derivations and discussion in Was 8.1

**Fig. 8.2** Schematic illustration of  $\Delta G_n^0$ , the free energy of formation of a spherical void consisting of  $n$  vacancies and the effect of thermal fluctuations on the critical size void embryo

# Void Nucleation Theory: Graphical depiction



Derivations and discussion in Was 8.1

We can solve for the critical embryo size,  $n^*$  or  $r^*$ , by:

$$\Delta G_n^0 = -nkT \cdot \ln(S_v) + (36\pi\Omega^2)^{1/3}\gamma n^{2/3}$$

$$0 = -nkT \cdot \ln(S_v) + (36\pi\Omega^2)^{1/3}\gamma n^{2/3}$$

Solve for  $n$

$$n^* = \frac{32\pi\gamma^3\Omega^2}{3(k_b T \ln S_v)^3}$$

Convert to  $r$

$$r^* = \frac{2\gamma\Omega}{(k_b T \ln S_v)}$$

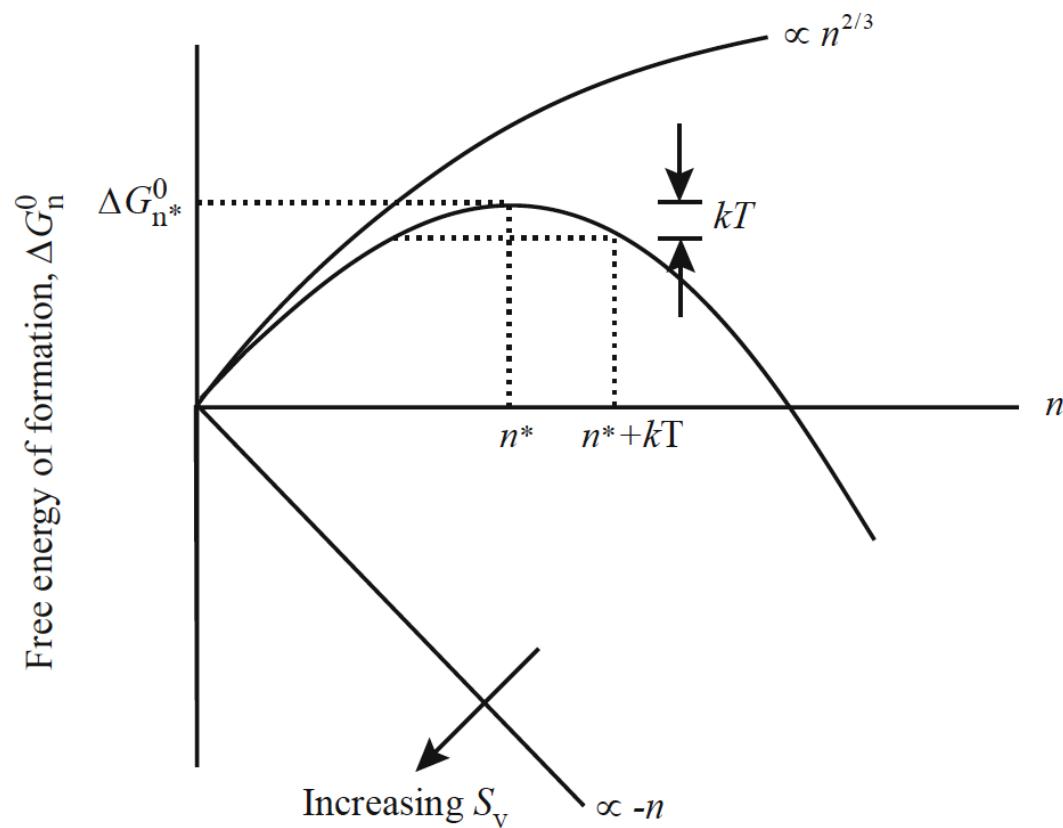
# Void Nucleation Theory: Graphical depiction



- Homogeneous nucleation:  
When supercritical particles are formed due to thermal fluctuations
- Heterogeneous nucleation:  
When external objects (surfaces, interfaces, impurities, defects, seeds) lower the barrier for nucleation

Derivations and discussion in Was 8.1

# Void Nucleation Theory: Graphical depiction



Derivations and discussion in Was 8.1

- Homogeneous nucleation:

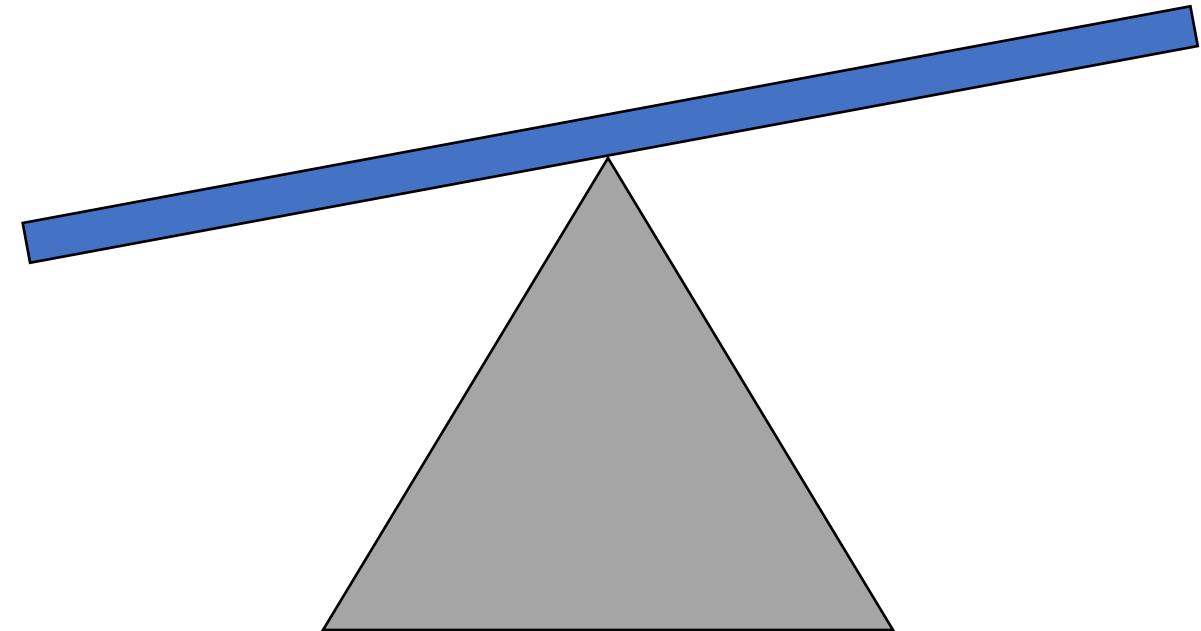
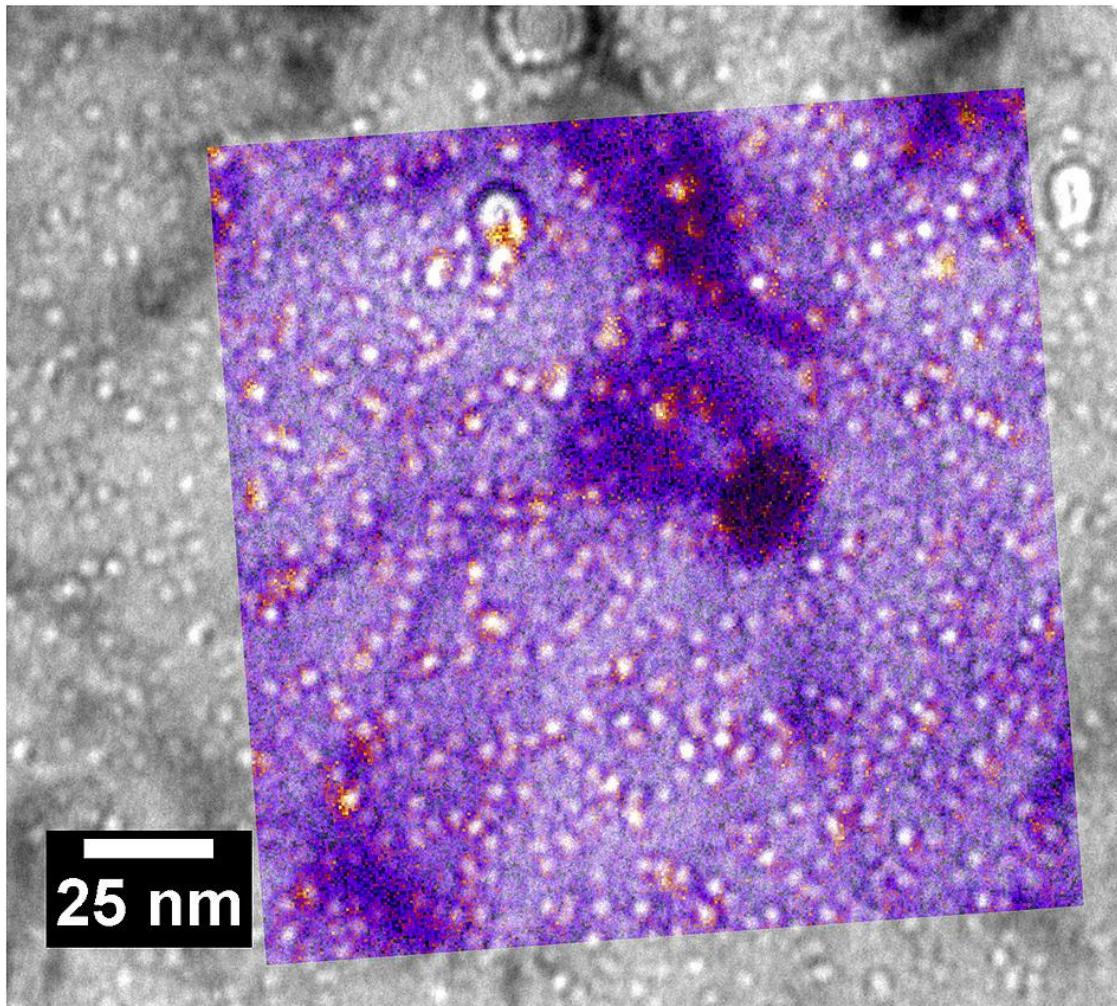
When supercritical particles are formed due to thermal fluctuations

- Heterogeneous nucleation:

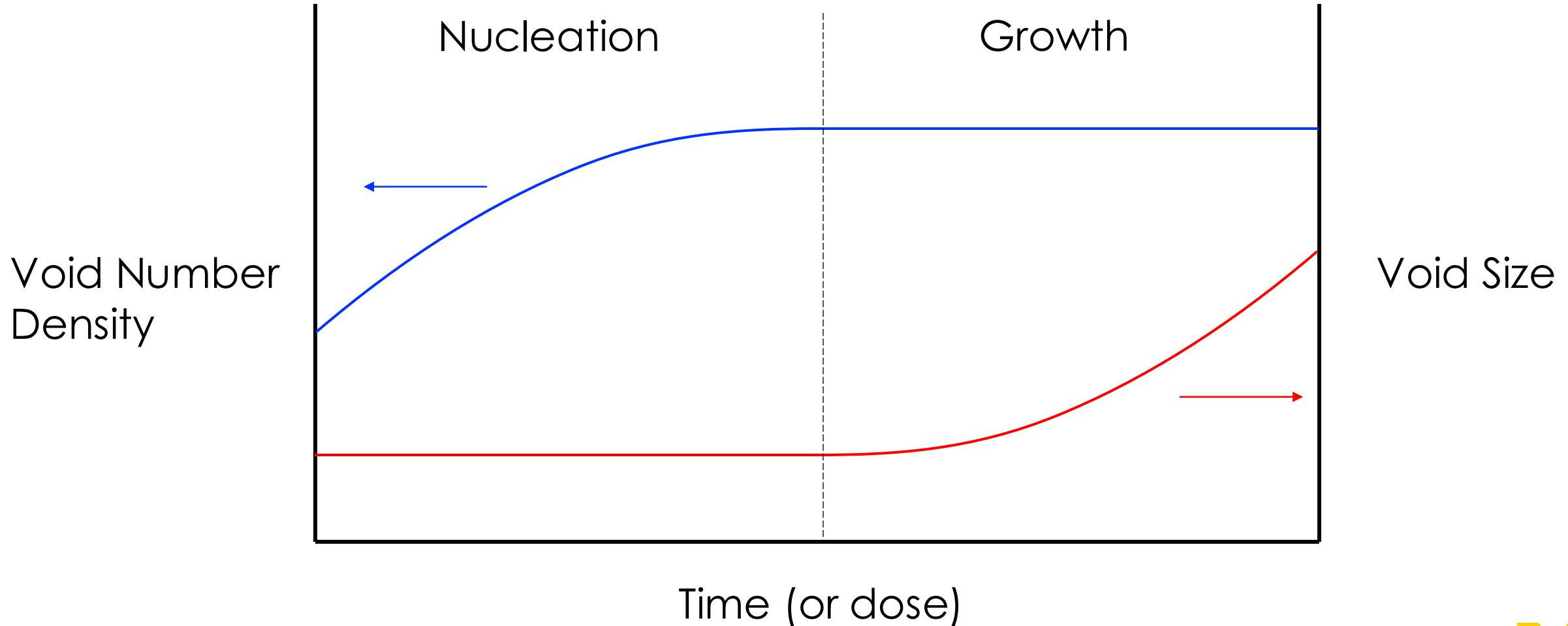
When external objects (surfaces, interfaces, impurities, defects, seeds) lower the barrier for nucleation

What happens to the graph with heterogeneous nucleation?

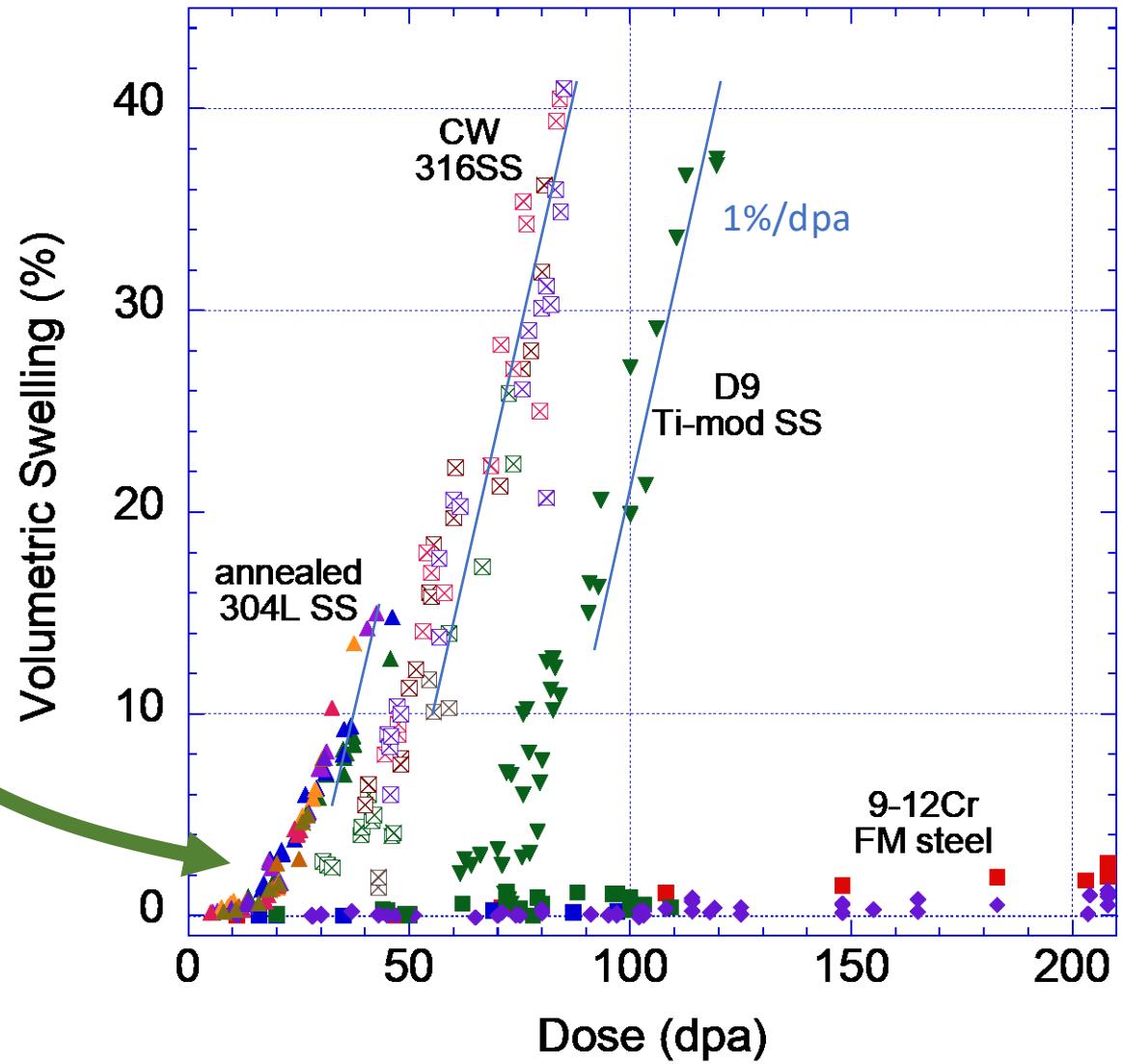
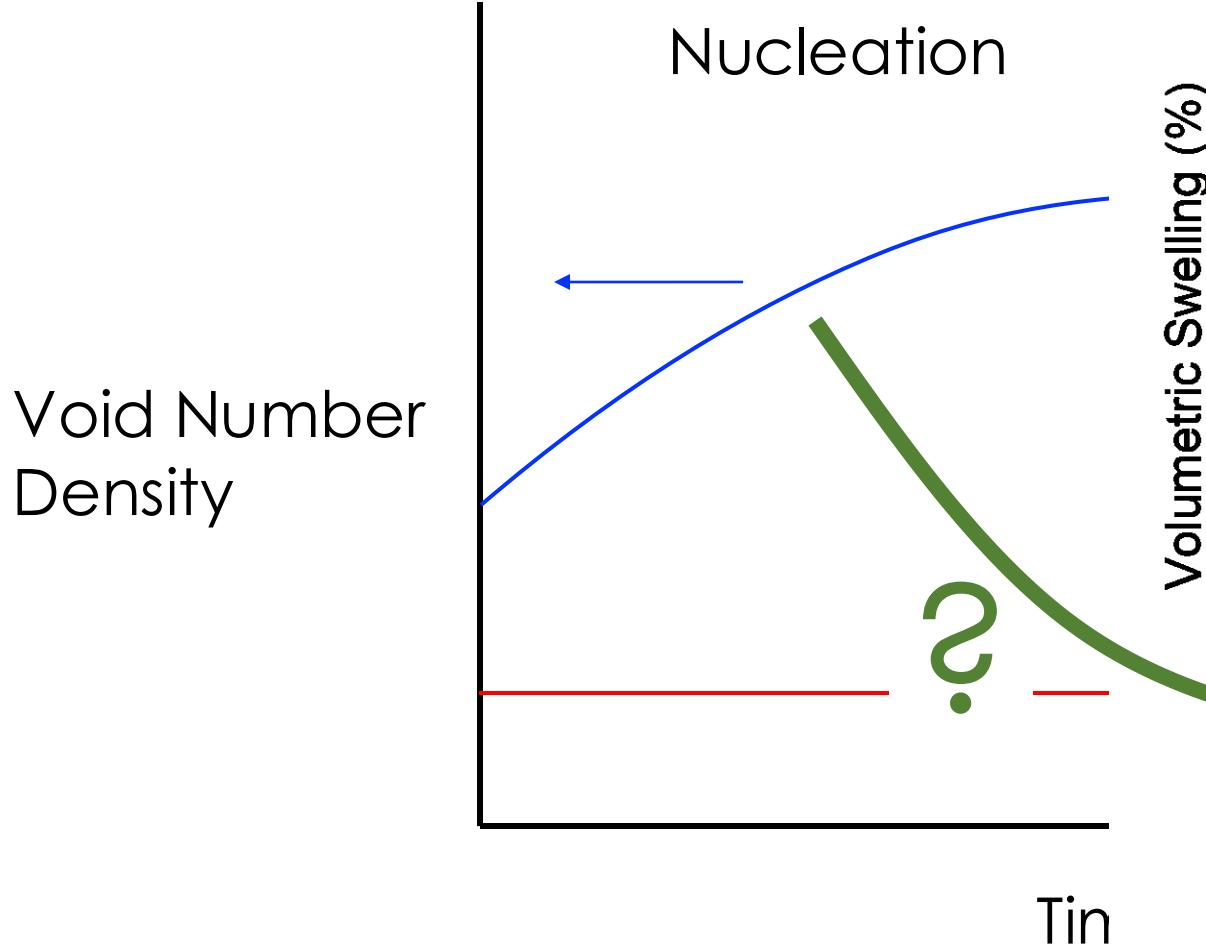
# Void Nucleation Theory: Balance



# Nucleation vs. Growth



# Nucleation vs. Growth



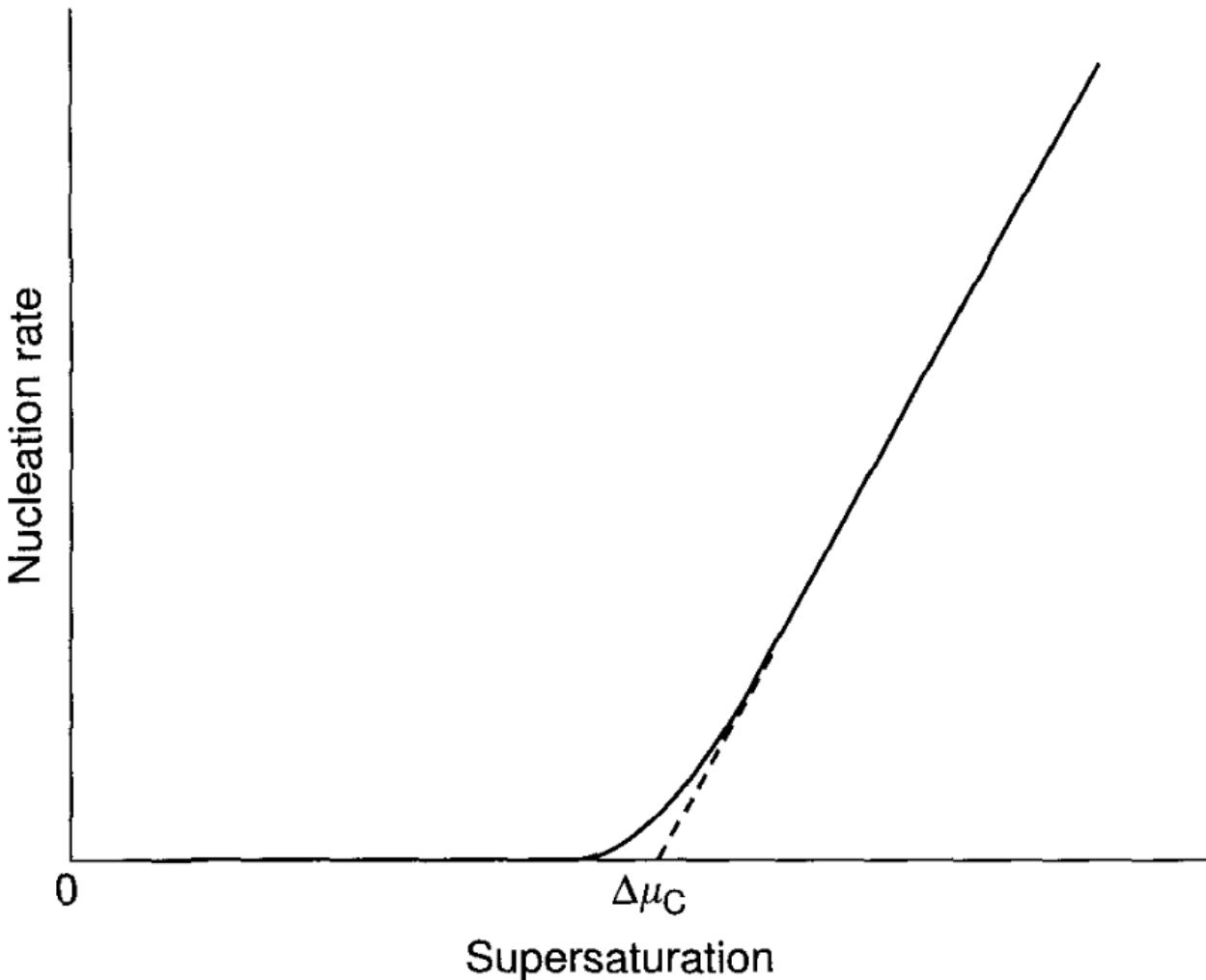
# Void Nucleation Rate

Nucleation rate can be generalized as:

$$J_0 \propto \exp\left(-\frac{\Delta G}{kT}\right) D_v C_v$$

But it depends on:

- Dose rate
- Temperature
- Sink density, etc.



# Void Nucleation Rate, $J_0$

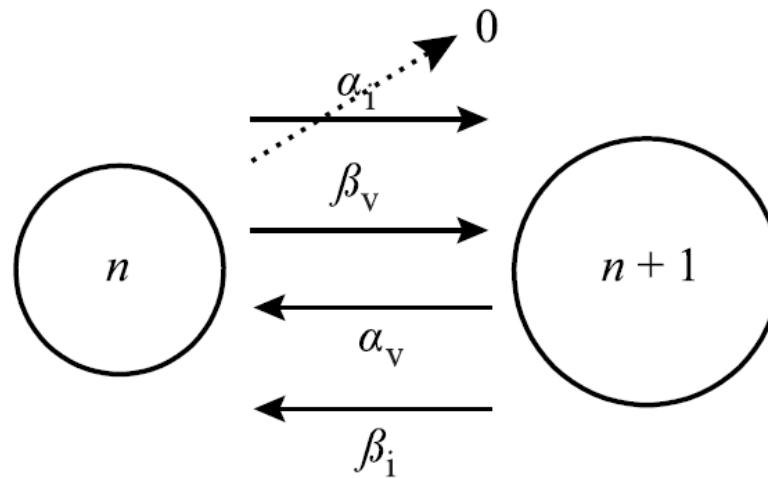
- Voids are three-dimensional clusters of vacancies formed by the following reactions
  1. **Cluster growth** by  $\nu$  absorption:  $\nu + \nu_j \rightarrow \nu_{j+1}$
  2. More generally, we consider **small cluster mobility**:  $\nu_j + \nu_k \rightarrow \nu_{j+k}$
  3. **Cluster shrinkage** by  $\nu$  emission:  $\nu_j \rightarrow \nu_{j-1} + \nu$ 
    - Depends on equilibrium  $\nu$  concentration at void surface -  $C_\nu^0$  from the rate of absorption of  $\nu$  by cavities and also depends on the binding energy between the  $\nu$  and the cluster
  4. **Cluster shrinkage** by  $i$  absorption:  $\nu_j + i_k \rightarrow \nu_{j-k}$ 
    - Depends on  $i$  and  $i_k$  concentrations
  5. Growth by  $i$  emission is neglected, e.g.  $C_i^0 \sim 0$



# Void Nucleation Rate

- The flux between any two sized voids, say  $n$  and  $n + 1$ :

$$J_n = \beta_v(n)\rho(n) - p(n+1)(\alpha_v(n+1) + \beta_i(n+1))$$



- $\beta_v(n)\rho(n)$  = rate of  $\nu$  absorption by clusters of size  $n$
- $\alpha_v(n+1)p(n+1)$  = rate of  $\nu$  emission by clusters of size  $n + 1$
- $\beta_i(n+1)p(n+1)$  = rate of  $i$  absorption by clusters of size  $n + 1$

# Void Nucleation Rate

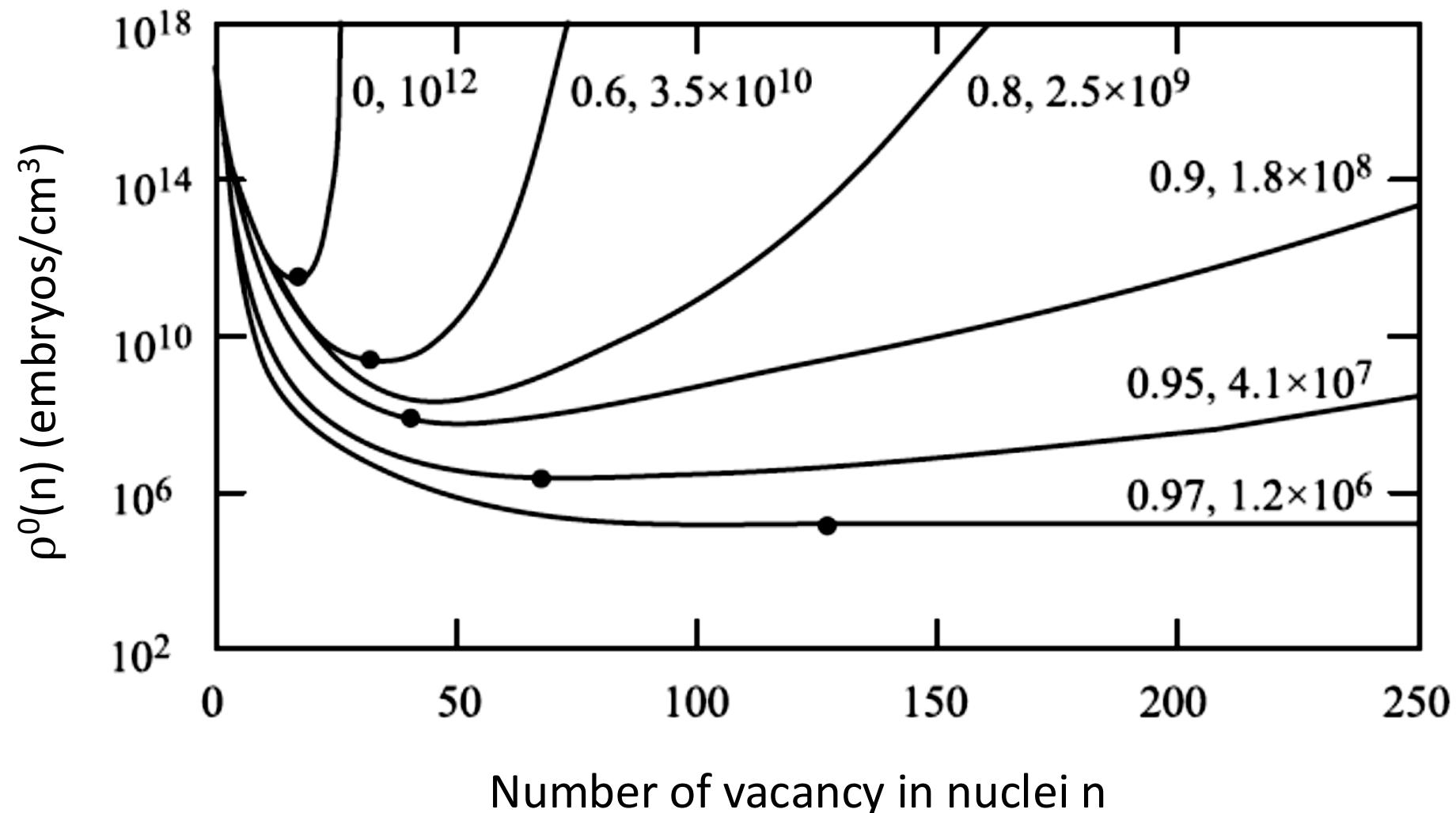
- Lengthy derivation covered in Was 8.1.2
- For sake of simplicity, the # of void embryos can be written as:

$$\frac{\rho^0(n)}{C_v} = e^{\sum_{k=1}^{n-1} \ln \left( \frac{\sqrt[3]{\frac{k}{k+1}}}{\left( \frac{C_v^{eq}}{C_v} e^{\left( \frac{8\pi\gamma}{\xi^3\sqrt{k+1}} - p \right) \frac{\Omega}{kT}} + \frac{D_i C_i}{D_v C_v} \right)} \right)}$$

- $C_v^{eq}/C_v$  is the inverse of vacancy supersaturation  $S_v^{-1}$
- $(D_i C_i)/(D_v C_v)$  is the arrival rate ratio between  $v$  and  $i$
- $\gamma$  is the surface energy of the cavity
- $p$  is the gas pressure in the cavity ( $p=0$  for voids!)



# Concentration of void embryo sizes w/ nucleation rate



# Void Nucleation Rate

- To obtain void nucleation as  $(D_i C_i)/(D_v C_v)$  approaches 1 requires higher vacancy supersaturation
- Strong dependence of nucleation on vacancy supersaturation

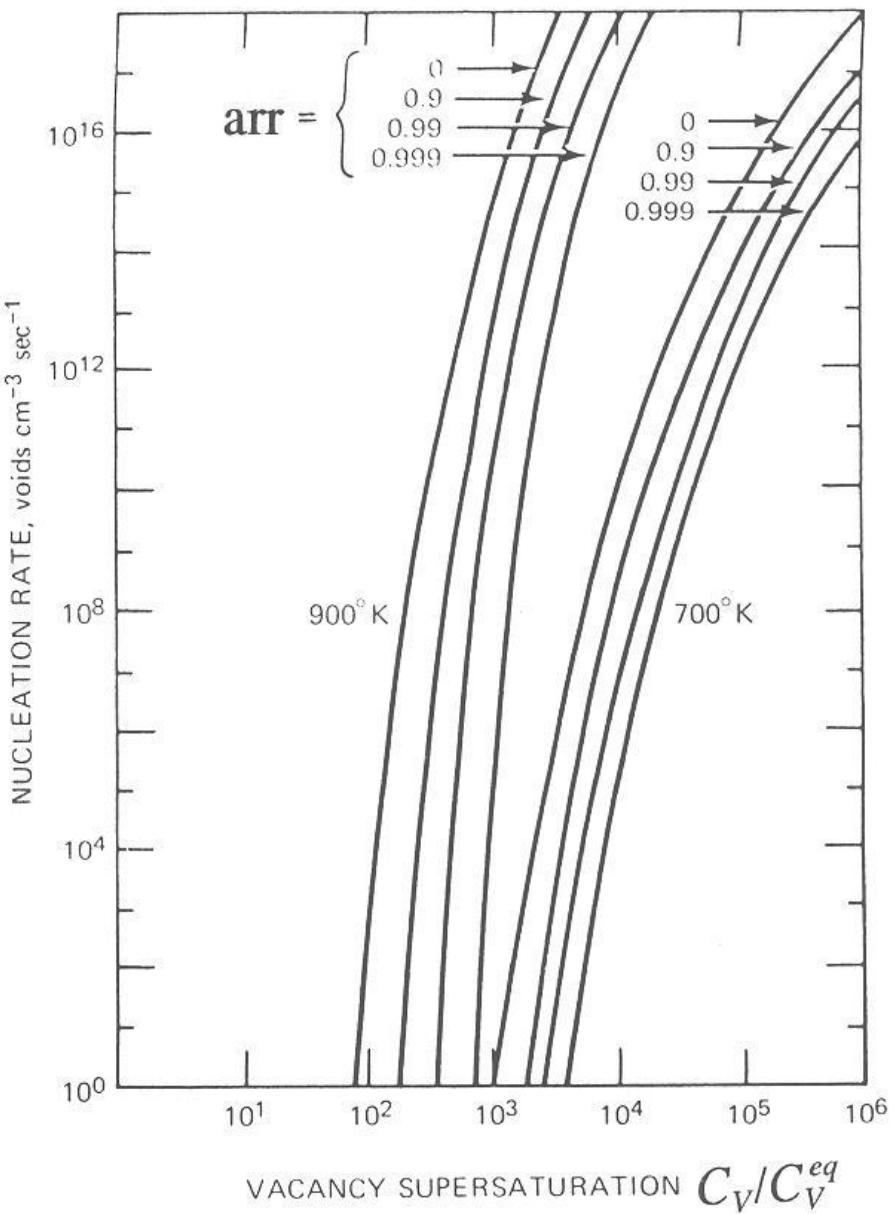
## Typical results:

$$T = 700 \text{ K}; C_v/C_v^{\text{eq}} = 10^4$$

$$\text{arr} = (D_i C_i)/(D_v C_v) = 0.99$$

$$J \sim 10^8 \text{ voids nucleated}/\text{cm}^3/\text{s}$$

- After 1 year,  $3 \times 10^{15}$  voids/cm<sup>3</sup>
- The voids are small, about the critical size



# Lecture Break

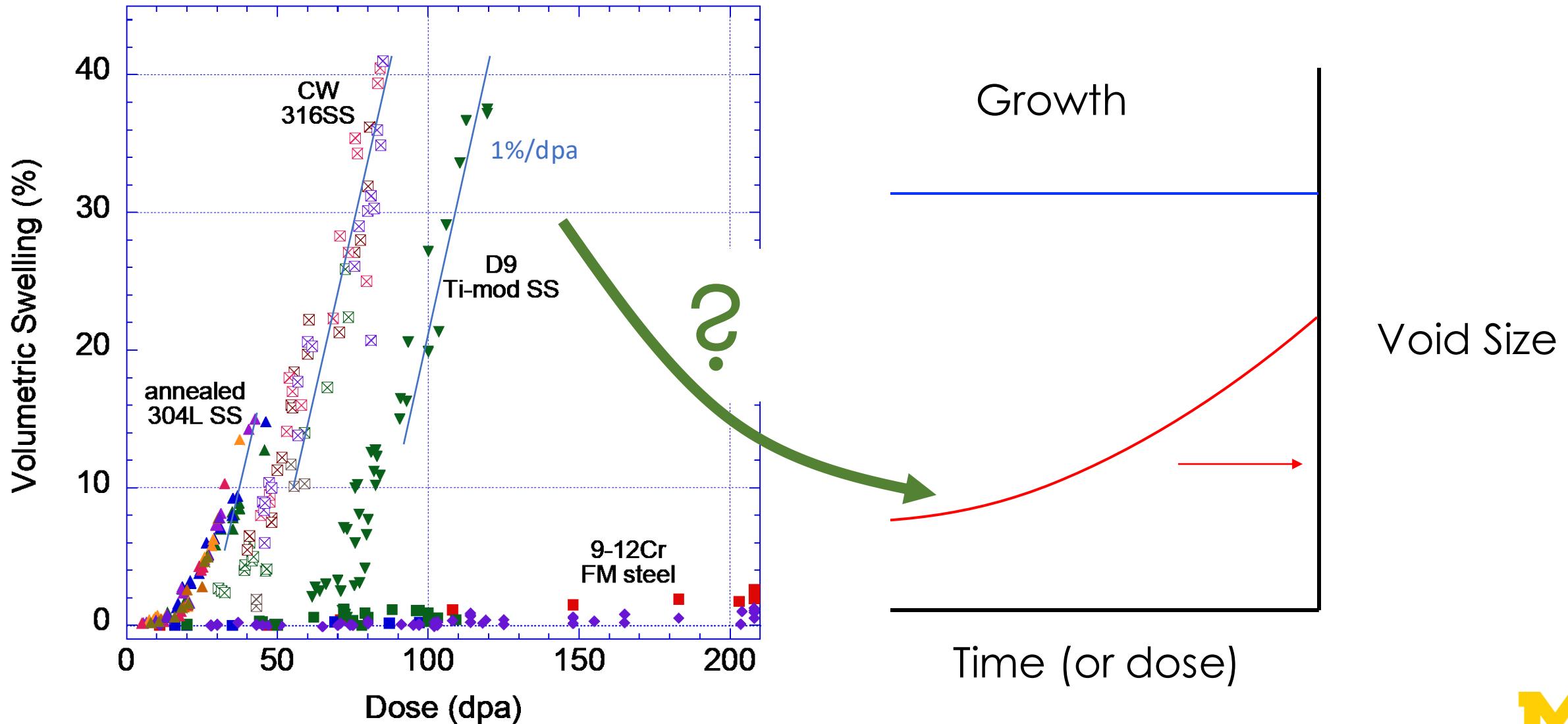
While the Stormy Kromer company is famous for its iconic, warm hats now handcrafted in **Ironwood, Michigan**, its namesake had another passion.

George "Stormy" Kromer, a former semi-pro baseball player himself, was also a notorious minor league manager. In 1951, while managing the Vincennes Velvets, he led the team to an embarrassing, nationally recognized losing streak.

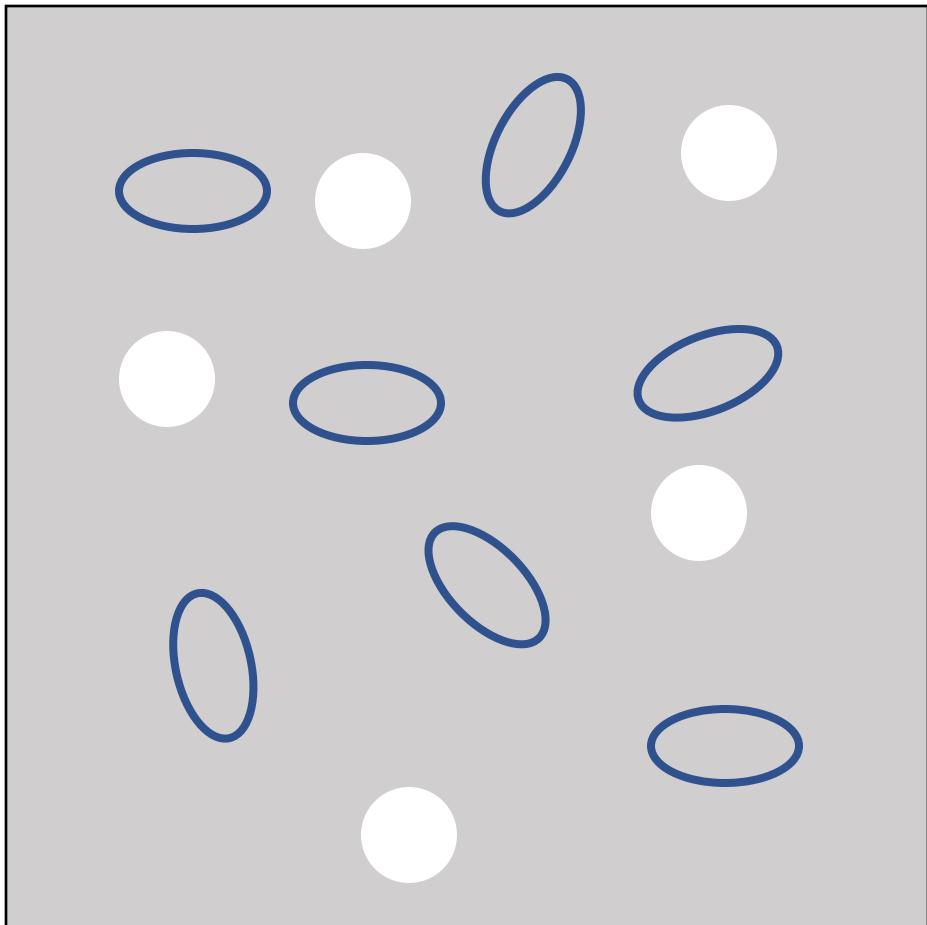
How many consecutive games did Stormy Kromer's team lose during that infamous streak?



# Nucleation vs. Growth



# Void Growth – Simple Model



Let's assume a material with nucleated voids and dislocation loops only, then the total sink strength can be defined as (from Table 5.2 Was):

$$k_v^2 = z_v \rho_d + 4\pi R \rho_v$$

$$k_i^2 = z_i \rho_d + 4\pi R \rho_v$$

# Void Growth

- For void growth, we need to know the net flux of vacancies to a void embryo. The net rate is thus a combination of the fluxes of interstitials and vacancies to a *nucleated* void, where:

$$J_{net}^V = J_v^V - J_i^V = 4\pi R \Omega D_v (C_v - C_v^V) - 4\pi R \Omega D_i (C_i - C_i^V)$$

$$J_{net}^V = dV/dt = 4\pi R \Omega (D_v C_v - D_i C_i)$$

$$dR/dt = \dot{R} = \frac{\Omega}{R} (D_v C_v - D_i C_i)$$

$$C_v^V = C_v^0 \exp\left(\frac{2\gamma\Omega}{Rk_b T}\right)$$



# Void Growth

$$\frac{\partial C_v}{\partial t} = K_0 - K_{iv} C_i C_v - K_{vs} C_v C_s = 0$$

$$C_v = \frac{-K_{is} C_s}{2K_{iv}} + \left[ \frac{K_0 K_{is}}{K_{iv} K_{vs}} + \frac{K_{is}^2 C_s^2}{4K_{iv}} \right]^{1/2}$$

Next slide

$$\frac{dR}{dt} = \dot{R} = \frac{\Omega}{R} (D_v C_v - D_i C_i)$$

$$C_i = \frac{-K_{vs} C_s}{2K_{iv}} + \left[ \frac{K_0 K_{vs}}{K_{iv} K_{is}} + \frac{K_{vs}^2 C_s^2}{4K_{iv}} \right]^{1/2}$$

Next slide

$$\frac{\partial C_i}{\partial t} = K_0 - K_{iv} C_i C_v - K_{vs} C_i C_s = 0$$



# Void Growth

Remember:

$$C_v = \frac{-K_{is}C_s}{2K_{iv}} + \left[ \frac{K_0K_{is}}{K_{iv}K_{vs}} + \frac{K_{is}^2C_s^2}{4K_{iv}} \right]^{1/2}$$

and

$$C_i = \frac{-K_{vs}C_s}{2K_{iv}} + \left[ \frac{K_0K_{vs}}{K_{iv}K_{is}} + \frac{K_{vs}^2C_s^2}{4K_{iv}} \right]^{1/2}$$

$$k_{jx}^2 = \frac{K_{jx}C_x}{D_j}$$

and

$$k_v^2 = z_v\rho_d + 4\pi R\rho_V$$

$$k_i^2 = z_i\rho_d + 4\pi R\rho_V$$

You can now pull all three equations above together to get:

$$C_v = \frac{D_v(4\pi R\rho_v + z_vp_d)}{2K_{iv}} (\sqrt{1+\eta} - 1)$$

$$C_i = \frac{D_i(4\pi R\rho_v + z_ip_d)}{2K_{iv}} (\sqrt{1+\eta} - 1)$$

Where:

$$\eta = \frac{4K_0K_{iv}}{D_iD_v(4\pi R\rho_v + z_vp_d)^2}$$



# Void Growth

- With everything defined,

$$C_v = \frac{D_v(4\pi R\rho_v + z_vp_d)}{2K_{iv}}(\sqrt{1+\eta} - 1)$$

$$C_i = \frac{D_i(4\pi R\rho_v + z_ip_d)}{2K_{iv}}(\sqrt{1+\eta} - 1)$$

$$\eta = \frac{4K_0 K_{iv}}{D_i D_v (4\pi R\rho_v + z_vp_d)^2}$$

$$\frac{dR}{dt} = \dot{R} = \frac{\Omega}{R}(D_v(C_v - C_v^V) - D_i C_i)$$

- We can now rewrite the growth law as:

$$\dot{R}R = \frac{\Omega}{2K_{iv}} D_i D_v (z_i \rho_d - z_v \rho_d)(\sqrt{1+\eta} - 1)$$



# Void Growth

$$R\dot{R} = K_o \Omega \left( \frac{z_i - z_v}{z_v} \right) \frac{z_v \rho_d}{(4\pi R \rho_v + z_v \rho_d)(4\pi R \rho_v + z_i \rho_d)} F(\eta)$$

- The **first term** is the main dpa-rate effect on void growth
- The **second term** is the “bias” term: if  $Z_i = Z_v$ , void growth is *impossible*
- The **third term** is the sink-strength balance term. Void growth is eliminated if there are too many or too few dislocations. Optimum growth occurs when the void sink term ( $4\pi R \rho_v$ ) and the dislocation sink term ( $z_v \rho_d$ ) are equal.
- The **fourth term** contains the effect of point defect recombination:

$$F(\eta) = 2(\sqrt{1+\eta} - 1)/\eta$$

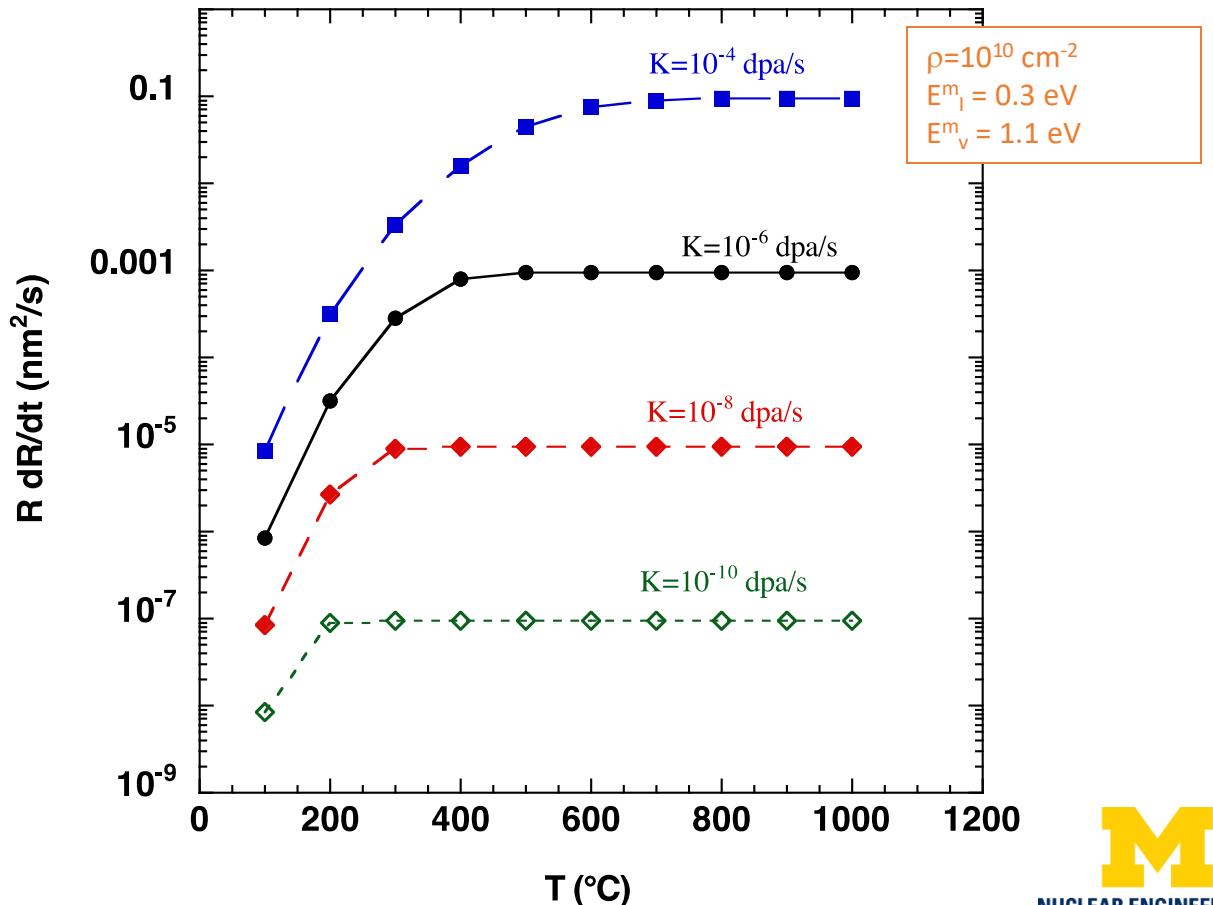
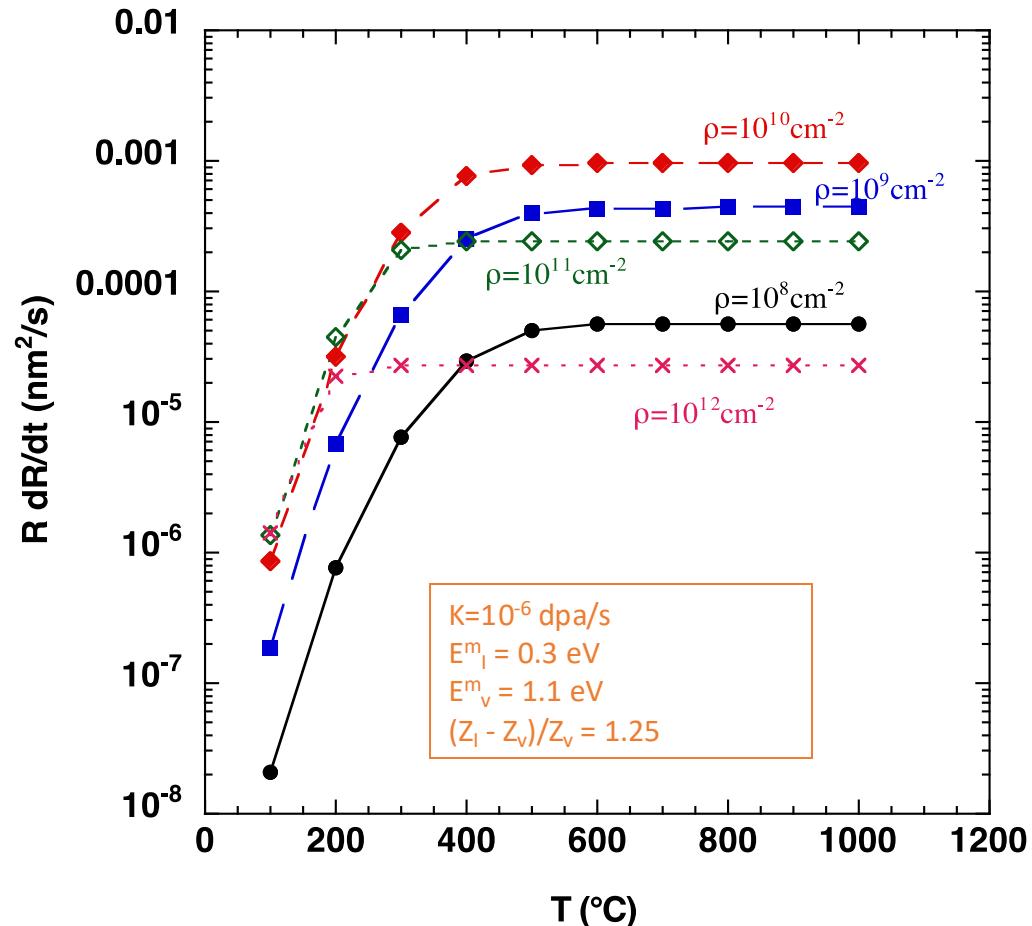
Since  $\eta$  decreases with increasing temperature and  $F$  decreases with increasing  $\eta$ :

- At high temperature,  $F \rightarrow 1$  and recombination does not effect void growth
- At low temperature,  $F \rightarrow 0$  and recombination prevents void growth.

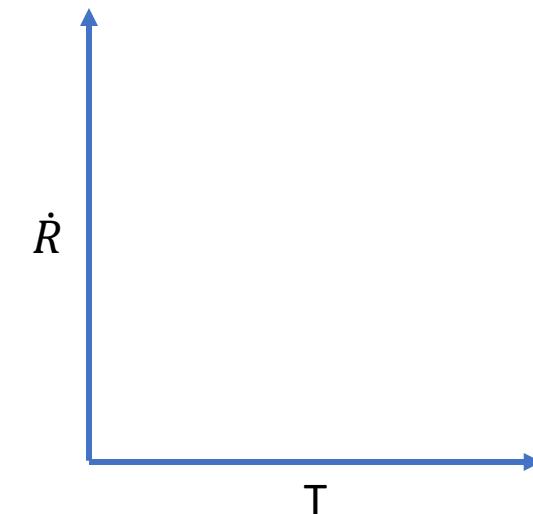
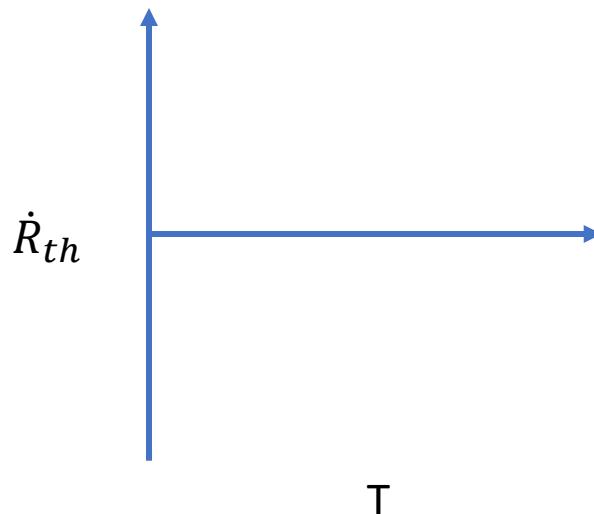
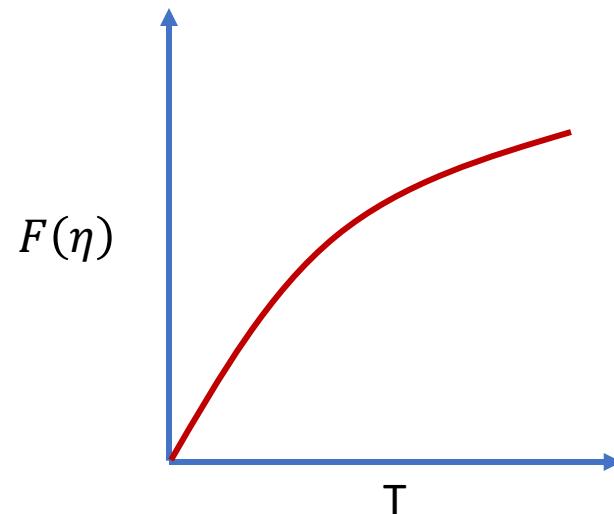


# Void Growth

$$R \dot{R} = K_o \Omega \left( \frac{z_i - z_v}{z_v} \right) \frac{z_v \rho_d}{(4\pi R \rho_v + z_v \rho_d)(4\pi R \rho_v + z_i \rho_d)} F(\eta)$$



# Void Swelling Temperature Dependence



At very high temperatures, void growth ceases because the vacancies “boil off” the voids. Repeating the previous derivation without neglecting  $C_v^0$  gives the following shrinkage rate that competes with the growth rate:

$$R\dot{R}_{th} = -\frac{D_v C_v^0 \Omega^2 z_v \rho_d}{kT(4\pi RN + z_v \rho_d)} (e^y - 1)$$



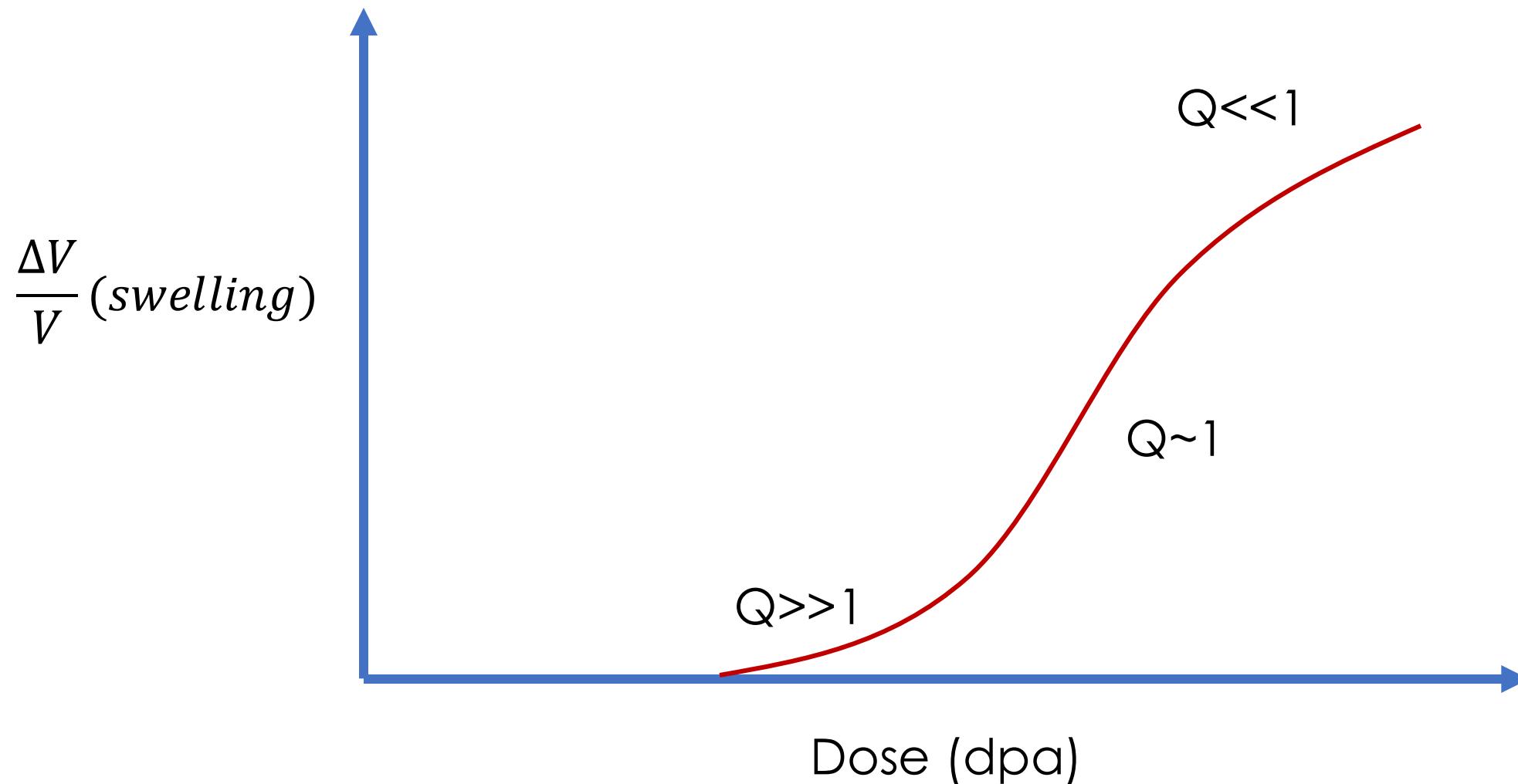
# Effect of Structure on Void Swelling

- Ferritic steels swell at rates ~0.2%/dpa
- Structure alone is not sufficient to explain the difference between  $\alpha$ -Fe (BCC) and  $\gamma$ -Fe (FCC)
- BCC vanadium alloys can swell at rates more like austenitic steels
- Difference is likely in the relative bias for point defects at sinks
- If the bias is removed:  $z_i = z_v$ , void growth is impossible
- Recall the 3<sup>rd</sup> term, put simply:

$$\left\{ \frac{z_v \rho_d}{(4\pi R \rho_v + z_v \rho_v)(4\pi R \rho_v + z_i \rho_d)} \right\} \frac{Q}{(1+Q)^2} \quad \text{Where: } Q = \frac{\rho_d z_i}{z_v 4\pi R \rho_v}$$

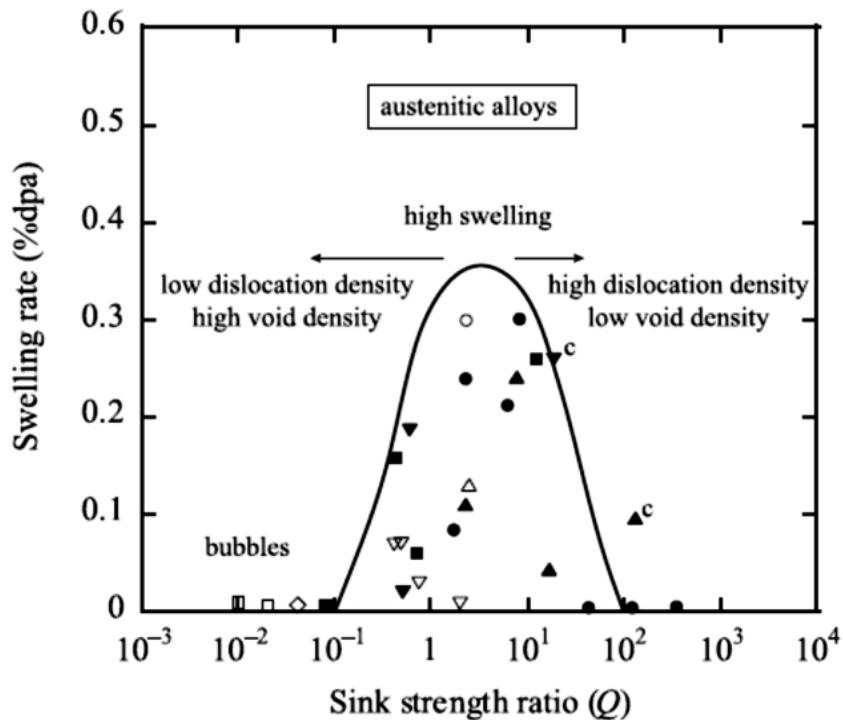


# The Q-factor for structure dependence on dose



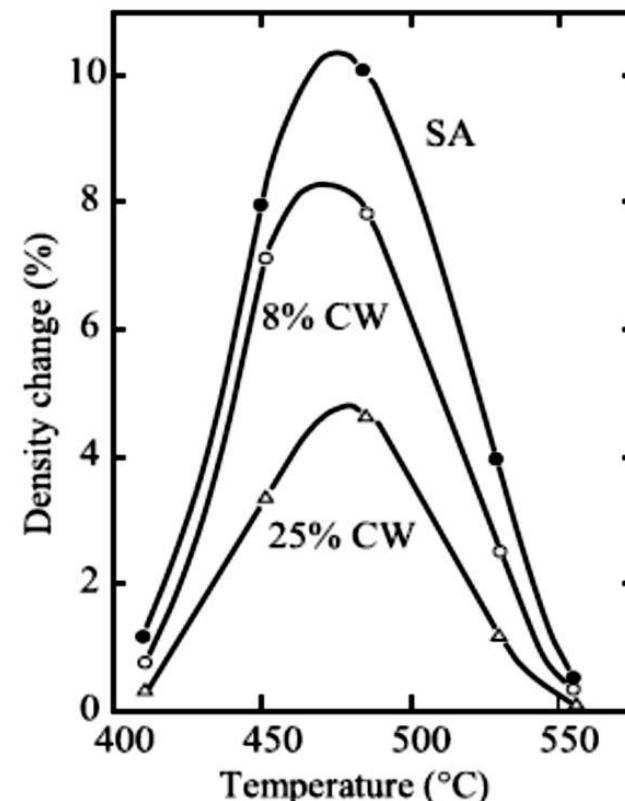
# Effect of Structure on Void Swelling

● Johnston et al.  
▲ Appleby et al.  
■ Packan and Farrell  
▼ Maziasz  
▽ Sprague et al.  
○ Westmoreland et al.  
△ Smidt et al.  
□ Tanaka et al.  
◊ Lee and Mansur  
c coalescence



*Experimentally observed swelling rates as a function of  $Q$  for austenitic stainless steels (Mansur LK (1994) J Nucl Mater 216:97–123)*

- Growth rate is maximum when  $Q \sim 1$
- Growth decreases for  $Q \neq 1$
- Observed experimentally
- CW reduces swelling because  $Q \gg 1$

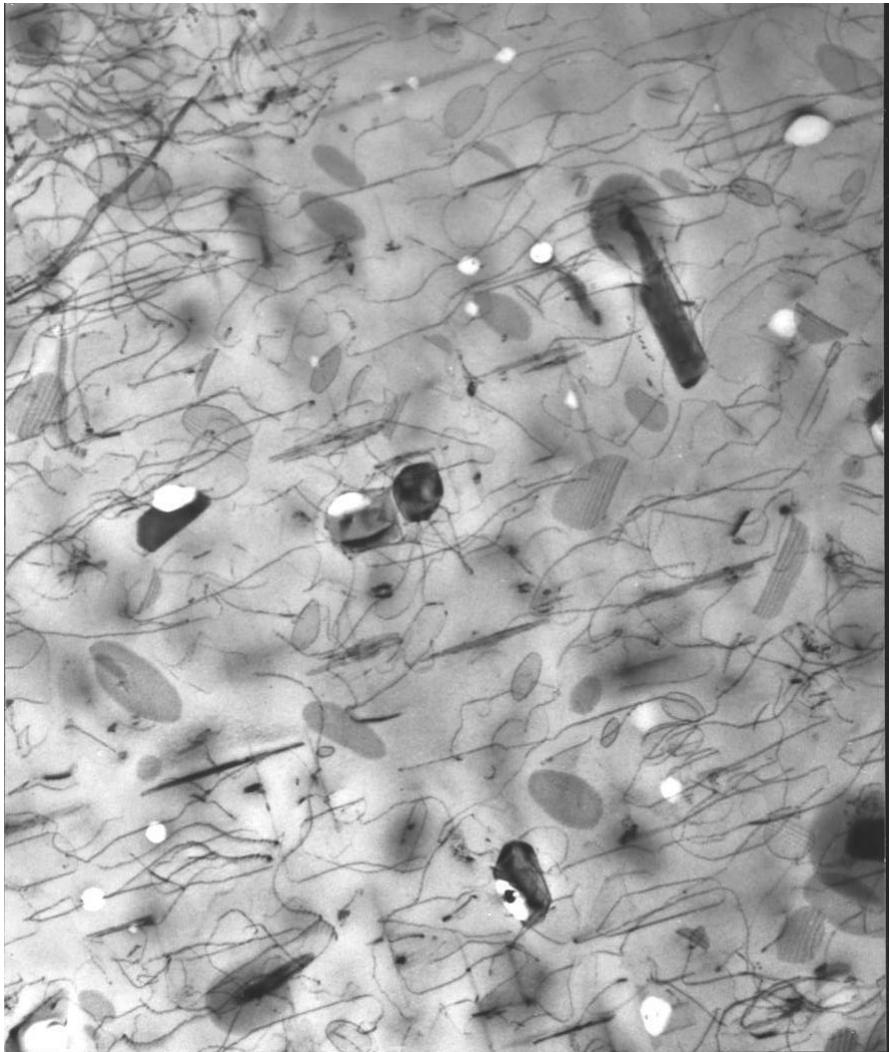


*Dependence of swelling on cold-work for various temperatures for 316 stainless steel irradiated in the RAPSODIE reactor to doses of 20–71 dpa (Dupouy JM, Lehmann J, Boutard JL (1978) In: Proceedings of the Conference on Reactor Materials Science, vol. 5, Alushta, USSR. Moscow, USSR Government, pp 280–296)*

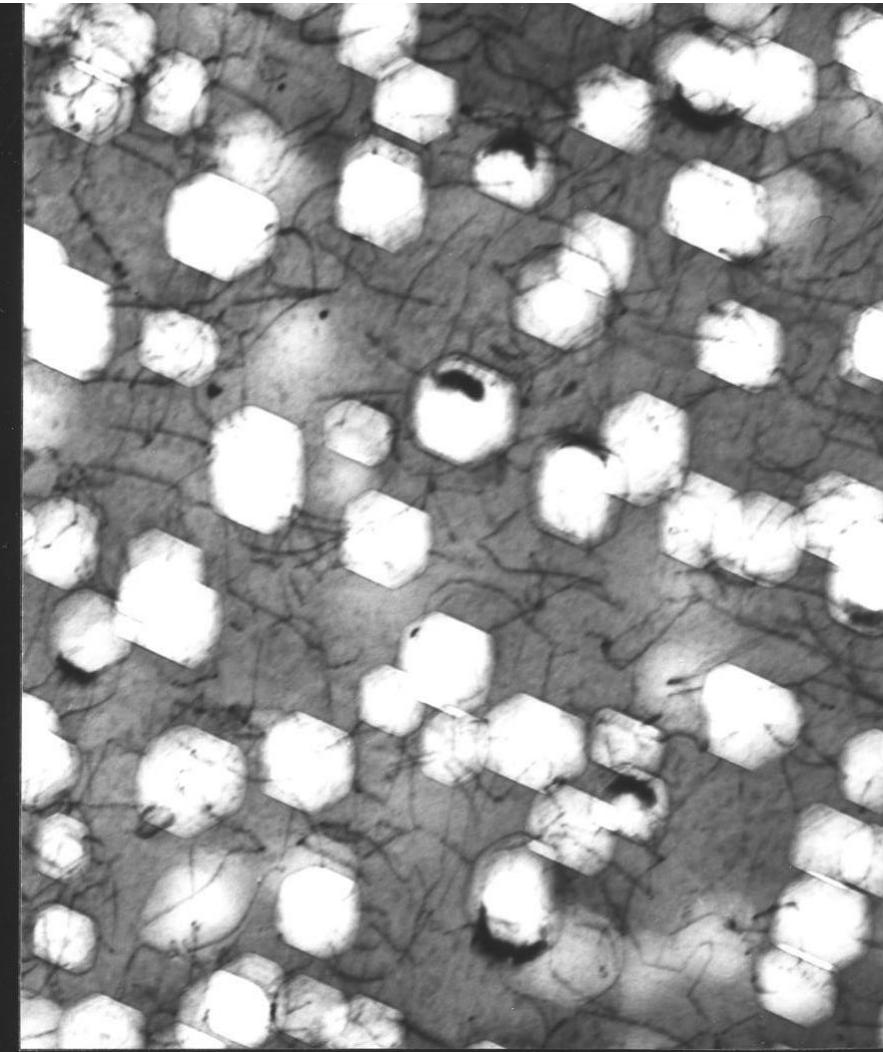


# Image of voids + effect of sink strength

Commercial 316 SS (high sink strength)



High-purity (low sink strength)

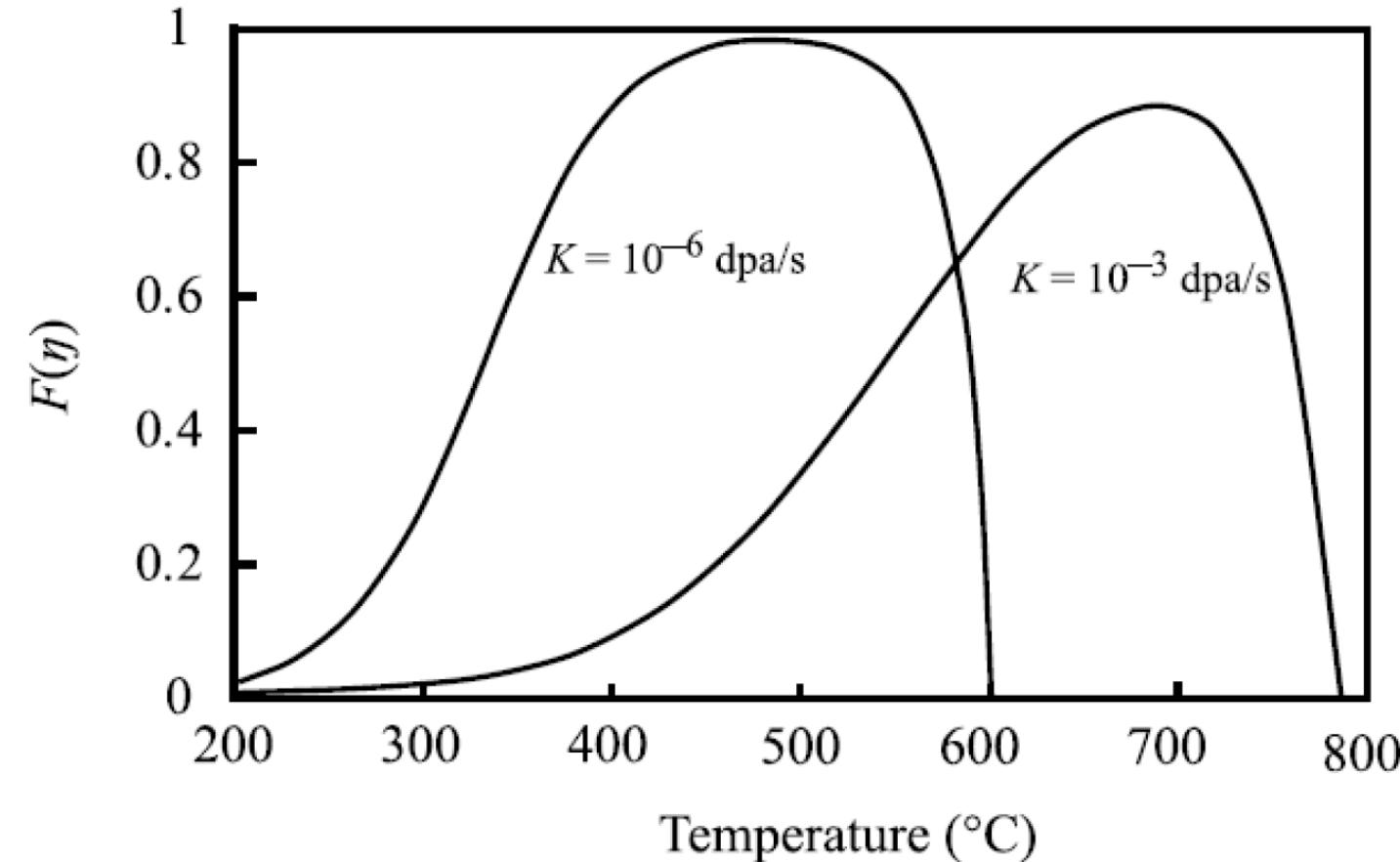


100 nm



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# Effect of Dose Rate



Dose rate is captured in the **fourth term** where:

$$F(\eta) = 2\left(\sqrt{1+\eta} - 1\right)/\eta$$

And:

$$\eta = \frac{4K_0 K_{iv}}{D_i D_v (4\pi R \rho_v + z_v p_d)^2}$$



# Effect of Dose Rate

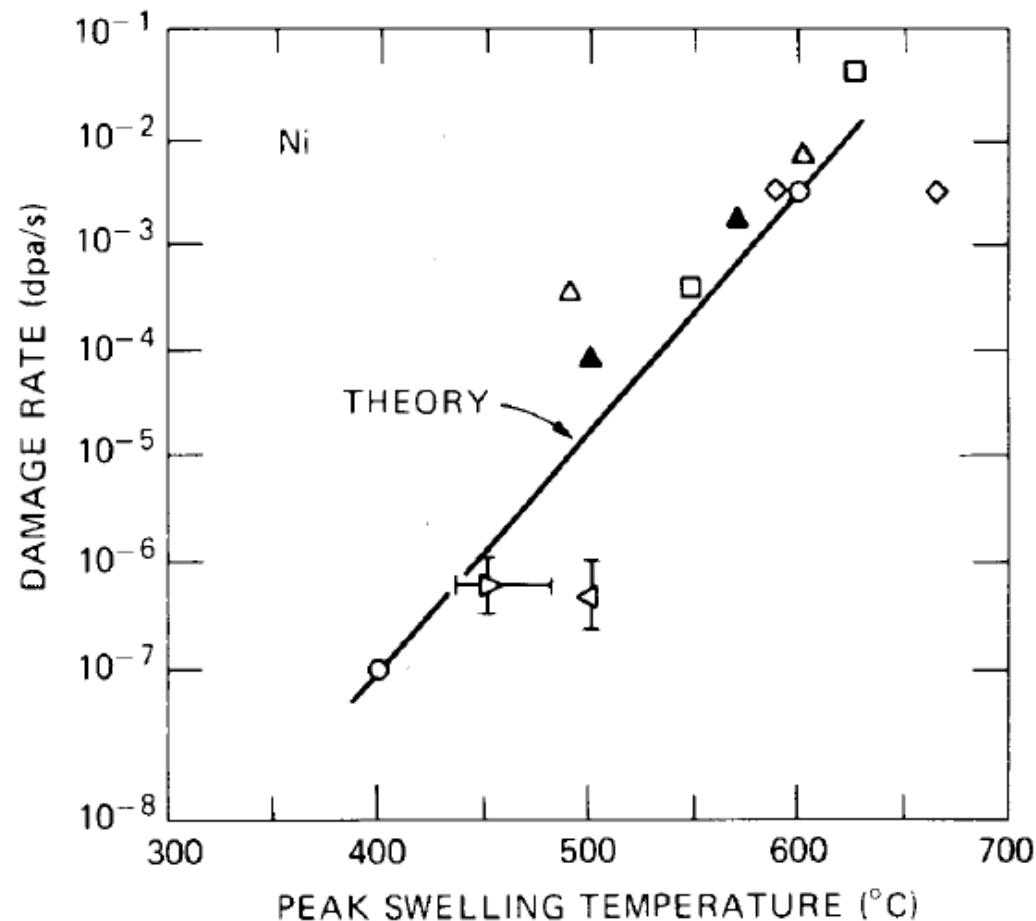


Figure 8.29. Compilation of experimental results for peak swelling temperature as a function of dose rate. Theoretically predicted trend is shown as the line. After Refs. 140 and 141.

Dose rate is captured in the **fourth term** where:

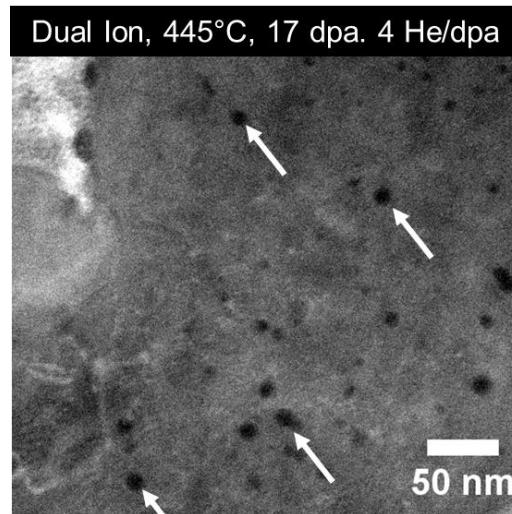
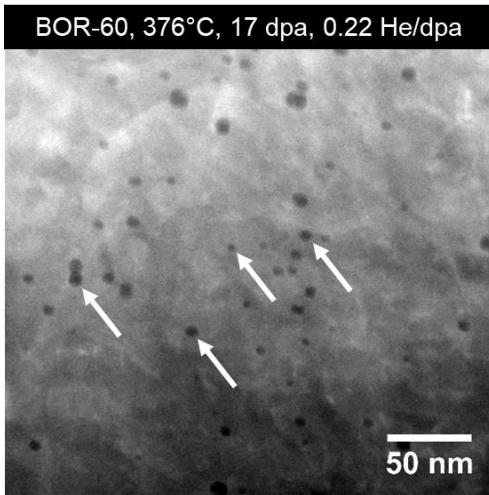
$$F(\eta) = 2\left(\sqrt{1+\eta} - 1\right)/\eta$$

And:

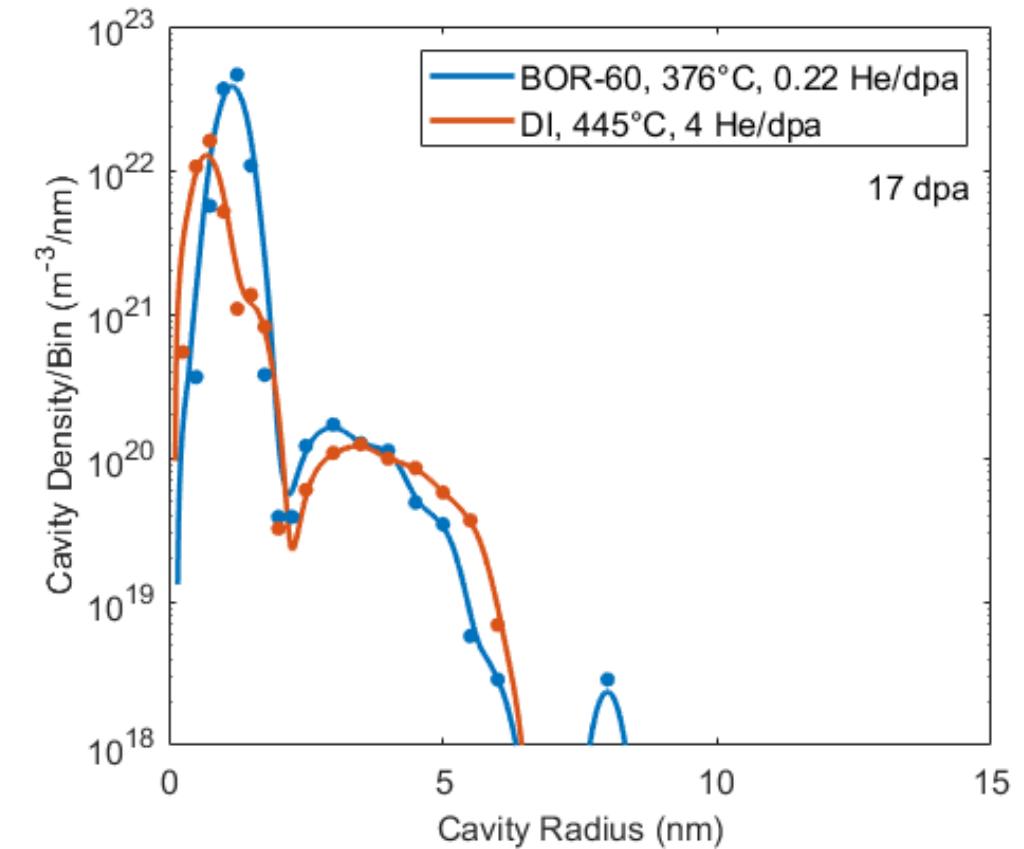
$$\eta = \frac{4K_0 K_{iv}}{D_i D_v (4\pi R \rho_v + z_v p_d)^2}$$

# Effect of Dose Rate – Real World Example

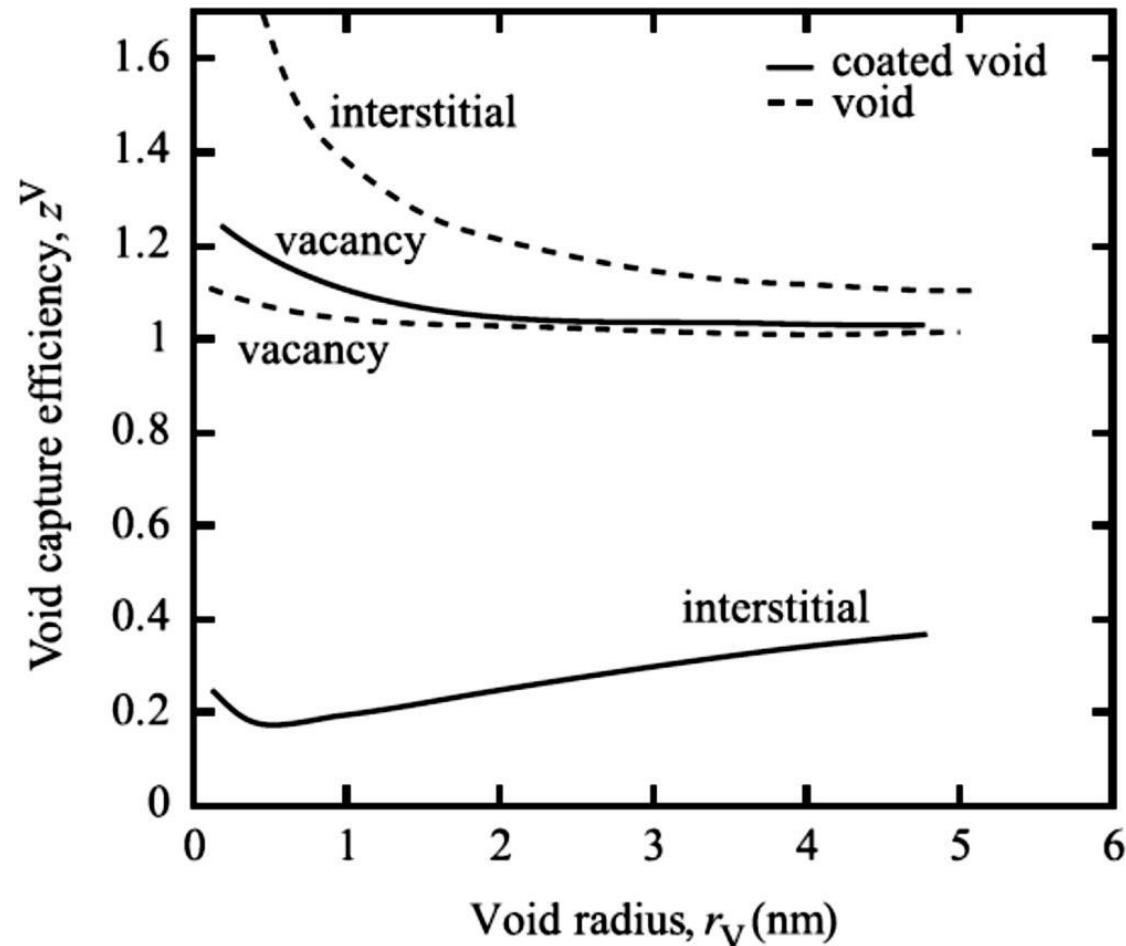
## STEM HAADF



$$T_2 - T_1 = \frac{\frac{kT_1^2}{E_v^m + 2E_v^f} \ln \left( \frac{G_2}{G_1} \right)}{1 - \frac{kT_1}{E_v^m + 2E_v^f} \ln \left( \frac{G_2}{G_1} \right)}$$

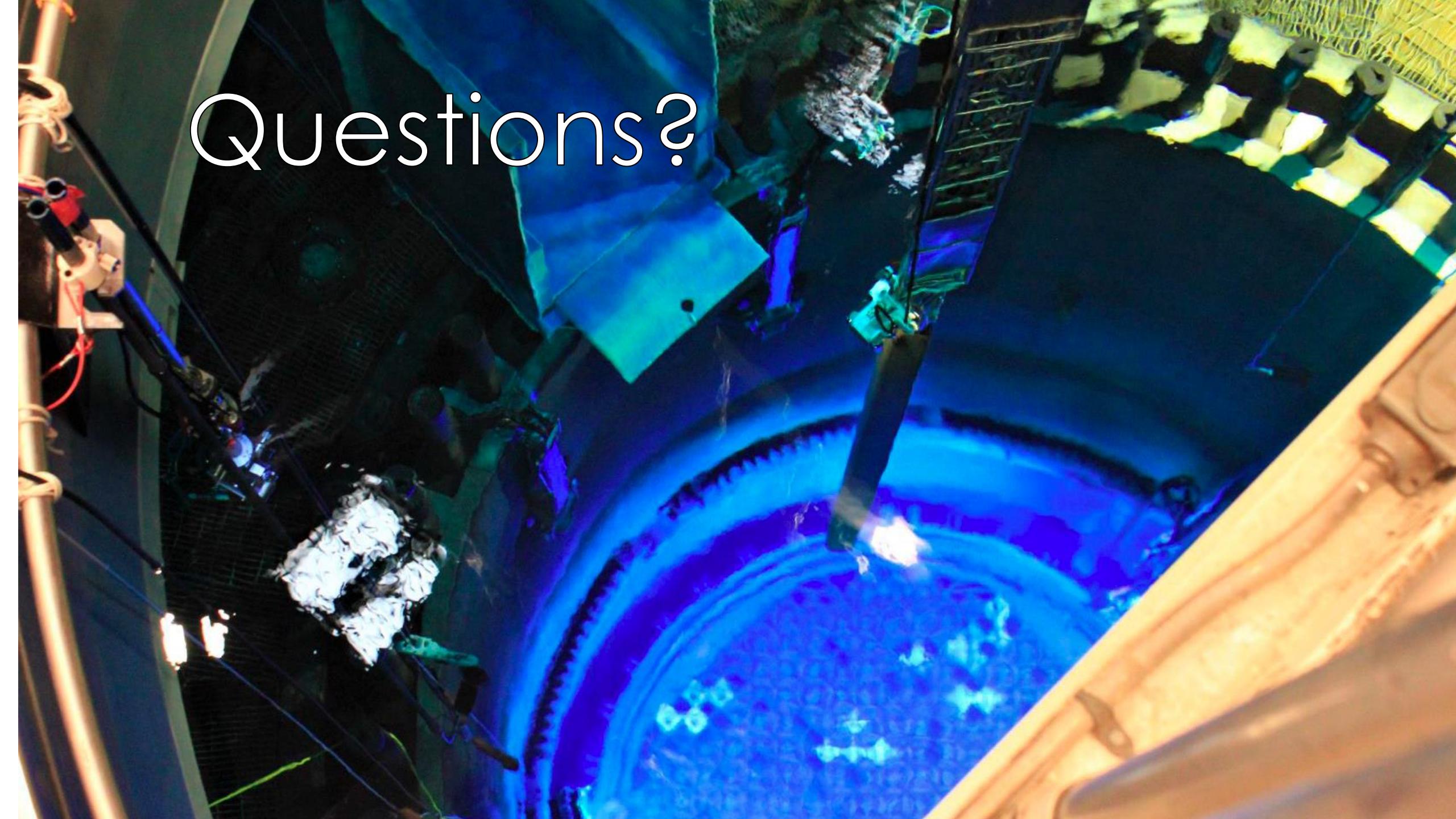


# Effect of void surface segregation on defect bias



- For a bare unpressurized void, **interstitial bias is greater than vacancy bias**. Voids will shrink
- If “shell” shear modulus or lattice parameter is greater than matrix shear modulus, **vacancy bias becomes greater than interstitial bias**
  - This effect can occur because of **radiation induced segregation**
- Thicker shells have a greater effect

Capture efficiency for point defects diffusion to a void and a coated void as a function of void radius  $RV$ . (W.G. Wolfer, L.K. Mansur, *The capture efficiency of coated voids*, Journal of Nuclear Materials, Volume 91, Issue 2, 1980, Pages 265-276)



Questions?

# HW4

- Q#3 can be treated separate (e.g. enrich/deplete concepts will be different) from Q#2
- In Q#4 should assume that loops will form, and thus the question is asking if you expect to see these loops as faulted or perfect loops
- In Q#5, keep the final expression in terms of constants ( $\Omega$ ,  $\gamma_{SFE}$ ,  $\sigma$ ,  $G$ ,  $b$ ).
  - Unity of  $a_o$  means  $a_o=1$  (simply, it can be ignored)
  - The equation for a cavity should be disc. This has been updated in Canvas.

