5.15 Show that, for a metal with a low sink density undergoing neutron irradiation at low temperature (T/T_m<0.2), when sinks contribute to interstitial annihilation, the vacancy and interstitial concentrations as a function of time can be written Eq. (5.11) as:

$$C_v = \left[\frac{K_0 K_{is} C_s t}{K_{iv}}\right]^{1/2}$$

$$C_i = \left[\frac{K_0}{K_{iv} K_{is} C_s t}\right]^{1/2}$$

(Hint: consider this case to be intermediary to the quasi-steady state and steady state cases such that dC_i/dt<0 and dC_v/dt>0 and write the point defect balance equations as inequalities)

$$\frac{dC_v}{dt} = K_0 - K_{iv}C_iC_v - K_{vs}C_vC_s$$
$$\frac{dC_i}{dt} = K_0 - K_{iv}C_iC_v - K_{is}C_iC_s$$

Assume K_{vs} = 0 since interstitials reach sinks first

When sinks contribute to interstitial annihilation, the system is between the quasi-steady state and steady state

The $K_{is}C_iC_s$ term makes dC_i/dt<0, and the corresponding decrease in C_i makes dC_v/dt>0, Then

$$K_0 - K_{iv}C_iC_v > 0$$
$$K_0 - K_{iv}C_iC_v - K_{is}C_iC_s < 0$$
$$K_0 > K_{iv}C_iC_v > K_0 - K_{is}C_iC_s$$

As an approximation for this time period:

$$K_{iv}C_iC_v = K_0 - \frac{1}{2}K_{is}C_iC_s$$

$$C_i = \frac{2K_0}{2K_{iv}C_v + K_{is}C_s}$$

$$\frac{dC_v}{dt} = K_0 - K_{iv}\left(\frac{2K_0}{2K_{iv}C_v + K_{is}C_s}\right)C_v$$

$$\left(2K_{iv}C_v + K_{is}C_s\right)\frac{dC_v}{dt} = K_0K_{is}C_s$$

Since the first term is much larger than the second, it dominates and the second term can be neglected:

$$2K_{iv}C_v\frac{dC_v}{dt} = K_0K_{is}C_s$$

$$2C_v dC_v = \frac{K_0K_{is}C_s}{K_{iv}}dt$$

$$C_v^{\ 2} = \frac{K_0K_{is}C_s t}{K_{iv}} + const.$$

Since this is an approximate solution, the time at which sinks begin to be of importance is arbitrary, so the integration constant can be set to 0.

$$C_v^{\ 2} = \frac{K_0K_{is}C_s t}{K_{iv}}$$

$$C_v = \sqrt{\frac{K_0K_{is}C_s t}{K_{iv}}}$$

$$C_i = \frac{2K_0}{2K_{is}C_v + K_{iv}C_s}\ ;\ \ \frac{2K_0}{2K_{iv}C_v} = \frac{K}{K_{iv}\sqrt{\frac{K_0K_{is}C_s t}{K_{iv}}}}$$

$$C_i = \sqrt{\frac{K_0}{K_{iv}K_{is}C_s t}}$$