Statistical Techniques. Lab 3

Plan for today

- Estimation
- Confidence intervals
- Statistical hypotheses testing
- p-value
- Z-test
- T-test

Estimation

Two forms of estimation:

Point estimation - most likely value of parameter (e.g., x-bar is point estimator of μ)

Interval estimation - range of values with known likelihood of capturing the parameter, i.e., a confidence interval (CI)

Estimation: task

For the following random samples, find the maximum likelihood estimate of θ:
$X_i \sim \text{Exponential}(\theta)$, and we have observed $(x_1, x_2, x_3, x_4,) = (1.23, 3.32, 1.98, 2.12)$.

Answer: 0.46

Confidence Interval for μ (σ Unknown)

- Assumptions
  - Population standard deviation is unknown
  - Population is normally distributed
- Confidence Interval Estimate

$$\bar{X} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$$

where $\bar{X}$ – mean of a sample, S – st. dev. of a sample, n – number of observations in a sample

Confidence interval: task

The following measurements were recorded for lifetime, in years, of certain type of machine: 3.4, 4.8, 3.6, 3.3, 5.6, 3.7, 4.4, 5.2, and 4.8. Assuming that measurements represent a random sample from a normal population, find a 99% confidence interval for the mean life time of the machine.

Answer: (3.37, 5.25)

Hypothesis testing

A statistical hypothesis is an assertion or conjecture concerning one or more populations

The alternative hypothesis H_1 usually represents the question to be answered or the theory to be tested, and thus its specification is crucial

The null hypothesis H_0 nullifies or opposes H_1 and is often the logical complement to H_1

Source: Walpole R. E. et al. Probability and statistics for engineers and scientists)

Hypothesis testing: example

Let's imagine we have a coin

I tossed a coin once and got *Head*
Then I tossed it twice and got *Head* too
We can say that our coin is special since it landed on Heads twice in a row!

In Statistics, the hypothesis is:
"Even though we got *2 Heads* in a row, our coin is no different from a normal coin"

Let's test this hypothesis by calculating p-value!

The full example is available through this link

Hypothesis testing: example

p-value is the probability of obtaining test results at least as extreme as the result actually observed, under the assumption that the null hypothesis is correct (according to this link)

So, p-values are determined by adding up the probabilities

1. Probability of obtaining the observed results (two heads) is 0.25
2. Probability of obtaining the results as extreme as the result observed (two tails) is 0.25
3. Probability of obtaining the results that are more extreme as the results observed is 0

p-value for 2 Heads = 0.25 + 0.25 + 0 = 0.5

Even though we got 2 Heads in a row, our coin is no different from a normal one

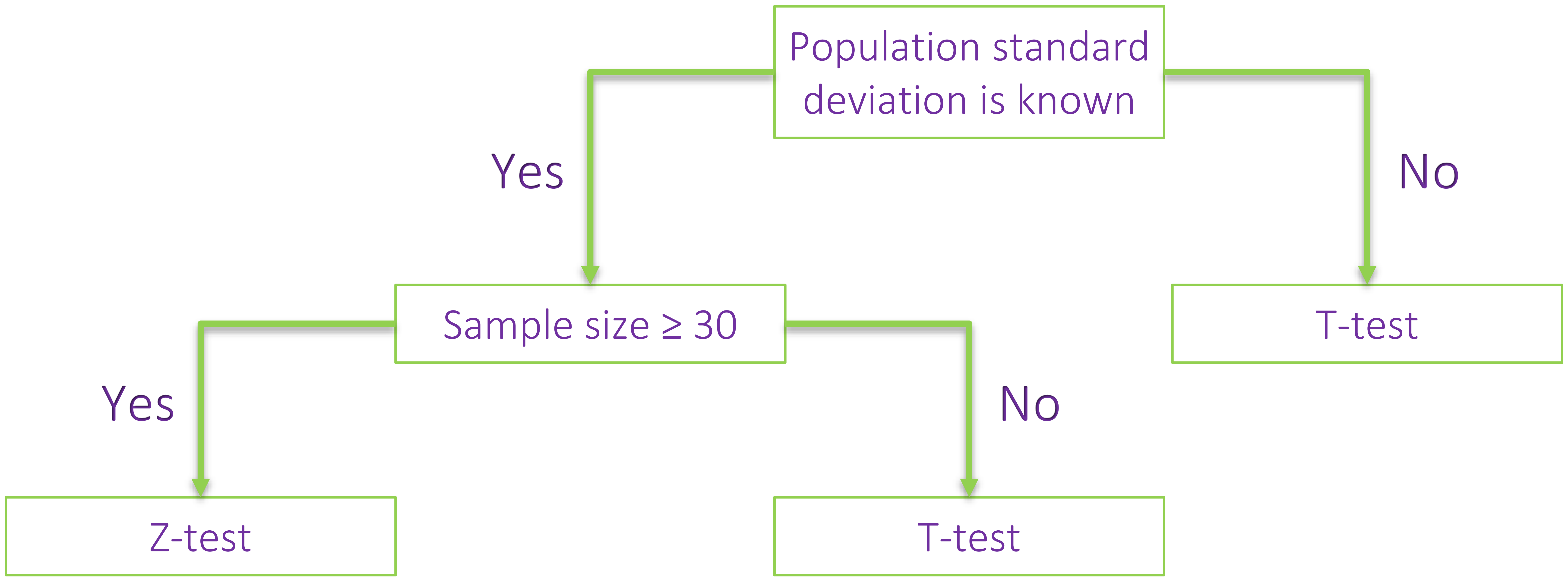
The full example is available through this link

Hypothesis testing: task

Using the previous example, formulate and test the hypothesis for the case of obtaining 4 Heads and 1 Tail.

Answer: p-value = 0.375

Select Appropriate Test



Z-test: tasks

1. An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a P-value in your answer.

Answer: p-value=0.1

2. The average height of females in the freshman class of a certain college has historically been 162.5 centimeters with a standard deviation of 6.9 centimeters. Is there reason to believe that there has been a change in the average height if a random sample of 50 females in the present freshman class has an average height of 165.2 centimeters? Use a P-value in your conclusion. Assume the standard deviation remains the same

Answer: p-value=0.0056

T-test: task

According to a dietary study, high sodium intake may be related to ulcers, stomach cancer, and migraine headaches. The human requirement for salt is only 220 milligrams per day, which is surpassed in most single servings of ready-to-eat cereals. If a random sample of 20 similar servings of a certain cereal has a mean sodium content of 244 milligrams and a standard deviation of 24.5 milligrams, does this suggest at the 0.05 level of significance that the average sodium content for a single serving of such cereal is greater than 220 milligrams? Assume the distribution of sodium contents to be normal.

Answer: yes

Break, 5 min

Coding part

Practice with Python

[Part 1]

[Part 2]

Thank you