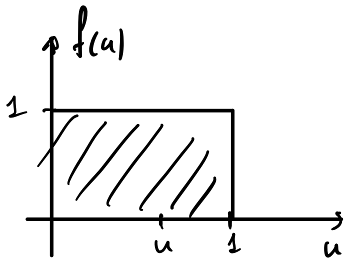
STDS. Week 7. Sampling MCMC.

May, 4 2021

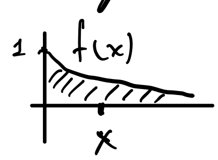
1. Inverse transform sampling (Inverse CDF)

Setup: We have Uniform in $[0,1]$

we want to get samples from Expon-l.(λ)



pdf: $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$



CDF: $F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0 & < 0 \end{cases}$

$$F(x) \triangleq \mathbb{P}(X \leq x) = \mathbb{P}(T(u) \leq x) = \mathbb{P}(u \leq T^{-1}(x)) =$$

$\boxed{\text{Assume } T(u) = X}$

$$= T^{-1}(x) \qquad F(x) = T^{-1}(x)$$

$$\underline{F^{-1}(x) = T(x)}$$

So, we need to inverse the CDF $F(x)$, plug-in $u \sim Unif(0,1)$ inside and the result will be a sample x from $F(x)$

Example. Continuation.

$$F(x) = 1 - e^{-\lambda x}, \quad x \geq 0$$

F^{-1}: $\quad u = 1 - e^{-\lambda x}; \quad 1 - u = e^{-\lambda x}, \quad -\lambda x = \ln(1-u)$

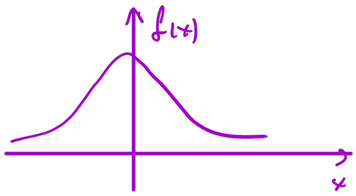
$$x = -\frac{\ln(1-u)}{\lambda}$$

② Accept-Reject Sampling

Inverse CDF is OK, but what if you have no access to it?

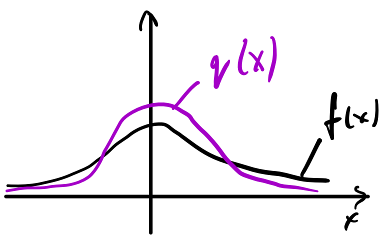
Suppose we have only a density $f(x)$ such that

p.d.f. for (x): $p(x) = \dfrac{f(x)}{\text{const } C}$, $C.\text{const} = \int_{-\infty}^{\infty} f(x)\,dx$



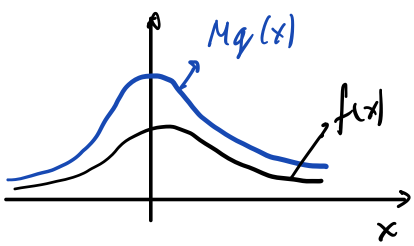
How to sample from $p(x)$?

Choose $q(x)$ that is close to $p(x)$ and easy to sample from



Scale $q(x)$ by a const M

Such that $q(x)$ above $f(x)$ $\forall x$



<u>Method:</u>

a proposal distr.

① Sample x from $q(x)$

② Accept with prob $\dfrac{f(x)}{M\,q(x)}$

Why?: $\mathbb{P}(x \mid A) = p(x)$?

$$\mathbb{P}(A) = \int_{-\infty}^{\infty} q(x)\,\frac{f(x)}{q(x)\cdot M}\,dx = \frac{1}{M}\int_{-\infty}^{+\infty} f(x)\,dx = \frac{C}{M}$$

$$\mathbb{P}(x \mid A) = \frac{P(A \mid x)\,q(x)}{P(A)} = \frac{\frac{f(x)}{Mq(x)}\cdot q(x)}{\frac{C}{M}} = = \frac{f(x)}{C} = p(x)$$

Discussion
indep...