STELLAR STRUCTURE

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June 2020

1 INTRODUCTION

The basics are covered and we are all ready to not only travel to the far stars but dig up their surface to study the real magic. Here, you should stop and contemplate the immense contribution by scientists to understand the processes in the most indirect ways possible. We will deal with stellar structures and processes happening throughout the lifetime of a star and why on the first place it happens.

2 TOPICS TO STUDY

1. Hydrostatic equilibrium, Timescales: dynamical, thermal, nuclear

2. Energy generation, thermonuclear reactions

3. Energy transport; opacity, radiative and convective transport

4. Equations of stellar structure

5. Virial Theorem, Pressure

6. Stellar properties as a function of mass, homology

6. Degeneracy: Chandrasekhar limit

3 Hydrostatic Equilibrium

We are discussing the hydrostatic equilibrium first. Stars are composed of plasma which being a fourth state is considered as a fluid. This whole struc-

ture is held together by gravity which is not opposed by one but three different types of pressure.
Three types of stellar pressure are :

1. Ideal Pressure - Collision b/w gas particles ($P_{ideal} \propto \rho T$)

2. Radiation Pressure - Collision b/w photons and matter ($P_{rad} \propto \rho T^4$)

3. Degeneracy Pressure - Result of resistance of electrons or neutrons against being compressed to a smaller volume

($P_{deg} \propto \rho^{5/3}$ when non-relativistic)
($P_{deg} \propto \rho^{4/3}$ when relativistic)

A star's interior stability is promised by the balancing pressure and gravity. The star is stabilized(i.e.,nuclear reactions are kept under control) by a pressure-temperature thermostat or is self - regulating.

We shall discuss first about a thin shell of material in the sphere. Let the shell be at a distance r in a sphere of M and R having thickness dr. Mass of the shell $dM = 4\pi r^3 \rho dr$. The shell is subject to gravitational force and pressure both acting inwards.

$$F_g = -\frac{GM\rho}{r^2}\,dr \qquad F_p = -\frac{dP}{dr}\,dr \tag{1}$$

We already know $F_p = P(r) - P(r+dr)$ as pressure at r is more than at r+dr. Using F = ma we can calculate acceleration of the shell.

$$(\rho dr)\ddot{r} = -[\frac{GM\rho}{r^2} + \frac{dP}{dr}]dr \tag{2}$$

Therefore, acceleration is

$$\ddot{r} = -[\frac{GM}{r^2} + \frac{dP}{\rho dr}] \tag{3}$$

This is the equation of motion of the shell. But when calculating equation for hydrostatic equilibrium of the star we nullify the acceleration part and equate the pressure applied and the gravitational force.

$dP.A = -GM(r)m/r^2 = -GM(r) \times (\rho A dr)/r^2$

$dP/dr = -GM(r)\rho(r)/r^2$

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \tag{4}$$

Changing in terms of dm we will have Lagrangian form of the equation

$dP/dm = (dP/dr)(dr/dm) = (dP/dr)(dm/dr)^{-1}$

$-(GM\rho)/(r^2) \times (1/4\pi r^2 \rho) = -(GM)/(4\pi r^4)$

The initial one was Euler form. Now, a simple trick and we get to know the central temperature of a star.
For starters, we assume the density to be uniform

$\rho = constant$
$dP/dr = -GM(r)\rho/r^2$
$M(r) = 4\pi r^3 \rho/3$
$dP/dr = -G(4\pi r^3 \rho^2)/3r^2$

$$\int_{P_c}^{0} dP = \int_0^R \frac{-4\pi G\rho^2}{3} r dr \tag{5}$$

$$P_c = \frac{4\pi G\rho^2}{6} R^2 = \frac{G\rho}{2R} \times \frac{4\pi\rho R^3}{3} \tag{6}$$

Therefore,

$$P_c = \frac{GM\rho}{2R} \tag{7}$$

As $P_c = P_{ideal}$

$$P_c = 1.69\rho N_a k T_c = \frac{GM\rho}{2R} \tag{8}$$

$$T_c = \frac{GM}{3.38 N_a k R} \tag{9}$$

Lagrangian form is better in case of stars, the mass parameter is the independent coordinate and others are a function of it. We label each mass shell by the mass m interior to it. Thus for a star of total mass M, the shell m = 0 is the one at the center of the star, the one m = M/2 is at the point that contains half the mass of the star, and the shell m = M is the outermost one.

4 The Virial Theorem and it's Implications

Hydrostatic equilibrium helps in linking gravitational potential energy and internal energy. The virial theorem provides an equation that clarifies the relationship between average over time of the total kinetic energy of distinct particles to that of the total potential of the system. We can also look in the alternative ways. Multiply the equation of hydrostatic equilibrium on both sides by V.

$$\int_0^P VdP = -\int_0^P \frac{GMV}{4\pi r^4}dm = -\int_0^P \frac{GM}{3r}dm \tag{10}$$

Here the gravitational potential will indicate the energy required to assemble the matter from infinity into a star.

$$\Omega = -\int_0^M \frac{GMdm}{r} \tag{11}$$

$$\int_0^{P(R)} VdP = [PV]_0^R - \int_0^{V(R)} PdV \tag{12}$$

as at boundary condition P = 0 and at centre V = 0 therefore the first term at right hand vanishes giving

$$\Omega = -3\int_0^{V(R)} PdV = -3\int_0^{V(R)} \frac{Pdm}{\rho} \tag{13}$$

This is the general form used. Using $P = \rho k_B T/\mu m_H = R\rho T/\mu$ which is applicable for ideal gas and substituting in the derived equation of virial theorem(22) we will get

$$\int_0^M \frac{RTdm}{\mu} = -\frac{\Omega}{3} \tag{14}$$

For a monatomic ideal gas, the internal energy per particle is $(3/2)k_bT$, so the internal energy per unit mass is $u = (3/2)RT/\mu$. We will get

$$\int_0^M \frac{2udm}{3} = -\frac{\Omega}{3} \qquad U = -\frac{\Omega}{2} \tag{15}$$

Therefore the total energy $E = U + \Omega$. Note that, since $\Omega < 0$, this implies that the total energy of a star made of ideal gas is negative, which makes sense given that a star is a gravitationally bound object. As we have only talked about whole radius condition we should also take one example where $R_s < R$. We will get

$$P_sV_s - \int_0^{V_s} \frac{P}{\rho} dm = \frac{\Omega_s}{3} \tag{16}$$

As can be concluded P_s and V_s are for pressure(exerted by enveloping sphere) and volume of shell s.

We know that $U = 3MR\overline{T}/2\mu$ and $\Omega = -\alpha GM^2/R$ which help us to know the average temperature of the star(solve the eqn).

$$\overline{T} = \frac{\alpha\mu GM}{3R^2} \tag{17}$$

The result $E = U + \Omega$ bears a significant resemblance to one that applies to orbits. In an orbit $K = mv^2/2 = GmM/2R$ and potential yielding to $-GmM/R$ which is the same. Therefore it hints at an alternative proof for a system of particles.

$$K = \frac{\sum_i^N m_i({\dot{x}_i}^2 + {\dot{y}_i}^2 + {\dot{z}_i}^2)}{2} \qquad W = -(G/2)\sum_{i,j} \frac{m_im_j}{|\vec{r_i} - \vec{r_j}|} \tag{18}$$

$$I = \sum_i^N m_im_i({x_i}^2 + {y_i}^2 + {z_i}^2) \qquad \frac{\mathrm{d}I}{\mathrm{d}t} = 2\sum_i^N m_i[x_i\ddot{x}_i + y_i\ddot{y}_i + z_i\ddot{z}_i] \tag{19}$$

$$\frac{\mathrm{d}^2I}{\mathrm{d}t^2} = 2\sum_i^N m_i({\dot{x}_i}^2 + {\dot{y}_i}^2 + {\dot{z}_i}^2 + x_i\ddot{x}_i + y_i\ddot{y}_i + z_i\ddot{z}_i) \tag{20}$$

Accordingly, we will use acceleration for solving further.

$$\ddot{\vec{r_i}} = -G\sum_j \frac{m_j(\vec{r_i} - \vec{r_j})}{|\vec{r_i} - \vec{r_j}|^3} \tag{21}$$

Now , let us substitute this value in the following eqn :-

$$\sum m_i(x_i\ddot{x}_i + y_i\ddot{y}_i + z_i\ddot{z}_i) = -G\sum_{i,j} \frac{m_im_j}{|\vec{r}_{ij}|^3}[x_i(x_i - x_j) + y_i(y_i - y_j) + z_i(z_i - z_j)] \tag{22}$$

$$= -G \sum_{i,j} \frac{m_i m_j}{|\overrightarrow{r}_{ij}|^3} [x_i^2 + y_i^2 + z_i^2 - x_i x_j - y_i y_j - z_i z_j] \tag{23}$$

$$= -G \sum_{i,j} \frac{m_i m_j}{|\overrightarrow{r}_{ij}|^3} [x_j^2 + y_j^2 + z_j^2 - x_i x_j - y_i y_j - z_i z_j] \tag{24}$$

$$= -\frac{G}{2} \sum_{i,j} \frac{m_i m_j}{|\overrightarrow{r}_{ij}|^3} [(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2] \tag{25}$$

Taking equation(17) (18), dividing by 2 and subtracting from the other we get perfect squares of r_i and r_j.

$$S = -\frac{G}{2} \sum_{i,j} \frac{m_i m_j}{|\overrightarrow{r}_{ij}|^3} (r_{ij})^2 = W \qquad \frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2K + W \tag{26}$$

5 Equation of State in Stars

$P_{total} = P_I + P_e + P_r$
$P_{total} = P_{gas} + P_r$

P_I = pressure of the ions
P_e = pressure of the electrons
P_r = radiation pressure

We have already discussed ideal pressure before but let us take a detailed recap.
$P_{ideal} = nkT$
$P_{ideal} = \rho/(\mu m_H) \times kT$
Here, m$_H isthemassofhydrogenand\mu$ stands for the average mass of particles.
R = k/m$_H$
$P_{ideal} = (R/\mu)\times\rho$T
We will here look at how to compute μ but a quick bit, hydrogen then helium are the most abundant elements followed by the rest. Therefore same is the case of mass fraction in stars.
μ will depend upon the composition of the gas and the state of ionization in the star.
neutral $\mu = 1$ fully ionized $\mu = 0.5$
For stars we will have to consider both neutrons and electrons for the density

calculation.

$$Hydrogen - Neutron = \frac{X\rho}{m_H} \qquad electron = \frac{X\rho}{m_H} \tag{27}$$

$$Helium - Neutron = \frac{Y\rho}{4m_H} \qquad electron = \frac{2Y\rho}{4m_H} \tag{28}$$

$$Others - Neutron = \frac{Z\rho}{Am_H} \qquad electron = \frac{A}{2}.\frac{Z\rho}{Am_H} \tag{29}$$

Where,
X hydrogen - mass m_H, one electron
Y helium - mass $4m_H$, two electrons
Z the rest, 'metals', average mass Am_H, approximately (A / 2) electrons per nucleus
Note, that we refer to metals as elements other than H and He in astronomy usually - so even C would be considered as one. Assuming A>>1.

$$n = \frac{\rho}{m_H}[2X + \frac{3}{4}Y + \frac{Z}{2}] \tag{30}$$

$$[2X + \frac{3}{4}Y + \frac{Z}{2}] = \mu^{-1} \tag{31}$$

5.0.1 Radiation Pressure

Let us take a look at radiation pressure.

Electromagnetic radiation exerts a minuscule pressure on everything, known as radiation pressure. In everyday situation the pressure is negligible, but in star it can become important given the vast quantities of photons emitted. Inside a star blackbody conditions exist making the radiation pressure proportional to the fourth power of temperature. As the temperatures rises internally the radiation's pressure increase thereby dominating other ones. In the most massive stars, the mass of the star is supported against gravity primarily by radiation pressure, a situation which ultimately sets the upper limit for how massive a star can become.

$$P_r = \frac{4\sigma T^4}{3c} \tag{32}$$

where, c = speed of the photons(contribution towards pressure) and σ is Stefan-Boltzman constant

6 Timescales

Stellar evolution is described by three time dependant equations, in which each deals with a different type of change and accordingly the timescale on which they change.

$$\tau = \frac{\phi}{\dot{\phi}} \tag{33}$$

Here, ϕ is a parameter which differs accordingly to the different types of timescale.

6.1 Dynamical Timescale

This describes the dynamical or structural change in the star. Therefore, the parameter we will be choosing is that of R, as that is the characteristic dimension of a spherically symmetric star. $\dot{\phi}$ is the v_{esc} (derivative of R in a gravitational field).

$$\tau_{dyn} = \frac{R}{v_{esc}} = \frac{R}{\sqrt{2GM/R}} = \sqrt{\frac{R^3}{2GM}} \tag{34}$$

Using average density $\overline{\rho}$,

$$\tau_{dyn} = \sqrt{\frac{R^3}{2GM}} \approx \frac{1}{\sqrt{G\overline{\rho}}} \tag{35}$$

Using solar scale,

$$\tau_{dyn} \approx 1000\sqrt{\frac{R^3}{R_{sun}}} \times \sqrt{\frac{M_{sun}}{M}} s \tag{36}$$

6.2 Thermal Timescale

The thermal timescale comes into play when thermal processes affect the internal energy of the star. Therefore the parameter in this case ϕ = U . Therefore, the derivative of this will be rate at which energy is radiated by the star which is $\dot{\phi}$ = L

$$\tau_{th} = \frac{U}{L} \approx \frac{GM^2}{LR} \tag{37}$$

Using solar scale,

$$\tau_{nuc} \approx 10^{15} \frac{M^2}{M_{sun}^2} \times \frac{L_{sun}}{L} \times \frac{R_{sun}}{R} s \tag{38}$$

6.3 Nuclear Timescale

The third equation deals with change in nuclear composition of the star. Now, nuclear composition goes through a lot of different processes which we will discuss later. The change in quantity by nuclear processes is a small fraction of the rest mass given by Einstein. Therefore, as $E = mc^2$ the parameter will be $\phi = \epsilon mc^2$. Again rate of change is nuclear luminosity(total rate at which nuclear reactions in the star release energy) $L_{nuc} = L$

$$\tau_{nuc} \approx \frac{\epsilon mc^2}{L_{nuc}} \approx \frac{\epsilon mc^2}{L} \tag{39}$$

Using solar scale,

$$\tau_{nuc} \approx \epsilon \times 4.5 \times 10^{20} \frac{M}{M_{sun}} \times \frac{L_{sun}}{L} s \tag{40}$$

Therefore,

$$\tau_{dyn} \ll \tau_{th} \ll \tau_{nuc} \tag{41}$$

7 Equations of stellar structure

Evolution of a star is a quasi - static process therefore the equilibrium is maintained (composition changes slowly). Let us discuss the equations of stellar equations both in Euler and Lagrangian form.

$$\frac{dP}{dr} = -\frac{\rho GM}{r^2} \qquad \frac{dP}{dm} = -\frac{GM}{4\pi r^4} \tag{42}$$

$$\frac{dm}{dr} = 4\pi r^4 \qquad \frac{dr}{dm} = \frac{1}{4\pi r^4} \tag{43}$$

$$\frac{dT}{dr} = \frac{-3}{4ac}\frac{k\rho}{T^3}\frac{F}{4\pi r^2} \qquad \frac{dT}{dr} = \frac{-3}{4ac}\frac{k}{T^3}\frac{F}{(4\pi r^2)^2} \tag{44}$$

$$\frac{dF}{dr} = 4\pi r^2 \rho q \qquad \frac{dF}{dm} = q \tag{45}$$

First represents hydrostatic equilibrium equation.
Second represents continuity equation.
Third represents radiative transfer equation.
Fourth represents thermal equilibrium equation.

7.1 Polytropes

It turns out that many of the observed properties of stars, except their lifetimes and radii, reflect chiefly the need to be in hydrostatic and thermal equilibrium, and not the energy source.
Polytropes is a concept used by physicists to understand the stellar structure of the stars. Polytropes are self-gravitating gaseous spheres in which the pressure depends on density in the form of $P = K\rho^{(n+1)/n}$. The n is known as polytropic index. Basically it is a solution of Lane - Emden equation which we will derive right now.

Let us begin with HSE equation
$dP/dr = -\rho GM/r^2$
$dP/dr \times r^2/\rho = -GM$

Differentiating agin wrt r

$$\frac{d}{dr}[\frac{r^2}{\rho}\frac{dP}{dr}] = -G\frac{dM}{dr} = -4\pi G r^2\rho \tag{46}$$

$$\frac{1}{r^2}\frac{d}{dr}[\frac{r^2}{\rho}\frac{dP}{dr}] = -4\pi G\rho \tag{47}$$

The Lane - Emden equation is a dimensionless form of Poisson's equation for the gravitational potential of a Newtonian self-gravitating, spherically symmetric, polytropic fluid(in this case of a star). So let us make this dimensionless. Central density is ρ_c and then $\rho = \rho_c\theta^n$.

$P = k(\rho_c\theta^n)^\lambda$

$$\frac{1}{r^2}\frac{d}{dr}[\frac{r^2}{\rho}\frac{dP}{dr}] = -4\pi G\rho \tag{48}$$

$$\frac{1}{r^2}\frac{d}{dr}[\frac{r^2}{\rho_c(\theta)^n}\frac{dk(\rho_c)^\lambda(\theta)^{n\lambda}}{dr}] = -4\pi G\rho_c(\theta)^n \tag{49}$$

Taking the constants out and replacing the λ with (n+1)/n we get

$$[\frac{P_c(n+1)}{4\pi G{\rho_c}^2}]\frac{1}{r^2}\frac{d}{dr}[\frac{r^2 d\theta}{dr}] = -\theta^n \tag{50}$$

As the quantity in brackets has a dimension of $length^2$, therefore we assign a value α^2 to the quantity in the brackets. We will also assign $r = \alpha\xi$ which leads to

$$\frac{1}{\xi^2}\frac{d}{d\xi}[\xi_2\frac{d\Theta}{d\xi}] = -\Theta^n \tag{51}$$

And tada we got the Lane - Emden equation. The two central boundary conditions are now $\Theta = 1$ and $d\Theta/d\xi = 0$ at $\xi = 0$

Putting n = 0 in the equation (43) we get $(d/d\xi)(\xi^2 d\Theta/d\xi) = -\xi^2$
Integrating wrt ξ we get

$$\Theta = -\frac{\xi^2}{6} - \frac{C}{\xi} + D \tag{52}$$

Applying the boundary conditions that $\Theta = 1$ and $d\Theta/d\xi = 0$ at $\xi = 0$, we immediately see that we must choose $C = 0$ and $D = 1$; so the solution is therefore $\Theta = 1 - (\xi^2/6)$
It is obvious the function is monotonically decreasing when $\xi > 0$ and becomes 0 at $\xi = \xi_1 = \sqrt{6}$

The radius at which ξ reaches zero is r = R, therefore $R = \alpha\xi$

Computing the mass of the star we get,

$$M = \int_0^R 4\pi r^2 \rho dr = 4\pi\rho_c \int_0^{\xi_1} \xi^2\Theta^n dr = -4\pi\rho_c \int_0^{\xi_1} \frac{d}{d\xi}[\xi^2\frac{d\Theta}{d\xi}] \tag{53}$$

We get

$$M = -4\pi\alpha^3\rho_c \times \xi_1^2[\frac{d\Theta}{d\xi}] \tag{54}$$

We also use this to calculate how much dense is the star centrally to mean density. $D_n = \rho_c/\overline{\rho} = \rho_c 4\pi R^3/3M$ Use the already mentioned derivations we get

$$D_n = -\frac{3d\Theta}{\xi_1 d\xi}^{-1} at\xi = \xi_1 \tag{55}$$

A second useful relationship is between mass and radius. We start by expressing the central density $_c$ in terms of the other constants and length scale α.

$$\rho_c = \frac{(n+1)K^{n/n-1}}{4\pi G\alpha^2} \tag{56}$$

Next we substitute this into the equation for the mass:

$$M = -4\pi\alpha^3 \times \xi_1^2 \frac{d\Theta}{d\xi} \times \frac{(n+1)K}{4\pi G^2}^{n/n-1} \tag{57}$$

This is at $\xi = \xi_1$ and doing substitution we relate mass and radius.
$M \approx R^{(n-3)/(n-1)}$

A third useful expression is for the central pressure. From the equation of state : $P_c = K\rho_c^{n+1/n}$

$$P_c = \frac{(4\pi G)^{1/n}}{n+1} \times \frac{GM}{-\xi^2(d\Theta/d\xi)}^{(n-1)/n} \times \frac{R}{\xi_1}^{3-n} \times \rho_c^{(n+1)/n} \tag{58}$$

$$P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3} \tag{59}$$

8 Physics of stellar fluids

In this section we will discuss deeply about different forms of pressure and how they affect the total pressure and the physics related to each pressure. The physics of stellar interior deals with :

Properties of gaseous systems
Radiation and it's effects
Interaction b/w gas and radiation

The equation of state is a relation b/w the exerted pressure, present temperature and density. For understanding the stars, let us assume that the gas is ideal as we have done before and here we will prove how the assumption is correct. For a gas to be ideal the particles have to be non-interactive and obeying the gas laws exactly. At the prevailing temperatures it is expected that coulomb interactions will take place. But here we will see that the kinetic energy of the particles is large when compared with coulombic forces.

$d = (Am_H/\overline{\rho})^{1/3} = (4\pi A m_H/3M)^{1/3} R$

Using the d(mean distance) in equation $E_c = ((Ze)(e)/4\pi\epsilon d)$ we get
$E_c/k_B\overline{T} << 1$ and substituting $\overline{T}$ with $\overline{T} = (\alpha/3) \times (\mu/R) \times (GM/R)$ (which we took care of while discussing virial theorem) and replace μ with A

Therefore,

$$\frac{E_c}{k_B\overline{T}} = \frac{1}{4\pi\epsilon}\frac{Z^2e^2}{G(Am_H)^{4/3}M^{2/3}} \approx 0.011\frac{Z^2}{A^{4/3}}\frac{M}{M_{sun}}^{-2/3} \tag{60}$$

An important point ,i.e., for Z = 1, A = 1 and for Z higher A = 2Z the ratio is well below 1 but the problem arises when M $\approx$M$_{sun}$ $\times$ 10^{-3} or lower the ratio is approx 1. Now the stars may not be of this mass but planets are like Jupiter. We can conclude from here that stars are composed of gases that show ideal features and as the planet's mass reduces the closer it is to being of a solid structure.

8.1 Kinetic Theory Model of Pressure

The pressure is the force exerted by a gas on the surface. Here we will use for pressure - momentum per unit time per unit area. The reason there is a momentum transfer is that particles in the gas are moving around at random, and that some of them will strike the walls of the vessel, bounce off, and transfer momentum. We can compute the pressure by computing this momentum transfer.

When a particle with an angle θ, momentum p encounters and bounces elastically off an immobile surface. The momentum transferred is $2pcos\theta$. Talking in collective sense, a beam of particles will encounter the surface with momentum p, number density n and velocity ν. It will give a surface strike rate at area dA - $n\nu cos\theta dA$. Total rate at which the beam transfers the momentum is :

$$\frac{d^2p_{surf}}{dtdA} = 2n\nu pcos^2\theta \tag{61}$$

Now, as the beam will travel in all direction we will take a strip in θ and $\theta + d\theta$ relative to the normal, where the number density will be denoted as $d(n)\theta/d\theta$. The solid angle of the strip taken will be $2\pi sin\theta d\theta$ and 4π for the total.
$d(n)\theta/d\theta = (1/2)nsin\theta$
Therefore the momentum transferred =

$$\frac{d^2p_{surf}}{dtdA} = np\nu\int_0^{\pi/2} cos^2\theta sin\theta d\theta = \frac{n\nu p}{3} \tag{62}$$

We let $dn(p)/dp$ be the number of particles with momenta between p and $p + dp$. The total pressure will be:

$$P = \int_0^\infty \frac{1}{3} \frac{d(n)p}{dp} p\nu dp \tag{63}$$

9 Nuclear Reactions in Stars

The nuclear process happening in the stars is the actual energy generator. Whenever we talk about nuclear processes we have to talk about the binding energy. It is known that the mass is not conserved in nuclear reactions - the difference depending on the binding energy of the interacting elements. Let us take a nuclear reaction:

$$I(A_i, Z_i) + J(A_j, Z_j) \rightleftharpoons K(A_k, Z_k) + L(A_l, Z_l) \tag{64}$$

Now, the energy Q released will be, $Q = (M_i + M_j - M_k - M_l)c^2$.
Taking it in a different form:
$Q = ((M_i - A_i m_H) + (M_j - A_j m_H) - (M_k - A_k m_H) - (M_l - A_l m_H))c^2 + (A_i m_H + A_j m_H - A_k m_H - A_l m_H)c^2$
Due to law of conservation of baryons in a reaction(baryons here deal with protons and neutrons) the 2nd part of right hand equation will become zero. Therefore yielding, $\Delta M(I) = (M_i - A_i m_H)c^2$, the change is known as mass excess(positive or negative).