MID2.

NRCM

1. Given SD, $\sigma = 11$ (entire state)

District A :- Sample size $(n_1) = 100$, Mean $(\bar{x}_1) = 210$

District B :- Sample size $(n_2) = 150$, Mean $(\bar{x}_2) = 220$

Now,

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Since population σ is known & samples is large (>30),

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$Z = \frac{210 - 220}{11\sqrt{\frac{1}{100} + \frac{1}{150}}} = \frac{-10}{1.420} = -7.04$$

at 5% level of significance,

$|Z_\alpha| = 1.96$

$|-7.04| > 1.96$ we reject H_0

$\therefore$ There is a significant difference between the mean yields

2. Given, Sample of men $n_1 = 400$

" " women $n_2 = 600$

men in favor $x_1 = 200$

women " " $x_2 = 325$

$P_1 = \frac{200}{400} = 0.5$ $\qquad$ $P_2 = \frac{325}{600} = 0.5417$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{200 + 325}{400 + 600} = 0.525$$

$\hat{q} = 1 - 0.525 = 0.475$

$$Z = \frac{P_1 - P_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.5 - 0.5417}{\sqrt{0.525 \times 0.475\left(\frac{1}{400} + \frac{1}{600}\right)}} = -1.295$$

at 5% significance,

$|Z_\alpha| = 1.96$

$|Z| < |Z_\alpha|$

$\therefore$ we accept H_0.

$\therefore$ There is no significant difference in the opinions of men & women.

3. Given,

university A : $n_1 = 400$, $\bar{x}_1 = 55$, $\cancel{\sigma_1 = 10}$ $S_1 = 10$

" B : $n_2 = 100$, $\bar{x}_2 = 57$, $\cancel{\sigma_2 = 15}$ $S_2 = 15$

Since σ is unknown but samples are large,

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{55 - 57}{\sqrt{\frac{10^2}{400} + \frac{15^2}{100}}} = \frac{-2}{\sqrt{2.5}} = -1.265$$

at 5% significance,

$|Z_\alpha| = 1.96$

$|Z| < |Z_\alpha|$

$\therefore$ The difference is not significant

$H_0 : \mu_1 = \mu_2$ (not significant)

$H_1 : \mu_2 \neq \mu_1$

4. Given,

$n = 1000$, rice eaters $(x) = 540$

Sample proportion $P_s = \frac{x}{n} = \frac{540}{1000} = 0.54$

$\alpha = 1\% = 0.01$

$H_0 : P = 0.5$

$H_1 : P \neq 0.5$

$Q = 1 - P = 0.5$

$$Z = \frac{P_s - P}{\sqrt{\frac{P \cdot Q}{1000}}} = \frac{0.54 - 05}{\sqrt{\frac{0.5 \times 0.5}{1000}}} = \frac{0.04}{\sqrt{0.00025}} = 2.53$$

$Z_\alpha = 2.58$

$Z_\alpha > |Z|$

$\therefore H_0$ is accepted

$\therefore$ we can assume both rice and wheat eaters are equally populated.

5. Given, $f(Z) = U + iV$

$f(Z) = (x^2 - y^2) + i(Kxy)$ is analytic

$U = x^2 - y^2$ $\qquad$ $V = Kxy$

$\frac{\partial U}{\partial x} = 2x$ $\qquad$ $\frac{\partial V}{\partial x} = Ky$

$\frac{\partial U}{\partial y} = -2y$ $\qquad$ $\frac{\partial V}{\partial y} = Kx$

For func^n to be analytic. must satisfy,

$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}$ & $\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$

5. Given, $f(z) = e^{x}(\cos ky + i \sin ky)$ is analytic

it is of form, $f(z) = u + iv$

$u = e^{x} \cos ky$ $\qquad$ $v = e^{x} \sin ky$

$\frac{\partial u}{\partial x} = e^{x} \cos ky$ $\qquad$ $\frac{\partial v}{\partial x} = e^{x} \sin ky$

$\frac{\partial u}{\partial y} = -e^{x} k \sin ky$ $\qquad$ $\frac{\partial v}{\partial y} = e^{x} k \cos ky$

The func^n to is analytic, then

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \bigg| \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$e^{x} \cos ky = e^{x} k \cos ky \qquad \bigg| \qquad -e^{x} k \sin ky = -e^{x} \sin ky$$

$$k = 1 \qquad \bigg| \qquad \boxed{k = 1}$$

$$\boxed{\therefore k = 1}$$

6. let, $f(z) = u(x,y) + iv(x,y)$

$\text{Re}\, f(z) = u(x,y)$, $\text{Im}\, f(z) = v(x,y)$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u^2 = 2|f'(z)|^2 \text{ to prove this}$$

Now we need to Diff w.r.t 'x' with u^2

$$\frac{\partial}{\partial x}(u^2) = 2u \frac{\partial u}{\partial x}$$

$$\frac{\partial^2}{\partial x^2}(u^2) = \frac{\partial}{\partial x}\left(2u \frac{\partial u}{\partial x}\right)$$

$$= 2\left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial^2 u}{\partial x^2} 2u$$

$$\frac{\partial^2}{\partial x^2}(u^2) = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + u \frac{\partial^2 u}{\partial x^2}\right] \text{ --- } (1)$$

$$f(z) = u + iv$$

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$$

Squared modulus,

$$|f'(z)|^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2$$

For analytic funcn,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x} \Rightarrow \frac{\partial v}{\partial x} = \frac{-\partial u}{\partial y}$$

$$|f'(z)|^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(-\frac{\partial u}{\partial y}\right)^2$$

$$2|f'(z)|^2 = 2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \quad \text{—} (2)$$

from (1) & (2)

$$\therefore \boxed{\frac{\partial^2}{\partial x^2}(u)^2 = 2|f'(z)|^2}$$

Given, $f(z) = \cos z$

7. Given, $f(z) = \cos z$

$z = x+iy$

$f(z) = \cos z = \cos(x+iy) = \cos x \cos iy - \sin x \sin iy$

$f(z) = \cos x \cosh y - i \sin x \sinh y$

$u = \cos x \cosh y$ $\qquad$ $v = -\sin x \sinh y$

$\frac{\partial u}{\partial x} = -\sin x \cosh y$ $\qquad$ $\frac{\partial v}{\partial x} = -\cos x \sinh y$

$\frac{\partial u}{\partial y} = \cos x \sinh y$ $\qquad$ $\frac{\partial v}{\partial y} = -\sin x \cosh y$

Now Cauchy-reimann,

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ $\qquad$ $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$-\sin x \cosh y = -\sin x \cosh y$ $\qquad$ $\cos x \sinh y = -(-\cos x \sinh y)$

$\therefore f(z)$ is analytic

Now, $f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$

$= -\sin x \cosh y + i(-\cos x \sinh y)$

$= -(\sin x \cosh y + i \cos x \sinh y)$

$\boxed{\sin(x+iy) = \sin x \cos iy + \cos x \sin iy}$

$= -(\sin x \cos iy + \cos x \sin iy)$

$= -\sin(x+iy)$

$\boxed{f'(z) = -\sin z}$

8. let $f(z) = u(x,y) + iv(x,y)$

$|f(z)|^2 = u^2 + v^2$

$f'(z) = u_x + i v_x$

$|f'(z)|^2 = u_x^2 + v_x^2$

Now u^2 diff w.r.t x & y

$\frac{\partial}{\partial x}(u^2) = 2uu_x$ $\qquad$ $\frac{\partial}{\partial y}(u^2) = 2uu_y$

$\frac{\partial^2}{\partial x^2}(u^2) = 2(u_x)^2 + 2uu_{xx}$ — (1) $\qquad$ $\frac{\partial^2}{\partial y^2}(u^2) = 2(u_y)^2 + 2uu_{yy}$ — (2)

(1) + (2)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u^2 = 2(u_x)^2 + 2uu_{xx} + 2(u_y)^2 + 2uu_{yy}$$

$$= 2(u_x^2 + u_y^2) + 2u(u_{xx} + u_{yy})$$

$f(z)$ is analytic, u is harmonic,

$u_{xx} + u_{yy} = 0$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u^2 = 2(u_x^2 + u_y^2) \quad \text{— (1)}$$

$u_x = v_y$

$u_y = -v_x$

Simly $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)v^2 = 2(v_x^2 + v_y^2)$ — (2)

(1) + (2)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 2(u_x^2 + u_y^2 + v_x^2 + v_y^2)$$

$$= 2(u_x^2 + (-v_x)^2 + v_x^2 + (u_x^2))$$

$$= 2(u_x^2 + v_x^2 + v_x^2 + u_x^2)$$

$$= 4(u_x^2 + v_x^2)$$

$$\therefore \boxed{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4(u_x^2 + v_x^2)}$$

9. Given, $u = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \sin y$ $\frac{\partial u}{\partial y} = e^x \cos y$

$\frac{\partial^2 u}{\partial x^2} = e^x \sin y$ $\frac{\partial^2 u}{\partial y^2} = -e^x \sin y$

By, Laplace eq^n, u to be harmonic must satisfy,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$e^x \sin y - e^x \sin y = 0$$

$\therefore$ u is harmonic

Let $f(z) = u + iv$

$$f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$\frac{\partial u}{\partial x}(z,0) = e^z \sin y = 0$ $\frac{\partial u}{\partial y}(z,0) = e^z \cos(0) = e^z$

$$f'(z) = 0 - i e^z$$

$$f'(z) = -i e^z$$

$$f(z) = \int -i e^z dz = -i e^z + c$$

$$f(z) = -i e^{x+iy} = -i e^x . e^{iy} = -i e^x(\cos y + i \sin y)$$

$$f(z) = -i e^x \cos y - i^2 \sin y \, e^x = e^x \sin y - i e^x \cos y$$

$$\therefore V = -e^x \cos y \; \& \; f(z) = -ie^z + c$$

10. Given, $u = \frac{x}{x^2+y^2}$

$\frac{\partial u}{\partial x} = \frac{x^2+y^2-(2x)x}{(x^2+y^2)^2}$ $\frac{\partial u}{\partial y} = \frac{-(2y)y}{(x^2+y^2)^2}$

let $f(z) = u + iv$

by milne thomson

$$f'(z) = \frac{\partial u}{\partial x}(z,0) - i\frac{\partial u}{\partial y}(z,0)$$

$$\frac{\partial u}{\partial x}(z,0) = \frac{z^2+0-2z^2}{(z^2+0)^2} = \frac{-z^2}{z^4} = \frac{-1}{z^2}$$

$$\frac{\partial u}{\partial y}(z,0) = \frac{-0}{(z^2+0)^2} = 0$$

$$f'(z) = -\frac{1}{z^2}$$

$$f(z) = \int -z^{-2}\,dz = -\frac{z^{-2+1}}{-2+1} + c = \frac{1}{z} + c$$

$$\therefore f(z) = \frac{1}{z} + c$$

11. Given, $u = \frac{1}{2}\log(x^2+y^2)$

u to be harmonic must satisfy Laplace eq,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} = \frac{1}{2}\frac{1}{x^2+y^2}2x \qquad \frac{\partial u}{\partial y} = \frac{1}{2}\frac{1}{x^2+y^2}2y$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2)-(2x)x}{(x^2+y^2)^2} \qquad \frac{\partial^2 u}{\partial y^2} = \frac{x^2+y^2-(2y)y}{(x^2+y^2)^2}$$

$$= \frac{y^2-x^2}{(x^2+y^2)^2} \qquad = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2} = 0$$

$\therefore u$ is harmonic

we have

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial y} = \frac{x}{x^2+y^2}$$

$$v = \int \frac{x}{x^2+y^2}\,dy$$

$$= \tan^{-1}\left(\frac{y}{x}\right) + g(x)$$

$$\cancel{\frac{\partial v}{\partial x} = \frac{-y}{x^2+y^2}}$$

$$\cancel{v = \int \frac{-y}{x^2+y^2}\,dx =}$$

$\frac{\partial}{\partial x}\left(\frac{1}{x}\right) = \frac{x^{-1}}{-1} = \frac{-2}{x^2}$

$(1+i)^2 = 1 + i^2 - 2i$

Now, $v = \tan^{-1}\left(\frac{y}{x}\right) + g(x)$

$$\frac{\partial v}{\partial x} = \frac{-y}{x^2+y^2} + g'(x)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial v}{\partial x} = -\frac{y}{x^2+y^2}$$

$\therefore g'(x) = 0$

$g(x) = c$

$\therefore v = \tan^{-1}\left(\frac{y}{x}\right) + c$

12) Given, $f(z) = u + iv$

$u - v = e^x(\cos y - \sin y)$

$\cancel{(u-v)^2 = e^{2x}(\cos^2 y + \sin^2 y - 2\sin y\cos y)}$

$F(z) = (i+1) f(z)$

$= (u-v) + i(u+v)$

$u - v = U_r$ & $u + v = V_r$

Now

$$\frac{\partial U_r}{\partial x} = e^x(\cos y - \sin y) \qquad \frac{\partial U_r}{\partial y} = e^x(-\sin y - \cos y)$$

Now, By milne thomson

$$f'(z) = \frac{\partial U_r}{\partial x}(z,0) - i\frac{\partial U_r}{\partial y}(z,0)$$

$$= e^z - i(e^z(-1))$$

$$f'(z) = e^z(1+i)$$

$$f(z) = e^z(1+i) + c$$

$F(z) = (1+i) f(z)$

$= (1+i)\left[e^z(1+i) + c\right]$

$= (1+i)^2\left[e^z + \frac{c}{1+i}\right]$

$\Rightarrow (1+i) f(z) = \cancel{(1+i)}\left[e^z(1+i) + c\right]$

$$\boxed{\therefore f(z) = e^z + \frac{c}{1+i}}$$

13) Given, $z+1$ on C, $z=0, z=1, z=i, z=i+1$

$$f(z) = z+1$$

$\therefore$ This is polynomial func^n of z.

Polynomials are analytic everywhere in the complex plane.

$\therefore$ No singularities (2)

By Cauchy's theorem,

$$\int_C f(z)\,dz = 0$$

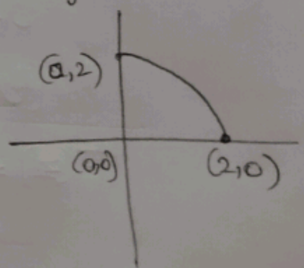
$\therefore$ Since $f(z)$ is analytic everywhere & C is a closed square, the integral is guaranteed to be 0.

14) Given, $\int_C (x-2y)\,dx + (y^2-x^2)\,dy$. C is $x^2+y^2=4$

$(0,0) \rightarrow (2,0)$

By graph $r=2$, $(2,0) \rightarrow (0,2)$

$$x^2+y^2 \le 4$$



By green's theorem,

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$P = x-2y$ $\qquad$ $Q = y^2-x^2$

$\dfrac{\partial P}{\partial y} = -2$ $\qquad$ $\dfrac{\partial Q}{\partial x} = -2x$

$$I = \iint_R \left(-2x-(-2)\right) dx\,dy$$

$$= \iint_R (2-2x)\,dx\,dy$$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$

r : 0 to 2

θ : 0 to $\frac{\pi}{2}$

$$I = \int_0^{\pi/2}\int_0^2 \left(2-2(r\cos\theta)\right) r \, dr \, d\theta$$

$$= \int_0^{\pi/2}\int_0^2 (2r - 2r^2\cos\theta) \, dr \, d\theta$$

$$= \int_0^{\pi/2} \left[r^2 - \frac{2}{3} r^3 \cos\theta \right]_0^2 d\theta$$

$$= \int_0^{\pi/2} \left(4 - \frac{16}{3}\cos\theta\right) d\theta$$

$$= \left[4\theta - \frac{16}{3}\sin\theta\right]_0^{\pi/2} = 2\pi - \frac{16}{3}$$

$$\therefore I = 2\pi - \frac{16}{3}$$

$$\therefore \int_C (x-2y)\,dx + (y^2-x^2)\,dy = 2\pi - \frac{16}{3}$$

15) Given, $\frac{z^2+4}{z-3}$ on C i) $|z|=5$ ii) $|z|=2$

$$z-3=0$$
$$z=3$$

$\therefore$ simple pole at $z=3$

i) $|z|=5$

The pole lies inside the circle $|z|=5$

By cauchy's I.F,

$$\int_C \frac{f(z)}{z-a} = 2\pi i f(a) \quad , \quad f(z) = z^2+4$$
$$f(a) = f(3) = 3^2+4 = 13$$

$$\int_C \frac{z^2+4}{z-3} = 2\pi i \times 13$$
$$= 26\pi i$$

ii) $|z|=2$

The pole is outside the circle $|z|=2$

$$\int_C f(z)\,dz = 0$$

16) Given $\frac{\cos \pi z^2}{(z-1)(z-2)^3}$ on $C: |z|=3$

$(z-1)(z-2)^3=0$ $\quad f(z) = \frac{\cos \pi z^2}{(z-1)(z-2)^3}$

$z=1, z=2$

$\therefore$ a simple pole at $z=1$ (inside)

a pole at $z=2$ of order 3 (inside)

By residue theorem,

$I = 2\pi i \times \sum \text{Res}$

at $z=1$,

$$\text{Res}(f,1) = \underset{z\to 1}{Lt} (z-1) \frac{\cos \pi z^2}{(z-1)(z-2)^3}$$

$$= \underset{z\to 1}{Lt} \frac{\cos \pi z^2}{(z-2)^3} = \frac{-1}{-1} = 1$$

at $z=2$, $n=3$

$$\text{Res}(f,2) = \frac{1}{2!} \underset{z\to 2}{Lt} \frac{d^2}{dz^2} (z-2)^3 \cdot \frac{\cos \pi z^2}{(z-1)(z-2)^3}$$

$$= \frac{1}{2} \underset{z\to 2}{Lt} \frac{d}{dz} \left(\frac{-\sin \pi z^2 \cdot 2z\pi (z-1) - \cos \pi z^2}{(z-1)^2} \right)$$

$$= \frac{1}{2} Lt \frac{d}{dz} \left(\frac{N}{D} \right)$$

$N = -\sin \pi z^2 \cdot 2\pi z (z-1) - \cos \pi z^2$

$N'(z) = -2\pi \left[(2z-1) \sin(\pi z^2) + (z^2 - z) \cos(\pi z^2) \cdot 2\pi z \right] - \left[-\sin(\pi z^2) \cdot 2\pi z \right]$

$N'(2) = -16\pi^2$ $\qquad N(2) = -1$

$D(z) = (z-1)^2$ $\qquad D(2) = 1$

$D'(z) = 2(z-1)$

$D'(2) = 2$

$$\phi = \frac{d}{dz}\left(\frac{N}{D}\right) = \frac{N'D - D'N}{D^2} = \frac{(-16\pi^2) - 2(-1)}{1} = -16\pi^2 + 2$$

$$\text{Res}(f,2) = \frac{1}{2}(-16\pi^2 + 2)$$

$Res(f,2) = 1 - 8\pi^2$

$I = 2\pi i \times [1 + 1 - 8\pi^2]$

$= 2\pi i \times [2 - 8\pi^2]$

$\therefore I = 4\pi i [1 - 4\pi^2]$

17) Given, $f(z) = \dfrac{1}{z^2 - z - 6}$ about $z = -1$

$z^2 - z - 6 = (z-3)(z+2)$

$f(z) = \dfrac{1}{(z-3)(z+2)} = \dfrac{A}{z-3} + \dfrac{B}{z+2}$

$1 = A(z+2) + B(z-3)$

$A = -B \quad , \; 1 = 2A - 3B$

$B = -\dfrac{1}{5}$

$A = \dfrac{1}{5}$

$f(z) = \dfrac{1}{5}\left(\dfrac{1}{z-3} - \dfrac{1}{z+2}\right)$

let $u = z+1 \Rightarrow z = u-1$

$f(z) = \dfrac{1}{5}\left(\dfrac{1}{u-4} - \dfrac{1}{u+1}\right)$

$\dfrac{1}{u-4} = \dfrac{1}{-4\left(1-\frac{u}{4}\right)} \quad w = \dfrac{u}{4}$

$= \dfrac{1}{-4}(1-w)^{-1} = \dfrac{-1}{4}\sum_{n=0}^{\infty}\left(\dfrac{u}{4}\right)^n$

$\dfrac{1}{1+u} = \dfrac{1}{1-(-u)} = \sum_{n=0}^{\infty}(-1)^n u^n$

$f(z) = \left[-\dfrac{1}{4}\sum_{n=0}^{\infty}\left(\dfrac{u}{4}\right)^n + \sum_{n=0}^{\infty}(-1)^n u^n\right]\dfrac{1}{5}$

$\therefore f(z) = \dfrac{-1}{5}\sum_{n=0}^{\infty}\left(\dfrac{1}{4^{n+1}} + (-1)^n\right)(z+1)^n$

18) Given $f(z) = \dfrac{z+3}{z(z^2-z-2)}$

$$z(z^2-z-2) = z(z-2)(z+1)$$

$$f(z) = \frac{z+3}{z(z-2)(z+1)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z+1}$$

$$z+3 = A(z-2)(z+1) + Bz(z+1) + C(z)(z-2)$$

$\boxed{B = \frac{5}{6}}$ $\boxed{A = \frac{-3}{2}}$ $\boxed{C = \frac{2}{3}}$

$$f(z) = \frac{-3}{2z} + \frac{5}{6(z-2)} + \frac{2}{3(z+1)}$$

i) $0 < |z| < 1$

$$\frac{5}{6(z-2)} = \frac{5}{6(-2)\left(1-\frac{z}{2}\right)} = \frac{-5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$$

$$\frac{2}{3(z+1)} = \frac{2}{3(z-(-1))} = \frac{2}{3}\sum_{n=0}^{\infty}(-1)^n(z)^n$$

$$\therefore f(z) = \frac{-3}{2z} - \frac{5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n + \frac{2}{3}\sum_{n=0}^{\infty}(-1)^n(z)^n$$

ii) $1 < |z| < 2$

$$\frac{5}{6(z-2)} = \frac{-5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$$

$$\frac{2}{3(z+1)} = \frac{2}{3z\left(1+\frac{1}{z}\right)} = \frac{2}{3}\sum_{n=0}^{\infty}\left(\frac{(-1)^n}{z^{n+1}}\right)$$

$$\therefore f(z) = \frac{-3}{2z} - \frac{5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n + \frac{2}{3}\sum_{n=0}^{\infty}\frac{(-1)^n}{z^{n+1}}$$

19) Given $f(z) = \frac{\cos(\pi z^2)}{(z-1)(z-2)}$ on c'le $|z| = \frac{3}{2}$

The poles are $z=1$ (inside)

$z=2$ (outside)

By Cauchy I.F,

$$\int_C \frac{f(z)}{z-a} dz = 2\pi i\, f(a)$$

$$f(z) = \frac{\cos \pi z^2}{z-2}, \quad a=1$$

$$f(1) = \frac{-1}{-1} = 1$$

$$\therefore \int_C \frac{\cos(\pi z^2)}{(z-1)(z-2)} dz = 2\pi i \times 1$$

$$= 2\pi i$$

20) Given $\frac{2z-1}{z(z+2)(2z+1)}$ on c'le $|z|=1$

The poles are $z=0$ (inside)

$z=-2$ (outside)

$z=-\frac{1}{2}$ (inside)

By residue theorem,

$$\int_C f(z)\, dz = 2\pi i \times \sum \text{Res}$$

at $z=0$

$$\text{Res}(z=0) = \underset{z\to 0}{Lt}\; z\, \frac{2z-1}{z(z+2)(2z+1)} = \underset{z\to 0}{Lt} \frac{2z-1}{(z+2)(2z+1)}$$

$$= \frac{-1}{2} = -\frac{1}{2}$$

at $z=-\frac{1}{2}$

$$\text{Res}(z=-\tfrac{1}{2}) = \underset{z\to -\frac{1}{2}}{Lt} \left(z+\tfrac{1}{2}\right) \cdot \frac{2z-1}{z(z+2)(2z+1)} = \frac{1}{2} \underset{z\to -\frac{1}{2}}{Lt} \frac{2z-1}{z(z+2)}$$

$$= \frac{1}{2} \frac{-2}{-\frac{3}{4}} = \frac{4}{3}$$

$$= (z-2)(z+1) = \frac{A}{z} + \frac{B}{z-2}$$

$$= A(z-2)(z+1) + Bz(z+1)$$

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$$I = 2\pi i \times \left[\frac{-1}{2} + \frac{4}{3}\right]$$

$$= 2\pi i \times \left[\frac{-3+8}{6}\right]$$

$$\therefore I = \frac{5\pi i}{3}$$