A-3

1. $L\left(e^{3t} - 2e^{-2t} + \sin 2t + \cos 3t + \sinh 3t - 2\cosh 4t + 9\right)$

$= L(e^{3t}) - 2L(e^{-2t}) + L(\sin 2t) + L(\cos 3t) + L(\sinh 3t) - 2L(\cosh 4t) + L(9)$

$= \frac{1}{s-3} - \frac{2}{s+2} + \frac{2}{s^2+4} + \frac{s}{s^2+9} + \frac{3}{s^2-9} - \frac{2s}{s^2-16} + \frac{9}{s}$

2. $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$

$L\left(\frac{e^{-at} - e^{-bt}}{t}\right) = \int_s^\infty \left(\frac{1}{s+a} - \frac{1}{s+b}\right) ds$

$= \left[\log(s+a) - \log(s+b)\right]_s^\infty$

$= \log\left(\frac{1+\frac{a}{s}}{1+\frac{b}{s}}\right)_s^\infty$

$= 0 - \log\left(\frac{s+a}{s+b}\right)$

$= \log\left(\frac{s+a}{s+b}\right)^{-1}$

$= \log\left(\frac{s+b}{s+a}\right)$

3. $L\left[\sin 2t \cos t\right]$

$= \frac{1}{2} L\left[2 \sin 2t \cos t\right]$

$= \frac{1}{2} L\left[\sin 3t + \sin t\right]$

$= \frac{1}{2}\left[\frac{3}{s^2+9} + \frac{1}{s+1}\right]$

4) $L\left(e^{-4t}\int_0^t \frac{\sin 3t}{t}\,dt\right)$

$g(t) = \int_0^t \frac{\sin 3t}{t}\,dt$ $\qquad L\left(\int_0^t \frac{\sin at}{t}\,dt\right) = \tan^{-1}\left(\frac{a}{s}\right)$

$L(g(t)) = \tan^{-1}\left(\frac{3}{s}\right) = G(s)$

$L\left(e^{-at} g(t)\right) = G(s+a)$

$= L\left(e^{-4t} g(t)\right)$

$= \tan^{-1}\left(\frac{3}{s+4}\right)$

---

5) $L^{-1}\left(\frac{s}{(s^2+4)(s^2+9)}\right)$

$\bar{f}(s) = \frac{s}{s^2+4}$ $\qquad \bar{g}(s) = \frac{1}{s^2+9}$

5) By convolution theorem, $L^{-1}\left(\frac{s}{(s^2+4)(s^2+9)}\right)$

$$L^{-1}\left(\frac{s}{(s^2+4)(s^2+9)}\right)$$

$$f(t) = L^{-1}\left(\frac{s}{s^2+4}\right) \qquad g(t) = L^{-1}\left(\frac{1}{s^2+9}\right)$$

$$f(t) = \cos 2t \qquad g(t) = \frac{1}{3}\sin 3t$$

By convolution theorem,

$$f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du$$

$$= \frac{1}{3}\int_0^t \cos 2u \sin 3(t-u)\, du$$

$$2\cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$= \frac{1}{6}\int_0^t \sin(3t-u) - \sin(5u-3t)\, du$$

$$= \frac{1}{6}\left(\left[-\frac{\cos(3t-u)}{-1}\right]_0^t - \left[\frac{-\cos(5u-3t)}{5}\right]_0^t\right)$$

$$= \frac{1}{6}\left(\left[\cos 2t - \cos 3t\right] - \left[\frac{-\cos 2t + \cos 3t}{5}\right]\right)$$

$$= \frac{1}{6}\left(\cos 2t - \cos 3t + \frac{\cos 2t - \cos 3t}{5}\right)$$

$$= \cos 2t - \cos 3t\left[\frac{1}{6} + \frac{1}{30}\right]$$

$$= \cos 2t - \cos 3t\left[\frac{1}{5}\right]$$

$$= \frac{1}{5}(\cos 2t - \cos 3t)$$

6) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t$, $y(0) = y'(0) = 0$

~~$D^2y + 2Dy + 2y = 5\sin t$~~

~~$y(D^2+2D+2) = 5\sin t$~~

$y'' + 2y' + 2y = 5\sin t$

$L[y'' + 2y' + 2y] = 5\,L[\sin t]$

~~$s^2 L(y) + 2$~~

$[s^2 L(y) - s\,y(0) - y'(0)] + 2[sL(y) - y(0)] + 2L(y) = \frac{5}{s^2+1}$

$s^2 L(y) + 2sL(y) + 2L(y) = \frac{5}{s^2+1}$

$L(y)[s^2+2s+2] = \frac{5}{s^2+1}$

$L(y) = \frac{5}{(s^2+1)(s^2+2s+2)}$

$\frac{5}{(s^2+1)(s^2+2s+2)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2s+2}$

$5 = (As+B)(s^2+2s+2) + (Cs+D)(s^2+1)$

$5 = As^3 + 2As^2 + 2As + Bs^2 + 2Bs + 2B + Cs^3 + Cs$
$+ Ds^2 + D$

$C+A=0,\ 2A+B+D=0,\ 2A+2B+C=0,\ 2B+D=5$

$\boxed{A=-C}$ $\boxed{B=\frac{C}{2}}$

$-2C+\frac{C}{2}+D=0$

$-3C+2D=0$

$\boxed{D=\frac{3}{2}C}$

$\frac{2C}{2}+\frac{3}{2}C=5$

$5C=10$

$\boxed{C=2}$

$\boxed{B=1}$ $\boxed{D=3}$

$\boxed{A=-2}$

$$\frac{5}{(s^2+1)(s^2+2s+2)}=\frac{-2s+1}{s^2+1}+\frac{2s+3}{(s^2+2s+2}$$

$$=\frac{-2s}{s^2+1}+\frac{1}{s^2+1}+\frac{2(s+1)}{(s+1)^2+1}+\frac{1}{(s+1)^2+1}$$

$$\boxed{y=-2\cos t+\sin t+2e^{-t}\cos t+e^{-t}\sin t}$$

7) $(D^2+4D+5)y=5$, $y(0)=y'(0)=0$

$y''+4y'+5y=5$

$L(y''+4y'+5y)=L(5)$

$[s^2L(y)-sy(0)-y'(0)]+4[sL(y)-y(0)]+5L(y)=\frac{5}{s}$

$s^2L(y)+4sL(y)+5L(y)=\frac{5}{s}$

$L(y)(s^2+4s+5)=\frac{5}{s}$

$$L(y)=\frac{5}{s(s^2+4s+5)}$$

$$\frac{5}{s(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$$5 = As^2 + 4As + 5A + Bs^2 + Cs$$

$$A+B=0 \,,\quad 4A+C=0 \,,\quad 5A=5$$

$$\boxed{B=-1} \quad \boxed{C=-4} \quad \boxed{A=1}$$

$$\frac{5}{s(s^2+4s+5)} = \frac{1}{s} + \frac{-(s+2)}{s^2+4s+5} - \frac{2}{(s+2)^2+1}$$

$$\boxed{y = 1 - e^{-2t}\cos t - 2e^{-2t}\sin t}$$

8) $L\{f(t)\} = \dfrac{9s^2 - 12s + 15}{(s-1)^3}$, $L\{f(3t)\}$

Given,

$$L[f(t)] = \frac{9s^2 - 12s + 15}{(s-1)^3} = \bar{f}(s)$$

by change of scale property

$$L[f(3t)] = \frac{1}{3}\bar{f}\left(\frac{s}{3}\right)$$

$$= \frac{1}{3}\,\frac{9\left(\frac{s}{3}\right)^2 - 12\left(\frac{s}{3}\right) + 15}{\left(\frac{s}{3} - 1\right)^3}$$

$$= \frac{1}{3} \times 27\left[\frac{s^2 - 4s + 15}{(s-3)^3}\right]$$

$$= 9\left[\frac{s^2 - 4s + 15}{(s-3)^3}\right]$$

9) $L^{-1}\left[\dfrac{s}{(s^2+1)(s^2+9)(s^2+25)}\right]$

$$\frac{s}{(s^2+1)(s^2+9)(s^2+25)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9} + \frac{Es+F}{s^2+25}$$

$$s = (As+B)(s^4 + 34s^2 + 225) + (Cs+D)(s^4 + 26s^2 + 25)$$
$$+ (Es+F)(s^4 + 10s^2 + 9)$$

$$s = As^5 + Bs^4 + A34s^3 + B34s^2 + A225s + B225 +$$
$$Cs^5 + Ds^4 + C26s^3 + D26s^2 + C25s + D25 +$$
$$Es^5 + Fs^4 + E10s^3 + F10s^2 + E9s + F9$$

$\cancel{A+B+C+D+E+F=0}$

$\boxed{A+C+E=0}$, $\boxed{B+D+F=0}$, $\cancel{A+C+E=0}$,

$\cancel{B+D+F=0}$

$\boxed{34A+26C+10E=0}$, $\boxed{34B+26D+F10=0}$

$\boxed{1=225A+25C+9E}$, $\boxed{225B+25D+9F=0}$

$\boxed{A=-(C+E)}$ $\qquad$ $\boxed{B=-(D+F)}$

$-34C-34E+26C+10E=0$ $\qquad$ $-34D-34F+26D+10F=0$

$-8C=24E$ $\qquad$ $-8D=24F$

$\boxed{C=-3E}$ $\qquad$ $\boxed{D=-3F}$

$\boxed{A=2E}$ $\qquad$ $\boxed{B=2F}$

$1=\overset{450}{\cancel{500}}E-75E+9E$ $\qquad$ $450F-75F+9F=0$

$\boxed{E=\frac{1}{384}}$ $\qquad$ $\boxed{F=0}$

$\boxed{A=\frac{1}{192}}$ $\qquad$ $\boxed{B=0}$

$\boxed{C=\frac{-1}{128}}$ $\qquad$ $\boxed{D=0}$

$$\frac{s}{(s^2+1)(s^2+9)(s^2+25)}=\frac{\frac{1}{192}s}{s^2+1}+\frac{\frac{-1}{128}s}{s^2+9}+\frac{\frac{1}{384}s}{s^2+25}$$

on apply L^{-1}

$$=\frac{1}{192}\cos t-\frac{1}{128}\cos 3t+\frac{1}{384}\cos 5t$$

$$\boxed{L^{-1}\left[\frac{s}{(s^2+1)(s^2+9)(s^2+25)}\right]=\frac{\cos t}{192}-\frac{\cos 3t}{128}+\frac{\cos 5t}{384}}$$

10) $\int_0^\infty t^2 e^{-4t} \sin 2t \, dt = \frac{11}{500}$

$$\int_0^\infty t^2 e^{-4t} \sin 2t \, dt$$

$$\boxed{L(f(t)) = \int_0^\infty e^{-st} f(t) dt}$$

$S = 4$, $f(t) = t^2 \sin 2t$

$$L(f(t)) = L\left(\int_0^\infty t^2 \sin 2t \, dt\right)$$

$$= \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right) \qquad \left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}\right]$$

$$= -\frac{d}{ds}\left(\frac{4s}{(s^2+4)^2}\right)$$

$$= -\left(\frac{4(s^2+4)^2 - 2(s^2+4)2s(4s)}{(s^2+4)^4}\right)$$

$(\because s=4)$

$$= \frac{+352\cancel{0}}{1600\cancel{0}} = \frac{11}{500}$$

$$\therefore L\left(\int_0^\infty t^2 e^{-4t} \sin 2t \, dt\right) = \frac{11}{500}$$