A-3

1. $L\left(e^{3t} - 2e^{-2t} + \sin 2t + \cos 3t + \sinh 3t - 2\cosh 4t + 9\right)$

$= L(e^{3t}) - 2L(e^{-2t}) + L(\sin 2t) + L(\cos 3t) + L(\sinh 3t)$
$\quad - 2L(\cosh 4t) + L(9)$

$= \frac{1}{s-3} - \frac{2}{s+2} + \frac{2}{s^2+4} + \frac{s}{s^2+9} + \frac{3}{s^2-9} - \frac{2s}{s^2-16} + \frac{9}{s}$

2. $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$

$L\left(\frac{e^{-at} - e^{-bt}}{t}\right) = \int_s^\infty \left(\frac{1}{s+a} - \frac{1}{s+b}\right) ds$

$= \left[\log(s+a) - \log(s+b)\right]_s^\infty$

$= \log\left(\frac{1+\frac{a}{s}}{1+\frac{b}{s}}\right)_s^\infty$

$= 0 - \log\left(\frac{s+a}{s+b}\right)$

$= \log\left(\frac{s+a}{s+b}\right)^{-1}$

$= \log\left(\frac{s+b}{s+a}\right)$

3. $L[\sin 2t \cos t]$

$= \frac{1}{2}L[2\sin 2t \cos t]$

$= \frac{1}{2}L[\sin 3t + \sin t]$

$= \frac{1}{2}\left[\frac{3}{s^2+9} + \frac{1}{s+1}\right]$

4) $L\left(e^{-4t}\int_0^t \frac{\sin 3t}{t}\,dt\right)$

$g(t) = \int_0^t \frac{\sin 3t}{t}\,dt$ $\qquad L\left(\int_0^t \frac{\sin at}{t}\,dt\right) = \tan^{-1}\left(\frac{a}{s}\right)$

$L(g(t)) = \tan^{-1}\left(\frac{3}{s}\right) = G(s)$

$L\left(e^{-at}\,g(t)\right) = G(s+a)$

$= L\left(e^{-4t}\,g(t)\right)$

$= \tan^{-1}\left(\frac{3}{s+4}\right)$

x

---

5) $L^{-1}\left(\frac{s}{(s^2+4)(s^2+9)}\right)$

$\bar{f}(s) = \frac{s}{s^2+4}$ $\qquad \bar{g}(s) = \frac{1}{s^2+9}$

6) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5\sin t$, $y(0) = y'(0) = 0$

$\cancel{D^2y + 2Dy + 2y = 5\sin t}$

$\cancel{y(D^2+2D+2) = 5\sin t}$

$y'' + 2y' + 2y = 5\sin t$

$L[y'' + 2y' + 2y] = 5\,L[\sin t]$

$\cancel{s^2 L(y) + 2}$

$[s^2 L(y) - s\,y(0) - y'(0)] + 2[sL(y) - y(0)] + 2L(y) = \frac{5}{s^2+1}$

$s^2 L(y) + 2sL(y) + 2L(y) = \frac{5}{s^2+1}$

$L(y)[s^2+2s+2] = \frac{5}{s^2+1}$

$L(y) = \frac{5}{(s^2+1)(s^2+2s+2)}$

$\frac{5}{(s^2+1)(s^2+2s+2)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2s+2}$

$5 = (As+B)(s^2+2s+2) + (Cs+D)(s^2+1)$

$5 = As^3 + \cancel{Cs^3} + 2As^2 + 2As + Bs^2 + 2Bs + 2B + Cs^3 + Cs$
$+ Ds^2 + D$

$C+A=0, \quad 2A+B+D=0, \quad 2A+2B+C=0, \quad 2B+D=5$

$\boxed{A=-C}$ $\qquad$ $\boxed{B=\frac{C}{2}}$

$\frac{2C}{2}+\frac{3}{2}C=5$

$5C=10$

$-2C+\frac{C}{2}+D=0$

$-3C+2D=0$

$\boxed{D=\frac{3}{2}C}$

$\boxed{C=2}$

$\boxed{B=1}$ $\quad$ $\boxed{D=3}$

$\boxed{A=-2}$

$$\frac{5}{(s^2+1)(s^2+2s+2)} = \frac{-2s+1}{s^2+1} + \frac{2s+3}{(s^2+2s+2}$$

$$= \frac{-2s}{s^2+1} + \frac{1}{s^2+1} + \frac{2(s+1)}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}$$

$$\boxed{y = -2\cos t + \sin t + 2e^{-t}\cos t + e^{-t}\sin t}$$

7) $(D^2+4D+5)y=5 \quad , \; y(0)=y'(0)=0$

$y''+4y'+5y=5$

$L(y''+4y'+5y) = L(5)$

$[s^2L(y) - sy(0) - y'(0)] + 4[sL(y) - y(0)] + 5L(y) = \frac{5}{s}$

$s^2L(y)+4sL(y)+5L(y) = \frac{5}{s}$

$L(y)(s^2+4s+5) = \frac{5}{s}$

$L(y) = \frac{5}{s(s^2+4s+5)}$

$$\frac{5}{s(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$$5 = As^2+4As+5A + Bs^2+Cs$$

$$A+B=0, \quad 4A+C=0, \quad 5A=5$$

$$\boxed{B=-1} \quad \boxed{C=-4} \quad \boxed{A=1}$$

$$\frac{5}{s(s^2+4s+5)} = \frac{1}{s} + \frac{-(s+2)}{s^2+4s+5} - \frac{2}{(s+2)^2+1}$$

$$\boxed{y = 1 - e^{-2t}\cos t - 2e^{-2t}\sin t}$$

8) $L\{f(t)\} = \dfrac{9s^2-12s+15}{(s-1)^3}$, $L\{f(3t)\}$

Given,

$$L[f(t)] = \frac{9s^2-12s+15}{(s-1)^3} = \bar{f}(s)$$

by change of scale property

$$L[f(3t)] = \frac{1}{3}\bar{f}\left(\frac{s}{3}\right)$$

$$= \frac{1}{3}\,\frac{9\left(\frac{s}{3}\right)^2-12\left(\frac{s}{3}\right)+15}{\left(\frac{s}{3}-1\right)^3}$$

$$= \frac{1}{3}\times 27\left[\frac{s^2-4s+15}{(s-3)^3}\right]$$

$$= 9\left[\frac{s^2-4s+15}{(s-3)^3}\right]$$

9) $L^{-1}\left[\dfrac{s}{(s^2+1)(s^2+9)(s^2+25)}\right]$

$$\frac{s}{(s^2+1)(s^2+9)(s^2+25)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9} + \frac{Es+F}{s^2+25}$$

$$s = (As+B)(s^4+34s^2+225) + (Cs+D)(s^4+26s^2+25)$$
$$+ (Es+F)(s^4+10s^2+9)$$

$$s = As^5+Bs^4+A34s^3+B34s^2+A225s+B225 +$$
$$Cs^5+Ds^4+C26s^3+D26s^2+C25s+D25 +$$
$$Es^5+Fs^4+E10s^3+F10s^2+E9s+F9$$

$\cancel{A+B+C+D+E+F=0}$

$\boxed{A+C+E=0}$, $\boxed{B+D+F=0}$, $\cancel{A+C+E=0}$,

$\cancel{B+D+F=0}$

$\boxed{34A+26C+10E=0}$, $\boxed{34B+26D+F10=0}$

$\boxed{1=225A+25C+9E}$, $\boxed{225B+25D+9F=0}$

$\boxed{A=-(C+E)}$

$\boxed{B=-(D+F)}$

$-34C-34E+26C+10E=0$

$-8C=24E$

$\boxed{C=-3E}$

$-34D-34F+26D+10F=0$

$-8D=24F$

$\boxed{D=-3F}$

$\boxed{A=2E}$

$\boxed{B=2F}$

$1=\overset{450}{\cancel{500}}E-75E+9E$

$\boxed{E=\frac{1}{384}}$

$\boxed{A=\frac{1}{192}}$

$\boxed{C=\frac{-1}{128}}$

$450\cancel{F}-75F+9F=0$

$\boxed{F=0}$

$\boxed{B=0}$

$\boxed{D=0}$

$$\frac{s}{(s^2+1)(s^2+9)(s^2+25)} = \frac{\frac{1}{192}s}{s^2+1} + \frac{\frac{-1}{128}s}{s^2+9} + \frac{\frac{1}{384}s}{s^2+25}$$

on apply L^{-1}

$$= \frac{1}{192}\cos t - \frac{1}{128}\cos 3t + \frac{1}{384}\cos 5t$$

$$\boxed{L^{-1}\left[\frac{s}{(s^2+1)(s^2+9)(s^2+25)}\right] = \frac{\cos t}{192} - \frac{\cos 3t}{128} + \frac{\cos 5t}{384}}$$

10) $\int_0^\infty t^2 e^{-4t} \sin 2t \, dt = \frac{11}{500}$

$\int_0^\infty t^2 e^{-4t} \sin 2t \, dt$

$$\boxed{L(f(t)) = \int_0^\infty e^{-st} f(t) dt}$$

$s = 4$, $f(t) = t^2 \sin 2t$

$L(f(t)) = L\left(\int_0^\infty t^2 \sin 2t \, dt\right)$

$= \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right)$ $\left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}\right]$

$= -\frac{d}{ds}\left(\frac{4s}{(s^2+4)^2}\right)$

$= -\left(\frac{4(s^2+4)^2 - 2(s^2+4)2s(4s)}{(s^2+4)^4}\right)$

$(\because s = 4)$

$= \frac{+352\cancel{0}}{1600\cancel{0}} = \frac{11}{500}$

$\therefore L\left(\int_0^\infty t^2 e^{-4t} \sin 2t \, dt\right) = \frac{11}{500}$