problem-2

Solve the differential eq^n $2xydy-(x^2-y^2+1)dx=0$.

Soln: Given D.E is $2xydy-(x^2-y^2+1)dx=0$.

Compare with $M(x,y)dx+N(x,y)dy=0$

Here

$M=-(x^2-y^2+1)$

diff part w.r.to 'y'.

$$\frac{\partial M}{\partial y}=-\frac{\partial}{\partial y}[x^2-y^2+1]$$

$$=-\left[\frac{\partial}{\partial y}(x^2)-\frac{\partial}{\partial y}(y^2)+\frac{\partial}{\partial y}(1)\right]$$

$$=-[0-2y+0]$$

$$\frac{\partial M}{\partial y}=2y$$

$N=2xy$.

diff part w.r.to 'x'

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(2xy)$$

$$=2y\frac{\partial}{\partial x}(x)$$

$$\frac{\partial N}{\partial x}=2y(1)$$

$$\frac{\partial N}{\partial x}=2y.$$

$\boxed{\frac{d}{dx}(x^n)=nx^{n-1}}$

$$\Rightarrow \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$\Rightarrow$ eq^n ① is an exact D.E

General soln of eq ① is

$$\int_{y\text{ con}} M(x,y)dx+\int_{\text{free form 'x'}} N(x,y)dy=c$$

$$-\int_{y\text{ const}}(x^2-y^2+1)dx+\int_{\text{free from 'x'}} 2xy\,dy=c.$$

$$-\left[\int x^2dx+(1-y^2)\int dx\right.+0=c$$

$$-\left[\frac{x^3}{3}+(1-y^2)x\right]=c \qquad \therefore \boxed{\int x^n\,dx=\frac{x^{n+1}}{n+1}}$$

$$\rightarrow \boxed{-\frac{x^3}{3}+(y^2-1)x=c.}$$

problem 2:

Solve $(y-xy^2)\,dx-(x+x^2y)\,dy=0$

Solⁿ :- Given,

D.E equⁿ is $(y-xy^2)\,dx-(x+x^2y)\,dy=0$ — (i)

Compare with $Mdx + Ndy = 0$

Here.

$M = (y - xy^2)$

diff part w.r.t 'y'

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(y - xy^2)$$
$$= 1 - 2xy.$$

$N = -(x + x^2y)$

diff partially w.r.t 'x'

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x + x^2y).$$
$$= (-x - x^2y).$$
$$= -1 - 2xy$$

$$\Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eq ① is non exact & it is of the form

$y f(x,y)\,dx + x\,g(xy)\,dy = 0.$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{(y - xy^2)x - (-x - x^2y)y}$$

$$= \frac{1}{xy - \cancel{x^2y^2} + xy + \cancel{x^2y^2}}$$

$$= \frac{1}{2xy}.$$

Multiply eq ① with $I.F. = \frac{1}{2xy}$ on B.S.

$$\frac{1}{2xy}(y - xy^2)dx - (x + x^2y)dy = 0$$

$$\left(\frac{y}{2xy} - \frac{xy^2}{2xy}\right)dx - \left(\frac{x}{2xy} + \frac{x^2y}{2xy}\right)dy = 0.$$

Here.

$$M = y.$$

$$M_1 = \left(\frac{1}{2x} - \frac{y}{2}\right)$$

diff parti w. r. to 'y'

$$\frac{\partial M_1}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{2x} - \frac{y}{2}\right)$$

$$= \frac{\partial}{\partial y}\left(\frac{1}{2x}\right) - \frac{\partial}{\partial y}\left(\frac{y}{2}\right)$$

$$\frac{\partial M_1}{\partial y} = -\frac{1}{2}.$$

$$N_1 = -\left(\frac{1}{2y} + \frac{x}{2}\right)$$

diff parti w. r. to 'x'.

$$\frac{\partial N_1}{\partial x} = \frac{\partial}{\partial x}-\left(\frac{1}{2y} + \frac{x}{2}\right).$$

$$= \frac{\partial}{\partial x}\left(\frac{-1}{2y}\right) + \frac{\partial}{\partial x}\left(\frac{x}{2}\right).$$

$$\frac{\partial N_1}{\partial x} = -\frac{1}{2}.$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}.$$

$\Rightarrow$ eq^n ② is an Exact D.E.

General sol^n of eq^n ② is.

$$\int M_1(x,y)\,dx + \int N_1(x,y)\,dy = c.$$

'y' constant $\qquad$ free from 'x'.

$$\int\left(\frac{1}{2x} - \frac{y}{2}\right)dx + \int -\left(\frac{1}{2x} + \frac{x}{2}\right)dy = c.$$

'y' constant $\qquad$ free from 'x'.

$$\frac{1}{2}\log x - \frac{y}{2}\int dx - \frac{1}{2}\int \frac{1}{y}\,dy + 0 = c.$$

$$\frac{1}{2}\log x - \frac{xy}{2} - \frac{1}{2}\log y = c \;//$$

Problem 3:

Solve the $x^2y\,dx - (x^3+y^3)\,dy = 0$

Sol^n: Given D.E is $x^2y\,dx - (x^3+y^3)dy = 0$ —— ①

compare with $M(x,y)dx + N(x,y)dy = 0$.

Here

$M = x^2y$

diff part w.r.to 'y'

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^2y).$$

$$= x^2 \frac{\partial}{\partial y}(y).$$

$$\frac{\partial M}{\partial y} = x^2(1).$$

$N = -(x^3+y^3)$.

diff par w.r.to 'x'

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-x^3-y^3)$$

$$= -\frac{\partial}{\partial x}(x^3) - y^3\frac{\partial}{\partial x}(1)$$

$$\frac{\partial N}{\partial x} = -[3x^2] - y^3(0)$$

$$\Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

eq ① non-exact D.E and

it is homogeneous D.E.

Then

$$I.F = \frac{1}{Mx+Ny} = \frac{1}{x^2y(x)+(-x^3-y^3)y}$$

$$I.F = \frac{1}{\cancel{x^3y}-\cancel{x^3y}-y^4} = -\frac{1}{y^4}$$

Mul eq ① with $-\frac{1}{y^4}$ on B.S.

$$\left[\frac{x^2y}{-y^4}\right]dx - \left[\frac{(x^3+y^3)}{-y^4}\right]dy = 0$$

$$\left(-\frac{x^2}{y^3}\right)dx + \left(\frac{x^3+y^3}{y^4}\right)dy = 0 \text{ —— ②}$$

compare with $M_1 dx + N_1 dy = 0$.

$M_1 = \frac{-x^2}{y^3}$

'y'

$\frac{\partial M_1}{\partial y} = -x^2 \frac{\partial}{\partial y}\left(\frac{1}{y^3}\right)$.

$\frac{\partial M_1}{\partial y} = -x^2\left[-3y^{-4}\right]$

$\frac{\partial M_1}{\partial y} = \frac{3x^2}{y^4}$

$N_1 = \frac{x^3}{y^4} + \frac{1}{y}$.

'x'.

$\frac{\partial N_1}{x} = \frac{1}{y^2}\frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial x}\left(\frac{1}{y}\right)$

$\frac{\partial N_1}{\partial x} = \frac{1}{y^4}(3x^2) + 0$.

$\frac{\partial N_1}{\partial x} = \frac{3x^2}{y^4}$

$\because \boxed{d(y^{-3}) = -3y^{-3-1}}$

$\Rightarrow \frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$.

$\because \boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$

$\Rightarrow$ eq (2) is an exact D.E.

General soln of eq (2) is $\int M_1 dx + \int N_1 dy = c$.

$$\int\left(\frac{-x^2}{y^3}\right)dx + \int\left(\frac{x^3}{y^4} + \frac{1}{y}\right)dy = c.$$

y'constant. free from 'x'

$$-\frac{1}{y^3}\int x^2 dx + 0 + \int \frac{1}{y}\, dy = c.$$

$$\Rightarrow -\frac{1}{y^3}\left(\frac{x^3}{3}\right) + \log y = c.$$

Problem 3 :- Solve. $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$

Solⁿ :- Given D. Eqn is $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$

$$M = 3x^2y^4 + 2xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(3x^2y^4 + 2xy\right)$$

$$= 3x^2\,4y^3 + 2x$$

$$= 12x^2y^3 + 2x.$$

$$N = 2x^3y^3 - x^2.$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(2x^3y^3 - x^2\right)$$

$$= 6x^2y^3 - 2x.$$

$$\cancel{= 2\left(3x^2y^3 + 2x\right)}$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial z}$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

eqn (1) is non exact D.Eqn

$$\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x} = 12x^2y^3 + 2x - 6y^3x^2 + 2x$$
$$= 6x^2y^3 + 4x$$

$$-\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-(6x^2y^3 + 4x)}{3x^2y^4 + 2xy}$$

$$= \frac{-2\cancel{(3x^2y^3 + 2x)}}{y\cancel{(3x^2y^3 + 2x)}}$$

$$= -\frac{2}{y} = f(y).$$

$$I.F = e^{\int f(y)dy} = e^{\int -\frac{2}{y}dy} = e^{-2\int \frac{1}{y}dy}$$
$$= e^{-2\log y}$$
$$= e^{\log y^{-2}}$$

$$I.F = y^{-2} = \frac{1}{y^2}$$

Multiply I.F with eqn (1) on b.s.

$$\frac{1}{y^2}(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$$

$$\left(3x^2y^2 + \frac{2x}{y}\right)dx + \left(2x^3y - \frac{x^2}{y^2}\right)dy = 0 \quad -(2)$$

$$M_1 = 3x^2y^2 + \frac{2x}{y}$$

$$\frac{\partial M_1}{\partial y} = 3x^2\frac{\partial}{\partial y}(x^2) + 2x\left(\frac{1}{y}\right)$$
$$= 6x^2y + 2x\left(-\frac{1}{y^2}\right)$$
$$= 6x^2y - \frac{2x}{y^2}.$$

$$N_1 = 2x^3y - \frac{x^2}{y^2}.$$

$$\frac{\partial N_1}{\partial x} = 2y\left(\frac{\partial}{\partial x}(x^3)\right) - \frac{1}{y^2}\frac{\partial}{\partial x}(x^2)$$
$$= 6x^2\cancel{x}y - \frac{2x}{y^2}$$
$$= 6x^2y - \frac{2x}{y^2}.$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}.$$

eqn (2) is exact D. eqn. and General solⁿ of eqⁿ (2) is

$$\int M_1 dx + \int N_1 dy = c.$$

'y' const free from 'x'.

$$\int \left(3x^2 y^2 + \frac{2x}{y}\right) dx + \int \left(2x^3 y - \frac{x^2}{y^2}\right) dy = c$$

$$3y^2 \int x^2 dx + \frac{2}{y} \int x\, dx + 0 + 0 = c.$$

$$\cancel{3} y^2 \frac{x^3}{\cancel{3}} + \frac{\cancel{2}}{y} \frac{x^2}{\cancel{2}} = c$$

$$x^3 y^2 + \frac{x^2}{y} = c \;//$$

Problem 4 :

Problem 5:

$\Rightarrow$ solve $x\dfrac{dy}{dx} + y = x^2y^6$.

Sol: Given D.E $x\dfrac{dy}{dx} + y = x^2y^6$.

divide with 'x' on B.S.

$$\frac{dy}{dx} + \frac{y}{x} = xy^6$$

divide y^6 on B.S

$$y^{-6}\frac{dy}{dx} + y^{-5}/x = x.$$

put $y^{-5} = t$

$$-5y^{-6}\frac{dy}{dx} = \frac{dt}{dx}$$

$$y^{-6}\frac{dy}{dx} = -\frac{1}{5}\frac{dt}{dx}$$

$$-\frac{1}{5}\frac{dt}{dx} + \frac{y^{-5}}{x} = x \quad \text{--- (1)}$$

Multiply with -5

$$\frac{dt}{dx} - 5\,y^{-5}/x = -5x.$$

$$\Rightarrow \frac{dt}{dx} = -\frac{5t}{x} = -5x.$$

Let

$P \Rightarrow -5/x$, $Q = -5x$.

$$\text{I.F} \gg e^{\int P dx} = e^{-\int 5/x\, dx} \Rightarrow e^{-5\log x}$$

$$\text{I.F} = e^{-5\log x} = \frac{x}{5} \Rightarrow x^{-5}$$

General soln of eq 1

$$t\,(I.F) = \int Q\,(I.F)\,dx + c.$$

$$t\,(x^{-5}) = \int -5x\,(x^{-5})\,dx + c$$

$$t\,(x^{-5}) = -5\int x^{-4}\,dx + c.$$

$$t\,(x^{-5}) = +5\,\frac{x^{-3}}{3} + c.$$

$$y^{-5}x^{-5} = \frac{5x^{-3}}{3} + c.$$

$$\frac{1}{y^5x^5} = \frac{5}{3}x^{-3} + c_{//}.$$

Problem 6:

Problem 5:

solve $(x^3+3xy^2)dx+(y^3+3x^2y)dy=0$

Sol^n: Given D.E is $(x^3+3xy^2)dx+(y^3+3x^2y)dy=0$.

compare with $M(x,y)dx+N(x,y)dy=0$.

Here.

$M=(x^3+3xy^2)$

diff. par. w.r.to 'y'

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}[x^3+3xy^2]$$

$$=\frac{\partial}{\partial y}(x^3)+3x\frac{\partial}{\partial y}(y^2)$$

$$\frac{\partial M}{\partial y}=0+3x(2y)$$

$N=(y^3+3x^2y)$

diff. p. w.r.to 'x'.

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}[y^3+3x^2y].$$

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}(y^3)+3y\frac{\partial}{\partial x}(x^2)$$

$$\frac{\partial N}{\partial x}=0+3y(2x).$$

$$\Rightarrow \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$\Rightarrow$ eq (1) is an exact D.E

General sol^n is $\underset{\text{'y' constant}}{\int M(x,y)\,dx}+\underset{\text{free from 'x'}}{\int N(x,y)\,dy}=c.$

$$\underset{\text{y 'const}}{\int (x^3+3xy^2)\,dx}+\underset{\text{free from 'x'}}{\int (y^3+3x^2y)\,dy}=c$$

$$\int x^3\,dx + 3y^2\int x'\,dx + \int y^3\,dy + 0 = c.$$

$$\frac{x^4}{4} + 3y^2\left[\frac{x^2}{2}\right] + \left[\frac{y^4}{4}\right] = c.$$

$$\therefore \boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$t = 67.8$ min.

(5) If the temperature of Air is $20^\circ c$ and the temperature of the body drops from $100^\circ c - 80^\circ c$. in 10 min what will be its temperature after 20min. when will be the temperature $40^\circ c$.

Soln:- By, Newton law of cooling

the temp of body $= 100^\circ c$.

at $t = 0$.

Air temperature $\theta_0 = 20^\circ c$.

$$\frac{\partial \theta}{\partial t} \propto (\theta - \theta_0)$$

$$\theta = \theta_0 + ce^{-kt} \quad \ldots\ldots \underline{1}.$$

eqn 1.

$$\Rightarrow 100 = 20 + ce^{-k(0)}.$$

$\Rightarrow 100 - 20 = c$

$\boxed{c = 80}$

after 10min the temperature cool down to 80.

by eqn 1.

$\theta = \theta_0 + ce^{-kt}$

$t = 10$ min.

$\theta_0 = 20$

$\theta_\theta = 80$

$\Rightarrow 80 = 20 + ce^{-k(10)}$ $\Rightarrow 80 - 20 = 80e^{-k(10)}$

$\frac{6\cancel{0}}{8\cancel{0}} = e^{-k(10)}$

$-10k = \ln(6/8)$

$k = -\frac{1}{10} \ln(3/4)$

$\boxed{k = 0.02}$

$\Rightarrow$ what will be the temp at 20 min.

$\theta = ?$

$\theta_0 = 20$

$t = 20$

$k = 0.02$.

eqn ① $\Rightarrow \theta = \theta_0 + ce^{-kt}$.

$\theta = 20 + 80e^{-(0.02)(20)}$

$\theta = 20 + 80e^{-(00.4)}$

$\theta = 20 + 80$.

$\boxed{\theta = 65.6}$

then time required for $\theta = 40$.

$\theta = \theta_0 + ce^{-kt}$

$40 = 20 + 80 \cdot e^{-(0.028)t}$

$\frac{2\cancel{0}}{4\cancel{0}} \cdot e^{-(0.028)}$

$$\ln\left(\frac{2}{8}\right) = e^{-(0.028)t}$$

$$\ln\left(\frac{1}{4}\right) = e^{-(0.028)t}$$

$$-1.38 = -(0.028)t$$

$$t = \frac{1.38}{0.028}$$

$$\boxed{t = 49.5}$$

6/3/25

Problem 1:

If 30 percentage of active substances disappears in 10 days, how long will it takes 90 percentage of it disappear.

Soln:

By the Natural law of decay we have $\frac{dy}{dx} = -kx$

and its soln $x = ce^{-kt}$ — (1)

The x_0 be a initial amount of the radio active Substances.

$t = 0, x = x_0$.

$eq^n ① \Rightarrow x_0 = ce^{-k(0)}$

$$\boxed{c = x_0}$$

30% of radioactive substance will dissappear in 10 days.

i.e. when $t = 10$ days, $x = x_0\left(1 - \frac{30}{100}\right)$

$$x = \frac{70}{100} x_0.$$

$$eq^n ① \Rightarrow x_0\left[\frac{7\cancel{0}}{10\cancel{0}}\right] = x_0 e^{-k(10)}$$

$$e^{-10k} \Rightarrow \frac{7}{10}.$$

$$-10k = \ln\left[\frac{7}{10}\right]$$

$$k = -\frac{1}{10}\ln\left[\frac{7}{10}\right]$$

$$\boxed{k = 0.035}$$

In how many days it will dissappear 90%.

i.e, when $t = ?$, $x = x_0\left(1 - \frac{90}{100}\right) \Rightarrow x_0\left(\frac{1}{10}\right)$,

$$eq^n ① \Rightarrow x_0\left(\frac{1}{10}\right) = x_0 e^{-kt}$$

$$e^{-kt} = \frac{1}{10}.$$

$$-kt = \ln\left[\frac{1}{10}\right]$$

$$t = -\frac{1}{k}\ln\left[\frac{1}{10}\right]$$

$$t = \frac{-1}{0.035}\ln\left[\frac{1}{10}\right]$$

$$\boxed{t = 65.7 \simeq 66 \text{ days}}$$

7/03/25

Solve $p^2 + 2py\cot x = y^2$.

Soln:- Given,

$$p^2 + 2py\cot x = y^2.$$

$$p^2 + (2y\cot x)p - y^2 = 0.$$

It is quadratic eqn in 'p'

here, $a = 1$, $b = 2y\cot x$, $c = -y^2$.

$$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$P = \frac{-(2y\cot x) \pm \sqrt{(2y\cot x)^2 - 4(1)(-y^2)}}{2(1)}$$

$$P = \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}.$$

$$P = \frac{-2y\cot x \pm \sqrt{4y^2(1+\cot^2 x)}}{2}.$$

$$\boxed{\operatorname{cosec}^2 x - \cot^2 x = 1}$$

$$P = \frac{-2y\cot x \pm 2y\operatorname{cosec} x}{2}.$$

$$P = -y\cot x \pm y\operatorname{cosec} x.$$

$$P = -y\cot x \mp y\operatorname{cosec} x.$$

$1 + \cos\theta = 2\cos^2 \theta/2.$

$\sin\theta = 2\sin\theta/2 \cos\theta/2.$

$$P = -y\left[\frac{\cos x}{\sin x} + \frac{1}{\sin x}\right]$$

$$P = -y\left[\frac{1+\cos x}{\sin x}\right]$$

$$P = -y\left[\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right] = -y\cot x/2.$$

$$P = -y\cot\frac{x}{2}.$$

but $p = \dfrac{dy}{dx}$

$$\frac{dy}{dx} = -y\cot\frac{x}{2}$$

$$\cancel{P = -y\cot x - y\,\mathrm{cosec}\,x}$$

$$\frac{dy}{y} = -\cot\frac{x}{2}\,dx.$$

Integrate on B.S.

$$\int\frac{dy}{y} = -\int\cot\frac{x}{2}\,dx + \log c_1$$

$$\log y = -\left[\frac{\log|\sin x/2|}{1/2}\right] + \log c_1$$

$$\log y = -2\log\left|\sin\frac{x}{2}\right| + \log c_1$$

$$\log y = \log\left[\sin\frac{x}{2}\right]^{-2} + \log c_1$$

$$\log y = \log\left[c_1 \cdot \frac{1}{\sin^{-2}\frac{x}{2}}\right]$$

$$\log y = \log\left[c_1 \cdot \mathrm{cosec}^2\frac{x}{2}\right].$$

$$\boxed{y = c_1\,\mathrm{cosec}^2\frac{x}{2}}$$

$$P = -y\cot x + y\,\mathrm{cosec}\,x.$$

$$P = -y\left[\frac{\cos x}{\sin x} - \frac{1}{\sin x}\right]$$

$$P = -y\left[\frac{\cos x - 1}{\sin x}\right] \qquad 1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

$$\sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}.$$

$$P = y\left[\frac{2\sin^2 x/2}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right] = y\tan\frac{x}{2}.$$

$$P = y\tan x/2.$$

$$\frac{dy}{dx} = y\tan\frac{x}{2} \qquad \text{but } p = \frac{dy}{dx}.$$

$$\frac{dy}{y} = \tan\frac{x}{2}\cdot dx.$$

Integrate on B.S

$$\int\frac{dy}{y} = \int\tan\frac{x}{2}\,dx + \log c_2.$$

$$\log y = \frac{\log\left|\sec\frac{x}{2}\right|}{1/2} + \log c_2.$$

$$\log y = \log\left|\sec^2\frac{x}{2}\right| + \log c_2.$$

$$\log y = \log\left[c_2\cdot\sec^2\frac{x}{2}\right]$$

$$\boxed{y = c_2\cdot\sec^2\frac{x}{2}.}$$

$\therefore$ The soln of given D.E is $\left(y - c_1\operatorname{cosec}^2\frac{x}{2}\right)\left(y - c_2\sec^2\frac{x}{2}\right) = 0$ //.

$$\frac{x}{y} = \int 2y \, dy + c$$

$$\frac{x}{y} = \frac{2y^2}{2} + c.$$

$$\frac{x}{y} = y^2 + c$$ //

problem 5:

Solve $x \log x \frac{dy}{dx} + y = 2 \log x$.

Soln:- Given D.E $x \log x \frac{dy}{dx} + y = 2 \log x$ — (1)

$$x \log x \frac{dy}{dx} = 2 \log x - y.$$

$$\frac{dy}{dx} = \frac{2 \log x - y}{x \log x}$$

$$\frac{dy}{dx} = \frac{2\log x}{x\log x} - \frac{y}{x\log x}$$

$$\frac{dy}{dx} + \left(\frac{1}{x\log x}\right)y = \frac{2\log x}{x\log x} = \frac{2}{x}.$$

$$P = \left(\frac{1}{x\log x}\right), \quad Q = \left(\frac{2}{x}\right)$$

$$I.F = e^{\int P dx} \Rightarrow e^{\int \frac{1}{x\log x} dx} = e.$$

$$\log x = t.$$

$$\frac{1}{x} dx = dt$$

$$e^{\int \frac{1}{t} dt}$$

$$= e^{\log_e t} \Rightarrow \log x = I.F.$$

General solⁿ.

$$y(I.F) = \int Q\,(I.F)\,dx + c$$

$$y(\log x) = \int \left(\frac{2}{x}\right) \log x \cdot dx + c.$$

$$\log x = t$$

$$\frac{1}{x} dx = dt.$$

$$y(t) = \int 2t\,dt + c$$

$$y(t) = \frac{\cancel{2}t^2}{\cancel{2}} + c$$

$$y\log x = \log x^2 + c$$