Mid-2
U-3

① $L^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\}$, By convolution

$f(s)=\frac{1}{s}$, $g(s)=\frac{1}{s^2+2s+2}$

$L^{-1}(f(s))=1 = f(t)$, $L^{-1}(g(s))=e^{-t}\sin t = g(t)$

$$L^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\}=\int_0^t 1.\, e^{-(t-u)}\sin(t-u)\,du$$

let $t-u=x$ $x=t$ to 0

$-du=dx$

$$=-\int_t^0 e^{-x}\sin x\,dx$$

$$=\int_0^t e^{-x}\sin x\,dx$$

$$\boxed{\int e^{ax}\sin bx\,dx=\frac{e^{ax}(a\sin bx-b\cos ax)}{a^2+b^2}}$$

$$=\left[\frac{e^{-x}(-1.\sin x-\cos(-1)x)}{1+1}\right]_0^t$$

$$=\left[\frac{e^{-x}(-\sin x-\cos x)}{2}\right]_0^t$$

$$=\frac{1}{2}\left[\left(e^{-t}(-\sin t-\cos t)\right)-\left(e^{0}(-\sin 0-\cos 0)\right)\right]$$

$$=\frac{1}{2}\left[1-e^{-t}(\sin t+\cos t)\right]$$

$$\therefore \boxed{L^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\}=\frac{1-e^{-t}(\sin t+\cos t)}{2}}$$

② $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$

$$L^{-1}\left(\frac{1}{s^2+a^2}\right)=\frac{1}{a}\sin at=f(t)$$

$$L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)=\int_0^t \frac{1}{a}\sin at\,dt$$

$$=\frac{1}{a}\int_0^t \sin at\,dt$$

$$=\frac{1}{a}\left[-\frac{\cos at}{a}\right]_0^t$$

$$=\frac{1}{a}\left[\frac{-\cos at-(-\cos 0)}{a}\right]$$

$$=\frac{1}{a^2}\left[1-\cos at\right]$$

$$\therefore \boxed{L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)=\frac{1-\cos at}{a^2}}$$

③ $L\left(e^{-4t}\int_0^t \frac{\sin 3t}{t}\,dt\right)$

$$L\left(\frac{\sin 3t}{t}\right)=\int_s^\infty \frac{3}{s^2+9}\,ds$$

$$=\left[\frac{-(2s)3}{(s^2+9)^2}\right]_s^\infty$$

$$=\left[\frac{-6s}{s^4+18s^2+81}\right]_s^\infty$$

$$=\left[\frac{-6s}{s^4\left(1+\frac{18}{s^2}+\frac{81}{s^4}\right)}\right]_s^\infty$$

$$=\frac{6s}{(s^2+9)^2}$$

③ $L\left(e^{-4t}\int_0^t \frac{\sin 3t}{t}\,dt\right)$

$L\left(\int_0^t \frac{\sin 3t}{t}\right) = \frac{\tan^{-1}\left(\frac{s}{3}\right)}{s} = f(s)$

$\left[\because L\left(\int_0^t \frac{\sin at}{t}\,dt\right) = \frac{\tan^{-1}\left(\frac{s}{a}\right)}{s}\right]$

$L\left(e^{-4t} f(s)\right) = L(s+a)$

$= L(s+4)$

$= \frac{\tan^{-1}(s+4/3)}{s+4}$

$$\boxed{\therefore L\left(e^{-4t}\int_0^t \frac{\sin 3t}{t}\,dt\right) = \frac{\tan^{-1}\left(\frac{s+4}{3}\right)}{s+4}}$$

④ $(D^2+4D+4)y = e^t,\ y(0)=y'(0)=0$

$y''+4y'+4y = e^{t} = \frac{1}{s-1}$

$s^2L(y) - sy(0) - y'(0) + 4(sL(y) - y(0))$

$+4L(y) = \frac{1}{s-1}$

$L(y)\left[s^2+4s+4\right] = \frac{1}{s-1}$

$L(y) = \frac{1}{(s-1)(s^2+4s+4)}$

$y = L^{-1}\left(\frac{1}{(s-1)(s^2+4s+4)}\right)$

$\frac{1}{(s-1)(s+2)^2} = \frac{A}{s-1} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$

$1 = As^2+4As+4A + B(s^2+3s+2) + C(s-1)$

$\boxed{1 = 4A+2B-C}$, $4A+3B+C=0$

$\boxed{A+B=0}$

$\boxed{A=-B}$

$-B+C=0$

$\boxed{C=B}$

$1 = -2B-C$

$1 = -9B$

$\boxed{B=-\frac{1}{9}}$, $\boxed{A=\frac{1}{9}}$, $\boxed{C=-\frac{7}{9}}$

$y = \frac{1}{9}e^{t} - \frac{1}{9}e^{-2t} - \frac{7}{9}$

$1 = A(s+2)^2 + B(s^2+s-2) + C(s-1)$

$s=1$, $\boxed{A=\frac{1}{9}}$

$s=-2$, $\boxed{C=\frac{-1}{3}}$

$1 = 4A-2B-C$

$1 = \frac{4}{9} - 2B + \frac{1}{3}$

$9 = 4-18B+3$

$\frac{2}{-18} = B$, $\boxed{B=-\frac{1}{9}}$

$$\boxed{y = \frac{1}{9}e^{t} - \frac{1}{9}e^{-2t} - \frac{1}{3}te^{-2t}}$$

$\left(\because L\left(\frac{1}{(s+a)^2}\right) = te^{-at}\right)$

⑤ $L^{-1}($

$\frac{s-2}{s^2+5s\ldots}$

$s-2=$

$s=-2,$

$s=3,$

⑥ $\frac{d^2}{dt^2}$

y''

$s^2L(y$

$L(y)$

$L(y)$

$y=L$

$\frac{5}{(s^2+1)(}$

$^2+3S+2)+C(S-1)$

$-3B + C = 0$

$-7B + C = 0$

$\boxed{C = 7B}$

$\boxed{C = -\frac{7}{9}}$

$)+C(S-1)$

e^{-2t}

⑤ $L^{-1}\left(\frac{S-2}{S^2+5S+6}\right)$

$$\frac{S-2}{S^2+5S+6} = \frac{A}{S+2} + \frac{B}{S+3}$$

$$S-2 = A(S+3) + B(S+2)$$

$S=-2$, $\cancel{A=5}$ $\boxed{A=-4}$

$S=-3$, $\cancel{B=1}$ $\boxed{B=5}$

$$\boxed{L^{-1}\left(\frac{S-2}{S^2+5S+6}\right) = -4e^{-2t} + 5e^{-3t}}$$

⑥ $\frac{d^2}{dt^2}y + 2\frac{dy}{dt} + 2y = 5\sin t,\ y(0)=y'(0)=0$

$$y'' + 2y' + 2y = \frac{5}{S^2+1}$$

$$S^2L(y) - Sy(0) - y'(0) + 2(SL(y) - y(0)) + 2L(y) = \frac{5}{S^2+1}$$

$$L(y)\left[S^2+2S+2\right] = \frac{5}{S^2+1}$$

$$L(y) = \frac{5}{(S^2+1)(S^2+2S+2)}$$

$$y = L^{-1}\left(\frac{5}{(S^2+1)(S^2+2S+2)}\right)$$

$$\frac{5}{(S^2+1)(S^2+2S+2)} = \frac{AS}{S^2+1} + \frac{BS+C}{S^2+2S+2}$$

$$5 = AS((S+1)^2+1) + (BS+C)(S^2+1)$$

$$5 = AS^3 + 2AS^2 + 2AS + BS^3 + CS^2 + BS + C$$

$\boxed{C=5}$, $A+B=0$

$2A + C = 0$ $\boxed{B = \frac{5}{2}}$

$\boxed{A = -\frac{5}{2}}$

$\frac{5}{2}S + 5$

$$y = -\frac{5}{2}\cos t + \frac{5}{2}\left[\cancel{\cos}\, e^{-t}\cos t + e^{-t}\sin t\right]$$

$$\therefore \boxed{y = -\frac{5}{2}\cos t + \frac{5}{2}e^{-t}(\cos t + \sin t)}$$

⑦ $L^{-1}\left\{\frac{S^2}{(S^2+4)(S^2+9)}\right\}$, By convolution

$f(S) = \frac{S}{S^2+4}$, $g(S) = \frac{S}{(S^2+9)}$

$f(t) = \cos 2t$, $g(t) = \cos 3t$

$$L^{-1}\left\{\frac{S^2}{(S^2+4)(S^2+9)}\right\} = \int_0^t \cos 2u \cos 3(t-u)\,du$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{1}{2}\int_0^t (\cos(5t-3u) + \cos(3u-t))\,du$$

$$= \frac{1}{2}\left[\frac{\sin(5t-3u)}{-3} + \frac{\sin(3u-t)}{3}\right]_0^t$$

$$= \frac{-1}{2}\left[\frac{\sin 5t}{-3} - \frac{\sin t}{3}\right]$$

$$= \frac{1}{6}(\sin 5t + \sin t)$$

(7) $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$

$f(s)=\frac{s}{s^2+4}$ $\quad g(s)=\frac{s}{s^2+9}$

$f(t)=\cos 2t$ $\quad g(t)=\cos 3t$

$L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}=\int_0^t \cos 2u \cos 3(t-u)\,du$

$2\cos A\cos B=\cos(A+B)+\cos(A-B)$

$=\frac{1}{2}\int_0^t \left(\cos(3t-u)+\cos(5u-3t)\right)du$

$=\frac{1}{2}\left[\frac{\sin 3t-u}{-1}+\frac{\sin 5u-3t}{5}\right]_0^t$

$=\frac{1}{2}\left[\left[\frac{\sin 2t}{-1}-\left(\frac{\sin 3t}{-1}\right)\right]+\left[\frac{\sin 2t}{5}+\frac{\sin 3t}{5}\right]\right]$

$=\frac{1}{2}\left[\left(-1+\frac{1}{5}\right)\sin 2t+\left(1+\frac{1}{5}\right)\sin 3t\right]$

$=\frac{-2}{5}\sin 2t+\frac{3}{5}\sin 3t$

$$\boxed{\therefore L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}=\frac{3}{5}\sin 3t-\frac{2}{5}\sin 2t}$$

(8) $(D^2+4D+5)y=5,\ y(0)=y'(0)=0$

$y''+4y'+5y=5$

$s^2L(y)-sy(0)-y'(0)+4(sL(y)-y(0))$
$+5L(y)=\frac{5}{s}$

$L(y)\left[s^2+4s+5\right]=\frac{5}{s}$

$L(y)=\frac{5}{s(s^2+4s+5)}$

$y=L^{-1}\left(\frac{5}{s(s^2+4s+5)}\right)$

$\frac{5}{s(s^2+4s+5)}=\frac{A}{s}+\frac{Bs+C}{s^2+4s+5}$

$5=As^2+4sA+5A+Bs^2+Cs$

$\boxed{A=1}$, $4A+C=0$

$\boxed{C=-4}$

$A+B=0$

$\boxed{B=-1}$

$\cancel{y=1-\frac{s+2}{}}$

$$\boxed{\therefore y=1-\left[e^{-2t}\cos t+2e^{-2t}\sin t\right]}$$