UNIT – II

① Solve $(D^2+1)y = \operatorname{cosec} x$ by the method of variation of parameters.

② Solve $D^2(D^2+4)y = 320(x^3+2x^2+e^x)$

③ Solve $(D^2+5D+6)y = \sin 4x \sin x$

④ Solve $(D-2)^2 y = 8(e^{2x}+\sin 2x+x^2)$

⑤ Solve $(D^2-4D+4)y = x^2\sin x + e^{2x}+3$

⑥ Solve $(D+2)(D-1)^2 y = e^{-2x} + 2\sinh x$

⑦ Solve $(D^2-2D+2)y = e^x \tan x$ by the method of variation of parameters.

⑧ Solve $(D^2+3D+2)y = xe^x\sin x$.

⑨ Solve $(D^2+4)y = e^x + \sin 2x + \cos 2x$.

⑩ Form the differential equations of LCR circuit.

UNIT – II

Linear differential Equations of Second order and Higher order

Def:– An eqn of the form

$$\frac{d^n y}{dx^n} + P_1(x)\cdot\frac{d^{n-1}(y)}{dx^{n-1}} + P_2(x)\cdot\frac{d^{n-2}(y)}{dx^{n-2}} + \cdots + P_n(x)y = Q(x)$$

where $P_1(x), P_2(x), P_3(x) \ldots P_n(x)$ and $Q(x)$ are all continuous and real valued functions of x is called a linear differential equation of order n.

Linear differential Equation with constant coefficients:–

The general linear differential eqn of order n is of the form

$$\frac{d^n y}{dx^n} + a_1\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_n y = f(x)$$

where $a_1, a_2, a_3 \ldots a_n$ are real constant

This eqn can also be written in operator form as

$$D^n y + D^{n-1}y\cdot a_1 + a_2 D^{n-2} y + \cdots + a_n y = f(x)$$

$$[D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n]\, y = f(x) \longrightarrow (1)$$

The Solution of eq (1) consists of two parts.

(1) Complementary function

(2) Particular Integral

i.e $\boxed{y = C.F + P.I}$. or $\boxed{y = y_c + y_p}$

where C.F is a complementary func.

P.I is a particular Integral

To find complementary function :—

We have to form the auxiliary eqn which is obtained by putting

$D = m$ and $f(x) = 0$ ∴ the auxiliary eqn ① is

$(m^n + a_1 m^{n-1} + a_2 m^{n-2} + \dots + a_n) = 0 \longrightarrow ②$

eq ② is an ordinary algebraic eqn in m of degree n

By solving this equation we get n roots (or values) for m.

Say $m_1, m_2, m_3 \dots m_n$

Roots of A.E, $f(m) = 0$	C.F (complementary function)
① m_1, m_2, m_3, m_4 - ie all roots are real and distinct $m_1 \neq m_2 \neq m_3 \dots$	① $c_1 e^{m_1 x} + c_2 e^{m_2 x} + \dots + c_n e^{m_n x}$
② $m_1, m_2, m_3 \dots m_n$ (i.e Two roots are real and equal and remaining root are real and different $m_1 = m_2, m_3 \neq m_4 \neq m_5$	② $(c_1 + c_2 x)e^{m_1 x} + c_3 e^{m_3 x} + c_4 e^{m_4 x} + \dots + c_n e^{m_n x}$

3) $m_1, m_1, m_1, m_4 \ldots m_n$ (3) $(c_1+c_2x+c_3x^2)e^{m_1x} + c_4e^{m_4x} + \ldots + c_ne^{m_nx}$

(i.e Three roots are real and equal and remaining roots are real and different

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(4) The roots of A.E are complex root say $\alpha+i\beta$ and $\alpha-i\beta$ and the remaining roots are real and different (4) $e^{\alpha x}[c_1\cos\beta x + c_2\sin\beta x] + c_3e^{m_3x} + \ldots + c_ne^{m_nx}$

(5) A pair of conjugate complex roots $\alpha \pm i\beta$ are repeated twice and the remaining roots are real and different (5) $e^{\alpha x}[(c_1+c_2x)\cos\beta x + (c_3+c_4x)\sin\beta x] + c_5e^{m_5x} + \ldots + c_ne^{m_nx}$

(1) Solve $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

Sol The given D.E is $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

can be written as $(D^2-8D+15)y = 0$ where $D = \frac{d}{dx}$

Here $f(D) = D^2-8D+15$

Auxiliary Equation is $f(m) = 0$

$$m^2-8m+15=0$$
$$m^2-5m-3m+15=0$$
$$m(m-5)-3(m-5)=0$$
$$(m-3)(m-5)=0$$
$$\therefore m = 3, 5$$

The roots are real and different

Hence the general solution is $y = c_1e^{m_1x} + c_2e^{m_2x} = c_1e^{3x} + c_2e^{5x}$

② $(D^2-8D+9)y=0$

So the auxiliary eqn is $f(m)=0$

$$m^2-8m+9=0$$

Here $a=1,\ b=-8,\ c=9$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-8)\pm\sqrt{(-8)^2-4.1.9}}{2(1)}=\frac{8\pm\sqrt{64-36}}{2}$$

$$m=\frac{8\pm\sqrt{28}}{2}=\frac{8\pm2\sqrt{7}}{2}=4\pm\sqrt{7}$$

$$m=4+\sqrt{7},\ 4-\sqrt{7}$$

So the roots are Real and different

Hence the general Solution is

$$y=c_1e^{m_1x}+c_2e^{m_2x}=c_1e^{(4+\sqrt{7})x}+c_2e^{(4-\sqrt{7})x}$$

③ Solve $(D^2-3D+4)y=0$

Sol the auxiliary eqn is $f(m)=0$, i.e $m^2-3m+4=0$

Here $a=1,\ b=-3,\ c=4$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{(-3)^2-4.1.4}}{2(1)}=\frac{3\pm\sqrt{9-16}}{2}$$

$$m=\frac{3\pm\sqrt{-7}}{2}=\frac{3\pm i\sqrt{7}}{2}=3/2\pm i\frac{\sqrt{7}}{2}=\alpha\pm i\beta$$

Hence the general solution is $y=e^{\alpha x}[c_1\cos\beta x+c_2\sin\beta x]$

i.e $y=e^{3/2x}\left[c_1\cos\frac{\sqrt{7}}{2}x+c_2\sin\frac{\sqrt{7}}{2}x\right]$ where c_1 and c_2 are Constant

③ Solve $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$

Sol Given eqn in the operator form is $(D^3-3D+2)y=0$

Let $f(D) = D^3-3D+2$

The A.E is $f(m)=0 \Rightarrow m^3-3m+2=0$

Synthetic Division

$$\begin{array}{c|cccc} m=1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

$(m-1)(m^2+m-2)=0$

$(m-1)(m^2+2m-m-2)=0$

$(m-1)(m(m+2)-1(m+2))=0$

$(m-1)(m-1)(m+2)=0$

$m = 1, 1, -2$

Since two roots are real and equal and 3rd root is different

The general solution is

$y = (C_1+C_2x)e^{m_1x} + C_3e^{m_3x}$

$$\boxed{y = (C_1+C_2x)e^{1x} + C_3e^{-2x}}$$

④ Solve $(D^4-2D^3-3D^2+4D+4)y=0$ where $D=\frac{d}{dx}$

Sol Given eqn is

$(D^4-2D^3-3D^2+4D+4)y=0$

The A.E is $f(m)=0$

$m^4-2m^3-3m^2+4m+4=0$

Synthetic Division

$$\begin{array}{c|ccccc} m=-1 & 1 & -2 & -3 & 4 & 4 \\ & 0 & -1 & 3 & 0 & -4 \\ \hline m=-1 & 1 & -3 & 0 & 4 & \boxed{0} \\ & 0 & -1 & 4 & -4 & \\ \hline & 1 & -4 & 4 & \boxed{0} & \end{array}$$

$$(m+1)(m+1)(m^2-4m+4)=0$$

$$(m+1)^2(m^2-2m-2m+4)=0$$

$$(m+1)^2((m-2)(m-2))=0$$

$$(m+1)^2(m-2)^2=0$$

$$m=-1,-1,2,2$$

The roots are $m=-1,-1,2,2$

Hence the general solution is given by

$$y=(c_1+c_2x)e^{-x}+(c_3+c_4x)e^{2x}$$

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Ass ①

① $(D^2+4)y=0$

② $(D^2-3D+4)y=0$

③ $(D^4+18D^2+81)y=0$

④ $(D^3-D)y=0$

⑤ $(D^2-6D+13)y=0$

Inverse operator:—

The operator D^{-1} is called inverse of the diff operator D

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Def: If f is any func of x then $D^{-1}y$ or $\frac{1}{D}f$ is called the integral of f.

① Theorem:—

If Q is any func of x and α is a constant, then a particular value of $\frac{1}{D-\alpha}Q$ is equal to $e^{\alpha x}\int Q(x)e^{-\alpha x}dx$

i.e P.I of $\frac{1}{D-\alpha}Q = e^{\alpha x}\int Q(x)e^{-\alpha x}dx$

$$\frac{1}{D+\alpha}Q = e^{-\alpha x}\int Q(x)e^{\alpha x}dx$$

find

① $\frac{1}{D}(x^2)$

$\Rightarrow \int x^2 dx = \frac{x^3}{3}$

② $D^2(x^4)$

$D(Dx^4) = D(4x^3) = 12x^2$

② Find the particular value of $\frac{1}{D+1}(x)$

$$\frac{1}{D+1}(x) = e^{-x}\int x e^{x}dx$$

$$= e^{-x}[xe^{x} - e^{x}] = e^{-x}e^{x}[x-1]$$

$$\boxed{\frac{1}{D+1}(x) = x-1}$$

③

$y_2 = y(0,2) =$

find the particular Solution of $\frac{1}{(D-2)(D-3)} e^{2x}$

Sol

$$\frac{1}{(D-2)(D-3)} e^{2x} = \frac{1}{D-2}\left[\frac{1}{D-3} e^{2x}\right]$$

Now $\frac{1}{D-3}e^{2x} = e^{3x}\int e^{2x}e^{-3x}dx = e^{3x}\int e^{-x}dx = \frac{e^{3x}\cdot e^{-x}}{-\cancel{x}}$

$$\frac{1}{D-3}e^{2x} = -e^{2x}$$

$$\therefore \frac{1}{(D-2)}\left[\frac{1}{(D-3)}e^{2x}\right] = \frac{1}{D-2}\left[-e^{2x}\right] = -e^{2x}\int e^{2x}e^{-2x}dx$$

$$= -e^{2x}\int e^{0}dx$$

$$= -xe^{2x}$$

$$\boxed{\frac{1}{(D-2)(D-3)}e^{2x} = -xe^{2x}}$$

① Solve the equation $(D^2-2D+2)y = e^x \tan x$

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Sol $(D^2-2D+2)y = e^x \tan x$

A.E is $f(m) = 0$

$m^2-2m+2=0$

$(m-1)(m-1)=0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{4-4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2}$

$m = \frac{2 \pm \sqrt{4i^2}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$

The roots are complex

Thus C.F $y_c = e^x [C_1 \cos x + C_2 \sin x]$

$$P.I \; y_p = \frac{e^x \tan x}{D^2-2D+2} = e^x \frac{1}{(D+1)^2-2(D+1)+2} \tan x$$

$$= e^x \frac{1}{D^2+\cancel{2D}+1-\cancel{2D}-\cancel{2}+\cancel{2}} \tan x$$

$$= e^x \frac{1}{D^2+1} \tan x$$

$$= e^x \frac{1}{D^2-i^2} \tan x$$

$$= e^x \frac{1}{(D+i)(D-i)} \tan x$$

$$\left\{ \begin{aligned} &\frac{1}{(D+i)(D-i)} = \frac{A}{D+i} + \frac{B}{D-i} \\ &D=i \Rightarrow 1 = A(D-i)+B(D+i) \\ &\quad 1 = B(2i) \\ &\quad B = 1/2i \\ &D=-i \Rightarrow 1 = A(-i-i)+B(0) \\ &\quad A = \frac{-1}{2i} \end{aligned} \right.$$

$$= \frac{e^x}{2i}\left[\frac{1}{D-i} - \frac{1}{D+i}\right] \tan x \longrightarrow ①$$

Now $= \frac{e^x}{2i}\left[\frac{1}{D-i}\tan x - \frac{1}{D+i}\tan x\right] \longrightarrow ①$

Now $\frac{1}{D-i}\tan x = e^{ix}\int \tan x \, e^{-ix}\,dx \quad \left\{\frac{1}{D-\alpha}x = e^{\alpha x}\int x \cdot e^{-\alpha x}\,dx\right\}$

$$= e^{ix}\int (\cos x - i\sin x)\frac{\sin x}{\cos x}\,dx$$

$$= e^{ix}\left[\int \left[\sin x - i\frac{\sin^2 x}{\cos x}\right]dx\right]$$

$$= e^{ix}\int\left[\left(\sin x - i\frac{(1-\cos^2 x)}{\cos x}\right)\right]dx \qquad \int \sec x\,dx = \log\tan(\pi/4 + x/2) + c$$

$$= e^{ix}\int [\sin x - i\sec x + i\cos x]\,dx$$

$$= e^{ix}\left[\int \sin x\,dx - i\int \sec x\,dx + i\int \cos x\,dx\right]$$

$$= e^{ix}\left[-\cos x - i\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) + i\sin x\right]$$

$$= -e^{ix}\left[(\cos x - i\sin x) + i\log\tan\left(\frac{\pi}{4}+x/2\right)\right]$$

$$= -e^{ix}\left[e^{-ix} + i\log\tan(\pi/4 + x/2)\right]$$

$$= -\left[1 + i e^{ix}\log\tan\left[\pi/4 + x/2\right]\right] \longrightarrow (2)$$

Replacing i by $-i$ in eq (2), we get

$$\frac{1}{D+i}\tan x = -\left[1 - i e^{-ix}\log\tan(\pi/4 + x/2)\right] \longrightarrow (3)$$

P.I $y_p = \frac{e^x}{2i}\left[\cancel{(-1)} - i e^{ix}\log\tan(\pi/4 + x/2)\cancel{+1} - i e^{-ix}\log\tan(\pi/4 + x/2)\right]$

$$= \frac{e^x}{2\cancel{i}}\left[(-\cancel{i})\left[e^{ix} + e^{-ix}\right]\log\tan(\pi/4 + x/2)\right]$$

$$= -\frac{e^x}{\cancel{2}}\left[\cancel{2}\cos x\right]\log\tan(\pi/4 + x/2)$$

$$y_p = -e^x\cos x\log\tan(\pi/4 + x/2)$$

$$y = y_c + y_p = e^x[c_1\cos x + c_2\sin x] - e^x\cos x\log\tan(\pi/4 + x/2)$$

1. Particular Integral :-

Given eqn is $f(D)y = Q(x) \longrightarrow (1)$

$$y = \frac{1}{f(D)} Q(x)$$

Clearly eq(1) is satisfied, if we take $y = \frac{1}{f(D)} Q(x)$

Thus particular Integral $= P.I = \frac{1}{f(D)} Q(x)$

Let $f(D) = (D-\alpha_1)(D-\alpha_2) \dots (D-\alpha_n)$

$$P.I = \frac{1}{f(D)} Q = \frac{1}{(D-\alpha_1)(D-\alpha_2)\dots(D-\alpha_n)} Q$$

$$= \left[\frac{A_1}{D-\alpha_1} + \frac{A_2}{D-\alpha_2} + \dots + \frac{A_n}{D-\alpha_n} \right] Q$$

(Resolving into partial fractions)

$$P.I = A_1 e^{\alpha_1 x} \int Q e^{-\alpha_1 x} dx + A_2 e^{\alpha_2 x} \int Q e^{-\alpha_2 x} dx + \dots + A_n e^{\alpha_n x} \int Q e^{-\alpha_n x} dx$$

(1) Solve $(D^2+a^2)y = \sec ax$

Sol Given eqn is $(D^2+a^2)y = \sec ax \longrightarrow (1)$

Let $f(D) = D^2+a^2$

The A.E is $f(m) = 0$ i.e $m^2+a^2 = 0$

$$m^2 = -a^2$$

$$m^2 = i^2a^2$$

$$m = \pm ai$$

$\therefore y_c = c_1 \cos ax + c_2 \sin ax$

and $y_P = \frac{1}{f(D)} Q(x)$

$$= \frac{1}{D^2+a^2} \sec ax$$

$$= \frac{1}{(D+ai)(D-ai)} \sec ax$$

$$= \frac{1}{2ai}\left[\frac{1}{D-ai} - \frac{1}{D+ai}\right] \sec ax$$

$$= \frac{1}{2ai}\left[\frac{1}{D-ai} \sec ax - \frac{1}{D+ai} \sec ax\right]$$

Now $\frac{1}{D-ai} \sec ax = e^{iax} \int \sec ax \, e^{-aix} dx$

$$= e^{aix} \int \frac{\cos ax - i \sin ax}{\cos ax} dx$$

$$= e^{aix}\left[\int [1 - i \tan ax] dx\right.$$

$$= e^{aix}\left[x - i \log\left(\frac{\sec ax}{a}\right) \cancel{dx}\right]$$

$$= e^{aix}\left[x + \frac{i}{a} \log(\cos ax)\right]$$

Similarly $\frac{1}{D+ai} \sec ax = e^{-iax} \int \sec ax \cdot e^{aix} dx$

$$= e^{-iax}\left[\int \frac{\cos ax + i \sin ax}{\cos ax}\right] dx$$

$$= e^{-iax}\left[\int 1 dx + i \int \tan ax \, dx\right]$$

$$= e^{-iax}\left[x - \frac{i}{a} \log(\cos ax)\right]$$

$$y_P = \frac{1}{2ai}\left[e^{iax}\left[x + \frac{i}{a}\log\cos ax\right] - e^{-iax}\left[x - \frac{i}{a}\log\cos ax\right]\right] \quad (4)$$

$$= \frac{1}{2ai}\left[x(e^{aix} - e^{-iax}) + \frac{i}{a}\log\cos ax\left[e^{aix} + e^{-iax}\right]\right]$$

$$= \frac{x}{\cancel{2}a\cancel{i}}(\cancel{2}i\sin ax) + \frac{\cancel{i}}{\cancel{2}a^2\cancel{i}}\left[\log\cos ax\,[\cancel{2}\cos ax]\right]$$

$$= \frac{x}{a}\sin ax + \frac{1}{a^2}\log(\cos ax)\cos ax.$$

The general solution of eq (1) is

$$y = y_C + y_P$$

$$y = C_1\cos ax + C_2\sin ax + \frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax \cdot \log(\cos ax)$$

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(2) Solve $(D^2+4D+3)y = e^{e^x}$

Sol Given eqn is $(D^2+4D+3)y = e^{e^x}$

A.E is $m^2+4m+3 = 0$

$(m+3)(m+1) = 0$

$m = -3, -1$

Roots are real and different

Hence C.F is $y_C = C_1e^{-3x} + C_2e^{-x}$

Now P.I is $y_P = \frac{1}{(D+3)(D+1)}e^{e^x} = \frac{1}{2}\left[\frac{1}{D+1} - \frac{1}{D+3}\right]e^{e^x}$

$\therefore y_P = \frac{1}{2}\left[\frac{1}{D+1}(e^{e^x}) - \frac{1}{D+3}(e^{e^x})\right] = \frac{1}{2}\left[P.I_1 - P.I_2\right]$

Now $P.I_1 = \frac{1}{D+1} e^{e^x} = e^{-x}\int e^{e^x}. e^x\, dx$ $\{$ put $e^x = t, e^x dx = dt \}$

$$P.I_1 = e^{-x}\int e^t dt \qquad \left\{ \frac{1}{D+a}Q = e^{-ax}\int Q e^{ax} dx \right\}$$

$$= e^{-x} e^t$$

$$P.I_1 = e^{-x} e^{e^x}$$

and $P.I_2 = \frac{1}{D+3} e^{e^x} = e^{-3x}\int e^{e^x} e^{3x} dx = e^{-3x}\int e^t . e^{2x} . e^x dx$

$$= e^{-3x}\int e^t . t^2 . dt$$

$$= e^{-3x}[t^2 e^t - 2t e^t + 2e^t]$$

$$= e^{-3x} e^t [t^2 - 2t + 2]$$

$$= e^{-3x} e^{e^x}[e^{2x} - 2e^x + 2]$$

Hence $y_P = \frac{1}{2}[P.I_1 - P.I_2]$

$$= \frac{1}{2} e^{e^x}[\cancel{e^{-x}} - \cancel{e^{-x}} + 2e^{-2x} - 2e^{-3x}]$$

$$= \frac{1}{2} e^{e^x}[\cancel{e^{-x}} + 2e^{-2x} - 2e^{-3x}]$$

$$= [e^{-2x} - e^{-3x}] e^{e^x}$$

$\therefore$ The general solution is $y = y_c + y_p$

i.e $y = c_1 e^{-3x} + c_2 e^{-x} + e^{e^x}(e^{-2x} - e^{-3x})$

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① Rules for finding particular integral in Some Special cases

1. P.I of $f(D)y = \phi(x)$ when $\phi(x) = e^{ax}$, where 'a' is constant

Case I: $y_p = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$ if $f(a) \neq 0$ · since

Case II: Let $f(a)=0$ Then $f(D) = (D-a)^k$

$\frac{1}{D-a} e^{ax} = x e^{ax}$ if $f(a)=0$ and $\frac{1}{D+a} e^{-ax} = x e^{-ax}$

$\frac{1}{(D-a)^2} e^{ax} = \frac{x^2}{2!} e^{ax}$ If $f(a)=0$ and $\frac{1}{(D+a)^2} e^{-ax} = \frac{x^2}{2!} e^{-ax}$

$\frac{1}{(D-a)^3} e^{ax} = \frac{x^3}{3!} e^{ax}$ If $f(a)=0$ and $\frac{1}{(D+a)^3} e^{-ax} = \frac{x^3}{3!} e^{-ax}$

$\frac{1}{(D-a)^K} e^{ax} = \frac{x^K}{K!} e^{ax}$ If $f(a)=0$ and $\frac{1}{(D+a)^K} e^{-ax} = \frac{x^k}{k!} e^{-ax}$

① Solve $(D^2-4D+13)y = e^{2x}$

Sol A·E is $m^2-4m+13=0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{16-52}}{2}$

$13 \times 4 = 52$, $52-16 = 36$

$m = \frac{4 \pm \sqrt{-36}}{2} = 2 \pm 3i$

The roots are complex

Thus $C.F = e^{2x}[C_1 \cos 3x + C_2 \sin 3x]$

and $P.I = \frac{e^{2x}}{D^2-4D+13}$ put $D=2$

$$= \frac{e^{2x}}{4-8+13} = \frac{e^{2x}}{9}$$

Hence the general solution is

$$y = C.F + P.I = e^{2x}[c_1 \cos 3x + c_2 \sin 3x] + \frac{e^{2x}}{9}$$

(2) $(D^2+16)y = e^{-4x}$

Sol A.E is $m^2 = -16$

$$m^2 = i^2 4^2$$

$$m = \pm i4$$

$$m = 0 \pm i4$$

So the roots are complex and conjugate.

Thus $C.F = e^{0.x}[c_1 \cos 4x + c_2 \sin 4x]$

$$C.F = c_1 \cos 4x + c_2 \sin 4x$$

Now $P.I = \frac{e^{-4x}}{D^2+16}$ put $D = -4$

$$P.I = \frac{e^{-4x}}{16+16} = \frac{e^{-4x}}{32}$$

Hence the G.S. is

$$y = C.F + P.I$$

$$y = c_1 \cos 4x + c_2 \sin 4x + \frac{e^{-4x}}{32}$$

③ Solve $(D^2+2D+1)y = e^{-x}$

Sol A.E is $m^2+2m+1=0$ (43)

$$(m+1)^2=0$$

$$m=-1,-1$$

The roots are real and equal

Thus $C.F = (c_1+c_2x)e^{m_1x}$

$$y_c = C.F = (c_1+c_2x)e^{-x}$$

$$P.I = y_p = \frac{e^{-x}}{(D+1)^2} = e^{-x}.\frac{x^2}{2!}$$ $\{\because f(-1)=0$, case of failure so $\left\{\frac{1}{(D+a)^2}e^{-ax} = \frac{x^2}{2!}e^{-ax}\right\}$

Thus the general solution is

$$y = y_c+y_p$$

$$\boxed{y = (c_1+c_2x)e^{-x} + e^{-x}.\frac{x^2}{2!}}$$

M.IMP Solve $(D+2)(D-1)^2y = e^{-2x}+2\sinh x$

Sol The given eqn is $(D+2)(D-1)^2y = e^{-2x}+2\sinh x$.

This is of the form $f(D)y = e^{-2x}+2\sinh x$

$$f(D) = (D+2)(D-1)^2$$

A.E is $f(m)=0$

$$(m+2)(m-1)^2=0$$

$$m=-2 \quad m=1,1$$

The two roots are real and equal and third roots is real and different

$\therefore$ complementary function is

$$y_c = c_1 e^{-2x} + (c_2 + c_3 x) e^{1x}$$

Now $P.I = \dfrac{e^{-2x} + 2\sinh x}{f(D)}$

$$P.I = \frac{e^{-2x} + 2\sinh x}{(D+2)(D-1)^2} \qquad \left\{ \sinh x = \frac{e^{x} - e^{-x}}{2} \right\}$$

$$P.I = \frac{e^{-2x} + \cancel{2}\left[\frac{e^{x} - e^{-x}}{\cancel{2}}\right]}{(D+2)(D-1)^2}$$

$$P.I = \frac{e^{-2x} + e^{x} - e^{-x}}{(D+2)(D-1)^2}$$

$$P.I = \frac{e^{-2x}}{(D+2)(D-1)} + \frac{e^{x}}{(D+2)(D-1)^2} + \frac{e^{-x}}{(D+2)(D-1)^2}$$

$$P.I = y_{P_1} + y_{P_2} + y_{P_3}$$

where $y_{P_1} = \dfrac{e^{-2x}}{(D+2)(D-1)^2}$ $\{ f(-2) = 0$, case of failure$\}$

$$= \frac{e^{-2x}}{(D+2)(-2-1)^2} = \frac{e^{-2x}}{(-3)^2(D+2)} = \frac{e^{-2x}}{9(D+2)}$$

$$\therefore y_{P_1} = \frac{x e^{-2x}}{9} \qquad \left\{ \frac{1}{(D+a)} e^{-ax} = \frac{x e^{-ax}}{1!} \right\}$$

and $y_{P_2} = \dfrac{e^{x}}{(D+2)(D-1)^2}$ $\{$ Here $f(1) = 0 \}$ $\qquad \left\{ \dfrac{1}{(D-a)^2} e^{+ax} = \dfrac{x^2}{2!} e^{ax} \right.$

$$= \frac{e^{x}}{(1+2)(D-1)^2} = \frac{e^{x}}{3(D-1)^2} = \frac{x^2 e^{x}}{3 \times 2} = \frac{x^2 e^{x}}{6}$$

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and $y_{P_3} = \dfrac{e^{-x}}{(D+2)(D-1)^2}$

$$= \frac{e^{-x}}{(-1+2)(-1-1)^2} = \frac{e^{-x}}{1(2)^2} = \frac{e^{-x}}{4}$$

$$P.I = y_{P_1} + y_{P_2} - y_{P_3}$$

$$y_P = P.I = \frac{xe^{-2x}}{9} + \frac{x^2e^x}{6} - \frac{e^{-x}}{4}$$

$\therefore$ General Solution is

$$y = y_c + y_P$$

$$\boxed{y = c_1e^{-2x} + (c_2 + c_3x)e^x + \frac{xe^{-2x}}{9} + \frac{x^2e^x}{6} - \frac{e^{-x}}{4}}$$

(2) Solve $(D^2+5D+6)y = e^x$

Sol A.E is $m^2+5m+6=0$

$$m^2+3m+2m+6=0$$

$$(m+3)(m+2)=0$$

$$m=-3,-2$$

The roots are real and different

Thus C.F $y_c = c_1e^{-3x} + c_2e^{-2x}$

Now $P.I = y_P = \dfrac{1}{D^2+5D+6}e^x$

$$y_P = \frac{1}{1+5+6}e^x = \frac{e^x}{12}$$

$\therefore$ The general solution is $y = y_c + y_P = c_1e^{-3x} + c_2e^{-2x} + \dfrac{e^x}{12}$

2) Particular Integral of $f(D)y = \phi(x)$ when $\phi(x) = \sin bx$ (or) $\cos bx$, where b is a constant.

Let $f(D)y = \sin ax$

$P.I\ (y) = \frac{1}{f(D)} \sin ax$, let $f(D) = \phi(D^2)$ Then

1) $P.I = \frac{\sin ax}{f(D^2)} = \frac{\sin ax}{f(-a^2)}$ provided $f(-a^2) \neq 0$

2) $P.I = \frac{\cos ax}{f(D^2)} = \frac{\cos ax}{f(-a^2)}$ provided $f(-a^2) \neq 0$

Case of failure :-

3) 1) $P.I = \frac{\cos ax}{D^2+a^2} = \frac{x}{2a} \sin ax$, If $f(-a^2) = 0$

2) $P.I = \frac{\sin ax}{D^2+a^2} = \frac{-x}{2a} \cos ax$, If $f(-a^2) = 0$

① Solve $(D^2-4D+3)y = \cos 2x$

Sol The given eqn is $(D^2-4D+3)y = \cos 2x$

This is of the form $f(D)y = \cos 2x$

Here $f(D) = (D^2-4D+3)$

A.E is $f(m) = 0 \Rightarrow$ $m^2-4m+3=0$

$m^2-3m-m+3=0$

$(m-3)(m-1)=0$

$m=3$, and $m=1$

The roots are real and different

The complementary function is

$$y_c = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$\boxed{y_c = C_1 e^{3x} + C_2 e^{x}}$$

Now $P.I = \dfrac{\cos 2x}{D^2 - 4D + 3} = \dfrac{\cos 2x}{-2^2 - 4D + 3}$ $\{\text{put } D^2 = -2^2 = 4\}$

$$= \frac{\cos 2x}{-4 - 4D + 3}$$

$$= \frac{\cos 2x}{-4D - 1}$$

$$= \frac{\cos 2x}{-(4D+1)} = \frac{-\cos 2x\,(4D-1)}{(4D+1)(4D-1)}$$

$$= \frac{(1-4D)\cos 2x}{4D^2 - 1^2}$$

$$= \frac{(1-4D)\cos 2x}{(4D)^2 - 1} = \frac{(1-4D)\cos 2x}{16(-2^2) - 1}$$

$$= \frac{(1-4D)\cos 2x}{-64 - 1}$$

$$= \frac{(4D-1)\cos 2x}{65}$$

$$= \frac{1}{65}\left[4D\cos 2x - \cos 2x\right]$$

$$= \frac{1}{65}\left[4(-\sin 2x \cdot 2) - \cos 2x\right]$$

$$= \frac{1}{65}\left[8\sin 2x - \cos 2x\right]$$

$$y_p = \frac{-1}{65}\left[8\sin 2x + \cos 2x\right]$$

Hence the general solution is

$$y = C.F + P.I = C_1 e^{x} + C_2 e^{3x} - \frac{1}{65}\left[\cos 2x + 8\sin 2x\right]$$

M.Imp

(2) Solve $(D^2+4)y = e^x + \sin 2x + \cos 2x$

Sol: Given eqn is $(D^2+4)y = e^x + \sin 2x + \cos 2x$

This of the form $f(D)y = e^x + \sin 2x + \cos 2x$

let $f(D) = D^2+4$

A.E is $f(m) = 0$

$m^2+4=0$

$m^2=-4$

$m^2 = i^2 2^2 = (2i)^2 = \pm 2i$

$m = \pm 2i$

The roots are $m = 0 \pm 2i = \alpha \pm i\beta$

The roots are complex

$C.F = c_1 \cos 2x + c_2 \sin 2x$

Now $P.I = \frac{1}{f(D)}[e^x + \sin 2x + \cos 2x]$

$y_p = \frac{1}{(D^2+4)}[e^x + \sin 2x + \cos 2x]$

$y_p = \frac{1}{D^2+4}e^x + \frac{\sin 2x}{D^2+4} + \frac{\cos 2x}{D^2+4}$

$y_p = P.I_1 + P.I_2 + P.I_3 \longrightarrow (1)$

where $P.I_1 = \frac{1}{D^2+4}e^x$ (put $D=1$)

$P.I_1 = \frac{1}{1^2+4}e^x = \frac{1}{5}e^x$

$P.I_2 = \frac{1}{D^2+4}\sin 2x = \frac{-x\cos 2x}{4}$ $\left\{ \begin{array}{l} f(-a^2)=0 \text{ Case of} \\ \text{failure, } \frac{\sin ax}{D^2+a^2} = \frac{-x}{2a}\cos ax \end{array} \right.$

$P.I_2 = -\frac{x}{4}\cos 2x$

$$P.I_3 = \frac{\cos 2x}{D^2+4} = \frac{x\sin 2x}{4} \quad \left\{ f(-a^2)=0 \text{ case of failure } \frac{\cos ax}{D^2+a^2} = \frac{x}{2a}\sin ax \right\}$$

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$$\text{eq}(1) \Rightarrow y_p = P.I_1 + P.I_2 + P.I_3$$

$$y_p = \frac{1}{5}e^x + \frac{x\cos 2x}{4} + \frac{x\sin 2x}{4}$$

Hence The general solution

$$y = y_c + y_p$$

$$y = c_1\cos 2x + c_2\sin 2x + \frac{e^x}{5} - \frac{x\cos 2x}{4} + \frac{x\sin 2x}{4}$$

*

(2) Solve $(D^2+5D+6)y = \sin 4x \sin x$.

Sol Given eqn is $(D^2+5D+6)y = \sin 4x \sin x$.

A.E is $m^2+5m+6=0$

$$m^2+3m+2m+6=0$$

$$m(m+3)+2(m+3)=0$$

$$(m+3)(m+2)=0$$

$$m_1=-3,\ m_2=-2$$

The roots are real and different

The C.F is $y_c = c_1e^{-3x} + c_2e^{-2x}$

Now P.I is $y_p = \frac{\sin 4x \sin x}{D^2+5D+6}$

$$= \frac{1}{2}\cdot\frac{2\sin 4x\sin x}{D^2+5D+6}$$

$$\{2\sin A \sin B = \cos(A-B) - \cos(A+B)\}$$

$$y_P = \frac{1}{2} \frac{\cos(4x-x) - \cos(4x+x)}{(D^2+5D+6)}$$

$$y_P = \frac{1}{2} \frac{\cos 3x - \cos 5x}{D^2+5D+6}$$

$$y_P = \frac{1}{2}\left[\frac{\cos 3x}{D^2+5D+6} - \frac{\cos 5x}{D^2+5D+6}\right]$$

$$y_P = \frac{1}{2}\left[y_{P_1} - y_{P_2}\right]$$

Here $y_{P_1} = \dfrac{\cos 3x}{D^2+5D+6}$ $\{$put $D^2 = -9\}$

$$y_{P_1} = \frac{\cos 3x}{-9+5D+6}$$

$$y_{P_1} = \frac{\cos 3x}{5D-3} \Rightarrow \frac{\cos 3x(5D+3)}{(5D-3)(5D+3)}$$

$$y_{P_1} = \frac{(5D+3)\cos 3x}{(5D)^2 - 3^2} = \frac{(5D+3)\cos 3x}{25D^2-9}$$

$$y_{P_1} = \frac{(5D+3)(\cos 3x)}{25(-9)-9} = \frac{(5D+3)\cos 3x}{-225-9} =$$

$$y_{P_1} = \frac{-1}{234}\left[5D\cos 3x + 3\cos 3x\right]$$

$$y_{P_1} = \frac{-1}{234}\left[5(-\sin 3x)\cdot 3 + 3\cos 3x\right]$$

$$y_{P_1} = \frac{15\sin 3x}{234} - \frac{\cancel{3}}{\cancel{234}_{78}}\cos 3x$$

$$y_{P_1} = \frac{15}{234}\sin 3x - \frac{1}{78}\cos 3x$$

$$y_{P_2} = \frac{\cos 5x}{D^2+5D+6}$$

$$y_{P_2} = \frac{\cos 5x}{-25+5D+6} = \frac{\cos 5x}{5D-19}$$

$$y_{P_2} = \frac{(5D+19)\cos 5x}{(5D-19)(5D+19)}$$

$$y_{P_2} = \frac{(5D+19)\cos 5x}{(5D)^2-(19)^2} = \frac{(5D+19)\cos 5x}{25(-25)-} = \frac{(5D+19)\cos 5x}{-625-361}$$

$$y_{P_2} = \frac{-1}{986}\left[5D\cos 5x + 19\cos 5x\right]$$

$$y_{P_2} = \frac{-1}{986}\left[5(-\sin 5x)5 + 19\cos 5x\right]$$

$$y_{P_2} = \frac{25}{986}\sin 5x - \frac{19}{986}\cos 5x$$

$$y_P = \frac{1}{2}\left[y_{P_1} - y_{P_2}\right]$$

$$= \frac{1}{2}\left[\left[\frac{15}{234}\sin 3x - \frac{1}{78}\cos 3x\right] - \left(\frac{25}{986}\sin 5x - \frac{19}{986}\cos 5x\right)\right]$$

$$y_P = \frac{15}{468}\sin 3x - \frac{1}{156}\cos 3x - \frac{25}{1972}\sin 5x + \frac{19}{1972}\cos 5x$$

Hence the general solution is

$$y = y_c + y_P$$

$$y = c_1 e^{-3x} + c_2 e^{-2x} + \frac{15}{468}\sin 3x - \frac{1}{156}\cos 3x - \frac{25}{1972}\sin 5x + \frac{19}{1972}\cos 5x$$

① P.I of $f(D)y = \phi(x)$ when $\phi(x) = x^K$ where is a +ve Integer

let P.I $\Rightarrow f(D)y = x^K$

$$P.I = \frac{x^K}{f(D)}$$

we frequently use the following rules:-

1) $\frac{1}{(1-D)} = (1-D)^{-1} = 1+D+D^2+D^3+ - - -$

2) $\frac{1}{1+D} = (1+D)^{-1} = 1-D+D^2-D^3+ - - -$

3) $\frac{1}{(1-D)^2} = (1-D)^{-2} = 1+2D+3D^2+4D^3+ - - -$

4) $\frac{1}{(1+D)^2} = (1+D)^{-2} = 1-2D+3D^2-4D^3+ - - -$

5) $\frac{1}{(1-D)^3} = (1-D)^{-3} = 1+3D+6D^2+10D^3+ - - -$

6) $\frac{1}{(1+D)^3} = (1+D)^{-3} = 1-3D+6D^2-10D^3+ - - -$

*

① Solve $\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+2x+4$ (48)

Sol Given eqn is $(D^2+D)y=x^2+2x+4$

A.E is $f(m)=0$

$$m^2+m=0$$
$$m(m+1)=0$$
$$m=0,-1$$

The roots are real and different

C.F is $y_c=c_1e^{m_1x}+c_2e^{m_2x}=c_1e^{0x}+c_2e^{-x}$

$$P.I=y_p=\frac{1}{f(D)}Q(x)=\frac{x^2+2x+4}{D^2+D}$$
$$=\frac{x^2+2x+4}{D(D+1)}$$
$$=\frac{1}{D}\left[\frac{x^2+2x+4}{D+1}\right]$$
$$=\frac{1}{D}(D+1)^{-1}(x^2+2x+4)$$
$$=\frac{1}{D}[(1-D+D^2-D^3+D^4+\cdots](x^2+2x+4)]$$
$$=\frac{1}{D}[(1-D+D^2)(x^2+2x+4)]$$, neglecting higher powers of D
$$=\frac{1}{D}\left[1(x^2+2x+4)-\frac{d}{dx}(x^2+2x+4)+\frac{d^2}{dx^2}(x^2+2x+4)\right]$$
$$=\frac{1}{D}[x^2+\cancel{2x}+4-\cancel{2x}-\cancel{2}+\cancel{2}+0]$$
$$=\frac{1}{D}(x^2+4)=\int(x^2+4)\,dx=\frac{x^3}{3}+4x$$

The general solution is $y=y_c+y_p$

$$y=c_1e^{0x}+c_2e^{-x}+\frac{x^3}{3}+4x$$

(2) IMP
Solve $D^2(D^2+4)y = 320(x^3+2x^2+e^x)$

Sol Given eqn is $D^2(D^2+4)y = 320(x^3+2x^2+e^x)$

let $f(D) = D^2(D^2+4)$

A.E is $f(m)=0$

$$m^2(m^2+4)=0$$

$$m=0,0,\quad m=\pm 2i$$

The two roots are real and repeated and two roots are complex conjugate numbers.

Thus C.F is $y_c = (c_1+c_2x)e^{mx} + e^{\alpha x}[c_3\cos\beta x + c_4\sin\beta x]$

$$y_c = (c_1+c_2x)e^{0.x} + e^{0.x}[c_3\cos 2x + c_4\sin 2x]$$

$$y_c = c_1 + c_2x + c_3\cos 2x + c_4\sin 2x$$

Now P.I $= \dfrac{1}{D^2(D^2+4)} \cdot 320(x^3+2x^2+e^x)$

$$y_p = \frac{1}{D^2(D^2+4)} \cdot 320(x^3+2x^2) + \frac{1}{D^2(D^2+4)} 320e^x$$

$$y_p = y_{p_1} + y_{p_2}$$

where $y_{p_1} = \dfrac{320(x^3+2x^2)}{D^2(D^2+4)}$

$$= \frac{\overset{80}{\cancel{320}}(x^3+2x^2)}{D^2.\cancel{4}\left[\frac{D^2}{4}+1\right]}$$

$$= \frac{80}{D^2}\left(1+\frac{D^2}{4}\right)^{-1}(x^3+2x^2)$$

$$= \frac{80}{D^2}\left[1-\frac{D^2}{4}+\left(\frac{D^2}{4}\right)^2-\left(\frac{D^2}{4}\right)^3+\cdots\right][x^3+2x^2]$$

$$y_{p_1} = \frac{80}{D^2}\left[1 - \frac{D^2}{4} + \frac{D^4}{16} - \frac{D^6}{64} + \cdots\right](x^3 + 2x^2)$$

$$y_{p_1} = \frac{80}{D^2}\left[1 - \frac{D^2}{4} + \frac{D^4}{16}\right](x^3 + 2x^2)$$

{neglecting higher powers of D}

$$= 80\left[\frac{1}{D^2} - \frac{1}{\cancel{D^2}}\cdot\frac{\cancel{D^2}}{4} + \frac{1}{\cancel{D^2}}\cdot\frac{D^{\cancel{4}\,2}}{16}\right](x^3 + 2x^2)$$

$$= 80\left[\frac{1}{D^2} - \frac{1}{4} + \frac{D^2}{16}\right](x^3 + 2x^2)$$

$$= 80\left[\frac{1}{D^2}(x^3 + 2x^2) - \frac{1}{4}(x^3 + 2x^2) + \frac{1}{16}\left[D^2(x^3 + 2x^2)\right]\right]$$

$$= 80\left[\frac{1}{D}\left[\int (x^3 + 2x^2)\,dx\right] - \frac{1}{4}x^3 - \frac{1}{2}x^2 + \frac{1}{16}\left[6x + 4\right]\right]$$

$$= 80\left[\int \left[\frac{x^4}{4} + \frac{2}{3}x^3\right]dx - \frac{1}{4}x^3 - \frac{1}{2}x^2 + \frac{3}{8}x + \frac{1}{4}\right]$$

$$= 80\left[\frac{x^5}{4\times 5} + \frac{2}{3}\cdot\frac{x^4}{4} - \frac{1}{4}x^3 - \frac{1}{2}x^2 + \frac{3}{8}x + \frac{1}{4}\right]$$

$$= 80\times\frac{x^5}{20} + 80\cdot\frac{x^4}{6} - \frac{80}{4}x^3 - \frac{80}{2}x^2 + 80\cdot\frac{3}{8}x + 80\cdot\frac{1}{4}$$

$$= 4x^5 + \frac{40x^4}{3} - 20x^3 - 40x^2 + 30x + 20$$

and $y_{p_2} = \frac{1}{D^2(D^2+4)}\,320e^x$ {put $D=1$}

$$= \frac{1}{1(1^2+4)}\,320e^x$$

$$y_{p_2} = \frac{320}{5}e^x = 64e^x$$

Hence the general solution is $y = y_c + y_{p_1} + y_{p_2}$

$$y = (c_1 + c_2x) + c_3\cos 2x + c_4\sin 2x + 4x^5 + \frac{40x^4}{3} - 20x^3 - 40x^2 + 30x + 20 + 64e^x$$

(2) Solve the differential eqn $(D^2-4D+4)y = 8(x^2+e^{2x}+\sin 2x)$

Sol The given eqn is $(D^2-4D+4)y = 8e^{2x}+8x^2+8\sin 2x$

AE is $f(m)=0$

$$m^2-4m+4=0$$
$$m^2-2m-2m+4=0$$
$$m(m-2)-2(m-2)=0$$
$$(m-2)^2=0$$
$$m=2,2.$$

The roots are real and equal.

$$y_c=(c_1+c_2x)e^{m_1x}$$
$$\boxed{y_c=(c_1+c_2x)e^{2x}}$$

Now $P.I = \frac{1}{(D^2-4D+4)}(8x^2+8e^{2x}+8\sin 2x)$

$$y_P=\frac{1}{(D-2)^2}[8x^2+8e^{2x}+8\sin 2x]$$
$$=\frac{8x^2}{(D-2)^2}+\frac{8e^{2x}}{(D-2)^2}+\frac{8\sin 2x}{(D-2)^2}$$

$= 8[D-2]^{-2}x^2+$

$y_P = y_{P_1}+y_{P_2}+y_{P_3}$

$y_{P_1}=\frac{8x^2}{(D-2)^2} = 8(D-2)^{-2}x^2$

wrong

$$y_P = y_{P_1}+y_{P_2}+y_{P_3}$$
$$y_{P_1}=\frac{8x^2}{(D-2)^2}$$
$$=\frac{8x^2}{(-2)^2[1-\frac{D}{2}]^2}$$
$$=\frac{\overset{2}{\cancel{8}}x^2}{\cancel{4}(1-\frac{D}{2})^2}$$
$$=2(1-\frac{D}{2})^{-2}x^2$$

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$$y_{P_1} = 2\left[1 + 2\left(\frac{D}{2}\right) + 3\left(\frac{D}{2}\right)^2 + \cdots\right](x^2)$$

$$y_{P_1} = 2\left[1 + D + \frac{3}{4}D^2\right]x^2 \quad \{\text{neglecting higher power of } D\}$$

$$y_{P_1} = 2\left[x^2 + 2x + \frac{3}{4}(2)\right]$$

$$y_{P_1} = 2x^2 + 4x + 3$$

and $$y_{P_2} = \frac{8e^{2x}}{(D-2)^2} \quad \{f(2) = 0, \text{ case of failure}$$

$$y_{P_2} = \frac{\overset{4}{\cancel{8}}\, x^2}{\cancel{2!}} e^{2x} \quad \left\{\frac{1}{(D-a)^2}e^{ax} = \frac{x^2}{2!}e^{ax}\right\}$$

$$y_{P_3} = \frac{8\sin 2x}{(D-2)^2}$$

$$y_{P_3} = \frac{8\sin 2x}{D^2 - 4D + 4} = \frac{\overset{2}{\cancel{8}}\sin 2x}{-\cancel{4} - \cancel{4}D + \cancel{4}} \quad \{\text{put } D^2 = -2^2 = -4$$

$$= -2\frac{\sin 2x}{D}$$

$$= -2\int \sin 2x\, dx$$

$$= -\cancel{2}\left(-\frac{\cos 2x}{\cancel{2}}\right)$$

$$\boxed{y_{P_3} = \cos 2x}$$

Hence the general solution is

$$y = y_C + y_P$$

$$y = (c_1 + c_2 x)e^{2x} + 2x^2 + 4x + 3 + 4x^2 e^{2x} + \cos 2x$$

① P.I of $f(D)y = \phi(x)$ when $\phi(x) = e^{ax}V$ where 'a' is a constant and V is a function of x

we will use this method to find P.I. when V is $\sin bx$ (or) $\cos bx$ (or) x^k or a polynomial of degree K.

In this case $P.I = \frac{1}{f(D)}[e^{ax}V] = e^{ax}\frac{1}{f(D+a)}(V)$

① Solve $(D^2-5D+6)y = e^x \sin x$

Sol Given eqn is $(D^2-5D+6)y = e^x \sin x$

Auxiliary Equation is $m^2-5m+6=0$

$(m-2)(m-3)=0$

$m_1=2, m_2=3$

C.F is $y_c = c_1 e^{2x}+c_2 e^{3x}$

$$P.I = y_p = \frac{1}{D^2-5D+6} e^x \sin x$$

$$= e^x \frac{1}{(D+1)^2-5(D+1)+6} \sin x$$

$$= e^x \frac{1}{D^2+2D+1-5D-5+6} \sin x$$

$$= e^x \frac{1}{D^2-3D+2} \sin x$$

$$= e^x \frac{1}{-1^2-3D+2} \sin x$$

$$= e^x \frac{1}{-3D+1} \sin x$$

$$= e^x \frac{1+3D}{(1-3D)(1+3D)} \sin x$$

$$= e^x \frac{1+3D}{1^2-9D^2} \sin x$$

$$= e^x \frac{(1+3D)}{1-9(-1)} \sin x$$

$$= \frac{e^x}{10}[\sin x + 3D\sin x]$$

$$= \frac{e^x}{10}[\sin x + 3\cos x]$$

The G.S. is $y = y_c + y_p$

$$y = c_1 e^{2x} + c_2 e^{3x} + \frac{e^x}{10}[\sin x + 3\cos x]$$

M.Q.MP

Solve:-

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$$(D^2-4D+4)y = x^2\sin x + e^{2x}+3$$

Sol A.E is $m^2-4m+4=0$

$$(m-2)^2=0$$

$$m=2,2$$

The roots are real & equal

$$y_c = (c_1+c_2x)e^{mx}$$

$$\boxed{y_c = (c_1+c_2x)e^{2x}}$$

$$P.I = \frac{1}{f(D)}x^2\sin x + e^{2x}+3$$

$$P.I = \frac{1}{D^2-4D+4}[x^2\sin x + e^{2x}+3]$$

$$P.I = \frac{1}{(D^2-4D+4)}(x^2\sin x) + \frac{1}{(D^2-4D+4)}e^{2x} + \frac{1}{(D^2-4D+4)}3.$$

$$P.I = \frac{1}{(D-2)^2}x^2\sin x + \frac{1}{(D-2)^2}e^{2x} + \frac{1}{(D-2)^2}3$$

$$P.I = y_{P_1}+y_{P_2}+y_{P_3}$$

Now $y_{P_1} = \frac{1}{(D-2)^2}x^2\sin x.$

$$y_{P_1} = \text{R.P} \frac{1}{(D-2)^2}x^2e^{ix}$$

$$y_{P_1} = \text{I.P of } \frac{1}{-2(1-\frac{D}{2})^2}x^2e^{+xi}$$

$$y_{P_1} = \text{I.P of } e^{ix}\frac{1}{(-2)^2(1-\frac{D+i}{2})^2}x^2$$

$$y_{P_1} = \text{I.P of } \frac{e^{ix}}{(-2)^2}\left[1-\frac{D+i}{2}\right]^{-2}x^2$$

~~on simplification, we get~~

$$\cancel{y_{P_1} = \frac{1}{625}\left[(220x+244)\cos x + (40x+33)\sin x\right]}$$

① P.I of $f(D)y = \phi(x)$ when $\phi(x) = x^m V$, m being a +ve integer and V is any function of x

Here V is either $\sin ax$ or $\cos ax$ only. It should not be of the form x^n or e^{ax}.

(i) $P.I = \frac{1}{f(D)} x^m \sin ax$

P.I = Imaginary part (I.P) of $\frac{1}{f(D)} x^m (\cos ax + i \sin ax)$

$P.I$ = I.P of $\frac{1}{f(D)} x^m e^{iax}$

2) $P.I = \frac{1}{f(D)} x^m \cos ax$

P.I = Real part (R.P) of $\frac{1}{f(D)} x^m (\cos ax + i \sin ax)$

$P.I$ = R.P of $\frac{1}{f(D)} x^m e^{iax}$.

Alternative method for finding P.I of $f(D)y = \phi(x)$ where $\phi(x) = x.V$ (when m=1) where V is a func of x.

$$P.I = \frac{1}{f(D)} (xV)$$

$$\boxed{P.I = \left[x - \frac{f'(D)}{f(D)}\right] \frac{1}{f(D)} V}$$

$$y_{P_1} = I.P \text{ of } \frac{e^{ix}}{4}\left[1 + 2\left(\frac{D+i}{2}\right)^{2} + 3\left(\frac{D+i}{2}\right)^{2} + 4\left(\frac{D+i}{3}\right)^{3}\right]x^2$$

$$y_{P_1} = I.P \text{ of } \frac{e^{ix}}{4}\left[1 + (D+i) + \frac{3}{4}(D^2 + 2Di + i^2)\right]x^2$$

$$y_{P_1} = I.P \text{ of } \frac{e^{ix}}{4}\left[x^2 + (2x + ix^2) + \frac{3}{4}(2 + 2(2x)i - x^2)\right]$$

$$= I.P \text{ of } \frac{e^{ix}}{4}\left[x^2 + ix^2 + 2x + \frac{3}{2} + 3xi - \frac{3}{4}x^2\right]$$

$$= I.P \text{ of } \frac{e^{ix}}{4}\left[\frac{1}{4}x^2 + 2x + \frac{3}{2} + ix^2 + 3xi\right]$$

$$= I.P \text{ of } \frac{1}{4}[\cos x + i\sin x]\left[\frac{1}{4}x^2 + 2x + \frac{3}{2} + ix^2 + 3xi\right]$$

$$= \frac{1}{4}\left[x^2\cos x + 3x\cos x + \frac{1}{4}x^2\sin x + 2x\sin x + \frac{3}{2}\sin x\right]$$

$$y_{P_1} = \frac{1}{4}\left[(x^2 + 3x)\cos x + \left(\frac{1}{4}x^2 + 2x + 3/2\right)\sin x\right]$$

$$\boxed{y_{P_1} = \frac{1}{4}\left[(x^2 + 3x)\cos x + \left(\frac{1}{4}x^2 + 2x + 3/2\right)\sin x\right]}$$

$$y_{P_2} = \frac{1}{(D-2)^2} e^{2x} \quad \{f(2)=0, \text{ case of failure}$$

$$\frac{e^{ax}}{(D-a)^2} = \frac{x^2}{2!} e^{ax}$$

$$\boxed{y_{P_2} = \frac{x^2}{2} e^{2x}}$$

$$y_{P_3} = \frac{1}{(D-2)^2} 3.$$

$$= \frac{1}{(D-2)^2} 3e^{0.x}$$

$$= \frac{1}{(0-2)^2} 3e^{0.x} \quad \{\text{put } D=0\}$$

$$= \frac{1}{(-2)^2} e^{0.\cancel{x}} \cdot 3$$

$$y_{P_3} = \frac{3}{4}$$

$$y_P = y_{P_1} + y_{P_2} + y_{P_3}$$

$$y_P = \boxed{\frac{1}{625}\left[\cancel{(220x+244)}\cos x + (40x+33)\sin x\right]} + \frac{x^2}{2}e^{2x} + \frac{3}{4}$$

General solution is

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x) e^{2x} + y_P .$$

M.IMP

① ** Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x \sin x$ (3)

(or) $(D^2+3D+2)y = xe^x \sin x$.

Sol Given eqn is $(D^2+3D+2)y = xe^x \sin x$

A.E is $f(m)=0$

$m^2+3m+2=0 \Rightarrow (m+2)(m+1)=0$

$m_1=-2,\ m_2=-1$.

The roots are Real and different

C.F is $y_c = c_1e^{-2x} + c_2e^{-x}$

Now P.I is $y_p = \frac{1}{D^2+3D+2} xe^x \sin x$

$$y_p = e^x \frac{1}{(D+1)^2+3(D+1)+2} x\sin x$$

$$= e^x \frac{1}{D^2+2D+1+3D+3+2} x\sin x$$

$$= e^x \frac{1}{D^2+5D+6} x\sin x \qquad \left\{ P.I = \frac{1}{f(D)} xV = \left[x - \frac{f'(D)}{f(D)}\right]\frac{1}{f(D)}V \right\}$$

$$= e^x \left[x - \frac{2D+5}{D^2+5D+6}\right] \frac{1}{D^2+5D+6} \sin x$$

$$= e^x \left[x - \frac{2D+5}{D^2+5D+6}\right] \frac{1}{-1^2+5D+6} \sin x$$

$$= e^x \left[x - \frac{2D+5}{D^2+5D+6}\right] \frac{1}{5D+5} \sin x$$

$$= \frac{e^x}{5} \left[x - \frac{2D+5}{D^2+5D+6}\right] \frac{D-1}{(D+1)(1-D)} \sin x$$

$$y_p = \frac{e^x}{5}\left[x - \frac{2D+5}{D^2+5D+6}\right]\frac{D-1}{D^2-1}\sin x$$

$$y_p = \frac{e^x}{5}\left[x - \frac{2D+5}{D^2+5D+6}\right]\frac{D-1}{-1-1}\sin x$$

$$y_p = \frac{e^x}{10}\left[x - \frac{2D+5}{D^2+5D+6}\right]\left[(1-D)\sin x\right]$$

$$y_p = \frac{e^x}{10}\left[x - \frac{2D+5}{D^2+5D+6}\right](\sin x - \cos x)$$

$$y_p = \frac{e^x x(\sin x - \cos x)}{10} - \frac{e^x}{10}\frac{(2D+5)(\sin x - \cos x)}{D^2+5D+6}$$

$$= \frac{e^x x(\sin x - \cos x)}{10} - \frac{e^x}{10}\left[\frac{2\cos x - 2(-\sin x) + 5\sin x - 5\cos x}{D^2+5D+6}\right]$$

$$= \frac{e^x x(\sin x - \cos x)}{10} - \frac{e^x}{10}\left[\frac{7\sin x - 3\cos x}{D^2+5D+6}\right]$$

$$= \frac{e^x x(\sin x - \cos x)}{10} + \frac{e^x}{10}\left[\frac{3\cos x - 7\sin x}{-1^2+5D+6}\right]$$

$$= \frac{e^x x}{10}(\sin x - \cos x) + \frac{e^x}{50}\frac{D+1}{(D+1)(D-1)}(3\cos x - 7\sin x)$$

$$= \frac{e^x x}{10}(\sin x - \cos x) + \frac{e^x}{50}\frac{D-1}{D^2-1^2}(3\cos x - 7\sin x)$$

$$= \frac{e^x x}{10}(\sin x - \cos x) + \frac{e^x}{50}\frac{D-1}{-1-1}(3\cos x - 7\sin x)$$

$$= \frac{e^x x}{10}(\sin x - \cos x) + \frac{e^x}{50\times 2}(1-D)(3\cos x - 7\sin x)$$

$$= \frac{e^{x}x}{10}(\sin x-\cos x)+\frac{e^{x}}{100}[3\cos x-7\sin x-3(-\sin x)+7\cos x]$$

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$$y_p = \frac{e^{x}x}{10}(\sin x-\cos x)+\frac{e^{x}}{100}[10\cos x-4\sin x]$$

$\therefore$ The general solution is $y = y_c + y_p$

$$\boxed{y = C_1 e^{-2x} + C_2 e^{-x} + \frac{e^{x}x}{10}(\sin x-\cos x)+\frac{e^{x}}{100}(10\cos x-4\sin x)}$$

(2) Working Rule

Linear Equations of the second order with variable coefficients

An eqn of the form

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y = R(x), \text{ where } P(x), Q(x), R(x)$$

are real valued func of x is called the linear eqn of the second order with variable coefficients.

1. General solution of $\frac{d^2y}{dx^2}+P.\frac{dy}{dx}+Q.y=R$ be the method of variation of parameters

Working Rule :-

1) To solve $\frac{d^2y}{dx^2}+P.\frac{dy}{dx}+Qy=R$ by the method of variation of parameters, follow these steps:-

1. Reduce the given eqn to the standard form, if necessary
2. Find the solution of $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$ and let the solution by $y_c = C_1 u(x) + C_2 v(x)$

3. Take P.I $= y_p = Au + Bv$, where A and B are func of x.

4. Find $W(u,v) = u \cdot \frac{dv}{dx} - v \cdot \frac{du}{dx}$

5. Find A and B using

$$A = -\int \frac{vR\,dx}{W(u,v)} = -\int \frac{vR\,dx}{u \cdot \frac{dv}{dx} - v\frac{du}{dx}}$$

$$B = \int \frac{uR\,dx}{W(u,v)} = \int \frac{uR\,dx}{u\frac{dv}{dx} - v\frac{du}{dx}}$$

6. Write the general solution of the given eqn as

$$y = y_c + y_p$$

$y = c_1 u(x) + c_2 v(x) + A x\, u(x) + B x\, v(x)$ where c_1 and c_2 are constant

MQMP

① ** Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. (5)

Sol Given eqn in the operator form is $(D^2+1)y = \operatorname{cosec} x$

Here $P=0, Q=1$, and $R = \operatorname{cosec} x$.

A.E is $m^2+1=0$

$m^2=-1$

$m^2=i^2$

$m=0\pm i = \alpha \pm i\beta$

The roots are complex conjugate numbers

$\therefore$ C.F is $y_c = e^{\alpha x}[C_1 \cos\beta x + C_2 \sin\beta x]$

$y_c = e^{0.x}[C_1\cos x + C_2 \sin x]$

$$\boxed{y_c = C_1\cos x + C_2 \sin x}$$

This is of the form $y_c = C_1 u(x) + C_2 v(x)$

Now Here $u(x) = \cos x$, $v(x) = \sin x$

Now $y_p = A u(x) + B v(x)$

$y_p = A\cos x + B \sin x$

Here $u = \cos x$, $v = \sin x$

$\frac{du}{dx} = -\sin x$, $\frac{dv}{dx} = \cos x$

$\therefore u.\frac{dv}{dx} - v.\frac{du}{dx} = \cos x(\cos x) - \sin x(-\sin x)$

$u.\frac{dv}{dx} - v.\frac{du}{dx} = \cos^2 x + \sin^2 x = 1$

$$\boxed{\therefore u.\frac{dv}{dx} - v.\frac{du}{dx} = 1}$$

A and B are given by

$$A = -\int \frac{VR}{U.\frac{dV}{dx} - V\frac{dU}{dx}}\,dx = -\int \frac{\sin x . \operatorname{cosec} x}{1}\,dx$$

$$A = -\int \sin x\left(\frac{1}{\sin x}\right) dx$$

$$A = -\int dx$$

$$\boxed{A = -x}$$

$$B = \int \frac{UR}{U.\frac{dV}{dx} - V\frac{dU}{dx}}\,dx = \int \frac{\cos x . \operatorname{cosec} x}{1}\,dx$$

$$= \int \cot x\,dx$$

$$\boxed{B = \log|\sin x|}$$

$$y_P = A\cos x + B\sin x$$

$$y_P = -x\cos x + \log|\sin x| . \sin x$$

Hence the general solution is given by

$$y = y_c + y_P$$

$$\boxed{y = c_1\cos x + c_2\sin x - x\cos x + \sin x\log|\sin x|}$$

(2) Solve $(D^2-2D+2)y = e^x \tan x$ by the method of variation of parameters. (56)

Sol Given eqn is $(D^2-2D+2)y = e^x \tan x$

A.E is $f(m)=0$

$$m^2-2m+2=0$$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{(2)^2-4(1)(2)}}{2(1)}$$

$$m = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$$

$$m = 1 \pm i$$

we have $\boxed{y_c = e^x [c_1 \cos x + c_2 \sin x]}$

$$y_c = c_1 e^x \cos x + c_2 e^x \sin x$$

This is of the form $y_c = c_1 u(x) + c_2 v(x)$

where $u(x) = e^x \cos x$, $v(x) = e^x \sin x$.

$$\frac{du}{dx} = \frac{d}{dx}(e^x \cos x) = e^x(-\sin x) + \cos x\, e^x$$

$$\frac{du}{dx} = e^x[\cos x - \sin x]$$

$$\frac{dv}{dx} = \frac{d}{dx}(e^x \sin x) = e^x \cos x + \sin x\, e^x$$

$$\frac{dv}{dx} = e^x[\cos x + \sin x]$$

$$u.\frac{dv}{dx} - v.\frac{du}{dx} = e^x \cos x[\cos x + \sin x]e^x - e^x \sin x . e^x[\cos x - \sin x]$$

$$= e^{2x}[\cos^2 x + \cancel{\cos x \sin x} - \cancel{\cos x \sin x} + \sin^2 x]$$

$$= e^{2x}[\cos^2 x + \sin^2 x]$$

$$\boxed{u.\frac{dv}{dx} - v.\frac{du}{dx} = e^{2x}}$$

$y_p = A\,u(x) + B\,v(x)$

using variation of parameters

$$A = -\int \frac{vR}{u\frac{dv}{dx} - v.\frac{du}{dx}}\,dx = -\int \frac{e^x \sin x \; e^x \tan x}{e^{2x}}\,dx$$

$$= -\int \frac{e^{2x}.\sin x}{e^{2x}}.\frac{\sin x}{\cos x}\,dx$$

$$= -\int \frac{\sin^2 x}{\cos x}\,dx$$

$$= -\int \frac{(1-\cos^2 x)}{\cos x}\,dx$$

$$= -\int \left[\frac{1}{\cos x} - \cos x\right] dx$$

$$= -\int [\sec x - \cos x]\,dx$$

$$= -\int \sec x\,dx + \int \cos x\,dx$$

$$A = -\log|\sec x + \tan x| + \sin x$$

$$B = \int \frac{uR}{u.\frac{dv}{dx} - v.\frac{du}{dx}}\,dx = \int \frac{e^x \cos x . e^x \tan x}{e^{2x}}\,dx$$

$$= \int \cos x . \frac{\sin x}{\cos x}\,dx$$

$$= \int \sin x\,dx$$

$$\boxed{B = -\cos x}$$

$\therefore\ y_p = A.u(x) + B\,v(x)$

$y_p = e^x \cos x\,[-\log|\sec x + \tan x| + \sin x] + (-\cos x)\,e^x \sin x$

$\therefore$ General solution is $y = y_c + y_p$

$$y = e^x[c_1 \cos x + c_2 \sin x] + e^x \cos x\,[\sin x - \log|\sec x + \tan x|] - e^x \cos x \sin x$$

① Solve $(D^2-4D+4)y = x\sin x + e^{2x} + 3.$

Sol Given $(D^2-4D+4)y = x\sin x + e^{2x} + 3$

The A.E is $m^2-4m+4=0$

$$m^2-2m-2m+4=0$$

$$m(m-2)-2(m-2)=0$$

$$(m-2)(m-2)=0$$

$$m=2,2$$

The roots are real and ~~root~~ equal.

$$y_c = (C_1 + C_2x)e^{2x}.$$

Now $y_p = \frac{1}{f(D)} x\sin x + e^{2x} + 3$

$$= \frac{1}{D^2-4D+4}\left[x\sin x + e^{2x} + 3\right]$$

$$y_p = \frac{x\sin x}{D^2-4D+4} + \frac{e^{2x}}{D^2-4D+4} + \frac{3}{D^2-4D+4}.$$

$$y_p = y_{p_1} + y_{p_2} + y_{p_3} \longrightarrow ①$$

$$y_{p_1} = \frac{1}{D^2-4D+4} x\sin x.$$

$$\left\{ P.I = \frac{1}{f(D)} x.V = \left[x - \frac{f'(D)}{f(D)}\right]\frac{1}{f(D)}.V \right\}$$

$$= \left[x - \frac{2D-4}{D^2-4D+4}\right]\frac{1}{D^2-4D+4}\sin x$$

$$= \left[x - \frac{2D-4}{D^2-4D+4}\right]\frac{\sin x}{-1^2-4D+4} \quad \{\text{put } D^2=-1^2\}$$

$$y_{P_1} = \left[x - \frac{2D-4}{D^2-4D+4}\right]\frac{\sin x}{3-4D}$$

$$y_{P_1} = \left[x - \frac{2D-4}{D^2-4D+4}\right]\frac{3+4D}{(3-4D)(3+4D)}\sin x$$

$$y_{P_1} = \left[x - \frac{2D-4}{D^2-4D+4}\right]\frac{3+4D}{3^2-(4D)^2}\sin x$$

$$y_{P_1} = \left[x - \frac{2D-4}{D^2-4D+4}\right]\frac{3+4D}{9-16(-1^2)}\sin x$$

$$y_{P_1} = \frac{1}{25}\left[x - \frac{2D-4}{D^2-4D+4}\right](3+4D)\sin x$$

$$y_{P_1} = \frac{1}{25}\left[x - \frac{2D-4}{D^2-4D+4}\right][3\sin x + 4D\sin x]$$

$$y_{P_1} = \frac{1}{25}\left[x - \frac{2D-4}{D^2-4D+4}\right][3\sin x + 4\cos x] \qquad \left\{D\sin x = \frac{d}{dx}\sin x = \cos x\right\}$$

$$y_{P_1} = \frac{1}{25}\left[x[3\sin x + 4\cos x] - \frac{2D-4}{D^2-4D+4}(3\sin x + 4\cos x)\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x - \left[\frac{2D[3\sin x + 4\cos x] - 4[3\sin x + 4\cos x]}{D^2-4D+4}\right]\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x - \left[\frac{6D\sin x + 8D\cos x - 12\sin x - 16\cos x}{D^2-4D+4}\right]\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x - \left[\frac{6\cos x + 8(-\sin x) - 12\sin x - 16\cos x}{D^2-4D+4}\right]\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x - \left[\frac{-10\cos x - 20\sin x}{D^2-4D+4}\right]\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x - (-10)\left[\frac{\cos x + 2\sin x}{D^2-4D+4}\right]\right]$$

$$y_{P_1} = \frac{1}{25}\left[3x\sin x + 4x\cos x + 10\frac{1}{D^2-4D+4}(\cos x + 2\sin x)\right] \{\text{put } D^2 = -1^2\}$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x + 10 \cdot \frac{1}{-1^2-4D+4}(\cos x + 2\sin x)\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x + 10\frac{1}{3-4D}(\cos x + 2\sin x)\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x + 10\frac{(3+4D)}{(3-4D)(3+4D)}[\cos x + 2\sin x]\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x + 10\frac{(3+4D)}{9-16D^2}(\cos x + 2\sin x)\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x + \frac{\overset{2}{\cancel{10}}(3+4D)}{\underset{5}{\cancel{25}}}(\cos x + 2\sin x)\right] \text{ put } D^2 = -1$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x + \frac{2}{5}[3\cos x + 6\sin x + 4D\cos x + 8D\sin x]\right]$$

$$= \frac{1}{25}\left[3x\sin x + 4x\cos x + \frac{2}{5}[3\cos x + 6\sin x - 4\sin x + 8\cos x]\right]$$

$$\boxed{y_{P_1} = \frac{1}{25}\left[3x\sin x + 4x\cos x + \frac{2}{5}[11\cos x + 2\sin x]\right]}$$

$$\begin{Bmatrix} D\sin x = \cos x \\ D\cos x = -\sin x \end{Bmatrix}$$

$$y_{P_2} = \frac{e^{2x}}{D^2-4D+4}$$

$$= \frac{1}{(D-2)^2}e^{2x} \qquad \left\{\frac{1}{(D-a)^2}e^{ax} = \frac{x^2}{2!}e^{ax}\right\}$$

$$y_{P_2} = \frac{x^2}{2!}e^{2x}$$

$$\boxed{y_{P_2} = \frac{x^2}{2}e^{2x}}$$

$$y_{P_3} = \frac{1}{(D-2)^2}3.e^{0.x} = \frac{1}{(0-2)^2}3e^{0.x} = \frac{1}{(-2)^2}3.(1) = \frac{3}{4}$$

$$\boxed{y_{P_3} = 3/4}$$

$$y_P = \frac{1}{25}\left[3x\sin x + 4x\cos x + \frac{2}{5}[11\cos x + 2\sin x]\right] + \frac{x^2 e^{2x}}{2} + \frac{3}{4}$$

(5) Hence the general solution is

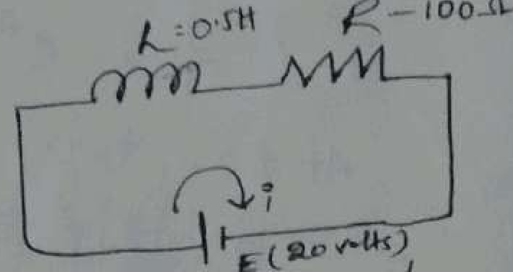
$$y = y_c + y_p.$$

$$y = (c_1 + c_2 x)e^{2x} + \frac{1}{25}[3x\sin x + 4x\cos x] + \frac{2}{125}[11\cos x + 2\sin x] + \frac{x^2 e^{2x}}{2} + \frac{3}{4}$$

*

① A resistance of 100 ohms an inductance of 0.5 Henry is connected in Series with a battery of 20 volts. Find the current in the circuit if initially there is no current in the circuit

Sol Let i be the current flowing in the circuit at any time t.



LR circuit

By Kirchoff's Law

$$E = R.i + L.\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}.i = \frac{E}{L}$$

Given $R = 100\Omega$
$L = 0.5$ Henry
$E = 20$ volts.

$$\frac{di}{dt} + \left(\frac{100}{0.5}\right) i = \frac{20}{0.5}$$

$$\frac{di}{dt} + 200i = 40 \longrightarrow ①$$

This is of the form $\frac{dy}{dx} + Py = Q$

$P = 200$, $Q = 40$

$$\Rightarrow I.F = e^{\int P dt} = e^{\int 200 dt} = e^{200t}$$

$$\boxed{I.F = e^{200t}}$$

Solution $i \times I.F = \int Q.(I.F) dt + c$

$$i \times e^{200t} = \int 40 e^{200t} dt + c$$

$$i e^{200t} = 40 \int e^{200t} dt + c$$

$$i e^{200t} = 40 . \frac{e^{200t}}{200} + c$$

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$$i . e^{200t} = \frac{1}{5} e^{200t} + c$$

$$i = \frac{1}{5} \cdot \frac{e^{200t}}{e^{200t}} + \frac{1}{e^{200t}}$$

$$i = \frac{1}{5} + c\, e^{-200t}$$

At $t = 0,\ i = 0$

$$0 = \frac{1}{5} + c \cdot e^{-200(0)}$$

$$0 = \frac{1}{5} + c$$

$$\boxed{c = -1/5}$$

$$\Rightarrow \quad i = \frac{1}{5} + \left(-\frac{1}{5}\right) e^{-200t}$$

$$\boxed{i = \frac{1}{5}\left[1 - e^{-200t}\right]}$$