UNITS & MEASUREMENTS

1. In a given system of units, 1 unit of mass = 2kg, 1 unit of length = 5m and 1 unit of time = 5s then in this system, 1N represents
   1) $\frac{5}{2}$ units of force 2) $\frac{2}{5}$ units of force 3) 2 units of force 4) $\frac{1}{2}$ units of force
2. The length of a linear rod is measured as 100.1cm. the approximate error in the measurement
   1) ±0.2% 2) ±0.5% 3) ±0.3% 4) ±0.1%
3. In the relation $P = \frac{\alpha}{\beta} = e^{\frac{-\alpha x}{nR\theta}}$, P is power, X is distance, n is number of moles, R is a gas constant and θ is temperature. The dimension of formula of β is
   1) $\left[M^0L^0T^0\right]$ 2) $\left[M^1L^0T^1\right]$ 3) $\left[M^0L^{-1}T^1\right]$ 4) $\left[M^0L^{-1}T^{-1}\right]$
4. Power delivered by a force is given by the relation $P = \frac{\alpha}{\beta}e^{-\beta t}$**,** where t is time, find the dimensional formula for α
   1) $\left[M^1L^2T^{-4}\right]$ 2) $\left[M^1L^2T^4\right]$ 3) $\left[M^{-1}L^{-2}T^4\right]$ 4) $\left[M^1L^{-2}T^4\right]$
5. Considering density, frequency and velocity as fundamental quantities, find the dimensions of angular momentum
   1) $[\rho]^2[f]^{-4}[v]^5$ 2) $[\rho]^2[f]^{-4}[v]^5$ 3) $[\rho][f]^4[v]^{-5}$ 4) $[\rho][f]^{-4}[v]^{-5}$
6. Solar constant and Stefan's constant have same dimensions in
   1) Mass 2) Length 3) Time 4) All the above
7. $\frac{he}{4\pi M}$ has the same dimensions as (h-plank's constant, e= change, m=mass)
   1) Magnetic moment 2) Magnetic induction 3) Angular momentum 4) Pole strength
8. Magnetic induction and magnetic flux differ in the dimensions of
   1) Mass 2) Electric current 3) Lengths 4) Time
9. Three of the quantities defined below has the same dimensional formula. Identify the
   i) $\sqrt{Energy / mass}$ ii) $\sqrt{pressure / density}$ ii) $\sqrt{force / linear\ density}$
   1) ii only 2) ii and iii only 3) iii only 4) i, ii and iii only
10. If $e, E_0, h\ and\ c$ respectively represents electronic charge, permittivity of free space, planks constant and speed of light then $\frac{e^2}{E_0hc}$ has the dimensions of
    A) angle B) relative density C) strain D) current
    1) A,B 2) D 3) A,B,C 4) A,B,C,D
11. If density (D), acceleration (a) and force are taken as basic quantities then the time period has dimensions
    1) $\frac{1}{6} in$ F 2) $\frac{-1}{6} in$ F 3) $\frac{2}{3} in$ F 4) All of the above
12. The liquid drop of density ρ, radius r and surfaces tension σ oscillates with time period T. which of the following expression for T^2 is correct
    1) $\rho r^3 / \sigma$ 2) $\rho\sigma / r^3$ 3) $r^3\sigma / \rho$ 4) none

13. The position of a particle at time 't' is given by the equation : $X(t)=\frac{V_0}{A}(1-e^{At})$

$V_0 = constant\ and\ A>0$

Dimensions of V_0 and A respectively are;

1) $M^0LT^0\ and\ T^{-1}$ 2) $M^0LT^{-1}\ and\ LT^{-2}$ 3) $M^0LT^{-1}\ and\ T$ 4) $M^0LT^{-1}\ and\ T^{-1}$

14. If L is the inductance, C is the capacitance and R is the resistance, then $R\sqrt{\frac{C}{L}}$ has the dimensions

1) $MLT^{-2}I^{-2}$ 2) ML^2T^2I 3) $ML^{-1}T^{-2}I^{-1}$ 4) $M^0L^0T^0I^0$

15. If units in two systems of measurements are in the ratio of the units of angular momentum in those two systems is

1) $2:3$ 2) 9:4 3) 1:1 4) 4:9

16. The reverberation period T of a room on its volume V, its surface area A and velocity of sound C, if K is dimensionless constant then T=

1) $\frac{KV}{CA}$ 2) $\sqrt{\frac{KV}{CA}}$ 3) $\frac{KV}{C\sqrt{A}}$ 4) $\frac{K}{C}\sqrt{VA}$

17. If the vibration frequency of atoms in a crystal depends on the atomic mass m, the atomic spacing r and compressibility c, then it is proportional to

1) $m^{1/2}$ 2) $r^{1/2}$ 3) C 4) all the above

18. The time period f a soap bubble is T which depends on pressure P density d and surface tension S then the relation for T is

1) $T\propto pds$ 2) $T\propto p^{-3/2}d^{1/2}s$ 3) $T\propto pd^{-2}s^{3/2}$ 4) $T\propto pd^2s^{1/3}$

19. If the velocity is represented by V, force by F and work by W. These quantities are taken as fundamental quantities then the correct dimensions of mass will be

1) W/V^2 2) W/V 3) F/V^2 4) V/W

20. Velocity of sound medium is given by $A^{1/2}d^{-1/2}$ where 'd' is density of medium. Then A represents

1) Elastic constant 2) Energy 3) Volume 4) Compressibility

21. A quantity X is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of a free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

1) resistance 2) charge 3) voltage 4) current

22. In a simple pendulum experiment, the maximum percentage error in the measurement of length is 2% and that in the observation of the time period is 3%. Then the maximum percentage error in determination of the acceleration due to gravity g is

1) 5% 2) 6% 3) 1% 4) 8%

23. The frequency of vibration of string is given by $V=\frac{p}{2l}\left[\frac{F}{m}\right]^{1/2}$. Here P is number of segment in the string and l is the length. The dimensional formula for m will be

1) $[M^0LT^{-1}]$ 2) $[ML^0T^{-1}]$ 3) $[ML^{-1}T^0]$ 4) $[M^0L^0T^0]$

24. Number of particles crossing unit area perpendicular to X-axis in unit time is given by $n=-D\frac{n_2-n_1}{x_2-x_1}$, where $n_1\ and\ n_2$ are number of particles per unit volume in the position $x_1\ and\ x_2$. Find dimensions of D called as diffusion constant

1) $[M^0LT^2]$ 2) $[M^0L^2T^{-4}]$ 3) $[M^0LT^{-3}]$ 4) $[M^0L^2T^{-1}]$

25. $X = 3YZ^2$ find dimensions of Y in (MKSA) system, if X and Z are the dimension of capacity and magnetic field respectively

1) $\left[M^{-3}L^{-2}T^{-4}A^{-1}\right]$ 2) $\left[ML^{-2}\right]$ 3) $\left[M^{-3}L^{-2}T^{4}A^{4}\right]$ 4) $\left[M^{-3}L^{-2}T^{8}A^{4}\right]$

26. The equation of a wave is given by : $Y = A\sin\omega\left[\frac{x}{v} - K\right]$ where ω is the angular velocity and v is the linear velocity. The dimension of K is :

1) LT 2) T 3) T^{-1} 4) T^2

27. If $V = \sqrt{\frac{rP}{P}}$, then dimensions of γ are :

1) $\left[M^0L^0T^0\right]$ 2) $\left[M^0L^0T^{-1}\right]$ 3) $\left[M^1L^0T^0\right]$ 4) $\left[M^0L^1T^0\right]$

28. Suppose refractive index μ is given as $\mu = A + \frac{\beta}{\lambda^2}$ where A and B are constant and λ is the wavelength, then dimension of B are same as that of:

1) wave length 2) volume 3) pressure 4) Area

29. The quantities A and B are related by the relation, $m = A/B$, where m is the linear density and A is the force. The dimensions of B are of

1) Pressure 2) work 3) latent heat 4) none of the above

30. If Q denote the charge on the plate of a capacitor of capacitance C then the dimensional formula for Q^2/C is

1) $\left[L^2M^2T\right]$ 2) $\left[LMT^2\right]$ 3) $\left[L^2MT^{-2}\right]$ 4) $\left[L^2M^2T^2\right]$

KEY

1-10	1	4	3	1	2	4	1	3	4	3
11-20	1	1	4	4	4	1	2	2	1	1
21-30	4	4	3	4	4	2	1	4	3	3

SOLUTIONS

1. 1N can be represented as kg m/s^2

$$1N = \frac{[M][L]}{[T]^2}$$

$$1N = \frac{(1kg)(1m)}{[1s]^2} = \frac{\left(\frac{1}{2}unit\ of\ mass\right)\left(\frac{1}{5}unit\ of\ length\right)}{\left(\frac{1}{5}unit\ of\ time\right)^2}$$

$$1N = \frac{1}{2}\times\frac{1}{5}\times\frac{5}{1}\times\frac{5}{1} \text{ unit of force}$$

$$1N = \frac{5}{2} \text{ unit of force}$$

2. Length = 100.1cm

So, looking the length, we can say it is measured with a instrument of least count = 0.1cm

Maximum error in reading = least count = 0.1cm,

$\Delta x = 0.1cm$

% error in reading = $\frac{\Delta x}{x} \times 100$

$= \frac{0.1}{100.1} \times 100$

$= 0.1\%$

3. In exponential function, exponent must be a number $\frac{\alpha x}{ne\theta} = \dim ensionless = 1$

$\Rightarrow [\alpha] = \frac{[nR\theta]}{[x]} = \frac{[n][R][\theta]}{[x]}$

$\Rightarrow [\alpha] = \frac{mol \times Jmol^{-1}K^{-1}.k}{L}$

$\Rightarrow [\alpha] = [JL^{-1}]$

$[P] = \frac{[\alpha]}{\beta} \Rightarrow \beta = \frac{[\alpha]}{[P]}$

$\Rightarrow \beta = \frac{[J\,L^{-1}]}{[J.T^{-1}]}$

$= [M^0 L^{-1} T^1]$

4. By principal of homogeneity and power to e ha to be dimensionless. We get

$[\beta t] = [M^0 L^0 T^0]$

$\Rightarrow [\beta] = [M^0 L^0 T^{-1}]$

$P = \frac{\alpha}{\beta} e^{-\beta t}$

$[M^1 L^2 T^{-3}] = \frac{\alpha}{[T^{-1}]} \times [M^0 L^0 T^0]$

$\alpha = [M^1 L^2 T^{-4}]$

5. $[L] = [\rho]^a [f]^b [V]^c$

$[M\,L^2 T^{-1}] = [M\,L^{-3}]^a [T^{-1}]^b [LT^{-1}]^c$

$[M\,L^2 T^{-1}] = [M^a\, L^{-3a+c} T^{-b-c}]$

Solving a=1, c=5, b=-4

$[L] = [\rho][f]^{-4}[V]^5$

6. Solar constant = $\frac{Energy}{area \times time} = \frac{[M^1 L^2 T^{-2}]}{[L^2] \times [T]}$

$= [M^1 L^0 T^{-3}]$

Stefan's constant = $\dfrac{\Delta E}{\Delta A \Delta T \theta 4} = \dfrac{M^1L^2T^{-2}}{\left[L^2\right]\left[T^1\right]\left[K^4\right]}$

$$\left[\sigma\right] = \left[M^1L^0T^{-3}K^{-4}\right]$$

7. $\dfrac{he}{4\pi m} = \dfrac{\left[M^1L^2T^{-1}\right]\left[M^0L^0T^1A^1\right]}{\left[M^1\right]} = \left[M^0L^2T^0A^1\right]$

Magnetic moment (M) = $2l \times m$

$$\left[M\right] = \left[M^0L^2T^0A^1\right]$$

8. Magnetic induction $\left(\vec{B}\right) = \dfrac{\phi}{A}$

$\left[B\right] = \left[M^1LT^{-2}A^{-1}\right]$

Magnetic flux $\left(\phi\right) = B \times A$

$$= \left[M^1L^2T^{-2}A^{-1}\right]$$

9. i) $\sqrt{\dfrac{Energy}{mass}} = \sqrt{\dfrac{M^1L^2T^{-2}}{M^1}} = \sqrt{L^2T^{-2}} = \left[L^1T^{-1}\right]$

ii) $\sqrt{\dfrac{\Pr essure}{density}} = \sqrt{\dfrac{M^1L^{-1}T^{-2}}{M^1L^{-3}}} = \sqrt{L^2T^{-2}} = \left[L^1T^{-1}\right]$

iii) $\sqrt{\dfrac{Force}{linear\ density}} = \sqrt{\dfrac{M^1L^1T^{-2}}{M^1L^{-1}}} = \sqrt{L^2T^{-2}} = \left[L^1T^{-1}\right]$

10. $\dfrac{e^2}{E_0h_c} = \dfrac{\left[M^0L^0T^1A^1\right]^2}{\left[M^{-1}L^{-3}T^4A^2\right]\left[M^1L^2T^{-1}\right]\left[L^1T^{-1}\right]}$

$= \dfrac{\left[T^2A^2\right]}{\left[T^2A^2\right]} = \left[M^0L^0T^0\right]$

A) Angle = $\dfrac{arc.length}{radius} = \dfrac{\left[M^0L^1T^0\right]}{\left[M^0L^1T^0\right]} = \left[M^0L^0T^0\right]$

B) Relative density = $\left[M^0L^0T^0\right]$

C) Strain = $\dfrac{change\ in\ \dim ention}{original \dim enstions} = \left[M^0L^0T^0\right]$

D) Current = $\dfrac{q}{T} = \left[M^0L^0T^0A^1\right]$

11. $\left[T\right] = \left[D\right]^x\left[a\right]^y\left[F\right]^z$

$\left[M^0L^0T^1\right] = \left[M^1L^{-3}\right]^x\left[LT^{-2}\right]^y\left[M^1L^1T^{-2}\right]^z$

Comparing $x + z = 0, -3x + y + z = 0, -2y - 2z = 1$

$x = -2, y = -2/3, z = 116$

$T=(D)^{-2}(a)^{-2/3}[F]^{1/6}$

$\boxed{1/6\ in\ F}$

12. $T=r^a\,\sigma^b\rho^c$

$[T]=[L]^a\left[MT^{-2}\right]^b\left[M\,L^{-3}\right]^c$

$\boxed{b=-1/2}, c=1/2, a=3/2$

$T=r^{3/2}\,\sigma^{-1/2}\,\rho^{1/2}$

$$T=\sqrt{\frac{r^3\rho}{\sigma}}$$

$$\boxed{T^2=\frac{r^3\rho}{\sigma}}$$

13. $x(t)=\dfrac{V_0}{A}\left(1-e^{At}\right)$

$[At]=cons\tan t=1$

$$A=\frac{1}{T}=\left[T^{-1}\right]$$

$$x(t)=\frac{V_0}{A}$$

$V_0=x(t)\times A$

$[V_0]=[L]\times\left[T^{-1}\right]$

$[V_0]=\left[LT^{-1}\right]$

14. $\sqrt{\dfrac{C}{L}}=\left[M^1L^2T^{-3}A^{-2}\right]\times\sqrt{\dfrac{M^{-1}L^{-2}T^4A^2}{M\,L^2T^{-2}A^{-2}}}$

$=\left[M^1L^2T^{-3}A^{-2}\right]\times\left[M^{-2}L^{-4}T^6A^4\right]$

$=\left[M^1L^2T^{-3}A^{-2}\right]\times\left[M^{-1}L^{-2}T^3A^2\right]$

$=\left[M^0L^0T^0A^0\right]$

15. Angular momentum

Units $=kg\,m^2$ / sec

$$\Rightarrow\frac{\frac{2}{3}\times\left(\frac{2}{3}\right)^r}{2/3}=\frac{4}{9}$$

16. $T=V^aA^bC^c$

$[T]=\left[L^3\right]^a\left[L^2\right]^b\left[LT^{-1}\right]^c$

$c=-1, b=-1, a=+1$

$T=V^1A^{-1}C^{-1}$

$$T=\frac{V}{AC},\quad k\to cons\tan t$$

$$T = \frac{KV}{AC}$$

17. $f = M^a r^b c^c$

$$\left[T^{-1}\right] = \left[M\right]^a \left[L\right]^b \left[M^{-1}LT^2\right]^c$$

$$a = c = -1/2$$

$$b = 1/2$$

$$f \propto M^{-1/2} r^{1/2} C^{-1/2}$$

$$f \propto r^{1/2}$$

18. $T \propto P^a d^b S^c$

$$\left[T^1\right] = \left[M^1 L^{-1} T^{-2}\right]^a \left[M^1 L^{-3}\right]^b \left[M^1 T^{-2}\right]^c$$

$$\left[M^0 L^0 T^{-1}\right] = \left[M^{a+b+c} L^{-a-3b} T^{-2a-2c}\right]$$

On comparing

$$a + b + c = 0, -a - 3b = 0, -2a - 2c = 1$$

$$b = 1/2, a = -3/2, c = 1$$

$$T \propto P^{-3/2} d^{1/2} S^1$$

19. $M \propto V^a F^b W^c$

$$\left[M^1\right] = \left[L^1 T^{-1}\right]^a \left[M^1 L^1 T^{-2}\right]^b \left[M^1 L^2 T^{-2}\right]^c$$

$$b = 0, c = 1, a = -2$$

$$M = V^{-2} F^0 W^1$$

$$M = W / V^2$$

20. $V \propto A^{1/2} d^{-1/2}$

$$\left[V\right] = \left[A\right]^{1/2} \left[d\right]^{-1/2}$$

$$\left[A\right]^{1/2} = \left[V\right]\left[d\right]^{-1/2}$$

$$\left[A\right] = \left[V\right]^2 \left[d\right]^1$$

$$\left[A\right] = \left[M^1 L^{-1} T - 2\right]$$

Elastic constant $= \dfrac{stress}{strain}$

$$E = \left[M^1 L^{-1} T^{-2}\right]$$

21. $x = \varepsilon_0 L \dfrac{\Delta V}{\Delta t}$

$$x=\frac{\left[M^{-1}L^{-3}T^{4}A^{2}\right]\left[L\right]\left[M^{1}L^{2}T^{-3}A^{-1}\right]}{\left[T^{1}\right]}$$

$$x=\left[A\right]$$

x = current

22. $T=2\pi\sqrt{\frac{L}{g}}$

$$T^2=4\pi^2\frac{L}{g}$$

$$g=4\pi^2\frac{L}{T^2}$$

The maximum percentage error $\frac{\Delta g}{g}=\frac{\Delta L}{L}+2\frac{\Delta T}{T}$

$$\frac{\Delta g}{g}=2\%+2\times 3$$

$$=2\%+6\%$$

$$\frac{\Delta g}{g}=8\%$$

23. $V=\frac{p}{2l}\left(\frac{F}{m}\right)^{1/2}$

Squaring the equation on either side,

$$V^2=\frac{P^2}{4l^2}\left(\frac{F}{m}\right)$$

$$\Rightarrow m=\frac{p^2F}{4l^2v^2}$$

$$\Rightarrow m=\frac{\left[ML^{1}T^{-2}\right]}{\left[L^{2}\right]\left[T^{-1}\right]^{2}}$$

$$\Rightarrow m=\left[M^{1}L^{-1}T^{0}\right]$$

24. n=no.of particle per unit area per unit time n_1 & n_2 = no.of particle per unit volume x_1 & x_2 are distance from some reference point. Dimension of ‘n’ = $\left[L^{-2}T^{-1}\right]$

Dimensional formula of n_1 & $n_2=\left[L^{-3}\right]$

Dimensional formula of x_1 & $x_2=\left[L\right]$.

$$n = -d\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$$

$$D = -n\left(\frac{x_2 - x_1}{n_2 - n_1}\right)$$

$$D = \frac{\left[L^{-2}T^{-1}\right]\left[L\right]}{\left[L^{-2}\right]}$$

$$\boxed{D = \left[M^0L^2T^{-1}\right]}$$

25. $x = 3yz^2$

Let $y = M^aL^bT^cQ^d$

$x = 3yz^2$

$$\Rightarrow y = \frac{x}{3z^2}$$

$$\Rightarrow y = \frac{\left[M^{-1}L^{-2}A^2T^4\right]}{\left[M^1A^{-1}T^{-2}\right]^2}$$

$Q = AT$

$$y = \left[M^{-3}L^{-2}T^{4Q^4}\right]$$

26. $\dfrac{x}{V}$ have the same dimension as of K.

Dimension formula of K=$\dfrac{x}{V}$

$$K = \frac{[L]}{\left[LT^{-1}\right]} = [T]$$

27. $V = \sqrt{\dfrac{r\rho}{\rho}}$

$$\Rightarrow r = \frac{r\rho}{\rho}$$

$$\Rightarrow r = \frac{\left[LT^{-1}\right]^2\left[M^1L^{-3}\right]}{\left[M^1L^{-1}T^{-2}\right]}$$

$$r = \left[M^0L^0T^0\right]$$

28. $\mu = A + \dfrac{B}{\lambda^2}$

$\mu \rightarrow$ dimension less quantity according to principle of homogeneity

$$\mu = \frac{B}{\lambda^2}$$

$$B = \lambda^2$$

$$B = L^2$$

$L^2 \rightarrow$ dimension of area

29. $m \rightarrow \left[M^1 L^{-1}\right]$

$$A \rightarrow \left[M^1 L^1 T^{-2}\right]$$

$$m = A / B$$

$$\Rightarrow B = A / m$$

$$\Rightarrow B = \frac{\left[M^1 L^1 T^{-2}\right]}{\left[M^1 L^{-1}\right]}$$

$$\Rightarrow B = \left[L^2 T^{-2}\right]$$

$\left[L^2 T^{-2}\right] \rightarrow$ dimensional formula of latest heat.

30. The term $\frac{Q^2}{2c}$ is the formula of energy stored in a capacitor so it has the dimension of energy. So,

$$\frac{Q^2}{c} = \left[M^1 L^2 T^{-2}\right]$$