ELLIPSE –

1. The Line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2+4y^2=4$ meets its auxiliary circle at the point M. Find the area of the triangle with vertices at A, M and the origin O.
a) $\frac{6}{5}$ b) $\frac{2}{5}$ c) $\frac{8}{5}$ d) $\frac{4}{5}$

2. If $x-2y+4=0$ is a tangent to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$, for some $b \in R$ then the distance between the foci of the ellipse is
a) 1 b) 2 c) 4 d) 6

3. Find the equation of the ellipse whose axes are the axes of coordinates and which passes through the point $\left(1,\frac{-3}{2}\right)$ and has eccentricity $\frac{1}{2}$
a) $3x^2-4y^2=12$ b) $-3x^2+4y^2=12$ c) $3x^2+4y^2=-12$ d) $3x^2+4y^2=12$

4. In an ellipse, the distance between its foci is 10 and minor axis is $4\sqrt{5}$. Then its eccentricity is.
a) $\frac{4}{\sqrt{5}}$ b) $\frac{-5}{3}$ c) $\frac{3}{\sqrt{5}}$ d) $\frac{\sqrt{5}}{3}$

5. The distance between the foci of an ellipse is 4 and the distance between its directrices is 16 then the length of its latus rectum is
a) 16 b) 6 c) 4 d) 12

6. If the line $x-2y=12$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,-\frac{9}{2}\right)$, then the length of the latus rectum of the ellipse is
a) 5 units b) $12\sqrt{2}$ units c) 9 units d) $8\sqrt{3}$ units

7. An ellipse has foci $(4,2),(2,2)$ and it passes through the point $P(2,4)$. The eccentricity of the ellipse is
a) $\tan\frac{\pi}{10}$ b) $\tan\frac{\pi}{12}$ c) $\tan\frac{\pi}{6}$ d) $\tan\frac{\pi}{8}$

8. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2}+\frac{y^2}{4a^2}=1$ and the co-ordinate axis is kab, then k is equal to
a) 1 b) 2 c) 3 d) 4

9. If the eccentricity of the ellipse $\frac{x^2}{a^2+1}+\frac{y^2}{a^2+2}=1$ is $\frac{1}{\sqrt{6}}$, then the latus rectum of the ellipse is
a) $\frac{5}{\sqrt{6}}$ b) $\frac{10}{\sqrt{6}}$ c) $\frac{8}{\sqrt{6}}$ d) none of these

10. The length of the major axis of the ellipse $(5x-10)^2+(5y+15)^2=\frac{(3x-4y+7)^2}{4}$ is
a) 10 b) $\frac{20}{3}$ c) $\frac{20}{7}$ d) 4

11. If the ellipse $\frac{x^2}{a^2-7}+\frac{y^2}{13-5a}=1$ is inscribed in a square of side length $\sqrt{2}a$, then a is equal to

a) $\frac{6}{5}$

b) $\left(-\infty,-\sqrt{7}\right)\cup\left(\sqrt{7},\frac{13}{5}\right)$

c) $\left(-\infty,-\sqrt{7}\right)\cup\left(\frac{13}{5},\sqrt{7}\right)$

d) no such a exists

12. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is inscribed in a rectangle whose length to breadth ratio is 2 : 1, then the area of the rectangle is

a) $4.\frac{a^2+b^2}{7}$ b) $4.\frac{a^2+b^2}{3}$ c) $12.\frac{a^2+b^2}{5}$ d) $8.\frac{a^2+b^2}{5}$

13. An ellipse passing through the origin has its foci (3, 4) and (6, 8). The length of its semi-minor axis is b. then the value of $b/\sqrt{2}$ is _________

14. Let the distance between a focus and the corresponding directrix of an ellipse be 8 and the eccentricity be ½. If the length of the minor axis is k, then $\sqrt{3}k/2$ is

15. The curve represented by $x=8(\cos t+\sin t), y=5(\cos t-\sin t)$, (where t is parameter) is

a) circle b) a parabola c) an ellipse 4) A hyperbola

16. If $M_1\ and\ M_2$ are the feet of the perpendiculars from the foci $S_1\ and\ S_2$ of the ellipse $\frac{x^2}{9}+\frac{y^2}{10}=1$ on the tangent at a point P on the ellipse, then $\frac{x^2}{9}+\frac{y^2}{10}=1$ on the tangent at a point P on the ellipse, then $(S_1M_1)(S_2M_2)$ is equal to

a) 16 b) 9 c) 4 d) 3

17. The curve represented by the equation $4x^2+16y^2-24x-32y-12=0$ is

a) a parabola

b) a pair of straight lines

c) an ellipse with eccentricity $\frac{1}{2}$

d) an ellipse with eccentricity $\frac{\sqrt{3}}{2}$

18. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is

a) 27 b) $\frac{27}{4}$ c) 18 d) $\frac{27}{2}$

19. If the tangent at a point on the ellipse $\frac{x^2}{27}+\frac{y^2}{3}=1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is

a) $3\sqrt{3}$ b) $\frac{9}{2}$ c) 9 d) $9\sqrt{3}$

20. The centre of the ellipse $\frac{(x+y-3)^2}{9}+\frac{(x-y+1)^2}{16}=1$ is

a) $(-1,2)$ b) $(1,-2)$ c) $(-1,-2)$ d) $(1,2)$

21. The locus of the point which divides the double ordinates of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the ratio 1 : 2 internally is

a) $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$ b) $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=\frac{1}{9}$ c) $\frac{9x^2}{a^2}+\frac{9y^2}{b^2}=1$ d) none of these

22. The equation of the ellipse whose axes are coincident with the coordinates axes and which touches the straight lines $3x-2y-20=0\,and\,x+6y-20=0$ is

a) $\frac{x^2}{40}+\frac{y^2}{10}=1$ b) $\frac{x^2}{5}+\frac{y^2}{8}=1$ c) $\frac{x^2}{10}+\frac{y^2}{40}=1$ d) $\frac{x^2}{40}+\frac{y^2}{30}=1$

23. Given two fixed points A and B and $AB=4$. Then simplest form of the equation to the locus of P such that $PA+PB=8$ is

a) $\frac{x^2}{16}+\frac{y^2}{12}=1$ b) $\frac{x^2}{16}+\frac{y^2}{9}=1$ c) $\frac{x^2}{9}+\frac{y^2}{16}=1$ d) $\frac{x^2}{12}+\frac{y^2}{21}=1$

24. The ellipse $9x^2+16y^2=144$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (8,0). Then the equation of the ellipse is:

a) $\frac{x^2}{12}+\frac{y^2}{4}=1$ b) $\frac{x^2}{64}+\frac{y^2}{12}=1$ c) $\frac{x^2}{36}+\frac{y^2}{12}=1$ d) $\frac{x^2}{12}+\frac{y^2}{64}=1$

25. P and Q are points with eccentric angles θ and $\left(\theta+\frac{\pi}{6}\right)$ on the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$ then the area (in sq. units) of the triangles OPQ (where O is the origin) is equal to

a) 4 b) 8 c) 2 d) 6

26. The auxiliary circle of a family of ellipses passes through the origin and makes intercepts of 8 and 6 units on the $x-$ an the y-axis, respectively. If the eccentricity of all such ellipses is $\frac{1}{2}$, then the locus of the focus will be

a) $\frac{x^2}{16}+\frac{y^2}{9}=25$

b) $4x^2+4y^2-32x-24y+75=0$

c) $\frac{x^2}{16}+\frac{y^2}{9}=25$

4) none of these

27. Number of points on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ from which pair of perpendicular tangents are drawn to the ellipse $3x^2+4y^2=12$ is

a) 4 b) 2 c) 9 d) 6

28. PSP^1 is focal chord of the ellipse $x^2+4y^2=4$ if $SP=2\;then\;7SP^1=$

a)16 b) 2 c)4 d) 8

29. The distances from the foci to a point $P(x_1,y_1)$ on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ are

a) $4\pm\frac{2}{3}y_1$ b) $5\pm\frac{4}{5}y_1$ c) $5\pm\frac{4}{5}x_1$ d) $4\pm\frac{\sqrt{7}y_1}{4}$

30. Length Latusrecpum to the ellipse $4(x-2y+1)^2+9(2x+y+2)^2=180$

a) $\frac{8}{3}$ b) 6 c) $\frac{4}{3}$ d) $\frac{2}{3}$

KEY

	1	2	3	4	5	6	7	8	9	10
1-10	**c**	**b**	**d**	**d**	**b**	**c**	**d**	**b**	**b**	**b**
11-20	**d**	**d**	**5**	**8**	**c**	**b**	**d**	**a**	**c**	**d**
21-30	**a**	**a**	**a**	**b**	**c**	**b**	**a**	**b**	**d**	**a**

SOLUTIONS :

1. Given ellipse $x^2+4y^2=4$

$\frac{x^2}{4}+y^2=1$

$\therefore a=2, b=1$

So $A=(2,0), B=(0,1)$

Equation of circle $x^2+y^2=4$

Slope of AB = $\frac{1-0}{0-2}=\frac{-1}{2}$

Slope of line n = $\frac{-1}{2}$

Equation of AB $y-y_1=n(x-x_1)$

$y-0=\frac{-1}{2}(x-2)$

$2y=-x+2$

$x+2y-2=0$

$x=-2y+2$

Put x value in circle equation $x^2+y^2=4$

$(2-2y)^2+y^2=4$

$4+4y^2-8y+y^2=4$

$5y^2-8y=0$

$y(5y-8)=0$

$y=0\, or\, y=\frac{8}{5}$

$\Rightarrow x=\frac{-6}{5}$

So $M=\left(\frac{-6}{5},\frac{8}{5}\right)$

We have $A=(2,0)\ M=\left(\frac{-6}{5},\frac{8}{5}\right) O=(0,0)$

Area of triangle AOM = $\frac{1}{2}|x_1y_2-x_2y_1|$

$=\frac{1}{2}\left|\left(\frac{-6}{5}\right)0-2\left(\frac{8}{5}\right)\right|=\frac{1}{2}\left|0-\frac{16}{5}\right|$

$= \frac{1}{2}.\frac{16}{5} = \frac{8}{5}$

2. Equation of ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$

Given $x-2y+4=0$ is tangent to ellipse

$\therefore n^2 = a^2l^2 + b^2m^2$

$16 = (4)(1)+b^2(4)$

$4b^2 = 16-4 = 12$

$b^2 = \frac{12}{4} = 3$

Eccentricity $e = \sqrt{\frac{a^2-b^2}{a^2}}$

$= \sqrt{\frac{4-3}{4}} = \sqrt{\frac{1}{4}}$

$e = \frac{1}{2}$

Distance between foci of ellipse in $2ae$

$\therefore 2(2)\left(\frac{1}{2}\right)$

= 2 units

3. Equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Given eccentricity $e = \frac{1}{2}$

Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through $\left(1,\frac{-3}{2}\right)$

$\therefore \frac{1^2}{a^2}+\frac{\left(\frac{-3}{2}\right)^2}{b^2}=1$

$\frac{1}{a^2}+\frac{9}{4b^2}=1 \Rightarrow \frac{1}{a^2}+\frac{9}{4\left[a^2\left(1-e^2\right)\right]}=1$

$\Rightarrow \frac{1}{a^2}+\frac{9}{4\left(a^2\left(\frac{9}{4}\right)\right)}=1$

$\Rightarrow \frac{1}{a^2}+\frac{3}{a^2}=1$

$\frac{4}{a^2}=1$

$a^2 = 4$

$\therefore b^2 = a^2\left(1-e^2\right)$

$= 4\left(1-\frac{1}{4}\right) = 4\left(\frac{3}{4}\right)$

$b^2 = 3$

Equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$

$3x^2+4y^2=12$

4. Given distance between foci is 10

$2ae=10$

$ae=5$ .....(1)

Length of minor axis is $4\sqrt{5}$

$2b=4\sqrt{5}$

$b=2\sqrt{5}$

$b^2=20$

$a^2\left(1-e^2\right)=20$

$a^2-a^2e^2=20$

$a^2-25=20$

$a^2=20+25=45$

Eccentricity $e=\sqrt{\dfrac{a^2-b^2}{a^2}}$

$=\sqrt{\dfrac{45-20}{45}}=\sqrt{\dfrac{25}{45}}$

$e=\dfrac{\sqrt{5}}{3}$

5. Given distance between foci is 4

$2ae=4$

$ae=2$......(1)

Distance between directrices is 16

$\dfrac{2a}{e}=16$

$\dfrac{a}{e}=8$ …….. (2)

$(1)\times(2)\Rightarrow ae\times\dfrac{a}{e}=8\times 2$

$a^2=16$

$a=4$

From (1) $ae=2$

$4e=2$

$e=\dfrac{1}{2}$

$b^2=a^2\left(1-e^2\right)$

$=16\left(1-\dfrac{1}{4}\right)=16\times\dfrac{3}{4}$

$b^2=12$

Length of latusrectum $\dfrac{2b^2}{a}=\dfrac{2(12)}{4}=6units$

6. $\because\left(3,-\dfrac{9}{2}\right) lies\, on \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\Rightarrow\dfrac{9}{a^2}+\dfrac{81}{4b^2}=1$ ......(1)

Equation of the tangent at $\left(3,-\dfrac{9}{2}\right) is \dfrac{3x}{a^2}+\dfrac{\left(-\dfrac{9}{2}y\right)}{b^2}=1$

& given equation of the tangent is : $x-2y=12 \Rightarrow \frac{x}{12}+\left(-\frac{y}{6}\right)=1$

On comparing these equations: $\frac{a^2}{3}=12 \Rightarrow a^2=36 \Rightarrow a=6$

$\frac{2b^2}{9}=6 \Rightarrow b^2=27 \Rightarrow b=3\sqrt{3}$

$\therefore$ Length of L.R = $\frac{2b^2}{a}=\frac{2\times 27}{6}=9$

7. Let (4,2) = S_1 $and\left(2,2\right)=S_2$ and eccentricity of ellipse is e

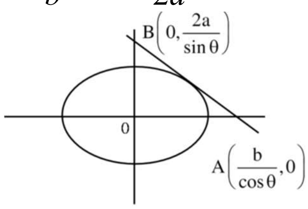
Then $S_1S_2=2ae\, and\, PS_1+PS_2=2a$ (where 2a is length of major axis)

$\Rightarrow e=\frac{S_1S_2}{PS_1+PS_2}=\frac{2}{2\sqrt{2}+2}$

$\Rightarrow e=\frac{1}{\sqrt{2}+1}=\sqrt{2}-1=\tan\frac{\pi}{8}$

8. Tangent

$\frac{x\cos\theta}{b}+\frac{y\sin\theta}{2a}=1$



So, area $\left(\Delta OAB\right)=\frac{1}{2}\times\frac{b}{\cos\theta}\times\frac{2a}{\sin\theta}$

$=\frac{2ab}{\sin 2\theta}\geq 2ab$

$\Rightarrow k=2$

9. Here, $a^2+2>a^2+1\, or\, a^2+1=\left(a^2+2\right)\left(1-e^2\right)$

Or $a^2+1=\left(a^2+2\right)\frac{5}{6}$

Or $6a^2+6=5a^2+10$

Or $a^2=10-6=4$ or $a=\pm 2$

Latus rectum $=\frac{2\left(a^2+1\right)}{\sqrt{a^2+2}}=\frac{2\times 5}{\sqrt{6}}=\frac{10}{\sqrt{6}}$

10. $\left(5x-10\right)^2+\left(5y+15\right)^2=\frac{\left(3x-4y+7\right)^2}{4}$

Or $\left(x-2\right)^2+\left(y+3\right)^2=\left(\frac{1}{2}\frac{3x-4y-7}{5}\right)^2$

Or $\sqrt{\left(x-2\right)^2+\left(y+3\right)^2}=\frac{1}{2}\frac{\left|3x-4y-7\right|}{5}$

It is an ellipse, whose focus is (2,-3), directrix is $3x-4y+7=0$, and eccentricity is ½.

Length of perpendicular from the focus to the directrix is

$=\frac{1}{2}\frac{\left|3\times 2-4\times\left(-3\right)+7\right|}{5}=5$

Or $\frac{a}{e}-ae=5$ or $2a-\frac{a}{2}=5$ or $a=\frac{10}{3}$

So, the length of the major axis is 20/3.

11. we know if perpendicular tangents are drawn on an ellipse from a point then the point lie on the director circle.

Here, $AB=2\sqrt{a^2+b^2}$

$\Rightarrow AB=2\sqrt{a^2+7+13-5a}$

$\Rightarrow AB=2\sqrt{a^2-5a+6}$ in ΔABC,

By Pythagoras theorem we get,

$(AB)^2=(AC)^2+(BC)^2$

$\Rightarrow 4\left(a^2-5a+6\right)=2a^2+2a^2$

$\Rightarrow a=\frac{6}{5}$

But if $a=\frac{6}{5}$, then the term $\left(a^2-7\right)$ becomes negative which is not possible.

$\therefore$ No such a exists.

12. Since mutually perpendicular tangents can be drawn from the vertices of the rectangle, all the vertices of the rectangle should lie on the director circle $x^2+y^2=a^2+b^2$

Let breadth = $2l$ and length = $4l$. Then, $l^2+(2l)^2=a^2+b^2$

$l^2=\frac{a^2+b^2}{5}$

Area $=4l\times 2l=8\left(\frac{a^2+b^2}{5}\right)$

13. The points are $A(3,4), B(6,8), and\, O(0,0)$. $OA+OB=2a$ (where a is semi-major axis)

$2a=5+10=15$

$\therefore a=\frac{15}{2}$

Now, $2ae=\sqrt{(6-3)^2+(8-4)^2}=5$

$e=\frac{1}{3}$

$\therefore b^2=\frac{225}{4}\left(1-\frac{1}{9}\right)=50$

$b=5\sqrt{2}$

$\Rightarrow \frac{b}{\sqrt{2}}=5$

14. Distance between focus and corresponding directrix of an ellipse $=\frac{a}{e}-ae=8$

$a\left(\frac{1}{e}-e\right)=8$

$a\left(2-\frac{1}{2}\right)=8$ ($\because$ e= ½)

$a\left(\frac{3}{2}\right)=8$

$a=\frac{16}{3}$

$b^2=a^2\left(1-e^2\right)$

$=\frac{256}{9}\left(1-\frac{1}{4}\right)$

$b^2=\frac{64}{3}$

$b=\frac{8}{\sqrt{3}}$

Length of minor axis = 2b = $\frac{16}{\sqrt{3}}$ =k

$\frac{\sqrt{3}k}{2}=8$

15. Here, $\frac{x}{8}=\cos t+\sin t.....(i)$

$\frac{y}{5}=\cos t-\sin t....(ii)$

Square (i) k(ii) then ad we get,

$\Rightarrow\left(\frac{x}{8}\right)^2+\left(\frac{y}{5}\right)^2=2\left(\sin^2 t+\cos^2 t\right)=2$

$\Rightarrow\frac{x^2}{64}+\frac{y^2}{50}=1,$ Which is an ellipse.

16. The product of perpendiculars from the foci at any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to $b^2\,or\,a^2$ accordingly as $a>b\,or\,a<b,$ respectively.

Therefore, $\left(S_1M_1\right)\left(S_2M_2\right)=9$.

17. The given equation can be rewritten as $4x^2-24x+36+16y^2-32y+16-36-16-12=0$

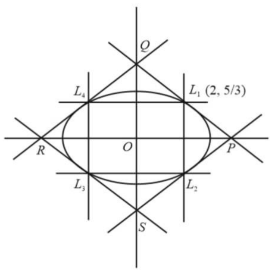
$\Rightarrow\left(2x-6\right)^2+\left(4y-4\right)^2=64$

$\Rightarrow\frac{\left(x-3\right)^2}{16}+\frac{\left(y-1\right)^2}{4}=1$

This represents an ellipse and $a^2=16,b^2=4$

$\therefore e=\sqrt{1-\frac{4}{16}}=\frac{\sqrt{3}}{2}$

18.



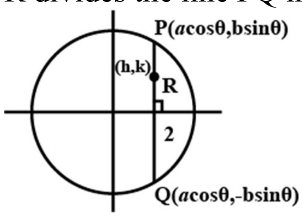
$\frac{x^2}{9}+\frac{y^2}{5}=1$

$e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$
$a^2=9, b^2=5$
Foci = $(\pm ae, 0)=(\pm 2, 0)$
Ends of latus rectum = $\left(\pm ae, \pm\frac{b^2}{a}\right)$ = $\left(\pm 2, \pm\frac{5}{3}\right)$
Tangent at 'L' is T = 0
$\frac{2\times x}{9}+\frac{5}{3}\times\frac{y}{5}=1$
It cuts coordinate axes at $P\left(\frac{9}{2},0\right)\&Q(0,3)$
Area of quadrilateral $PQRS=4$ (area of triangle OPQ) = $4\left(\frac{1}{2}\times\frac{9}{2}\times 3\right)=27$ sq.units

19. Equation of tangent to ellipse $\frac{x}{\sqrt{27}}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$
Area bounded by line and coordinate axis
$\Delta=\frac{1}{2}.\frac{\sqrt{27}}{\cos\theta}.\frac{\sqrt{3}}{\sin\theta}=\frac{9}{\sin 2\theta}$
Δ will be minimum when $\sin 2\theta=1$
$\Delta_{\min}=9$

20. Note that the given equations are major axis and minor axis of the given ellipse
$\left.\begin{array}{l}x+y-3=0\\ x-y+1=0\end{array}\right\}$
Therefore, centre is the point of intersection of these equations,
$\frac{x}{1-3}=\frac{y}{-3-1}=\frac{1}{-1-1}$
Then, the centre of the ellipse = $(1,2)$

21. R divides the line PQ in the ratio 1 : 2 then,



$h=\frac{1\times a\cos\theta+2\times a\cos\theta}{3}\Rightarrow h=a\cos\theta$
$\Rightarrow\frac{h}{a}=\cos\theta.....(1)$
And, $k=\frac{1\times(-b\sin\theta)+2\times b\sin\theta}{3}$
$k=\frac{b\sin\theta}{3}$
$\Rightarrow\frac{3k}{b}=\sin\theta......(2)$
Squaring and adding equations 1 and we get ;
$\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$

22. Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$$

WE know that the general equation of the tangent to the ellipse is

$$y=mx\pm\sqrt{a^2m^2+b^2} \quad (i)$$

Since $3x-2y-20=0 \, or \, y=\frac{3}{2}x-10$

Is tangent to the ellipse, comparing with (i),

$$m=\frac{3}{2} and \, a^2m^2+b^2=100$$

Or $a^2\times\frac{9}{4}+b^2=100$

Or $9a^2+4b^2=400 \quad (ii)$

Similarly, since $x+6y-20=0 \, i.e.,$

$$y=-\frac{1}{6}x+\frac{10}{3}$$

Is tangent to the ellipse, comparing with (i),

$$m=\frac{1}{6} and \, a^2m^2+b^2=\frac{100}{9}$$

Or $\frac{a^2}{36}+b^2=\frac{100}{9}$

Or $a^2+36b^2=400 \quad (iii)$

Solving (ii) and (iii), we get $a^2=40 \, and \, b^2=10.$

Therefore, the required equation of the ellipse is $\frac{x^2}{40}+\frac{y^2}{10}=1$

23. $PA+PB=8$, $AB=4$

Here $AB<K$ then $PA+PB=k$ locus of P is an ellipse

A,B, are foci, distance between foci = $2ae=4$

$ae=2$

$SP+S^1P=8$

But $SP+S^1P=2a$

$2a=8$

$a=4$

$b^2=a^2(1-e^2)$

$=a^2-a^2e^2$

$=16-4=12$

$\therefore$ equation of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{16}+\frac{y^2}{12}=1$$

24. Equation of ellipse $9x^2+16y^2=144$

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$a^2 = 16 \;\; b^2 = 9$

$a = 4 \quad b = 3$

General equation of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It passes through (8,0)

$\therefore \frac{(8)^2}{a^2} + \frac{0}{b^2} = 1$

$\frac{64}{a^2} = 1$

$a^2 = 64$

The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through $(4,3)$ also

$\therefore \frac{(4)^2}{64} + \frac{3^2}{b^2} = 1$

$\frac{16}{64} + \frac{9}{b^2} = 1$

$\frac{1}{4} + \frac{9}{b^2} = 1$

$\frac{9}{b^2} = 1 - \frac{1}{4} = \frac{3}{4}$

$b^2 = \frac{36}{3}$

$b^2 = 12$

$\therefore$ Equation of ellipse in $\frac{x^2}{64} + \frac{y^2}{12} = 1$

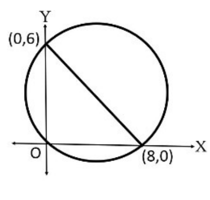
25. Let, $P = (4\cos\theta, 2\sin\theta)\, and\, Q = \left(4\cos\left(\theta + \frac{\pi}{6}\right), 2\sin\left(\theta + \frac{\pi}{6}\right)\right)$

$$\Rightarrow \text{Area if } \Delta OPQ = \frac{1}{2}\begin{vmatrix} 4\cos\theta & 2\sin\theta & 1 \\ 4\cos\left(\theta + \frac{\pi}{6}\right) & 2\sin\left(\theta + \frac{\pi}{6}\right) & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 4\left(\cos\theta\sin\left(\theta + \frac{\pi}{6}\right) - \cos\left(\theta + \frac{\pi}{6}\right)\sin\theta\right)$$

$$= 4\sin\left(\theta + \frac{\pi}{6} - \theta\right) = 4\sin\frac{\pi}{6} = 2 sq.units$$

26.



The centre of the family of ellipse is (4,3) and the distance of focus from the center is ae = $\frac{5}{2}$.

Hence, the locus is

$$(x-4)^2+(y-3)^2=\frac{25}{4}$$
$$=4x^2+4y^2-32x-24y+75=0$$

27. Equation of Ellipse $3x^2+4y^2=12$

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

Equation of director circle to the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ is $x^2+y^2=12$

The director circle will cut the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at four points

Hence number of points is 4

28. Equation of ellipse $x^2+4y^2=4$

$$\frac{x^2}{4}+\frac{y^2}{1}=1$$
$$a^2=4 \quad b^2=1$$
$$a=2 \quad b=1$$

Length of latusrectum = $\frac{2b^2}{a} \ =\frac{2(1)}{2}$

L.L.R = 1

$$l=\frac{L.L.R}{2}$$
$$l=\frac{1}{2}$$
$$\frac{1}{sp}+\frac{1}{sp^1}=\frac{2}{l}$$
$$\frac{1}{2}+\frac{1}{sp^1}=\frac{2}{\frac{1}{2}}$$
$$\frac{1}{sp^1}=4-\frac{1}{2}=\frac{7}{2}$$
$$sp^1=\frac{2}{7}$$
$$7sp^1=2$$

29. Equation of ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ b > a

$$a^2=9 \ b^2=16$$
$$a=3 \quad b=4$$

Eccentricity $e=\sqrt{\frac{b^2-a^2}{b^2}}$

$$=\sqrt{\frac{16-9}{16}}=\frac{\sqrt{7}}{4}$$

Focal distances from $P(x_1,y_1)$ to the given ellipse is $b\pm ey_1=4\pm\frac{\sqrt{7}}{4}y_1$

30. Equation of ellipse $4(x-2y+1)^2+9(2x+y+2)^2=180$

$$\frac{4(x-2y+1)^2}{180}+\frac{9(2x+y+2)^2}{180}=1$$

$$\frac{(x-2y+1)^2}{45}+\frac{(2x+y+2)^2}{20}=1$$

$$\frac{\left[\frac{x-2y+1}{\sqrt{1+4}}\right]^2}{45}+\frac{\left[\frac{2x+y+2}{\sqrt{4+1}}\right]^2}{20}=1$$

$$\therefore \frac{x^2}{9}+\frac{y^2}{4}=1$$

$a^2=9 \quad b^2=4$

$a=3 \quad b=2$

Length of latusrectum = $\frac{2b^2}{a}$

$$\frac{2(4)}{3}=\frac{8}{3}$$