

1. If $A = \{2x / x \in N \text{ and } x < 3\}$, $B = \{x / x^2 - 4x + 3 = 0 \text{ and } x > 1\}$, then $A \times B =$
 - 1) $\{(4,3), (2,3)\}$
 - 2) $\{(2,4), (2,3)(4,3)\}$
 - 3) $\{(1,4), (2,3), (2,2)\}$
 - 4) $\{(1,2), (1,3)(2,3)\}$
2. If $A = \{1, 2, 3\}$, $B = \{x\}$, then $(A \times B) \cup (B \times A) =$
 - 1) $\{(1,x), (2,x), (3,x)\}$
 - 2) $\{(x,1), (x,2), (x,3)\}$
 - 3) $\{(1,x), (2,x), (3,x), (x,1), (x,2), (x,3)\}$
 - 4) None
3. If $n(A \times B) = 15$, $n(A) = 3$, then $n(B) =$
 - 1) 12
 - 2) 5
 - 3) 45
 - 4) 3
4. If $n(A) = n$ then $n\{(x, y, z); x, y, z \in A, x \neq y, y \neq z, z \neq x\} =$
 - 1) n^3
 - 2) $n(n-1)^2$
 - 3) $n^2(n-2)$
 - 4) $n^3 - 3n^2 + 2n$
5. Let $X = \{1, 2, 3, 4, 5\}$. Then the number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty is
 - 1) 2^5
 - 2) 5^3
 - 3) 5^2
 - 4) 3^5
6. Range of $\{(1, x), (1, y), (2, x), (2, y), (3, z)\}$ is
 - 1) $\{1, 2, 3\}$
 - 2) $\{x, y, z\}$
 - 3) $\{1, x\}$
 - 4) $\{1, 2, 3, x, y, z\}$
7. Let $R = \{(1,3), (4,2), (2,4), (2,3), (3,1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is
 - 1) a function
 - 2) reflexive
 - 3) not symmetric
 - 4) transitive
8. Let $R = \{(3,3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,12), (3,6),\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is
 - 1) reflexive and transitive only
 - 2) reflexive only
 - 3) an equivalence relation
 - 4) reflexive and symmetric only
9. For, $n, m \in N$, $n | m$ means that n is a factor of m, the relation $|$ is
 - 1) reflexive and symmetric
 - 2) transitive and symmetric
 - 3) reflexive, transitive and symmetric
 - 4) reflexive, transitive and not symmetric
10. Let $x, y \in Z$ and suppose that a relation R on Z is defined by $x R y$ if and only if $x \leq y$ then
 - 1) R is partial order
 - 2) R is an equivalence relation
 - 3) R is reflexive and symmetric
 - 4) R is symmetric and transitive

11. A and B are two sets having 3 and 4 elements respectively and having 2 elements in common The number of relations which can be defined from A to B is
 1) 2^5 2) $2^{10} - 1$ 3) $2^{12} - 1$ 4) none of these
12. Let $R = \{(x, y) : x, y \in A, x + y = 5\}$ are where $A = \{1, 2, 3, 4, 5\}$ then
 1) R is not reflexive, symmetric and not transitive
 2) R is an equivalence relation
 3) R is reflexive, symmetric but not transitive
 4) R is not reflexive, not symmetric but transitive
13. For $x, y \in R$, define a relation R by $x R y$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then R is
 1) an equivalence relation 2) R is symmetric
 3) R is transitive 4) none of these
14. If R and S are two symmetric relations then
 1) RoS is a symmetric relation 2) SoR is a symmetric relation
 3) RoS^{-1} is symmetric relation 4) RoS is a symmetric relation if and only if $RoS = SoR$
15. f is a relation on the set R of real numbers defined as $(a, b) \in f \Rightarrow 1 + ab > 0$. Then f is
 1) transitive, reflexive but not symmetric
 2) reflexive, symmetric but not transitive
 3) reflexive, symmetric, transitive
 4) not reflexive, not symmetric, not transitive
16. If $A = \{1, 2, 3\}$, the number of symmetric relation in A is
 1) 3 2) 8 3) 328 4) 63
17. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)R(c, d)$ if $ad(b + c) = bc(a + d)$, then R is
 1) symmetric only 2) reflexive only 3) transitive only 4) an equivalence relation
18. Let $P = \{(x, y) / x^2 + y^2 = 1, x, y \in R\}$ Then P is
 1) reflexive 2) symmetric 3) transitive 4) anti-symmetric
19. Let R be an equivalence relation on a finite set A having n elements Then the number of ordered pairs in R is
 1) Less than n 2) greater than or equal to n
 3) less than or equal to n 4) None of these
20. Which one of the following relation on R is an equivalence relation
 1) $aR_1b \Leftrightarrow |a| = |b|$ 2) $aR_2b \Leftrightarrow a \geq b$
 3) $aR_3b \Leftrightarrow a \text{ divides } b$ 4) $aR_4b \Leftrightarrow a < b$
21. let R be a relation on a set A such that $R = R^{-1}$ then R is
 1) reflexive 2) symmetric 3) transitive 4) None of these
22. Let R be a relation on the set N be defined by $\{(x, y) \mid x, y \in N, 2x + y = 41\}$ then R is
 1) reflexive 2) symmetric 3) transitive 4) None of these

23. The void relation on a set A is
 1) reflexive 2) symmetric and transitive
 3) reflexive and symmetric 4) reflexive and transitive
24. Let R be a relation on the set N of natural numbers denoted by $nRm \Leftrightarrow n$ is a factor of m (i.e. n/m)
 Then R is
 1) reflexive and symmetric 2) transitive and symmetric
 3) equivalence 4) reflexive, transitive but not symmetric
25. Which of the following is not correct for relation R on the set of real numbers
 1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric
 2) $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive
 3) $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric
 4) $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric
26. Two points P and Q in a plane are related if $OP = OQ$ where O is a fixed point this relation is
 1) reflexive but not symmetric 2) symmetric but not transitive
 3) an equivalence relation 4) None of these
27. The relation R defined in $A = \{1, 2, 3\}$ by aRb if $|a^2 - b^2| \leq 5$ which of the following is false
 1) $R = \{(1, 1)(2, 2)(3, 3)(2, 1)(1, 2)(2, 3)(3, 2)\}$
 2) $R^{-1} = R$
 3) Domain of R = $\{1, 2, 3\}$
 4) Range of R = $\{5\}$
28. Let a relation R on the Set N of natural numbers be defined as $(x, y) \in R$ if and only if
 $x^2 - 4xy + 3y^2 = 0$ for all $x, y \in N$. The relation R is
 1) Reflexive 2) Symmetric 3) transitive 4) An equivalence relation
29. Given the relation $R = \{(1, 2)(2, 3)\}$ on the set $A = \{1, 2, 3\}$. The minimum number of ordered pairs which when added to R make it an equivalence relation is
 1) 5 2) 6 3) 7 4) 8
30. Let X be a family of sets and R be a relation on X defined by A is disjoint from B then R is
 1) Reflexive 2) Symmetric 3) anti symmetric 4) Transitive

KEY SHEET-2

| <u>1</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>5</u> | <u>6</u> | <u>7</u> | <u>8</u> | <u>9</u> | <u>10</u> |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1 | 3 | 2 | 4 | 4 | 2 | 3 | 1 | 4 | 1 |
| <u>11</u> | <u>12</u> | <u>13</u> | <u>14</u> | <u>15</u> | <u>16</u> | <u>17</u> | <u>18</u> | <u>19</u> | <u>20</u> |
| 4 | 1 | 4 | 4 | 2 | 4 | 4 | 2 | 2 | 1 |
| <u>21</u> | <u>22</u> | <u>23</u> | <u>24</u> | <u>25</u> | <u>26</u> | <u>27</u> | <u>28</u> | <u>29</u> | <u>30</u> |
| 2 | 4 | 2 | 4 | 2 | 3 | 4 | 1 | 3 | 2 |

HINTS

1. $A = \{2x : x \in N, x < 3\} = \{2, 4\}, B = \{x : x^2 - 4x + 2 = 0, x > 1\} = \{3\} \dots A \times B = \{(2, 3), (4, 3)\}$
2. $A \times B = \{(1, x), (2, x), (3, x), B \times A = \{(x, 1), (x, 2), (x, 3)\}$
3. $n(A \times B) = n(A)n(B) \Rightarrow 15 = 3n(B) = 5.$
4. There are n choice for the first coordinate, $n-1$ choices for second coordinate and $n-2$ choice for the third coordinate, hence $n(\{x, y, z \in A, x \neq y \neq z\}) = n(n-1)(n-2) = n^3 - 3n^2 + 2n$
5. If $a \in X$ then one of the following 4 cases hold $a \notin Y, a \notin Z$
 $Y \cap Z \neq \emptyset \Rightarrow$ only 3 cases hold \therefore The number of required pairs $= 3^5$
6. Range = $\{x, y, z\}$
7. $(2, 4), (2, 3) \in R \Rightarrow 2$ has two image $\Rightarrow R$ is not a function
 $(1, 1) \notin R = R$ is not reflexive; $(2, 3) \in R, (3, 2) \notin R \Rightarrow R$ is not symmetric
8. $A = \{3, 6, 9, 12\}$ and $(3, 3), (6, 6), (9, 9), (12, 12) \in R \Rightarrow R$ is reflexive
 $(6, 12) \in R$ but $(12, 6) \notin R \Rightarrow R$ is not symmetric
 $(12, 6) \in R \Rightarrow R$ is not symmetric
 $(3, 6) \in R, (6, 12) \in R \Rightarrow (3, 12) \in R \therefore R$ is transitive
9. Since n is a factor of n , so the relation is reflexive. If, however, n is a factor m , m is not necessarily a factor of n , so the relation is not symmetric. On the hand $n \mid m$ and $m \mid l$ imply $n \mid l$, so the relation is transitive
10. Since $x \leq x$ for all $x \in Z$ so R is reflexive but is not symmetric as $(1, 2) \in R$ and $(2, 1) \notin R$. Also R is transitive as $x \leq y, y \leq z \Rightarrow x \leq z$. R is antisymmetric for if $x \leq y$ and $y \leq x$ then $x = y$ Hence R is partial order
11. Conceptional
12. $R = \{(1, 4), (4, 1), (2, 3), (3, 2)\}$, so R is not reflexive as $(1, 1) \notin R$. Also R is symmetric by definition and R is not transitive as $(1, 4) \in R, (4, 1) \in R$ but $(1, 1) \notin R$
13. Since $x - x + \sqrt{2} = \sqrt{2}$ which is an irrational number so $x R x$ for all $x \in R$. Hence R is reflexive. R is not symmetric as $(\sqrt{2}, 1) \in R$ but $(1, \sqrt{2}) \notin R$. Again R is not transitive since $(\sqrt{2}, 1) \in R$ and $(1, 2\sqrt{2}) \in R$ but $(\sqrt{2}, 2\sqrt{2}) \notin R$
14. Since R and S are symmetric relation so $R^{-1} = R$ and $S^{-1} = S$
 But $(R \circ S)^{-1} (R \circ S)^{-1} = S^{-1} \circ R^{-1} = S \circ R$ Thus $R \circ S$ is symmetric if and only if $R \circ S = S \circ R$
15. For $(a, b) \in f, 1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow (b, a) \in f \Rightarrow f$ is symmetric
16. The number of symmetric relation = $2^{3(3+1)/2} - 1 = 2^6 - 1 = 63$
- 17.

For $(a, b), (c, d) \in N \times N; (a, b) R (c, d) \Rightarrow ad(b+c) = bc(a+d)$

Reflexive : Since $ab(b+a) = ba(a+b) \forall ab \in N. \therefore (a, b) R (a, b). \therefore R$ is reflexive.

Symmetric : For $(a, b), (c, d) \in N \times N$, let $(a, b) R (c, d)$.

$\therefore ad(b+c) = bc(a+d) \Rightarrow bc(a+d) = ad(b+c) \Rightarrow cb(d+a) = da(c+b) \Rightarrow (c, d) R (a, b).$

$\therefore R$ is symmetric.

Transitive : For $(a, b), (c, d), (e, f) \in N \times N$, let $(a, b) R (c, d), (c, d) R (e, f)$

$\therefore ad(b+c) = bc(a+d), cf(d+e) = de(c+f)$

$\Rightarrow adb + adc = bca + bcd \rightarrow (1)$ and $cf d + cfe = dec + def \rightarrow (2)$

$(1) ef + (2) ab \Rightarrow adbef + adcef + cfdab + cfeab = bcaef + bcdef + decab + defab$

$\Rightarrow adcf(b+e) = bcde(a+f) \Rightarrow af(b+e) = be(a+f) \Rightarrow (a, b) R (e, f).$

$\therefore R$ is transitive. Hence R is an equivalence relation.

18. $x^2 + y^2 = 1 \Rightarrow (x, y) \in R$
 $y^2 + x^2 = 1 \Rightarrow (y, x) \in R$
 \therefore symmetric
19. $n(A) = n \Rightarrow (A \times A) = n^2$
 $\Rightarrow n^2 \geq n$
20. $(a, b) \in R_1 \Rightarrow |a| = |a| \Rightarrow |a| = |b|$
i) $|a| = |a| \Rightarrow (a, a) \in R_1$ (Syme)
ii) $|a| = |b| \Rightarrow |b| = |a| \Rightarrow (b, a) \in R_1$ (Tr)
iii) $|a| = |b| \wedge |b| = |c| \Rightarrow |a| = |c| \Rightarrow (a, c) \in R_1$ (Tr)
21. $R = R^{-1} \Rightarrow R \circ R^{-1} = 1 \Rightarrow I$ is symmetric relation
22. $(x, y) \in R \Rightarrow 2x + y = 41$
i) $(x, x) \in R \Rightarrow 2x + x = 41 \Rightarrow x = \frac{41}{3} \notin N(x)$
ii) $(x, y) \in R \Rightarrow 2x + y = 41$; But $2x + x \neq 41 \Rightarrow (y, x) \notin R(x)$
iii) $(x, y) \in R \Rightarrow 2x + y = 41 \wedge (y, z) \in R \Rightarrow 2y + z = 41 \Rightarrow 2x + z \neq 41(x)$

23. Void relation is Null set
 As then is no elements if can't be reflexive
24. $(n, m) \in R \Rightarrow n$ is a factor of m
 Z is a factor of $6 \Rightarrow (6, 2) \notin R$ Not symmetric

25. $(x, y) \in R \Rightarrow 0 \leq |x - y| \leq 1$
i) $|x - x| = 0 \Rightarrow (x, x) \notin R$ not ref
ii) $(x, y) \in R \Rightarrow 0 \leq |x - y| \leq 1$
 $0 \leq |y - x| \leq 1 \Rightarrow (y, x) \in R$ (symmetric)
iii) $(x, y) \in R \wedge (y, z) \in R \Rightarrow 0 < |x - y| \leq 1 \wedge 0 \leq |y - z| \leq 1$
 $\Rightarrow |x - z| = |x - y + (y - z)|$
 $|x - z| \leq |x - y| + |y - z|$

26. $(P, Q) \in R \Rightarrow OP = OQ$



- i)* $OP = OP \Rightarrow (P, P) \in R$
ii) $OP = OQ \Rightarrow OQ = OP \Rightarrow (Q, P) \in R$
iii) $(P, Q) \in R \wedge (Q, R) \in R \Rightarrow OP = OQ \wedge OQ = OR \Rightarrow (P, R) \in R$

27. $A = \{1, 2, 3\}$ and $(a, b) \in R \Rightarrow |a^2 - b^2| \leq 5$

$$a = 1 \wedge b = 1 \Rightarrow |a^2 - b^2| = 0$$

$$a = 1 \wedge b = 2 \Rightarrow |a^2 - b^2| = (1 - 4) = 3$$

$$= \{0, 3, 8, \dots\} \neq \{5\}$$

$$a = 1 \wedge b = 3 \Rightarrow |a^2 - b^2| = \pi |1 - 9| = 8$$

28. $x^2 - 4xy + 3y^2 = 0 \Rightarrow x^2 - xy - 3xy + 3y^2 = 0$
 $x(x - y) - 3y(x - y) = 0 \Rightarrow (x - y)(x - 3y) = 0$
 $x = y$ or $x = 3y$

Lie thins Range

- i) $\text{Ref } (x, x) \in R \Rightarrow x^2 - 4x + x^2 = 0$
 ii) Sym of $(x, y) \in R \Rightarrow (x - y)(x - 3y) = 0$
 $\Rightarrow (y - x)(By - 3x)$ need not be zero \Rightarrow Not Sym
 $(3, 1) \in R$ but $(1, 3) \notin R$
 iii) Trans let $(x, y) \in R \wedge (y, z) \in R$
 clearly $(9, 3) \in R \wedge (3, 1) \in R$
 $9^2 - 4(9)3 + 3(3)^2 = 0 \wedge 3^2 - 4(3)1 + 3(1)^2 = 0$
 But $(9, 1) \notin R (\because 9^2 - 4(9) + 3 = 81 - 36 + 3 \neq 0)$

29. $A = \{1, 2, 3\}$

$R = \{(1, 2)(2, 3)\}$

To make it reflexive add $(1, 1) (2, 2) (3, 3)$

Now to make it symmetric we must add $(2, 1) (3, 2)$

$\Rightarrow R = \{(1, 2)(2, 3)(1, 1)(2, 2)(3, 3)\}$

$\Rightarrow R = \{(1, 2)(2, 3)(2, 1)(3, 2)(1, 3)(3, 1)(1, 1)(2, 2)(3, 3)\}$

\therefore minimum no of elements to be ass is 7

30. $(A, B) \in R \Rightarrow A \cap B = \phi$

i) $A \cap A \neq \phi (A, A) \notin R \Rightarrow$ not Ref

ii) $A \cap B \neq \phi \Rightarrow B \cap A = \phi \Rightarrow$ symmetric

iii) $A \cap B = \phi \wedge B \cap C = \phi \Rightarrow A \cap C$ need not be empty

Ex: $= \{1, 2, 3\} B = \{4, 5, 6\} \wedge C = \{1, 3, 7\}$

$A \cap B = \phi$ $B \cap C = \phi$

But $A \cap C = \{1, 3\} \neq \phi$

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