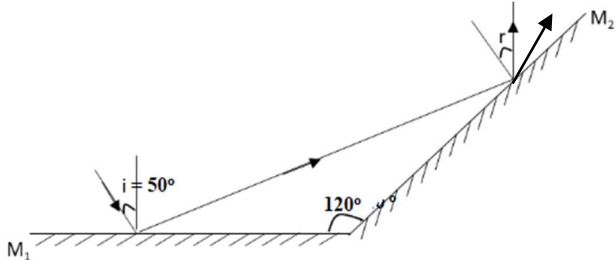
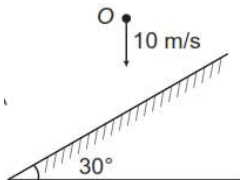
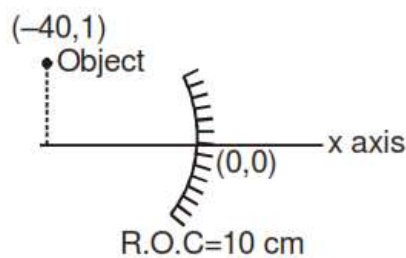


## GEOMETRICAL OPTICS

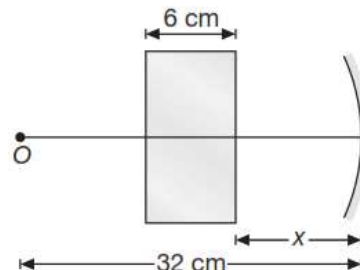
### MULTIPLE CHOICE QUESTIONS

- If  $\hat{n}_i$  (unit vector along incident ray) is  $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ . If mirror is kept along y-z plane then unit vector along reflected ray is
  - $\hat{n}_r = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$ .
  - $\hat{n}_r = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$ .
  - $\hat{n}_r = -\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$ .
  - $\hat{n}_r = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ .
- Two mirrors,  $M_1$  and  $M_2$ , are placed in contact at an angle of  $120^\circ$  as shown in figure. A ray is incident at  $50^\circ$  to the normal to  $M_1$ . In what direction does the light ray leave  $M_2$ ?
 
  - The angle of reflection to  $M_2$  is  $\theta_r = 120^\circ$
  - The angle of reflection to  $M_2$  is  $\theta_r = 70^\circ$
  - The angle of reflection to  $M_2$  is  $\theta_r = 20^\circ$
  - The angle of reflection to  $M_2$  is  $\theta_r = 50^\circ$
- An object is falling vertically downwards with velocity 10 m/s, then the image velocity (in terms of  $\hat{i}$  and  $\hat{j}$ )
 
  - $(5\sqrt{3}\hat{i} + 5\hat{j})$  m/s
  - $(\sqrt{3}\hat{i} + 10\hat{j})$  m/s
  - $(2\sqrt{3}\hat{i} - 5\hat{j})$  m/s
  - $(5\hat{i} + 10\hat{j})$  m/s
- There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object then the distance which the image will move.
  - 10 cm
  - 20 cm
  - 30 cm
  - 40 cm
- A fish at a depth of 12 cm in water is viewed by an observer on the bank of a lake. To what height the image of the fish is raised? (Refractive index of the lake water =  $4/3$ )
  - 9 cm
  - 12 cm
  - 3.8 cm
  - 3 cm
- An equiconvex lens has radius of curvature 'R', The refractive index of the material of the lens when its numerical value 'R' and focal length 'f' are same is
  - 1.25
  - 1.75
  - 2
  - 1.5
- An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance of 5 cm, then the image becomes triple its size. The nature of image, type of mirror used and radius of curvature of the mirror are
  - Virtual, Convex mirror and 60 cm
  - Real, Concave mirror and 60 cm
  - Virtual, Convex mirror and 6 cm
  - Real, Concave mirror and 6 cm

8. As shown in the figure, a spherical concave mirror with its pole at (0, 0) and principal axis along x-axis. There is a point object at (-40 cm, 1 cm), the position of image is:



- 1)  $(x = \frac{-40}{7} \text{ cm}, y = \frac{-1}{7} \text{ cm})$
  - 2)  $(x = \frac{-2}{7} \text{ cm}, y = \frac{-10}{7} \text{ cm})$
  - 3)  $(x = \frac{-1}{7} \text{ cm}, y = \frac{-20}{7} \text{ cm})$
  - 4)  $(x = \frac{-1}{7} \text{ cm}, y = \frac{-4}{7} \text{ cm})$
9. A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index  $\mu = 1.5$  and thickness 6 cm is inserted between object and mirror. From the mirror the position of final image when the distance  $x = 5$  cm is



- 1) 20 cm
- 2) 32 cm
- 3) 19 cm
- 4) 17 cm

10. The speed and wavelength of light in medium if wavelength of light is  $\lambda = 780$  nm (in air) and in a medium of refractive index  $\mu = 1.55$ .

- 1)  $3 \times 10^8$  m/s and 460 nm
- 2)  $1.94 \times 10^8$  m/s and 503 nm
- 3)  $2.3 \times 10^8$  m/s and 780 nm
- 4)  $3 \times 10^8$  m/s and 620 nm

11. A ray of light moving along vector  $(3\sqrt{2}\hat{i} - 3\hat{j} - 3\hat{k})$  under goes refraction at an interface of two media, which is y-z plane. The refractive index for  $x \leq 0$  is one while for  $x \geq 0$  it is  $\sqrt{2}$ . Then

- a) refracted ray bend towards y-axis
- b) refracted ray bend towards x-axis

- c) the unit vector along the refracted ray is  $\frac{\sqrt{6}\hat{i} - \hat{j} - \hat{k}}{2}$

- d) the unit vector along the refracted ray is  $\frac{\sqrt{6}\hat{i} - \hat{j} - \hat{k}}{\sqrt{8}}$

- 1) a, c
- 2) b, d
- 3) a, b and c
- 4) a, d

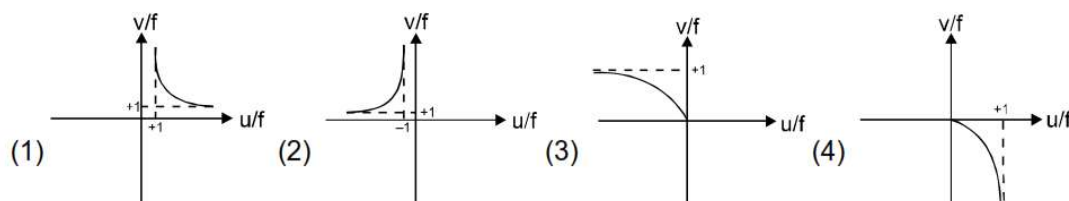
12. A small telescope has an objective lens of focal length 144 cm and eye piece of focal length 6.0 cm. What is the magnifying power of the telescope and the separation between objective and eye piece?

- 1) 15 and 100 cm
- 2) 24 and 150 cm
- 3) 14 and 120 cm
- 4) 10 and 90 cm

13. A long sighted person has a minimum distance of distinct vision of 50 cm. He wants to reduce it to 25 cm. He could use a –

- 1) convex lens of focal length of 50 cm
- 2) convex lens of focal length of 25 cm
- 3) concave lens of focal length of 50 cm
- 4) concave lens of focal length of 25 cm

14. A real inverted image in a concave mirror is represented by (u, v, f are co-ordinates)

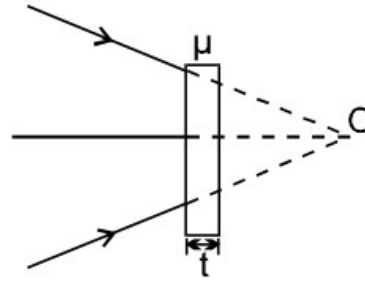


15. A bubble in glass slab ( $\mu=1.5$ ) when viewed from one side appears at 5 cm and 2 cm from other side then thickness of slab is:

- 1) 3.75 cm
- 2) 23 cm
- 3) 10.5 cm
- 4) 1.5 cm

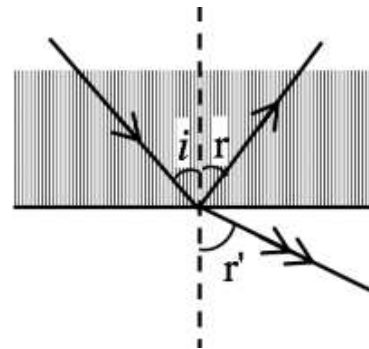
16. A beam of light is converging towards a point. A plane parallel plate of glass of thickness  $t$ , refractive index  $\mu$  is introduced in the path of the beam. The convergent point is shifted by (assume near normal incidence):

- 1)  $t\left(1 - \frac{1}{\mu}\right)$  away
- 2)  $t\left(1 + \frac{1}{\mu}\right)$  away
- 3)  $t\left(1 - \frac{1}{\mu}\right)$  nearer
- 4)  $t\left(1 + \frac{1}{\mu}\right)$  nearer



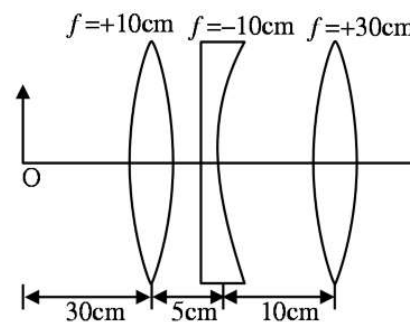
17. A ray of light passes from a denser medium to a rare medium at an angle of incidence  $i$ . The reflected and refracted rays make an angle  $90^\circ$  with each other. The angle of reflection and refraction are respectively  $r$  and  $r'$ . The critical angle is given by

- 1)  $\sin^{-1}(\cot r)$
- 2)  $\tan^{-1}(\sin i)$
- 3)  $\sin^{-1}(\tan r')$
- 4)  $\sin^{-1}(\tan r)$



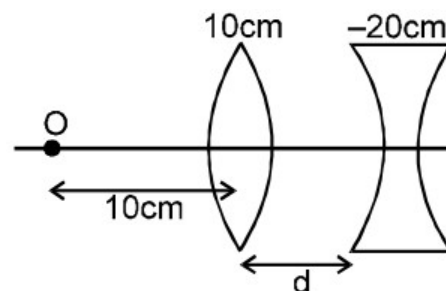
18. Find the distance of the image from object O, formed by the combination of lenses in the figure:

- 1) 75 cm
- 2) 10 cm
- 3) 20 cm
- 4) infinity



19. What should be the value of distance  $d$  so that final image is formed on the object itself? (focal lengths of the lenses are written on the lenses)

- 1) 10 cm
- 2) 20 cm
- 3) 5 cm
- 4) None of these



20. A prism of  $6^\circ$  angle gives a deviation of  $3^\circ$ . The refractive index of material of prism is:

- 1) 1.5
- 2) 2
- 3) 0.5
- 4) 1.8

### NUMERICAL TYPE QUESTIONS

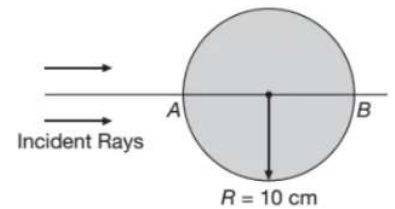
21. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle  $30^\circ$  at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is  $n$  times of 10, the  $n$  value is \_\_\_\_\_

22. A fish is rising up vertically inside a pond with velocity  $4 \text{ cms}^{-1}$  and notices a bird, which is diving vertically downward and its velocity appears to be  $16 \text{ cms}^{-1}$  (to fish). the actual velocity of the diving bird, in  $\text{cms}^{-1}$ , (if refractive index of water is  $4/3$ ) \_\_\_\_\_

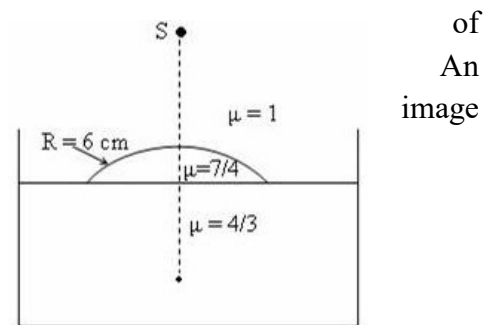
23. A man of height 2 m is standing on level road where because of temperature variation the refractive index of air is varying as  $\mu = \sqrt{1 + ay}$ , where  $y$  is height from road. If  $a = 2 \times 10^{-6} \text{ m}^{-1}$ . Then find the maximum distance, in km, till which he can see on the road.

24. The height of a candle flame is 5 cm. A lens produces an image of this flame 15 cm high on screen. Without touching the lens, the candle is moved over a distance of  $l = 1.5 \text{ cm}$  away from the lens and sharp image of the flame 10 cm high is obtained again after shifting the screen. Calculate the focal length of the lens, in cm.

25. A parallel paraxial beam of light is incident on a glass sphere of radius 10 cm along its diameter AB from one side as shown. If all the rays after refraction converge at the point B then calculate the refractive index of the glass sphere.

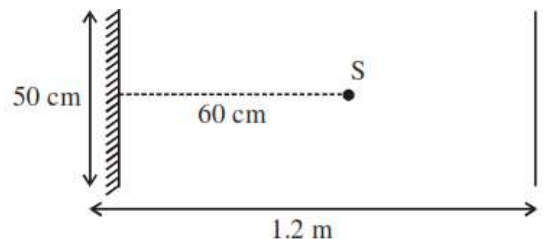


26. Water (with refractive index  $= 4/3$ ) in a tank of 18 cm deep. Oil of refractive index  $7/4$  lies on water making a convex surface of radius curvature, ' $R$ ' = 6 cm as shown. Consider oil to act as a thin lens. object ' $S$ ' is placed 24 cm above water surface. The location of its is at ' $x$ ' cm above the bottom of the tank. Then ' $x$ ' is

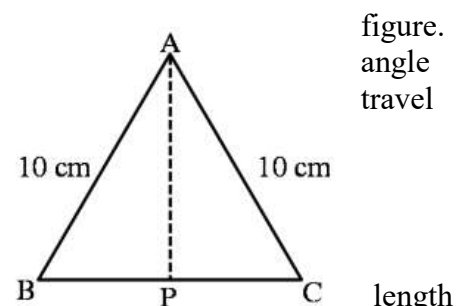


27. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is \_\_\_\_\_ cm.

28. A point source of light  $S$ , placed at a distance 60 cm in front of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see the figure). The distance between the extreme points where he can see the image of the light source in the mirror is \_\_\_\_\_ cm.



29. Cross-section view of a prism is the equilateral triangle ABC in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (midpoint of BC) to A is \_\_\_\_\_  $\times 10^{-10} \text{ s}$ .



30. A converging lens of focal length 5.0 cm is placed in contact with a diverging lens of focal length 10.0 cm. Find the combined focal length of the system.

\_\_\_\_\_

## KEY-DPP (GEOMETRICAL OPTICS)

1-10	3)	2)	1)	2)	4)	3)	2)	1)	4)	2)
11-20	2)	2)	1)	1)	3)	1)	4)	1)	1)	1)
21-30	3	9	2	9	2	2	15	50	5	10

## SOLUTIONS

1. Use vector form of law of reflection:  $\hat{n}_r = \hat{n}_i - 2(\hat{n}_i \cdot \hat{n})\hat{n}$

Normal unit vector to the YZ plane mirror:  $\hat{n} = \hat{i}$

Given that:

$$\hat{n}_i = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$

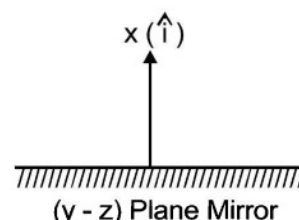
From vector form of law of reflection

$$\Rightarrow \hat{n}_r = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) - 2\left\{\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{i}\right\}\hat{i}$$

$$\Rightarrow \hat{n}_r = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) - \frac{2}{\sqrt{3}}(\hat{i})$$

$$\Rightarrow \hat{n}_r = \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} + \hat{k})$$

**Observation:** Only the sign of normal unit vector changes, others remain unchanged.

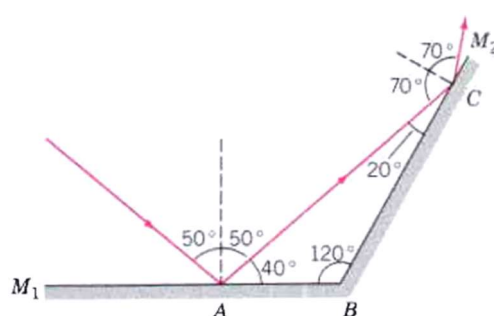


2. From the law of reflection, the angle of incidence = reflection at  $M_1$  is  $50^\circ$ .

The angle made by the reflection ray to the plane of  $40^\circ$ .

In triangle ABC, the angle at C is  $180^\circ - (40^\circ + 120^\circ) =$

The angle of incidence to  $M_2$  is  $70^\circ$ , so this also the reflection is  $70^\circ$ .

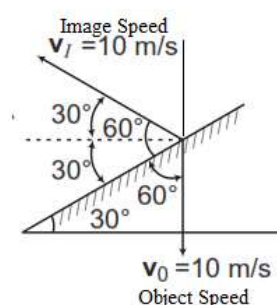


angle of  
 $M_1$  is  
 $20^\circ$ .  
angle of

3. Image Velocity  $V_{\text{image}} = 10 \cos 30^\circ \hat{i} + 10 \sin 30^\circ \hat{j}$

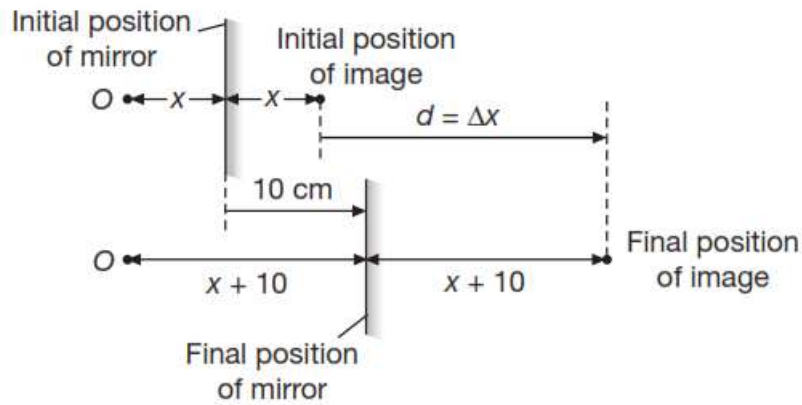
$$\Rightarrow V_{\text{image}} = 10 \left(\frac{\sqrt{3}}{2}\right)\hat{i} + 10 \left(\frac{1}{2}\right)\hat{j}$$

$$\Rightarrow V_{\text{image}} = (5\sqrt{3}\hat{i} + 5\hat{j}) \text{ m/s}$$



4. In a plane mirror, image distance = object distance from the mirror.

$$\Rightarrow |x_{im}| = |x_{om}|$$



From figure,  $2(x + 10) = 2x + d$

$$\Rightarrow d = 20 \text{ cm.}$$

5. Given real depth ( $x$ ) = 12 cm

We know that,  $\mu = \left( \frac{\text{Real depth}(x)}{\text{Apparent depth}(y)} \right) \Rightarrow \frac{4}{3} = \left( \frac{12}{y} \right) \Rightarrow y = 9 \text{ cm}$

Height of image raised is  $x - y = 12 - 9 = 3 \text{ cm}$ .

6. Give that:  $R_1 = R_2 = R$

Focal length = radius of curvature

From lens makers formula

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{R} = (\mu - 1) \left( \frac{1}{R} + \frac{1}{R} \right) \Rightarrow (\mu - 1) = \frac{1}{2} \Rightarrow \mu = \frac{3}{2} = 1.5.$$

7. Since the image formed on the screen, it is real.

Real object and **real image** imply **concave mirror**.

Applying  $m = \frac{f}{f - u}$  or  $-2 = \frac{f}{f - u}$  ..... (i)

According to question:

When candle is shifted by a distance of 5 cm, then the image becomes triple its size.

After shifting  $-3 = \frac{f}{f - (u + 5)}$  ..... (ii)

From equations (i) and (ii), we get

$$f = -30 \text{ cm or } R = 60 \text{ cm}$$

8. According to sign convention,

$u = -40 \text{ cm}$ ,  $h_1 = +1 \text{ cm}$ ,  $f = -5 \text{ cm}$

From Mirror formula:  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-40} = \frac{1}{-5}$

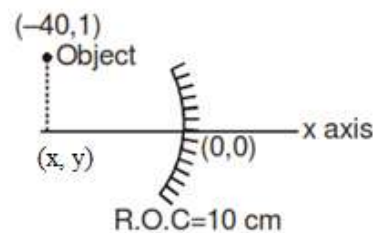
$$\Rightarrow \frac{1}{v} = \frac{1}{40} + \frac{1}{-5} \Rightarrow v = \frac{-40}{7} \text{ cm}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{-v}{u}$$

$$\Rightarrow h_2 = \left( \frac{-v}{u} \right) h_1$$

$$\Rightarrow h_2 = - \frac{(-40/7)}{-40} = - \frac{1}{7} \text{ cm}$$

Therefore, the position of image is  $\left( \frac{-40}{7} \text{ cm}, - \frac{1}{7} \text{ cm} \right)$ .



9. The normal shift produced by a glass slab is given by

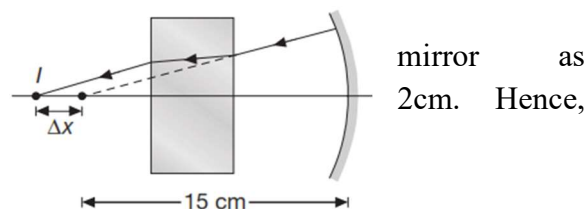
$$\Delta x = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2 \text{ cm}$$

i.e., for the mirror the object is placed at a distance  $(32 - \Delta x) = 30 \text{ cm}$  from it.

Applying mirror formula i.e.,  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ , we get  $\Rightarrow \frac{1}{v} - \frac{1}{30} = \frac{1}{-10} \Rightarrow v = -15 \text{ cm}$ .

**When  $x = 5 \text{ cm}$**

The light falls on the slab after being reflected from the shown. But the slab will again shift it by a distance  $\Delta x =$  the final image is formed at a distance  **$(15+2) = 17 \text{ cm}$** .



10. (i) Speed of light in medium  $v = \frac{c}{\lambda} = \frac{3 \times 10^8}{1.55} = 1.94 \times 10^8 \text{ m/s}$

(ii)  $\lambda_{\text{medium}} = (\lambda_{\text{air}})/\mu = \frac{780}{1.55} = \mathbf{503 \text{ nm}}$ .

11.  $|3\sqrt{2}\hat{i} - 3\hat{j} - 3\hat{k}| = \sqrt{(3\sqrt{2})^2 + 3^2 + 3^2} = 6$

Angle between normal x-axis and incidence ray is  $(3\sqrt{2}\hat{i} - 3\hat{j} - 3\hat{k}) \cdot (\hat{i}) = 6 \cdot 1 \cdot \cos\theta$

$$\Rightarrow \frac{3\sqrt{2}}{6} = \cos\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

Applying Snell's law at P,  $\sin\theta = \mu \sin r$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sqrt{2} \sin r \Rightarrow \sin r = 1/2 \Rightarrow r = 30^\circ$$

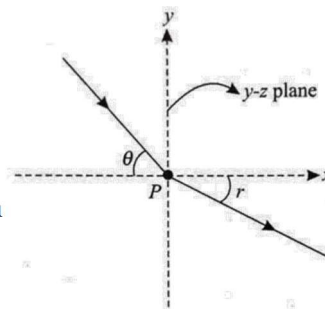
Let the angle made by refracted ray with y-axis and z-axis is  $\alpha$  then

$$\cos^2 \alpha + \cos^2 \alpha + \cos^2 r = 1$$

$$2 \cos^2 \alpha = 1 - \frac{3}{4} \Rightarrow \cos \alpha = \frac{1}{\sqrt{8}}$$

$$\hat{u}_{\text{refracted}} = \cos r \hat{i} - \cos \alpha \hat{j} - \cos \alpha \hat{k}$$

$$\Rightarrow \hat{u}_{\text{refracted}} = \cos 30^\circ \hat{i} - \frac{1}{\sqrt{8}} \hat{j} - \frac{1}{\sqrt{8}} \hat{k} = \frac{\sqrt{3}}{2} \hat{i} - \frac{1}{\sqrt{8}} \hat{j} - \frac{1}{\sqrt{8}} \hat{k} = \frac{(\sqrt{6}\hat{i} - \hat{j} - \hat{k})}{\sqrt{8}}$$



12.  $M = \frac{f_{\text{object}}}{f_{\text{eye-piece}}} = \frac{144}{6} = \mathbf{24}$

and  $L = f_{\text{object}} + f_{\text{eye-piece}} = 144 + 6 = \mathbf{150 \text{ cm}}$ .

13.  $u = -25 \text{ cm}$  and  $v = -50 \text{ cm}$  (minimum distance of distinct vision is  $v_{\text{image}}$ )

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-50} - \frac{1}{-25} = \frac{1}{f}$$

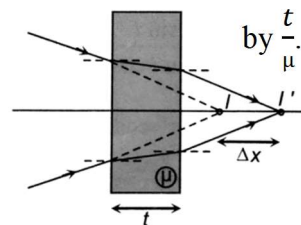
$$\Rightarrow f = \mathbf{50 \text{ cm}} \text{ and nature of the lens is convex.}$$

14. For real and inverted image by concave mirror:  $v = -v$ ,  $u = -u$  and  $f = -f \Rightarrow \frac{u}{f}$  and  $\frac{v}{f}$  are +ve.

15.  $5+2 = \frac{t_1}{1.5} + \frac{t_2}{1.5} \Rightarrow 7 \times 1.5 = t_1 + t_2 \Rightarrow \mathbf{10.5}$

16. When glass plate of thickness  $t$  is introduced, the optical path increases

So the convergence point shifts by  $\Delta x = t \left(1 - \frac{1}{\mu}\right)$  nearer to glass and shift takes place by the direction of a ray.



slab

17.  $r + r' + 90^\circ = 180^\circ \Rightarrow r' = 90^\circ - r = 90^\circ - i$

$$n_1 \sin i = n_2 \sin r'$$

$$n_1 \sin i = n_2 \sin (90^\circ - i) \Rightarrow n_1 \sin i = n_2 \cos i = \tan i = n_2/n_1$$

$$\text{Now } \sin C = n_2/n_1 = \tan i$$

$$\Rightarrow C = \sin^{-1}(\tan i) = \sin^{-1}(\tan r) \text{ (since incident angle and reflected angle are same)}$$

18. i)  $\frac{1}{v_1} + \frac{1}{30} = \frac{1}{10} \Rightarrow \frac{1}{v_1} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30}$

$$\Rightarrow v_1 = 15 \text{ cm.}$$

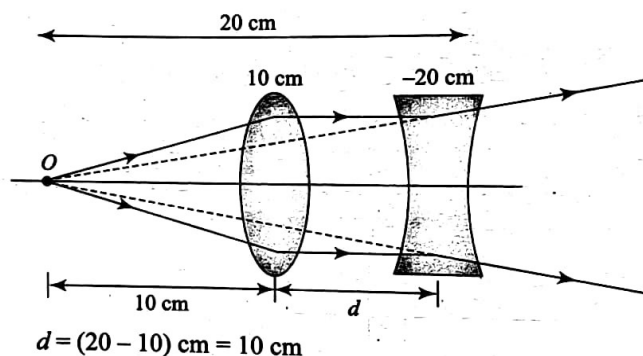
ii)  $\frac{1}{v_2} + \frac{1}{10} = \frac{1}{10} \Rightarrow \frac{1}{v_2} = \frac{1}{10} - \frac{1}{10} = 0$

$$\Rightarrow v_2 = \infty \text{ cm}$$

Similarly, iii)  $v_3 = 30 \text{ cm}$

The distance of the image from object O is:  **$Ov_3 = 75 \text{ cm.}$**

- 19.



20. Given that,

$$A = 6^\circ \text{ and } \delta = 3^\circ$$

$$\delta = A(\mu - 1) \Rightarrow 3^\circ = 6^\circ (\mu - 1) \Rightarrow \frac{1}{2} = (\mu - 1) \Rightarrow \mu = 3/2 = 1.5.$$

### Numerical Type-Solutions

21. Let  $d$  is the distance covered by the ray after reflection from surface

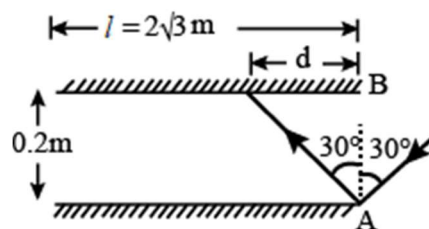
$$\tan 30^\circ = \frac{d}{0.2}$$

$$\frac{d}{0.2} = \frac{1}{\sqrt{3}} \Rightarrow d = \frac{2}{10\sqrt{3}}$$

Total number of reflections =

$$\Rightarrow n = \left( \frac{\text{Mirror length}(l)}{\text{distance covered by reflected ray}(d)} \right)$$

$$\Rightarrow n = \left( \frac{2\sqrt{3}}{\frac{2}{10\sqrt{3}}} \right) = 30 = 3 \times 10$$



a



22. Let at some instant bird is at a height of  $x$  from the water surface diving downwards with  $v$  cm/s.

and it is

distance

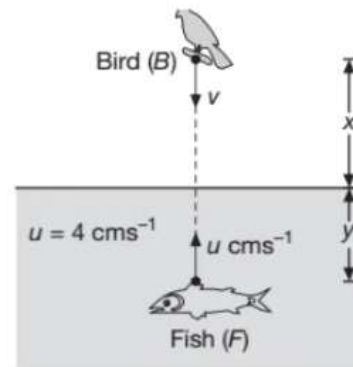
At this instant fish is at a depth  $y$  below water surface. Then the between fish and image of bird will be  $s = \mu x + y$  where  $\mu = 4/3$   
 $\Rightarrow s = 4/3 x + y$

Differentiating w.r.t time, we get

$$\left(\frac{ds}{dt}\right) = \frac{4}{3} \left(\frac{dx}{dt}\right) + \left(\frac{dy}{dt}\right) \Rightarrow 16 = \frac{4}{3}(v) + u, \text{ where } u = 4 \text{ cm/s}$$

$$4v + 3(4) = 48 \Rightarrow v = 9 \text{ cm/s}$$

The actual velocity of diving bird is **9 cm/s**.



23. Since,  $\theta = 90 - i$

Let P be point on the trajectory of the ray.

Slope of tangent at point P =  $\tan \theta$

$$\text{Slope} = \frac{dy}{dx} = \tan \theta = \tan (90 - i) = \cot i$$

$$\Rightarrow \tan \theta = \cot i$$

At the surface,  $y = 0$ . Given that  $\mu = \sqrt{1 + ay}$

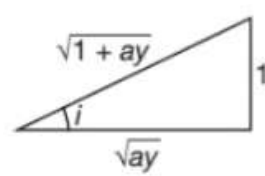
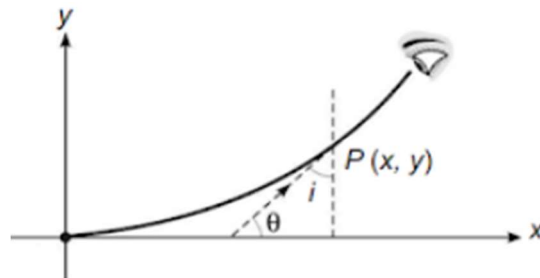
$$\Rightarrow \mu_0 = 1.$$

From Snell's law,

$$\mu \sin i = \text{constant} \Rightarrow \mu_0 \sin 90^\circ = \mu \sin i$$

$$\Rightarrow 1 = \sqrt{1 + ay} \sin i$$

$$\Rightarrow \sin i = \frac{1}{\sqrt{1 + ay}} \Rightarrow \cot i = \sqrt{ay}$$



$$\Rightarrow \int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx$$

$$\Rightarrow x = 2\sqrt{\frac{y}{a}}$$

Substituting  $y = 2 \text{ m}$  and  $a = 2 \times 10^{-6} \text{ m}^{-1}$ , we get

$$x_{\max} = 2000 \text{ m} = 2 \text{ km}$$

Lateral magnification in first case is  $-3$ , so

if  $u = -x$  then  $v = +3x$

$$\text{Since } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{+3x} + \frac{1}{x} = \frac{1}{f}$$

$$\Rightarrow \frac{4}{3x} = \frac{1}{f}$$

$$\Rightarrow x = \frac{4f}{3} \quad \dots(1)$$

In the second case magnification is  $-2$ , so now we have

$$u = -(x+1.5), v = 2(x+1.5)$$

$$\text{Since } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{2(x+1.5)} + \frac{1}{(x+1.5)} = \frac{1}{f}$$

$$\Rightarrow \frac{3}{2(x+1.5)} = \frac{1}{f}$$

$$\Rightarrow f = \frac{2}{3}(x+1.5)$$

Solving equations (1) and (2), we get

$$f = 9 \text{ cm}$$

24.

25. For refraction formula, we use  
 $u \rightarrow \infty, R = +10 \text{ cm}, v = 2R = +20 \text{ cm}$   
 $\mu_1 = 1$  and  $\mu_2 = \mu$   
 Since,  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$   
 $\Rightarrow \frac{\mu}{20} = \frac{\mu - 1}{10}$   
 $\Rightarrow \mu = 2\mu - 2$   
 $\Rightarrow \mu = 2$

26. For refraction at spherical surfaces

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$u = -24 \text{ cm}, R = 6 \text{ cm}, \mu_1 = 1, \mu_2 = 7/4$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{7/4}{v} - \frac{1}{-24} = \frac{7/4 - 1}{6}$$

After solving, we get  $v = 21 \text{ cm}$

The image acts as a source for refraction at oil

Now for refraction at oil water interface,  $u = 21$

$\mu_1 = 7/4, \mu_2 = 4/3, v' = ?$  (Plane of Oil-water interface)

From refraction at spherical surfaces

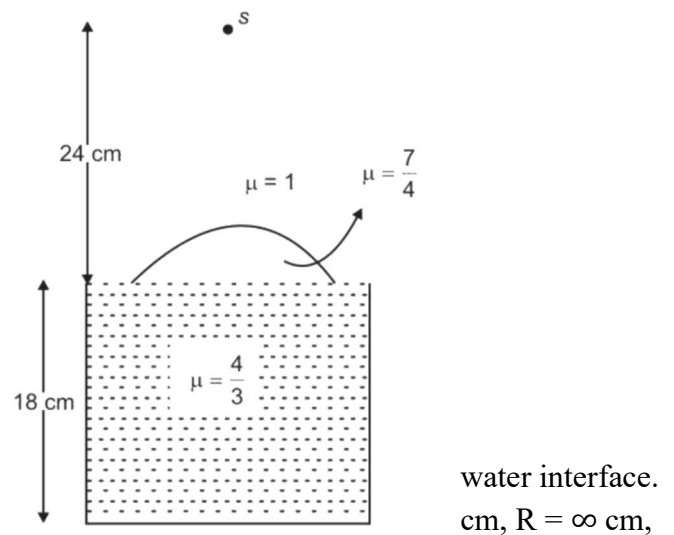
$$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{4/3}{v'} - \frac{7/4}{21} = \frac{7/4 - 4/3}{\infty}$$

$$\frac{4/3}{v'} - \frac{7/4}{21} = 0$$

$v' = 16 \text{ cm}$  This is the distance from oil water interface.

Height of image from bottom of the tank =  $18 - 16 \text{ cm} = 2 \text{ cm}$ .



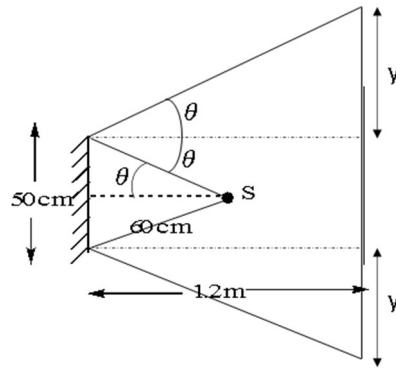
27. We know, the relationship between the linear magnification  $m$ , the object distance  $u$  and the focal length  $f$  for convex mirror ( $f > 0, u < 0$ ) is  $m = \left(\frac{f}{u+f}\right)$ .
- +  $m = \left(\frac{f}{-10+f}\right)$  ..... (1)
- $m = \left(\frac{f}{-20+f}\right)$  ..... (2)

$$(1) / (2) \Rightarrow -1 = \left( \frac{f-20}{f-10} \right) \Rightarrow -f+10 = f-20 \Rightarrow 2f=30 \Rightarrow f=15 \text{ cm.}$$

28. Using properties of similar triangles,

$$\frac{25}{60} = \frac{y}{1.2} = \tan \theta$$

$$\Rightarrow y = 30/60 = 1/2 = 0.5 \text{ m} = \mathbf{50 \text{ cm}}$$



29. Given that, angle of incidence ( $\theta_i$ ) = prism angle ( $A$ ) =  $60^\circ$  ..... (1)

$$\delta_{\min} = 2i - A = 2 \times 60^\circ - 60^\circ = 60^\circ \text{ ..... (2)}$$

Formula for minimum deviation

$$\mu = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \mu = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \mu = \frac{\sin(A)}{\sin\left(\frac{A}{2}\right)} = \sin 60^\circ / \sin 30^\circ = \left( \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \right) = \sqrt{3}$$

$$\text{Refractive index } (\mu) = \frac{\text{speed of light in vacuum}(c)}{\text{speed of light in medium}(v)}$$

$$v = \frac{3 \times 10^8}{\sqrt{3}}$$

$$h = AP = (10 \text{ cm}) \sin 60^\circ = (10 \times 10^{-2}) \left( \frac{\sqrt{3}}{2} \right) = 5\sqrt{3} \times 10^{-2} \text{ m}$$

$$\text{The time taken by light to travel from P (midpoint of BC) to A is } t = \frac{h}{v} = \frac{5\sqrt{3} \times 10^{-2}}{\frac{3 \times 10^8}{\sqrt{3}}}$$

$$= \mathbf{5 \times 10^{-10} \text{ s.}}$$

30. The focal length of the combination of lenses is given by  $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Here,  $f_1 = +5.0 \text{ cm}$  and  $f_2 = -10.0 \text{ cm}$

$$\text{Therefore, the combined focal length } F \text{ is given by } \frac{1}{F} = \frac{1}{5.0} - \frac{1}{10.0} = + \frac{1}{10.0} \text{ cm}$$

i.e., this combination behaves as a converging lens of focal length **10.0 cm**.