NARAYANA EDUCATIONAL INSTITUTIONS

DAILY PRACTICE TEST:: DEFINITE INTEGRALS

MATHS – II B

1. $\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}=$

1) $1+\sqrt{5}$ 2) $-1+\sqrt{5}$ 3) $-1+\sqrt{2}$ 4) $1+\sqrt{2}$

2. $\underset{n\to\infty}{Lt}\frac{1^9+2^9+3^9+\cdots+n^9}{n^{10}}=$

1) $\frac{1}{2}$ 2) $\frac{1}{5}$ 3) $\frac{1}{10}$ 4) $\frac{1}{15}$

3. $\underset{n\to\infty}{Lt}\left[\frac{1}{1+n^3}+\frac{4}{8+n^3}+\frac{9}{27+n^3}+\cdots+\frac{1}{2n}\right]=$

1) $\frac{1}{2}\log 2$ 2) $\frac{1}{3}\log 3$ 3) $\frac{1}{3}\log 2$ 4) $\frac{1}{2}\log 3$

4. $\underset{n\to\infty}{Lt}\left[\frac{n^{\frac{1}{2}}}{n^{\frac{3}{2}}}+\frac{n^{\frac{1}{2}}}{(n+3)^{\frac{3}{2}}}+\frac{n^{\frac{1}{2}}}{(n+6)^{\frac{3}{2}}}+\cdots+\frac{n^{\frac{1}{2}}}{\{n+3(n-1)\}^{\frac{3}{2}}}\right]=$

1) $\frac{1}{3}$ 2) $\frac{1}{5}$ 3) $\frac{1}{10}$ 4 $\frac{1}{2}$

5. $\underset{n\to\infty}{Lt}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}=$

1) $e^{(\pi-4)/2}$ 2) $2e^{(\pi-4)/2}$ 3) $\frac{e^{(\pi-4)/2}}{2}$ 4)none

6. $\underset{n\to\infty}{Lt}\left[\frac{\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{n+n}}{n\sqrt{n}}\right]=$

1) $\frac{2\left(2\sqrt{2}-1\right)}{3}$ 2) $\frac{\left(2\sqrt{2}-1\right)}{3}$ 3) $\frac{\left(2\sqrt{2}+1\right)}{3}$ 4) *none*

7. $\int_0^{\pi}e^x\sin x\,dx=$

1) $\frac{1}{2}e^{\pi}$ **2)** $e^{\pi}+1$ 3) $\frac{1}{2}\left(e^{\pi}-1\right)$ 4) $\frac{1}{2}\left(e^{\pi}+1\right)$

8. $\int_0^1 x\left(\mathrm{Tan}^{-1} x\right)^2 dx =$

$1\ \frac{\pi^2}{6}-\frac{\pi}{3}+\frac{1}{2}\log 2$ 2) $\frac{\pi^2}{16}+\frac{\pi}{4}+\frac{1}{2}\log 2$ 3) $\frac{\pi^2}{16}-\frac{\pi}{4}+\frac{1}{2}\log 2$ 4) *none*

9. $\int_0^1 \mathrm{Sin}^{-1}x\, dx =$

1) $\frac{\pi+2}{2}$ 2) $\frac{\pi-2}{2}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi-2}{3}$

10. Evaluate $\int_0^{\pi/2} x\cot x\, dx$

1) $\frac{\pi}{2}\log\left(\frac{1}{2}\right)$ 2) $2\pi\log 2$ 3) $\frac{\pi}{2}\log 2$ 4) $\pi\log\left(\frac{1}{2}\right)$

11. *Evaluate* $\int_0^\infty \frac{\tan^{-1} x}{x\left(1+x^2\right)}dx$

1) $\frac{2}{\pi}\log 2$ 2) $\frac{\pi}{2}\log 2$ 3) $\frac{1}{2}\log 2$ 4) $\frac{1}{\pi}\log 2$

12. $\int_0^1 \frac{x}{1+\sqrt{x}}dx =$

1) $2\left[\frac{5}{6}+\log 2\right]$ 2) $2\left[\frac{5}{6}-\log 2\right]$ 3) $\left[\frac{5}{6}-\log 2\right]$ 4) *none*

13. $\int_{1/2}^1 \mathrm{Sin}^{-1}\sqrt{x}\, dx =$

1) $\frac{\pi-1}{4}$ 2) $\frac{\pi-1}{8}$ 3) $\frac{\pi-2}{4}$ 4) $\frac{\pi-2}{8}$

14. $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}} =$

1) $\pi/2$ 2) $\pi/3$ 3) $\pi/4$ 4 *none*

15. $\int_0^a x^3\left(ax-x^2\right)^{3/2} dx =$

1) $-\frac{9\pi a^7}{2048}$ 2) $\frac{3\pi a^7}{2048}$ 3) $\frac{9\pi a^7}{2048}$ 4) $\frac{9\pi a^7}{2345}$

16 $\int_0^{1/\sqrt{2}} \frac{\text{Sin}^{-1} x}{\left(1-x^2\right)^{\frac{3}{2}}} dx =$

1) $\frac{\pi}{4}-\frac{1}{2}\log 2$ 2) $\frac{\pi}{4}+\frac{1}{2}\log 2$

3) $\frac{\pi}{2}-\frac{1}{2}\log 2$ 4) $\frac{\pi}{4}-\frac{1}{4}\log 2$

17. $\int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}} dx =$

1) 0 2) $\pi/2$ 3) $\pi/4$ 4) π

18.. $\int_0^1 X^2(1-x)^5\, dx =$

1) $1/186$ 2) $1/168$ 3) $1/68$ 4) $1/135$

19. $\int_{-a}^{b} \frac{\sqrt[3]{x+a}}{\sqrt[3]{x+a}+\sqrt[3]{x+a}} dx$

1) $a-b$ 2) $a+b$ 3) $(a+b)/2$ 4) $b-a$

20. If [x]denotes the greatest integer function then the value of $\int_{0.5}^{4.5}[x]dx+\int_{-1}^{1}|x|dx$

1) 9 2) 8 3) 7 4) 6

21. If [x] denotes the greatest integer $\leq x, then\, evaluate\ \int_0^2 [x^2]dx$

1) $(5+\sqrt{3}+\sqrt{2})$ 2) $5-\sqrt{3}-\sqrt{2}$ 3) $(5+\sqrt{3}-\sqrt{2})$ 4) $(5-\sqrt{3}+\sqrt{2})$

22. If [y] denotes the greatest integer $\leq$ y, then evaluate $\int_0^{\infty}\left[\frac{2}{e^x}\right]dx$

1) 2 2) $\log_e 2$ 3) $\frac{2}{e}$ 4) $\log_e\left(\frac{2}{e}\right)$

23. If [x] denotes the greatest integer $\leq$ x, then evaluate $\int_1^2\left\{\left[x^2\right]-\left[x\right]^2\right\}dx$

1) $4-\sqrt{3}$ 2) $4-\sqrt{3}-\sqrt{2}$ 3) $4+\sqrt{2}-\sqrt{3}$ 4) $4-2+\sqrt{3}$

24. Evaluate $\int_0^1 x.|x-\frac{1}{2}|dx$

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4 $\frac{1}{12}$

25. Evaluate $\int_0^{2n\pi}[|\sin x|-\{|\frac{1}{2}\sin x|\}\,dx$

1) $-2n$ 2) n 3) $2n$ 4) none of the above

26. $\int_{-1}^{1} \frac{\cosh x}{1+e^x} dx =$

1) 0 2) $\frac{e^2+1}{2e}$ 3) $\frac{e^2-1}{2e}$ 4) *none*

27. Evaluate $\int_{-a}^{a} \log_e\left(x+\sqrt{1+x^2}\right) dx$

1) $2a$ 2) 0 3) $2\log_e a$ 4) $\log_e 2a$

28.. Evaluate $\int_{-\pi/2}^{\pi/2} \left(3\sin x + \sin^3 x\right) dx$

1 3 2) 2 3) 0 4) $\frac{10}{3}$

29. Evaluate $\int_{-\pi}^{\pi} \frac{x\cos x}{\left(1+\sin^2 x\right)} dx$

1) 2π 2) π 3) $\frac{\pi}{2}$ 4) 0

30. Evaluate $\int_{\log(1/2)}^{\log 2} \sin\left(\frac{e^x-1}{e^x+1}\right) dx$

1) $\cos\frac{1}{3}$ 2) $2\log 2$ 3) $2\cos\left(\frac{1}{2}\right)$ 4) 0

31. The integral $\int_{-a}^{a} \frac{\sin^2 x}{1-x^2} dx, 0<a<1,$ *is equivalent to*

1) 0 2) $\int_0^{2a} \frac{\sin^2 x}{1-x^2}$ 3) $2\int_0^{a} \frac{\sin^2 x}{1-x^2} dx$ 4) $2a\int_0^{a} \frac{\sin^2 x}{1-x^2} dx$

32. Evaluate $\int_{-1}^{1} \frac{dx}{\left(1+x^2\right)^2}$

1) $\frac{5\pi}{7}$ 2) 0 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{4}+\frac{1}{2}$

33. Evaluate $\int_{-2}^{3} \left|x^2-x\right| dx$

1) $2\frac{1}{2}$ 2) $4\frac{1}{2}$ 3) $9\frac{1}{2}$ 4) $16\frac{1}{2}$

34. Evaluate $\int_0^{\pi/24} \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} dx$

1) $\sqrt{2}\log\sqrt{2}$ 2) $2\log 2$ 3) $\frac{1}{2}\log 2$ 4 $\frac{1}{2}\log\sqrt{2}$

35. Evaluate $\int_0^{\pi} \left|\sin^4 x\right| dx$

1) $\frac{2\pi}{3}$ 2) $\frac{3\pi}{8}$ 3) $\frac{5\pi}{8}$ 4) $\frac{5\pi}{7}$

36. Evaluate $\int_0^{\pi/2}\left|\sin\left(x-\frac{\pi}{4}\right)\right|dx$

1) $2+\sqrt{2}$ 2) $2-\sqrt{2}$

3) $-2+\sqrt{2}$ 4) 0

37. $\int_{-1}^{1}(x-[2x])dx =$

1)1 2)0 3)2 4)4

38. $\int_{0}^{\pi/4}\sin x \cdot d(x-[x]) =$

1) 1/2 2) $1-1/\sqrt{2}$ 3) 1 4) *none*

39. $\int_{0}^{\infty}\left[\frac{2}{e^x}\right]dx =$

1) $\log_e 2$ 2) e^2 3)0 4)2/e

40. $\int_{0}^{\pi/4}\sec^6 x\,dx =$

1)8/15 2)28/15 3)35/8 4)44/15

41. $\int_{0}^{1}x^4(1-x)5/2\,dx =$

1) $\frac{1384}{45045}$ 2) $\frac{84}{5045}$ 3) $\frac{384}{45045}$ 4) $\frac{284}{45045}$

42. $\int_{0}^{\pi}\cos^5\frac{x}{2}dx =$

1)16/15 2)8/35 3)16/35 4)4/35

43. $\int_{0}^{\pi/2}\cos^4 x\sin^8 x\,dx =$

1) $\frac{5\pi}{2048}$ 2) $\frac{7\pi}{2048}$ 3) $\frac{9\pi}{2048}$ 4) $\frac{11\pi}{2048}$

44. $\underset{x\to 0}{Lt}\left(\frac{\int_0^x \tan^2 t\, sec^2 t\, dt}{x^3}\right) =$

1)0 2)1 3)1/3 41/2

45. $\underset{x\to 0}{Lt}\dfrac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}=$

1) 1/3) 2)1 3)2/3 4) none

46. Let

$f\ be\ a\ real\ valued\ function\ such\ that\ f(2)=2\ and\ f^1(2)=1.Then,\lim_{x\to 2}\dfrac{\int_2^{f(x)}4t^3dt}{x-2}\ equals$

1) 6 2) 16 3) 32 4) 44

47. Evaluate $\lim_{x\to 1+0}\dfrac{\int_1^x|t-1|\,dt}{\sin(x-1)}$

1) -1 **2)** 0 3) 1 4) 2

48. The points of local maxima of the function $f(x)=\int_0^{x^2}\left(\dfrac{t^2-5t+4}{2+e^t}\right)dt\ in\ the\ interval\ [-2,2]\ are:$

1)x=-2 and x=1 2)x=-1 and x=1 3)x=0 and x=1 4)x=1 and x=2

49. $If\ f(x)=\int_{x^2}^{x^3}\dfrac{dt}{\log t},x>0\ and\ x\neq 1;then$

1)f(x) is an increasing function 2) f(x) is an decreasing function

3)f(x) has minimum value at x=1 4)None of these

50. If $I=\int_0^1 e^{x^2}dx, then$

1)I<1 2)I>e 3)1<I<e 4) None of these

51. $If\ I_1=\int_1^2\dfrac{dx}{x}\ and\ I_2=\int_1^2\dfrac{dx}{\sqrt{1+x^2}},then$

1) $I_1=I_2$ 2) $I_1<I_2$ 3) $I_1>I_2$ 4) $I_2>2I_1$

52. If $I_1=\int_x^1\dfrac{dt}{(1+t^2)}\ and\ I_2=\int_1^{1/x}\dfrac{dt}{(1+t^2)},\ then\ for\ x>0$

1) $I_1<I_2$ 2) $I_1>I_2$ 3) $I_1=I_2$ 4) $2I_2=I_1$

MATHS – II B SOLUTIONS

1. Required limit $\underset{n\to\infty}{Lt}\dfrac{1}{n}\sum_{r=1}^{2n}\dfrac{r/n}{\sqrt{1+r^2/n^2}}=\int_0^2\dfrac{x}{\sqrt{1+x^2}}dx=\left[\sqrt{1+x^2}\right]_0^2=\sqrt{5}-1$

2. $\underset{n\to\infty}{Lt}\dfrac{1^9+2^9+3^9+\cdots+n^9}{n^{10}}=\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\dfrac{r^9}{n^{10}}=\underset{n\to\infty}{Lt}\dfrac{1}{n}\sum_{r=1}^{n}\left(\dfrac{r}{n}\right)^9=\int_0^1 x^9dx=\left[\dfrac{x^{10}}{10}\right]_0^1=\dfrac{1}{10}$

3. $$\underset{n\to\infty}{Lt}\left[\frac{1}{1+n^3}+\frac{4}{8+n^3}+\frac{9}{27+n^3}+\cdots+\frac{1}{2n}\right]$$

$$=\underset{n\to\infty}{Lt}\left[\frac{1^2}{1^3+n^3}+\frac{2^2}{2^3+n^3}+\frac{3^2}{3^3+n^3}+\cdots+\frac{n^2}{n^3+n^3}\right]=\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{r^2}{r^3+n^3}$$

$$=\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n}\frac{(r/n)^2}{(r/n)^3+1}=\int_0^1\frac{x^2}{x^3+1}dx$$

$$=\frac{1}{3}\int_0^1\frac{3x^2}{1+x^3}dx=\frac{1}{3}\left[\log\left|1+x^3\right|\right]_0^1=\frac{1}{3}[\log 2-\log 1]=\frac{1}{3}\log 2$$

4. $$\underset{n\to\infty}{Lt}\left[\frac{n^{\frac{1}{2}}}{n^{\frac{3}{2}}}+\frac{n^{\frac{1}{2}}}{(n+3)^{\frac{3}{2}}}+\frac{n^{\frac{1}{2}}}{(n+6)^{\frac{3}{2}}}+\cdots+\frac{n^{\frac{1}{2}}}{\{n+3(n-1)\}^{\frac{3}{2}}}\right]=$$

$$\underset{n\to\infty}{Lt}\sum_{r=1}^{n-1}\frac{n^{\frac{1}{2}}}{[n+3r]^{3/2}}=\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n-1}\frac{1}{[1+3r/n]^{3/2}}=\int_0^1\frac{1}{(1+3x)^{3/2}}dx$$

$$\int_0^1(1+3x)^{-3/2}dx=\left[\frac{(1+3x)^{-1/2}}{(-1/2)(3)}\right]_0^1=\frac{-2}{3}\left[\frac{1}{\sqrt{1+3x}}\right]_0^1=\frac{-2}{3}\left[\frac{1}{2}-1\right]=\frac{1}{3}$$

$$Let\, y=\underset{n\to\infty}{Lt}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$$

$$\log y=\underset{n\to\infty}{Lt}\frac{1}{n}\left[\log\left(1+\frac{1}{n^2}\right)+\log\left(1+\frac{2^2}{n^2}\right)+\cdots+\log\left(1+\frac{n^2}{n^2}\right)\right]$$

$$=\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)=\int_0^1\log\left(1+x^2\right)dx=\left[x\cdot\log\left(1+x^2\right)\right]_0^1-\int_0^1\frac{2x^2}{1+x^2}dx$$

$$=\log 2-2\int_0^1\left(1-\frac{1}{1+x^2}\right)dx=\log 2-2\left[x-\mathrm{Tan}^{-1}x\right]_0^1=\log 2-2\left[1-\frac{\pi}{4}\right]$$

5. $$=\log 2-2+\frac{\pi}{2}=\log 2+\log e^{\left(\frac{\pi}{2}-2\right)}=\log\left(2\cdot e^{\left(\frac{\pi-4}{2}\right)}\right)\Rightarrow y=2\cdot e^{\left(\frac{\pi-4}{2}\right)}$$

$$\underset{n\to\infty}{Lt}\left[\frac{\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{n+n}}{n\sqrt{n}}\right]=\underset{n\to\infty}{Lt}\sum_{r=1}^{n}\frac{\sqrt{n+r}}{n\sqrt{n}}=\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n}\sqrt{1+r/n}$$

6. $$=\int_0^1\sqrt{1+x}\,dx=\frac{2}{3}\left[(1+x)^{3/2}\right]_0^1=\frac{2}{3}\left[2^{3/2}-1\right]=\frac{2\left(2\sqrt{2}-1\right)}{3}$$

7. $$\int_0^{\pi}e^x\sin x\,dx=\left[\frac{e^x}{2}(\sin x-\cos x)\right]_0^{\pi}=\frac{e^x}{2}(0+1)-\frac{e^0}{2}+\frac{1}{2}=\left(e^{\pi}+1\right)$$

$$\int_0^1 x\left(\mathrm{Tan}^{-1}x\right)^2 dx=\left[\frac{x^2}{2}\left(\mathrm{Tan}^{-1}x\right)^2\right]_0^1-\int\frac{x^2}{2}\cdot 2\,\mathrm{Tan}^{-1}x\cdot\frac{1}{1+x^2}dx=\frac{1}{2}\left(\frac{\pi}{4}\right)^2-\int_0^1 \mathrm{Tan}^{-1}x\left(1-\frac{1}{1+x^2}\right)dx$$

$$=\frac{\pi^2}{32}-\int_0^1 \mathrm{Tan}^{-1}x\,dx+\int_0^1\frac{\mathrm{Tan}^{-1}}{1+x^2}dx=\frac{\pi^2}{32}-\left[x\cdot\mathrm{Tan}^{-1}x-\frac{1}{2}\log\left(x^2+1\right)\right]_0^1+\left[\frac{1}{2}\left(\mathrm{Tan}^{-1}x^2\right)\right]_0^1$$

8. $$=\frac{\pi^2}{32}-\left[\frac{\pi}{4}-\frac{1}{2}\log 2\right]+\frac{1}{2}\cdot\left(\frac{\pi}{4}\right)^2=\frac{\pi}{16}-\frac{\pi}{4}+\frac{1}{2}\log 2=\frac{1}{2}\log 2+\frac{\pi^2}{16}-\frac{\pi}{4}$$

9. $$\int_0^1 \mathrm{Sin}^{-1}x\,dx=\left[\left(\mathrm{Sin}^{-1}x\right)x\right]_0^1-\int_0^1\frac{1}{\sqrt{1-x^2}}x\,dx=\frac{\pi}{2}+\int_0^1\frac{-x}{\sqrt{1-x^2}}dx=\frac{\pi}{2}+\int_1^0\frac{t\,dt}{\sqrt{t^2}}\quad\left[1-x^2=t^2\right]$$

$$=\frac{\pi}{2}+\int_1^0 dt=\frac{\pi}{2}+\left[t\right]_1^0=\frac{\pi}{2}-1=\frac{\pi-2}{2}$$

$$\int_0^{\pi/2} x\cot x\,dx=\left[x\cdot\log\left(\sin x\right)\right]_0^{\pi/2}-\int_0^{\pi/2}1.\log\left(\sin x\right)dx$$

$$=0-\int_0^{\pi/2}\log\left(\sin x\right)dx$$

$$=-\left(-\frac{\pi}{2}\log 2\right)$$

10. $$=\frac{\pi}{2}\log 2$$

$$\int_0^{\infty}\frac{\tan^{-1}x}{x\left(1+x^2\right)}dx$$

put $x=\tan t$ *so that* $dx=\sec^2 t\,dt$*.Then*

11. $$I=\int_0^{\pi/2}\frac{t}{\tan t\cdot\sec^2 t}\cdot\sec^2 t\,dt=\int_0^{\pi/2}t\cot dt$$

$$=\left[t\log\sin t\right]_0^{\pi/2}-\int_0^{\pi/2}1.\log\sin t\,dt$$

$$=-\int_0^{\pi/2}\log\sin t\,dt=-\left(-\frac{\pi}{2}\log 2\right)=\frac{\pi}{2}\log 2$$

12.

$$Put\ \sqrt{x}=t\Rightarrow\frac{1}{2\sqrt{x}}dx=dt\Rightarrow dx=2t\,dt\qquad x=0\Rightarrow t=0,x=1\Rightarrow t=1$$

$$\int_0^1\frac{x}{1+\sqrt{x}}dx=\int_0^1\frac{t^2}{1+t}\cdot 2t\,dt=2\int_0^1\frac{t^2}{1+t}dt=2\int_0^1\left(1-t+t^2-\frac{1}{1+t}\right)dt=2\left[t-\frac{t^2}{2}+\frac{t^2}{3}-\log\left(1+t\right)\right]_0^1$$

$$=2\left[1-\frac{1}{2}+\frac{1}{3}-\log 2\right]=2\left(\frac{5}{6}-\log 2\right)$$

13.

$$Put\sqrt{x}=t.Then\,\frac{1}{2\sqrt{x}}\cdot dx=dt\Rightarrow dx=2tdt;\,x=1/2,1\Rightarrow t=\frac{1}{\sqrt{2}},1$$

$$\int_{1/2}^{1}\mathrm{Sin}^{-1}\sqrt{x}\,dx=2\int_{1/\sqrt{2}}^{1}t\,\mathrm{Sin}^{-1}t\,dt=2\left[\frac{t^2\,\mathrm{Sin}^{-1}t}{2}\right]_{1/\sqrt{2}}^{1}-2\int_{1/\sqrt{2}}^{1}\frac{1}{\sqrt{1-t^2}}\frac{r^2}{2}dt$$

$$=\frac{\pi}{2}-\frac{\pi}{8}+\int_{1/\sqrt{2}}^{1}\left[\sqrt{1-t^2}-\frac{1}{\sqrt{1-t^2}}\right]dt=\frac{3\pi}{8}+\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\mathrm{Sin}^{-1}t-S\,\mathrm{in}^{-1}t\right]_{1/\sqrt{2}}^{1}$$

$$=\frac{3\pi}{8}+\frac{1}{2}\cdot\frac{\pi}{2}-\frac{\pi}{2}-\frac{1}{2\sqrt{2}}\cdot\frac{1}{\sqrt{2}}-\frac{1}{2}\cdot\frac{\pi}{4}+\frac{\pi}{4}=\frac{3\pi}{8}+\frac{\pi}{4}-\frac{\pi}{2}-\frac{1}{4}-\frac{\pi}{8}+\frac{\pi}{4}=\frac{\pi}{4}-\frac{1}{4}=\frac{\pi-1}{4}$$

14.

$$Put\;x=a\sin\theta$$

$$\int_0^a\frac{dx}{x+\sqrt{a^2-x^2}}dx=\int_0^{\pi/2}\frac{1}{a\sin\theta+a\cos\theta}\cdot a\cos\theta\,d\theta=\int_0^{\pi/2}\frac{\cos\theta}{\sin\theta+\cos\theta}d\theta=\frac{\pi}{4}$$

15.

$$\int_0^a x^3\left(ax-x^2\right)^{3/2}dx=\int_0^{\pi/2}a^3\sin^6\theta\left(a^2\sin^2\theta-a^2\sin^4\theta\right)^{3/2}2a\sin\theta\cos\theta d\theta\;Put x=a\sin^2\theta$$

$$dx=2a\sin\theta\cos\theta d\theta$$

$$x=0,a\Rightarrow\theta=0,\pi/2$$

$$=2a^7\int_0^{\pi/2}\sin^6\theta\sin^3\theta\cos^3\sin\theta\cos\theta d\theta=2a^7\int_0^{\pi/2}\sin^{10}\theta\cos^4\theta d\theta$$

$$=2a^7\times\frac{3}{14}\times\frac{1}{12}\times\frac{9}{10}\times\frac{7}{8}\times\frac{5}{6}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}=\frac{9\pi a^7}{2048}$$

16

$$\int_0^{1/\sqrt{2}}\frac{\mathrm{Sin}^{-1}x}{\left(1-x^2\right)^{\frac{3}{2}}}dx=\int_0^{1/\sqrt{2}}\frac{\mathrm{Sin}^{-1}x}{\left(1-x^2\right)\sqrt{1-x^2}}dx$$

$$Put\,\mathrm{Sin}^{-1}x=t.Then\,\frac{1}{\sqrt{1-x^2}}dx=dt.\,x=0,\frac{1}{\sqrt{2}}\Rightarrow t=0,\frac{\pi}{4}$$

$$\therefore\int_0^{1/\sqrt{2}}\frac{\mathrm{Sin}^{-1}x}{\left(1-x^2\right)\sqrt{1-x^2}}dx=\int_0^{\pi/4}\frac{t}{\left(1-\sin^2t\right)}dt=\int_0^{\pi/4}\frac{t}{\cos^2t}dt=\int_0^{\pi/4}t\sec^2tdt=\left[t\tan t-\log|\sec t|\right]_0^{\pi/4}$$

$$=\left[\frac{\pi}{4}\cdot1-\log\left|\sec\frac{\pi}{4}\right|\right]-\left[0-\log|\sec0|\right]=\frac{\pi}{4}-\log\sqrt{2}+\log1=\frac{\pi}{4}-\frac{1}{2}\log2$$

17.

$$I=\int_0^{\pi/2}\frac{1}{1+\sqrt{\cot x}}dx=\int_0^{\pi/2}\frac{1}{1+\sqrt{\tan x}}dx=\int_0^{\pi/2}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+1}dx\Rightarrow2I=\int_0^{\pi/2}dx=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

18. $$\int_0^1 X^2(1-x)^5\,dx=\int_0^1 (1-x)^2\cdot x^5 dx=\int_0^1 \left(x^5-2x^6+x^7\right)\,dx=\left[\frac{x^6}{6}-\frac{2x^7}{7}+\frac{x^8}{8}\right]_0^1=\frac{1}{6}-\frac{1}{7}+\frac{1}{8}=\frac{1}{168}$$

19. $$\int_{-a}^{b}\frac{\sqrt{x+a}}{\sqrt[3]{x+a}+\sqrt[3]{b-x}}dx=\int_a^b\frac{\sqrt[3]{-a+b-x+a}}{\sqrt[3]{-a+b-x+a}+\sqrt[3]{b-(-a+b-x)}}dx=\int_a^b\frac{\sqrt[3]{b-x}}{\sqrt[3]{b-x}+\sqrt[3]{a+x}}dx$$

$$2I=\int_{-a}^{b}dx[x]_{-a}^{b}=b+a\qquad \therefore I=\frac{a+b}{2}$$

20. $$I=\int_{0.5}^{4.5}[x]dx+\int_{-1}^{1}|x|dx=\int_{0.5}^{1}[x]dx+\int_1^2[x]dx+\int_2^3[x]dx+\int_3^4[x]dx+\int_4^{4.5}[x]dx+2\int_0^1|x|dx$$

$$=0+1\int_1^2 dx+2\int_2^3 dx+3\int_3^4 dx+4\int_4^{4.5}dx+\left[x^2\right]_0^1=1+2+3+4(0.5)+1=9$$

21. $$I=\int_0^2\left[x^2\right]dx$$

$$=\int_0^1\left[x^2\right]dx+\int_1^{\sqrt{2}}\left[x^2\right]dx+\int_{\sqrt{2}}^{\sqrt{3}}\left[x^2\right]dx+\int_{\sqrt{3}}^{2}dx$$

$$=\int_0^1 0.dx+\int_1^{\sqrt{2}}1.dx+\int_{\sqrt{2}}^{\sqrt{3}}2.dx+\int_{\sqrt{3}}^{2}3.dx$$

$$\left[\begin{array}{c}\because 0\le x<1\Rightarrow 0\le x^2<1\Rightarrow\left[x^2\right]=0;\\ 1\le x<\sqrt{2}\Rightarrow 1\le x^2<2\Rightarrow\left[x^2\right]=1;\\ \sqrt{2}\le x<\sqrt{3}\Rightarrow 2\le x^2<3\Rightarrow\left[x^2\right]=2;\\ and\ \sqrt{3}\le x<2\Rightarrow 3\le x^2<4\Rightarrow\left[x^2\right]=3;\end{array}\right]$$

$$=0+[x]_1^{\sqrt{2}}+[2x]_{\sqrt{2}}^{\sqrt{3}}+[3x]_{\sqrt{3}}^{2}$$

$$=\left(\sqrt{2}-1\right)+\left(2\sqrt{3}-2\sqrt{2}\right)+\left(6-3\sqrt{3}\right)=\left(5-\sqrt{3}-\sqrt{2}\right)$$

$$I=\int_0^{\infty}\left[\frac{2}{e^x}\right]dx=\int_0^{\log 2}\left[\frac{2}{e^x}\right]dx+\int_{\log 2}^{\infty}\left[\frac{2}{e^x}\right]dx$$
$$=\int_0^{\log 2}1dx+\int_{\log 2}^{\infty}0.dx$$

22.

$$\left[\begin{array}{l}\because 0<x\le \log 2\Rightarrow 1<e^x\le 2\Rightarrow 1\le \frac{2}{e^x}<2\Rightarrow\left[\frac{2}{e^x}\right]=1;\\ \quad Also, \log 2<x\Rightarrow 2<e^x\Rightarrow \frac{2}{e^x}<1\Rightarrow 0<\frac{2}{e^x}<1\\ \quad \sin ce\,\frac{2}{e^x}\,can\,never\,have\,values\,less\,than\,0,\\ \quad So,\forall x\,such\,that\,\log 2<x<\infty we\,have\left[\frac{2}{e^x}\right]=0\\ \end{array}\right.$$

$$=\log[x]_0^{\log 2}+0=\log_e 2$$

$$I=\int_1^2\left\{\left[x^2\right]-[x]^2\right\}dx$$
$$=\int_1^2\left[x^2\right]dx-\int_1^2[x]^2dx$$
$$=\int_1^{\sqrt{2}}\left[x^2\right]dx-\int_{\sqrt{2}}^{\sqrt{3}}[x]^2dx+\int_{\sqrt{3}}^{2}\left[x^2\right]dx-\int_1^2[x]^2dx$$
$$=\int_1^{\sqrt{2}}1.dx+\int_{\sqrt{2}}^{\sqrt{3}}2.dx+\int_{\sqrt{3}}^{2}3dx-\int_1^2 1^2dx$$
$$\left[\begin{array}{c}\because 1\le x<\sqrt{2}\Rightarrow 1\le x^2<2\Rightarrow\left[x^2\right]=1;\\ \sqrt{2}\le x<\sqrt{3}\Rightarrow 2\le x^2<3\Rightarrow\left[x^2\right]=2,\\ \sqrt{3}\le x<2\Rightarrow 3\le x^2<4\Rightarrow\left[x^2\right]=3\\ and\,1\le x<2\Rightarrow[x]=1\end{array}\right]$$

23.

$$\Rightarrow[x]_1^{\sqrt{2}}+2[x]_{\sqrt{2}}^{\sqrt{3}}+3[x]_{\sqrt{3}}^{2}-[x]_1^2$$
$$=\left(\sqrt{2}-1\right)+2\left(\sqrt{3}-\sqrt{2}\right)+3\left(2-\sqrt{3}\right)-(2-1)=4-\sqrt{3}-\sqrt{2}$$

$$I=\int_0^1 x\,|\,x-\frac{1}{2}\,|\,dx=\int_0^{1/2} x\left|x-\frac{1}{2}\right|dx+\int_{1/2}^1 x\left|x-\frac{1}{2}\right|dx$$

$$=\int_0^{1/2} x.\left\{-\left(x-\frac{1}{2}\right)\right\}dx+\int_{1/2}^1 x\cdot\left(x-\frac{1}{2}\right)dx$$

$$\left[\begin{array}{l}\because 0\le x<\frac{1}{2}\Rightarrow\left(x-\frac{1}{2}\right)<0\\ \qquad\Rightarrow\left|x-\frac{1}{2}\right|=-\left(x-\frac{1}{2}\right)\\ and\frac{1}{2}\le x\le 1\Rightarrow\left(x-\frac{1}{2}\right)\ge 0\\ \qquad\Rightarrow\left|x-\frac{1}{2}\right|=\left(x-\frac{1}{2}\right)\end{array}\right]$$

$$=-\int_0^{1/2}\left(x-\frac{1}{2}x\right)dx+\int_{1/2}^1\left(x^2-\frac{1}{2}x\right)dx$$

$$=-\left[\frac{x^3}{3}-\frac{x^2}{4}\right]_0^{1/2}+\left[\frac{x^3}{3}-\frac{x^2}{4}\right]_{1/2}^1$$

24.

$$=-\left[\frac{1}{24}-\frac{1}{16}\right]+\left[\left(\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{24}-\frac{1}{16}\right)\right]=\frac{1}{8}$$

$$I=\int_0^{2n\pi}[\,|\sin x\,|-\{|\,\frac{1}{2}\sin x\,|\}\,dx$$

$$=2n\int_0^{\pi}\left[\left|\sin x\right|-\frac{1}{2}\left|\sin x\right|\right]$$

$$\left[\begin{array}{c}U\sin g\ prop.18\\ \because f(x)=\left|\sin x\right|-\left|\frac{1}{2}\sin x\right|is\,a\\ periodic\ fubnction\ of\ period\,\pi\end{array}\right]$$

25.

$$=n\int_0^{\pi}\left|\sin x\right|dx=n\int_0^{\pi}\sin x\,dx$$

$$\left[\begin{array}{l}\because 0\le x\le\pi\Rightarrow\sin x\ge 0\\ \qquad\Rightarrow\left|\sin x\right|=\sin x\end{array}\right]$$

$$=2n\int_0^{\pi/2}\sin x\,dx$$

$$=2n\left[-\cos x\right]_0^{\pi/2}=2n$$

$$I=\int_{-1}^{1}\frac{\cosh x}{1+e^x}dx=\int_{-1}^{1}\frac{\cosh(-x)}{1+e^{-x}}dx=\int_{-1}^{1}\frac{\cosh x\cdot e^x}{e^x+1}dx$$

$$2I=\int_{-1}^{1}\frac{\cosh\left(I+e^x\right)}{1+e^x}dx=\int_{-1}^{1}\cosh x\,dx=2\int_0^1\cosh x\,dx=2\left(\sinh x\right)_0^1=2\left(\frac{e-1/e}{2}\right)=\frac{\left(e^2-1\right)}{e}$$

26.

$$\Rightarrow I=\frac{e^2-1}{2e}$$

$$Let\ f(x) = \log_e\left(x+\sqrt{1+x^2}\right).Then,$$

$$f(x) = \log_e\left\{-(x)+\sqrt{1+(-x^2)}\right\}$$

$$= \log_e\left\{\sqrt{1+x^2-x}\right\} = \log_e\left\{\frac{\left(\sqrt{1+x^2-x}\right)\left(\sqrt{1+x^2+x}\right)}{\left(\sqrt{1+x^2+x}\right)}\right\}$$

27. $$= \log\left\{\frac{1}{\sqrt{1+x^2+x}}\right\} = \log\left\{\left(x+\sqrt{1+x^2}\right)^{-1}\right\}$$

$$= -\log\left(x+\sqrt{1+x^2}\right) = -f(x)$$

$$\therefore f(x)\, is\, an\, odd\, function\, and\, so,$$

$$\int_{-0}^{q} \log_e\left(x+\sqrt{1+x^2}\right)dx = \int_{-a}^{a} f(x)dx = 0$$

$$Letf(x) = 3\sin x + \sin^3 x.Then$$

$$f(-x) = 3\sin(-x) + \sin^3(-x) = -3\sin x = -\sin^3 x = -\left(3\sin x + \sin^3 x\right)$$

28. $$= -f(x)$$

$$\therefore f(x)\, is\, an\, odd\, function\, and\, so,$$

$$\int_{-\pi/2}^{\pi/2}\left(3\sin x + \sin^3 x\right)dx = \int_{-\pi/2}^{\pi/2} f(x)dx = 0$$

$$Let\ f(x) = \frac{x\cos x}{\left(1+\sin^2 x\right)}.Then,$$

29. $$f(-x) = \frac{(-x)\cos(-x)}{\left[1+\sin^2(-x)\right]} = \frac{x\cos x}{1+\sin^2 x} = -f(x)$$

$$\therefore f(x)\, is\, an\, odd\, function, and\, so$$

$$\int_{-\pi}^{\pi}\frac{x\cos x}{1+\sin^2 x}dx = \int_{-\pi}^{\pi} f(x)dx = 0$$

$$I = \int_{\log\left(\frac{1}{2}\right)}^{\log 2}\sin\left(\frac{e^x-1}{e^x+1}\right)dx = \int_{-\log 2}^{\log 2}\sin\left(\frac{e^x-1}{e^x+1}\right)dx$$

$$Let\ f(x) = \sin\left(\frac{e^x-1}{e^x+1}\right).Then,$$

30. $$f(-x) = \sin\left(\frac{e^{-x}-1}{e^{-x}+1}\right) = \sin\left(\frac{1-e^x}{1+e^x}\right)$$

$$= \sin\left(\frac{e^x-1}{e^x+1}\right) = -f(x)$$

$$\therefore f(x)\, is\, an\, odd\, function\, and\, so$$

$$I = \int_{-\log 2}^{\log 2} f(x)dx = 0$$

$$Let f\left(x\right)=\frac{\sin^2 x}{1-x^2}.\ Then,$$

$$f\left(x\right)=\frac{\sin^2\left(-x\right)}{1-\left(-x\right)}=\frac{\left(-\sin x\ \right)^2}{1-x^2}=\frac{\sin^2 x}{1-x^2}=f\left(x\right)$$

31. $\therefore f\left(x\right) is\ an\ even\ \ function\ \ and\ \ so,$

$$\int_{-a}^{a}\frac{\sin^2 x}{1-x^2}dx=\int_{-a}^{a}f\left(x\right)dx=2\int_{0}^{a}f\left(x\right)dx$$

$$=2\int_{0}^{a}\frac{\sin^2 x}{1-x^2}dx$$

$$I=\int_{-1}^{1}\frac{dx}{\left(1+x^2\right)^2}=2\int_{0}^{1}\frac{dx}{\left(1+x^2\right)^2}$$

$Put\ \ x=\tan\ t\ \ so\ \ that\ \ dx=\sec^2\ \ t\ \ dt. Then,$

32. $$I=2\int_{0}^{\frac{\pi}{4}}\frac{\sec^2 t\,dt}{\sec^4 t}=2\int_{0}^{\frac{\pi}{4}}\cos^2 t\,dt\ =\int_{0}^{\frac{\pi}{4}}\left(1+\cos 2t\right)dt$$

$$=\left[t+\frac{\sin 2t}{2}\right]_{0}^{\pi/4}=\frac{\pi}{4}+\frac{1}{2}$$

$$\left(x^2-x\right)\geq 0\Leftrightarrow x\left(x-1\right)\geq 0\Leftrightarrow x\geq 1\,or\ x\leq 0$$

$$and\ \left(x^2-x\right)<0\Leftrightarrow x\left(x-1\right)<0\Leftrightarrow 0<x<1$$

$$\therefore When\ x\geq 1\,or\ x\leq 0\ then\left|x^2-x\right|=\left(x^2-x\right)$$

$$and\ when\ 0<x<1\ then\left|x^2-x\right|=-\left(x^2-x\right)=\left(x-x^2\right)$$

Now

$$I=\int_{-2}^{3}\left|x^2-x\right|dx=\int_{-2}^{0}\left|x^2-x\right|dx+\int_{0}^{1}\left|x^2-x\right|dx+\int_{1}^{3}\left|x^2-x\right|dx$$

$$=\int_{-2}^{0}\left(x^2-x\right)dx+\int_{0}^{1}\left(x-x^2\right)dx+\int_{1}^{3}\left(x^2-x\right)dx$$

$$=\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_{-2}^{0}+\left[\frac{x^2}{2}-\frac{x^3}{23}\right]_{0}^{1}+\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_{1}^{3}$$

$$=\left\{0-\left(-\frac{8}{3}-2\right)\right\}+\left\{\left(\frac{1}{2}-\frac{1}{3}\right)-0\right\}+\left\{\left(9-\frac{9}{2}\right)-\left(\frac{1}{3}-\frac{1}{2}\right)\right\}$$

33. $$=\frac{57}{6}=\frac{19}{2}=9\frac{1}{2}$$

$$I=\int_0^{\pi/4}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\,dx$$

$$=\int_0^{\pi/4}\frac{\sqrt{\sin^2 x+\cos^2 x-2\sin x\cos x}}{\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}}dx$$

$$=\int_4^{\pi/4}\frac{|\sin x-\cos x|}{|\sin x+\cos x|}dx$$

$$=\int_0^{\pi/4}\frac{-(\sin x-\cos x)}{(\sin x+\cos x)}dx$$

$$\left[\begin{array}{c}\because 0\le x\le\frac{\pi}{4}\Rightarrow \cos x>0,\sin x\ge 0\, and \sin x\le\cos x\\ \Rightarrow \sin x-\cos x\le 0\, and \sin x+\cos x\ge 0\\ \Rightarrow |\sin x-\cos x|=-(\sin x-\cos x)\, and\, |\sin x+\cos x|=(\sin x+\cos x)\end{array}\right]$$

34. $$=\int_0^{\pi/4}\frac{(\cos x-\sin x)}{(\sin x+\cos x)}dx$$

$$=\left[\log|\sin x+\cos x|\right]_0^{\pi/4}=\log\sqrt{2}=\frac{1}{2}\log 2$$

$$I=\int_0^{\pi}\left|\sin^4 x\right|dx=\int_0^{\pi}\sin^4 x\;dx$$

35. $$\left[\begin{array}{c}\because 0\le x\le\pi\Rightarrow\sin x\ge 0\Rightarrow\sin^4 x\ge 0\\ \Rightarrow\left|\sin^4 x\right|=\sin^4 x\end{array}\right]$$

$$=\int_0^{\pi/2}\left[\sin^4 x+\sin^4(\pi-x)\right]dx$$

$$=2\int_0^{\pi/2}\sin^4 x\,dx=2\times\frac{3\times 1}{4\times 2}\times\frac{\pi}{2}=\frac{3\pi}{8}$$

$$I=\int_0^{\pi/2}\left|\sin\left(x-\frac{\pi}{4}\right)\right|dx$$

$$=\int_0^{\pi/4}\left|\sin\left(x-\frac{\pi}{4}\right)\right|dx+\int_{\pi/2}^{\pi/2}\left|\sin\left(x-\frac{\pi}{4}\right)\right|dx$$

$$=\int_0^{\pi/4}-\sin\left(x-\frac{\pi}{4}\right)dx+\int_{\pi/4}^{\pi/2}\sin\left(x-\frac{\pi}{4}\right)dx$$

$$\left[\begin{array}{l}\because 0\le x<\frac{\pi}{4}\Rightarrow\left(x-\frac{\pi}{4}\right)<0\Rightarrow\sin\left(x-\frac{x}{4}\right)<0\\ \qquad\qquad\qquad\Rightarrow\left|\sin\left(x-\frac{\pi}{4}\right)\right|=-\sin\left(x-\frac{\pi}{4}\right)\\ \frac{\pi}{4}\le x\le\frac{\pi}{2}\Rightarrow 0\le\left(x-\frac{\pi}{4}\right)\le\frac{\pi}{4}\\ 0\le\sin\left(x-\frac{\pi}{4}\right)\le\sin\frac{\pi}{4}\Rightarrow 0\le\sin\left(x-\frac{\pi}{4}\right)\le\frac{1}{\sqrt{2}}\\ \Rightarrow\left|\sin\left(x-\frac{\pi}{4}\right)\right|=\sin\left(x-\frac{\pi}{4}\right)\end{array}\right]$$

$$=-\left[\cos\left(x-\frac{\pi}{4}\right)\right]_0^{\frac{\pi}{4}}+-\left[\cos\left(x-\frac{\pi}{4}\right)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

36. $$=\left[\cos\left(x-\frac{\pi}{4}\right)\right]_0^{\frac{\pi}{4}}-\left[\cos\left(x-\frac{\pi}{4}\right)\right]_{\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$-\left[\cos 0-\cos\left(-\frac{\pi}{4}\right)\right]-\left[\cos\left(\frac{\pi}{4}\right)-\cos 0\right]$$

$$\left(1-\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{\sqrt{2}}-1\right)=2-\frac{2}{\sqrt{2}}=2-\sqrt{2}$$

$$\int_{-1}^{1}\left(x-[2x]\right)dx=-\int_{-1}^{1}[2x]dx=-\left\{\int_{-1}^{-1/2}-2+\int_{-1/2}^{0}-1+\int_{0}^{1/2}0+\int_{1/2}^{1}1\right\}$$

37. $$=-\left\{-2\left(-\frac{1}{2}+1\right)-1\left(\frac{1}{2}\right)+\left(1-1/2\right)\right\}=1$$

38. $$If\ 0<x<\pi/4\,then\left[x\right]=0.\therefore\int_0^{\pi/4}\sin x\cdot d\left(x-\left[x\right]\right)=\int_0^{\pi/4}\sin x\,dx=1-\frac{1}{\sqrt{2}}$$

39. $$If\ x<\log 2\,then\left(\frac{2}{e^x}\right)=1\,and\ if\ x>\log 2\,then\left(\frac{2}{e^x}\right)=0.\therefore\int_0^{\infty}\left[\frac{2}{e^x}\right]dx=\int_0^{\log 2}1dx+\int_{\log 2}^{\infty}0dx=\log_e 2$$

40. $$\int_0^{\pi/4} \sec^6 x\,dx = \frac{\left(\sqrt{2}\right)^{6-2}}{6-1} + \frac{6-2}{6-1} I_4 = \frac{4}{5} + \frac{4}{5}\left[\frac{\left(\sqrt{2}\right)^{4-2}}{4-1} + \frac{4-2}{4-1} I_2\right]$$
$$= \frac{4}{5} + \frac{4}{5}\left[\frac{2}{3} + \frac{2}{3}\int_0^{\pi/4} \sec^2 x\,dx\right] = \frac{4}{5} + \frac{8}{15} + \frac{8}{15}\left[\operatorname{Tan} x\right]_0^{\pi/4} = \frac{4}{5} + \frac{16}{15} = \frac{28}{15}$$

41. $$\int_0^1 x^4 (1-x)^{5/2}\,dx = \int_0^{\pi/2} \sin^8\theta\left(1-\sin^2 d\theta\right)^{5/2} 2\sin\theta\cos\theta\,d\theta$$
$$= 2\int_0^{\pi/2} \sin^9\theta\cdot\cos^6\theta\,d\theta = 2\cdot\frac{8}{15}\times\frac{6}{13}\times\frac{4}{11}\times\frac{2}{9}\times\frac{1}{7} = \frac{384}{45045}$$

42. $Put\ x/2 = t. Then\ dx = 2dt.\ x = 0,\pi \Rightarrow t = 0,\pi/2$
$$\therefore \int_0^{\pi} \cos^5\frac{x}{2}\,dx = \int_0^{\pi/2} \cos^5 t(2dt) = 2\int_0^{\pi/2} \cos^5 t\,dt = 2\cdot\frac{4}{5}\cdot\frac{2}{3}\cdot 1 = \frac{16}{15}$$

43. $$\int_0^{\pi/2} \cos^4 x\sin^8 x\,dx = \frac{7}{12}\cdot\frac{5}{10}\cdot\frac{3}{8}\cdot\frac{1}{6}\int_0^{\pi/2} \cos^4 x\,dx = \frac{7}{4}\cdot\frac{1}{2}\cdot\frac{1}{8}\cdot\frac{1}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = \frac{7\pi}{2048}$$

44. $$\underset{x\to 0}{Lt} = \frac{\int_0^x \tan^2 t\cdot sec^2 t\,dt}{x^3} = \underset{x\to\infty}{Lt}\frac{\tan^2 x\cdot\sec^2 x}{3x^2} = \frac{1}{3}$$

45. $$\underset{x\to 0}{Lt}\frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3} = \underset{x\to 0}{Lt}\frac{\sin x\cdot 2x}{3x^2} = \underset{x\to 0}{Lt}\frac{2}{3}\cdot\frac{\sin x}{x} = \frac{2}{3}$$

$Let\ g(x)=\int_2^{f(x)} 4t^3\, dt\ h(x)=x-2.\ Then$

$g(2)=\int_2^{f(2)} 4t^3\, dt=\int_2^{2} 4t^3 dt=0 \qquad [\because f(2)=2]$

$and\ \ h(2)=2-2=0$

46. $$\therefore \lim_{x\to 2}\left\{\frac{\int_2^{f(x)} 4t^3 dt}{x-2}\right\}=\lim_{x\to 2}\frac{g(x)}{h(x)}\left[\frac{0}{0}\ form\right]$$

$$=\lim_{x\to 2}\frac{g'(x)}{h'(x)} \qquad \left[L'^{Hospital's\ \ Rule}\right]$$

$$=\lim_{x\to 2}\left\{\frac{4\left[f(x)\right]^3 . f'(x)}{1}\right\}=4\times\left[f(2)\right]^3\times f'(2)$$

$$=4\times 2^3\times 1=32$$

$$\lim_{x\to 1+0}\frac{\int_1^x |t-1|\, dt}{\sin(x-1)}$$

$$=\lim_{h\to 0}\frac{\int_1^{1+h} |t-1|\, dt}{\sin(1+h-1)}$$

47. $$=\lim_{h\to 0}\frac{\int_1^{1+h} (t-1)\, dt}{\sin h} \qquad \left[\begin{array}{l}\because 1\le t\le 1+h\\ \Rightarrow t-1\ge 0\Rightarrow |t-1|=(t-1)\end{array}\right]$$

$$=\lim_{h\to 0}\frac{(1+h-1)}{\cos\ h} \qquad \left[U\sin g\quad L'\quad Hospital's\ Rule\ \right]$$

$$=\lim_{h\to 0}\frac{h}{\cos\ h}=\frac{0}{1}=0$$

$$f(x)=\int_0^{x^2}\left(\frac{t^2-5t+4}{2+e^t}\right)dt$$

$$\Rightarrow f(x)=\left(\frac{x^4-5x^2+4}{2+e^{x^2}}\right)\cdot 2x$$

$$Now,\ f(x)=0\Rightarrow 2x\left(\frac{x^4-5x^2+4}{2+e^{x^2}}\right)=0\Rightarrow 2x\left(x^4-5x^2+4\right)=0$$

$$\Rightarrow 2x\left(x^2-1\right)\left(x^2-4\right)=0$$

$$\Rightarrow 2x(x-1)(x+1)(x-2)(x+2)=0$$

$$\Rightarrow x=0\,or\,x=1\,or\,x=-1\,or\,x=2\,or\,x=-2$$

$\therefore x=0, x=1, x=-1, x=2$ *and* $x=-2$ *are the candidates for local* min*ima.*

For ery small $+ve$ *value of* h, *we have:*

$(i)\, f(0-h)<0$ *and* $f(0+h)>0$

$\therefore x=0$ *is a point of* min*ima*

$(ii)\ f(1-h)>0$ *and* $f(1+h)<0$

$\therefore x=1$ *is a point of* max*ima*

$(iii)\ f(-1-h)>0$ *and* $f(-1+h)<0$

$\therefore x=-1$ *is a point of* max*ima*

$(iv)\, f(2-h)<0$ *and* $f(2+h)>0$

$\therefore x=2$ *is a point of* min*ima*

48. $(v)\, f(-2-h)<0$ *and* $f(-2+h)>0$

$\therefore x=-2$ *is a point of* min*ima*

Thus, $x=-1$ *and* $x=1$ *are the points of local* max*ima*

49.

$$f(x)=\int_{x^2}^{x^3}\frac{dt}{\log t}, x>0\,and\,x\neq 1;$$

$$\Rightarrow f(x)=\frac{1}{\log\left(x^3\right)}\cdot 3x^2-\frac{1}{\log\left(x^2\right)}\cdot 2x$$

$$\begin{bmatrix}\because F(x)=\int_{h(x)}^{g(x)} f(t)\,dt \\ \Rightarrow F'(x)=f\left[g(x)\right]g^{'(x)}-f\left[h(x)\right]h^{'}(x)\end{bmatrix}$$

$$=\frac{3x^2}{3\log x}-\frac{2x}{2\log x}=\frac{x^2}{\log x}-\frac{x}{\log x}=\frac{x(x-1)}{\log x}$$

Now, $0<x<1\Rightarrow x>0, (x-1)<0$ *and* $\log x<0$

$1<x\Rightarrow x>0,\ (x-1)>0$ *and* $\log x>0\Rightarrow f(x)>0$

$\therefore f(x)>0\,\forall x>0$

i.e., $f(x)$ *is an incresing function.*

Also, since $f(x)$ *does not exist at* $x=1$, *so* $f(x)$ *cannot have a* min*imumvalue at* $x=1$

$$0 < x < 1$$
$$\Rightarrow 0 < x^2 < 1$$
$$\Rightarrow e^0 < e^{x^2} < e^1 \text{ since } e > 1$$
$$\Rightarrow 1 < e^{x^2} < e$$
$$\Rightarrow \int_0^1 1.dx < \int_0^1 e^{x^2} dx < \int_0^1 e.dx$$
$$\Rightarrow [x]_0^1 < \int_0^1 e^{x^2} dx < e[x]_0^1$$

50. $1 < / < e$

$$\sqrt{1+x^2} > x$$
$$\Rightarrow \frac{1}{x} > \frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow \int_1^2 \frac{dx}{x} > \int_1^2 \frac{dx}{\sqrt{1+x^2}}$$

51. $\Rightarrow I_1 = I_2$

$$I_1 = \int_x^1 \frac{dt}{\left(1+t^2\right)}$$
$$Put\, t = \frac{1}{u}\, so\, that\, dt = -\frac{1}{u^2}\, du. Then,$$
$$I_1 = -\int_{1/x}^1 \frac{\frac{1}{u^2} du}{\left(1+\frac{1}{u^2}\right)} = -\int_{1/x}^1 \frac{du}{\left(1+u^2\right)} = \int_1^{1/x} \frac{du}{\left(1+u^2\right)}$$
$$= \int_1^{1/x} \frac{dt}{\left(1+t^2\right)}$$

52. $I_{2.}$