TOPIC : HYPERBOLA (DPP)

1. If m is a variable ,the locus of the point of intersection of the lines $\frac{x}{3}-\frac{y}{2}=m$ and $\frac{x}{3}+\frac{y}{2}=\frac{1}{m}$ is
   1) A parabola 2) An ellipse 3) A hyperbola 4) Straight line
2. The number of tangents to $\frac{x^2}{9}-\frac{y^2}{4}=1$ through (6,2) is
   1) 0 2) 1 3) 2 4) 3
3. If H(x,y) =0 represents the equation of a hyperbola A (x, y)=0 ,C(x, y)=0 the joint equation of its asymptotes and the conjugate hyperbola respectively, then for any point (a, b) in the plane H(a, b), A(a, b) and C(a, b) are in
   1) A.P 2)G.P 3) H.P 4)A.G.P
4. For the hyperbola $\frac{x^2}{cos^2\alpha}-\frac{y^2}{sin^2\alpha}=1$ which of the following remains constant when α varies ?
   1) Abscissa of foci 2) eccentricity 3)Diractrix 4)Abscissa of vartices
5. Product of perpendiculas from foci of $\frac{x^2}{4}-\frac{y^2}{9}=1$ to $y=mx+\sqrt{4m^2-9}$ where $m>\frac{3}{2}$ is
   1) 4 2) $\frac{36}{13}$ 3) 3 4) 9
6. If the normal at θ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the transverse axis at G, than $AG.A'G=$
   1) $a^2(e^4sin^2\theta-1)$ 2) $a^2(e^4sec^2\theta+1)$ 3) $b^2(e^4sec^2\theta-1)$ 4) None
7. If e_1 and e_2 are eccentricities of the hyperbola $xy=c^2$ and $x^2-y^2=a^2$ then $e_1^2+e_2^2=$
   1) 4 2) 1 3) $e_1^2-e_2^2$ 4)2 $e_1^2e_2^2$
8. The locus of the middle of the normal chords of the rectangular hyperbola $x^2-y^2=a^2$ is
   1) $(x^2+y^2)^3+4a^2x^2y^2=0$ 2) $(x^2-y^2)^3+4a^2x^2y^2=0$
   3) $(x^2+y^2)^3-4a^2x^2y^2=0$ 4) $(x^2-y^2)^3-4a^2x^2y^2=0$
9. Area of the triangle formed by any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with its asymptotes is
   1)abc 2)4ab 3)ab 4) a^2b^2
10. If the ordinate of any point P on the hyperbola $9x^2-16y^2=144$ is produced to cut the Asymptotes in the points Q and R. then the product PQ.PR equal to
    1) 16 2) 9 3) 25 4) 7
11. From any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ tangents are drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$ then area cut-off by the chord of contact on the asymptotes is equal to
    1) $\frac{a}{2}$ 2)ab 3)2ab 4)4ab
12. The line ax+by+c=0, meets the hyperbola $4x^2-y^2=4p^2$ at P and Q . the point intersection of the tangents at P and Q is
    1) $\left(\frac{p^2a}{c},\frac{4p^2b}{c}\right)$ 2) $\left(\frac{-p^2a}{c},\frac{4p^2b}{c}\right)$ 3) $\left(\frac{-p^2a}{c},\frac{-4p^2b}{c}\right)$ 4) $\left(\frac{p^2a}{c},\frac{-4p^2b}{c}\right)$
13. A tangent to the hyperbola $y=\frac{x+9}{x+5}$ passes through the origin is
    1) $x+25y=0$ 2) $5x+y=0$ 3) $5x-y=0$ 4) $x-25y=0$

14. Tangents drawn from a point on the circle $x^2 + y^2 = 9$ to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$, then

Tangents are at angle

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{2\pi}{3}$

15. Equations of the common tangent of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$ are

1) $y = \pm x \pm \sqrt{a^2 - b^2}$ 2) $y = \pm x \pm 2\sqrt{a^2 + b^2}$

3) $y = \pm x \pm \sqrt{a^2 + b^2}$ 3) $y = \pm x \pm 2\sqrt{a^2 - b^2}$

16.The angle between lines joining the origin to the points of the line $\sqrt{3}\,x + y - 2 = 0$ and

hyperbola $y^2 - x^2 = 4$ is

1) $tan^{-1}(\frac{2}{\sqrt{3}})$ 2) $\frac{\pi}{6}$ 3) $tan^{-1}(\frac{\sqrt{3}}{2})$ 4) $\frac{\pi}{2}$

17. The equation of the tangent parallel to $y = x$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$

1) $x - y + 1 = 0$ 2) $x - y - 2 = 0$ 3) $x + y + 5 = 0$ 4) $x + y + 6 = 0$

18. Equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$. which is bisected at the point (6, 2) Is

1) $16x - 75y = 418$ 2) $75x - 16y = 418$ 3) $25x - 4y = 400$ 4) None of these

19. Number of common tangents with finite slope to the curve $xy = c^2$ and $y^2 = 4ax$ is

1) 0 2) 1 3) 2 4) 4

20. The set of integral values of m for which the line $x + y = m$ cuts hyperbola $xy = 1$ at integral Points

1)$\{1, -1\}$ 2) $\{-2,2\}$ 3)$\emptyset$ 4) None of these

21. The number of normals to hyperbole $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ form an external point is =_____

22. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle

, O being the centre of the hyperbola , then the least value of eccentricity is = _____

23. Two tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having slopes $m_1 and\ m_2$ intersects the axes at four concylic points then the value of $m_1 m_2$ is = _____

24. The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0\ is$ k then k^2=_____

25. If the tangents and normal to a rectangular hyperbole $x^2 - y^2 = 4$ cut off intercepts

$a_1 a_2 + b_1 b_2$= _____

26. If e is eccentricity of a hyperbola whose asymptotes are $3x + 4y = 2\ and\ 4x - 3y + 5 = 0$

Than e^2 =_____

27. The area of Δ formed by the tangent at any point on t the hyperbola $\frac{x^2}{4} - \frac{y^2}{1} = 1$ and the

Asymptotes is = _____

28. If the product of the length of perpendicular drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ its asymptotes is $\frac{k}{3}$ then k = _____

29. The latus rectum of the hyperbole $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ is P then $\frac{2}{9}$ p =_____

30.The eccentricity of the conjugate hyperbola of the hyperbola $x^2 - 3y^2 = 1$ is = _____

KEY

S .NO	1	2	3	4	5	6	7	8	9	10
1 ╾ 10	3	1	1	1	4	1	1	2	3	2
11 ╾ 20	4	2	1	2	1	3	1	2	2	2
21 ╾ 30	4	1	1	2	0	2	2	2	1	2

SOLUTION & EXPLANTION

1. Eliminate parameter m , $\left(\frac{x}{3} - \frac{y}{2}\right)\left(\frac{x}{3} + \frac{y}{2}\right) = 1$
2. $s_{11} > 0 \Rightarrow$ point lie inside (i.e in the region not containing the centre)
3. H+C = 2A
4. A$\epsilon = cos^2\alpha + sin^2\alpha = 1 \Rightarrow \forall\alpha$
5. Product of perpendicular from the foci to any tangent is b^2
6. Normal at $\theta \Rightarrow \frac{ax}{sec\theta} - \frac{by}{tan\theta} = a^2e^2$ G = $(a\epsilon^2 sec\theta, 0)$ =>AG=$a\epsilon^2 sec\theta - a$,AG=$ae^2 sec\theta + a$
7. $\epsilon_1 = \epsilon_2 = \sqrt{2}$
8. Normal $\frac{x}{sec\theta} + \frac{y}{tan\theta} = 2a$ - - - - - -(1)

⇨ Chord Equation whose midpoint (x_1, y_1) is $xx_1 - yy_1 = x_1^2 - y_1^2$ - - - - - - - - - (2)
And elinilnating θ using 1& 2

9.Area of $\Delta^{le} = \frac{n^2\sqrt{h^2 - a}}{am^2 - 2hlm + bl^2}$

10.P(asec $\theta, b\ tan\theta$)

Asymptotes are $\frac{x}{a} + \frac{y}{b} = 0, \frac{x}{a} - \frac{y}{b} = 0$

Than Q=$(asec\theta, -bsec\theta)$

R = $(asec\theta, -bsec\theta)$

PQ.PR=$b^2(sec\theta + \tan\theta)(sec\theta - \tan\theta) = b^2$

11.Equation of chord of contact of P to given hyperbole is $\frac{x}{a}sec\theta - \frac{y}{b}tan\theta = 2$ ----------(1)

$\frac{x}{a}sec\theta - \frac{y}{b}tan\theta = 2$ - - - - - - - -(1)

EQ of asymptotes are $y = \pm\frac{b}{a}x$ - - - - - -(2)

Solve (1) and(2)

12. Hyperbole $\frac{x^2}{p^2}-\frac{y^2}{4p^2}=1$;

Point of intersection of tangents $(\frac{-a^2l}{n},\frac{b^2m}{n})=(\frac{-p^2a}{c},\frac{4p^2b}{a})$

13.$\left[\frac{dy}{dx}\right](x_1y_1)=\frac{-4}{(x+5)^2}$ Equation of the tangent is $(y-y_1)=\frac{-4}{(x_1+5)^2}(x-x_1)$

The tangent is passing through origin then we get the point of contact and hence we get the tangent function

14. $x^2+y^2=9$ is director circle of given hyperbola =>$\theta=\frac{\pi}{2}$

15.Slope form of tangent $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $y=mx+\sqrt{a^2m^2-b^2}$

Which is also tangent to $\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1\ then\ m=\pm1$.

16.Use homogenisation and $tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}$

17.Let the EQ of tangents is $y=x+k\ condition\ c^2=a^2m^2-b^2, k=\pm1$

Tangents are $x-y\pm1=0$.

18.Use $s_1=s_{11}$

19.Use tangents condition to both cases

20.Solving $x(m-x)=1; x^2-mx+1=0$

=>$x=\frac{m\pm\sqrt{m^2-4}}{2}\in Z$.

21.Number of normal drawn to hyperbola is 4.

22. $tan30°=\frac{btan\theta}{asec\theta}=>cosec^2\theta=\frac{3b^2}{a^2}=>cosec^2\theta=3(e^2-1)$

Now $cosec^2\theta\geq1\quad 3(e^2-1)\geq,1=>e>,\frac{2}{\sqrt{3}}$.

23. Equation of tangent are $y=m_1x+\sqrt{a^2m_1^2-b^2}$,

$y=m_2x+\sqrt{a^2m_2^2-b^2}\ \ OA\times OB=OC\times OD=>m_1m_2=-1$.

24.$x^2-y^2-4x+4y+16=0\ is$ rectangular hyperbola =>$e^2=k^2=2$.

25. EQ of tangent is $\frac{x}{a_1}+\frac{y}{b_1}=1$

EQ of normal is $\frac{x}{a_2}+\frac{y}{b_2}=1$

Now $m_1m_{2=-1=>}{}_{a_1a_2+b_1b_2=0.}$

26.$m_1m_2=-1=>e=\sqrt{2}$ =>$e^2=2$.

27.Area of $\Delta^{le} = ab = 2.$

28. $\frac{x^2}{2} - \frac{y^2}{1} = 1, use \frac{a^2b^2}{a^2+b^2}$

$\frac{k}{3} = \frac{2}{3} => k = 2.$

29. EQ of hyperbola is $\frac{(x-1)^2}{16} - \frac{(y+1)^2}{9} = 1$

$LL' = \frac{2b^2}{a} = \frac{9}{2} = p => \frac{2p}{9} = 1.$

30.EQ of hyperbola is $\frac{x^2}{1} - \frac{y^2}{1/3} = 1$

$e_1 = \frac{\sqrt{1+\frac{1}{3}}}{1} = \frac{2}{\sqrt{3}}$

Now $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1 => e_2 = 2$