NARAYANA EDUCATIONAL INSTITUTIONS

DPP :: FUNCTIONS

MATHS

1. $f:R\to R\ f(x)=\dfrac{x^5}{1+x^5}$ then f is_____

1) one-one but not onto
2) on to but not one-one
3) bijection
4) neither one-one nor onto

2. the function $f(x)=\dfrac{a^x+a^{-x}}{a^x-a^{-x}}$ is _____

1) an even function
2) an old function
3) neither even nor odd
4) none

3. If $f(x)=\dfrac{3x+4}{2x-3}$ then fof of (3)=_____

1) 3
2) $\dfrac{13}{3}$
3) 13
4) $\dfrac{1}{3}$

4. $g(x)=3\sqrt{x^2+1}\ f(x)=\sqrt{x^3-2}$ then $fog(-4)$

1) $\sqrt{15}$
2) $3\sqrt{63}$
3) $3\sqrt{17}$
4) $\sqrt{63}$

5. $f(x)=\log\left(\dfrac{1+x}{1-x}\right)$ then $f\left(\dfrac{x_1+x_2}{1+x_1x_2}\right)$

1) 0
2) $f(x_1)f(x_2)$
3) $f(x_1)+f(x_2)$
4) $f(x_1x_2)$

6. If $f(x)=\log\left(\dfrac{1+x}{1-x}\right)\ g(x)=\dfrac{3x+x^3}{1+3x^2}$ then $fog(x)$

1) $-f(x)$
2) $3f(x)$
3) $(f(x))^3$
4) none

7. The domain of the function $f(x)=\sqrt{16-9x^2}$

1) $\left[-\dfrac{4}{3},\dfrac{4}{3}\right]$
2) $\left[\dfrac{4}{3},\dfrac{4}{3}\right]$
3) $R-\left[-\dfrac{4}{3},\dfrac{4}{3}\right]$
4) $R-\left[\dfrac{-4}{3},\dfrac{4}{3}\right]$

8. The domain of the function $f(x)=\dfrac{1}{\sqrt{64-x^2}}$ is_____

1) (-8 8)
2) $[-8,8]$
3) $R-(-8,8)$
4) $R-[-8,8]$

9. The domain of the function $f(x)-\dfrac{1}{\sqrt{x^2-81}}$ is _____

1) $[-9,9]$
2) (-9 9)
3) $R-$(-9 9)
4) $R-[-9,9]$

10. Domain of the function $f(x)=\log|x|$ is_____

1) R
2) $R-\{0\}$
3) R^+
4) None

11. Domain of the function $f(x)=\dfrac{x}{\sqrt{|x|-x}}$ is _____

1) $[0\infty)$
2) $(-\infty 0)$
3) $(-\infty 0]$
4) $[1\infty)$

12. Domain of the function $f(x)=\frac{1}{\log(1-x)}$ is

1) $(-\infty 1)$ 2) $(-\infty 0)\cup(01)$ 3) (1∞) 4) $(-\infty -1)$

13. The range of $f(x)=\sin^2 x+\cos^4 x$ is_____

1) $\left[\frac{1}{4}1\right]$ 2) $\left[\frac{3}{4}1\right]$ 3) $\left[\frac{1}{4}\frac{3}{4}\right]$ 4) $[01]$

14. The range of $f(x)=\frac{1}{2-\cos 3x}$ is _____

1) $\left[0\frac{1}{3}\right]$ 2) $\left[\frac{1}{3}1\right]$ 3) $\left[\frac{1}{3}3\right]$ 4) $\left[01\right]$

15. Find the range of $f(x)=\frac{1}{2-\cos 3x}$ is _____

1) $\left[0\frac{1}{3}\right][0$ 2) $\left[\frac{1}{3}3\right]$ 3) $\left[\frac{1}{3}3\right]$ 4) $\left[01\right]$

16. The range of the function $f(x)=\frac{x^2-x+1}{x^2+x+1}$

1) $\left[\frac{1}{3}3\right]$ 2) $\left[\frac{1}{3}1\right]$ 3) $[1\ 3]$ 4) none

17. If $a^2+b^2+c^2=1$ then the range of $ab+bc+ca$ is_____

1) $\left[-\frac{1}{2}1\right]$ 2) $\left[-\frac{1}{2}\frac{1}{2}\right]$ 3) $[-1\ 1]$ 4) $[0\ 1]$

18. If $f(x)=\frac{2x+3}{3x-2}$ then $f^{-1}(x)=$

1) $\frac{5x+3}{3x-2}$ 2) $\frac{5x+3}{3-2x}$ 3) $\frac{3x-2}{5x+3}$ 4) $\frac{3-2x}{5+3x}$

19. If $f(x)=x-x^2+x^3-x^4+.....$ and $|x|<1$ then $f^{-1}(1)=$

1) $\frac{x}{x-1}$ 2) $\frac{x-1}{x}$ 3) $\frac{x}{1+x}$ 4) $\frac{x}{1-x}$

20. The function $f(x)$ is defined as $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$ when $x\neq 0$ then the function is

1) $f(x)=x^2-2$ 2) $f(x)=x^2+1$ 3) $f(x)=x^2+2$ 4) $f(x)=x+\frac{1}{x}$

21. $f(x)=\frac{\cos^2 x+\sin^4 x}{\sin^2 x+\cos^4 x}$, $f(x)$ $x\in R$ then $f(2002)$

1) 1 2) 2 3) 3 4) 4

22. If $f(x)=\frac{2^x+2^{-x}}{2}$ then $f(x+y)+f(x-y)$

1) $2f(x)f(y)$ 2) $f(x)f(y)$ 3) $\frac{1}{2}f(x)f(y)$ 3) none

23. $f(x)f\left(\frac{1}{x}\right)=4$ and $f(x)>0$ for $x>0$ $f(2)=8$ then $f(2)=8$

1) 25 2) 5 3) 50 4) 32

24. Let $g(x)=1+x-[x]$ and $f(x)=\begin{cases}-1 & x<0\\ 0 & x=0\\ 1 & x>0\end{cases}$ then for all x $f[g(x)]$ is equal to

1) x 2) 1 3) $f(x)$ 4) $g(x)$

25. $f:R\to R$ is given by $f(x)=\frac{4^x}{4^x+2}$ for all $x\in R$ the $f\left(\frac{1}{1997}\right)+f\left(\frac{2}{1997}\right)+.....+f\left(\frac{1996}{1997}\right)=$

1) 997 2) 1998 3) 1997 4) 998

26. $f:R\to R$ is given by $f(x)=\frac{9^x}{9^x+3}$ for all $x\in R$ the $f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+.....+f\left(\frac{1995}{1996}\right)=$

1) 997.5 2) 998 3) 997 4) 998.5

27. If $f\left(\frac{x^2-1}{x^2+1}\right)=2$ for all x. Then

1) $f(x)=x^2\,(x\in R)$ 2) $f(x)=2(-1\le x\le 1)$

3) $f(x)=\frac{x^2-1}{x^2+2}(-1\le x<1)$ 4) none of the above

28. The range of function $f(x)=4^x+2^x+4^{-x}+2^{-x}+3$ is

1) $\left[\frac{3}{4}\ \infty\right)$ 2) $\left(\frac{3}{4}\ \infty\right)$ 3) $(7\ \infty)$ 4) $[7\ \infty]$

29. The domain of the function $f(x)=\log_{2x-1}(x-1)$ is

1) $(1\ \infty)$ 2) $\left(\frac{1}{2}\ \infty\right)$ 3) $(0\ \infty)$ 4) none of these

30. $f:R\to R$, $g:R\to R$ defined by $f(x)=2x+1, g(x)=x^2+1$. Then arrange the following is ascending order

a) $fog(2)$ b) $gof(3)$ c) $gof(-1)$ d) $fog(-2)$

KEY

MATHS

	1	2	3	4	5	6	7	8	9	10
1-10	1	2	2	1	3	2	1	1	4	2
11-20	2	2	2	3	2	1	1	1	3	1
21-30	1	3	3	1	4	1	2	1	1	2

1. $f(x)=f(y)$

$\frac{x^5}{1+x^5}=\frac{y^5}{1+y^5}\Rightarrow x=y$ f is one-one

$f(x)\neq 1\Rightarrow$ co-domain = Range

$\Rightarrow f$ is not onto

2. $f(+x)=\frac{a^x+a^{-x}}{a^x-a^{-x}}$

$f(-x)=\frac{a^{-x}+a^x}{a^{-x}-a^x}=\left[\frac{a^x+a^{-x}}{a^x-a^{-x}}\right]=-f(x)$

$f(x)$ is odd function

3. If $f(x)=\frac{ax+b}{cx-a}$ $fofof(x)=f(x)$

$f(x)=\frac{3x+4}{2x-3}$

$fofof(3)=f(3)=\frac{9+4}{6-3}=\frac{13}{3}$

4. $fog(-4)=f(g(-4))=f\left(3\sqrt{17}\right)=\sqrt{15}$

5. $f(x)=\log\left(\frac{1+x}{1-x}\right)=2\tanh^{-1}x$

$f\left(\frac{x_1+x_2}{1+x_1x_2}\right)=2\tanh^{-1}\left(\frac{x_1+x_2}{1+x_1x_2}\right)$

$=2\tanh^{-1}x_1+2\tanh^{-1}x_2=f(x_1)+f(x_2)$

6. $f(x)=\log\left(\frac{1+x}{1-x}\right)=2\tanh^{-1}x$

$f\left(\frac{3x+x^3}{1+3x^2}\right)=2\tanh^{-1}\left(\frac{3x+x^3}{1+3x^2}\right)$

$=2.3\tanh^{-1}x=3\left(2\tanh^{-1}x\right)=3f(x)$

7. $16-9x^2\geq 0$

$9x^2-16\leq 0$

$x\varepsilon\left[\frac{-4}{3}\ \frac{4}{3}\right]$

8. $64-x^2\succ 0$

$x^2-64\prec 0$

$x\varepsilon(-8\ 8)$

9. $x^2-81\succ 0$

$(x+9)(x-9) \succ 0$

$x \varepsilon R - [-9\ 9]$

10. $f(x) = \log|x|$

$|x| \neq 0 \Rightarrow x = 0$

$x \varepsilon R - \{0\}$

11. $|x| - x \succ 0$

$|x| \prec x \Rightarrow x \varepsilon (-\infty\ 0)$

Note for all $x \in (0\ \infty)$

$|x| - x = 0$

12. $f(x) = \dfrac{1}{\log(1-x)}$ $\quad 1-x \succ 0$

$1 \succ x \Rightarrow x \prec 1$

$1 - x \neq 1 \quad x \neq 0$

$\therefore x \in (-\infty\ 0) \cup (0\ 1)$

13. $f(x) = \sin^2 x + \cos^4 x$

$f(x) = 1 - \cos^2 + \cos^4 x$

$= 1 - \cos^2 x[1 - \cos^2 x]$

$= 1 - \sin^2 x \cos^2 x$

$= 1 - \dfrac{1}{4}\sin^2 2x$

Range of $\sin^2 2x$ is $[0\ 1]$

Range of $f(x) = \left[1 - \dfrac{1}{4} - 0\right]$

$\left[\dfrac{3}{4} 1\right]$

14. $f(x) = \dfrac{x^2}{x^2+1}$

$1 - \dfrac{1}{x^2+1} \leq 1$

$0 \prec \dfrac{1}{x^2+1} \leq 1$

Range of $f(x) = [01)$

15. $f(x) = \dfrac{1}{2 - \cos 3x}$

We know $-1 \leq \cos 3x \leq 1$

Range of $f(x) = \left[\dfrac{1}{2+1}, \dfrac{1}{2-1}\right] = \left[\dfrac{1}{3} 1\right]$

16. $y = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$x^2(y-1) + x(y+1) + y - 1 = 0$

$x \in R$

$\Delta \geq 0$

$(-3y^2 + 10y - 3) + y - 1 = 0$

$3y^2 - 10y + 3 \leq 0$

$(3y-1)(y-3) \leq 0$

$y \in \left[\frac{1}{3} 3\right]$

17. If $a^2+b^2+c^2=p$ the range of $ab+bc+ca$ is

$\left[\frac{-p}{2} p\right] \Rightarrow \left[\frac{-1}{2} 1\right]$

$p=1$

18. $y=\frac{2x+3}{3x-5}$

$3xy-5y=2x+3$

$3xy-2x=3+5y$

$x=\frac{3+5y}{3x-2}$

$f^{-1}(x)=\frac{3+5y}{3x-2}$

19. $f(x)=x-x^2+x^3-x^4.....$

$=\frac{x}{1-(-x)}$

$=\frac{x}{1+x}$

20. $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

$f(x)=x^2-2$

21. $f(x)=\frac{\cos^2 x+\sin^4 x}{\sin^2 x+\cos^4 x}$

$=\frac{1-\sin^2+\left(1-\cos^2 x\right)^2}{\sin^2 x+\cos^4 x}$

$=\frac{1-\sin^2+1-2\cos^2 x+\cos^4 x}{\sin^2 x+\cos^4 x}$

$=\frac{2-\sin^2-2+2\sin^2 x+\cos^4 x}{\sin^2 x+\cos^4 x}$

$=\frac{\sin^2 x+\cos^4 x}{\sin^2 x+\cos^4 x}=1$

22. $f(x+y)+f(x-y)=\frac{2^{x+y}+2^{-(x-y)}}{2}+\frac{2^{x-y}+2^{-(x-y)}}{2}$

$=\frac{2^x 2^y+2^{-x}2^{-y}+2^x 2^{-y}+2^{-x}2^y}{2}$

$=\frac{1}{2} f(x) f(y)$

23. $f(x) f\left(\frac{1}{x}\right)=4$

$f(2) f\left(\frac{1}{2}\right)=4$

$8 f\left(\frac{1}{2}\right)=4$

$f\left(\frac{1}{2}\right)=\frac{1}{2}$ $f(2)=8$ and

$f\left(\frac{1}{2}\right)=\frac{1}{2}\Rightarrow f(x)=2x^2$ $f(5)=2\times 25=50$

24. $x-[x]\geq 0$ for all $x\in R$

$1+x-[x]\geq 1$ for all $x\in R$ $f[1+x-[x]]=1$

$f(g(x))=1\forall x$

25. $f(x)=\frac{4^x}{4^x+2}$ $f(1-x)=\frac{4^{1-x}}{4^{1-x}+2}=\frac{4}{4+2(4^x)}=\frac{2}{2+4^x}$

$f(x)+f(1-x)=1$

$\left[f\left(\frac{1}{1997}\right)+\left(\frac{1996}{1997}\right)+f\left(\frac{2}{1997}\right)+f\left(\frac{1995}{1990}\right)\right]+.....+\left[f\left(\frac{1998}{1997}\right)+f\left(\frac{1999}{1997}\right)\right]$

$=1+1+.....+998 \text{ times } =998$

26. $f(x)=\frac{9^x}{9^x+3}\Rightarrow f(x)+f(1-x)=1$

$\left[f\left(\frac{1}{1996}\right)+f\left(\frac{1995}{1996}\right)\right]+\left[f\left(\frac{2}{1996}\right)+f\left(\frac{1994}{1996}\right)\right]+.....+$

$\left[f\left(\frac{1997}{1996}\right)+f\left(\frac{999}{1996}\right)\right]+f\left(\frac{998}{1996}\right)$

$=(1+1+1+....997)\text{ times }+f\left(\frac{1}{2}\right)$

$997+0.5=995.5$

27. $f\left(\frac{x^2-1}{x^2+1}\right)=2\forall x$

f is constant function $\therefore f(x)=2$

28. $f(x)=4^x+2^x+4^{-x}+2^{-x}+3$

Let $2^x=3$ $a^2+a+\frac{1}{a^2}+\frac{1}{a}+3$

$\left(a^2+\frac{1}{a^2}\right)+\left(a+\frac{1}{a}\right)+3$ $y^2+y+1\succ 0$ since $\Delta\angle 0$

Least value of $y^2+y+1=\frac{4-1}{4}=\frac{3}{4}$ Range $=\left[\frac{3}{4}\infty\right]$

29. $f(x)=\log_{2x-1}{}^{(x-1)}$

$x-1\succ 0$ $2x-1\succ 0$

$x\succ 1$ $x\succ\frac{1}{2}$

$x\in(1\ \infty)$

30. $fog(x)=f\left(x^2+1\right)=2x^2+3$

$gof(x)=g(2x+1)=(2x+1)^2+1=4x^2+4x+2$

$fog(2)=11, gof(3)=5^0, gof(-1)=2$ $fog(-1)=5$