PHYSICS-SYSTEM OF PARTICLES AND RIGID BODY DYNAMICS CENTRE OF MASS

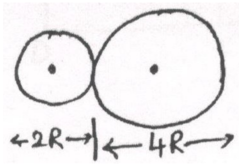
PHYSICS **MAX.MARKS: 100**

Section-I
(Single Correct Answer Type)

This section contains 20 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

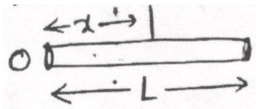
1. Two uniform discs made of same material and thickness of radii R and 2R are joined as shown in figure. Locate Centre of mass



1) $\frac{17R}{7}$ 2) $\frac{17R}{5}$

3) $\frac{12R}{7}$ 4) $\frac{12R}{5}$

2. The linear mass density of a rod of length L is given by $\rho = \rho_o(1+\frac{x}{L})$. Where ρ_o constant, x is distance from the left end. Locate Centre of mass from left end.

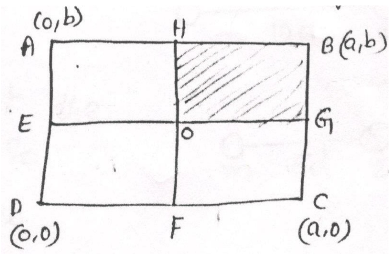


1) $\frac{L}{9}$ 2) $\frac{3L}{9}$ 3) $\frac{5L}{9}$ 4) $\frac{3L}{7}$

3. A uniform rectangular thin sheet ABCD of mass M hsas length a and breadth b as shown in figure. If The shaded portion HBGO is cutoff , the co-ordinates of centre of mass of remaining portion will be

1) $\left(\frac{2a}{3},\frac{2b}{3}\right)$ 2) $\left(\frac{5a}{3},\frac{5b}{3}\right)$

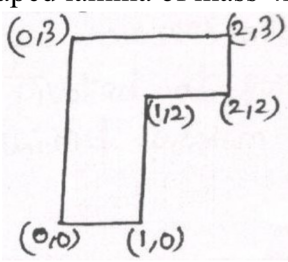
3) $\left(\frac{3a}{4},\frac{3b}{4}\right)$ 4) $\left(\frac{5a}{12},\frac{5b}{12}\right)$



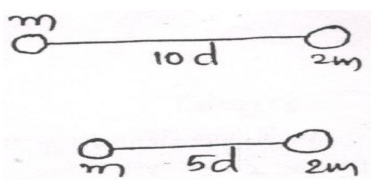
4. The co-ordinates of Centre of mass of uniform flag shaped lamina of mass 4Kg are

1) (1.25m, 1.5m) 2) (1m, 1.75m)

3) (0.75m, 0.75m) 4) (0.75m, 1.75m)



5. Two spheres of masses m and 2m are initially at rest and are separated by a distance of 10d. Due to mutual force of attraction they approach each other. When separation between them is 5d, the acceleration of Centre of mass



1) Zero 2) 2g

3) 3g 4) 4g

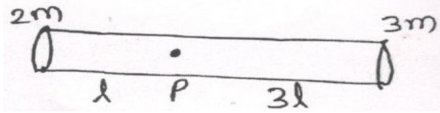
6. Let F be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about

The origin. Then

1) $\vec{r}.\vec{\tau}=0$ and $\vec{F}.\vec{\tau}=0$ 2) $\vec{r}.\vec{\tau}=0$ and $\vec{F}.\vec{\tau}\neq 0$

3) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}=0$ 4) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}\neq 0$

7. A rigid massless rod of length $4l$ has two masses attached at each end as shown in figure. The rod is pivoted at point P on the horizontal axis. When released from initial position its instantaneous angular acceleration will be



1) $\frac{g}{2l}$ 2) $\frac{7g}{3l}$

3) $\frac{g}{13l}$ 4) $\frac{7g}{29l}$

8. A fly wheel having moment of inertia $2Kgm^2$ about its vertical axis rotates at the rate of 60rpm about this axis the torque which can stop the wheel's rotation in one minute would be

1) $\frac{\pi}{10}N-m$ 2) $\frac{-\pi}{15}N-m$ 3) $\frac{2\pi}{15}N-m$ 4) $\frac{2\pi}{18}N-m$

9. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it , its kinetic energy as a function of θ, Where θ is the angle by which it has rotated is given as $K\theta^2$, If its moment of inertia is I then angular acceleration of the disc is

1) $\frac{K}{2I}\theta$ 2) $\frac{K}{I}\theta$ 3) $\frac{K}{4I}\theta$ 4) $\frac{2K}{I}\theta$

10. Consider two uniform discs of the same thickness and different radii $R_1=R$ and $R_2=\alpha R$ made of same material. If the ratio of moments of inertia about their axes is $\frac{I_1}{I_2}=\frac{1}{16}$ then the value of α is

1) $\sqrt{2}$ 2) 2   3) 4   4) $2\sqrt{2}$

11. A fly wheel of moment of inertia 10kgm^2 is rotating at 50 rad/s. It must be brought to stop in 10s. What is the required average power.

1) $750w$ 2) $1250w$ 3) $1500w$ 4) $500w$

12. A solid body rotates with angular velocity $\vec{\omega}=3t\,\hat{i}+2t^2\,\hat{j}$ rad/s. Find the angle between the vectors of angular velocity and the angular acceleration at time t = 1sec.
1

1) $\text{Cos}^{-1}\left[\frac{5}{17\sqrt{3}}\right]$ 2) $\text{Cos}^{-1}\left[\frac{17}{13\sqrt{5}}\right]$ 3) $\text{Cos}^{-1}\left[\frac{17}{5\sqrt{13}}\right]$ 4) $\text{Cos}^{-1}\left[\frac{13}{17\sqrt{5}}\right]$

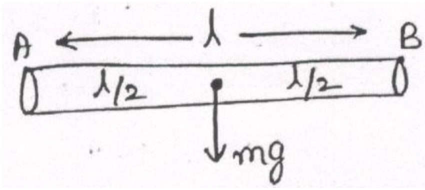
13. A disc start rotating about its axis with angular acceleration $\alpha=3\lambda t^2$, where λ is constant. After how much time the angle between acceleration and velocity vector of a point on the rim will be 30°?

1) $\frac{3}{\lambda}$ 2) $\frac{\sqrt{3}}{\lambda}$ 3) $\left(\frac{\sqrt{3}}{\lambda}\right)^{\frac{1}{3}}$ 4) $\left(\frac{\sqrt{3}}{\lambda}\right)^{\frac{1}{4}}$

14. A thin uniform circular disc of mass M and radius R rotating in horizontal plane about an axis passing through its Centre and perpendicular to its plane with an angular velocity ω. Another disc of same dimensions but of mass $\frac{M}{4}$ is placed on the first disc co – axially. The angular velocity of the system is

1) $\frac{2}{3}\omega$ 2) $\frac{4}{5}\omega$ 3) $\frac{3}{4}\omega$ 4) $\frac{1}{3}\omega$

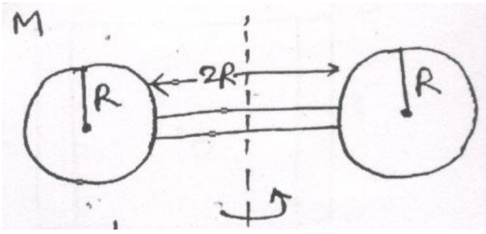
15. A uniform rod of length l and mass m is free to rotate in vertical plane about A. The rod initially in horizontal position is released. The initial angular acceleration of the rod is (moment of inertia of rod about A is $\frac{ml^2}{3}$)



1) $\frac{3g}{2l}$ 2) $\frac{2l}{3g}$

3) $\frac{3g}{2l^2}$ 4) $\frac{mgl}{2}$

16. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M . The moment of inertia of the system about the axis passing perpendicularly through the centre of rod is



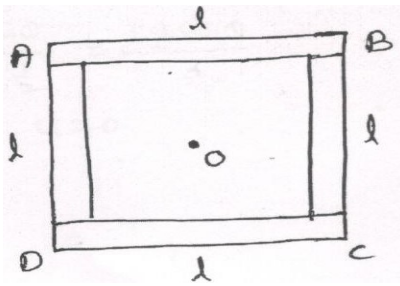
1) $\frac{152}{15}MR^2$ 2) $\frac{17}{15}MR^2$

3) $\frac{137}{15}MR^2$ 4) $\frac{209}{15}MR^2$

17. A circular disc x of radius R is made from an iron plate of thickness t and another disc y of radius 4R Is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between I_x and I_y is

1) $I_y = 64\, I_x$ 2) $I_y = 32\, I_y$ 3) $I_y = 16\, I_x$ 4) $I_y = I_x$

18. Four thin rods of same mass M and same length l form a square as shown infigure. Moment of inertia of this system about an axis through centre O and perpendicular to its plane is



1) $\frac{4}{3}Ml^2$ 2) $\frac{1}{3}Ml^2$

3) $\frac{1}{6}Ml^2$ 4) $\frac{2}{3}Ml^2$

19. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its Centre is placed over another disc of moment of inertia I_1 is rotating with an angular velocity ω about the same axis . The final angular velocity of the combination of disc is

1) ω 2) $\frac{I_1\omega}{I_1+I_2}$ 3) $\frac{(I_1+I_2)\omega}{I_2}$ 4) $\frac{I_2\omega}{I_1+I_2}$

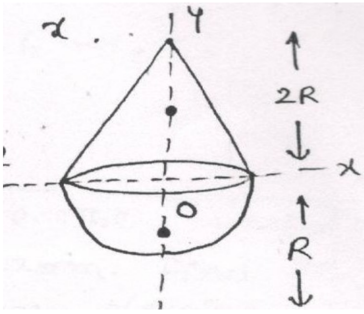
20. The ratio of times taken by solid sphere and that taken by disc of the same mass and radius to roll down a rough inclined plane from rest, from same height is

1) 15: 14 2) $\sqrt{15}:\sqrt{14}$ 3) 14 : 15 4) $\sqrt{14}:\sqrt{15}$

(Numerical Value Answer Type)

This section contains 10 questions. The answer to each question is a **Numerical value. If the Answer in the decimals, Mark nearest Integer only. Have to Anaswer any 5 only out of 10 questions and question will be evaluated according to the following marking scheme:**
Marking scheme : +4 for correct answer, -1 in all other cases.

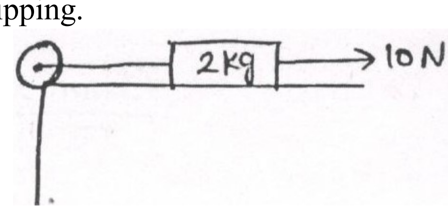
21. A right circular cone of radius R and height 2R is placed on a hemisphere of radius R. Centre of mass of the combined mass from O is $\frac{R}{x}$, find x.



22. A Wheel starting from rest is uniformly accelerated at 2rad/s^2 for 5s. It is allowed to rotate uniformly for the next 10s and is finally brought to rest in the next 5s. Find the total angle rotated by the wheel.

23. A wheel of radius 10cm starts from rest and rotate under constant angular acceleration 6 rad/s^2 . Find acceleration of a point on the rim after time $\frac{\sqrt{2}}{3}$ sec.

24. Consider the situation as shown in figure. The moment of inertia of wheel is 2kgm^2 and radius is 0.25m. Find the angular acceleration of the wheel assuming no slipping.



25. Two balls of masses 3m and 2m are attached to the ends of a rod of length L. If the rod rotates with an angular speed ω about an axis passing through the Centre of mass of system and perpendicular to the plane the angular moment of the system is $\frac{x\, m\omega L^2}{5}$. Find x

26. From a circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through Centre is _______________ MR^2

27. The moment of inertia of a wheel, initially at rest is 8kg m^2 . In order to produce rotational Kinetic energy of 1600J an angular acceleration of 5 rad/s^2 must be applied about axis of wheel for a duration of _______________seconds.

28. A hallow cylinder of mass M and radius R is rotating about its axis of symmetry and hallow sphere of same mass and radius is rotating about an axis passing through its Centre. If torques of equal magnitude are applied them then ratio of angular accelerations produced is

29. A particle starts from the point (0, 8) m and moves with uniform velocity of 3i m/s. After 5 seconds the angular velocity of the particle about the origin will be

30. If radius of earth is doubled without changing its mass what will be the length of day (in hrs)

KEY

1-10	2	3	4	4	1	1	4	2	4	2
11-20	2	3	4	2	1	3	1	1	2	4
21-30	16	150	1	1	6	4	4	1	0	96

1. $m\ \alpha\ A \Rightarrow m\ \alpha\ R^2$

$m_1 = KR^2 = m$

$m_2 = K(2R)^2 = K(4R^2) = 4m$ $\qquad x_1 = R\ ,\ x_2 = 2R+2R = 4R$

$$x_m = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{17R}{5}$$

2. For small element $\rho = \dfrac{dm}{dx}$

$dm = \rho\, dx$

$dm = \rho_0(1+\frac{x}{L})dx$

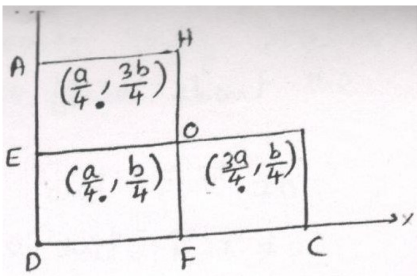
$$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int x\rho_0\left(1+\frac{x}{L}\right)dx}{\int \rho_0\left(1+\frac{x}{L}\right)dx} = \frac{\rho_0\int_0^L\left(x+\frac{x^2}{L}\right)dx}{\rho_0\int_0^L\left(1+\frac{x}{L}\right)dx}$$

$$= \frac{\rho_0\left[\frac{x^2}{2}+\frac{x^3}{3L}\right]_0^L}{\rho_0\left[x+\frac{x^2}{2L}\right]_0^L} = \frac{\frac{L^2}{2}+\frac{L^3}{3L}}{L+\frac{L^2}{2L}} = \frac{5L}{9}$$

3. $m_1 = m_2 = m_3 = m_4 = \dfrac{m}{4}$

$$x_{cm} = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3} = \frac{\frac{m}{4}\left(\frac{a}{4}\right)+\frac{m}{4}\left(\frac{a}{4}\right)+\frac{m}{4}\left(\frac{3a}{4}\right)}{\frac{m}{4}+\frac{m}{4}+\frac{m}{4}}$$

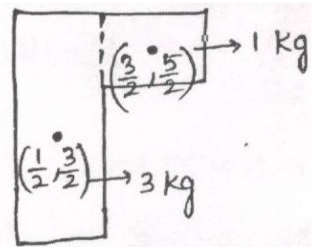
$$x_{cm} = \frac{\frac{5a}{4}}{3} = \frac{5a}{12}$$



$$y_{cm}=\frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3} = \frac{\frac{m}{4}\left(\frac{3b}{4}\right)+\frac{m}{4}\left(\frac{b}{4}\right)+\frac{m}{4}\left(\frac{b}{4}\right)}{\frac{m}{4}+\frac{m}{4}+\frac{m}{4}}$$

$$y_{cm}=\frac{\frac{5b}{4}}{3}=\frac{5b}{12}$$

4. $$x_{cm}=\frac{m_1x_1+m_2x_2}{m_1+m_2}$$



$$x_{cm}=\frac{3\left(\frac{1}{2}\right)+1\left(\frac{3}{2}\right)}{3+1}=\frac{6}{8}=\frac{3}{4}=0.75\text{m}$$

$$y_{cm}=\frac{m_1y_1+m_2y_2}{m_1+m_2}$$

$$y_{cm}=\frac{3\left(\frac{3}{2}\right)+1\left(\frac{5}{2}\right)}{3+1}=\frac{14}{8}=\frac{7}{4}=1.75\text{ m}$$

5. Initially spheres at rest. So

$\overrightarrow{a_{cm}}=0$

Internal forces doesn't affect the position of Centre of mass. So

$$\overrightarrow{F_{net}}=0\text{. So }\overrightarrow{a_{cm}}=0$$

6. Torque $\vec{\tau}=r\,F\sin\theta=\vec{r}\times\vec{F}$

Torque is $\perp^r$ to both $\vec{r}$ and $\vec{F}$ so the angle between $\vec{r}$ and $\vec{\tau}$ is 90° and $\vec{F}$ and $\vec{\tau}$ is 90°.

$\vec{r}\,.\,\vec{\tau}=r\,\tau\cos\theta=0$

$\vec{F}\,.\,\vec{\tau}=F\,\tau\cos\theta=0$

7. $I=2m(l)^2+3m(3l)^2=29ml^2$

$\tau_{net}=(3m)g(3l)-(2m)g(l)=9mgl-2mgl=7mgl$

$\tau_{net}=I\alpha$

$7mgl=29ml^2\alpha$

$$\alpha=\frac{7g}{29l}$$

8. $\omega_0=60$ rpm , $\omega=0$

$$\alpha=\frac{\omega-\omega_0}{t}=\frac{0-60\text{ rpm}}{1\text{ min}}=-60\text{ rot/min}^2$$

1 rot = 2π rad, 1 min = 60s

$$\alpha=\frac{-60\times2\pi}{3600}\text{rad/s}^2=\frac{-\pi}{30}\text{ rad/s}^2$$

Torque

$\tau=I\alpha$

$$\tau=2\times\frac{-\pi}{30}$$

$$\tau=\frac{-\pi}{15}\text{ N-m}$$

9. rotational KE = $\frac{1}{2}I\omega^2$

Given KE = $K\theta^2$

$\frac{1}{2}I\omega^2 = K\theta^2$

$\frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = \frac{d}{dt}\left(K\theta^2\right)$

$\frac{1}{2}I.2\omega.\frac{d\omega}{dt} = K.2\theta.\frac{d\theta}{dt}$

$I\omega\alpha = 2K\theta.\omega$

$I\alpha = 2K\theta$

$\alpha = \frac{2K}{I}\theta$

10. For disc $I = \frac{mr^2}{2}$

$I\alpha\, mr^2$

$\rho = \frac{m}{v} \Rightarrow m = \rho V = \rho Al$

$m\alpha r^2$

$I\alpha\, r^2.r^2 \Rightarrow I\alpha r^4$

$\frac{I_1}{I_2} = \left(\frac{r_1}{r_2}\right)^4$

$\frac{1}{16} = \left(\frac{r_1}{r_2}\right)^4$

$\left(\frac{1}{2}\right)^4 = \left(\frac{r_1}{r_2}\right)^4 \Rightarrow \frac{r_1}{r_2} = \frac{1}{2} \Rightarrow r_2 = 2r_1$

$r_2 = 2R = \alpha R$

$\boxed{\alpha = 2}$

11. $\omega_0 = 50\, rad/s\,;\, \omega = 0, \quad I = 10\, kg - m^2, t = 10s$

$\alpha = \frac{\omega - \omega_0}{t} = \frac{0-50}{10} = -5\, rad/s^2$

$\theta = \omega_{0t} + \frac{1}{2}\alpha t^2$

$\theta = 50(10) + \frac{1}{2}(-5)(10)^2 = 500 - 250 = 250\, rad$

Torque = $\tau = I\alpha = 10(5) = 50$

Work done $w = \tau\theta = 50(250) = 12500\, J$

Average power $P = \frac{w}{t} = \frac{12500}{10} = 1250W$

12. $\vec{\omega} = 3t\hat{i} + 2t^2\hat{j}$

$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = 3\hat{i} + 4t\hat{j}$

$\vec{\omega}.\vec{\alpha} = \left(3t\hat{i} + 2t^2\hat{j}\right).\left(3\hat{i} + 4t\hat{j}\right) = 9t + 8t^3$

$t = 1s$ $\quad$ $\vec{\omega} = 3t\,\hat{i} + 2t^2\hat{j}$

$\vec{\omega}.\vec{\alpha} = 9 + 8 = 17$ $\quad$ $t = 1s$

$\left|\vec{\omega}\right| = \sqrt{3^2 + 2^2} = \sqrt{13}$ $\quad$ $\vec{\omega} = 3\hat{i} + 2\hat{j}$

$\vec{\alpha} = 3\hat{i} + 4t\hat{j}$

$t = 1s$

$\vec{\alpha} = 3\hat{i} + 4\hat{j}$ $\quad$ $\vec{\omega}.\vec{\alpha} = \left|\vec{\omega}\right|\left|\vec{\alpha}\right|\cos\theta$

$\left|\vec{\alpha}\right| = \sqrt{3^2 + 4^2} = 5$ $\quad$ $\cos\theta = \frac{\vec{\omega}\,.\,\vec{\alpha}}{\left|\vec{\omega}\right|\left|\vec{\alpha}\right|} = \frac{17}{5\sqrt{13}}$

$\theta = \cos^{-1}\left[\frac{17}{5\sqrt{13}}\right]$

13. $\alpha = 3\lambda t^2$

$\frac{d\omega}{dt} = 3\lambda t^2$

$d\omega = 3\lambda t^2 dt$

$\int d\omega = 3\lambda \int t^2 dt$

$w = \lambda t^3$

Centripetal acceleration $a_c = \omega^2 r = \lambda^2 t^6 r$

Tangential acceleration $a_t = r\alpha = 3\lambda t^2 r$

$\tan\theta = \frac{a_c}{a_t} \Rightarrow \tan 30 = \frac{\lambda^2 t^6 r}{3\lambda t^2 r} = \frac{\lambda t^4}{3}$

$\frac{1}{\sqrt{3}} = \frac{\lambda t^4}{3} \Rightarrow t = \left(\frac{\sqrt{3}}{\lambda}\right)^{1/4}$

14. $I_1 = \frac{1}{2}MR^2$ $\quad$ $I_2 = \frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2$

$I_2 = \frac{1}{2}MR^2 + \frac{1}{8}MR^2 = \frac{5}{8}MR^2$

$\omega_1 = \omega, \quad \omega_2 = ?$

From conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$

$$\frac{1}{2}MR^2\omega = \frac{5}{8}MR^2\omega_2$$

$$\omega_2 = \frac{4}{5}\omega$$

15. At A $\tau = r \times F$

$$\tau = \left(\frac{l}{2}\right) \times mg = \frac{mgl}{2}$$

$$\tau = I\alpha$$

$$\frac{mgl}{2} = \frac{ml^2}{3}.\alpha$$

$$\alpha = \frac{3g}{2l}$$

16. $I = I_G + Ma^2$

$$I = \frac{2}{5}MR^2 + M(2R)^2 = \frac{2}{5}MR^2 + 4MR^2 = \frac{22MR^2}{5}$$

For two spheres $I_1 = \frac{22MR^2}{5} \times 2 = \frac{44MR^2}{5}$

For rod $I_2 = \frac{Ml^2}{12} = \frac{M(2R)^2}{12} = \frac{MR^2}{3}$

$$I = I_1 + I_2$$

$$I = \frac{44MR^2}{5} + \frac{MR^2}{3} = \frac{137\,MR^2}{15}$$

17. $I_x = \frac{MR^2}{2} = \frac{\rho VR^2}{2} = \frac{\rho(A \times t)R^2}{2} = \frac{\rho\pi R^2 tR^2}{2} = \frac{\rho\pi tR^4}{2}$

$$I_y = \frac{\rho(A \times t)R^2}{2} = \frac{\rho\left(\pi \times (4R)^2 \times \frac{t}{4}\right)16R^2}{2} = \frac{\rho\left(\pi \times 4R^2 t\right)R^2 \times 16}{2}$$

$$I_y = 32\rho\pi tR^4$$

$$\frac{I_x}{I_y} = \frac{\frac{\rho\pi tR^4}{2}}{32\rho\pi tR^4} = \frac{1}{64}$$

$$I_y = 64\,I_x$$

18. For each rod $I = I_G + Ma^2$

$$I = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{12} + \frac{Ml^2}{4}$$

For four rods $I = 4\left[\frac{Ml^2}{12} + \frac{Ml^2}{4}\right]$

$$I = \frac{Ml^2}{3} + Ml^2 = \frac{4Ml^2}{3}$$

19. Initial angular momentum $L_i = I_1\omega$

Final angular momentum $L_f = \left(I_1 + I_2\right)\omega^1$

$$I_1\omega = \left(I_1 + I_2\right)\omega^1$$

$$\omega^1 = \frac{I_1\omega}{I_1 + I_2}$$

20. $a = \dfrac{g\sin\theta}{1 + \dfrac{k^2}{r^2}}$

For sphere

$$I = \frac{2}{5}Mr^2$$

$$MK^2 = \frac{2}{5}Mr^2$$

$$\frac{K^2}{r^2} = \frac{2}{5}$$

$$a_s = \frac{g\sin\theta}{1 + \frac{2}{5}} = \frac{5g\sin\theta}{7}$$

For disc

$$I = \frac{1}{2}Mr^2$$

$$MK^2 = \frac{1}{2}Mr^2$$

$$\frac{K^2}{r^2} = \frac{1}{2}$$

$$a_d = \frac{g\sin\theta}{1 + \frac{1}{2}} = \frac{2g\sin\theta}{3}$$

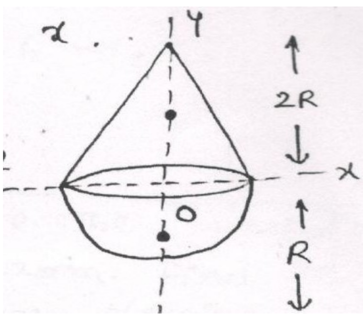
$$s = ut + \frac{1}{2}at^2 \rightarrow \textit{from rest } u = 0$$

$$s = \frac{1}{2}at^2$$

$$t^2 \alpha \frac{1}{a} \Rightarrow t\ \alpha\ \frac{1}{\sqrt{a}}$$

$$\frac{t_s}{t_d} = \sqrt{\frac{a_d}{a_s}} = \sqrt{\frac{2g\sin\theta/3}{5g\sin\theta/7}} = \sqrt{\frac{14}{15}}$$

21. Volume of cone $V = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi R^2\left(2R\right)$



$V = \frac{2}{3}\pi R^3 =$ Volume of hemisphere

So $m_1 = m, \quad m_2 = m$

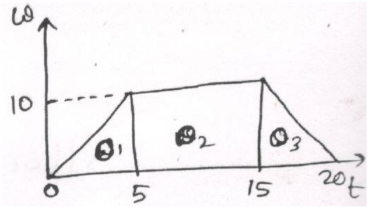
For cone $y_1 = \frac{h}{4} = \frac{2R}{4} = \frac{R}{2}$

For hemisphere $y_2 = \frac{3R}{8} = \frac{-3R}{8}\left(\text{downwards}\right)$

$$y_{un} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m\left(\frac{R}{2}\right) + m\left(\frac{-3R}{8}\right)}{m+m} = \frac{R}{16} = \frac{R}{x}$$

$x = 16$

22. $\omega_0 = 0, \quad \alpha = 12 \ rad/s^2 \quad t = 5s$



$$\alpha = \frac{\omega - \omega_0}{t} \Rightarrow \omega = 10\, rad/s$$

$$\omega = \frac{\theta}{t} \Rightarrow \theta = \omega t$$

Area of the graph gives angular displacement.

$$\theta_1 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 5 \times 10 = 25\, rad$$

$$\theta_2 = l \times b = 10 \times 10 = 100\, rad$$

$$\theta_3 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 5 \times 10 = 25\, rad$$

$$\theta = \theta_1 + \theta_2 + \theta_3 = 150\, rad$$

$\theta = 150$

23. $R = 10\, cm = 0.1\, m$

$\alpha = 6\, rad/s^2$

$$t = \frac{\sqrt{2}}{3} \sec$$

$\omega = \omega_0 + \alpha t$

$$\omega = 0 + 6\left(\frac{\sqrt{2}}{3}\right) = 2\sqrt{2}\, rad/s$$

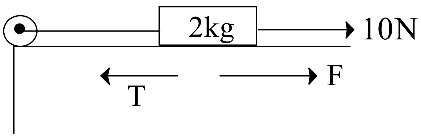
Centripetal acceleration $a_c = \omega^2 R = \left(2\sqrt{2}\right)^2 \times 0.1 = 0.8\, m/s^2$

Tangential acceleration $a_t = R\alpha = 0.1 \times 6 = 0.6\, m/s^2$

$$a = \sqrt{a_c^{\,2} + a_t^{\,2}} = \sqrt{0.8^2 + 0.6^2} = 1\, m/s^2$$

$a = 1$

24.



$F_{net} = F - T$

$ma = 10 - T$

$2a = 10 - T$

Torque

$T \times R = I\alpha$

$T \times 0.25 = 2\alpha$

$T = 8\alpha$

$a = R\alpha$

$a = 0.25\, \alpha$

$$a = \frac{\alpha}{4}$$

Now 2a = 10 – T

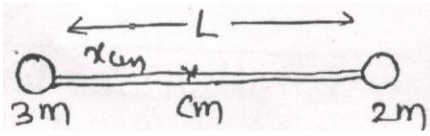
$$2\left(\frac{\alpha}{4}\right)=10-8\alpha$$

$$8\alpha+\frac{\alpha}{2}=10$$

$$17\alpha=20$$

$$\alpha=\frac{20}{17}=1.17\ rad/s^2$$

25. $$x_{cm}=\frac{m_1x_1+m_2x_2}{m_1+m_2}$$



$$x_{cm}=\frac{(3m\times 0)+(2m\times L)}{m_1+m_2}=\frac{2mL}{5m}=\frac{2L}{5}$$

$$x_1=\frac{2L}{5} \qquad x_2=L-x_1=L-\frac{2L}{5}=\frac{3L}{5}$$

$$V_1=x_1\omega=\frac{2L\omega}{5} \qquad V_2=x_2\omega=\frac{3L\omega}{5}$$

Angular momentum of system

$$L=L_1+L_2=m_1v_1x_1+m_2v_2x_2$$

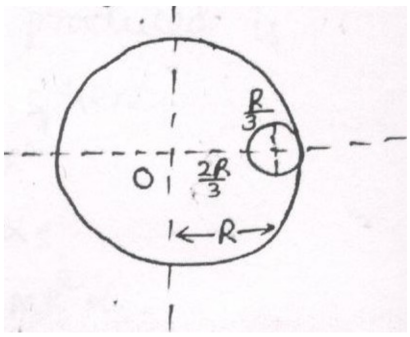
$$L=3m.\frac{2L\omega}{5},\frac{2L}{5}+2m.\frac{3L\omega}{5}.\frac{3L}{5}$$

$$L=\frac{12m\omega L^2}{25}+\frac{18m\omega L^2}{25}$$

$$L=\frac{30\ m\omega L^2}{25}=\frac{6m\omega L^2}{5}=\frac{xm\omega L^2}{5}$$

$$x=6$$

26. For bigger disc area density



$$\rho=\frac{M}{A}=\frac{9M}{\pi R^2}$$

For removed disc $m=\rho\times A=\frac{9M}{\pi R^2}\times\pi\left(\frac{R}{3}\right)^2$

$$m=M$$

For bigger disc $I_1=\frac{MR^2}{2}=\frac{9MR^2}{2}$

For smaller removed disc

$$I_2=I_G+M_a{}^2$$

$$I=I_1-I_2 \qquad I_2=\frac{M\left(\frac{R}{3}\right)^2}{2}+M\left(\frac{2R}{3}\right)^2$$

$$I = \frac{9MR^2}{2} - \frac{MR^2}{2} = 4MR^2$$

$$I = 4\,MR^2$$

$$I_2 = \frac{MR^2}{18} + \frac{4MR^2}{9} = \frac{9MR^2}{18}$$

$$I_2 = \frac{MR^2}{2}$$

27. $I = 8\,kg\,m^2$

$KE_{rot} = 1600\,J$

$\omega_0 = 0$

$\alpha = 5\,rad/s^2$

$$KE_{rot} = \frac{1}{2}I\omega^2$$

$$1600 = \frac{1}{2}I\omega^2 \qquad \omega = \omega_0 + \alpha t$$

$$3200 = 8\omega^2 \qquad \omega = 0 + \alpha t$$

$$\omega^2 = 400 \qquad t = \frac{\omega}{\alpha} = \frac{20}{5} = 4$$

$$\omega = 20\,rad/\sec$$

$$t = 4$$

28. For hallow cylinder

$$I = \frac{1}{2}MR^2$$

$$\tau_C = I\alpha_C$$

$$\tau_C = \frac{1}{2}MR^2\alpha_C$$

For hallow sphere

$$I = \frac{2}{3}MR^2$$

$$\tau_S = I\alpha_s$$

$$\tau_S = \frac{2}{3}MR^2\alpha_S$$

Given $\tau_C = \tau_S$

$$\frac{1}{2}MR^2\alpha_C = \frac{2}{3}MR^2\alpha_S$$

$$\frac{\alpha_C}{\alpha_S} = \frac{4}{3} = 1.33$$

$$\frac{\alpha_C}{\alpha_S} = 1$$

29. $x = v.t$

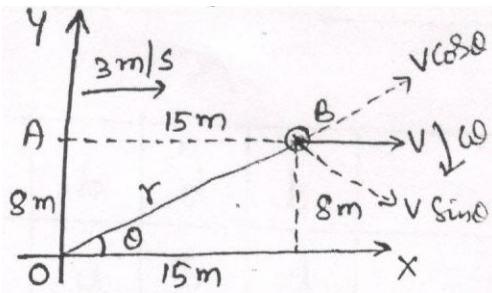
$$x = 3\times 5 = 15\,m$$

$$r = \sqrt{15^2 + 8^2} = 17m$$

$$\sin\theta = \frac{8}{r} = \frac{8}{17}$$

$$v = r\omega$$

$$v\sin\theta = r\omega \Rightarrow \omega = \frac{v\sin\theta}{r}$$



$$\omega = \frac{3\left(\frac{8}{17}\right)}{17} = \frac{24}{289} = 0.08\, rad / s$$

$$\omega = 0$$

30. For earth $I = \frac{2}{5}MR^2$

L = constant

$$I_1\omega_1 = I_2\omega_2$$

$$\frac{2}{5}MR^2\left(\frac{2\pi}{T_1}\right) = \frac{2}{5}M\left(2R\right)^2 \times \frac{2\pi}{T_2}$$

$$\frac{1}{T_1} = \frac{4}{T_2}$$

$$T_2 = 4 \times T_1 = 4 \times 24\ hr$$

$$T_2 = 96\ hrs$$