TOPIC: VECTORS

1. If $|\vec{a}|=2, |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$ then $|\vec{a}.\vec{b}|$ is equal to _______

   1) 6 2) 4 3) 3 4) 5

2. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$, then angle between $\vec{a}$ & $\vec{b}$ is ______(in degrees)

   1) 30^0 2) 45^0 3) 60^0 4) 75^0

3. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to _______

   1) 4 2) 5 3) 6 4) 7

4. Let $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$. If area of parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ & $\vec{b}$ is $8\sqrt{3}$ square units, than $\vec{a}.\vec{b}$ is equal to _____

   1) 8 2) 6 3) 4 4) 2

5. In a triangle ABC, If $|BC|=3, |CA|=5$ and $|BA|=7$ then projection of vector BA on BC is equal to _______

   1) $\frac{11}{2}$ 2) $\frac{13}{2}$ 3) $\frac{15}{2}$ 4) $\frac{17}{2}$

6. Given that $\vec{A}+\vec{B}=\vec{C}$. If $|\vec{A}|=4, |\vec{B}|=5, |\vec{C}|=\sqrt{61}$ then angle between $\vec{A}$ and $\vec{B}$ is _______

   1) 30^0 2) 60^0 3) 90^0 4) 120^0

7. What is the angle between two vector forces of equal magnitude such that their resultant is one – third of either of original forces

   1) $\cos^{-1}\left(\frac{-17}{18}\right)$ 2) $\cos^{-1}\left(\frac{-1}{3}\right)$ 3) 45^0 4) 120^0

8. Two vectors $\vec{A}$ and $\vec{B}$ have precisely equal magnitudes. For the magnitude of $\vec{A}+\vec{B}$ to be larger than the magnitude of $\vec{A}-\vec{B}$ by a factor n . What must be the angle between them is

   1) $\theta=2\cot^{-1}\left(\frac{1}{n}\right)$ 2) $\theta=2\,\mathrm{Tan}^{-1}\left(\frac{1}{n}\right)$ 3) 60^0 4) 120^0

9. If $\vec{A}$ is perpendicular to $\vec{B}$ then

   1) $\vec{A}\times\vec{B}=0$ 2) $\vec{A}.(\vec{A}+\vec{B})=A^2$ 3) $\vec{A}.\vec{B}=AB$ 4) $\vec{A}.(\vec{A}+\vec{B})=A^2+\vec{A}\vec{B}$

10. The resultant C of $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ .Also $|\vec{A}|=|\vec{C}|$. The angle between $\vec{A}$ and $\vec{B}$ is _____in rad

    1) $\frac{\pi}{4}$ 2) $\frac{5\pi}{4}$ 3) $\frac{7\pi}{4}$ 4) $\frac{3\pi}{4}$

11. The vector sum of the two forces is perpendicular to their vector differences. In that case, the forces are

    1) Cannot be predicted 2) Always perpendicular
    3) Are equal to each other in magnitude 4) Are not equal to each other in magnitude

12. If $0.5\hat{i}+0.8\hat{j}+c\hat{k}$ is a unit vector, then $'c'$ is_____

    1) $\sqrt{0.89}$ 2) 0.2 3) 0.3 4) $\sqrt{0.11}$

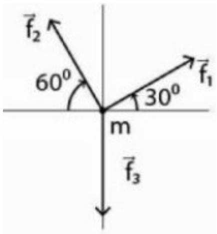
13. A point P is moving in uniform circular motion with radius $3m$. Let at some instant the acceleration of the point is $\vec{a} = 6\hat{i} - 4\hat{j}\ m/s^2$, the position vector is $\vec{r}$ and velocity vector $\vec{v}$ .Choose correct statement

1) $\vec{v}.\vec{a} = 0 \,\&\, \vec{r}\times\vec{a} \neq 0$ 2) $\vec{v}.\vec{a} \neq 0 \,\&\, \vec{r}\times\vec{a} \neq 0$ 3) $\vec{v}.\vec{a} = 0 \,\&\, \vec{r}\times\vec{a} = 0$ 4) $\vec{v}.\vec{a} \neq 0 \,\&\, \vec{r}\times\vec{a} = 0$

14. Let $A_1 + A_2 = 5A_3$, $A_1 - A_2 = 3A_3$, $A_3 = 2\hat{i} + 4\hat{j}$ then $\frac{|A_1|}{|A_2|} =$_______

1) 4 2) 5 3) 6 D) 7

15. A box of mass m is in equilibrium under the application of three forces as shown below. If magnitude of f_1 is $10\,N$,what is the magnitude of f_3_______



1) $5\,N$ 2) $15\,N$ 3) $20\,N$ 4) $30\,N$

16. The position vector of a particle moving in a plane is given by $\vec{r} = a\cos\omega t\,\hat{i} + b\sin\omega t\,\hat{j}$ where a, b and ω are constant. The acceleration of particle is directed along the vector

1) $-a\hat{i} + b\hat{j}$ 2) $b\hat{i} + a\hat{j}$ 3) $-\vec{r}$ 4) $\frac{d\vec{r}}{dt}$

17. If the angle between vectors A and B is θ then the value of product $(B \times A).A$ is______

1) $BA^2\cos\theta$ 2) $BA^2\sin\theta$ 3) Zero 4) None of these

18. The square of the resultant of two equal forces is three times of their product. The angle between the forces is ____

1) π 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{2\pi}{3}$

19. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitude of $\vec{A}+\vec{B}$ is equal to 2 times of magnitude of $\vec{A}-\vec{B}$, then angle between $\vec{A}$ and $\vec{B}$ is ______

1) $\sin^{-1}\frac{3}{5}$ 2) $\sin^{-1}\frac{1}{3}$ 3) $\cos^{-1}\frac{3}{5}$ 4) $\cos^{-1}\frac{1}{3}$

20. What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$

1) $\sqrt{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$ 2) $2\left(\hat{i}+\hat{j}+\hat{k}\right)$ 3) $\sqrt{2}\left(\hat{i}+\hat{j}\right)$ 4) $\left(\hat{i}+\hat{j}\right)$

21. The resultant of two forces $2P$ and $\sqrt{2}P$ is $\sqrt{10}P$ then angle between forces is_____

1) 30^0 2) 60^0 3) 45^0 4) 90^0

22. The direction cosines of a vector $\vec{A}$ are $\cos\alpha = \frac{4}{5\sqrt{2}}$, $\cos\beta = \frac{1}{\sqrt{2}}$, $\cos\gamma = \frac{3}{5\sqrt{2}}$,then vector $\vec{A}$ is______

1) $4\hat{i} + \hat{j} + 2\hat{k}$ 2) $4\hat{i} + 5\hat{j} + 3\hat{k}$ 3) 0 4) None of these

23. If projection of $2\hat{i} + 4\hat{j} - 2\hat{k}$ on $\hat{i} + 2\hat{j} + \alpha\hat{k}$ is zero then value of α will be

1) 2 2) 3 3) 4 4) 5

24. If vectors $\overrightarrow{a_1} = x\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{a_2} = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to vector $x\hat{i} + y\hat{j} + z\hat{k}$ is _____

1) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ 2) $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$ 3) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$ 4) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

25. In a triangle ABC, If $|BC| = 8$, $|CA| = 7$, $|AB| = 10$ then projection of vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to____

1) $\frac{25}{4}$ 2) $\frac{85}{14}$ 3) $\frac{127}{20}$ 4) $\frac{115}{16}$

26. If $\vec{b} = 3\hat{i} + 4\hat{j}$, $\vec{a} = \hat{i} - \hat{j}$. The vector having the same magnitude as that of $\vec{b}$ and parallel to $\vec{a}$ is _____

1) $\frac{5}{\sqrt{2}}\left(\hat{i} - \hat{j}\right)$ 2) $\frac{5}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ 3) $5\left(\hat{i} - \hat{j}\right)$ 4) $5\left(\hat{i} + \hat{j}\right)$

27. Two vectors $\vec{a}$ and $\vec{b}$ are at an angle of 60^0 with each other their resultant makes an angle of 45^0 with $\vec{a}$. If $\left|\vec{b}\right| = 2$ units then $\left|\vec{a}\right|$ is _____

1) $\sqrt{3}$ 2) $\sqrt{3}-1$ 3) $\sqrt{3}+1$ 4) $\frac{\sqrt{3}}{2}$

28. Find at least one vector perpendicular to $3\hat{i} - 4\hat{j} + 7\hat{k}$

1) $\hat{i} + 2\hat{j} + 3\hat{k}$ 2) $\hat{i} - \hat{j} - 3\hat{k}$ 3) $\hat{i} + 2\hat{j} + \frac{5}{7}\hat{k}$ 4) Zero

29. Two vectors have magnitude 5 units and 12 units respectively, find their cross product if the angle between them is 30^0

1) 30units 2) 40units 3) 50units 4) 60units

30. The sum and difference of two perpendicular vectors of equal lengths are ____

1) Perpendicular to each other and of equal lengths
2) Perpendicular to each other and of different lengths
3) Of equal lengths and have an acute angle between them
4) Of equal lengths and have an obtuse angle between them

PHYSICS

KEY:

PHYSICS										
1-10	1	3	2	4	1	2	1	2	2	4
11-20	3	4	3	1	3	3	3	2	3	4
21-30	3	2	4	4	2	1	2	3	1	1

HINTS

1. Given $\left|\vec{a}\right| = 2$, $\left|\vec{b}\right| = 5$

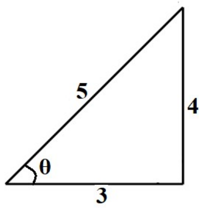
$\left|\vec{a} \times b\right| = 8$

$ab \sin\theta = 8$

$2 \times 5 \times \sin\theta = 8$

$\sin\theta = \frac{4}{5}$

$\vec{a}.\vec{b} = ab\cos\theta = 6$



2. Given $\left(\vec{a} + 3\vec{b}\right) \perp \left(7\vec{a} - 5\vec{b}\right)$

$\left(\vec{a} + 3\vec{b}\right).\left(7\vec{a} - 5\vec{b}\right) = 0$

$7a^2 - 15b^2 + 16\ \vec{a}.\ \vec{b} = 0$ ………(1)

$\left(\vec{a}-4\vec{b}\right).\left(7\vec{a}-2\vec{b}\right)=0$

$7a^2+8b^2-30\ \vec{a}.\vec{b}=0$ ………..(2)

From 1 & 2 It is clear that $a=b$

$\therefore \cos\theta=\dfrac{b}{2a}=\dfrac{1}{2}$

$\theta=60^0$

3. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=2\hat{i}+4\hat{j}-\hat{k}-\lambda\hat{i}+2\hat{j}+3\hat{k}$

$=\left(2-\lambda\right)i+6j-2k$

Given $\vec{a}.\vec{b}=ab\cos\theta$

$a\cos\theta=\dfrac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}=1$

$\vec{a}.\vec{b}=2-\lambda+12-2$

$=2-\lambda+10$

$=12-\lambda$

$\therefore \vec{a}.\vec{b}=\left|b\right|$

$12-\lambda=\sqrt{\left(2-\lambda\right)^2+6^2+\left(-2\right)^2}$

$\left(12-\lambda\right)^2=\left(4+\lambda^2-4\lambda+40\right)$

$\cancel{\lambda^2}+144-24\lambda=\cancel{\lambda^2}-4\lambda+44$

$20\lambda=100$

$\lambda=5$

4. Given area of parallelogram $\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix}$

$\vec{a}\times\vec{b}=i\left(4\alpha\right)+8j-4\alpha k$

$\vec{a}\times\vec{b}=\sqrt{16\alpha^2+64}+16\alpha^2$

$8\sqrt{3}=\sqrt{32\alpha^2+64}$

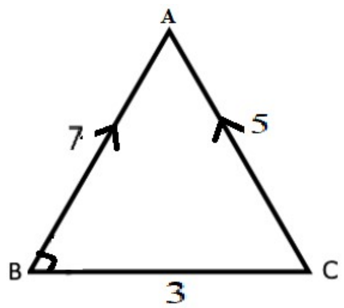
$64\times3=32\alpha^2+64$

$32\alpha^2=128$

$\alpha^2=4$

$\alpha=2$

5.



Projection of BA on BC

$=BA\cos\underline{|ABC}$

$=7\times\dfrac{\left(7^2+3^2-5^2\right)}{2\times7\times3}$

$$= 7 \times \left| \frac{49 + 9 - 25}{2 \times 7 \times 3} \right.$$

$$= \frac{11}{2}$$

6. $\vec{A} + \vec{B} = \vec{C}$

$$\left(\vec{A} + \vec{B}\right).\left(\vec{A} + \vec{B}\right) = \vec{C}.\vec{C}$$

$$A^2 + B^2 + 2AB\cos\theta = C^2$$

$$4^2 + 5^2 + 2 \times 4 \times 5 \times \cos\theta = \left(\sqrt{61}\right)^2$$

$$41 + 40\cos\theta = 61$$

$$40\cos\theta = 20$$

$$\cos\theta = \frac{1}{2} = 60^0$$

7. $\left(\frac{1}{3}\right)^2 = 1^2 + 1^2 + 2 \times 1 \times 1 \times \cos\theta$

$$\frac{1}{9} = \left(1 + \cos\theta\right) \times 2$$

$$\frac{1}{18} = 1 + \cos\theta$$

$$\cos\theta = \frac{1}{18} - 1 = -\frac{17}{18}$$

8. Given $\left|\vec{A} + \vec{B}\right| = n\left|\vec{A} - \vec{B}\right|$

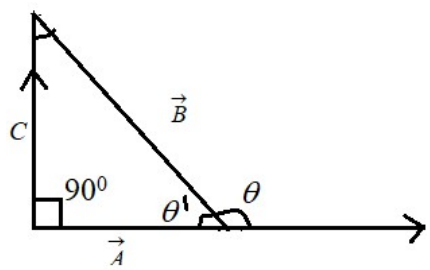
$$2A\cos\frac{\theta}{2} = n \times 2A\sin\frac{\theta}{2}$$

$$\mathrm{Tan}\frac{\theta}{2} = \frac{1}{n}$$

$$\frac{\theta}{2} = \mathrm{Tan}^{-1}\frac{1}{n}$$

9. $\vec{A}.\vec{B} = 0, \vec{A} \times \vec{B} \neq 0$

10.

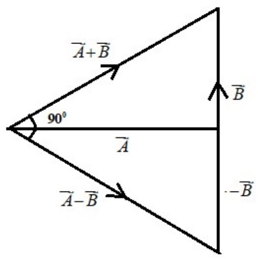


$$\mathrm{Tan}\,\theta' = \frac{C}{A} = \mathrm{Tan}\,45^0$$

$$\theta' = \frac{\pi}{4}$$

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

11.



$\left(\vec{A}+\vec{B}\right).\left(\vec{A}-\vec{B}\right)=0$

$A^2-B^2=0$

$A^2=B^2$

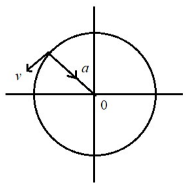
$\left|\vec{A}\right|=\left|\vec{B}\right|$

12. Let unit vector $=\frac{1}{2}\hat{i}+\frac{4}{5}\hat{j}+c\hat{k}$

$\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{4}{5}\right)^2+c^2}=1$

$c^2=\frac{11}{100}\Rightarrow c=\sqrt{0.11}$

13.



From fig $\vec{v}\perp\vec{a}$ so $\vec{v}.\vec{a}=0$

$a\parallel r$ so $r\times a=0$

14. Let $\overrightarrow{A_1}+\overrightarrow{A_2}=5\overrightarrow{A_3}$ ……….(1)

$\overrightarrow{A_1}-\overrightarrow{A_2}=3\overrightarrow{A_3}$ ………..(2)

Add eqn 1&2 subtract eqn 1&2 to get $\overrightarrow{A_1},\overrightarrow{A_2}$

15. From fig Resolve components

$F_2\cos 60=F_1\cos 30$

$F_2\times\frac{1}{\cancel{2}}=10\times\frac{\sqrt{3}}{\cancel{2}}$

$F_2=10\sqrt{3}$

Along vertical

$F_2\sin 60+F_1\sin 30=F_3$

$F_3=10\sqrt{3}\times\frac{\sqrt{3}}{2}+10\times\frac{1}{2}$

$=20N$

16. $\vec{a}=-\omega^2\vec{r}$

so, acceleration is along $-\vec{r}$

17. $\left(\vec{B}\times\vec{A}\right).\vec{A}=\left(\vec{B}\times\vec{A}\right)A\cos\theta=0$ $\left(\theta=90^0\right)$

18. Given $F_R^2=3AB$

$A^2+B^2+2AB\cos\theta=3AB$

$\cos\theta=\frac{1}{2}$

$\theta = \frac{\pi}{3}$

19. $\left|\vec{A}+\vec{B}\right| = 2\left|\vec{A}-\vec{B}\right|$

$\cos\theta = \frac{3\left(A^2+B^2\right)}{10AB} = \frac{3\times 2A^2}{10A^2} = \frac{3}{5}$

20. Projection of $\vec{A}$ on $\vec{B} = \frac{\vec{A}.\vec{B}}{\left|\vec{B}\right|}\hat{B} = \frac{\left(\hat{i}+\hat{j}+\hat{k}\right).\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}.\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}} = \frac{\cancel{2}\left(\hat{i}+\hat{j}\right)}{\cancel{2}} = \hat{i}+\hat{j}$

21. $\sqrt{10}p = \sqrt{(2p)^2+\left(\sqrt{2}p\right)^2} + 2\times 2p.\sqrt{2}p.\cos\theta$

$\cos\theta = \frac{1}{\sqrt{2}}, \theta = 45^0$

22. Given $\cos\alpha = \frac{4}{5\sqrt{2}}$, $\cos\beta = \frac{1}{\sqrt{2}}.\frac{5}{5}$, $\cos\gamma = \frac{3}{5\sqrt{2}}$

$\overrightarrow{A} = 4\hat{i}+5\hat{j}+3\hat{k}$

23. Given projection of $\vec{A}$ on $\vec{B}$ is zero i.e A should be $\perp$ to B

$\vec{A}.\vec{B} = 0$

$\left(2\hat{i}+4\hat{j}-2\hat{k}\right).\left(\hat{i}+2\hat{j}+\alpha\hat{k}\right) = 0$

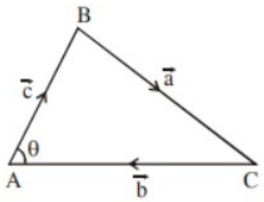
$\alpha = 5$

24. Given $\overrightarrow{a_1}$ & $\overrightarrow{a_2}$ are collinear

$\frac{x}{1} = \frac{-1}{y} = \frac{1}{z}$

unit vector in direction of $x\hat{i}+y\hat{j}+z\hat{k} = \frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

25.



let $a = 8$

$b = 7$

$c = 10$

$\cos\theta = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{7^2+10^2-8^2}{2\times 7\times 10}$

$= \frac{85}{140} = \frac{17}{28}$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$

$= \left|\overrightarrow{AB}\right|\cos\theta = 10\times\frac{17}{28} = \frac{85}{14}$

26. Let, third vector $\vec{c}$

$\therefore \vec{c} = c\ \hat{c} = b\ \hat{a}$ (Given $c = b$)

$= b\frac{\vec{a}}{|\vec{a}|}$ $\qquad |b| = \sqrt{3^2 + 4^2} = 5$

$= \frac{5(i-j)}{\sqrt{2}}$

27. We know that $\text{Tan}\,\alpha = \frac{b\sin\theta}{a + b\cos\theta}$

$1 = \frac{2\sin 60}{a + 2\cos 60}$

$1 = \frac{\cancel{2} \times \frac{\sqrt{3}}{\cancel{2}}}{a + \cancel{2} \times \frac{1}{\cancel{2}}}$

$1 = \frac{\sqrt{3}}{a+1}$

$a = \sqrt{3} - 1$

28. Let $x\hat{i} + y\hat{j} + z\hat{k}$ is perpendicular to $3\hat{i} - 4\hat{j} + 7\hat{k}$ then their dot product should equal to zero .so $3x - 4y + 7z = 0$, option verification by taking $x = 1, y = 2, z = \frac{5}{7}$. So required equation is $\hat{i} + 2\hat{j} + \frac{5}{7}\hat{k}$

29. $\vec{A} \times \vec{B} = AB\sin\theta = 5 \times 12 \times \sin 30 = 5 \times \cancel{12} \times \frac{1}{\cancel{2}} = 30$ units

30. Conceptual

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