CIRCLES - DPP

MATHEMATICS

1. IF the lines $X+2Y-5=0$ & $2X-3Y+4=0$ lies along diameters of circles of area 9π, then equation of the circle is

1) $X^2+Y^2-2X-4Y-4=0$ 3) $X^2+Y^2+2X-4Y-4=0$

3) $X^2+Y^2+2X+4Y-4=0$ 4) $X^2+Y^2-2X+4Y-4=0$

2. The centre of the circle which passes through the vertices of triangle formed by the lines $Y=0$, $Y=X$ and $2x+3y=10$ is

1) $\left(-\frac{5}{2},-\frac{1}{2}\right)$ 2) $\left(\frac{5}{2},-\frac{1}{2}\right)$ 3) $\left(-\frac{1}{2},-\frac{1}{2}\right)$ 4) $\left(\frac{5}{2},\frac{1}{2}\right)$

3. The coordinate of the point on the circles $X^2+Y^2-2X-4Y-11=0$ farthest from the origin are

1) $\left(2+\frac{8}{\sqrt{5}},1+\frac{4}{\sqrt{5}}\right)$ 2) $\left(1+\frac{8}{\sqrt{5}},2+\frac{4}{\sqrt{5}}\right)$

3) $\left(1+\frac{4}{\sqrt{5}},2+\frac{8}{\sqrt{5}}\right)$ 4) $\left(1-\frac{8}{\sqrt{5}},2+\frac{4}{\sqrt{5}}\right)$

4. The Tangent to circle $X^2+Y^2=5$ at $(1,-2)$ also touches the circle $X^2+Y^2-8X+6Y+20=0$ then coordinate of the corresponding point of contact

1) $(7,1)$ 2) $(-3,-1)$ 3) $(3,-1)$ 4) $(5,0)$

5. If $3X+Y=0$ is a tangent to ole whose centre is $(2,-1)$ then find the equation of other tangent to the circle from the origin is

1) $X+3Y=0$ 2) $Y-3X=0$ 3) $3X+Y=0$ 4) $X-3Y=0$

6. If $X+Y+K=0$ is a tangent to the circle $X^2+Y^2-2X-4Y+3=0$ then K=

1) ± 20 2) $-1,-5$ 3) ± 2 4) 4

7. The equation of the normal to the circle $X^2+Y^2-2X=0$, parallel to the line $X+2Y=3$

1) $2X-Y+3=0$ 2) $X+2Y-1=0$ 3) $X+2Y-2=0$ 4) $X-2Y+1=0$

8. The length of the tangent drawn from any point on the circle $x^2+y^2+2gx+2fy+\alpha=0$ to the circle $x^2+y^2+2gx+2fy+\beta=0$ is

1) $\sqrt{\beta-\alpha}$ 2) $\sqrt{\alpha-\beta}$ 3) $\sqrt{\alpha\beta}$ 4) $\sqrt{\alpha/\beta}$

9. The tangents drawn from the origin to the circle $x^2+y^2-2px-2qy+q^2=0$ are perpendicular if

1) $p^2+q^2=1$ 2) $p^2-q^2=1$

3) $p^2-q^2=0$ 4) $p^2+q^2=0$

10. The locus of point of intersection of the tangents to the circle $X^2+Y^2=a^2$ at point whose parametric angles differ by 60^0 is

1) $3\left(x^2+y^2\right)=2a^2$ 2) $4\left(x^2+y^2\right)=3a^2$ 3) $3\left(x^2+y^2\right)=4a^2$ 4) $x^2+y^2=3a^2$

11. The sum of the minimum & maximum distance of the point (4, -3) to the circle $x^2+y^2+4x-10y-7=0$ is

1) 10 2) 12 3) 16 4) 20

12. The equation of the tangent to the circle $x^2+y^2+4x-4y+4=0$ which makes equal intercepts on the positive coordinate axes is

1) $x+y=2$ 2) $x+y=\sqrt{2}$

3. $x+y=2\sqrt{2}$ 4. $x-y=\sqrt{2}$

13. The line x=y touches a circle at the point (1, 1) . If the circle also passes through the point (1, -3) , then its radius is

1) $3\sqrt{2}$ 2) 3 3) $2\sqrt{2}$ 4) 2

14. Let the tangents drawn to the circle $x^2+y^2=16$ from the point P(0,h) meet the X-axis at point A & B . If the Area of ΔPAB is minimum, then h =

1) $4\sqrt{2}$ 2) $4\sqrt{3}$ 3) $3\sqrt{2}$ 4) $3\sqrt{3}$

15. The circle touches Y-axis at (0, 3) and makes an intercept of 2 units on the positive X-axis . Intercept made by the circle on the line $\sqrt{10}\,x-3y=1$ in units is

1) 3 2) 6 3) $2\sqrt{10}$ 4) 10

16. Tangents are drawn to $x^2+y^2=1$ from any orbitary points P on the line $2x+y-4=0$ the corresponding chord of contact pass through a fixed point then that point is

1) $\left(\frac{1}{4},\frac{1}{2}\right)$ 2) $\left(\frac{1}{2},\frac{1}{4}\right)$ 3) $\left(\frac{-1}{2},\frac{1}{4}\right)$ 4) $\left(\frac{1}{2},\frac{-1}{4}\right)$

17. The equation of the circle which has two normal $(x-1)(y-2)=0$ and a tangents $3x+4y=6$ is

1) $x^2+y^2-2x-4y+4=0$ 2) $x^2+y^2+2x-4y+5=0$

3) $(x-1)^2+(y-2)^2=5$ 4) $(x-1)^2+(y-2)^2=4$

18. If $A\left(\frac{\pi}{3}\right),B\left(\frac{\pi}{6}\right)$ are the points on the circle represented in parametric form with centre (0,0) & radius 12 then the length of chord of AB is

1) $6\left(\sqrt{6}-\sqrt{2}\right)$ 2) $6\left(\sqrt{6}-\sqrt{3}\right)$ 3) $\sqrt{2}\left(\sqrt{3}-2\right)$ 4) $6\left(\sqrt{3}-2\right)$

19. A point moves such that the sum of square of its distance from the sides of square of side unity is equal to 9 . The locus of such point is a circle

1) Inscribed in a square 2) Circum scribing the square

3) Inside the square 4) Containing the square

20. If two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points, then find the values of r

1) r=2 2) r<2 3) r>84 4) $2<r<8$

21. The centre of a set of circles, each of radius 3, lies on the circle $x^2+y^2=25$ The locus of any point in the set is

1) $4\le x^2+y^2\le 64$ 2) $x^2+y^2\le 25$

3) $x^2+y^2\ge 25$ 4) $3\le x^2+y^2\le 9$

22. Given $A(0,6),B(4,0),C(-3,0),D(0,-2)$ con-cyclic points, the orthocentre of ΔABC is

1) $(2,0)$ 2) $(0,-2)$ 3) $(0,2)$ 4. $(2,2)$

23. If the radius of the circle $(x-1)^2+(y-2)^2=1$ and $(x-7)^2+(y-10)^2=4$ are increasing uniformly write to time as 0.3 and 0.4unit/sec .Then they will touch each other at time equal to

1) 45 sec 2) 90 sec 3) 11 sec 4) 135 sec

24. The Area of the Quadrilateral formed by the tangent from the point (4,5) to the circle $x^2+y^2-4x-2y-11=0$ with a pair of radius joining the points of contact of these tangents is

1) 4 2) 6 3) 8 4) 1

25. The Area of the triangle formed by the tangents drawn from P (4, 4) to the circle $x^2+y^2-2x-2y-7=0$ and the chord of contact of P write to S=0 is

1) $\frac{9}{2}$ 2) $\frac{81}{10}$ 3) $\frac{3}{2}$ 4) $\frac{81}{4}$

26. If A(2, c) & B(d, 2) are two points such that the polar of one point write to circle $X^2+Y^2=16$ passes through the other then c+d=

1) 4 2) 6 3) 8 4) 10

27. The locus of midpoint of chord of the circles $X^2+Y^2-2PX=0$ passing through the origin is

1) $X^2+Y^2+2PX=0$ 2) $X^2+Y^2-PX=0$

3) $X^2+Y^2+PX=0$ 4) $X^2+Y^2-4PX=0$

28. The locus of the midpoint of the chords of the circle $X^2+Y^2=4$ which is subtends a right angle at the origin is

1) $X+Y=2$ 2) $X^2+Y^2=1$ 3) $X^2+Y^2=2$ 4) X+Y=1

29. The Area of the triangle formed by tangent, normal at $\left(1,\sqrt{3}\right)$ to the circle $X^2+Y^2=4$ and X-axis is

1) $4\sqrt{3}$ 2) $\frac{7}{2}\sqrt{3}$ 3) $2\sqrt{3}$ 4) $\frac{1}{2}\sqrt{3}$

30. The locus of center of circle which touches $(Y-1)^2+X^2=1$ externally and also touches X-axis is

1) $\left\{X^2=4Y, Y\geq O\right\}\cup\left\{(0,Y), Y<O\right\}$ 2) $X^2=Y$

3) $Y=4X^2$ 4) $Y^2=4X\cup(0,Y), Y\in R$

KEY

S NO	1	2	3	4	5	6	7	8	9	10
1-10	1	2	2	3	4	2	2	1	3	3
11-20	4	3	3	1	3	2	1	1	4	4
21-30	1	3	2	3	1	3	2	3	3	1

HINTS

1. Centre of the circle = point of intersection of the diameters

= (1, 2).

Let r be radius of circle   Area of circles = 9π

- $\pi r^2 = 9\pi$
- $r = 3$

Equation of circle $(X-1)^2 + (Y-2)^2 = 3^2$

2. Given lines $Y = 0$ _____ (1)

$Y = X$ ______ (2)

$2X + 3Y = 10$ ______(3)

Solving 1 & 2 is 0 (0, 0)

Solving 2 & 3 is A (2, 2)

Solving 1 & 3 is B (5, 0) Let $X^2 + Y^2 + 2gx + 2fy + C = 0$ be the equation of the circle passing through 0 , A & B

- C = 0 , 4+4+4g+4f=0 , 25+10g+C=0
- g = $\frac{-5}{2}$, f = $\frac{1}{2}$

3. The required point lies on the normal to the circle through the

Origin , i.e . on the line $2X = Y$ which gives

$X^2 + 4X^2 - 2X - 8X - 11 = 0 \Rightarrow 5X^2 - 10X - 11 = 0$ 2. $X = 1 \pm \frac{4}{\sqrt{5}}$ & $Y = 2\left(1 \pm \frac{11}{\sqrt{5}}\right)$

And the coordinates of required point farthest from the origin are

$\left(1 + \frac{4}{\sqrt{5}}, 2 + \frac{8}{\sqrt{5}}\right)$

4. Equation of tangent to $X^2 + Y^2 = 5$ at (1, -2) is $X - 2Y - 5 = 0$

Solving second circle, we get $(2Y+5)^2 + Y^2 - 8(2Y+5) + 6Y + 20 = 0$

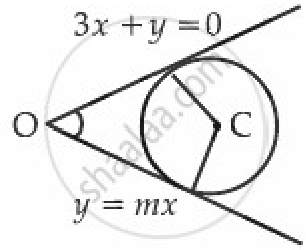
$$5Y^2 + 10Y + 5 = 0$$

$$(Y+1)^2 = 0$$

$$Y = -1 \quad X = -2 + 5 = 3$$

The point of contact on second circle is (3, -1)

5. Centre (2, -1)



$3X+Y=0$

$a=3, b=1, c=0$

$$r=d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|6-1+0|}{\sqrt{3^2+1}}=\frac{5}{\sqrt{10}}$$

Let $Y=MX$ be to circle than r=d $MX-Y=0$

$$\frac{5}{\sqrt{10}}=\frac{|2M+1|}{\sqrt{3^2+1^2}} \qquad 5=|2M+1|$$

$$\pm 5=2M+1$$

$$\begin{array}{ll} 5=2M+1 & -5=2M+1 \\ 6=2M & -5-1=2M \end{array} \qquad 3=M$$

$$Y=3X \qquad -3=M$$

$$Y-3X=0$$

6. Radius =Distance from the center C(1, 2) to $X+Y+K=0$

$$r=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \qquad \sqrt{1+4-3}$$

$$=\frac{|1+2+K+1|}{\sqrt{2}} \qquad |K+3|=2$$

$$K+3=\pm 2$$

$$K=-1,-5$$

7. Slope of line is $=\frac{-1}{2}$,so slope of normal is $\frac{-1}{2}$ normal passes through the center of the circle here center is (1, 0)

Therefore, equation of normal is $Y-0=\frac{-1}{2}(X-1)$

$$X+2Y-1=0$$

8. Let (h,k) be any point on the first circle , then $h^2+k^2+2gh+2fk+\alpha=0$ ______(1)

Length of the tangent from (h,k) to second circle is $\sqrt{h^2+k^2+2gh+2fk+\beta}=\sqrt{\beta-\alpha}$ $(\because 1)$

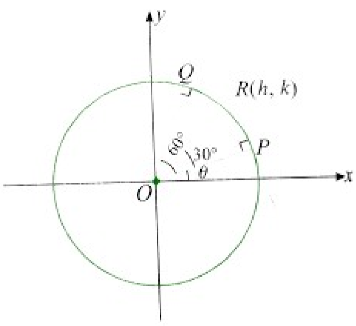
9. Equation of given circle can be written as

$(x-p)^2+(y-q)^2=p^2$ This has (p,q) as the Centre & P as the radius showing that it touching y-axis . So one of the tangents from the origin to the circle is y-axis

Other tangents from the origin to the circle must be x-axis, which is possible if $q=\pm p$

$$p^2-q^2=0$$

10. Let the points on the circle



whose parametric angles differ by 60^0 be $P(a\cos\theta, a\sin\theta)$

$Q\left(a\cos(\theta+60), a\sin\left(\theta+60^\circ\right)\right)$

Tangents at points P and Q intersect at R(h,k) In figure $\underline{|POQ}=60$ & $\underline{|POQ}=30^\circ$ In triangle OPR

$$OP=OR\cos 30^\circ$$

$a=\sqrt{h^2+k^2}\,\frac{\sqrt{3}}{2}$ $\qquad 2a=\sqrt{h^2+k^2}\sqrt{3}$

SOBS

$$4a^2=3\left(h^2+k^2\right)$$

11. Let P(4, -3), centre of circle is c (-2, 5),r=6

minimum distance = cp-r=10-6=4

maximum distance = cp+r=10+6=16 sum of minimum & maximum distance = 20

12. Centre (-2,2) r=2 equation of line having equal intercept on the positive axes is X+Y+K=0, K<0 is a tangent to the circle $\frac{|-2+2+k|}{\sqrt{2}}=2$ $\qquad k=\pm 2\sqrt{2}$

$k=-2\sqrt{2}$ $\qquad (\because k<0)$ $\qquad$ Equation of circle is $x+y=2\sqrt{2}$

13. The equation of circle with touches the line X-Y=0 at (1, 1) is

$(X-1)^2+(Y-1)^2+\lambda(X-Y)=0$ P.T (1, -3)

$$0+16+\lambda(1+3)=0$$

$$\lambda=-4$$

Equation ole is

$$(X-1)^2+(Y-1)^2-4(X-Y)=0$$

$$X^2+Y^2-6X+2Y+2=0$$

$$r=2\sqrt{2}$$

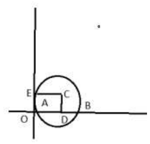
14. Equation of tangent to circle $X^2+Y^2=16$ is $y=mx\pm 4\sqrt{1+m^2}$ equation of tangent is $y=mx\pm h$

This tangent meet X-axis at $A\left(\frac{h}{m},0\right) \& B\left(\frac{-h}{m},0\right)$ Area of

$$\triangle PAB=\frac{1}{2}\begin{vmatrix} h & \frac{h}{m} & -\frac{h}{m} & 0 \\ h & 0 & 0 & h \end{vmatrix} =\frac{1}{2}\left|\frac{-h^2}{m}-\frac{h^2}{m}\right|=\frac{h^2}{m}=\frac{16(1+m^2)}{m}=$$

$m=1$ Then $h=4\sqrt{1+1}=4\sqrt{2}$

15. AB=2, AD=1 CD=3=OE



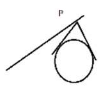
$$AC=\sqrt{(AD)^2+(CD)^2} =\sqrt{1+3^2}$$

$$=\sqrt{10}=CE$$

$$\therefore C\left(\sqrt{10},3\right)$$

The line $\sqrt{10}X-3Y=1$ Passing through the centre $C\left(\sqrt{10},3\right)$ of the circle & hence diameter & the required intercept is twice the radius the circle

16. Variable pt on line $2X+Y-4=0$



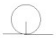
is p(t,4-2t), t $\in$ R Equation of chord of contact of circle $X^2+Y^2=1$ write to point p is $tX+(4-2t)Y=1$ or

$(4Y-1)+t(X-2Y)=0$ is a family of straight line $4Y-1=0$

$X-2Y=0$ $\quad\quad\quad\quad$ $Y=\dfrac{1}{4}$

$X=\dfrac{1}{2}$ $\quad\quad$ $Q\left(\dfrac{1}{2},\dfrac{1}{4}\right)$

17. $(X-1)(Y-2)=0$



$X=1, Y=2$ is centre of circle $(1,2)$

$r=d$ $\quad\quad$ $r=\perp$

distance from centre $(1,2)$ to $3X+4Y-6=0$ $\quad\quad$ $r=\left|\dfrac{3+8-6}{\sqrt{9+16}}\right|=\dfrac{5}{5}=1$

Equation of circle is

$$(X-1)^2+(Y-2)^2=1^2$$

$$X^2+Y^2-2X-4Y+4=0$$

18. $C(0,0), r=12$ $\quad\quad$ $A\left(\dfrac{\pi}{3}\right)=\left(X_1+r\cos\theta, Y_1+r\sin\theta\right)$

$$=\left(6,6\sqrt{3}\right)$$

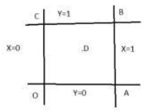
$$B\left(\frac{\pi}{6}\right)=\left(0+12.\frac{\sqrt{3}}{2},0+12.\frac{1}{2}\right)=\left(6\sqrt{3},6\right)$$

Length of chord

$$AB=\sqrt{\left(6\sqrt{3}-6\right)^2+\left(6\sqrt{3}-6\right)^2}$$

$$=6\left(\sqrt{6}-\sqrt{2}\right)$$

19. Let P(X,Y) be a any point on



the locus & OABC be a

the square of side unity

then $X^2+Y^2+(X-1)^2+(Y-1)^2=9$

$$X^2+Y^2-X-Y-\frac{7}{2}=0$$

Centre $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ & $\quad r=2$

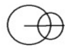
the circle contains the square

20. We have circle $(X-1)^2+(Y-3)^2=r^2$ having centre $C_1(1,3)$ & $r_1=r$ and circle $X^2+Y^2-8X+2Y+8=0$ having centre $C_2(4,-1)$, $r_2=3$ intersect in two distinct points of

$|r_1-r_2|<c_1c_2<r_1+r_2$

$|r-3|<5<r+3$ $\quad 2<r<8$

21. OA=2,OB=8 Distance of



any point (X,Y) on

a circle with centre C lying on the circle $X^2+Y^2=25$ and radius 3 from the origin lies between OA & OB Hence $4 \le X^2+Y^2 \le 64$

22. A(0,6) B(4,0) C(-3,0) D(0,-2) Equation of Altitude through A is X=0 Equation of the Altitude through B is $X+2Y-4=0$ Orthocentre of $\triangle ABC$ is $(0,2)$

23. $c_1(1,2), c_2(7,10)$ $\quad c_1c_2=10$ $\quad r_1=1$

$r_2=2$ $\quad r_1+r_2=3$ $\quad c_1c_2>r_1+r_2$ Radius of two circles at time 't' are (1+0.3t) and (2+0.4t)

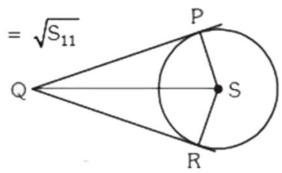
For the two circles to touches each other

$$(AB)^2=\left[(1+0.3t)\pm(2+0.4t)\right]^2$$

$$100=(3+0.7t)^2\,(or)\,(1+0.1t)^2$$

t=10,t=90

24. Let P(4,5), centre of circle C(2,1) Radius of circle ,



r=4 PA=Length of tangent $=\sqrt{S_{11}}$

$$=\sqrt{16+25-16-10-11}=2$$

$$Area\ of\ PACB=2(Area\ of\ \triangle PAC)$$

$$=2.\frac{1}{2}\times 2\times 4=8$$

25. We have $X^2+Y^2-2X-2Y+7=0$, P(4,4)

$r=3, S_{11}=9$

Area of circle $=\dfrac{rs_{11}^{\frac{3}{2}}}{s_{11}+r^2}=\dfrac{3\left(3^2\right)^{\frac{3}{2}}}{9+9}=\dfrac{9}{2}$

26. A(2,C)   B(d,2)

$X^2+Y^2=16$

Polar of a passes through B=A,B are conjugate points write to given circle

$S_{12}=0 \qquad X_1X_2+Y_1Y_2=16$

2(d)+C(2)=16

C+d=8

27. Let $p(x_{1,}y_1)$ be a point on a locus equation of mid point of centre of contact is $S_1=S_{11}$

$xx_1+yy_1-p(x+x_1)=x_1^2+y_1^2-2px_1$

point (0,0) $x_1^2+y_1^2-px_1=0$

$x_1^2+y_1^2-px=0$

28. Given circle is $X^2+Y^2=4$ ______(1) Let (h,k) be a locus equation of midpoint of chord of contact is $S_1=S_{11}$

$XX_1+YY_1-4=X_1^2+Y_1^2-4$

$\dfrac{hX+KY}{h^2+k^2}=1$ ______(2) Homogeneous of equation with help of equation (2) $x^2+y^2=4\left(\dfrac{hx+ky}{h^2+k^2}\right)^2$

$$\left[\left(h^2+k^2\right)^2-4h^2\right]x^2-ghkxy+\left[\left(h^2+k^2\right)-4k^2\right]y^2=0$$

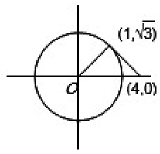
$\theta=90$

a+b=0

$\left(h^2+k^2\right)-4h^2+\left(h^2+k^2\right)^2-4k^2=0$

$h^2+k^2=2 \qquad x^2+y^2=2$

29. $x^2+y^2=4$ C(0,0) , r=2 equation of tangent at



$A(1,\sqrt{3})$ is $x+\sqrt{3}y=4$

If the tangent cuts X-axis at P then x=4,y=0 & hence P=(4,0)

$AP=\sqrt{(1-4)^2+\left(\sqrt{3}-0\right)^2}=\sqrt{12}$ Area of

$\Delta OAP \quad =\frac{1}{2}(OA)(AP) \quad =\frac{1}{2}(2)\sqrt{12}=2\sqrt{3}$

30. Let C(h,k) be centre $r_1=r$

& $c_2 = (0,1), r_2 = 1$ $\quad c_1c_2 = r_1 + r_2$

$$\sqrt{(h-0)^2 + (k-1)^2} = r + 1$$

$r = |k|$ sublines equation

$$\sqrt{h^2 + (k-1)^2} = |k|$$

SOBC

$$h^2 + k^2 + 1 - 2k = k^2 + 1 + 2|k|$$

$$h^2 = 2k + 2|k|$$

Case 1. $k \geq 0$ Case 2.

$k < 0$ $\quad h^2 = 2k + 2k$ $\quad h^2 = 2k - 2k$

$h^2 = 4k$ $\quad h = 0$ $\quad y^2 = 4x, y \geq 0$

$x = 0$

$(0, y), y < 0$