NARAYANA EDUCATIONAL INSTITUTIONS

TOPIC: MATRICES & DETERMINANTS

MATHS-A

1. $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ and $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$ and $Tr(A)-Tr(B)$ has the value equal

A) 0 B) 1 C) 2 D) None

2. For each real $x, -1<x<1$, Let $A(x)$ be the matrix $(1-x)^{-1}\begin{bmatrix}1 & -x\\ -x & 1\end{bmatrix}$ and $z=\dfrac{x+y}{1+xy}$ then

A) $A(z)=A(x).A(y)$ B) $A(z)=A(x)-A(y)$ C) $A(z)=A(x)+A(y)$ D) $A(z)=A(x)\left[A(y)\right]^{-1}$

3. $A=[aij]_{4\times4}$ such that $aij=\begin{cases}2 & when\ \ i=i\\ 0 & when\ \ i\neq j\end{cases}$ then $\left\{\dfrac{\det(adj(adjA))}{7}\right\}$ is where $\{\ \}$ represents fractional part function

A) $\dfrac{1}{7}$ B) $\dfrac{2}{7}$ C) $\dfrac{3}{7}$ D) None of these

4. $F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$ where $\alpha\in R$ then $\left[F(\alpha)\right]^{-1}=$

A) $f(\alpha^{-1})$ B) $f(-\alpha)$ C) $f(2\alpha)$ D) None of the above

5. $P=\begin{bmatrix}\cos\pi/6 & \sin\pi/6\\ -\sin\pi/6 & \cos\pi/6\end{bmatrix}, A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^1$ then $P^1Q^{2009}P=$

A) $\begin{bmatrix}1 & \sqrt{3}/2\\ 0 & 2009\end{bmatrix}$ B) $\begin{bmatrix}1 & 2009\\ 0 & 1\end{bmatrix}$ C) $\begin{bmatrix}\sqrt{3}/2 & 2009\\ 0 & 1\end{bmatrix}$ D) $\begin{bmatrix}\sqrt{3}/2 & -1/2\\ 0 & 2009\end{bmatrix}$

6. If $\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^3 & b^3 & c^3\end{vmatrix}=(a-b)(b-c)(c-a)(a+b+c)$ where a,b,c are all different the determinant

$\begin{vmatrix}1 & 1 & 1\\ (x-a)^2 & (x-b)^2 & (x-c)^2\\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b)\end{vmatrix}$ vanishes when

A) $a+b+c=0$ B) $x=\dfrac{1}{3}(a+b+c)$ C) $x=\dfrac{1}{2}(a+b+c)$ D) $x=a+b+c$

7. The value of $\begin{vmatrix}-1 & 2 & 1\\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1\\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1\end{vmatrix}=$

A) zero B) $-16\sqrt{2}$ C) $-8\sqrt{2}$ D) None of these

8. $\Delta=\begin{vmatrix}3 & 4 & 5 & x\\ 4 & 5 & 6 & y\\ 5 & 6 & 7 & z\\ x & y & z & o\end{vmatrix}=0$ then

A) x,y,z are in A.P B) x,y,z are in G.P C) x,y,z are in H.P D) None of these

9. $\begin{vmatrix}x & 2 & x\\ x^2 & x & 6\\ x & x & 6\end{vmatrix}=Ax^4+Bx^3+Cx^2+Dx+E$ then the value of $5A+4B+3C+2D+E$ is equal to

A) zero B) -16 C) 16 D) -11

10. $f\left(x\right)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ Tanx & x & 1\end{vmatrix}$ value of $\lim_{x\to 0}\frac{f\left(x\right)}{x}=$

A) 1 B) -1 C) zero D) None of these

11. If the value of the determinant $\begin{vmatrix}a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{vmatrix}$ is a positive then $\left(a,b,c>0\right)$

A) $abc>1$ B) $abc>-8$ C) $abc<-8$ D) $abc>-2$

12. Let $\overline{a}=x_r i+y_r j+z_r k, r=1,2,3$ be mutually perpendicular unit vectors given the value of $\begin{vmatrix}x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\\ z_1 & z_2 & z_3\end{vmatrix}$

A) zero B) ± 1 C) ± 2 D) ± 3

13. The system of equation

$ax-y-z=a-1$

$x-ay-z=a-1$ has no solutions if a is

$x-y-az=a-1$

A) either -2 (or) 1 B) -2 C) 1 D) not -2

14. If $pqr\neq 0$ and the system of equation

$\left(p+a\right)x+by+cz=0$ $\quad ax+\left(q+b\right)y+cz=0$

$ax+by+\left(r+c\right)z=0$ has a Non trival Solution then value of $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}$

A) -1 B) 0 C) 1 D) 2

15. If the system, of linear equation $x+y+z=6, x+2y+3z=14, \quad 2x+5y+\lambda z=\mu$ Has a unique solution then

A) $\lambda\neq 8$ B) $\lambda=8,\mu\neq 36$ C) $\lambda=8,\mu=36$ D) None of these

16. If $C<1$ and the system of equation $x+y-z=0, 2x-y-cz=0, bx+3by-cz=0$ is consistent then the possible real values of 'b' are

A) $b\in\left(-3,3/4\right)$ B) $b\in\left(\frac{-3}{2},4\right)$ C) $b\in\left(\frac{-3}{4},3\right)$ D) None of these

17. $a=\dfrac{x}{y-z}, b=\dfrac{y}{z-x}, c=\dfrac{z}{x-y}$ where x, y, z are not all zero then the value of $ab+bc+ca$

A) 0 B) 1 C) -1 D) None of these

18. The value of λ for which the following the system of equations does not have a solution $x+y+z=6, 4x+\lambda y-\lambda z=0, 3x+2y-4z=-8$

A) 3 B) -3 C) 0 D) 1

19. The value of $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$

A) $k(a+b)(b+c)(c+a)$ B) $kabc\left(a^2+b^2+c^2\right)$

C) $k(a-b)(b-c)(c-a)$ D) $k(a+b-c)(b+c-a)(c+a-b)$

20. $\begin{vmatrix} \sin\pi/6 & \sin\pi/4 & \sin\pi/3 \\ \cos\pi/6 & \cos\pi/4 & \cos\pi/3 \\ \sin\pi/3 & \sin\pi/2 & \sin\pi/3 \end{vmatrix} =$

A) $\frac{1}{4}\left(2+\sqrt{6}\right)$ B) $2-3\sqrt{2}$ C) $2-\sqrt{6}$ D) $\frac{1}{4}\left(2-3\sqrt{2}+\sqrt{6}\right)$

21. A is a skew symmetric matrix the trace A is__________

22. If A,B,C are $n\times n$ matrix and $\det(A)=2, \det(B)=5$ and $\det(C)=5$ then the value of $\left[\det\left(A^2\right)BC^{-1}\right]=$ _______ where $[\ ]$ represents greatest integer function is

23. The matrix $A=\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is nilpotent of Index $=$________

24. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} =$________

25. The number of Non trival solutions of the system $x-y+z=0, x+2y-z=0, 2x+y+3z=0$ is_________

26. If $y=\begin{vmatrix} \sin x & cox & \sin x \\ \cos x & -\sin x & \cos x \\ x & 1 & 1 \end{vmatrix}$ then $\dfrac{dy}{dx}=$__________

27. The rank of the matrix $\begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{vmatrix}$ is 1 been $a=$_______

28. $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $adJA=\begin{bmatrix} 6 & -2 & -6 \\ -4 & 2 & x \\ y & -1 & -1 \end{bmatrix}$ then $x+y=$ ________

29. $f(x)=\begin{vmatrix}\sec x & \cos x\\ \cos^2 x & \cos^2 x\end{vmatrix}$ then $\int_0^{\pi/2} f(x)\,dx=$ ________

30. $A=\begin{bmatrix}0 & a\\ 0 & 0\end{bmatrix}$ and $(A+I)50-50A=\begin{bmatrix}b & c\\ d & e\end{bmatrix}$ then the value of $b+c+d+e$ is _______

KEY

01)	**C**	02)	**A**	03)	**A**	04)	**B**	05)	**B**
06)	**B**	07)	**B**	08)	**A**	09)	**D**	10)	**C**
11)	**B**	12)	**B**	13)	**B**	14)	**A**	15)	**A**
16)	**C**	17)	**C**	18)	**A**	19)	**C**	20)	**D**
21)	**0**	22)	**4**	23)	**2**	24)	**0**	25)	**0**
26)	**1**	27)	**-6**	28)	**6**	29)	**0**	30)	**2**

HINTS

1. $Tr(A)+2Tr(B)=-1$ Let $Tr(A)=x, Tr(B)=y$

$2Tr(A)-Tr(B)=3$

$x+2y=-1, 2x-y=3\Rightarrow$ Slove $x=1, y=-1$

$Tr(A)-Tr(B)\Rightarrow x-y=1+1=2$

2. $A(x).A(y)=(1-x)^{-1}(1-y)^{-1}\begin{bmatrix}1 & -x\\ -x & 1\end{bmatrix}\begin{bmatrix}1 & -y\\ -y & 1\end{bmatrix}$

$=(1+xy-(x+y))^{-1}\begin{bmatrix}1+xy & -(x+y)\\ -(x+y) & 1+xy\end{bmatrix}$

$=\left(1-\dfrac{x+y}{1+xy}\right)^{-1}\begin{bmatrix}1 & -\dfrac{x+y}{1+xy}\\ \dfrac{-(x+y)}{1+xy} & 1\end{bmatrix}$

$=A(z)$

3. $|A|=2^4=16$ $A=\begin{bmatrix}2 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 2\end{bmatrix}$

$|adJ(adJA)|=|A|^{(n-1)^2}=16^{(4-1)^2}=16^9=(2^4)^9=2^{36}=\left\{\dfrac{(7+1)^{12}}{7}\right\}=\dfrac{1}{7}$

$=A(Z)$

4. $F\left(\alpha\right)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}, adJ\left[F\left(\alpha\right)\right]=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$

$$\det\left(f\left(\alpha\right)\right)=1,\left[F\left(\alpha\right)\right]^{-1}=\frac{AdJA}{\det A}=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f\left(-\alpha\right)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\therefore\left[f\left(\alpha\right)\right]^{-1}=f\left(-\alpha\right)$$

5. $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}, A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

$$P^T=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}, PP^T=I$$

$$Q=PAP^T$$
$$P^TQ=P^TPA.P^T=I.AP^T\Rightarrow P^TQ=AP^T$$
$$P^TQ^{2009}.P\Rightarrow P^TQ.Q^{2008}P$$
$$\Rightarrow AP^T.Q^{2008}.P\Rightarrow A.P^TQ.Q^{2007}.P$$
$$\Rightarrow A^2P^TQ^{2007}.P$$
$$\vdots$$
$$A^{2009}\left(P^TP\right)=A^{2009}$$
$$P^T.Q^{2009}P=A^{2009}=\begin{bmatrix}1 & 2009\\ 0 & 1\end{bmatrix}$$

6. $\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^3 & b^3 & c^3\end{vmatrix}=abc\begin{vmatrix}\frac{1}{a} & \frac{1}{b} & \frac{1}{c}\\ 1 & 1 & 1\\ a^2 & b^2 & c^2\end{vmatrix}=\begin{vmatrix}bc & ca & ab\\ 1 & 1 & 1\\ a^2 & b^2 & c^2\end{vmatrix}=\begin{vmatrix}1 & 1 & 1\\ a^2 & b^2 & c^2\\ ab & bc & ca\end{vmatrix}$

$$D=\begin{vmatrix}1 & 1 & 1\\ \left(x-a\right)^2 & \left(x-b\right)^2 & \left(x-c\right)^2\\ \left(x-b\right)\left(x-c\right) & \left(x-c\right)\left(x-a\right) & \left(x-a\right)\left(x-b\right)\end{vmatrix}=\left(a-b\right)\left(b-c\right)\left(c-a\right)\left(3x-a-b-c\right)$$

Now given a,b,c are different then $D=0\Rightarrow x=\frac{1}{3}\left(a+b+c\right)$

7. $\Delta = \begin{vmatrix} -4-2\sqrt{2} & -2\sqrt{2} & 0 \\ 4\sqrt{2} & 4\sqrt{2} & 0 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$ expand $= -\sqrt{16}$

8. $\Delta = \begin{vmatrix} 0 & 0 & 0 & x+z-2y \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix} R_1 \to R_1 + R_3 - 2R_2$

$= -(x+z-2y)\begin{vmatrix} 4 & 5 & 6 \\ 5 & 6 & 7 \\ x & y & z \end{vmatrix} \begin{matrix} c_1 \to c_1 + c_3 - 2c_2 \\ c_2 \to c_2 - c_3 \end{matrix}$

$= -(x+z-2y)\begin{vmatrix} 0 & -1 & 6 \\ 0 & -1 & 7 \\ x-2y+z & y-z & z \end{vmatrix}$

$= -(x+z-2y)^2 \begin{vmatrix} -1 & 6 \\ -1 & 7 \end{vmatrix}$

$\Delta = 0$

$(x-2y+2)^2 = 0$

$2y = x+z \Rightarrow AP$

9. $5A + 4B + 3C + 2D + E = \Delta(1) + \Delta^1(1)$

$\Delta(1) = 0 \Rightarrow R_2, R_3$ Identical

$\Delta^1(1) = -11$

10. $f^1(x) = \begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ Tanx & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ Tanx & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$

$f^1(0) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$

$\underset{x \to 0}{Lt} \dfrac{f(x)}{x} = \underset{x \to 0}{Lt} \dfrac{f^1(x)}{1} = f^1(0) = 0$

11. $\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = abc - (a+b+c) + 2$, Let $(abc)^{1/3} = x \Rightarrow abc = x^3$

$$\begin{aligned} \Delta > 0 &\Rightarrow abc + 2 > a+b+c \\ &\Rightarrow x^3 + 2 > 3x \\ &\Rightarrow x^3 - 3x + 2 > 0 \end{aligned} \quad \left[\begin{aligned} A.M + G.M &\Rightarrow \frac{a+b+c}{3} > (abc)^{1/3} \\ &\Rightarrow a+b+c > 3(abc)^{1/3} \end{aligned} \right]$$

$$\Rightarrow (x-1)^2(x+2) > 0 \Rightarrow x > -2$$

$$\Rightarrow (abc)^{1/3} > -2 \Rightarrow abc > (-2)^3$$

$$\Rightarrow abc > -8$$

12. $\Delta = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

$$\Delta^2 = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \times \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$= \begin{vmatrix} x_1^2+y_1^2+z_1^2 & x_1x_2+y_1y_2+z_1z_2 & x_1x_3+y_1y_3+z_1z_3 \\ x_1x_2+y_2y_1+z_2z_1 & x_2^2+y_2^2+z_2^2 & x_2x_3+y_2y_3+z_2z_3 \\ x_1x_3+y_1y_3+z_1z_3 & x_2x_3+y_2y_3+z_2z_3 & x_3^2+y_3^2+z_3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \Rightarrow \Delta = \pm 1$$

13. For no solution $\begin{vmatrix} a & -1 & -1 \\ 1 & -a & -1 \\ 1 & -1 & -a \end{vmatrix} = 0$

$$a(a^2-1) - 1(a-1) + 1(1-a) = 0$$

$$a(a^2-1) - 2a + z = 0$$

$$a(a-1)(a+1) - 2(a-1) = 0$$

$$(a-1)(a^2+a-2) = 0$$

$$(a-1)(a+2)(a-1) = 0$$

$$(a-1)^2(a+2) = 0$$

$$a = 1, 1, -2$$

But $a = 1$, There are infinite solution, when $a = -2$

$-2x - y - z = -3$

$x + 2y - z = -3$

$x - y + 2z = 3$ adding we get $0 = -9$

Which is not true hence no solution at $a = -2$

14. $\Delta = \begin{vmatrix} p+a & b & c \\ a & q+b & c \\ a & b & r+c \end{vmatrix} = 0$ $\begin{matrix} R_1 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$$\begin{vmatrix} p+a & b & c \\ -p & q & o \\ -p & o & r \end{vmatrix} = 0 \Rightarrow pqc + \left[q\left(p+a\right)+bp\right]r = 0$$

Dividing on bothsides pqr

$$\Rightarrow \frac{a}{p} + \frac{b}{q} + \frac{c}{r} = -1$$

15. Given equations has a unique solution

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda - 8 \neq 0, \lambda \neq 8$$

16. $$\begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0 \quad \begin{aligned} &\Rightarrow c + bc - 6b + b + 2c + 3bc = 0 \\ &\Rightarrow 3c + 4bc - 5b = 0 \\ &\Rightarrow c = \frac{5b}{4b+3} \end{aligned}$$

$$c < 1 \Rightarrow \frac{5b}{4b+3} - 1 < 0 \Rightarrow \frac{b-3}{4b+3} < 0 \Rightarrow b \in \left(\frac{-3}{4}, 3\right)$$

17. $x - ay + az = 0$

$bx + y - bz = 0$

$-cx + cy + z = 0$

$$\Delta = \begin{vmatrix} 1 & -a & a \\ b & 1 & -b \\ -c & c & 1 \end{vmatrix} = 0 \Rightarrow ab + bc + ca = -1$$

18. $$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

19. $$\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix} = \begin{vmatrix} ka & k^2 & 1 \\ kb & k^2 & 1 \\ kc & k^2 & 1 \end{vmatrix} + \begin{vmatrix} ka & a^2 & 1 \\ kb & b^2 & 1 \\ kc & c^2 & 1 \end{vmatrix} = +0 + k \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= k\left(a-b\right)\left(b-c\right)\left(c-a\right)$$

20. $A^T = -A$ then $Tr\left(A\right) = 0$

21. Write the values and expand the determinant

22. $|A| = 2, |B| = 5, |C| = 5$

$\det\left(A^2BC^{-1}\right) = \left|A^2BC^{-1}\right|$

$$= \frac{|A^2||B|}{|C|} = \frac{4 \times 5}{5} = 4$$

23. $A^2 = 0 \Rightarrow INDEX = 2$

24. $\frac{1}{abc}\begin{vmatrix}1 & a^3 & abc\\ 1 & b^3 & abc\\ 1 & c^3 & abc\end{vmatrix}$

$\frac{abc}{abc}\begin{vmatrix}1 & a^3 & 1\\ 1 & b^3 & 1\\ 1 & c^3 & 1\end{vmatrix}=0$

25. $\begin{vmatrix}1 & -1 & 1\\ 1 & 2 & -1\\ 2 & 1 & 3\end{vmatrix}\begin{matrix}=1(6+1)+1(3+2)+1(1-4)\\ =7+5-3=9\neq 0\end{matrix}$

$\therefore$ no of non trival solutions $=0$

26. $\frac{dy}{dx}=\begin{vmatrix}\cos x & -\sin x & \cos x\\ \sin x & -\sin x & \cos x\\ x & 1 & 1\end{vmatrix}+\begin{vmatrix}\sin x & \cos x & \sin x\\ -\sin x & -\cos x & -\sin x\\ x & 1 & 1\end{vmatrix}+\begin{vmatrix}\sin x & \cos x & \sin x\\ \cos x & -\sin x & \cos x\\ 1 & 0 & 0\end{vmatrix}$

$=0+0+1\begin{vmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{vmatrix}$

$=\cos^2 x+\sin^2 x=1$

27. $A=\begin{vmatrix}-1 & 2 & 5\\ 0 & 0 & a+6\\ 0 & 0 & a+6\end{vmatrix}\begin{matrix}R_2+2R_1\to R2\\ R_3\to R_3+R_1\end{matrix}$

$\begin{vmatrix}-1 & 2 & 5\\ 0 & 0 & a+6\\ 0 & 0 & 0\end{vmatrix} R_3\to R_3-R_2 \text{ Rank}=1$

$a+6=0$

$a=-6$

28. $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 3 & 0\\ 0 & 1 & 2\end{bmatrix}$, cafacter of $A=\begin{bmatrix}+6 & -4 & +2\\ -2 & +2 & -1\\ 6 & +4 & -1\end{bmatrix}^T$

$=\begin{bmatrix}6 & -2 & -6\\ -4 & 2 & 4\\ 2 & -1 & -1\end{bmatrix}, x=y, y=2$

$x+y=4+2=6$

29. $f(x)=\cos x-\cos^3 x$ $\qquad$ $\begin{matrix}Let \sin x=t\\ \cos x dx=dt\\ U.L\Rightarrow 1=t\\ L.L\Rightarrow 0=t\end{matrix}$

$\int_0^{\pi/2}(\cos x-\cos^3 x)dx=\int_0^{\pi/2}\cos x\sin^2 x\,dx$

$$= \int_0^1 t^2 dt = \left[\frac{t^3}{3}\right]_0^1$$

$$= \frac{1}{3} - 0 = \frac{1}{3} \cong 0.33 \cong 0$$

30. $$A^2 = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$(A+I)^2 = A^2 + 2AI + I^2 = 2AI + 1$$

$$(A+I)^3 = 3A + I......(A+I)^{50} = 50A + I$$

$$(A+I)^{50} - 50A = I$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} b & c \\ d & e \end{pmatrix}$$

$$b = 1, c = 0, d = 0, e = 1$$

$$b + c + d + e = 1 + 0 + 0 + 1 = 2$$