NARAYANA EDUCATIONAL INSTITUTIONS

DPP (THEORY OF EQUATION)

1. consider the equation $x^2+2x-n=0$ where n $\in$ N and n $\in[5,100]$. The total number of different values of n so that the given equation has integral roots is
   1) 8 2) 3 3) 6 4) 4
2. The least value of the expression $x^2+4y^2+3z^2-2x-12y-6z+14$ is _______
   1) 1 2) 0 3) -1 4) 2
3. If the roots of the equation $x^3+kx^2+56x-64=0$ are in G.P then the value of k____
   1) 12 2) 14 3) -14 4) -12
4. If the roots of the equation $ax^3-12x^2+11x-3=0$ are in A.P then a =_____
   1) 2 2) 4 3) 0 4) 1
5. If the roots of the equation $x^3+ax^2+bx+c=0$ are in H.P then the mean root is _______
   1) $\frac{3c}{a}$ 2) $\frac{3c}{b}$ 3) $-\frac{3c}{b}$ 4) $-\frac{3c}{a}$
6. The equation whose roots are exceed by 2 then those of $8x^3-4x^2+6x+1=0$ is
   1) $8x^3-52x^2+118x+91=0$ 2) $8x^3+52x^2+118x+91=0$
   2) $8x^3-52x^2+118x-91=0$ 4) $8x^3-52x^2-118x+91=0$
7. If f (x) =0 has a repeated root k then another equation having k as root is _______
   1) f(2x) =0 2) f (-x) =0 3) f^1 (x) =0 4) f^{11} (x) =0
8. If $\alpha,\beta.\gamma$ are the roots of the equation $x^3-2x^2+7x+5=0$ then the values of $\alpha^4+\beta^4+\gamma^4$ is _____
   1) -36 2) 36 3) -38 4) 38
9. The number of real roots of $x^9-3x^8+4x^5-4x^2+1=0$ is _____
   1) atmost 4 2) atmost 5 3) atmost 3 4) atmost 2
10. The number of positive roots of $x^5+3x^4-2x^2-3x+1=0$ is _____
    1) 0 2) 2 3) 3 4) 4
11. The degree of the equation $\left(x^{n/2}+1\right)^2=\left(2x-1\right)$ where n $\in$N and n>2 and n is odd positive integer is___
    1) 1 2) $3n$ 3) n 4) 2n
12. f (x) =0 is a R.E of first type odd degree then a factor of f (x) is _____
    1) $x+1$ 2) $x-1$ 3) $x+2$ 4) $x-2$
13. To eliminate rth term from the beginning in the transformed equation f (x) = 0 we have to diminish the roots of $f\left(x\right)=0$ by 'h' where______
    1) $f^{n+r+1}(h)=0$ 2) $f^{n-r+1}(h)=0$ 3) $f^{n-r-1}(h)=0$ 4) $f^{n-r}(h)=0$
14. If α,β,γ and δ be the roots of $3x^4-27x^3+36x^2-5=0$ then the equation whose roots are $-1/\alpha,-1/\beta,-1/\gamma$ *and* $\frac{-1}{\delta}$ is _____
    1) $5x^4-36x^2-27x+3=0$ 2) $5x^4+36x^2-27x-3=0$
    3) $5x^4-36x^2-27x-3=0$ 4) $5x^4+36x^2+27x+3=0$

15. $\sum\alpha=\frac{5}{3},\sum\alpha\beta=\frac{7}{3},\alpha\beta\gamma=\frac{1}{3}$ then the equation whose roots are α,β,γ is_____

1) $3x^3-5x^2-7x-1=0$ 2) $3x^3+5x^2+7x+1=0$

3) $3x^3-5x^2-7x-1=0$ 4) $3x^3-5x^2+7x-1=0$

16. If α,β,γ are the roots of the equation $x^3+ax^2+bx+c=0$ *then* $\sum\alpha^2\beta^2$ =____

1) b^2-2ac 2) b^2+4ac 3) b^2+2ac 4) b^2-4ac

17. If α,β,γ are the roots of $x^3-14x^2+56x+k=0$ *then* $\sum\alpha(\beta+\gamma)==$____

1) 110 2) 112 3) 28 4) 26

18. If f (x) = $2x^4-13x^2+ax+b$ is divisible by x^2-3x+2 then (a, b) =

1) $(-9,-2)$ 2) (6,4) 3) (9,2) 4) (2,9)

19. If a, b and c are the roots of $x^3+qx+r=0$ then $(a-b)^2+(b-c)^2+(c-a)^2=$

1) $-6q$ 2) $-4q$ 3) $6q$ 4) 4q

20. If α,β,γ are the roots of $x^3-x^2+8x-6=0$ *then* $(1+\alpha^2)\,(1+\beta^2)\,(1+\gamma^2)=$

1) 12 2) 74 3) 24 4) -74

21. If the roots of the equation $x^3+3px^2+3qx-8=0$ are in a geometric progressions then $-\frac{q^3}{p^3}=$

22. The equation $x^5-5x^3+5x^2-1=0$ has three equal roots. If α,β are the other two roots of this equation , then $\alpha+\beta+\alpha\beta=$

23. If the roots of $x^5-ax^4+bx^3-cx^2+dx-1=0$ are all positive such that their arithmetic mean and geometrical mean are equal then a+b+c+d=____

24. Let $\propto$ be a root of the equation $x^4+x^2+1=0$ then the value of $\alpha^{1011}+\alpha^{2022}-\alpha^{3033}=$ __

25. If the sum of all the roots of the equation $e^{2x}-11e^x-45e^{-x}+\frac{81}{2}=0$ is $\log_e p$ then p= ____

26. The Quotient obtained when $3x^4-x^3+2x^2-2x-4$ is divided by x^2-4 is ax^2+bx+c *then* $a+b+c=$___

27. The product of multiple roots of $x^4-2x^3-11x^2+12x+36=0$ is K then K+7=____

28. If the roots of $x^3-14x^2+56x-64=0$ are in G.P then the middle root is ____

29. If f(x) = x^3+x^2+x+1 then the coefficient of x in f (x+5) is _____

30. If α is a positive root of the equation $x^4+x^3-4x^2+x+1=0$ *then* $\alpha+\frac{1}{\alpha}=$______

KEY

HINTS

QNO	1	2	3	4	5	6	7	8	9	10
KEY	**1**	**1**	**3**	**2**	**3**	**3**	**3**	**3**	**2**	**2**
QNO	11	12	13	14	15	16	17	18	19	20
KEY	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**1**	**2**
QNO	21	22	23	24	25	26	27	28	29	30
KEY	**8**	**-2**	**30**	**1**	**45**	**16**	**1**	**4**	**86**	**2**

1. $x^2+2x+1=n+1$

$(x+1)^2=n+1$

$x+1=\pm\sqrt{n+1}$ $\qquad$ $x=-1\pm\sqrt{n+1}$

$x=-1\pm\sqrt{n+1}$ is a perfect square

$\because n+1=3^2, 4^2, 5^2.....10^2$

no of values of n=8

2. $(x^2-2x+1)+4(y^2-3y+\frac{9}{4})+3\ (z^2-2z+1)+1$

$(x-1)^2+4(y-3/2)^2+3(z-1)^2+1$

least value = 1

3. $s_3=\frac{a}{r}.\ a.ar=-\frac{a_3}{a_0}$

$a^3=64\Rightarrow a=4$

$\therefore 4^3+k(4)^2+56(4)-64=0\ \therefore k=-14$

4. Roots are in AP $2b^3+27a^2d=9abc$

$a=a,\ b=-12\ ,c=11,\ d=-3$ $\qquad$ $\therefore a=4$

5. Mean root = $\frac{3s_3}{s_2}$

6. $f(x)=8x^3-4x^2+6x+1=0$

$f(x-2)=0$

7. CONCEPTUAL

8. $p_1=-2, p_2=7, p_3=5, p_4=0$

Newton's law:

$s_1+p_1=0$

$s_2+s_1p_1+2p_2=0$

$s_3+s_2p_1+s_1p_2+3p_3=0$

$s_4+s_3p_1+s_2p_2+s_1p_3+4p_4=0$

$\therefore \alpha^4+\beta^4+\gamma^4=s_4=$ -38

9. Descarte's rule of signs is number of real roots = atmost k [(No. if sign changes in f(x) +No. of sign changes in f (-x)]

10. Descarte's rule of signs' No. of positive roots= No. of sign changes in f(x)

11. $x^n+2x^{n/2}+1=4x^2-4x+1$

$2x^{n/2}=(4x^2-4x-x^n)$

$s.o.b.s$

$4x^n=(4x^2-4x-x^n)^2$

$\because$ The degree = 2n

12. CONCEPTUAL

13. CONCEPTUAL

14. $f\left(-\frac{1}{x}\right)=0$

15. $x^3-\sum\propto(x^2)+\sum\propto\beta(x)-\propto\beta r=0$

16. $\sum(\propto\beta)^2={S^2}_2-2S_1S_3$

17. $\sum \propto (\beta + y) = 2S_2$

18.

$$\begin{array}{c|ccccc} 3 & 2 & 0 & -13 & a & b \\ & 0 & 6 & 18 & 3 & 0 \\ -2 & 0 & 0 & -4 & -12 & -2 \\ \hline & 2 & 6 & 1 & a-9 & b-2 \end{array}$$

$a-9=0;\ b-2=0$

$a=9, b=2$

19. $(a-b)^2+(b-c)^2+(c-a)^2$

$=2\left[(a+b+c)^2-2(ab+bc+ca\right]-2(ab+bc+ca)$

$= 2S^2{}_1-6S_2$

20. $(1+\alpha^2)(1+\beta^2)\ (1+\gamma^2)$

$=1+(\alpha^2+\beta^2+\gamma^2)\ +\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2\gamma^2$

$=1+S_1^2-2S_2+S_2^3-2S_1S_3+S_3^2$

21.

$S_3=\frac{a}{r}.a.ar=\frac{-a_3}{a_0}$

$a^3=8$

$a=2$ is root

$\because 2^3+3p(2)^2+3q(2)-8=0$

12p+6q=0

6q= -12p

$-\frac{q}{p}=2$

22. $f(x)=x^5-5x^3+5x^2-1=0$

$f^1(x)=5x^4-15x^2+10x=0$

$f^{11}(x)=20x^3-30x+10=0$

$f(1)=0\ \ f^{11}(1)=0$

1 is multiple roots of order '3'

$$\begin{array}{c|cccccc} 1 & 1 & 0 & -5 & 5 & 0 & -1 \\ & 0 & 1 & 1 & -4 & 1 & 1 \\ \hline 1 & 1 & 1 & -4 & 1 & 1 & \underline{|0} \\ & 0 & 1 & 2 & -2 & -1 & \\ \hline 1 & 1 & 2 & -2 & -1 & \underline{|0} & \\ & 0 & 1 & 3 & 1 & & \\ \hline & 1 & 3 & 1 & \underline{|0} & & \end{array}$$

$x^2+3x+1=0$

$\alpha+\beta=-3, \alpha\beta=1$

$\alpha+\beta+\alpha\beta=-3+1=-2$

23. $\alpha,\beta,\gamma,\delta,\lambda$ are the roots

$$\frac{\alpha+\beta+\gamma+\delta+\lambda}{5}=\sqrt[5]{\alpha\,\beta\,\gamma\,\delta\,\lambda}$$

$\alpha+\beta+\gamma+\delta+\lambda=5$

$(x-1)(x-1)(x-1)(x-1)(x-1)=\left(x^2-2x+1\right)\left(x^3-3x^2+3x-1\right)$

$= x^5-5x^4+10x^3+10x^2+5x-1=0$

Comparing with $x^5-ax^4+bx^3-cx^2+dx-1=0$

$\therefore a=5, b=10, c=10, d=5$

$a+b+c+d=30$

24. ω is root of $x^4+x^2+1=0$

$\because \alpha=\omega$ $\qquad$ $\because \alpha^{1011}+\alpha^{2022}-\alpha^{3033}$

$=(\omega^3)^{337}+(\omega^3)^{674}-(\omega^3)^{1011}$

$=1+1-1=1$

25. $Let\, e^x=y$

$$y^2-11y-\frac{45}{y}+\frac{81}{2}=0$$

$y^3-11y^2+\dfrac{81}{2}y-45=0$ $\qquad$ $S_3=y_1.y_2.y_3=45$

$e^{x_1+x_2+x_3}=45$

$x_1+x_2+x_3=\log 45$

$\log_e p=\log_e 45$

$p=45$

26.

	3	-1	2	-2	-4
0	0	0	0	0	0
4	0	0	12	-4	56
	3	-1	14	-6	54

Quotient is $3x^2-x+14$

$\therefore a+b+c=16$

27. $f(x)=x^4-2x^3-11x^2+12x+36=0$

$f^1(x)=4x^3-6x^2-22x+12$ $\;f(-2)=0$ $\;f^1(-2)=0$

- 2 is multiple root

$f(3)=0$

$f^1(3)=0$

3 is multiple root

product of multiple roots k=-2(3) = -6

28. $S_3=\dfrac{a}{r}a.ar=64$

$a=4$

29. $f(x+5)=x^3+16x^2+86x+156=0$

30. 1 is positive root of f (x) =0

$\because \alpha=1$