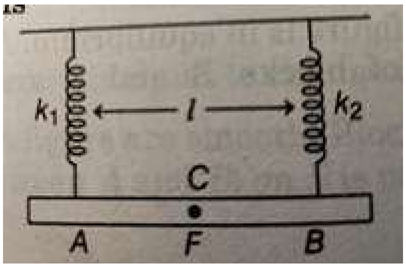
TOPIC :WORK POWER ENERGY

PHYSICS

1. Two light vertical springs with spring constants k_1 and k_2 are separated by a distance l. Their upper ends are fixed to the ceiling and their lower ends to the ends A and B of a light horizontal rod AB.A vertical downward force F is applied at point C on the rod. AB will remain horizontal in equilibrium, if the distance AC is

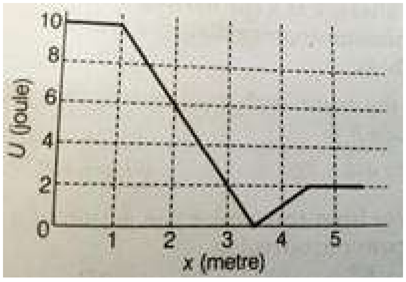


1) $\frac{lk_1}{k_2}$ 2) $\frac{lk_1}{k_2+k_1}$ 3) $\frac{lk_2}{k_1}$ 4) $\frac{lk_2}{k_1+k_2}$

2. A pump is required to lift 800 kg of water per minute from a 10m deep well and eject it with speed of 20 m/s. The required power (in watts) of the pump will be

1) 6000 2) 4000 3) 5000 4) 8000

3. A body with mass 1 kg moves in one direction in the presence of a force which is described by the potential energy graph. If the body is released from rest at $x = 2m,$ than its speed its speed when it crosses $x = 5m$ is (Neglect dissipative forces)

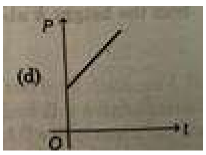


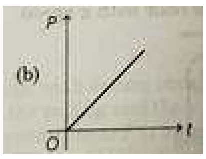
1) $2\sqrt{2}ms^{-1}$ 2) 1 ms^{-1} 3) 2 ms^{-1} 4) $3ms^{-1}$

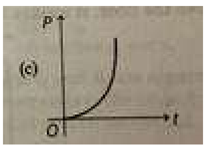
4. A particle of mass m moves from rest under the action of a constant force F which acts for two seconds. The maximum power attained is

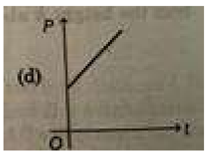
1) $2Fm$ 2) $\frac{F^2}{m}$ 3) $\frac{2F}{m}$ 4) $\frac{2F^2}{m}$

5. A body moves under the action of a constant force along a straight line. The instantaneous power developed by this force with time t is correctly represented by

1) 

2) 

3) 

4) 

6. A block of mass 5 kg is raised from the bottom of the lake to a height of 3 m without change in kinetic energy at any instant. If the density of the block is 3000 kg m^{-3}, then the work done by the external force is equal to

1) $100J$ 2) $150J$ 3) $50J$ 4) $75J$

7. A particle moves under the action of a force $F = 20\hat{i} + 15\hat{j}$ along a straight line $3y + ax = 5$, where α is a

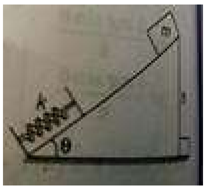
Constant. If the work done by the force F is zero, then the value of α is

1) $\frac{4}{9}$ 2) $\frac{9}{4}$ 3) 3 4) 4

8. An open knife of mass m is dropped from a height h on a wooden floor. If the blade penetrates up to the depth d into the wood, the average resistance offered by the wood to the knife edge is

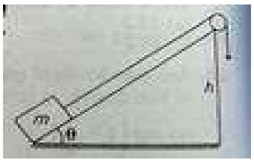
1) $mg\left(1+\frac{h}{d}\right)$ 2) $mg\left(1+\frac{h}{d}\right)^2$ 3) $mg\left(1-\frac{h}{d}\right)$ 4) $mg\left(1+\frac{d}{h}\right)$

9. A body of mass m is released from a height h on a smooth inclined plane that is shown in the figure. The following can be true about the velocity of the block knowing that the wedge is fixed



1) u is highest when it just touches the spring
2) u is highest when it compresses the spring by some amount
3) u is highest when the spring comes back to natural position
4) u is highest at the maximum compression

10. A block of mass m is directly pulled up slowly on a smooth inclined plane of height h and inclination θ with the help of a string parallel to the incline. Which of the following statement is incorrect for the block when it moves up from the bottom to the top of the incline?



1) Work done by the normal reaction force is zero
2) Work done by the string is mgh
3) Work done by gravity is mgh
4) Net work done on the block is zero

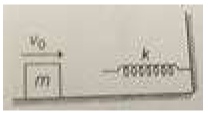
11. Two indentical cylindrical vessels with their bases at the same level, each contains a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other h_2. The area of either bases is A. The work done by the gravity in equalizing the levels when the vessels are interconnected is

1) $A\rho g\left[\frac{(h_1-h_2)}{2}\right]$ 2) $A\rho g\left[\frac{(h_1+h_2)}{2}\right]^2$ 3) $A\rho g\left[\frac{(h_1-h_2)}{4}\right]$ 4) $A\rho g\left[\frac{(h_1-h_2)}{2}\right]^2$

12. The rate of doing work by force acting on a particle of mass m moving along x-axis depends on position x of particle and is equal to $2x$. The velocity of particle is given by expression

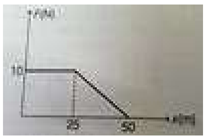
1) $\left[\frac{3x^2}{m}\right]^{1/3}$ 2) $\left[\frac{3x^2}{2m}\right]^{1/3}$ 3) $\left[\frac{2mx}{9}\right]^{1/3}$ 4) $\left[\frac{mx^2}{3}\right]^{1/2}$

13. A block of mass m moving with velocity u_0 on a smooth horizontal surface hits the spring of constant k as shown. The maximum compression in spring is



1) $\sqrt{\frac{2m}{k}}.\upsilon_0$ 2) $\sqrt{\frac{m}{k}}.\upsilon_0$ 3) $\sqrt{\frac{m}{2k}}.\upsilon_0$ 4) $\frac{m}{2k}.\upsilon_0$

14. An object of mass 5 kg is acted upon by a force that varies with position of the object as shown. If the object starts from rest at a point $x = 0$. What is its speed at $x = 50m$?

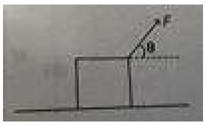


1) 12.2 ms^{-1} 2) 182 ms^{-1} 3) 16.4 ms^{-1} 4) 20.4 ms^{-1}

15. A force $\left(F = \frac{-ky}{\sqrt{x^2+y^2}}\hat{i} + \frac{kx}{\sqrt{x^2+y^2}}\hat{j}\right)N$ is applied on a block, whose position is $P(x, y)$. The block moves on a circular path of radius R. Find the work done by the force F in a complete rotation.

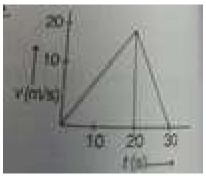
1) $2\pi Rk$ 2) πRk 3) $4\pi Rk$ 4) $Zero$

16. A block of mass m is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. If the block travels with a uniform velocity and has a displacement d and the coefficient of friction is μ, then the work done by the applied force is



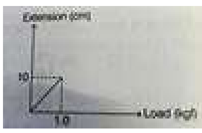
1) $\frac{\mu mgd}{\cos\theta + \mu\sin\theta}$ 2) $\frac{\mu mgd\cos\theta}{\cos\theta + \mu\sin\theta}$ 3) $\frac{\mu mgd\sin\theta}{\cos\theta + \mu\sin\theta}$ 4) $\frac{\mu mgd\cos\theta}{\cos\theta - \mu\sin\theta}$

17. $\upsilon - t$ graph of an object of mass 1 kg is shown. Select the wrong statement.



1) Work done on the object in 30 s is zero
2) The average acceleration of the object is zero
3) The average velocity of the object is zero
4) The average force on the object is zero

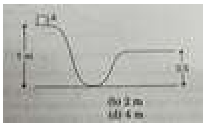
18. The pointer reading versus load graph for a spring balance is as shown



The spring constant is

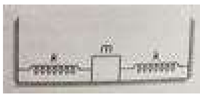
1) $\frac{15\,kgf}{cm}$ 2) $\frac{5\,kgf}{cm}$ 3) $\frac{0.1\,kgf}{cm}$ 4) $\frac{10\,kgf}{cm}$

19. The figures shows a particle sliding on a frictionless track, which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?



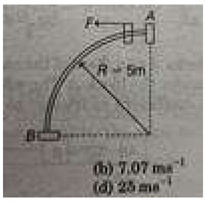
1) $1\ m$ 2) $2\ m$ 3) $3\ m$ 4) $4\ m$

20. A block of mass m is attached to two unstretched springs of spring constants k each as shown. The block is displaced towards right through a distance x and is released. The speed of the block as it passes through the mean position will be



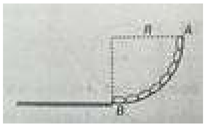
1) $x\sqrt{\frac{m}{2k}}$ 2) $x\sqrt{\frac{2k}{m}}$ 3) $x\frac{m}{k}$ 4) $x\frac{2k}{m}$

21. A bead of mass $\frac{1}{2}\ kg$ starts from rest from A to move in a vertical plane along a smooth fixed quarter ring of radius 5m, under the action of a constant horizontal force $F = 5N$ as shown. The speed of bead as it reaches the point B is (Take, $g = 10\ ms^{-2}$)



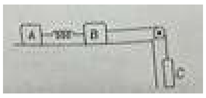
1) 14.14 ms^{-1} 2) 7.07 ms^{-1} 3) 4 ms^{-1} 4) 25 ms^{-1}

22. A smooth chain AB of mass m rests against a surface in the form of a quarter of a circle of radius R. If it is released from rest, the velocity of the chain after it comes over the horizontal part of the surface is



1) $\sqrt{2gR}$ 2) $\sqrt{gR}$ 3) $\sqrt{2gR\left(1-\frac{2}{\pi}\right)}$ B 4) $\sqrt{2gR(2-\pi)}$

23. In the diagram shown, the blocks A and B are of the same mass M and the mass of the block C is M_1. Friction is present only under the block A. The whole system is suddenly released from the state of rest. The minimum coefficient of friction to keep the block A in the state of rest is equal to



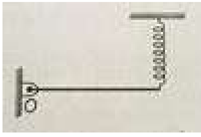
1) $\frac{M_1}{M}$ 2) $\frac{2M_1}{M}$ 3) $\frac{M_1}{2M}$ 4) None of these

24. The potential energy ϕ in joule of a particle of mass 1 kg moving in $x-y$ plane obeys the law, $\phi = 3x + 4y$. Here, x and y are in metres. If the particle is at rest at $(6m, 8m)$ at time 0, then the work

done by conservative force on the particle from the initial position to the instant when it crosses the x-axis is

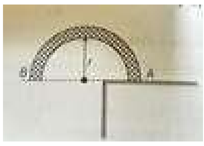
1) $25J$ 2) $-25J$ 3) $50J$ 4) $-50J$

25. A rod of mass M hinged at O is kept in equilibrium with a spring of stiffness k as shown in figure. The potential energy stored in the spring is



1) $\frac{(mg)^2}{4k}$ 2) $\frac{(mg)^2}{2k}$ 3) $\frac{(mg)^2}{8k}$ 4) $\frac{(mg)^2}{k}$

26. A uniform chain of length πr lies inside a smooth semicircular tube AB of radius r. Assuming a slight disturbance to start the chain in motion, the velocity with which it will emerge from the end **of the tube will be**



1) $\sqrt{gr\left(1+\frac{2}{\pi}\right)}$ 2) $\sqrt{2gr\left(\frac{2}{\pi}+\frac{\pi}{2}\right)}$ 3) $\sqrt{gr(\pi+2)}$ 4) $\sqrt{\pi gr}$

27. When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is

1) $aL^2 + bL^3$ 2) $\frac{1}{2}\left(aL^2 + bL^3\right)$ 3) $\frac{aL^2}{2} + \frac{bL^3}{3}$ 3) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

28. The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)J$

The total mechanical energy of the particle is $2J$. Then , the maximum speed (in m/s) is

1) $\sqrt{2}$ 2) $1/\sqrt{2}$ 3) 2 4) $3/\sqrt{2}$

29. A body is displaced from $x = 4$ m to $x = 2$ m along the x-axis. For the forces mentioned in Column I, match the corresponding work done is Column II.

Column I	Column II
(a) $F = 4\hat{i}$	(p) positive
(b) $F = \left(4\hat{i} - 4\hat{j}\right)$	(q) negative
© $F = -4\hat{i}$	(r) zero
(d) $F = \left(-4\hat{i} - 4\hat{j}\right)$	(s) $\lvert W \rvert = 8$ units

30. The kinetic energy of a projectile at its highest position is K. If the range of the projectile is four times the height of the projectile, then the initial kinetic energy of the projectile is

1) $\sqrt{2K}$ 2) $2K$ 3) $4K$ 4) $2\sqrt{2K}$

KEY

1-10	1	2	1	4	2	1	4	1	3	4
11-20	2	1	2	1	1	2	1	3	1	2
21-30	1	3	2	3	3	2	3	4	Qprs	1

PHYSICS HINTS AND SOLUTIONS

1. $\in (Moments\ about\ C) = O$

$\left(K_1 x\right) AC = \left(K_2 x\right) Bc$

$\frac{AC}{BC} = \frac{K_2}{K_1}$

$AC + BC = \ell$

Solving these two equations

We get $AC = \left(\frac{k_2}{k_1 + k_2}\right)\ell$

2. $P = \frac{W}{t} = \frac{mgh + \frac{1}{2}mv^2}{t}$

$= \frac{(800\times10\times10) + \frac{1}{2}\times800\times(20)^2}{60}$

$= 4000\ W$

3. $K_i + U_i = K_f + U_f$

$\therefore\ 0 + 6 = \frac{1}{2}\times1\times v^2 + 2$

$\therefore\quad v = 2\sqrt{2}\ m/s$

4. $a = \frac{F}{m}$

$v = at = \left(\frac{F}{m}\right)(2)$

$P = F.v = \frac{2F^2}{m}$

5. $a = \frac{F}{m}$

$v = at = \frac{F}{m}t$

$P = F.v = \left(\frac{F^2}{m}\right)t$

Or $\ P\ \alpha\ t$

i.e. $P-t$ graph is a straight line passing through origin.

6. Upthrust = (Volume immersed) (density of liquid)g

$$=\left(\frac{5}{3000}\right)(1000)(10)$$

$$=\frac{50}{3}N$$

Weight $=50N$

$\therefore$ Applied force (upwards) = weight – upthrust

$$=50-\frac{50}{3}=\frac{100}{3}N$$

$$W=Fs=\frac{100}{3}\times 3$$

$$=100J$$

7. $Let\theta_1$ is the angle of F with positive x-axis.

$$\therefore \quad \tan\theta_1=\frac{F_y}{F_x}=\frac{15}{20}=\frac{3}{4}=m_1$$

Slope of given line, $m_2=-\frac{\alpha}{3}$

$W=0$ *if* $F\perp s$ *or* $m_1\ m_2=-1$

$$\therefore \quad \left(\frac{3}{4}\right)\left(\frac{\alpha}{3}\right)=-1$$

$$\therefore \quad \alpha=4$$

8. Decrease in potential energy = Work done against friction

$$\therefore \quad mg(h+d)=F.d$$

Here, F = average resistance

$$\Rightarrow \quad F=mg\left(1+\frac{h}{d}\right)$$

9. Speed (and hence the kinetic energy) will increase as long as $mg\ \sin\theta>kx$.

10. $(a)\ W_N=NS\ \cos 90^0=0$

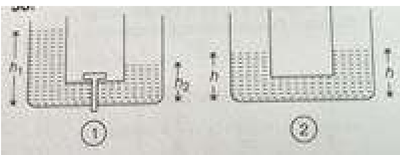
$(b)\ W_T\ =TS\ \cos 0^0=(mg\sin\theta)\left(\frac{h}{\sin\theta}\right)=mgh$

$(c)\ W_{mg}=(mg)(h)\ \cos 180^0=-mgh$

$(d) W_{Total}=\Delta K=K_f-K_i=0$

As block is moved slowly or $K_f=K_i$

11.



Since, the volume remains same.

$Ah_1+Ah_2=Ah+Ah$

$$\Rightarrow \qquad h=\frac{h_1+h_2}{2}$$

$$U_1=\frac{\left[\left(\rho A h_1\right) g h_1+\left(\rho A h_2\right) g h_2\right]}{2}$$

$$U_2=\frac{\left(\rho A h_2\right) g h_2}{2}\times 2$$

$$W_{gr}=-\left(U_2-U_1\right)=U_1-U_2$$

$$=\frac{\rho A g}{2}\left[\left(h^2{}_1+h_2^2\right)-2h^2\right]$$

$$=\frac{\rho A g}{2}\left[\left(h_1^2+h_2^2\right)-2\left(\frac{h_1+h_2}{2}\right)^2\right]$$

$$=\rho A g\left[\frac{h_1+h_2}{2}\right]^2 \qquad =$$

12. $P=2x=Fv=mav=m\left(v\frac{dv}{dx}\right)v$

$$\therefore \qquad v^2 dv=\frac{1}{m}2x\,dx$$

Integrating, we get

$$\therefore \qquad v=\left(\frac{3x^2}{m}\right)^{\frac{1}{3}}$$

13. $E_i=E_f$

$$\frac{1}{2}mv_0^2=\frac{1}{2}kx_{maz}^2 \Rightarrow x_{\max}=\sqrt{\frac{m}{k}}v_0$$

14. Change in kinetic energy = work done

=Area under $F-x$

$$\therefore \frac{1}{2}\times 5\times v^2=10\times 25+\frac{1}{2}\times 25\times 10=375$$

$$\therefore \qquad v=1224\ ms^{-1}$$

15. $\tan\theta=\frac{F_y}{F_x}=-\frac{x}{y}$

Or $\quad \frac{dy}{dx}=-\frac{x}{y}\ or\ \int y dx=-\int x\ dx$

Or $\quad x^2+y^2=c$ is a circular curve.

Thus, the force is along tangent to the path of the block. The magnitude of the force is

$$F=\sqrt{F_x^2+F_y^2}=k$$

$$\therefore \qquad W=\int F\ ds=k\int ds$$

$$=k2\pi R=2\pi Rk$$

16. $N = mg - \sin\theta$

Block moves with uniform velocity. Hence, net force = 0

Or $F\cos\theta = \mu N = \mu\left(mg - F\sin\theta\right)$

$\therefore \quad F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$

$$W = Fd\ \cos\theta = \frac{\mu\, mgd\cos\theta}{\cos\theta + \mu\sin\theta}$$

17. Average velocity $= \dfrac{Total\ displacement}{Time}$

$$= \frac{Area\ under\ v-t\ graph}{Time}$$

Since, area $\neq 0$

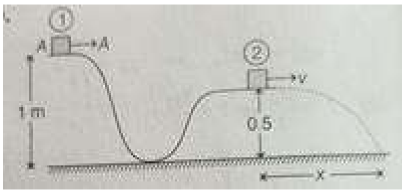
$\therefore$ Average velocity $\neq 0$

18. $F = kx\ or\ k =$ slope of $F - x$ graph (F along y-axis)

Here, F is along x-axis.

So, $k = \dfrac{10}{10} = 0.1\dfrac{kgf}{cm}$

19.



$mg\left(1\right) = \dfrac{1}{2}mv^2 + mg\left(0.5\right)$

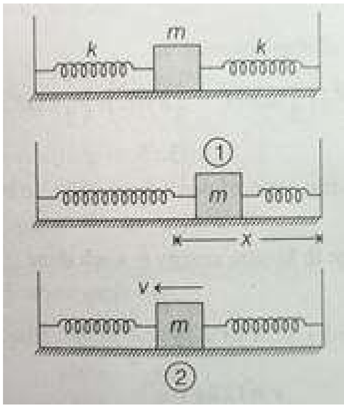
$v^2 = 2g\left(1 - 0.5\right) = g = 10$

$\Rightarrow \quad v = \sqrt{10}\, m/s$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 0.5}{10}} = \sqrt{\frac{1}{10}}$$

$Now, x = vt = 1m$

20.



$\dfrac{1}{2}kx^2 + \dfrac{1}{2}kx^2 = \dfrac{1}{2}mv^2$

$$kx^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2k}{m}}x$$

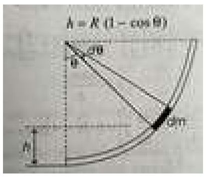
21. $$W_{All} = \frac{1}{2}mv^2$$

$$\therefore \quad W_F + W_{mg} + W_N = \frac{1}{2}mv^2$$

$$\therefore \quad (5\times 5) + \left(\frac{1}{2}\times 10\times 5\right) + 0 = \frac{1}{2}\times\frac{1}{2}\times v^2$$

$$\therefore \quad v = 14.14\ m/s$$

22. $$dm = \left(\frac{m}{\pi/2}\right)d\theta = \left(\frac{2m}{\pi}\right)d\theta$$



$$dU_i = (dm)gh = \frac{2mgR}{\pi}(I - \cos\theta)d\theta$$

$$\therefore \quad U_i = \int_0^{\pi/2} dU_i = \frac{2mgR}{\pi}\left(\frac{\pi}{2} - 1\right)$$

$$= mgR\left(1 - \frac{2}{\pi}\right)$$

$$Now, \quad U_i + K_i = U_f + K_f$$

$$\therefore \quad mgR\left(1 - \frac{2}{\pi}\right) = 0 + \frac{1}{2}mv^2$$

$$or \qquad v = \sqrt{2gR\left(1 - \frac{2}{\pi}\right)}$$

23. Let X_m is maximum elongation of spring. Then, increase in potential energy of spring = decrease in potential energy of C

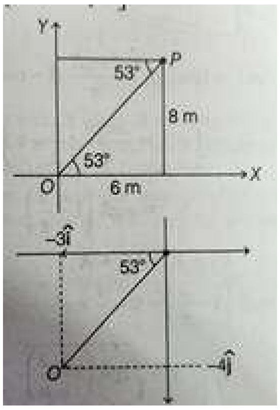
$$\therefore \quad \frac{1}{2}KX_m^2 = M_1 g X_m$$

$$or \quad KX_m = \text{ maxmimum spring force}$$

$$= 2M_{1g} = \mu_{\min} Mg$$

$$\therefore \quad \mu_{\min} = \frac{2M_1}{M}$$

24. $$F = -\left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j}\right] = \left(-3\hat{i} - 4\hat{j}\right)$$



Since, particle was initially at rest. So, it will move in the direction of force.
We can see that initial velocity is in the direction of PO. So, the particle will cross the X -axis at origin.

$$K_i + U_i = K_f + U_f$$

$$\therefore 0 + (3\times 6 + 4\times 8) = K_f + (3\times 0 + 4\times 0)$$

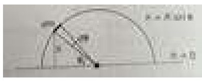
$$or \qquad K_f = 50J$$

25. $\sum(Moment\ about\ O) = 0$

$$\therefore \qquad (kx)l = mg\left(\frac{1}{2}\right) or\ x = \frac{mg}{2k}$$

$$U = \frac{1}{2}kx^2 = \frac{(mg)^2}{8k}$$

26. $dm = \left(\frac{m}{\pi}\right)d\theta$



$$\therefore \qquad dU = (dm)gh = \left(\frac{m}{\pi}d\theta\right)gr\ \sin\theta$$

$$U_i = \int_0^{\pi} dU = \frac{2mgr}{\pi}$$

$$Now, \qquad K_i + U_i = K_f + U_f$$

$$\therefore \qquad 0 + \frac{2mgr}{\pi} = \frac{1}{2}mv^2 - mg\left(\frac{\pi r}{2}\right)$$

$$\therefore \qquad v = \sqrt{2gr\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$$

27. We know that, change in potential energy of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^f F.dr$$

$$Given, \qquad F = ax + bx^2$$

We know that, work done in stretching the rubber band by L is $|dW| = |Fdx|$

$$|W| = \int_0^L \left(ax + bx^2\right) dx$$

$$= \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$$

$$= \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2}\right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3}\right]$$

$$= |W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

28. $V(x) = \frac{x^4}{4} - \frac{x^2}{2}$

$$F = -\frac{dV_{(x)}}{dx} = -\left[x^3 - x\right] = 0$$

$$\Rightarrow x\left(x^2 - 1\right) = 0$$

$$x = 0, x = \pm 1$$

$$\Rightarrow \frac{d^2V_{(x)}}{dx} = 3x^2 1$$

$$At\ x = \pm 1, \frac{d^2V_{(x)}}{dx} = +ve, i.e.$$

$at\ x = \pm 1. PE$ is minimum

$$V_{\min} = -\frac{1}{4}$$

$$E = K_{\max} + V_{\min}$$

$$\Rightarrow \quad 2 = \frac{1}{2} \times 1 \times v_{\max}^2 - \frac{1}{4}$$

$$\Rightarrow \quad V_{\max} = \frac{3}{\sqrt{2}} m/s$$

29. B,D,C,A

30. At highest point

$$\frac{1}{2} m u_x^2 = K$$

$$\therefore \quad u_x = \sqrt{\frac{2K}{m}}$$

$$R = 4H$$

$$\frac{2u_x u_y}{g} = \frac{4u_y^2}{2g}$$

$$\therefore \quad u_y = u_x = \sqrt{\frac{2K}{m}}$$

$$\begin{aligned} Now, \quad K_i &= \frac{1}{2}mu^2 \\ &= \frac{1}{2}m\left(u_x^2 + u_y^2\right) \\ &= \frac{1}{2}m\left(\frac{2K}{m} + \frac{2K}{m}\right) \\ &= 2K \end{aligned}$$