NARAYANA
EDUCATIONAL INSTITUTIONS

TOPIC: COMPLEX NUMBERS

MATHEMATICS MAX. MARKS: 100

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

61. If $Z+2|Z|=\pi+4i$ *then* $\text{Im}(Z)=$

1) $\sqrt{\pi^2+16}$ 2) π 3)4 4) $\sqrt{\pi^2-16}$

62. If ω is a cube root of unity and $(1+\omega^2)^{11}=a+b\omega+c\omega^2$ then (a,b,c) equals

1)(1,1,1) 2)(1,1,0) 3)(1,0,1) 4)(0,1,1)

63. If $x^2+y^2=1$ *and* $x\neq-1,$ *then* $\dfrac{1+y+ix}{1+y-ix}=$

1)1 2)2 3) $x+iy$ 4) $y+ix$

64. If $Z\in C$ *and* $2Z=|Z|+i$ *then* $Z=$

1) $\dfrac{\sqrt{3}}{6}+\dfrac{1}{2}i$ 2) $\dfrac{\sqrt{3}}{6}+\dfrac{1}{3}i$ 3) $\dfrac{\sqrt{3}}{6}+\dfrac{1}{4}i$ 4) $\dfrac{\sqrt{3}}{6}+\dfrac{1}{6}i$

65. If ω is a complex cube root of unity and $(1+\omega^4)^n=(1+\omega^8)^n$ then least positive integral value of n=

1) 2 2) 3 3) 4 4) 6

66. If $|Z-1|=|Z+1|=|Z-2i|$ then the value of $|Z|$ is

1) 1 2) 2 3) 5/4 4) ¾

67. If $|\omega|=2$ then the set of points $x+iy=\omega-\dfrac{1}{\omega}$ lie on

1)circle 2)parabola 3)hyperbola 4)ellipse

68. An equation of straight line joining the complex numbers a and ib (where a,b$\in R$ *and* $a,b\neq 0$)is

1) $Z\left(\dfrac{1}{a}-\dfrac{i}{b}\right)+\overline{Z}\left(\dfrac{1}{a}+\dfrac{i}{b}\right)=2$ 2) $Z(a+ib)=2ab$

3) $Z(a-ib)+\overline{Z}(a+ib)=2(a^2+b^2)$ 4) $Z(a-ib)=ab$

69. Suppose Z_1,Z_2,Z_3 are three complex numbers and $\Delta=\dfrac{1}{-4i}\begin{vmatrix}1 & \overline{Z_1} & Z_1\\ 1 & \overline{Z_2} & Z_2\\ 1 & \overline{Z_3} & Z_3\end{vmatrix}$ then

1) $|\text{Re}(\Delta)|=0$ 2) $\text{Im}(\Delta)=0$ 3) $|\text{Re}(\Delta)|\geq 0$ 4) $\text{Im}(\Delta)\leq 0$

70. If Z=x+iy and $0\leq\sin^{-1}\left(\dfrac{Z-4}{2i}\right)\leq\dfrac{\pi}{2}$ then

1) $x=4,0\leq y\leq 2$ 2) $0\leq x\leq y,0\leq y\leq 2$ 3) $x=0,0\leq y\leq 2$ 4) $x=9,0\leq y\leq 2$

71. Principle argument of $Z=\dfrac{i-1}{i(1-\cos\frac{2\pi}{7})+\sin\frac{2\pi}{7}}$ is

1) $\dfrac{\pi}{8}$ 2) $\dfrac{2\pi}{28}$ 3) $\dfrac{13\pi}{28}$ 4) $\dfrac{17\pi}{28}$

72. $a>0, and\ Z|Z|+aZ+3i=0$ then Z is

1) 0 2) a positive real number 3) a negative real number 4) Purely imaginary

73. If $\Delta=\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}=x+iy$ then

1) $x=3, y=1$ 2) $x=1, y=3$ 3) $x=0, y=3$ 4) $x=0, y=0$

74. The in equality $|Z-i|<|Z+i|$ represent the region

1) $\operatorname{Re}(Z)>0$ 2) $\operatorname{Re}(Z)<0$ 3) $\operatorname{Im}(Z)>0$ 4) $\operatorname{Im}(Z)<0$

75. Suppose $a<0\ and\ Z_1, Z_2, Z_3, Z_4$ be the fourth roots of a. Then $Z_1^{\ 2}+Z_2^{\ 2}+Z_3^{\ 2}+Z_4^{\ 2}=$

1) $a+|a|$ 2) $|a|-a$ 3) $-a^2$ 4) a^2

76. If α, β are the roots of $x^2+px+q=0$ and ω is a cube roots of unity then $(\omega\alpha+\omega^2\beta)(\omega^2\alpha+\omega\beta)=$

1) p^2 2) 3q 3) p^2-3q 4) p^2-4q

77. If $8iz^3+12z^2-18z+27i=0$ then

1) $|z|=\frac{3}{2}$ 2) $|z|=\frac{2}{3}$ 3) $|z|=1$ 4) $|z|=\frac{3}{4}$

78. If $Z_K=Cos\left(\frac{1<k\pi}{10}\right)+i\sin\left(\frac{k\pi}{10}\right) then\ Z_1Z_2Z_3Z_4=$

1)-1 2)2 3)-2 4)1

79. If Z is purely imaginary and $\operatorname{Im}(Z)<0\ then\ \arg\left(i\overline{Z}\right)+\arg(Z)=$

1) π 2) $\frac{\pi}{2}$ 3)0 4) $\frac{-\pi}{2}$

80. If $Z\in C$ the minimum value of $|Z|+|Z-i|$ is attained at

1)exactly one point 2)exactly two points

3)infinite no .of points 4)exactly three points

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 10 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.** **Have to Answer any 5 only out of 10 questions** and question will be evaluated according to the following marking scheme:

Marking scheme: +4 for correct answer, -1 in all other cases.

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81. Suppose Z_1, Z_2, Z_3 are vertices of an equilateral triangle whose circumcentre $-3+4i\ then\ |z_1+z_2+z_3|=$---------------------

82. The number of complex numbers Z such that $(1+i)z=i|Z|$ is ------------

83. Let Z_1, Z_2 be two non-zero complex numbers such that $|Z_1+Z_2|=|Z_1-Z_2|\ then\ \frac{Z_1}{\overline{\overline{Z_1}}}+\frac{Z_2}{\overline{\overline{Z_2}}}=$---------

84. If $\overline{Z}=3\bar{i}+\frac{25}{Z+3i}\ then\ |Z|$ cannot exceed is __________

85. If $x=2+5i$ then the value of $x^3-5x^2+33x-19$ is equal to -----------

86. If $(4+i)(Z+\overline{Z})-(3+i)(Z-\overline{Z})+26i=0$ then the value of $|z|^2$ is-----------

87. If $\omega(\neq 1)$ is a cube root of unity then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}=$

88. If the complex number z lies on the boundary of the circle of radius 5 and centre is at 4 then the greatest value of $|z+1|$ is ------------

89. If $x+iy=\dfrac{3}{\cos\theta+i\sin\theta+2}$ then $4x-x^2-y^2$ reduces to -----

90. If $z=i(1+\sqrt{3})$ then $z^4+2z^3+4z^2+5=$

MATHS

61	62	63	64	65	66	67	68	69	70
3	2	4	1	2	4	4	1	2	1
71	72	73	74	75	76	77	78	79	80
4	4	4	3	1	3	1	1	2	3
81	82	83	84	85	86	87	88	89	90
15	0	0	8	10	17	0	5	3	5

61. $z=(\pi-2|z|)+4i$

$\pi-2|z|$ is real, $\text{Im}(z)=4$

62. $a+b\omega+c\omega^2=(1+\omega^2)^{11}=(-\omega)^{11}=1+\omega$

$\therefore a=1, b=1, c=0$

63. Let $z=y+ix$ then $\bar{z}=y-ix$

Now $x^2+y^2=1 \Rightarrow z\bar{z}=1$

$$\frac{1+y+ix}{1+y-ix}=\frac{z\bar{z}+z}{1+\bar{z}}=\frac{z(\bar{z}+1)}{1+\bar{z}}=z=y+ix$$

64. $|2z|^2=|z^2|+1$

$\Rightarrow 3|z|^2=1 \Rightarrow |z|=\dfrac{1}{\sqrt{3}}$

$\therefore z=\dfrac{\sqrt{3}}{6}+\dfrac{1}{2}i$

65. $(1+\omega)^n=(1+\omega^2)^n$

$\Rightarrow (-\omega^2)^n=(-\omega)^n$

$\Rightarrow \omega^n=1$

$\therefore n=3$

66. Z is the centre of the circle passing through $1+0i$, $-1+0i$ and $0+2i$ clearly centre lies on the y-axis

If $z=0+ai$ is the centre then $\sqrt{1+a^2}=|a+2|$

$\Rightarrow 1+a^2=a^2+4a+4$

$\Rightarrow a=\dfrac{3}{4}$

$\therefore |z|=\dfrac{3}{4}$

67. $|\omega|=2 \Rightarrow \omega=2.e^{i\theta}=2(\cos\theta+i\sin\theta)$

$\Rightarrow x+iy=\omega-\dfrac{1}{\omega}=2.e^{i\theta}-\dfrac{1}{2}e^{-i\theta}$

$\therefore x=\dfrac{3}{2}\cos\theta,\ y=\dfrac{5}{2}\sin\theta$

$$\Rightarrow \frac{x^2}{\left(\frac{-9}{4}\right)}+\frac{y^2}{\left(\frac{25}{4}\right)}=1$$

Which represents an ellipse.

68. An equation of straight line joining a and ib in $\begin{vmatrix} z & \bar{z} & 1 \\ a & a & 1 \\ ib & -ib & 1 \end{vmatrix} = 0$

$\Rightarrow z(a+ib) - \bar{z}(a-ib) - 2iab = 0$

$\Rightarrow z\left(\frac{1}{a} - \frac{i}{b}\right) + \bar{z}\left(\frac{1}{a} + \frac{i}{b}\right) = 2$

69. $\bar{\Delta} = \frac{1}{-4i}\begin{vmatrix} 1 & \bar{z_1} & z_1 \\ 1 & \bar{z_2} & z_2 \\ 1 & \bar{z_3} & z_3 \end{vmatrix} = \Delta$

$\Rightarrow \Delta$ is purely real $\Rightarrow I_m(z) = 0$

70. $\frac{z-4}{2i} = \frac{x-4}{2i} + \frac{y}{2}$

$for\ 0 \le \sin^{-1}\left(\frac{z-4}{2i}\right) \le \frac{\pi}{2}$

$i.e.\ x - 4 = 0,\ 0 \le \frac{y}{2} \le 1$

$\Rightarrow\ x = 4,\ 0 \le y \le 2$

71. $i(1 - \cos\frac{2\pi}{7}) + \sin\frac{2\pi}{7}$

$= 2\sin\frac{\pi}{7}(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7})$

Also $1 - i = \sqrt{2}(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$

$\therefore Z = \frac{\sqrt{2}}{2\sin\frac{\pi}{7}}\left[\cos\frac{17\pi}{28} + i\sin\frac{17\pi}{28}\right]$

$\therefore Arg(Z) = \frac{17\pi}{28}$

72. $Z = \frac{-3i}{|z| + a} \Rightarrow Z\ is\ purely\ imaginary$

73. use $C_2 \to C_2 + 3i.C_3$

74. $|z - i| = |z + i|$ represents the real axis

As z=i satisfies $|z - i| < |z + i|$

$\therefore |z - i| < |z + i|$ represents $I_m(z) > 0$

75. $Let\ a = -b^4\ \ where\ b > 0$

Then $z^4 = a = b^4(-1)$

$\Rightarrow z = b(\pm\cos\frac{\pi}{4} \pm i\sin\frac{\pi}{4})$

$\therefore z_1^2 + z_2^2 + z_3^2 + z_4^2 = 2b^2\left[\cos\frac{\pi}{2} - i\sin\frac{\pi}{2} + \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]$

$= 0$

$= a + |\bar{a}|\ \ (\because a < 0)$

76. $x^2+px+q=0 \Rightarrow \alpha+\beta=-p \;and\; \alpha\beta=q$

$$\left(\omega\alpha+\omega^2\beta\right)(\omega^2\alpha+\omega\beta)=\alpha^2+\beta^2+\left(\omega+\omega^2\right)\alpha\beta$$
$$=\alpha^2+\beta^2-\alpha\beta$$
$$=\left(\alpha+\beta\right)^2-3\alpha\beta$$
$$=p^2-3q$$

77. $8iz^3+12z^2-18z+27i=0$

$\Rightarrow 8iz^3-12i^2z^2-18z-27i=0$

$\Rightarrow 4iz^2(2z-3i)-9(2z-3i)=0$

$\Rightarrow (4iz^2-9)(2z-3i)=0$

$\therefore z^2=\dfrac{9}{4i}\;(or)\;z=\dfrac{3i}{2}$

78. We have $Z^k=\omega^k \; where \; \omega=\cos\dfrac{\pi}{10}+i\sin\dfrac{\pi}{10}$

Thus, $Z_1Z_2Z_3Z_4=\omega\omega^2\omega^3\omega^4=\omega^{10}$

$$\omega^{10}=(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10})^{10}=\cos\pi+i\sin\pi$$
= -1+0
= -1

79. Let $z=-it$ where t>0, then $i\bar{z}=i(i.t)=-t$

$\therefore \arg(i\bar{z})+\arg(z)=\pi-\dfrac{\pi}{2}=\dfrac{\pi}{2}$

80. $1=|i|=|z+(i-z)|\le|z|+|i-z|$

$\Rightarrow |z|+|z-i|\ge 1$

The minimum value 1 is attained at all points $z=it$ where $t\in[0,1]$

81. If a triangle is equilateral then centroid and circumcentre coincides.

$\therefore \dfrac{1}{3}\left(z_1+z_2+z_3\right)=-3+4i$

$\Rightarrow |z_1+z_2+z_3|=3\sqrt{9+16}=15$

82. $|(1+i)z|=|i|z||$

$\Rightarrow \sqrt{2}|z|=|z|$

$\Rightarrow (\sqrt{2}-1)|z|=0$

$\Rightarrow |z|=0$

$\Rightarrow z=0$

83. $|z_1+z_2|^2=|z_1-z_2|^2$

$\Rightarrow |z_1|^2+|z_2|^2+z_1\overline{z_2}+\overline{z_1}z_2=|z_1|^2+|z_2|^2-z_1\overline{z_2}-\overline{z_1}z_2$

$\Rightarrow 2(z_1\overline{z_2}+\overline{z_1}z_2)=0$

$\Rightarrow \dfrac{z_1}{\overline{z_2}}+\dfrac{z_2}{\overline{z_2}}=0$

84. $(\bar{z}-3i)(z+3i)=25$

$|z-3i|^2=25\;(or)\;|z-3i|=5$

Now $|z|=|z-3i+3i|\le|z-3i|+|3i|=5+3=8$

85. x=2+5i $\quad\Rightarrow x-2=5i$

$\quad\quad\Rightarrow x^2-4x+4=-25$

$\Rightarrow x^2 - 4x + 29 = 0$

Now divide $x^3 - 5x^2 + 33x - 19$ *by* $x^2 - 4x + 29$

Then remainder is 10.

86. *Let* $z = x + iy$

Then $2(4+i)x - (3+i)2iy + 26i = 0$

$4x + y = 0, \;\; x - 3y + 13 = 0$

$x = -1, \;\; y = 4$

$\therefore \; |z|^2 = 17$

87. Use $R_2 \to R_2 - R_1 - R_3$

88. As -1 lies on the circle $|z-4| = 5$, the real number $|z+1|$ is maximum when z is the other endpoint of the diameter

89. $\dfrac{1}{x+iy} = \dfrac{\cos\theta + 2}{3} + \dfrac{i\sin\theta}{3}$

$\Rightarrow \; \dfrac{x-iy}{x^2+y^2} = \dfrac{\cos\theta + 2}{3} + \dfrac{i\sin\theta}{3}$

$\therefore \; \dfrac{x}{x^2+y^2} = \dfrac{\cos\theta + 2}{3}$

Also $x^2 + y^2 = \dfrac{9}{5 + 4\cos\theta}$

$$\therefore \; 4x - (x^2+y^2) = \left[\frac{4(\cos\theta+2)}{3} - 1\right](x^2+y^2)$$
$$= 3$$

90. $z = i(1+\sqrt{3}) = -1 + \sqrt{3}i = 2\omega$

$$\therefore \; z^4 + 2z^3 + 4z^2 + 5 = (2\omega)^4 + 2(2\omega)^3 + 4(2\omega)^2 + 5$$
$$= 5$$