NARAYANA EDUCATIONAL INSTITUTIONS

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INTEGRATION-DPP

SECTION-A

1. $\int \frac{\sin x dx}{\sin x - \cos x} =$

1) $\frac{x}{2} + \text{In}(\cos x - \sin x) + c$ 2) $\frac{x}{2} + \frac{1}{2} In(\cos x + \sin x) + c$

3) $\frac{x}{2} - \frac{1}{2} In(\cos x + \sin x) + c$ 4) $\frac{x}{2} + \frac{1}{2} In(\sin x - \cos x) + c$

2. $\int \frac{xdx}{(x^2+1)(x^2+3)} =$

1) $\frac{1}{2} In\left(\frac{x^2+1}{x^2+3}\right) + c$ 2) $\frac{1}{2} In\left(\frac{x^2+3}{x^2+1}\right) + c$

3) $\frac{1}{4} In\left(\frac{x^2+1}{x^2+3}\right) + c$ 4) $\frac{1}{4} In\left(\frac{x^2+3}{x^2+1}\right) + c$

3. $\int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^2 x}} dx =$

1) $\frac{1}{3} \cos^{-1}\left(\cos^{\frac{3}{2}} x\right) + c$ 2) $\frac{1}{3} \sin^{-1}\left(\sin^{\frac{3}{2}} x\right) + c$

3) $\frac{2}{3} \sin^{-1}\left(\cos^{\frac{3}{2}} x\right) + c$ 4) $-2/3 \sin^{-1}\left(\cos^{3/2} x\right) + c$

4. $\int \frac{e^{In\left(1+\frac{1}{x^3}\right)}}{x^2 + \frac{1}{x^2}} dx =$

1) $\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$ 2) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$

3) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{2x}\right) + c$ 4) $\frac{1}{2} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$

5. $\int \frac{x + \sin x}{1 + \cos x} dx =$

1) $x \sin \frac{x}{2} + c$ 2) $x \cos \frac{x}{2} + c$

3) $x \tan \frac{x}{2} + c$ 4) $x \cot \frac{x}{2} + c$

6. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx =$

1) $\sin 2x + c$ 2) $-\frac{1}{2} \sin 2x + c$ 3) $\frac{1}{2} \sin 2x + c$ 4) $-\sin 2x + c$

7. If $\int \frac{x^2}{\sqrt{1-x}} dx = A(1-x)^{\frac{1}{2}} + B(1-x)^{\frac{3}{2}} + C(1-x)^{\frac{5}{2}} + D$ Then A+B+C=

1) $-\frac{8}{15}$ 2) $-\frac{16}{15}$ 3) $-\frac{14}{15}$ 4) $\frac{1}{15}$

8. $\int \frac{dx}{\cos^2 x + \sqrt{2}\sin 2x} =$

1) $\sqrt{\tan x} - \frac{(\tan x)^{\frac{3}{2}}}{3} + c$ 2) $\sqrt{\tan x} + \frac{(\tan x)^{\frac{3}{2}}}{3} + c$

3) $\sqrt{\tan x} - \frac{(\tan x)^{\frac{5}{2}}}{5} + c$ 4) $\sqrt{\tan x} + \frac{(\tan x)^{\frac{5}{2}}}{5} + c$

9. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}} = Ax\cos^{-1}x + B\sqrt{1-x^2} + c$, then A+B=

1) $\frac{1}{2}$ 2) 0 3) $-\frac{1}{2}$ 4) -1

10. $\int \frac{dx}{\sqrt{1+\cos ec^2 x}} dx =$

1) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + c$ 2) $\sin^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + c$ 3) $\cos^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + c$ 4) $\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + c$

11. $\int \sqrt{1+\cos ec\ x dx} =$

1) $2\sin^{-1}\sqrt{\cos x} + c$ 2) $2\cos^{-1}\sqrt{\sin x} + c$ 3) $2\sin^{-1}\sqrt{\sin x} + c$ 4) $2\cos^{-1}\sqrt{\cos x} + c$

12. $\int \sqrt{1+\sin 2x}\, dx =$

1) $\sin x + \cos x + c \forall x \in R$ 2) $\sin x - \cos x + c \forall x \in R$

3) $\sin x - \cos x + c, x \in \left[\frac{-\pi}{4}, \frac{3\pi}{4}\right]$ 4) $\cos x - \sin x + c, x \in \left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$

13. $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}} =$

1) $\frac{(x^4+1)^{\frac{1}{4}}}{x} + c$ 2) $-\frac{(x^4+1)^{\frac{1}{4}}}{x} + c$ 3) $\frac{\sqrt{(x^4+1)}}{x} + c$ 4) $-\frac{\sqrt{(x^4+1)}}{x} + c$

14. $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx =$

1) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + c$ 2) $\sqrt{2}\sin^{-1}(\sin x + \cos x) + c$

3) $\sin^{-1}(\sin x - \cos x) + c$ 4) $\sin^{-1}(\sin x + \cos x) + c$

15. $\int \frac{\cos^3 + \cos^5 x}{\sin^2 x + \sin^4 x} dx =$

1) $\sin x - 6\tan^{-1}\sin x + c$ 2) $\sin x - 2\cos ecx + c$

3) $\sin x - 2\cos ecx - 6\tan^{-1} x + c$ 4) $\sin x - 2\cos ecx + 5\tan^{-1}\sin x + c$

16. If $\int \frac{\sin x dx}{\sin(x-\alpha)} = Ax + B\ln\ \sin(x-\alpha) + c$, then (A,B)=

1) $(\sin\alpha, \cos\alpha)$ 2) $(\cos\alpha, \sin\alpha)$ 3) $(-\sin\alpha, \cos\alpha)$ 4) $(-\cos\alpha, \sin\alpha)$

17. $\int \frac{(x^2-1)dx}{x^2\sqrt{2x^4-2x^2+1}} =$

1) $\frac{\sqrt{2x^4-2x^2+1}}{x^2} + c$ 2) $\frac{\sqrt{2x^4-2x^2+1}}{x^3} + c$

3) $\frac{\sqrt{2x^4-2x^2+1}}{x} + c$ 4) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2} + c$

18. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx = A\sqrt{1-x} + B\sin^{-1}\sqrt{x} + C\sqrt{x-x^2} + D$, where A+B+C=

1) -1 2) -2 3) -3 4) -4

19. $\int \frac{dx}{(\sin^3 x \cos^5 x)^{\frac{1}{4}}} =$

1) $2\tan^{\frac{1}{4}} x + c$ 2) $2\cot^{\frac{1}{4}} x + c$ 3) $4\tan^{\frac{1}{4}} x + c$ 4) $4\cot^{\frac{1}{4}} x + c$

20. $\int \frac{x^2+1}{x^4+1} dx =$

1) $\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + c$ 2) $\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$

3) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + c$ 4) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$

21. If $\int \frac{\cos 9x + \cos 6x}{2\cos 5x - 1} dx = A\sin 4x + B\sin x + c, then\ A + B =$

1) $\frac{1}{2}$ 2) $\frac{3}{4}$ 3) $\frac{5}{4}$ 4) $\frac{7}{4}$

22. $\int \frac{\cos 5x + 5\cos 3x + 10\cos x}{\cos 6x + 6\cos 4x + 15\cos^2 x + 10} dx =$

1) In $\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + c$ 2) In $\tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$ 3) $\frac{1}{2} In \tan\left(\frac{x}{2} - \frac{\pi}{4}\right) + c$ 4) $\frac{1}{2} In \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$

23. $\int \frac{dx}{(x\tan x + 1)^2} =$

1) $\frac{\tan x}{x\tan x + 1} + c$ 2) $\frac{\cot x}{x\tan x + 1} + c$ 3) $\frac{-\tan x}{x\tan x + 1} + c$ 4) $\frac{-1}{x\tan x + 1} + c$

24. $\int \sqrt{2 + \tan^2 x} dx = In(\tan x + \sqrt{2 + \tan^2 x}) + f(x) + c$, where $f(x) =$

1) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)$ 2) $\cos^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)$ 3) $\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)$ 4) $\sin^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)$

25. $\int \sin^{-1} \frac{2x+2}{\sqrt{4x^2+8x+13}} dx = (Ax+b)\tan^{-1}\left(\frac{2x+2}{3}\right) + c\, In(4x^2+8x+13) + D$,then A+B+C=

1) $\frac{5}{2}$ 2) $\frac{5}{3}$ 3) $\frac{5}{4}$ 4) 2

26. $\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx =$

1) $\frac{1}{6m}(2x^{3m} + 3x^{2m} + 6x^m)^{1/m} + c$ 2) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{m+1/m} + c$

3) $\frac{1}{6(m)}(2x^{3m} + 3x^{2m} + 6x^m)^{m+1/m} + c$ 4) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{1/m} + c$

27. $\int \frac{dx}{x\sqrt{x^2+2x-1}} =$

1) $\cos^{-1}\left(\frac{1-x}{\sqrt{2}x}\right) + c$ 2) $\cos^{-1}\left(\frac{1+x}{\sqrt{2}x}\right) + c$ 3) $\sin^{-1}\left(\frac{1-x}{\sqrt{2}x}\right) + c$ 4) $\sin^{-1}\left(\frac{1+x}{\sqrt{2}x}\right) + c$

28. Let $f(x) = \cos^{-1}\frac{\cos x}{\cos\alpha}$ and $g(x) = \cosh^{-1}\left(\frac{\sin x}{\sin\alpha}\right) \int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} dx =$

1) $f(x)\cos\alpha + g(x)\sin\alpha + c$ 2) $f(x)\sin\alpha + g(x)\sin\alpha + c$

3) $f(x)\cos\alpha - g(x)\sin\alpha + c$ 4) $f(x)\sin\alpha - g(x)\cos\alpha + c$

29. $\int \frac{\left(\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x\right)dx}{\sqrt{\sin^3 x \cos^3 x \sin(x-\alpha)}} = a\sqrt{\cos\alpha \tan x - \sin\alpha} + b\sqrt{\cos\alpha - \sin\alpha \cot x}$ then ab =

1) $4\cos ec 2\alpha$ 2) $8\cos ec 2\alpha$ 3) $2 s ec\alpha$ 4) $4 s ec\alpha$

30. $\int \frac{dx}{(4+3x^2)\sqrt{3-4x^2}} =$

1) $\frac{1}{5}\tan^{-1}\frac{2x}{\sqrt{3-4x^2}}+c$ 2) $\frac{1}{10}\tan^{-1}\frac{5x}{2\sqrt{3-4x^2}}+c$

3) $\frac{1}{5}\tan^{-1}\frac{5x}{2\sqrt{3-4x^2}}+c$ 4) $\frac{1}{10}\tan^{-1}\frac{5x}{\sqrt{3-4x^2}}+c$

SECTION-A - (KEY)
MATHS

1-10	D	C	D	B	C	B	B	D	B	D
11-20	**C**	**C,D**	**B**	**A**	**C**	**B**	**D**	**B**	**C**	D
20-30	C	D	A	A	C	B	A	C	B	B

SOLUTIONS

1. $\int \frac{\sin x dx}{\sin x - \cos x} = \frac{1}{2}\int \frac{\sin x - \cos x + \cos x + \sin x}{\sin x - \cos x}dx$

$$= \frac{1}{2}\int\left(1+\frac{\cos x + \sin x}{\sin x - \cos x}\right)dx$$

$$=\frac{1}{2}(x + In(\sin x - \cos x) + c$$

2. $\int \frac{xdx}{(x^2+1)\left(x^2+3\right)} = \int \frac{x}{2}\left(\frac{1}{x^2+1}-\frac{1}{x^2+3}\right)x$

$$=\frac{1}{4}\int\left(\frac{2x}{x^2+1}-\frac{2x}{x^2+3}\right)dx$$

$$=\frac{1}{4}\left[In(x^2+1) - In(x^2+3)\right]+c$$

$$=\frac{1}{4}In\left(\frac{x^2+1}{x^2+3}\right)c$$

3. $\int\sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}} = \int \frac{\sqrt{\cos x}.\sin x dx}{\sqrt{1-\cos^3 x}}, t = \cos^{\frac{3}{2}} x$

$$=-\frac{2}{3}\int\frac{dt}{\sqrt{1-t^2}} = -\frac{2}{3}\sin^{-1} t + c$$

$$=-\frac{2}{3}\sin^{-1}\left(\cos^{\frac{3}{2}} x\right)+c$$

4. $$\int \frac{e^{ln\left(1+\frac{1}{x^2}\right)}}{x^2+\frac{1}{x^2}}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}, t = x-\frac{1}{x}$$

$$=\int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}+c$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)+c$$

5. $$\int \frac{x+\sin x}{1+\cos x}dx = \int\left(\frac{x}{1+\cos x}+\frac{\sin x}{1+\cos x}\right)dx$$

$$=\left(\int \frac{x}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right)dx$$

$$=\int \frac{d}{dx}\left(x\tan\frac{x}{2}\right)dx = x\tan\frac{x}{2}+c$$

6. $$\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$$

$$=\int \frac{(\sin^4 x-\cos^4 x)(\sin^4 x+\cos^4 x)dx}{(\sin^2 x+\cos^2 x)-2\sin^2 x-\cos^2 x}$$

$$=\int (\sin^4 x-\cos^4 x)dx = \int (\sin^2 x-\cos^2 x)dx$$

$$=\int -\cos\ 2xdx = -\frac{1}{2}\sin 2x+c$$

7. $$\int \frac{x^2}{\sqrt{1-x}}dx = -2\int\left(1-2t^2-t^4\right)dt, t^2\ =1-x$$

$$=-2\left(t-\frac{2}{3}t^3+\frac{t^5}{5}\right)+c$$

$$= -2(1-x)^{\frac{1}{2}}+\frac{4}{3}(1-x)^{\frac{3}{2}}-\frac{2}{5}(1-x)^{\frac{5}{2}}+D$$

A+B+C=$-2+\frac{4}{3}-\frac{2}{5}=\frac{-16}{15}$

8. $$\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = \frac{1}{2}\int \frac{dx}{(\cos x)^{\frac{7}{2}}\sqrt{\sin x}}$$

$$=\frac{1}{2}\int \frac{dx}{\cos^4 x\sqrt{\tan x}}, t=\tan x$$

$$=\frac{1}{2}\int \frac{1+t^2}{\sqrt{t}}dt = \frac{1}{2}\int\left(t^{\frac{1}{2}}+t^{\frac{3}{2}}\right)dt$$

$$=\sqrt{t}+\frac{t^{\frac{5}{2}}}{5}+c=\sqrt{\tan x}+\frac{(\tan x)^{\frac{5}{2}}}{5}+c$$

9. $$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx, x=\cos 2\theta$$

$$=\int \theta d(\cos 2\theta)=\theta\cos 2\theta-\frac{\sin 2\theta}{2}+c$$

$$=\frac{x}{2}\cos^{-1}x-\frac{1}{2}\sqrt{1-x^2}+c_1 \rightarrow A=\frac{1}{2}, A+B=0$$

10. $$\int\frac{dx}{\sqrt{1+\cos ec^2 x}}=\int\frac{\sin x dx}{\sqrt{\sin^2 x+1}}=\int\frac{\sin x dx}{\sqrt{2-\cos^2 x}}$$

$$=-\int\frac{dt}{\sqrt{2-t^2}}, t=\cos x$$

$$=\cos^{-1}\frac{t}{\sqrt{2}}+c=\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)+c$$

11. $$\int\sqrt{1+\cos ec}dx=\int\frac{\sqrt{\sin x+1}}{\sqrt{\sin x}}dx$$

$$=2\int\frac{\sqrt{t^2+1}}{\sqrt{1-t^4}}dt, \sin x=t^2, \cos x dx=2tdt$$

$$=2\int\frac{dt}{\sqrt{1-t^2}}dt=2\sin^{-1}t+c$$

$$=2\sin^{-1}\sqrt{\sin x}+c$$

12 . $$I=\sqrt{1+\sin 2x}dx=\int\sqrt{(\sin x+\cos x)^2}dx$$

$$=\int|\sin x+\cos x|dx$$

$$=\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)>0, x\in\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$$

$$\therefore I=\int(\sin x+\cos x)dx=\sin x-\cos x+c$$

Further,

$$\sin x+\cos x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)<0, x\in\left[\frac{3\pi}{4},\frac{7\pi}{4}\right]$$

$$=I=\int-(\sin x+\cos x)dx=\cos x-\sin x+c$$

13. $$\int\frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}=\int\frac{dx}{x^5(1+\frac{1}{x^4})^{\frac{3}{4}}}, t=1+\frac{1}{x^4}$$

$$=-\frac{1}{4}\int\frac{dt}{t^{\frac{3}{4}}}=-t^{\frac{1}{4}}+c$$

$$=-\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+c$$

14. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx=\int\frac{\left(\sin x+\cos x\right)dx}{\sqrt{\sin x\cos x}}$

$$=\sqrt{2}\frac{\left(\sin x+\cos x\right)dx}{\sqrt{2\sin x\cos x}}$$

$$=\sqrt{2}\frac{\left(\sin x+\cos x\right)dx}{\sqrt{1-\left(\sin x-\cos x\right)^2}}$$

$$=\sqrt{2}\sin^{-1}\left(\sin x-\cos x\right)+c$$

15. $\int\frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx=\int\frac{\left(\cos^2 x+\cos^4 x\right)\cos x dx}{\sin^2 x+\sin^4 x}$

$$=\int\frac{1-t^2+\left(1-t^2\right)^2}{t^2+t^4}dt, t=\sin x$$

$$=\int\frac{2-3t^2+t^4}{t^2+t^4}dt=\int 1+\frac{2-4t^2}{t^2\left(1+t^2\right)}dt$$

$$=\int\left(1+\frac{2}{t^2}-\frac{6}{t^2+1}\right)dt=t-\frac{2}{t}-6\tan^{-1}t+c$$

$=\sin x-2\operatorname{cosec}x-6\tan^{-1}\sin x+c$

16. $\int\frac{\sin xdx}{\sin(x-\alpha)}, x-\alpha=t$

$$=\int\frac{\sin(\alpha+t)dt}{\sin t}=\int(\sin\alpha\cot\alpha+\cos\alpha)dt$$

$$=\sin\alpha In\sin t+\cos\alpha t+c_1$$

$$=\sin\alpha In\sin(x-\alpha)+\cos\alpha(x-\alpha)+c_1$$

$$=\ \sin\alpha In\sin(x-\alpha)+\cos\alpha+c$$

$$\therefore(A,B)=(\cos\alpha,\sin\alpha)$$

17. $\int\frac{(x^2-1)dx}{x^2\sqrt{2x^4-2x^2+1}}=\int\frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{\sqrt{2-\frac{2}{x^3}+\frac{1}{x^4}}}, t=2-\frac{2}{x^2}+\frac{1}{x^4}$

$$==\frac{1}{4}\int\frac{dt}{\sqrt{t}}=\frac{1}{2}\sqrt{t}+c$$

$$=\frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$$

$$==\frac{2x^4-2x^2+1}{2x^2}+c$$

18. $\int\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}dx = \sin^2\theta, dx = 2\sin\theta\cos\theta d\theta$

$$=\int\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}, 2\sin\theta\cos\theta d\theta$$

$$=2\int(1-\sin\theta\sin\theta d\theta$$

$$=2\sin\theta(1-\cos 2\theta)d\theta$$

$$=-2\cos\theta-\theta+\frac{\sin 2\theta}{2}+D$$

$$=-2\sqrt{1-x}-\sin^{-1}\sqrt{x}+\sqrt{x-x^2}+D$$

A+B+C=-2-1+1=-2

19. $\int\dfrac{dx}{(\sin^2 x\cos^2 x)^{\frac{1}{4}}}=\int\dfrac{dx}{\sin^{\frac{3}{4}}x\cos^{\frac{5}{4}}x}, \tan x = t$

$$\int t^{\frac{3}{4}}dx = 4t^{\frac{1}{4}} = 4\tan^{\frac{1}{4}}x = 4\tan^{\frac{1}{4}}x + c$$

20. $\int\dfrac{x^2+1}{x^4+1}dx=\int\dfrac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx=\int\dfrac{d\left(x-\frac{1}{x}\right)}{\left(x-\frac{1}{x}\right)^2+2}$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)+c$$

21. $\int\dfrac{\cos 9x+\cos 6x}{2\cos 5x-1}dx=\int\dfrac{2\cos\frac{15x}{2},\cos\frac{3x}{2}}{2\left(2\cos^2\frac{5x}{2}-1\right)-1}dx$

$$=\int\frac{2\cos 3.\frac{5x}{2}.\cos\frac{3x}{2}}{4\cos^2\frac{5x}{2}-3}dx$$

$$=\int\frac{\left(2\cos^3\frac{5x}{2}-3\cos\frac{5x}{2}\right)}{4\cos^2\frac{5x}{2}-3}\cos\frac{3x}{2}$$

(Using $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$)

$$=\int 2\cos\frac{5x}{2}\cos\frac{3x}{2}dx.$$

22. cos6x+6cos4x+15cos2x+10

=cos6x+cos4x+5(cos4x+cos2x)+10(cos2x+1)

$$= \frac{1}{2} In\left(\frac{1+\sin x}{\cos x}\right)+c$$

$$= \frac{1}{2} In\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)+c$$

23. $$\int\frac{dx}{\left(x\tan x+1\right)^2}=\int\frac{\cos^3 xdx}{\left(x\sin x+\cos x\right)^2}$$

$$=\int-\frac{\cos x}{x}d\left(\frac{1}{x\sin x+\cos x}\right)$$

$$=-\frac{\cos x}{x}\left(\frac{1}{x\sin x+\cos x}\right)+\int \frac{1}{x\sin x+\cos x}d\left(\frac{\cos x}{x}\right)$$

$$=-\frac{1}{x\left(x\tan x+1\right)}-\int\frac{1}{x\left(x\tan x+1\right)}+\frac{1}{x}+c$$

$$=\frac{\tan x}{x\tan x+1}+c$$

24. $$\int\sqrt{2+\tan^2 xdx}=\int\frac{2+\tan^2 x}{\sqrt{2+\tan^2 x}}dx$$

$$=\int\frac{\sec^2 xdx}{\sqrt{2+\tan^2 x}}+\int\frac{dx}{\sqrt{1+\sec^2 x}}$$

$$= In\left(\tan x+\sqrt{2+\tan^2 x}\right)+\int\frac{d\left(\sin x\right)}{\sqrt{2-\sin^2 x}}$$

$$= In\left(\tan x+\sqrt{2+\tan^2 x}\right)+\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right)+c$$

25. $$\int\sin^{-1}\frac{2x+2}{\sqrt{4x^2+8x+13}}=\int\tan x^{-1}\left(\frac{2x+2}{3}\right)dx$$

$$=\frac{3}{2}\int\tan^{-1}t.dt, t=\frac{2x+2}{3}=\frac{3}{2}\left[t\tan^{-1}-\int\frac{t}{1+t^2}dt\right]$$

$$=\frac{3}{2}t\tan^{-1}t-\frac{3}{4}In(1+t^2)+D_1$$

$$=\left(x+1\right)\tan^{-1}\left(\frac{2x+2}{3}\right)-\frac{3}{4}In(4x^2+18x+13)+D$$

$$\therefore A+B+C=1+1-\frac{3}{4}=\frac{5}{4}$$

26 $$l=\frac{1}{6M}\int t^{\frac{1}{m}}dt=\frac{1}{6m}\frac{t^{1+\frac{1}{m}}}{1+\frac{1}{m}}+c$$

$$=\frac{1}{6\left(m+1\right)}\left(2x^{3m}+3x^{2m}+6x^m\right)^{(m+1)}+c$$

27. $\int \frac{dx}{x\sqrt{x^2+2x+1}} = \int \frac{1}{\sqrt{1+\frac{2}{x}-\frac{1}{x^2}}}\frac{dx}{x^2}, t=\frac{1}{x}$

$$= -\int \frac{dt}{\sqrt{1+2t-t^2}} = -\int \frac{dt}{\sqrt{2-(t-1)^2}}$$

$$= \cos^{-1}\left(\frac{t-1}{\sqrt{2}}\right)+c = \cos^{-1}\left(\frac{1-x}{\sqrt{2}x}\right)+c$$

28. $I = \int \frac{\sin(x-\alpha)dx}{\sqrt{\sin^2 - x\sin^2\alpha}}$

(Using sin(A+B)sin(A-B)=$\sin^2$A-$\sin^2$B)

$$= \cos\alpha \frac{\sin xdx}{\sqrt{\cos^2\alpha-\cos^2 x}} - \sin\alpha\int\frac{\cos xdx}{\sqrt{\sin^2 x-\sin^2\alpha}}$$

$$= \cos\alpha\cos^{-1}\frac{\cos x}{\cos\alpha} - \sin\alpha\cosh^{-1}\left(\frac{\sin x}{\sin\alpha}\right)+c$$

29. $I = \int \frac{dx}{\sqrt{\cos^2 x(\sin x\cos\alpha - \cos x\sin\alpha)}} + \frac{dx}{\sqrt{\sin^1 x(\sin x\cos\alpha-\cos x\sin\alpha)}}$

$$= \int \frac{\sec^2 xdx}{\sqrt{\cos\alpha\tan x-\sin\alpha}} + \frac{\cos ec^3 dx}{\sqrt{\cos\alpha-\sin\alpha\cot x}}$$

$$= \frac{2}{\cos\alpha}\sqrt{\cos\alpha\tan x-\sin\alpha} + \frac{2}{\sin\alpha}\sqrt{\cos\alpha-\sin\alpha\cot x+c}$$

$$ab = \frac{2}{\cos\alpha}.\frac{2}{\sin\alpha} = 8\cos ec2a$$

30. $x = \frac{\sqrt{3}}{2}\sin\theta \to I = 2\int\frac{d\theta}{9\sin^2\theta+16}$

$$= 2\int \frac{\frac{1}{\cos^2}d\theta}{9\tan^2\theta+16\sec^2\theta} \text{(Using } \sec^2\theta\text{=1+}\tan^2\theta\text{)}$$

$$= 2\int\frac{\sec^2\theta d\theta}{1+25\tan^2\theta} = \frac{1}{10}\tan^{-1}\frac{5}{4}\tan\theta+c$$

$$= \frac{1}{10}\tan^{-1}\frac{5x}{2\sqrt{3-4x^2}}+c$$

SECTION-B

1. If $f(x) = \int \frac{3x^4-1}{(x^4+x+1)^2}dx$ and $f(0)=0,$ then $f(-1)=$

2. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax+B\log(9e^{2x}-4)+C, then |4A| =$

3. If $\int \frac{x^2-1}{(x^2+1)\sqrt{1+x^4}}dx = \frac{1}{a}\tan^{-1}\frac{\sqrt{x^2+1/x^2}}{\sqrt{2}}+C$, then the value of a is _______

4. Let $f(x)=\int\left(\frac{\cos x}{x}-\log x^{\sin x}\right)dx$ and $f(1)=0$, then the value of $f(\pi/2)$ is _____

5. *If* $f(x)=\int\frac{1}{(x+1)\sqrt{x^2-1}}dx$ and f(1)=0, then the value of $5(f(3/2))^2$ is _____

6. If $\int\frac{(x-x^3)^{1/3}}{x^4}dx = a\left(\frac{1}{x^2}-1\right)^b + c$, then the value of $1/|ab|$ *is* ___________

7. If $\int\frac{1}{\left[(x-1)^3(x+2)^5\right]^{1/4}}dx = a\left(\frac{x-1}{x+2}\right)^b + c$, then the value of 1/ab is ___________

8. If $f(x)=\int\frac{\sqrt{\tan x}}{\sin x\cos x}dx$, $f(0)=3$, then the value of $f(\pi/4)$ is _______

9. If $\int\frac{5x^4+4x^5}{(x^5+x+1)^2}dx = f(x)+c$, then the value of $1/f(1)$ is_______

10. If $\int\frac{\cos^2 x+\sin 2x}{(2\cos x-\sin x)^2}dx = \frac{\cos x}{2\cos x-\sin x}+ax+b$ In $|2\cos x-\sin x|+c$, then $|a+2b|$ is________

INTEGER TYPE SECTION II (key)

Q.NO.	1	2	3	4	5	6	7	8	9	10
ANSW.	1	6	2	0	1	2	3	5	3	1

SOLUTIONS

SECTION-B

1. (1) $\int\frac{3x^4-1}{x^2\left(x^3+1+x^{-1}\right)^2}dx = \int\frac{3x^2-x^{-2}}{\left(x^3+1+x^{-1}\right)^2}dx$

$$=\frac{-x}{x^4+x+1}+c$$

$f(o)=o \Rightarrow c=0 \Rightarrow f(-1)=1$

2. (6) We have $\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax+B\,In(9e^{2x}-4)+C$

Differentiating both sides w.r.t.x, we get

$$\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}=A+\frac{18Be^x}{9e^x-4e^{-x}}$$

$$\Rightarrow \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}=A+\frac{18Be^x}{9e^x-4e^{-x}}$$

$$\Rightarrow \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}=\frac{(9A+18B)e^x-4Ae^{-x}}{9e^x-4e^{-x}}$$

$$\Rightarrow 9A+18B=4; -4A=6$$

$\Rightarrow A=-3/2, B=\left(4+\frac{27}{2}\right)\frac{1}{18}=\frac{35}{36}, C$ can have any real value.

3. (2) $I=\int \frac{x^2-1}{(x^2+1)\sqrt{\sqrt{x^4+1}}}dx$

$$=\int \frac{x^2(1-1/x^2)}{(x^2+1/x)\sqrt{x^2+1/x^2}}dx$$

$$=\int \frac{(1-1/x^2)dx}{(x+1/x)\sqrt{(x+1/x)^2-2}}$$

Putting $x+(1/x)=t$, we have $I=\int \frac{dt}{t\sqrt{t^2-2}}$

Again putting $t^2-2=y^2, 2t\,dt=2y\,dy,$ we get

$$I=\int \frac{ydy}{(y^2+2)y}=\frac{1}{\sqrt{2}}\tan^{-1}\frac{y}{\sqrt{2}}$$

$$=\frac{1}{2}\tan^{-1}\frac{\sqrt{x^2+1/x^2}}{\sqrt{2}}+c$$

4. (0) $\int\left(\frac{\cos x}{x}-\log x^{\sin x}\right)dx$

$$\int \frac{\cos x}{x}dx=\int \sin x\log x dx$$

$$=\cos x\log x-\int -\sin x\log x dx-\int \sin x\log x dx$$

(integration 1st integral by parts)

$$=\cos x\log x+c$$

$$f(1)=0+c=0\Rightarrow c=0$$

$$\Rightarrow f(\pi/2)=0$$

5. (1) let $1=\int \frac{1}{(x+1)\sqrt{x^2-1}}dx$

Putting x+1=$\frac{1}{t} and dx=\frac{-1}{t^2}dt, we\ get$

$$I=\int \frac{1}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2-1}}\left(-\frac{1}{t^2}\right)dt$$

$$= -\int \frac{dt}{\sqrt{1-2t}} = -\int (1-2t)^{1/2}\,dt$$

$$= \int \frac{(1-2t)^{1/2}}{(-2)\left(\frac{1}{2}\right)} + c = \sqrt{1-2t} + c$$

$$\sqrt{1-\frac{2}{x+1}} + c = \sqrt{\frac{x-1}{x+1}} + c$$

$f(1) = 0 \Rightarrow c = 0.$ then

$$f\left(\frac{3}{2}\right) = \sqrt{\frac{1}{5}} \Rightarrow 5\left(f\left(\frac{3}{2}\right)\right)^2 = 1.$$

(2) I= $\int f\left(\frac{3}{2}\right) = \sqrt{\frac{1}{5}} \Rightarrow 5\left(f\left(\frac{3}{2}\right)\right)^2 = 1.$

6. (2) I= $\int \frac{\left(x-x^3\right)^{1/3}}{x^4}dx = \int \frac{\left(\frac{1}{x^2}-1\right)^{1/3}}{x^3}dx$

Putting $\frac{1}{x^2} = t, \frac{1}{x^3}dx = -\frac{dt}{2}, we\,get$

I= $\frac{1}{2}\int t^{1/3}dt = -\frac{3}{8}t^{4/3} + c = -\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3} + c$

$\Rightarrow a = -3/8, b = 4/3 \Rightarrow ab = -1/2 \Rightarrow 1/|ab| = 2$

7. (3) I= $\int \frac{1}{\left[(x-1)^3(x+2)^5\right]^{1/4}}dx$

$$\int \frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}dx$$

Let $\frac{x-1}{x+2} = t \Rightarrow \frac{3dx}{(x+2)^2} = dt$

$$= \frac{1}{3}\int \frac{1}{t^{3/4}}dt$$

$$= \frac{1}{3}\left(\frac{t^{1/4}}{1/4}\right) + c = \frac{4}{3}t^{1/4} + c = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$$

$\Rightarrow a = 4/3\,and\,b = 1/4 \Rightarrow ab = 1/3$

8. $(5)\,I = \int \frac{\sqrt{\tan x}}{\sin x\cos x}dx = \int \frac{\sqrt{\tan x}}{\tan x}\sec^2 x\,dx$

$$\int \frac{1}{\sqrt{t}} dt, \text{ where } t = \tan x$$

$$I = 2t^{1/2} + c = 2\sqrt{\tan x} + c$$

9. (3) Let I= $\int \frac{(5x^4 + 4x^5)}{\left(x^5 + x + 1\right)^2} dx$

Dividing above and below by x^{10}, we get

$$= \int \frac{\left(\frac{5}{x^6} + \frac{4}{x^5}\right) dx}{\left(1 + \frac{1}{x^4} + \frac{1}{x^5}\right)^2}$$

Putting $1 + \frac{1}{x^4} + \frac{1}{x^5} = t,$

$\left(-\frac{4}{x^5} - \frac{5}{x^6}\right) dx = dt \; or \left(\frac{4}{x^5} + \frac{5}{x^6}\right) dx = -dt$, we get

$$I = -\int \frac{dt}{t^2} = \frac{1}{t} + c = \frac{1}{\left(1 + \frac{1}{x^4} + \frac{1}{x^5}\right)} + c$$

$$= \frac{x^5}{\left(x^5 + x + 1\right)} + c$$

$$f(1) = 1/3$$

10. (1) Let I= $\int \frac{\cos^2 x + \sin\ 2x}{\left(2\cos x - \sin^2 x\right)} dx$

$$\int \frac{\cos x + 2\sin x \cos x}{\left(2\cos x - \sin x\right)^2} dx$$

Integrating by parts, taking $\cos\ x$ as the first and

$\frac{\left(\cos x + 2\sin x\right)}{\left(2\cos x - \sin x\right)^2} dx$ as the second function, we have

$$\text{I} = \cos x \left\{\frac{1}{2\cos x - \sin x}\right\} - \int \frac{-\sin x dx}{\left(2\cos x - \sin x\right)}$$

$$= \cos x \left\{\frac{1}{2\cos x - \sin x}\right\} + \frac{\sin x dx}{\left(2\cos x - \sin x\right)}$$

$$\frac{\cos x}{\left(2\cos x - \sin x\right)}$$

$$+ \int \frac{-\frac{1}{5}\left(2\cos x - \sin x\right) - \frac{2}{5}\left(-2\sin x - \cos x\right)}{\left(2\cos x - \sin x\right)}$$

$$\left[N^r = \lambda D^r + \mu \frac{d}{dx} D^r\right]$$

$$\frac{\cos x}{\left(2\cos x - \sin x\right)}$$

$$-\frac{1}{5}\int dx - \frac{2}{5}\int \frac{\left(-2\sin x - \cos x\right)}{\left(2\cos x - \sin x\right)} dx$$

$$= \frac{\cos x}{2\cos x - \sin x} - \frac{1}{5}x - \frac{2}{5} In\left|2\cos x - \sin x\right| + c$$

$$\Rightarrow a = -\frac{1}{5}, b = -\frac{2}{5} \Rightarrow \left|a + 2b\right| = 1.$$