TOPIC: STATISTICS

1) If the airthmetic mean of the observations $x_1, x_2, x_3 ......x_n$ is 1 then airthmetic mean of $\frac{x_1}{k}, \frac{x_2}{k}, \frac{x_3}{k} ...... \frac{x_n}{k}$ (k>0) is….

1) greater than 1 2) less than 1 3) equal to 1 4) none of these

2) The average score of boys in an examination of a school is 71 and that of girls is 73. The average score of school in that examination is 71.8. The ratio of the number of boys to the number of girls appeared in the examinations is

1) 3:2 2) 3:4 3) 1:2 4) 2:1

3) A student obtain 75%,80% and 85% in three subjects. If the marks of another subject is added, then his average cannot be less than

1) 60% 2) 65% 3) 80% 4) 90%

4) The mean of 100 items is 49. It is discovered that three items which should have been 60,70,80 were wrongly read as 40,20,50 respectively. The correct mean is...

1) 48 2) 82.5 3) 50 4) 80

5) The median of a set of nine distinct observations is 20.5 . If each of the last four observations of the set is increased by 2, then the median of the new set is

1) increased by 2 2) decreased by 2

3) Two times the original median 4) Remains same

6) If in a moderately asymmetrical distribution the mode and the mean of the data are 6λ and 9λ respectively. Then the median is…

1) 8λ 2) 7λ 3) 6λ 4) 5λ

7) If mode of a data exceeds its mean by 12 then mode exceeds the median is

1) 4 2) 8 3) 6 4) 10

8) If the A.M between two numbers exceeds their G.M by 2 and the G.M. exceeds their H.M by $\frac{8}{5}$ then the numbers are….

1) 12,3 2) 9,16 3) 4,16 4) 5,14

9) Let a, b, c, d and e are the observations with mean 'm' and standard deviation 's'. The standard deviation of the observations a+k, b+k, c+k, d+k, e+k is…

1) ks 2) s 3) s + k 4) $\frac{s}{k}$

10) Let $x_1, x_2, ...x_n$ be 'n' observations .Let $\omega_i = lx_i$ +k for i =1,2,….n where l and k are constants .The mean of x_i is 48and their S.D is 12. Also mean of ω_i's is 55 and S.D of ω_i's is 15.The values of l and k should be

1) l =2.5 , k= 5 2) $l = -1.25$, k=5

3) l =2.5 , k= -5 4) l =1.25 , k= -5

11) The mean of 100 observations is 50 and their S.D is 5. Then sum of squares of all the observations is.

1) 50000 2) 250000 3) 252500 4) 255000

12) In an experiment with 15 observations on x, the following results were available. $\sum x^2 = 2830, \sum x = 170$. One observation 20 was found to be wrong and was replaced by correct value 30. Then the corrected variance is

1) 78.00 2) 188.66 3) 177.33 4) 8.33

13) The S.D of some temperature data in c^0 is 5. If the data were converted in to ${}^0F$,the new variance would be

1) 81 2) 57 3) 36 4) 25

14) Consider the numbers 1,2,3,4,5. If '2' is added to each number, the variance of the numbers so obtained is

1) 1 2) 6 3) 2 4) 9

15) Consider first 6 positive integers. If we multiply each number by -2 and add 2 to each number, the variance of numbers so obtained is—

1) 11.6 2) 21.7 3) 32.3 4) 10.8

16) The mean of 8 observations is 8 and their variance is 24.25. If six of the given observations are 12,3,18,8,2,10. Then other observations are

1) 7,5 2) 5,6 3) 3,4 4) 4,7

17) If the standard deviation of the numbers 6,7,a and 12 is $\frac{\sqrt{91}}{4}$ then which of the following is true

1) $3a^2 - 34a + 91 = 0$ 2) $3a^2 - 23a + 44 = 0$

3) $3a^2 - 50a + 200 = 0$ 4) $3a^2 - 32a + 84 = 0$

18) If the mean and S.D of 9 observations $x_1, x_2, x_3....x_9$ are equal to 43 and 5,then the mean of $(x_1-3)^2, (x_2-3)^2, ...(x_9-3)^2$ is------

1) 400 2) 1625 3) 1537 4) 480

19) If the mean of 4,7,2,8,6 and 'a' is 7. Then the mean deviation from mean is…

1) 2 2) 1 3) 3 4) 4

20) The variance of first 50 odd natural numbers is

1) 433 2) 836 3) 833 4) 436

21) Runs scored by a batsman in 10 innings are 38,70,48,34,42,55,63,46,54,44. The mean deviation about median is…

1) 8.6 2) 6.4 3) 10.6 4) 9.6

22) The means of two groups of observations A and B are $\bar{x}, \bar{y}$, S.D's are respectively 2 and 3. In order that the group A is to be more consistent than group B then $\frac{\bar{y}}{\bar{x}} <$ …..

1) $\frac{3}{2}$ 2) $\frac{5}{1}$ 3) $\frac{2}{3}$ 4) $\frac{6}{5}$

23) If α, β are respectively the mean deviation about mean and variance of first five prime numbers then the ordered pair (α, β) is……..

1) (2.27,10.42) 2) (2.27,10.24) 3) (2.72,10.24) 4) (2.72,10.42)

24) If S_1 and S_2 are the variances of first $2k$ and k (k>1)natural numbers respectively then $\frac{S_1}{S_2}$ lies in the interval

1) $[4, \alpha]$ 2) $(1, 4]$ 3) $(4, 5]$ 4) $[7, \alpha)$

25) The coefficient of variation of the first 5 prime number is----

1) $\frac{400}{7}$ 2) $\frac{406}{7}$ 3) $\frac{416}{7}$ 4) $\frac{425}{8}$

26) In a discrete data $\frac{1}{4}^{th}$ of the observations are equal to 'a' , another $\frac{1}{4}^{th}$ of the observations are equal to '$-a$' .Out of the remaining half of them are equal to 'b' and the rest are equal to '$-b$' .If the variance of all the observations is "ab" then

1) $a^2 = 4b^2$ 2) a=-2b 3) a=b 4) a= $-3b$

27) The mean and S.D of 100 observations $x_1, x_2, ...x_{100}$ were calculated as 40 and 5.1 respectively by a student who took by mistake 50 instead of 40 for one observation .Then correct value of $\sum_{i=1}^{100} x_i^2$

1) 3990 2) 161701 3) 162601 4) 4000

28) In a data with 15 number of observations $x_1, x_2 x_3 ... x_{15}$ $\sum_{i=1}^{15} x_i^2 = 3600$ and $\sum_{i=1}^{15} x_i = 175$ .If the value of one observation 20 was found wrong and was replaced by its correct value 40 then corrected variance of that data is—

1) 151 2) 149 3) 145 4) 144

29) If the coefficient of variation and variance of a frequency distribution are 7.2 and 3.24 respectively then its mean is

1) 45 2) 25 3) 20 4) 16

30) Two distributions A and B have the same mean. If their coefficients of variation are 6 and 2 respectively and σ_A, σ_B are their standard deviations then

1) $\sigma_A = 3\sigma_B$ 2) $3\sigma_A = \sigma_B$ 3) $\sigma_A = 2\sigma_B$ 4) $2\sigma_A = \sigma_B$

31) Let '0' is the mean deviation of first five odd natural numbers about their mean and p is the mean deviation of first five prime numbers about their mean then $p - 0 =$-------

1) 0.3 2) 0.32 3) 0.23 4) 0.2

PAPER SETTER –HYD-CHINTAL

1-10	4	1	1	3	4	1	2	3	2	4
11-20	3	1	1	3	1	2	3	2	3	3
21-30	1	1	3	2	1	3	2	1	2	1
31	2									

Hints:

1) $\frac{x_1 + x_2 + x_3 + ... + x_n}{n} = 1 \Rightarrow x_1 + x_2 + x_3 + ... + x_n = n$

Require A.M = $\frac{\frac{x_1}{k} + \frac{x_2}{k} + \frac{x_3}{k} + ... + \frac{x_n}{k}}{n} = \frac{1}{k}\left(\frac{x_1 + x_2 + x_3 + ... + x_n}{n}\right) = \frac{1}{k}$

2) Let there be n_1 boys and n_2 girls

Let $\overline{X_1}$ and $\overline{X_2}$ are average scores of boys and girls.

Let $\overline{X}$ is average of both boys and girls .

$\overline{X_1}$ =71, $\overline{X_2}$ =63, $\overline{X}$ =71.8

$\overline{X} = \dfrac{n_1\overline{X_1} + n_2\overline{X_2}}{n_1 + n_2} \Rightarrow 71.8 = \dfrac{n_1(71) + n_2(73)}{n_1 + n_2}$

(71.8) n_1+(71.8) n_2= $n_1(71) + n_2(73)$

0.8 n_1=1.2 n_2 $\Rightarrow$ $\dfrac{n_1}{n_2} = \dfrac{12}{8} = \dfrac{3}{2}$

3) Marks from 3 subjects out of 300 = 75+80+85=240

If the marks of another subject is added then the marks will be ≥ 240 out of 400

Minimum average marks = $\dfrac{240}{4} = 60\%$

4) Sum of 100 items = 49 x100=4900

Sum of items added = 60+70+80=210

New sum =4900+210-110=5000

Correct mean = $\dfrac{5000}{100} = 50$

5) n=9,median term = $\left(\dfrac{9+1}{2}\right)^{th}$ term = 5th term

Now the last four observations are increased by 2. since the median is 5th observation, which remains unchanged, there will be no change in median

6) Mode=3 Median-2Mean

6λ =3 Median - 18λ $\Rightarrow$ Median = 8λ

7) Mode – Mean =12

Mode=3Median – 2Mean

Mode-Mean=3(Median – Mean)

12 = 3(Median – Mean) $\Rightarrow$ Median –Mean =4

Again

Mode – Mean = 2(Median- Mean) = 2 x 4=8

8) A – G =2 $\rightarrow$ (1)

G – H = $\dfrac{8}{5} \rightarrow (2)$

G^2 = A.H=(G+2)(G- $\dfrac{8}{5}$)

(or)G=8 (or) ab =64 $\rightarrow$ (3)

From (1) we get A=10

a + b =20 $\rightarrow$ (4)

solving (3) and (4) we get a=4 and b=16 (or) a=16 and b=4

9) Mean =m= $\dfrac{a+b+c+d+e}{5}$

$\sum x_i$ =a+ b+ c+ d+ e=5m

New mean = $\dfrac{a+k+b+k+c+k+d+k+e+k}{5} = \dfrac{(a+b+c+d+e)+5k}{5} = m+k$

$$\text{S.D}= \sqrt{\frac{\sum(x_i^2 + k^2 + 2kx_i)}{n} - (m^2 + k^2 + 2mk)}$$

$$= \sqrt{\frac{\sum x_i^2}{n} - m^2 + \frac{2k\sum x_i}{n} - 2mk}$$

$$= \sqrt{\frac{\sum x_i^2}{n} - m^2 + 2mk - 2mk}$$

$=\sqrt{\dfrac{\sum x_i^2}{n}-m^2}$ =S

10) Given $\omega_i = lx_i + k$,

$\overline{x_i}$ =48

S.D of x_i's =12, $\overline{\omega_i} = 55$ and S.D of ω_i's $= 15$

Then $\overline{\omega_i} = l\overline{x_i} + k$

$55 = 48l + k \rightarrow (1)$

Now S.D of ω_i's = l (S.D of x_i's)

$15 = l(12) \Rightarrow l = \dfrac{15}{12}$ =1.25

From equ (1) we get

K=55-1.25 x 48= -5

11) $\overline{x} = 50$,n=100 and $\sigma = 5$

$$50 = \frac{\sum x_i}{100} \Rightarrow \sum x_i = 5000$$

Now $S^2 = \dfrac{\sum x_i^2}{n} - (\overline{x})^2 \Rightarrow 25 = \dfrac{\sum x_i^2}{100} - (50)^2 \Rightarrow 2525 = \dfrac{\sum x_i^2}{100} \Rightarrow \sum x_i^2 = 252500$

12) $\sum x = 170$, $\sum x^2 = 2830$

The increase in $\sum x$ is 10 then

$\sum x^1$ =170+10=180

The increase in $\sum x^2$ is 900-400=500 then

$\sum x^2$ =2830+500 = 3330

Variance $= \dfrac{1}{n}\sum x^2 - \left(\dfrac{1}{n}\sum x^1\right)^2 = \dfrac{1}{15}\text{x}3330 - \left(\dfrac{1}{15}\text{x}180\right)^2$ =222-144=78

13) Given $\sigma_c = 5$

Relation between ^0C and ^0F is given by

F= $\dfrac{9C}{5} + 32$

$\therefore \sigma_F = \dfrac{9}{5}.\sigma_c = \dfrac{9}{5}\text{x}5 = 9$

14) Given numbers are 1,2,3,4,5

If '2' is added to each number

New Observations are

3,4,5,6,7

$\sum x_i$ =3+4+5+6+7=25

$\sum x_i^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = (1^2 + 2^2 + ... + 7^2) - (1^2 + 2^2) = \dfrac{7\text{x}8\text{x}15}{6} - 5$ =135

Var = $\dfrac{\sum x_i^2}{n}$ - $\left(\dfrac{\sum x_i}{n}\right)^2$ = $\dfrac{135}{5}$ - $\left(\dfrac{25}{5}\right)^2$ =27-25=2

15) The first 6 positive integers are

1,2,3,4,5,6

On multiplying each number by -2 we get

-2, -4, -6, -8, -10, -12

On adding '2' in each number we get

0, -2, -4, -6, -8, -10

$\sum x_i$ =0-2-4-6-8-10=-30

$\sum {x_i}^2$ =220

Var= $\frac{\sum x_i^2}{n}-(\bar{x})^2=\frac{220}{6}-(\frac{-30}{6})^2=\frac{35}{3}=11.6$

16) Mean $\left(\bar{x}\right)$= $\frac{12+3+18+8+2+10+a+b}{8}$ =8

53+a+b=64

a + b=11----(1)

$\sum {x_i}^2 = 12^2+3^2+18^2+8^2+2^2+110^2+a^2+b^2=a^2+b^2+645$

Variance $\left(\sigma^2\right)$= $\frac{\sum x_i^2}{n}$ - $\left(\frac{\sum x_i}{n}\right)^2$ =24.25

= $\frac{a^2+b^2+645}{8}$ - $\left(8\right)^2$ =24.25

= a^2+b^2+645 =706 $\Rightarrow$ $a^2+b^2=61$ --------(2)

From (1) & (2) ab=30

$(a-b)^2=$ $a^2+b^2-2ab=1$ $\Rightarrow$ a – b =1 --------(3)

a = 6, b= 5

17) $\bar{x}=$ $\frac{6+7+a+12}{4}=\frac{25+a}{4}$

$\sigma=\sqrt{\frac{\sum x_i^2}{n}-\left(\bar{x}\right)^2}$

$\frac{\sqrt{91}}{4}=\sqrt{\frac{36+49+a^2+144}{4}-\left(\frac{25+a}{4}\right)^2}$

S.o. b. s

$\frac{91}{16}=\frac{229+a^2}{4}$ - $\frac{\left(25+a\right)^2}{16}$

$91=4(229+a^2)-(25+a)^2$

$91=916+4a^2-625-a^2-50a$

$3a^2-50a+200=0$

18) Mean =43

$\frac{x_1+x_2+\ldots\ldots x_9}{9}=43$

$x_1+x_2+\ldots\ldots x_9$ =387 -----(1)

S. D=5

$5^2=\frac{{x_1}^2+{x_2}^2+\ldots .{x_9}^2}{9}-(43)^2$

${x_1}^2+{x_2}^2+\ldots .{x_9}^2=9(5^2+43^2)$

= 9(25+1849) = 9 (1874) =16,866

Required Mean = $\frac{(x_1-3)^2+(x_2-3)^2+\ldots+(x_9-3)^2}{9}$

$=\frac{({x_1}^2+{x_2}^2+\ldots .{x_9}^2)+9\text{x}9-6(x_1+x_2+\ldots x_9)}{9}$

$=\frac{16,886+81-6(387)}{9}=\frac{16947-2322}{9}=\frac{14625}{9}$ =1625

19) Mean = $\frac{4+7+2+8+6+a}{6}=7$ $\Rightarrow$ a=15

Mean deviation about mean $= \dfrac{\sum_{i=1}^{6}|x_i - 7|}{6}$

$$= \frac{|4-7|+|7-7|+|2-7|+|8-7|+|6-7|+|15-7|}{6}$$

$$= \frac{3+0+5+1+1+8}{6} = 3$$

20) Variance of first n odd

Natural numbers $= \dfrac{n^2-1}{3}$

Required variance $= \dfrac{(50)^2-1}{3} = \dfrac{2499}{3} = 833$

21) Ascending order of data is
34,38,42,44,46,48,54,55,63,70

Median (M) $= \dfrac{46+48}{32} = 47$

Mean deviation about median $= \dfrac{\sum|x_i - M|}{n} = \dfrac{\sum|x_i - 47|}{10} = \dfrac{13+9+5+3+1+1+7+8+16+23}{10} = 8.6$

22) $CV_A = \dfrac{\sigma_x}{\bar{x}} \text{ x } 100 = \dfrac{2}{\bar{x}} \text{ x } 100$, $CV_B = \dfrac{\sigma_y}{\bar{y}} \text{ x } 100 = \dfrac{3}{\bar{y}} \text{ x} 100$

A is more consistent than B

$$CV_A < CV_B \Rightarrow \frac{2}{\bar{x}}\text{x}100 < \frac{3}{\bar{y}}\text{x}100 \Rightarrow \frac{2}{\bar{x}} < \frac{3}{\bar{y}} \Rightarrow \frac{\bar{y}}{\bar{x}} < \frac{3}{2}$$

23) Mean of first five prime numbers $= \dfrac{2+3+5+7+11}{5} = \dfrac{28}{5} = 5.6$

Mean deviation about mean $= \dfrac{|2-5.6|+|3-5.6|+|5-5.6|+|7-5.6|+|11-5.6|}{5}$

$$= \frac{3.6+2.6+0.6+1.4+5.4}{5} = \frac{13.6}{5} = 2.72$$

Variance$= \dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2 = \dfrac{4+9+25+49+121}{5} - (5.6)^2 = 41.6 - 31.36 = 10.24$

24) Variance of 2k natural numbers $= S_1 = \dfrac{2k(2k+1)(4k+1)}{6(2k)} - \left(\dfrac{2k(2k+1)^2}{2 \text{x} 2k}\right)$

$$= (2k+1)\left[\frac{4k+1}{6} - \frac{2k+1}{4}\right]$$

$$= (2k+1)\ [8k+2-6k-3] = \frac{4k^2-1}{12}$$

Variance of first k natural numbers $= S_2 = \dfrac{k(k+1)(2k+1)}{6 \text{x} k} - \left(\dfrac{k(k+1)}{2 \text{x} k}\right)^2 = (k+1)\left[\dfrac{2k+1}{6} - \dfrac{k+1}{4}\right]$

$$= \frac{k+1}{12}[4k+2-3k-3] = \frac{k+1}{12}(k-1) = \frac{k^2-1}{12}$$

$$\frac{S_1}{S_2} = \frac{4k^2-1}{k^2-1} = 4+\frac{3}{k^2-1}, (k>1)$$

$$\frac{S_1}{S_2} \in (1,4]$$

25) $\bar{x} = \frac{2+3+5+7+11}{5} = \frac{28}{5}$

$\sum x_i^{\ 2} = 4+9+25+49+121 = 208$

$\sigma = \sqrt{\frac{\sum x_i^2}{5} - (\bar{x})^2} = \sqrt{\frac{208}{5} - \left(\frac{28}{5}\right)^2} = \sqrt{\frac{1040-784}{25}} = \frac{16}{5}$

Coefficient of variation $= \frac{\sigma}{x}(\ 200) = \frac{16}{28}(100) = \frac{400}{7}$

26) Given $\frac{1}{4}^{th}$ observation = a

Another $\frac{1}{4}^{th}$ observation = - a

$\frac{1}{4}^{th}$ observation =b, $\frac{1}{4}^{th}$ observation = - b

$\bar{x} = \frac{a-a+b-b}{n}$ =0

Variance =ab= $\frac{\sum x_i^{\ 2}}{n}$ - $\overline{(x)}^2 = \frac{\sum x_i^{\ 2}}{n}$

ab = $\frac{\frac{n}{4}(a^2+b^2)+\frac{n}{4}(b^2+b^2)}{n} \Rightarrow ab = \frac{a^2+b^2}{2} = a^2+b^2-2ab = 0 \Rightarrow (a-b)^2$ =0 $\Rightarrow$ a=b

27) $\sigma^2 = \frac{\sum x_i^2}{100} - (\bar{x})^2$

$(5.1)^2 = \frac{\sum x_i^2}{100}$ - $(40)^2$

$\Rightarrow \sum x_i^{\ 2}$ =26.01+1600) x100

= 2601+16000- $(50)^2 + (40)^2$

= 2601+160000-2500+1600

= 161701

28) $\sum_{i=1}^{15} x_i^{\ 2} = 3600, \sum_{i=1}^{15} x_i = 175$ and n=15

Replacing observation 20 by 40

$\sum_{i=1}^{15} x_i = 175-20+40$ =195

$\sum_{i=1}^{15} x_i^{\ 2} = 3600-(20)^2+(40)^2$ =4800

Corrected variance $= \frac{\sum x_i^{\ 2}}{n}$ - $\left(\frac{\sum x_i}{n}\right)^2 = \frac{4800}{15} - \left(\frac{195}{15}\right)^2$ =320-169=151

29) Coefficient of variation =7.2

Variance =3.24

$\sigma^2 = 3.24 \Rightarrow \sigma = \sqrt{3.24} = 1.8$

C.V= $\frac{\sigma}{\bar{x}}$**x**100 $\Rightarrow$ $7.2 = \frac{1.8}{\bar{x}}$x100 $\Rightarrow$ $\bar{x} = \frac{1.8}{7.2}$x100 = 25

30) C.V is the ratio of S.D and mean

Given mean of both distribution is same

$CV_A = \frac{\sigma_A}{\mu_A}, CV_B = \frac{\sigma_B}{\mu_B}$

$\frac{CV_A}{CV_B} = \frac{\sigma_A}{\mu_A}x\frac{\sigma_B}{\mu_B}$ $\Rightarrow$ $\frac{CV_A}{CV_B} = \frac{\sigma_A}{\sigma_B}$ $[\because \mu_A = \mu_B]$

$\frac{6}{2} = \frac{\sigma_A}{\sigma_B} \Rightarrow \sigma_A = 3.\ \sigma_B$

31) $\overline{x_1} = \frac{1+3+5+7+9}{5} = 5$, $\overline{x_2} = \frac{2+3+5+7+11}{5} = \frac{28}{5}$

O=$\frac{|1-5|+|3-5|+|5-5|+|7-5|+|9-5|}{5} = \frac{12}{5}$

P=$\frac{\left|2-\frac{28}{5}\right|+\left|3-\frac{28}{5}\right|+\left|5-\frac{28}{5}\right|+\left|7-\frac{28}{5}\right|+\left|11-\frac{28}{5}\right|}{5} = \frac{68}{25}$

P-O=$\frac{68}{25}$-$\frac{12}{5}$=$\frac{8}{25}$=0.32