SERIES & SEQUENCE : DPP

MATHEMATICS (IIA)

1. The 1025^{th} term in the sequence 1, 22, 4444, 88888888, ... is
1) 2^9 2) 2^{10} 3) 2^{11} 4) 2^{12}

2. If $1 + \lambda + \lambda^2 + \ldots + \lambda^n = (1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})$, then the value 'n' is (where $n \in N$)
1) 32 2) 16 3) 31 4) 15

3. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in AP such that $|a| < 1, |b| < 1$, and $|c| < 1$, then x, y, z are in
1) AP 2) GP 3) HP 4) none of these

4. The coefficient of x^{203} in the expansion of $(x-1)(x^2-2)(x^3-3)\ldots\ldots(x^{20}-20)$ is
1) -35 2) 21 3) 13 4) 15

5. If the sum of 'n' terms of the series $\frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \ldots\ldots$ is S_n, then S_n exceeds 199 for all 'n' greater than
1) 99 2) 50 3) 199 4) 100

6. The numbers $3^{2\sin 2x - 1}$, 14, $3^{4-2\sin 2x}$ from first three terms of an AP, its 5^{th} term is equal to
1) -25 2) -12 3) 40 4) 53

7. Let $S = \frac{8}{5} + \frac{16}{65} + \ldots + \frac{128}{2^{18}+1}$, then
1) $S = \frac{1088}{545}$ 2) $S = \frac{545}{1088}$ 3) $S = \frac{1056}{545}$ 4) $S = \frac{545}{1056}$

8. The sum of the infinite terms of the series $\frac{5}{3^2.7^2} + \frac{9}{7^2.11^2} + \frac{13}{11^2.15^2} + \ldots\ldots$ is
1) $\frac{1}{18}$ 2) $\frac{1}{36}$ 3) $\frac{1}{54}$ 4) $\frac{1}{72}$

9. If an AP, $a_7 = 9$ if $a_1\, a_2\, a_7$ is least, the common difference is
1) $\frac{13}{20}$ 2) $\frac{23}{20}$ 3) $\frac{33}{20}$ 4) $\frac{43}{20}$

10. The roots of equation $x^2 + 2(a-3)x + 9 = 0$ lie between -6 and 1 and 2, $h_1, h_2, \ldots, h_{20}$, $[a]$ are in HP, where $[a]$ denotes the integral part of a and 2, $a_1, a_2, \ldots\ldots, a_{20}$, $[a]$ are in AP, then $a_3 h_{18}$ is equal to
1) 6 2) 12 3) 3 4) 10

11. Given that $0 < x < \frac{\pi}{4}$ and $\frac{\pi}{4} < y < \frac{\pi}{2}$ and $\sum_{k=0}^{\infty} (-1)^k \tan^{2k} x = a$, $\sum_{k=0}^{\infty} (-1)^k \cot^{2k} y = b$, then $\sum_{k=0}^{\infty} \tan^{2k} x \cot^{2k} y$ is
1) $\frac{1}{a} + \frac{1}{b} - \frac{1}{ab}$ 2) $a + b - ab$ 3) $\frac{1}{\frac{1}{a} + \frac{1}{b} - \frac{1}{ab}}$ 4) $\frac{ab}{a+b-1}$

12. The sum of the series $\sqrt{3} + 3\sqrt{2} + 6\sqrt{3} + \ldots\ldots$ up to 16 terms
1) $335923\,(\sqrt{18} + \sqrt{3})$ 2) $335923\,\sqrt{18}$ 3) $335923\,\sqrt{3}$ 4) none of these

13. Sum of certain number of terms n, of the series $\frac{2}{9}, \frac{-1}{3}, \frac{1}{2}, \ldots$ is $\frac{55}{72}$, then $n =$
1) 4 2) 5 3) 6 4) 1

14. Let $P = 3^{\frac{1}{3}} . 3^{\frac{2}{9}} . 3^{\frac{3}{27}} \ldots . \infty$, then $P^{\frac{1}{3}}$ is equal to :

1) $3^{\frac{1}{3}}$ 2) $3^{\frac{1}{4}}$ 3) $3^{\frac{1}{2}}$ 4) $3^{\frac{1}{6}}$

15. If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a, b, c are non-zero real numbers, then a, b, c are in :
1) A.P 2) G.P 3) H.P 4) none of these

16. Find the sum of first 24 terms of the A.P a_1, a_2, a_3 …….. if it is known that
$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$
1) 600 2) 700 3) 900 4) none of these

17. If S_1, S_2, S_3 are the sums of $n, 2n, 3n$ terms respectively of an A.P., then $\frac{S_3}{(S_2 - S_1)} =$
1) 1 2) 2 3) 3 4) none of these

18. If $1 . 3 + 3 . 3^2 + 5 . 3^2 + 7 . 3^4 + \ldots$ upto 'n' terms is equal to $3 + (n-1) . 3^b$, then $b =$
1) $n - 1$ 2) $n + 1$ 3) $2n + 1$ 4) none of these

19. The value of $S = \frac{5}{1^2 . 4^2} + \frac{11}{4^2 . 7^2} + \frac{17}{7^2 . 10^2} + \ldots \infty$ is
1) 1 2) 1/2 3) 1/3 4) 1/4

20. The sum of $(x+2)^{n-1} + (x+2)^{n-2}(x+1) + (x+2)^{n-3}(x+1)^2 + \ldots + (x+1)^{n-1}$ is equal to
1) $(x+2)^{n-2} - (x+1)^n$ 2) $(x+2)^{n-2} - (x+1)^{n-1}$ 3) $(x+2)^n - (x+1)^n$ 4) none of these

21. If $a_i > 0$, $i = 1, 2, 3, \ldots, 50$ and $a_1 + a_2 + a_3 + \ldots + a_{50}$, then the minimum value of $\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_{50}}$ is equal to

22. If $\sum_{n=1}^{k} \left[\frac{1}{3} + \frac{n}{90}\right] = 21$, where $[x]$ denotes the integral part of x, then 'k' is equal to

23. Let $S_k : k = 1, 2, \ldots, 100$ denotes the sum of the infinite G.P, whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$ is :

24. The length of three unequal edges of a rectangular solid block are in G.P. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. The length of the longest edge is

25. Let A_n be the sum of the first n terms of the geometric series $704 + \frac{704}{2} + \frac{704}{4} + \frac{704}{8} + \ldots$ and B_n be the sum of the first n terms of the geometric series $1984 - \frac{1984}{2} + \frac{1984}{4} - \frac{1984}{8} + \ldots$
If $A_n = B_n$, then the value of 'n' is (where $n \in N$)

26. The value of $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1 = 220$, then the value of 'n' equals

27. If $\sum_{r=1}^{n} \frac{r^4 + r^2 + 1}{r^4 + r} = \frac{675}{26}$, then '$n$' is equal to

28. If x, y, z and w are positive integers such that $x + 2y + 3z + 4w = 50$, then maximum value of $\left(\frac{x^2 y^4 z^3 w}{16}\right)^{1/10}$ is

29. If $\sum_{k=1}^{n} \left(\sum_{m=1}^{k} m^2 \right) = an^4 + bn^3 + cn^2 + dn + e$, then $a + b + c + d + e =$

30. Let $U_n = \frac{(n+1)!}{(n+3)!}$, $n \in N$, if $S_n = \sum_{n=1}^{n} U_n$, then $\lim_{n \to \infty} S_n$ equals

HINTS & SOLUTIONS

1-10	**2**	**3**	**3**	**3**	**3**	**4**	**1**	**4**	**3**	**2**
11-20	**3**	**1**	**2**	**2**	**3**	**3**	**3**	**2**	**3**	**3**
21-30	**50**	**8**	**3**	**12**	**5**	**10**	**25**	**5**	**1**	**1**

1. The number of digits in each term of the sequence are 1, 2, 4, 8, ... which are in GP

Let 1025^{th} term is 2^n,

Then $1 + 2 + 4 + 8 + \ldots + 2^{n-1} < 1025 \le 1 + 2 + 4 + 8 + \ldots + 2^n$

$\Rightarrow 1.\dfrac{(2^n - 1)}{(2-1)} < 1025 \le 1.\dfrac{(2^{n+1}-1)}{(2-1)}$

$\Rightarrow 2^n - 1 < 1025 \le 2^{n+1} - 1$ (or) $2^{n+1} \ge 1026 > 1024$ …….. (i)

$\Rightarrow 2^{n+1} > 2^{10}$ (or) $n + 1 > 10$

$\therefore$ $n > 9$

$\therefore$ n = 10 (which is always satisfy equation (i),

$\therefore$ 1025^{th} term is 2^{10}.

2. $\because$ LHS $= \dfrac{1(1-\lambda^{n+1})}{(1-\lambda} = \left(\dfrac{1-\lambda^{n+1}}{1-\lambda}\right)$, and RHS $= (1 + \lambda)(1 + \lambda^2)(1 + \lambda^4)(1 + \lambda^8)(1 + \lambda^{16})$

$= \dfrac{(1-\lambda)(1+\lambda)(1+\lambda^2)(1+\lambda^4)\ (1+\lambda^8)\ (1+\lambda^{16})}{(1-\lambda)} = \dfrac{(1-\lambda^{32})}{(1-\lambda)}$

$\Rightarrow \dfrac{1-\lambda^{n+1}}{1-\lambda} = \dfrac{1-\lambda^{32}}{1-\lambda}$

$\Rightarrow 1 - \lambda^{n+1} = 1 - \lambda^{32}$

$\therefore$ $n + 1 = 32 \Rightarrow n = 31$

3. $x = \dfrac{1}{1-a},\ y = \dfrac{1}{1-b},\ z = \dfrac{1}{1-c}$ (or) $a = 1 - \dfrac{1}{x},\ b = 1 - \dfrac{1}{y},\ c = 1 - \dfrac{1}{z}$

$\because$ *a, b, c* are in AP

$1 - \dfrac{1}{x},\ 1 - \dfrac{1}{y},\ 1 - \dfrac{1}{z}$ are in AP

$\Rightarrow -\dfrac{1}{x},\ -\dfrac{1}{y},\ -\dfrac{1}{z}$ are in AP (or) $\dfrac{1}{x},\ \dfrac{1}{y},\ \dfrac{1}{z}$ are in AP $\therefore$ *x, y, z* are in HP

4. We have $(x - 1)(x^2 - 2)(x^3 - 3)(x^4 - 4) \ldots\ldots (x^{20} - 20)$ ……. (i)

$= x^{190}(x-1)\left(x - \dfrac{2}{x}\right)\left(x - \dfrac{3}{x^2}\right)\left(x - \dfrac{4}{x^3}\right)\ldots\ldots\left(x - \dfrac{20}{x^{19}}\right)$

$$= x^{190}\left\{\begin{matrix} x^{20} - x^{19}\left(1 + \dfrac{2}{x} + \dfrac{3}{x^2} + \dfrac{4}{x^3} + \dfrac{5}{x^4} + \ldots\right) + x^{18}\left(\left(\dfrac{2}{x} + \dfrac{3}{x^2} + \dfrac{4}{x^3} + \ldots\right) + \left(\dfrac{6}{x^3} + \dfrac{8}{x^4} + \dfrac{10}{x^5} + \ldots\right) + \dfrac{12}{x^5}\right) \\ -x^{17}\left(\dfrac{6}{x^3} + \dfrac{8}{x^4} + \ldots\ldots\right) + x^{16}(\ldots\ldots) \end{matrix}\right\}$$

$\therefore$ coefficient of x^{203} in Eq … (i)

$= -7 + 6 + 10 + 12 - 8 = 13$

5. $\therefore$ n^{th} term $T_n = \dfrac{1 + 2 + 3 + \ldots\ldots + n}{1^3 + 2^3 + 3^3 + \ldots\ldots n^3}$

$= \dfrac{2}{\left[\dfrac{n(n+1)}{2}\right]^2} = \dfrac{2}{n(n+1)}$

$= 2\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$

Putting n = 1,2, 3, 4, ……, n

$\therefore T_1 + T_2 + T_3 + \ldots\ldots + T_n = 2\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots+\frac{1}{n}-\frac{1}{n+1}\right)$

$= 2\left(1-\frac{1}{n+1}\right)$

$S_n = 2\left(1-\frac{1}{n+1}\right)$

$\therefore S_n + 199 = \frac{2n}{n+1} + 199 > 199$ for all 'n'

6. Since, $3^{2\sin 2x-1}$, 14, $3^{4-2\sin 2x}$ are in AP

$\therefore 28 = 3^{2\sin 2x}\cdot\frac{1}{3}+3^4\cdot\frac{1}{3^{2\sin 2x}}$

Put $3^{2\sin 2x} = t$

$\therefore 28 = \frac{t}{3}+\frac{81}{t}$

$\Rightarrow \quad 84t = t^2 + 243$

$\Rightarrow \quad t^2 - 84t + 243 = 0$

$(t-81)(t-3) = 0$

$\therefore \quad t = 3, 81$

$3^{2\sin 2x} = 3^1, 3^4$

$\therefore \quad 2\sin 2x = 1, 4$

$\text{Sin } 2x = \frac{1}{2}, 2 \;(\because \sin 2x \neq 2)$

$\therefore \quad \sin 2x = \frac{1}{2}$

$\therefore$ first term $= 3^{1-1} = 1$

Second term = 14

Third term = 27

Here, common difference = 13

$\therefore$ fifth term = 1 + 4 x 13 = 53.

7. $\because S = \sum_{r=1}^{16n}\frac{8r}{(4r^4+1)}$

$= \sum_{r=1}^{16}\frac{8r}{(2r^2-2r+1)(2r^2+2r+1)}$

$= 2\sum_{r=1}^{16}\left(\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right)$

$= 2\left(\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+\frac{1}{13}+\ldots\ldots+\frac{1}{481}-\frac{1}{545}\right)$

$= 2\left(1-\frac{1}{545}\right) = \frac{1088}{545}$

8. $T_n = \frac{5+(n-1)4}{[3+(n-1)4]^2[7+(n-1)4]^2}$

$= \frac{1}{8}\left\{\frac{1}{(4n-1)^2}-\frac{1}{(4n+3)^2}\right\}$

$\therefore S_n = T_1 + T_2 + T_3 + \ldots\ldots + T_n$

$= \frac{1}{8}\left\{\frac{1}{3^2}-\frac{1}{7^2}+\frac{1}{7^2}-\frac{1}{11^2}+\frac{1}{11^2}-\frac{1}{15^2}+\ldots\ldots+\ldots+\frac{1}{(4n-1)^2}-\frac{1}{(4n+3)^2}\right\}$

$= \frac{1}{8}\left[\frac{1}{3^2}-\frac{1}{(4n+3)^2}\right]$

$$\therefore\ S_\infty = \frac{1}{8}\left\{\frac{1}{9} - 0\right\} = \frac{1}{72}$$

9. Let 'd' be the common difference

$\because$ $a_7 = 9$

$\therefore$ $a_1 + 6d = 9$

Let $D = a_1\, a_2\, a_7$

$= (9 - 6d)(9 - 5d)\, 9$

$$= 270\left\{\left(d - \frac{33}{20}\right)^2 - \frac{9}{400}\right\}$$

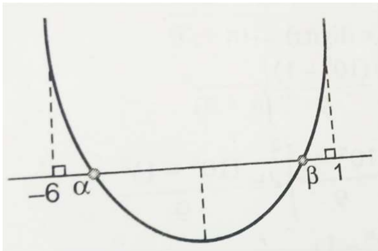
For least value of D,

$$d - \frac{33}{20} = 0$$

$$d = 33/20$$

10. $\because f(x) = x^2 + 2(a - 3)x + 9$

(i) $f(-6) > 0$ (ii) $f(1) > 0$ (iii) $D \geq 0$ (iv) $-6 < \dfrac{-b}{2a} < 1$



Then we get $6 \leq a < 6.25$

$\therefore$ $[a] = 6$

Now, 2, h_1, h_2, ….., h_{20}, $[a]$ are in HP

i.e., 2, h_1, h_2, ….. h_{20}, 6 are in HP

$$\therefore\ \frac{1}{h_{18}} = \frac{1}{2} + 18\left(\frac{\frac{1}{6} - \frac{1}{2}}{21}\right)$$

$$\Rightarrow\ h_{18} = \frac{14}{3}$$

And 2, a_1, a_2, ….., a_{20}, 6 are in AP

$$\therefore\ a_3 = 2 + 3\left(\frac{6-2}{21}\right) = \frac{18}{7}$$

$$\therefore\ a_3\, h_{18} = \frac{18}{7} \cdot \frac{14}{3} = 12$$

11. $a = \dfrac{1}{1 + \tan^2 x}$

$\Rightarrow \tan^2 x = \dfrac{1}{a} - 1$ and $b = \dfrac{1}{1 + \cot^2 y}$

$= \cot^2 y = \dfrac{1}{b} - 1$

Then $\displaystyle\sum_{k=0}^{\infty} \tan^{2k} x \cot^{2k} y$

$$= \frac{1}{1 - \tan^2 x \cot^2 y} = \frac{1}{1 - \left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)}$$

$$= \frac{ab}{ab - (1-a)(1-b)}$$

$$= \frac{ab}{a + b - 1} \quad \text{(or)} \quad \frac{1}{\frac{1}{a} + \frac{1}{b} - \frac{1}{ab}}$$

12. $a=\sqrt{3}$, $r=\dfrac{3\sqrt{2}}{\sqrt{3}}=\sqrt{3}\times\sqrt{2}=\sqrt{6}$

$\because\ n=16$

$\therefore$ Required sum = $\dfrac{\sqrt{3}[(\sqrt{6})^{16}-1]}{(\sqrt{6}-1)}=\dfrac{\sqrt{3}(6^{8}-1)}{(\sqrt{6}-1)}$

$=\dfrac{\sqrt{3}}{(\sqrt{6}-1)}(1679615)$

$= 335923\ (\sqrt{18}+\sqrt{3}\ )$

13. Here, $a=\dfrac{2}{9}$, $r=-\dfrac{1}{3}\times\dfrac{9}{2}=-\dfrac{3}{2}$

$S_{\text{n}}=\dfrac{a(1-r^{n})}{(1-r)}=\dfrac{55}{72}$

$\Rightarrow\ \dfrac{2}{9}\dfrac{\left[1-\left(-\dfrac{3}{2}\right)^{n}\right]}{\left(1-\left(\dfrac{-3}{2}\right)\right)}=\dfrac{55}{72}$

$\Rightarrow 1-\left(\dfrac{-3}{2}\right)^{n}=\dfrac{55}{72}\times\dfrac{9}{2}\times\dfrac{5}{2}=\dfrac{275}{32}$

$\Rightarrow 1-\dfrac{275}{32}=\left(\dfrac{-3}{2}\right)^{n}$

$\Rightarrow \dfrac{-243}{32}=\left(\dfrac{-3}{2}\right)^{n}\Rightarrow\left(\dfrac{-3}{2}\right)^{5}=\left(\dfrac{-3}{2}\right)^{n}$

$\Rightarrow\ n=5$

14. $P=3^{S}$ where $S=\dfrac{1}{3}+\dfrac{2}{9}+\dfrac{3}{27}+\ .\ .\ .\ .\ \infty$

Which is an infinite arithmetic-geometric series with $a=1$, $d=1$ for A.P and $b=\dfrac{1}{3}$, $r=\dfrac{1}{3}$, for G.P

$\therefore\qquad S_{\infty}=\dfrac{ab}{1-r}+\dfrac{dbr}{(1-r)^{2}}$

$==\dfrac{\frac{1}{3}}{1-\frac{1}{3}}+\dfrac{1.\frac{1}{3}.\frac{1}{3}}{\left(1-\frac{1}{3}\right)^{2}}=\dfrac{1}{2}+\dfrac{1}{4}=\dfrac{3}{4}\qquad\Rightarrow\quad P=3^{5}=3^{\frac{3}{4}}$

15. $(2a)^2+3(b)^2+4(c)^2-(2a)(3b)-(3b)(4c)-4(c)(2a)=0$

if is the form $\quad x^2+y^2+z^2-xy-yz-zx=0$

or $\quad \frac{1}{2}[(x-y)^2+(y-z)^2+(z-x)^2]=0$

$\Rightarrow\quad x-y=0,\ y-z=0,\ z-x=0\qquad\Rightarrow x=y=z$

$\Rightarrow\quad 2a=3b=4c$

$\Rightarrow\quad \dfrac{a}{\frac{1}{2}}=\dfrac{b}{\frac{1}{3}}=\dfrac{c}{\frac{1}{4}}=k$

$\Rightarrow\quad a=\dfrac{k}{2},\ b=\dfrac{k}{3},\ c=\dfrac{k}{4}$

Therefore, $\dfrac{1}{a},\dfrac{1}{b},\dfrac{1}{c}$ *and* $\dfrac{2}{k},\dfrac{3}{k},\dfrac{4}{k}$ which are in AP

$\Rightarrow$ *a, b, c* are in H.P

16. sum of the terms equidistant from the beginning = sum of the terms equidistant from end
= sum of first and last term

Since, $a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$

Here we have

$(a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow \quad (a_1 + a_{24}) + (a_1 + a_{24}) + (a_1 + a_{24}) = 225$

$\Rightarrow \quad 3\,(a_1 + a_{24}) = 225$

$\Rightarrow \quad a_1 + a_{24} = 75$

Consider, $\quad S_n = \frac{n}{2}[a + 1]$

Therefore, $\quad S_{24} = \frac{24}{2}[a_1 + a_{24}] = 12 \times 75 = 900$

17. Here, $\quad S_1 = \frac{n}{2}[2a + (n-1)\,d]$

$S_2 = n[\,2a + (2n-1)d\,]$

Now, $\quad S_2 - S_1 = na\,(3n-1)\frac{nd}{2} = \frac{n}{2}[2a + (3n-1)\,d\,]$

$S_3 = \frac{3n}{2}[\,2a + (3n-1)\,d\,]$, $\qquad$ thus $\frac{S_3}{S_2 - S_1} = 3$

18. here, $S = 1.3 + 3.3^2 + 5.3^3 + 7.3^4 + \ldots\ldots. + (2n-1)3^n$ …….. (i)

$3S = 1\,.\,3^2 + 3\,.\,3^3 + + \ldots\ldots\ldots.. + (2n-3)\,3^n + (2n-1)\,3^{n+1}$ …….. (ii)

Eq. (i) – (ii) , then we get

$-2S = 1.3 + 2\,[3^2 + 3^3 + \ldots\ldots + 3^n] - (2n-1)\,3^{n+1}$

$S = \left(\frac{2n-1}{2}\right)3^{n+1} - \frac{2.3^2\,(3^{n-1)} - 1)}{2(3-1)} - \frac{1.3}{2}$

$= 3 + (n-1)\,3^{n+1}$

It is given that $= 3 + (n-1)\,3^b \Rightarrow \; b = n + 1$

19. $S = \frac{5}{1^2.4^2} + \frac{11}{4^2.7^2} + \frac{17}{7^2.10^2} + \ldots\ldots.\infty$

$3S = \frac{3.5}{1^2.4^2} + \frac{3.11}{4^2.7^2} + \frac{3.17}{7^2.10^2} + \ldots\ldots.\infty$

$\Rightarrow 3S = \frac{(4-1)(4+1)}{1^2\,.\,4^2} + \frac{(7-4)(7+4)}{4^2\,.\,7^2} + \frac{(10-7)(10+7)}{7^2\,.\,10^2} + \ldots\ldots\infty$

$\Rightarrow S = \frac{1}{3}$

20. $S = (x+2)^{n-1} + (x+2)^{n-2}\,(x+1) + (x+2)^{n-3}\,(x+1)^2 + \ldots\ldots + (x+1)^{n-1}$

Given series is G.P with first term $(x + 2)^{n-1}$, common ratio $\frac{x+1}{x+2}$ and number of terms n.

$$\therefore S = \frac{(x+2)^{n-1}\left[1 - \left(\frac{x+1}{x+2}\right)^n\right]}{1 - \frac{x+1}{x+2}}$$

$$= (x+2)^n\left[1 - \left(\frac{x+1}{x+2}\right)^n\right] = (x+2)^n - (x+1)^n$$

21. $\because$ AM $\geq$ HM

$$\therefore \frac{a_1 + a_2 + a_3 + \ldots + a_{50}}{50} \geq \frac{50}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_{50}}\right)} \quad \text{(or)} \quad \frac{50}{50} \geq \frac{50}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_{50}}\right)}$$

$$\Rightarrow \frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_{50}} \geq 50$$

Hence, minimum value of $\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_{50}}$ is 50

22. If $\left[\frac{1}{3}+\frac{n}{90}\right]=0 \Rightarrow 0\leq\frac{1}{3}+\frac{n}{90}<1$

$\Rightarrow \quad 1 \leq n < 60$

and if $\left[\frac{1}{3}+\frac{n}{90}\right] = 1 \Rightarrow 1 \leq \frac{1}{3}+\frac{n}{90} < 2$

$\Rightarrow \quad 60 \leq n < 150$

$$\because \sum_{n=1}^{k}\left[\frac{1}{3}+\frac{n}{90}\right] = 21$$

$$\Rightarrow \quad \sum_{n=1}^{59}\left[\frac{1}{3}+\frac{n}{90}\right] + \sum_{n=60}^{k}\left[\frac{1}{3}+\frac{n}{90}\right] = 21$$

$$\Rightarrow \quad 0 + \sum_{n=60}^{k}\left[\frac{1}{3}+\frac{n}{90}\right] = 21$$

Which is possible

Only when $k=80\left(\begin{array}{l}\because 60 \leq n < 150 \\ \therefore \left[\frac{1}{3}+\frac{n}{90}\right] = 1\end{array}\right)$

23. $\sum_{k=2}^{100} |(k^2-3k+1)S_k| \quad for \quad k=2,\ |(k^2-3k+1)||S_k|=1$

$$\sum_{k=3}^{100}\left|\frac{k-1}{(k-2)!}+\frac{k-1+1}{(k+1)!}\right|$$

$$= \sum_{k=3}^{100} \frac{1}{(k-3)!}+\frac{1}{(k-2)!}-\frac{1}{(k-2)!}-\frac{1}{(k-1)!}$$

$$= \sum_{k=3}^{100}\left(\frac{1}{(k-3)!}-\frac{1}{(k-1)!}\right)$$

$$S=1+\left(1-\frac{1}{2!}\right)+\left(\frac{1}{1!}-\frac{1}{3!}\right)+\left(\frac{1}{2!}-\frac{1}{4!}\right)+\left(\frac{1}{3!}-\frac{1}{5!}\right)+\left(\frac{1}{4!}-\frac{1}{6!}\right)+\ldots\ldots$$

$$+\left(\frac{1}{94!}-\frac{1}{96!}\right)+\left(\frac{1}{95!}-\frac{1}{97!}\right)+\left(\frac{1}{96!}-\frac{1}{98!}\right)+\left(\frac{1}{97!}-\frac{1}{99!}\right)$$

$$= 2-\frac{1}{98!}-\frac{1}{99!}$$

So, $\quad E=\frac{100^2}{100!}+3-\frac{1}{98!}-\frac{1}{99.98!}=\frac{100^2}{100!}+3-\frac{100}{99!}$

$$= \frac{100}{100.99!}+3-\frac{100}{99!} = 3$$

24. Let a, ar, ar^2 be the edges of rectangular solid block

Then, volume = 216 cm^3

$\Rightarrow \quad a\,(ar)\,(ar)^2 = 216$

$\Rightarrow \quad (ar)^3 = 216 \qquad \Rightarrow \qquad ar = 6$ ……. (i)

Now, total surface area = 252 cm^2

$\Rightarrow \quad 2\,[a\,(ar) + ar\,(ar)^2 + a\,(ar^2)\,] = 252$

Using eq. (i), we get

$\Rightarrow \quad 2\,(6a + 36r + 36) = 252$

$\Rightarrow \quad 12\,(a + 6r + 6) = 252$

$\Rightarrow \quad a + 6r = 15 \quad \ldots\ldots \text{(ii)}$

$\Rightarrow \quad a+6\times\left(\dfrac{6}{a}\right)=15 \qquad$ [from (i)]

$\Rightarrow \quad a^2 - 15a + 36 = 0$

$\Rightarrow \quad a = 3,\ 12 \qquad$ [using (ii)]

If $a = 3$, $r = 2$ and $a = 12$, $r = ½$ then edges are 3, 6, 12 (or) 12, 6, 3

Therefore the length of longest edge is 12 cm

25. $A_n = 704+\dfrac{704}{2}+\dfrac{704}{4}+$ .... to n terms

$$= \frac{704\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}=704\times 2\left(1-\left(\frac{1}{2}\right)^n\right)$$

$B_n = 1984 - \dfrac{1984}{2}+\dfrac{1984}{4}+$......to n terms

$$= \frac{1984\left(1-\left(\frac{-1}{2}\right)^n\right)}{1-\left(\frac{-1}{2}\right)} = 1984\times\frac{2}{3}\left(1-\left(\frac{-1}{2}\right)^n\right)$$

Now, $A_n = B_n \Rightarrow 704\times 2\left(1-\left(\frac{1}{2}\right)^n\right) = 1984\times\frac{2}{3}\left(1-\left(\frac{-1}{2}\right)^n\right)$

$$33\left(1-\left(\frac{1}{2}\right)^n\right) = 31\left(1-\left(\frac{-1}{2}\right)^n\right)$$

$$33-31=33\left(\frac{1}{2}\right)^n - 31\left(\frac{-1}{2}\right)^n$$

$2^{n+1} = 33 - 31\,(-1)^n$

$\Rightarrow n = 5$

26. $\displaystyle\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}1$

$$= \sum_{i=1}^{n}\sum_{j=1}^{i} j$$

$$=\sum_{i=1}^{n}\frac{i\,(i+1)}{2}$$

$$= \frac{1}{2}\sum_{i=1}^{n}(i^2+i)$$

$$=\frac{n(n+1)(n+2)}{6}= 220$$

$\therefore\ n = 10$

27. $T_r = \dfrac{r^4 + r^2 + 1}{r^4 + r}$

$= \dfrac{(r^2 + r + 1)(r^2 - r + 1)}{r(r+1)(r^2 - r + 1)}$

$= \dfrac{r^2 + r + 1}{r(r+1)}$

$= 1 + \dfrac{1}{r} - \dfrac{1}{r+1}$

So,

$T_1 = 1 + \dfrac{1}{1} - \dfrac{1}{2}$

$T_2 = 1 + \dfrac{1}{2} - \dfrac{1}{3}$

$T_3 = 1 + \dfrac{1}{3} - \dfrac{1}{4}$

….. ……. ……..

$T_n = 1 + \dfrac{1}{n} - \dfrac{1}{n+1}$

$\therefore\ S = n + 1 - \dfrac{1}{n+1} = \dfrac{675}{26}$

$\therefore\ 26\,(n+1)^2 - 26 = 675\,(n+1)$

$\Rightarrow 26\,(n+1)^2 - 675\,(n+1) - 26 = 0$

$\Rightarrow 26\,(n+1)\,[\,n + 1 - 26] + [\,(n+1) - 26\,] = 0$

$\Rightarrow (n - 25)\,(26n + 27) = 0$

$\therefore\ n = 25$

28. We have $x + 2y + 3z + 4w = 50$

Using the fact A.M $\geq$ G.M., we get

$$\frac{2\left(\frac{x}{2}\right) + 4\left(\frac{y}{2}\right) + 3\left(\frac{z}{1}\right) + 1\left(\frac{4w}{1}\right)}{2+4+3+1} = \frac{50}{10} \geq \left[\left(\frac{x}{2}\right)^2 \left(\frac{y}{2}\right)^4\ (z)^3\ (4w)\right]^{1/10}$$

$$\Rightarrow\ 5 \geq \left[\left(\frac{x^2}{2^2}\right)\left(\frac{y^4}{2^4}\right)(z)^3\,(2^2\,w)\right]^{1/10}$$

$$\Rightarrow\ 5 \geq \left(\frac{x^2\,y^4\,z^3\,w}{16}\right)^{1/10}$$

29. $\displaystyle\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = \sum_{k=1}^{n}(1^2 + 2^2 + 3^2 + \ldots\ldots + k^2)$

$= \displaystyle\sum_{k=1}^{n}\frac{k(k+1)(2k+1)}{6}$

$= \dfrac{1}{6}\displaystyle\sum_{k=1}^{n}(2k^3 + 3k^2 + k)$

$= \dfrac{1}{3}\left\{\dfrac{n(n+1)}{2}\right\}^2 + \dfrac{1}{2}\left\{\dfrac{n(n+1)(2n+1)}{6}\right\} + \dfrac{1}{6}\left\{\dfrac{n(n+1)}{2}\right\}$

$\dfrac{1}{12}\{n^4 + 4n^3 + 5n^2 + 2n\}$

$\therefore\ a = \dfrac{1}{12},\ b = \dfrac{1}{3},\ c = \dfrac{5}{12},\ d = \dfrac{1}{6},\ e = 0$

So, $a + b + c + d + e = 1$

30. $U_n = \dfrac{1}{(n+2)(n+3)}$

$$=\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$

$$S_n = \sum_{n=1}^{n}\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$

$$=\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right).....+\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$

$$=\frac{1}{3}-\frac{1}{n+3}$$

$$\therefore \lim_{n\to\infty} S_n = \frac{1}{3}$$