SYSTEM OF CIRCLE

Maths - B

1. The radical center of three circles described on the three sides 4x-7y+10=0 , x+y-5=0 and 7x+4y-15=0 of triangle as diameter is
   1) (3, 2) 2) (1, 2) 3) (0, 4) 4) (1, 1)
2. The locus of the center of a circle of a which cuts orthogonally the circle $x^2+y^2-20x+4=0$ and which touches $x=2$ is
   1) $y^2=16x+4$ 2) $x^2=16y$ 3) $x^2=16y+4$ 4) $y^2=16x$
3. The coordinates of the center of the circle which intersects circles $x^2+y^2+4x+7=0$, $2x^2+2y^2+3x+5y+9=0$ $and\ x^2+y^2+y\neq 0$ orthogonally are
   1) (-2, 1) 2) (-2,-1) 3) (2, -1) 4) (2, 1)
4. The equation of the circle passing through the point (2a,0) and whose radical axis is $x=\frac{a}{2}$ with respect to the circle $x^2+y^2=a^2$, will be
   1) $x^2+y^2-2ax=0$ 2) $x^2+y^2+2ax=0$ 3) $x^2+y^2+2ay=0$ 4) $x^2+y^2-2ay=0$
5. If the lengths of tangents drawn to the circles $x^2+y^2-8x+40=0$, $5x^2+5y^2-25x+80=0$, $x^2+y^2-8x+16y+160=0$ from the point P are equal, then P=
   1) $\left(8,\frac{15}{2}\right)$ 2) $\left(-8,\frac{15}{2}\right)$ 3) $\left(8,\frac{-15}{2}\right)$ 4) $\left(-8,\frac{-15}{2}\right)$
6. If A, B, C are the centers of three circles touching externally then the radical center of the circles is
   1) the orthocenter 2) the circumcenter 3) the in center 4) the center of ΔABC
7. You are given $n(n\geq 3)$ circles having different radical axes and radical centers. The value of n for which the number of radical axes is equal to the number of radical centers is
   1) 3 2) 4 3) 5 4) 8
8. If two circles cut a third circle S orthogonally, then the radical axis of the two circles
   1) Touches 2) does not meet 3) passes through the center of S 4) none
9. r_1, r_2 are the radii of two non-interesecting circles having A, B as centers. If P is the midpoint of AB then the perpendicular distance of P from their radical axis is
   1) $\frac{r_1^2+r_2^2}{2AB}$ 2) $\frac{r_1^2-r_2^2}{2AB}$ 3) $\frac{2AB}{r_1^2-r_2^2}$ 4) $none$
10. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its center is
    1) $x^2+y^2-3ax-4by+(a^2+b^2+p^2)=0$ 2) $2ax+2by-\left(a^2-b^2+p^2\right)=0$
    3) $x^2+y^2-2ax-3by+(a^2+b^2+p^2)=0$ 4) $2ax+2by-\left(a^2+b^2+p^2\right)=0$
11. The length of the common chord of the circles $x^2+y^2+ax+by+c=0$ and $x^2+y^2+bx+ay+c=0$ is is
    1) $\sqrt{\frac{(a+b)^2-8c}{2}}$ 2) $\sqrt{\frac{(a-b)^2-8}{2}}$ 3) $\sqrt{\frac{(a-b)^2+8c}{2}}$ 4) $\sqrt{\frac{(a+b)^2+8c}{2}}$

12. The equation of the circle cutting orthogonally the circles $x^2+y^2+2x+8=0$, $x^2+y^2-8x+8=0$ and which touches the line $x-y+4=0 is$

1) $x^2+y^2+4y=0$ 2) $x^2+y^2+8y+8=0$, 3) $x^2+y^2+16y-8=0$, 4) $x^2+y^2+16y-16=0$,

13. The condition that the two circles which passes through the points (0, a), (0,-a) and touch the line $y=mx+c$ will cut orthogonally is

1) $c^2=a^2\left(1+m^2\right)$ 2) $c^2=a^2\left(2+m^2\right)$ 3) $c^2=a^2\left(3+m^2\right)$ 4) $c^2=a^2\left(4+m^2\right)$

14. A circle passes through the points (3,4) and cuts the circle $x^2+y^2=a^2$ orthogonally ;the locus of its centre is a striaght line .If the distance of this striaght line from the origin is 25,then $a^2=$

1) 250 2) 225 3) 100 4) 25

15. The locus of centres of all circles which touch the line $x=2a$ and cut the circle $x^2+y^2=a^2$ orthogonally is

1) $y^2+4ax-5a^2=0$ 2) $y^2+4ax+5a^2=0$ 3) $y^2=4ax+5a^2$ 4) $y^2=4ax-5a^2$

16. The line $2x+4y=1$ intersects the circle $x^2+y^2=4$ at A and B. If the equation of the circle on AB as diameter is $x^2+y^2+2gx+2+y+c=0$ then $10\left(g+f\right)=$

1) -3 2) 4 3) 7 4) 3

17. If the common chord of the circles $x^2+\left(y-\lambda\right)^2=16$ and $x^2+y^2=16$ subtend a right angle at the origin ,then $\lambda=$

1) 8 2) $\pm4\sqrt{2}$ 3) $4\sqrt{2}$ 4) none

18. $x^2+y^2+6x+8y-7=0$ and a circle passing through (0,0) and touching $y=x$ have a common chord.Then that chord always passes through the point

1) $\left(\frac{1}{2},\frac{1}{2}\right)$ 2) $\left(-\frac{1}{2},-\frac{1}{2}\right)$ 3) $\left(1,1\right)$ 4) none

19. Equation to a system of circles is $2\left(x^2+y^2\right)+\lambda x-\left(1+\lambda^2\right)y-10=0$. Then number of circles belonging to the system that are orthogonal to $x^2+y^2+4x+6y+3=0$ is

1) 2 2) 1 3) 0 4) none

20. The length of tangents from $\left(\alpha,\beta\right)$ to both circles $x^2+y^2-4x-5=0$ and $x^2+y^2+6x-2y+6=0$ are eqaul then

1) $2\alpha+10\beta+11=0$ 2) $2\alpha-10\beta+11=0$ 3) $10\alpha-2\beta+11=0$ 4) $10\alpha+2\beta+11=0$

21. $P=\left(x_1,y_1\right)$ *and* $Q=\left(x_2,y_2\right)$ and O is the origin.The length of the common chord of circles drawn on OP and OQ as diameter is

1) $PQ\left(x_1y_2-x_2y_1\right)$ 2) $\frac{x_1y_2-x_2y_1}{PQ}$ 3) $\frac{PQ}{x_1y_2-x_2y_1}$ 4) none

22. The equation to the radical axis of the system of circles $x^2+y^2-2ax+2by+\lambda\left(ax-by\right)=0$ *is*

1) $ax+by=0$ 2) $ax-by=0$ 3) $\lambda\left(ax+by\right)=0$ 4) none

23. The radical centre of the circles $x^2+y^2+a_rx+b_ry+c=0, r=1,2,3$ is

1) (a,b) 2) (b,a) 3) (0,0) 4) none

24. The circle orthogonal to the three circles $x^2+y^2a_ix+b_iy+c=0\left(i=1,2,3\right)$ is

1) $x^2+y^2-b_ix-a_iy-c=0$ 2) $x^2+y^2=c$

3) $x^2+y^2=a_i+b_i$ 4) $x^2+y^2=c^2$

25. The common chord of $x^2+y^2-4x-4y=0$ *and* $x^2+y^2=16$ subtends at the origin an angle equal to

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

26. The perpendicular ditance from the point (1,2) to the radical axis of the circles $x^2+y^2+6x-46=0$ and $x^2+y^2-2x-6y-6=0$ is

27. If the locus of the centre of the circle which cuts the circles $x^2+y^2+4x-2y-4=0$ and $x^2+y^2-4x-6y-3=0$ orthogonally is $lx+my+n=0$ then $l+m+n=$

28. The slope of radical axes of the circles $x^2+y^2+3x+4y-5=0$ and $x^2+y^2-5x+5y-6=0$ is

29. If the radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ and $2x^2+2y^2+3x+8y+2c=0$ touches the circle $x^2+y^2+2x+2y+1=0$ then f =

30. If p in the radical centre of the three circles $(x-1)^2+(y-2)^2=225,(x-4)^2+(y-1)^2=225,(x-5)^2+(y-4)^2=225$ then p_x+p_y is

KEY

1-10	2	4	2	1	3	3	3	3	2	4
11-20	1	3	2	2	1	1	2	1	1	3
21-30	2	2	3	2	4	2	11	8	2	6

SOLUTIONS

1. Since the radical centre of the three circles described on the sides of a triangle as diameters is the orthocentre of the triangle .
   Radical centre = orthocentre
   Orthocentre is the P.I of $4x-7y+10=0$ and $7x+4y-15=0$
   Orthocentre =(1,2)
   Radiacal centre =(1,2)

2. Let the circle be
   $x^2+y^2+2gx+2fy+c=0$ (i)
   It cuts the circle $x^2+y^2-20x+4=0$ orthogonally.
   $\because 2(-10g+0\times f)=c+4$
   $\Rightarrow -20g=c+4$ (ii)
   Circle (i) touches the line $x=2, i.e. x+0y-2=0$
   $\therefore \left|\dfrac{-g+0-2}{\sqrt{1^2+0^2}}\right|=\sqrt{g^2+f^2-c}$
   $\Rightarrow (g+2)^2=g^2+f^2-c$
   $\Rightarrow 4g+4=f^2-c$ (iii)
   Eliminating c from (ii) and (iii), we get
   $-16g+4=f^2+4 \Rightarrow f^2+16g=0$
   Hence ,the locus of $(-g,-f)$ is $y^2-16x=0$.

3. The required point is the radical centre of the given circles.

4. Verify the options

Let $s \equiv x^2+y^2-2ax=0$

$s^1 \equiv x^2+y^2-a^2=0$

R.A"s in $s-s^1=0$

$x^2+y^2-2ax-x^2-y^2+a^2=0$

$-2ax+a^2=0$

$2ax=a^2$

$2x=a$

$x=\frac{a}{2}$

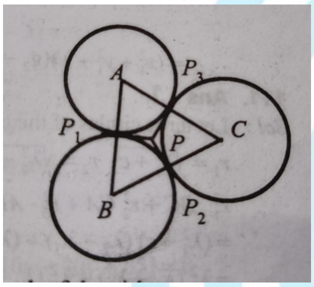
5. Given circles are $x^2+y^2-8x+40=0 \to (1), x^2+y^2-5x+16=0 \to (2)$

$x^2+y^2-8x+16y+160=0 \to (3)$

Radical axis of (1) and (2) is $3x-24=0 \Rightarrow x=8$

Radical axis of (1) and (3) is $16y+120=0 \Rightarrow y=-15/2 \therefore$ Required point $p=(8,-15/2)$.

6. Let p be the radical centre of the circles.



Let p_1, p_2, p_3 be the points of contact of the circles as shown in the figures

Now pp_1 is perpendicular to $\overline{AB}, PP_2$ is perpendicular to $\overline{BC}, PP_3$ is

Perpendicular to $\overline{CA}$. Also , p is the radical centre $\Rightarrow PP_1=PP_2=PP_3$.

$\therefore$ p is the incentre of ΔABC

7. Number of radical axes from n circle is nC_2 .Number of radical centres from n circles is nC_3

$nC_2=nC_3 \Rightarrow n=2+3=5$

8. The centre of the circles $s=0$ which cuts two given circles orthogonally lies on the radical axis of the given circles .Hence the R.A. through the centre of $S=0$.

9. Let A,B be the centre and r_1, r_2 be the radi of the circles $x^2+y^2+2\lambda_1 x+c=0$ and $x^2+y^2+2\lambda_2 x+c=0$ respectively.

$\therefore A(-\lambda_1,0)B(-\lambda_2,0)$. $r_1^2=\lambda_1^2-c, r_2^2=\lambda_2^2-c$. Midpoint of $\overline{AB}$ is $p\left(\frac{-\lambda_1-\lambda_2}{2},0\right)$

Distance from p to the R.A. $x=0$ is $\left|\frac{-\lambda_1-\lambda 2}{2}\right|=\frac{\lambda_1+\lambda_2}{2}, AB=\lambda_1-\lambda_2$

Perpendicualr distance from p to the $=\dfrac{\lambda_1+\lambda_2}{2}=\dfrac{(\lambda_1+\lambda_2)(\lambda_1-\lambda_2)}{2(\lambda_1-\lambda_2)}=\dfrac{\lambda_1^2-\lambda_2^2}{2AB}=\dfrac{r_1^2-r_2^2}{2AB}$

10. Equation of a point circle with centre (a,b) is $(x-a)^2+(y-b)^2=0$ i.e.

$x^2+y^2-2ax-2by+a^2+b^2=0$ .Given circle is $x^2+y^2-p^2=0$

Locus of the centre is radical axis is $s-s^1=0 \Rightarrow 2ax+2by-p^2-a^2-b^2=0$

11. Common chord is $(a-b)x+(b-a)y=0 \Rightarrow x-y=0$

$C=(-a/2,-b/2)$

$BC=\dfrac{b-a}{2\sqrt{2}}, AC=\sqrt{\dfrac{a^2+b^2-4c}{4}}$

$AB^2=AC^2-BC^2=\dfrac{a^2+b^2-4c}{4}-\dfrac{(b-a)^2}{8}=\dfrac{a^2+b^2-8c+2ab}{8}=\dfrac{(a+b)^2-8c}{8}$

$AB=\dfrac{\sqrt{(a+b)^2-8c}}{2\sqrt{2}}$ $\therefore$ Length of common chord $=\sqrt{\dfrac{(a+b)^2-8c}{2}}$

12. Let the required circle be $s \equiv x^2+y^2+2gx+2fy+c=0$

S=0 cuts $x^2+y^2+2x+8=0$ and $x^2+y^2-8x+8=0$ orthogonally

$\therefore 2g(1)=c+8$ and $2g(-4)=c+8 \Rightarrow g=0, c=-8$

S = 0 touches $x-y+4=0 \Rightarrow \sqrt{g^2+f^2-c}=\dfrac{|-g+f+4|}{\sqrt{2}} \Rightarrow \sqrt{f^2+8}=\dfrac{|f+4|}{\sqrt{2}}$

$\Rightarrow 2(f^2+8)=(f+4)^2 \Rightarrow 2f^2+16=f^2+8f+16 \Longrightarrow f(f-8)=0 \Rightarrow f=0\, or\, 8.$

$\therefore$ Required circle equation is $x^2+y^2+16y-8=0.$

13. Let $x^2+y^2+2gx+2fy+k=0 \rightarrow (1)$ be the circle passing through (0,a), (0,-a) and touching the line $y=mx+c.$

(1) passes through $(0,a) \Rightarrow 0+a^2+0+2fa+k=0 \Rightarrow a^2+2af+k=0 \rightarrow (2)$

(1) passes through $(0,-a) \Rightarrow 0+a^2+0-2fa+k=0 \Rightarrow a^2-2af+k=0 \rightarrow (3)$

$(2)-(3) \Rightarrow 4af=0 \Rightarrow f=0;\ (2) \Rightarrow a^2+k=0 \Rightarrow k=-a^2$

Centre of $(1) is (-g,-f), radius\, of\, (1) is \sqrt{g^2+f^2-k}$

(1) touches $y=mx+c \Rightarrow \left|\dfrac{-mg+f+c}{\sqrt{1+m^2}}\right|=\sqrt{g^2+f^2-k} \Rightarrow \dfrac{|-mg+c|}{\sqrt{1+m^2}}=\sqrt{g^2-k}$

$\Rightarrow (-mg+c)^2=(g^2-k)(1+m^2) \Rightarrow m^2g^2-2mgc+c^2=g^2+g^2m^2-k-km^2$

$\Rightarrow g^2+2gmc-c^2+a^2+a^2m^2=0$

Let g_1, g_2 be the roots of this equation .Then $g_1g_2=-c^2+a^2+a^2m^2$.

The two circles cut orthogonally $\Rightarrow 2g_1g_2+0=-2a^2 \Rightarrow g_1g_2=-a^2 \Rightarrow -c^2+a^2+a^2m^2=-a^2$

$\Rightarrow c^2=2a^2+a^2m^2 \Rightarrow c^2=a^2(2+m^2)$. $\therefore$ The required condition is $c^2=a^2(2+m^2)$.

14. Let $s \equiv x^2+y^2+2gx+2fy+c=0$

$s^1 \equiv x^2 + y^2 - a^2 = 0$

$s = 0$ *and* $s^1 = 0$ cuts orthogonally $c = a^2$

S=0 passes through (3,4)

$9 + 16 + 6g + 8f + c = 0$

$6g + 8f + a^2 + 25 = 0 \left(\therefore c = a^2 \right)$

The locus of centre $(-g, -f)$ is

$6(-x) + 8(-y) + a^2 + 25 = 0$

$6x + 8y - a^2 - 25 = 0 \rightarrow (1)$

The distance from origin to (1) is 25

$\frac{\left| -a^2 - 25 \right|}{\sqrt{76 + 64}} = 25 \Rightarrow a^2 + 25 = 250 = a^2 = 225$

15. Let $p(x_1, y_1)$ be the point in the locus .

The equation of a circle with centre (x_1, y_1) *is* $x^2 + y^2 - 2x_1x - 2y_1y + k = 0 \rightarrow (1)$

Radius of (1) is $\sqrt{x_1^2 + y_1^2 - k}$

If (1) touches the line $x = 2a$ *then* $|x_1 - 2a| = \sqrt{x_1^2 + y_1^2 - k} \Rightarrow (x_1 - 2a)^2 = x_1^2 + y_1^2 - k$

If (1) cuts $x^2 + y^2 = a^2$ orthogonally then $0 + 0 = k - a^2 \Rightarrow k = a^2$

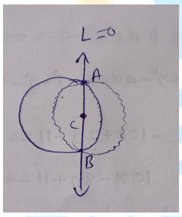
$\therefore (x_1 - 2a)^2 = x_1^2 + y_1^2 - a^2 \Rightarrow x_1^2 + 4a^2 - 4ax_1 = x_1^2 + y_1^2 - a^2 \Rightarrow y_1^2 + 4ax_1 = 5a^2$

The equation to the locus of p *is* $y^2 + 4ax = 5a^2$

16. Let $s \equiv x^2 + y^2 - 4 = 0$

And $L = 2x + 4y = 1$

Equation of a circle having AB as diameter is



$S + \lambda L = 0$

$x^2 + y^2 - 4 + \lambda(2x + 4y - 1) = 0 \qquad \rightarrow (i)$

$x^2 + y^2 + 2\lambda x + 4\lambda y - 4 - \lambda = 0$

centre $(c) = (-\lambda, -2\lambda)$

Since centre $(-\lambda, -2\lambda)$ *lies on line* $2x + 4y = 1$

$2(-\lambda) + 4(-2\lambda) = 1$

$-2\lambda - 8\lambda = 1$

$\lambda = -\frac{1}{10}$

Sub $\lambda = -\frac{1}{10}$ *in eq* (i)

$$x^2+y^2-4-\frac{1}{10}(2x+4y-1)=0$$

$$x^2+y^2-4-\frac{2}{10}x-\frac{4}{10}y+\frac{1}{10}=0$$

$$x^2+y^2-\frac{2}{10}x-\frac{4}{10}y-\frac{39}{10}=0$$

Compare $x^2+y^2+2gx+2fy+c=0$

Hence $g=-\frac{1}{10},\ f=-\frac{2}{10},\ c=-\frac{39}{10}$

Now $10(g+f)=10\left(-\frac{1}{10}-\frac{2}{10}\right)==-3$

17. Equation to common chord of $x^2+y^2-2\lambda y+\lambda^2-160=0 \quad .....(1)$ *and* $x^2+y^2-16=0 \quad ....(2)$

is $S-S'=0 \Rightarrow 2\lambda y-\lambda^2=0 \Rightarrow y=\frac{\lambda}{2}$

Hemogenising (2) with (3) we have

$$x^2+y^2=16\left(\frac{2y}{\lambda}\right)^2 \quad .....(4)$$

The pair of lines are $\perp \Leftrightarrow$ *coeft.of* x^2 + *coeft.of* $y^2=0$

$$\Rightarrow\ 1+1-16\left(\frac{4}{\lambda^2}\right)=0 \ \Rightarrow 2\lambda^2=64 \Rightarrow \lambda=\pm4\sqrt{2}$$

18. Equation to the point circle of (0,0) is $x^2+y^2=0$.

Equation to the tangent is $x-y=0$

$\therefore$ Any circle passing (0,0) and touching $x-y=0$ *is* $x^2+y^2+\lambda(x-y)=0 \quad .....(1)$

Given circle is $x^2+y^2+6x+8y-7=0 \quad .....(2)$

Common chord of (1) and (2) is $6x+8y-7-\lambda(x-y)=0 \quad .....(3)$

Line (3) always passes through the points of intersection of the lines

$6x+8y-7=0$ *and* $x-y=0 \Rightarrow$ *i.e.* point $\left(\frac{1}{2},\frac{1}{2}\right)$

19. Circles are orthoganal $\Leftrightarrow \frac{\lambda}{2}(2)-\left(\frac{1+\lambda^2}{2}\right)(3)=\frac{-10}{2}+3$

$\Leftrightarrow \quad 3\lambda^2-2\lambda-1=0 \Rightarrow (3\lambda+1)(\lambda-1)=0 \ \lambda=\frac{-1}{3},\ 1 \Rightarrow \lambda$ has two values

20. Let $s \equiv x^2+y^2-4x-5=0$

$s^1 \equiv x^2+y^2+6x-2y+6=0$

Equation of R.A.'s is $s-s^1=0$

$$x^2+y^2-4x-5-x^2-y^2-6x+2y-6=0$$

$$-10x+2y-11=0$$

$10x-2y+11=0$

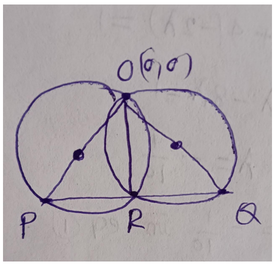
Since the point (α,β) lies on R.A $10x-2y+11=0$

$10\alpha-2\beta+11=0$

21. $O(0,0),p(x_1,y_1),Q(x_2,y_2)$

Let OR lie the common chord .

From the diagram



Area of $\Delta OPR=\frac{1}{2}\times PQ\times OR$

$\frac{1}{2}|x_1y_2-x_2y_1|=\frac{1}{2}\times PQ\times OR$

$OR=\frac{x_1y_2-x_2y_1}{PQ}$

22. $x^2+y^2-2ax+2by+\lambda(ax-by)=0$

This is in the form $s+\lambda L=0$

R.A.'s in $L=ax-by=0$.

23. $x^2+y^2+a_rx+b_ry+c=0$, $r=1,2,3$

Verify the options (3)

Let $0(0,0)$

$\sqrt{s_{11}}=\sqrt{0+0+0+0+c}=\sqrt{c}$

$\sqrt{s_{11}^1}=\sqrt{0+0+0+0+c}=\sqrt{c}$

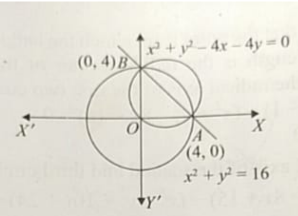
$\sqrt{s_{11}^{11}}=\sqrt{0+0+0+0+c}=\sqrt{c}$

$\therefore\ \sqrt{s_{11}}=\sqrt{s_{11}^1}=\sqrt{s_{11}^{11}}$

$\therefore$ Radical centre is $(0,0)$

24. Verify the options and check them orthogonal condition $2g\,g^1+2f\,f^1=c+c^1$

25.



The equation of the common chord of the circles $x^2+y^2-4x-4y=0$ and $x^2+y^2=16$ $is\,x+y=4$

Which meets $x^2+y^2=16\,at\,A(4,0)\,and\,B(0,4)$. Obviously $OA\perp OB$. Hence the common chord AB makes a right angle at the centre of circle $x^2+y^2=16$.

26. $s=x^2+y^2+6x-46=0$

$s^1=x^2+y^2-2x-6y-6=0$

R.A."s is $s-s^1=0$

$x^2+y^2+6x-46-x^2-y^2+2x+6y+6=0$

$8x+6y-40=0\Rightarrow 4x+3y-20=0$

The perpendicular distance from (1,2) to R.A. $4x+3y-20=0$

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|4+6-20|}{\sqrt{16+9}}=\frac{10}{5}=2$$

27. $s=x^2+y^2+4x-2y-4=0$

$s^1=x^2+y^2-4x-6y-3=0$

Locus of the centre of the circle is R.A."s of $s=0\ and\ s^1=0$ R.A."s is $s-s^1=0$

$x^2+y^2+4x-2y-4-x^2-y^2+4x+6y=3=0$

$8x+4y-1=0$

$compare\ \ lx+my+n=0$

$Here\,l=8,m=4,n=-1$

$Now\ l+m+n=8+4-1=11$

28. $s=x^2+y^2+3x+4y-5=0$

$s^1=x^2+y^2-5x+5y-6=0$

R.A's is $s-s^1=0$

$x^2+y^2+3x+4y-5-x^2-y^2+5x-5y+6=0$

$8x-y+1=0$

$$Slope=\frac{-a}{b}=\frac{-8}{-1}=8$$

$s=x^2+y^2=2gx+2fy+c=0$

29. $s^1=x^2+y^2+\frac{3}{2}x+4y+c=0$

$R.A.'s\ is\,s-s^1=0$

$$x^2+y^2+2gx+2fy+c-x^2y^2-\frac{3}{2}x-4y-c=0$$

$$\left(2g-\frac{3}{2}\right)x+(2f-4)y=0$$

$(4g-3)x+4(f-2)y=0\rightarrow(1)$

This line touches the circle $x^2+y^2+2x+2y+1=0\rightarrow(2)$

$centre(c)=(-1,-1),radicals(r)=\sqrt{1+1-1}=1$

$radius(r)=$ The perpendicular distance from centre $(-1,-1)$ to eq (1)

$$1=\frac{\left|(-1)(49-3)+(-1)4(f-2)\right|}{\sqrt{(4g-3)^2+16(f-2)^2}}$$

$$8(4g-3)(f-2)=0$$

$$g=\frac{3}{4}(or)\,f=2$$

30. centers of given cricles are

$Let\ A(1,2),\ B(4,1)\ C(5,4)\, and\ r_1=r_2=r_3=15$

$if\ r_1=r_2=r_3$ then radical center is circumcentre of $\Delta^l ABC$

Find the circumcentre

$\therefore p(3,3)$

$p_x+p_y=3+3=6$