NARAYANA EDUCATIONAL INSTITUTIONS

PERMUTATIONS

MATHS
SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

61. The number of ways in which 5 different prizes be given among 11 candidates, each candidate receiving any number of pizes are
1) 11^5 2) 5^{11} 3) $^{11}C_5$ 4) $^{11}P_5$

62. If all the letters of the word "NIDHI" are arranged in alphabetical order than the Rank of the word NIDH, is
1) 43 2) 54 3) 55 4) 56

63. Number of 7 digit telephone numbers that can be created if the first three digits must be 426, 427 (or) 428 and the right most digit is divisible by 3, is K thousand then K is equal to
1) 12 2) 9 3) 6 4) 3

64. Number of ways in which 5 boys and 5 girls can be seated alternatively on a round table if a particular boys and a particular girl are never adjacent to each other in any arrangement is
1) 228 2) 552 3) 1584 4) 1728

65. A class has three teachers Mr. P, Ms. Q and Mrs. R and six students A, B, C, D, E, F number of ways in which they can be seated in a line of 9 chair is, if between any two teachers there are exactly two students, is K(6!) then the value of K is
1) 18 2) 12 3) 24 4) 6

66. A class is composed of 2 brothers and 6 other boys. In how many ways can all the boys be seated at a round table so that the 2 brothers are not seated besides each other is :
1) 720 2) 1440 3) 3600 4) 4320

67. Rajdhani express going from Bombay to Delhi stops at 5 intermediate stations. 10 passengers enter the train during the journey with ten different ticket of two classes. The number of different sets of tickets they may have had is :
1) $^{15}C_{10}$ 2) $^{20}C_{10}$ 3) $^{30}C_{10}$ 4) None

68. Eleven animals of a circus have to be placed in eleven cages (one in each cage). If 4 of the cages are too small for 6 of the animals, then find the number of the ways of caging all the animals
1) 604800 2) 8400 3) 608400 4) None

69. A natural number has prime factorization given by $n = 2^x 5^y 5^z$ where y and z are such that y + z = 5 and $y^{-1} + z^{-1} = \frac{5}{6}, y > z$ then the number of odd divisors of n, including 1, is
1) 11 2) 6 3) 6x 4) 12

70. How many ways are there to seat n married couples $(n \geq 3)$ around a table such that men and women alternate and each women is not adjacent to her husband
1) $n!(n-1)!-2(n-1)!$ 2) $(n-1)!(n!-2)$ 3) n! – 2 4) None of these

71. Number of derangement of all the digits of number 1234567 such that even digits occupy even places and odd digits occupy odd places is
1) 12 2) 14 3) 16 4) 18

72. If A = {1, 2, 3, 4, 5}, B = {a, b, c, d} then the number of onto functions that can be defined from A to B is
1) 120 2) 240 3) 625 4) 1024

73. If A = {1, 2, 3, 4}, B = {a, b, c, d, e} then the number of all possible constant functions from A to B is
1) 9 2) 4 3) 5 4) 16

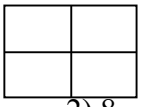
74. IF A = {x/x is a prime number and x < 30} find the number of different rational numbers whose numerator and denominator belong to A
1) $^{10}P_2$ 2) $^{10}P_2 + 1$ 3) $^{10}C_2$ 4) $^{10}C_2 + 1$

75. Find the number of words that can be made out of the letters of the word MOBILE when consonants always occupy odd places
1) 36 2) 6 3) 9 4) None

76. Total number of words that can be formed using all letters of the word BRIJESH that neither begins with I nor ends with B is equal to
1) 3720 2) 4920 3) 3600 4) 4800

77. In a three – storey building, there are four rooms on the ground floor, two on the first and two on the second floor if the rooms are to be allotted to six persons, one person occupying one room only. The number of ways in which this can be done so that no floor remains empty is
1) $^{8}P_0 - 2(6!)$ 2) $^{8}P_6$ 3) $^{8}P_5\,(6!)$ 4) None of these

78. The number of ways to fill each of the four cells of the table with a district natural number such that the sum of the numbers is 10 and the sum of the numbers placed diagonally are equal is



1) 4 2) 8 3) 24 4) 6

79. N dices are rolled. The number of possible outcomes in which at least one of the dice shows on even number is 189, then n =
1) 3 2) 4 3) 5 4) None of these

80. The number of permutation of the letters of the word HINDUSTAN such that neither the pattern 'HIN' nor 'DUS' nor 'TAN' appears, are
1) 166674 2) 169194 3) 166680 4) 181434

SECTION – II
(Numerical Value Answer Type)

This section contains 10 questions. Answer any Five Questions. The answer to each question is a Numerical values comprising of positive or negative decimal numbers (place value ranging from Thousands Place to Hundredths Place).
Eg: 1234.56, 123.45, -123.45, -1234.56, -0.12, 0.12 etc.
Marking scheme: +4 for correct answer, 0 in all other cases.

81. 10 different toyes are to be distributed among 10 children. If the total number of ways of distributing all these toys so that exactly two children do not get any toy.is K(10!), then the value of k is

82. Find the rank of the word "TANYA"

83. If the letters of the word "SOMYA" are arranged alphabetically in all possible manner, then find the number of words that appear before the word "SOMYA".

84. Let N denotes the number of odd integers between 550 and 800 using the digits 4, 5, 6, 7 and 9. Find the sum of the digits in N.

85. Find the number of natural numbers which are less than 2×10^8 and which can be written by means of the digit 1 and 2.

86. The letters of the word QUESTION are arranged in all possible ways the number of arrangements in which there are exactly 2 letters between 'Q' and 'S'.

87. The number of one-one functions from a set A containing 2 elements to a set B containing 3 elements is

88. The number of four digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is

89. Find the number of ways in which 8 different flowers can be strung to form a garland so that four particular flowers are never separated.

90. A variable name in certain computer language must be either an alphabet or an alphabet followed by a decimal digit. The total number of different variable names that can exist in that language is equal to

PERMUTATIONS

Key

MATHS

61-70	1	3	1	4	1	3	3	1	4	1
71-80	4	2	3	2	1	1	1	2	1	2
81-90	375	40	87	6	766	7200	6	310	288	286

Hints

61. First prize can be given away in 11 ways
Second prize can be given away in 11 ways
Third prize can be given away in 11 ways
Fourth prize can be given away in 11 ways
Fifth prize can be given away in 11 ways
$\therefore$ required no. of ways = $11 \times 11 \times 11 \times 11 \times 11 = 11^5$

62. Given word is NIDHI
Given word in the Dictionary order DHIIN

$$\underline{D}\_\_\_\_ = \frac{4!}{2!} = 12$$

$$\underline{H}\_\_\_\_ = \frac{4!}{2!} = 12$$

$$\underline{I}\_\_\_\_ = 4! = 24$$

$$\underline{N}\,\underline{D}\_\_\_ = \frac{3!}{2!} = 3$$

$$\underline{N}\,\underline{H}\_\_\_ = \frac{3!}{2!} = 3$$

$$\underline{N}\,\underline{I}\,\underline{D}\,\underline{H}\,\underline{I} = 0! = 1$$

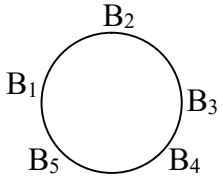
Rank of the word NIDHI = 55

63. $\underbrace{\_\_\_}_{3\ ways}\ \underbrace{\_\_\_}_{each\ place\ in\ 10\ ways}\ \underbrace{\_\_\_}_{filled\ in\ 4\ ways\ (0,3,6,9)}$

Required no. of ways = (3) (10^3) (4) = 12000

$\therefore$ K = 12

64.



5 boys can be seated in 4! Ways

Now for G_1 can not sit between B_1 and B_2 or B_1 and B_5 and therefore can set only in 3 ways and the remaining 4 girls in 4! Ways.

Total ways = 4!(3)4! = 24 x 3 x 24 = 1728

65. i) T S S T S S T S S

ii) S T S S T S S T S

iii) S S T S S T S S T

required no. of ways = 3.3! 6! = (18) 6!

$\Rightarrow$ K(6!) = 18(6!)

$\therefore$ K = 18

66. required no. of ways = 7! – 6! . 2! = 7.6! – 6! . 2

= (7 – 2)6! = 5 x 6! = 5 x 720 = 3600

67. No. of different tickets of two classes = $^6C_2$. 2 = 30

B S_1 S_2 S_3 S_4 S_5 D

Hence no. of Different sets of tickets = $^{30}C_{10}$

68. Let the 6 animals be placed in 7 of longer cages this can be done in $^7P_6$ ways

In each of these ways one larger cage is left vacant.

The remaining five animals can be placed in the remaining five cages in 5! ways.

$\therefore$ The required number of ways is $^7P_6$ x 5! = 604800

69. y + z = 5

$\frac{1}{y}+\frac{1}{z}=\frac{5}{6}$ y > z

$\Rightarrow$ y = 3, x = 2

$\Rightarrow n = 2^x.3^3.5^2 = (2.2.2.....)(3.3.3)(5.5)$

Number of odd divisors = 4 x 3 = 12

70. Number of ways of seating 'n' couples in an alternating manner, i.e., one women one man and so on… = (n – 1)! n!

Number of ways couples can be seated together always is (n – 1)! x 2!

$\therefore$ required number of ways = n!(n – 1)! – 2(n – 1)!

71. $D_4 \times D_3 = 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)\times 3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)$

= (12 – 4 + 1) x (3 – 1) = 9 x 2 = 18

72. A = {1, 2, 3, 4, 5}, B = {a, b, c, d}

n(A) = m = 5, n(B) = n = 4

No. of onto functions = $n^m-\left({}^nc_1(n-1)^m-{}^nc_2(n-2)^m+{}^nc_3(n-3)^m-{}^nc_4(n-4)^m\right)$

$= 4^5-\left({}^4c_1 3^5-{}^4c_2 2^5+{}^4c_3 1^5-{}^4c_4 0^5\right) = 4^5-\left(4\times 3^5-6.2^5+4-0\right)$

= 1024 – (972 – 192 + 4) = 1024 – (784) = 240

73. $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d, e\}$
$n(A) = 4$, $n(B) = 5$
No. of constant functions = no. of elements in set B = 5

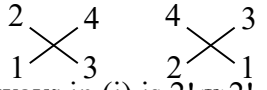
74. Here $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
a rational number is made by taking any two numbers in any order.
$\therefore$ The required number of rational numbers = ${}^{10}P_2 + 1$ (including 1)

75. The word MOBILE has three even places and three odd places. It has 3 constants and 3 vowels. In three odd places. We have to fix up 3 consonants this can be done in
${}^3P_3 = 6$ ways
in the remaining three places, we have to fix up the remaining three this can be done in
${}^3P_3 = 6$ ways
$\therefore$ The total no. of ways = 6 x 6 = 36

76. Total number of words without any restriction is 7!
Total number of words Beginning with I is 6!
Total number of words ending with B is 6!
Total number of words beginning with I and ending with B is 5!
Required number of words = 7! – 6! – 6! + 5! = 7! – 2(6)! + 5 ! = 7.6.5! – 12.5! + 5!
= (42 – 12 + 1)5! = 31 x 120 = 3720

77. The number of ways of allotment without any restriction is ${}^8P_6$ now it is possible that all rooms of 2nd floor or 3rd floor are not occupied. Thus there are two ways in which one floor remains unoccupied.
Hence the number of ways of allotment in which a floor is unoccupied is 2 x 6!
Number of ways in which none of the floor remains unoccupied is ${}^8P_6 – 2(6!)$

78. The natural numbers are 1, 2, 3, 4 clearly in one diagonal we have to place 1, 4 and in the other 2, 3



(i) (ii)

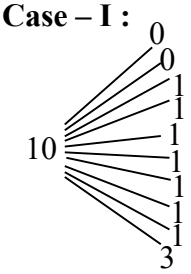
The number of ways in (i) is 2! x 2! = 4
The number of ways in (ii) is 2! x 2! = 4
$\therefore$ The total number of ways = 4 + 4 = 8

79. Total number of out comes = 6^n
Number of outcomes when no dice shows an even number
i.e., when all the n dice show odd number = 3^n
$\therefore$ Number of ways when at least one dice shows an even number = $6^n – 3^n$
But given $6^n – 3^n = 189$
Put $n = 3 \Rightarrow 6^3 – 3^3 = 189$
$\therefore$ $n = 3$

80. Total number of permutations = $\frac{9!}{2!}$
Number of those containing ‘HIN’ = 7!
Number of those containing ‘DUS’ = $\frac{7!}{2!}$
Number of those containing ‘TAN’ = 7!
Number of those containing ‘HIN’ and ‘DUS’ = 5!
Number of those containing ‘HIN’ and ‘TAN’ = 5!
Number of those containing ‘TAN’ and ‘DUS’ = 5!
Number of those containing ‘HIN’ ‘DUS’ and ‘TAN’ = 3!

Required number $= \frac{9!}{2!} - \left(7! + 7! + \frac{7!}{2}\right) + 3 \times 5! - 3! = 169194$

81. **Case – I :**



$= \frac{10!}{7!3!2!}$

Case – II : 0 0 1 1 1 1 1 1 2 2

Total $= 10! \times \left(\frac{10!}{7!3!2!} + \frac{10!}{6!2!2!2!2!}\right)$

$= 10!(60 + 315) = 375(10!)$

$\Rightarrow$ K(10!) = 375(10!)

$\Rightarrow$ K = 375

82. T A N Y A

A A N T Y

A _ _ _ _ = 4! = 24

N _ _ _ _ $= \frac{4!}{2!} = 12$

T A A _ _ = 2! = 2

T A N A _ = 1! = 1

T A N Y A = 0! = 01

40

83. S O M Y A

A M O S Y

A _ _ _ _ = 4! = 24

M _ _ _ _ = 4! = 24

O _ _ _ _ = 4! = 24

S A _ _ _ = 3! = 6

S M _ _ _ = 3! = 6

S O A _ _ = 2! = 2

87 words

S O M A Y = 0! = 1

84. **Case 1** :- Consider the numbers between 550 – 599

4, 5, 6, 7, 8, 9

1st place in one way (i.e. 5)

2nd place in 5 ways (i.e. 5, 6, 7, 8, 9)

3rd place in 3 ways (i.e. 5, 7, 9)

Number of number in this case = 1.5.3 = 15

Case 2 :- 600 – 799 (both inclusive)

Number of numbers = 2.6.3 = 36

Total = 15 + 36 = 51

Sum of digits in N = 6

85. Number of 1 digit number = 2

Number of 2 digit number = 2 x 2 = 2^2

Number of 3 digit number = 2 x 2 x 2 = 2^3

:

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Number of 8 digit number = 2 x 2 x ….. x 2 = 2^8

Number of 9 digit number = 1 x 2 x 2 x ….. x 2 (8 times) = 2^8

Total numbers = $2 + 2^2 + 2^3 + \ldots\ldots + 2^8 + 2^8 = \frac{2\left(2^8-1\right)}{2-1} + 2^8 = 2^9 - 2 + 2^8 = 766$

86.

1. Q _ _ S _ _ _ _	1. S _ _ Q _ _ _ _
2. _ Q _ _ S _ _ _	2. _ S _ _ Q _ _ _
3. _ _ Q _ _ S _ _	3. _ _ S _ _ Q _ _
4. _ _ _ Q _ _ S _	4. _ _ _ S _ _ Q _
5. _ _ _ _ Q _ _ S	5. _ _ _ _ S _ _ Q

Total ways = 5 + 5 = 10

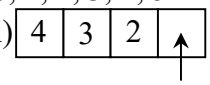
remaining 6 letters (U, E, T, I, O, N) can be arranged among themselves in 6! Ways

required number of ways = 6! x 10 = 720 x 10 = 7200

87. n(A) = 2, n(B) = 3

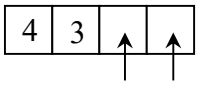
No. of one – one functions = ${}^{n(B)}P_{n(A)} = {}^3P_2 = 3 \times 2 = 6$

88. Following are the cases in which the 4-digit numbers strictly greater than 4321 can be formed using digits 0, 1, 2, 3, 4, 5

Case (i) 

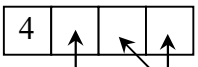
= 4 numbers

2/3/4/5 $\rightarrow$ 4 ways

Case (ii) 

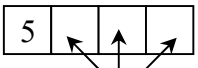
= 3 x 6 = 18 numbers

3/4/5 $\rightarrow$ 3ways 0/1/2/3/4/5 $\rightarrow$ 6ways

Case (iii) 

= 2 x 6 x 6 = 72 numbers

4/5 $\rightarrow$ 2ways 0/1/2/3/4/5 $\rightarrow$ 6ways

Case (iv) 

= 6 x 6 x 6 = 216 numbers

0/1/2/3/4/5 $\rightarrow$ 6ways

required total = 4 + 18 + 72 + 216 = 310

89. Four particular flowers can be treated as 1 unit

1 unit of four flowers, remaining four flowers = 5 units of flowers

5 units of flowers can be string in a garland = $\frac{(5-1)!}{2} = \frac{4!}{2}$

1 unit of four flowers can be arranged among themselves in = 4! Ways

required number of ways = $\frac{4!}{2} \times 4!$ = 12 x 24 = 288

90. Total number of variables if only alphabet is used is 26

Total number of variables if alphabet and digits both are used is 26 x 10 = 260

$\therefore$ The total number of variables = 26 + 260 = 286