TOPIC: DIFFERENTIAL EQUATIONS

MATHS-1B

1. The differential equation representing all the tangents to the parabola $y^2=2x$ is

1) $xy_1-2yy_1=2$ 2) $2xy_1^2-2yy_1+1=0$

3) $2x^2y_1-2yy_1+1=0$ 4) $xy_1^2-yy_1+1=0$

2. Order and degree of the differential equation $\left(1+y_2^{1/3}\right)^{1/2}=\left(y_3y_1\right)^{1/3}$ respectively are

1) 3,6 2) 3,4 3) 3,8 4) 3,12

3. Solution of $y-x\frac{dy}{dx}=3\left[1-x^2\frac{dy}{dx}\right]$ is

(1) $(y+3)(1+3x)=cx$ (2) $(y-3)(1-3x)=cx$

(3) $(y-3)(1+3x)=cx$ (4) $(y+3)(1-3x)=cx$

4. If the solution of the differential equation $\frac{dy}{dx}=\frac{ax+3}{2y+f}$ represents a circle then the value of 'a' is

(1) 2 (2) -2 (3) 3 (4) -4

5. The solution of D.E $\ln\left(\frac{dy}{dx}\right)=ax+by$ is

(1) $e^{by}=e^{ax}+c$ (2) $\frac{e^{by}}{b}=\frac{e^{ax}}{a}+c$ (3) $\frac{e^{-ax}}{a}=\frac{e^{by}}{b}+c$ (4) $-\frac{e^{-by}}{b}=\frac{e^{ax}}{a}+c$

6. The solution of D.E $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

(1) $x\sqrt{1-y^2}-y\sqrt{1-x^2}=c$ (2) $x\sqrt{1-y^2}+y\sqrt{1-x^2}=c$

(3) $x\sqrt{1+y^2}+y\sqrt{1+x^2}=c$ (4) None of these

7. The differential equation $\left(1+y^2\right)x\,dx=\left(1+x^2\right)y\,dy$ represents a family of

(1) Ellipses of constant eccentricity (2) Ellipses of variable eccentricity

(3) Hyperbolas of constant eccentricity (4) Hyperbolas of variable eccentricity

8. The solution of D.E $\frac{dy}{dx}=\sqrt{3x+y+4}$ is

(1) $\sqrt{3x+y+4}-\ln\left|\sqrt{3x+y+4}+3\right|=x+c$ (2) $2\sqrt{3x+y+4}-\ln\left|\sqrt{3x+y+4}+3\right|=x+c$

(2) $2\left(\sqrt{3x+y+4}-\ln\left|\sqrt{3x+y+4}+3\right|\right)=x+c$ (4) $2\left(\sqrt{3x+y+4}-\ln\left|\sqrt{3x+y+4}+3\right|\right)=c$

9. The solution of D.E $xdx+ydy=x\left(xdy-ydx\right)$ is

(1) $y+1=c\sqrt{x^2+y^2}$ (2) $y+1=2c\left(x^2+y^2\right)$

(3) $\frac{y}{x^2+y^2}=c(1+x)$ (4) $\frac{x}{x^2+y^2}=c(1+y)$

10. The solution of D.E $\frac{xdx-ydy}{xdy-ydx}=\sqrt{\frac{1+x^2-y^2}{x^2-y^2}}$ is

(1) $\left|\sqrt{x^2-y^2}+\sqrt{1+x^2-y^2}\right| = c\dfrac{|x-y|}{\sqrt{x^2-y^2}}$ (2) $\left|\sqrt{x^2-y^2}+\sqrt{1+x^2-y^2}\right| = c\left|\dfrac{x+y}{\sqrt{x^2-y^2}}\right|$

(3) $\left|\sqrt{x^2+y^2}+\sqrt{1-x^2+y^2}\right| = c\dfrac{|x-y|}{\sqrt{x^2-y^2}}$ (4) $\left|\sqrt{x^2+y^2}+\sqrt{1-x^2+y^2}\right| = c\dfrac{|x+y|}{\sqrt{x^2+y^2}}$

11. The solution of D.E $xdy - ydx - \sqrt{x^2-y^2}\,dx = 0$ is

(1) $\sin^{-1}\left(\dfrac{x}{y}\right) = cx$ (2) $\sin^{-1}\left(\dfrac{y}{x}\right) = cx$

(3) $\sin^{-1}\left(\dfrac{y}{x}\right) = \ln|cx|$ (4) None of these

12. The solution of D.E $\dfrac{x+y\dfrac{dy}{dx}}{y-x\dfrac{dy}{dx}} = \dfrac{x\cos^2\left(x^2+y^2\right)}{y^3}$ is

(1) $\tan\left(x^2+y^2\right) = \dfrac{y^2}{x^2}+c$ (2) $\tan\left(x^2+y^2\right) = \dfrac{x+c}{y}$

(3) $\tan\left(x^2+y^2\right) = \dfrac{x^2}{y^2}+c$ (4) $\cot\left(x^2+y^2\right) = \dfrac{x^2}{y^2}+c$

13. The solution of D.E $\dfrac{dy}{dx} = \cos(x-y)$ is

(1) $y+\cot\left(\dfrac{x-y}{2}\right) = c$ (2) $x+\cot\left(\dfrac{x-y}{2}\right) = c$

(3) $x+\tan\left(\dfrac{x-y}{2}\right) = c$ (4) None of these

14. The solution of D.E $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$ represents a family of

(1) Circles (2) Parabola (3) Ellipses (4) Hyperbolas

15. The solution of D.E $\left(x\dfrac{dy}{dx}-y\right)\tan^{-1}\left(\dfrac{y}{x}\right) = x$ given that $y(1)=0$ is

(1) $\sqrt{x^2+y^2} = e^{\frac{y}{x}\tan^{-1}\left(\frac{y}{x}\right)}$ (2) $\sqrt{x^2-y^2} = e^{\frac{y}{x}\tan^{-1}\left(\frac{y}{x}\right)}$

(3) $\sqrt{x^2+y^2} = e^{\frac{-y}{x}\tan^{-1}\left(\frac{y}{x}\right)}$ (4) $\sqrt{x^2+y^2} = e^{\frac{x}{y}\tan^{-1}\left(\frac{x}{y}\right)}$

16. The solution of D.E $(y+x+5)dy = (y-x+1)dx$ is

(1) $\log\left[(y+3)^2+(x+2)^2\right]+\tan^{-1}\left(\dfrac{y+3}{y+2}\right) = c$ (2) $\log\left[(y+3)^2+(x-2)^2\right]+\tan^{-1}\left(\dfrac{y-3}{x-2}\right) = c$

(3) $\log\left[(y+3)^2+(x+2)^2\right]+2\tan^{-1}\left(\dfrac{y+3}{x+2}\right) = c$ (4) $\log\left[(y+3)^2+(x+2)^2\right]-2\tan^{-1}\left(\dfrac{y+3}{x+2}\right) = c$

17. Let $x(t)$ be the solution of $t\left(1+t^2\right)dx = \left(x+xt^2-t^2\right)dt$ given that $x(1) = \dfrac{-\pi}{4}$, t>0 then

$\dfrac{-4}{\pi}\lim\limits_{x\to\infty} x^1(t) =$

(1) 1 (2) 2 (3) 3 (4) 4

18. If $\int_a^x t\,y(t)\,dt = x^2 + y(x)$ then y as a function x is

(1) $y = 2-\left(2+a^2\right)e^{\frac{x^2-a^2}{2}}$ (2) $y = 1-\left(2+a^2\right)e^{\frac{x^2-a^2}{2}}$

(3) $y = 2-\left(1+a^2\right)e^{\frac{x^2-a^2}{2}}$ (4) $y = c+2-\left(2-a^2\right)e^{\frac{x^2-a^2}{2}}$

19. If $y(x)$ be the solution of D.E $\left(1+x^2\right)\frac{dy}{dx}+4xy=\frac{1}{1+x^2}$ with $y(0)=0$ then $\left(\int_0^1 f(x)dx\right)^{-1} =$

(1) 1 (2) 2 (3) 3 (4) 4

20. The solution of D.E $\frac{dy}{dx}=-\left(\frac{x+y\cos x}{1+\sin x}\right)$ is

(1) $y=\frac{x^2+c}{1+\sin x}$ (2) $y=\frac{2c-x^2}{1+\sin x}$ (3) $y=\frac{2c-x^2}{2(1+\sin x)}$ (4) None of these

21. The solution of D.E $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$ is

(1) $y=\left(2\sqrt{x}+c\right)e^{-2\sqrt{x}}$ (2) $y=e^{2\sqrt{x}}\left(2\sqrt{x}+c\right)$

(3) $y=\left(2\sqrt{x}+c\right)e^{\sqrt{x}}$ (4) None of these

22. The solution of D.E $ye^y dx=\left(y^3+2xe^y\right)dy$ when $y(0)=1$

(1) $y=x^2\left(e^{-1}-e^{y}\right)$ (2) $x=y^2\left(e^{-1}-e^{-y}\right)$ (3) $y=x^2\left(e^{-1}-e^{-x}\right)$ (4) $x=y\left(e^{-1}-e^{-y}\right)$

23. The solution of D.E $\frac{dy}{dx}=\frac{1}{xy\left[x^2\sin y^2+1\right]}$ is

(1) $x^2\left(\cos y^2-\sin y^2-2ce^{-y^2}\right)=2$ (2) $y^2\left(\cos x^2-\sin y^2+2ce^{-y^2}\right)=2$

(3) $x^2\left(\cos y^2-\sin y^2-e^{-y^2}\right)=4$ (4) None

24. The solution of D.E $x\cos x\left(\frac{dy}{dx}\right)+y\left(-x\sin x+\cos x\right)=1$ is

(1) $x+y=cx\sec x$ (2) $xy=(x+c)\sec x$ (3) $x+y=(x+c)\sec x$ (4) None

25. The solution of D.E $\frac{dt}{dx}=\frac{t\left[\frac{d}{dx}\left(g(x)\right)\right]-t^2}{g(x)}$ is

(1) $t=\frac{g(x)}{x}+c$ (2) $t=\frac{g(x)}{x^2}+c$ (3) $t=\frac{g(x)}{x+c}$ (4) $t=g(x)+x+c$

26. The solution of D.E $y\sin x\frac{dy}{dx}=\cos x\left(\sin x-y^2\right)$ is

(1) $y=\frac{\sin^2 x}{2}+c$ (2) $y\sin^2 x=x+c$ (3) $y\left(\sin^2 x\right)=\frac{x^3}{3}+c$ (4) None

27. The solution of $\sec^2\theta d\theta+\tan\theta\left(1-r\tan\theta\right)dr=0$ is

(1) $\cot\theta=r-1+ce^{r}$ (2) $\tan\theta=1-r+ce^{-r}$ (3) $\cot\theta=r-1+ce^{-r}$ (4) None of these

28. The solution of D.E $\left(x\cos y - y\sin y\right)dy + \left(x\sin y + y\cos y\right)dx = 0$

(1) $x\cos y + y\sin y - \sin y = ce^{-x}$
(2) $x\sin y + y\cos y - \sin y = ce^{-x}$
(3) $x\sin y + y\cos y - \sin y = ce^{-x}$
(4) $x\cos y + y\cos y + \cos y = ce^{-x}$

29. Let $y(x)$ be the solution of $\frac{1}{x}\frac{dy}{dx} + ye^{x} = e^{(1-x)e^{x}}$ such that $y(1)=0$ then $y(0)+\frac{e}{2}=$

(1) 1
(2) -1
(3) 2
(4) 0

30. The solution of D.E $2x\frac{dy}{dx} = y + 6x^{5/2} - 2\sqrt{x}$ $\left(x>0\right)$ is

(1) $y = \frac{3}{2}x^{5/2} - \sqrt{x}\ln x + c\sqrt{x}$
(2) $y = \frac{3}{2}x^{7/2} - 2\sqrt{x}\ln x + c\sqrt{x}$
(3) $y = \frac{5}{2}x^{7/2} - 2\sqrt{x}\ln\sqrt{x} + c$
(4) None of these

KEY										
MATHS-1B										
1-10	2	1	2	2	4	2	4	3	1	2
11-20	3	3	2	4	1	3	2	1	4	3
21-30	1	2	1	2	3	4	1	3	4	1

HINTS & SOLUTIONS
MATHS

1. The equation of the tangent to $y^2 = 2x$ is of the form

$$y = mx + \frac{1}{2m} \quad \Rightarrow y_1 = m$$

$$\therefore D.E\ is\ \ y = xy_1 + \frac{1}{2y_1} \Rightarrow 2xy_1^2 - 2yy_1 + 1 = 0$$

2. $\left(1+y_2^{1/3}\right)^{1/2} = \left(y_3y_1\right)^{1/3} \Rightarrow \left(1+y_2^{1/3}\right)^3 = \left(y_3y_1\right)^2$

$\Rightarrow \quad 1 + y_2 + 3y_2^{1/3}\left(1+y_2^{1/3}\right) = \left(y_3y_1\right)^2$

$\Rightarrow \quad 3y_2^{1/3}\left(y_3y_1\right)^{2/3} = \left(y_3y_1\right)^2 - y_2 - 1$

$\Rightarrow \quad 27y_2\left(y_3y_1\right)^2 = \left[\left(y_3y_1\right)^2 - y_2 - 1\right]^3$

3. $y - x\frac{dy}{dx} = 3 - 3x^2\frac{dy}{dx}$

$\Rightarrow \quad y - 3 = \left(x - 3x^2\right)\frac{dy}{dx} \quad \Rightarrow \int\frac{dx}{x(1-3x)} = \int\frac{dy}{y-3}$

4. $\int(ax+3)\,dx = \int(2y+f)$

$\frac{ax^2}{2}+3x=y^2+fy+c \Rightarrow \frac{ax^2}{2}-y^2+3x-fy-c=0$ *represents a circle then coe. of* x^2 = *coe.of* y^2

$$\therefore \frac{a}{2}=-1 \Rightarrow a=-2$$

5. $\ln\left(\frac{dy}{dx}\right)=ax+by \Rightarrow \frac{dy}{dx}=e^{ax+by}=e^{ax}.e^{by}$

$\int e^{-by}dy=\int e^{ax}dx$

6. $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}} \Rightarrow \int\frac{dy}{\sqrt{1-y^2}}+\int\frac{dx}{\sqrt{1-x^2}}=0$

$\Rightarrow \sin^{-1}x+\sin^{-1}y=\sin^{-1}c$

$\Rightarrow \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)=\sin^{-1}(c)$

7. $\int\frac{2x}{1+x^2}dx=\int\frac{2y}{1+y^2}dy$

$\log\left(1+x^2\right)=\log\left(1+y^2\right)+\log c$

$\left(1+x^2\right)=c+cy^2 \Rightarrow x^2-cy^2=c-1$

8. Let $3x+y+4=t^2 \Rightarrow 3+\frac{dy}{dx}=2t\frac{dt}{dx} \Rightarrow \frac{dt}{dx}=2t\frac{dt}{dx}-3$

$\therefore 2t\frac{dt}{dx}-3=t \Rightarrow 2t\frac{dt}{dx}=t+3$

$\int\frac{2t}{t+3}dt=\int dx$

9. Put $x=r\cos\theta,\ y=r\sin\theta \Rightarrow r=\sqrt{x^2+y^2},\ \theta=\tan^{-1}\left(\frac{y}{x}\right)$

$r^2=x^2+y^2$

$2rdr=2xdx+2ydy \Rightarrow xdx+ydy=rdr$

$x(xdy-ydx)=r\cos\theta\left[r\cos\theta(r\cos\theta)-r\sin\theta(-r\sin\theta)\right]d\theta$

$=r^2\cos\theta d\theta$

$\therefore\ rdr=r^3\cos\theta d\theta \Rightarrow \int\frac{1}{r^2}dr=\int\cos\theta d\theta$

$\frac{-1}{r}=\sin\theta+k \Rightarrow \frac{-1}{\sqrt{x^2+y^2}}+\frac{y}{\sqrt{x^2+y^2}}=c$

10. Put $x=r\sec\theta,\ y=r\tan\theta \Rightarrow r^2=x^2-y^2 \quad 2rdr=2xdx-2ydy$

$xdy-ydx\left[r\sec\theta.r\sec^2\theta-r\tan\theta.\ r\sec\theta\tan\theta\right]d\theta$

$\Rightarrow xdx-ydy=rdr$

$=r^2\sec\theta\left(\sec^2\theta-\tan^2\theta\right)d\theta$

$=r^2\sec\theta d\theta$

$$\frac{rdr}{r^2\sec\theta d\theta}=\frac{1+r^2}{r^2}\ \Rightarrow\int\frac{dr}{\sqrt{1+r^2}}=\int\sec\theta d\theta$$

$$\log\left|r+\sqrt{1+r^2}\right|=\log\left|\sec\theta+\tan\theta\right|+\log c$$

11. $xdy-ydx=\sqrt{x^2-y^2}\,dx\quad\Rightarrow\dfrac{xdy-ydx}{\sqrt{x^2-y^2}}=dx$

$$\Rightarrow\frac{xdy-ydx}{x\sqrt{1-\left(\frac{y}{x}\right)^2}}=dx\Rightarrow\frac{\frac{xdy-ydx}{x^2}}{\sqrt{1-\left(\frac{y}{x}\right)^2}}=\frac{dx}{x}$$

$\dfrac{y}{x}=t$

$$\therefore\sin^{-1}\left(\frac{y}{x}\right)=\log x+\log c\Rightarrow\sin^{-1}\left(\frac{y}{x}\right)=\log|cx|$$

12. $\dfrac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=\dfrac{x\cos^2\left(x^2+y^2\right)}{y^3}\Rightarrow\dfrac{xdx+ydy}{ydx-xdy}=\dfrac{x\cos^2\left(x^2+y^2\right)}{y^3}$

$$\sec^2\left(x^2+y^2\right)\frac{1}{2}(2xdx+2ydy)=\frac{ydx-xdy}{y^2}.\left(\frac{x}{y}\right)$$

$$\therefore\frac{1}{2}\int\sec^2\left(x^2+y^2\right).d\left(x^2+y^2\right)=\int\left(\frac{x}{y}\right).d\left(\frac{x}{y}\right)$$

$$\Rightarrow\ \frac{1}{2}\tan\left(x^2+y^2\right)=\frac{\left(\frac{x}{y}\right)^2}{2}+\frac{c}{2}$$

$$\Rightarrow\ \tan\left(x^2+y^2\right)=\frac{x^2}{y^2}+c$$

13. Put

$$x-y=t\Rightarrow1-\frac{dy}{dx}=\frac{dt}{dx}\Rightarrow1-\frac{dt}{dx}=\cos t$$

$$\frac{dt}{dx}=1-\cos t\Rightarrow\int\frac{1}{2\sin^2 t/2}dt=\int dx\Rightarrow\frac{1}{2}\frac{-\cot(t/2)}{\frac{1}{2}}+c=x$$

$$x+\cot\left(\frac{x-y}{2}\right)=c$$

14. Put $\vartheta=\dfrac{y}{x}\ \Rightarrow y=\vartheta x\Rightarrow\dfrac{dy}{dx}=\vartheta+x\dfrac{d\vartheta}{dx}$

$$\vartheta + x\frac{d\vartheta}{dx} = \frac{1-\vartheta}{1+\vartheta} \Rightarrow x\frac{d\vartheta}{dx} = \frac{1-\vartheta}{1+\vartheta} - \vartheta = \frac{1-\vartheta-\vartheta-\vartheta^2}{1+\vartheta}$$

$$\int \frac{1+\vartheta}{1-2\vartheta-\vartheta^2} d\vartheta = \int \frac{dx}{x} \Rightarrow \frac{-1}{2}\log\left|1-2\vartheta-\vartheta^2\right| = \ln x + \log c$$

$$\log x^2 + \log\left|1-\frac{2y}{x}-\frac{y^2}{x^2}\right| = \log c \;\Rightarrow x^2 - 2xy - y^2 = c \;\left(h^2 > ab\right)$$

15. $\left(\frac{dy}{dx}-\frac{y}{x}\right)\tan^{-1}\left(\frac{y}{x}\right) = 1 \;\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{1}{\tan^{-1}\left(\frac{y}{x}\right)}$

$$let\;\vartheta = \frac{y}{x} \Rightarrow \vartheta + x\frac{d\vartheta}{dx} = \vartheta + \frac{1}{\tan^{-1}\vartheta} \Rightarrow \int \tan^{-1}(\vartheta).d\vartheta = \int \frac{dx}{x}$$

$$\tan^{-1}\vartheta.\vartheta - \int \frac{1}{1+\vartheta^2}\vartheta.d\vartheta = \log x + c$$

$$\vartheta\tan^{-1}(\vartheta) - \frac{1}{2}\log\left(1+\vartheta^2\right) = \log x + c$$

$$\Rightarrow \frac{y}{x}\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left(1+\frac{y^2}{x^2}\right) = \log x + c$$

$$put\;\; x=1,\; y=0 \;\Rightarrow c=0 \qquad \therefore \frac{y}{x}\tan^{-1}\frac{y}{x} = \log\left[\sqrt{\frac{x^2+y^2}{x^2}}.x\right]$$

$$\Rightarrow e^{y/x\tan^{-1}(y/x)} = \sqrt{x^2+y}$$

16. Solving $Y - X + X = 0,\; Y + X + 5 = 0 \quad we\;get\;(h,k) = (-2,-3)$

$$\therefore \frac{dY}{dX} = \frac{Y-X}{Y+X} \Rightarrow \;put\; v = \frac{Y}{X}$$

$$x\frac{dv}{dX} = \frac{-v^2+1}{v+1}$$

$$\frac{v+1}{v^2+1}dv + \frac{dX}{X} = 0$$

$$\frac{1}{2}\log\left(v^2+1\right) + \tan^{-1}(v) + \log|X| = c$$

$$\log\left(v^2+1\right) + \log X^2 + 2\tan^{-1} v = c$$

$$\log\left(X^2+Y^2\right) + 2\tan^{-1}\left(\frac{Y}{X}\right) = c$$

17. $\frac{dx}{dt} = \frac{x}{t} - \frac{t}{1+t^2}$

$$\frac{dx}{dt}-\frac{x}{t}=\frac{-t}{1+t^2} \qquad I.F=e^{-\int\frac{1}{t}dt}=e^{\log\left(\frac{1}{t}\right)}=\frac{1}{t}$$

$$G.S \quad x\left(\frac{1}{t}\right)=\int\frac{1}{t}\left(\frac{-t}{1+t^2}\right)dt \Rightarrow \frac{x}{t}=-\tan^{-1}+c$$

$$put\ t=1,\ x=\frac{-\pi}{4}\Rightarrow \quad \frac{-\pi}{4}=\frac{-\pi}{4}+c\Rightarrow c=0$$

$$\therefore\ x=-t\ \tan^{-1}t\Rightarrow x^1(t)=\frac{-t}{1+t^2}-\tan^{-1}t$$

$$\frac{-\pi}{4}\lim_{x\to\infty}\left[\frac{-t}{1+t^2}-\tan^{-1}(t)\right]=\frac{-4}{\pi}\times\frac{-\pi}{4}=2$$

18. $\int_a^x ty(t)\,dt=x^2+y(x)$

$$xy(x)=2x+y^1(x)\Rightarrow\frac{dy}{dx}-xy=-2x$$

$$I.F=e^{\int -xdx}=e^{\frac{-x^2}{2}}\Rightarrow G.S\ \ ye^{\frac{-x^2}{2}}=\int -2xe^{\frac{-x^2}{2}}dx$$

$$e^{\frac{-x^2}{2}}=t$$

$$ye^{\frac{-x^2}{2}}=\int 2dt \qquad e^{\frac{-x^2}{2}}(-x)dx=dt$$

$$ye^{\frac{-x^2}{2}}=2e^{\frac{-x^2}{2}}+c\Rightarrow y=2+cye^{\frac{-x^2}{2}}\rightarrow(1)$$

$$\int_a^x ty(t)\,dt=x^2+y(x)$$

$$If\ \ x=a\ \ 0=a^2+y\Rightarrow y=-a^2\Rightarrow from(1) \qquad -a^2=2+ce^{\frac{-a^2}{2}}$$

$$ce^{\frac{-a^2}{2}}=-2-a^2$$

$$c=-\left(2+a^2\right)e^{\frac{-a^2}{2}}$$

$$\therefore\ y=2-\left(2+a^2\right)e^{\frac{x^2-a^2}{2}}$$

19. $\frac{dy}{dx}+y\left(\frac{4x}{1+x^2}\right)=\frac{1}{\left(1+x^2\right)^2}$

$$I.F=e^{\int\frac{4x}{1+x^2}dx}=e^{2\log\left(1+x^2\right)}=\left(1+x^2\right)^2$$

$$G.S\ \ y\left(1+x^2\right)^2=x+c\Rightarrow put\ x=0,\ y=0\ \ then\ c=0\Rightarrow y=\frac{x}{\left(1+x^2\right)^2}$$

$$\int_0^1\frac{x}{\left(1+x^2\right)^2}=\frac{1}{2}\quad\int_0^1\frac{2x}{\left(1+x^2\right)^2}dx=\frac{1}{2}\left(\frac{-1}{1+x^2}\right)_0^1$$

$$=\frac{-1}{2}\left[\frac{1}{2}-1\right]=\frac{1}{4}\Rightarrow\left(\int_0^1 f(x)\,dx\right)^{-1}=4$$

20. $\frac{dy}{dx}+\frac{y\cos x}{1+\sin x}=\frac{-x}{1+\sin x}$

$$I.F = e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1+\sin x$$

$$G.S \quad y(1+\sin x) = \int \frac{-x}{1+\sin x}.(1+\sin x)dx = \frac{-x^2}{2}+c$$

$$y = \frac{2c-x^2}{1+\sin x}$$

21. $\frac{dy =}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$$I.F = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$$

$$G.S \quad ye^{2\sqrt{x}} = \int \frac{e^{-2\sqrt{x}}}{\sqrt{x}}.e^{2\sqrt{x}}dx = 2\sqrt{x}+c$$

$$y = \left(2\sqrt{x}+c\right)e^{-2\sqrt{x}}$$

22. $\frac{dy}{dx} = \frac{y^3+2xe^y}{ye^y} = \frac{y^2}{e^y} + \frac{2x}{y} \Rightarrow \frac{dx}{dy} - x\left(\frac{2}{y}\right) = \frac{y^2}{e^y}$

$$I.F = e^{\int \frac{-2}{y}dy} = e^{-2\log y} = \frac{1}{y^2}$$

$$G.S \quad x\left(\frac{1}{y^2}\right) = \int \frac{y^2}{e^y}.\frac{1}{y^2}dy$$

$$\frac{x}{y^2} = -e^{-y} + c$$

$$put \ x = 0, \quad y = 1 \Rightarrow 0 = -e^{-1} + c \Rightarrow c = e^{-1}$$

$$\therefore \frac{x}{y^2} = -e^{-y} + e^{-1} \Rightarrow x = y^2(e^{-1} - e^{-y})$$

23. $\frac{dx}{dy} = xy\left[x^2\sin y^2 + 1\right] = x^3 y\sin y^2 + xy$

$$\frac{dx}{dy} - xy = x^3 y\sin y^2 \Rightarrow \frac{1}{x^3}\frac{dx}{dy} - \frac{y}{x^2} = y\sin y^2$$

$$put \ \frac{-1}{x^2} = z \Rightarrow \frac{2}{x^3}.\frac{dx}{dy} = \frac{dz}{dy}$$

$$\frac{1}{2}\frac{dz}{dy} + y.z = y\sin y^2 \Rightarrow \frac{dz}{dy} + 2yz = 2y\sin y^2$$

$$I.F = e^{\int 2ydy} = e^{y}$$

$$G.S \ z.e^{y^2} = \int 2y\sin y^2.e^{y^2}dy$$

$$\frac{-1}{x^2}e^{y^2} = \frac{e^{y^2}}{2}\left[\sin y^2 - \cos y^2\right] + c$$

$$\frac{-2}{x^2} = \sin y^2 - \cos y^2 + ce^{y^2} \Rightarrow 2 = x^2\left[\cos y^2 - \sin y^2 - 2ce^{-y^2}\right]$$

24. $\frac{dy}{dx} = \frac{1}{xcox} - y\frac{(-x\sin x + \cos c)}{x\cos x}$

$$\frac{dy}{dx}+y\frac{(-x\sin x+\cos c)}{x\cos x}=\frac{1}{x\cos x}$$

$$I.F=e^{\int\frac{(-x\sin x+\cos c)}{x\cos x}dx}=e^{\log|x\cos x|}=x\cos x$$

$$G.S\ \ y(x\cos x)=\int\frac{1}{x\cos x}x\cos x dx$$

$$xy\cos x=x+c\Rightarrow xy=(x+c)\sec x$$

25. $\frac{dt}{dx}=t\frac{g(x)}{g(x)}-\frac{t}{g(x)}$

$$\frac{dt}{dx}-t\frac{g(x)}{g(x)}=\frac{t^2}{g(x)}\Rightarrow\frac{-1}{t^2}\frac{dt}{dx}+\frac{1}{t}\frac{g^1(x)}{g(x)}=\frac{1}{g(x)}$$

$$put\ \ \frac{1}{t}=z\Rightarrow\frac{-1}{t^2}\frac{dt}{dx}=\frac{dz}{dx}$$

$$\frac{dz}{dx}+z\frac{g^1(x)}{g(x)}=\frac{1}{g(x)}$$

$$I.F=e^{\int\frac{g^1(x)}{g(x)}dx}=g(x)$$

$$G.S\ \ z.g(x)=\int\frac{1}{g(x)}.g(x)dx$$

$$\Rightarrow\frac{1}{t}g(x)=x+c$$

$$\Rightarrow t=\frac{g(x)}{x+c}$$

26. $y\sin x\frac{dy}{dx}=\cos x\left(\sin x-y^2\right)$

$$put\ \ y^2=z\Rightarrow 2y\frac{dy}{dx}=\frac{dz}{dx}\Rightarrow\frac{1}{2}\sin x\frac{dz}{dx}=\cos x(\sin x-z)$$

$$\frac{dz}{dx}=\frac{2\cos x(\sin x-z)}{\sin x}\Rightarrow\frac{dz}{dx}=2\cos x-2z\cot x$$

$$\frac{dz}{dx}+2z\cot x=2\cot x$$

$$I.F\ =e^{\int 2\cot x dx}=e^{2\log(\sin x)}=\sin x$$

$$G.S\ \ \ y\left(\sin^2 x\right)=\int 2\cos x.\sin^2 x dx=2\frac{\sin^3 x}{3}+c$$

27. $\sec^2\theta d\theta=\tan\theta(r\tan\theta-1)dr$

$$\frac{d\theta}{dr}=\frac{r\tan^2\theta}{\sec^2\theta}-\frac{\tan\theta}{\sec^2\theta}\Rightarrow\frac{d\theta}{dr}+\frac{\tan\theta}{\sec^2\theta}=r\sin^2\theta$$

$$\Rightarrow \cos ec^2\theta\frac{d\theta}{dr}+\cot\theta=r \qquad \because \cot\theta=z$$

$$-\cos ec^2\theta\frac{d\theta}{dr}=\frac{dz}{dr}$$

$$-\frac{dz}{dr}+z=r\Rightarrow\frac{dz}{dr}-z=-r$$

$$I.F \;=e^{\int -1dr}=e^{-r}$$

$$G.S \quad z.e^{-r}=\int e^{-r}\left(-r\right)dr$$

$$\cot\theta\left(e^{-r}\right)=-e^{-r}\left(-r+1\right)+c\Rightarrow\cot\theta=r-1+ce^{r}$$

28. $let \;\; x\sin y+y\cos y=t$ D.w.r to x

$$\Rightarrow x\cos y\frac{dy}{dx}+\sin y-y\sin\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow\left(x\cos y-y\sin y\right)\frac{dy}{dx}+\sin y+\cos y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow\left(x\cos y-y\sin y\right)\frac{dy}{dx}=-\left(x\sin y+y\cos y\right)$$

$$\frac{dt}{dx}-\left(\sin y+\cos y\frac{dy}{dx}\right)=-t$$

$$let \;\; \sin y=z\Rightarrow\cos y\frac{dy}{dx}=\frac{dz}{dx}$$

$$\frac{dt}{dx}-\left(z+\frac{dz}{dx}\right)+t=0$$

$$\frac{d}{dx}\left(t-z\right)+\left(t-z\right)=0 \qquad let \;\; t-z=u$$

$$\frac{du}{dx}+u=0 \qquad \frac{d}{dx}\left(t-z\right)=\frac{du}{dx}$$

$$\frac{du}{dx}=-u\Rightarrow\frac{1}{4}du=-du$$

$$\log u=-x+c\Rightarrow\log\left(t-z\right)+z=c$$

$$\Rightarrow\log\left(x\sin y+y\cos y-\sin y\right)=c-x$$

Given

$$x\sin y+y\cos y-\sin y=ce^{-x}$$

29. $\frac{dy}{dx}+y\left(xe^{x}\right)=x.e^{e^{x}(1-x)}$

$$I.F \;= e^{\int xe^{x}dx} = e^{e^{x}(x-1)}$$

$$G.S \;\; y.e^{e^{x}(x-1)} = \int xe^{e^{x}(1-x)}e^{e^{x}(x-1)}dx$$

$$y.e^{e^{x}(x-1)} = \frac{x^{2}}{2}+c \;\; put \;\; x=1, \;\; y=0 \Rightarrow 0=\frac{1}{2}+c \Rightarrow c=\frac{-1}{2}$$

$$y(0)e^{-1}=\frac{-1}{2} \Rightarrow y(0)=\frac{-e}{2}$$

$$y(0)+\frac{e}{2}=0$$

$$\therefore \;\; y.e^{e^{x}(x-1)} = \frac{x^{2}}{2}\frac{-1}{2}$$

30. $\dfrac{dy}{dx}=\dfrac{y}{2x}+3x^{3/2}-\dfrac{1}{\sqrt{x}} \Rightarrow \dfrac{dy}{dx}-\dfrac{y}{2x}=3x^{3/2}-\dfrac{1}{\sqrt{x}}$

$$I.F = e^{\int \frac{-1}{2x}dx} = e^{\frac{-1}{2}\log x} = \frac{1}{\sqrt{x}}$$

$$G.S \;\; y\left(\frac{1}{\sqrt{x}}\right)=\int\left(3x^{3/2}-\frac{1}{\sqrt{x}}\right).\frac{1}{\sqrt{x}}dx=\int\left(3x-\frac{1}{x}\right)dx$$

$$y=\frac{3}{2}x^{5/2}-\sqrt{x}\ln x+c\sqrt{x}$$