SEC: SR AZ TOPIC : RANDOM VARIABLES AND DISTRIBUTIONS

1. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains Exactly 3 balls is

1. $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ 2. $55\left(\frac{2}{3}\right)^{10}$ 3. $220\left(\frac{1}{3}\right)^{12}$ 4. $22\left(\frac{1}{3}\right)^{11}$

2. One hundred identical coins each with probability p of showing up heads, are tossed once. If $0 < p < 1$ and probability of heads showing on 50 coins is equal to that of showing on 51 coins then the value of p is

1. $\frac{1}{2}$ 2. $\frac{49}{101}$ 3. $\frac{50}{101}$ 4. $\frac{51}{101}$

3. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards.
Then P (X = 1) + P (X = 2) equals

1. 52/169 2. 25/169 3. 49/169 4. 24/169

4. The probability that a candidate secures a seat in Engineering through EAMCET is $\frac{1}{10}$. Seven candidates are selected at random from a centre. The probability that exactly two will get seats is

1. $15\ (0.1)^2\ (09)^5$ 2. $20\ (0.1)^2\ (0.9)^5$ 3. $21\ (0.1)^2\ (0.9)^5$ 4. $(0.1)^2\ (0.9)^5$

5. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3$ k, then K is equal to

1. $\frac{17}{2}$ 2. $\frac{17}{4}$ 3. $\frac{17}{8}$ 4. 4

6. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is:

1. $\frac{15}{2^{13}}$ 2. $\frac{15}{2^{12}}$ 3. $\frac{15}{2^{8}}$ 4. $\frac{15}{2^{14}}$

7. For a binomial variable X if n=5, $P(X=1)=8P(X=3)$, then p =

1. $\frac{4}{5}$ 2. $\frac{1}{5}$ 3. $\frac{1}{3}$ 4. $\frac{2}{3}$

8. A random variable x has the following probability distribution:

x	0	1	2	3	4
P(x)	k	2k	4k	6k	8k

The value of $P(1 < x < 4 \mid x \le 2)$ is L then 7L is equal to

1. $\frac{4}{7}$ 2. $\frac{2}{3}$ 3. $\frac{3}{7}$ 4. $\frac{4}{5}$

9. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is :

1. $\frac{17}{3^5}$ 2. $\frac{13}{3^5}$ 3. $\frac{11}{3^5}$ 4. $\frac{10}{3^5}$

10. Let X be a rrandom variable having binomial distribution B(7 , p). If $P(x=3) = 5P(x=4)$, then the sum of the mean and the variance of X is

1. $\frac{105}{16}$ 2. $\frac{7}{16}$ 3. $\frac{77}{36}$ 4. $\frac{49}{16}$

11. The probability that a student is not a swimmer is 1/4. The probability that out of five students atleast four are swimmers is

1. $\frac{18}{31}$ 2. $\frac{1}{88}$ 3. $\frac{81}{128}$ 4. $\frac{18}{181}$

12. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then 54 $P(X \le 2)$ is equal to

1. $\frac{73}{27}$ 2. $\frac{146}{27}$ 3. $\frac{146}{81}$ 4. $\frac{126}{81}$

13. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X=1) = \frac{4}{243}$, then $P(X = 4 or 5)$ is equal to

1. $\frac{5}{9}$ 2. $\frac{64}{81}$ 3. $\frac{16}{27}$ 4. $\frac{145}{243}$

14. Suppose on an average 1 house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, the probability that exactly 5 houses will have a firing during a year

1. $\frac{1}{15e^2}$ 2. $\frac{14}{15e^2}$ 3. $\frac{4}{15e^2}$ 4. e^{-2}

15. If X is a poisson variate such that $P(X=1) = P(X=2)$, then $P(X=4) =$

1. $\frac{1}{2e^2}$ 2. $\frac{1}{3e^2}$ 3. $\frac{2}{3e^2}$ 4. $\frac{1}{e^2}$

16. If a random variable X has the probability distribution given by $P(X=0) = 3C^3$, $P(X=2) = 5C - 10C^2$ and $P(X=4) = 4C - 1$ then the variance of that distribution is

1. $\frac{68}{9}$ 2. $\frac{22}{9}$ 3. $\frac{612}{81}$ 4. $\frac{128}{81}$

17. A pair of dice is thrown at a time. X is the maximum of the two numbers shown on the dice. Then mean OF x is

1. $\frac{151}{36}$ 2. $\frac{161}{36}$ 3. $\frac{141}{36}$ 4. $\frac{131}{36}$

18. The random variable takes the values 1,2,3, ……m. If $P(X=n)=\frac{1}{m}$ to each n, then the variance of X is

1. $\frac{(m+1)(2m+1)}{6}$ 2. $\frac{m^2-1}{12}$ 3. $\frac{m+1}{2}$ 4. $\frac{m^2+1}{12}$

19. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to:

1. $\frac{40}{243}$ 2. $\frac{128}{625}$ 3. $\frac{80}{243}$ 4. $\frac{32}{625}$

20. In a poisson distribution, the variance is m. The sum of terms in odd places in the distribution is
1. $e^{-m}\tan h(m)$ **2.** $e^{-m}\cos h(m)$ **3.** $e^{-m}\sin h(m)$ **4.** $e^{-m}\cot h(m)$

21. Let a random variable X have a bionomial distribution with mean 8 and variance 4. If $p(x\le 2)=\frac{k}{2^{16}}$, then k is _____.

22. A bag contains 30 while balls and 10 red balls. 16 balls are drawn one by one randomly from the bag, with replacement. If X be the number of white balls drawn, then $\left(\frac{mean\ of\ X}{s\tan dard\ \ deviation\, of\ X}\right)$ is _____.

23. The probability of hitting a target is 1/3. The least number of times to fire so that the probability of hitting the target atleast once is more than 90% is _____.

24. A Pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is _____.

25. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\sigma^2$ is equal to _____.

26. A pair of dice is rolled 4 times. The probability of getting doubled exactly 2 times is K then 216/5k is _____.

27. The random variable x follows binomial distribution B(n , p) for which the difference of the mean and the variance is 1. If $2P(X=2)=3P(X=1)$, then $n^2P(X>1)$ is equal to _____.

28. A random variable x takes the values 0,1,2,3 and its mean is 1.3. If $P(x=3)=2P(x=1)$ and $P(x=2)=0.3$, then $P(x=0)=$ k then 20K =_____.

29. The probability function of a random variable X is given by $P(X=k)=ck^2$, where c is a constant and $k \in \{1,2,3,4\}$. If σ^2 is the variance of X and μ is the mean of X, then $\sigma^2+\mu^2$ is _____.

30. If X is a Poisson variate such that $2P(X=1)=5P(X=5)+2P(X=3)$. Then the variance is _____.

KEY

1	2	3	4	5	6	7	8	9	10
1	**4**	**2**	**3**	**2**	**1**	**2**	**1**	**3**	**3**
11	12	13	14	15	16	17	18	19	20
3	**2**	**3**	**3**	**3**	**4**	**2**	**2**	**4**	**2**
21	22	23	24	25	26	27	28	29	30
137	**7**	**6**	**123**	**56**	**5**	**11**	**8**	**12**	**2**

HINTS

1. p=1/3
   q=2/3
   n=12
   k=3
   $p(\propto=k) = {}^{n}c_k\ P^K \varepsilon^{n-k}$

$$= {}^{12}c_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^9$$

$$= \frac{55}{3}\left(\frac{2}{3}\right)^{11}$$

2. n = 100
   $p(\alpha=50) = p(\alpha=51)$

$$p = \left(\frac{51}{101}\right)$$

3.

A	NA
4	**48**

$$P(x=1) = 2.\frac{{}^{4}c_1}{{}^{52}c_1}.\frac{{}^{48}c_1}{{}^{52}c_1} = \frac{24}{169}$$

$$P(x=2) = \frac{{}^{4}c_2}{{}^{52}c_2} = \frac{1}{169}$$

$$P(x=1)+P(x=2) = \frac{25}{169}$$

4. $P=\frac{1}{10}, q=\frac{9}{10}, n=7$

$$P(x=2) = {}^{7}c_2\left(\frac{1}{10}\right)^2\left(\frac{7}{10}\right)^5$$

$= 21(0.1)^2(0.9)^5$

5. n = 5

$P = \frac{1}{4}, q = \frac{3}{4}$

$P(X = 0) + P(X = 1) + P(X = 2)$

$^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3\left(\frac{3}{4}\right)^3\left(\frac{17}{8}\right)$

$K = \frac{17}{8}$

6. $^nc_7 = {}^nc_9 \Rightarrow n = 7 + 9 = 16$

$p = \frac{1}{2}, q = \frac{1}{2}$

$p(x = 2) = {}^{16}c_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{14}$

$= \frac{8.15}{216} = \frac{15}{2^{13}}$

7. n = 5

$P(x = 1) = 8\ P(x = 3)$

$^5c_1 pq^4 = 8 \quad {}^5c_3\ p^3q^2$

$q^2 = 16p^2$

$q = 4p = 1 - p$

$P = \frac{1}{5}$

8. $21K = 1$

$K = \frac{1}{21}$

$P(1 < x < 4) \mid x \le 2$

$= \frac{P(x = 2)}{P(x = 0) + P(x = 1) + P(x = 2)}$

$= \frac{4k}{7k} = \frac{4}{7}$

9. $P = \frac{1}{3}, q = \frac{2}{5}, n = 5$

$P(x = 4) + P(x = 5)$

$^5c_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) + {}^5c_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0$

$= \frac{11}{35}.$

10. n = 7

$P(x = 3) = 5\ P(x = 4)$

$^7c_3p^3q^4 = {}^{57}c_4p^4q^3$

$q = 5p = 1 - p$

$p = \frac{1}{6}, q = \frac{5}{6}$

$$np + npq = \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42+35}{36} = \frac{77}{36}.$$

11. $P = \frac{3}{4}, q = \frac{1}{4}, n = 5$

$$P(x=4) + P(x=5) = \frac{81}{128}.$$

12. $np = 4,\ npq = \frac{4}{3}$

$$q = \frac{1}{3}, p = \frac{2}{3}, n = 6$$

$$54(p(x=0) + p(x=1) + p(x=2))$$

$$54\left({}^{6}c_0 \left(\frac{1}{3}\right)^6 + {}^{6}c_1 \left(\frac{4}{3}\right)^1 \left(\frac{1}{3}\right)^5 + {}^{6}c_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4 \right)$$

$$= \frac{146}{27}$$

$$np = \alpha$$

$$pnq = \frac{\alpha}{3}$$

$$q = \frac{1}{3}, p = \frac{2}{3}$$

$$p(x=1) = \frac{4}{243}$$

13. ${}^{n}c_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^{n-1} = \frac{4}{243}$

$$2\frac{{}^{n}c_1}{3^{n_7}} = \frac{4}{243}$$

$$p(x=4) + p(x=5)$$

$${}^{6}c_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{2}\right)^2 + {}^{6}c_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1$$

$$\frac{43^2}{36} = \frac{16}{27}.$$

$$p = \frac{1}{1000}$$

$$n = 2000$$

$$\lambda = np = 2$$

14. $$p(\alpha = k) = \frac{\lambda k}{\lfloor k} e^{-\lambda}$$

$$p(x = 5) = \frac{2^5}{\lfloor 5} e^{-2}$$

$$= \frac{32}{120} e^{-2}$$

$$= \frac{4}{15e^{-2}}$$

15. $$p(\alpha = k) = e^{-\lambda} \frac{\lambda^k}{\lfloor k}$$

$$p(\alpha = 1) = p(\alpha = 2)$$

$$e^{-\lambda} \frac{\lambda}{\lfloor 1} = e^{-\lambda} \frac{\lambda^2}{\lfloor 2}$$

$$\lambda = 2$$

$$p(x = 4) = e^{-\lambda} \frac{16}{24} = \frac{2}{3e^2}$$

16. $$p(x = 0) = 3c^3$$

$$p(x = 1) = 5c - 10c^2$$

$$p(x = 2) = 4c - 1$$

$$p(x = 0) + p(x = 1) + p(x = 2) = 1$$

$$3c^2 - 10c^2 + 9c - 2 = 0$$

$$c = 1, 2, \frac{1}{3}$$

$$c = \frac{1}{3}$$

0	**1**	**2**
1/9	**5/9**	**1/9**

$$\text{Variance} = \frac{128}{81}$$

17.

1	2	3	4	5	6
$\boxed{\frac{1}{36}}$	$\boxed{\frac{3}{36}}$	$\boxed{\frac{5}{36}}$	$\boxed{\frac{7}{36}}$	$\boxed{\frac{9}{36}}$	$\boxed{\frac{4}{36}}$

Mean = $\frac{1+6+15+24+45+36}{36} = \frac{161}{36}$

18.

1	2	3	-------	m
1/m	1/m	1/m	-------	1/m

Mean = $\frac{1+2+3+------+m}{m}, \frac{m(m+1)}{2m} = \frac{m+1}{2}$

Variance = $\frac{1^2+2^2+-----m^2}{m} - \frac{(m+1)^2}{4}$

$= \frac{m(m+1)(2m+1)}{6m} - \frac{(m+1)^2}{4}$

$= \frac{(m+1)(m-1)}{12} = \frac{m^2-1}{12}$

19. n = 5

P(x = 1) = 0.4096

P(x = 2) = 0.2048

$\frac{P(x=2)}{P(x=1)} = \frac{1}{2}$

$\frac{{}^5c_2 p^2 q^3}{{}^5c_1 pq^4} = \frac{1}{2}$

$\frac{2p}{q} = \frac{1}{2}$

$4p = q \Rightarrow p = \frac{1}{5}.$

20. P(x = k) = $e^{\lambda} \frac{\lambda^k}{\lfloor k}$

$e^{-\lambda}\left(\frac{1}{\lfloor 0} + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^4}{\lfloor 4} + \frac{\lambda^6}{\lfloor 6} ----\right)$

$e^{-m}\left(1 + \frac{m^2}{\lfloor 2} + \frac{m^4}{\lfloor 4} + \frac{m^6}{\lfloor 6} ----\right)$

$e^{-m}\cosh(m)$

21. np = 8, npq = 4

$q = \frac{1}{2}, p = \frac{1}{2}, n = 16$

P(x = 0) + P(x = 1) + P(X = 2) = $\frac{k}{2^{16}}$

$$\frac{{}^{16}c_0 + {}^{16}c_1 + {}^{16}c_2}{2^{16}} = \frac{k}{2^{16}}$$

$$k = 137$$

22. W = 30, R = 10 , T = 40

$$P(w) = \frac{30}{40} = \frac{3}{4} : n = 16$$

$$\mu = np = 16.\frac{3}{4} = 12$$

$$SD = \sqrt{npq} = \sqrt{16\frac{3}{4}.\frac{1}{4}} = \sqrt{3}$$

$$\frac{12}{\sqrt{3}} = 4\sqrt{3}$$

$$p = \frac{1}{3}, q = \frac{2}{3}$$

$$1 - p(x = 0) > \frac{9}{10}$$

$$\frac{1}{10} > p(x = 0)$$

23. $$\frac{1}{10} > \left(\frac{2}{3}\right)^n$$

$$\left(\frac{2}{3}\right)^n < \frac{1}{10}$$

$$10.2^n < 3^n$$

$$n = 6$$

24. $$n = 5, p = \frac{4}{36} = \frac{1}{9}, q = \frac{8}{9}$$

$$p(x = 4) + p(x = 5) = \frac{k}{3^{11}}$$

$${}^5c_4\left(\frac{1}{9}\right)^4\left(\frac{8}{9}\right) + {}^5c_5\left(\frac{1}{9}\right)^5\left(\frac{8}{9}\right)^0 = \frac{k}{3^{11}}$$

$$\frac{40+1}{95}$$

$$\frac{41}{3^{10}} = \frac{k}{3^{11}}$$

$$k = 123$$

25.

$X = x_i$	0	1	2	3
$p(X = x_i)$	$\frac{{}^4c_0.{}^6c_3}{{}^{10}c_3}$	$\frac{{}^4c_1.{}^6c_2}{{}^{10}c_3}$	$\frac{{}^4c_1.{}^6c_1}{{}^{10}c_3}$	$\frac{{}^4c_3.{}^6c_6}{{}^{10}c_3}$
p	1/6	1/2	3/10	1/10

$$\mu = 0 + \frac{1}{2} + \frac{3}{10} + \frac{3}{10} = \frac{12}{10}$$

$$\sigma^2 = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{10} - \left(\frac{12}{10}\right)^2$$

$$100\sigma^2 = 56$$

$$n = 4, p = \frac{1}{6}, q = \frac{5}{6}$$

$$k = 2$$

26. $$p(x = 2) = {}^4c_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= \frac{25}{216}$$

$$np - npq = 1$$

$$np^2 = 1$$

$$= p(x = 2) = 3p(x = 1)$$

$$= {}^nc_2 p^2 q^{n-2} = 3\,{}^nc_1 p^1 q^{n-1}$$

$$(n-1)p^2 = 3pq$$

$$(n-1)p^2 = p(3-3p)$$

$$np^2 - p^2 = 3p - 3p^2$$

$$1 - p^2 = 3p - 3p^2$$

$$2p^2 - 3p + 2 = 0$$

$$p = 2, \frac{1}{2}$$

$$p = \frac{1}{2}, n = 4, q = \frac{1}{2}$$

$$p(x > 1) = 1 - [p(x = 0) + p(x = 1)]$$

$$= 1 - [{}^4c_0 p^0 q^4 + {}^4c_1 p^1 q^3]$$

$$= 1 - \frac{1+4}{16}$$

27.

$$= 1 - \frac{5}{16} = \frac{11}{16}$$

$$n^2 p(x > 1) = 11.$$

28.

x	0	1	2	3
p	1	m	0.3	2m
	0.4	0.1	0.3	0.2

$\mu = 1.3$

$l + m + 0.3 + 2m = 1$

$l + 3m = 0.7$

$0 + m + 0.6 + 6\,m = 1.3$

$7\,m = 0.7$

$M = 0.1\ ,\ l = 0.4.$

29.

$$\sum_{k}^{4} ck^2 = 1$$

$$c(1+4+9+16) = 1$$

$$c = \frac{1}{30}$$

1	2	3	4
1/30	4/30	9/30	16/30

$$\sigma^2 + \mu^2 = \frac{1+16+81+256}{30}$$

$$= \frac{354}{30} = 11.8 = 12$$

$$p(x=k) = \frac{\lambda^k}{\lfloor k} e^{-\lambda}$$

$$2\,p(x=1) = 5\ \ p(x=5) + 2\,p(x=3)$$

$$2\frac{\lambda^1}{\lfloor 1} e^{-\lambda} = 5\frac{\lambda^5}{\lfloor 5} e^{-\lambda} + 2\frac{\lambda^3}{\lfloor 3} e^{-\lambda}$$

30.

$$2 = \frac{\lambda^4}{24} + \frac{\lambda^2}{3}$$

$$\lambda^4 + 8\lambda^2 - 48 = 0$$

$$\lambda^2 = 4$$

$$\lambda = 2$$

$$\sqrt{\lambda} = \sqrt{2}$$