NARAYANA EDUCATIONAL INSTITUTIONS

DPP: THE PLANE

1. Find the equation of the plane passing through the point $(1,1,-1)$ and perpendicular to the planes $x+2y+3z-7=0$ and $2x-3y+4z=0$ is

1. $17x+2y-7z=26$ 2. $17x+2y-7z=-26$

3. $17x-2y+7z=26$ 4. $17x-2y-7z=-26$

2. Find the equation of the plane passing through the points $(2,1,-1)$, $(-1,3,4)$ and perpendicular to the plane $x-2y+4z=10$

1. $18x-17y+4z=49$ 2. $18x-17y-4z=49$

3. $18x-17y+4z=-49$ 4. $18x+17y+4z=49$

3. The plane $x-2y+3z=0$ is rotated through an right angle about the line of intersection with the plane $2x+3y-4z-5=0$, find the equation of the plane in its new position is ______

1. $22x+5y-4z-35=0$ 2. $22x-5y+4z-35=0$

3. $22x-5y-4z+35=0$ 4. $20x-3y+4z-7=0$

4. The image of the point with position vector $\vec{i}+3\vec{k}$ in the plane $x+y+z-1=0$ is

1. $(-1,-2,1)$ 2. $(-1,2,-1)$ 3. $(-1,3,1)$ 4. $(1,-3,-1)$

5. If the plane $x+y+z=1$ is rotated through an angle 90^0 about the line of intersection with the plane $x-2y+3z=0$ then the new eqn of the plane is

1. $x-8y+7z=-2$ 2. $x+8y-7z=-2$ 3. $x-8y+7z=5$ 4. $8x-y+3z=5$

6. The volume of the tetrahedron included between the plane $6x+4y+5z=60$ and the coordinate planes is

1. 300 2. 200 3. 100 4. 500

7. A tetrahedron has vertices $O(0,0,0)$, $A(1,2,1)$, $B(2,1,3)$ and $C(-1,1,2)$ then the angle between the faces OAB and OAC is ___

1. $\theta=\cos^{-1}\left(\sqrt{\frac{3}{35}}\right)$ 2. $\theta=\cos^{-1}\left(\sqrt{\frac{2}{35}}\right)$

3. $\theta=\cos^{-1}\left(\sqrt{\frac{4}{27}}\right)$ 4. $\theta=\sin^{-1}\left(\sqrt{\frac{1}{35}}\right)$

8. The plane $2\lambda x-(1+\lambda)y+3z=0$ passes through the intersection of the planes

1. $2x-y=0$ and $y+3z=0$
2. $2x-y=0\, and\, y-3z=0$
3. $2x+3z=0\, and\, y=0$
4. $2x-3y=0\, and\, z=0$

9. If for a plane the intercepts on the coordinate axes are 8, 4 and 4 then the distance from origin to the centroid the triangle formed by the points on the coordinates axes is

1. $4\sqrt{\frac{2}{3}}$
2. $8\sqrt{\frac{2}{3}}$
3. $\frac{7}{3}$
4. $7\sqrt{\frac{3}{5}}$

10. Equation of obtuse angle bisector of the planes is $2x-y-2z-6=0$, $3x+2y-6z-12=0$ is

1. $5x-13y+4z-6=0$
2. $5x-13y-4z+6=0$
3. $2x-4y+5z-7=0$
4. $2x-4y+5z-11=0$

11. Let the points on the plane P be equidistant from the points (-4, 2, 1) and $(2,-2,3)$ then the distance between the plane P and the plane $6x-4y+2z=7$ is ______

1. $\frac{9}{2\sqrt{14}}$
2. $\frac{5}{2\sqrt{14}}$
3. $\frac{3}{2\sqrt{14}}$
4. $\frac{1}{\sqrt{14}}$

12. Let P be a plane passing through the points $(0,0,0)$ $(4,1,1)$ $(5,0,1)$ and R be any point $(2,1,6)$ then the image of R in the plane P is

1. $(4,3,-4)$
2. $(4,-3,-4)$
3. $(3,-4,4)$
4. $(3,4,4)$

13. The plane which bisects the angle between the two given planes $2x-y+2z-4=0$ *and* $x+2y+2z-2=0$ which is at a distance of $\frac{2}{\sqrt{10}}$ from origin is

1. $x+3y=2$
2. $x-3y=2$
3. $3x+y+4z=6$
4. $3x-y+4z=6$

14. The plane which bisects the line segment joining the points $(-2,-2,4)$ and $(2,6,-2)$ at right angle passes through which one of the following points?

1. $(-1,1,-1)$
2. $(1,0,2)$
3. $(-1,-1,1)$
4. $(0,-1,2)$

15. Consider the 3 planes

$\pi_1;2x-6y-4z=2$

$\pi_2;4x-2y+5z=4$

$\pi_3;3x-9y-6z=1$

Then which of the following is not correct

1. $\pi_1\, and\, \pi_2$ are perpendicular
2. $\pi_1\, and\, \pi_3$ re parallel

3. Perpendicular distance from O(0, 0, 0) to π_1 is $\frac{1}{\sqrt{14}}$

4. Sum of the intercepts made by π_3 on coordinate axes is $\frac{1}{9}$

16. The image of the point (-1, 3, 4) in the plane $x-2z=0$ is (x_1, y_1, z_1) then $y_1 =$ _____
1) 1 2) 3 3) 4 4) 2

17. If the plane $2x+y-5z=0$ is rotated about its line of intersection with the plane $3x-y+4z-7=0$ by an angle of $\frac{\pi}{2}$ then the plane after rotation is

1. $8x-y+3z=14$
2. $8x+y+5z=14$
3. $8x-y+3z=14$
4. $7x+y+3z=14$

18. If the plane P passes through the intersection of two mutually perpendicular planes $2x+ky-4z-3=0$ and $5x-2y+z-5=0$ and intercepts 2 units on x axis then intercepts on z axis is ______
1) $\frac{8}{21}$ 2) $\frac{-10}{21}$ 3) $\frac{10}{17}$ 4) $\frac{-10}{17}$

19. A plane P is drawn $\perp$ to the two planes $2x-2y+z=0$ and $x-y+2z-4=0$ and passes through the point Q(1, -1, 1). if the distance of the plane P from the point R (2, a, a) is $\sqrt{2}$ where $(a \neq 0)$ then $(QR)^2 =$ _____

1. 27 2. 32 3. 35 4. 15

20. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$ $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the points $(-1, -1, 1)$ then the plane equation is _____
1) $2x+3y+4z+1=0$ 2) $2x-3y+4z+1=0$
3) $x+3y+z+3=0$ 4) $x+y+z+1=0$

21. If the point $(1, 1, \lambda)$ and $(-3, 0, 1)$ be equidistant from the plane $3x+4y-12z+13=0$ then find the value of $[\lambda]$ if $(\lambda > 1)$ here $[.]$ denotes the greatest integer function.

22. Foot of the perpendicular drawn from the origin to the plane $x+3y-4z=52$ is (h, k, l) then $h+k+l=$ ____

23. If a plane meets the coordinate axes at A, B and C such that centroid of the triangle is (1, 2, 4) then the area of the triangle formed by the plane with x-axis, y-axis is _____

24. The image of the point (1, 3, 4) w.r.t plane $2x-y+z+3=0$ is $P(\alpha, \beta, \gamma)$ then the distance between the centroid of the triangle formed by the plane on coordinate axes is G. if PG=k then $\left[\sqrt{k}\right] =$ __ . Where $[.]$ is the greatest integer function

25. The equation of the plane that contains the point $(1,-1,2)$ and $\perp$ to each of the planes $2x+3y-2z=5$ and $x+2y-3z=8$ is $ax+by+cz=d$ then $\left|\frac{a+c}{b}\right|=$ _____

26. If the plane $2x-y+2z+3=0$ has the distances $\frac{2}{3}$ and $\frac{1}{3}$ units from the planes $4x-2y+4z+\lambda=0$ and $2x-y+2z+\mu=0$ respectively then the maximum value of $\frac{\lambda+\mu}{2}$ is equal to ______

27. The equation of a plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$ and at a distance of $\frac{2}{\sqrt{3}}$ from the point (3, 1, -1) is $ax+by+cz=17$ then $a+c=$ ______

28. Let the plane $x+3y-2z+6=0$ meet the coordinate axes at the points A, B, C. If the orthocenter of ΔABC is $\left(\alpha,\beta,\frac{6}{7}\right)$ then $\left|\frac{\beta}{\alpha}\right|=$

29. Let Q be the Foot of the perpendicular drawn from the point P(1, 2, 3) to the plane $x+2y+z=14$ If R is a point on the plane such that $\angle PRQ=45^0$ then area of ΔPRP^1 where P^1 is the image of P w.r.t the same plane

30. Equation of a plane through the line of intersection of planes $2x+3y-4z=1$ *and* $3x-y+z+2=0$ which makes intercept 4 on the positive x axis is $2x+3y-4z-1+\lambda(3x-y+z+2)=0$ then $|2\lambda|=$ __

KEY

01 - 10	1	4	1	1	1	1	1	2	1	1
11 - 20	1	1	2	1	4	2	3	2	3	4
2 1- 30	2	0	9	5	1	7	6	3	6	1

SOLUTIONS

1. The eqn of the plane passing through the point (x_1,y_1,z_1) and parallel to lines whose dr's are (a_1,b_1,c_1) and (a_2,b_2,c_2) is.

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$ie \begin{vmatrix} x-1 & y-1 & z+1 \\ 1 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}=0$$

$$\Rightarrow 17x+2y-7z=26$$

2. The eqn of the plane passing through the points (x_1, y_1, z_1) (x_2, y_2, z_2) and parallel to the line whose dr's (a, b, c)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a & b & c \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z+1 \\ -1-2 & 3-1 & 4+1 \\ 1 & -2 & 4 \end{vmatrix} = 0$$

$\Rightarrow 18x + 17y + 4z = 49$

3. The eqn of any plane through the intersection of the planes $x - 2y + 3z = 0$ *and* $2x + 3y - 4z - 5 = 0$ is $(x - 2y + 3z) + \lambda(2x + 3y - 4z - 5) = 0$

$(1 + 2\lambda)x + (3\lambda - 2)y + (3 - 4\lambda)z - 5\lambda = 0$ ____(i)

it is given that angle between the planes is 90^0

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow (1 + 2\lambda)1 + (3\lambda - 2)(-2) + (3 - 4\lambda)3 = 0$

$1 + 2\lambda - 6\lambda + 4 + 9 - 12\lambda = 0$

$\Rightarrow 16\lambda = 14$

$\boxed{\lambda = 7/8}$ *put in* (i)

required plane $22x + 5y - 4z - 35 = 0$

4. Let (α, β, γ) be the image point

$$\frac{\alpha - 1}{1} = \frac{\beta - 0}{1} = \frac{\gamma - 3}{1} = \frac{-2(1 + 0 + 3 - 1)}{1^2 + 1^2 + 1^2}$$

$$\frac{\alpha - 1}{1} = \frac{\beta}{2} = \frac{\gamma - 3}{1} = \frac{-6}{3}$$

$\alpha = -1, \quad \beta = -2, \quad \gamma = 1$

5. The new position of the plane is

$(x - 2y + 3z) + \lambda(x + y + z - 1) = 0$

$\Rightarrow (1 + \lambda)x + (\lambda - 2)y + (3 + \lambda)z - \lambda = 0$

Given that this is $\perp$ to $x + y + z = 1$

$\therefore (1 + \lambda)1 + (\lambda - 2)1 + (3 + \lambda)1 = 0$

$\Rightarrow 1 + \lambda + \lambda - 2 + 3 + \lambda = 0$

$3\lambda + 2 = 0 \quad \lambda = -2/3$

$\therefore$ New position of the plane is

$$(x - 2y + 3z) - \frac{2}{3}(x + y + z - 1) = 0$$

$3x - 6y + 9z - 2x - 2y - 2z + 2 = 0$

$x - 8y + 7z = -2$

6. Eqn of the plane in the intercept from is

$\frac{x}{10}+\frac{y}{15}+\frac{z}{12}=1 \quad which\ meets\ the$

coordinate axes at the points $A(10,0,0)\ B(0,15,0)\ and\ C(0,0,12)$ the coordinate of the origin $(0,0,0)$

$\therefore$ volume of the tetrahedron $OABC=\frac{1}{6}\begin{vmatrix}10 & 0 & 0\\ 0 & 15 & 0\\ 0 & 0 & 12\end{vmatrix}$

$$=\frac{1}{6}\left|10(15\times 12)\right|=300$$

7. Vector perpendicular to the face OAB is $\overrightarrow{OA}\times\overrightarrow{OB}$

$=\left(\vec{l}+2\vec{l}+\vec{k}\right)\times\left(2\vec{l}+\vec{j}+3\vec{k}\right)$

$=5\vec{l}-\vec{j}-3\vec{k}$

vector perpendicular to the face $OAC\ is\ \overrightarrow{OA}\times\overrightarrow{OC}=3\vec{i}-3\vec{j}+3\vec{k}$

Since the angle between the face = angle between there is normal

$\cos\theta=\frac{5(3)+(-1)(-3)+(-3)(3)}{\sqrt{5^2+1^2+3^2}\ \sqrt{3^2+3^2+3^2}}$

$=\frac{15+3-9}{\sqrt{35}\ \sqrt{27}}=\frac{9}{\sqrt{35}\ 3\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{35}}$

$\theta=\cos^{-1}\left(\sqrt{\frac{3}{35}}\right)$

8. Given eqn can be written as

$2\lambda x+y-\lambda y+3z=0$

$(2x-y)\lambda+(-y+3z)=0$

in the eqn of the plane passes through the intersection of planes $2x-y=0\ and\ y-3z=0$

9. Plane eqn $\frac{x}{8}+\frac{y}{4}+\frac{z}{4}=1$

let points on the axes be $A(8,0,0)\ B(0,4,0)\ \&\ C(0,0,4) centroid\ G\left(\frac{8}{3},\frac{4}{3},\frac{4}{3}\right)$

Now $OG=\sqrt{\frac{64}{9}+\frac{16}{9}+\frac{16}{9}}=\sqrt{\frac{96}{9}}=\sqrt{\frac{32}{3}}=4\sqrt{\frac{2}{3}}$

10. Here $d_1\ d_2=(-6)(-12)>0$

$a_1a_2+b_1b_2+c_1c_2=\ 2.3+(-1)(2)+(-2)(-6)$

$=\ 6-2+12>0$

$\therefore$ obtuse angular bisection is

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \frac{(a_2x+b_2y+c_2z+b_2)}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\frac{(2x-y-2z-1)}{\sqrt{2^2+1^2+2^2}} = \frac{(3x+2y-6z-12)}{\sqrt{3^2+2^2+6^2}}$$

$$7(2x-y-2z-6) = 3(3x-2y-6z-12)$$

$$14x-7y-14z-42 = 9x+6y-18z-36$$

$$5x-13y+4z-6=0$$

11. Let the point on the plane be $p(x,y,z)$ given that the points on the plane P are equidistant from $(-4,2,1)$ *and* $(2,-2,3)$

$ie\ (x+4)^2+(y-2)^2+(z-1)^2 = (x-2)^2+(y+2)^2+(z-3)^2$

solving we get plane p as $3x-2y+z+1=0$

Given plane (parallel) $6x-4y+2z-7=0$

$$6x-4y+2z+2=0$$

distance between plans $= \dfrac{|-7-2|}{\sqrt{6^2+4^2+2^2}}$

$$= \frac{9}{2\sqrt{14}}$$

12. Eqn of the plane $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & y & z \\ 4 & 1 & 1 \\ 5 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \boxed{x+y-5z=0}$$

using image theorem let (α,β,γ) *be the image of* $R(2,1,6)$ *w.r.t plane*

$$\frac{\alpha-2}{1} = \frac{\beta-1}{1} = \frac{\gamma-6}{5} = \frac{-2(2+1-5(6))}{1^2+1^2+(-5)^2}$$

$$\alpha-2 = \beta-1 = \frac{\gamma-6}{5} = \frac{-2(27)}{27}$$

$$\Rightarrow (\alpha,\beta,\gamma) = (4,3,-4)$$

13. Eqn of the given planes are $2x-y+2z-4=0$ ____ (i)

$$x+2y+2z-2=0 \text{ ____ } (ii)$$

angular bisectors are $\dfrac{(2x-y+2z-4)}{\sqrt{2^2+1^2+2^2}} = \dfrac{\pm(x+2y+2z-2)}{\sqrt{1^2+2^2+2^2}}$

bisectors are $x-3y=2$ & $3x+y+4z-6=0$

$\perp$ *distance from* $0(0,0,0)$ *to* $x-3y-2=0$ *is* $\dfrac{2}{\sqrt{10}}$

14. Let the points be $A(-2, -2, 4)$ $B(2, 6, -2)$

The midpoint of the line joining AB is $P(0,2,1)$

DR's AB line $(-2-2,\ -2-6,\ 4+2)=(-4, -8, 6)$
$=(a,b,c)$

eqn of the line $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$\Rightarrow\ -4(x-0)\ -8(y-2)\ +6(z-1)=0$

$-4x-8y+16+6z-6=0$

$-4x-8y+6z+10=0$

$2x+4y-3z-5=0$

by checking $(-1, 1, -1)$ *safisties the plane eqn*

$2(-1)+4(1)-3(-1)-5$

$-2+4+3-5=0$

15. If the planes $a_1x+b_1y+c_1z=d_1,\ a_2x+b_2y+c_2z=d_2$ are parallel then

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Here $\pi_1: 2x-6y-4z=2$

$\pi_2: 4x-2y+5z=4$

$\pi_3: 3x-9y-6z=1$

π_1 *and* π_3 are parallel since $\frac{2}{3}-\frac{-6}{-9}=\frac{-4}{-6}$

$\frac{2}{3}-\frac{2}{3}=\frac{2}{3}$

Perpendicular distance from O (0, 0, 0) to $ax+by+cz=0$ is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}$

Equation of the plane in intercepts from is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

π_3 Plane can be written as $\frac{x}{\frac{1}{3}}+\frac{y}{\frac{-1}{9}}+\frac{z}{\frac{-1}{6}}=1$

Sum of intercepts = $\frac{1}{3}-\frac{1}{9}-\frac{1}{6}=\frac{6-2-3}{18}=\frac{1}{18}$

16. Using image theorem

Let $(h,k,l)=(x_1,y_1,z_1)$

$$\frac{x_1+1}{1}=\frac{y_1-3}{0}=\frac{z_1-4}{-2}=\frac{-2(-1-2(4))}{1^2+0^2+(-2)^2}$$

$$\frac{y_1-3}{0}=\frac{-2(-9)}{5}$$

$$y_1-3=0 \Rightarrow y_1=3$$

17. Given plane equation $2x+y-5z=0$ let the equation of the plane formed be

$$(2x+y-5z)+\lambda(3x-y+4z-7)=0$$

$$\Rightarrow \quad x(2+3\lambda)+(1-\lambda)y+(-5+4\lambda)z-7\lambda=0 \_\_\_\_\_\_\_(1)$$

this plane is perpendicular to $2x+y-5z=0$

Using $a_1a_2+b_1b_2+c_1c_2=0$

$$2(2+3\lambda)+1(1-\lambda)-5(-5+4\lambda)=0$$

$$4+6\lambda+1-\lambda+25-20\lambda=0$$

$$-15\lambda+30=0$$

$\Rightarrow \lambda=2$ sub in -------(1)

required plane is $8x-y+3z=14$

18. Planes $2x+ky-4z-3=0$ and $5x-2y+z-5=0$

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow 2.5+k(-2)-4(1)=0$$

$$10-2k-4=0$$

$$6-2k=0 \quad \Rightarrow k=3$$

Now equation of the plane passing through the intersection of $P_1 \& P_2$ planes is $P_1+\lambda P_2=0$

$$(2x+3y-4z-3)+\lambda(5x-2y+z-5)=0$$

$$(2+5\lambda)+\lambda(3-2\lambda)+z(-4+\lambda)-(3+5\lambda)=0$$

$$(2+5\lambda)x+(3-2\lambda)y+(-4+\lambda)z=(3+5\lambda)$$

$$\frac{x}{\frac{(3+5\lambda)}{(2+5\lambda)}}+\frac{y}{\frac{(3+5\lambda)}{(3-2\lambda)}}+\frac{z}{\frac{(3+5\lambda)}{-4+\lambda}}=1$$

Given x intercept is 2 units

$$\Rightarrow \frac{3+5\lambda}{2+5\lambda}=2$$

$3+5\lambda=4+10\lambda$

$5\lambda=-1$

$\lambda=\frac{-1}{5}$

Z intercept = $\dfrac{3+5\left(\frac{-1}{5}\right)}{-4-\frac{1}{5}}=\dfrac{3-1}{\frac{-21}{5}}=\dfrac{2}{\frac{-21}{5}}=\dfrac{-10}{21}$

19. Eqn of plane passing through $Q(1,-1,1)$ *and* $\perp$ *to the planes* $2x-2y+z=0$ and $x-y+2z-4=0$ *is*

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}=0$$

plane is $x+y=0$

given point $R(2,a,a)$ where $a\neq 0$ perpendicular distance from plane is $\sqrt{2}$

$\frac{|2+a|}{\sqrt{1+1}}=\sqrt{2}$

$|2+a|=2$

$|2+a|=\pm 2$

$case\ (i)\quad 2+a=2$

$\because a\neq 0$

$case\ (ii)\quad 2+a=-2$

$\boxed{a=-4}$

$\because a\neq 0 \Rightarrow a=-4$

$\because$ Point $R(2,a,a)=(2,-4,-4)$

$QR=\sqrt{(2-1)^2+(-4+1)^2+(-4-1)^2}$

$=\sqrt{1+9+25}=\sqrt{35}$

$(QR)^2=35$

20. By the data the point are coplanar condition for co-planarity of four points (x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3) and (x_4, y_4, z_4) is

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 2 & -(\lambda^2 + 1) \\ 2 & 1 - \lambda^2 & 0 \\ 1 - \lambda^2 & 2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow -(\lambda^2 + 1)\left(4 - (1 - \lambda^2)(1 - \lambda^2)\right) = 0$$

$$\lambda^2 + 1 \neq 0 \quad \Rightarrow \quad 4 - (1 - \lambda^2)^2 = 0$$

$$(1 - \lambda^2)^2 = 4$$

$$(1 - \lambda^2) = \pm 2$$

$$1 - \lambda^2 = 2$$

$$\lambda^2 \neq -1$$

$$or \quad 1 - \lambda^2 = -2$$

$$\lambda^2 = 3$$

$$\boxed{\lambda = \pm\sqrt{3}}$$

$\because$ Point is $(-3, 1, 1)$ $(1, -3, 1)$ $(1, 1, -3)$

Equation of the plane is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x + 3 & y - 1 & z - 1 \\ 4 & -4 & 0 \\ 4 & 0 & -4 \end{vmatrix} = 0$$

Solving we get $x + y + z + 1 = 0$

21. It is given that the points $(1,1,\lambda)$ *and* $(-3,0,1)$ are equidistant from the plane

$3x+4y-12z+13=0$

$$\frac{|3(1)+4(1)-12(\lambda)+13|}{\sqrt{3^2+4^2+(-12)^2}}=\frac{|3(-3)+4(0)-12(1)+13|}{\sqrt{3^2+(4)^2+(-12)^2}}$$

$\Rightarrow |20-12\lambda|=8$

$20-12\lambda=\pm 8$

$20-12\lambda=8$ / $20-12\lambda=-8$

$12-12\lambda$ / $28=12\lambda$

$\boxed{\lambda=1}$ $\boxed{\lambda=7/3}$ $\because \lambda>1$

$Now \left[\frac{7}{3}\right]=2$

22. Foot of the $\perp$ draw from the origin to the plane

$ax+by+cz=d\ is\ \left[\frac{ad}{a^2+b^2+c^2},\frac{bd}{a^2+b^2+c^2},\frac{cd}{a^2+b^2+c^2}\right]$

Foot of the $\perp$ be $(h,k,l)=\left(\frac{1.52}{1^2+3^2+4^2},\frac{3.52}{1^2+3^2+4^2},\frac{-4.52}{1^2+3^2+4^2}\right)$

$=\left(\frac{52}{26},\frac{3.52}{26},\frac{-4.52}{26}\right)$

$=(2,6,-8)\ \ Now\ h+k+l=0$

23. Let the eqn the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Then $A(a,o,o)$, $B(o,b,o),C(o,o,c)$ are the points on the coordinate axes

Centroid of the triangle $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)=(1,2,4)$

$\Rightarrow a-3,\ b=6\ c=12$

plane is $\frac{x}{3}+\frac{y}{6}+\frac{z}{12}=1$

Now area of the Δ/e formed by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ with X axis, Y axis is $\frac{1}{2}|ab|$ squints

with X axis, Y axis is $\frac{1}{2}|ab|$ squints

$\therefore Area\ with\ X\ axis,\ Y\ axis=\frac{1}{2}|3.6|$

$=9\ squnits$

24. Image of the point $(1,3,4)$ w.r.t plane $2x-y+z+3=0$ *is* $P(\alpha,\beta,\gamma)$

$$\frac{\alpha-1}{2}=\frac{\beta-3}{-1}=\frac{\lambda-4}{1}=\frac{-2(2(1)-3+4+3)}{2^2+1^2+1^2}$$

$solving\ \ P(\alpha,\beta,\gamma)=(-3,5,2)$

centroid of the Δ / e formed by the plane o9n the coordinate axes is $G=(-1/2,\ 1,\ -1)$

$$Now\ GP = k = \frac{125}{4}$$

$$\left[\sqrt{k}\right] = \left[\sqrt{\frac{125}{4}}\right] = [5.59]$$

$$= 5$$

25. The eqn of the plane is

$$\begin{vmatrix} x-1 & y+1 & z-2 \\ 2 & 3 & -2 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$$(x-1)(-9+4)-(y+1)\ (-6+2)+(z-2)(4-3)=0$$

$$(x-1)(-5)-(y+1)(+4)+(z-2)(1)=0$$

$$-5x+5+4y+4+z-2=0$$

$$-5x+4y+z+7=0$$

$$5z-4y-z-7=0$$

$$\frac{a+c}{b} = \frac{5-1}{-4} = -1$$

composing with $ax+by+cz=d$

$a=5,\ b=-4,\ c=-1,\quad d=7$

$$\Rightarrow \left|\frac{a+c}{b}\right| = 1$$

26. Given plane are $2x-y+2z-3=0$ ____ (1)

$4x-2y+4z+\lambda=0$ ____ (2)

$2x-y+2z+\mu=0$ ____ (3)

distance between (i) & (ii) is $\frac{2}{3}$

$$\Rightarrow \frac{|\lambda+2(3)|}{\sqrt{4^2+2^2+4^2}} = \frac{2}{3}$$

$$\frac{|\lambda-6|}{6} = \frac{2}{3} \Rightarrow |\lambda-6| = 4$$

$$\lambda-6=\pm 4$$

$$\lambda-6=4/\lambda-6=-4$$

$$\boxed{\lambda=10}\ \boxed{\lambda=2}$$

distance between planes (i) & (iii) is

$$\frac{|\mu-3|}{\sqrt{2^2+1^2+2^2}} = \frac{1}{3}$$

$$\frac{|\mu-3|}{3} = \frac{1}{3} \Rightarrow |\mu-3|=1$$

$$\mu-3=\pm 1$$

$$\mu-3=1/\mu-3=-1$$

$$\boxed{\mu=4} \quad \boxed{\mu=2}$$

Now for max value of $\frac{\lambda+\mu}{2}$ *take* $\lambda=10$ & $\mu=4$

$$\Rightarrow \frac{10+4}{2}=7$$

27. Equation of the plane passing through the intersection of the planes $x+2y+3z-2=0$ and $x-y+z-3=0$ is

$(x+2y+3z-2)+\lambda(x-y+z-3)=0$

$\Rightarrow x(1+\lambda)+y(2-\lambda)+z(3+\lambda)-(2+3\lambda)=0$

Given Distance from $(3,1,-1)$ is $\frac{2}{\sqrt{3}}$

$$\Rightarrow \frac{|3(1+\lambda)+1(2-\lambda)-1(3+\lambda)-(2+3\lambda)|}{\sqrt{1+\lambda^2+(2-\lambda)^2+(3+\lambda)^2}}=\frac{2}{\sqrt{3}}$$

$$\Rightarrow \frac{|3+3\lambda+2-\lambda-3-\lambda-2-3\lambda|}{\sqrt{3\lambda^2+4\lambda+14}}=\frac{2}{\sqrt{3}}$$

$$\Rightarrow \frac{|-2\lambda|}{\sqrt{3\lambda^2+4\lambda+14}}=\frac{2}{\sqrt{3}}$$

Solving $\lambda=-\frac{7}{2}$ sub in 1

Plane is $5x-11y+z=17$ comparing $ax+by+cz=17$

$a+c=5+1=6$

28. Plane $x+3y-2z=-6$

$$\frac{x}{-6}+\frac{y}{-2}+\frac{z}{3}=1$$

Let P be the orthocenter $=\left(\alpha,\beta,\frac{6}{7}\right)$

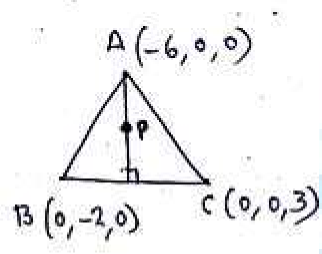
$Ap \perp Bc$

Direction ratios $Ap = \left(\alpha + 6, \beta, \frac{6}{7} \right)$

Direction ratios $Bc = (0,2,3)$

$Ap \perp Bc$

$\Rightarrow O(\alpha + 6) + 2\beta + \frac{18}{7} = 0$



$2\beta = -\frac{18}{7}$

$\beta = -\frac{9}{7}$

$Bp \perp Ac$

Direction ratios of $BP = \left(\alpha - 0, \beta + 2, \frac{6}{7} \right)$

Direction ratios of $AC = (0+6, 0-0, 3-0) = (6,0,3)$

$BP \perp AC$

$6(\alpha) + 0.(\beta + 2) + 3\left(\frac{6}{7} \right) = 0$

$6\alpha + \frac{18}{7} = 0$

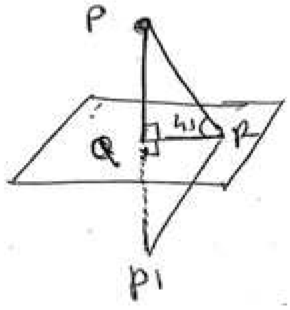
$6\alpha = \frac{-18}{7} \Rightarrow \quad \alpha = \frac{-3}{7}$

Now $\left| \frac{\beta}{\alpha} \right| = \left| \frac{\frac{-9}{7}}{\frac{-3}{7}} \right| = 3$

29. $P(1,2,3)$

Plane Equation $x + 2y + z - 14 = 0$

Give $\angle PRQ = 45^0$



$\Rightarrow \angle RPQ = 45^0$

$\Rightarrow PQ = QR$

$$PQ = \frac{|1+2(2)+3-14|}{\sqrt{1^2+2^2+1^2}}$$

$$= \frac{|-6|}{\sqrt{6}} = \sqrt{6} = PQ$$

$\because$ ^1P Is the image of P w.r.t the plane

$\Delta PRP^1 = 2\Delta PQR$

$$= 2\frac{1}{2}\times\sqrt{6}\times\sqrt{6}$$

= 6 Units

30. The plane $2x+3y-4z-1+\lambda(3x-y+z+2)=0$ passes through (4, 0, 0)

$\because$ x Intercept is 4

$2(-4)-1+\lambda(3(4)+2)=0$

$8-1+\lambda(14)=0$ $14\lambda=-7$

$\lambda = \frac{-1}{2}$

$\because$ $|2\lambda| = 1$