

QUADRATIC EQUATION AND EXPRESSION

MATHEMATICS-A

1. If $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} = 6$ and $\frac{a}{c} + \frac{b}{d} + \frac{c}{a} + \frac{d}{b} = 8$, then the value of $\frac{a}{b} + \frac{c}{d}$ is/are
 1) 4 2) 3 3) $\frac{1}{4}$ 4) $\frac{1}{2}$
2. If p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$ then p & q are the roots of the equation
 1) $x^2 - 2x + 8 = 0$ 2) $x^2 - 2x + 16 = 0$ 3) $x^2 - 2x + 2 = 0$ 4) $x^2 - 2x + 36 = 0$
3. Let $\alpha, \beta (\alpha > \beta)$ be the roots of the equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$, then

$$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$$
 1) 13 2) 14 3) 15 4) 16
4. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3$ find the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$
 1) $3x^2 - 19x + 3 = 0$ 2) $3x^2 - 19x - 3 = 0$ 3) $3x^2 + 19x - 3 = 0$ 4) $3x^2 + 19x + 3 = 0$
5. If the roots of equation $3x^2 + 5x + 1 = 0$ are $(\sec \theta_1 - \tan \theta_1)$ and $(\operatorname{cosec} \theta_2 - \cot \theta_2)$ then find the equation whose roots are $(\sec \theta_1 + \tan \theta_1)$ and $(\operatorname{cosec} \theta_2 + \cot \theta_2)$
 1) $3x^2 - 5x + 1 = 0$ 2) $3x^2 + 5x - 1 = 0$ 3) $x^2 - 5x + 3 = 0$ 4) $x^2 + 5x + 3 = 0$
6. If a, b, c are in A.P. and one root of the equation $ax^2 + bx + c = 0$ is 2 then find the other root
 1) $\frac{5}{2}$ 2) $\frac{-5}{2}$ 3) $\frac{-5}{4}$ 4) $\frac{5}{4}$
7. Find a quadratic equation whose product of the roots x_1 & x_2 is equal to 4 and satisfy the relation

$$\frac{x_1}{x_1 - 1} + \frac{x_2}{x_2 - 1} = 2$$
 1) $x^2 - 2x + 4 = 0$ 2) $x^2 - 3x + 4 = 0$ 3) $x^2 - 2x - 4 = 0$ 4) $x^2 - 3x - 4 = 0$
8. Let $\alpha, \beta \in \mathbb{R}$ if α^2, β^2 are the roots of the quadratic equation $x^2 - px + 1 = 0$ and $\alpha^2\beta$ are the roots of quadratic equation $x^2 - px + 8 = 0$ then find α, β
 1) $\alpha = \frac{1}{2}, \beta = 4$ 2) $\alpha = 4, \beta = \frac{-1}{2}$ 3) $\alpha = 4, \beta = \frac{1}{2}$ 4) $\alpha = \frac{1}{2}, \beta = -4$
9. If α and β are the roots of equation $ax^2 + bx + c = 0$ then find the roots of the equation

$$ax^2 - bx(x-1) + c(x-1)^2 = 0$$
 in terms of α and β
 1) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ 2) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$ 3) $\frac{\alpha}{1+\alpha}, \frac{\beta}{1+\beta}$ 4) $\frac{1+\alpha}{\alpha}, \frac{1+\beta}{\beta}$
10. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and a, b, c are root zero real numbers then find the value of $\frac{a^3 + b^3 + c^3}{abc}$
 1) 2 2) 3 3) 4 4) 1

11. If equation $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root then find the value of a and b.
 1) $a = 7, b = 8$ 2) $a = -7, b = 8$ 3) $a = -7, b = -8$ 4) $a = 7, b = -8$
12. The sum of the non real roots of $(x^2 + x - 2)(x^2 + x - 3) = 12$ is
 1) 0 2) 1 3) -1 4) 2
13. The number of irrational roots of the equation $\frac{4x}{x^2 + x + 3} + \frac{5x}{x^2 - 5x + 3} = -\frac{3}{2}$ is
 1) 4 2) 0 3) 1 4) 2
14. If $x^2 + ax - 3x - (a + 2) = 0$ has real and distinct roots, then the minimum value of $\frac{a^2 + 1}{a^2 + 2}$
 1) 1 2) 0 3) $\frac{1}{2}$ 4) $\frac{1}{4}$
15. α, β are the roots of $ax^2 + bx + c = 0$ and α^2, β^2 are the roots of $a^2x^2 + b^2x + c^2 = 0$ then a, b, c are in
 is
 1) G.P. 2) H.P 3) A.P. 4) None
16. Solve for x $(5 + 2\sqrt{6})^{x^2 - 3} + (5 - 2\sqrt{6})^{x^2 - 3} = 10$
 1) ± 3 2) $\sqrt{2}$ 3) $-\sqrt{2}$ 4) ± 2
17. Find the value of 'm' for which exactly one root of the equation $x^2 - 2mx + m^2 - 1 = 0$ lies in the interval $(-2, 4)$
 1) $[-3, -1) \cup (3, 5]$ 2) $(-3, -1) \cup (3, 5)$ 3) $(-3, -1] \cup [3, 5)$ 4) $[-3, -1] \cup [3, 5]$
18. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots of the equation $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P. the integral value of (p, q) respectively are
 1) $(-2, -32)$ 2) $(-2, 3)$ 3) $(-6, 3)$ 4) $(-6, 32)$
19. If a, b, c are real and $a \neq b$ the root of the equation $2(a - b)x^2 - 11(a + b + c)x - 3(a - b) = 0$ are
 1) Real and equal 2) real and unequal 3) Purely imaginary 4) None
20. If for all real values of a one root of the equation $x^2 - 3x + f(a) = 0$ is double the other f(x) is equal to
 1) $2x$ 2) x^2 3) $2x^2$ 4) $2\sqrt{x}$
21. If the roots of the quadratic equation $(4p - p^2 - 5)x^2 - (2p - x)x + 3p = 0$ lies on either side of unity then number of integral value of p is
 1) 1 2) 2 3) 3 4) 4
22. If $b_1 b_2 = 2(c_1 + c_2)$ then at least one of the equation $x^2 + b_1 x + c_1 = 0$ and $x^2 + b_2 x + c_2 = 0$ has
 1) Imaginary roots 2) Real roots 3) Purely imaginary root 4) None
23. If α, β are the roots of $x^2 - px + q = 0$ and α', β' are the roots of $x^2 - p'x + q' = 0$ then the value of $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$ is
 1) $2[p^2 - 2q + p'^2 - 2q' - pp']$ 2) $2[p^2 - 2q - p'^2 - 2q' + qq']$
 3) $2[p^2 - 2q - p'^2 - 2q' + pp']$ 4) $2[p^2 - 2q - p'^2 - 2q' - qq']$
24. If the equation $x^2 - 3px + 2q = 0$ and $x^2 - 3ax + 2b = 0$ have a common root and the other roots of the second equation is the reciprocal of the other roots of the first then $(2q - 2b)^2$
 1) $36pa(q - b)^2$ 2) $18pa(q - b)^2$ 3) $36pq(p - q)^2$ 4) $18pq(p - a)^2$
25. The number of real solution of $|x| + 2\sqrt{5 - 4x - x^2} = 16$
 1) 6 2) 1 3) 0 4) 4

26. If roots of $x^2 - (a-3)x + a = 0$ are such that at least one of them is greater than 2 then
 1) $a \in [7, 9]$ 2) $a \in [7, \infty]$ 3) $a \in [9, \infty]$ 4) $a \in [7, 9]$
27. Let $n \in \mathbb{Z}$ and ΔABC be a right triangle with right angle at C if $\sin A$ and $\sin B$ are the roots of the quadratic equation $(5n+8)x^2 - (7n-20)x + 120 = 0$. Then find the value of n
 1) 56 2) 55 3) 66 4) 67
28. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$ then $\lim_{n \rightarrow \infty} \sum_{r=1}^n \alpha^r + \lim_{n \rightarrow \infty} \sum_{r=1}^n \beta^r$ is equal to
 1) $\frac{1}{12}$ 2) $\frac{1}{13}$ 3) $\frac{1}{15}$ 4) $\frac{1}{14}$
29. Let $P(x)$ be a quadratic polynomial such that $P(0) = 1$. If $P(x)$ leaves remainder 4 when divided by $x - 1$ and it leaves remainder 6 when divided by $x + 1$ then
 1) $P(2) = 11$ 2) $P(2) = 19$ 3) $P(-2) = 19$ 4) $P(-2) = 11$
30. If the quadratic equation $4^{\sec^2 \alpha} x^2 + 2x + \left(\beta^2 - \beta + \frac{1}{2} \right) = 0$ has real roots then the value of $\cos^2 \alpha + \cos^{-1} \beta =$
 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{3} + 1$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{2} - 1$

MATHEMATICS-A

1-10	1	2	4	1	4	3	1	2	3	2
11-20	3	3	4	3	1	4	2	1	2	3
21-30	2	2	1	3	3	3	3	1	3	2

SOLUTIONS

01. Let $\frac{a}{b} = x, \frac{b}{c} = y, \frac{c}{d} = z, \frac{d}{a} = w$
 i.e., $x + y + z + w = 6$ and $xy + yz + zw + wx = 8$
 $(x+z) + (y+w) = 6 \rightarrow (1)$ and $(x+z)(y+w) = 8 \rightarrow (2)$
 So from (1) and (2)
 $(x+z)$ and $(y+w)$ are roots of the equation $t^2 - 6t + 8 = 0$
 $t = 2, 4 \Rightarrow \frac{a}{b} + \frac{c}{d} = 2$ (or) 4
02. $x^2 + (p+q)x + pq = 0$
 $p+q = 2 \Rightarrow (p+q)^2 = 4$
 $p^2 + q^2 + 2pq = 4$
 $(p^2 + q^2)^2 = (4 - 2pq)^2$
 $p^4 + q^4 + 2p^2q^2 = 16 + 4p^2q^2 = 16pq$
 Let $pq = x$
 $272 + 2x^2 = 17 + 4x^2 - 16x$
 $2x^2 - 16x - 256 = 0$

$$x^2 - 8x - 128 = 0$$

$$x^2 - 16x + 8x - 128 = 0$$

$$x(x - 16) + 8(x - 16) = 0$$

$$(x - 16)(x + 8) = 0$$

$$x = -8 \text{ (not positive)}$$

$$x = 16, \text{ (since p, q are positive)}$$

$$pq = 16, p + q = 2$$

$$x^2 - 2x + 16 = 0$$

$$03. \frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}} \\ \frac{(P_{16} - P_{15})(P_{15} - P_{14})}{P_{13}P_{14}} \dots (1)$$

$$\boxed{P_n = \alpha^n - \beta^n} \quad P_n - P_{n-1} = (\alpha^n - \beta^n) - (\alpha^{n-1} - \beta^{n-1})$$

$$= \alpha^{n-1}(\alpha - 1) - \beta^{n-1}(\beta - 1)$$

$$= 4(\alpha^{n-2} - \beta^{n-2})$$

$$P_n - P_{n-1} = 4P_{n-2}$$

$$P_{16} - P_{15} = 4P_{14}$$

$$P_{15} - P_{14} = 4P_{13}$$

$$\text{From 1 } \frac{4P_{14} - 4P_{13}}{P_{13} \cdot P_{14}} = 16$$

$$04. \text{ we have } \alpha^2 = 5\alpha - 3 \text{ and } \beta^2 = 5\beta - 3$$

$$\text{Hence } \alpha, \beta \text{ are roots of } x^2 = 5x - 3$$

$$x^2 - 5x + 3 = 0$$

$$\alpha + \beta = 5, \alpha\beta = 3$$

$$\text{Sum} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{25 - 6}{3} = \frac{19}{3}$$

$$\text{Product} = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

So the required equation is

$$x^2 - (\text{sum})x + \text{product} = 0$$

$$x^2 - \frac{19}{3}x + 1 = 0$$

$$3x^2 - 19x + 3 = 0$$

$$05. \text{ Let } \sec \theta_1 + \tan \theta_1 = \alpha, \text{ then } \sec \theta_1 + \tan \theta_1 = \frac{1}{\beta}$$

$$\text{and } \operatorname{cosec} \theta_2 - \cot \theta_2 = \beta \text{ then } \operatorname{cosec} \theta_2 + \cot \theta_2 = \frac{1}{\beta}$$

$$\text{Hence, we have to find the equation having roots } \frac{1}{\alpha} \text{ and } \frac{1}{\beta}$$

$$\text{Replacing } x \text{ by } \frac{1}{x} \text{ in the given equation we get required equation}$$

$$3\left(\frac{1}{x}\right)^2 + 5\left(\frac{1}{x}\right) + 1 = 0$$

$$x^2 + 5x + 3 = 0$$

06. Given equation $ax^2 + bx + c = 0$

$$x = 2 \quad 4a + 2b + c = 0$$

a, b, c are in A.P.

$$2b = a + c$$

$$4a + a + c + c = 0$$

$$5a + 2c = 0$$

$$\boxed{\frac{c}{a} = \frac{-5}{2}}$$

Let α be the after root

$$\text{Product of roots } 2\alpha = \frac{c}{a}$$

$$2\alpha = \frac{-5}{2}$$

$$\alpha = \frac{-5}{4}$$

07. $\frac{x_1}{x_1 - 1} + \frac{x_2}{x_2 - 1} = 2$

$$\text{Since } x_1 x_2 = 4$$

$$x_2 = \frac{4}{x_1}$$

$$\frac{x_1}{x_1 - 1} + \frac{4/x_1}{4/x_1 - 1} = 2$$

$$\frac{x_1}{x_1 - 1} + \frac{4}{4 - x_1} = 2$$

$$4x_1 - x_1^2 + 4x_1 - 4 = 2(x_1 - 1)(4 - x_1)$$

$$8x_1 - x_1^2 + 4 = 8x_1 - 2x_1^2 - 8 + 2x_1$$

$$x_1^2 - 2x_1 + 4 = 0$$

$$x^2 - 2x + 4 = 0$$

08. $x^2 - px + 1 = 0, \rightarrow \alpha, \beta^2$ are roots

$$\text{product of roots } \alpha, \beta^2 = 1 \dots (1)$$

$$\text{and } x^2 - qx + 8 = 0 \rightarrow \alpha^2 \beta = 8 \dots (2)$$

$$(1) \times (2) (\alpha \beta^2)(\alpha^2 \beta) = 1 \times 8$$

$$\alpha^3 \beta^3 = 2^3 \Rightarrow \alpha \beta = 2$$

$$\text{From (1) } \alpha \beta^2 = 1$$

$$\alpha \beta \cdot \beta = 1 \Rightarrow \left(\beta = \frac{1}{2} \right)$$

$$\text{From (2) } \alpha^2 \beta = 8$$

$$\alpha \cdot \alpha \beta = 8 \Rightarrow \alpha = 4$$

$$\text{Sum of roots } \alpha^2 + \beta^2 = p$$

$$4 + \frac{1}{4} = P \Rightarrow P = \frac{17}{4}$$

$$q = \alpha^2 + \beta = 16 + \frac{1}{2} q = \frac{33}{2}$$

09. $ax^2 - bx(x-1) + c(x-1)^2 = 0$

$$\frac{ax^2}{(x-1)^2} - \frac{bx}{x-1} + c = 0$$

Now α is a root of $ax^2 + bx + c = 0$

Then let $\alpha = \frac{x}{1-x}, x = \frac{\alpha}{\alpha+1}$

Hence the roots of $\frac{\alpha}{1+\alpha}, \frac{\beta}{1+\beta}$

10. Given equation $ax^2 + bx + c = 0, bx^2 + cx + a = 0$ have a common root

$$(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$$

$$(cb - a^2) = (ac - b^2)(ba - c^2)$$

$$c^2b^2 + a^4 - 2a^2bc = a^2bc - ac^3 - ab^3 + b^2c^2$$

$$a^3 + b^3 + c^3 = 3abc$$

$$\frac{a^3 + b^3 + c^3}{abc} = 3$$

11. $x^2 + ax + 12 = 0 \text{ --- (1)}$

$$x^2 + bx + 15 = 0 \text{ --- (2)}$$

$$x^2 + (a+b)x + 36 = 0 \text{ --- (3)}$$

$$(1) + (2) \quad 2x^2 + (a+b)x + 27 = 0$$

Now subtract $x^2 - 9 = 0$

$$x = \pm 3$$

Let common positive root is 3

$$a + 12 + 3a = 0 \Rightarrow a = -7$$

$$9 + 3b + 15 = 0, b = -8$$

12. Put $x^2 + x = y$

$$(y-2)(y-3) = 12$$

$$y^2 - 5y - 6 = 0$$

$$(y-6)(y+1) = 0$$

When $y = 6$, we get $x^2 + x - 6 = 0$

$$(x+3)(x-2) = 0, x = -3, 2$$

When $y = -1$, we get $x^2 + x + 1 = 0$

ω, ω^2 are roots non real roots

$$\text{Sum } \omega + \omega^2 = -1$$

13. $\frac{4}{x+1+\frac{3}{x}} + \frac{5}{x-5+\frac{3}{x}} = -\frac{3}{2}$

Put $x + \frac{3}{x} = y$

$$\frac{4}{y+1} + \frac{5}{y-5} = \frac{-3}{2}$$

$$4(y-5) + 5(y+1) = \frac{-3}{2}(y+1)(y-5)$$

$$8y - 40 + 10y + 10 = -3(y^2 - 4y - 5)$$

$$3y^2 + 6y - 45 = 0 \quad x + \frac{3}{x} = 3$$

$$y^2 + 2y - 15 = 0 \quad x^2 + 3 = 3x$$

$$(y+5)(y-3) = 0 \quad x^2 - 3x + 3 = 0$$

$$y = -5, 3 \quad x = \frac{3 \pm \sqrt{3}i}{2}$$

$$x^2 + 5x + 3 = 0 \quad \text{Imaginary roots}$$

$$x = \frac{-5 \pm \sqrt{13}}{2}$$

Two irrational roots

14. $b^2 - 4ac > 0$

$$(a-3)^2 + 4(1)(a+2) > 0$$

$$a^2 - 6a + 9 + 4a + 8 > 0$$

$$a \in \mathbb{R}$$

$$\frac{a^2+1}{a^2+2} = 1 - \frac{1}{a^2+2} \geq \frac{1}{2}$$

15. $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

$$\alpha^2, \beta^2 \text{ are roots of } a^2x^2 + b^2x + c^2 = 0$$

$$\alpha^2 + \beta^2 = \frac{-b^2}{a^2}, \alpha^2\beta^2 = \frac{c^2}{a^2}.$$

$$(\alpha + \beta)^2 - 2\alpha\beta = \frac{-b^2}{a^2}$$

$$\frac{2b^2}{a^2} = \frac{2c}{a}$$

$$b^2 = ac$$

a, b, c are in G.P.

16. $(5+2\sqrt{6})(5-2\sqrt{6}) = 1$

$$(5-2\sqrt{6}) = \frac{1}{(5+2\sqrt{6})}$$

$$(5+2\sqrt{6})^{x^2-3} + \left(\frac{1}{5-2\sqrt{6}}\right)^{x^2-3} = 10$$

$$\text{Put } (5+2\sqrt{6})^{x^2-3} = k$$

$$k + \frac{1}{k} = 10$$

$$k^2 - 10k + 1 = 0$$

$$k = 10 \pm \frac{4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

$$(5 + 2\sqrt{6})^{x^2-3} (5 + 2\sqrt{6}) = (5 + 2\sqrt{6})^{\pm 1}$$

$$x^2 = 3 = \pm 1$$

$$x^2 = 4, x^2 = 2$$

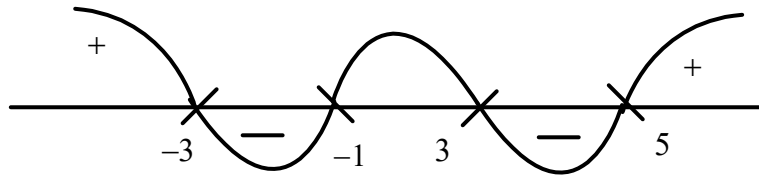
$$x = \pm 2, x = \pm \sqrt{2}$$

17. Let $f(x) = x^2 - 2mx + m^2 - 1$ are exactly one root of $f(x) = 0$ lies in $(-2, 4)$ we can take $D > 0$ and $f(-2)f(4) < 0$

$$(I) \Delta > 0 \quad (-2m)^2 - 4(m^2 - 1) > 0 \Rightarrow 4 > 0$$

$$m \in \mathbb{R} \dots (1)$$

$$(II) f(-2)f(4) = 0$$



$$(4 + 4m + m^2 - 1)(16 - 8m + m^2 - 1) < 0$$

$$(m^2 + 4m + 3)(m^2 - 8m + 15) < 0$$

$$(m+1)(m+3)(m-3)(m-5) < 0$$

$$m \in (-3, -1) \cup (3, 5) \dots (2)$$

$$\text{From (1) and (2) } m \in (-3, -1) \cup (3, 5)$$

18. Let r be the common root of the G.P. then $\beta = ar, \gamma = ar^2, \delta = ar^3$

$$\alpha + \beta = a + ar = 1$$

$$a(1+r) = 1 \dots (1)$$

$$\alpha\beta = p \Rightarrow a, ar = p$$

$$a^2r = p \dots (2)$$

$$\gamma + \delta = ar^2 + ar^3 = 4$$

$$ar^2(1+r) = 4$$

$$rq = p$$

$$a^2r^5 = q$$

$$\frac{ar^2(1+r)}{a(1+r)} = \frac{4}{1}, r^2 = 4$$

$$r = \pm 2$$

If we take $r = 2$ then a is not integer

If we take $r = -2$ in equation (1)

We get $a = -1$

From (2) $a^2r = p$

$$(-1)^2(2) = p$$

$$p = 2$$

From (3) $a^2r^5 = q$

$$(-1)^2(2)^5 = q$$

$$q = -32$$

$$(p, q) = (-2, -32)$$

19. We have Δ

$$2(a-b)x^2 - 11(a+b+c)x - 3(a-b) = 0$$

$$\Delta = (-11(a+b+c))^2 - 4(2)(a-b)(3(a-b))$$

$$121(a+b+c)^2 + 24(a-b)^2 > 0$$

Roots are real & unequal

20. Let α be the one of $x^2 - 3ax + f(a) = 0$

$$\alpha + 2\alpha = 3a \Rightarrow 3\alpha = 3a$$

$$\alpha = a$$

$$\alpha \cdot 2\alpha = f(a)$$

$$f(a) = 2\alpha^2,$$

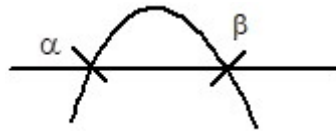
$$= 2a^2$$

$$f(x) = 2x^2$$

21. Since the coefficient of $x^2 = 4p - p^2 - 5 < 0$

Let graph is open down word according to the question must lies between the roots $f(1) > 0$

$$4p - p^2 - 5 - 2p + 13p > 0$$



$$-p^2 + 5p - 4 > 0$$

$$p^2 - 5p + 4 < 0$$

$$(p-1)(p-4) < 0$$

$$1 < p < 4$$

Integral value of p

$$p = 2, 3$$

Number of integral value of p is 2

22. Let D_1 & D_2 be discriminant of $x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ respectively then

$$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$$

$$= b_1^2 + b_2^2 - 2b_1b_2$$

$$\boxed{b_1b_2 = 2(c_1 - c_2)}$$

$$= (b_1 - b_2)^2 \geq 0$$

$$D_1 \geq 0 \text{ (or) } D_2 \geq 0 \text{ (or) } D_1 \text{ and } D_2$$

Both are positive hence at least one root of its equation has real roots.

23. α, β are roots of $x^2 - px + q = 0$

$$\alpha + \beta = p, \alpha\beta = q$$

$$\alpha', \beta' \text{ are roots of } x^2 - p'x + q' = 0$$

$$\alpha' + \beta' = p', \alpha'\beta' = q'$$

$$\text{Now } (\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$$

$$\begin{aligned}
& 2(\alpha^2 + \beta^2) + 2(\alpha'^2 + \beta'^2) - 2\alpha'(\alpha + \beta) - 2\beta'(\alpha + \beta) \\
&= 2\left[(\alpha + \beta)^2 - 2\alpha\beta + (\alpha' + \beta')^2 - 2\alpha'\beta' - (\alpha + \beta)(\alpha' + \beta')\right] \\
&= 2[p^2 - 2q + p'^2 - 2q' - pp']
\end{aligned}$$

24. According to the equation α, β are the root of $x^2 - 3px + 2q = 0$ and $\alpha, \frac{1}{\beta}$

$$\alpha + \beta = 3p, \alpha\beta = 2q$$

$$\alpha + \frac{1}{\beta} = 3a, \frac{\alpha}{\beta} = 2b$$

$$(2q - 2b)^2 = \left(\alpha\beta - \frac{\alpha}{\beta}\right)^2 = \alpha^2 \left(\beta - \frac{1}{\beta}\right)^2$$

$$= \frac{\alpha}{\beta} \cdot \alpha\beta(\alpha + \beta) - \left(\alpha + \frac{1}{\beta}\right)^2$$

$$= (2b)(2q)(3p - 3a)^2$$

$$= 36bq(p - a)^2$$

25. $2\sqrt{5 - 4x - x^2} = 16 - |x|$

$$4(5 - 4x - x^2) = 256 - 32|x| + x^2$$

$$5x^2 + 32|x| + 16|x| + 236 = 0$$

$$5x^2 - 16x + 236 = 0 \text{ when } x \geq 0$$

$$\text{and } 5x^2 + 48x + 236 = 0 \text{ when } x < 0$$

In both cases equation has non real roots

26. $x^2 - (a - 3)x + a = 0$

$$\Delta = (a - 3)^2 - 4a = a^2 - 10a + 9 = (a - 1)(a - 9)$$

Case (i): Both roots are greater than 2

$$D \geq 0, f(2) > 0, -\frac{\beta}{2A} > 2$$

$$(a - 1)(a - 9) \geq 0$$

$$4 - (a - 3)2 + a > 0, \frac{a - 3}{2} > 2$$

$$(a - 1)(a - 9) \geq 0, a < 10, a > 7$$

$$a \in (-\infty, 1] \cup [9, \infty)$$

$$a \in [9, 10] \dots (1)$$

Case (ii) One root is greater than 2 and the other root is less than (or) equal to 2

$$f(2) \leq 0$$

$$(1) \text{ and } (2) \ a \in [9, 10) \cup [10, \infty) \ a \in [9, \infty)$$

$$4 - (a - 3)2 + a \leq 0$$

$$a \geq 10 \dots (2)$$

27. We have $\sin A + \sin B = \frac{7n - 20}{5n + 8} \dots (1)$

$$\sin A \cdot \sin B = \frac{120}{5n+8} \dots (2)$$

$$\sin^2 A + \sin^2 B = 1$$

$$(\sin A + \sin B)^2 = 2 \sin A \sin B = 1$$

$$x^2 - 65n - 66 = 0$$

$$(n-66)(n+1) = 0$$

$$n = 66$$

$$28. \alpha + \alpha^2 + \alpha^3 \dots \infty + \beta + \beta^2 + \beta^3 \dots \infty$$

$$\frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta}$$

$$= \frac{\alpha - \alpha\beta + \beta - \alpha\beta}{(1-\alpha)(1-\beta)}$$

$$= \frac{\alpha + \beta - 2\alpha\beta}{1-\beta-\alpha+\alpha\beta}$$

$$= \frac{25}{375} - 2\left(\frac{-2}{375}\right)$$

$$1 - \left(\frac{25}{375}\right) + \left(\frac{-2}{375}\right)$$

$$= \frac{1}{12}$$

$$29. P(x) = ax^2 + bx + c$$

$$P(x) = a(0)^2 + b(0) + c = 1$$

$$c = 1$$

$$p(x) = ax^2 + bx + 1$$

$$p(x) = (x-1)g(x) + 4$$

$$p(x) - 4 = (x-1)g(x)$$

$$p(x) - 4 = ax^2 + bx = 3 = (a-1)g(x)$$

$$\text{Put } x = 1$$

$$a(1)^2 + b(1) - 3 = 3(1-1)g(x)$$

$$a + b - 3 = 0$$

$$a + b = 3 \dots (1)$$

$$P(x) = (x+1)h(x) + 6$$

$$P(x) - 6 = (x+1)h(x)$$

$$ax^2 + bx + 1 - 6 = ax^2 + bx - 5 = (x+1)h(x)$$

$$a(-1)^2 + b(-1) - 5 = (-1+1)h(x)$$

$$a - b - 5 = 0$$

$$a - b = 5 \dots (2)$$

$$a + b = 3$$

$$2a - 8$$

$$a = 4, b = -1$$

$$p(x) = ax^2 + bx + c$$

$$= 4x^2 - x + 1$$

$$p(-2) = 4(-2)^2 - (-2) + 1$$

$$16 + 2 + 1 = 19$$

$$p(-2) = 19$$

$$30. \quad 4^{\sec^2 x} + 2x + \left(\beta^2 - \beta + \frac{1}{2}\right) = 0$$

Real roots $\Delta \geq 0$

$$4 - 4 \cdot 4^{\sec^2 \alpha} \left(\beta^2 - \beta + \frac{1}{2}\right) \geq 0$$

$$\Rightarrow 4^{\sec^2 \alpha} \left(\beta^2 - \beta + \frac{1}{2}\right) \geq 1$$

$$\Rightarrow 4^{\sec^2 \alpha} \left(\beta^2 - \beta + \frac{1}{2}\right) \leq 1$$

$$4^{\sec^2 \alpha} \geq 4, \beta^2 - \beta + \frac{1}{2} = \left(\beta + \frac{1}{2}\right)^2 + \frac{1}{4} \geq \frac{1}{4}$$

The equation satisfy only when $4^{\sec^2 \alpha} = 4, \beta^2 - \beta + \frac{1}{2} = \frac{1}{4}$

$$\sec^2 \alpha = 1, \left(\beta - \frac{1}{2}\right)^2 = 0$$

$$\cos^2 \alpha = 1, \beta = \frac{1}{2}$$

$$\cos^2 \alpha + \cos^{-1} \beta = 1 + \cos^{-1} \frac{1}{2} = \frac{\pi}{3} + 1$$

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