# DPP- AREAS-MAINS MODEL

## MATHS-2B

1. The area of the region bounded by the curves $y=|x-2|, x=1, x=3$ and the x—axis is
a) 4 b) 2 c) 3 d) 1
2. The area of the region bounded by $y=|x-1|$ and $y=1$ is
a) 2 b) 1 c) $\frac{1}{2}$ d) none of those
3. Area bound by lines $y=2+x, y=2-x$ and $x=2$ is
a) 3 b) 4 c) 8 d) 16
4. Area enclosed between the curve $y^2(2a-x)=x^3$ and line $x=2a$ above x-axis is
a) $\pi a^2$ b) $\frac{3\pi a^2}{2}$ c) $2\pi a^2$ d) $3\pi a^2$
5. Area bounded by the curve $xy-3x-2y-10=0$, x-axis and the lines $x=3, x=4$ is
a) 16 log 2-3 b) 16 log 2-13 c) 16 log 2+3 d) none of these
6. The area of the triangle formed by the tangent to the hyperbola $xy=a^2$ and coordinate axes is
a) $a^2$ b) $2a^2$ c) $3a^2$ d) $4a^2$
7. If a curve $y=a\sqrt{x}+bx$ passes through the point (1, 2 ) and the area bound by the curve, line $x=4$ and x-axis is 8 square units, then
a) $a=3, b=-1$ b) $a=3, b=1$ c) $a=-3, b=1$ d) $a=-3, b=-1$
8. Let m be the area of the smaller region bounded by the curve $y^2=4x$ and the lines $y=-x$ and $x=4$, which lies in the IV quadrat, then the value of 3m is ____
a) 16 b) 8 c) 4 d) 2
9. The are enclosed by the parabola $y^2=4ax$ and the straight line $y=2ax,$ is
a) $\frac{a^2}{3} sq.units$ b) $\frac{1}{3a^2} sq.units$ c) $\frac{1}{3a} sq.units$ d) $\frac{2}{3a} sq.units$
10. The area bounded by the curve $x=at^2, y=2at$ and the x-axis in $1\le t\le 3$ is
a) $26a^2$ b) $8a^2$ c) $\frac{26a^2}{3}$ d) $\frac{104a^2}{3}$
11. Let A be the area bounded by the curve $y=(x-1)|x|$, the x-axis and the ordinates $x=-1$ and $x=1$ then ____
a) -1 b) 0 c) 1 d) 2
12. The area between the curve $y=2x^4-x^2$, the axis and the ordinates of 2 minima of the curve is
a) $\frac{7}{120}$ b) $\frac{9}{120}$ c) $\frac{11}{120}$ d) None of these
13. The slope of the tangent to a curve $y=f(x)$ at $(x, f(x))$ is $2x+1$. If the curve passes through the point $(1,2)$, then the area of the region bounded be the curve, the x-axis and the line $x=1$ is
a) $\frac{5}{6}$ b) $\frac{6}{5}$ c) 6 d) $\frac{1}{6}$
14. The area bounded by $y=e^{|x|}$, y-axis and $y=e$ is _____
a) $e-1$ b) $2(e-1)$ c) $2e-1$ d) $2e$

15. Ratio of the area cut off a parabola by any double ordinate is that of the corresponding rectangle contained by that double ordinate and its distance from the vertex is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) 1

16. The area bounded by the curves $x = a\cos^3 t, y = a\sin^3 t$ is

a) $\frac{3\pi a^2}{8}$ b) $\frac{3\pi a^2}{16}$ c) $\frac{3\pi a^2}{32}$ d) $3\pi a^2$

17. For which of the following values of m, the area of the region bounded by the curve $y = x - x^2$ and the line $y = nx$ equals $= \frac{9}{2}$

a) -4 b) -2 c) 2 d) 4

18. If the ordinate $x = a$ divide the area bounded by the curve $y = \left[1 + \frac{8y}{x^2}\right]$, x-axis and the ordinates $x = 2, x = 4$ into 2 equal parts, then a =

a) 8 b) $2\sqrt{2}$ c) 2 d) $\sqrt{2}$

19. The area of the region lying inside $x^2 + (y-1)^2 = 1$ and out side $c^2x^2 + y^2 = c^2$, where $c = \left(\sqrt{2} - 1\right)$ is

a) $\left(4 - \sqrt{2}\right)\frac{\pi}{4} + \frac{1}{\sqrt{2}}$ b) $\left(4 + \sqrt{2}\right)\frac{\pi}{4} - \frac{1}{\sqrt{2}}$ c) $\left(4 + \sqrt{2}\right)\frac{\pi}{4} + \frac{1}{\sqrt{2}}$ d) None of these

20. Let a and b respectively be the points of local maximum and minimum of the function $f(x) = 2x^3 - 3x^2 - 12$. If A is the total area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, then 4A is equal to __________.

a) 114 b) 124 c) 116 d) none of these

21. The area of the region formed by $x^2 + y^2 - 6x - 4x + 12 \le 0, y \le x$ and $x \le \frac{5}{2}$ is

a) $\frac{\pi}{6} - \frac{\sqrt{3}+1}{8}$ b) $\frac{\pi}{6} + \frac{\sqrt{3}-1}{8}$ c) $\frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$ d) None of these

22. If the area bounded by the curves $y = x - bx^2$ and $y = \frac{1}{b}x^2$, where b>0 is maximum, then b=

a) 0 b) 1 3) 2 4) None of these

23. Let $f(x) = \max imum\left[x^2, \left(1 - x^2\right), 2x(1-x)\right]$ where $0 \le x \le 1$. The area of the region bounded by the curves $y = f(x)$,x-axis, x=0 and x=1 is

a) $\frac{17}{27}$ b) $\frac{14}{27}$ c) $\frac{19}{27}$ d) None of these

24. The are of the closed figure bounded by $x = -1$ and $x = 2$ and $y = \begin{cases} -x^2 + 2, & x \le 1 \\ 2x - 1, & x > 1 \end{cases}$ and the abscissa axis is

a) $\frac{16}{3} sq.units$ b) $\frac{10}{3} sq.units$ c) $\frac{13}{3} sq.units$ d) $\frac{7}{3} sq.units$

25. The area of the region $S = \left\{(x, y): \; y^2 \le 4x \le y + 2\right\}$ is A then 24A=_________

26. The area (in Sq. units) of the region enclosed between the parabola $y^2 = 4x$ and line $x - y = 3$ is ____

27. The area bounded by the lines $y = \left||x-1| - 2\right|$ and $y = 2$ is _____

28. The area (in Sq. units of the region $\left\{(x, y): x \ge 0, x + y \le 3, x^2 \le 4y \; and \; y \le 1 + \sqrt{x}\right\}$ is ____

29. The area bounded curve $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is ____

30. The area bounded by the curve $y = \frac{1}{\sqrt{x}}$ and $x = 4, x = 9$ is __________

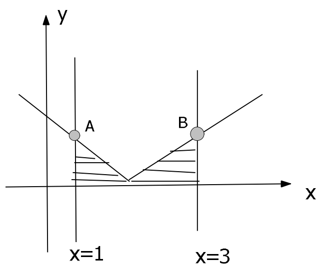
MATHS-2A key

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| d | b | b | b | c | b | a | b | c | d |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| c | a | a | b | c | a | b | b | a | a |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| c | b | a | a | 343 | 64/3 | 8 | 5/2 | 1/3 | 2 |

## AREAS-HINTS

## MATHS-2B

1.




$$x=1$$
$$y=2-x=2-1=1$$
$$y=1 \quad A(1,1)$$

$$x=3$$
$$y= x-2 = 3-2=1$$
$$y=1 \quad B(3,1)$$

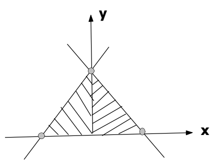
Area = $\frac{1}{2}\times1\times1+\frac{1}{2}\times1\times1=1$

2. The point of integration

$$2x^2=x+3$$
$$2x^2-x-3=0$$
$$x=-1,\frac{3}{2}$$

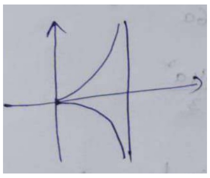
$$A=\int_{-1}^{3/2}(x+3-2x^2)dx=\left(\frac{x^2}{2}+3x-\frac{2x^3}{3}\right)_{-1}^{3/2}$$
$$A=\frac{19}{13}$$
$$3A=19$$

3.




$$\frac{1}{\cancel{2}}\times\cancel{2}\times4=4$$

4.



$x = 2a$ is an asymptote

$$A = 2\int_0^{2a} \frac{x^{3/2}}{\sqrt{2a-x}}dx$$

Put $x = 2a\sin^2\theta$

$dx = 4a\sin\theta\cos\theta d\theta$

$$A = \frac{3\pi a^2}{2}$$

5.

$$xy - 3x - 2y - 10 = 0$$

$$(x-2)y = 3x+10$$

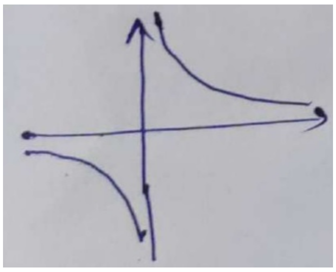
$$y = \frac{3x+10}{x-2}$$

$x = 3, x = 4$

$x - axis$

$$\int_3^4 \frac{3x+10}{x-2}dx = 16\log 2 + 3$$

6. $xy = 16$ and coordinate axes area is = 2(16) = 32



7. $y = a\sqrt{x} + bx$

$x = 4$

$x - axis$

$$\int_0^4 (a\sqrt{x} + bx)dx = 8$$

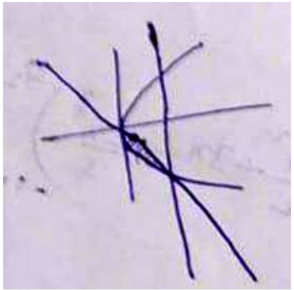
$$2a + 3b = 3$$

Line passing (1,2) $\Rightarrow$ $a + b = 2$

By solving $a = 3$

$b = -1$

8.



$$\int_0^4 2\sqrt{x}dx = \frac{32}{3}$$

$$\text{Area} = \frac{32}{3} - \frac{1}{\cancel{2}} \times \cancel{4}^2 \times 4 = \frac{8}{3}$$

$3A = 8$

9. Conceptual $y^2 = 4ax, y = mx$

$$\text{Area} = \frac{8a^2}{3m^3}$$

$$\text{Area} = \frac{1}{3a}$$

10.

$t = 1 \qquad x = a, \qquad y = 2a$

$t = 3 \qquad x = 9a, \qquad y = 6a \qquad\qquad y^2 = 4ax$

$$\int_a^{9a} \sqrt{4ax}\,.dx = \frac{104a^2}{3}$$

11. $$\left|\int_{-1}^{0} (x-1)(-x)\,dx\right| + \left|\int_{-1}^{0} (x-1)x\,dx\right| = \frac{5}{3\times 2} + \frac{1}{6} = 1$$

12. $y = 2x^4 - x^2$

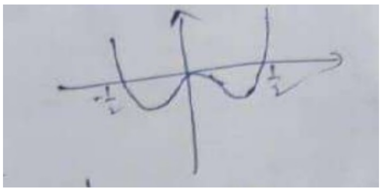
$$\frac{dy}{dx} = 8x^3 - 2x = 0$$

$x = 0, \qquad x = \pm\frac{1}{2}$

$$A = \int_{-\frac{1}{2}}^{\frac{1}{2}} (2x^2 - x^2)dx$$

$$= \int_0^{\frac{1}{2}} (2x^2 - x^2)dx$$

$$= \frac{7}{120}$$



13.

$$\frac{dy}{dx} = 2x + 1$$

$$\int dy = \int (2x+1)dx$$

$y = x^2 + x + c$

$(1,2) \qquad c = 0$

$y = x^2 + x$

$$\int_0^1 (x^2 + x)dx = \frac{5}{6}$$

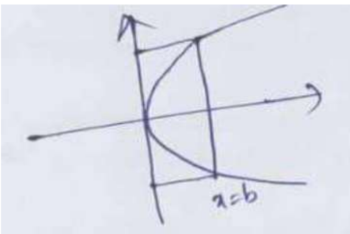
14. $$\int_{-1}^{0} e^{-x}\,dx + \int_0^1 e^x\,dx = 2(e-1)$$

$(\therefore\ y = e^{|x|}, y = e$

$|x| = 1$

$x = \pm 1)$

15. Conceptual




$A_1 =$ area b/w $y^2 = 4ax$ and $x = b$

$$= 2\int_0^b \sqrt{4ax}\,dx$$

$$= \frac{8}{3}a^{1/2}b^{3/2}$$

$A_2 =$ Area of ABCD

$$= 2\sqrt{4ab} \times b$$

$$= 4a^{1/2}b^{3/2}$$

$$A_1 : A_2 = \frac{8}{3} : 4$$

$$= 2 : 3$$

16. Conceptual $x^{2/3} + y^{2/3} = a^{2/3}$ area $= \dfrac{3\pi a^2}{8}$

17. By substitution m values from options and verify

18. $\int_2^4 y\,dx = \int_2^4 (1 + \frac{8}{x^2})\,dx = 4$

$$\therefore A = 2\int_2^a y\,dx$$

$$4 = 2\int_2^a (1 + \frac{8}{x^2})\,dx$$

$$a = 2\sqrt{2}$$

19. $\left.\begin{array}{l} x^2 + (y-1)^2 = 1 \\ c^2x^2 + y^2 = c^2 \end{array}\right\} (y-1)^2 = \dfrac{y^2}{c^2}$

$$y = \frac{c}{1+c}, \frac{c}{c-1}$$

$$x^2 = 1 - \frac{y^2}{c^2}$$

$$x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

$$\text{Area} = 2\left[\int_0^{1/\sqrt{2}} c\sqrt{1-x^2}\,dx - \int_0^{1/\sqrt{2}} (1 - \sqrt{1-x^2})dx\right]$$

$$= \frac{\sqrt{2}\pi}{4} - \frac{\sqrt{2}}{2}$$

$\therefore$ Hence required area = $\pi - \frac{\sqrt{2}\pi}{4} + \frac{\sqrt{2}}{2}$

20.

$f(x) = 2x^3 + 3x^2 + 12x$

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

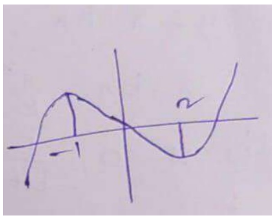
$x = 2, -1$

$f''(2) > 0 \qquad b = 2$

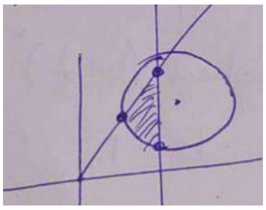
$f''(-1) < 0 \qquad a = -1$

$$A = \left|\int_{-1}^{0} (2x^3 - 3x^2 - 12x)dx\right| + \left|\int_{0}^{2} (2x^3 - 3x^2 - 12x)dx\right|$$

$4A = 114$



21.



Given $(x-3)^2 + (y-2)^2 = 1$

$(y-2)^2 = 1 - (x-3)^2$

$y = 2 + \sqrt{1-(x-3)^2}$

$$\int_{2}^{\frac{5}{2}} (line - circle)dx$$

$$= \int_{2}^{5/2} x - (2 + \sqrt{1-(x-3)^2})$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$$

22. $y = x - bx^2, y = \frac{1}{b}x^2 \qquad b > 0$

$x - bx^2 = \frac{x^2}{b}$

Simplify $x = 0, \quad \frac{b}{b^2+1}$

$$\int_{0}^{\frac{b}{b^2+1}} \left(x - bx^2 - \frac{x^2}{b}\right)dx = \left(\frac{x^2}{2} - \frac{bx^3}{3} - \frac{x^3}{3b}\right)_0^{\frac{b}{b^2+1}}$$

$$= \frac{b^2}{6(1+b^2)^2} = \frac{1}{6\left(\frac{1}{b}+b\right)^2}$$

$$b+\frac{1}{b} \geq 2 \Rightarrow \left(b+\frac{1}{b}\right)^2 \geq 4$$

Here area in max when $\left(b+\frac{1}{b}\right)^2$ is min so $\left(b+\frac{1}{b}\right)^2 = 4 \;\Rightarrow\; b=1$ $\quad (\therefore b>0)$

23. $y = x^2 ------(1)$

$y = (1-x^2) ------(2)$

$y = 2x(1-x) ------(3)$

Solving (1) & (3) $\Rightarrow x = 0, \frac{2}{3}$

" (2) & (3) $\Rightarrow x = \frac{1}{3}, 1$

$$f(x) = \begin{cases} (1-x)^2, & 0 \leq x \leq \frac{1}{3} \\ 2x(1-x), & \frac{1}{3} \leq x \leq \frac{2}{3} \\ x^2, & \frac{2}{3} \leq x \leq 1 \end{cases}$$

$$\therefore A = \int_0^{1/3} (1-x)^2\,dx + \int_{1/3}^{2/3} 2x(1-x)\,dx + \int_{2/3}^{1} x^2\,dx = \frac{17}{27}$$

24. $f(x) = \int_{-1}^{1} (-x^2+2)dx + \int_{1}^{2} (2x-1)dx \quad = \frac{16}{3}$

25. $y^2 = 4x,\; 4x = y+2$

$y^2 = y+2$

$y^2 - y - 2 = 0$

$y = -1, 2$

$$\int_{-1}^{2} \left(\frac{y+2}{4} - \frac{y^2}{4}\right) dy = \frac{27}{24}$$

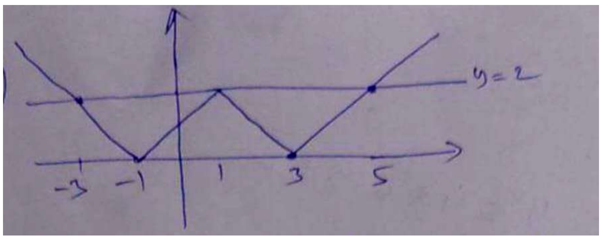
$24A = 27$

26. $y^2 = 4x,\; x - y = 3$

Simplify $y = -2, 6$

$$\int_{-2}^{6} (y+3-\frac{y^2}{4})dy = \frac{64}{3}$$

27.



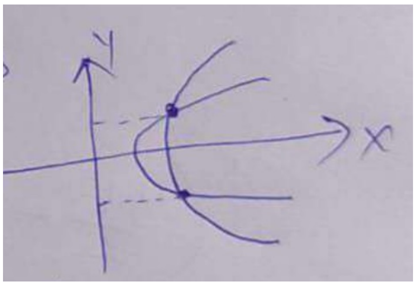
$$A = \frac{1}{\cancel{2}} \times \cancel{2} \times 4 + \frac{1}{\cancel{2}} \times \cancel{2} \times 4 = 8$$

28. Give $x + y \le 3 - - - (1)$, $x^2 \le 4y - - - (2)$, $y \le 1 + \sqrt{x} - - - (3)$, $x \ge 0 - - - (4)$ Solving above equation to get point of intesseition

$$\text{Area} = \int_0^1 (1 + \sqrt{x})dx + \int_1^2 (3 - x)dx + \int_0^2 (x^2)dx \quad = \frac{5}{2}$$

29. $y^2 = 2x - 1$, $y^2 = 4x - 3$ $\qquad x = 1$, $y = \pm 1$

$$\int_{-1}^{1} \left( \frac{y^2 + 3}{4} - \frac{y^2 + 1}{2} \right) dx = \frac{1}{3}$$




30. $\int_4^9 \left( \frac{1}{\sqrt{x}} \right) dx = \left( 2\sqrt{x} \right)_4^9 = 2$