TOPIC: NUCLEAR PHYSICS (DPP)

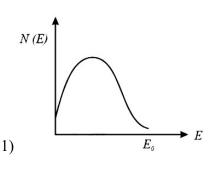
- 1. Half-life of radioactive substance is 3.20 h. What is the time taken for 75% of substance to be used?

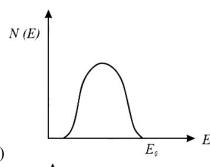
 1) 6.4 h

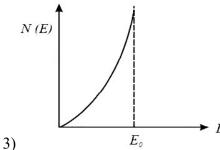
 2) 12 h

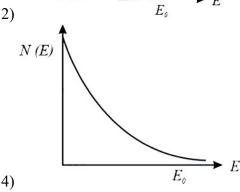
 3) 4.18 days

 4) 1.2 days
- 2. The energy spectrum of β particles [number N (E) as a function of β energy E] emitted from a radioactive source is









- 3. The mean lives of a radioactive substance are 1620 years and 405 years for α and β emission respectively. If it is decaying by both α and β emission simultaneously, then the time during which three-fourths of the sample will decay is
 - 1) 643 years
- 2) 449 years
- 3) 528 years
- 4) 279 years
- 4. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially, they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $\frac{1}{e}$ after a time
 - $1)\,\frac{1}{10\,\lambda}$
- $2) \frac{1}{11\lambda}$
- $3) \frac{11}{10\lambda}$
- 4) $\frac{1}{9\lambda}$
- 5. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially, both have an equal number of nuclei. After *n* half lives of A, rate of disintegration of both are equal. The value of *n* is
 - 1)4

2) 2

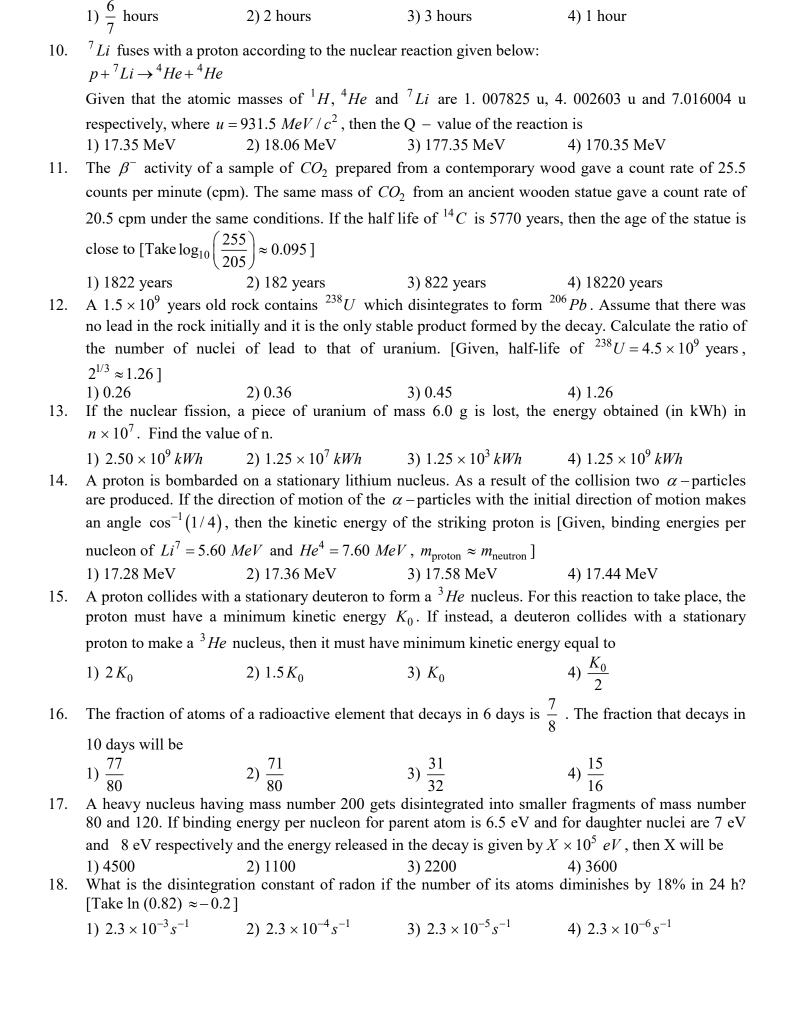
3) 1

- 4) 5
- 6. The binding energy per nucleon of deuterium and helium nuclei are 1.1 MeV and 7.0 MeV respectively. When two deuterium nuclei fuse together to form a helium nucleus, the energy released in the fusion is
 - 1) 2.2 MeV
- 2) 23. 6 MeV
- 3) 28.0 MeV
- 4) 30. 2 MeV
- 7. The ratio of the nuclear radius, of an atom with mass number A and ${}_{2}^{4}He$ is $\left(14\right)^{1/3}$. What is the value of A?
 - 1) 56

2)80

3) 79

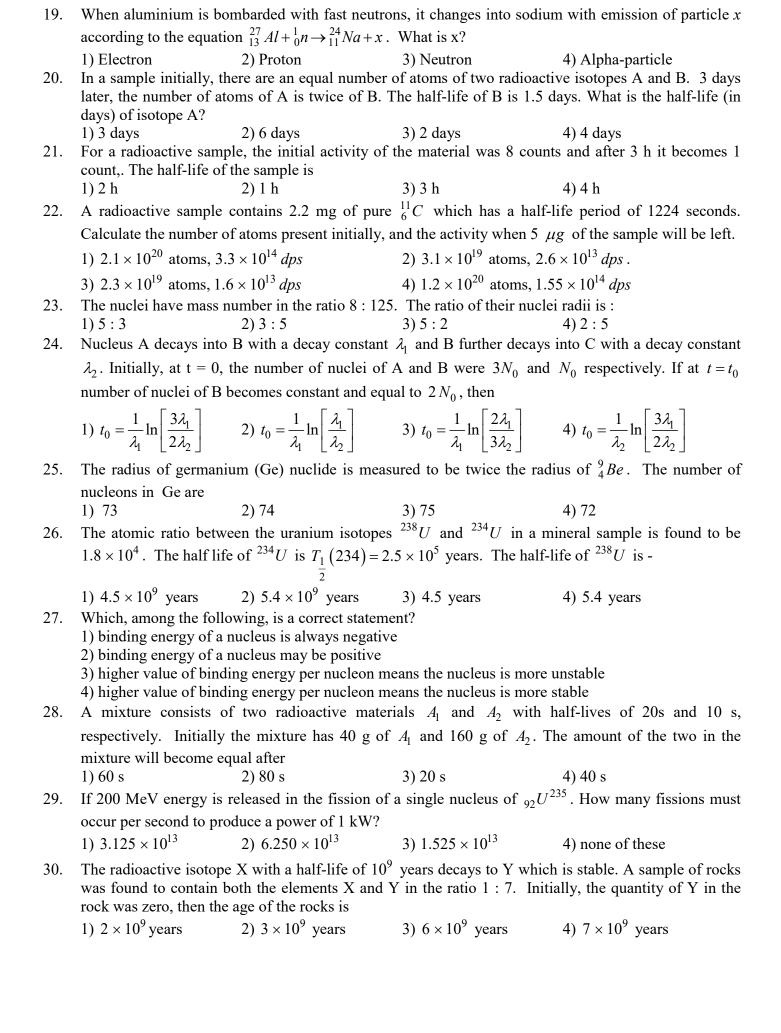
- 4) 30
- 8. After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. The initial activity of the sample (in dps) is
 - 1) 6000
- 2) 9000
- 3) 3000
- 4) 24000



The rate of disintegration of a radioactive substance falls from $\frac{40}{3}$ dps to $\frac{5}{3}$ dps in 6 hours. The half-

9.

life of the radioactive substance is



| 1 – 10 | 1 | 1 | 2 | 4 | 3 | 2 | 1 | 4 | 2 | 1 |
|---------|---|---|---|---|---|---|---|---|---|---|
| 11 – 20 | 1 | 1 | 2 | 1 | 1 | 3 | 3 | 4 | 4 | 1 |
| 21 – 30 | 2 | 4 | 4 | 1 | 4 | 1 | 4 | 4 | 1 | 2 |

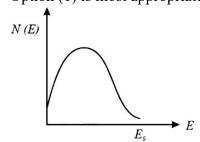
SOLUTIONS

1.
$$N_0 \xrightarrow{1} \frac{N_0}{2} \xrightarrow{N_0} \frac{N_0}{4}$$

Remaining substance after two half lives is $\frac{N_0}{4}$ (or)

The substance used during this time = $N_0 - \frac{N_0}{4} = 3\frac{N_0}{4} = 75\% N_0$

2. In β – decay, energy emission is continuous and energy is shared between β – particle and daughter nuclei. Thus certain β – particles have zero energy and certain have maximum energy. Option (1) is most appropriate



3.
$$\lambda = \lambda_{\alpha} + \lambda_{\beta}$$

$$= \frac{1}{T_{\alpha}} + \frac{1}{T_{\beta}} \left[\because \lambda = \frac{1}{T} \right]$$

$$= \frac{1}{1620} + \frac{1}{405} \text{ [given, } T_{\alpha} = 1620 \text{ yr and } T_{\beta} = 405 \text{ yr]}$$

$$= \frac{5}{1620} \text{ yr}^{-1}$$

$$\frac{3}{4} th \text{ sample will decay, i.e., remaining } \frac{1}{4} th$$

$$N = N_0 \left(\frac{1}{2}\right)^n$$

$$\frac{N_0}{4} = N_0 \left(\frac{1}{2}\right)^n$$

$$\Rightarrow n = 2$$

$$\therefore t = nT_{\frac{1}{2}} = n\frac{\ln 2}{\lambda}$$
$$= 2 \times \frac{0.693}{\frac{5}{1620}} = 499 \ yr$$

4.
$$\frac{1}{9\lambda}$$
Here,
$$\frac{N_{x_1}(t)}{N_{x_2}(t)} = \frac{1}{e}$$

$$Or \frac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}} = \frac{1}{e}$$

(Because initially, both have the same number of nuclei, N_0)

Or
$$e = \frac{e^{-\lambda t}}{e^{-10\lambda t}} = e^{9\lambda t}$$

$$9\lambda t = 1$$

$$t = \frac{1}{9\lambda}$$

5. Let
$$\lambda_A = \lambda :: \lambda_B = 2\lambda$$

If N_0 is total number of atoms in A and B at t = 0, then initial rate of disintegration of $A = \lambda N_0$, and initial rate of disintegration of $B = 2\lambda N_0$

As
$$\lambda_B = 2\lambda_A$$

$$T_B = \frac{1}{2}T_A$$

i.e., half-life of B is half the half-life of A.

After one half-life of A

$$\left(-\frac{dN}{dt}\right)_A = \frac{\lambda N_0}{2}$$

Equivalently, after two half lives of B

$$\left(-\frac{dN}{dt}\right)_{R} = \frac{2\lambda N_{0}}{4} = \frac{\lambda N_{0}}{2}$$

Clearly,
$$\left(-\frac{dN}{dt}\right)_A = -\left(\frac{dN}{dt}\right)_B$$

After n = 1 i.e, one half-life of A

6. The fusion reaction is a given below:

$$_{1}H^{2} + _{1}H^{2} \rightarrow _{2}He^{4} + Q$$

The binding energy of the reacting nuclei

$$=1.1 \times 2 + 1.1 \times 2 = 4.4 \,MeV$$

The binding energy of the product nuclei

$$= 7.0 \times 4 = 28.0 \; MeV$$

Hence, the energy released in the fusion reaction,

$$Q = 28.0 - 4.4 = 23.6 \; MeV$$

7. From the relation $r \propto A^{1/3}$.

We have,
$$\frac{r_2}{r_1} = \left(\frac{A_2}{A_1}\right)^{1/3}$$

Or
$$\left(\frac{A_2}{4}\right)^{1/3} = (14)^{1/3}$$

Therefor, $A_2 = 56$

8. Activity reduces from 6000 dps to 3000 dps in 140 days. It implies that half-life of the radioactive sample is 140 days. In 280 days (or two half-lives) activity will remain $\frac{1}{4}$ th of the initial activity.

Hence the initial activity of the sample is

$$4 \times 6000 \, dps = 24000 \, dps$$

$$9. A = A_0 e^{-\lambda t}$$

$$\Rightarrow 100 = 800 e^{-\lambda(6 \times 60)}$$

$$\Rightarrow e^{-350x} = \frac{1}{8}$$

$$\Rightarrow -360\lambda = \ln\left(\frac{1}{8}\right) = -\ln 8$$

$$\Rightarrow \lambda = \frac{\ln\left(2^3\right)}{360} = \frac{\ln 2}{120}$$

$$\Rightarrow T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{(\ln 2/120)} = 120 \text{ minutes}$$

$$\Rightarrow T_{1/2} = 2 \text{ hours}$$

10. The total mass of the initial particles

$$m_i = 1.007825 + 7.016004$$

$$m_i = 8.023829 u$$

And the total mass of final particles

$$m_f = 2 \times 4.002603 = 8.005206 \ u$$

Difference between the initial and final mass of particles

$$= m_i - m_f = 8.023829 - 8.005206$$

$$= 0.018623u$$

The Q value is given by

$$Q = (\Delta m)c^2$$

$$= 0.018623 \times 931.5 = 17.35 \ MeV$$

11.
$$r = 20.5 \ cpm, r_0 = 25.5 \ cpm$$

$$r_0 \propto N_0 \text{ and } r \propto N$$

$$\therefore \frac{r_0}{r} = \frac{N_0}{N}$$

Also,
$$t = \frac{2.303}{\lambda} \log \left(\frac{N_0}{N} \right) = \frac{2.303}{\lambda} \log \left(\frac{r_0}{r} \right)$$

$$t = \frac{2.303 \times 5770}{0.693} \log \left(\frac{25.5}{20.5} \right) = 1822 \text{ years}$$

12. Let N_0 be the initial number of uranium nuclei. After time t, let N_U be the number of uranium nuclei and N_{Pb} be the number of lead nuclei.

The number of half-lives passed is

$$n = \frac{t}{T_{half}} = \frac{1.5 \times 10^9}{4.5 \times 10^9} = \frac{1}{3}$$

$$N_U = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

$$N_{Pb} = N_0 - N_0 \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

$$\frac{N_{Pb}}{N_{U}} = 2^{\frac{1}{3}} - 1 = 0.26$$

13.
$$E = \Delta mc^2$$

$$= 0.5 \times 10^{-3} \times \left(3 \times 10^{8}\right)^{2}$$

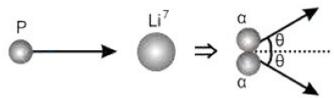
$$=4.5 \times 10^{13} \,\mathrm{J}$$

$$E = \frac{4.5 \times 10^{13}}{3.6 \times 10^6} \text{ kWh}$$

$$=1.25 \times 10^7 \, kWh$$

14. Q value of reaction is,

$$Q = (2 \times 4 \times 7.06 - 7 \times 5.6) MeV = 17.28 MeV$$



Applying conservation of energy for collision,

$$K_p + Q = 2K_{\alpha} \qquad \dots (i)$$

(Here, K_p and K_{α} are the kinetic energies of proton and α – particle respectively)

From the conservation of linear momentum (As there is no external force) ...(ii)

[Here
$$P = \sqrt{2 \ mk}$$
]

$$\Rightarrow K_p = 16 K_\alpha \cos^2 \theta = \left(16 K_\alpha\right) \left(\frac{1}{4}\right)^2 \qquad \text{(as } m_\alpha = 4 m_p\text{)}$$

$$\therefore K_{\alpha} = K_p \qquad \dots (iii)$$

Solving eqs. (i) and (iii) with $Q = 17.28 \ MeV$

We get
$$K_p = 17.28 \ MeV$$

15. In case 1:

Conservation of linear momentum : $mv_0 = 3 mv$

$$\Rightarrow v = \frac{v_0}{3}$$

Conservation of mechanical energy: $\frac{1}{2}mv_0^2 + \frac{1}{2}3m\left(\frac{v_0}{3}\right)^2 + U_0$

Here $\,U_0\,$ is the minimum energy required for the reaction to happen

$$\Rightarrow U_0 = K_0 - \frac{K_0}{3} = \frac{2K_0}{3}$$

In case 2:

Conservation of linear momentum : $2mv'_0 = 3mv' \implies v' = \frac{2}{3}v'_0$

Conservation of mechanical energy: $\frac{1}{2}2m(v_0')^2 = \frac{1}{2}3m(\frac{2}{3}v_0')^2 + \frac{2K_0}{3}$

$$\Rightarrow m{v_0'}^2 - \frac{2}{3}m{v_0'}^2 = \frac{2K_0}{3}$$

$$\Rightarrow \frac{1}{3}mv_0'^2 - \frac{2K_0}{3}$$

$$\Rightarrow \frac{1}{2} 2m v_0^{\prime 2} = 2K_0$$

16. $\frac{7}{8}$ fraction decays in 6 days

 $\frac{1}{8}$ fraction is active after 6 days

By using,
$$N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$N = \frac{N_0}{2^n}$$

$$\frac{N_0}{8} = \frac{N_0}{2^n}$$

$$n = 3$$

$$6 \text{ days} = \text{half lives}$$

$$t_{\frac{1}{2}} = 2$$
 days

10 days =
$$5t_{1/2}$$

$$N = \frac{N_0}{2^5} = \frac{N_0}{32}$$

So decay =
$$N_0 - \frac{N_0}{32} = \frac{31}{32} N_0$$

17.
$${}^{200}X \rightarrow {}^{80}Y + {}^{120}Z$$

Energy released =
$$80 \times 7 + 120 \times 8 - 200 \times 6.5$$

$$= 220 MeV$$

$$= 2200 \times 10^5 \, eV$$

18. Undisintegrated part

$$\frac{N}{N_0} = (100 - 18)\% = 82\%$$

Using relation
$$N = N_0 \left(e^{-\lambda t} \right)$$

$$\frac{82}{100} = e^{-(24 \times 60 \times 60 \lambda)}$$

$$\therefore 24 \times 60 \times 60 \times \lambda = \log\left(\frac{100}{82}\right)$$

Or
$$\lambda = 2.3 \times 10^{-6} \, \text{s}^{-1}$$

19. The mass number of x = 27 + 1 - 24 = 4 and its atomic number = 13 + 0 - 11 = 2Hence particle x is the helium nucleus, which is called an alpha particle

20.

$$t = 0 \qquad \qquad \begin{array}{c} A & B \\ N_0 & N_0 \end{array}$$

$$t_0 = 3 \text{ days } 2N \text{ N}$$

$$2N = N_0 \left(0.5\right)^{t_0/\tau_1}$$

$$N = N_0 \left(0.5 \right)^{t_0/\tau_2}$$

$$2 = (0.5)^{t_0} \left(\frac{1}{\tau_1} - \frac{1}{\tau_2} \right)$$

$$0.5^{-1} = (0.5) \left(\frac{3}{\tau_1} - 2 \right)$$

$$\Rightarrow -1 = \frac{3}{\tau_1} - 2$$

$$\therefore \tau_1 = 3 \text{ days}$$

21. Here $A_0 = 8$ counts, A = 1 count and t = 3h

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^n$$
or $\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n \Rightarrow n = 3$
So, $T_{1/2} = \frac{t}{n} = \frac{3}{3} = 1h$

22. Number of atoms present initially

$$= \frac{6.023 \times 10^{23} \times 2.2 \times 10^{-3}}{11}$$
$$= 1.2 \times 10^{20} \text{ atoms}$$

No. of atoms present in $5 \mu g$ of the sample

$$N = \frac{6.023 \times 10^{23} \times 5 \times 10^{-6}}{11} = 2.74 \times 10^{17} \text{ atoms}$$

Activity of the sample = λN

$$= \frac{0.693}{\frac{T_1}{2}} \times N$$

$$= \frac{0.693 \times 2.74 \times 10^{17}}{1224}$$

 $=1.55 \times 10^{14}$ disintegrations/second

23. Here,

$$R = R_0 A^{1/3}$$

Since,
$$R = R_0 A^{1/3}$$

$$\therefore \frac{R_1}{R_2} = \frac{A_1^{1/3}}{A_2^{1/3}} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}}$$
$$= \left(\frac{8}{125}\right)^{1/3}$$
$$= \frac{2}{5}$$

 \therefore The ratio of their radii = 2:5

24.

$$e^{\lambda_1 t_0} = \frac{3\lambda_1}{2\lambda_1}$$

$$\Rightarrow t_0 = \frac{1}{\lambda_1} \ln \left[\frac{3\lambda_1}{2\lambda_2} \right]$$

25. Let the radius of ${}_{4}^{9}Be$ nucleus be r. Then, radius of germanium (Ge) nucleus will be 2r.

Radius of nucleus is given by

$$R = R_0 A^{1/3}$$

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\Rightarrow \frac{r}{2r} = \left(\frac{9}{A_2}\right)^{1/3} \qquad (\because A_1 = 9)$$

$$\Rightarrow \left(\frac{1}{2}\right)^3 = \frac{9}{A_2}$$

Hence,
$$A_2 = 9 \times (2)^3 = 9 \times 8 = 72$$

Thus, in germanium (Ge) nucleus number of nucleons is 72.

26. In radioactive equilibrium

$$\lambda_1 N_1 = \lambda_2 N_2$$

$$\frac{N_1}{T_1} = \frac{N_2}{T_2}$$

$$T_2 = \frac{N_2}{N_1} \times T_1$$

$$T_2 = 1.8 \times 10^4 \times 2.5 \times 10^5$$

$$T_2 = 4.5 \times 10^9 \text{ years}$$

27. It has been observed that total mass of nucleus is always less than the sum of the masses of its nucleons. The energy difference between the nucleus and its constituent particles due to their mass difference is termed as the binding energy of the nucleus.

In other words, we can say that to break the nucleus into its constituent particles, some energy is needed to be supplied. This energy is termed as binding energy of the nucleus. More is the binding energy per nucleon, more is the energy to break the nucleus and hence we can say the more stable the nucleus is.

28. Let after time ts, A_1 and A_2 will become equal in the mixture.

As
$$N = N_0 \left(\frac{1}{2}\right)^n$$

Where n is the number of half-lives

For
$$A_1$$
, $N_1 = N_{01} \left(\frac{1}{2}\right)^{t/20}$

For
$$A_2$$
, $N_2 = N_{01} \left(\frac{1}{2}\right)^{t/10}$

According to question, $N_1 = N_2$

$$\frac{40}{2^{t/20}} = \frac{160}{2^{t/10}}$$

$$2^{t/10} = 4(2^{t/20})$$
 or $2^{t/10} = 2^2 2^{t/20}$

$$2^{t/10} = 2^{\left(\frac{t}{20} + 2\right)}$$

$$\frac{t}{10} = \frac{t}{20} + 2 \text{ or } \frac{t}{10} - \frac{t}{20} = 2$$
Or $\frac{t}{20} = 2 \text{ or } t = 40 \text{ s}$

29. We know that $1 \, kW = 1 \times 10^3 \, Js^{-1}$

Also,
$$1.6 \times 10^{-9} \ J = 1 \ eV$$

$$\therefore 200 \ MeV = 200 \times 1.6 \times 10^{-19} \times 10^6 \ J$$

$$Number of fissions = \frac{Power}{Energy released}$$

$$= \frac{10^3}{200 \times 1.6 \times 10^{-13}} = 3.125 \times 10^{13}$$

30. 3×10^9 years

$$\frac{N}{N_0} = \frac{1}{1+7} = \frac{1}{8}$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{8}$$

$$\therefore n = 3$$

$$t = nT = 3 \times 10^9$$
 years