SEC: SR CAO AZ **JEE MAINS REVISION ASSIGNMENT** **MATHS**

BINOMIAL THEOREM

1. If a is real and the 4th term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, for each $x \in R-\{0\}$, then the values of a and n are respectively

1) $5, \frac{1}{2}$ 2) $6, -\frac{1}{2}$ 3) $3, \frac{1}{3}$ 4) $6, \frac{1}{2}$

2. If the $(r+1)^{th}$ term in the expansion of $\left(\frac{a^{1/3}}{b^{1/6}}+\frac{b^{1/2}}{a^{1/6}}\right)^{21}$ has equal exponents of both a and b, then value of r is

1) 8 2) 9 3) 10 4) 11

3. If A and B are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then

1) $A=B$ 2) $A=2B$ 3) $2A=B$ 4) $3A=2B$

4. If x^{2k} occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$, then

1) $n-2k$ is a multiple of 2
2) $n-2k$ is a multiple of 3
3) $k=0$
4) $n-2k$ is a multiple of 6

5. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then value of $C_0 + 2C_1 + 3C_2 + ...(n+1)C_n$ is

1) 2^{n-1} 2) $n\left(2^{n-1}\right)$ 3) $n\left(2^{n-1}\right)+2^n$ 4) $(n+1)2^n$

6. The coefficient of the middle term in the binomial expansion of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals

1) $-\frac{3}{10}$ 2) $\frac{10}{3}$ 3) $-\frac{5}{3}$ 4) $\frac{3}{5}$

7. The number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$ is

1) 96 2) 97 3) 98 4) 99

8. The greatest value of the term independent of x, as α varies over R, in the expansion of $\left(x\cos\alpha+\frac{\sin\alpha}{x}\right)^{20}$ is

1) ${}^{20}C_9\left(\frac{1}{2}\right)^9$ 2) ${}^{20}C_{15}\left(\frac{1}{2}\right)^{15}$ 3) ${}^{20}C_{19}\left(\frac{1}{2}\right)^{19}$ 4) ${}^{20}C_{10}\left(\frac{1}{2}\right)^{10}$

9. The remainder when 2^{2024} is divided by 17 is

1) 1 2) 2 3) 8 4) 12

10. If $\left(1+x+x^2\right)^{48}=a_0+a_1x+a_2x^2+....+a_{96}x^{96}$, then value of $a_0-a_2+a_4-a_6+....a_{96}$ is

1) -1 2) 0 3) 1 4) 48

11. Let $(1+x)^n=C_0+C_1x+C_2x^2+....+C_nx^n$ and $\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+...+n\frac{C_n}{C_{n-1}}=\frac{1}{k}n(n+1)$, then the value of k is

1) 2 2) 3 3) 6 4) 12

12. The last term in the binomial expansion of $\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3.9^{\frac{1}{3}}}\right)^{\log_3 8}$, then the 5$^{\text{th}}$ term from the beginning is

1) ${}^{10}C_6$ 2) $2\left({}^{10}C_4\right)$ 3) $\frac{1}{2}\left({}^{10}C_4\right)$ 4) $-{}^{10}C_6$

13. Coefficient of $\frac{1}{x}$ in the expansion of $(1+x)^n(1+1/x)^n$ is

1) ${}^{2n}C_{n-1}$ 2) ${}^{2n}C_n$ 3) 1 4) 0

14. If coefficient of x^2,x^3,x^4 in the expansion of $\left(1+\frac{x}{a}\right)^6$ are in A.P then a equals

1) $\left(4+\sqrt{7}\right)/3$ 2) $\left(4+\sqrt{3}\right)/3$ 3) $2-\sqrt{3}$ 4) $2+\sqrt{3}$

15. Suppose F is the fractional part of $M=\left(\sqrt{13}+\sqrt{11}\right)^6$, then value of $M(1-F)$ is

1) 128 2) 64 3) 32 4) 16

16. The coefficient of x^7 in the expansion of $\left(1-x-x^2+x^3\right)^6$ is

1) 132 2) 144 3) -132 4) -144

17. The coefficient of x^n in the expansion of $\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2+....+\frac{1}{n!}x^n\right)^2$

1) $\frac{2^n}{n!}$ 2) $\frac{2^n}{n}$ 3) $n!$ 4) $\frac{1}{n!}$

18. Let a_n denote the term independent of x in the expansion of $\left[x+\frac{\sin(1/n)}{x^2}\right]^{3n}$, then $\underset{x\to\infty}{Lt}\frac{(a_n)n!}{{}^{3n}P_n}$ equals

1) 0 2) 1 3) e 4) $e/\sqrt{3}$

19. The coefficient of x^{60} in $(1+x)^{51}\left(1-x+x^2\right)^{50}$ is

1) ${}^{50}C_{20}$ 2) $-\left({}^{50}C_{20}\right)$ 3) ${}^{51}C_{20}$ 4) $-\left({}^{51}C_{20}\right)$

20. The value of $2C_0+\frac{2^2}{2}C_1+\frac{2^3}{3}C_2+....+\frac{2^{11}}{11}C_{10}$, is

1) $\frac{1}{11}\left(2^{11}-1\right)$ 2) $\frac{1}{11}\left(3^{11}-1\right)$ 3) $\frac{1}{11}\left(11^3-1\right)$ 4) $\frac{1}{11}\left(11^2-1\right)$

21. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}}+x^2\log_{10}x\right)^8$ is 5600, then the value of x is ____

22. If $\omega \neq 1$, is a cube root of unity, then sum of the series $S=\sum_{r=0}^{100} {}^{100}C_r\left(2+\omega^2\right)^{100-r}\omega^r$ is equal to ____

23. The expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$ is a polynomial of degree ____

24. Coefficient of x^4 in $\left(1+x+x^2+x^3\right)^{11}$ is ____

25. Sum of the coefficients of x^3 and x^6 in the expansion of $\left(x^2-\frac{1}{x}\right)^9$ is ____

26. If $\left(1+x+x^2\right)^8=a_0+a_1x+a_2x^2+....+a_{16}x^{16}$, then a_5 equals ___

27. The expression $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$ is a polynomial of degree ___

28. If sum of the coefficients in the expansion of $\left(x+\frac{1}{x}\right)^n$ is 128, then coefficient of x in the expansion of $\left(x+\frac{1}{x}\right)^n$ is ______

29. Coefficient of the term independent of x in the expansion of $\left(x+\frac{1}{x}\right)^4\left(x-\frac{1}{x}\right)^{12}$ is ____

30. If sum of the coefficients of x^7 and x^4 in the expansion of $\left(\frac{x^2}{a}-\frac{b}{x}\right)^{11}$ is zero, then ab = ____

KEY

1	**4**	2	**2**	3	**2**	4	**2**	5	**3**
6	**1**	7	**2**	8	**4**	9	**1**	10	**3**
11	**1**	12	**2**	13	**1**	14	**1**	15	**2**
16	**4**	17	**1**	18	**1**	19	**1**	20	**2**
21	**10**	22	**1**	23	**7**	24	**990**	25	**0**
26	**504**	27	**9**	28	**35**	29	**198**	30	**1**

SOLUTIONS

1. We have

$$T_4 = T_{3+1} = {}^nC_3 (ax)^{n-3}\left(\frac{1}{x}\right)^3$$

$$= {}^nC_3 a^{n-3} x^{n-6} = \frac{5}{2}$$

As this is true for each $x \in R - \{0\}$, we get

$$n-6=0 \text{ and } {}^nC_3 a^{n-3} = \frac{5}{2}$$

$$\Rightarrow \quad n=6 \text{ and } {}^nC_3 a^3 = \frac{5}{3}$$

$$\therefore \quad a^3 = \frac{5}{2}\times\frac{3!3!}{6!} = \frac{5}{2}\times\frac{1}{20} = \frac{1}{8}$$

$$\Rightarrow \quad a = \frac{1}{2}$$

Thus, $n = 6, a = \frac{1}{2}$

2. We have,

$$T_{r+1} = {}^{21}C_r \left(\frac{a^{1/3}}{b^{1/6}}\right)^{21-r}\left(\frac{b^{1/2}}{a^{1/6}}\right)^r$$

$$= {}^{21}C_r \frac{a^{7-r/3}}{b^{7/2-r/6}}.\frac{b^{r/2}}{a^{r/6}} = {}^{21}C_r a^{7-r/2} b^{2r/3-7/2}$$

Since exponents of a and b in the $(r+1)^{th}$ term are equal

$$7-\frac{r}{2} = \frac{2r}{3}-\frac{7}{2} \Rightarrow \frac{21}{2} = \frac{7}{6}r \Rightarrow r = 9$$

3. We know that coefficient of x^r in the expansion of $(1+x)^m$ is ${}^mC_r$.

Thus, $A = {}^{2n}C_n$ and $B = {}^{2n-1}C_n$

We have $\frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{(2n)!}{n!n!}\frac{(n!)(n-1)!}{(2n-1)!} = \frac{2n}{n} = 2$

$\Rightarrow \quad A = 2B$

4. T_{r+1} the $(r+1)^{th}$ term in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$ is given by

$$T_{r+1} = {}^{n-3}C_r (x)^{n-3-r}\left(\frac{1}{x^2}\right)^r = {}^{n-3}C_r x^{n-3-3r}$$

As x^{2k} occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$, we must have $n-3-3r = 2k$ for some non-negative integer r.

$\Rightarrow \quad 3(1+r) = n-2k \qquad \Rightarrow n-2k$ is a multiple of 3

5. We have

$$C_0x + C_1x^2 + C_2x^3 + ... + C_nx^{n+1} = x(1+x)^n$$

Differentiating both the sides, we get

$$C_0 + 2C_1x + 3C_2x^2 + ... + (n+1)C_nx^n$$

$$= (1+x)^n + nx(1+x)^{n-1} \quad \text{--- (1)}$$

Putting $x = 1$, we get

$$C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$$

$$= 2^n + n(1)2^{n-1} = (n+2)2^{n-1}$$

6. Middle term in the expansion of $(1+\alpha x)^4$ is ${}^4C_2(\alpha x)^2 = 6\alpha^2x^2$ and the middle term in the expansion of $(1-\alpha x)^6$ is ${}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$.

We are given $6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = 0$ or $\alpha = -\frac{3}{10}$

7. T_{r+1}, the $(r+1)^{th}$ in the expansion of $\left(5^{1/6} + 2^{1/8}\right)^{100}$ is given by

$$T_{r+1} = {}^{100}C_r\left(5^{1/6}\right)^{100-r}\left(2^{1/8}\right)^r$$

No. of rational terms = $\left[\frac{n}{LCM\ of\ \{l,k\}}\right]$

$$= \left[\frac{100}{LCM\ of\ \{6,8\}}\right]$$

$$= \left[\frac{100}{24}\right] = 4$$

$\therefore$ No. of irrational terms = 101 – 4 = 97

8. T_{r+1}, the $(r+1)^{th}$ term in the expansion of $\left(x\cos\alpha + \frac{\sin\alpha}{x}\right)^{20}$ is ${}^{20}C_r(x\cos\alpha)^{20-r}\left(\frac{\sin\alpha}{x}\right)^r$

$$= {}^{20}C_rx^{20-2r}(\cos\alpha)^{20-r}(\sin\alpha)^r$$

For this term to be independent of x, we set $20 - 2r = 0$

$$\Rightarrow r = 10$$

Let β = Term independent of x, then

$$\beta = {}^{20}C_{10}(\cos\alpha)^{10}(\sin\alpha)^{10}$$

$$= {}^{20}C_{10}[\cos\alpha\sin\alpha]^{10}$$

$$= {}^{20}C_{10}\left[\frac{1}{2}\sin 2\alpha\right]^{10} = {}^{20}C_{10}\left(\frac{1}{2}\right)^{10}.(1)$$

Thus, the greatest possible value of β is ${}^{20}C_{10}\left(\frac{1}{2}\right)^{10}$.

9. We have $2^4 = 16 = 17-1$

$= 2^{2024} = \left(2^4\right)^{506} = (17-1)^{506}$

$= {}^{506}C_0.(17)^{506} - {}^{506}C_1(17)^{505} + ...- {}^{506}C_{505}(17) + {}^{506}C_{506}(1)$

$= 17m+1$

Where m is some positive integer.
Thus, the required remainder is 1.

10. Putting $x = i$, we get,

$\left(1+i+i^2\right)^{48} = a_0 + a_1 i + a_2 i^2 + ... + a^{98} i^{96}$

$\Rightarrow \quad i^{48} = (a_0 - a_2 + a_4 - ....) + i(a_1 - a_3 + ...)$

Equating the real parts we get

$a_0 - a_2 + a_4 - a_6 + ... + a_{96} = 1$

11. $r\dfrac{C_r}{C_{r-1}} = r\dfrac{n!}{r!(n-r)!}\dfrac{(r-1)!(n-r+1)!}{n!} = n-r+1$

$\Rightarrow \quad \sum_{r=1}^{n} r\dfrac{C_r}{C_{r-1}} = \sum_{r=1}^{n}(n-r+1) = \dfrac{1}{2}n(n+1)$, K = 2

12. ${}^nC_n\left(-\dfrac{1}{\sqrt{2}}\right)^n = \left(3^{-1-2/3}\right)^{\log_3 8} = 3^{-5\log_3 2} = \dfrac{1}{2^5}$

$\Rightarrow \quad n = 10$

$\therefore \quad 5^{\text{th}}$ term from the beginning $= {}^{10}C_4\left(\sqrt{2}\right)^{10-4}\left(-\dfrac{1}{\sqrt{2}}\right)^4$

$= 2\left({}^{10}C_4\right)$

13. We have $(1+x)^n\left(1+\dfrac{1}{x}\right)^n = \dfrac{(1+x)^{2n}}{x^n}$

$\therefore$ Coefficient of x^{-1} in $(1+x)^n\left(1+\dfrac{1}{x}\right)^n$

= Coefficient of x^{n-1} in $(1+x)^{2n}$ = ${}^{2n}C_{n-1}$

14. Coefficient of x^r in the expansion of $\left(1+\dfrac{x}{a}\right)^6 = {}^6C_r\left(\dfrac{1}{a}\right)^r$

According to given condition ${}^6C_2(1/a)^2, {}^6C_3(1/a)^3, {}^6C_4(1/a)^4$ are in A.P

$\therefore \quad 2\left({}^6C_3\right)\left(\dfrac{1}{a}\right)^3 = {}^6C_2\left(\dfrac{1}{a}\right)^2 + {}^6C_4\left(\dfrac{1}{a}\right)^4$

$\Rightarrow \quad 2(20)a = 15\left(a^2+1\right)$

$\Rightarrow \quad 3a^2 - 8a + 3 = 0$

$\Rightarrow \quad a = \left(4 \pm \sqrt{7}\right)/3$

15. Let $N=\left(\sqrt{13}-\sqrt{11}\right)^6$

As $\left(\sqrt{13}-\sqrt{11}\right)=\dfrac{2}{\sqrt{13}+\sqrt{11}}$, we get $0<N<1$

Also, $M+N=\left(\sqrt{13}+\sqrt{11}\right)^6+\left(\sqrt{13}-\sqrt{11}\right)^6$,

$$=2\left[{}^6C_0\left(\sqrt{13}\right)^6+{}^6C_2\left(\sqrt{13}\right)^4\left(\sqrt{11}\right)^2+{}^6C_4\left(\sqrt{13}\right)^2\left(\sqrt{11}\right)^4+{}^6C_6\left(\sqrt{11}\right)^6\right]$$

$=M+N$ is an integer, say, J

Let $M=K+F$, where K is the greatest integer contained in M.

We have

$J=M+N=K+N+F$

As $0<N<1, 0<F<1$, we get $0<N<F<2$

Also, $N+F$ is an integer

$\Rightarrow \quad N+F=1 \Rightarrow 1-F=N$

Thus, $M(1-F)=\left(\sqrt{13}+\sqrt{11}\right)^6\left(\sqrt{13}-\sqrt{11}\right)^6=2^6=64$

16. $\left(1-x-x^2+x^3\right)^6=(1-x)^6\left(1-x^2\right)^6$

Coefficient of x^7 in $(1-x)^6\left(1-x^2\right)^6$

= Coefficient of x^7 in $\left[1-{}^6C_1x+{}^6C_2x^2-....+{}^6C_6x^6\right]\times\left[1-{}^6C_1x^2+{}^6C_2x^4-....+{}^6C_6x^{12}\right]$

$$=\left(-{}^6C_1\right)\left(-{}^6C_3\right)+\left(-{}^6C_3\right)\left({}^6C_2\right)+\left(-{}^6C_5\right)\left(-{}^6C_1\right)=-144$$

17. Coefficient of x^n in

$$\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2+....+\frac{1}{n!}x^n\right)$$
$$\left(1+\frac{1}{1!}x+\frac{1}{2!}x^2+....+\frac{1}{n!}x^n\right)$$

$$=\frac{1}{n!}+\frac{1}{1!(n-1)!}+\frac{1}{2!(n-2)!}+....+\frac{1}{(n-1)!1!}+\frac{1}{n!}$$

$$=\frac{1}{n!}\left[{}^nC_0+{}^nC_1+{}^nC_2+....{}^nC_{n-1}+{}^nC_n\right]=\frac{2^n}{n!}$$

18. $T_{r+1}={}^{3n}C_r x^{3n-r}\left[\sin\left(\frac{1}{n}\right)\right]^r x^{-2r}$

For this term to be independent of x, set

$3n-r-2r=0 \Rightarrow r=n$

$\therefore \quad a_n={}^{3n}C_n \sin^n\left(\frac{1}{n}\right)$

Now, $\dfrac{n!a_n}{{}^{3n}P_n}=\sin^n\left(\dfrac{1}{n}\right)\to 0$ as $n\to\infty$

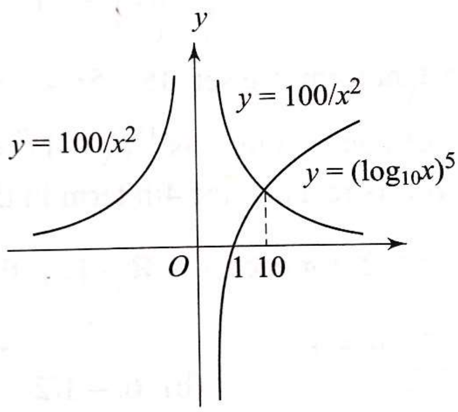
19. The coefficient of x^{60} in $(1+x)^{51}\left(1-x+x^2\right)^{50}$

= the coefficient of x^{60} in $(1+x)\left(1-x^3\right)^{50} = {}^{50}C_{20}$

20. Given expression is equal to

$$\int_0^2\left(C_0+C_1x+C_2x^2+.....C_{10}x^{10}\right)dx=\int_0^2(1+x)^{10}\,dx=\frac{1}{11}\left(3^{11}-1\right)$$

21. Note that for $\log_{10}x$ to be defined, $x>0$



We have $T_6=T_{5+1}={}^8C_5\left(\frac{1}{x^{8/3}}\right)^{8-5}\left(x^2\log_{10}x\right)^5$

$\Rightarrow\ 5600=\frac{8!}{5!3!}\left(\frac{1}{x^8}\right)x^{10}\left(\log_{10}x\right)^5$

$\Rightarrow\ 5600=56x^2\left(\log_{10}x\right)^5$

$\Rightarrow\ 100=x^2\left(\log_{10}x\right)^5$

$\Rightarrow\ \frac{100}{x^2}=\left(\log_{10}x\right)^5$

The curves, $y=\frac{100}{x^2}$

$y=\left(\log_{10}x\right)^5$

Intersect in just one point
See figure, This points is (10, 1)
Therefore, $x=10$

22. We have $S=\left(2+\omega^2+\omega\right)^{100}=1^{100}=1$

23. Using $(a+b)^5+(a-b)^5$

$=2\left[a^5+{}^5C_2a^3b^2+{}^5C_4ab^4\right]$

We get, $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$=2\left[x^5+10x^3\left(x^3-1\right)+5x\left(x^3-1\right)^2\right]$

Which is a polynomial of degree 7.

24. We have

$1+x+x^2+x^3=1+x+x^2(1+x)=(1+x)\left(1+x^2\right)$

$\Rightarrow\ \left(1+x+x^2+x^3\right)^{11}=(1+x)^{11}\left(1+x^2\right)^{11}$

$=\left(1+{}^{11}C_1x+{}^{11}C_2x^2+{}^{11}C_3x^3+{}^{11}C_4x^4+...\right)\left(1+{}^{11}C_1x^2+{}^{11}C_2x^4+....\right)$

Thus, coefficient of x^4 in $\left(1+x+x^2+x^3\right)^{11}$

$=(1)\left({}^{11}C_2\right)+\left({}^{11}C_2\right)\left({}^{11}C_1\right)+\left({}^{11}C_4\right)(1)$

$= 55 + (55)(11) + 330 = 990$

25. $T_{r+1} = {}^9C_r \left(x^2\right)^{9-r}\left(-\frac{1}{x}\right)^r = (-1)^r \left({}^9C_r\right) x^{18-3r}$

For coefficient of x^3, x^6 we set $18 - 3r = 3, 6$

$\Rightarrow \quad r = 5, 4$

$\therefore$ sum of coefficients of x^3 and x^6

$= (-1)^5 \left({}^9C_4\right) + (-1)^4 \left({}^9C_5\right) = 0$

26. We have $\left(1 + x + x^2\right)^8 = \sum\limits_{\substack{p,q \ge 0 \\ p+q \le 8}} \frac{8!}{p!q!(8-p-q)!} x^p \left(x^2\right)^q$

For coefficient of x^5, we set $p + 2q = 5$

This is possible if $p = 5, q = 0, p = 3, q = 1, p = 1, q = 2$

Thus, coefficient of x^5 is $\frac{8!}{5!3!} + \frac{8!}{3!4!} + \frac{8!}{2!5!} = 504$

27. Use $(x+a)^6 + (x-a)^6 = 2\left[x^6 + {}^6C_2 x^4 a^2 + {}^6C_4 x^2 a^4 + {}^6C_6 a^6\right]$

Putting $a = \sqrt{x^3 - 1}$, we find it to be a polynomial of 9.

28. Sum of the coefficients in the expansion of $\left(x + \frac{1}{x}\right)^n$ is equal to 2^n.

$\therefore \quad 2^n = 128 = 2^7 \Rightarrow n = 7$

Now, T_{r+1}, the $(r+1)^{th}$ term in the expansion of $\left(x + \frac{1}{x}\right)^7$ is

$T_{r+1} = {}^7C_r x^{7-r} \left(\frac{1}{x}\right)^r = {}^7C_r x^{7-2r}$

For the coefficient of x, we set $7 - 2r = 1 \Rightarrow r = 3$

Thus, coefficient of x in the expansion of $\left(x + \frac{1}{x}\right)^7$ is ${}^7C_3 = 35$.

29. $\left(x + \frac{1}{x}\right)^4 \left(x - \frac{1}{x}\right)^{12} = \left(x + \frac{1}{x}\right)^4 \left(x - \frac{1}{x}\right)^4 \left(x - \frac{1}{x}\right)^8$

$= \left(x^2 - \frac{1}{x^2}\right)^4 \left(x - \frac{1}{x}\right)^8$

$= \left({}^4C_0 x^8 - {}^4C_1 x^4 + {}^4C_2 - {}^4C_3 \frac{1}{x^4} + {}^4C_4 \frac{1}{x^8}\right) \times \left({}^8C_0 x^8 - {}^8C_1 x^6 + {}^8C_2 x^4 - .... + {}^8C_8 \frac{1}{x^8}\right)$

$\therefore$ Coefficient of the term independent of ‘x’

$= \left({}^4C_0\right)\left({}^8c_8\right) - \left({}^4C_1\right)\left({}^8C_6\right) + \left({}^4C_2\right)\left({}^8C_4\right) - \left({}^4C_3\right)\left({}^8C_2\right) + \left({}^4C_4\right)\left({}^8C_0\right) = 198$

30. $T_{r+1} = {}^{11}C_r \left(\frac{x^2}{a}\right)^{11-r} \left(-\frac{b}{x}\right)^r$

$= {}^{11}C_r (-1)^r a^{r-11} b^r x^{22-3r}$

For coefficient of x^4 and x^7, we put $22-3r=4$ and $22-3r=7$

$\Rightarrow$ r = 6 and r = 5

We are given

$${}^{11}C_6 (-1)^6 a^{-5} b^6 + {}^{11}C_5 (-1)^5 a^{-6} b^5 = 0$$

$$\Rightarrow \frac{b^6}{a^5} = \frac{b^5}{a^6} \Rightarrow ab = 1$$