NARAYANA EDUCATIONAL INSTITUTIONS

PARABOLA

MATHS **MAX.MARKS: 100**

Section-I
(Single Correct Answer Type)

This section contains 20 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** option can be correct.

Marking Scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

1 The point on the parabola $y^2=8x$ at which the normal is inclined at 60^0 to the x- $axis$ has the co-ordinates

1) $\left(6,-4\sqrt{3}\right)$ 2) $\left(6,\ 4\sqrt{3}\right)$ 3) $\left(-6,-4\sqrt{3}\right)$ 4) $\left(-6,\ 4\sqrt{3}\right)$

2. The point on the parabola $y^2=8x$ at which the normal is parallel to the line $x-2y+5=0$ is

1) $\left(-\frac{1}{2},\ 2\right)$ 2) $\left(\frac{1}{2},\ -2\right)$ 3) $\left(2,-\frac{1}{2}\right)$ 4) $\left(-2,\ \frac{1}{2}\right)$

3. The point on the Parabola $y^2=36x$ whose ordinate is three times the abscissa is

1) $(0,\ 0),\ (4,\ 12)$ 2) $(1,\ 3),\ (4,\ 12)$ 3) $(4,\ 12)$ 4) none of these

4. The co-ordinates of the extremities of the latusrectum of the parabola $5y^2=4x$ are

1) $(1\sqrt{5},\ 2\sqrt{5}),\ (-1\sqrt{5},\ 2\sqrt{5})$ 2) $(1\sqrt{5},\ 2\sqrt{5}),\ (1\sqrt{5},\ -2\sqrt{5})$

3) $(1\sqrt{5},\ 4\sqrt{5}),\ (1\sqrt{5},\ -4\sqrt{5})$ 4) $(1\sqrt{5},-\ 4\sqrt{5}),\ (-1\sqrt{5},\ 2\sqrt{5})$

5. If the tangent to the Parabola makes an angle of 45^0 with x-$axis$ then the point of contact is

1) $(a\sqrt{2},\ a\sqrt{2})$ 2) $(a\sqrt{4},\ a\sqrt{4})$ 3) $(a\sqrt{2},\ a\sqrt{4})$ 4) $(a\sqrt{4},\ a\sqrt{2})$

6. If $(2,0)$ is the vertex and y-$axis$ id the directrix of a parabola the its focus is

1) $(2,\ 0)$ 2) $(-2,\ 0)$ 3) $(4,\ 0)$ 4) $(-4,\ 0)$

7. The ends of Latus rectum of Parabola $x^2+8y=0$ are

1) $(-4,-2)$and$(4,\ 2)$ 2) $(4,-2)$and$(-4,\ 2)$

3) $(-4,-2)$and$(4,\ -2)$ 4) $(4,\ 2)$and $(-4,\ 2)$

8. The tangent drawn at any point P to the Parabola $y^2=4ax$ meet the directrix at the point K, then the angle which KP subtends at its focus is

1) 30^0 2) 45^0 3) 60^0 4) 90^0

9. The equation of the Parabola with its vertex at the origin, $axis$ on the y-$axis$ and passing through the points $(6,\ -3)$ is

1) $y^2 = 12x+6$ 2) $x^2 = 12y$ 3) $x^2 = -12y$ 4) $y^2 = -12x+6$

10. The equation of the Tangent to the Parabola $y^2 = 9x$ which goes through the points $(4,\ 10)$ is

1) $x + 4y + 1 = 0$ 2) $9x + 4y + 4 = 0$

3) $x - 4y + 36 = 0$ 4) $9x - 4y + 4 = 0$

11. The point on the Parabola $y^2 = 18x$ for which the ordinate is three times the abscissa is

1) $(6,\ 2)$ 2) $(-2,\ -6)$ 3) $(3,\ 18)$ 4) $(2,\ 6)$

12. If $lx + my + n = 0$ is a Tangent to the Parabola $x^2 = y$, then condition of Tangent is

1) $l^2 = 2mn$ 2) $l = 4m^2n^2$ 3) $m^2 = 4\ln$ 4) $l^2 = 4m$

13. If the line $lx + my + n = 0$ is a tangent to the parabola $y^2 = 4ax$ then locus of its point of contact is
    1) A Straight line 2) A circle 3) A parabola 4) two straight lines
14. The equation of common tangent to the circle $x^2 + y^2 = 2$ and parabola $y^2 = 8x$ is
    1) $y = x+1$ 2) $y = x+2$ 3) $y = x-2$ 4) $y = -x+2$
15. The equation of the tangent to the parabola $y^2 = 16x$ which is perpendicular to the line $y = 3x+7$ is
    1) $y - 3x + 4 = 0$ 2) $3y - x + 36 = 0$ 3) $3y + x - 36 = 0$ 4) $3y + x + 36 = 0$
16. The equation of the parabola whose vertex is $(-1,-2)$, *axis* is vertical and which passes through the point $(3,6)$ is
    1) $x^2 + 2x - 2y - 3 = 0$ 2) $2x^2 = 3y$ 3) $x^2 - 2x - y + 3 = 0$ 4) *none of these*
17. If the parabola $y^2 = 4x$ and passes through the point (1, -2) then the tangent at this point is
    1) $x + y - 1 = 0$ 2) $x - y - 1 = 0$ 3) $x + y + 1 = 0$ 4) $x - y + 1 = 0$
18. The two parabola $y^2 = 4x$ and $x^2 = 4y$ intersect at a point p, whose abscissa is not zero, such that
    1) They both touch each other at p
    2) They cut at right angles at p
    3) They tangents to each curve at p make complementary angles with the x-axis
    4) None of these
19. If the axis of a parabola is horizontal and it passes through the points (0, 0), (0,-1) and (6, 1) then its equation is
    1) $y^2 + 3y - x - 4 = 0$ 2) $y^2 - 3y + x - 4 = 0$ 3) $y^2 - 3y - x - 4 = 0$ 4) *None of these*
20. The equation of the parabola whose vertex and focus are (0, 4) and (0, 2) respectively is
    1) $y^2 - 8x = 32$ 2) $y^2 + 8x = 32$ 3) $x^2 + 8y = 32$ 4) $x^2 - 8y = 32$

(Numerical Value Answer Type)

This section contains 10 questions. The answer to each question is a **Numerical value. If the Answer in the decimals, Mark nearest Integer only. Have to answer any 5 only out of 10 questions and question will be evaluated according to the following marking scheme:**
Marking scheme: +4 for correct answer, -1 in all other cases.

21. If the vertex of a parabola is at origin and directrix is $x + 5 = 0$ then the latus rectum is
22. The Latusrectum of a Parabola whose directrix is $x + y - 2 = 0$ and focus is (3, -4)
23. The angle between the Tangents drawn from the points (1, 4) to the Parabola $y^2 = 4x$ is
24. If the Parabola $y^2 = 4ax$ passes through (-3, 2) then length of its Latusrectum is
25. The focal distance of a point on the Parabola $y^2 = 16x$ whose ordinate is twice the abscissa is
26. A Parabola passing through the point (-4,-2) has its vertex at the origin and y-*axis* as its axis. The Latusrectum of the Parabola is
27. The angle of intersection between the curves $x^2 = 4(y+1)$ and $x^2 = -4(y+1)$ is
28. The equation of Latusrectum of a Parabola is $x + y = 8$ and the equation of a Tangent at the vertex is $x + y = 12$ then length of the Latusrectum
29. If the line $2x + y + k = 0$ is normal to the parabola $y^2 = -8x$ the value of k will be
30. Angle between the curves $y^2 = 4(x+1)$ and $x^2 = 4(y+1)$ is

KEY
MATHEMATICS

1)	**1**	**2)**	**2**	**3)**	**1**	**4)**	**2**	**5)**	**4**
6)	**3**	**7)**	**3**	**8)**	**4**	**9)**	**3**	**10)**	**3**
11)	**4**	**12)**	**4**	**13)**	**3**	**14)**	**2**	**15)**	**4**
16)	**1**	**17)**	**3**	**18)**	**3**	**19)**	**4**	**20)**	**3**
21)	**20**	**22)**	**4.24**	**23)**	**1**	**24)**	**1**	**25)**	**8**
26)	**8**	**27)**	**0**	**28)**	**11**	**29)**	**24**	**30**	**2**

SOLUTIONS

1. Normal at (h,k) to the parabola $y^2 = 8x$ is

   y-k =$-\frac{k}{4}(x-h)$ Gradient = tan 60^0 = $\sqrt{3}$= $\frac{-k}{4}$

   k=$-4\sqrt{3}$ and h=6

   Hence required point is h=(6,$-4\sqrt{3}$)6

2. Let point be (h,k)

   Normal y-k=$-\frac{k}{4}(x-h)$

   (or)

   $kx-4y+kh+4k=0$ Gradient = $\frac{-k}{4}=\frac{1}{2}, k=2$

   Substituting (h,k) and k=-2

   We get h=$\frac{1}{2}$ Hence point is $\left(\frac{1}{2},-2\right)$

   Satisfies the parabola $y^2 = 8x$

3. $y_1 = 3x,$

   According to given condition $9x_1^{\ 2} = 36x_1$

   $x_1 = 4,0; y_1 = 12,0$

   Hence the points are (0,0) and (4,12)

4. $y^2 = 4\left(\frac{1}{5}\right)x; a = \frac{1}{5}$

   focus is $\left(\frac{1}{5},0\right)$ and co-ordinates of latus rectum are

   $y^2 = \frac{4}{25}$

   $y = \pm\frac{2}{5}$ or end points of latus rectum are

$\left(\frac{1}{5},\pm\frac{2}{5}\right)$

5. Parabola $y^2 = ax$ i.e, $y^2 = 4\left(\frac{a}{4}\right)x \longrightarrow$ (i)

$\because$ Let point of contact is $\left(x_{1,}y_1\right)$

$$y - y_1 = \frac{2\left(\frac{a}{4}\right)}{y_1}$$

$$y = \frac{a}{2y}(x) - \frac{ax_1}{2y_1} + y_1$$

Here, $m = \frac{a}{2y_1} = \tan 45^0$

$$\frac{a}{2y_1} = 1$$

$x_1 = \frac{a}{2}$ from(i), $x_1 = \frac{a}{4}$

Point is $\left(\frac{a}{4},\frac{a}{2}\right)$

6. vertex $= (2,0)$

Focus is $(2+2,0) = (4,0)$

7. $x^2 = -8y, a = -2$

So, focus $= (0,-2)$

End of latus rectum $= (4,-2),(-4,-2)$

Trick: Since the ends of rectus rectum lie on parabola,

So only points $(-4,-2)$ and $(4,-2)$ satisfy the parabola.

8. Since the axis of parabola is y-axis

Equation of parabola $x^2 = 4ay$, equation of parabola is $x^2 = -12y$

Science it passes through $(6,-3)$

$36 = -12a \Rightarrow a = -3$

9. Given that $y^2 = 9x$

Here, $a = \frac{9}{4}$

Now, equation of tangent the parabola $y^2 = 9x$ is

$$y = mx + \frac{\frac{9}{4}}{m}$$

If this tangent goes through the point $(4,10)$,

Then $10 = 4m + \frac{9}{4m}$

$\Rightarrow (4m-9)(4m-1) = 0$

$\Rightarrow m = \frac{9}{4}, \frac{1}{4}$

Equation of tangents are , $4y = x + 36$ and $y = -2x - k$ or

$x - 4y + 36 = 0$ and $9x - 4y + 4 = 0$

10. Let $y = 3x$

then $(3x)^2 = 18x$

$9x^2 = 18x$

$x = 2$ and $y = 6$

11. Given that $lx + my + n + o \rightarrow 1$ $x^2 = y$ $\rightarrow 2$

The point of intersection of the line and parabola are obtained by solving 1&2 simultaneously

Substituting the values of x

from 1& 2, we get $\left(\frac{my + n}{l}\right)^2 = y$

$\Rightarrow m^2y^2 + n^2 + 2mny = yl^2$

$m^2y^2 + (2mn - l^2)y + n^2 = 0 \rightarrow 3$

if line 3 touches the parabola(ii),then discriminant=o

$(2mn - l^2) = 4m^2n^2$

$\Rightarrow 4m^2n^2 + l^4 - 4mnl^2 = 4m^2n^2$

$l =^2 4mn$

12. Equation of tangent to parabola $ty = x + at^2 \rightarrow (1)$

Clearly , $lx + my + n = 0$ represents the same line

Hence, $\frac{1}{l} = -\frac{t}{m} = \frac{at^2}{n}$

$t = \frac{-m}{l}, t^2 = \frac{n}{la}$

Eliminating t, we get, $m^2 = \frac{nl}{a}$

i.e,,an equation of parabola

13. $y^2 = 8x, \therefore 4a = 8, a = 2$

Any tangent of parabola ,is $y = mx + \frac{a}{m}$ (or)

$mx - y + \frac{2}{m} = 0$

If it is a tangent to the circle $x^2 + y^2 = 2$,

Then perpendicular from center (0,0) is equal to radius $\sqrt{2}$

$$\frac{2/m}{\sqrt{m^2+1}}$$

(or)

$$\frac{4}{m^2}=2\left(m^2+1\right)$$

$$m^4+m^2-2=0$$

$$\left(m^2+2\right)\left(m^2-1\right)=0$$

$m=\pm 1$ Hence the common tangent are

$y=\pm\left(x+2\right)\therefore y=x+2$

14. Line perpendicular to give line, $3y+x=\lambda$

$$y=\frac{-1}{3}x+\frac{\lambda}{3}$$

Here, $m=\frac{-1}{3},c=\frac{\lambda}{3}$

If we compare $y^2=16x$ with $y^2=4ax$

Then $a=4$

Condition for tangency is

$$c=\frac{a}{m}\Rightarrow\frac{\lambda}{3}=\frac{4}{\left(-1/3\right)}\Rightarrow\lambda=36$$

Required equation is ; $x+3y+36=0$

15. $\left(x+1\right)^2=4a\left(y+2\right)$

Passes through (3,6)

$16=4a.8$

$$a=\frac{1}{2}$$

$$\left(x+1\right)^2=2\left(y+2\right)$$

16. Solving $x^2=4y$ and $y^2=4x$

We get $x=0,y=0$ and $x=4,y=4$

Therefore the co-ordinates of p are (4,4).

The equations of the tangents to the two parabolas at (4,4) are

$$2x-y-4=0\rightarrow(1)$$

$$x-2y+4=0\rightarrow(2)$$

Now, $m_1=$ Slope of(1)=2

$m_2=$ Slope of(2) $=\frac{1}{2}$

$m_1m_2=1$ i.e ; $\tan\theta_1\tan\theta_2=1$

17. There will be no constant term in a curve

Which passes through (0,0). So none is correct .

18. vertex (0,4); focus (0,2)

$\therefore a = 2$

Hence parabola is $(x-0)^2 = -4.2(y-4)$

. i.e. $x^2 + 8y = 32$

19. principal $axis$, of parabolas are $x - axis$,

Therefore angle between them is 90^0.

20. Here, $p(at^2, 2at)$ and $s(a,0)$

If the tangent at $p, ty = x + at^2$

Meets the directrix

$x = -a$ at k,

Then k $= \left(-a, \dfrac{at^2 - a}{t}\right)$

$m_1 =$ Slope of $sp = \dfrac{2at}{a(t^2-1)}$

$m^2 =$ Slope of Sk $= \dfrac{a(t^2-1)}{-2at}$

Clearly

$m_{1,}m_2 = -1$

$\therefore \angle psk = 90^0$

21. S $= (5,0)$

Therefore, latus rectum $= 4a = 20$

22. Distance between focus and direct is $= \left|\dfrac{3-4-2}{\sqrt{3}}\right| = \dfrac{\pm 3}{\sqrt{3}}$

Hence latus rectum $= 3\sqrt{2}$

(Science latus rectum is two times the distance between focus and direction)

23. Let point be (h,k)

But 2h=k

then $k^2 = 16h$

$4h^2 = 16h$

$h = 0, h = 4$

$k = 0, k = 8$

$(0,0),(4,8)$. Hence focal distances are respectively $0 + a = 4$

$4 + 4 = 8(\because a = 4)$

24. Let the equation of parabola is $x^2 = 4ay$

but $a = \frac{4}{-2} = -2$

The equations $x^2 = -8y$ and

Latus rectum $= 4a = 8$

25. The point $(-3,2)$ will satisfy the equation $y^2 = 4ax$

$4 = -12a$

$4a = \frac{-4}{3} = \frac{4}{3}$ (taking positive sign)

26. Clearly , $a = \left|\frac{-8}{\sqrt{1+1}}\right| - \left|\frac{-12}{\sqrt{1+1}}\right| = \frac{4}{\sqrt{2}}$

Length of latus rectum $= 4a = 4 \times \frac{4}{\sqrt{2}} = 8\sqrt{2}$

27. $\because$ parabola passes through the point (1,-2)

Then $4 = 4a \Rightarrow a = 1$

Formula for tangent

$yy_1 = 2a\left(x + x_1\right)$

$-2y = 2\left(x+1\right)$

Required tangent is $x + y + 1 = 0$

28. There will be constant term in a curve which passes through (0,0). So none is correct.

29. Any tangent $y^2 = 4x$ is $y = mx + \frac{1}{m}$

Since it passes through (1,4),

We have $4 = m + \frac{1}{m}$

$\Rightarrow m^2 - 4m + 1 = 0$

$\Rightarrow m_1 + m_2 = 4, m_1 m_2 = 1$

$\Rightarrow m_1 - m_2 = 2\sqrt{3}$

If θ is the required angle, then

$\tan\theta = \frac{2\sqrt{3}}{1+1} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

30. Point of intersection $(0,1)$

$\frac{dy}{dx} = \frac{2x}{4}$ and $\frac{-2x}{4}$

$m_1 = 0, m_2 = 0$

$\theta = 0^0$

29. Any tangent $y^2 = 4x$ is $y = mx + \frac{1}{m}$

Since it passes through (1,4),

We have $4 = m + \frac{1}{m}$

$\Rightarrow m^2 - 4m + 1 = 0$

$\Rightarrow m_1 + m_2 = 4, m_1 m_2 = 1$

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$m_1 = 0, m_2 = 0$