SETS & RELATIONS – I

1. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted physics course and those whose number is divisible by 5 opted chemistry course. Then the number of students who did not opt for any of the three course is:

A) 42 B) 102 C) 1 D) 38

2. Let Z be the set of integers. If $A=\left\{x \in Z: 2^{(x+2)(x^2-5x+6)}=1\right\}$ and $B=\{x \in Z:-3<2x-1<9\}$, Then the number of subsets of the set $A \times B$, is:

A) 2^{18} B) 2^{10} C) 2^{12} D) 2^{15}

3. Let $S=\{1,2,3, ...,100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is:

A) $2^{50}(2^{50}-1)$ B) $2^{100}-1$ C) $2^{50}-1$ D) $2^{50}+1$

4. Two sets A and B as under $A=\{(a,b) \in R \times R:|a-5|<1 \ and \ |b-5|<1\}$

$B=\left\{(a,b) \in R \times R: 4(a-6)^2+9(b-5)^2 \leq 36\right\}$ Then

A) $A \subset B$ B) $A \cap B=\phi(an\,empty\,set)$

C)neither $A \subset B$ nor $B \subset A$ D) $B \subset A$

5. Let A and B be two sets containing four and two elements respectively. Then the number of subjects of the set $A \times B$ each having at least three elements is

A)219 B)256 C)275 D)510

6. If $X=\left(4^n-3n-1: n \in N\right)$ and $Y=\{9(n-1): n \in N\}$; where N is the set of natural numbers, then $X \cup Y$ is equal to

A) N B) Y-X C) X D) Y

7. Let $X=\{1,2,3,4,5\}$. The number of different ordered pairs (Y,Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is

A) 5^2 B) 3^5 C) 2^5 D) 5^3

8. Let R be the real numbers.

Statement I $A=\{(x,y) \in R \times R: y-x \ is \ an \ \ integer\}$ is an equivalence relation on R.

Statement II $B=\{(x,y) \in R \times R: x=ay \ for \ some \ \ lational \ number \ \alpha \ \}$

is an equivalence relation on R.

A) Statement I is true , Statement II is true; statement II is not a correct explanation of statement I

B) Statement I is true , Statement II is false

C) Statement I is false , Statement II is true

D) Statement I is true , Statement II is true; statement II is a correct explanation of statement I

Condition for equivalence relation A relation which is symmetric, reflexive and transitive is equivalence relation.

9. Consider the following relation R on the set or real square matrices of order 3.

$R=\{(A,B): A=P^{-1}BP \text{ for some invertible matrix } p\}$

Statement I R is an equivalence relation

Statement II for any two invertible 3x3 matrices M and N, $(MN)^{-1}=N^{-1}M^{-1}$.

A) Statement I is false, Statement II is true

B) Statement I is true, Statement II is true; Statement II is correct explanation of statement I

C) Statement I is true, Statement II is true; Statement II is not a correct explanation of statement I

D) Statement I is true, Statement II is false

Condition for equivalence relation A relation which is symmetric, reflexive and transitive is equivalence relation.

10. Consider the following relations

$R=\{(x,y)\,|\,x,y \text{ are real numbers and } x=wy \text{ for some rational number } w\}$;

$S=\left\{\left(\frac{m}{n},\frac{p}{q}\right)\,|\,m,n,p \text{ and } q \text{ are integers such that } n,q\neq 0 \text{ and } qm=pn\right\}$.

Then,

A) R is an equivalence relation but S is not an equivalence relation

B) Neither R nor S is an equivalence relation

C) S is an equivalence relation but R is not an equivalence relation

D) R and S both are equivalence relations

11. If A,B and C are three sets such that $A\cap B=A\cap C$ and $A\cup B=A\cup C$, then

A) $A=C$ B) $B=C$ C) $A\cap B=\phi$ D) $A=B$

12. Let W denotes the words in the English dictionary define the relation R by

$R=\{(x,y)\in W\times W: \text{the words } x \text{ and } y \text{ have atleast one letter in common}\}$.

Then R is,

A) reflexive, symmetric and not transitive

B) reflexive, symmetric and transitive

C) reflexive, not symmetric and transitive

D) not reflexive, symmetric and transitive

13. Let $R=\{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A=\{3,6,9,12\}$. The relation is

A) reflexive and symmetric only

B) an equivalence relation

C) reflexive only D) reflexive and transitive only

14. Let $R=\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A=\{1,2,3,4\}$. the relation R is

A) a function B) transitive C) not symmetric D) reflexive

15. Let $A=\{\theta : 2\cos^2\theta+\sin\theta \le 2\}$ and $B=\{\theta : \pi/2 \le \theta \le 3\pi/2\}$. Then $A\cap B$. is

A) $\left[\frac{\pi}{2},\pi\right]\cup\left[\frac{7\pi}{6},\frac{3\pi}{2}\right]$ B) $\left[\frac{\pi}{2},\frac{5\pi}{6}\right]\cup\left[\pi,\frac{3\pi}{2}\right]$ C) $\left[\frac{\pi}{2},\frac{3\pi}{4}\right]\cup\left[\frac{7\pi}{6},\frac{3\pi}{2}\right]$ D) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

16. If $A=\{x:|x|<2\}$, $B=\{x:|x-5|\le 2\}$, $C=\{x:|x|>x\}$, then $A\cap B$ and $A-C$ are respectively

A) $(-2,0),(3,7)$ B) $(-2,2),\phi$ C) $\phi,[0,2)$ D) $[3,7],(-2,0)$

17. In a certain city, only two news papers A and B are published. It is known that 25% of the city population reads A and 20% reads B. while 8% reads A and B. It is also known that 30% of those who read A but not B. look into advertisements and 40% of those who read B but not A look into advertisement while, 50% of those who read both A and B, look into advertisements. What % of the population read on advertisement ?

A) 40.2% B) 14.3% C) 13.9% D) 25.8%

18. N is the set of natural numbers. The relation R is defined on $N\times N$ as follows:
(a,b)R(c,d)$\Leftrightarrow$ ad(b+c)=bc(a+d). Then R is

A) Reflexive but not symmetric , not transitive

B) reflexive and symmetric but not transitive

C) symmetric but not reflexive

D) equivalence relation

19. Suppose $A_1, A_2, ...A_3$ are thirty sets each with five elements and $B_1, B_2, ...B_n$ are n sets each with three elements.

Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$

Assume that each element of S belongs to exactly ten of the $A_i^{'}$s and exactly to nine of the $B_j^{'}$s. Find n.

A) 45 B) 150 C) 15 D) 30

20. If A and B be two sets containing 3 and 6 elements respectively. What can be the minimum and maximum number of elements in $A \cup B$.

A) 3,9 B) 6,9 C) 3,6 D) 0,9

21. Let A and B be two sets then $(A \cup B)^c \cup (A^c \cap B) =$

A) A^c B) B^c C) ϕ D) U

22. A set contains (2n+1) elements. The number of subsets of this set containing more than n elements is equal to

A) 2^{n-1} B) 2^n C) 2^{n+1} D) 2^{2n}

23. The set $(A \cup B \cup C) \cap (A \cap B^{'} \cap C^{'})^{'} \cap C^{'}$ is equal to

A) $B \cap C^{'}$ B) $A \cap C$ C) $B \cup C^{'}$ D) $A \cap C^{'}$

24. If $aN = \{ax / x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then

A)d=bc B)c=bd C)b=cd D)none

25. In a class of 55 students the numbers of students studying different subjects are 23 in mathematics, 24 in physics, 19 in chemistry, 12 in mathematics and physics, 9 in mathematics and chemistry, 7 in physics and chemistry and 4 in all the three subjects. The numbers of students who have taken exactlyone subject is

A)6 B)13 C)16 D) 22

26. Let $R = \{(2,3),(3,4)\}$ be a relation defined on the set $A = \{1,2,3,4\}$. The minimum number of ordered pairs required to be added in R so that enlarged relation becomes an equivalence relation is

A) 5 B) 6 C) 8 D) 14

27. Let R be a relation such that $R = \{(1,4),(3,7),(4,5),(4,6),(7,6)\}$, then $(R^{-1} o R)^{-1} =$

A) $\{(1,1),(3,3),(4,4),(7,7),(4,7),(7,4),(4,3)\}$

B) $\{(1,1),(3,3),(4,4),(7,7),(4,7),(7,4),\}$

C) $\{(1,1),(3,3),(4,4)\}$ D) ϕ

28. A relation R on the set of non zero complex numbers is defined by $z_1 R z_2 \Leftrightarrow \frac{z_1 - z_2}{z_1 + z_2}$ is real, then R is

A) Reflexive B) Symmetric C) Transitive D) Equivalence

29. Let R be an equivalence relation defined on a set containing numbers of 6 elements. Then the minimum numbers of ordered pairs that R should contain

A) 6 B) 12 C) 6^6 D) 36

30. Let R and S be two non-void relations on set A which of the following statements is false.

A) R and S are transitive $\Rightarrow R \cup S$ is transitive

B) R and S are transitive $\Rightarrow R \cap S$ is transitive

C) R and S are Symmetric $\Rightarrow R \cup S$ is symmetric

D) R and S are Symmetric $\Rightarrow R \cap S$ is symmetric

KEY

1-10	D	D	A	A	A	D	B	B	C	C
11-20	B	A	D	C	B	C	C	D	A	B
21-30	A	D	A	A	D	C	B	D	A	A

SOLUTIONS

1. Let n(A)=number of students opted Mathematics=70,

n(B)= number of students opted Physics=46,

n(C)= number of students opted Chemistry=28,

$n(A \cap B) = 23,$

$n(B \cap C) = 9,$

$n(A \cap C) = 14,$

$n(A \cap B \cap C) = 4,$

Now $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

=70+46+28-23-9-14+4=102

So number of students not opted for any course

=Total- $n(A \cup B \cup C)$

=140-102=38

2. $A = \left\{ X \in Z : \ 2^{(x+2)(x^2-5x+6)} = 1 \right\}$

$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow X = -2, 2, 3$

$A = \{-2, 2, 3\}$

$B = \{X \in Z : -3 < 2x - 1 < 9\}$

$B = \{0, 1, 2, 3, 4\}$

$A \times B$ has is 15 elements so number of subjects of $A \times B$ is 2^{15}.

3. $S = \{1, 2, 3......100\}$

=Total non empty subjects $-$subsets with product of element is odd

$= 2^{100} - 1 - 1\left[(2^{50} - 1)\right]$

$= 2^{100} - 2^{50}$

$= 2^{50}(2^{50} - 1)$

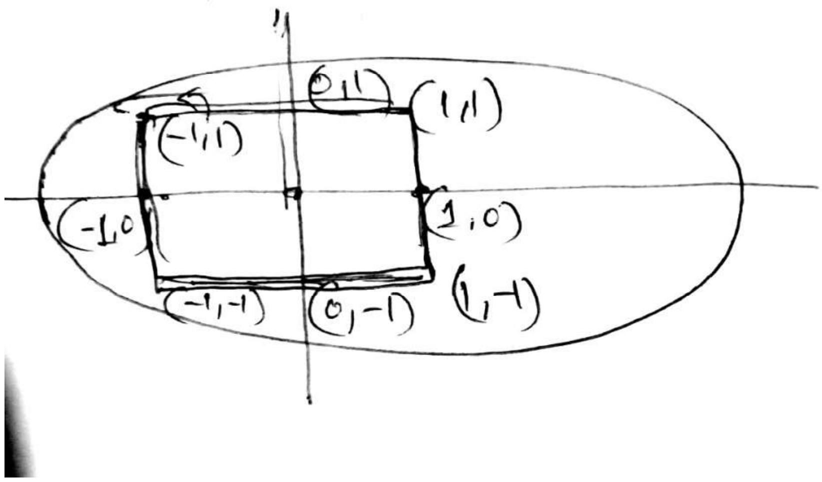
4. $A = \{(a,b) \in R \times R : |a-5| < 1 \ \ and \ \ |b-5| < 1\}$

Let a-5=x, b-5=y

Set A contain all points inside

$|x| < 1, |y| < 1$

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \leq 36\}$



Set B contain all points inside or on

$$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$$

$(\pm 1, \pm 1)$ lies inside the ellipse

$\Rightarrow A \subset B$

5. Number of element of set (AXB)=4x2=8

Total number of subset of (AxB)= 2^8 =256

Number of subset containing 0 elements= $8_{c_0} = 1$

Number of subset containing 1 elements each= $8_{c_1} = 8$

Number of subset containing 2 elements each= $8_{c_2} = 28$

$\therefore$ number of subset having at least 3 elements

=256-1-8-28

=256-37

=219

6. $\therefore X = \{4^n - 3n - 1 : n \in N\}$

$X = \{0, 9, 54, 243, ...\}$ $[put \;\; n = 1, 2, 3, ...]$

$Y = \{9(n-1) : n \in N\}$

$Y = \{0, 1, 18, 27, ...\}$

It is clear that $X \subset Y$.

$\therefore X \cup Y = Y$

7. Given A set $X = \{1, 2, 3, 4, 5\}$

To find the number of different ordered pairs (Y, Z) such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z = \phi$.

Since, $Y \subseteq X$, $Z \subseteq X$, Hence we can only use the elements of X to construct sets Y and Z.

n(y)	Number of ways to make y	Number of ways to make Z such that $\boxed{Y \cap Z = \phi}$
0	$\boxed{^5C_0}$	$\boxed{2^5}$
1	$\boxed{^5C_1}$	$\boxed{2^4}$
2	$\boxed{^5C_2}$	$\boxed{2^3}$
3	$\boxed{^5C_3}$	$\boxed{2^2}$
4	$\boxed{^5C_4}$	$\boxed{2^1}$
5	$\boxed{^5C_5}$	$\boxed{2^0}$

Let us explain anyone of the above 6 rows say third row

In third row Number of elements y=2

$\therefore$ Number of ways to select of x can be part of $y = {}^5C_2$ ways

Now, if y contains any 2 elements, then these 2 elements cannot be used in any way to construct Z. because we want $Y \cap Z = \phi$. And from the remaining 3 elements which are not present in y, 2^3 subsets can be made each of which can be equal to Z and still $Y \cap Z = \phi$ will be true.

Hence, total number of ways to construct sets y and z such that $Y \cap Z = \phi$

$= {}^5C_0 \times 2^5 + {}^5C_1 \times 2^{5-1} + \ldots\ldots {}^5C_5 \times 2^{5-5}$

$= (2+1)^5 = 3^5$

8. Statement I

$A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$

(a) Reflexive

1. $xRx : (x - x)$ is an integer.

i.e. true

$\therefore$ Reflexive

2. Symmetric

$xRy : (x - y)$ *Is an integer*

$\Rightarrow -(y - x)$ *is an integer.*

$\Rightarrow (y - x)$ *is an integer*

$\Rightarrow yRx$

$\therefore$ *symmetric*

(c) Transitive

xRy *and* yRz

$(x - y)$ *is an integer and* $(y - z)$ *is an integer*

$\Rightarrow (x - y) + (y - z)$ *is an integer.*

$\Rightarrow (x - z)$ *is an integer.*

$\Rightarrow xRz$

$\therefore$ *Transitive*

Hence, A ia na equivalence relation.

Statement II

$B = \{(x, y) \in R \times R : x = ay \text{ for some rational number } \alpha \}$

If

$\alpha = \frac{1}{2}$, *then for reflexive, we have*

$xRx \Rightarrow x = \frac{1}{2}x$, *Which is not true,* $\quad \forall x \in R - \{0\}$.

$\therefore$ B is not reflexive on R.

Hence, B is not an equivalence relation on R.

Hence, statement 1 is true, statement 2 is false.

9. Given, $R = \{(A, B) : A = P^{-1}BP \text{ for some invertible matrix } P\}$

For statement I

(i) reflexive $ARA \Rightarrow A = P^{-1}AP$, which is true only, if P= I .

Since, $A = P^{-1}BP$ for some invertible matrix P.

$\therefore$ *we can assume* $p = I$.

$\Rightarrow ARA \Rightarrow A = I^{-1}AI$

$\Rightarrow A = A$

$\Rightarrow$ R is reflexive

Note here, due to some invertible matrix, P is used(reflexive) but if for all invertible matrix is used, then R is not reflexive.

(ii) Symmetric

$ARB \Rightarrow A = P^{-1}BP$

$\Rightarrow PAP^{-1} = P(P^{-1}BP)P^{-1}$

$\Rightarrow PAP^{-1} = (PP^{-1})B(PP^{-1})$

$\therefore B = PAP^{-1}$

Since, for some invertible matrix P, we can let $Q = p^{-1}$

$\therefore B = (P^{-1})^{-1}AP^{-1}$

$\Rightarrow B = Q^{-1}AQ$

$\Rightarrow B = Q^{-1}AQ$

$\Rightarrow BRA$

$\Rightarrow R$ *is symmetric.*

(iii) Transitive

ARB and BRC

$\Rightarrow A = P^{-1}BP$

And $B = P^{-1}CP$

$\Rightarrow A = P^{-1}\left(P^{-1}CP\right)P$

$= \Rightarrow A = (P^{-1})^2 C(P)^2$

So, ARC, for some $P^2 = P$

$\Rightarrow R$ *is transitive*

So, R is an equivalence relation.

For statement II It is always true that $(MN)^{-1} = N^{-1}M^{-1}$

Hence, both statements are true but second is not the correct explanation of first.

10. Given, relation R is defined as $R = \begin{Bmatrix} (x, y) \mid x, y \text{ are real numbers and} \\ x = wy \quad \text{for some rational number } w \end{Bmatrix}$

(i) Reflexive $xRx \Rightarrow x = wx$

$\therefore w = 1 \in \text{rational number}$

The relation R is reflexive.

(ii) symmetric $xRy \Rightarrow yRx$ as oR1

But 1 Ro$\Rightarrow 1 = w.(0)$

Which is not true for any rational number.

The relation R is not symmetric.

Thus, R is not equivalence relation.

Now, for relation S which is defined as

$$S = \left\{\frac{m}{n}, \frac{p}{q} \mid m, n, p \text{ and } q \in \text{integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$$

(i) Reflexive $\frac{m}{n} R \frac{m}{n} \Rightarrow mn = mn \quad [true]$

The relation S is reflexive.

(ii) symmetric $\frac{m}{n} R \frac{p}{q} \Rightarrow mq = n\,p$

$\Rightarrow np = mq \Rightarrow \frac{p}{q} R \frac{m}{n}$

The relation S is symmetric.

(iii) $\frac{m}{n} R \frac{p}{q}$

and $\frac{p}{q} R \frac{r}{s}$

$\Rightarrow mq = mp$

$and \quad ps = rq$

$\Rightarrow mq.ps = np.rq$

$\Rightarrow ms = nr$

$\Rightarrow \frac{m}{n} = \frac{r}{s}$

$\Rightarrow \frac{m}{n} R \frac{r}{s}$

The relation S is transitive. Hence, the relation S is equivalence relation.

11. Given $A \cap B = A \cap C \quad and \quad A \cup B = A \cup C$

$\therefore \quad B = C$

12. Let $W = \{CAT, TOY, YOU......\}$ Clearly, R is reflexive and symmetric but not transitive.

$W = \left\{ {}_{CAT}R_{TOY}, {}_{TOY}R_{YOU} \Rightarrow {}_{CAT}{}^{R}{}_{YOU} \right\}$

13. Since for every elements of A, there exists elements $(3,3),(6,6),(9,9),(12,12) \in R \Rightarrow R$ is reflexive relation .

Now, $(6,12) \in R$ but $(12,6) \notin R,$ so it is not a symmetric relation.

Also, $(3,6),(6,12) \in R$

$\Rightarrow (3,12) \in R$

$\therefore$ R is transitive relation.

14. Given, $R = \{(1,3),(4,2),(2,4),(2,3),(3,1)\}$

Is a relation on the set A={1,2,3,4}.

(a) Since,$(2,4) \in R$ and $(2,3) \in R$. So, R is not a function.

(b) Since,$(1,3) \in R$ and $(3,1) \in R$. But$(1,1) \notin$ R . So, R is not transitive.

(c) Since,$(2,3) \in R$ but (3,2) $\notin$R. So, R is not symmetric.

(d) Since,(1,1), (2,2) (3,3) ,(4,4) $\notin$R. So, R is not reflexive.

15. Since $2\cos^2\theta + \sin\theta \le 2$

$\Rightarrow 2(1-\sin^2\theta) + \sin\theta \le 2$

$\Rightarrow 2\sin^2\theta - \sin\theta \le 0$

$\Rightarrow \sin\theta\left(\sin\theta - \frac{1}{2}\right) \ge 0$

$\therefore \sin\theta \le 0 \quad and \quad \sin\theta \ge \frac{1}{2}$

Now the value of θ which lie in the interval $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

$\left\{ since \;\; B = \left\{ \theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \right\} \right\}$ and satisfy $\sin\theta \le 0$ are given by $\pi \le \theta \le \frac{3\pi}{2}$ and the values of θ

which lie in the interval $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$ $\left\{ since \;\; B = \left\{ \theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \right\} \right\}$ and satisfy $\sin\theta \ge \frac{1}{2}$ are given

by $\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$

$\therefore A \cap B = \left\{ \theta : \pi \le \frac{3\pi}{2} \right\}$ and $A \cap B = \left\{ \theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6} \right\}$

Hence $\therefore A \cap B = \left\{ \theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6} \;\; or \;\; \pi \le \theta \le \frac{3\pi}{2} \right\}$

$= \left\{ \theta : \theta \in \left[\frac{\pi}{2}, \frac{5\pi}{6} \right] \cup \left[\pi, \frac{3\pi}{2} \right] \right\}$

16. $A = \{x : |x| < 2\}$

$= \{x : -2 < x < 2\}$

$A = (-2, 2)$

$B = \{x : |x - 5| \le 2\}$

$= \{x : -2 \le x - 5 \le 2\} = \{x : 3 \le x \le 7\}$

$B = (3, 7)$

$C = \{x : |x| > x\}$

$= \{x : x > x \;\; and \;\; -x > x\}$

$= \{x : x > -x > x\}$

$= \{x : x < 0\}$

$C = (-\infty, 0)$

$\therefore (i) A \cap B = \{x : x \in A \;\; and \;\; x \in B\} = \phi$

$(iv) A - C = \{x : x \in A \;\; and \;\; x \notin C\} = (0, 2)$

17. Let C= Set of people who read paper A

D= Set of people who read paper B

Given $n(C) = 25, n(D) = 20, n(C \cap D) = 8$

$n(C\cap D^{'})=n(C)-n(C\cap D)$

$25-8=17$

But given total people who read A but not B=30%

$\therefore$ % of people reading A but not B=30% of 17

$=\frac{30\times 17}{100}=\frac{(51)}{10}$

And $n(C^{'}\cap D)=n(D)-n(C\cap D)$

=20-8=12

But given total people who read B but not A=40%

$\therefore$ % of people reading B but not A =40% of 12

$$=\frac{40\times 12}{100}=\frac{(24)}{5}$$

Are given total people who read A and B=50%

$\therefore$ % of people reading both A and B=50% of 8

$$=\frac{50\times 8}{100}=4$$

$\therefore$ % of people reading an advertisement

$=\frac{51}{10}+\frac{24}{5}+4=13.9\%$

18. Reflexive.

Since $(a,b)R(a,b)\Leftrightarrow ab(b+a)=ba(a+b)\forall a,b\in N$ is true. Hence R is reflexive.

Symmetric.

$(a,b)R(c,d)\Leftrightarrow ad(b+c)=bc(a+d)$

$\Leftrightarrow bc(a+d)=ad(b+c)$

$\Leftrightarrow cb(d+a)=da(c+b)$

$\Leftrightarrow (c,d)R(a,b)$

Hence R is symmetric.

Transitive. Since $(a,b)R(c,d)\Leftrightarrow ad(b+c)=bc(a+d)$

$\Leftrightarrow \frac{b+c}{bc}=\frac{a+d}{ad}$

$\Leftrightarrow \frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a}$

$$\Leftrightarrow \frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$$

$$(a,b)R(c,d)\Leftrightarrow \frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d} \quad ....(1)$$

And similarly,

$$(c,d)R(e,f)\Leftrightarrow \frac{1}{c}-\frac{1}{d}=\frac{1}{e}-\frac{1}{f} \quad ....(2)$$

From (1) and (2),

$$(a,b)R(c,d) \text{ and } (c,d)R(e,f)\Leftrightarrow \frac{1}{a}-\frac{1}{b}=\frac{1}{e}-\frac{1}{f}\Leftrightarrow (a,b)R(e,f)$$

Hence R is transitive.

Hence R is equivalence relation.

19. Given A's are thirty sets with five elements each, so

$$\sum_{i=1}^{30} n(A_i)=5\times 30=150 \quad ......(i)$$

If the m distinct elements in S and each element of S belongs to exactly 10 of the $A_i^{'}$'s so

We have $\sum_{i=1}^{30} n(A_i)=10m \quad ......(ii)$

$\therefore$ From (i) and (ii) ,we get 10m=150

m=15 ....(iii)

Similarly

$\sum_{j=1}^{n} n(B_j)=3n$ and $\sum_{j=1}^{n} n(B_j)=9m$

3n=9m

$n=\frac{9m}{3}$=3m=$3\times 15=45$ $\quad [from(iii)]$

Hence n=45.

20. we have $n(A\cup B)=n(A)+n(B)-n(A\cap B), n(A\cup B)$ is minimum or maximum according as $n(A\cap B)$ Maximum or minimum respectively.

Case I. If $n(A\cap B)$ is minimum, i.e. $n(A\cap B)$=0 such that

$A=\{a,b,c,d,e,f\}$ *and* $B=\{g,h,i\}$

$n(A\cup B)=n(A)+n(B)$

$=6+3=9$

Case II. If $n(A \cap B)$ is maximum i.e. $n(A \cap B)$=3 such that

$A = \{a, b, c, d, e, f\}$ *and* $B = \{d, a, c\}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

=6+3-3

=6

21. $(A \cup B)^c \cup (A^c \cup B)$

$= (A^c \cap B^c) \cup (A^c \cup B)$

$= (A^c \cup A^c) \cap (A^c \cup B) \cap (B^c \cup A^c) \cap (B^c \cup B)$

$= A^c \cap [A^c \cup (B \cap B^c)] \cap \cup$

$= A^c \cap (A^c \cup \phi) \cap \cup$

$= A^c \cap A^c \cap \cup$

$= A^c \cap \cup$

$= A^c$

22. Let the original set contains 2n+1 elements, then subsets of this set containing more than n elements means subjects containing $(n+1)$ elements, $(n+2)$ elements......$(2n+1)$ elements.

$\therefore$ Required number of subjects

$= {}^{2n+1}c_{n+1} + {}^{2n+1}c_{n+2} + \ldots\ldots\ldots + {}^{2n+1}c_{2n} + {}^{2n+1}c_{2n+1}$

$= {}^{2n+1}c_n + {}^{2n+1}c_{n+1} + \ldots\ldots\ldots + {}^{2n+1}c_1 + {}^{2n+1}c_{0.}$

$= {}^{2n+1}c_0 + {}^{2n+1}c_1 + \ldots\ldots\ldots + {}^{2n+1}c_{n-1} + {}^{2n+1}c_{n.}$

$= \frac{1}{2}\left[(1+1)^{2n+1}\right]$

$= \frac{1}{2}\left(2^{2n+1}\right) = 2^{2n}.$

23. $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$

$= [(A \cap A') \cup (B \cup C)] \cap C' = (B \cup C) \cap C'$

$= (B \cap C') \cup (C \cap C') = B \cap C'$

24. We have $bN = \{bx \mid x \in N\} =$ the set of positive integral multiples of b and $cN = \{cx \mid x \in N\} =$ the set of positive integral multiples of c.

$\therefore bN \cap cN =$ the set of positive integral multiples of $bc = bcN [\because b \text{ and } c \text{ are relatively prime}]$

Hence, $d = bc$

25.

$n(M) = 23, n(P) = 24, n(C) = 19,$

$n(M \cap C) = 9,\ n(P \cap C) = 7, n(M \cap P \cap C) = 4.$

We have to find , $n(M \cap P' \cap C'),$

$n(P \cap M' \cap C'), n(C \cap M' \cap P')$

Now $n(M \cap P' \cap C') = n[M \cap (P \cup C)']$

$= n(M) - n[M \cap (P \cup C)]$

$= n(M) - n[(M \cap P) \cup (M \cup C)]$

$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$

$= 23 - 12 - 9 + 4 = 27 - 21 = 6.$

$n(P \cap M' \cap C') = n[P \cap (M \cup C)']$

$= n(P) - n[P \cap (M \cup C)]$

$= n(P) - n[(P \cap M) \cup (P \cap C)]$

$= n(P) - n[P \cap M] - n(P \cap C) + n(P \cap M \cap C)$

$= 24 - 12 - 7 + 4 = 9,\ n(C \cap M' \cap P') =$

$n(C) - n(C \cap P) - n(C \cap M) + n(C \cap P \cap M)$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

26.

Given $R = \{(2,3),(3,4)\}$

To make it reflexive,enlarge R as following

$R = \{(1,1),(2,2),(3,3),(4,4),(2,3),(3,4)\}$

Hence four more ordered pairs are added.

To make it symmetric, enlarge R as following

$R = \{(1,1),(2,2),(3,3),(4,4),$

$(2,3),(3,4),(3,2),(4,3)\}$

Hence two more ordered pairs are added.

Finally to make it transitive, we enlarge R to

$\{(1,1),(2,2),(3,3),(4,4),(2,3),(3,2),(3,4),$

$(4,3),(2,4)(4,2)\}$. Hence two more ordered

pairs are added. $\therefore$ Total 8 ordered pair must be

added to make the relation R an equivalence.

27. $R = \{(1,4),(3,7),(4,5),(4,6),(7,6)\}$

$R^{-1} = \{(4,1),(7,3),(5,4),(6,4),(6,7)\}$

28. i) $z_1 R z_1 \Rightarrow \frac{z_1 - z_1}{z_1 + z_1} \forall z_1 \in C \Rightarrow 0$

$\therefore$ R is a reflexive.

ii) $z_1 R z_2 \Rightarrow \frac{z_1 - z_2}{z_1 + z_2}$ is real

$\Rightarrow \left(\frac{z_2 - z_1}{z_2 + z_2}\right)$ *is real* $\Rightarrow \left(\frac{z_2 - z_1}{z_2 + z_1}\right)$ *is real*

$\Rightarrow z_2 R z_1 \forall z_1,\ z_2 \in C \therefore R$ *is a symmetric*

iii)let $z_1 = a_1 + ib_1,\ z_2 = a_2 + ib_2,\ z_3 = a_3 + ib_3$

when $a_1 b_1, a_2 b_2, a_3 b_3 \in R$

now $z_1 R z_2 \Rightarrow \frac{z_1 - z_2}{z_1 + z_2}$ *is real*

$\Rightarrow \frac{(a_1 - a_2) + i(b_1 - b_2)}{(a_1 + a_2) + i(b_1 + b_2)} \times \frac{(a_1 + a_2) - (b_1 + b_2)}{(a_1 + a_2) - (b_1 + b_2)}$ *is real*

$\Rightarrow (a_1 + a_2)(b_1 - b_2) - (a_1 - a_2)(b_1 + b_2) = 0$ (for purely real, imaginary part=0)

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$, similarly $z_2 R z_3 \Rightarrow \frac{a_2}{b_2} = \frac{a_3}{b_3}$

$z_1 R z_2 \;\; and \;\; z_2 R z_3 \Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} \;\; and \;\; \frac{a_2}{b_2} = \frac{a_3}{b_3}$

$\Rightarrow \frac{a_1}{b_1} = \frac{a_3}{b_3} \;\; z_1 R z_3 \;\; is \;\; transitive.$

Hence R is an equivalence relation.

29. The minimum number of ordered pairs that R should contain 6 elements.

30. Let $A = \{1,2,3\}$ and $R = \{(1,1)(1,2)\}$ and $S = \{(2,2)(2,3)\}$ clearly R and S are transitive relations on A now

$R \cup S = \{(1,1)(2,2)(1,2)(2,3)\}, R \cup S$ is not transitive as $(1,3) \notin R \cup S$

BASICS

1. The product of all the solution of the equation $(x-2)^2 - 3|x-2| + 2 = 0$ is

A) 2 B)-4 C)0 D)none of these

2. The number of solution of the equation $\log(-2x) = 2\log(x+1)$ is

A) zero B)1 C)2 D)none

3. Greatest integer less than or equal to the number $\log_2 15 . \log 1/6 \log_3 1/6$

A) 2 B)3 C)2 D)1

4. The number of solutions $|[x] - 2x| = 4$ is (where [.] denotes greatest integer function)

A)2 B)4 C)3 D)infinite

5. The solution set of the in equation $1 + \log_{\frac{1}{3}}(x^2 + x + 1) > 0$ is

A) $(-\infty,-2) \cup (1,\infty)$ B) $[-1,2]$ C) $[-2,1]$ D) $[-\infty,\infty]$

6. Number of values of x satisfying the equations $5\{x\} = x + [x]$ and $[x] - \{x\} = \frac{1}{2}$ is

A)1 B)2 C)3 D)4

7. If $|x^2 - 9| + |x^2 - 4| = 5,$ then the set values of x is

A) $(-\infty,-3) \cup (3,\infty)$ B) $(-\infty,-2) \cup (3,\infty)$ C) $(-\infty,3)$ D) $(-3,-2) \cup (2,3)$

8. If $\frac{|x+2| - x}{x} < 2,$ then the set of values of x is

A) $(-\infty,1) \cup (2,\infty)$ B) $(-\infty,-2) \cup (3,\infty)$ C) $(-\infty,-1) \cup (0,\infty)$ D)none of these

9. Solution of the inequality $\log_e^2[2x]-\log[2x]\le 0$ is

A) $[1,3]$ B) $[0,3]$ C) $\{1,2\}$ D) $\left[\frac{1}{2},\frac{3}{2}\right)$

10. Solution set $|x^2-5x+7|+|x^2-5x-14|=21$ is

A) $[-2,7]$ B) $(-\infty,-2]\cup[7,\infty)$ C) $R-\{4,-4,0\}$ D) $\{0\}$

11. The set of real value (s) of p for which the $|2x+3|+|2x-3|\,px+6s$ has exactly two solutions is

A) $[-2,7]$ B) $(-4,4]$ C) $R-\{-4,4\}$ D) $\{0\}$

12. $e^{e^{\ell n \ell n 3}}$ is simplified to

A) B) $\ell n3$ C) 3 D) $\ell n(\ell n3)$

13. $N=\dfrac{81^{\frac{1}{\log_2 9}}+3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409}\left(\sqrt{7}^{\frac{2}{\log_{25} 7}}-(125)^{\log_{25} 6}\right)$, then $\log_2$ N has value

A) 0 B) 1 C) -1 D) None of these

14. If $\ell n(x+z)+\ell n(x-2y+z)=2\ell n(x-z)$, then

A) $y^2=\dfrac{2xz}{x+z}$ B) $y^2=xz$ C) $2y=x+z$ D) $\dfrac{x}{z}=\dfrac{x-y}{y-z}$

15. If x,y,z$\in$ R then system $x+y+z=2, 2xy-z^2=4$

A) Has only one real solution B) has no real solution

C) has only two real solution D) has infinite solutions.

16. If X satisfies $|x-1|+|x-2|+|x-3|\ge 6$, then

A) $0\le x\le 4$ B) $x\le -2$ or $x\ge 4$ C) $x\le 0$ or $x\ge 4$ D) None of these

17. The equation $||x-1|+a|=4$ can have real solutions for x, if a belongs to the interval

A) $(-\infty,5]$ B) $(-\infty,-4]$ C) $(4,\infty)$ D) $[-5,5]$

18. The least integer value of x, which satisfy $|x|+\left|\dfrac{x}{x-1}\right|=\dfrac{x^2}{|x-1|}$, is

A) 0 B) 1 C) 2 D) 3

19. Solve the following for x. $\dfrac{|x+3|+x}{x+2}>1$

A) $(-5,-2)$ B) $(-5,-1)$ C) $(-5,-1)\cup(1,\infty)$ D) $(-5,-2)\cup(-1,\infty)$

20. What will be the number of digits in the sum of all integral values of x in interval $[-4,100]$ satisfying

$\left|2x-\sqrt{(2x-1)^2}\right|=1$

A) 2 B) 3 C) 4 D) 5

21. Solve the following equations for x. $\left|x^2-9\right|+\left|x^2-4\right|=5$

A) $[-3,-2]\cup[2,3]$ B) $(-\infty,-3]\cup[-2,2]\cup[3,\infty)$

C) $[-3,3]$ D) $[0,\infty)$

22. Let $u=(\log_2 x)^2-6(\log_2 x)+12,$ where x is a real number, then the equation $x^u=256$ *has*

A) No solution for x B) Exactly one solution for x

C) Exactly two distinct solutions for x D) Exactly three distinct solutions for x

23. The product of all values of x which make the following statement true

$(\log_3 x)(\log_5 9)-(\log_x 25)+\log_3 2=\log_3 54,$ is

A) $\sqrt{5}$ B) 5 C) $5\sqrt{5}$ D) 25

24. Suppose n be an integer greater than 1, let $a_n=\dfrac{1}{\log_n 2002}$. suppose $b=a_2+a_3+a_4+a_5$

And $c=a_{10}+a_{11}+a_{12}+a_{13}+a_{14}$, then $(b-c)$ equals

A) $\dfrac{1}{1001}$ B) $\dfrac{1}{1002}$ C) -1 D) -2

25. Solve $\left|\left|x-1\right|-2\right|<5$.

A) $(-6,8)$ B) $(-6,4)\cup\{8\}$ C) $(0,8)$ D) $(-6,-4)\cup(4,8)$

26. Solve $x>\sqrt{(1-x)}$.

A) $(0,1)$ B) $\left(\dfrac{\sqrt{5}-1}{2},1\right]$ C) $\left(-\infty,\dfrac{-\sqrt{5}-1}{2}\right)$ D) $\left(\dfrac{\sqrt{5}-1}{2},\infty\right)$

27. For what value of x, $\left|\tan x+\cot x\right|<\left|\tan x\right|+\left|\cot x\right|$ is true.

A) $\left(0,\dfrac{\pi}{4}\right]$ B) $\left(0,\dfrac{\pi}{2}\right)$ C) ϕ D) $\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$

28. Find all possible values of $\dfrac{x^2+1}{x^2-2}$.

A) $\left(-\infty,\dfrac{-1}{2}\right]$ B) $(0,\infty)$ C) $\left(-\infty,\dfrac{-1}{2}\right]\cup(0,\infty)$ D) $\left(-\infty,\dfrac{-1}{2}\right]\cup(1,\infty)$

29. Solve $\left|x^2-4x+3\right|=x+1$.

A) -1 B) – 4 C) $\{-1,-2\}$ D) $(-3,-1)$

30. Solve $|x-1|-|2x-5|=2x$.

A) ϕ B) $[1,\infty)$ C) $\left[1,\frac{5}{2}\right)$ D) – 4

KEY

1-10	C	B	C	B	C	A	D	D	D	A
11-20	B	C	A	D	A	C	B	A	D	C
21-30	A	B	C	C	A	B	C	D	A	D

SOLUTIONS

1. Since x=0is one of the solution so the product will be zero.

2. $\log(-2x)=2\log(x+1)$

$-2x>0\Rightarrow x<0$ ....(i)

$x+1>0\Rightarrow x>-1$ .....(ii)

From (i) & (ii),we get $x\in(-1,0)$

$\therefore -2x=(x+1)^2\Rightarrow x^2+4x+1=0\Rightarrow x=\dfrac{-4\pm2\sqrt{3}}{2}$

So only one solution lies in (-1, 0)

3. $\log_2\log_{\frac{1}{6}}$ $2\log_3\dfrac{1}{6}=$

$=\dfrac{10(3\times5)}{\log 3}=1+log_3 5>2$ (but<2)

4. *case I*

$[x]-2x=4$ ....(i)

$\Rightarrow[x]-2([x]+\{x\})=4$

$\Rightarrow[x]+2\{x\}+4=0$ .....(ii)

$\therefore 0\le 2\{x\}<2$

$\therefore 0\le -2[x]-4<2$

$\Rightarrow [x] = -4, -5$

$\therefore$ *from* (i) *we get* $x = -4, \frac{-9}{2}$

case II

$[x] - 2x = -4$ (from (iii))

$[x] = 2x - 4$

$\Rightarrow [x] = 2([x] + \{4\}) - 4$

$\Rightarrow 2\{x\} = 4 - [x]$ ....(iv)

$\therefore 0 \le 2\{x\} < 2$

$\therefore 0 \le 4 - [x] < 2$

$\Rightarrow 2 < [x] \le 4$

$\Rightarrow [x] = 3, 4$

$\therefore$ *from* (i) *we get* $x = 4, \frac{7}{2}$

$\therefore$ *Number of solutions* $\left|[x] - 2x\right| = 4$ *are* 4

5. $\log_{\frac{1}{3}}\left(X^2 + x + 1\right) > -1 \Rightarrow x^2 + x + 1 < 3$

$\Rightarrow x^2 + x - 2 < 0 \Rightarrow (x+2)(x-1) < 0 \Rightarrow X \in (-2, 1)$

6. $5\{x\} = x + [x]$ ....(i)

$[x] - \{x\} = \frac{1}{2}$ ....(ii)

$\therefore 0 \le [x] < 1$

$\Rightarrow 0 \le [x] - \frac{1}{2} < 1$ $(by\,(ii))$

$\Rightarrow [x] = 1$ $\therefore \{x\} = \frac{1}{2}$

$\therefore$ *from* (i) *we get* $\frac{5}{2} = x + 1$

$\therefore x = \frac{3}{2}, (one\,value)$

7. $\left|x^2 - 9\right| + \left|x^2 - 4\right| = 5$

$$\left|x^2-9\right|+\left|x^2-4\right|=\left|\left(x^2-9\right)-\left(x^2-4\right)\right|$$

$$\Rightarrow \left(x^2-9\right)\left(x^2-4\right)\le 0\left\{\therefore |a|+|b|=|a-b| \Leftrightarrow a.b\le 0\right\}$$

$$\Rightarrow x\in[-3,-2]\cup[2,3]$$

8. $x\ne 0\, case I\ when\, x\ge -2$

$$\frac{|x+2|-x}{2}<2\Rightarrow \frac{2}{x}<2\Rightarrow \frac{1}{x}<1=(x-1)/x>0$$

$$x\in[-2,0]\cup[1,\infty] \qquad ....(i)$$

$case II\ when x<-2$

$$\frac{|x+2|-x}{x}<2\Rightarrow \frac{-2-2x}{x}<2\Rightarrow \frac{1+x}{x}+1>0$$

$$\Rightarrow (1+2x)/x>0\Rightarrow x\in(-\infty,-2) \qquad ....(ii)$$

$$\therefore\ from\,(i)\,and\,(ii)\,we get\ x\in(-\infty,0)\cup(1,\infty)$$

9. $0<\log_e[2x]\le 1$

$$1\le[2x]\le e\Rightarrow [2x]=1,2\Rightarrow 1\le 2x<3$$

$$\frac{1}{2}\le x\le \frac{3}{2}$$

10. $|a|+|b|=|a-b|$

$$\Rightarrow ab\le 0$$

$$\left(x^2-5x+7\right)\left(x^2-5x-14\right)\le 0$$

$$(x-7)(x+2)\le 0$$

$$\Rightarrow x\in[-2,7]$$

11. When the line y=px+6 has slope 4 then the line will be parallel to the line y=4x and when p=0line will coincide with y=6.In between these 2 position (that is when 0<p<4) both the curve will intersect at 2 points, similarl the case with y=-4x also

12. $Use\ A^{\log_A B}=B$

$$e^{\ln(ln3)=\ln 3}$$

$$\therefore e^{e^{\ln(\ln 3)}}=e^{\ln 3}=3$$

13. $$N=\frac{\left(3^4\right)^{\log_{3^2}5}+3^{3\log_3\sqrt{6}}}{409}\left[7^{\log_7 25}-\left(5^3\right)^{\log_5{2^6}}\right]$$

$$N = \frac{3^{\log_3 25} + 3^{\log_3 \sqrt{6}^3}}{409}\left[25 - 6\sqrt{6}\right]$$

$$N = \frac{\left(25 + 6\sqrt{6}\right)\left(25 - 6\sqrt{6}\right)}{409}$$

$$N = 1$$

$$lag_2 N = \log_2 1 = 0$$

14. $\ln\left(x+z\right) + \ln\left(x - 2y + z\right) = 2\ln\left(x - z\right)$

$$\ln\left(x+z\right)\left(x-2y+z\right) = \ln\left(x-z\right)^2$$

$$x^2 - 2xy + 2zx - 2yz + z^2 = x^2 + z^2 - 2zx$$

$$\Rightarrow y = \frac{2xz}{z+x} \quad or \ \frac{x}{z} = \frac{x-y}{y-z}$$

15. $AM \geq GM$

$$\frac{x+y}{2} \geq \sqrt{xy}$$

$$\left(\frac{2-z}{2}\right) \geq \sqrt{xy}$$

$$\frac{2-z}{2} \geq \left(\frac{y+z}{2}\right)^{\frac{1}{2}}$$

$$\frac{4 + z^2 - 4z}{4} \geq \frac{4+z^2}{2}$$

$$\left(z+2\right)^2 \leq 0 \ \therefore z = -2$$

$$x + y = 4$$

$$xy = 4$$

$$x - y = 0$$

$$\therefore x = 2, y = 2 \, and \ z = -2$$

Only one real solution

16. $\left|x-1\right| + \left|x-2\right| + \left|x-3\right| \geq 6,$

Case I When $x < 1,$

$$-\left(x-1\right) - \left(x-2\right) - \left(x-3\right) \geq 6$$

$$-3x + 6 \geq 6$$

$$x \leq 0 \quad ....(i)$$

Case II When $1 < x < 2$,

$(x-1)-(x-2)-(x-3) \geq 6$

$4-x \geq 6$

$x \leq -2$

$\therefore$ No solution.

Case III When $2 < x < 3$,

$x-1+x-2-x+3 \geq 6$

$x \geq 6$

$\therefore$ No solution.

Case IV When $x > 3$

$\Rightarrow x-1+x-2+x-3 \geq 6$

$\Rightarrow 3x \geq 12$

$\Rightarrow x \geq 4$

From Eqs. (i) and (iv), we get

$x \in (-\infty, 0] \cup [4, \infty)$

17. $||x-1|+a| = 4$

$|x-1|+a = 4, -4$

$|x-1| = 4-a \text{ or } -4-a$

Can have real solution only, If either

$4-a \geq 0 \text{ or } -4-a \geq 0$

$\Rightarrow a \leq 4 \text{ or } a \leq -4$

$\therefore a \in (-\infty, -4] \text{ or } a \in (-\infty, 4]$

18. $|x| + \left|\dfrac{x}{x-1}\right| = \dfrac{x^2}{|x-1|}$

As, $|a|+|b| = |a+b|$ only, when $ab \geq 0$

And $|x| + \left|\dfrac{x}{x-1}\right| = \left|x + \dfrac{x}{x-1}\right|$ is true only, if $x.\dfrac{x}{x-1} \geq 0$

$\Rightarrow x \in \{0\} \cup (1, \infty)$

$\therefore$ Least value of x=0.

19. $\dfrac{|x+3|+x}{x+2} > 1$

$$\Rightarrow \quad \frac{|x+3|+x-x-2}{x+2}>0$$

$$\Rightarrow \quad \frac{|x+3|-2}{x+2}>0$$

$$\Rightarrow \quad \frac{x+3-2}{x+2}>0, x>-3$$

$$or \ \frac{-x-3-2}{x+2}>0, x<-3$$

$$\Rightarrow \quad \frac{x+1}{x+2}>0, \ or \ -\frac{(x+5)}{x+2}>0$$

$\Rightarrow \ x>-1 \ \ and \ \ x>-2,$

$\Rightarrow \ x>-5 \ \ and \ \ x<-2$

Or

$x<-1 \ \ and \ \ x<-2,$

$x<-5 \ \ and \ \ x>-2$

$\therefore \ x \in (-1, \infty) \cup x \in (-5, -2)$

20. $\big|2x-|2x-1|\big|=1$

Only, when $2x-1 \geq 0$

i.e. $\ |2x-(2x-1)|=1$

$\Rightarrow 1=1$

$\therefore \ x \geq \frac{1}{2}, \ x \in l \ \ and \ \ x \in [-4, 100]$

Sum of integer values of $x=1+2+3+..........+100$

$$=\frac{100}{2}(1+100)$$

$$=5050$$

Number of digits in 5050 is 4.

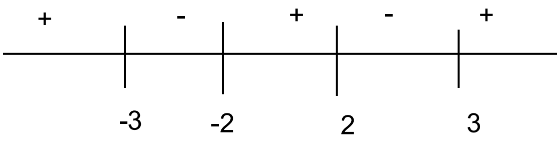
21. $|x^2-9|+|x^2-4|=5$

Using, $|a|+|b|=|a-b|$

$\Rightarrow \quad ab \leq 0$

$\therefore \quad (x^2-9)(x^2-4) \leq 0$

$(x-3)(x+3)(x-2)(x+2) \leq 0$



$\Rightarrow \quad x \in [-3,-2] \cup [2,3]$

22. Given, $x^4 = 256$

$\Rightarrow \log_x 256 = 4$

$\Rightarrow \log_x 2^8 = 4$

$\Rightarrow 8\log_x 2 = 4$

$\Rightarrow 4 = \dfrac{8}{\log_2 x},$

Now, put $\log_2 x$ =t in

$4 = (\log_2 x)^2 - 6(\log_2 x) + 12$

$\Rightarrow \dfrac{8}{t} = t^2 - 6t + 12$

$\Rightarrow t^3 - 6t^2 + 12t - 8 = 0$

$(t-2)^3 = 0$

$\Rightarrow \quad t = 2$

$\Rightarrow \log_2 x = 2$

$\Rightarrow 2^2 = x = 4$

Hence, solution for x is only one.

23. $(\log_3 x)(\log_5 9) - (\log_x 25) + \log_3 2 = \log_3 54,$

$\Rightarrow \dfrac{\log x}{\log 3} \times \dfrac{2\log 3}{\log 5} - \dfrac{2\log 5}{\log x} + \dfrac{\log 2}{\log 3} = \dfrac{3\log 3 + \log 2}{\log 3}$

$\Rightarrow \dfrac{2\log x}{\log 5} - 2\dfrac{\log 5}{\log x} = 3$

$2\log_5 x - \dfrac{2}{\log_5 x} = 3$

$2t - \dfrac{2}{t} = 3 \quad [where, t = \log_5 x]$

$2t^2 - 3t - 2 = 0$

$(t-2)(2t+1) = 0$

$t_1 = 2, t_2 = -\frac{1}{2}$

$\Rightarrow l\,og_5 x = 2$

$l\,og_5 x = -\frac{1}{2}$

$5^2 = x_1, 5^{-1/2} = x_2$

$x_1 \times x_2 = 25 \times \frac{1}{\sqrt{5}} = 5\sqrt{5}$

24. $b - c = \frac{1}{\log_2 2002} + \frac{1}{\log_3 2002} + \frac{1}{\log_4 2002} + \frac{1}{\log_5 2002} -$

$\left(\frac{1}{\log_{10} 2002} + \frac{1}{\log_{11} 2002} + \frac{1}{\log_{12} 2002} + \frac{1}{\log_{13} 2002} + \frac{1}{\log_{14} 2002}\right)$

$= \log_{2002} 2 + \log_{2002} 3 + \log_{2002} 4 + \log_{2002} 5 - \left(\log_{2002} 10 + \log_{2002} 11 + \log_{2002} 12 + \log_{2002} 13 + \log_{2002} 14\right)$

$= \log_{2002}(2 \times 3 \times 4 \times 5) - \log_{2002} 11 \times 12 \times 13 \times 14$

$= \log_{2002}\left(\frac{2 \times 3 \times 4 \times 5}{10 \times 11 \times 12 \times 13 \times 14}\right)$

$= \log_{2002} \frac{1}{2002} = -1$

25. $\left||x-1| - 2\right| < 5$

$\Rightarrow -5 < |x-1| - 2 < 5$

$\Rightarrow -3 < |x-1| < 7$

$\Rightarrow |x-1| < 7$

$\Rightarrow -7 < x - 1 < 7$

$\Rightarrow -6 < x < 8$

26. Given inequality can be solved by squaring both sides. But some times squaring gives extraneous solutions which do not satisfy the original equality. Before squaring we must restrict x for which terms in the given inequality are well defined.

$x > \sqrt{(1-x)}$. Here x must be positive.

Here $\sqrt{(1-x)}$ is defined only when $1 - x \geq 0$ *or* $x \leq 1$ (1)

Squaring given inequality but sides $x^2 > 1 - x$

$$\Rightarrow x^2+x-1>0 \Rightarrow \left(x-\frac{-1-\sqrt{5}}{2}\right)\left(x-\frac{-1+\sqrt{5}}{2}\right)>0$$

$$\Rightarrow x<\frac{-1-\sqrt{5}}{2} \quad or\, x>\frac{-1+\sqrt{5}}{2} \qquad (2)$$

From (1) and (2) $x \in \left(\frac{\sqrt{5}-1}{2}, 1\right] \quad (as\ x\ is\ +ve)$

27. Since $\tan x$ *and* $\cot x$ have always the same sign,

$|\tan x+\cot x|<|\tan x|+|\cot x|$ does not hold true for any value of x.

28. Let y$=\frac{x^2+1}{x^2-2} \Rightarrow yx^2-2y=x^2+1 \Rightarrow x^2=\frac{2y+1}{y-1}$

Now $x^2 \geq 0 \Rightarrow \frac{2y+1}{y-1} \geq 0 \Rightarrow y \leq -1/2\ or\ \ y>1$

29. $|x^2+4x+3|=x+1 \ \Rightarrow |(x+1)(x+3)|=x+1$

$\Rightarrow |(x+1)(x+3)|=(x+1)(x+3),\ when (x+1)(x+3) \geq 0$

$or\ \ x \leq -3\ \ or\ \ x \geq -1$

Hence given equation reduces to $(x+1)(x+3)=x+1$

$\Rightarrow x=-1(x=-2\ is\ rejected\ as\ x \leq -3\ \ or\ x \geq -1)$

$|(x+1)(x+3)|=-(x+1)(x+3),\ when (x+1)(x+3)<0 \quad \Rightarrow -3<x<-1$

Hence given equation reduces to $-(x+1)(x+3)=(x+1)$

$\Rightarrow -4\ \ which\ is\ rejected\ as\ -3<x<-1$

30. Let $f(x)=|x-1|-|2x-5|$

$A.$	$B. f(x)$	$C. f(x) \geq 4$	$D. A \cap C$
$x<1$	$1-x-(5-2x)$	$x-4=2x \Rightarrow x=-4$	$x=-4$
$1 \leq x \leq 5/2$	$x-1-(5-2x)$	$3x-6=2x \Rightarrow x=6$	*no such x exists*
$x>5/2$	$x-1-(2x-5)$	$4-x=2x \Rightarrow x=4/3$	*no such x exists*

Hence solutions set is $\{-4\}$