Integration of Trig

Monday, February 24, 2025 6:13 PM

(2/24/25)

Outline for Today's session (2/24/25)

1. Trigonometric Identities recap
   - Review of Trig concepts & the *magic hexagon* :>
   - Differentiation & Integration of Trigonometric Functions.
2. Logarithmic & Exponential Functions (if may oras pa)

As always,

- yellow highlight : problem / sample problem
- blue highlight : theorem
- purple text : (usually) answer to problem. may not be consistent

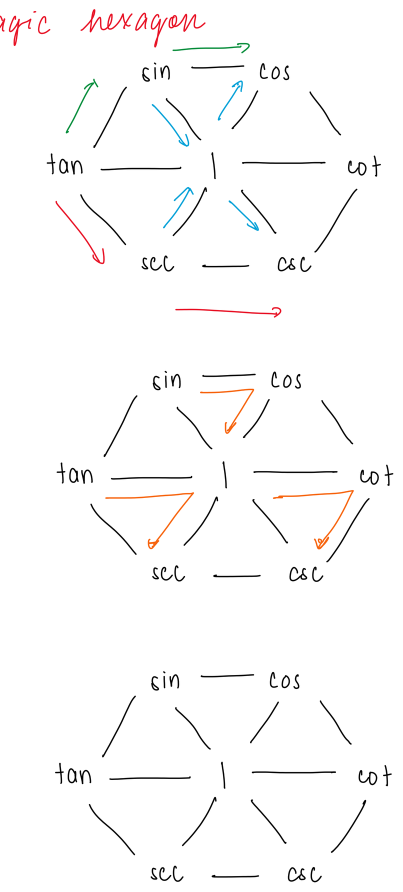
Content of ff. page

1. Theorems
   - Trig identities
   - Deriv & Antideriv of Trig
2. Problems
   - located on the lower half of the page

Review of Trig Identities

1. Trigonometric Identities pt. 1

magic hexagon



* colored arrows used for visualization purposes only

1. Quotient

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\sin\theta = \dfrac{\cos\theta}{\cot\theta}$, etc.

$\tan\theta = \dfrac{\sec\theta}{\csc\theta}$, $\sec\theta = \dfrac{\csc\theta}{\cot\theta}$, etc.

2. Reciprocal

$\sec\theta = \dfrac{1}{\cos\theta}$, $\sin\theta = \dfrac{1}{\csc\theta}$, etc.

3. Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1^2$

$\tan^2\theta + 1^2 = \sec^2\theta$

$1^2 + \cot^2\theta = \csc^2\theta$

2. Trigonometric Identities pt. 2

1. Sum / Difference (red mark : probably won't be used as much)

$\sin(x \pm y) = \sin x \cos y \mp \sin y \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

* $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

2. Double (derived from sum identities)

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

I never found a reason to use half-angle, cuz pampagulo sya sa integral imo.

Derivatives & Antiderivatives of Trig

- $\dfrac{d}{dx}(\sin x) = \cos x$
- $\dfrac{d}{dx}(\cos x) = -\sin x$
- $\dfrac{d}{dx}(\tan x) = \sec^2 x$
- $\dfrac{d}{dx}(\cot x) = -\csc^2 x$
- $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$
- $\dfrac{d}{dx}(\sec x) = \sec x \tan x$
- $\int \cos x \, dx = \sin x + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \sec^2 x \, dx = \tan x + C$
- $\int \csc^2 x \, dx = -\cot x + C$
- $\int \csc x \cot x \, dx = -\csc x + C$
- $\int \sec x \tan x \, dx = \sec x + C$

NOT IMPORTANT!

Don't bother reading if you don't to learn any more unnecessary stuff

why is $\dfrac{d}{dx}(\sin x) = \cos x$?

[sol] by definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Also, via squeeze theorem

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(watch a video on YT or smth)

so, setting $f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1) + \sin h \cos x}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sin x(\cos h - 1)(\cos h + 1)}{h(\cos h + 1)} + \frac{\sin h}{h} \cdot \cos x \right)$$

$$= \lim_{h\to 0} \left(\frac{\sin x(-\sin^2 h)}{h(\cos h + 1)} + \frac{\sin h}{h} \cdot \cos x \right)$$

$$= \lim_{h\to 0} \left[-\frac{\sin h}{h} \cdot \frac{\sin x \cdot \sin h}{\cos h + 1} + \frac{\sin h}{h} \cdot \cos x \right)$$

$$= -1 \cdot \cancel{\frac{(\sin x)(0)}{2}} + 1 \cdot \cos x = \boxed{\cos x}$$

- there's prolly more intuitive stuff in YT or Khan academy, ngl.

PROBLEMS

1. [PS3 - #2]

$\int \sin 2\pi x \, dx$

2. [PS3 - #6]

$\int \sin^3 x \cos x \, dx =$

3. [PS3 - #18]

$\int x \sin^3 x^2 \cos x^2 =$

4. [PS3 - #16]

$\int (1 + \tan^2 x) \sec^2 x \, dx$

5. [PS3 - #30]

$\int \sin 3x \sin 6x \, dx$

6. [PS3 - #31]

$\int \dfrac{1 + \cos 4y}{1 - \cos 4y} \, dy$