

Basic integration of trigonometric functions.

Evaluate the given integrals.

1.) $\int \cos(3x + 1) dx$

2.) $\int \sin 2\pi x dx$

3.) $\int \csc^2 \pi x dx$

4.) $\int \sec 2x \tan 2x dx$

5.) $\int \sin(3 - 2x) dx$

6.) $\int \sin^2 x \cos x dx$

7.) $\int \cos^4 x \sin x dx$

8.) $\int x \sec^2 x^2 dx$

9.)

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

10.) $\int \csc(1 - 2x) \cot(1 - 2x) dx$

11.) $\int \sqrt{1 + \sin x} \cos x dx$

12.)

$$\int \frac{\sin x}{\sqrt{1 + \cos x}} dx$$

13.) $\int \sin \pi x \cos \pi x dx$

14.) $\int \sin^2 \pi x \cos \pi x dx$

15.) $\int \sin \pi x \cos^2 \pi x dx$

16.) $\int (1 + \tan^2 x) \sec^2 x dx$

17.) $\int (1 + \tan^2 x)^2 \sec^2 x dx$

18.) $\int x \sin^3 x^2 \cos x^2 dx$

19.) $\int x \sin^4(x^2 - \pi) \cos(x^2 - \pi) dx$

20.)

$$\int \frac{\sec^2 x}{\sqrt{1 + \tan x}} dx$$

21.)

$$\int \frac{\csc^2 2x}{\sqrt{2 + \cot 2x}} dx$$

22.)

$$\int \frac{\cos(1/x)}{x^2} dx$$

23.)

$$\int \frac{\sin(1/x)}{x^2} dx$$

24.) $\int x^2 \tan(x^3 + \pi) \sec^2(x^3 + \pi) dx$

25.) $\int (x \sin^2 x - x^2 \sin x \cos x) dx$

26.)

$$\int \frac{\sec^3 x \tan x}{(1 + \sec^3 x)^2} dx$$

27.)

$$\int \frac{\cos^2 \varphi}{\sin^4 \varphi} d\varphi$$

28.) $\int (6 \cos^2 x + \sin^2 x)^{\frac{1}{2}} \cos x \sin x dx$

29.) $\int (3 \sin^2 x - \cos^2 x)^{-\frac{1}{2}} \sin x \cos x dx$

30.) $\int \sin 3x \sin 6x dx$

31.)

$$\int \frac{1 + \cos 4y}{1 - \cos 4y} dy$$

32.)

$$\int \frac{\tan w}{1 - \tan^2 w} dw$$

33.) $\int \cos \beta (1 - \cos 2\beta)^3 d\beta$

Differentiation of Logarithmic and Exponential Functions.Use the differentiation formula $\frac{d}{dx} \ln x = \frac{1}{x}$ and $\frac{d}{dx} e^x = e^x$ to solve for $\frac{dy}{dx}$.

34.) $y = \ln 4x$

35.) $y = \ln(2x + 1)$

36.) $y = \ln(x^3 + 1)$

37.) $y = \ln[(x + 1)^3]$

38.) $y = \ln \sqrt{1 + x^2}$

39.) $y = (\ln x)^3$

40.) $y = \ln|x^4 - 1|$ for $x \neq \pm 1$

41.) $y = \ln \ln x$

42.) $y = (2x + 1)^2 \ln(2x + 1)$

43.) $y = \ln \left| \frac{x+2}{x^3-1} \right|$ for $x \neq 2$ and $x \neq 1$

44.) $y = \frac{1}{\ln x}$

45.) $y = \ln^4 \sqrt{x^2 + 1}$

46.) $y = \sin \ln x$

47.) $y = \cos \ln x$

48.) $y = 3e^{2x+1}$

49.) $y = 2e^{-4x}$

50.) $y = e^x \ln x$

51.) $y = x^2 e^x$

52.) $y = e^{\sqrt{x}+1}$

53.) $y = \frac{1}{2}(e^x - e^{-x})$

54.) $y = e^{\sqrt{x}} \ln \sqrt{x}$

55.) $y = (3 - 2e^{-x})^3$

56.) $y = (e^{x^2} + 1)^2$

57.) $y = (e^{2x} - e^{-2x})^2$

58.) $y = (x^2 - 2x + 2)e^x$

59.) $y = x^2 e^x - x e^{x^2}$

60.) $y = \frac{e^{2x}-1}{e^{2x}+1}$

61.) $y = e^{4 \ln x}$

62.) $y = \ln e^{3x}$

63.) $y = \sin e^{2x}$

64.) $y = e^{\sin 2x}$

65.) $y = e^{-2x} \cos x$

66.) $y = \ln \cos e^{2x}$

67.) $y = \log(1 - 4 \tan x)$

68.) $y = \log \sin \frac{x}{a}$

69.) $y = 10^{2x}$

70.) $y = 10^{\cos 2x}$

71.) $y = 3^{-4x}$

Logarithmic Differentiation.Instead of using the Product Rule or the Quotient Rule from Differential Calculus, use what is called logarithmic differentiation: from the given $y = f(x)$, take the logarithm of both sides as in $\ln y = \ln f(x)$; apply the laws of logarithms to expand the expression, then differentiate term by term on both sides; finally, isolate $\frac{dy}{dx}$.

72.) $y = (x^2 + 1)^2(x - 1)^5 x^3$

73.) $y = x(x + a)(x + b)(x + c)$

74.) $y = \frac{x(x^2+1)}{x^2-1}$

75.) $y = \frac{x^3(x-1)}{(x+1)^4}$

76.) $y = \frac{(x+1)(x-1)^2}{(x+1)^4}$

77.) $y = \frac{(x-1)(2x-1)}{(x+1)^3}$

78.) $y = \frac{x^4(x-1)}{(x+2)(x^2+1)}$

79.) $y = \left[\frac{(x-1)(x-2)}{(x-3)(x-4)} \right]^2$

80.) $y = \sqrt{\frac{1-x^2}{1+x^2}}$

81.) $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Integrals yielding logarithmic functions. Even if the integrand of each of the following integrals is not a transcendental function, evaluate the integral to obtain a logarithmic function as an antiderivative.

82.)

$$\int \frac{2}{3y-4} dy$$

83.)

$$\int \frac{4 dt}{5t+2}$$

84.)

$$\int \frac{x^2}{x^3+4} dx$$

85.)

$$\int \frac{v}{6v^2-1} dv$$

86.)

$$\int \frac{2x-5}{x^2-5x+3} dx$$

87.)

$$\int \frac{y-3}{y^2-6y+1} dy$$

88.)

$$\int \frac{(u^2+1)^2}{u^3} du$$

$$89.) \int \frac{y}{(1+y^2)^4} dy$$

$$90.) \int \frac{x^3}{(x^4+16)^2} dx$$

$$91.) \int \frac{v+3}{v-1} dv$$

$$92.) \int \frac{v(v^2-1)}{v^2+1} dv$$

Integration of Transcendental Functions. Evaluate the given integral. Some may involve more than one type of elementary transcendental functions.

$$93.) \int \frac{\ln y}{y} dy$$

$$94.) \int \frac{d\beta}{\beta \ln^4 \beta}$$

$$95.) \int \frac{(1+2 \ln x)^4}{x} dx$$

$$96.) \int \frac{x^2 \ln^3(1+x^3)}{1+x^3} dx$$

$$97.) \int e^x(4-e^x)^{\frac{3}{2}} dx$$

$$98.) \int (1+e^{-2y})^{\frac{1}{2}} e^{-2y} dy$$

$$99.) \int \frac{e^t}{\sqrt{1+5e^t}} dt$$

$$100.) \int \frac{e^{4v}}{(1+3e^{4v})^2} dv$$

$$101.) \int e^{2x} \sin^4 e^{2x} \cos e^{2x} dx$$

$$102.) \int \frac{dy}{e^y \sin^2 e^{-y}}$$

$$103.) \int \frac{\cos \ln x}{x} dx$$

$$104.) \int \frac{\cos^3 x}{1-\sin x} dx$$

$$105.) \int \frac{\cos \beta}{2+3 \sin \beta} \beta$$

$$106.) \int \frac{\sin 2t}{4-3 \cos 2t} dt$$

$$107.) \int \frac{\sin 2t \cos 2t}{1+3 \cos 4t} dt$$

$$108.) \int \frac{\csc^2 x}{1+\cot x} dx$$

$$109.) \int \frac{\sec^2 y}{1+\tan y} dy$$

$$110.) \int \frac{\sec^2 3x}{1+4 \tan 3x} dx$$

$$111.) \int \frac{\sec x \tan x}{2 \sec x + 3} dx$$

$$112.) \int \frac{\csc x \cot x}{1-\csc x} dx$$

$$113.) \int \frac{e^y}{e^y-1} dy$$

$$114.) \int \frac{e^{3x}}{4+e^{3x}} dx$$

$$115.) \int \frac{e^{2y}}{e^y-1} dy$$

$$116.) \int \frac{e^{2t}+e^{2t}}{e^{2t}-e^{-2t}} dt$$

$$117.) \int \frac{dx}{x \ln x}$$

$$118.) \int \frac{dy}{y(1+2 \ln y)}$$

$$119.) \int \frac{\sin 2x}{1+\sin^2 x} dx$$

$$120.) \int \frac{\sec^2 x \tan x}{4+\tan^2 x} dx$$