

```
fun append (xs,ys) =  
  if xs=[]  
  then ys  
  else (hd xs)::append(tl xs,ys)  
  
fun map (f,xs) =  
  case xs of  
    [] => []  
  | x::xs' => (f x)::(map(f,xs'))  
  
val a = map (increment, [4,8,12,16])  
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

Programming Languages

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Functions (Formally)

Function bindings: 3 questions

- Syntax: `fun x0 (x1 : t1, ... , xn : tn) = e`
 - (Will generalize in later lecture)
- Evaluation: ***A function is a value!*** (No evaluation yet)
 - Adds **x0** to environment so *later* expressions can *call* it
 - (Function-call semantics will also allow recursion)
- Type-checking:
 - Adds binding **x0 : (t1 * ... * tn) -> t** if:
 - Can type-check body **e** to have type **t** in the static environment containing:
 - “Enclosing” static environment (earlier bindings)
 - **x1 : t1, ..., xn : tn** (arguments with their types)
 - **x0 : (t1 * ... * tn) -> t** (for recursion)

More on type-checking

```
fun x0 (x1 : t1, ... , xn : tn) = e
```

- New kind of type: $(t_1 * \dots * t_n) \rightarrow t$
 - Result type on right
 - The overall type-checking result is to give x_0 this type in rest of program (unlike Java, not for earlier bindings)
 - Arguments can be used only in e (unsurprising)
- Because evaluation of a call to x_0 will return result of evaluating e , the return type of x_0 is the type of e
- The type-checker “magically” figures out t if such a t exists
 - Later lecture: Requires some cleverness due to recursion
 - More magic after hw1: Later can omit argument types too

Function Calls

A new kind of expression: 3 questions

Syntax: **$e_0 (e_1, \dots, e_n)$**

- (Will generalize later)
- Parentheses optional if there is exactly one argument

Type-checking:

If:

- **e_0** has some type **$(t_1 * \dots * t_n) \rightarrow t$**
- **e_1** has type **t_1** , ..., **e_n** has type **t_n**

Then:

- **$e_0 (e_1, \dots, e_n)$** has type **t**

Example: **`pow(x, y-1)`** in previous example has type **`int`**

Function-calls continued

$e_0(e_1, \dots, e_n)$

Evaluation:

1. (Under current dynamic environment,) evaluate e_0 to a function **fun** x_0 ($x_1 : t_1, \dots, x_n : t_n$) = e
 - Since call type-checked, result *will be* a function
2. (Under current dynamic environment,) evaluate arguments to values v_1, \dots, v_n
3. Result is evaluation of e in an environment extended to map x_1 to v_1, \dots, x_n to v_n
 - (“An environment” is actually the environment where the function was defined, and includes x_0 for recursion)