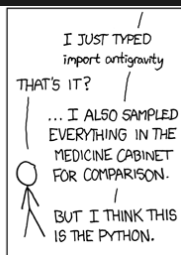
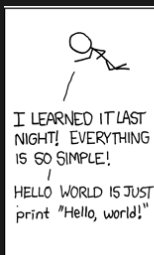
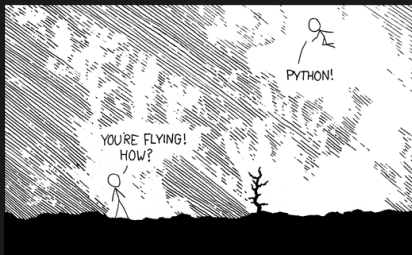


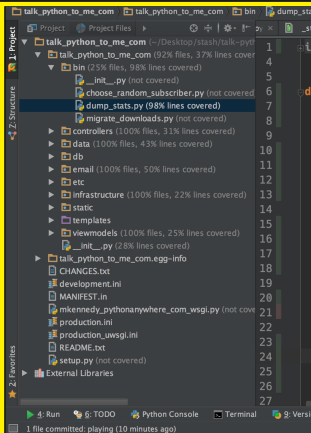
Fundamentals of Machine Learning for Computer Vision (IMCV)

Lecture Zero – September 30th, 2021

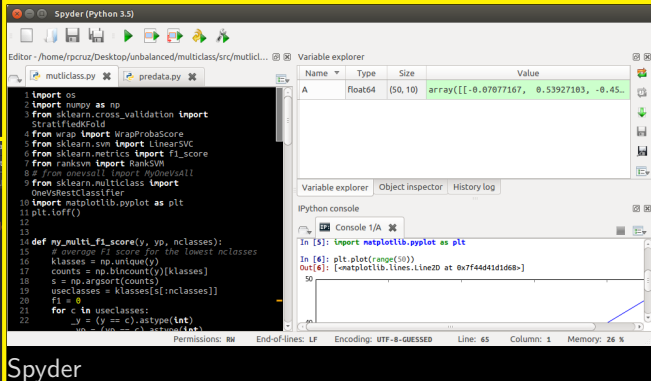
Wilson Silva wilson@fe.up.pt



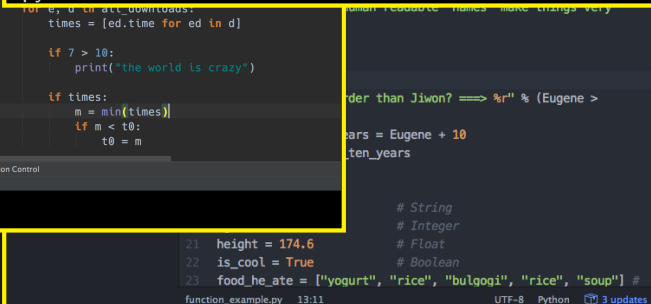
Editors



PyCharm



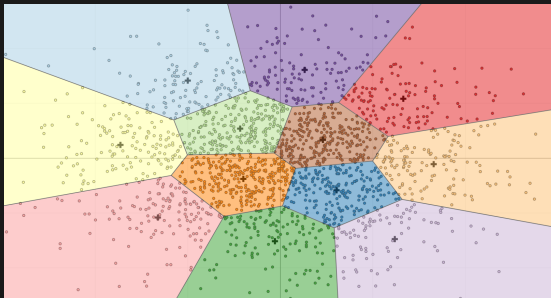
Spyder



Atom

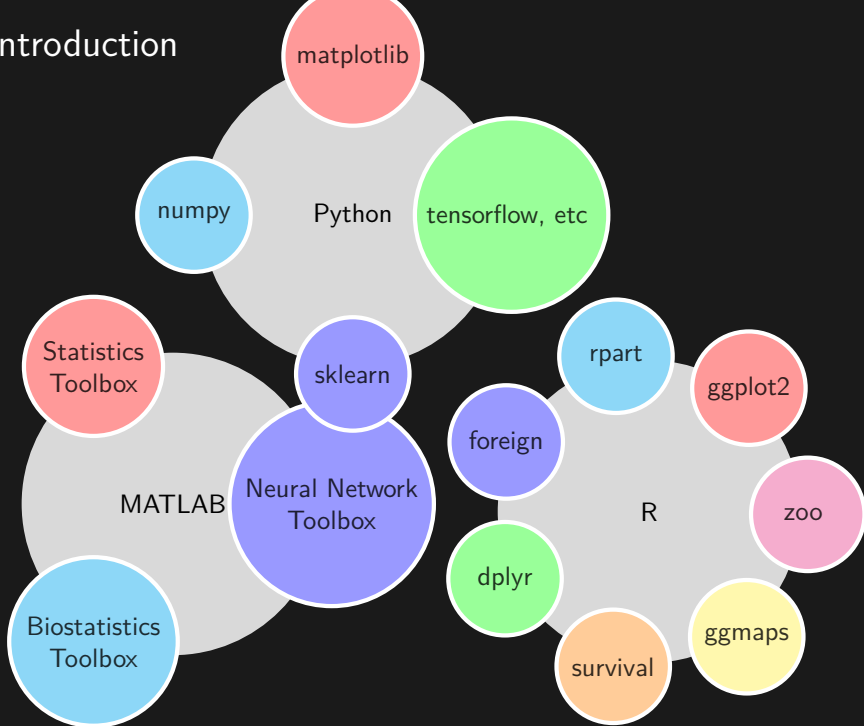
Agenda

Theory	Practice
~Introduction~	
Numpy	Implement Euclidean distance
Cycles/functions	K-means algorithm
File handling	Use a real dataset
Matplotlib	Plots
Classes/modules	—
Scikit-learn	Use its k-means
~Finishing touches~	



Introduction

Introduction



Introduction

- Furthermore, Python is a **general** purpose language
 - It is used in everything from user interfaces to web servers
 - Linear algebra is **not** a primary citizen

Introduction: Syntax

Python

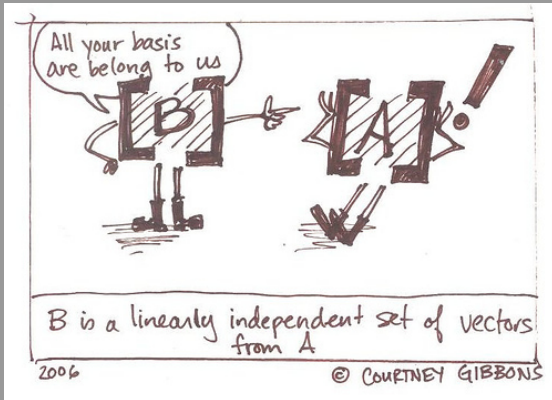
```
1 def is_prime(n):  
2     for m in range(2, n):  
3         if n % m == 0:  
4             return False  
5     return True
```

MATLAB

```
1 function ret = is_prime(n)  
2     ret = true;  
3     for m=1:(n-1)  
4         if mod(n, m) == 0  
5             ret = false;  
6             break  
7         end  
8     end  
9 end
```

Warning: In Python you **must** properly indent your code.

NumPy: linear algebra package



NumPy

The *de-facto* package for linear algebra is `numpy`.

Usually, it is shortened to `np`:

```
1 import numpy as np
```

NumPy features two main structures:

- `np.ndarray` — array of multiple dimensions
- `np.matrix` — matrix (two dimensions)

NumPy vs MATLAB

Summary of NumPy for MATLAB users ^{1 2}

MATLAB	NumPy
<code>size(a)</code>	<code>a.shape</code>
<code>a(1:5,:)</code>	<code>a[0:5,:]</code> or <code>a[0:5]</code> or <code>a[:5]</code>
<code>a(end-4:end,:)</code>	<code>a[-5:]</code>
<code>a.'</code>	<code>a.transpose()</code> or <code>a.T</code>
<code>a * b</code>	<code>a.dot(b)</code>
<code>a .* b</code>	<code>a * b</code>

The main conceptual difference is that Numpy supports **arithmetic broadcasting**. That is, you can do the following element-wise multiplication: $(6,3) * (6,1)$. It automatically assumes you want to multiply by column. In MATLAB, you would have to use `bsxfun(@times,r,A)` or first use `repmat()`.

¹<https://docs.scipy.org/doc/numpy-dev/user/numpy-for-matlab-users.html>

²<http://mathesaurus.sourceforge.net/matlab-numpy.html>

NumPy Example

Create $B_{ij} = \begin{cases} 5, & \text{if } A_{ij} > 1 \\ 0, & \text{otherwise.} \end{cases}$

Suggestions:

```
B = A.copy()
B[A > 1] = 5
B[A <= 1] = 0
```

```
B = np.zeros(A.shape)
B[A > 1] = 5
```

```
B = (A > 1) * 5
```

```
B = np.asarray(
    [[5 if aij > 1 else 0 for aij in ai] for
     ai in A])
```

It is probably a good idea to finish this off with:

```
B = B.astype(np.int8)
```

to save memory!

API

Python Lists

1. Create a list – `l = [1, 5, 2]`
2. Access item – `l[1]`
3. Modify item – `l[1] = 7`
4. Add item – `l.append(8)`, `l += [9, 10]`

NumPy

1. Create ndarray from list – `np.array()`
2. Create ndarray of zeros/ones – `np.zeros()`, `np.ones()`
3. Sample Uniform(0,1) – `np.rand`, `np.random.random()`
4. Sample Normal(0,1) – `np.randn`, `np.random.normal()`
5. Horizontal concatenation – `np.vstack()`, `np.c_[]`
6. Vertical concatenation – `np.hstack()`, `np.r_[]`
7. Any concatenation – `np.concatenate(list, axis)`

Project: Implement Euclidean distance

Get the project files from MS Teams.

1. Implement a function called **euclidean_distance(a, b)**

- Given two vectors **a** and **b**, computes the square-root of the squared difference between the two vectors
- $d = \|x_1 - x_2\|_2$
- TIP: you might want to use vectorisation!

2. Use it for the two example vectors given in the script

`v1 = [1.1, 2.5, 4.4, 0.1, 2.3, 3.4]`

`v2 = [2.0, 2.2, 1.0, 1.0, 2.5, 3.4]`

Possibly useful functions:

`np.random.randn`, `np.mean`, `np.std`, `np.sqrt`, `np.square`, `np.linalg.norm`

Project: Implement Euclidean distance (solution)

```
1# Implement the euclidean_distance function here:
2def euclidean_distance(a, b):
3    # Computes and returns the Euclidean distance between vectors
    # 'a' and 'b'
4    distance = np.sqrt(np.sum(np.square(a - b)))
5    return distance
6
7# Check if it is working! Here are two vectors:
8v1 = np.array([1.1, 2.5, 4.4, 0.1, 2.3, 3.4])
9v2 = np.array([2.0, 2.2, 1.0, 1.0, 2.5, 3.4])
10
11# Call the euclidean_distance function below to compute the
    distance between them:
12dist = euclidean_distance(v1, v2)
13
14print('Distance between v1 and v2: ', dist) # (For these vectors
    , it should be something like 3.6482...)
```

Cycles & Functions

Loops

Python has some interesting functions that can help you in loops.

```
1 for n in range(10, 30, 2):  
2     print(n)  
3 # 10 12 14 16 18 20 22 24 26 28  
4  
5 for i, n in enumerate(range(10, 20, 2)):  
6     print((i, n))  
7 # (0, 10) (1, 12) (2, 14) (3, 16) (4, 18)  
8  
9 for a, b in zip(range(10, 20), range(40, 50)):  
10    print((a, b))  
11 # (10, 40) (11, 41) (12, 42) (13, 43) (14, 44) (15, 45) (16, 46)  
    (17, 47) (18, 48) (19, 49)
```

You will probably use these functions a lot in your loops: `range`, `enumerate` and `zip`.

List Comprehension and Functools

List comprehension

```
1 [i**2 for i in range(5)]
```

```
1 [i for i in range(5) if i % 2 ==  
  0]
```

Functools

```
1 map(lambda x: x**2, range(5))
```

```
1 filter(lambda x: x % 2 == 0,  
         range(5))
```

```
1 functools.reduce(lambda a,b: a+b  
                  , range(5)) # sum numbers  
                               (10)
```

- List comprehension is considered the most Pythonic approach.

- But...



Project: Implement the k-means algorithm

Given data X_{ij} , the user first defines K clusters.

The algorithm:

1. Random cluster centroids are selected randomly using the MacQueen method;
2. Each observation is assigned the closest cluster:
 - $c_i(t+1) = \arg \min_k \|x_i - \tilde{c}_k\|_2$
3. Compute the new centroid of each cluster (using the Euclidean distance you implemented before):
 - $\forall k, j: \tilde{c}_{kj} = \frac{1}{\sum_{i=1}^N \mathbb{1}_{c_i=k}} \sum_{i=1}^N \mathbb{1}_{c_i=k} X_{ij}$
4. Repeat steps 2-3 for a set number of iterations.

Please place this inside a `kmeans`: $\mathbb{Z}, \mathbb{R}^{N \times M} \rightarrow \mathbb{Z}^N$ function, which maps K and X to its respective clusters.

Note: you can also implement a convergence condition that stops the loop as soon as the clusters stabilise.

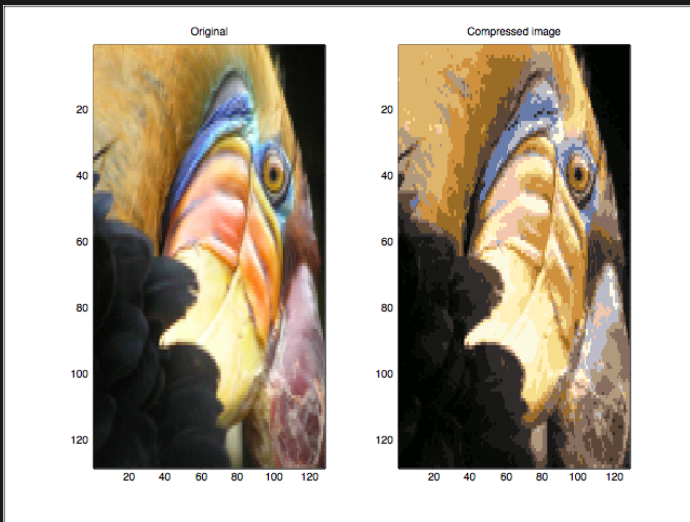
Project: Implement the k-means algorithm (solution)

```
1 def k_means(X, k=3, n_iterations=10):
2     # (...)
3     # The k_means algorithm is iterative. Start a loop here for '
4         n_iterations':
5         for ii in range(n_iterations):
6             # In each loop, for each data point:
7             for index in range(X.shape[0]):
8                 # Compute the Euclidean distance between the data
9                     point and each centroid:
10                 distance = [euclidean_distance(X[index], cc) for cc
11                             in centroids]
12
13                 # Then, assign each data point to the cluster with
14                     the nearest centroid:
15                 membership[index] = np.argmin(distance)
16
17             # Now, recompute the centroids of each cluster, computing
18                 the mean of the cluster data points:
19             for cc in range(k):
20                 centroids[cc] = np.mean(X[membership == cc], axis=0)
21     # (...)
```

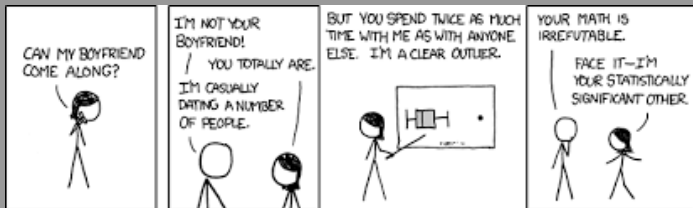
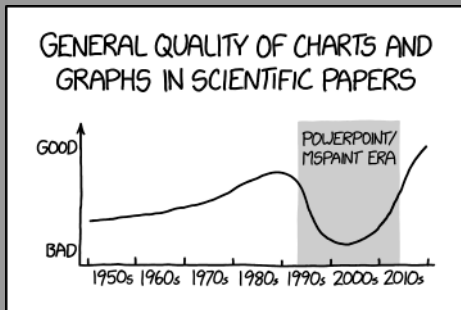
Kmeans: usage example

K-means can be used for e.g. image compression...

24 (8×3) bits \rightarrow k=64 (6 bits)



Matplotlib



Matplotlib

The *de-facto* package for plotting graphics in python is matplotlib.

Matplotlib contains an API³ called pyplot that is inspired in MATLAB.

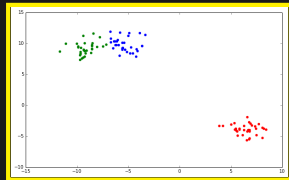
See: http://matplotlib.org/1.4.3/api/pyplot_api.html

³API = Application Programmer's Interface

Matplotlib

Example using synthetically created data:

```
1 from matplotlib.pyplot as plt
2 from sklearn.datasets import make_blobs
3 X, y = make_blobs(centers=3)
```



```
1 plt.scatter(X[y == 0, 0], X[y == 0, 1], color='red')
2 plt.scatter(X[y == 1, 0], X[y == 1, 1], color='green')
3 plt.scatter(X[y == 2, 0], X[y == 2, 1], color='blue')
4 plt.show()
```

OR

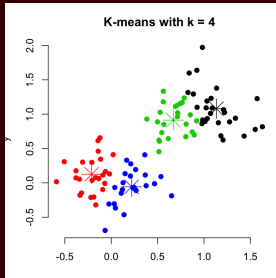
```
1 colors = ['red', 'green', 'blue']
2 plt.scatter(X[:, 0], X[:, 1], color=[colors[_y] for _y in y])
3 plt.show()
```

OR

```
1 colors = plt.cm.rainbow(np.linspace(0, 1, 3))
2 plt.scatter(X[:, 0], X[:, 1], color=colors[y])
3 plt.show()
```

Project: k-Means in a simulated dataset

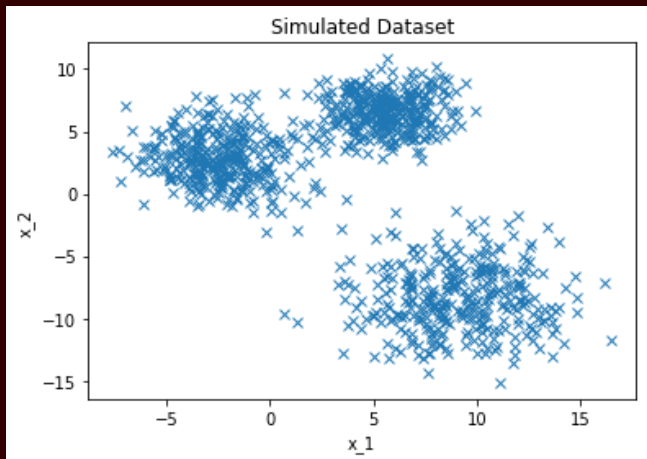
1. Use matplotlib to **plot the simulated dataset**
2. Use the implemented k-Means function to **cluster the dataset** (note: use $k = 3$)
3. Use matplotlib again, to **plot the k-Means clustering result** (use a different color for the points of each cluster)
4. At last, try to **plot the movement of the centroids** over the iterations



Project: k-Means in a simulated dataset (solution)

```
1# Now, to test it, we shall use a simulated dataset:
2X, _ = additional_stuff.simulated_dataset()
3
4# We can use matplotlib to visualise the simulated data points:
5# First, we plot the data points:
6pl.plot(X[:,0], X[:,1], 'x')
7# Then, some additional stuff:
8pl.title('Simulated Dataset')
9pl.xlabel('x_1')
10pl.ylabel('x_2')
11pl.show()
12
13# Now, we can apply our implementation of 'k-means' to this
   dataset:
14membership, centroids, centroids_history = k_means(X, k=3)
```

Project: k-Means in a simulated dataset (solution)



Project: k-Means in a simulated dataset (solution)

```
1# Using matplotlib, we can plot the result:
2# First, plot the data points:
3colors = ['red', 'green', 'blue']
4pl.scatter(X[:, 0], X[:, 1], marker='x', color=[colors[int(_y)]
        for _y in membership])
5# Then, plot the final cluster centroids:
6pl.scatter(centroids[:,0], centroids[:,1], 'o', color='black')
7# And some additional stuff:
8pl.title('k-Means Clustering results')
9pl.xlabel('x_1')
10pl.ylabel('x_2')
11pl.show()
```

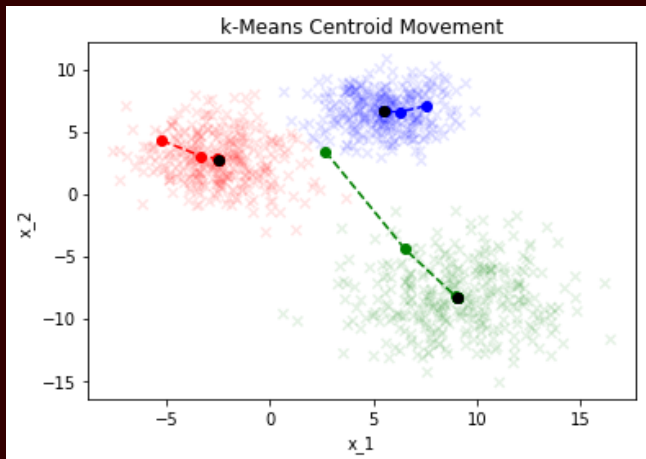
Project: k-Means in a simulated dataset (solution)



Project: k-Means in a simulated dataset (solution)

```
1# We can also see the movement of each centroid during the
   iterations:
2# Plot the data points again (use alpha to make them transparent)
   :
3pl.scatter(X[:, 0], X[:, 1], marker='x', alpha=0.1, color=[colors
   [int(_y)] for _y in membership])
4# And plot each centroid's movement:
5pl.plot(centroids_history[:,0,0], centroids_history[:,0,1], '--',
   marker='o', color='red')
6pl.plot(centroids_history[:,1,0], centroids_history[:,1,1], '--',
   marker='o', color='green')
7pl.plot(centroids_history[:,2,0], centroids_history[:,2,1], '--',
   marker='o', color='blue')
8# Then, plot the final cluster centroids:
9pl.plot(centroids[:,0], centroids[:,1], 'o', color='black')
10# And some additional stuff:
11pl.title('k-Means Centroid Movement')
12pl.xlabel('x_1')
13pl.ylabel('x_2')
14pl.show()
```

Project: k-Means in a simulated dataset (solution)



File Handling

Open files using Numpy

Open a CSV file and add some random noise.

```
1 import numpy as np
2 d = np.loadtxt('file.csv', [np.float64, np.float64, np.int8],
3                 delimiter=',', skiprows=1)
4 X = d[:, :-1]
5 y = d[:, -1]
```


Open files using Numpy

Open a CSV file and add some random noise.

```
1 import numpy as np
2 d = np.loadtxt('file.csv', [np.float64, np.float64, np.int8],
3                  delimiter=',', skiprows=1)
4 X = d[:, :-1]
5 y = d[:, -1]
```

By default numpy always works with `np.float64`. But you may want to change dtype when opening a CSV file or creating a vector in order to use less memory. The following types are available:

<code>np.int</code>	8, 16, 32 and 64 bits
<code>np.uint</code>	8, 16, 32 and 64 bits
<code>np.float</code>	16, 32 and 64 bits
<code>np.complex</code>	64 and 128 bits

Complex numbers are represented using two 32-bits or 64-bits floats.

By default numpy always works with `np.float64`. If you work with GPUs, it's a good idea to convert everything to 32-bits.

Open images using Scikit-image

```
1 from skimage.io import imread
2 img = imread('filename.png')
3
4 print(img.shape)
5 # (512, 512, 3)  # if color (RGB)
```

Pandas

If you are used to R `data.frame` then you are going to miss things like accessing columns by column name.

There is a widely used data-frame package for Python called `pandas`.

(This package is particularly useful when dealing with time series because it supports a lot of timeseries functionality we are not going to cover.)

Opening is faster than numpy...

```
1 import pandas as pd
2 df = pd.read_csv('titanic.csv')
3 df = pd.read_excel('a.xlsx')
4
5 df.groupby('gender').mean()
6 #                age      height
7 # gender
8 # female    53.870850    158.201459
9 # male      53.491803    158.971695
10
11 df.to_csv('blabla.csv')
12 df.to_excel('blabla.xlsx')
```

Indexing

```
1 df['nome-coluna']
2
3 df.loc['nome-linha']
4 df.ix[5] # nro linha
5 df.ix[:, 5] # linha, coluna
```

Save Session

There are two ways to save your session/variables to a file:

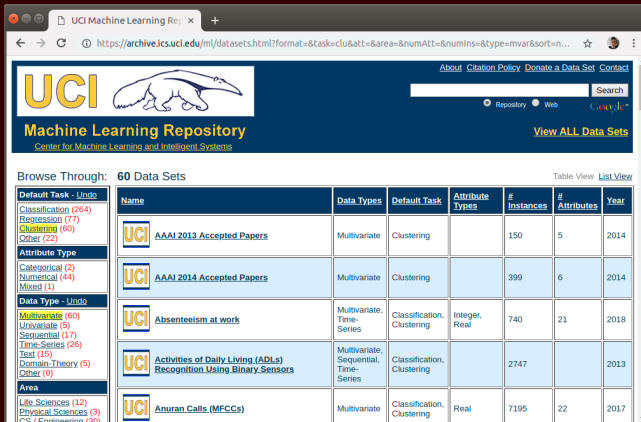
- `np.save`: numpy binary format
- `pickle`: binary file (python version-specific)
- `json`: text file (universal)

Pickle is the native Python approach:






```
1 import pickle
2 pickle.dump(d, open('save.pickle', 'wb'))
3 # and then you can just load it again:
4 d = pickle.load(open('save.pickle', 'rb'))
```

Project: Use a real dataset

We are going to use the Iris dataset from “UCI Machine Learning”



The screenshot shows the UCI Machine Learning Repository website. The header includes the UCI logo, a search bar, and navigation links. The main content area displays a list of datasets under the heading "Browse Through: 60 Data Sets". On the left, there are filters for Default Task, Attribute Type, Data Type, and Area. The dataset list table has columns for Name, Data Types, Default Task, Attribute Types, # Instances, # Attributes, and Year.

Name	Data Types	Default Task	Attribute Types	# Instances	# Attributes	Year
 AAAI 2013 Accepted Papers	Multivariate	Clustering		150	5	2014
 AAAI 2014 Accepted Papers	Multivariate	Clustering		399	6	2014
 Absenteeism at work	Multivariate, Time-Series	Classification, Clustering	Integer, Real	740	21	2018
 Activities of Daily Living (ADLs) Recognition Using Binary Sensors	Multivariate, Sequential, Time-Series	Classification, Clustering		2747		2013
 Anuran Calls (MFCCs)	Multivariate	Classification, Clustering	Real	7195	22	2017

It includes 150 samples of flowers of three classes (iris-setosa, iris-versicolor, iris-virginica) and includes measurements of sepal width and length and petal width and length.

Project: Use a real dataset

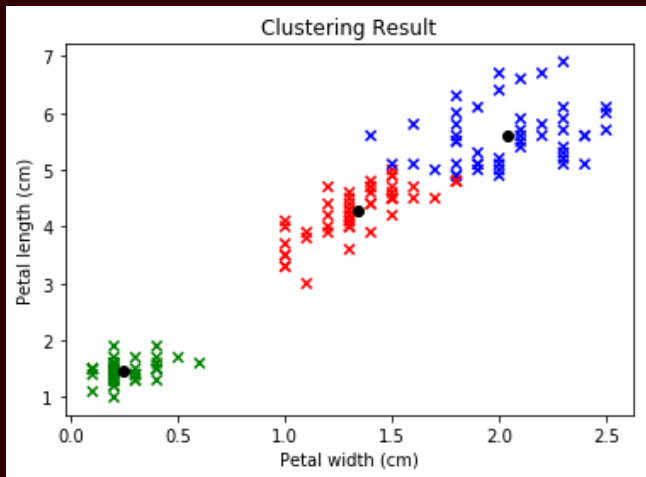
Tasks:

1. **Open 'iris.txt'** using pandas `read_csv()` function;
2. **Check the dataset** using Spyder's Variable Explorer;
3. Make a **numpy array with just two features** of the dataset;
4. Use k-means to **cluster the dataset** (use $k = 3$);
5. Use matplotlib to **plot the results**.

Project: Use a real dataset (solution)

```
1# We can also apply our k-means algorithm to a real dataset.
2# To do that, we can use pandas to import the dataset:
3data = pd.read_csv('iris.txt')
4
5# To use k-means, let's select just two columns of the dataset:
6X = np.array(data[['petal width', 'petal length']])
7
8# And perform k-Means clustering with k = 3 (no. of classes)
9membership, centroids, _ = k_means(X, k=3)
10
11# And plot the results:
12colors = ['red', 'green', 'blue']
13pl.scatter(X[:, 0], X[:, 1], marker='x', color=[colors[int(_y)]
14          for _y in membership])
14pl.plot(centroids[:,0], centroids[:,1], 'o', color='black')
15pl.title('Clustering Result')
16pl.xlabel('Petal width (cm)')
17pl.ylabel('Petal length (cm)')
18pl.show()
```

Project: Use a real dataset (solution)



Project: Use an image

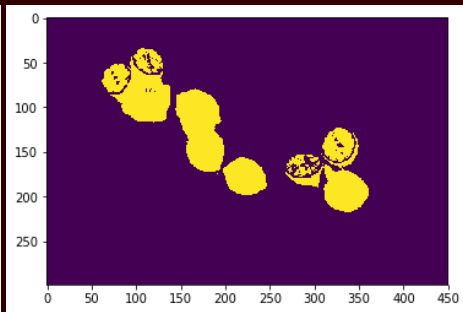
Tasks:

1. **Open the image** 'lemons.jpg' using scikit-image `io.imread()` function;
2. **Visualise the image** using matplotlib `imshow()` function;
3. Change the shape from $(w, h, c) \rightarrow (w \times h, c)$ with numpy's `reshape()` function;
4. **Use k-means on the reshaped image;**
5. Reshape the k-means result back: $(w \times h, c) \rightarrow (w, h, c)$;
6. **Visualise the result** using the `imshow()` function;
7. **Save the result** using scikit-image: `io.imsave(filename, image)`.

Project: Use an image (solution)

```
1# First, we use scikit-image to import the image:
2image = io.imread('lemons.jpg')
3
4# Visualise the image using matplotlib:
5pl.imshow(image)
6pl.show()
7
8# Each pixel should be considered a single sample (in this case,
  with three
9# features — RGB). We need to restructure the image accordingly:
10image_resaped=image.reshape((image.shape[0]*image.shape[1], 3))
11
12# Now, we finally apply k-means to it:
13membership, _ , _ = k_means(image_resaped, k=2)
14
15# Reshaping and visualising the result:
16result = membership.reshape((image.shape[0], image.shape[1]))
17pl.imshow(result)
18pl.show()
19
20# Save the result as an image:
21io.imsave('result.jpg', result)
```

Project: Use an image (solution)



Classes & Modules

Classes

```
1 class Animal:
2     def talk(self):
3         raise NotImplementedError('Implement me')
4
5 class Cat(Animal):
6     def talk(self):
7         print("Miau")
8
9 class Human(Animal):
10    def talk(self):
11        print("Bla bla")
12
13 Cat().talk()  # what is the output?
```

Two notes:

- Python is very dynamic: there are no formal contracts like abstract methods
- in Python, the reference to the object itself is passed explicitly

Modules

Different ways to access a module:

- ```
1 import fib
 2 fib.whatever()
```

- ```
1 from fib import whatever
  2 whatever()
```

- ```
1 from fib import *
 2 whatever()
```

(import everything into your namespace — usually not recommended)

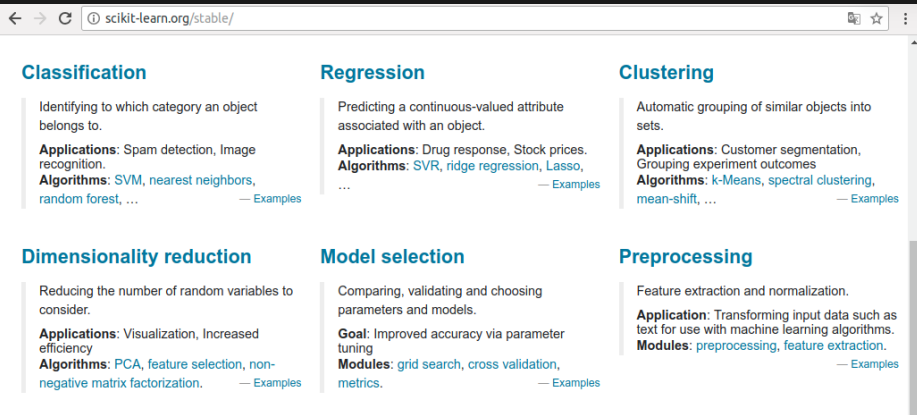
Inside each directory, you ~~must~~ may have a `__init__.py` file where you can put initialization code.

# Scikit-learn



# Scikit-learn

Scikit-learn has literally **everything!**



The screenshot shows the Scikit-learn website with a navigation bar at the top containing a back arrow, a forward arrow, a refresh icon, and the URL "scikit-learn.org/stable/". The main content area is divided into six columns, each with a title and a list of topics. The topics are: Classification, Regression, Clustering, Dimensionality reduction, Model selection, and Preprocessing. Each column also includes a brief description, applications, and algorithms.

← → ↻ ⓘ scikit-learn.org/stable/ ⓘ ☆ ⋮

## Classification

Identifying to which category an object belongs to.

**Applications:** Spam detection, Image recognition.

**Algorithms:** SVM, nearest neighbors, random forest, ... — Examples

## Regression

Predicting a continuous-valued attribute associated with an object.

**Applications:** Drug response, Stock prices.

**Algorithms:** SVR, ridge regression, Lasso, ... — Examples

## Clustering

Automatic grouping of similar objects into sets.

**Applications:** Customer segmentation, Grouping experiment outcomes

**Algorithms:** k-Means, spectral clustering, mean-shift, ... — Examples

## Dimensionality reduction

Reducing the number of random variables to consider.

**Applications:** Visualization, Increased efficiency

**Algorithms:** PCA, feature selection, non-negative matrix factorization. — Examples

## Model selection

Comparing, validating and choosing parameters and models.

**Goal:** Improved accuracy via parameter tuning

**Modules:** grid search, cross validation, metrics. — Examples

## Preprocessing

Feature extraction and normalization.

**Application:** Transforming input data such as text for use with machine learning algorithms.

**Modules:** preprocessing, feature extraction. — Examples



# Scikit-learn interface

|                     |                                                                                                                                                                                                                             |
|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Estimator:</b>   | <p>Implements the fit method to learn from data.<br/>For Supervised Learning:</p> <pre>1 estimator = estimatorObj.fit(data, labels)</pre> <p>and Unsupervised Learning:</p> <pre>1 estimator = estimatorObj.fit(data)</pre> |
| <b>Predictor:</b>   | <pre>1 labels = predictorObj.predict(data)</pre> <p>May implement <code>predict_proba</code> to return the degree of certainty.</p>                                                                                         |
| <b>Transformer:</b> | <p>Filters or modifies the data:</p> <pre>1 new_data = transformerObj.transform(data)</pre>                                                                                                                                 |
| <b>Model:</b>       | <p>Measures goodness of fit:</p> <pre>1 score = modelObj.score(data, labels)</pre>                                                                                                                                          |

# Project: Apply K-means with sklearn

## Tasks:

1. Find the sklearn documentation for the K-means algorithms;
2. Go to the code you have previously written for the Iris dataset and substitute your k-means function by the KMeans object of sklearn. You should:
  - 2.1 Create the object.
  - 2.2 Estimate the parameters with the data (`fit(.)`).
  - 2.3 For each sample predict the assigned cluster (`predict(.)`).
3. Plot and check you have obtained an equivalent result.

# Project: Apply K-means with sklearn

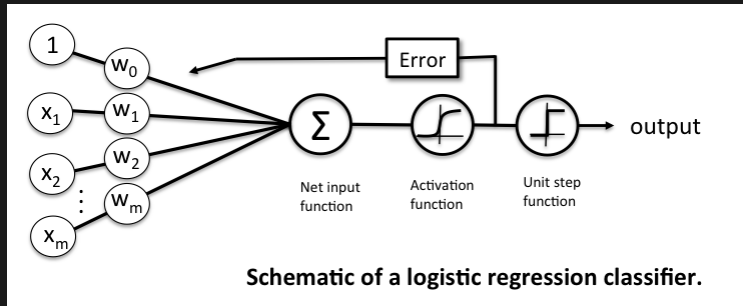
## Tasks:

1. Find the sklearn documentation for the K-means algorithms;
2. Go to the code you have previously written for the Iris dataset and substitute your k-means function by the KMeans object of sklearn. You should:
  - 2.1 Create the KMeans object.
  - 2.2 Estimate the parameters with the data (fit(.)).
  - 2.3 For each sample predict the assigned cluster (predict(.)).
3. Plot and check you have obtained an equivalent result.

## Solution:

```
1 from sklearn.cluster import KMeans as kmeans
2 model = kmeans(n_clusters=3)
3 model.fit(X)
4 membership = model.predict(X)
5 centroids = model.cluster_centers_
```

# Classification with Logistic Regression



# Project: Apply Logistic Regression for Classification

We now focus on the problem of categorizing data.

1. Find the sklearn documentation for the Logistic Regression algorithm;
2. Split the data into training and testing.
3. Create the LogisticRegression object and estimate the parameters with the training data.
4. Obtain the accuracy of the model in the training and testing data (score(.)).

Further exercises:

5. Run the same code again. Did you obtain the same result?
6. Try the same exercise with another classifier.

Useful functions: `train_test_split`, `LogisticRegression`, `fit`, `score`

# Project: Apply Logistic Regression for Classification (solution)

Solution:

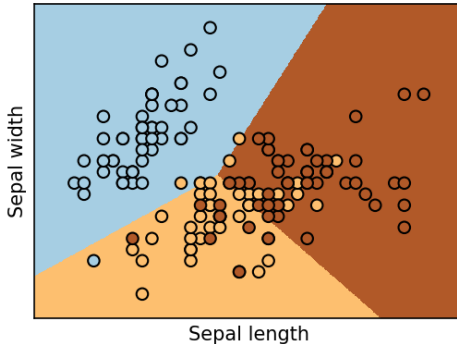
```
1 from sklearn.linear_model import LogisticRegression
2 X_train, X_test, y_train, y_test = train_test_split(X, y)
3 logreg = LogisticRegression()
4 logreg.fit(X_train, y_train)
5
6 print("Train set: ", logreg.score(X_train, y_train))
7 print("Test set: ", logreg.score(X_test, y_test))
```

Note: If you want to obtain the same exact results you need to remove the randomness in the process. Some common sources of variation are the train and test split and the initial parameters of the logistic regression.

Note: Because different classifiers have the same interface, we can simply substitute the 3rd line by another model. In this way we can quickly estimate the performance of multiple models for any machine learning problem we want to solve.

# Plotting the Feature Space

Using the provided function (`plot_fs()`) plot the feature space. This function might be useful for other projects!



# Finishing Touches

## Pointers





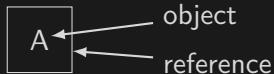
# References

In programming languages when calling a function, there are two possible behaviors:

- pass by value (or copy)
- pass by reference

Python uses what some people like to call 'pass-by-object-reference'. Everything is an object and the reference to the object is copied (not the object itself).

```
1 def fn(b):
2 print(id(b))
3 print(a is b)
4 a = []
5 print(id(a))
6 fn(a)
```



Output:

```
1 139663236727240
2 139663236727240
3 True
```

# References

```
1 a = [1,2,3]
2 b = a
3 b[1] = 7
4 print(a)
```

What is the value of a?

```
1 a = [1,2,3]
2 b = a
3 b = [1,7,3]
4 print(a)
```

What is the value of a?

```
1 a = 2
2 b += 5
3 print(a)
```

What is the value of a?



Wilson Silva [wilson@fe.up.pt](mailto:wilson@fe.up.pt)