Functional Analysis HW7

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Problem 31.

Since $\left|f(\hat{k})\right| = \left|\int_{\mathbb{R}^n} e^{-2\pi ikx} f(k) dk\right| \leq \int_{\mathbb{R}^n} |f(x)|\, dx = \|f\|_1$,

$$\|f\|_{H^s} = \left(\int_{\mathbb{R}^n} (1 + |2\pi k|^2)^s \left|\hat{f}(k)\right|^2 dk\right)^{\frac{1}{2}}$$

$$\leq \|f\|_1 \left(\int_{\mathbb{R}^n} (1 + |2\pi k|^2)^s dk\right)^{\frac{1}{2}}$$

Since, $\left(\int_{\mathbb{R}^n} (1 + |2\pi k|^2)^s dk\right)^{\frac{1}{2}}$ is finite if $s < -\frac{n}{2}$, $L^1(\mathbb{R}^n) \subset H^s(\mathbb{R}^n)$ if $s < -\frac{n}{2}$.

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