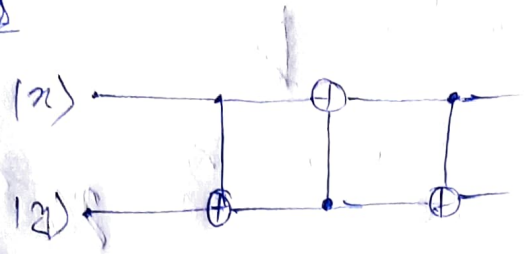
Assignment ①

Is it possible to

Question ① Implement Swap Gate using single qubit gates only.

Answer



Hence the matrix of this swap gate looks like = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M$

If possible let ∃ 1-input - 1-output qubit gates A & B ~~such that~~, with matrices ~~such~~

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ & $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ resp.

So $A \otimes B = M$.

$\Rightarrow$ $\begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix} = M$.

$\Rightarrow$ $a_{11} b_{12} = 0 \Rightarrow a_{11} = 0$ or $b_{12} = 0$ [∵ C is field]

If $a_{11} = 0 \Rightarrow a_{11} b_{11} = 0$ [∵ C is a field] — But $a_{11} b_{11} = 1$

If $b_{12} = 0 \Rightarrow a_{21} b_{12} = 0$ [∵ C is a field] — But $a_{21} b_{12} = 1$

— Contradiction.

So, it's not possible to implement swap gate via single input qubit gates.

AND- Gate Using Toffoli-Gate :-

Truth table.

x	y	z			z1 = z ⊕
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

So, if we put $Z=0$ then we can get easily the AND Gate.

So, the Basic concept of $x \oplus 0 = x$

Hence, AND Gate $\oplus$ 0 = AND Gate

If we $\oplus$ 1 instead of 0 then it will be NAND Gate.

In a

Very Similar manner we can also implement other gates.

X-Gate $\oplus$ 0 = X-Gate.

X-Gate $\oplus$ 1 = NOT-X-Gate.