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Proof of "NO Cloning Theorem"

Statement: It is impossible to clone perfectly given any unknown quantum state, using some unitary operation.

Proof: Suppose, we start with a quantum Machine with two slots: i) A : data slot; starts a unknown but pure quantum state $|\psi_1\rangle$.

ii) B : target slot; starts a standard pure slot $|\psi_2\rangle$

∴ The initial ~~slot~~ state of the machine is $= |\psi_1\rangle|\psi_2\rangle$.

Suppose, U is an unitary operation st. $U(|\psi_1\rangle|\psi_2\rangle) = |\psi_1\rangle|\psi_1\rangle$

Suppose ∃ two pure states $|\psi\rangle$ & $|\phi\rangle$ st

$U(|\psi\rangle|\psi_2\rangle) = |\psi\rangle|\psi\rangle$ & $U(|\phi\rangle|\psi_2\rangle) = |\phi\rangle|\phi\rangle$

Let us consider the inner product of $|\psi\rangle$ & $|\phi\rangle$.

$\langle\psi|\phi\rangle = \langle\psi|\phi\rangle\langle\psi_2|\psi_2\rangle$. [∵ $\langle\psi_2|\psi_2\rangle = 1$].

$= \langle\psi_2|\langle\psi||\phi\rangle|\psi_2\rangle. = \langle\psi_2|\langle\psi|U^*U|\phi\rangle|\psi_2\rangle.$ [U is unitary]

$= (\langle\psi_2|\langle\psi|U^*)(U(|\phi\rangle|\psi_2\rangle)).$

$= (\langle\psi|\langle\psi|)(|\phi\rangle|\phi\rangle) = \langle\psi|\phi\rangle^2.$

⇒ $\langle\psi|\phi\rangle = 0$ or 1. if $\langle\psi|\phi\rangle = 0$ ~~then~~, i.e, ψ & ϕ is independent or $\langle\psi|\phi\rangle = 1$ or $\psi = \phi$.

Hence, a cloning can be possible only for orthogonal states.

∴ In general; cloning is not possible. (P)