Deformations of Hopf-Ore Smash Products

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Hopf-Ore Smash Products

Definition (Ore Extensions)

For a k-alg R, an **Ore extension** $R[x;\sigma,\delta]$ has product from $k[x]$ with relation $xr = \sigma(r)x + \delta(r)\ \forall r \in R$
where $\sigma \in \mathrm{Aut}R$ and δ a σ-derivation.

Examples: Skew poly rings, Weyl alg, UEA of solvable Lie alg/$\mathbb{C}$

Definition (Hopf-Ore Extension)

If H is a Hopf alg, $H[x;\sigma,\delta]$ is a **Hopf-Ore Extension** if it is both an Ore extension and a Hopf alg with H a sub-Hopf alg, and $\triangle(x) = x \otimes * + * \otimes x$.

Definition (Poincare-Birkhoff-Witt Deformation)

A filtered alg $\mathcal{H}$ is a **PBW deformation** of its homogeneous version if it has the PBW property.

Let H be a Hopf alg, $H[x;\sigma,\delta]$ a Hopf-Ore ext, R a Koszul alg.

Question

Under what conditions does a PBW deformation of $R\#H$ extend to a PBW deformation of $R \# H[x;\sigma,\delta]$?

Techniques:

- Conditions for PBW deformations à la Shepler & Witherspoon
- Hochschild cohomology tools for twisted tensor products

Example

Consider the Koszul alg $R = \mathbb{C}[y]$ and the Hopf alg $H = \mathbb{C}$.
A Hopf-Ore ext $H[x] = \mathbb{C}[x]$ acts on R by $x \cdot y = 0$.
Which PBW deformations of $R\#H[x]$ arise
from PBW deformations of $R\#H$?

Sweedler algebra as Hopf-Ore Extension smash with skew poly ring