Deformations of Hopf-Ore Smash Products

Brandon Mather

TORA, April 2025

Hopf-Ore Smash Products

Definition (Ore Extensions)

For a k-alg R, an **Ore extension** $R[x; \sigma, \delta]$ has product from $k[x]$
with relation $xr = \sigma(r)x + \delta(r) \; \forall r \in R$
where $\sigma \in \mathrm{Aut}R$ and δ a σ-derivation.

Examples: Skew poly rings, Weyl alg, UEA of solvable Lie alg$/\mathbb{C}$

Definition (Hopf-Ore Extension)

If H is a Hopf alg, $H[x; \sigma, \delta]$ is a **Hopf-Ore Extension** if it is both
an Ore extension and a Hopf alg with H a sub-Hopf alg, and
$\triangle(x) = x \otimes * + * \otimes x$.

Definition (Poincare-Birkhoff-Witt Deformation)

A filtered alg $\mathcal{H}$ is a **PBW deformation** of its homogeneous
version if it has the PBW property.

Let H be a Hopf alg, $H[x; \sigma, \delta]$ a Hopf-Ore ext, R a Koszul alg.

Question

Under what conditions does a PBW deformation of $R \# H$ extend to a PBW deformation of $R \# H[x; \sigma, \delta]$?

Techniques:

- Conditions for PBW deformations à la Shepler & Witherspoon
- Hochschild cohomology tools for twisted tensor products

Example

The 4-dimn'l Sweedler alg, H_4, is a Hopf-Ore ext of the grp alg $\mathbb{C}[\mathbb{Z}/2\mathbb{Z}]$ and acts on the Koszul alg $\mathbb{C}[u, v]$. A natural question is what PBW deformations of $\mathbb{C}[u, v] \# H_4$ arise from PBW deformations of $\mathbb{C}[u, v] \# \mathbb{C}[Z/2\mathbb{Z}]$.