Deformations of Hopf-Ore Smash Products

Brandon Mather

TORA, April 2025

Hopf-Ore Smash Products

Definition (Ore Extensions)

For a $k-$alg R, **Ore extension** $R[x; \sigma, \delta]$ has product from $k[x]$ with relation $xr = \sigma(r)x + \delta(r)$ where $\sigma \in \text{Aut}R$ and δ a $\sigma-$derivation.

Definition

If H is a Hopf algebra, $H[x; \sigma, \delta]$ is a **Hopf-Ore Extension** if it is both an Ore extension and a Hopf Alg with R a sub-Hopf algebra, and $\triangle(x) = x \otimes 1 + g \otimes x$ some $g \in G(R)$.

Definition

A filtered alg $\mathcal{H}$ is a **PBW deformation** of its homogeneous version if it has the PBW property.

Let H be a Hopf alg, R a Koszul alg

Question

Given a PBW deformation of $R\#H$, under what conditions do we also get a PBW deformation of $R\#H[x;\sigma,\delta]$?

Techniques:

- Explicit PBW conditions for $R\#H$ given by Shepler & Witherspoon
- Hochschild cohomology techniques specific to twisted tensor products

Example

Consider the Koszul alg $R = \mathbb{C}[x]$ and the Hopf alg $H = \mathbb{C}[y]$, a Hopf-Ore extension of $\mathbb{C}$, H acts on R by $y \cdot x = 0$. $R\#H$ is iso to $\mathbb{C}[x][y; \mathrm{id}, \mathrm{id}]$, which is not PBW, what could the PBW deformations be?