Hopf Algebra Actions on Quantum Planes

Brandon Mather

University of North Texas

GAS November 2023

History

- (1939) Heinz Hopf works on homology of a compact Lie group leading to Hopf algs.
- (1969) Moss Sweedler writes seminal book "Hopf Algebras".
- (1986) Vladimir Drinfeld gives an address at ICM popularizing quantum groups.
- (1992) Susan Montogomery writes seminal book "Hopf Algebras and Their Actions on Rings".
- (Modern Day) Researchers work towards classification problems and actions of Hopf algs.

Goal

Understand actions of Hopf algs on noncommutative algs.

Quantum Polynomial Ring

Definition

Let $Q = (q_{ij})$ be an $n \times n$ matrix,
each entry q_{ij} a root of unity and $q_{ii} = q_{ij}q_{ji} = 1$.
A **quantum polynomial ring** is a ring

$$\mathbb{C}_Q[x_1, \ldots, x_n] = \mathbb{C}[x_1, \ldots, x_n]/\langle x_j x_i - q_{ij} x_i x_j \rangle.$$

For example, $\mathbb{C}_{-1}[u, v] = \mathbb{C}[u, v]/\langle uv + vu \rangle$

Sweedler's Hopf Algebra

The unique 4-dim'l non-commutative, non-cocommutative Hopf alg given by Moss Sweedler in "Hopf Algebras" (1969):

$H_4 = \left\langle g, x \mid g^2 = 1, x^2 = 0, xg = -gx \right\rangle$ with operations

Sweedler's Hopf Algebra

The unique 4-dim'l non-commutative, non-cocommutative Hopf alg given by Moss Sweedler in "Hopf Algebras" (1969):

$H_4 = \left\langle g, x \mid g^2 = 1, x^2 = 0, xg = -gx \right\rangle$ with operations

$$\triangle(g) = g \otimes g, \ \triangle(x) = x \otimes 1 + g \otimes x,$$

Sweedler's Hopf Algebra

The unique 4-dim'l non-commutative, non-cocommutative Hopf alg given by Moss Sweedler in "Hopf Algebras" (1969):

$H_4 = \left\langle g, x \mid g^2 = 1, x^2 = 0, xg = -gx \right\rangle$ with operations

$$\triangle(g) = g \otimes g,\ \triangle(x) = x \otimes 1 + g \otimes x,$$

$$\varepsilon(g) = 1,\ \varepsilon(x) = 0,$$

Sweedler's Hopf Algebra

The unique 4-dim'l non-commutative, non-cocommutative Hopf alg given by Moss Sweedler in "Hopf Algebras" (1969):

$H_4 = \left\langle g, x \mid g^2 = 1, x^2 = 0, xg = -gx \right\rangle$ with operations

$$\triangle(g) = g \otimes g,\ \triangle(x) = x \otimes 1 + g \otimes x,$$

$$\varepsilon(g) = 1,\ \varepsilon(x) = 0,$$

$$S(g) = g^{-1},\ S(x) = -x.$$

Actions of Sweedler's Algebra

H_4 acts on the quantum plane $\mathbb{C}_{-1}[u, v]$ by

$$g \cdot u = u, \quad g \cdot v = -v, \quad x \cdot u = 0, \quad x \cdot v = u$$

giving the representation

$$g \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad x \mapsto \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Kac-Paljutkin Algebra

The unique 8-dim'l non-commutative, non-cocommutative Hopf alg given by G. Kac and V. Paljutkin in "Finite Ring Groups" (1966):

$H_8 =$

$$\left\langle x, y, z \mid x^2 = y^2 = 1, xy = yx, zx = yz, zy = xz, z^2 = \frac{1}{2}(1+x+y-xy) \right\rangle$$

with operations

Kac-Paljutkin Algebra

The unique 8-dim'l non-commutative, non-cocommutative Hopf alg given by G. Kac and V. Paljutkin in "Finite Ring Groups" (1966):

$H_8 =$

$$\left\langle x, y, z \mid x^2 = y^2 = 1, xy = yx, zx = yz, zy = xz, z^2 = \tfrac{1}{2}(1+x+y-xy) \right\rangle$$

with operations

$$\triangle(x) = x \otimes x,\ \triangle(y) = y \otimes y,$$

$$\triangle(z) = \tfrac{1}{2}(1 \otimes 1 + 1 \otimes x + y \otimes 1 - y \otimes x)(z \otimes z),$$

Kac-Paljutkin Algebra

The unique 8-dim'l non-commutative, non-cocommutative Hopf alg given by G. Kac and V. Paljutkin in "Finite Ring Groups" (1966):

$H_8 =$

$$\left\langle x, y, z \mid x^2 = y^2 = 1, xy = yx, zx = yz, zy = xz, z^2 = \tfrac{1}{2}(1+x+y-xy) \right\rangle$$

with operations

$$\triangle(x) = x \otimes x, \ \triangle(y) = y \otimes y,$$

$$\triangle(z) = \tfrac{1}{2}(1 \otimes 1 + 1 \otimes x + y \otimes 1 - y \otimes x)(z \otimes z),$$

$$\varepsilon(x) = \varepsilon(y) = \varepsilon(z) = 1,$$

Kac-Paljutkin Algebra

The unique 8-dim'l non-commutative, non-cocommutative Hopf alg given by G. Kac and V. Paljutkin in "Finite Ring Groups" (1966):

$H_8 =$

$$\left\langle x, y, z \mid x^2 = y^2 = 1, xy = yx, zx = yz, zy = xz, z^2 = \tfrac{1}{2}(1+x+y-xy) \right\rangle$$

with operations

$$\triangle(x) = x \otimes x, \ \triangle(y) = y \otimes y,$$

$$\triangle(z) = \tfrac{1}{2}(1 \otimes 1 + 1 \otimes x + y \otimes 1 - y \otimes x)(z \otimes z),$$

$$\varepsilon(x) = \varepsilon(y) = \varepsilon(z) = 1,$$

$$S(x) = x, \ S(y) = y, \ S(z) = z.$$

Actions of Kac-Paljutkin Algebra

H_8 acts on $\mathbb{C}_q[x,y]$ where $q^2 = -1$ by

$$x \mapsto \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Actions of Kac-Paljutkin Algebra

H_8 acts on $\mathbb{C}_q[x,y]$ where $q^2 = -1$ by

$$x \mapsto \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

And on $\mathbb{C}_Q[x_1, x_2, x_3, x_4]$ for

$q_{12} = q_{34}^{-1}, \;\; q_{13} = q_{24}^{-1}, \;\; q_{14}^2 = 1, \;\; q_{23}^2 = -1$ by

$$x \mapsto \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Actions of Kac-Paljutkin Algebra

H_8 acts on $\mathbb{C}_q[x, y]$ where $q^2 = -1$ by

$$x \mapsto \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

And on $\mathbb{C}_Q[x_1, x_2, x_3, x_4]$ for

$q_{12} = q_{34}^{-1}, \quad q_{13} = q_{24}^{-1}, \quad q_{14}^2 = 1, \quad q_{23}^2 = -1$ by

$$x \mapsto \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

And on $\mathbb{C}_{-1}[u, v]$ by

$$x \mapsto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Described by Piotr Kulish and Nicolai Reshetikhin in "Quantum linear problem for the sine-Gordon equation and highest weight representations" (1983), leading Vladimir Drinfeld to quantum groups

$$U_q(\mathfrak{sl}_2) =$$

$$\left\langle E, F, K, K^{-1} \mid EF - FE = (q - q^{-1})^{-1}\left(K - K^{-1}\right), KEK^{-1} = q^2 E, \right.$$
$$\left. KFK^{-1} = q^{-2} F, KK^{-1} = K^{-1}K = 1 \right\rangle$$

with operations:

Quantized Universal Enveloping Algebra

Described by Piotr Kulish and Nicolai Reshetikhin in "Quantum linear problem for the sine-Gordon equation and highest weight representations" (1983), leading Vladimir Drinfeld to quantum groups

$$U_q(\mathfrak{sl}_2) =$$

$$\Big\langle E, F, K, K^{-1} \mid EF - FE = (q-q^{-1})^{-1}\left(K - K^{-1}\right), KEK^{-1} = q^2 E,$$
$$KFK^{-1} = q^{-2}F, KK^{-1} = K^{-1}K = 1\Big\rangle$$

with operations:

$$\triangle(E) = E \otimes 1 + K \otimes E,\ \triangle(F) = F \otimes K^{-1} + 1 \otimes F,$$

$$\triangle(K) = K \otimes K,\ \triangle(K^{-1}) = K^{-1} \otimes K^{-1},$$

Quantized Universal Enveloping Algebra

Described by Piotr Kulish and Nicolai Reshetikhin in "Quantum linear problem for the sine-Gordon equation and highest weight representations" (1983), leading Vladimir Drinfeld to quantum groups

$$U_q(\mathfrak{sl}_2) =$$

$$\Big\langle E, F, K, K^{-1} \mid EF - FE = (q - q^{-1})^{-1}\left(K - K^{-1}\right), KEK^{-1} = q^2 E,$$

$$KFK^{-1} = q^{-2}F, KK^{-1} = K^{-1}K = 1\Big\rangle$$

with operations:

$$\triangle(E) = E \otimes 1 + K \otimes E,\ \triangle(F) = F \otimes K^{-1} + 1 \otimes F,$$

$$\triangle(K) = K \otimes K,\ \triangle(K^{-1}) = K^{-1} \otimes K^{-1},$$

$$\varepsilon(E) = \varepsilon(F) = 0,\ \varepsilon(K) = \varepsilon(K^{-1}) = 1,$$

Quantized Universal Enveloping Algebra

Described by Piotr Kulish and Nicolai Reshetikhin in "Quantum linear problem for the sine-Gordon equation and highest weight representations" (1983), leading Vladimir Drinfeld to quantum groups

$$U_q(\mathfrak{sl}_2) =$$

$$\Big\langle E, F, K, K^{-1} \mid EF - FE = (q - q^{-1})^{-1}\left(K - K^{-1}\right), KEK^{-1} = q^2 E,$$

$$KFK^{-1} = q^{-2}F, KK^{-1} = K^{-1}K = 1\Big\rangle$$

with operations:

$$\triangle(E) = E \otimes 1 + K \otimes E,\ \triangle(F) = F \otimes K^{-1} + 1 \otimes F,$$

$$\triangle(K) = K \otimes K,\ \triangle(K^{-1}) = K^{-1} \otimes K^{-1},$$

$$\varepsilon(E) = \varepsilon(F) = 0,\ \varepsilon(K) = \varepsilon(K^{-1}) = 1,$$

$$S(E) = -K^{-1}E,\ S(F) = -FK,\ S(K) = K^{-1},\ S(K^{-1}) = K.$$

$U_q(\mathfrak{sl}_2)$ acts on $\mathbb{C}_q[x, y]$ with $q^2 = -1$ by the representation

Actions of $U_q(\mathfrak{sl}_2)$

$U_q(\mathfrak{sl}_2)$ acts on $\mathbb{C}_q[x, y]$ with $q^2 = -1$ by the representation

$$E \mapsto \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad F \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Actions of $U_q(\mathfrak{sl}_2)$

$U_q(\mathfrak{sl}_2)$ acts on $\mathbb{C}_q[x, y]$ with $q^2 = -1$ by the representation

$$E \mapsto \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad F \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$K \mapsto \begin{bmatrix} q & 0 \\ 0 & q^{-1} \end{bmatrix} \quad K^{-1} \mapsto \begin{bmatrix} q^{-1} & 0 \\ 0 & q \end{bmatrix}$$

Hopf Algebra

A Hopf algebra is a bialgebra H over a field with an antipode $S : H \to H$ where the bialgebra operations are

A Hopf algebra is a bialgebra H over a field with an antipode

$S : H \to H$ where the bialgebra operations are

$\triangledown : H \otimes H \to H$,

so that the following commute:

Associativity:

$$\begin{tikzcd} H \otimes H \otimes H \arrow[r, "\triangledown \otimes id"] \arrow[d, "id \otimes \triangledown"'] & H \otimes H \arrow[d, "\triangledown"] \\ H \otimes H \arrow[r, "\triangledown"'] & H \end{tikzcd}$$

Hopf Algebra

A Hopf algebra is a bialgebra H over a field with an antipode

$S : H \to H$ where the bialgebra operations are

$\bigtriangledown : H \otimes H \to H,$

$\eta : \mathbb{C} \to H,$

so that the following commute:

Associativity:

$$\begin{tikzcd} H \otimes H \otimes H \arrow[r, "\bigtriangledown \otimes id"] \arrow[d, "id \otimes \bigtriangledown"'] & H \otimes H \arrow[d, "\bigtriangledown"] \\ H \otimes H \arrow[r, "\bigtriangledown"'] & H \end{tikzcd}$$

Unit:

$$\begin{tikzcd} & H \otimes H \arrow[dd, "\bigtriangledown"] & \\ \mathbb{C} \otimes H \arrow[ur, "\eta \otimes id"] \arrow[dr, "="'] & & H \otimes \mathbb{C} \arrow[ul, "id \otimes \eta"'] \arrow[dl, "="] \\ & H & \end{tikzcd}$$

Hopf Algebra

A Hopf algebra is a bialgebra H over a field with an antipode

$S : H \to H$ where the bialgebra operations are

$\nabla : H \otimes H \to H,$ $\triangle : H \to H \otimes H,$

$\eta : \mathbb{C} \to H,$

so that the following commute:

Associativity:

$$\begin{tikzcd} H \otimes H \otimes H \arrow[r, "\nabla \otimes id"] \arrow[d, "id \otimes \nabla"'] & H \otimes H \arrow[d, "\nabla"] \\ H \otimes H \arrow[r, "\nabla"'] & H \end{tikzcd}$$

Coassociativity:

$$\begin{tikzcd} H \arrow[r, "\triangle"] \arrow[d, "\triangle"'] & H \otimes H \arrow[d, "\triangle \otimes id"] \\ H \otimes H \arrow[r, "id \otimes \triangle"'] & H \otimes H \otimes H \end{tikzcd}$$

Unit:

$$\begin{tikzcd} & H \otimes H \arrow[d, "\nabla"] & \\ \mathbb{C} \otimes H \arrow[ur, "\eta \otimes id"] \arrow[r, "="'] & H & H \otimes \mathbb{C} \arrow[ul, "id \otimes \eta"'] \arrow[l, "="] \end{tikzcd}$$

Hopf Algebra

A Hopf algebra is a bialgebra H over a field with an antipode $S : H \to H$ where the bialgebra operations are

$\nabla : H \otimes H \to H$, $\quad \triangle : H \to H \otimes H$,

$\eta : \mathbb{C} \to H$, $\quad \varepsilon : H \to \mathbb{C}$

so that the following commute:

Associativity:

$$\begin{tikzcd} H \otimes H \otimes H \arrow[r, "\nabla\otimes id"] \arrow[d, "id\otimes\nabla"'] & H \otimes H \arrow[d, "\nabla"] \\ H \otimes H \arrow[r, "\nabla"'] & H \end{tikzcd}$$

Coassociativity:

$$\begin{tikzcd} H \arrow[r, "\triangle"] \arrow[d, "\triangle"'] & H \otimes H \arrow[d, "\triangle\otimes id"] \\ H \otimes H \arrow[r, "id\otimes\triangle"'] & H \otimes H \otimes H \end{tikzcd}$$

Unit:

$$\begin{tikzcd} & H \otimes H \arrow[dd, "\nabla"] & \\ \mathbb{C} \otimes H \arrow[ur, "\eta\otimes id"] \arrow[dr, "="'] & & H \otimes \mathbb{C} \arrow[ul, "id\otimes\eta"'] \arrow[dl, "="] \\ & H & \end{tikzcd}$$

Counit:

$$\begin{tikzcd} & H \arrow[dl, "="'] \arrow[dr, "="] \arrow[dd, "\triangle"] & \\ \mathbb{C} \otimes H & & H \otimes \mathbb{C} \\ & H \otimes H \arrow[ul, "\varepsilon\otimes id"] \arrow[ur, "id\otimes\varepsilon"'] & \end{tikzcd}$$

Product and Coproduct compatibility:

$$\begin{tikzcd} H \otimes H \arrow[r, "\nabla"] \arrow[d, "\triangle\otimes\triangle"'] & H \arrow[r, "\triangle"] & H \otimes H \\ H \otimes H \otimes H \otimes H \arrow[rr, "id\otimes\tau\otimes id"'] & & H \otimes H \otimes H \otimes H \arrow[u, "\nabla\otimes\nabla"'] \end{tikzcd}$$

Hopf Algebra Diagrams

Product and Coproduct compatibility:

$$\begin{tikzcd} H \otimes H \arrow[r, "\nabla"] \arrow[d, "\triangle\otimes\triangle"'] & H \arrow[r, "\triangle"] & H \otimes H \\ H \otimes H \otimes H \otimes H \arrow[rr, "id\otimes\tau\otimes id"'] & & H \otimes H \otimes H \otimes H \arrow[u, "\nabla\otimes\nabla"'] \end{tikzcd}$$

Unit and Counit compatibility:

$$\begin{tikzcd} \mathbb{C} \arrow[r, "\eta"] \arrow[dr, "\eta\otimes\eta"'] & H \arrow[d, "\triangle"] \\ & H \otimes H \end{tikzcd} \qquad \begin{tikzcd} H \arrow[r, "\varepsilon"] & \mathbb{C} \\ H \otimes H \arrow[u, "\nabla"] \arrow[ur, "\varepsilon\otimes\varepsilon"'] & \end{tikzcd}$$

Product and Coproduct compatibility:

$$\begin{tikzcd}
H \otimes H \arrow[r, "\nabla"] \arrow[d, "\Delta\otimes\Delta"'] & H \arrow[r, "\Delta"] & H \otimes H \\
H \otimes H \otimes H \otimes H \arrow[rr, "id\otimes\tau\otimes id"'] & & H \otimes H \otimes H \otimes H \arrow[u, "\nabla\otimes\nabla"']
\end{tikzcd}$$

Unit and Counit compatibility:

$$\begin{tikzcd}
\mathbb{C} \arrow[r, "\eta"] \arrow[dr, "\eta\otimes\eta"'] & H \arrow[d, "\Delta"] \\
& H \otimes H
\end{tikzcd}
\qquad
\begin{tikzcd}
H \arrow[r, "\varepsilon"] & \mathbb{C} \\
H \otimes H \arrow[u, "\nabla"] \arrow[ur, "\varepsilon\otimes\varepsilon"'] &
\end{tikzcd}$$

Antipode:

$$\begin{tikzcd}
H \otimes H \arrow[rr, "id\otimes S"] & & H \otimes H \arrow[d, "\nabla"] \\
H \arrow[u, "\Delta"] \arrow[d, "\Delta"'] \arrow[r, "\varepsilon"] & \mathbb{C} \arrow[r, "\eta"] & H \\
H \otimes H \arrow[rr, "S\otimes id"] & & H \otimes H \arrow[u, "\nabla"']
\end{tikzcd}$$

Hopf Algebra Actions

Let H be a Hopf alg and A an alg with a map $\alpha : H \otimes A \to A$. Then we say H acts on A by α if the following the diagrams commute:

Hopf Algebra Actions

Let H be a Hopf alg and A an alg with a map $\alpha : H \otimes A \to A$. Then we say H acts on A by α if the following the diagrams commute:

$$\begin{tikzcd} H \otimes H \otimes A \arrow[r, "\nabla \otimes id"] \arrow[d, "id \otimes \alpha"] & H \otimes A \arrow[d, "\alpha"] \\ H \otimes A \arrow[r, "\alpha"] & A \end{tikzcd} \qquad \begin{tikzcd} \mathbb{C} \otimes A \arrow[r, "\eta \otimes id"] \arrow[dr, "="'] & H \otimes A \arrow[d, "\alpha"] \\ & A \end{tikzcd}$$

Hopf Algebra Actions

Let H be a Hopf alg and A an alg with a map $\alpha : H \otimes A \to A$. Then we say H acts on A by α if the following the diagrams commute:

$$
\begin{tikzcd}
H \otimes H \otimes A \arrow[r, "\nabla \otimes id"] \arrow[d, "id \otimes \alpha"] & H \otimes A \arrow[d, "\alpha"] \\
H \otimes A \arrow[r, "\alpha"] & A
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbb{C} \otimes A \arrow[r, "\eta \otimes id"] \arrow[dr, "="'] & H \otimes A \arrow[d, "\alpha"] \\
& A
\end{tikzcd}
$$

$$
\begin{tikzcd}
H \otimes A \otimes A \arrow[r, "\nabla"] \arrow[d, "\triangle \otimes id \otimes id"'] & H \otimes A \arrow[r, "\alpha"] & A & A \otimes A \arrow[l, "\nabla"'] \\
H \otimes H \otimes A \otimes A \arrow[rrr, "id \otimes \tau \otimes id"] & & & H \otimes A \otimes H \otimes A \arrow[u, "\alpha \otimes \alpha"']
\end{tikzcd}
$$

Hopf Algebra Actions

Let H be a Hopf alg and A an alg with a map $\alpha : H \otimes A \to A$. Then we say H acts on A by α if the following the diagrams commute:

$$\begin{tikzcd}
H \otimes H \otimes A \arrow[r, "\nabla \otimes id"] \arrow[d, "id \otimes \alpha"] & H \otimes A \arrow[d, "\alpha"] \\
H \otimes A \arrow[r, "\alpha"] & A
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbb{C} \otimes A \arrow[r, "\eta \otimes id"] \arrow[dr, "="'] & H \otimes A \arrow[d, "\alpha"] \\
 & A
\end{tikzcd}$$

$$\begin{tikzcd}
H \otimes A \otimes A \arrow[r, "\nabla"] \arrow[d, "\triangle \otimes id \otimes id"'] & H \otimes A \arrow[r, "\alpha"] & A & A \otimes A \arrow[l, "\nabla"'] \\
H \otimes H \otimes A \otimes A \arrow[rrr, "id \otimes \tau \otimes id"] & & & H \otimes A \otimes H \otimes A \arrow[u, "\alpha \otimes \alpha"']
\end{tikzcd}$$

$$\begin{tikzcd}
H \arrow[r, "\varepsilon"] \arrow[d, "\eta"'] & \mathbb{C} \arrow[r, "\eta"] & A \\
H \otimes A \arrow[urr, "\alpha"'] & &
\end{tikzcd}$$

Questions

- Which actions on algs by Hopf algs factor through a group action?

Questions

- Which actions on algs by Hopf algs factor through a group action?
- Which Hopf algs act on AS-regular algs?

Questions

- Which actions on algs by Hopf algs factor through a group action?
- Which Hopf algs act on AS-regular algs?
- When are the invariant subrings from Hopf alg actions Artin-Schelter Gorenstein?

Questions

- Which actions on algs by Hopf algs factor through a group action?
- Which Hopf algs act on AS-regular algs?
- When are the invariant subrings from Hopf alg actions Artin-Schelter Gorenstein?
- If H is semisimple and finite dimensional and A a semiprime alg, is $A\#H$ semiprime?

Questions

- Which actions on algs by Hopf algs factor through a group action?
- Which Hopf algs act on AS-regular algs?
- When are the invariant subrings from Hopf alg actions Artin-Schelter Gorenstein?
- If H is semisimple and finite dimensional and A a semiprime alg, is $A\#H$ semiprime?
- If B is a Koszul alg, are there nontrivial PBW deformations of $B\#U_q(\mathfrak{sl}_\mathfrak{n})$?

Thank You!

References I

[1] G. I. Kats. "Finite Ring Groups". In: Dokl. Akad. Nauk SSSR 147 (), pp. 21–24.

[2] Kenneth Chan et al. "Quantum binary polyhedral groups and their actions on quantum planes". en. In: arXiv:1303.7203 (July 2014). arXiv:1303.7203 [math]. URL: http://arxiv.org/abs/1303.7203.

[3] Miriam Cohen and Davida Fishman. "Hopf algebra actions". en. In: Journal of Algebra 100.2 (May 1986), pp. 363–379. ISSN: 00218693. DOI: 10.1016/0021-8693(86)90082-7.

[4] Juan Cuadra, Pavel Etingof, and Chelsea Walton. "Finite dimensional Hopf actions on Weyl algebras". en. In: arXiv:1509.01165 (July 2016). arXiv:1509.01165 [math]. URL: http://arxiv.org/abs/1509.01165.

[5] E. Kirkman, J. Kuzmanovich, and J.J. Zhang. "Gorenstein subrings of invariants under Hopf algebra actions". en. In: Journal of Algebra 322.10 (Nov. 2009), pp. 3640–3669. ISSN: 00218693. DOI: 10.1016/j.jalgebra.2009.08.018.

[6] Susan Montgomery. Hopf algebras and their actions on rings. Vol. 82. Providence, R.I.; 4: Published for the Conference Board of the Mathematical Sciences by the American Mathematical Society, 1993. ISBN: 0-8218-0738-2.

[7] Chelsea Walton and Sarah Witherspoon. "Poincaré–Birkhoff–Witt deformations of smash product algebras from Hopf actions on Koszul algebras". en. In: Algebra & Number Theory 8.7 (Oct. 2014), pp. 1701–1731. ISSN: 1944-7833, 1937-0652. DOI: 10.2140/ant.2014.8.1701.