Deformations of Hopf-Ore Smash Products

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Hopf-Ore Smash Products

Definition (Ore Extensions)

For a k-alg R, an **Ore extension** $R[x;\sigma,\delta]$ is $R[x]$ as a set with non-comm multiplication preserving (filtered) degree.

This forces the relation $x\,r = \sigma(r)\,x + \delta(r)\ \forall r \in R$
for some $\sigma \in \mathrm{Aut}R$ and δ a σ-derivation.
Examples: Skew poly rings, Weyl algs, UEA of solvable Lie algs/$\mathbb{C}$

Definition (Hopf-Ore Extension)

If H is a Hopf alg, $H[x;\sigma,\delta]$ is a **Hopf-Ore Extension** if it is both an Ore extension and a Hopf alg with H a sub-Hopf alg, and $\triangle(x) = x \otimes * + * \otimes x$.

Definition (Poincaré-Birkhoff-Witt Deformation)

A filtered alg $\mathcal{H}$ is a **PBW deformation** of its homogeneous version if it satisfies a PBW property.

Let H be a Hopf alg, $H[x;\sigma,\delta]$ a Hopf-Ore ext, R a Koszul alg.

Question

Under what conditions does a PBW deformation of $R\#H$ extend to a PBW deformation of $R \# H[x;\sigma,\delta]$?

Techniques:

- Work in a tensor alg over a non-comm ring modulo relations
- Conditions for PBW deformations à la Shepler & Witherspoon
- Hochschild cohomology tools for twisted tensor products

Example

The 4-dim Sweedler alg, H_4, is a Hopf-Ore ext of the group alg $k[\mathbb{Z}/2\mathbb{Z}]$ and acts on the Koszul alg $k[u,v]$.
A natural question:
which PBW deformations of $k[u,v]\#H_4$ arise
from PBW deformations of $k[u,v]\#k[\mathbb{Z}/2\mathbb{Z}]$?