Deformations of Hopf-Ore Smash Products

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Hopf-Ore Smash Products

Definition (Ore Extensions)

For a k-alg R, **Ore extension** $R[x; \sigma, \delta]$ has product from $k[x]$ with relation $xr = \sigma(r)x + \delta(r)$ $\forall r \in R$ where $\sigma \in \mathrm{Aut} R$ and δ a $\sigma-$derivation.

Examples: Skew poly rings, Weyl alg, UEA of solvable Lie alg

Definition (Hopf-Ore Extension)

If H is a Hopf algebra, $H[x; \sigma, \delta]$ is a **Hopf-Ore Extension** if it is both an Ore extension and a Hopf Alg with R a sub-Hopf algebra, and $\triangle(x) = x \otimes 1 + g \otimes x$ some $g \in G(R)$.

Definition (Poincare-Birkhoff-Witt Deformation)

A filtered alg $\mathcal{H}$ is a **PBW deformation** of its homogeneous version if it has the PBW property.

Let H be a Hopf alg, $H[x; \sigma, \delta]$ a Hopf-Ore ext, R a Koszul alg.

Question

Under what conditions does a PBW deformation of $R\#H$ extend to a PBW deformation of $R\#H[x; \sigma, \delta]$?

Techniques:

- PBW conditions for $R\#H$ given by Shepler & Witherspoon
- Hochschild cohomology tools for twisted tensor products

Example

Consider the Koszul alg $R = \mathbb{C}[x]$ and the Hopf alg $H = \mathbb{C}$, $H[y] = \mathbb{C}[y]$ a Hopf-Ore extension of H, $H[y]$ acts on R by $y \cdot x = 0$. Which PBW deformations of $R\#H[y] = \mathbb{C}[x]\#\mathbb{C}[y]$ arise from PBW dedormations of $R\#H = \mathbb{C}[x]\#\mathbb{C}$?