

Deformations of Hopf-Ore Smash Products

Brandon Mather
University of North Texas

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Hopf-Ore Smash Products

Definition (Ore Extensions)

For a k -alg R , an **Ore extension** $R[x; \sigma, \delta]$ is $R[x]$ as a set with non-comm multiplication preserving (filtered) degree.

This forces the relation $xr = \sigma(r)x + \delta(r) \forall r \in R$
for some $\sigma \in \text{Aut} R$ and δ a σ -derivation.

Examples: Skew poly rings, Weyl algs, UEA of solvable Lie algs/ \mathbb{C}

Definition (Hopf-Ore Extension)

If H is a Hopf alg, $H[x; \sigma, \delta]$ is a **Hopf-Ore Extension** if it is both an Ore extension and a Hopf alg with H a sub-Hopf alg, and $\Delta(x) = x \otimes * + * \otimes x$.

Definition (Poincaré-Birkhoff-Witt Deformation)

A filtered alg \mathcal{H} is a **PBW deformation** of its homogeneous version if it satisfies a PBW property.

Let H be a Hopf alg, $H[x; \sigma, \delta]$ a Hopf-Ore ext, R a Koszul alg.

Question

Under what conditions does a PBW deformation of $R \# H$ extend to a PBW deformation of $R \# H[x; \sigma, \delta]$?

Techniques:

- Work in a tensor alg over a non-comm ring modulo relations
- Conditions for PBW deformations à la Shepler & Witherspoon
- Hochschild cohomology tools for twisted tensor products

Example

The 4-dim Sweedler alg, H_4 , is a Hopf-Ore ext of the group alg $k[\mathbb{Z}/2\mathbb{Z}]$ and acts on the Koszul alg $k[u, v]$. A natural question: which PBW deformations of $k[u, v] \# H_4$ arise from PBW deformations of $k[u, v] \# k[\mathbb{Z}/2\mathbb{Z}]$?