Hopf Algebra Actions on Quantum Planes

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History

- (1939) Heinz Hopf works on homology of a compact Lie group leading to Hopf algs.
- (1969) Moss Sweedler writes seminal book "Hopf Algebras".
- (1986) Vladimir Drinfeld gives an address at ICM popularizing quantum groups.
- (1992) Susan Montogomery writes seminal book "Hopf Algebras and Their Actions on Rings".
- (Modern Day) Researchers work towards classification problems and actions of Hopf algs.

Goal

Understand actions of Hopf algs on noncommutative algs.

Quantum Polynomial Ring

Definition

Let $Q = (q_{ij})$ be an $n \times n$ matrix,
each entry q_{ij} a root of unity and $q_{ii} = q_{ij}q_{ji} = 1$.
A **quantum polynomial ring** is a ring

$$\mathbb{C}_Q[x_1, \ldots, x_n] = \mathbb{C}[x_1, \ldots, x_n]/\langle x_j x_i - q_{ij} x_i x_j \rangle.$$

For example, $\mathbb{C}_{-1}[u, v] = \mathbb{C}[u, v]/\langle uv + vu \rangle$

Sweedler's Hopf Algebra

The unique 4-dim'l non-commutative, non-cocommutative Hopf alg given by Moss Sweedler in "Hopf Algebras" (1969):

$H_4 = \left\langle g, x \mid g^2 = 1, x^2 = 0, xg = -gx \right\rangle$ with operations

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$$\varepsilon(g) = 1,\ \varepsilon(x) = 0,$$

$$S(g) = g^{-1},\ S(x) = -x.$$

Actions of Sweedler's Algebra

H_4 acts on the quantum plane $\mathbb{C}_{-1}[u, v]$ by

$$g \cdot u = u, \quad g \cdot v = -v, \quad x \cdot u = 0, \quad x \cdot v = v$$

giving the representation

$$g \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad x \mapsto \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Kac-Paljutkin Algebra

The unique 8-dim'l non-commutative, non-cocommutative Hopf alg given by G. Kac and V. Paljutkin in "Finite Ring Groups" (1966):

$H_8 =$

$$\left\langle x, y, z \mid x^2 = y^2 = 1, xy = yx, zx = yz, zy = xz, z^2 = \tfrac{1}{2}(1+x+y-xy) \right\rangle$$

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Actions of Kac-Paljutkin Algebra

H_8 acts on $\mathbb{C}_q[x, y]$ where $q^2 = -1$ by

$$x \mapsto \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

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And on $\mathbb{C}_Q[x_1, x_2, x_3, x_4]$ for

$q_{12} = q_{34}^{-1}, \;\; q_{13} = q_{24}^{-1}, \;\; q_{14}^2 = 1, \;\; q_{23}^2 = -1$ by

$$x \mapsto \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

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And on $\mathbb{C}_{-1}[u,v]$ by

$$x \mapsto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad y \mapsto \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad z \mapsto \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Quantized Universal Enveloping Algebra

Described by Piotr Kulish and Nicolai Reshetikhin in "Quantum linear problem for the sine-Gordon equation and highest weight representations" (1983), leading Vladimir Drinfeld to quantum groups

$$U_q(\mathfrak{sl}_2) =$$

$$\Big\langle E, F, K, K^{-1} \mid EF - FE = (q - q^{-1})^{-1}\left(K - K^{-1}\right), KEK^{-1} = q^2 E, \\ KFK^{-1} = q^{-2} F, KK^{-1} = K^{-1}K = 1 \Big\rangle$$

with operations:

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with operations:

$$\triangle(E) = E \otimes 1 + K \otimes E,\ \triangle(F) = F \otimes K^{-1} + 1 \otimes F,$$
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$$\varepsilon(E) = \varepsilon(F) = 0,\ \varepsilon(K) = \varepsilon(K^{-1}) = 1,$$

$$S(E) = -K^{-1}E,\ S(F) = -FK,\ S(K) = K^{-1},\ S(K^{-1}) = K.$$

Actions of $U_q(\mathfrak{sl}_2)$

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$$E \mapsto \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad F \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$K \mapsto \begin{bmatrix} q & 0 \\ 0 & q^{-1} \end{bmatrix} \quad K^{-1} \mapsto \begin{bmatrix} q^{-1} & 0 \\ 0 & q \end{bmatrix}$$

Hopf Algebra

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$S : H \to H$ where the bialgebra operations are

$\bigtriangledown : H \otimes H \to H,$

so that the following commute:

Associativity:

$$\begin{tikzcd}
H \otimes H \otimes H \arrow[r, "\bigtriangledown \otimes id"] \arrow[d, "id \otimes \bigtriangledown"'] & H \otimes H \arrow[d, "\bigtriangledown"] \\
H \otimes H \arrow[r, "\bigtriangledown"'] & H
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A Hopf algebra is a bialgebra H over a field with an antipode

$S : H \to H$ where the bialgebra operations are

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Unit:

$$\begin{tikzcd} & H \otimes H \arrow[d, "\nabla"] & \\ \mathbb{C} \otimes H \arrow[ur, "\eta \otimes id"] \arrow[dr, "="'] & & H \otimes \mathbb{C} \arrow[ul, "id \otimes \eta"'] \arrow[dl, "="] \\ & H & \end{tikzcd}$$

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Coassociativity:

$$\begin{tikzcd} H \arrow[r, "\triangle"] \arrow[d, "\triangle"'] & H \otimes H \arrow[d, "\triangle \otimes id"] \\ H \otimes H \arrow[r, "id \otimes \triangle"'] & H \otimes H \otimes H \end{tikzcd}$$

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\end{tikzcd}$$

Unit:

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 & H \otimes H \arrow[dd, "\bigtriangledown"] & \\
\mathbb{C} \otimes H \arrow[ur, "\eta \otimes id"] \arrow[dr, "="'] & & H \otimes \mathbb{C} \arrow[ul, "id \otimes \eta"'] \arrow[dl, "="] \\
 & H &
\end{tikzcd}$$

Counit:

$$\begin{tikzcd}
 & H \arrow[dl, "="'] \arrow[dr, "="] \arrow[dd, "\bigtriangleup"] & \\
\mathbb{C} \otimes H & & H \otimes \mathbb{C} \\
 & H \otimes H \arrow[ul, "\varepsilon \otimes id"] \arrow[ur, "id \otimes \varepsilon"'] &
\end{tikzcd}$$

Hopf Algebra Diagrams

Product and Coproduct compatibility:

$$\begin{tikzcd} H \otimes H \arrow[r, "\nabla"] \arrow[d, "\triangle \otimes \triangle"'] & H \arrow[r, "\triangle"] & H \otimes H \\ H \otimes H \otimes H \otimes H \arrow[rr, "id \otimes \tau \otimes id"'] & & H \otimes H \otimes H \otimes H \arrow[u, "\nabla \otimes \nabla"'] \end{tikzcd}$$

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$$\begin{tikzcd} \mathbb{C} \arrow[r, "\eta"] \arrow[dr, "\eta\otimes\eta"'] & H \arrow[d, "\triangle"] \\ & H\otimes H \end{tikzcd}
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Antipode:

$$\begin{tikzcd}
H \otimes H \arrow[rr, "id\otimes S"] & & H \otimes H \arrow[d, "\nabla"] \\
H \arrow[u, "\triangle"] \arrow[r, "\varepsilon"] \arrow[d, "\triangle"'] & \mathbb{C} \arrow[r, "\eta"] & H \\
H \otimes H \arrow[rr, "S\otimes id"] & & H \otimes H \arrow[u, "\nabla"']
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$$\begin{tikzcd}
H \arrow[r, "\varepsilon"] \arrow[d, "\eta"'] & \mathbb{C} \arrow[r, "\eta"] & A \\
H \otimes A \arrow[urr, "\alpha"'] & &
\end{tikzcd}$$

Questions

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- Which actions on algs by Hopf algs factor through a group action?
- Which Hopf algs act on AS-regular algs?
- When are the coinvariant subrings from Hopf alg actions Artin-Schelter Gorenstein?
- If H is semisimple and finite dimensional and A a semiprime alg, is $A\#H$ semiprime?
- If B is a Koszul alg, are there nontrivial PBW deformations of $B\#U_q(\mathfrak{sl}_\mathfrak{n})$?

Thank You!

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