

## Quiz 2

Name:

Suppose we have  $t$  hash functions  $h_1, h_2, \dots, h_t$  mapping from objects to  $b$  buckets  $\{0, \dots, b-1\}$ . For  $j$  in  $[1, t]$ , let  $s_j$  be a hash function from objects to  $\{+1, -1\}$  and let  $c_j$  be an array of counters of size  $b$ . While processing the stream, each time we encounter an item  $oi$ , update the counter  $c_j[h_j[oi]] += s_j[oi]$ .

Then, we estimate the count of  $oi$  with  $\text{median}_{j=1, \dots, t}$  of  $c_j[h_j[oi]] * s_j[oi]$ .

$$\rightarrow c_j[h_j[oi]] * s_j[oi]$$

Let

$$t = 5, b = 5$$

hash function  $h$ :

	<b>o1</b>	<b>o2</b>	<b>o3</b>	<b>o4</b>	<b>o5</b>	<b>o6</b>	<b>o7</b>
<b>h1</b>	3	1	1	4	1	0	2
<b>h2</b>	3	3	0	2	0	4	0
<b>h3</b>	4	1	4	2	4	2	2
<b>h4</b>	3	1	1	0	4	1	4
<b>h5</b>	1	0	4	0	3	3	2

hash functions  $s$ :

	<b>o1</b>	<b>o2</b>	<b>o3</b>	<b>o4</b>	<b>o5</b>	<b>o6</b>	<b>o7</b>
<b>s1</b>	1	1	-1	-1	-1	-1	1
<b>s2</b>	-1	-1	1	-1	-1	1	1
<b>s3</b>	-1	-1	1	1	-1	-1	-1
<b>s4</b>	1	-1	1	-1	-1	1	-1
<b>s5</b>	-1	1	1	-1	-1	1	-1

Data stream (the numbers below are the indexes  $i$  of object ID  $oi$ ):

1	1	2	6	5	2	2	5	2	5	7	1	4	4
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$\rightarrow$  Estimate the count of objects  $\{o1, o2, o3\}$ .

Input : 1

	0	1	2	3	4
c1	0	0	0	1	0
c2	0	0	0	-1	0
c3	0	0	0	0	-1
c4	0	0	0	1	0
c5	0	-1	0	0	0

Input : 1

	0	1	2	3	4
c1	0	0	0	2	0
c2	0	0	0	-2	0
c3	0	0	0	0	-2
c4	0	0	0	2	0
c5	0	-2	0	0	0

Input : 2

	0	1	2	3	4
c1	0	1	0	2	0
c2	0	0	0	-3	0
c3	0	-1	0	0	-2
c4	0	-1	0	2	0
c5	1	-2	0	0	0

Input : 6

	0	1	2	3	4
c1	-1	1	0	2	0
c2	0	0	0	-3	1
c3	0	-1	-1	0	-2
c4	0	0	0	2	0
c5	1	-2	0	1	0

Input : 5

	0	1	2	3	4
c1	-1	0	0	2	0
c2	-1	0	0	-3	1
c3	0	-1	-1	0	-3
c4	0	0	0	2	-1
c5	1	-2	0	0	0

Input : 2

	0	1	2	3	4
c1	-1	1	0	2	0
c2	-1	0	0	-4	1
c3	0	-2	-1	0	-3
c4	0	-1	0	2	-1
c5	2	-2	0	0	0

Input :2

	0	1	2	3	4
c1	-1	2	0	2	0
c2	-1	0	0	-5	1
c3	0	-3	-1	0	-3
c4	0	-2	0	2	-1
c5	3	-2	0	0	0

Input : 5

	0	1	2	3	4
c1	-1	1	0	2	0
c2	-2	0	0	-5	1
c3	0	-3	-1	0	-4
c4	0	-2	0	2	-2
c5	3	-2	0	-1	0

Input : 2

	0	1	2	3	4
c1	-1	2	0	2	0
c2	-2	0	0	-6	1
c3	0	-4	-1	0	-4
c4	0	-3	0	2	-2
c5	4	-2	0	-1	0

Input : 5

	0	1	2	3	4
c1	-1	1	0	2	0
c2	-3	0	0	-6	1
c3	0	-4	-1	0	-5
c4	0	-3	0	2	-3
c5	4	-2	0	-2	0

Input : 7

	0	1	2	3	4
c1	-1	1	1	2	0
c2	-2	0	0	-6	1
c3	0	-4	-2	0	-5
c4	0	-3	0	2	-4
c5	4	-2	-1	-2	0

Input : 1

	0	1	2	3	4
c1	-1	1	1	3	0
c2	-2	0	0	-7	1
c3	0	-4	-2	0	-6
c4	0	-3	0	3	-4
c5	4	-3	-1	-2	0

Input : 4

	0	1	2	3	4
c1	-1	1	1	3	-1
c2	-2	0	-1	-7	1
c3	0	-4	-1	0	-6
c4	-1	-3	0	3	-4
c5	3	-3	-1	-2	0

Input : 4

	0	1	2	3	4
c1	-1	1	1	3	-2
c2	-2	0	-2	-7	1
c3	0	-4	0	0	-6
c4	-2	-3	0	3	-4
c5	2	-3	-1	-2	0

o1 -> median of  $\langle 3, 7, -1, 0, 3 \rangle = 3$  (count = 3)

o2 -> median of  $\langle 1, 7, 4, 3, 2 \rangle = 3$  (count = 4)

o3 -> median of  $\langle -1, -2, -6, -3, 0 \rangle = -3$  (count = 0)