PHYS-183 : Day #5

- Up until now, forces and Newton's laws of motion have been able to describe the motion of any object that we have studied.

- Although we have only worked in one-dimension, Newton's laws work in any number of dimensions you want.

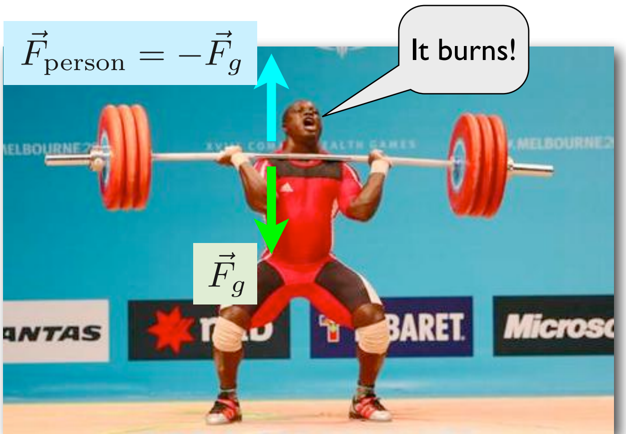
- However, when studying biological objects, Newton's laws do not tell us the entire story

- If we lift a heavy object, then after the object comes to a rest, the net force on the object is zero.

- However, our body will quickly get tired even if the net force is zero.

- Obviously forces are not enough to describe this situation.

- We also need to understand where the forces come from



When the weights are not moving, there is no net force, but the person still gets tired

In this chapter we will see that **energy is the source of forces**.

Work:

- When an object has a constant new force, then the acceleration of the object can be written (via Newton's 2nd law) as $a = F_{\rm net}/m$

- The larger the force, and the longer the force acts on an object, the faster that object will move as a function of time:

$$v(t) = \frac{F_{\rm net}}{m}t + v(0) \longleftarrow \text{initial velocity}$$

- Using $v(t)^2 = v(0)^2 + 2a\left[x(t) - x(0)\right]$ from the last chapter we can also write the velocity as a function of the distance the object travels [assuming v(0)=0]:

$$v(t) = \sqrt{2\frac{F_{\rm net}}{m}\left[x(t) - x(0)\right]}$$

- We see that the net force, acting over a distance $d = x(t) - x(0)$ increases the velocity of an object.

Def: The **work** done on an object by a constant force over a displacement Δx is defined to be

$$\boxed{W_F = F\Delta x}$$ (constant force along direction of x)

Horse pulling a sled with people

Ex. Horse pulling a sled:

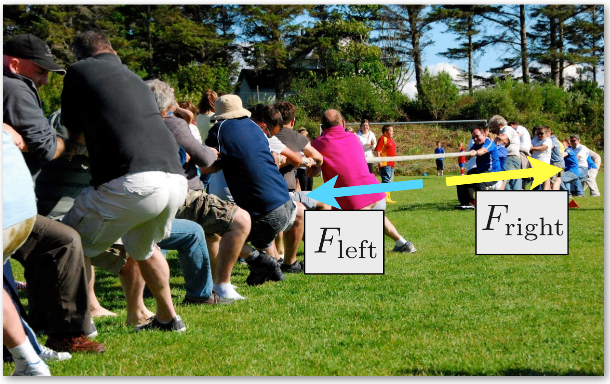
Suppose a horse pulls a sled with a constant force of 100 N over a distance of 5 km. What is the amount of work done by the horse on the sled?

$$W_F = F\Delta x = 100\ \mathrm{N} \times 5000\ \mathrm{m}$$
$$= \boxed{5 \times 10^5\ \mathrm{N-m}}$$

- The units of work are N-m, however work is so important that we call 1 N-m a **Joule** (J).

Ex. Tug-of-war:

Suppose a group of 10 people pull a rope to the left, each with an average force of 220 N, and 10 people pull to the right with an average force of 210 N each. During that time, the rope moves a distance of 3 m to the left, what is the net work done by all 20 people?



Tug-of-war: People try to pull the other side toward them.

Solution:

- Add up the total work done by each of the people on left:

$$W_{\text{left}} = 10 \cdot (220 \text{ N} \times 3 \text{ m}) = 6600 \text{ J}$$

- Add up the total work done by each of the people on right:

$$W_{\text{right}} = 10 \cdot (-210 \text{ N} \times 3 \text{ m}) = -6300 \text{ J}$$

(negative since F is in opposite direction of +x)

- The net work is therefore: $W_{\text{net}} = W_{\text{left}} + W_{\text{right}} = \boxed{300 \text{ J}}$

- We could have also calculated the net force on the rope and multiplied by the distance.

- This definition of work is different than how we describe “work” in everyday life.

- If the people on the right of the rope pulled with 10 N more force each, then the rope would go no where and there would be no work done.

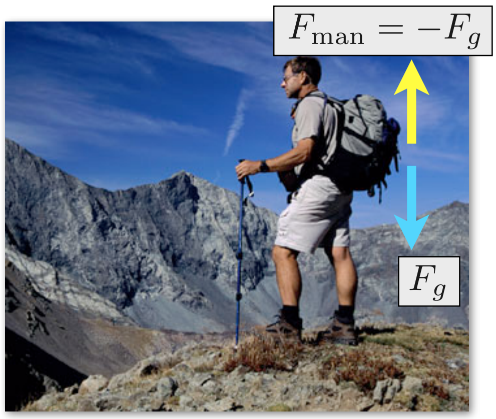
 - But the people would still get tired from pulling and we would say that “it took a lot of work”

- This definition of work is different than how we describe “work” in everyday life.

- Standing still with a heavy backpack on will make you very tired, but you do not do any work to keep the backpack off the ground

- Be very careful when calculating work

- Work in physics requires a non-zero net force applied over some displacement $\Delta x \neq 0$



This man does no work holding up the backpack. The net force is zero and there is no displacement

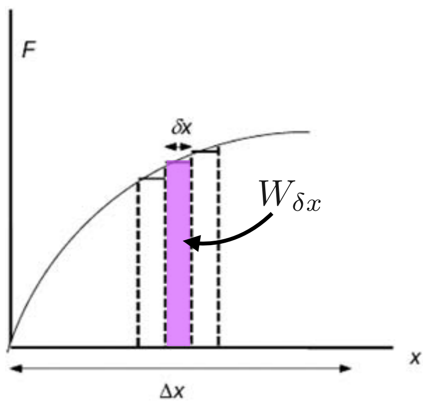
- So far our definition of work only applies to forces that are constant.

- But we have already seen examples of forces that are not constant:

The force of friction for liquids: $F_{\text{laminar}} \propto v$ and $F_{\text{turbulent}} \propto v^2$

The force of due to a spring: $F_{\text{spring}} \propto -x$

- To calculate the work done by a non-constant force we must divide the displacement into small subintervals δx and calculate the average force $\bar{F}_{\delta x}$ in each subinterval



- The work done in each subinterval is therefore:

$$W_{\delta x} = \bar{F}_{\delta x}\delta x$$

- This is the area under the force vs. displacement graph

- To get the net work, we add up each little section of the total displacement

$$W = \sum_{\delta x} W_{\delta x} = \sum_{\delta x} \bar{F}_{\delta x}\delta x$$

- If we take our subintervals to be very very small, we can use the rules of calculus to find the work done

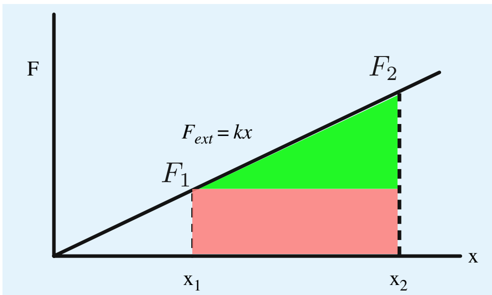
$$W = \int_{x_i}^{x_f} F(x)dx$$

Ex: Calculate the work done by stretching a spring from a position x_1 to x_2 :

(graphical method)

- To stretch the spring, we must use a force that is equal an opposite to the spring force

$$\Rightarrow \quad F = kx$$



$$W = A = \underbrace{kx_1(x_2 - x_2)}_{\text{Rectangle}} + \underbrace{\frac{1}{2}(x_2 - x_1)(kx_2 - kx_1)}_{\text{Triangle}} = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

(calculus)

$$W = \int_{x_1}^{x_2} kx dx = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

since $F_{\text{spring}} = -F$

$$W_{\text{spring}} = -W = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

- The work done can be positive or negative

If the objects work is positive → The object did the work

If the objects work is negative → Something did work on the object

- Since we stretched the spring in the last example, we did work on the spring so our work is positive, but the springs work is negative since we did work on the spring.

Kinetic Energy and the Work-Energy Theorem:

- Previously we used Newton's laws with a constant force to find the velocity of an object as a function of both time and position

- Here we will find the velocity of an object using the new concept of work

- Suppose at position x_1 the object has velocity v_1 and at x_2 velocity v_2. Since we know the distance, velocities, and acceleration $a = F_{\mathrm{net}}/m$ we can express their relationship as

(A) $$v_2^2 = v_1^2 + 2\left(\frac{F_{\mathrm{net}}}{m}\right)(x_2 - x_1)$$

- We can also calculate how much work was done by this constant force from x_1 to x_2 :

(B) $$W_{\mathrm{net}} = F_{\mathrm{net}}(x_2 - x_1)$$

- Lets plug equation (B) into equation (A)

$$v_2^2 = v_1^2 + \frac{2W_{\mathrm{net}}}{m}$$

and then solve for the net work done by the force

$$W_{\mathrm{net}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W_{\text{net}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

- From this equation we can define a new quantity called the translational **kinetic energy:**

$$\boxed{T = \frac{1}{2}mv^2}$$

(also measured in J)

- Here, translational means "moving from one place to another"
- The kinetic energy is the amount of energy that is required to get an object of mass m moving at a velocity v.

- Because the kinetic energy is proportional to the velocity squared, the kinetic energy is always positive.

- How much is 1J of energy? Equivalent to dropping a 1kg mass on your hand from 10cm high

Ex. Kinetic energy of moving car:

What is the kinetic energy of a 500kg car moving at 120 km/hr?

- First need to convert km/hr to m/s:

$$120\ \text{km/hr} \times 1000\ \text{m/km} * \frac{1}{3600}\ \text{hr/sec} = 33.3\ \text{m/s}$$

Finally: $T = \frac{1}{2}(500\ \text{kg})(33.3\ \text{m/s})^2 = \boxed{277222\ \text{J}}$

- Equivalent to dropping 277222 1kg masses on my hand from 10cm high (ouch!!!)

Ex. Kinetic energy of blood cells in your body:

The average adult human body has 3×10^{13} red blood cells, each with a mass of 4.5×10^{-14} kg. If the average velocity of blood is 0.9 m/s, what is the net kinetic energy of all of the red blood cells in your body?

- For an individual blood cell: $T = \frac{1}{2}(4.5 \times 10^{-14}\ \text{kg})(0.9\ \text{m/s})^2 = 1.8 \times 10^{-14}\ \text{J}$

- Therefore, the net kinetic energy is: $T_{\text{net}} = 3 \times 10^{13} \cdot 1.8 \times 10^{-14}\ \text{J} = \boxed{0.55\ \text{J}}$

Physics for Life Scientists (PHYS-183), Paul D. Nation

- We can now define the net work it terms of the change in kinetic energy:

$$W_{\text{net}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = T_2 - T_1 = \Delta T \qquad \text{(Work-Energy Theorem)}$$

➡ The net work done on an object is equal to the change in its kinetic energy

- If the work is positive, then the kinetic energy increases (faster velocity)

- If the work is negative, then the kinetic energy decreases (slower velocity)

- The work done on an object can change its kinetic energy, and kinetic energy can do work on another object

Energy is a measure of an objects ability to do work

Ex. High Jumper:

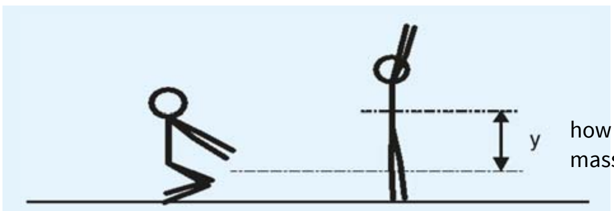
Using the work-energy theorem, estimate how high a person can jump from rest.

Solution:

Once a person leaves the ground, the person is undergoing free-fall with the only force due to gravity. Therefore, the person must maximize their initial velocity $v(0)$ to jump as high as possible

Since the person is at rest, the initial velocity is provided by the force generated in the persons legs

$$W = Fy = \frac{1}{2}mv_0^2$$



how far the center of mass moves

- At the top of the persons jump, the velocity is zero, and we can calculate the height h that the person can reach

$$v(t)^2 = v(0)^2 - 2gh = 0 \quad \Rightarrow \quad h = \frac{v(0)^2}{2g}$$

- Using the work-energy theorem, we can get rid of the initial velocity term

$$\text{displacement during jump up} \longrightarrow h = \frac{Fy}{mg} \longleftarrow \text{displacement of center of mass before leaving ground}$$

- We now need to estimate the force F and how big y can be.

  - At most y= 1/3 of the persons height
  - $F \approx mg$

- Therefore, the height of a persons jump from standstill is $h \approx \dfrac{y}{3}$

- For a 1.8m tall person, with center of mass at 0.9m, this implies that the person can raise their center of mass to 1.5m.

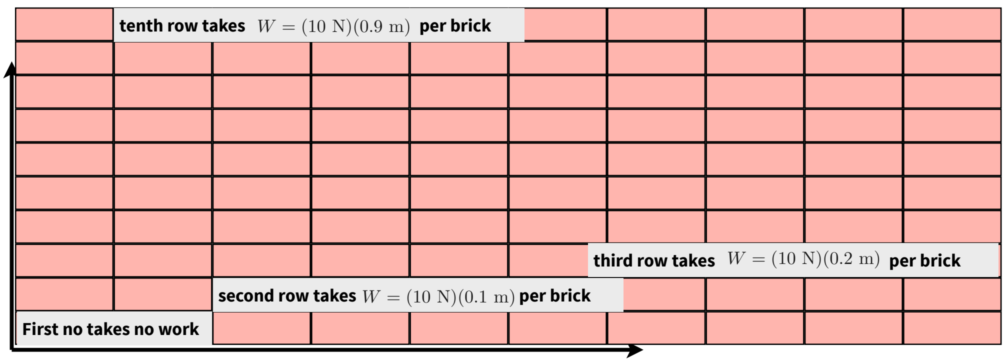
→ The person can only jump 1.5 m high.

- Is there a way to make the person jump higher?

Increase initial velocity!

Ex. In-Class Problem:

Suppose you need to build a brick wall that is 1m high and 3m long, out of bricks that are 10cm high, 30cm wide, and weigh 10N. How much work must you do to build the wall?



Total work for 1st row: $W = 10 * (10\ \mathrm{N})(0.0\ \mathrm{m}) = 0\ \mathrm{N}$

Total work for 2nd row: $W = 10 * (10\ \mathrm{N})(0.1\ \mathrm{m}) = 10\ \mathrm{N}$

Total work for 3rd row: $W = 10 * (10\ \mathrm{N})(0.2\ \mathrm{m}) = 20\ \mathrm{N}$

$\vdots$

Total work for entire wall: $W = 10\ \mathrm{N}(1+2+3+4+5+6+7+8+9) = \boxed{450\ \mathrm{N}}$