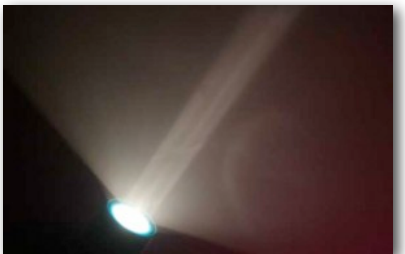
PHYS-183 : Day #29

Geometric Optics:

- Last week we studied the basic properties of EM waves

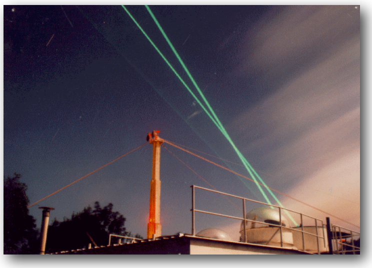
- Visible light, X-rays, microwaves, radio waves,... are all EM waves with the only difference being the frequency and wavelength of the waves.

- In our everyday world, the visible light that we see seems to always travel in a straight line.



Light from a flashlight

Light moving in a straight line results in shadows



Laser light used in pollution monitoring.

- However, other EM waves such as microwaves easily bend around objects, and therefore your cell phone has a signal even without a straight path to the cell phone tower.

- Light moves in a straight line due to its small wavelength: $\lambda \sim 5 \times 10^{-7}\ \mathrm{m}$

- This wavelength is smaller than the dimensions of typical everyday objects.

- For any EM wave, if the wavelength is smaller than any objects the wave interacts with, then the EM waves will move in a straight line.

- The study of light waves moving in straight lines is called **Geometric Optics**.

<u>Optical Properties of Materials:</u>

- Last week we saw that <u>every</u> EM wave in vacuum travels at the so called **speed of light in vacuum**

$$c = 3 \times 10^8 \ \mathrm{m/s}$$

- If light travels through a material, then the speed of light in the material will in general be different from *c*.

- The speed of light in a material *v* is determined by the **index of refraction** *n*:

$$\boxed{v = \frac{c}{n}}$$

Material (20°C Unless Specified)	*Refractive Index*
Diamond	2.42
Glass (crown)	1.52
Benzene	1.50
Quartz (fused)	1.46
Water	1.33
Air (1 atm, 0°C)	1.0003

[a]Measured at a wavelength of 589 nm (yellow sodium light).

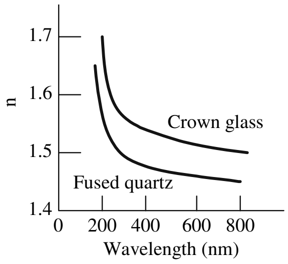
- The index of refraction is always larger than one -> the speed of light in a material is always slower than the speed of light in vacuum.

- The speed of light inside a material also depends on the frequency (or wavelength) of the light.
    - Obviously in vacuum this is not the case.

- This phenomena is called **dispersion**, and it is important when the light is made up of many different frequencies (wavelengths).

- **White light** is the general name for light made up of many different colors (frequencies or wavelengths).

- So far we have been talking about light that moves through a material. What happens if light gets absorbed by the material?



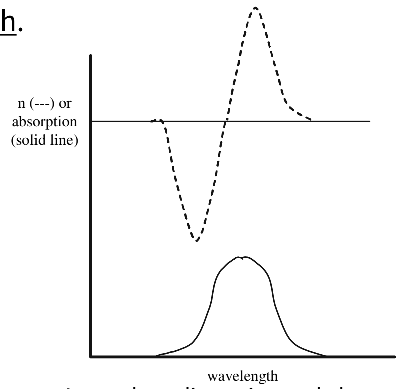
Dispersion of light in quartz and crown glass as function of wavelength.

- Near wavelengths (frequencies) where the EM waves are absorbed, and the material is therefore opaque, the <u>index of refraction increases with wavelength</u>.

- This process is called **anomalous dispersion**, meaning "not normal" dispersion

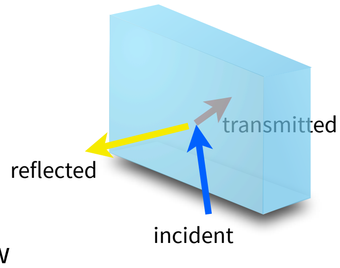
- Anomalous dispersion happens when the frequency of light is on resonance with an absorption frequency of the material.

- Therefore, absorption and dispersion are very closely related.



Anomalous dispersion and absorption

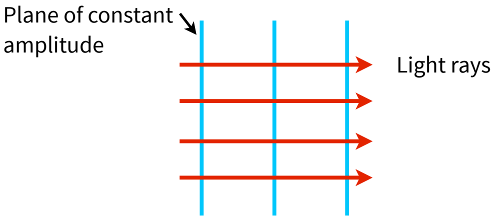
Light at an Interface:



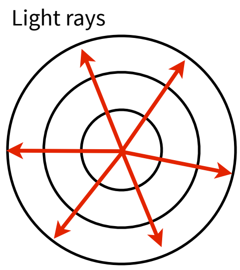
- When light hits the interface between two different materials with no absorption, the wave is partially reflected and partially transmitted into the second material.

- Because light travels in straight line in a uniform material, we can follow the light using **light rays** to follow the lights path.

- There are two main type of light waves:



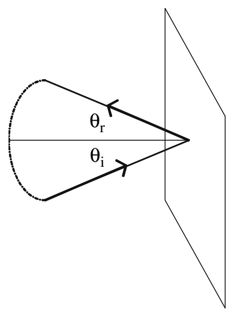
Plane Wave: Planes of constant amplitude are perpendicular to the light rays. Light rays do not spread apart.



Spherical Wave: Light rays come from a single point source and planes of constant amplitude are spherical. Light rays spread apart.

- If the interface between two materials is a smooth plane surface, then a plane wave will undergo **specular reflection**, where all of the reflected rays are also parallel.

- If the incoming ray makes an angle θ_i with a vector normal to the surface, then the reflected ray leaves with an angle θ_r that is equal to the incoming angle.



Law of reflection for a smooth interface.

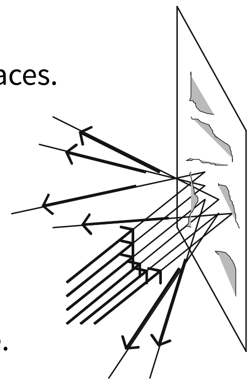
$$\boxed{\theta_i = \theta_r}$$

- This is known as the **law of reflection**.

- The reflected ray is in the same plane as the incident ray and the normal vector of the surface.

- This is what happens to light reflecting off of mirrors, windows, or shiny surfaces.

- If the surface of the object is rough, then the law of reflection breaks down and the light undergoes **diffuse reflection**.



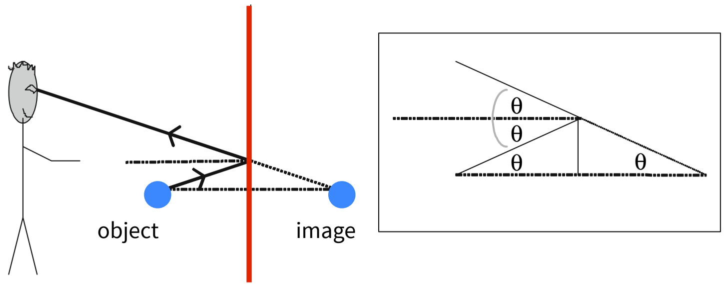
Diffuse reflection

- Only when there is specular reflection does the reflected light form an image.

- Suppose we have a single point source of light in front of a mirror.

- Some reflected light will reflect off the mirror and into your eye.

- This light appears to come from a single point located behind the mirror.



- This image is called a **virtual image** because the light does not actually come from the observed object.

- In contrast, a **real image** is an image generated from light rays that actually pass through the object.

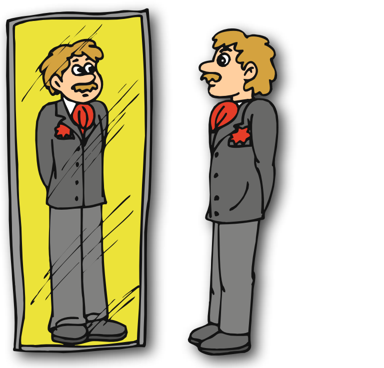
- Using geometry and the law of reflection we can show that the distance from the object to the mirror, called the **object distance**, is equal to the distance from the mirror to the image called the **image distance**.

- The distance the image appears behind the mirror is the same as the distance from the mirror to the object.

- If the source of light is an extended object, such as a light bulb, or light reflecting off a person, then each point on the object can be treated like a point source.

- The entire object will create a virtual image behind the mirror.

- If the surface of the interface is flat and smooth, then the image will be the same size as the object, and as far behind the mirror as the object is in front.



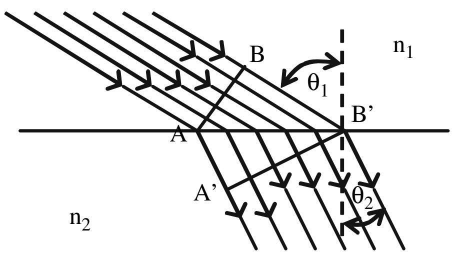
- However, the object will with the left and right sides reversed.

- Now lets consider the part of the light rays that are transmitted into the second material

- We will assume that the second material has a larger index of refraction so light travels slower in the second material

- Suppose the light rays come in at an angle θ_1

- In this situation the light rays in the second material bend toward the normal vector to the surface.



Light ray transmission when n2>n1

- The light rays bend due to **Fermat's Principle**: Light always takes the path that requires the least amount of time.

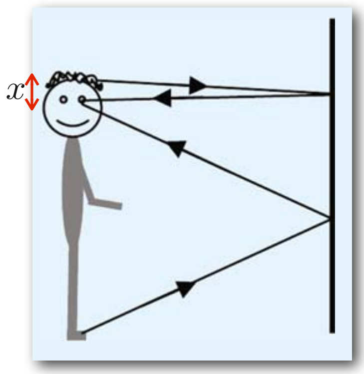
- The path with the shortest time, changes when the index of refraction changes

Light hitting an interface at a non-zero angle will always bend, also called **refracted**.

- The relationship between the incoming angle θ_1 and the refracted angle θ_2 is called **Snell's Law** or the **Law of Refraction**.

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

<u>**Ex.**</u> Find the minimum height of a plane mirror needed for a person with height H to see her entire self in the mirror.



<u>**Solution:**</u>

- In order to see her feet, the woman must have a light ray leave her feet and hit her eye. From the law of reflection, this ray must hit the mirror midway between her feet and her eyes.

$$d_1 = (H - x)/2$$

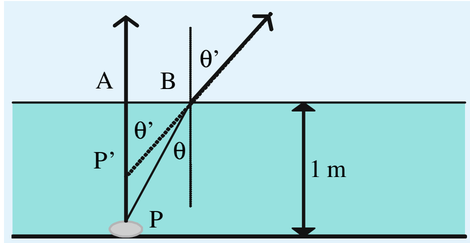
- In order for the woman to see the top of her head a light ray from the top must hit the mirror half way down to her eyes.

$$d_2 = x/2$$

- Therefore, the total height of the mirror should be: $d = d_1 + d_2 = \boxed{H/2}$

- The bottom of the mirror must start at the height d_1

Ex. How far below the surface of a like will a rock actually 1m below the surface appear to a person viewing it from above at some small angle θ'?



Solution:

- Because we are going from a high index of refraction to a low index of refraction, the light ray leaving the water will bend away from the normal vector.

$$n_{\mathrm{H_2O}} \sin\theta = n_{\mathrm{air}} \sin\theta'$$

- Since the viewing angle is assumed to be small, we can use the small angle approximation for the sine function: $\sin\theta \approx \theta$

$$n_{\mathrm{H_2O}}\theta = n_{\mathrm{air}}\theta' \quad \Rightarrow \quad \theta = \frac{n_{\mathrm{air}}}{n_{\mathrm{H_2O}}}\theta' = \frac{1}{1.33}\theta'$$

- When the light hits the persons eye, it will look as if the rock is at location P'

- To find the distance from AP' we must use trigonometry:

$$AB = AP'\tan\theta' = AP\tan\theta \quad \Rightarrow \quad AP'\theta' = AP\theta$$

$$AP' = AP(\theta/\theta') = 1(1/1.33) \approx \boxed{0.75\ \mathrm{m}}$$

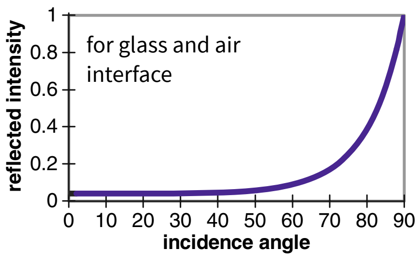
Transmitted and Reflected Intensity:

- As we have seen, when light is incident on an interface between two different materials, a part of the light ray is reflected, and the other part is transmitted.

- Calculating how much of the waves original intensity gets reflected and transmitted turns out to require more mathematics than we have.

- For a light ray that is normal to a plane surface with original intensity I_0 , the fraction that is reflected is given by

$$\boxed{\frac{I_r}{I_0} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2}$$



- The fraction of the wave that is transmitted is therefore:

$$\boxed{I_t/I_0 = 1 - I_r/I_0}$$

- If the light hits at an angle, then the reflected intensity increases with increasing angle.

- As the angle approaches 90deg, all of the light gets reflected

Almost all light gets reflected at large angles