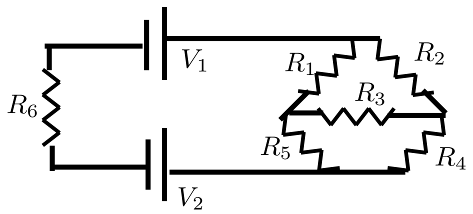
Direct Current Circuits

- So far we have dealt with circuits composed of one voltage source, and capacitors and resistors that can be simplified into a single equivalent capacitor or resistor.

- What happens if there is more than one voltage source, or the circuit is too complex to simplify?

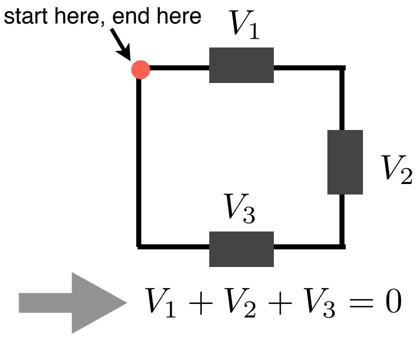


- To solve a general circuit requires **Kirchoff's Law's**:

(1) $\oint \vec{E} \cdot \vec{dl} = 0$ Potential around any loop =0

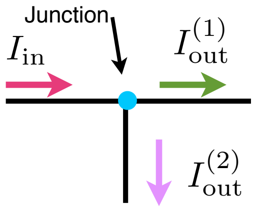
- We have seen this rule already. It comes from the conservative nature of the E-field.

- Some potentials must obviously be negative

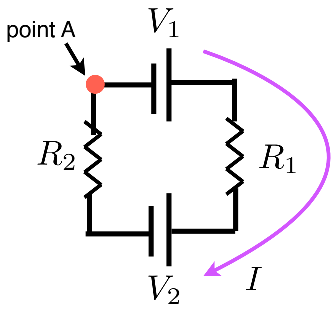


(2) Charge conservation: Charge cannot piled up.

at any junction: $I_{\text{in}} = \sum_k I_{\text{out}}^{(k)}$



ex simple circuit:



- What is the magnitude and direction of current?

- **Step #1:** Decide the current direction in the circuit loop:
  - Direction you pick does not matter
  - Once you pick directions, you cannot change
  - Now we have already satisfied Kirchoff #2

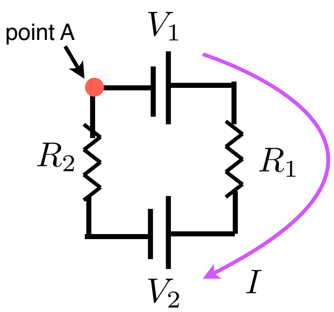
We will assume current goes clockwise here:

- **Step #2:** Do Kirchoff's 2nd law: $\sum_i V_i = 0$ along any closed loop
  - Pick a starting point along the circuit ("point A")
  - We must now decide what the sign of the potential will be across each device:

    if going from low -> high potential = + sign

    if going from high -> low potential = - sign

  - Go around loop <u>in the same direction</u> as the current goes.



- Start at "A" and go clockwise:
  - Potential at V1 goes from low (-) to high(+) therefore potential is +V1
  - Next go across R1, potential goes from high to low since current is going in that direction and current is (+)-charges

$$V_{R_1} = -IR_1$$

  - Across V2, go from (+) to (-) which means that we get -V2
  - R2 is the same as R1, going from high -> low

$$V_{R_2} = -IR_2$$

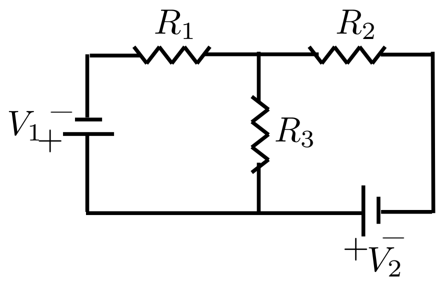
- Now use Kirchoff's 2nd law: add up all the potentials and set =0

$$V_1 - IR_1 - V_2 - IR_2 = 0 \quad \Rightarrow \quad V_1 - V_2 = I\left(R_1 + R_2\right)$$

- Resistors add up since they are in series (as expected)
- Current I is negative if V2>V1 (since R1 &R2 >0):

If current is negative then it goes in the opposite direction you picked in step #1 (Counter clockwise in this example)

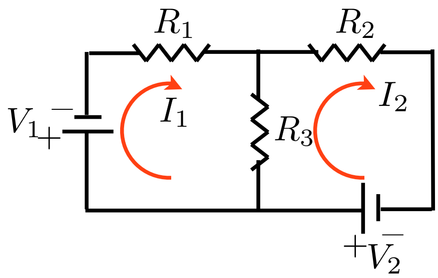
ex 2-loop circuit:



- What is the current I1, I2, I3, through each resistor?

  - Want magnitude and direction

- **Step #1:** Decide the direction of current going through each loop.



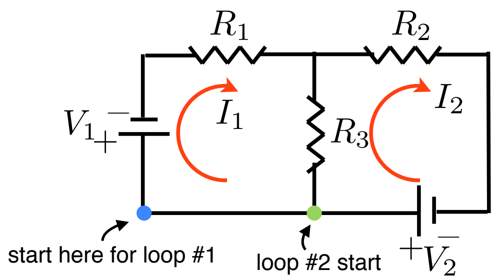
- There is a different current assigned to each closed loop

- Choose the same direction for each current to make life easy. You just get minus sign if you are wrong.

- Again, we have already satisfied Kirchoff's Law #1.

-Do not need to define I3, we see that I3 depends on I1 and I2

- **Step #2** : Kirchoff #2



- pick starting point for loop #1 and go around in same direction as current I1

$$-V_1 - I_1R_1 - I_1R_3 + I_2R_3 = 0$$

I2 in opposite direction as I1

- If current goes in opposite direction that you go around loop, then you must give minus sign to each voltage drop

- do the same for loop #2: $-I_2R_3 + I_1R_3 - I_2R_2 + V_2 = 0$

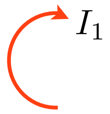
I1 in opposite direction as I2

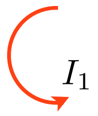
- Solve for I1 and I2 (two equations, two unknowns)

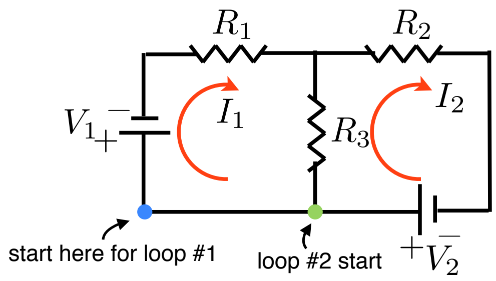
$$I_1\left(R_1 - R_3\right) + R_3I_2 = V_1$$

$$I_1R_2 - I_2\left(R_3 + R_2\right) = -V_2$$

- If I1 is positive, then you picked the correct direction:

- If I1 is positive: 

- If I1 is negative: 



- This solution method is sensitive to the sign (direction) of current

- I3 can be solved since we know I1 and I2:

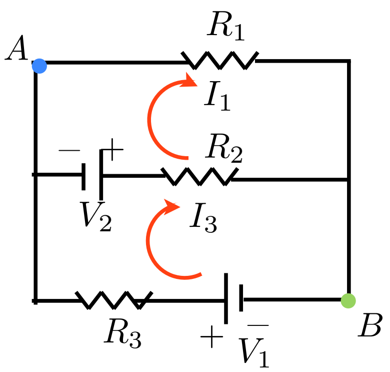
  - Assume: $I_1 = +3\ \mathrm{A} \qquad I_2 = +1\ \mathrm{A}$

$$I_3 = I_1 - I_2 = 2\ \mathrm{A}$$

since I2 in opposite direction

- I3 in same direction as I1 in this example

Ex. Solving Circuits (prob. 26.63):



$$R_1 = 10\ \Omega \qquad R_2 = 20\ \Omega \qquad R_3 = 30\ \Omega$$

$$V_1 = 15\ \mathrm{V} \qquad V_2 = 9\ \mathrm{V}$$

Question: What is the power through each of the resistors if the resistors obey Ohm's Law?

- For Ohmic resistors we know: $P = I^2 R$

- Must find the currents flowing through each resistor.

Step #1: Pick the directions for the currents I_1 and I_3

- Current through resistor #2 (I_2) will obviously be a combination of I_1 and I_3

Step #2: Pick a starting point, and go around each loop in the direction of the currents from step #1.

For current #1 (starting at A): $-I_1 R_1 - I_1 R_2 + I_3 R_2 - V_2 = 0$

For current #3 (starting at B): $V_1 - I_3 R_3 + V_2 - I_3 R_2 + I_1 R_2 = 0$

- We are left with two equations and two unknown variables (I1 & I3)

-We could solve the equation for current #1 to get I1 in terms of I3 and other given variables, and then plug into the equation for current #3.

-This gets kind of messy, so we will use **Cramer's Rule** instead**:**

- Lets write our system of equations in matrix form:

$$\begin{bmatrix} -(R_1 + R_2) & R_2 \\ R_2 & -(r_2 + R_3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_2 \\ -(V_1 + V_2) \end{bmatrix}$$

- Plug in values for resistors and voltages:

$$\begin{bmatrix} -30 & 20 \\ 20 & -50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -24 \end{bmatrix}$$

Going to call this matrix M

- Need to find the determinant of matrix M

$$M = \begin{bmatrix} -30 & 20 \\ 20 & -50 \end{bmatrix} \qquad \text{det M} = (-30 \times -50) - (20 \times 20) = 1100$$

- Cramer's rule says I1 can be found by replacing the first column of M with the RHS voltage values and dividing by the det M

$$I_1 = \frac{\text{det} \begin{bmatrix} 9 & 20 \\ -24 & -50 \end{bmatrix}}{\text{det M}} = \frac{30}{1100} = \frac{3}{110} \qquad \boxed{I_1 = \frac{3}{110} \text{ A}}$$

- To find I3, replace the second column of M, and repeat:

$$I_3 = \frac{\text{det} \begin{bmatrix} -30 & 9 \\ 20 & -24 \end{bmatrix}}{\text{det M}} = \frac{540}{1100} = \frac{54}{110} \qquad \boxed{I_3 = \frac{54}{110} \text{ A}}$$

- Since I3 >I1, the current I2 is simply I3-I1:

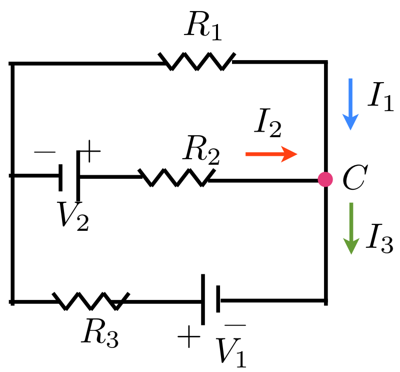
$$\boxed{I_2 = I_3 - I_1 = \frac{54}{110} - \frac{3}{110} = \frac{51}{110} \text{ A}}$$

- Having found the currents, the power is easy to evaluate:

$$\boxed{\begin{aligned} P_1 &= I_1^2 R_1 = 0.0074 \text{ W} \\ P_2 &= I_2^2 R_2 = 4.30 \text{ W} \\ P_3 &= I_3^2 R_3 = 7.23 \text{ W} \end{aligned}}$$

- To double check our answers, at point C, I1+I2 = I3 by Kirchoff #2 **OK!**

- The potential drop from the left side of the circuit to the right i the same along each branch. Verify the potential is the same across each branch:



Top: $V = -I_1 R_1 = -0.2727 \text{ V}$

Middle: $V = 9 - I_2 R_2 = -0.2727 \text{ V}$

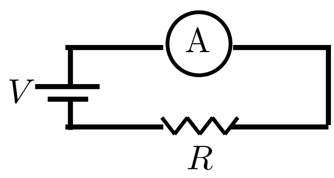
Bottom: $V = R_3 I_3 - V_1 = -0.2727 \text{ V}$

sign changes here since I3 goes right->left and we go left->right

Ammeters & Voltmeters

- Now that we are able to calculate the currents and voltages in circuits, we would like to compare our results with the actual values obtained from experiment

- To measure how much current is flowing through a particular part of the circuit we use an **ammeter**:

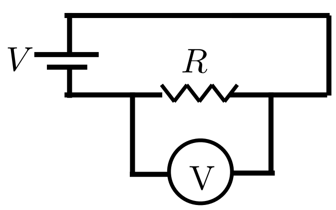


- The ammeter must be in the circuit loop so the current goes though the ammeter.

- Ammeters have very low resistance so that they do not change the value of the current.

$$V = IR \rightarrow I = V/R$$

- To measure the voltage (potential) across a circuit element, we use a **voltmeter** in parallel:



- The ends of the voltmeter must be on opposite sides of the circuit element since only potential differences matter

- Voltmeters have extremely high resistance so almost no current flows through them.