Electric Fields & Gauss' Law

- We have seen how the force between two or more particles can be expressed via Coulomb's Law:

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \qquad \text{or} \qquad \vec{F}_{1,\text{net}} = \sum_i F_{1i}$$

[give brief example with three charges]

- Can use this to find the force given q1 and qi.
- Magnitude and sign of q1 will change answer.
- We want some way to express the force at a given point p for any charge q at p
- We can eliminate q1 from the calculation:

Electric field due to charge q2

$$\frac{\vec{F}}{q_1} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{r} = \vec{E}(\vec{r})$$

- Direction of E-field does not depend on the sign of q1:

<u>ex: q1=-1e</u>

$$\vec{F}(r) = -\frac{1}{4\pi\epsilon_0}\frac{eq_2}{r^2}\hat{r}$$

$$\frac{\vec{F}(r)}{-e} = \vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{r}$$

Same

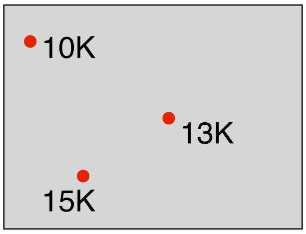
<u>ex: q1=+e</u>

$$\vec{F}(r) = \frac{1}{4\pi\epsilon_0}\frac{eq_2}{r^2}\hat{r}$$

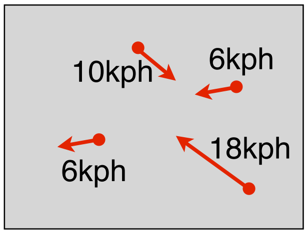
$$\frac{\vec{F}(r)}{e} = \vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{r}$$

- Electric field (or any field) is <u>defined at each point</u> in space.

Temperature (Scalar, just a number) **Wind speed (Vector, num. + direction)**

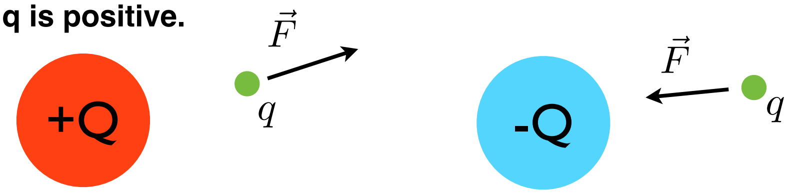


A point or arrow at every location



- We can always calculate the force from the E-field: $\boxed{\vec{F} = q \cdot \vec{E}(\vec{r})}$

- The direction of the electric field is such that **the E-field points in the same direction as the force if q is positive.**

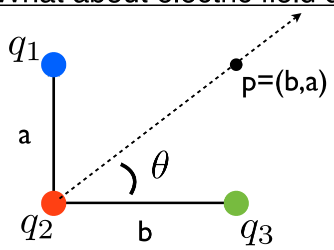


Since E-field is in same direction as F when q is positive:

$\Rightarrow$ $\boxed{\text{Electric field points away from positive charges and toward negative charges}}$

“Out of the positive, **<u>in</u>** to the **<u>n</u>**egative”

<u>Ex. What about electric field due to many charges?</u>



What is the magnitude and direction of E at p?

- Can use superposition principle since E=F/q.

- Do easy stuff first

E_{q_1} in x-direction only: $E_{q_1}(p) = \frac{kq_1}{b^2}\hat{x}$

E_{q_3} in y-direction only: $E_{q_3}(p) = \frac{kq_3}{a^2}\hat{y}$

- E-field from q2 must be calculated at point p, and then decomposed into x/y components

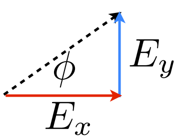
$$|E_{q_2}(p)| = \frac{kq_2}{a^2+b^2} \qquad \tan\theta = \frac{a}{b} \quad \Rightarrow \quad \theta = \arctan\frac{a}{b}$$

$$\Rightarrow \quad E_{q_2}(p) = \frac{kq_2}{a^2+b^2}\cos\theta\hat{x} + \frac{kq_2}{a^2+b^2}\sin\theta\hat{y}$$

- Add all up components

$$E_{q_2}(p) = \left[\frac{kq_1}{b^2} + \frac{kq_2}{a^2+b^2}\cos\theta\right]\hat{x} + \left[\frac{kq_3}{a^2} + \frac{kq_2}{a^2+b^2}\sin\theta\right]\hat{y}$$

- find angle



$$\tan\psi = \frac{E_y}{E_x}$$

- Electric field tells us something about a point charge or charge distribution

- The electric field can be represented graphically

- Electric field is mapped out by placing a **test charge** q at every point and calculating the E-field.

- Test charge has very small positive charge that does not change E-field.

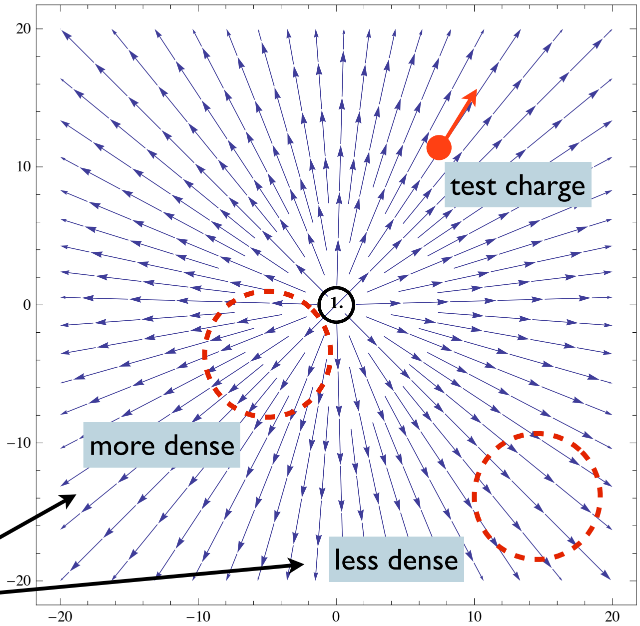
- E-field lines tell us direction of force, but not magnitude (strength) of the force.

- Strength of E-field (force) is given by the density (# of E-field) of lines in a given region

$$E \propto \frac{1}{r^2} \quad \text{(for point charge)}$$

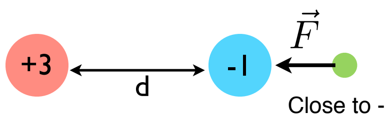
E-field lines should be more dense near point charge

Electric field from a single point-charge



- Force on negative charges is in opposite direction of E-field

- Lets look at E-field from more than one charge:



Close to -1, test charge is attracted due to 1/r^2

What is direction of force on test charge far away?

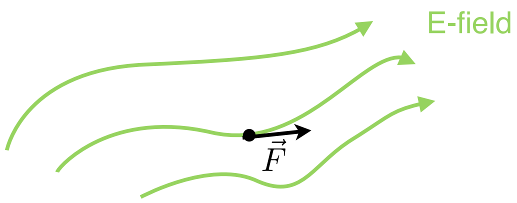
Note: ask the class for guess

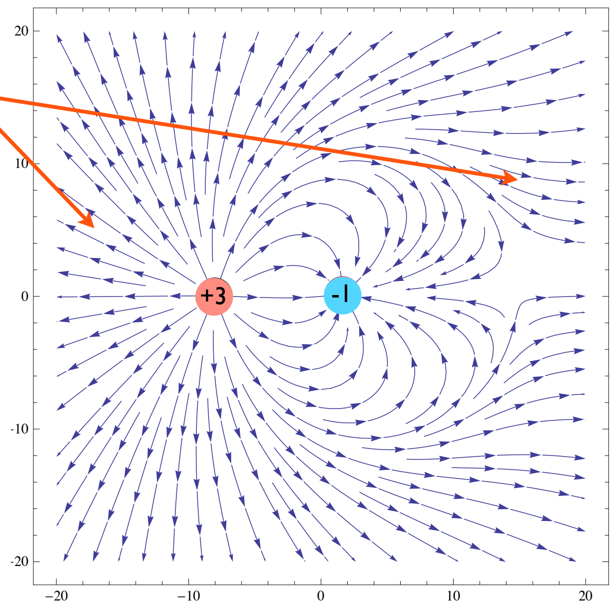
- Far away, (+3)+(-1) looks (not exactly) like +2 charge → E-field always points outward

- Number of lines give you a feel of the strength

- E-field (force) is directed outward far away

- Force is tangential to E-field lines





- Charges do not follow field line in general!

Field-lines are not trajectories

- Electric field points outward everywhere except in the middle of the two charges

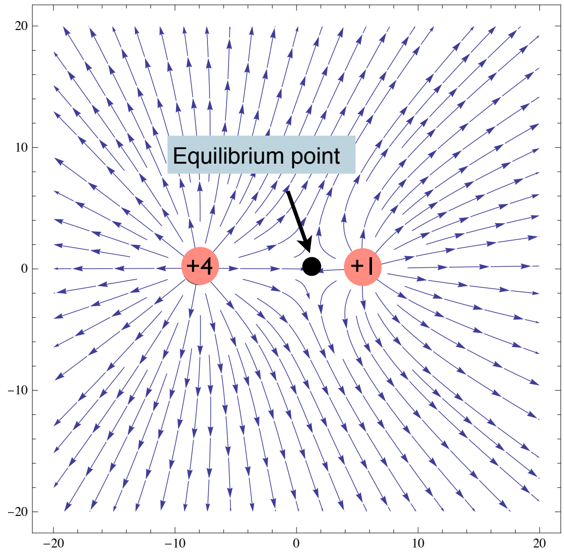
- +4 charges pushes strongly on a test charge in the middle, but eventually +1 wins since

$$\vec{E} \propto 1/r^2$$

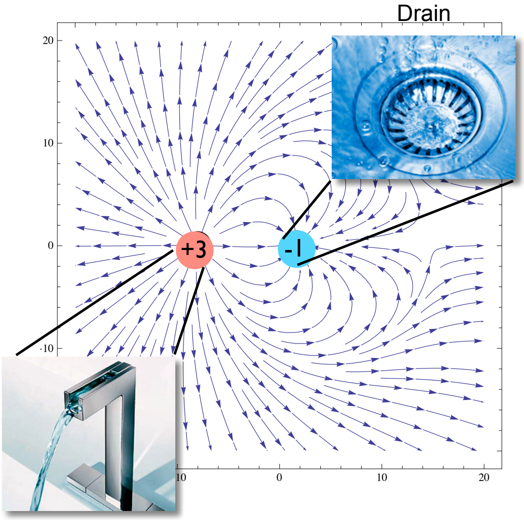
- There exists an equilibrium point in between the two charges where E=0 (near +1)

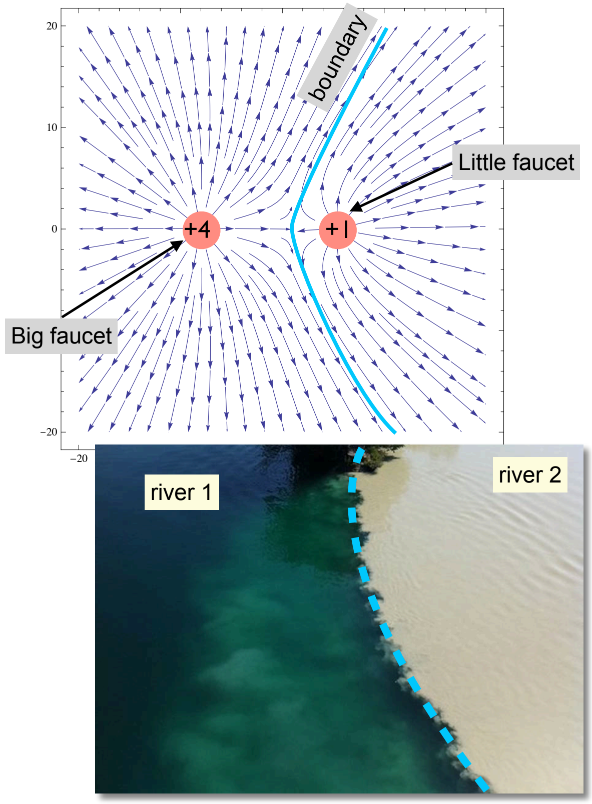
- There is always an equilibrium point between two charges with the same sign.

Electric field from 2 positive charges



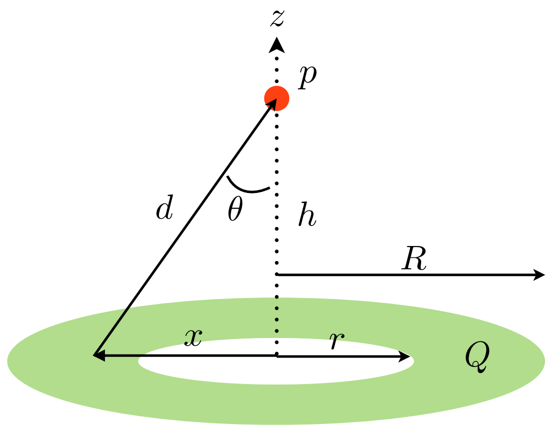
- (+) charges are sources of E-field, (-) charges are sinks; Think of E-field as a fluid.





2 rivers in Switzerland

Ex. Electric field from a charged disc:



A thin disc with radius R=10cm, and a hole in the middle with r=4cm, has a total charge Q=7nC. What is the electric field on the z-axis at point P as distance of h=30cm from the from the center of the disc?

- First, this is a continuous charge distribution -> we must find the charge density σ

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi\left(R^2 - r^2\right)}$$

- We must add up the electric field from all infinitesimal charge elements dQ:

$$E = \frac{kQ}{d^2} \rightarrow dE = \frac{kdQ}{d^2} = \frac{k\sigma dA}{d^2}$$

recall: $Q = \sigma A \rightarrow dQ = \sigma dA$

- Looking at the figure, we see that this example has some symmetry. Lets use it.

- Since we can rotate the disc around the z-axis and not change the problem, this tells use that t**here can be no net force in any direction but the z-direction.**

$$dE_z = \frac{k\sigma dA\cos\theta}{d^2} = \frac{kh\sigma dA}{d^3}$$ since $\cos\theta = h/d$

- Since the problem circular, express the area in terms of polar coordinates:

$$dA = xdxd\phi$$

- express distance d in terms of h and x:

$$dE_z = \frac{kh\sigma dA}{\left(x^2 + h^2\right)^{3/2}}$$

- Write out integral noting that there is no ϕ dependence in the problem

$$E_z = \int_0^{2\pi} d\phi \int_r^R \frac{kh\sigma x}{\left(h^2 + x^2\right)^{3/2}} dx$$

- can be further simplified:

$$E_z = 2\pi kh\sigma \int_r^R \frac{x}{\left(h^2 + x^2\right)^{3/2}} dx$$

- solving integral:

$$E_z = 2\pi kh\sigma \left[\frac{1}{\sqrt{r^2 + h^2}} - \frac{1}{\sqrt{R^2 + h^2}}\right]$$

- Replace σ with Q/A:

$$\boxed{E_z = \frac{2kQh}{R^2 - r^2}\left[\frac{1}{\sqrt{r^2 + h^2}} - \frac{1}{\sqrt{R^2 + h^2}}\right]}$$

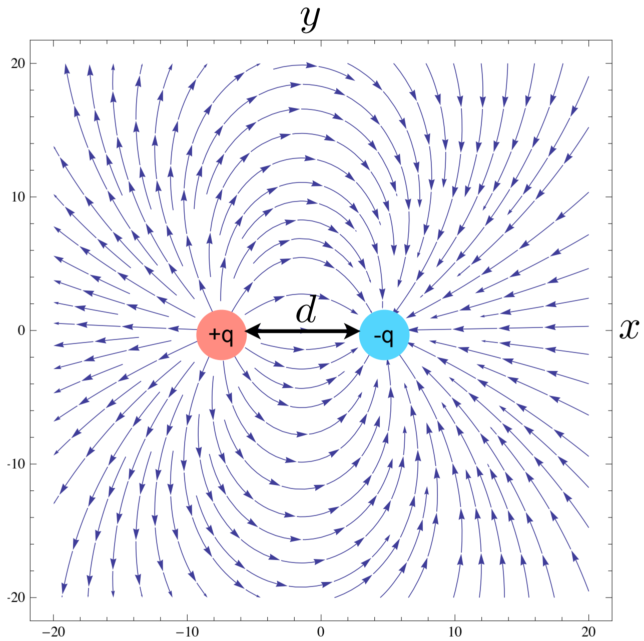
- Finally, plug in numerical values

$$R = 0.1 \text{ m}$$
$$r = 0.04 \text{ m}$$
$$Q = 7 \times 10^{-9} \text{ C}$$
$$h = 0.3 \text{ m}$$
$$k = 9 \times 10^9 \text{ C}^2/\text{N} - \text{m}^2$$

$$\boxed{E_z = 630 \ \frac{\text{N}}{\text{C}}}$$

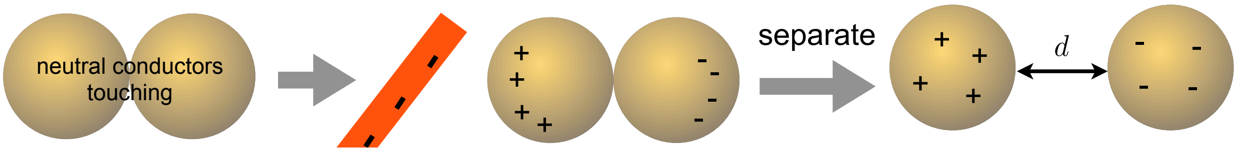
Electric Dipoles:

- Let us consider two charges, one with charge +q and the other -q, separated by a distance d

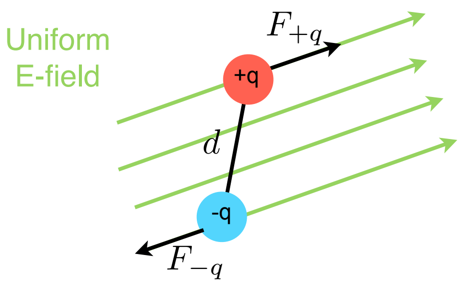


- What happens far away?
    - Total charge is zero
    - Arrows do not point in or out far away.
- E-field from point charge: 1/r^2
    - Since zero net charge, E-field should fall off faster than 1/r^2
    - E-field from dipole: 1/r^3
    - Can be calculated for the two axes with symmetry (see textbook).
- Dipoles are everywhere: H20, antennas,... (remember water movie)

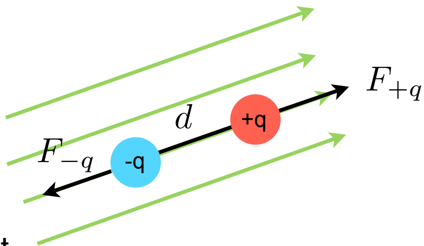
- Easy to create dipole with conductors by induction:



Electric Dipole in E-field:



- (+) charge will experience force in direction of E-field
- (-) charge will experience force opposite of E-field
- E-field produces a torque on the dipole.
- Dipole rotates clockwise (in this example)
- Dipole wants to align with the E-field



- Torque reverses direction if dipole goes past alignment.
- recall that torque $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = |r||F|\sin\theta$
- also $\vec{F} = q\vec{E}$ so $\vec{\tau} = q\vec{r} \times \vec{E}$
- define "electric dipole moment": $\vec{p} = q\vec{d}$
- dipole moment always points from -q to +q
- torque on dipole: $\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = |p||E|\sin\theta$

<u>dipole moment</u>