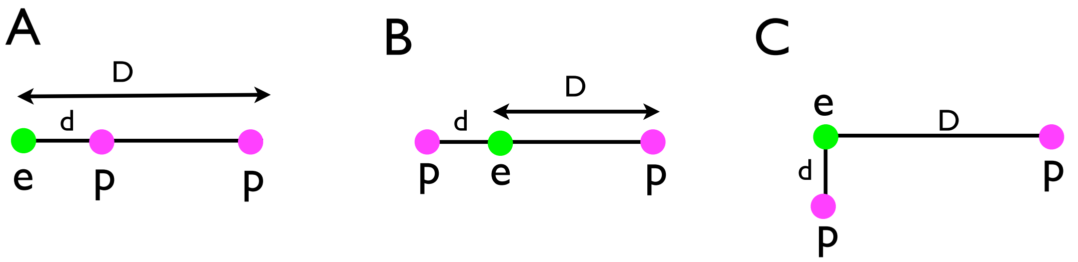
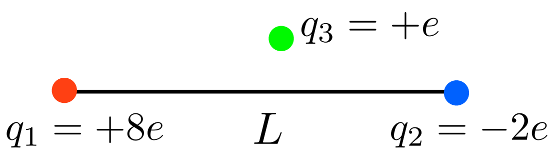
Conceptual Question:

Rank (high to low) the following situations in terms of the magnitude of the net force on the electron (e) from the protons (p).



ANS: A,C,B   A & B are limiting cases of C! (or can us Pythagorean Theorem)

Ex. Equilibrium position of charges



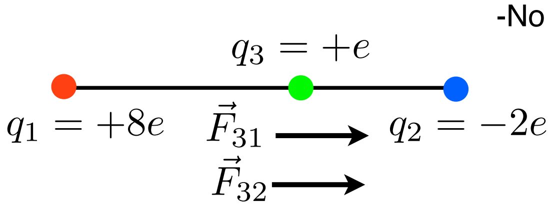
- Where should I put q3 so it is in equilibrium?
- Is the equilibrium stable?

Recall: **Equilibrium** means the net force is zero

$$\Rightarrow \quad \vec{F}_{3,\text{net}} = 0 \quad \Rightarrow \quad \vec{F}_{31} = -\vec{F}_{32}$$

Three situations:

#1: Put q3 in between q1 & q2:



-No need to do any calculations, just think about it.

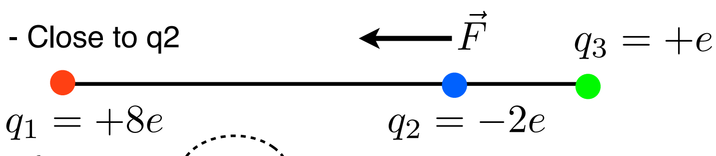
- Both forces in same direction; **no equilibrium**

#2: Put q3 to the **left** of q1 by distance d:

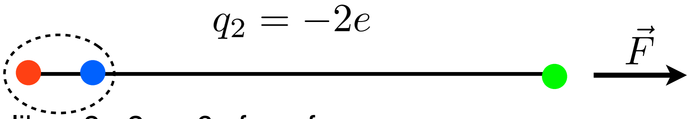
- Repulsive force from q1 always beats out attractive force from q2 since F~1/r^2; **no equilibrium**.

#3: Put q3 to the **right** of q2:

- Close to q2



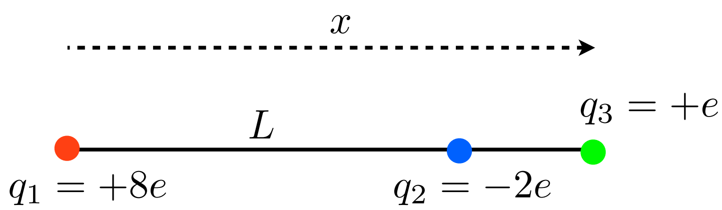
- Far from q2



Combined look like +8e-2e=+6e from far away

Forces now in opposite directions; can **have equilibrium**

Calculate location of equilibrium point:



- Know forces are in opposite directions so only need magnitudes.

$$|F_{31}| = k\frac{8e^2}{x^2} = k\frac{2e^2}{(x-L)^2} = |F_{32}|$$

$$\frac{4}{x^2} = \frac{1}{(x-L)^2} \quad \Rightarrow \quad 2 = \frac{x}{x-L}$$

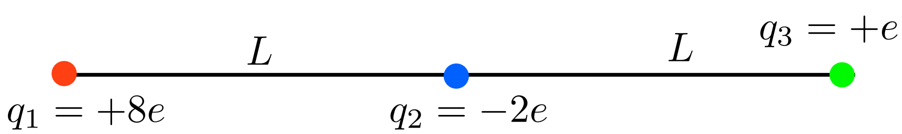
$$\boxed{x = 2L}$$

Is equilibrium point stable?:

- Recall that a equilibrium point is **stable** if any small displacement from equilibrium results in a force that is in the direction opposite that of the displacement

Two situations: move q3 slightly to the left or right.

Note: ask the class; give a few minutes to discuss

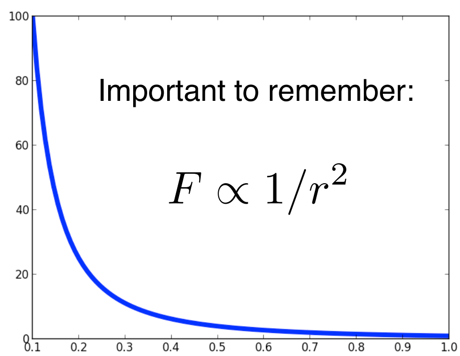


Case #1: deviation to the left:

- Both $\vec{F}_{32}$ and $\vec{F}_{31}$ increase, but

$\vec{F}_{32}$ grows faster than than $\vec{F}_{31}$ since q2 is closer to q3

net force directed **AWAY** from equilibrium



Case #2: deviation to the right:

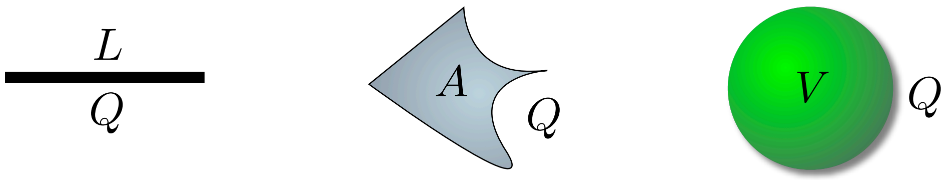
- Both $\vec{F}_{32}$ and $\vec{F}_{31}$ decrease, but

$\vec{F}_{32}$ drops faster than $\vec{F}_{31}$;q3 pushed to the right net force directed **AWAY** from equilibrium

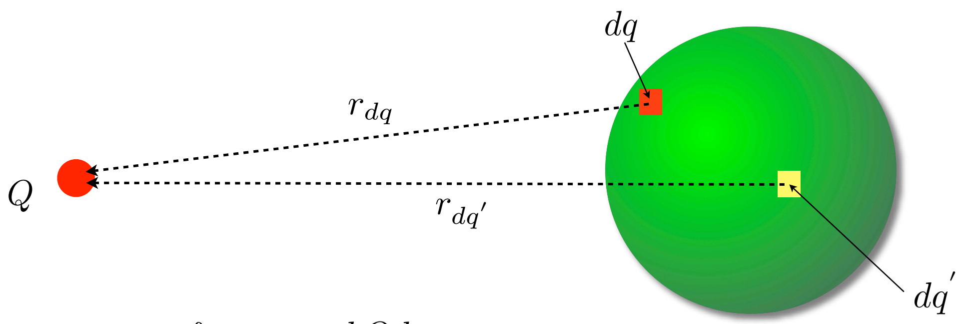
Continuous charge distributions:

- So far have discussed only individual charges; works ok for a small number of charges
- What if we what to calculate the force from a large number of charges?
- If charges are distributed uniformly over some region of space then we can use charge distributions; similar to mass densities when calculating masses and moments of inertia.
- Now dealing with continuous variables; sums replaced by integrals

	1D	2D	3D
Geometry:			
Charge density:	$\lambda = \frac{Q}{L}$	$\sigma = \frac{Q}{A}$	$\rho = \frac{Q}{V}$
Total charge:	$Q = \int_L \lambda dl$	$Q = \int_A \sigma dA$	$Q = \int_V \rho dV$



- Force is calculated by adding up the contributions dq from all charges:



$$\vec{F}_{net} = \int_{\text{all charges}} \frac{kQdq}{r_{dq}^2}\hat{r}$$

distance to charge element dq

- Recall that we have charge densities now, not individual charges.
- Rewrite dq as charge density times volume for 3D case, length 1D, area 2D

$$dq = \rho dV$$

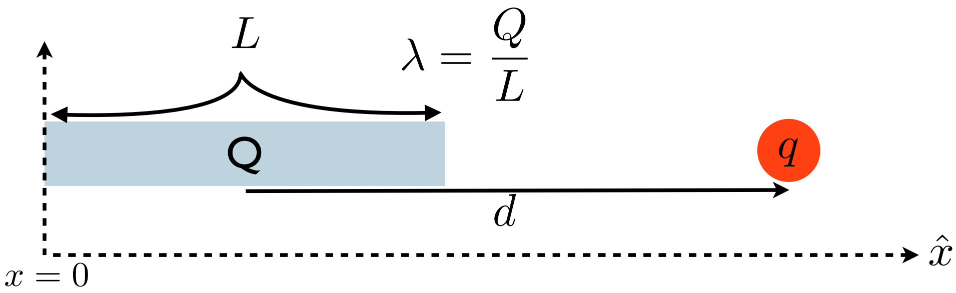
- k and Q are fixed and can be taken outside integral, distance r depends on which dV you are considering -> must stay inside integral
- if charge is uniform, then charge density can also come outside of integral

$$\vec{F}_{net} = kQ\rho \int_V \frac{dV}{r_{dV}^2}\hat{r}$$

distance to volume element dV

Ex: Force from a uniformly charged rod:

What is the electrostatic force on a charge q from a uniformly charged rod with total charge +Q and length L with its center a distance d from +q?



- Force is directed entirely in x-direction

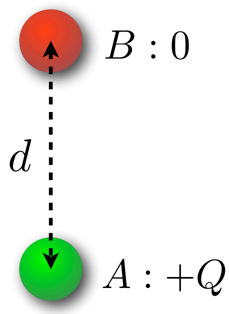
- Need to add up contribution to force from all charge elements dq:

$$\vec{F} = \frac{q\lambda\hat{x}}{4\pi\epsilon_0}\int_0^L \frac{1}{\left(d+\frac{L}{2}-x\right)^2}dx$$

$$\vec{F} = \frac{q\lambda\hat{x}}{4\pi\epsilon_0}\left.\frac{1}{d+\frac{L}{2}-x}\right|_0^L = \frac{q\lambda\hat{x}}{4\pi\epsilon_0}\left[\frac{1}{d-L/2}-\frac{1}{d+L/2}\right] = \boxed{\frac{qQ}{4\pi\epsilon_0}\frac{1}{d^2-\frac{L^2}{4}}\hat{x}}$$

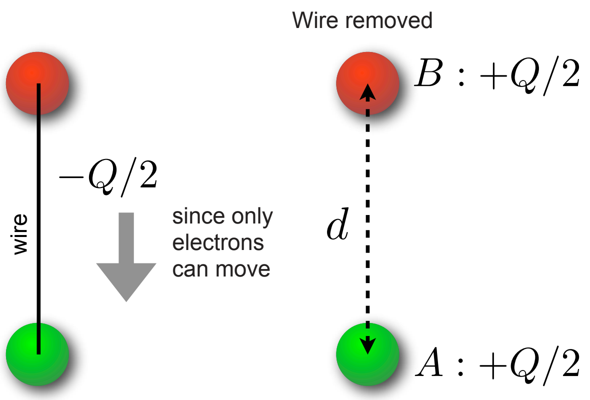
Sample Problems for Electrostatics:

Ex: Charged conductors



Two identical, electrically isolated conducting **spheres** A and B are separated by a (center-to-center) distance (d) that is large compared to the spheres. Sphere A has a positive charge of +Q, and sphere B is electrically neutral. Initially, there is no electrostatic force between the spheres. (Assume that there is no induced charge on the spheres because of their large separation.)

I). Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?



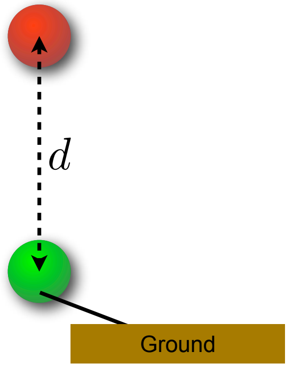
To calculate force from spheres, we could integrate over the volume of the spheres, or we can use the **Shell Theorem:**

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center **(it is a point charge)**.

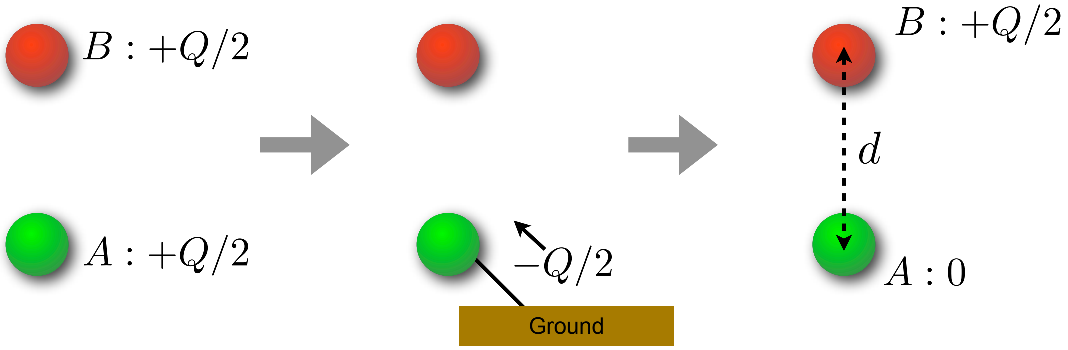
(A Sphere is nothing but a collection of shells, one inside the next)

$$\boxed{F = \frac{k}{4}\frac{Q^2}{d^2}}$$

II).Next, suppose sphere *A* is grounded momentarily, and then the ground connection is removed. What now is the electrostatic force between the spheres?



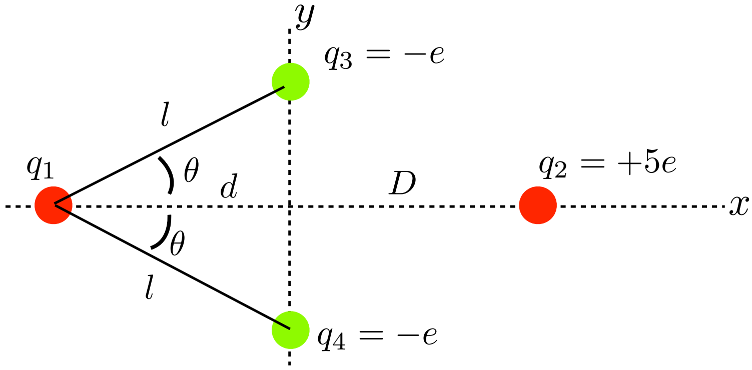
Remember that the “ground” is a place where you can put as much charge as you want -> everything connected to the ground looses charge



$$\boxed{F = k\frac{\frac{Q}{2}\cdot 0}{d^2} = 0}$$

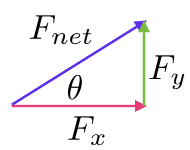
Ex: 4 Charges

Suppose I have 4 charges arranged as follows:



Useful stuff to know

$$\frac{d}{l} = \cos\theta$$



$$F_x = F_{net}\cos\theta$$

1) What is the distance D such that the net force on charge q1 is zero?

- I did not tell you what the charge q1 actually is ... you do not need it
  - Forces in y-direction always cancel out -> just worry about x-direction
  - Force from q3 & q4 is always opposite that from q2; does not depend on q1 (need magnitudes only)

- Lets remove q1 from the calculation by writing F/q:

x-component only

$$\left|\frac{F_{13}}{q_1}\right| = \left|\frac{F_{14}}{q_1}\right| = k\frac{e}{\left(\frac{d}{\cos\theta}\right)^2} \qquad \left|\frac{F^x_{13}}{q_1}\right| = \left|\frac{F^x_{14}}{q_1}\right| = k\frac{e}{d^2}\cos^3\theta \qquad \left|\frac{F_{12}}{q_1}\right| = k\frac{5e}{(D+d)^2}$$

F13+F14=F12 for equilibrium:

$$\frac{2ke\cos^3\theta}{d^2} = \frac{5ke}{(D+d)^2} \quad \Rightarrow \quad \frac{2\cos^3\theta}{d^2} = \frac{5}{(D+d)^2}$$

- Now solve for D:

$$\frac{(D+d)^2}{d^2} = \frac{5}{2}\frac{1}{\cos^3\theta} \quad \Rightarrow \quad D+d = \pm\sqrt{\frac{5}{2}\frac{d^2}{\cos^3\theta}}$$

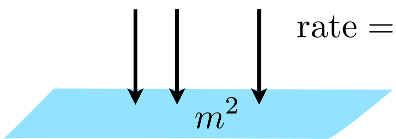
$$D = \pm\sqrt{\frac{5}{2}\frac{d^2}{\cos^3\theta}} - d$$ only want positive solution $$\boxed{D = \sqrt{\frac{5}{2}\frac{d^2}{\cos^3\theta}} - d}$$

1) If q3 & q4 move closer to the x-axis, what happens to the length of D?

- If q3 and q4 move closer to the x-axis, then the x-direction component of the force from q3 and q4 increases -> **D must get smaller** so that force from q2 can increase by same amount

Ex: Electrical current

- Earth is constantly being hit by cosmic ray protons coming from space. If all protons made it to the ground, then the average rate of protons would be 1500 protons/m^2-sec. What is the total electric current from these protons over the entire Earth?



$$\text{rate} = \frac{1500e}{m^2 - \text{sec}}$$

- First must convert the rate from e to Coulombs since current is Coulombs/sec:

$$1\text{C} = 6.2\times10^{18}\text{e} \quad \Rightarrow \quad \frac{1500e}{m^2-\text{sec}}\cdot\frac{1\text{C}}{6.2\times10^{18}e} = 2.4\times10^{-16}\frac{\text{C}}{m^2-\text{sec}}$$

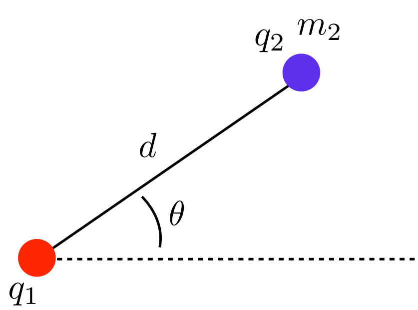
- Must find total surface area of Earth: $5.1\times10^8\ \text{km}^2$

- Convert area to m^2: $5.1\times10^8\ \text{km}^2\cdot1\times10^6\frac{\text{m}^2}{\text{km}^2} = 5.1\times10^{14}\text{m}^2$

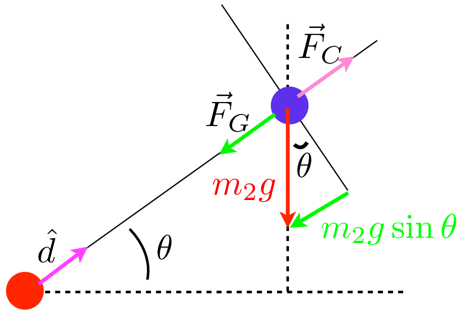
- Calculate rate over the entire surface area:

$$I_{\text{total}} = 2.4\times10^{-16}\frac{C}{m^2-\text{sec}}\cdot5.1\times10^{14}\text{m}^2 = \boxed{0.12\frac{\text{C}}{\text{sec}} = 0.12\ \text{A}}$$

Ex: Force Balance



- Suppose a charge q2 with mass m2 is connected to a charge q, with the same sign charge as q2, by a wire. If q2 is free to move along this wire, find the distance d such that the force of gravity is balanced by the repulsive electrostatic force.



- First find the gravitational force on q2 directed along the wire:

$$\vec{F}_G = -m_2 g \sin\theta \hat{d}$$

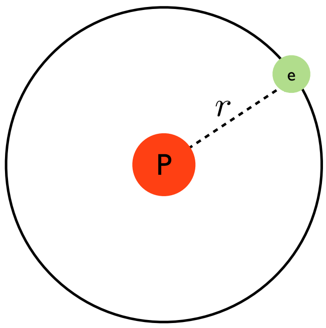
- Calculate repulsive Coulomb force:

$$\vec{F}_C = k\frac{q_1 q_2}{d^2}\hat{d}$$

- Find distance d where forces cancel each other:

$$k\frac{q_1 q_2}{d^2} = m_2 g \sin\theta \quad \Rightarrow \quad \boxed{d = \sqrt{\frac{kq_1 q_2}{m_2 g \sin\theta}}}$$

Ex: Atomic Forces

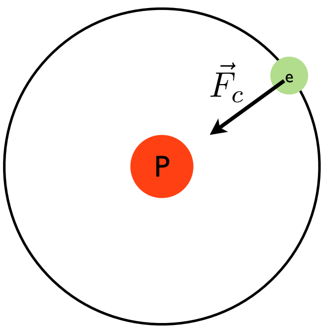


- In a simple model of Hydrogen, a single electron orbits around a proton at a distance of r.

1) Find the velocity of the the electron

- Recall that an object in circular orbit feels centripetal force.

$$F_c = m_e a_c = m_e \frac{v_e^2}{r}$$



- Centripetal force due to coulomb force between e & p

$$m_e \frac{v_e^2}{r} = k\frac{e^2}{r^2}$$

- Solve for v:

$$\boxed{v_e = \sqrt{\frac{ke^2}{m_e r}}}$$

2) What is the electrical current due to the electron at any point in the orbit?

- Must find time it takes to complete one orbit: $t_{\text{orbit}} = 2\pi r/v_e = 2\pi\sqrt{\frac{m_e r^3}{ke^2}}$

- Find current: $I = \frac{e}{t_{\text{orbit}}} = \frac{1.6\times 10^{-19}\ \text{C}}{t_{\text{orbit}}} = \boxed{1.6\times 10^{-19}\cdot\frac{1}{2\pi}\sqrt{\frac{ke^2}{m_e r^3}}\ \text{A}}$