Current and Resistors

- So far we have discussed a lot about charges, but have not talked much about how they move.

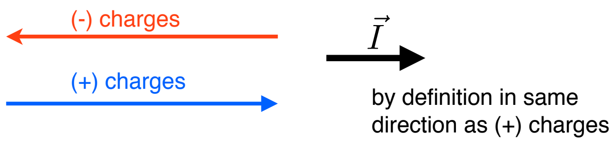
- The key quantity needed in the description of charge motion is the **Electric Current**:

$$I = \frac{dq}{dt} \quad \Rightarrow \quad q(t) = \int_0^t I(t')dt' \qquad I = [A = \frac{C}{\text{sec}}]$$

- In this chapter we will discuss currents that move in a single direction only, these are called **direct currents**, and are the currents produced in batteries, and used in cars, computers,....

- Currents are generated in conductors since charges can move.

- It is the electrons that move in a conductor, but in physics we define the charge to be in the direction that positive charges move. (because $100 guy got it wrong)



- Although electrons move, can always think of (+) charges as moving in opposite direction

- If we apply a potential difference across a conductor, then an E-field is created inside the conductor

- As in any conductor, the electrons try to move around until they cancel the E-field

- However, if the potential (E-field) is constant, than the electrons can never cancel the E-field and they will keep moving around

- Therefore, there is a relationship between the current (charges moving) and the applied voltage.

- Often times there is a linear relationship between current and potential: we call this **Ohm's "Law"**

- Here we will do a simple derivation of Ohm's law since the full version requires quantum mechanics.

- Suppose I have a Cu wire at 300K, then the electron velocity is: $v_e \approx 10^6 \ \mathrm{m/s}$

- The time it takes before an electron bumps into a Cu atom is called the **mean free-path**

$$\tau \approx 10^{-14} \ \mathrm{sec}$$

- Also, I can calculate the number of free electrons per m^3 in Cu: $n \approx 10^{29} \ 1/\mathrm{m}^3$

- Now suppose we apply a voltage across our Cu wire:

$\Rightarrow$ $F = eE$ (I am only worried about magnitudes here so forget the minus sign at the moment)

- We also know that F=ma, so: $a = \dfrac{F}{m_e}$ $\Rightarrow$ $v_d = a\tau = \dfrac{eE}{m_e}\tau$ (**drift velocity**)

- The drift velocity tells us how fast the electrons move due to the E-field.

If E$\uparrow$ then Vd$\uparrow$ and if $\tau\uparrow$ then Vd$\uparrow$

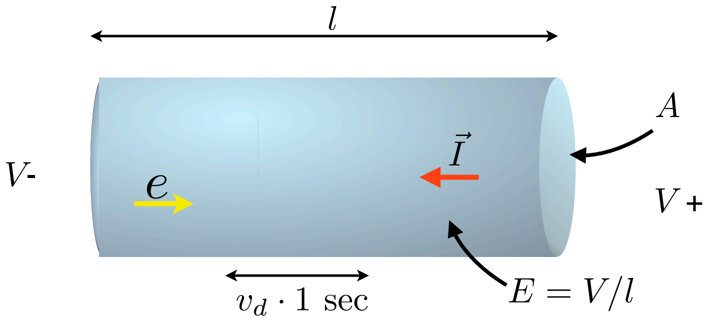
- Consider a 10m Cu wire with potential difference V=10V -> E=1 V/m

in this case: $v_d = \dfrac{1.6 \times 10^{-19} \ \mathrm{C} \cdot 3 \times 10^{-14}}{10^{-30} \ \mathrm{kg}} \approx 5 \times 10^{-3} \ \mathrm{m/s}$

- Electrons move 10 billion times faster due to thermal motion compared to the E-field.

- In fact, a turtle moves faster than the electrons!

- Lets now calculate the current through a conductor:



depends only on the material

$$I = v_d A n e = \frac{e^2 n \tau}{m_e} A E$$

$= \sigma$ **"conductivity"**

- ex. Cu @ 300K: $\sigma \approx 10^8$

- Can rearrange to solve for potential and define resistance:

$$I = \sigma A \frac{V}{l} \Rightarrow V = \frac{l}{\sigma A} I = RI \qquad R = \frac{l}{\sigma A}$$ **"resistance"**

- Can also define the "**resistivity**": $\rho = 1/\sigma$

-We now have **Ohm's Law**:

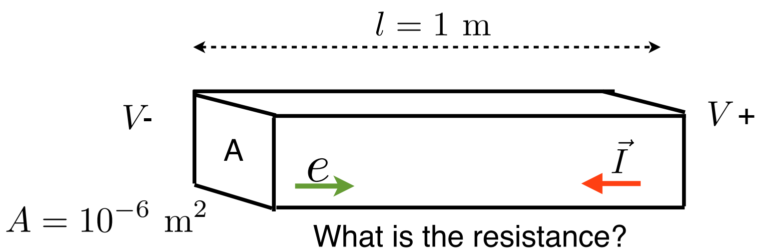
$$\boxed{V = IR} \quad \text{with} \quad R = \frac{l}{\sigma A} = \frac{l\rho}{A} \quad [\Omega]$$ (Ohm)

- We see that resistance is proportional to length, inversely proportional to area

- Think of current as flowing water, bigger pipe means easier flow; longer pipe more resistance

- Ohm's law also works for many insulators

- Suppose I am given a wire:



$V = 1\ \mathrm{V}$

What is the resistance?

- We know I and A, so only need σ

  - For good conductors, Au, Ag, Cu: $\sigma \approx 10^8 \rightarrow \rho = 10^{-8}$ $\quad R = 10^6 \rho = 10^{-2}\ \Omega$

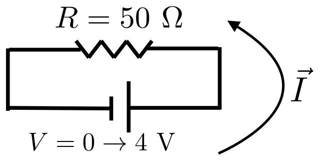
  - For good insulators, Glass, quartz: $\sigma \approx 10^{-12} - 10^{-16} \rightarrow \rho = 10^{12} - 10^{16}$

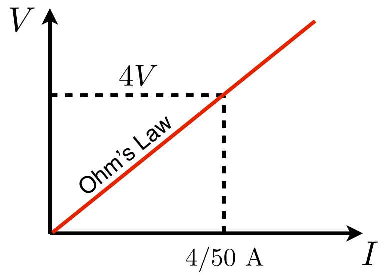
  assuming $\rho = 10^{14} \Rightarrow R = 10^{20}\ \Omega$ Huge resistance

- We can use Ohm's law to get the current:

$$I_{\mathrm{Cu}} = 100 \text{ A} \quad ; \quad I_{\mathrm{Glass}} = 10^{-20} \text{ A}$$

- Suppose I have the following circuit:





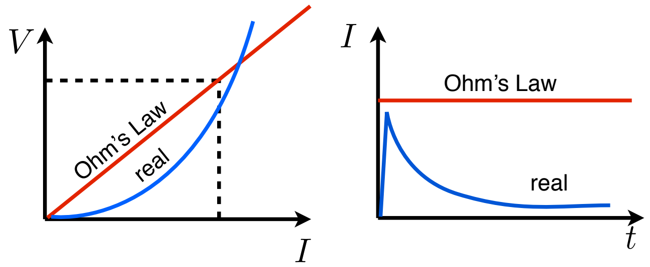
- Conductivity σ is a strong function of temperature:

  - higher temperature -> higher resistance

- Ohm's law does not hold very well in general

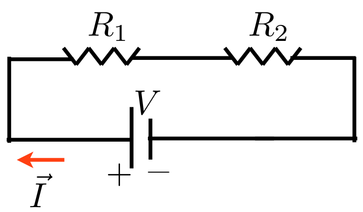
Light bulbs: light bulbs get hotter as current flows, this is why they glow!

$$R_{\mathrm{cold}} \approx 7\ \Omega$$
$$R_{\mathrm{hot}} \approx 50\ \Omega$$



Circuits of Resistors:

Resistors in series:



- Current through each resistor is the same

  - Think of water through a pipe: "what goes in must come out"

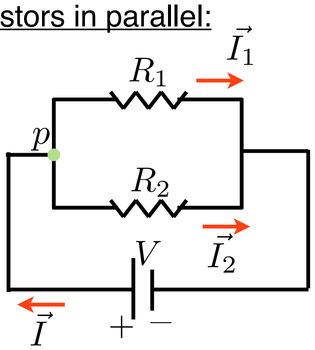
$$V_1 = IR_1 \qquad V_2 = IR_2$$

- We know that the total potential V=V1+V2:

$$V = V_1 + V_2 = IR_1 + IR_2 = I\left(R_1 + R_2\right) = IR_{eq}$$

- For resistors in series: $\boxed{R_{eq}^{\mathrm{series}} = \sum R_i^{\mathrm{series}}}$

Resistors in parallel:



- At point P, the current splits into two components:

$$I = I_1 + I_2$$

- Like capacitors in parallel, the potential across both resistors is the same

$$V = I_1 R_1 \qquad V = I_2 R_2$$

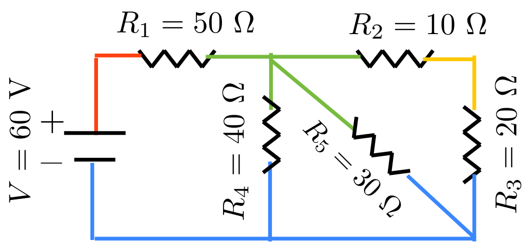
- Can evaluate the total current in the circuit:

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}}$$

- For resistors in parallel: $\boxed{\frac{1}{R_{eq}^{\text{para}}} = \sum \frac{1}{R_i^{\text{para}}}}$

- Can always combine resistors in series or parallel into a single resistor

Ex. Simplifying resistors:



- What is the equivalent resistance of the 5 resistors?

- Use color for potentials (as before)

-> R4 & R5 in parallel

-> R2 & R3 in series

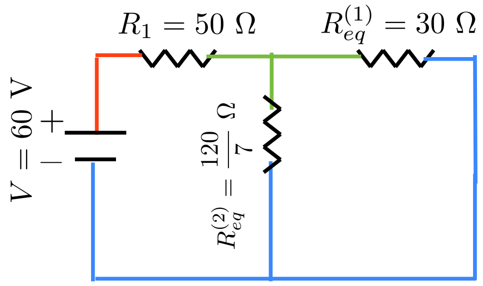
- R5 is connected in the middle of R1,R2 & R4

-> we cannot combine R1 and R4, or R1 and R2 yet

- Start with R2 and R3 (or R4 and R5): $R_{eq}^{\text{series}} = R_2 + R_3 = 30\ \Omega$

- Now do R4 and R5: $\frac{1}{R_{eq}^{\text{para}}} = \frac{1}{R_4} + \frac{1}{R_5} = \frac{7}{120} \quad \rightarrow \quad R_{eq}^{\text{para}} = \frac{120}{7}\ \Omega$

- Redraw the circuit:



- Now $R_{eq}^{(1)}$ and $R_{eq}^{(2)}$ are in parallel, do them next:

$$\frac{1}{R_{eq}^{(3)}} = \frac{1}{R_{eq}^{(1)}} + \frac{1}{R_{eq}^{(2)}} = \frac{11}{120} \quad \rightarrow \quad R_{eq}^{(3)} = \frac{120}{11}\ \Omega$$

- We are left with R1 in series with $R_{eq}^{(3)}$

$$R_{eq}^{\text{total}} = R_1 + R_{eq}^{(3)} = 50\ \Omega + \frac{120}{11}\ \Omega = \boxed{\frac{670}{11}\ \Omega}$$