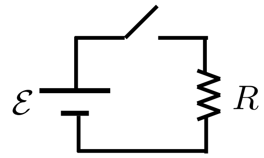
- Imagine that I have a circuit with a switch that closes at time t=0



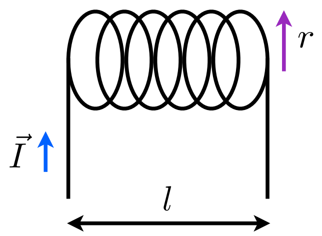
- Initially no B-field since I=0
- After switch is closed, current is generated producing a time-changing B-field until steady-state is reached
- current -> B-field

- time-changing current -> time-changing B-field -> induced EMF
- The **self-inductance L** is the inductance from the circuit onto itself. $[L] = H$
- Self-inductance can be big or small, but it <u>always</u> exists.

$$\phi_B = LI \quad \Rightarrow \quad \mathcal{E}_{\text{ind}} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$$

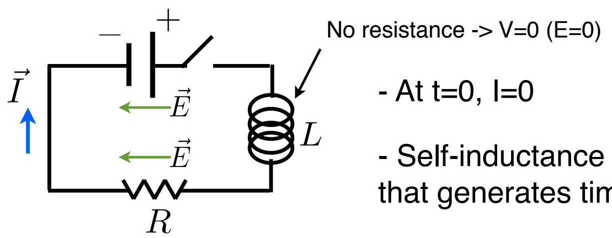
geometric properties

<u>ex. solenoid:</u>

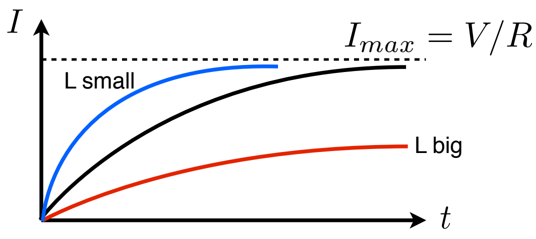


- assume N-loops: $B = (N/l)\mu_0 I$
- attach open surface, B-field goes through N times

$$\phi_B = \underbrace{\pi r^2 N}_{\text{total area}} \cdot (N/l)\mu_0 I \quad \Rightarrow \quad \boxed{L = \frac{\pi r^2 N^2 \mu_0}{l}}$$



- At t=0, I=0
- Self-inductance fights time-changing current that generates time-changing B-field



- Want to find equation of motion for circuit
- Cannot use KVL, must use Faraday!

  (Your textbook is wrong!!!)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$$

inductor term

- Go around loop in direction of current: $0 + IR - V = -L\frac{dI}{dt}$

- Rewrite: $V - L\frac{dI}{dt} = IR$ V=IR only after $\frac{dI}{dt} = 0$

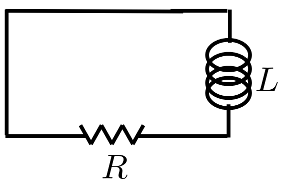
Inductor is opposing the voltage

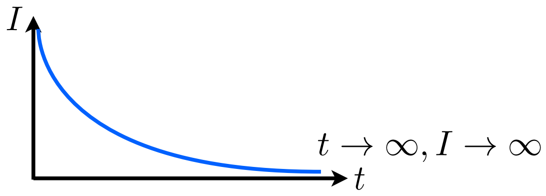
- Circuit satisfies the equation: $L\dfrac{dI}{dt} + IR - V = 0$

- We have seen how to solve equations of this type, but I give you the answer:

$$I = I_{\max}\left[1 - e^{-\frac{Rt}{L}}\right] = I_{\max}\left[1 - e^{-\frac{t}{\tau}}\right] \qquad \begin{array}{l} I_{\max} = V/R \\ \tau = L/R \end{array}$$

- Now remove battery: t=0, V=0 & I=Imax





- New equation: $L\dfrac{dI}{dt} + IR = 0 \rightarrow I = I_{\max}e^{-Rt/L} = I_{\max}e^{-t/\tau}$

- Current is still generated, even after battery is removed
    - Work is still being done to move charges

- Energy for this work must be stored in the B-field of the inductor.

- Lets find how much energy is dissipated in the resistor after the battery is removed:

$$U = \int_0^\infty I^2 R dt = I_{max}^2 R \underbrace{\int_0^\infty e^{-2Rt/L} dt}_{L/2R} = \frac{1}{2} L I_{max}^2$$

- B-field is entirely inside of the solenoid
    - Plug-in L and I for inductor into energy

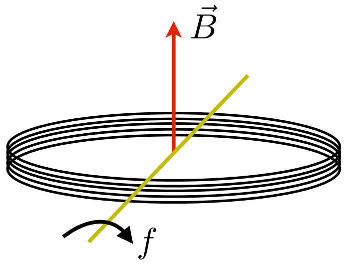
solenoid volume

$$\frac{1}{2}LI^2 = \frac{B^2}{2\mu_0}\pi r^2 l$$

- Can define **magnetic field energy density:**

$$\tilde{U}_B = \frac{B^2}{2\mu_0} \qquad J/m^3$$

- Must integrate over all volume to get total energy stored in B-field ex. 29.39:



$$N = 100000 \qquad f = 150 \text{ Hz}$$
$$B = 0.3 \text{ G} \qquad R = 1500\ \Omega$$
$$r = 0.25 \text{ m}$$

a) What is maximum current in loops?

$$\phi_B = NAB\cos(\omega t) \qquad \text{(A=loop area)}$$

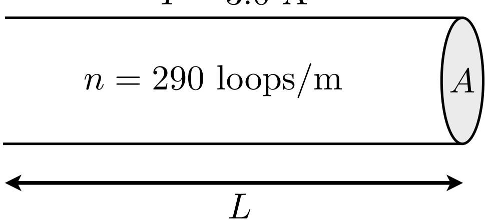
$$-\frac{d\phi_B}{dt} = NAB\omega\sin(\omega t) = \mathcal{E}_{\text{ind}}$$

$$\mathcal{E}_{\text{ind}} = IR \rightarrow I = \frac{NAB\omega\sin\omega t}{R} \quad \Rightarrow \quad \boxed{I_{\text{max}} = \frac{NAB\omega}{R} = 0.37 \text{ A}}$$

b) What is the power emitted given that $I_{\text{avg}} = 1/\sqrt{2}I_{\text{max}}$

$$P = I_{\text{avg}}^2 R = \frac{1}{2}I_{\text{max}}^2 R \quad \Rightarrow \quad \boxed{P = 102.7 \text{ W}}$$

ex. 29.67:



Energy stored in inductor: $U = 2.8$ J

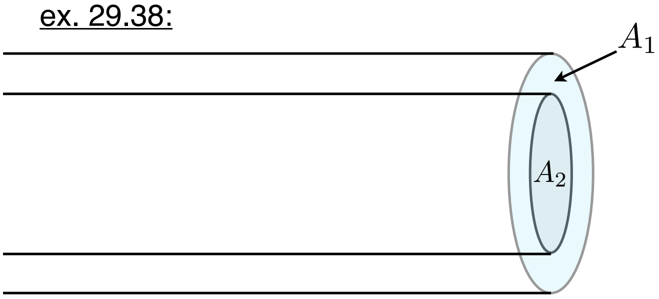
What is the area A?

- From magnetic field energy density: $U = \dfrac{B^2}{2\mu_0}V \leftarrow$ solenoid volume $\quad V = AL$

- B-field inside of solenoid is trivial (we saw it a lot): $B = n\mu_0 I$

- Can now solve for area: $\boxed{A = \dfrac{2U}{n^2\mu_0 I^2 L} = 1.96\ m^2}$

ex. 29.38:



- Consider two solenoids

- Both loops have same length l and number of loops N and resistance R

- Current in outer solenoid is:

$$I_1 = I_0 \cos(\omega t)$$

- What is the B-field inside the #2 solenoid?

- We need to find the flux through solenoid #2:

$$\phi_B = B_1 A_2 \qquad B_1 = n\mu_0 I_1 = n\mu_0 I_0 \cos(\omega t)$$

- Only change in flux matters: $\mathcal{E}_{\text{ind}} = -\dfrac{d\phi_B}{dt} = n\mu_0 A_2 I_0 \omega \sin(\omega t)$

$$I_2 = \frac{\mathcal{E}_{\text{ind}}}{R} = \frac{n\mu_0 A_2 I_0 \omega}{R} \sin(\omega t)$$

- Now can solve for B2:

$$B_2 = n\mu_0 I_2 \quad \Rightarrow \quad \boxed{B_2 = \frac{n^2\mu_0^2 A_2 I_0 \omega}{R} \sin(\omega t)}$$