PHYS-183 : Day #23

- Last week we discussed the forces between electric charges.

- The force between any two charged particles can described via Coulombs law:

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}$$

- We also saw that these forces arise from the Electric field, a vector field, generated by the charged particles

$$\vec{E}(\vec{r}) = \frac{\vec{F}}{q_1} = k\frac{q_2}{r^2}\hat{r}$$

- When looking at the force of gravity, we saw that the force on an object was related to the change in its potential energy as a function of position.

$$F = -\frac{\Delta U(x)}{\Delta x}$$

- We now want to find the potential energy associated with electric charges.

- We are going to learn about two new concepts:

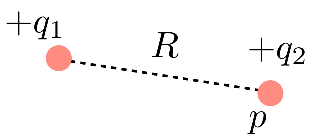
"Electrostatic Potential Energy" : U

"Electrostatic Potential" : V

These are independent concepts (as we will see)

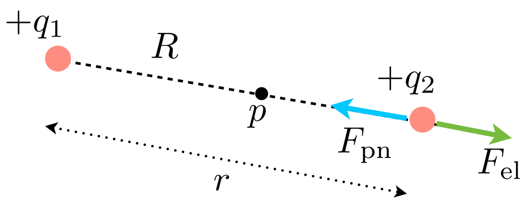
Electrostatic Potential Energy:

- Consider the following situation:



- It is clear that I had to do work to bring the charges together.
    - The charges repel since they are the same sign.
    - Like trying to push in a spring
    - If charges were connected by a string, and I cut it, the charges would fly apart.

- I had put work into this problem. This work is **Electrostatic Potential Energy**
- How much work do I have to do?
- If space is empty, it takes no work to put q1



- I Paul want to bring q2 in from far away to position p at a constant velocity.
- I have to push and push again Coulomb force
- Work is positive since F in same direction as charge is moving

- The work I do from infinity to R is thus:

$$W_{pn} = \int_{\infty}^{R} F_{pn} dr = -\int_{\infty}^{R} F_{el} dr = \int_{R}^{\infty} F_{el} dr$$

Since $\vec{F}_{el} = -\vec{F}_{pn}$

- We know what $\vec{F}_{el}$ is, also, force is in same direction as dr (since R -> infinity)

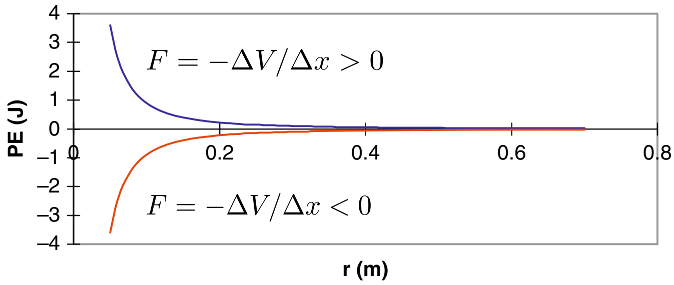
$$W_{pn} = kq_1q_2 \int_{R}^{\infty} \frac{dr}{r^2} = -\,kq_1q_2 \frac{1}{r}\Big|_{R}^{\infty} = \frac{kq_1q_2}{R} + \text{constant}$$

- This is the **electrostatic potential energy** U:

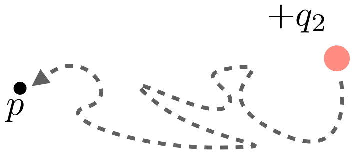
$$\boxed{U = \frac{kq_1q_2}{R} \quad [J]}$$

- This is a scalar valued function (not a field): only one number.
- If both q1 and q2 positive or negative -> work is positive.
- If charges have opposite sign -> work is negative since charges attractive.

- Minus the change in potential energy gives us the force between the particles.

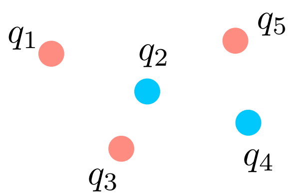


- Electric force is **<u>conservative force</u>**: Work does not depend on path taken



- You should convince yourself that if I took a strange path to get to point P, that the total work remains the same

- What if I have a collection of charges?



- How do I find the the total amount of work?

- Bring in charges on at a time and add up the work from each contribution.

- Again, the electric potential energy has only one value for a given charge distribution.

- Now that we have found the electric potential between two charges, we can now write the total energy of the two particle system:

$$E = KE_1 + KE_2 + U_{\text{mech}} + \frac{kq_1q_2}{r} = \text{constant}$$

<u>Ex.</u> Find the total energy of a hydrogen atom if the electron moves in a circular path around a stationary proton. Give the answer as a function of the electron radius *r*.

- The total energy is the kinetic energy of the electron plus the potential energy of the electron-proton pair

$$E = \frac{1}{2}m_e v^2 + k\frac{(+e)(-e)}{r}$$

- The only force on the electron is the Coulomb force, so this must generate the centripetal acceleration

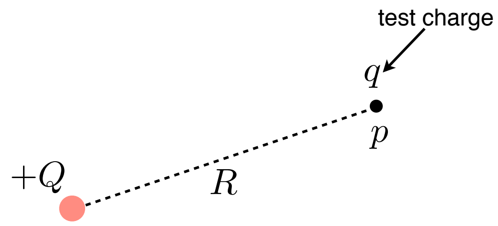
$$F = k\frac{e^2}{r^2} = m_e\frac{v^2}{r}$$

- Solving for mv^2, we have:

$$E = \frac{1}{2}k\frac{e^2}{r} - k\frac{e^2}{r} = -\frac{k}{2}\frac{e^2}{r}$$

-The energy of the electron is completely determined by the radius of the electrons motion.

Electrostatic Potential:



- We already know what the electrostatic potential energy is:

$$U = \frac{kqQ}{R}$$

- "**Electrostatic potential**" - Work per unit charge need to bring charge from infinity to point P.

$$V_p = \frac{W}{q} = \frac{U}{q} = \frac{kQ}{R} \quad \Rightarrow \quad \boxed{V_p = \frac{kQ}{R} \quad \left[\frac{J}{C}\right] = [V] \quad \text{Volts}}$$

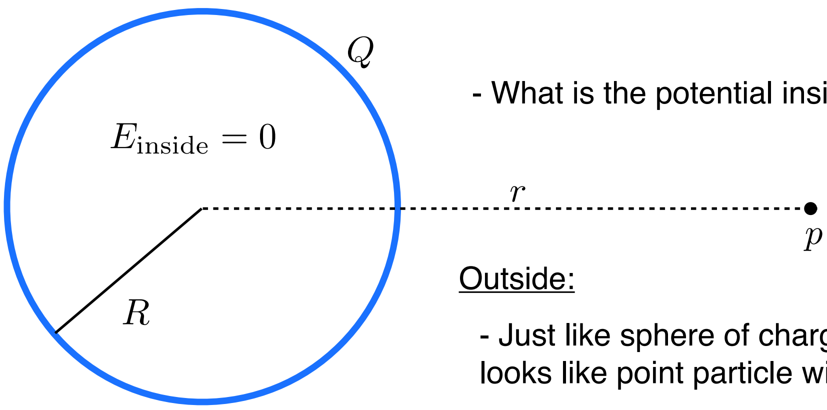
- Or: $\boxed{W = qV}$ - Work is the charge times the electrostatic potential

- Because potential from point charge is proportional to 1/r, potential is zero at infinity: $V_\infty = 0$

- Notice that if Q is positive -> potential is everywhere positive in space.

- Negative Q produces negative potential everywhere in space.

- Consider a uniformly charged spherical shell:



- What is the potential inside and outside of shell?

Outside:

- Just like sphere of charge, E-field from shell looks like point particle with charge *Q*

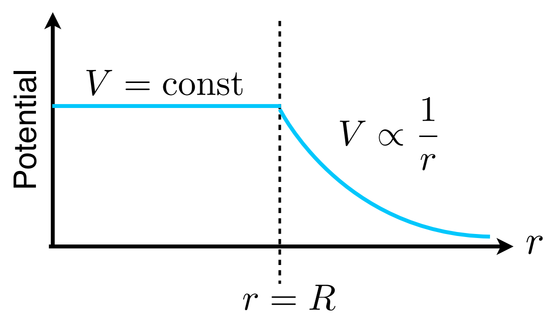
$$V_p = \frac{W}{q} = \frac{F}{q}r = Er = \frac{kQ}{r}$$

Inside:

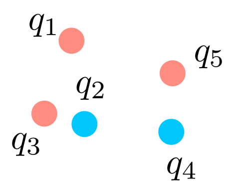
- Remember E=0 inside shell of charge!

- No E-field -> no force -> no work -> constant potential

(Remember integration constant)

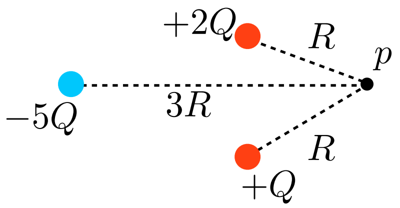


DO NOT CONFUSE U AND V



- U has <u>only</u> <u>one</u> value for collection of charges
- V has a <u>different value at every point</u> in space

<u>Ex. Three point charges:</u>



- What is the potential at point P?

- Recall: $V_p = \dfrac{kQ}{r}$

- We can always <u>use superposition to add up individual potential terms</u>

$$V_p = V_p^{+Q} + V_p^{+2Q} + V_p^{-5Q}$$

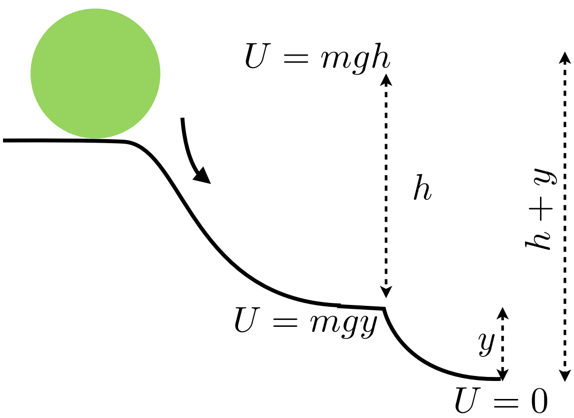
- Problem is simple now:

$$V_p^{+Q} = \frac{kQ}{R} \qquad V_p^{+2Q} = \frac{2kQ}{R} \qquad V_p^{-5Q} = \frac{-5kQ}{(3R)}$$

$$\boxed{V_p = \frac{4}{3}\frac{kQ}{R}}$$

<u>What is Electrostatic Potential?</u>

- Recall from gravity, the change in potential energy of an object above the Earth is: $\Delta U_G = mgh$



- This is a measure of the amount of energy (work) that was required to push the ball up the hill to height h.

- This is also equal to the kinetic energy if the ball rolled back down the hill to h=0

- But what if h does not measure entire height of the hill?

$$\Delta U_G = mg(h+y) - mgy = mgh$$

- The change in potential does not care where U=0 is, only the difference matters.

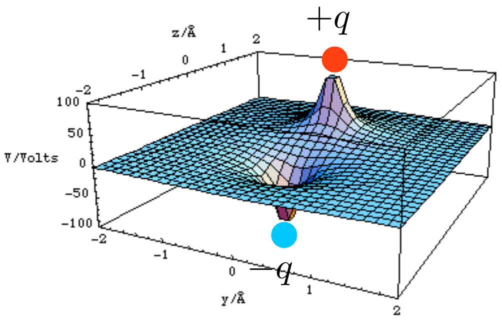
- If $\Delta h = 0$ then the change in potential is zero, for any mass.

- The mass doesn't matter, only the change in height matters, lets divide out the mass:

$$\frac{U_G}{m} = gh = V_G$$

This is the "**Gravitational Potential**": it is a measure of the amount of work required that does not require knowing the mass.

Electric potential works the exact same:



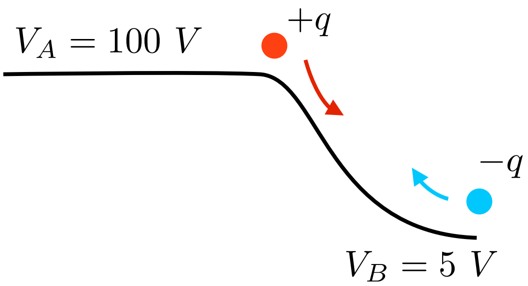
Potential from two oppositely charged particles

"Electrostatic Potential"

$$\frac{U_{\rm el}}{e} = \frac{kQ}{r} = V$$

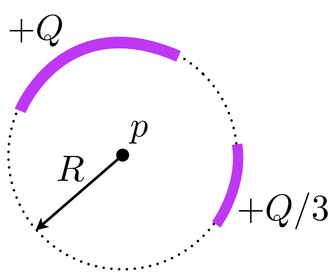
- The electrostatic potential is a measure of the amount of work required to move a particle that does not depend on the charge

The electric potential at a point p is the external work needed to move a unit positive charge from infinitely far away to that point along any path.



- You can think of (+) charges as rolling down the potential hill to lower V. (-) charges go up the hill to higher potentials.

<u>Ex. Charge at equal distance:</u>



- What is the potential at P from two sections of charge at a distance R from P?

- We don't know E so should probably add up V from each piece of charge.

- However we have a special situation: all of the charge is at the same distance away from P.

- Again, let us use super-position principle.

$$V_{\rm net} = \sum_i V_i = \sum_i \frac{kQ_i}{R} = \frac{k}{R}\sum_i Q_i = \frac{kQ_{\rm net}}{R}$$

When all charge is same distance away: $V_p = \dfrac{kQ_{\rm total}}{R}$

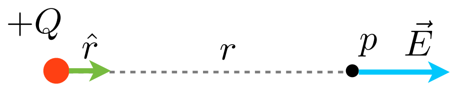
For the above example: $\boxed{V_p = \frac{4}{3}\frac{kQ}{R}}$

- The potential is the work per unit charge.

- The work is proportional to the electric force which itself is proportional to the E-field.

- We now want to relate the electrostatic potential to the E-field.

- Single point charge:



$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$ vector function

- We have already derived the potential: $V_p = \dfrac{kQ}{r}$ scalar function

- Want to show that E is derivative of V: $\dfrac{dV}{dr} = -\dfrac{kQ}{r^2}$

- We want a vector equation, so multiply both sides by $\hat{r}$:

$$\frac{dV}{dr}\hat{r} = -\frac{kQ}{r^2}\hat{r} = -\vec{E} \quad \Rightarrow \quad \boxed{\vec{E} = -\frac{dV}{dr}\hat{r}}$$

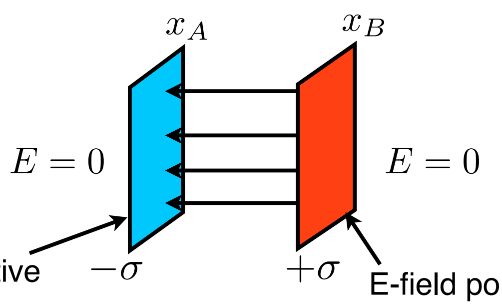
If you know the potential <u>everywhere</u> in space then you can find the E-field.

<u>Ex.</u> $V(x) = 10^5 x \qquad x \in [0, 0.01 \text{ m}]$ V=0 everywhere else

- What is the electric field?

$$\boxed{\vec{E} = -\frac{\partial V}{\partial x} = -10^5\hat{x}} \qquad x \in [0, 0.01 \text{ m}] \qquad E_y = E_z = 0$$

- Inside this region, the E-field has a constant value



This side should be negative

E-field points away from positive charges, so this side should be positive

- Next time we will see that this example is very important in the real world