PHYS-183 : Day #17

Waves:

- So far in this class we have focused on Newton’s laws of motion for systems that we can describe as a single object.

- Now we turn to the equally important concept of waves.

- Perhaps the most familiar wave to many people are water waves.

- However, there are many other important physical processes that can also be modeled as waves: Sounds, light, cell phone signals, ...

- At the most basic level, the laws of physics tell us that <u>everything</u> must behave as a wave

- Waves and their properties will be the focus for most of the remaining semester.

The damped harmonic oscillator :

- The harmonic oscillator is defined by the linear restoring force: $F = -kx$

- The characteristic motion of the variation in the oscillators position as a function of time is given by

$$x(t) = A\cos(\omega_0 t)$$

$$A = \text{amplitude}$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \text{angular frequency}$$

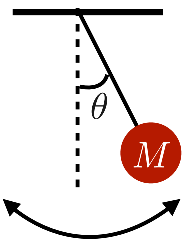
- The oscillators **natural frequency** f_0 and period T can both be defined from the angular frequency

$$f_0 = \frac{1}{T} = \frac{\omega_0}{2\pi}$$

- The natural frequency f_0 is the frequency of the oscillator with no external forces acting on it.

- With no external forces, the energy of the oscillator is conserved, with the energy constantly being exchanged between kinetic and potential energy.

- A good example of a system that behaves as a harmonic oscillator is the simple-pendulum created by hanging a mass from a massless string with length L



- If the angle θ is small (<10deg), then the angle of the mass from the vertical is given by

$$\theta(t) = \theta_{\text{max}}\cos(\omega_0 t)$$

- The frequency of the pendulum is independent of the mass:

$$\omega_0 = \sqrt{\frac{L}{g}}$$

Friction :

- Of course, in the real world, there is always friction.

- To a good approximation, the frictional force is proportional to the velocity of the oscillator.

$$F_{\text{net}} = -kx - bv \qquad b = \text{fiction constant}$$

friction term

- What happens to the motion of the harmonic oscillator when there is friction present?

- If the friction term is small, then we might guess that the motion will still be harmonic, but the amplitude will decrease

- This is correct. As a function of time, the motion can be expressed as

$$x(t) = \left(Ae^{-\frac{bt}{2m}}\right)\cos(\omega_{\text{damp}}t)$$

- The frequency of the oscillator has been modified by the friction force

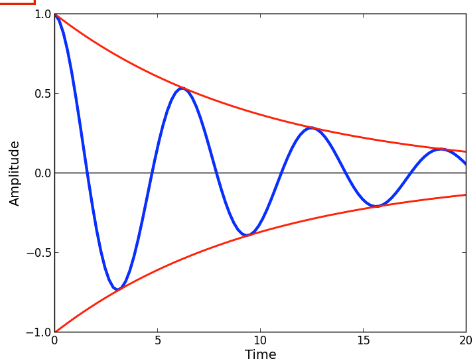
$$\boxed{\omega_{\text{damp}} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

- If there is <u>no friction</u>, then the energy of the oscillator is constant

$$E = 1/2kA^2$$

- Replacing the amplitude with the one found above:

$$\boxed{E = \frac{1}{2}kA^2e^{-\frac{bt}{m}}}$$



- Although the energy is lost exponentially, the period of the oscillation remains the same:

$$T = \frac{2\pi}{\omega_{\text{damp}}}$$

<u>Ex.</u> A 0.2kg mass is attached to a spring with a spring constant of k=40N/m and with a friction constant b=0.02kg/s. Assuming the spring is initially stretched by 0.1m, what is the amount it takes for the oscillator to lose 1/2 of its energy?

- The energy as a function of time is given by $E(t) = \frac{1}{2}kA^2e^{-\frac{bt}{m}} = E(0)e^{-\frac{bt}{m}}$

- If the energy has been reduced to 1/2 of its initial value then we have: $\frac{1}{2} = e^{-\frac{bt}{m}}$

- Solving for the time: $t = -\frac{m}{b}\log\left(\frac{1}{2}\right)$

- Plugging in the numbers: $t = \frac{0.2}{0.02}\log(2) = 10\log(2) = \boxed{6.9\text{s}}$

Driven harmonic oscillators:

- In order to keep the amplitude constant, we must add energy to oscillator from some external force

- A harmonic oscillator that has an external force and friction acting on it is called a **damped-driven harmonic oscillator** (“damped”=friction)

$$F_{\rm net} = -kx - bv + F_{\rm ext}$$

- If the external force is sinusoidal with an angular frequency $\omega_{\rm ext}$ then, after a sufficiently long time, the oscillator reaches a **steady-state** where the motion remains periodic

$$x(t) = A(\omega_0, \omega_{\rm ext})\cos(\omega_{\rm ext}t + \phi)$$

- The amplitude of the oscillation depends on <u>both</u> the natural and external frequencies.

- There is also now a phase angle ϕ, indicating that the external force and spring force may not oscillate the same in time.

- <u>In steady-state, the amplitude of the oscillation is fixed</u>. This means that the energy lost due to friction is balanced by the energy gained from the external force.

- How do we control the amount of energy from the external force to the oscillator?

- The amplitude of the external force will be important, but the frequency $\omega_{\rm ext}$ of the external force is also important

- For a fixed amplitude force, the amount of energy input into the oscillator depends on how close the external frequency $\omega_{\rm ext}$ is to the oscillators natural frequency ω_0 .

- Why? Because the power (energy/time) is given by $P = \vec{F}_{\rm ext} \cdot \vec{v}_{\rm osc}$

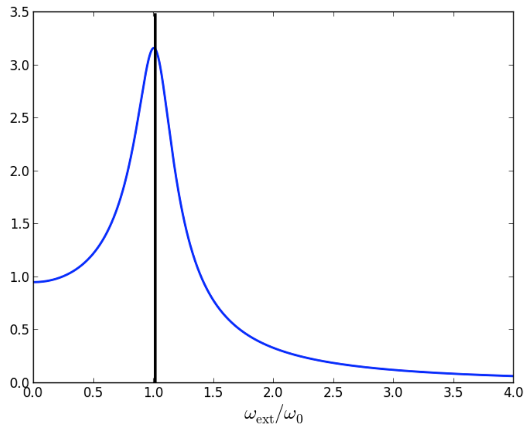
  - If $\omega_{\rm ext}$ and ω_0 are not close, then $\vec{F}_{\rm ext} \cdot \vec{v}_{\rm osc} \rightarrow 0$

- Mathematically, the amplitude is given by

$$A = \frac{F_{\rm ext}/m}{\sqrt{(\omega_0^2 - \omega_{\rm ext}^2)^2 + \left(\frac{b\omega_{\rm ext}}{m}\right)^2}}$$

- The maximum occurs when $\omega_{\rm ext} = \omega_0$

- This condition is called **resonance**.

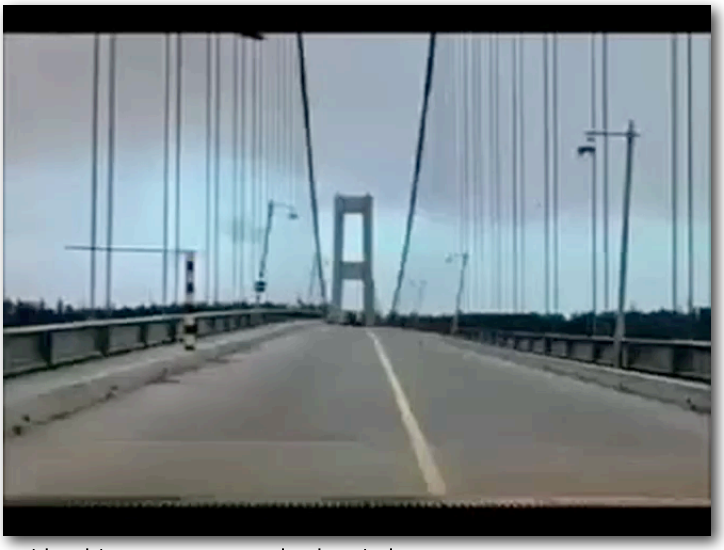


- When an oscillator is driven on resonance, the amplitude of the motion becomes extremely large.

- Changing a channel on the radio or television is an example of changing resonance frequencies.

- Resonance is particularly important for buildings and bridges.

- If the building experiences a force near one of the resonance frequencies of the building, then the building can shake itself apart.



Bridge driven on resonance by the wind.

<u>Wave concepts:</u>

- There are many kinds of waves.

- Mechanical waves are vibrations that travel through a material such as water and air (sound waves), waves along a string (musical instruments), or seismic waves (from earthquakes).

- A general property of all waves (except electromagnetic waves) is that they travel through material with a characteristic speed over long distances.

- Although a wave travels, the actual molecules and atoms of the material do not move, instead they vibrate about their equilibrium positions at a different speed.

- Molecules and atoms in the material do not travel in the same direction as the wave.

- Here, the motion of the rope is vertical, but the wave pulses made by the man are moving <u>in the tangential direction</u>.

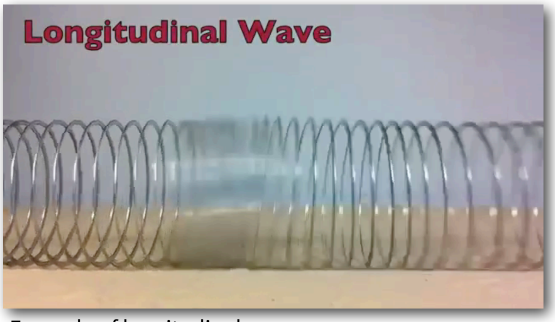
- Such waves are called **transverse waves**.

Example of transverse waves

- If we replace the rope with a spring, then the oscillations in the spring are <u>in the same direction</u> as the wave moves

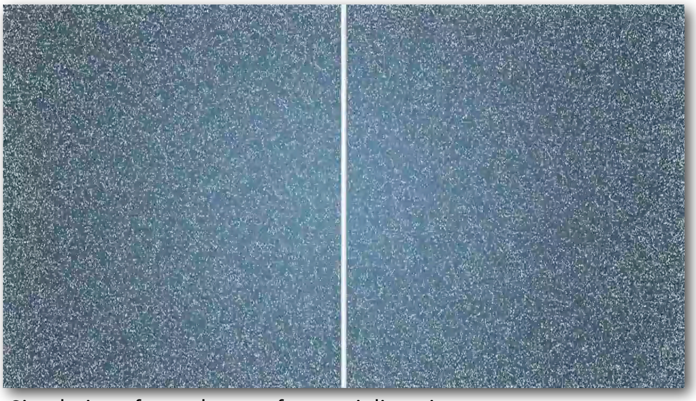
- These kind of waves are called **longitudinal waves**.

- Some waves are transverse, some are longitudinal, and some are a combination of both.



Example of longitudinal waves.

- Waves on the ocean are both transverse and longitudinal, but sound waves in air are longitudinal only.



Simulation of sound waves from a violin string

- Waves that can be described by sinusoidal functions are called **harmonic waves**.

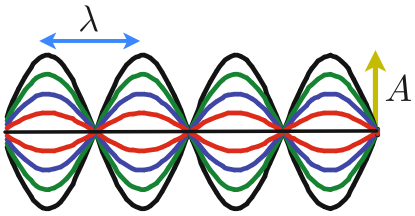
- There are two general kinds of waves:

Standing waves are waves that have a constant position

Traveling waves are waves that have move in a specific direction as a function of time.

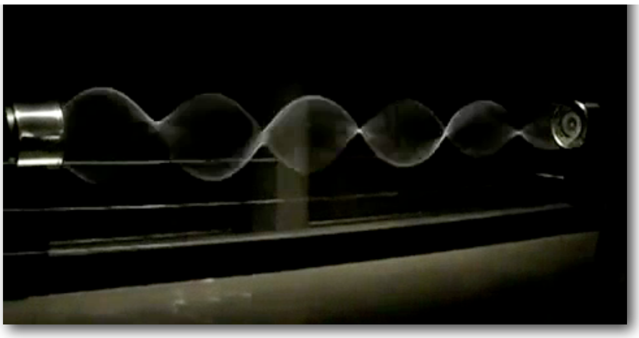
<u>Standing Waves:</u>

- One way to generate standing waves is to drive a harmonic oscillator on resonance



$A =$ wave amplitude

$\lambda =$ wavelength



- Instead of wavelength λ we can also define the **wave number:**

$$k = 2\pi/\lambda$$

- If we take a picture of the string, then the displacement as a function of position x is:

$$y(x) = A\sin(kx)$$

-As we move along the x-direction, every time $kx = \dfrac{2\pi x}{\lambda}$ increases by 2π then the amplitude is the same

Traveling Waves:

- Our standing wave on the string is actually a specific example of a traveling wave.

- Standing waves are actually waves that move so that their shape does not change.

- We can describe a general traveling wave by using an expression where the amplitude changes as both a function of space and time.

$$y(x,t) = A\sin(kx - \omega t) \qquad\qquad (\omega = 2\pi f)$$

- Using a constant time (ex. t=0) then this equation is the same as that of the standing wave amplitude.

- Different values for t (ex. t=1,2,..) just shift the the argument of the sine function.

- If we fix the position x instead of time, then we see that the amplitude oscillates up and down with amplitude *A* and frequency ω

$$y(0,t) = A\sin(-\omega t)$$

- Each element of the string moves up and down only.

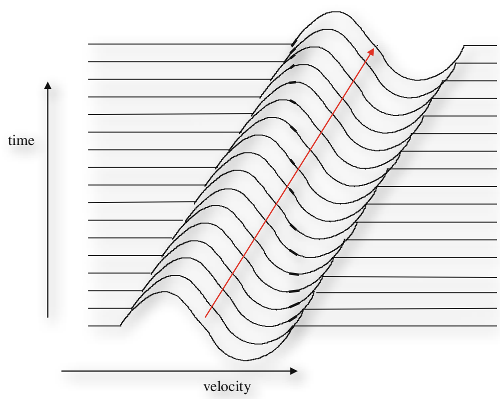
- Since the wave is traveling, it must have a velocity. This velocity can be found using the waves parameters ω and k

$$v = \frac{x}{t} = \lambda f = \frac{2\pi f}{2\pi/\lambda} = \frac{\omega}{k}$$

- The entire wave y(x,t) moves along the +x-direction with velocity $v = \omega/k$

$y(x,t) = A\sin(kx - \omega t)$ ⟵ moves in +x-direction

$y(x,t) = A\sin(kx + \omega t)$ ⟵ moves in -x-direction



Ex. A traveling wave on a string is described by the equation:

$$y(x,t) = 0.025\sin(1.5x - 200t)$$

what is the waves amplitude, wavelength, frequency, period, and velocity if x and y are measured in meters, and t in sec?

Solution:

- The amplitude of the wave can be obtained directly: $A = 0.025\ \mathrm{m}$

- To get the wavelength we must convert from wave number: $\lambda = 2\pi/k = 2\pi/(1.5) = 4.2\ \mathrm{m}$

- Frequency must come from angular frequency: $f = \omega/2\pi = 200/2\pi = 31.8\ \mathrm{Hz}$

- The period is just the inverse of the frequency: $T = 1/f = 1/31.8 = 0.03\ \mathrm{s}$

- The velocity can be calculated from ω and k : $v = \omega/k = 200/1.5 = 133\ \mathrm{m/s}$

What is moving in a wave?:

- We have seen that each point in a wave does not move in the same direction as the wave, but instead moves harmonically in the tangential direction

$$y(x,t) = A\sin(kx - \omega t)$$

- **Key idea**: Each point in the wave moves exactly like a harmonic oscillator.

- Therefore, the question is what exactly is moving when a wave moves?

- **Waves transfer energy from one place to another.**

- For example, given a external force acting on one end of a string, a wave transfers the energy from this force to the other end of the string at the velocity $v = \omega/k$

-As a wave moves down a string, the amplitude at each location changes, causing a change in the kinetic energy of the harmonic oscillator at each point.

Maximum kinetic energy when $y = 0$

No kinetic energy when $y = \pm A$