Capacitors

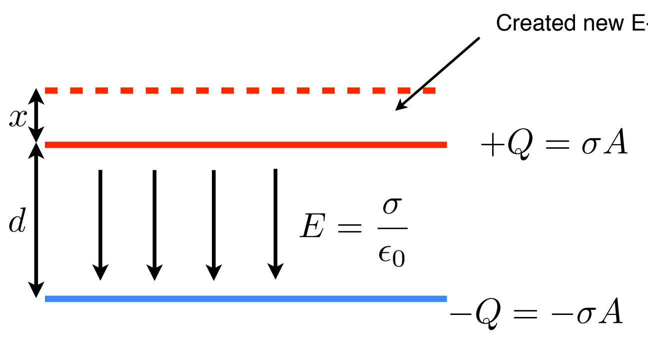
- Recall from last time: In order to bring a collection charges together I must do work.

- This work we defined as the **Electrostatic Potential Energy**

- The question is now: where is this energy stored?

- As we will show, this energy is stored in the electric field.

- Suppose I have two-parallel charged plates:



- E-field is uniform inside (as we showed)

- Now suppose I move the top plate up a distance x

- It is clear that I must do work since the upper plate and lower plate attract each other

- New E-field created has exactly the same strength as original E-field since charge on plates did not change.

- To create new E-field, I must do work.

- What is the amount of work I must do to create this E-field?

$$W = \int \vec{F} \cdot d\vec{l} = Fx$$ since F and x in same direction and F is constant

- What is the force that I must do?

  - First guess: $F_{el} = QE$

  - Makes since that if I have a charge Q and an E-field then the force I must do is QE

  - True most of the time, but not here!

- Lets zoom in on the top plate:

$$E = 0 \qquad +Q$$

$$E = \frac{\sigma}{\epsilon_0}$$

- Charge must be on surface

- Plates are conductors so E-field inside is going to be zero.

- E-field on Q is average of E-field

$$F_{el} = \frac{1}{2} QE$$

-Now we can calculate the work we have to do:

$$W = \frac{1}{2} QEx = \frac{1}{2} \underbrace{\sigma AEx \cdot \frac{\epsilon_0}{\epsilon_0}}_{= E} = \frac{1}{2}\epsilon_0 E^2 Ax$$

$Q = \sigma A$

- Volume of new E-field created

- Can now define the **field energy density**:

$$\frac{W}{\mathrm{vol}} = \frac{1}{2}\epsilon_0 E^2$$

- This result can be shown for <u>any</u> charge distribution

- Therefore, instead of adding up the work due to individual charges, we can define the electrostatic potential energy as:

$$\boxed{U = \int_{\mathrm{allspace}} \frac{1}{2}\epsilon_0 E^2 dV}$$

- Potential energy now thought of as the work needed to create an E-field in a given volume

- Energy is **in** the E-field

- If you know E-field everywhere you can do integration over all space

- For the parallel plates, we know E-field outside of plates is zero -> volume is just volume inside

$$U = \int_{\text{allspace}} \frac{1}{2}\epsilon_0 E^2 dV = \frac{1}{2}\epsilon_0 E^2 Ad$$ Since spacing of original distance was d

- since $E = \sigma/\epsilon_0$, we get: $U = \dfrac{\sigma^2 Ad}{2\epsilon_0}$

- We also know that $Q = \sigma A$ and $V = Ed$

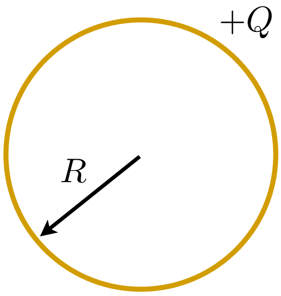
potential not volume!

- So finally, after substitution: $\boxed{U = \frac{1}{2}QV}$ (for the plates)

- Now we define a new quantity: Capacitance $C = \dfrac{Q}{V}$ $\left[\dfrac{C}{V} = F\right]$ Farads

  - The charge on an object divided by the potential of the object.
  - Capability to hold charge for a given potential

- Consider a charged conducting sphere:



- What is the capacitance?

- We know what the potential of a charged sphere is:

$$V = \frac{Q}{4\pi\epsilon_0 R}$$

- Therefore, by definition: $C = \dfrac{Q}{V} = 4\pi\epsilon_0 R$

(Capacitance of a single sphere)

<u>Some examples:</u>

1 F	R = 9x10^9 m	To moon and back 11 times
700 μF	R = 6.4x10^6 m	Earth
3x10^-12 F	R = 2.4x 10^-3 m	100 Won coin

- If they all has same potential, then largest one has most charge.

- Lets look at capacitance another way: consider two spheres with equal, but opposite sign charges:

- By definition: $C_B = \dfrac{Q_B}{V_B}$

- But sphere A has negative charge!

- Recall that potential is the work per unit charge to

  - I put charge +q in my pocket, start from infinity and go to sphere B. The work per unit charge that I must do is the potential.

  - Sphere B is repulsive -> positive work, but sphere A is attractive so the work is less than if it was just sphere B.

Sphere A nearby causes the potential on B to drop!

$$\uparrow C_B = \frac{Q_B}{V_B \downarrow}$$

- Sphere A has a big impact on the capacitance of sphere B.
  - Cannot consider just sphere B when defining capacitance

- We will change the definition of capacitance:

Given **two** conductors with the **same** magnitude of charge Q but different sign:

$$C = \frac{Q}{V}$$ Potential difference <u>between</u> conductors

- We will always think about the capacitance between two objects <u>**not**</u> single objects

- This is not an artificial example: charge must be conserved so objects typically come with equal but opposite signed charges.

- What is the capacitance of the two parallel plates for this new definition?

$$C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \frac{\sigma A \epsilon_0}{\sigma d} = \frac{A\epsilon_0}{d}$$

Capacitance depends only on geometry!

- Linearly proportional to area: obviously, and inversely proportional to the distance between plates.

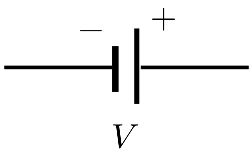
Capacitors in Electrical Circuits:

- Now we will look at capacitors in **electrical circuits**:

  An electrical circuit consists of conducting paths formed by wires that connect various circuit elements together.

Two basic elements we have learned so far:

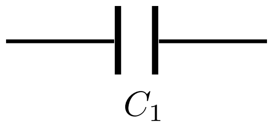
Voltage Source:



- Makes a **fixed** potential difference

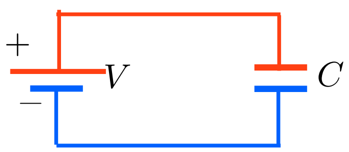
- Since only potential differences matter set V=0 at negative side -> positive side has potential = V

Capacitor:



- Stores energy in electric field due to charge stored on plates

Ex. Basic voltage + capacitor circuit



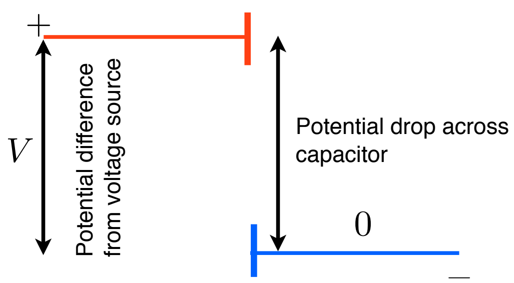
- All the parts of the circuit touching the + side of V have the same potential = V

- All the parts of the circuit touching the - side of V have the same potential = 0

- The only change in voltage happens across a circuit element

- The potential changes across V, and the potential changes across C.

- Recall that we can think of potential as the height of a mountain:



- Initial “Height” of circuit is V

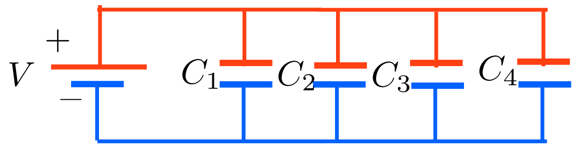
- Here, the height must drop to zero across C since it is the only element between V & 0.

- The total charge one the capacitor is:

$$Q = CV$$

Ex. Capacitors in parallel

- Of course, we can have more than one capacitor.
  - We can have capacitors in parallel (side by side)



- Each capacitor has the same voltage drop across it
- Capacitance of each element can be different

- The charge on each capacitor is thus:

$$Q_1 = C_1 V \qquad Q_2 = C_2 V \qquad Q_3 = C_3 V \qquad Q_4 = C_4 V$$

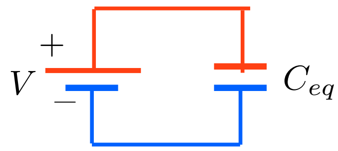
- Since the voltage V is the same for all, lets combine to find the total charge:

$$Q_{\text{total}} = C_1 V + C_2 V + C_3 V + C_4 V = (C_1 + C_2 + C_3 + C_4)\, V = C_{\text{eq}} V$$

- **Capacitors in parallel are equivalent (eq) to a single capacitor with capacitance:**

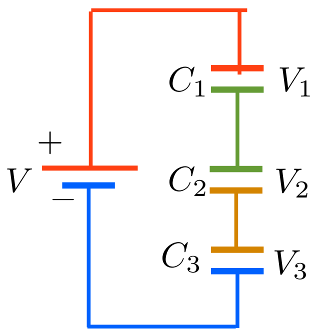
$$\boxed{C_{eq}^{\text{parallel}} = \sum C_i^{\text{parallel}}}$$

Example above becomes

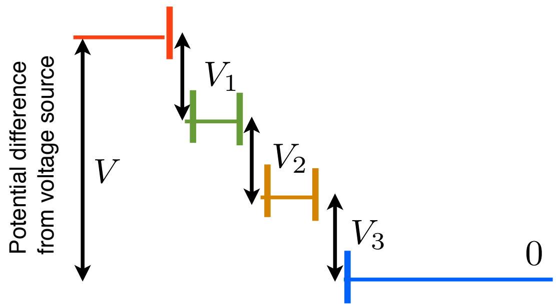


Ex. Capacitors in series

- Can also have capacitors **in series:** one after the other

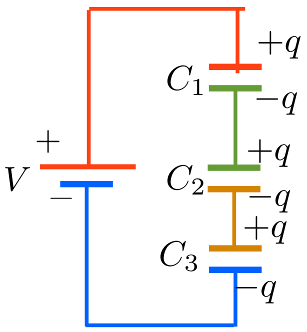


- No longer have just two different voltages.
- Now have three separate voltage drops



- Remember that only voltages across circuit elements matter

- What is the charge on each capacitor?
  - If there is no voltage source initially, all parts of the circuit are neutral
  - Once voltage is connected, a charge builds up on the capacitors



- The voltage causes a charge +q on the plate of C1 at voltage V

- An induced charge of -q is then created on the opposite plate.

- Because the circuit was originally neutral, and not net charge can be created, this induced -q causes +q to move to the opposite plate on C2.

- This happens for <u>every</u> capacitor in the series.

- Since we know $V = V_1 + V_2 + V_3$, each capacitor has charge q and $V = q/C$:

$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{q}{C_{eq}}$$

- We see that **a collection of capacitors in series is again equivalent to a single capacitor with capacitance:**

$$\boxed{\frac{1}{C_{eq}^{\text{series}}} = \sum \frac{1}{C_i^{\text{series}}}}$$

Example above becomes