Sphere #2:

- Same thing as sphere #1, but must add up charge over entire sphere #2.

$$Q_2^{\mathrm{enc}} = \frac{4}{3}\rho_2 \pi R^3$$

- E-field at P like point charge a distance 3R/2 away.

$$\boxed{E = \frac{4}{3}\rho_2 \pi R^3 \frac{1}{4\pi\epsilon_0 (3R/2)^2} = \frac{4\rho_2 R}{27\epsilon_0}}$$

- To find ratio, set E1=E2 and solve:

$$\frac{\rho_1 R}{6\epsilon_0} = \frac{4\rho_2 R}{27\epsilon_0} \quad \Rightarrow \quad \boxed{\frac{\rho_1}{\rho_2} = \frac{8}{9}}$$

Electric Potential

- We are going to learn about two new concepts:

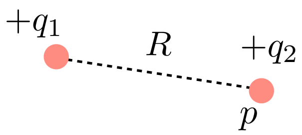
"Electrostatic Potential Energy" : U

"Electrostatic Potential" : V

These are independent concepts (as we will see)

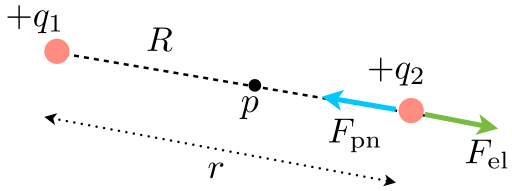
Electrostatic Potential Energy:

- Consider the following situation:



- It is clear that I had to do work to bring the charges together.
  - The charges repel since they are the same sign.
  - Like trying to push in a spring
  - If charges were connected by a string, and I cut it, the charges would fly apart.

- I had put work into this problem. This work is Electrostatic Potential Energy
- How much work do I have to do?
- If space is empty, it takes no work to put q1



- I Paul Nation want to bring q2 in from infinity (far away)
- I have to push and push
- Work is positive since F in same direction as charge is moving

- The work I do from infinity to R is thus:

$$W_{pn} = \int_{\infty}^{R} \vec{F}_{pn} \cdot \vec{dr} = -\int_{\infty}^{R} \vec{F}_{el} \cdot \vec{dr} = \int_{R}^{\infty} \vec{F}_{el} \cdot \vec{dr}$$

Since $\vec{F}_{el} = -\vec{F}_{pn}$

- We know what $\vec{F}_{el}$ is, also, force is in same direction as dr (since R -> infinity) -> $\cos\theta = 1$

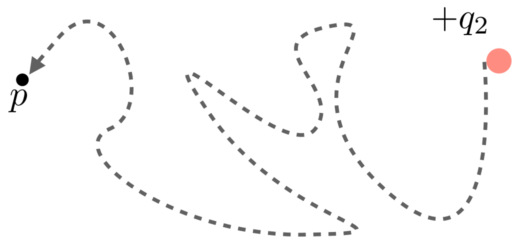
$$W_{pn} = \frac{q_1 q_2}{4\pi\epsilon_0} \int_{R}^{\infty} \frac{dr}{r^2} = -\frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r}\Big|_{R}^{\infty} = \frac{q_1 q_2}{4\pi\epsilon_0 R}$$

- This is the electrostatic potential energy U:

$$\boxed{U = \frac{q_1 q_2}{4\pi\epsilon_0 R} \quad [J]}$$

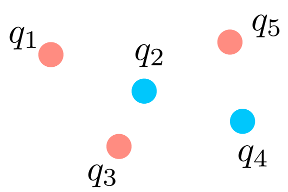
- This is a scalar valued function (not a field): only one number.
- If both q1 and q2 positive or negative -> work is positive.
- If charges have opposite sign -> work is negative since charges attractive.

- You should convince yourself that if I took a strange path to get to point P, that the total work remains the same



Electric force is **conservative force**:
Work does not depend on path taken

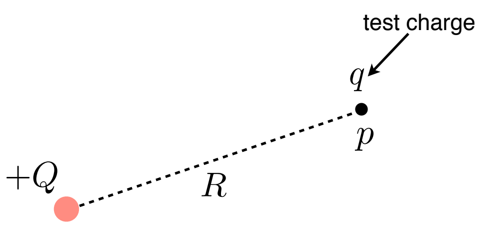
- What if I have a collection of charges?



- How do I find the the total amount of work?

- Bring in charges on at a time and add up the work from each contribution.

Electrostatic Potential:



- We already know what the electrostatic potential energy is:

$$U = \frac{qQ}{4\pi\epsilon_0 R}$$

- "**Electrostatic potential**" - Work per unit charge need to bring charge from infinity to point P.

$$V_p = \frac{W}{q} = \frac{U}{q} = \frac{Q}{4\pi\epsilon_0 R}$$

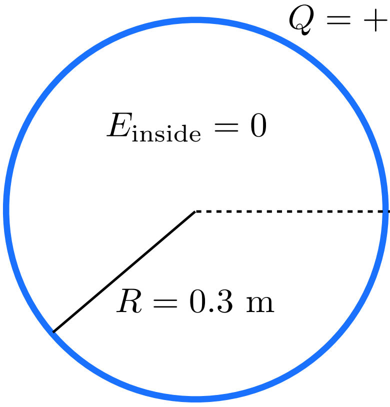
$$\boxed{V_p = \frac{Q}{4\pi\epsilon_0 R} \quad \left[\frac{J}{C}\right] = [V] \ \text{Volts}}$$

- Because potential from point charge is proportional to 1/r, potential is zero at infinity: $V_\infty = 0$

- Notice that if Q is positive -> potential is everywhere positive in space.

- Negative Q produces negative potential everywhere in space.

- Consider a uniformly charged spherical shell:



- What is the potential inside and outside of shell?

Outside:

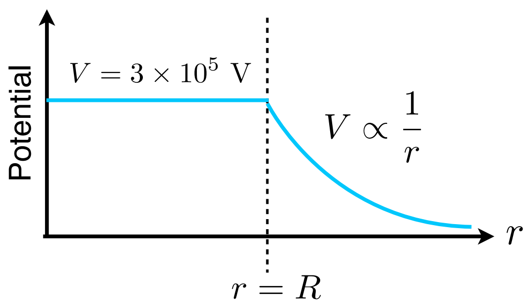
$$V_p = \int_r^\infty \underbrace{\frac{\vec{F}_{el}}{q}}_{=\vec{E}} \cdot dr = \int_r^\infty \vec{E} \cdot dr = \frac{Q}{4\pi\epsilon_0 r}$$

@r=R: 3x10^5 V   @r=3R: 1x10^5 V   @r=100R: 3000 V

- Recall: $W_{pn} = qV$ If q=1C -> 3x10^5 J of work: equivalent of 1.5 times up Namsan Tower

Inside:

- Remember E=0 inside shell of charge!
- No E-field -> no force -> no work -> constant potential



Equipotential Surfaces:

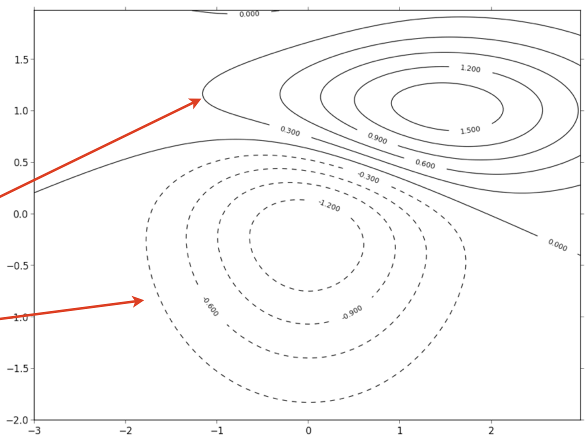
- From the potential: $V_p = \frac{Q}{4\pi\epsilon_0 R}$ it is clear that the potential is constant over a spherical surface.

- Surfaces over which the potential is constant are called “**Equipotential Surfaces**”

- We can plot these surfaces like mountains are plotted in topographic maps

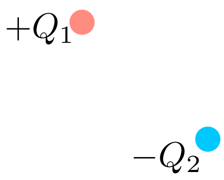
Potential is constant on this line

Potential is also constant here, but with different value than other lines



Equipotential surfaces **NEVER** cross!

- What is the potential at point P from more than one charge?



- As usual, use superposition: just add up individual potentials

$$V_p = V_p^{Q_1} + V_p^{Q_2}$$

positive potential

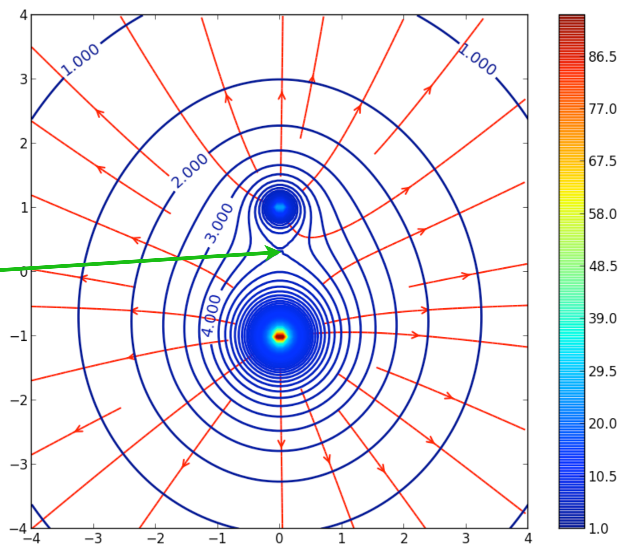
negative potential

- Consider (+1) and (+4) charges:
  - Everywhere potential is positive
  - Close to charges, surfaces are spheres
  - Far away, looks like single (+5) charge-> spherical surfaces
  - In between charges, surfaces are strange
  - One surface has point where E=0

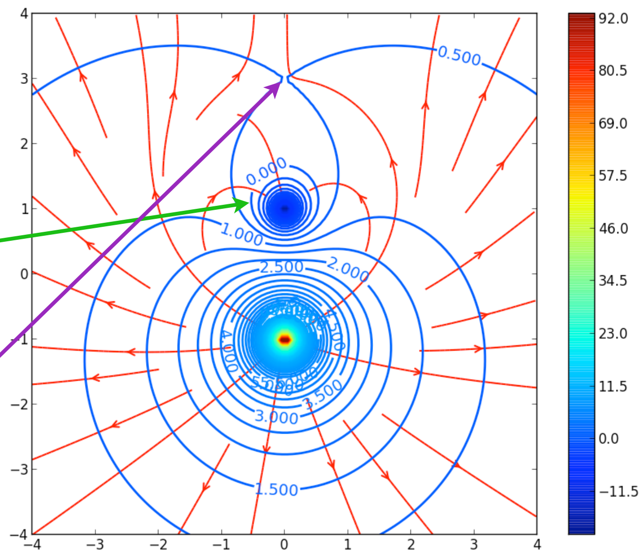
    DOES NOT MEAN V=0

    - V>0 everywhere
  - E=0 -> no force, can rest from pushing
  - E-field lines <u>ALWAYS</u> perpendicular to surfaces.

Equipotential & E-field from (+1) and (+4) Charges

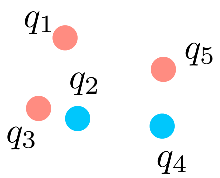


- Consider (-1) and (+4) charges:
- Far away, looks like single (+3) charge-> spherical surfaces
- Positive potential near (+4)
- Negative potential near (-1)
- Surface where potential V=0
  - E-field is <u>NOT</u> zero here!
- V=0 surface means that a if I bring test charge from infinity to that point, total work is zero
- Also have E=0 point (we did this example in class)



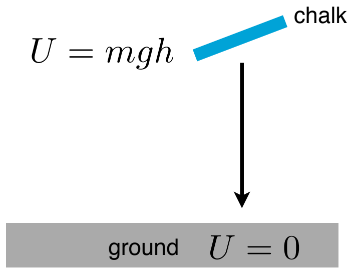
- Why do we need potential? If we know E-field, don't we know everything?
  - Yes!, but E-field can become very very complicated.
  - It is easier to work with potentials if you do not need trajectories, and only want to know what the change in kinetic energy is.

DO NOT CONFUSE U AND V



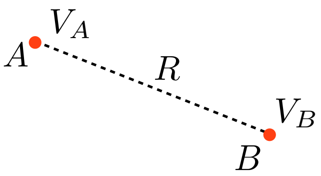
- U has <u>only</u> <u>one</u> value for collection of charges
- V is different everywhere

- For gravity, objects will go from higher potential to lower potential



- Likewise, positive charges go from high -> low potential
- Negative charges go from low -> high potential

- Suppose I have two points A & B, and I specify the potentials there:



- By definition: $V_A = \int_A^\infty \vec{E} \cdot d\vec{r}$; $V_B = \int_B^\infty \vec{E} \cdot d\vec{r}$

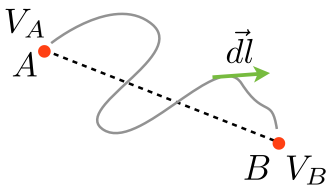
- Therefore, potential difference from A to B is:

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{r}$$

$$V_B - V_A = \Delta V = -\int_A^B \vec{E} \cdot d\vec{r} \quad \text{(used in book)}$$

- If there is no E-field between A & B then the potential is the same at A & B

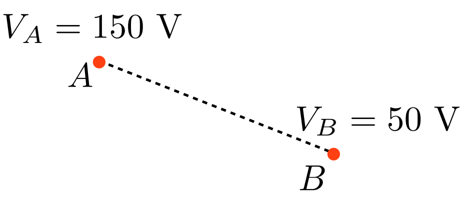
- Since the electrostatic force is conservative, we can choose any path
  - $\vec{dr}$ is radial direction only -> works for straight paths only



- define general line element
- $\vec{dl}$ is perpendicular to path taken

$$V_A - V_B = \int_A^B \vec{E} \cdot \vec{dl}$$

- Lets assume the following:



- If I have charge +q in my pocket and I go from infinity -> B, I must do work: $W_{pn} = qV_B = 50q$
- Infinity -> A: $W_{pn} = qV_A = 150q$
- If I first to go B, then I need to do extra work to get to A

- If I release +q at A, then it wants to go back to B since lower potential there.
  - Energy is released when going back to B

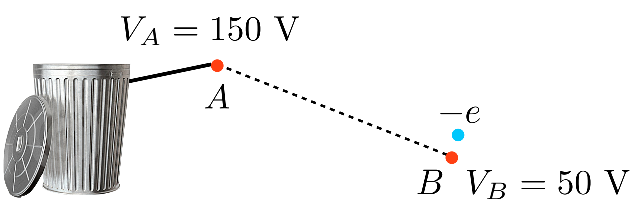
$$\Delta U = q\left(V_A - V_B\right) = -q\Delta V$$

  - Since force is conservative, potential energy must be converted to kinetic energy

$$q\left(V_A - V_B\right) = K_B - K_A \quad \text{or} \quad \Delta K = -q\Delta V$$

- **Any** piece of metal is an equipotential surface!
  - As long as no charge is moving inside
  - Charges free to move in conductor -> move until in equilibrium -> no force
  - No force -> E=0 -> potential is constant
  - Charges in conductor move around until they kill the E-field.

- I can attach a piece of metal to point A:



- Trash can is everywhere 150V
- Suppose I release electron at B
- Electron speeds up toward A

- E-field is very complicated -> Very hard to calculate trajectories
- If I only want to know KE then I can use potentials

$$q\left(V_A - V_B\right) = K_B - K_A$$

- If KB=0:

$$100e = \frac{1}{2} m_e v_A^2 \qquad v_A \approx 5.3 \times 10^5\ \mathrm{m/s}$$

- Recall that all potentials defined relative to infinity where V=0 since V~1/r
- However, it actually doesn't matter where you set V=0

$$q\left(V_A - V_B\right) = K_B - K_A$$

- Answer is same if VB=0 and VA=100 or if VA=10 and VB=-90,....

ONLY DIFFERENCES IN POTENTIAL MATTER

- In engineering, the Earth is typically assumed to be V=0