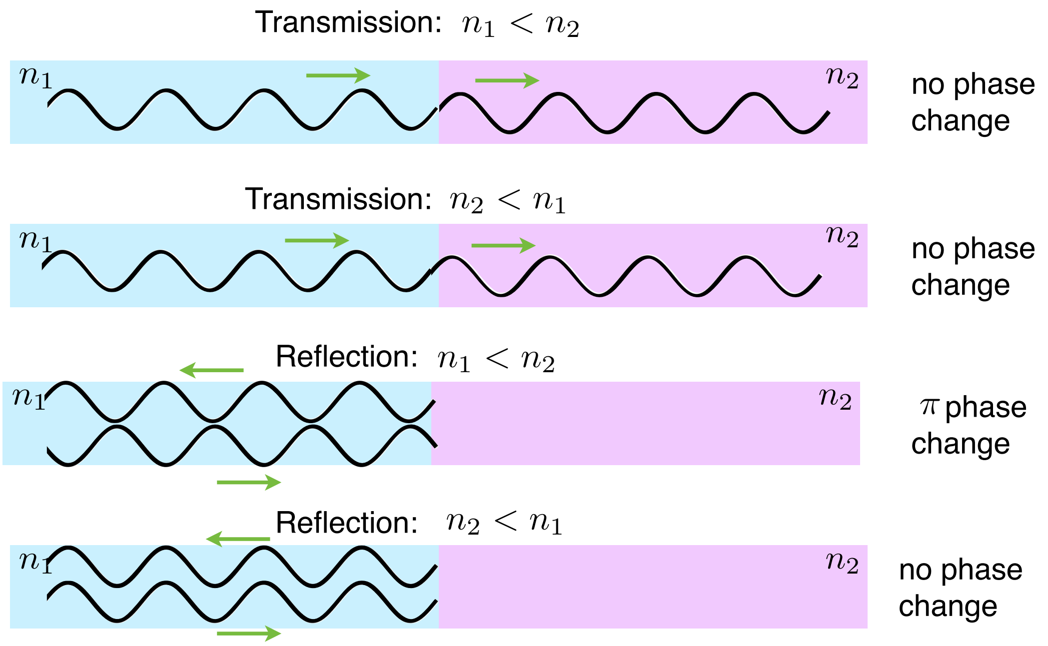
Thin-film Interference:

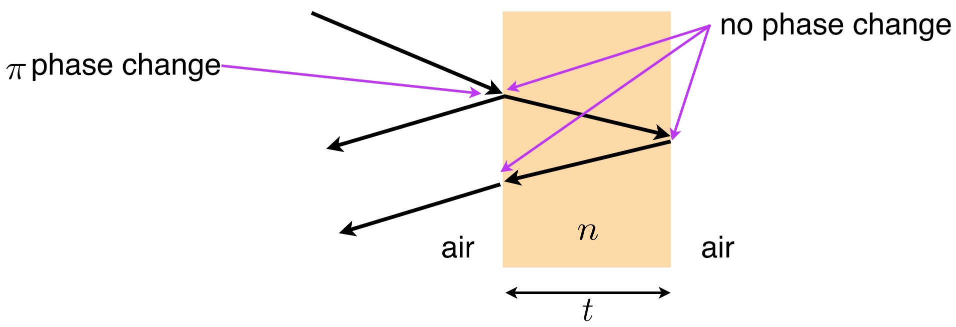
- A **thin film** is a material that light can travel though that is only a few wavelengths in thickness

- To understand the interference in thin-films we must first look at what happens to the phase of a light ray at the boundary between two materials with different refractive indices.



- In general, at the interface between to different media <u>there is always transmission and reflection</u>

- Suppose we have the following thin-film setup where $n > n_{\text{air}}$:



- We are only interested in waves that are reflected back to the left

    - Waves reflected off surface of n have been sifted by half a wavelength (π)

    - Waves that are transmitted, and then reflected off the back of n, have a phase shift determined by the extra distance traveled $\Delta x = 2t$

- The reflected and transmitted waves constructively interfere when

$$\Delta x = 2t = \left(m + \frac{1}{2}\right)\lambda_n \quad m = 0, 1, \ldots$$

wavelength in medium "n"

- But we also know that the wavelength inside the medium is related to the wavelength in air via $\lambda_n = \lambda_{\text{air}}/n$

$$\rightarrow \quad \boxed{\left(m + \frac{1}{2}\right)\frac{\lambda_{\text{air}}}{n} = 2t} \qquad \text{Constructive interference}$$

- This formula also indicates that there is a minimum thickness that the medium must have for interference to occur:

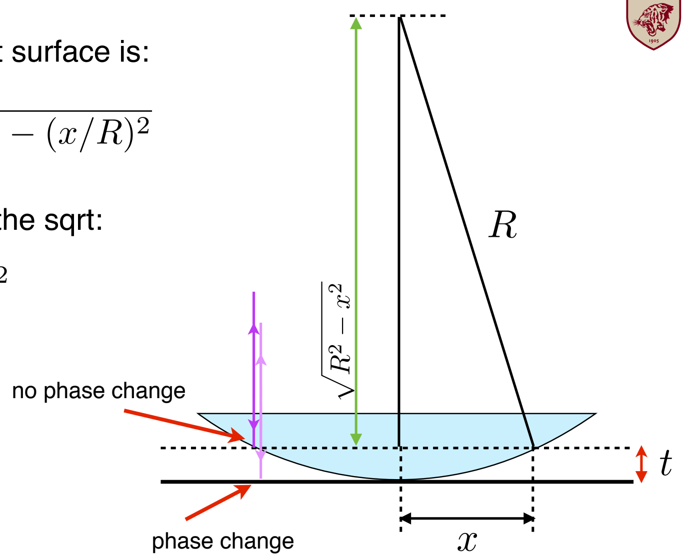
$$t_{\text{min}} = \frac{\lambda_{\text{air}}}{4n}$$

Newton's Rings:

- Similar to thing-film interference, we can have interference of light off of two surfaces, a spherical surface and a flat surface, called **Newton's rings**.

- Distance between curved surface and flat surface is:

$$t = R - \sqrt{R^2 - x^2} = R - R\sqrt{1 - (x/R)^2}$$



- Typically x<<R so we can Taylor expand the sqrt:

$$\sqrt{1 - (x/R)^2} \approx 1 - \frac{1}{2}\left(\frac{x}{R}\right)^2$$

- The distance now can be written as:

$$t \approx \frac{1}{2}\frac{x^2}{R}$$

- The total path difference is 2*t

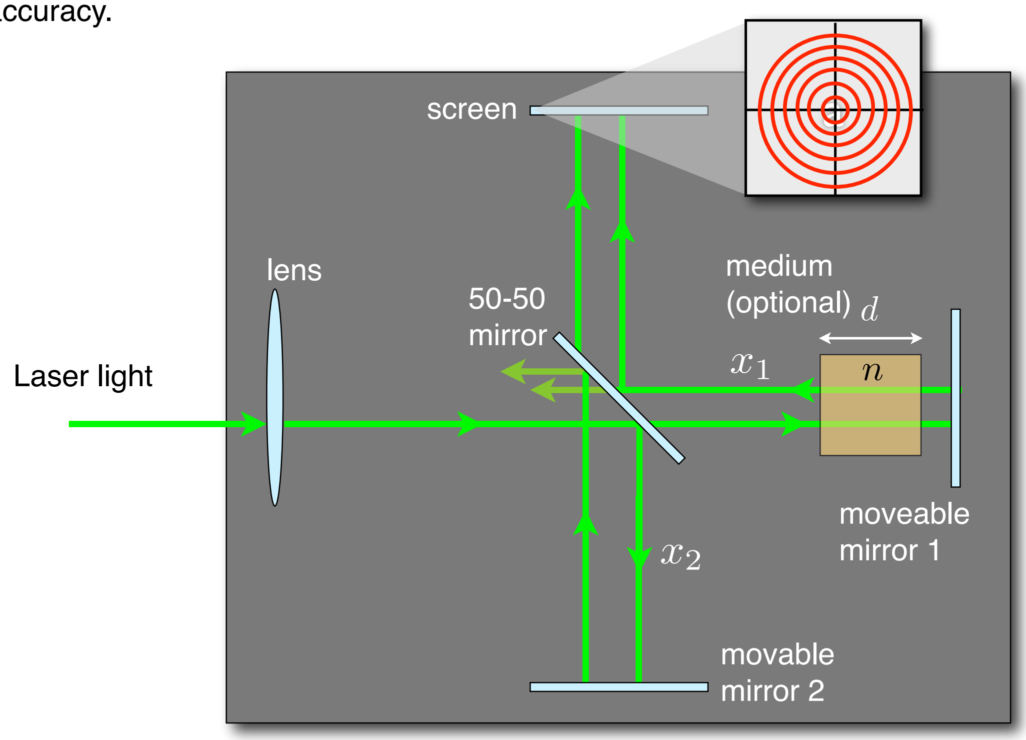
- Because of the pi phase-shift due to the reflection off the flat surface, constructive interference requires an additional 1/2 wavelength:

$$\boxed{\left(m + \frac{1}{2}\right)\lambda = 2t = \frac{x_m^2}{R}}$$

radius of m-th bright ring from constructive interference

Interferometer:

- Because the interference of light depends only on the change in length Δx , we can use an **interferometer** to measure lengths, or changes in length, will very high accuracy.



- Light from mirrors #1 & #2 hits the screen and interferes depending on the difference in path lengths:

$$\Delta x = 2x_1 - 2x_2 = 2\left(x_1 - x_2\right)$$

- Factor of 2 comes from the fact that the light is reflected, so it goes twice the distance.

- Light reflecting from mirrors also has pi phase-sift, but this does not matter since it happens for both mirrors so there is no difference.

- The condition for constructive interference is still: $\Delta x = m\lambda$ $m = 0, \pm 1, \pm 2, \ldots$

- If, for example, mirror #2 is moved by a distance $\Delta x_2 = \lambda/2$:

    - Interference pattern on screen shift by one fringe.

    - Keeping track of fringes allows for measuring distances smaller than one wavelength

- Suppose we now add the medium

- The path length difference, in terms of wavelength, will change since the wavelength inside the medium is $\lambda_n = \lambda/n$

- The number of wavelengths inside the medium is $N_{\mathrm{med}} = \dfrac{2d}{\lambda_n} = \dfrac{2dn}{\lambda}$

- If the medium was not there, then the total number of wavelengths would be:

$$N_{\mathrm{air}} = \frac{2d}{\lambda}$$

- Therefore, the difference in the number of wavelengths is:

$$N_{\mathrm{med}} - N_{\mathrm{air}} = \frac{2dn}{\lambda} - \frac{2d}{\lambda} = \frac{2d}{\lambda}(n-1)$$

- When placed along path x2, we get a shift of one fringe for every wavelength shift in path difference.

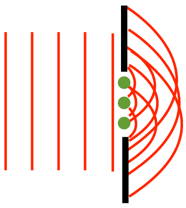
- If we know the index of refraction of material, we can also solve for the thickness d.

Diffraction:

- Recall that Huygen's principle says that every part of a wave acts like a point source for the new light waves.

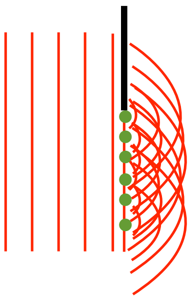
- If the light goes through an opening with a width that is comparable to the wavelength then:

coherent light



spherical light from point sources

- We also get something interesting when light hits a corner:



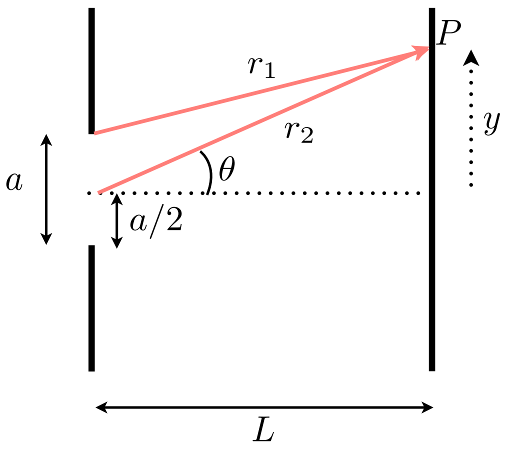
Light gets bent around the edge.

This is why your shadow outside is not completely dark.

- This spreading of the wave on the other side of the opening is called **diffraction.** It cannot be explained without wave optics.

Single-slit Diffraction:

- We will consider the following setup:



- We again have coherent light coming from the left with wavelength λ
- Will assume $L \gg a$
- One wave starts on edge, other at a/2
- Analysis is essentially the same as the double slit interference problem
- We will only look for dark fringes here

- The extra length for r2 is given by: $\sin\theta = \dfrac{\Delta x}{a/2} \rightarrow \Delta x = \dfrac{a}{2}\sin\theta$

- For dark fringes we must have: $\Delta x = \dfrac{a}{2}\sin\theta = \dfrac{\lambda}{2} \rightarrow a\sin\theta = \lambda$

- Although we have picked only two waves, this is valid for any pair of waves separated by a/2, and thus works for the entire slit

- Our previous result gives us the location of the first dark fringe

- To get the second dark fringe we must consider 4 waves separated by a/4

-This gives $\sin\theta = \dfrac{\Delta x}{a/4} \rightarrow \Delta x = \dfrac{a}{4}\sin\theta$

- The condition for destructive interference is: $\Delta x = \dfrac{a}{4}\sin\theta = \dfrac{\lambda}{2} \rightarrow a\sin\theta = 2\lambda$

- This can be generalized to find any dark fringe we want:

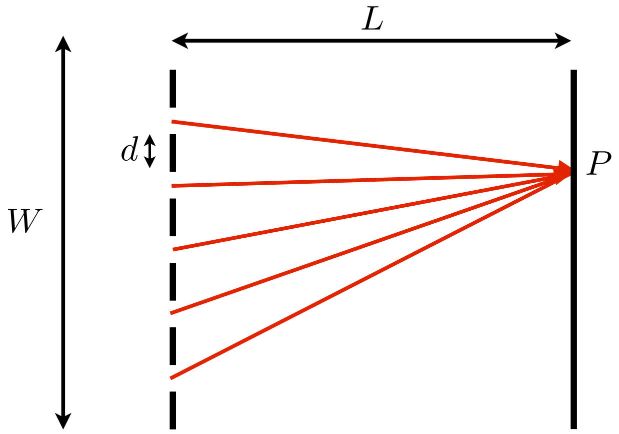
$$\boxed{a\sin\theta = m\lambda \qquad m = 1, 2, 3, \ldots}$$

- If the screen is far away then $\sin\theta \approx y/L$ (like previously) and we have:

$$\frac{ay}{L} = m\lambda \rightarrow y = \frac{m\lambda L}{a}$$

Multiple-slit Diffraction:

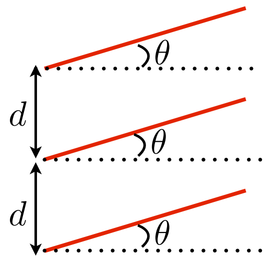
- Our previous results can also be extended to the case where there are multiple slits



- If separation between slits is d, then the total number of slits is given by

$$N = W/d$$

- We will assume that L>>d, so that all the light rays are parallel when leaving the slits



- Then we have the standard condition for constructive interference

$$d \sin\theta = m\lambda$$

Diffraction from Circular Opening:

- Suppose instead of going through a slit, the light went through a circular opening with diameter d

- This is a common situation; microscopes, telescopes,...

- Where is the first dark fringe located in this case?

- In this case we get the following result:

$$d \sin\theta = 1.22\lambda \rightarrow \sin\theta = 1.22\frac{\lambda}{d}$$

- The only difference is a numerical factor (1.22) that we will not derive.

- This formula defines an angle called the **Rayleigh Angle** (or Rayleigh Criteria):

$$\boxed{\theta_R = \arcsin\left(1.22\frac{\lambda}{d}\right)}$$

- This is the minimal angle that can be resolved when using light with wavelength λ and an opening with diameter d