Electron Double-Slit Experiment:

- Is there a particle that is small enough and goes slow enough for us to observe its wave-like properties?

- The de Broglie wavelength is $\lambda_{\rm dB} = \dfrac{h}{mv}$

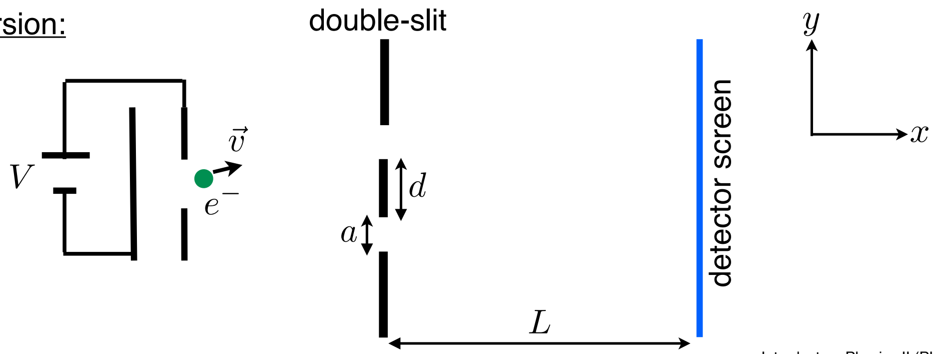
    - We want the smallest mass particle, and low velocities.

    - Lightest particle is the electron $m_e = 9.11 \times 10^{-31}\ \mathrm{Kg}$

- Can we observe interference and diffraction (wave properties) from electrons?

- Yes! Actually discovered by accident in 1927 by Davisson and Germer.

Modern version:



- This is the same double-slit experiment we looked at earlier.

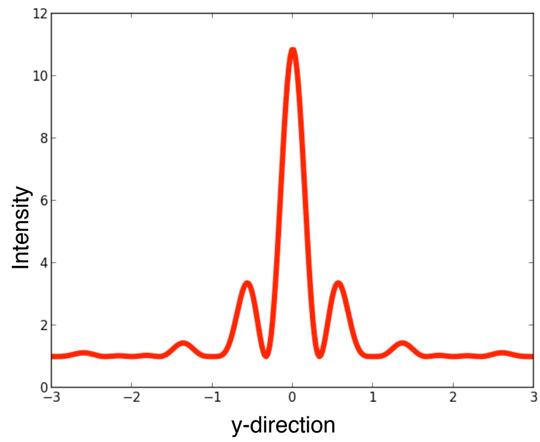
- If the slit spacing is about the same distance as the de Broglie wavelength of the electrons, then they should act like waves and interfere.

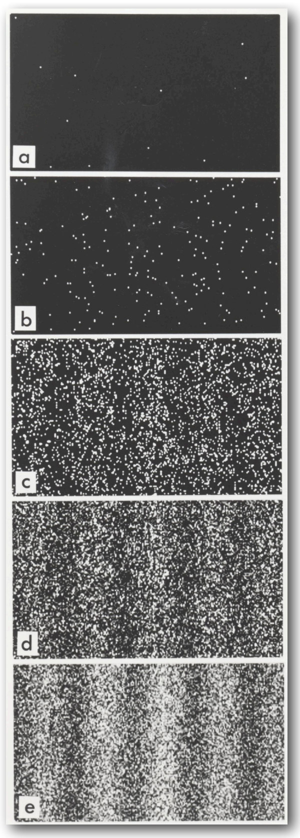
- We saw that, if the screen is far away, then the bright lines are at:

$$\Delta y = m \cdot \lambda_{\rm dB} L/d \qquad m = 0, 1, 2, \ldots$$

- The intensity (brightness) is given by:

$$I(y) = I_{\max} \cos^2\left(\frac{\pi d}{\lambda L} y\right) \left[\frac{\lambda L}{\pi a y} \sin\left(\frac{\pi a y}{\lambda L}\right)\right]^2$$





- The interference pattern is observed even when one electron at a time goes through the double-slit.

- If the electron is a particle, then it must go through only one slit
    - No interference pattern would be observed.

- Since we see interference, electrons behave like waves, not particles.

→ Electrons go through **both** slits at the same time! ←

- However, in contrast, the electron is a particle (spot) at the detector screen.

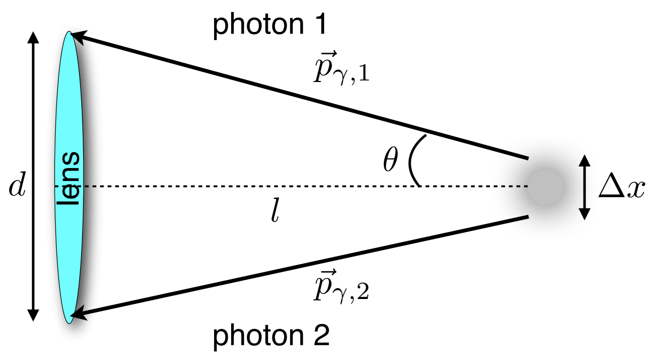
- This is what we mean by Wave-Particle Duality; Electrons can be particles or waves, depending on how your experiment is setup

Uncertainty relations:

- The idea of electrons going through both holes may be hard to believe.

- What if we try to measure exactly which hole each electron goes through.

- Suppose we want to use light to determine if an electron is in a region with width Δx



- The width Δx cannot be made smaller than the limit set by diffraction:

$$\Delta x = \frac{\lambda}{2\sin\theta}$$

- Suppose photon 1 interacts with the electron and then comes into the lens at the edge. The x-component of the momentum is

$$p_{1,x} = p_{\gamma,1}\sin\theta = \frac{h}{\lambda}\sin\theta$$

- From momentum conservation, the electron must have the same momentum, but in the opposite direction

- For photon 2 we have the same result: $p_{2,x} = p_{\gamma,2}\sin\theta = \frac{h}{\lambda}\sin\theta$

- We cannot know which direction the photon came from, but the worst cases are photon 1 and photon 2. The uncertainty in the electrons momentum is thus:

$$\Delta p = \frac{2h}{\lambda}\sin\theta$$

- Substituting in the equation for Δx:

$$\Delta x \Delta p = h$$

- If we want to know the position of the electron with higher and higher accuracy then we are forced to lose information about the electrons moment.

- Since $p = mv = m\frac{dx}{dt}$ the location of the electron at a later time will more uncertain

- Although we looked at a simple example, this relationship can be generalized into the **Heisenberg Uncertainty Principle:**

$$\boxed{\Delta x \Delta p \geq \frac{\hbar}{2}} \qquad \hbar = \frac{h}{2\pi}$$

- The uncertainty relation show that there is a lower limit to how well I can know <u>both</u> position and momentum for an object <u>at the same time</u>.

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One of the most important principles in ALL of physics!!!

- This uncertainty relationship is just one of many. Another important one is the **Energy-Time Uncertainty Relationship**:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- This equation says that classical energy conservation can be violated but only for a short time.

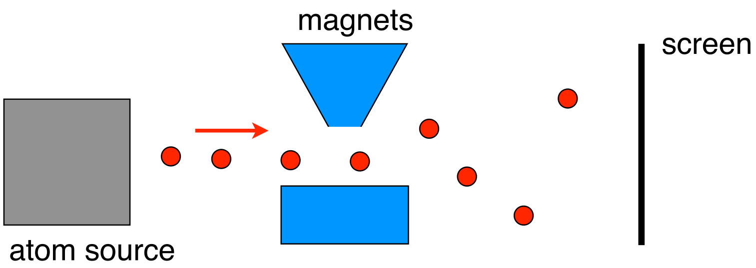
- The more energy you borrow, the shorter the time you have to use it.

Stern-Gerlach Experiment:

- The energy of photons is not the only thing that is quantized in quantum mechanics

- Here we will look at the angular momentum of an atom

- Consider a stream of neutral Silver atoms moving through an inhomogeneous magnetic field in the z-direction



- Recall from our discussion on magnetic dipoles that the magnetic moment of an atom is proportional to its angular momentum L:

$$\vec{\mu} = -\frac{e}{2m}\vec{L}$$

- In a B-field, the potential energy of the magnetic dipole can be written as

$$U = -\vec{\mu} \cdot \vec{B}$$

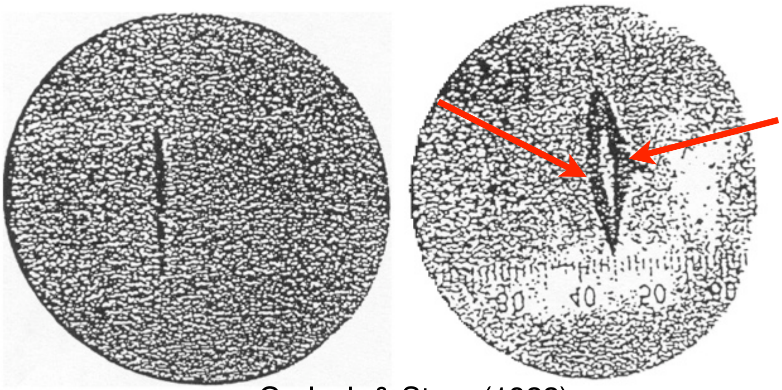
- Since the magnetic field changes in the z-direction only, the force on the atoms moving through the B-field is

$$F_z = -\nabla U = -\frac{\partial}{\partial z}\left(-\vec{\mu}\cdot\vec{B}\right) = \mu_z \frac{\partial B}{\partial z}$$

- Classically μ_z can have any value between $\pm|\vec{\mu}|$ -> the screen will have a continuous line of spots.

- Stern & Gerlach (1920): Only two spots occur on the detector screen.

→ The z-component of magnetic moment, and thus angular momentum, is quantized and has only two values



Gerlach & Stern (1922)

Fermions & Bosons:

- The angular momentum of all particles is quantized.

- This quantized angular momentum is called **Spin**

- Particles are grouped into two groups based on the values that their spin can have:

Bosons	Spin = $m \times \hbar$	$m = 1, 2, \ldots$	(Integer spin)
Fermions	Spin = $k \times \hbar$	$k = \frac{1}{2}, \frac{3}{2}, \ldots$	(Half-integer spin)

- In the Stern-Gerlach experiment, the particles were spin 1/2: Spin = $\pm\frac{1}{2}\hbar$

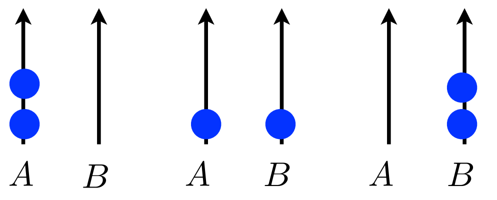
$+\frac{1}{2}\hbar$ = "**spin-up**" $-\frac{1}{2}\hbar$ = "**spin-down**"

- Electrons, protons, and neutrons (everything that makes up matter) are all spin-1/2 particles

- Light (Photons) are Bosons with spin = $1\hbar$

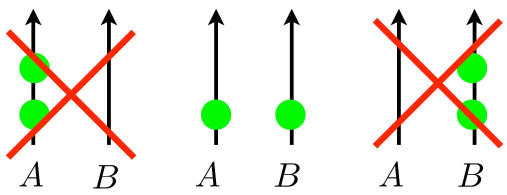
- The difference between Bosons and Fermions can be shown by considering two difference quantum states A & B

- If we have two Bosons, then the particles can be arranged in the following ways



Bosons can share the same quantum state

- But if the particles are two Fermions then we have the following

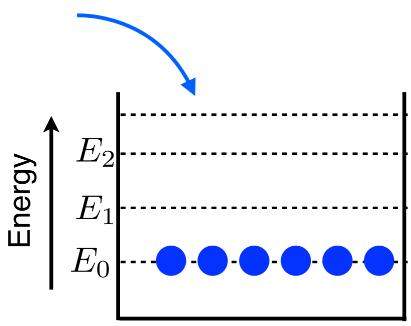


Fermions can never share the same quantum state

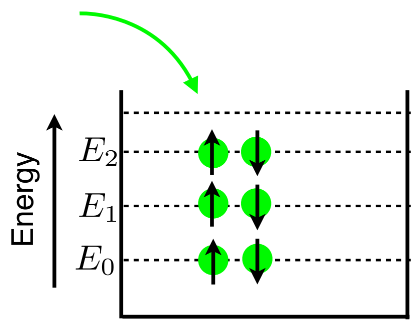
Pauli Exclusion Principle: No two Fermions can have the same quantum state at the same time and spatial position.

This is why your butt stays in the chair instead of falling through it!

- Suppose I dump a bunch of Bosons or Fermions into a container with several energy levels and at temperature T=0.



Since Bosons can be together (in fact they like it), they will all go to the lowest energy level.



Since Fermions cannot share quantum states, they will stack up to higher and higher energy levels. Here we show spin=1/2 particles that can share the same energy level provided one particle is spin-up and the other spin-down.