Wave Optics

- We have seen that Maxwell's equations have wave-like solutions that transmit energy at the speed of light.

- Two ways to look at light waves:

  Geometric Optics: Wavelength is small compared to typical length scales

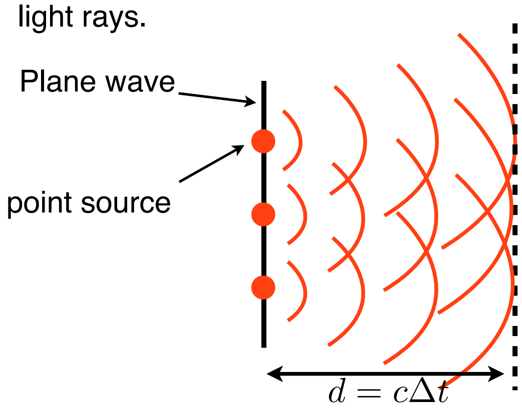
  Wave Optics: Wavelength of light is same size, or larger, and typical lengths

<u>Wave Optics:</u>

- Visible light: 450-750nm

- How do light waves travel?

Huygen's Principle: Every point on a moving light wave acts like a point source of new light rays.



- Point sources always radiate spherical waves in radial direction.

Fermat's Principle: The path taken by light is the path that takes <u>the least</u> amount of time.

- In a single medium (i.e. vacuum, air, glass,..) light always travels along a straight path.

- If light goes from one medium to another, then the light will become bent so that the path satisfies Fermat's Principle.

- How much the light is bent is determined by the **index of refraction** of the two materials:

$$n = \frac{c}{v}$$

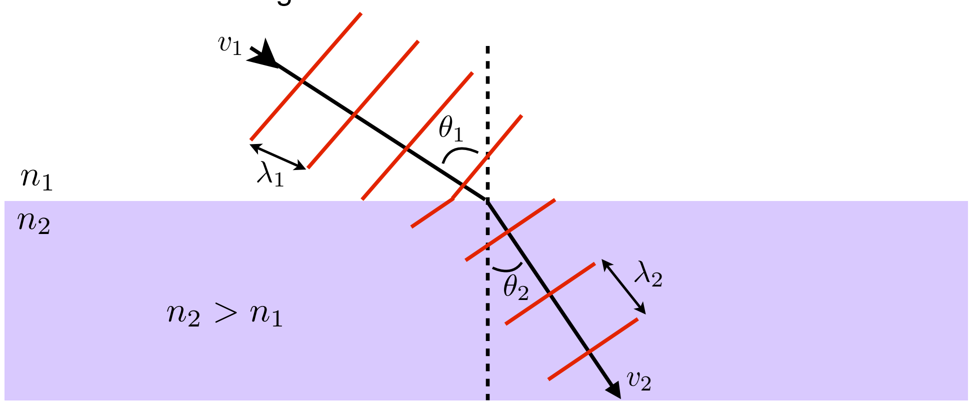
speed of light in vacuum

speed of light in medium

$$n_{\text{air}} \approx n_{\text{vac}} = 1$$
$$n_{\text{H}_2\text{O}} = 1.33$$
$$n_{\text{dia}} = 2.42$$

- In different mediums, only the velocity and wavelength of light changes. Frequency remains constant.

- Suppose we have the following situation:



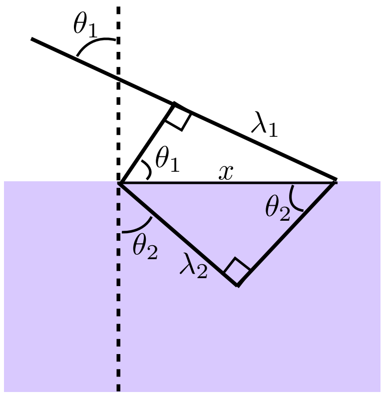
- Time between waves in medium #1 is: λ_1/v_1

- Time between waves in medium #2 is: λ_2/v_2

- These times must be the same, otherwise waves would be created, or disappear, at boundary. Since waves carry energy -> energy not conserved; created or destroyed

$$\Rightarrow \quad \frac{\lambda_1}{v_1} = \frac{\lambda_2}{v_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$$

- At the boundary we have the following:



$$\sin\theta_1 = \frac{\lambda_1}{x} \qquad \sin\theta_2 = \frac{\lambda_2}{x}$$

$$\rightarrow \quad \frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

but we have $v_1 = \dfrac{c}{n_1}$ and $v_2 = \dfrac{c}{n_2}$

- We have arrived at **Snell's Law**:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Also have: $\lambda_n = \lambda / n$ $\quad \lambda =$ speed of light in vacuum

$$f_n = \frac{v}{\lambda_n} = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f$$

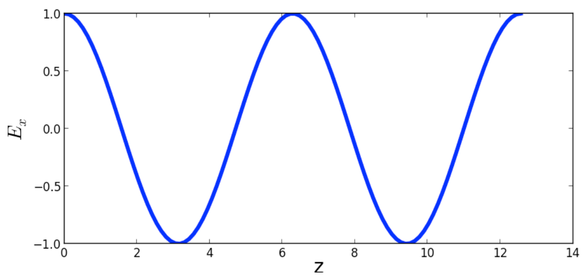
- Frequency does not change due to energy conservation (also Huygen's Principle)

<u>Interference:</u>

- Recall that, for plane waves, we wrote down the E-field solution as:

$$\vec{E} = E_0 \hat{x} \cos(kz - \omega t - \phi)$$

- If we look at the wave at a fixed time, then as a function of z, the wave oscillates sinusoidally:



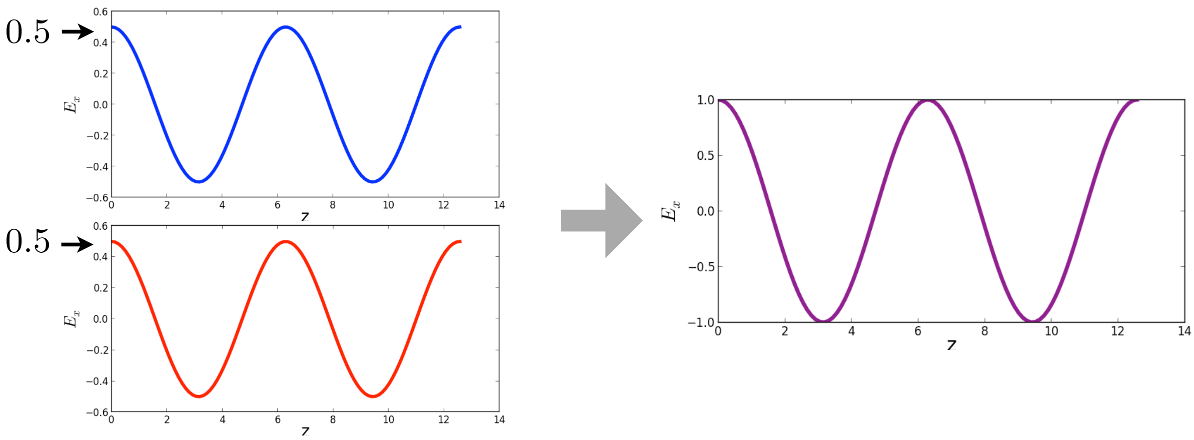
- Suppose we have two waves, then what is the total E-field?

- Recall that the E-field (and B-field) obey the principle of superposition. We can just add the two fields together to get the total

- Suppose the phase difference between the two waves satisfies $m \times 2\pi$, then the waves are <u>in phase</u> and the difference in path length between the two waves satisfies

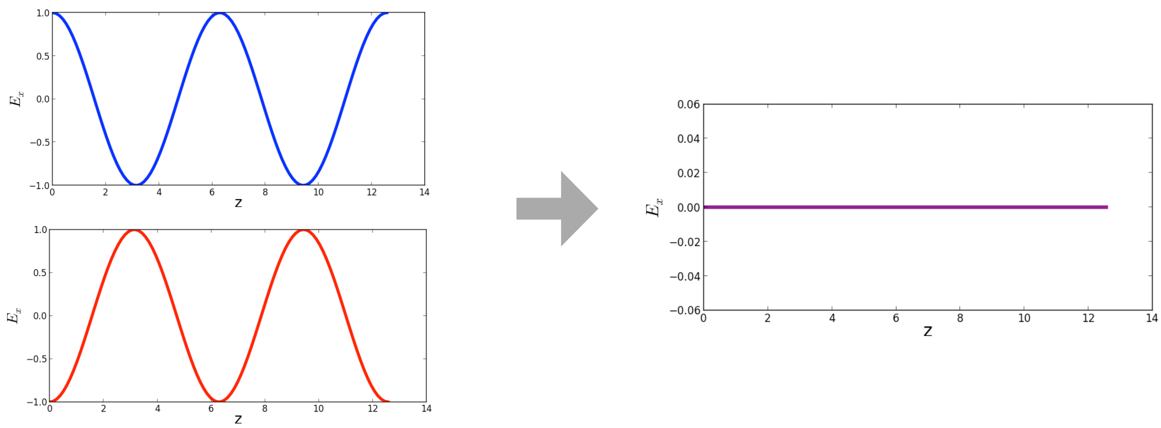
$$\boxed{\Delta x = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots \quad \text{Constructive interference}}$$

and the wave **constructively interfere**.



- If the phase difference between the waves is $m \times 2\pi + 2\pi/2$ then the <u>waves are pi out of phase with each other</u> and deconstructively interfere according to

$$\boxed{\Delta x = \left(m + \frac{1}{2}\right)\lambda \qquad \text{Deconstructive interference}}$$



- In general, we start out with light that constructively interferes.

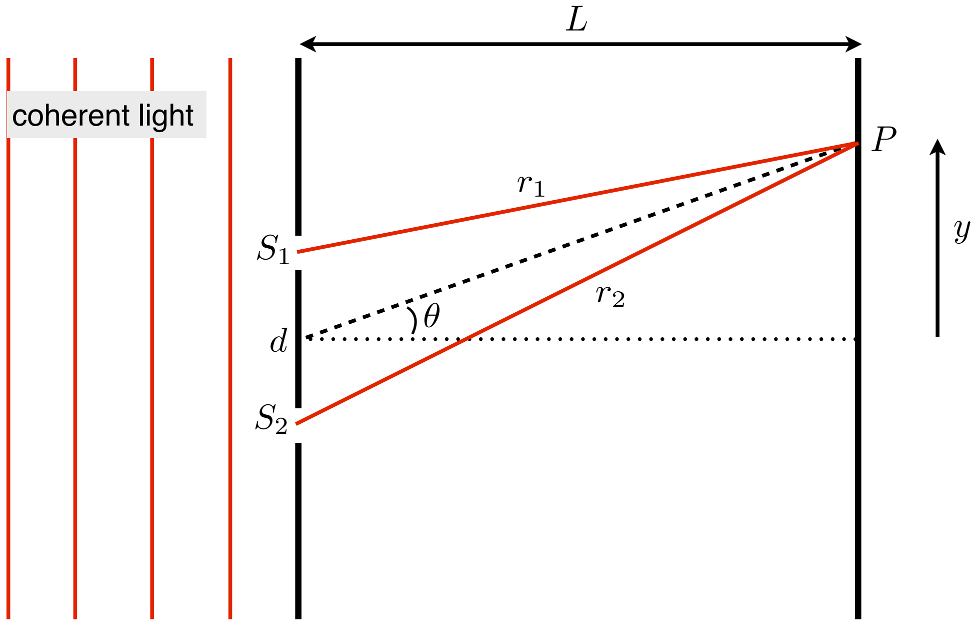
- Can create destructive interference by changing the distance that light has to travel, or by changing the speed of light by using different media.

- Constructive or destructive interference depends only on the difference in path length Δx

Double-Slit Interference:

- By far the most important interference experiment is the Young's double slit experiment carried out in 1801.

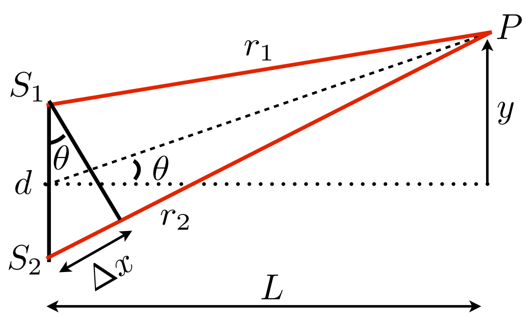
- This experiment uses **monochromatic coherent light**: Light made up of waves that are all in phase, and that have all the same wavelength.



- Recall that only the path difference $\Delta x = r_2 - r_1$ matters for interference

$$\sin\theta = \frac{\Delta x}{d}$$

$$\Rightarrow \quad \Delta x = d\sin\theta$$



- For constructive interference:

$$\Delta x = d\sin\theta = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

$m = 0$ central maximum

$m = \pm 1$ 1st bright fringe

- For destructive interference:

$$\Delta x = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

$m = 0$ 1st dark fringe

- If the screen is very far away, $L \gg y$ then $\theta \ll 1$

$$\frac{y}{L} = \tan\theta = \frac{\sin\theta}{\underset{\to 1}{\cos\theta}} \approx \sin\theta$$

- Therefore, the location of the bright fringes is given by:

$$d\frac{y}{L} = m\lambda \rightarrow y = \frac{mL\lambda}{d}$$

- Dark fringes are located at: $y = \dfrac{(m+1/2)\,L\lambda}{d}$

- We have found the location of the bright and dark fringes, but we do not know the amplitude at each point.

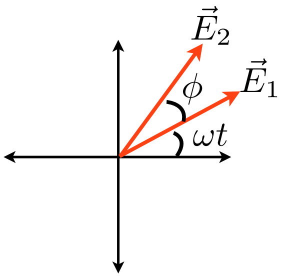
- To get the amplitude we must calculate the intensity along the screen.

- At point P, the E-field from slit S1 is: $\vec{E}_1 = E_{\max}\sin(\omega t)$

- At point P, the E-field from slit S2 is: $\vec{E}_2 = E_{\max}\sin(\omega t + \phi)$

- We have set the phase of E1 equal to zero since only phase differences matter.

- Since the E-field is a vector, we have the following situation at time t:



- The total E-field is thus: $\vec{E}_{\text{tot}} = 2E_{\max}\cos(\phi/2)$

- Intensity is proportional to E^2

$$\frac{I}{I_{\max}} = \frac{E^2}{E_{\max}^2}$$

$$\Rightarrow \quad I = 4I_{\max}\cos^2(\phi/2)$$

- The phase is related to the path length difference via: $\phi = \dfrac{2\pi\Delta x}{\lambda}$

- But we also know that $\Delta x = d\sin\theta$, thus we have

$$\boxed{I(\theta) = 4I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)}$$

- Or if $L \gg y,\ \theta \ll 1$: $$I(y) = 4I_{\max}\cos^2\left(\frac{\pi dy}{\lambda L}\right)$$

- Example $d = \lambda = I_{\max} = 1$: