PHYS-183 : Day #14

- Last time we derived the Bernoulli Equation that described the conservation of energy for an ideal , incompressible fluid

$$P+\frac{1}{2}\rho v^2+\rho g y=\text{constant}$$

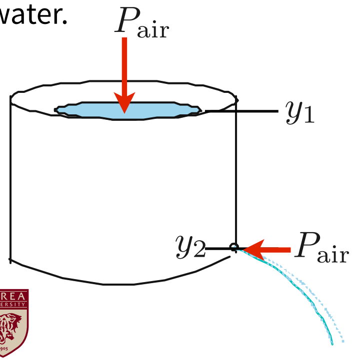
- We also saw that if we ignore the gravitational potential energy ($\Delta y=0$), then we have the equation

$$P+\frac{1}{2}\rho v^2=\text{constant}$$

- Now we want to look at the situation where the pressures at two different locations are equal

$$\Rightarrow \frac{1}{2}\rho v_1^2+\rho g y_1=\frac{1}{2}\rho v_2^2+\rho g y_2=\text{constant}$$

- We will use this equation to look at the speed of water leaving from the bottom of a container of water.



- We will assume y2-y1 does not change a lot

- Here, the pressure at the top of the water is the same as the pressure at the hole on the bottom

- The water at position y1 has no velocity

- In this case, Bernoulli's equation (with constant pressure) becomes:

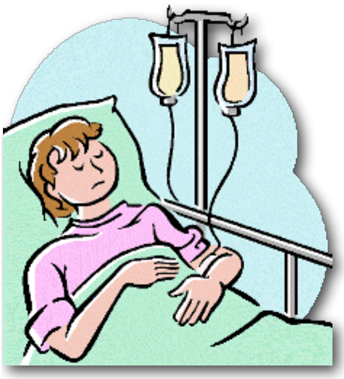
$$\rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Total energy/volume at top   Total energy/volume at bottom

- Solving for v2 we have: $v_2 = \sqrt{2g(y1 - y2)}$

- Remember the velocity for an object in free fall: $v(t) = \sqrt{2gh}$

Key Idea: The water has the same speed as it would if it was falling vertically due to the gravitational acceleration.



Ex. Suppose a bag of saline solution (salt water) is used as an intravenous drip (IV drip). If the bag is placed 1m above the persons arm and the tube connecting the bag to the person is 1mm in diameter, what is the flow rate Q before the tube is attached to the person? Assume an ideal fluid.

Solution:

- Since the tube is not attached to the person, the pressure on the bag is the same as the pressure on the end of the tube.

- In this case, the velocity of the saline coming out of the tube is the same as if it was in free fall

$$v(t) = \sqrt{2g(y_{\mathrm{bag}} - y_{\mathrm{arm}})} = \sqrt{2 \cdot 9.8 \cdot 1} = 4.4 \text{ m/s}$$

- The question ask for the flow rate $Q = Av$ so we just need to find the area of the tube.

$$Q = \pi r^2 v = \pi(0.0005)^2(4.4) = \boxed{3.5 \times 10^{-6} \text{ m}^3/\text{s} = 3.5 \text{ cm}^3/\text{s}}$$

- Because the tube is so small, our answer is about 10 times bigger than it should be.

- We ignored the viscosity "stickiness" of water. We will look at this next week.

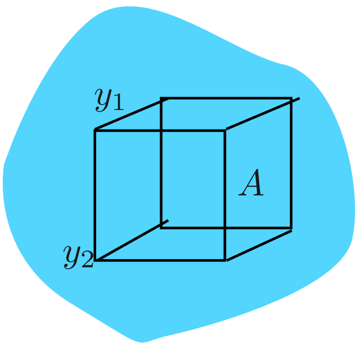
The Effects of Gravity in Hydrostatic Equilibirum:

- Previous we have seen that, when there are no net external forces, the pressure inside a fluid is the same everywhere (**Pascal's Theorem**).

- However, gravity is always around us, so we must consider the effects of the force of gravity on the pressure inside a fluid.

- Also, we will see that the weight of an object inside a fluid is reduced by the effects of fluid pressure.

- Suppose I pick a small cubic volume inside a fluid in hydrostatic equilibrium



- Since we have equilibrium, we have no net fluid flow so the velocities at y1 and y2 are zero.

- Then Bernoulli's equation tells us that:

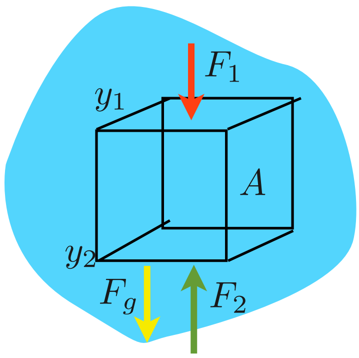
$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

- Or, rearranging terms:

$$P_2 = P_1 + \rho g(y_1 - y_2) = P_1 + \rho g h$$

- If I multiply both sides by the area A, then I get a relationship for the forces in hydrostatic equilibrium

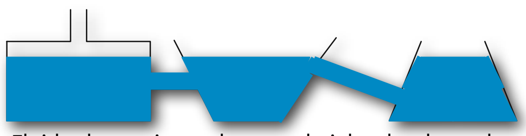
$$P_2 A = P_1 A + \underbrace{(\rho A h)}_{\text{mass of fluid inside the volume}} g \rightarrow F_2 = F_1 + F_g$$



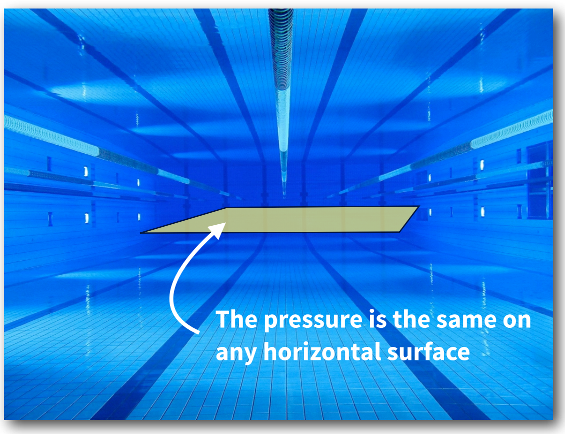
- In hydrostatic equilibrium, the pressures P1 and P2 are called **hydrostatic pressures**.

<u>Key Idea:</u> The pressure difference $\Delta P = P_2 - P_1$ depends <u>only</u> on the fluid density and the height difference between y1 and y2.

- In hydrostatic equilibrium, the pressure in a fluid is a constant if the height does not change, it <u>does not</u> depend on the shape of the container the liquid is in.



Fluids always rise to the same height, the shape does not matter



- Our hydrostatic equation only tells us the difference between P1 and P2, but does not tell us the actual pressure values

$$P_2 = P_1 + \rho g h$$ (we can not solve this unless we know P1 or P2)

- One easy pressure to use is the pressure due to the pressure due to the air (which is a fluid) in the atmosphere at sea level.

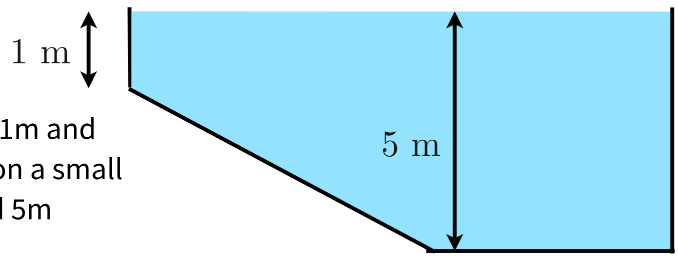
- This is called **atmospheric pressure**: $P_{\text{atm}} = 1.01 \times 10^5 \text{ Pa}$

- Setting P1 to the atmospheric pressure we have

$$P = P_{\text{atm}} + \rho g h$$

- When using P_{atm} the other pressure *P* is called the **absolute pressure**.

- This atmospheric pressure is the standard reference pressure used to calculate the value of the absolute pressure *P*



Ex. A swimming pool starts out with a depth of 1m and then goes to a depth of 5m. Find the pressure on a small balloon when held at the bottom of the 1m and 5m sections.

Solution:

- Since the swimming pool is open to the atmosphere, we know know that we must use the atmospheric pressure to calculate the absolute pressure at the two locations.

$$P = P_{\text{atm}} + \rho g h = 10^5 + 10^3 \cdot 9.8 \cdot h$$

- Therefore, at 1m and 5m we have:

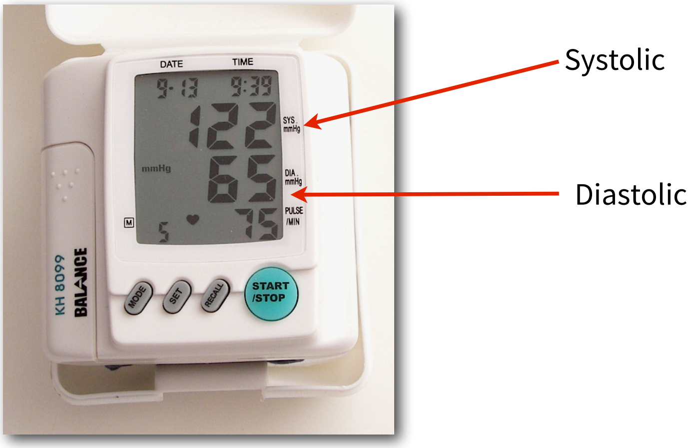
$$\boxed{\begin{aligned} P_{1\text{m}} &= 1.1 \times 10^5 \text{ Pa} \\ P_{5\text{m}} &= 1.5 \times 10^5 \text{ Pa} \end{aligned}}$$

- When swimming underwater your ears begin to hurt at ~3m due to the increase in pressure.

- The blood pressure in your body depends on two things:

(1) Your heartbeat. The blood pressure is higher when you heart is pumping (**systolic pressure**) and lower when your heart is relaxed (**diastolic pressure**).

- This is why your blood pressure is always measured using two numbers



- Your blood pressure is not measured in Pa, instead, the doctor uses a different unit (for historical reasons) called mm Hg, or millimeters of mercury.

$$1\ \text{mmHg} = 133.3\ \text{Pa}$$

(2) The blood pressure in your body depends on the height at which you measure it.

- The blood near your feet has a <u>higher pressure</u> because of all the pressure from the blood higher up in your body.

- The blood in your head has <u>lower pressure</u> because nothing is applying pressure from the top.

- Lets suppose a persons average blood pressure measured near the heart is $13.2 \times 10^3\ \text{Pa}$

- What is the persons blood pressure at their feet 1.3m lower than the heart?

$$P_2 - P_1 = \Delta P = \rho g h \qquad \rho_{\text{blood}} = 1060\ \text{kg/m}^3$$

$$\Delta P = 1060 \cdot 9.8 \cdot 1.3 = 13.5 \times 10^3\ \text{Pa}$$

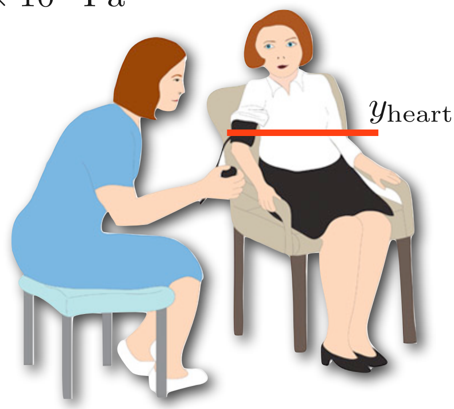
$$\Rightarrow \quad P_{\text{feet}} = (13.2 + 13.5) \times 10^3 = 26.7 \times 10^3\ \text{Pa}$$

- A persons blood pressure at their feet is nearly twice as big as the pressure near the heart.

- What is the persons blood pressure in their head, a distance 0.25m higher?

$$\Delta P = 1060 \cdot 9.8 \cdot 0.25 = 2.6 \times 10^3 \text{ Pa}$$

$$\Rightarrow P_{\text{head}} = (13.2 - 2.6) \times 10^3 = 10.6 \times 10^3 \text{ Pa}$$



- Because blood pressure changes if you go above or below the height of your heart, your blood pressure is always measured at the same height as your heart.

- If you experience a fast acceleration upward, for example in an airplane or a fast elevator, then the increased pressure can drain the blood from your head.

- Normally, your heart can pump blood to a maximum height

$$P_{\text{heart}} = \rho g h \rightarrow h = P_{\text{heart}}/(\rho g) = 1.27 \text{ m}$$

above the height of your heart.

- Assuming the pressure from the heart remains fixed, then if you experience a fast acceleration a then the new maximum height h' is

$$P_{\text{heart}} = \rho(a+g)h' = \rho g h \quad \Rightarrow \quad h' = h\frac{g}{(a+g)}$$

- Setting the maximum height h' to be the height of a persons head above the heart (0.25m), we can solve for a

$$a = g\frac{(h-h')}{h'} = g\frac{1.27-0.25}{0.25} = 4.1 \times g$$

- If you accelerate upward with an acceleration faster than 4.1 times the acceleration of gravity, your heart can not pump blood to your head.

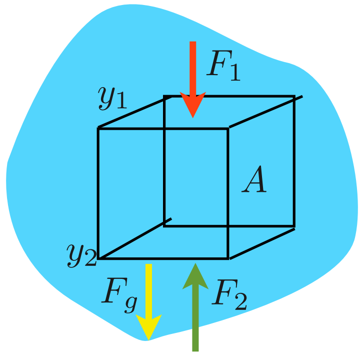
- Eventually, a person will black out from the lack of oxygen (blood)

More than 4 times gravity will cause loss of blood to the head

Archimedes Principle:

- We have seen that is hydrostatic equilibrium, the weight of a volume of fluid is exactly balanced by the net upward force due to the fluid pressure.

$$F_2 - F_1 = F_g > 0$$

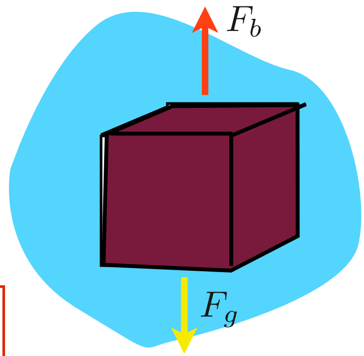


- This net upward force from the fluid is equal to the weight of the fluid inside of the volume

- Suppose we want to find the force acting on an object with mass *m* inside of the fluid.

- Like before, there will be the force of gravity on the object, but there will also be an upward force from the fluid pressure called the **buoyant force** F_b .

- To find the buoyant force, we imagine replacing the volume of fluid with our object with the same volume



- The pressure on the object will be equal to the pressure that was on the volume of fluid.

- Therefore, the buoyant force does not change

$$\boxed{F_b = \rho_{\text{fluid}} g V \qquad V = \text{Volume of object in fluid}}$$

- The buoyant force is determined only my the mass of the fluid $m = \rho_{\text{fluid}} V$ inside the volume *V* formed by the volume.

$$F_b = \rho_{\text{fluid}} g V$$

- This is known as **Archimedes Principle**.

- However, the force of gravity on the object depends on the objects mass

$$F_g = m_{\text{obj}} g = \left(\frac{m_{\text{obj}}}{V}\right) g V = \rho_{\text{obj}} g V$$

- The object will float if the buoyant force is larger

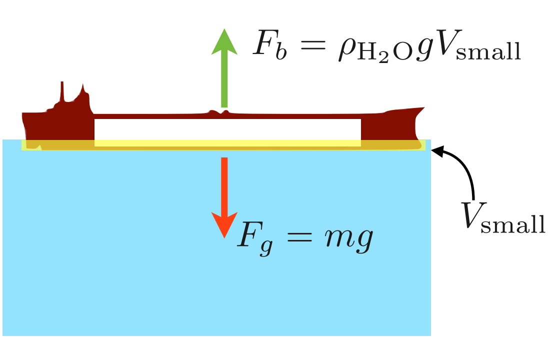
$$F_b - F_g = (\rho_{\text{fluid}} - \rho_{\text{obj}}) g V > 0 \quad \Rightarrow \quad \rho_{\text{fluid}} > \rho_{\text{obj}}$$

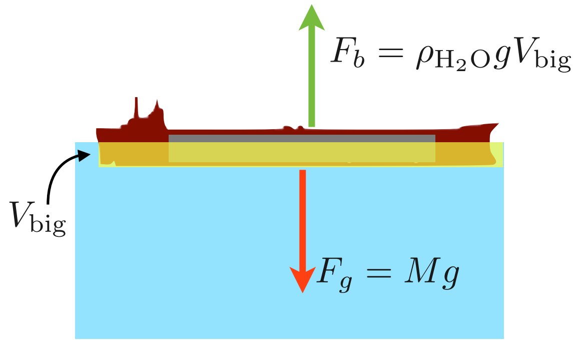
- The object will sink if gravitational force is larger $\quad \Rightarrow \quad \rho_{\text{obj}} > \rho_{\text{fluid}}$

- Remember that V is the volume of the object that is in the fluid.

- An object that is floating will have a reduced buoyant force, but the gravitational force is still equal to $F_g = m_{\text{obj}} g$

- An object floats on the surface of the liquid when the volume *V* of the object in the liquid is such that the buoyant force is equal in magnitude to the gravitational force (net force = 0)





- Or, if the fluid you are in is very dense, big $\rho_{\rm fluid}$, then you need a smaller volume in the fluid to float

$$\rho_{H_2O} = 1000 \text{ kg/m}^3$$

$$\rho_{\text{Dead Sea}} = 1240 \text{ kg/m}^3$$

- The Dead Sea is 24% denser than water due to the large amount of salt.

Floating on the Dead Sea in Israel. Large density due to salt.

- For an object with weight F=mg, because the buoyant force points in the opposite direction, the net downward force will be effectively reduced

$$F_{\rm net} = mg - \rho_{\rm liquid} g V$$

An object in a fluid <u>effectively weighs less</u> than it does outside of the fluid because of the buoyant force.

<u>Ex.</u> An 800N man has a volume of $V = 0.076 \text{ m}^3$ when underwater in a swimming pool. Calculate his effective weight in the pool. Calculate his effective weight when in the ocean.

<u>Solution:</u>

$$F_{\rm eff,pool} = 800 - \rho_{H_2O} g(0.076) = 800 - 1000 \cdot 9.8 \cdot 0.076 = \boxed{55 \text{ N}}$$

$$F_{\rm eff,ocean} = 800 - \rho_{\rm ocean} g(0.076) = 800 - 1025 \cdot 9.8 \cdot 0.076 = \boxed{37 \text{ N}}$$

- We see that a small increase in the density of the fluid, 2.5% in this example, reduced the mans effective weight by 33%.

Ex. Suppose there is an iceberg that is 168m above the surface of the ocean. If the iceberg is shaped like a cylinder, find how deep below the water the iceberg gets.

Most of a typical iceberg is actually underwater.

Solution:

- The iceberg is floating because the gravitational and buoyant forces are cancelled.

- So we need to find the volume *V* at which this happens

- First we need to find the density of the ice and ocean water:

$$\rho_{\mathrm{ocean}} = 1025\ \mathrm{kg/m^3} \qquad \rho_{\mathrm{ice}} = 917\ \mathrm{kg/m^3}$$

- Now we just set the buoyant force equal to the gravitational force:

$$F_b = \rho_{\mathrm{ocean}} A d g = F_g = \rho_{\mathrm{ice}} A (d + 168) g \qquad d = \text{distance underwater}$$

- We can cancel *Ag* from both sides to get: $1025d = 917(d + 168)$

$$\boxed{d = 1426\ \mathrm{m}}$$

- About 10 times more underwater than above.