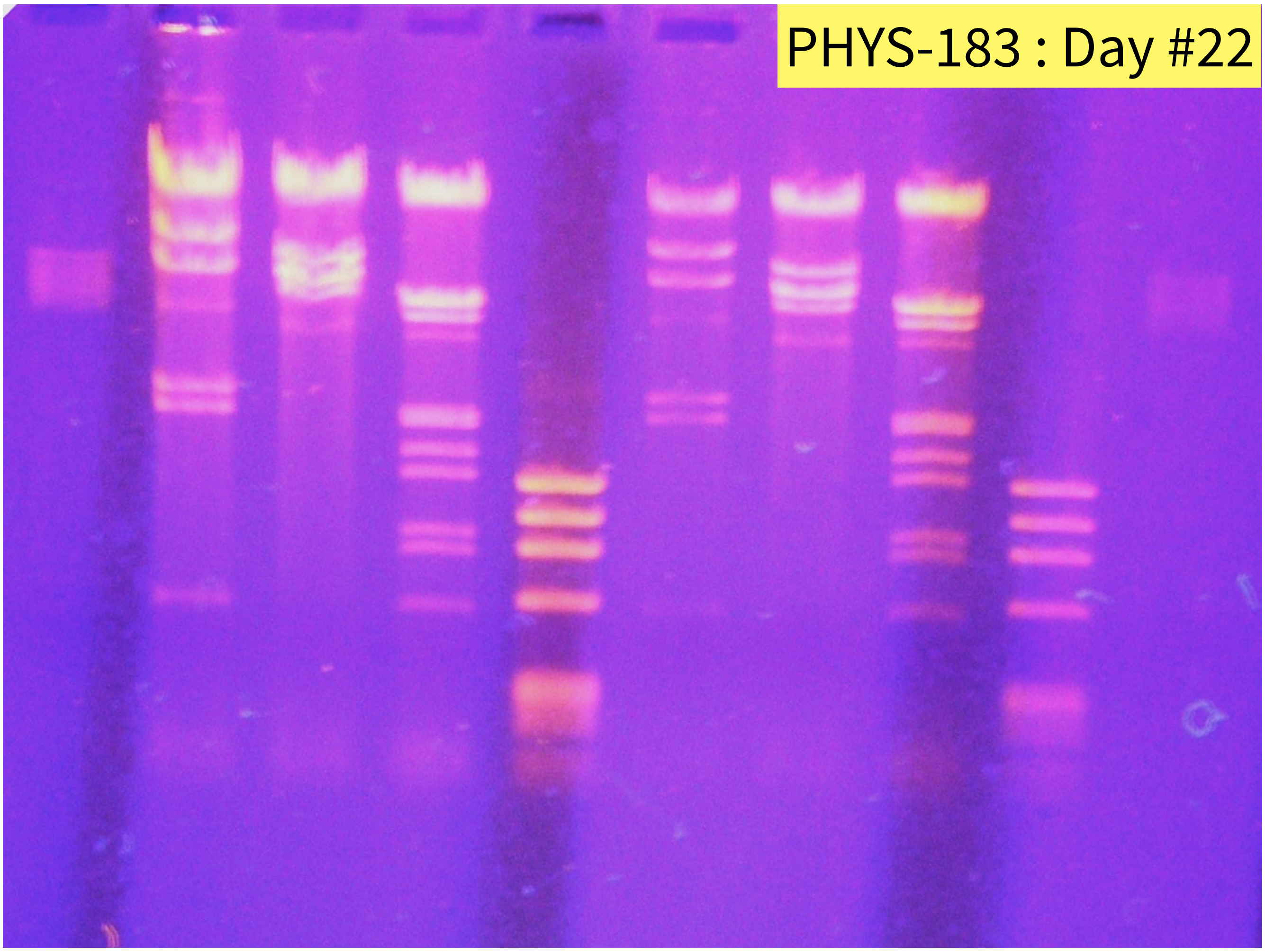


Electric Fields

- We have seen how the force between two or more particles can be expressed via Coulomb's Law:

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r} \qquad \text{or} \qquad \vec{F}_{1,\mathrm{net}} = \sum_i F_{1i}$$

- Can use this to find the force given q1 and qi.
- Magnitude and sign of q1 will change answer.
- We want some way to express the force at a given point p for any charge q at p
- We can eliminate q1 from the calculation:

Electric field due to charge q2

$$\frac{\vec{F}}{q_1} = k\frac{q_2}{r^2}\hat{r} = \vec{E}(\vec{r})$$

- Direction of E-field does not depend on the sign of q1:

<u>ex: q1=-1e</u>

$$\vec{F}(r) = -k\frac{eq_2}{r^2}\hat{r}$$

$$\frac{\vec{F}(r)}{-e} = \vec{E}(r) = k\frac{q_2}{r^2}\hat{r}$$

Same

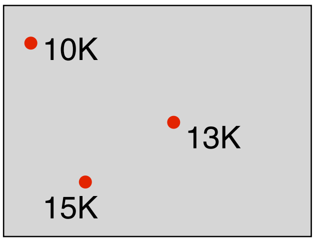
<u>ex: q1=+e</u>

$$\vec{F}(r) = k\frac{eq_2}{r^2}\hat{r}$$

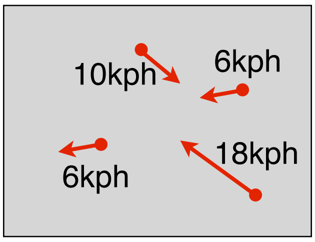
$$\frac{\vec{F}(r)}{e} = \vec{E}(r) = k\frac{q_2}{r^2}\hat{r}$$

- Electric field (or any field) is defined at each point in space.

Temperature (Scalar, just a number) **Wind speed (Vector, num. + direction)**

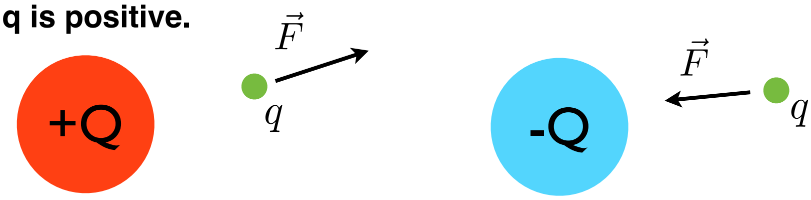


A point or arrow at every location



- We can always calculate the force from the E-field: $\boxed{\vec{F} = q \cdot \vec{E}(\vec{r})}$

- The direction of the electric field is such that **the E-field points in the same direction as the force if q is positive.**

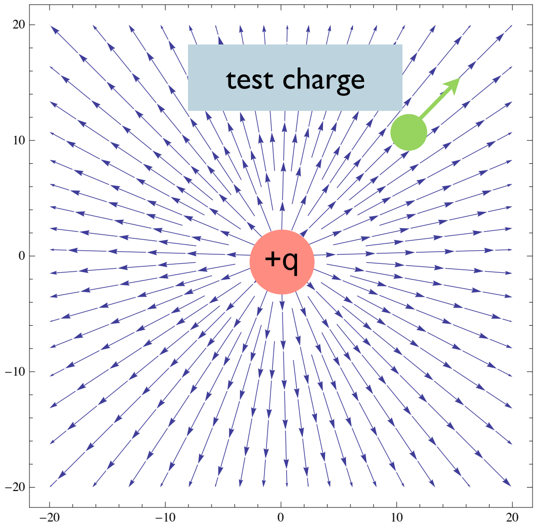


Since E-field is in same direction as F when q is positive:

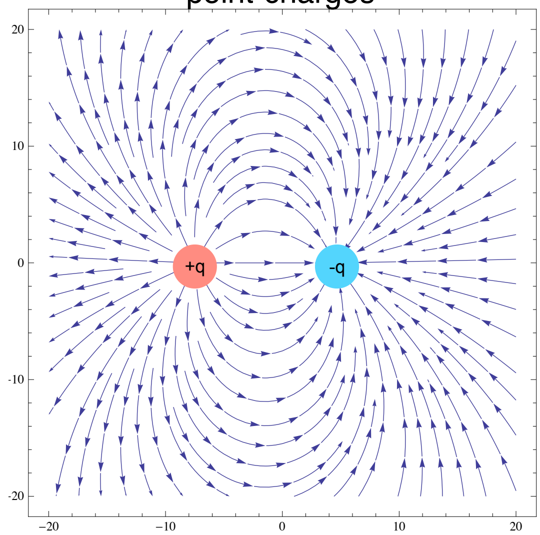
Electric field points away from positive charges and toward negative charges

"Out of the positive, **in** to the **n**egative"

Electric field from a single point-charge



Electric field from two oppositely charged point-charges



- Electric fields are real and can carry energy, momentum, and angular momentum.

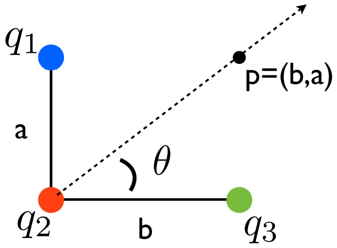
- Fields exist with or without the test charge. Test charges must be small so that their E-fields are small compared to the one you want to measure.

- Electric field from a point charge is spherically symmetric.

- Electric fields also obey the principle of superposition

$$\vec{E}_{\rm net} = \sum \vec{E}_i$$

Ex. What about electric field due to many charges?



What is the magnitude and direction of E at p?

- Can use superposition principle since E=F/q.
- Do easy stuff first

E_{q_1} in x-direction only: $E_{q_1}(p) = \dfrac{kq_1}{b^2}\hat{x}$

E_{q_3} in y-direction only: $E_{q_3}(p) = \dfrac{kq_3}{a^2}\hat{y}$

- E-field from q2 must be calculated at point p, and then decomposed into x/y components

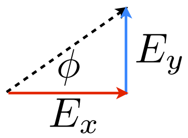
$$|E_{q_2}(p)| = \frac{kq_2}{a^2+b^2} \qquad \tan\theta = \frac{a}{b} \quad \Rightarrow \quad \theta = \arctan\frac{a}{b}$$

$$\Rightarrow \quad E_{q_2}(p) = \frac{kq_2}{a^2+b^2}\cos\theta\hat{x} + \frac{kq_2}{a^2+b^2}\sin\theta\hat{y}$$

- Add all up components

$$E_{q_2}(p) = \left[\frac{kq_1}{b^2} + \frac{kq_2}{a^2+b^2}\cos\theta\right]\hat{x} + \left[\frac{kq_3}{a^2} + \frac{kq_2}{a^2+b^2}\sin\theta\right]\hat{y}$$

- find angle



$$\tan\psi = \frac{E_y}{E_x}$$

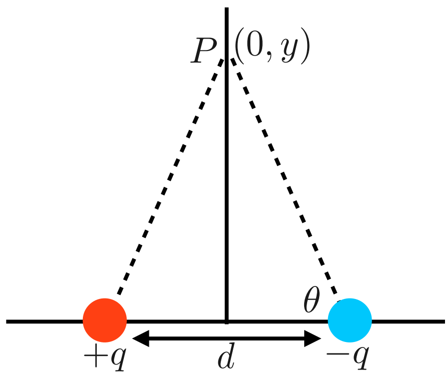
Ex. What is the E-field for a point P along the y-axis from a pair of oppositely charged particles on the x-axis?

- The magnitude of the electric field is the same for both particles:

$$E = \frac{kq}{y^2 + (d/2)^2}$$

- The y-component from the +q E-field cancels the y-component of the field from the -q charge.

- The net E-field must therefore point in the x-direction.



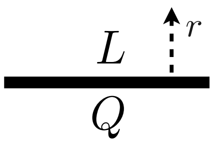
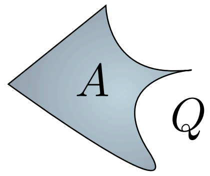
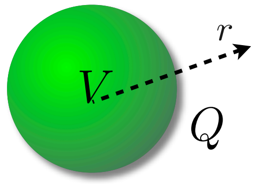
- Because the field goes out of positive and into negative, we know the direction of the field is to the right.

- The magnitude of the net E-field is given by:

$$E_{\text{net}} = 2E\cos\theta = 2E\frac{(d/2)}{\sqrt{y^2+(d/2)^2}} = \frac{kqd}{(y^2+(d/2)^2)^{3/2}}$$

Continuous charge distributions:

- So far have discussed only individual charges; works ok for a small number of charges
- What if we what to calculate the force from a large number of charges?
- If charges are distributed uniformly over some region of space then we can use charge distributions; similar to mass densities when calculating masses and moments of inertia.
- Now dealing with continuous variables

	1D	2D	3D
Geometry:			
Charge density:	$\lambda = \dfrac{Q}{L}$	$\sigma = \dfrac{Q}{A}$	$\rho = \dfrac{Q}{V}$
Electric field:	$E = 2k(\lambda/r)$	$E = 2\pi k\sigma$	$E = kQ/r^2$

- Calculations from continuous charge distributions are more complicated to solve mathematically.

- One important case is an uncharged or charged, conducting metal object in an E-field.

- Suppose we put a net charge on a conducting object. How does the charge distribute itself?

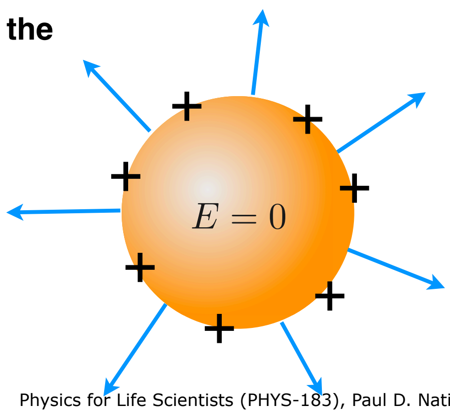
- The charges will experience Coulomb forces between each other causing them to repel.

- The charges will move until they reach equilibrium and there are no net forces.

- In equilibrium, there is no E-field inside of the conductor!

    - If there was, then there would be a net force, and thus no equilibrium.

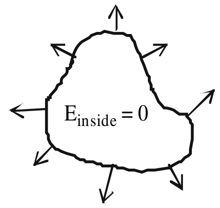
In a conductor, there is no E-field inside the conductor after reaching equilibrium



- In equilibrium, The E-field just outside the conductor is perpendicular to the conductors surface.

- If not, then the charges would have a net force

For a conductor, the E-field just outside the conductor after reaching equilibrium is perpendicular to the surface of the conductor



E-field inside any conductor is zero, and the E-field near the conductor is perpendicular.

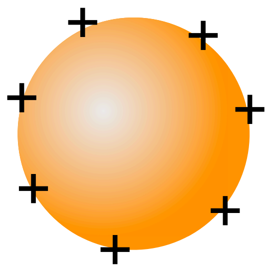
- These statements are true for any shaped conductor

- What about the charge distribution?

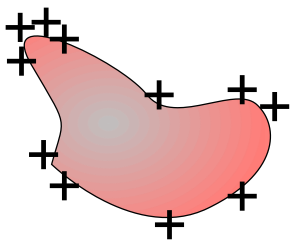
- Charge must be distributed so that the E-field satisfies the two properties that we have mentioned.

- For simple objects with symmetry such as spheres, cylinders, and large plane surfaces, the charge is distributed uniformly.

- For more complicated objects, the charge is not uniform.
    - The charge will build up where the object has to most curvature (i.e. at points)



Uniform charge distribution

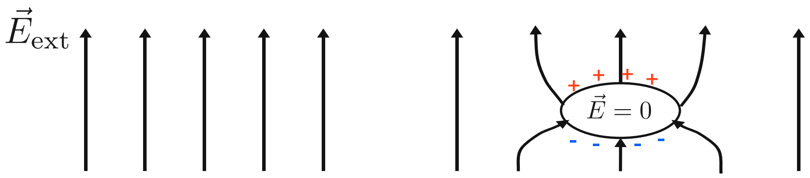


Non-uniform charge distribution

- What happens if you put a neutral conductor in an E-field?

- Again, in equilibrium, the E-field must be perpendicular at the surface and zero inside.

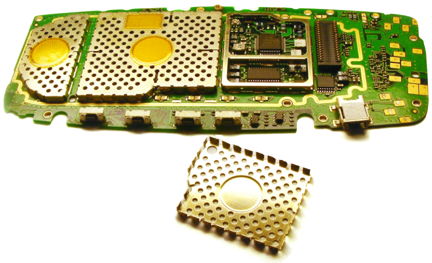
Charges in the conductor move so that they generate an electric field inside the conductor that cancels the external E-field



- The external E-field is modified by the conductor so that the E-field is perpendicular on the surface.

- Since the E-field inside is always zero, we can use conductors to **electrically shield** things from the E-field.

- Very important for electrical equipment (mobile phones, computers, satellites,...)



Electric shielding on a mobile phone

Electrophoresis:

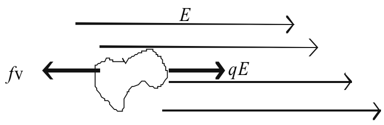
- **Electrophoresis** is the forced movement of charged macroscopic molecules in an electric field.

- If the macromolecule has a charge q, and is in a constant E-field, then the net force on the molecule will be

$$\vec{F} = q\vec{E}$$

- As the object accelerates, there will be a friction force that opposes the motion.

$$F_f = -f\vec{v}$$



- When the forces balance, the molecule will be moving at a constant velocity:

$$\vec{v} = \frac{q\vec{E}}{f}$$

-The velocity divided by the E-field is called the **Electrophoretic mobility** *U*:

$$U = \frac{v}{E} = \frac{q}{f}$$

-This value is an intrinsic property of the macromolecule depending on its charge and friction properties

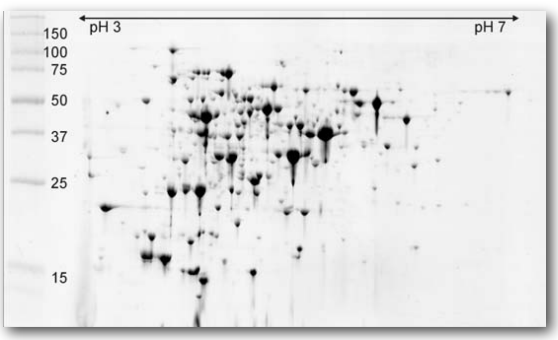
- Unlike the objects we have encountered so far, the charge on a macromolecule is not fixed, but rather depends on the pH of the fluid in which the molecule is placed.

- By adjusting the pH, the charge can be made positive, negative, or neutral.

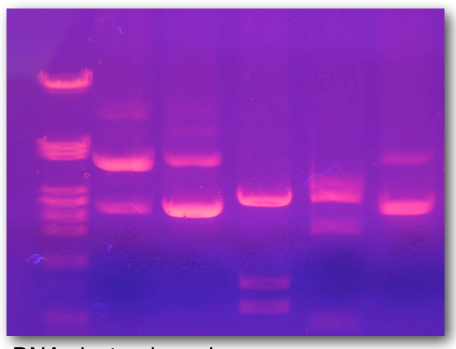
- The pH where the molecule is neutral is called the **isoelectric point**.
    - If pH > isoelectric point then molecule is negatively charged.
    - If pH < isoelectric point then molecule is positively charged.

- The electrophoretic mobility is related to the particles mass through the friction term.

- Therefore, by playing with the strength of the E-field, the pH of the fluid, and the time that the E-field is applied, we can obtain, for example, the mass of the macromolecules.



Electrophoresis of bacterial proteins



DNA electrophoresis