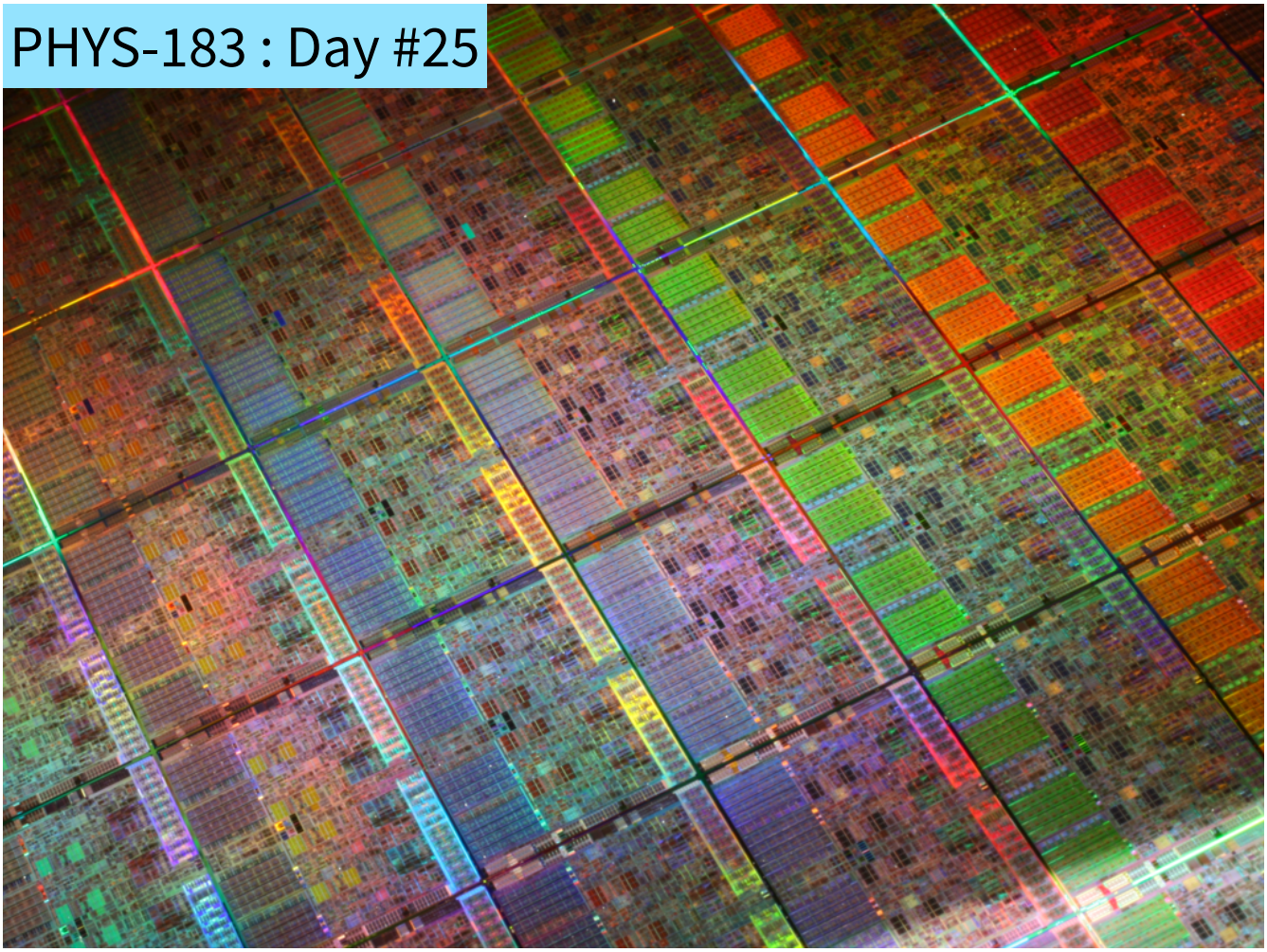
PHYS-183 : Day #25



Electrical Current and Resistance:

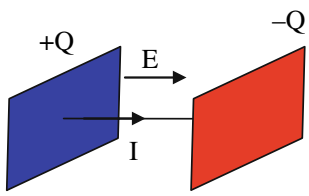
- So far in our study of electricity and electrical forces, we have confined ourselves to the study of electrostatics where the charges are not moving.

- This week we will look at new phenomena that occur when charges are free to move.

- The flow of charges is called **electric current** and will be the focus of our discussion.

- When we looked at cell membranes, we said that they could be modeled as a simple parallel plate capacitor.

- We also saw that cells have channels that transport charges in and out of the cell.



Capacitor + wire model of a cell membrane and membrane channel.

- In our capacitor model, these channels can be modeling as a conducting wire that allows charges to flow from one plate to another.

- The electric field along the wire will cause the free electrons to move.

- The rate of flow for the charge along the wire is the **electric current**:

$$I = \frac{\Delta Q}{\Delta t}$$

- The direction of the current is chosen to be in the direction opposite the flow of the electrons.

  - The current can be thought of as the flow of positive charges.

- The unit for the electrical is called the ampere (*A*), $1\ \mathrm{A} = 1\ \mathrm{C/s}$

- The electric current flowing through the wire in this situation is not constant

- As charge is transferred, the E-field is reduced [$E = 4\pi k(Q/A)$] and the current flow is reduced

- Later we will see how to calculate the time-dependent current flow.

- It is important to point out that, in order for charges to move, there must be an E-field inside the conducting wire.

- But last time we said that there was no E-field inside a conductor in equilibrium.

- To get the charges moving, there is obviously a net force and therefore this situation is not equilibrium!

- A capacitor that is discharging causes a current that decreases with time. How do we generate a current that remains constant?

- We have seen that a charged capacitor stores energy in the E-field: $\dfrac{U_\mathrm{C}}{(\mathrm{Vol})} = \dfrac{1}{4\pi k}\dfrac{E^2}{2}$

- As the capacitor discharges, the E-field decreases and the stored potential energy is converted into kinetic energy of the moving electrons.

- Therefore, to generate a constant current, we need some source of energy to keep the magnitude of the E-field constant as the charges move.

- The devices that generate this fixed amplitude E-field are called **voltage sources**

  - The best known voltage source is a battery.

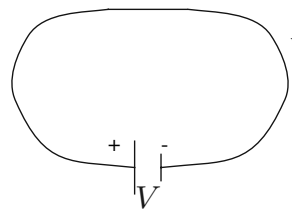
  - Batteries convert chemical energy to electric energy to keep the voltage across the device constant.

  - Batteries have both a positive side and a negative side, like a capacitor.

  - To get charges moving, we need to put a conducting wire across the two sides called **terminals**.

Battery with a fixed potential difference of 1.5V

- A basic circuit consisting of a battery with a uniform wire connecting its terminals is shown to the right.



Circuit consisting of a single voltage source (battery)

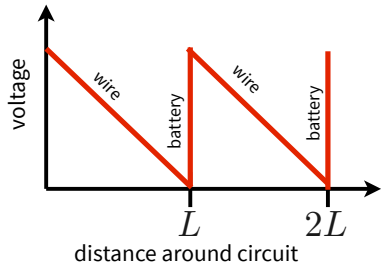
- Voltage sources are always drawn with a longer line on the (+)-side and shorter line on the (-)-side.

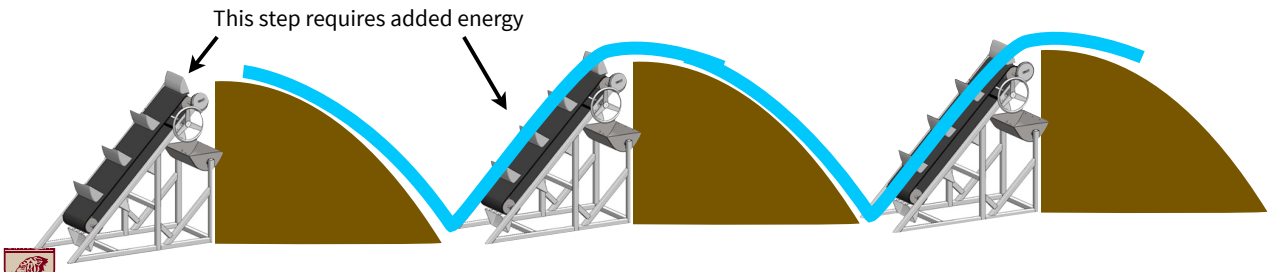
- Since the voltage is a constant, the E-field across the wire with length L is given as $E = V/L$

- The negative side of the battery is set to be the zero voltage, so the voltage across the wire drops linearly from the maximum voltage to zero voltage.

  - The voltage across the battery increases from the conversion of chemical to electrical energy.

  - The flow of current around the wire is very similar to the flow of water down a hill.





- We now would like to understand how this current is generated by considering the microscopic picture of a collection of free electrons and the forces that act on them.

- If there is no electric field, then the only force on the electrons is a random thermal force that causes diffusion along the wire.

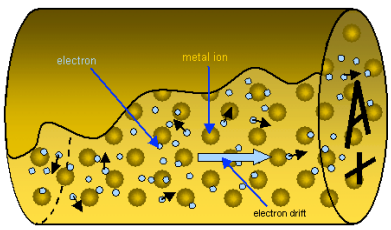
- This motion is random, with a very high speed $v_{\text{diff}} \sim 10^6 \text{ m/s}$ and the electrons randomly hit the the metal atoms inside the wire.

- Unlike the speed, the average velocity is zero! Therefore there is no net flow of charge and no electric current

- When an electric field is applied, the electrons will now have experience an acceleration in the opposite direction of the electric field

$$\vec{a} = \frac{\vec{F}}{m_e} = -\frac{e\vec{E}}{m_e}$$

- This acceleration lasts until the electron hits a metal atom and the electron moves off in a new random direction

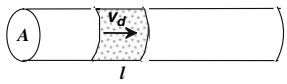


Motion of electrons due to applied E-field

- Because the electrons move so fast, the time in between collisions with metal atoms, τ , is very short

The average velocity of the electrons is very slow, about 10^{-3} $\mathrm{m/s}$, and is called the **drift velocity**.

$$\vec{v}_{\text{drift}} = \vec{a}t = \frac{-e\vec{E}}{m_e}\tau = \frac{eV\tau}{m_e L}$$



Free charge in a wire with area *A* and length *l* with a drift velocity v_{drift}

- If the number of free electrons per unit volume in the wire is called *n* and the wire has a cross-sectional area *A,* then the net free charge in a short length of the wire *l* is

$$\Delta Q = nAle$$

- To find the current, we need to divide by the amount of time it takes for the charges to move across this length of wire:

$$I\frac{\Delta Q}{\Delta t} = \frac{nAle}{l/v_{\text{drift}}} = nAev_{\text{drift}}$$

- Expressing the drift velocity in terms of the voltage, the current becomes:

$$I = \frac{ne^2\tau A}{m_e L}V$$

- We now define a new property called the **conductance** σ that is a measure of how easily charges move in a particular kind of conductor

$$\sigma = \frac{ne^2\tau}{m_e}$$

- We can now rewrite the current as

$$I = \sigma\frac{A}{L}V = GV$$

where *G* is known as the **conductance**. The conductance depends on the material <u>and</u> the geometry of the wire.

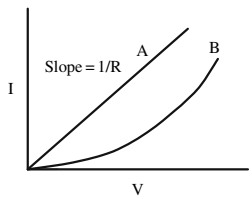
- Solving for the voltage *V*, this equation can be rewritten in terms of the **resistance** *R*

$$\boxed{V = IR} \qquad R = \frac{1}{G} = \frac{1}{\sigma}\frac{L}{A} = \rho\frac{L}{A}$$

- The **resistivity** ρ of the material is the inverse of the materials conductivity: $\rho = 1/\sigma$

- We see that the current flowing through a wire is proportional to the voltage across the wire.

- This linearity of current with applied voltage is known as **Ohm's Law**.



Ohm's Law is a linear I-V curve

- Ohm's Law also states that the resistivity of the material remains constant, independent of the applied voltage.

- Ohm's Law is not a "real" law like Newton's laws, it can be broken.

- Most materials in the real-world are do not obey Ohm's law

- The resistance is measured in Ohms (Ω): $1\ \Omega = 1\ \mathrm{V/A}$

- Resistivity is then measured in Ohm-meters $\Omega \cdot \mathrm{m}$

- The conductivity is inverse Ohm's called Siemens (S): $1/\Omega \equiv S$

- As we could guess, the resistivity is the lowest in conductors such as metals, and highest in insulators where charges can not move.

Material	*Resistivity, ρ ($\Omega \cdot m$)*
Conductors	
Aluminum	2.8×10^{-8}
Copper	1.7×10^{-8}
Iron	$10. \times 10^{-8}$
Mercury	$96. \times 10^{-8}$
Silver	1.6×10^{-8} ← Lowest
Tungsten	5.6×10^{-8}
Ionic materials	
Water (distilled)	$\sim 2 \times 10^{5}$
Fresh water	$\sim 5 \times 10^{2}$
Sea water	~0.3
Cytoplasm	~0.5
Fatty tissue	~15
Semiconductors	
Germanium	~0.5
Silicon	$\sim 2. \times 10^{3}$
Insulators	
Air (dry)	4×10^{13}
Glass	$10^{10} - 10^{14}$
Rubber	$10^{13} - 10^{16}$

Resistivity of various common materials

<u>Ex:</u> How much current flows through fresh water contained in an insulating tube 0.1m long and 0.05m in diameter when a 100V potential is applied across it?

<u>Solution:</u>

- The current through the tube of water is given by $I = V/R$

- Since we are not given the resistance we must calculate it from the resistivity and the geometry of the tube

$$R = \rho \frac{L}{A} = 5 \times 10^{2} \frac{0.1}{\pi (0.05/2)^{2}} = 5 \times 10^{2}(51) = 25500\ \Omega$$

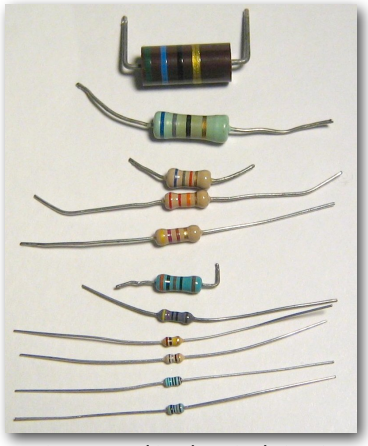
- Given the voltage, the current can now be calculated directly

$$I = \frac{100\ \mathrm{V}}{25500\ \Omega} = \boxed{3.9 \times 10^{-3}\ \mathrm{A}}$$

- In electrical circuits, resistors are simple devices that are manufactures to have a fixed specified amount of resistance

- When used in a a circuit drawing, the symbol for a resistor is

- For example, a circuit with a battery and two resistors, one after the other, is shown below.

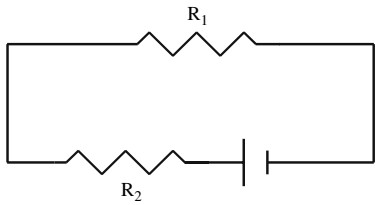


Resistors used in electrical circuits

Electrical Energy and Power:

- In a simple circuit, the terminals of the battery are maintained at a constant voltage by the chemical energy stored in the battery.

- If we plot the electric potential as a function of length along the wire , we saw that is decreases linearly to zero as we approach the negative side of the battery (set to V=0).



Circuit diagram with two resistors and a single battery

- The change in potential energy for a positive charge, going from positive to zero voltage is

$$\Delta PE = \Delta QV$$

- Because the charge moving through the wire in a time Δt is given by $\Delta Q = I\Delta t$, the rate at which electrical energy is lost is given by the **electrical power** *P*:

$$\boxed{P = \frac{\Delta PE}{\Delta t} = \frac{\Delta Q}{\Delta t}V = IV}$$

- The electrical power is measured in Watts (W), like mechanical power.

- Because the potential is held constant, the kinetic energy is constant along the wire.

  - However, some energy is always being lost due to collisions between the moving electrons and the atoms of the conductor.

  - The lost energy appears as heat coming from the circuit.

  - The electrical energy is said to be lost because we can not take the heat and convert it into useful work.

  - This heat generation is one of the fundamental issues that prevents electronic devices from becoming smaller and faster.

Cooling device used to remove heat from a computer processor.

- Conversely, electronics can be designed specifically to create heat.

A space heater creates a big resistance to generate heat

- Using Ohm's law, we can also express the electrical power as a function of voltage or current only

$$\boxed{P = IV = I^2R = \frac{V^2}{R}}$$

- It is important to remember that these last two expressions are only valid when Ohm's law is valid.

<u>Ex:</u> Calculate the power consumed for the previous example of fresh water in a tube

<u>Solution:</u>

- Since we know the current, voltage, and resistance, we can use any of the formulas to calculate the power

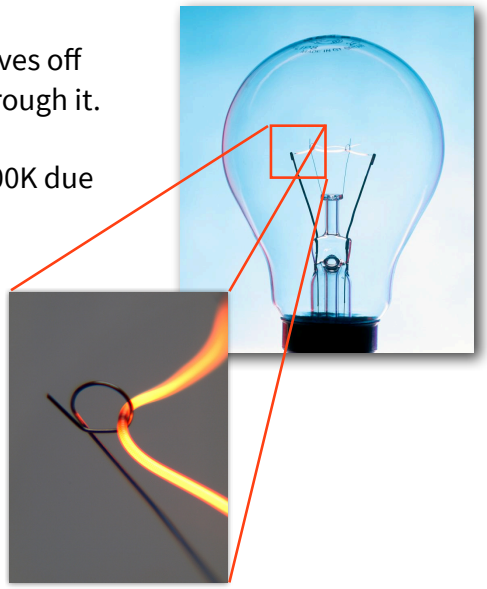
$$P = \frac{V^2}{R} = \frac{100^2}{25500} = \boxed{0.4\ \text{W}}$$

- Some resistors are designed to release their energy as radiation... these are called incandescent lightbulbs

- This type of lightbulb uses a piece of tungsten metal that gives off both heat and radiation when electrical current is passed through it.

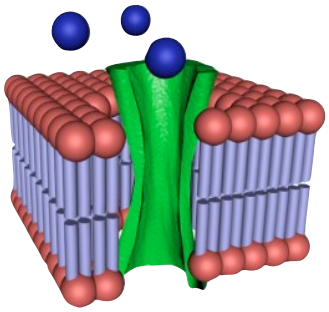
- The tungsten metal in a standard light bulb is heated to 3000K due to the resistance and gives off white light.

- All lights, including fluorescent and LED lights all use energy at a rate given by the power of the bulb



- The light given off by a light bulb is direct evidence for electrical current flow through the bulb.

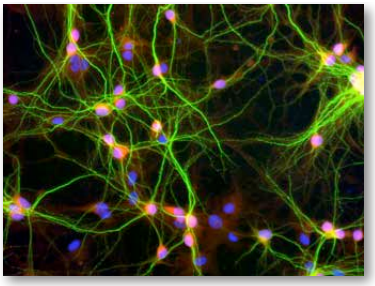
- Last time we saw that a cell membrane has many membrane channels that allow things like sodium Na+ and potassium K+ ions to go in and out of the cell.



Membrane channel letting ions into and out of cells.

- The motion of these ions is also an electrical current just like the electrons moving in a circuit

- This type of electrical current is responsible for a large part of the signaling that goes on inside of all living organisms (the other is chemical signaling)



Neurons inside a human brain

- The best example are neurons inside of your brain that communicate via the electrical current generated from the movement of Na+ and K+ ions.

- I want to prove to you that ions such as Na+ do actually create a current just like that of moving electrons my creating a lightbulb that uses moving Na+ charges to generate light.