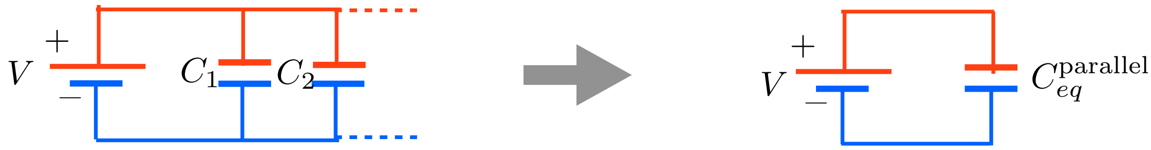
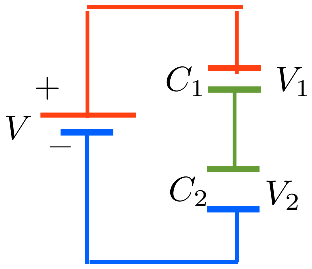
- Recall from last time:

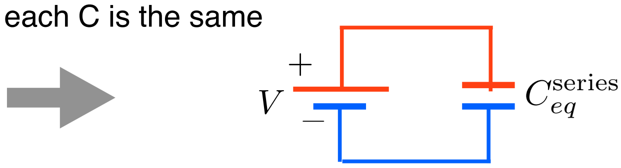
Voltage across each C is the same



Equivalent capacitance given by: $C_{eq}^{\mathrm{parallel}} = \sum C_i^{\mathrm{parallel}}$



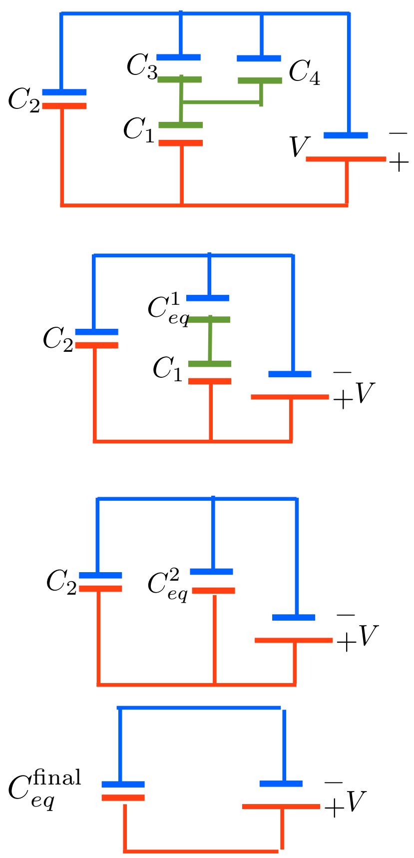
Charge on each C is the same



Equivalent capacitance given by: $\dfrac{1}{C_{eq}^{\mathrm{series}}} = \sum \dfrac{1}{C_i^{\mathrm{series}}}$

- Can use these to simplify a collection of capacitors into a single equivalent capacitor

Ex. Simplifying Capacitors I:



- Want to reduce problem to single capacitor. What is the equivalent capacitance?

- Want to reduce problem to single capacitor. What is the equivalent capacitance?

- Need to begin with C3 and C4. Cannot do C1 and C3 first since C4 is connected in the middle.

$$C_{eq}^1 = C_3 + C_4$$

- Now we need to do the capacitors in series since nothing is in parallel at the moment

$$\frac{1}{C_{eq}^2} = \frac{1}{C_1} + \frac{1}{C_{eq}^1} \rightarrow C_{eq}^2 = \frac{C_1 C_{eq}^1}{C_1 + C_{eq}^1}$$

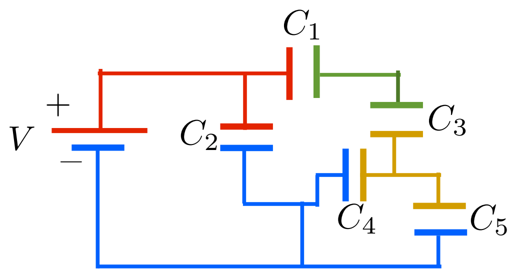
- Finally, add up capacitors in parallel:

$$C_{eq}^{\mathrm{final}} = C_2 + C_{eq}^2 = C_2 + \frac{C_1 C_{eq}^1}{C_1 + C_{eq}^1}$$

$$\boxed{C_{eq}^{\mathrm{final}} = C_2 + \frac{C_1 (C_3 + C_4)}{C_1 + C_3 + C_4}}$$

- Equations for equivalent capacitance become quite complicated fast
    - If your given numbers for the capacitance use them for simplification!

<u>Ex. Simplifying Capacitors II:</u>



$C_1 = 3.0\ \mathrm{nF}$ $\quad C_2 = 2.1\ \mathrm{nF}$

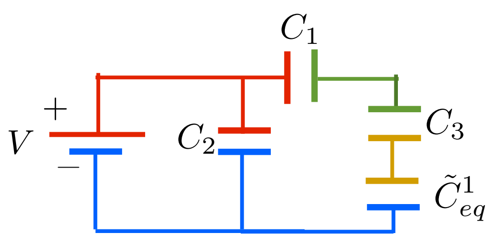
$C_3 = 2.0\ \mathrm{nF}$ $\quad C_4 = 4.7\ \mathrm{nF}$

$C_5 = 1.3\ \mathrm{nF}$

- Lets use color to find the capacitors in series and parallel.
  - Entire voltage V across C2 -> everything is in parallel to C2
  - Voltage across C4 and C5 is the same (yellow -> blue) thus they are in parallel.
  - Voltage across C1 and C3 are different so they are in series.

- Do C4 & C5 first: $\quad \tilde{C}^1_{eq} = C_4 + C_5 = 6\ \mathrm{nF}$

- Redraw circuit

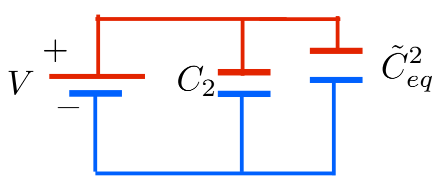


$\tilde{C}^1_{eq} = 6\ \mathrm{nF}$

- Now C1, C3, and $\tilde{C}^1_{eq}$ are in series and we must calculate those before doing C2.

$$\frac{1}{\tilde{C}^2_{eq}} = \frac{1}{C_1} + \frac{1}{C_3} + \frac{1}{\tilde{C}^1_{eq}} = \frac{1}{3\ \mathrm{nF}} + \frac{1}{2\ \mathrm{nF}} + \frac{1}{6\ \mathrm{nF}} = \frac{1}{1\ \mathrm{nF}}$$

- Redraw circuit: $\quad \tilde{C}^2_{eq} = 1\ \mathrm{nF}$

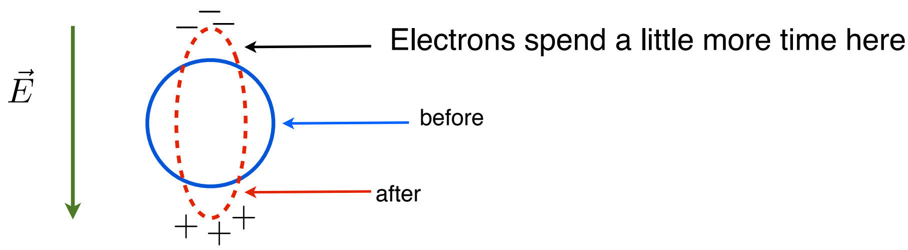


- Total capacitance is now just sum over last two:

$$\tilde{C}^{\mathrm{total}}_{eq} = C_2 + \tilde{C}^2_{eq} = 2.1\ \mathrm{nF} + 1\ \mathrm{nF} = \boxed{3.1\ \mathrm{nF}}$$

Dielectrics:

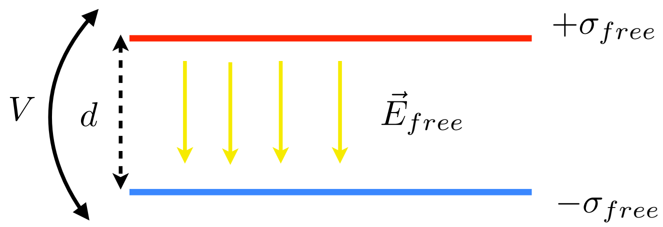
- E -fields can induce dipoles in insulating materials
- Recall that electrons are not free to move in an insulator
- Suppose I apply an E-field to a round atom or molecule



- This is a result of induction and is called **Polarization**

- Materials that behave like dipoles under E-fields are called **dielectrics**

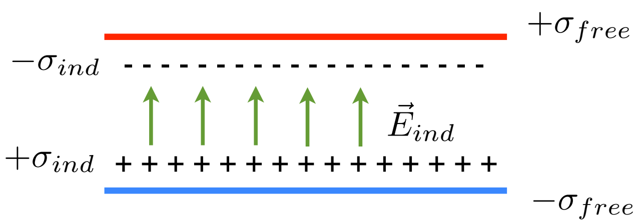
- Suppose I have two charged plates, charged by a voltage source V:



$$E_{free} = \frac{\sigma_{free}}{\epsilon_0}$$

-Now I remove V, and add a dielectric inside:

-Negative charges of dielectric go toward positive plate; postive charges toward negative plate



$$E_{ind} = \frac{\sigma_{ind}}{\epsilon_0}$$

- Dipoles create constant E-field <u>in opposite direction</u> from $\vec{E}_{free}$

- The total electric field inside the plates is just the vectorial sum of the 2 fields:

$$\vec{E}_{total} = \vec{E}_{free} + \vec{E}_{ind}$$

- Since E_ind is in opposite direction, the total <u>magnitude</u> of the E-field is:

$$|E_{total}| = |E_{free}| - |E_{ind}|$$

since opposite direction

- The induced charge can never be as large as the free charge so we assume:

$$\sigma_{ind} = \beta\sigma_{free} \qquad \beta \ll 1 \quad \Rightarrow \quad E_{ind} = \frac{\beta\sigma_{free}}{\epsilon_0}$$

- Therefore, the total E-field is now: $E_{total} = E_{free} - \beta E_{free} = \underbrace{(1-\beta)}_{= 1/\kappa} E_{free}$

- κ is called the **"dielectric constant".**

- The total electric field inside the capacitor will therefore always be:

$$\boxed{\vec{E} = \frac{\vec{E}_{free}}{\kappa}}$$

- We can also write the E-field inside the capacitor as:

$$E = \frac{1}{\kappa}E_{free} = \frac{\sigma_{free}}{\kappa\epsilon_0} = \frac{\sigma_{free}}{\epsilon}$$

where $\epsilon = \kappa\epsilon_0$ is called the "**electric permittivity**" of the dielectric.

permittivity of free space

- > can just replace ϵ_0 with ϵ.

- Sample dielectric constant values:

Vacuum	1.0
Air	1.000059 (always assumed to be = 1)
Paper	3
Glass	5

- Some materials are dipoles even without an E-field, these permanent dipoles can be extremely strong:

Water	80.4

- What does a dielectric do to the capacitance?

$$\uparrow C = \frac{Q}{V} = \frac{Q}{\downarrow Ed}$$

- Dielectrics always decrease the E-field -> Capacitance always increases.

$\Rightarrow$ Using a dielectric allows me to store more charge on capacitor

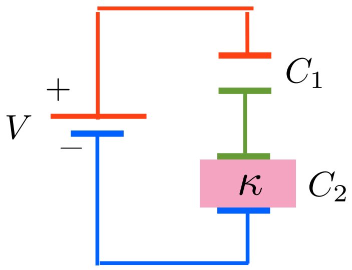
- The capacitance with a dielectric can be expressed as:

$$C_\kappa = \frac{Q}{V} = \frac{Q}{E_{total}d} = \frac{\kappa Q}{E_{free}d} = \kappa\frac{Q}{E_{free}d} = \kappa C$$

- Capacitance is just κ times the capacitance without the dielectric

$$\boxed{C_\kappa = \kappa C}$$

<u>Ex. Dielectrics I:</u> - Suppose I have two capacitors C1 and C2 with same area A and separation d, and C2 is filled with a dielectric with constant κ



a) What is the charge on each capacitor?

- Recall that the charge on each capacitor in series is the same

$$Q = C_{eq}^{\text{series}}V$$

- Must find equivalent capacitance:

$$\frac{1}{C_{eq}^{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{\kappa C_1} \Rightarrow C_{eq}^{\text{series}} = \frac{\kappa C_1}{\kappa + 1}$$

- Therefore: $Q = \frac{\kappa C_1}{\kappa + 1}V$ but $C_1 = \frac{\epsilon_0 A}{d}$ $\boxed{Q = \frac{\epsilon_0 A}{d}\frac{\kappa}{\kappa + 1}V}$

b) What is the total energy inside the capacitors?

$$U = \frac{1}{2}QV \quad \text{but since we know total charge:} \quad Q = C_{eq}V$$

$$U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\frac{\kappa C_1}{\kappa + 1}V^2 = \boxed{\frac{\epsilon_0 A}{2d}\frac{\kappa}{\kappa + 1}V^2}$$

c) What is the E-field inside C2?

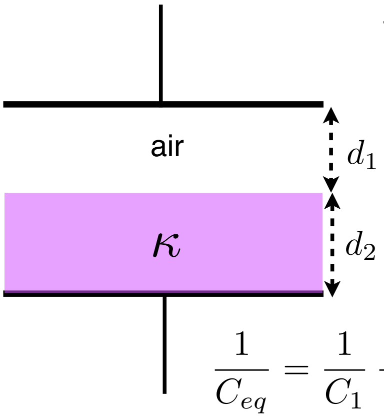
- Recall for E-field in capacitor with dielectric: $E = \frac{1}{\kappa} E_{free} = \frac{\sigma}{\kappa \epsilon_0} = \frac{Q}{\kappa \epsilon_0 A}$

- We have already calculated charge though all capacitors: $Q = \frac{\epsilon_0 A}{d} \frac{\kappa}{\kappa + 1} V$

Therefore: $E = \frac{1}{\kappa \epsilon_0 A} \frac{\epsilon_0 A}{d} \frac{\kappa}{\kappa + 1} V = \boxed{\frac{V}{d} \frac{1}{\kappa + 1}}$

<u>Ex. Dielectrics II:</u>

- Suppose I am given the following capacitor with area A. What is the capacitance?



- This looks like two capacitors in series, one with no dielectric, one with dielectric

$$C_1 = \frac{\epsilon_0 A}{d_1} \qquad C_2 = \kappa \frac{\epsilon_0 A}{d_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{\epsilon_0 A} + \frac{d_2}{\kappa \epsilon_0 A} = \frac{\kappa d_1 + d_2}{\kappa \epsilon_0 A} \qquad \boxed{C_{eq} = \frac{\kappa \epsilon_0 A}{\kappa d_1 + d_2}}$$