PHYS-183 : Day #6

- From last time, we saw that the net work done on an object is related to the change in kinetic energy

$$W_{\rm net}=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2=\Delta T$$

- We also made the general statement that: **Energy, by definition, is the ability to do work.**

- The kinetic energy, associated with an objects motion, can do work.
    - Is there any other kind of energy that also can be used to do work?

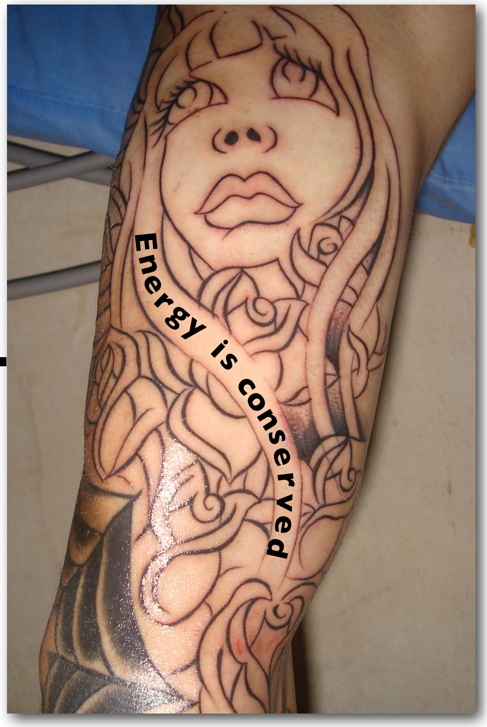
- Today we discuss the **potential energy (U):** The energy of interaction associated with the position of an object.
    - The amount of potential energy of an object depends on its location. In one-dimension U=U(x)
    - We will look at the gravitational potential energy due to the gravitational interaction of an object with Earth.
    - We will also look at the elastic potential energy due to a spring.
    - There are many different kinds of potential energy: thermal, electrical, magnetic, chemical, nuclear,...

- Although there are many types of <u>both</u> potential and kinetic energy it is important to remember:

Energy can be converted from one kind to another, but energy can never be destroyed!

- "**Conservation of Energy**" - The most important law in the universe

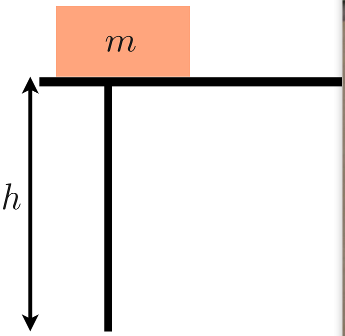
- If you remember anything from this class please remember this!



<u>Gravitational Potential Energy:</u>

- Consider a box of mass m sitting on the edge of a table that is at a height h above the floor

- If the box falls from the table, then gravity will do work on the box:



$$W_{\rm grav} = mgh$$

$$mg = F$$
$$h = \Delta x$$

- Force and displacement in same direction

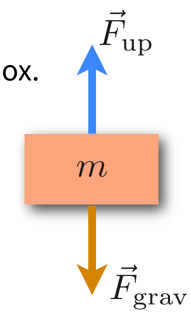
- Using the work-energy theorem, we can calculate the kinetic energy immediately before the box hits the ground:

$$T = mgh$$ (Since initial velocity was zero)

- Once the box hits the ground, there is a <u>very strong</u> force upward on the box, that stops the box over a very small displacement so that the final kinetic energy of the box is zero

$$W_{\rm ground} = F_{\Delta x} = -W_{\rm grav}$$

- To lift the box back up to the top of the table, we need to do positive work on the box.



- When we lift the box, both our force and the force of gravity act on the box

- One possible way to lift the box up to the table is to move the box at a constant velocity up to the table:

(1) Start with a force $\vec{F}_{\rm up}$ slightly bigger than $\vec{F}_{\rm grav}$ to get an upward velocity

(2) For the rest of the trip $F_{\rm up} = F_{\rm grav}$ so that the velocity is constant (no net force)

(3) At the top, reduce $F_{\rm up} < F_{\rm grav}$ to slow the box down to zero.

$$\Rightarrow \quad W_{\rm up} = mgh \qquad W_{\rm grav} = -mgh$$

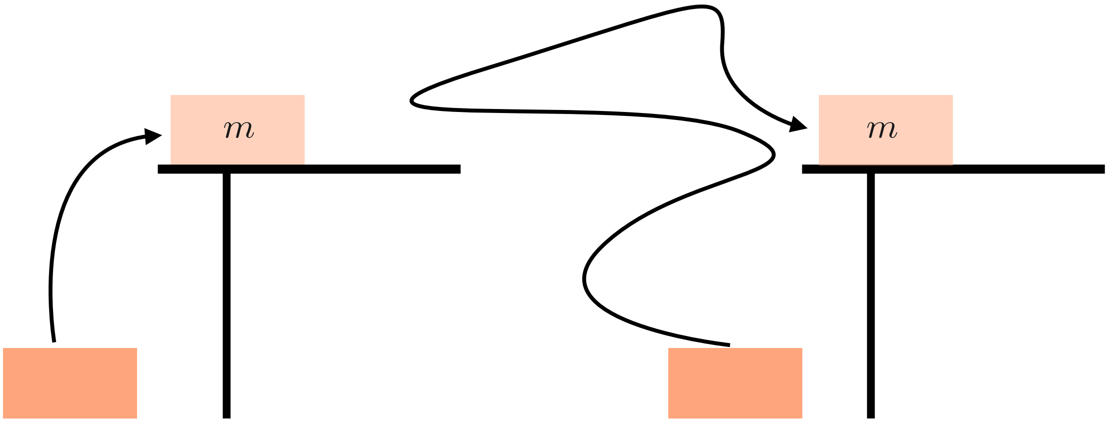
(opposite directions)

- What is the net work done on the box? $W_{\text{net}} = W_{\text{up}} + W_{\text{grav}} = mgh - mgh = 0$

- But this must be zero since both the initial and final velocity of the box are zero!

$$W_{\text{net}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - 0 = 0$$

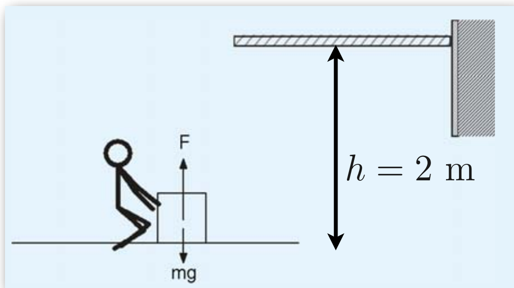
- This tells me that no matter how I take the box from the ground to the table, the net work must be zero



Both of these paths have the exact same about of work... zero work!

Ex. Lifting a box:

Suppose that a 3kg box is lifted vertically from the ground and thrown exactly (v=0 at top) onto a counter that is 2m off the ground. For the first meter, the upward force is equal to twice the force of gravity is applied and then the box is let go. Find the work done on the box and the maximum speed



Solution:

The work done by the person on the box is:

$$W_{\text{up}} = 2mgd = 2(3\text{ kg})(9.8\text{ m/s}^2)(1\text{ m}) = 59\text{ J}$$

The work done by gravity is:

$$W_{\text{grav}} = mgh = (3\text{ kg})(-9.8\text{ m/s}^2)(2\text{ m}) = -59\text{ J}$$

The reason that these numbers are equal and opposite is because the box made it exactly to 2m and then had zero velocity.

The maximum velocity and be calculated via the work-energy theorem over the first 1m

$$W_{\text{net}} = 2mgd - mgd = mgd = (3\text{ kg})(9.8\text{ m/s}^2)(1\text{ m}) = 29.5\text{ J} = \frac{1}{2}(3\text{ kg})\text{v}^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2(29.5\text{ J})}{3\text{ kg}}} = \boxed{4.5\text{ m/s}}$$

- For an object with mass *m* with height *y* above some reference level (y=0) the **gravitational potential energy** is defined to be:

$$\boxed{U = mgy}$$

- When an object changes height from y_1 to y_2, the force of gravity causes a change in the potential energy of the object

$$\Delta U = U_{\text{final}} - U_{\text{initial}} = mg(y_2 - y_1)$$

that is equal in magnitude to the amount of work done by gravity.

If $(y_2 - y_1) > 0$ (increase in height) then $\Delta U > 0$ but since $\vec{F}_{\text{grav}}$ points down $W_{\text{grav}} < 0$

If $(y_2 - y_1) < 0$ then $\Delta U < 0$ and $\vec{F}_{\text{grav}}$ is in same direction so $W_{\text{grav}} > 0$

- Therefore the work done by gravity is equal in magnitude, but opposite, in sign from *U*

$$\boxed{W_{\text{grav}} = -\Delta U}$$

- If gravity is the only force acting on the object, we can use the work-energy theorem

$$W_{\text{grav}} = \Delta T = T_2 - T_1 = -\Delta U = -U_2 + U_1$$

- Lets move all the (1) terms on the left, and all (2) terms on the right

$$T_1 + U_1 = T_2 + U_2$$

- Each side represents the total energy of our object at a fixed position (1) or (2)

➡ The total energy at each point is the same!

- Since the points (1) and (2) are arbitrary, this is statement is true for ALL points.

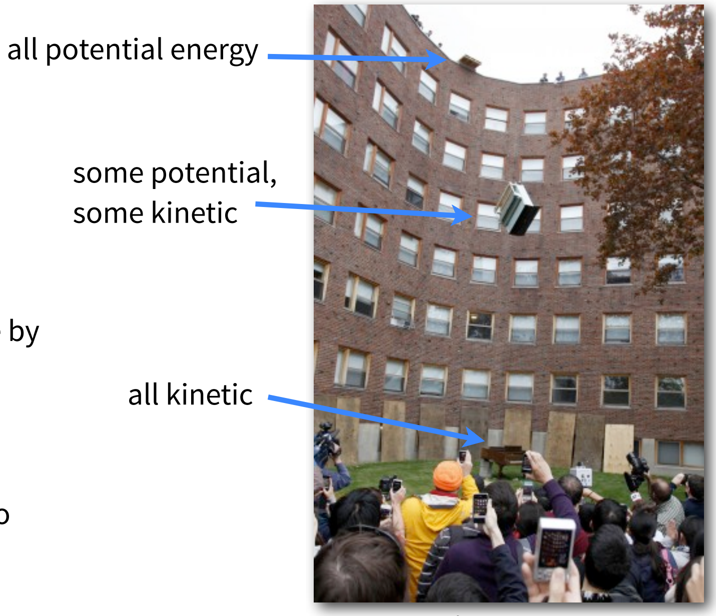
- If we call the total energy of our object $E = T + U$

The total energy of our system (object) is a constant of the motion

$$E = T + U = \text{constant}$$

(The total energy is conserved)

- Although the total energy is conserved, energy can be converted from kinetic to potential, or potential to kinetic.



Total energy E=T+U is always the same

- There is another important thing to remember about potential energy

$$W_{\mathrm{grav}} = \Delta T = -\Delta U = -(U_2 - U_1)$$

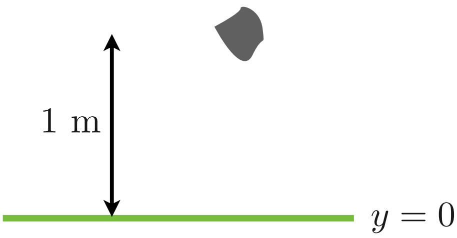
- The change in kinetic energy (or work done by gravity) depends only on the difference in potential energies

- This means that we can set set the height to be y=0 anywhere that we want.

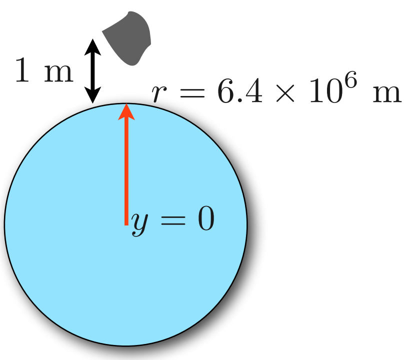
Ex. Falling Rock:

What is the kinetic energy of a 2kg rock that is dropped from 1m?

Ground is y=0



Center of Earth is y=0



- Since rock is at rest, the kinetic energy is just the minus the change in potential energy

Ground is y=0: $T = -\Delta U = -mg(0) + mg(1) = 19.6\ \mathrm{J}$

Center of Earth is y=0: $T = -\Delta U = -mg(6.4 \times 10^6) + mg(6.4 \times 10^6 + 1) = 19.6\ \mathrm{J}$

- Result is exactly the same since only the difference in heights matters

Potential Energy of a Spring:

- In our last class, we calculated the work done by a spring after it is stretched from x1 to x2:

$$W_{\mathrm{spring}} = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

- Just like we did for the gravitational potential energy, we can define the springs potential energy to be the negative of the work

$$\boxed{U_{\mathrm{spr}} = \frac{1}{2}kx^2}$$

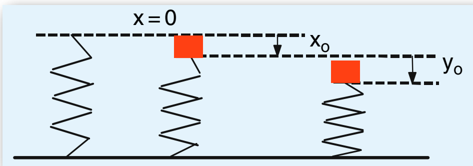
- If a mass *m* is attached to the end of the spring then we can also use the work-energy theorem to show that the total energy of the spring+mass is conserved

$$E = T_1 + U_{\mathrm{spr},1} = T_2 + U_{\mathrm{spr},2}$$

- If there are many different kinds of potentials (gravitational, spring, electrical, ...) then the conservation of energy can easily be extended.

$$E = T + U_{\mathrm{grav}} + U_{\mathrm{spr}} + U_{\mathrm{elec}} + \cdots = \mathrm{constant}$$

Ex. Spring:



- A spring is placed vertically and a 0.1kg mass is placed on the top, compressing the spring by $x_0 = 0.04\ \mathrm{m}$. The mass is then pulled down another $y_0 = 0.05\ \mathrm{m}$ and released with an initial velocity downward of $v_0 = 1\ \mathrm{m/s}$

- (1) Find the maximum compression of the spring relative to x=0, and (2) the maximum velocity.

Solution:

- We don't know the spring constant k, so the first thing to do is find k

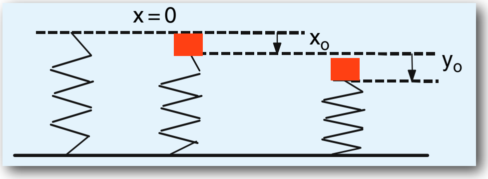
- After the mass is put on the spring, and the mass stops moving, then the net force is zero and the system is in equilibrium

- Therefore, the force of gravity must be equal in magnitude to the spring force

$$mg = kx_0 \quad \Rightarrow \quad k = \frac{mg}{x_0} = \frac{(0.1\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{0.04\ \mathrm{m}} = \boxed{24.5\ \mathrm{N/m}}$$

- Since the net force is zero at x_0 , this is the equilibrium point for the spring.

- Since x_0 is equilibrium, the mass will oscillate around that point



- From the equilibrium point, the spring was stretched an additional length of $y_0 = 0.05\ \mathrm{m}$, so the spring has potential energy

- Also, the spring has an initial velocity of $v_0 = 1\ \mathrm{m/s}$ so there is also kinetic energy

$$E = \frac{1}{2}mv_0^2 + \frac{1}{2}ky_0^2$$

Distance from equilibrium

- We do not need the gravitational potential because the weight of the mass has been removed by measuring the springs displacement from equilibrium (I will show you we do not have to do this)

- When the spring is stretched as far as possible, called the amplitude *A* of the spring ,the total energy must all be potential energy (kinetic energy = 0).

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}ky_0^2$$

$$A = \sqrt{y_0^2 + \frac{mv_0^2}{k}} = \sqrt{0.05^2 + \frac{0.1 \cdot 1^2}{24.5}} = \boxed{0.08\ \mathrm{m}}$$

- Since we want the total distance from x=0: $x_{\mathrm{total}} = x_0 + A = 0.04\ \mathrm{m} + 0.08\ \mathrm{m} = \boxed{0.12\ \mathrm{m}}$

Answer to part #1

- What if we didn't use the equilibrium position x_0, but used x=0 to find the maximum displacement?

- Now we must include the gravitational potential when calculating the total energy and the amplitude where the kinetic energy is zero

$$\frac{1}{2}kA^2 - mgA = \frac{1}{2}kx^2 - mgx + \frac{1}{2}mv_0^2 \qquad x = x_0 + y_0 = 0.09\ \mathrm{m}$$

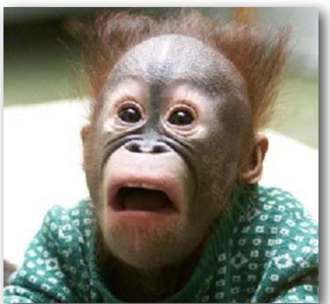
- Since we did not use equilibrium position, we now need to solve ugly quadratic equation

$$A = \frac{mg \pm \sqrt{(mg)^2 + mkv_0^2 - 2mgkx + (kx)^2}}{k}$$

- But the answer is still the same:

not physical

$$\boxed{x = 0.12} \quad \text{or} \quad x = -0.04$$



Always use equilibrium position

Always use the equilibrium position for calculations!

- Now we need to calculate the maximum velocity.

- We know that all of the total energy is potential energy when the spring is stretched all the way

$$E = \frac{1}{2}kA^2$$

- But we also know that the maximum velocity should be when all of the energy is kinetic energy

$$E = \frac{1}{2}mv_{\max}^2$$

- Since the total energy always is the same:

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2 \quad \Rightarrow \quad v_{\max} = \sqrt{\frac{k}{m}}A$$

- This is the exact same answer week got last week using Newton's laws!

$$v_{\max} = \left(\frac{2\pi}{T}\right)A = \omega A = \sqrt{\frac{k}{m}}A$$

Answer to part #2

- Plugging in the numbers for our problem: $v_{\max} = \sqrt{\frac{24.5\ \mathrm{N/m}}{0.1\ \mathrm{kg}}}(0.08\ \mathrm{m}) = \boxed{1.25\ \mathrm{m/s}}$

Forces from Energy:

- So far we have always used forces to calculate work, and then work to calculate the energy of a system

- But, we can go directly between forces (Newton's Laws) and energy

- It is possible to find the force on an object by looking at how the potential energy changes as a function of position

-Energy is just a number, forces are vectors, so in more than one-dimension it is easier to first calculate energies

- Suppose that the potential energy is a function of displacement only $U = U(x)$ just like for gravity and a spring

$$W = F\Delta x = -\Delta U(x)$$

- The work done on an object depends **only** on the displacement, and not on the path of the object, its velocity,...

- If this is true, then the force that does the work is called a **conservative force**

- The force of friction is a **non-conservative force**, the work depends on the path, and velocity

Conservative force ⟷ $U(x)$

(work depends only on displacement) (potential energy depends only on position)

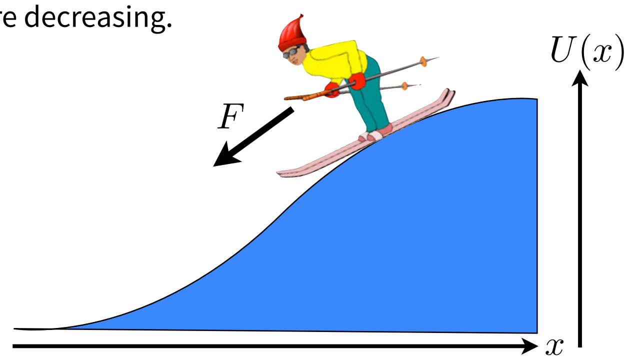
-For conservative forces, the force on an object is found if we know how the potential <u>changes</u> in the x-direction

$$\boxed{F = -\frac{\Delta U(x)}{\Delta x}}$$

Key Idea: For conservative forces, the potential energy $U(x)$ contains all of the information about the forces on an object.

-Because of the minus sign, we see that if U(x) increases and Δx increases, then the force will be negative (in opposite direction). Same thing if both are decreasing.

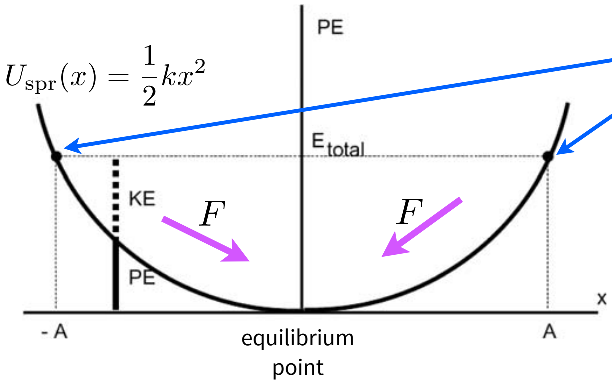
The force on an object always moves the system toward lower potential energy!



-If we take the displacement to be very very small, then we can use calculus to show that

$$\boxed{F = -\frac{dU(x)}{dx} = -\text{slope}}$$

<u>Ex. Spring:</u>



- All energy is potential energy when spring is stretched all the way

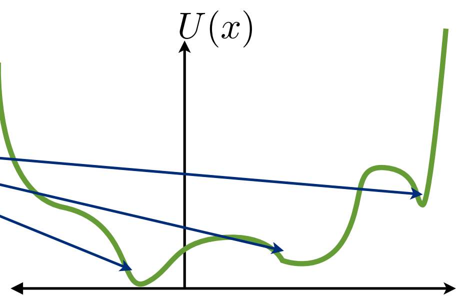
- Points on U vs. x graph where E=U are called **turning points**; objects can not be displaced any more.

- The force always points toward the equilibrium point

$$F = -\frac{dU(x)}{dx} = -\frac{d}{dx}\frac{1}{2}kx^2 = -kx$$

Near the bottom of <u>any</u> potential, the force on an object is the same as a spring.

- All the details are hidden inside the spring constant



Power:

- When work is done on an object, or by an object, the rate at which the work is done, or using the work-energy theorem, the rate at which energy is changed is important.

- The rate at which work is done is called the **power** where $\boxed{P = \dfrac{\Delta W}{\Delta t}}$

- If the work is the result of a constant force then using $W = F\Delta x$

$$P = \frac{F\Delta x}{\Delta t} = F\frac{\Delta x}{\Delta t} = Fv$$

- If force and velocity are in same direction, then power is positive, and kinetic energy will increase

- If power is negative, then force is in opposite direction to velocity and the object slows down, losing kinetic energy

- The units of power are called **Watts (W)**: $1\ \mathrm{W} = 1\ \mathrm{J/s}$

Ex. Jeju Wind Turbine:

- What is the average power from one wind turbine on Jeju-do?

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta T}{\Delta t} = \frac{(1/2)mv^2}{\Delta t}$$

- Need to find average wind velocity and mass of air that goes into the wind turbine

$v_{\mathrm{avg}} = 3.1\ \mathrm{m/s}$ (Jeju avg. wind)

$\rho_{\mathrm{air}} = 1.29\ \mathrm{kg/m^3}$

$r_{\mathrm{turbine}} = 20\ \mathrm{m}$ (small turbine)

Wind turbines on Jeju island.

- We can not find the mass of air, but we can find the mass/time

$$\frac{m}{\Delta t} = \frac{\rho V}{\Delta t} = \rho A v = \pi r^2 \rho v$$

small increase in *v*, huge increase in *P*

- The power can be written now as $P = \dfrac{1}{2}(\pi r^2 \rho) v^3 = 24146\ \mathrm{W}$

- But in the real-world we only get about 40% of the total energy: $\boxed{P_{\mathrm{real}} = 9658\ \mathrm{W}}$

One wind-turbine powers, on average, 743 13-Watt lightbulbs