# Quantum Physics



- At the beginning of the 20th century, many physicists believed that they were close to knowing everything about the universe.

- Thought there were only two small unanswered questions:

  #1: Why do atoms release energy at only specific frequencies?

  #2: How much energy in contained in the light emitted from something at a fixed temperature?

**Blackbody radiation:**

- A **Blackbody** is an object that absorbs and emitted EM waves perfectly for all frequencies of light.

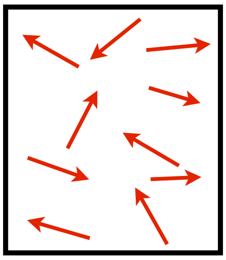
- A **blackbody radiator** is in thermal equilibrium, emits radiation, and is characterized only by its temperature T.

  - It does not depend on the material or shape.

  - It emits the same amount of energy that it absorbs at each frequency.

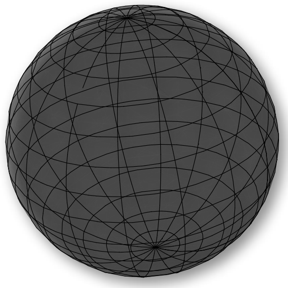
- Examples: gas in a box at fixed temperature, black holes, the universe.

Gas in Box



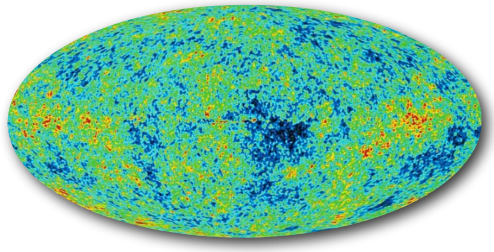
$T$

Black Hole



$T \propto 1/M$

Universe



$T = 2.7\ \mathrm{K}$

- It is known from experiment that the intensity (power/area) of the light emitted from a black body, as a function of temperature, is given by the Stefan-Boltzmann Law:

Stefan-Boltzmann constant: $5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$

$$I = \int_0^\infty \epsilon(\lambda) d\lambda = \sigma T^4$$

Spectral emittance

- Intensity goes as the 4th-power of the temperature.
- We want to find an expression for the spectral emittance $\epsilon(\lambda)$



-From experiment we know:

$$\epsilon(\lambda) \propto e^{-b/\lambda T} \qquad \text{small wavelengths.}$$

$$\epsilon(\lambda) \propto \lambda^{-4} \qquad \text{large wavelengths.}$$

-From classical thermodynamics, the spectral emittance can be calculated to be:

$$\epsilon(\lambda) = \frac{a}{\lambda^5} e^{-b/\lambda T}$$ **Wien's Law**

- a & b are constants

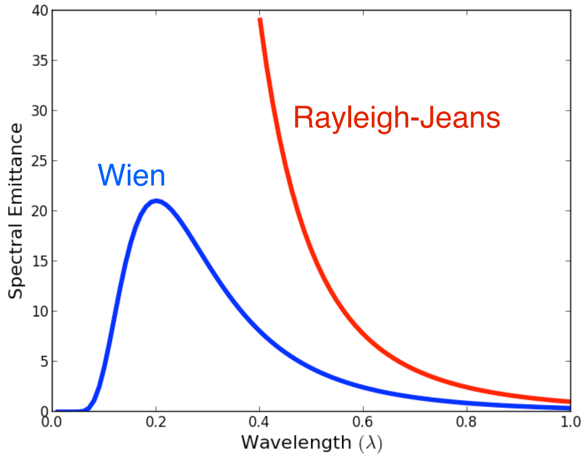
-Works only at small wavelengths.

-The peak of $\epsilon(\lambda)$ depends only on the temperature: $\lambda_{\mathrm{max}} T = 2.9 \times 10^{-3} \mathrm{K-m}$

-From classical electromagnetism we get the **Rayleigh-Jeans Law**:

$$\epsilon(\lambda) = \frac{2\pi c k_{\mathrm{B}} T}{\lambda^4}$$

-Works only at large wavelengths, and is a disaster as small wavelengths.




- Breakdown of R-J Law at small wavelengths is called the "**Ultraviolet Catastrophe**"

- Stefan-Boltzmann law then gives infinite energy that is obviously wrong

- Classical physics can not solve this problem

- Solution to the problem requires **Quantum Mechanics**

- Quantum mechanics comes from the word "quantized"

- Here, it is the light that is quantized. This means that the light comes in little pieces that cannot be broken into smaller pieces.

- The energy of each quantized unit of light, for a given frequency f is:

$$E = hf = hc/\lambda \qquad h = 6.626 \times 10^{-34}\ Js$$ **Planck's Constant**



- Planck's idea: Light behaves like a collection of "springs", and each spring can only have energies that satisfy:

$$E = nhf \qquad n = 0, 1, 2, \ldots$$

where f is the frequency of the "spring".

- With Planck's hypothesis we can recalculate the spectral emittance:

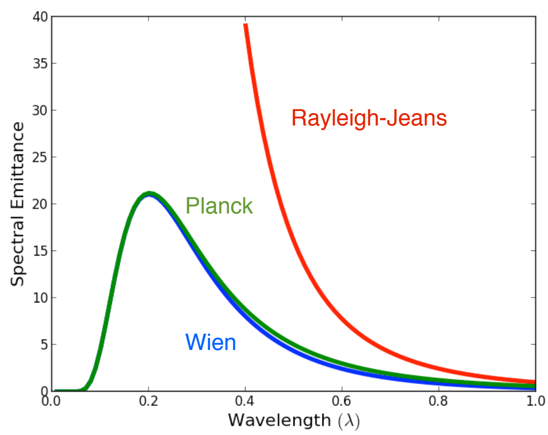
$$\boxed{\epsilon(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{\exp\left(hc/\lambda k_{\mathrm{B}}T\right) - 1}}$$

**[Nobel Prize 1918]**

-This can also be written in terms of frequency:

$$\epsilon(f) = \frac{2\pi h}{c^2}\frac{f^3}{\exp\left(hf/k_{\mathrm{B}}T\right) - 1}$$

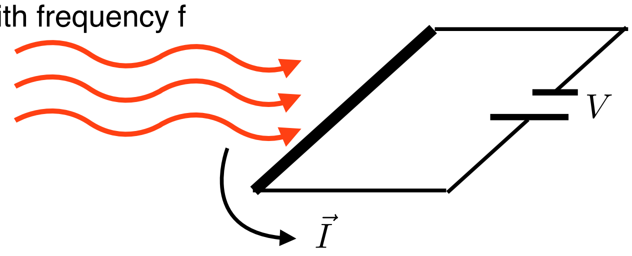
- Quantizing the energy of light means it is harder to excite the high frequency, high energy "springs" and thus corrects the ultraviolet catastrophe



## Photoelectric Effect:

- Although Planck used the quantization of light to solve the ultraviolet problem, the first proof that light comes in discrete units comes from the **Photoelectric effect**

- Suppose we have the following setup




- If the voltage V=0, then light is able to knock electrons off the metal in the wire and generate a current.

- There is a maximum voltage $V = V_{\rm stop}$ where the current vanishes for all light intensities

- From experiment we know:

  #1: The frequency of the light must be higher than a minimum frequency

  #2: Increasing the intensity of light increases the current, but not the kinetic energy



- Using classical physics:

  - Current should be generated for all frequencies **Wrong**

  - Kinetic energy of electrons should increase with increasing intensity **Wrong**

- The kinetic energy of the electron is related to the stoping voltage via

$$\Delta K = 0 - K_{\rm max} = -eV_{\rm max} - 0 = -\Delta U$$

$$\Rightarrow \quad K_{\rm max} = eV_{\rm max} = \frac{1}{2}m_e v^2$$

- Quantum mechanical solution due to Einstein (1905):

  - Light comes in quantized chucks call **Photons,** where each photon has energy $E = hf$

  - Kinetic energy of electron is the energy of the photon minus the energy needed to break the electron from the metal $\phi$ , called the **Work Function**.

$$\boxed{K_{\rm max} = hf - \phi}$$

- Since the kinetic energy cannot be negative, this also gives us the minimum frequency $f_{\min} = \phi/h$

- Now plugging in to solve for the stopping voltage:

$$\boxed{eV_{\max} = hf - \phi}$$

- This formula gave Einstein the **Nobel Prize in 1921** after his predictions were experimentally confirmed.

**<u>Compton Scattering:</u>**

- We have seen that light hitting a metal generates a current in a metal.

- Obviously the photons of light are interacting with the electrons in the metal.

- Here we will investigate what happens when a single photon hits an electron

- Using classical wave optics, we can use Huygen's principle to predict that the incoming and outgoing photons have the same wavelength and energy.



- Experiments show that the outgoing photon always have longer wavelengths when scattered

→ lower energy since $E = hf = \dfrac{hc}{\lambda}$

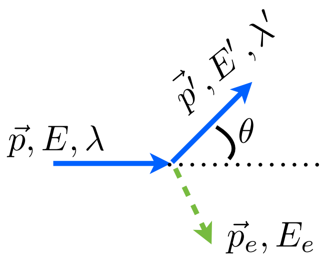
- To solve this problem we need to use both energy and moment conservation:

after before

$\vec{p}' + \vec{p}_e = \vec{p}$ (Momentum Conservation)

$E' + E_e = E + m_e c^2$ (Energy Conservation)

energy of the electron at rest




- Key to the problem: Recall from Chapter 31 that the energy and momentum of light are related by the speed of light

$$E = pc$$

- Using the quantized form the of photons energy we have

$$E = hf = \frac{hc}{\lambda} = pc \rightarrow \boxed{p = \frac{h}{\lambda}}$$

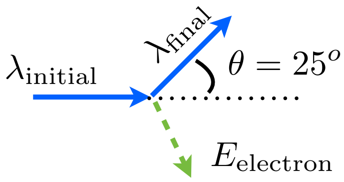
- Solving for the wavelength of the photon after the collision we have:

$$\boxed{\lambda' = \lambda + \frac{h}{m_e c}\left(1-\cos\theta\right)}$$ **[Nobel Prize 1927]**

- The wavelength is always larger if the electron scatters off the electron

- The factor $\lambda_e = h/m_e c$ is a fundamental length for the electron called the **Compton Wavelength** of the electron.

Ex. Suppose X-Rays with energy E=400 KeV undergo Compton scattering. If the scattering angle is 25° then a) What is the energy of the scattered photon? b) What is the kinetic energy of the scattered electron?




**a)** $$\lambda_{\text{final}} = \lambda_{\text{initial}} + \frac{h}{m_e c}\left(1-\cos\theta\right)$$

- Need to find the initial wavelength:

Good to remember this relationship

$$\lambda_{\text{initial}} = \frac{1240\ \text{eV}-\text{nm}}{4\times 10^5\ \text{eV}} = 0.0031\ \text{nm} = 3.1\times 10^{-12}\ \text{m}$$



- Final wavelength is then:

$$\lambda_{\text{final}} = 3.1\times 10^{-12} + \underbrace{2.43\times 10^{-12}}_{h/m_e c}\left[1-\cos\left(\underbrace{0.44}_{25^o = 0.44\ \text{rad}}\right)\right] = \boxed{3.3\times 10^{-12}\ \text{m}}$$

**b)** Using energy conservation and $E = hc/\lambda$ :

$$E^{\text{photon}}_{\text{initial}} = 6.4\times 10^{-14}\ \text{J}$$

$$E^{\text{photon}}_{\text{final}} = 6.0\times 10^{-14}\ \text{J}$$

$$E^{\text{electron}}_{\text{final}} = \boxed{0.4\times 10^{-14}\ \text{J}}$$

## Wave-Particle Duality:

- In chapter 34 we saw that light behave as a wave, it exhibits interference, it undergoes diffraction, etc...

- But in this chapter we have seen that light can also behave like a a particle; a discrete packet of energy.

- Therefore, light exhibits what we call **Wave-Particle Duality**; it can be both a wave and a particle at the same time.

- Whether you see the wave-like properties, or the particle-like properties, depends on the experiment you are doing.

- If like behaves as both a particle and a wave, can other things such as electrons, molecules, and even people also behave like a wave?

- The answer is yes! Any object with momentum also has an associated wavelength called the **de Broglie wavelength [Nobel Prize 1929]:**

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

- The wavelength becomes smaller if the mass or velocity is increased

- For everyday objects (you, me, apples, cars,...) it is impossible to see wave effects

<u>ex. Neutron @ 1m/s (very very slow neutron):</u>

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \mathrm{J/s}}{1.67 \times 10^{-27}\ \mathrm{Kg} \cdot 1\ \mathrm{m/s}} = 4 \times 10^{-7}\ \mathrm{m}$$

<u>ex. Paul @ 1m/s:</u>

Smaller than the smallest distance in physics

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \mathrm{J/s}}{192\ \mathrm{Kg} \cdot 1\ \mathrm{m/s}} = 3.5 \times 10^{-36}\ \mathrm{m}$$