PHYS-183 : Day #3

Simulation of blood flow

Applications of Newton’s Laws in 1D

- We have seen that Newton's Laws provide the simple rules that control the motion of every object that has mass.

- Now we want to use these rules to find the position, velocity, or acceleration of objects moving in 1D.

- Typically, we know what the forces are on an object and can use Newton's 2nd Law: $\vec{F} = m\vec{a}$

  - We can predict the future motion of the object

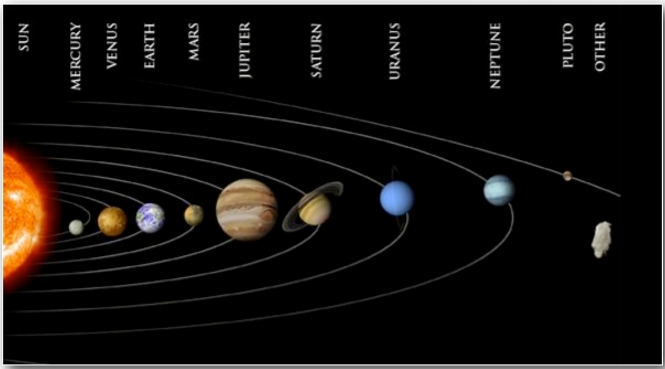
  - We can go backward in time to find the objects previous motion.

- In addition to Newton's 2nd Law we also need the position and velocity of the object at one specific time.

- Typically we know this information at $t = 0$, and therefore this data is called the **initial conditions** of the system.

<u>Ex. The planets</u>

- We know all forces
- Can measure position and velocity of planets to get initial conditions



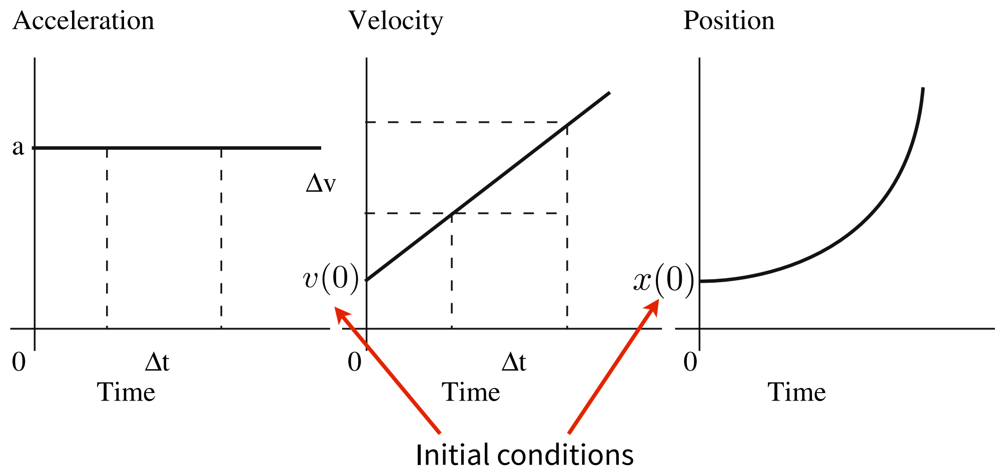
- In general, using Newton's second law requires finding solutions to differential equations.

- However, if the force is constant, like it is for gravity near the surface of the Earth,

$$\vec{F} = m\vec{g}_{\text{Earth}}$$

then we can easily solve for the motion.

- If F is constant, then the acceleration, velocity, and position, as a function of time look like:



-If the Force is constant, then I immediately know:

$$\Delta v = v(t_f) - v(t_i) = a\Delta t = a\,(t_f - t_i)$$

- We usually set $t_i = 0$ and call $t_f = t$ to make the equations simpler.

$$\boxed{v(t) = v(0) + at}$$

-If I know what the velocity is at t=0, then I know the velocity for <u>all</u> other times.

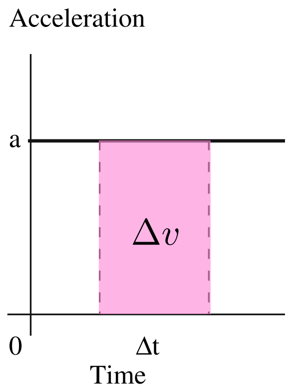
-We can also use a graphical method to relate velocity and acceleration:

- Recall that, if acceleration is constant:

$$\Delta v = a\Delta t$$

width of rectangle

height of rectangle

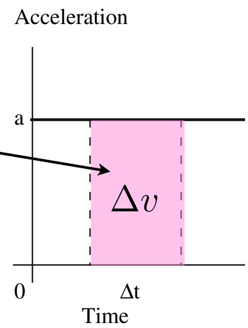


- This also works if acceleration is not constant since:

$$\Delta v = \bar{a}\Delta t$$

- Since in general $\Delta v = \bar{a}\Delta t$ we have for the average acceleration:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{\text{area under accel. vs. time graph}}{\Delta t}$$



- We have found a formula for velocity as a function of time, what about position?

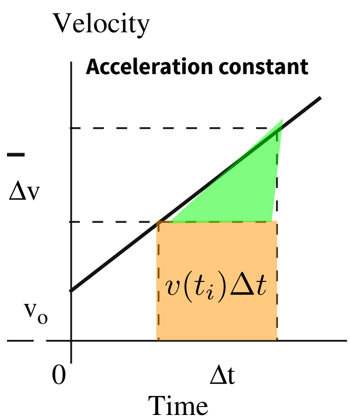
- Since the force is constant so is the acceleration, so the velocity is always increasing or decreasing at the same rate

$$\Rightarrow \quad \Delta x = \bar{v}\Delta t$$

- Just like average acceleration, for average velocity we can use the area under the curve

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\text{area under velocity. vs. time graph}}{\Delta t}$$

$$\text{area} = \underbrace{v(t_i)\Delta t}_{\text{rectangle}} + \underbrace{\frac{1}{2}\Delta t\left[v(t_f) - v(t_i)\right]}_{\text{triangle}} = \frac{1}{2}\left[v(t_f) + v(t_i)\right]\Delta t$$



- For the special case when acceleration is constant: $\bar{v} = \frac{1}{2}\left[v(t_f) + v(t_i)\right]$

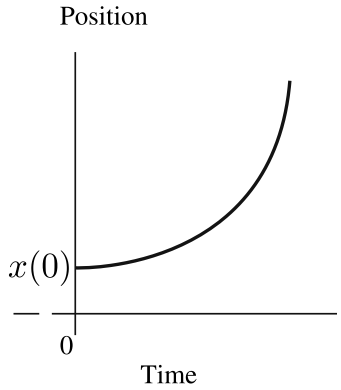
- Again, let $t_i = 0$ and $t_f = t$, then $\bar{v} = \dfrac{v(0) + v(t)}{2}$

- but we also know that $\bar{v} = \dfrac{x(t) - x(0)}{t}$ ➡ $x(t) = x(0) + \dfrac{v(0) + v(t)}{2}t$

- Finally, since we know that $v(t) = v(0) + at$

$$\boxed{x(t) = x(0) + v(0)t + \frac{1}{2}at^2}$$

- For constant acceleration, the position of an object as a function of time is a parabola



- By playing around with the equations, we can get a third important equation that does not depend on time

$$\boxed{v(t)^2 = v(0)^2 + 2a\left[x(t) - x(0)\right]}$$

- These three equations are also valid when the acceleration is <u>not</u> constant

  - If acceleration is not constant, then replace acceleration (a) with average acceleration ($\bar{a}$)

	Equation	*Variables*
1.	$v(t) = v_0 + at$	v, a, t
2.	$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	x, a, t
3.	$v^2 = v_0^2 + 2\,a\,(x - x_0)$	v, a, x

- You have to pick one of these equations based on what you want to calculate and what you know

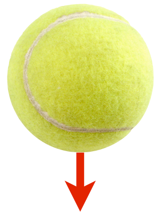
You should memorize these formulas

Ex. Free Fall: Constant Acceleration

Problem: A tennis ball is thrown straight up with an initial velocity of $12\ \mathrm{m/s}$, How high does the ball go, and how long does it take to come back to its starting position?

Solution:

- Since the tennis ball has mass, the gravitational force from the Earth is always pulling the ball back down with a constant acceleration g_{Earth}



$F = mg_{\mathrm{Earth}}$

- When the ball is at its maximum height, the velocity must be zero

- Given this information, we must use equation #3 to find the distance

$$v_{\mathrm{top}}^2 = v(0)^2 + 2(-g_{\mathrm{Earth}})(x_{\mathrm{top}} - x_{\mathrm{bottom}})$$

since up is positive direction, acceleration, which points down, must be negative

- Plugging in the known values:

$$0 = (12\ \mathrm{m/s})^2 - 2(9.8\ \mathrm{m/s^2})(\mathrm{x_{top}} - \mathrm{x_{bottom}})$$

$$(x_{\mathrm{top}} - x_{\mathrm{bottom}}) = \boxed{7.3\ \mathrm{m}}$$

- To calculate how long it takes, we can use two different methods:

- Since the ball returns to its same position we have $x(t_f) - x(0) = 0$

- We can use equation #2:

$$v(0)t - \frac{1}{2}g_{\mathrm{Earth}}t^2 = 0 \quad \Rightarrow \quad v(0) - \frac{1}{2}g_{\mathrm{Earth}}t = 0$$

- Therefore we get: $t = 2\dfrac{v(0)}{g_{\mathrm{Earth}}} = 2\dfrac{12\ \mathrm{m/s}}{9.8\ \mathrm{m/s^2}} = \boxed{2.4\ \mathrm{s}}$

- Or we can calculate how long the ball takes to go to the top and how long it takes to come down

- Using equation #1: $t_{\mathrm{up}} = \dfrac{v_0}{g_{\mathrm{Earth}}} = \boxed{1.2\ \mathrm{s}}$

- This is half of the total time

It takes the same amount of time for the ball to get to the top as it does to come back to the bottom

<u>Ex. Two ball drop:</u>

Problem: A ball is dropped from a height of 20 m to the ground. A time Δt later , a second ball is thrown toward the ground with an initial velocity of 10 m/s. If both balls hit the ground as the same time, what is Δt ?

Solution:

- The first thing to do is find the time t_1 it takes for the first ball to hit the ground

- Since we know the height, but not the final velocity, the only equation we can use is <u>equation #2.</u>

- If we set the ground to be $x = 0$, and assume +x-direction is upward:

$$0\ \mathrm{m} = 20\ \mathrm{m} - \frac{1}{2}(9.8\ \mathrm{m/s^2})\mathrm{t}_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{2 * 20\ \mathrm{m}}{9.8\ \mathrm{m/s^2}}} = \boxed{2.0\ \mathrm{s}}$$

- We now need to find the time time for the second ball, again using equation #2:

$$0\ \mathrm{m} = 20\ \mathrm{m} - (10\ \mathrm{m/s})\mathrm{t}_2 - \frac{1}{2}(9.8\ \mathrm{m/s^2})\mathrm{t}_2^2$$

- Solving for t_2 after multiplying both sides by -1 :

$$t_2 = \frac{-10 \pm \sqrt{10^2 - 4 * 4.9 * (-20)}}{2 * 4.9} = \boxed{1.2\ \mathrm{s}} \ \text{ or } \ -\cancel{3.3}\ \mathrm{s}$$

- The difference between t_1 and t_2 must be Δt : $\Delta t = t_1 - t_2 = 2.0\ \mathrm{s} - 1.2\ \mathrm{s} = \boxed{0.8\ \mathrm{s}}$

Motion in a Viscous Fluid:

- So far in this class we have not worried about **Friction**: the force that opposes the motion of objects that slide against each other.

- The friction force always is in the opposite direction of the velocity.

    - If the friction force is larger than all other forces, then the object will slow down and stop moving.

- The magnitude of the friction force depends on the velocity of the object

    - Faster moving objects have a stronger friction force

- Here we will study the biologically important example of motion in a fluid with friction.

- A fluid is any liquid or gas (i.e. air, water, alcohol, blood,...)

- For objects in a fluid, there are two different types of motion that depend on the objects size and the properties of the fluid it is moving in

Forces on Macroscopic objects:

- First we will discuss the forces on macroscopic objects in fluids
    - **Macroscopic** objects are things that can be seen with the human eye.

- Any macroscopic object in a fluid has three forces acting on it:

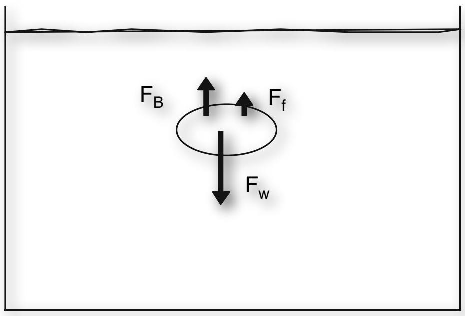
    (1) The gravitational force (weight) $F_w = mg$ that allows acts downward

    (2) A buoyant force F_b that always acts upward (the fluid pushes the object to the surface)

    (3) A **drag force** (frictional force) F_f that always points in the opposite direction as the velocity

- In this chapter we will always assume that F_b is small, so the object moves downward

➡ F_f is always in the opposite direction of the gravitational force



- An object moving in a fluid is surrounded by a thin layer of fluid called the **boundary layer** that moves with the object

- The object must push the fluid around it as it moves

- How the fluid moves around the object is determined by the objects **Reynolds Number:**

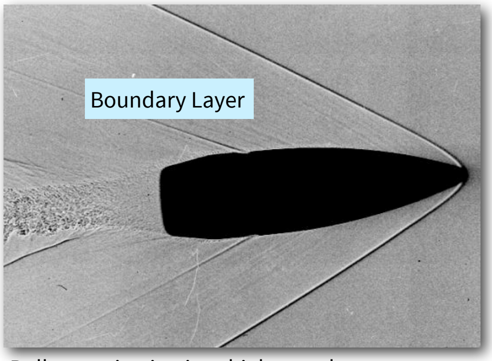
$$\mathcal{R} = \frac{L\rho v}{\eta}$$

$L =$ the size of the object
$v =$ the velocity of the object
$\rho =$ the density of the fluid
$\eta =$ fluid viscosity

fluid "stickiness / thickness"

- Thick fluids have high viscosity, gases have low viscosity

- The Reynolds numbers has no units



Bullet moving in air at high speed


Honey has a high viscosity

- In the same liquid, the larger the object the higher the Reynolds number.

- We will worry about things like fluid density later in the semester

Situation	*Reynolds Number*
Person swimming	1,000,000
Large flying bird	100,000
Flying mosquito	100
Swimming bacteria	0.0001

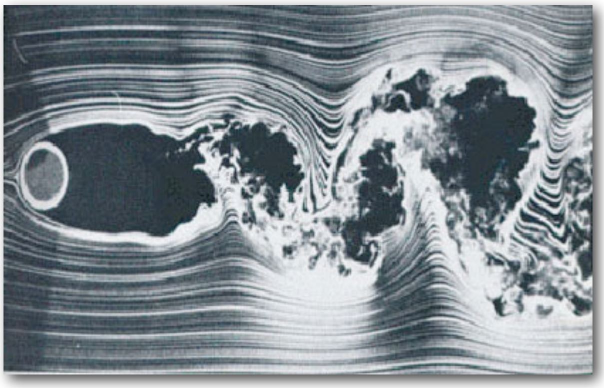
Sample Reynolds numbers

- If $\mathcal{R} \gg 1$ then the boundary layer flow is **turbulent**
  - Turbulent flow is chaotic (out of control)

- For turbulent fluid flow, the frictional force is proportional to the objects velocity squared:

$$\boxed{F_f = \frac{1}{2} C\rho A v^2 \qquad (\mathcal{R} \gg 1)}$$

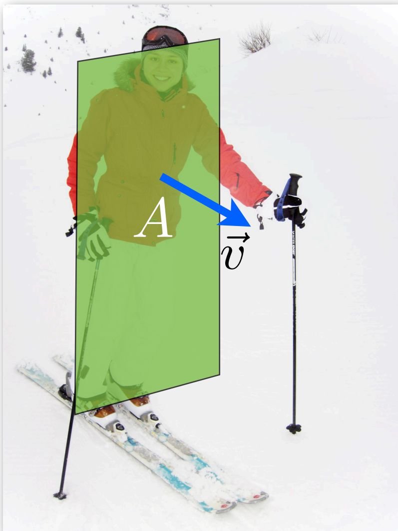
$C \approx 1$
$\rho =$ fluid density
$A =$ objects cross-sectional area
$v =$ objects velocity

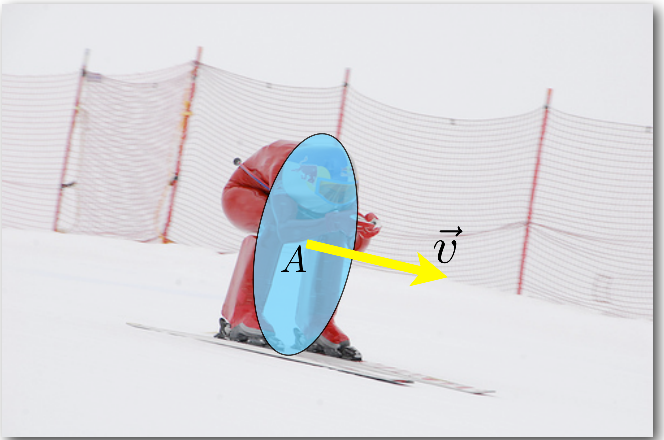

Turbulent fluid flow

- <u>Key Idea:</u> Double the velocity, friction is four times higher

- What is the cross-sectional area?

- It is the area of the object that the fluid “sees” when the object is moving



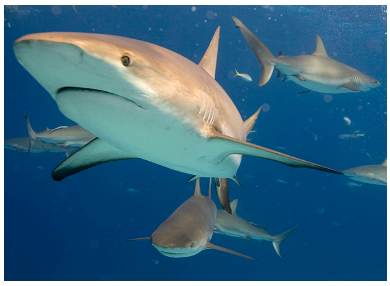


- Minimizing the area of the object decreases the total friction force.

- Most important for objects moving in liquids (ex. fish, sharks)

    - Water is over 800 times denser than air

    - Objects should minimize the area at much as possible



- If $\mathcal{R} \ll 1$ then the boundary layer flow is **laminar**

    -The fluid flow around the object is very smooth

-For laminar fluid flow, the frictional force is proportional to the objects velocity (not v^2):

$$\boxed{F_F = fv \qquad (\mathcal{R} \ll 1)}$$

depends on size and shape of object, and viscosity of fluid

Laminar fluid flow

-For spherical objects, f is straightforward to calculate:

$$f = 6\pi\eta r$$

$$r = \text{radius of sphere}$$

- When $\mathcal{R} \gg 1$ the viscosity ("stickiness") of the fluid does not matter

$$F_f = \frac{1}{2} C \rho A v^2 \qquad (\mathcal{R} \gg 1)$$

- Objects can continue moving for a long time before slowing down after the external force has turned off

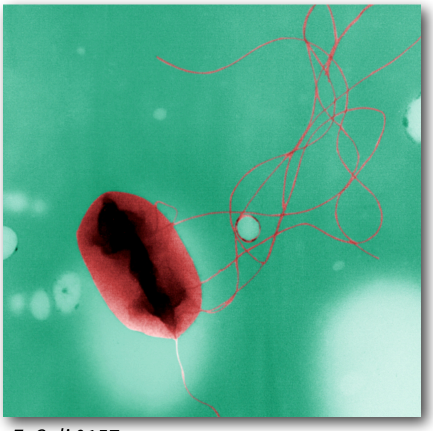
- When $\mathcal{R} \ll 1$ the density of the fluid does not matter

$$F_f = (6\pi\eta r)v \qquad (\mathcal{R} \ll 1)$$

- After external force is turned off, the object immediately comes to a stop

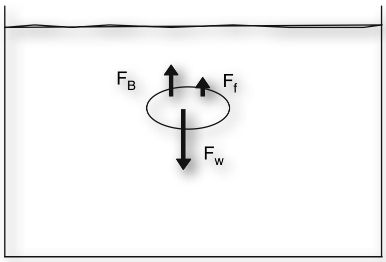
<u>Ex</u>. In water, anything less than $\sim 150\ \mu\mathrm{m}$ in size satisfies $\mathcal{R} \ll 1$, in air the size must be less than $\sim 40\ \mu\mathrm{m}$

- A swimming bacteria (such as *E. Coli*) must continuously push themselves to move around in water



E. Coli 0157

<u>Free Fall:</u>



- Let us again return to the case where an object experiences only three force: gravity, buoyant force, and friction force

- Here we use Newton's 2nd law to describe the motion:

$$F_{\text{net}} = ma = -mg_{\text{E}} + F_b + F_f$$

Free fall at terminal velocity

$$F_g = mg_{\text{E}}$$

- An object initially at rest (v=0) will accelerate downward until the net force is zero, and the object has a constant velocity

$$F_{\text{net}} = 0$$

- Since velocity is constant, a=0

$$mg_{\text{E}} = F_b + F_f$$

- If we know the Reynolds number, we can plug in the friction force and solve for this constant velocity called the **terminal velocity**

- The terminal velocity is the maximum velocity an object can have in a fluid since $F_{\text{net}} = 0$

- For large objects, that are <u>not smooth</u>, $\mathcal{R} \gg 1$ and the terminal velocity is given by:

$$mg_{\text{E}} = F_b + \frac{1}{2} C\rho A v_{\text{terminal}}^2 \quad \Rightarrow \quad \boxed{v_{\text{terminal}} = \sqrt{\frac{2(mg_{\text{E}} - F_b)}{C\rho A}}}$$

- The bigger the object area A , the lower the terminal velocity

Large area A

Small area A

- When $\mathcal{R} \ll 1$, or when the object is smooth, the terminal velocity can be solved using $F_F = f v_{\text{terminal}}$

$$\boxed{v_{\text{terminal}} = \frac{mg_{\text{E}} - F_b}{f}}$$

(The final answer depends on the shape of the object, size, and viscosity of the fluid)

Situation	*Terminal Velocity (m/s)*
Sky diver	100 (225 mph)
Person sinking in water	1
Pollen (~0.04 mm diameter) in air	0.05
Algae spores (same diameter as pollen) in water	0.00005

Sample terminal velocities

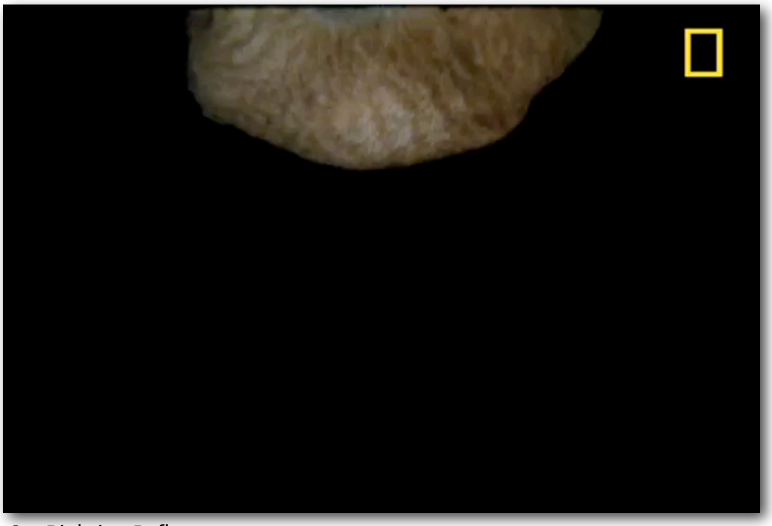
Ex. How to cats survive falls from high places?

- Suppose a person and a cat jumped off of Namsung Tower

- The person would die, but the cat would live. Why?

Two Reasons:

(1) Cats always land on their feet:



Cat Righting Reflex

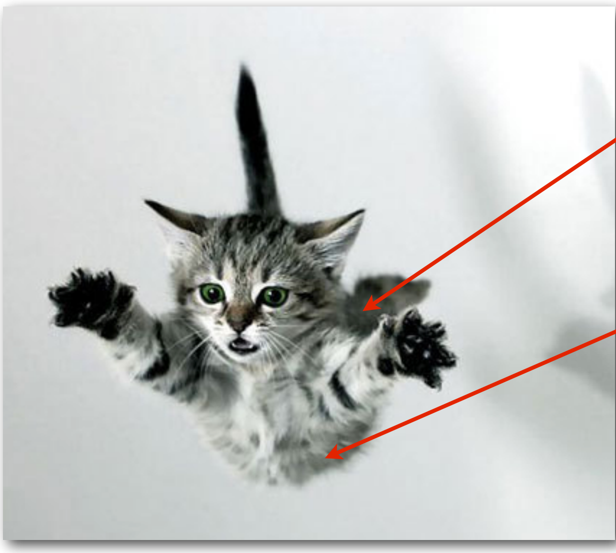
(1) Cats have an a very low terminal velocity:

$$v_{\text{cat terminal velocity}} = 27\ \text{m/s}$$

$$v_{\text{human terminal velocity}} = 54\ \text{m/s}$$

Cats fall at half the velocity that humans do!

- Cats try to create as much surface area as possible and are furry:



Fur makes the cats area seem even larger. Fur creates lots of friction force

Cats spread out just like skydivers to maximize their area

Cats can survive drops from over 40 stories with only a broken leg!