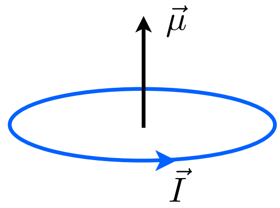
- B-field depends on N/L (density of loops) not the total number of loops

  - Loops far away do not have the same effect as closer loops since the dipole field falls off fast.

Magnetic Properties of Materials:

- We saw that a current loop with area A has a magnetic dipole moment: $\mu = IA$

- Atoms also have currents generated by the orbiting electrons



- For a very simple atom model this current is:

$$I = e/T = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

- Use original definition of magnetic moment: $\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$

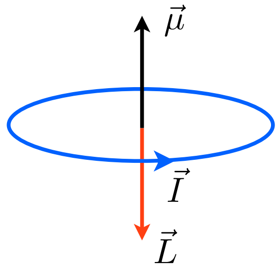
- The electron also has angular momentum: $\vec{L} = \vec{r} \times \vec{p} = rp = rm_e v$

- After solving for velocity in terms of magnetic moment:

$$L = \frac{2m_e \mu}{e}$$

- Both L and magnetic moment are vectors. Point in opposite direction since electrons move in opposite direction of current

$$\boxed{\vec{\mu} = -\frac{e}{2m_e}\vec{L}}$$

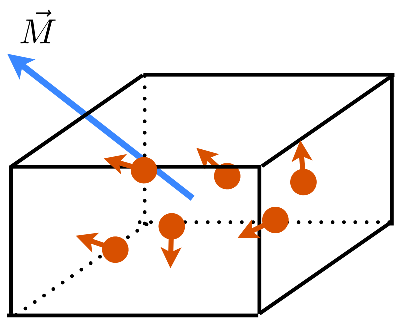


- We have already seen that a magnetic dipole will experience a torque when placed in an external B-field.

- If atoms act like little magnets, then they should also experience some sort force that changes the direction of their magnetic moments.

- Can define the **Magnetization** $\vec{M}$ of a material as the net dipole moment per unit volume

- Magnetization points in the direction of net magnetic moment from atomic moments.



- The material generates its own magnetic field:

$$\mu_0 \vec{M} = \mu_0 \frac{IA}{V} = \mu_0 \frac{I}{l} = B \qquad (Bl = \mu_0 I)$$

- If external B-field applied, total B-field is:

$$\vec{B} = \vec{B}_{\text{ext}} + \mu_0 \vec{M}$$

- It is easier (better) to deal with the **magnetic field strength:**

$$\vec{H} \equiv \vec{B}_{\text{ext}}/\mu_0 \qquad [H] = A/m \qquad \text{recall:} \quad [E] = V/m$$

- Therefore, for a material we have: $\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right)$

- How does the magnetization $\vec{M}$ depend on the external field?

- For many materials, the magnetization depends linearly on the field strength: $\vec{M} = \chi_{\text{m}} \vec{H}$

- The proportionality constant χ_{m} is called the **magnetic susceptibility**.

- χ_{m} can be both positive and negative:

$\chi_{\text{m}} < 0$ -> **diamagnetic** material

$\chi_{\text{m}} > 0$ -> **paramagnetic** material

- In diamagnetic materials, the magnetization generated is in opposite direction of B-field
    - If external B-field goes away then effect goes away

- Paramagnetic materials have magnetization that points in same direction as external B-field

- In terms of magnetic susceptibility the total B-field is:

$$\vec{B} = \mu_0 \left(\vec{H} + \chi_m \vec{H} \right) = \mu_0 \left(1 + \chi_m\right) \vec{H}$$

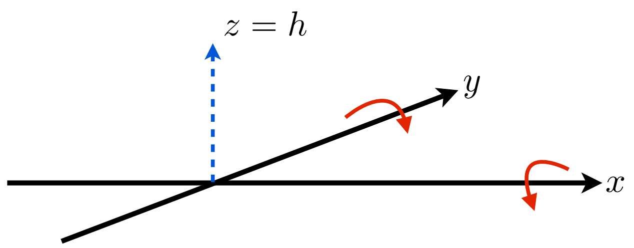
- Just like we did in the E-field, we can define **relative magnetic permeability**

$$\kappa_m = 1 + \chi_m \quad \Rightarrow \quad \mu = \left(1 + \chi_m\right) \mu_0 = \kappa_m \mu_0$$

NOT ALL MATERIALS ARE LINEAR IN $\vec{H}$

Example Problems for B-field:

ex. 28.31 Suppose I have a wire with current I in the x-direction, and a wire with current I pointing in the y-direction. What is the B-field above where the wires meet at z=h?

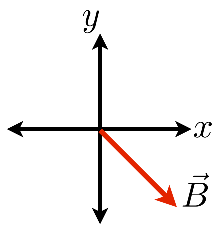


- Recall that we can add together the B-field from each wire separately

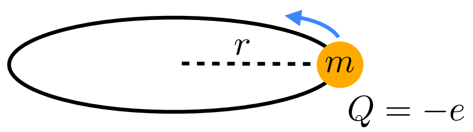
- The B-field from the x-wire: $B = \dfrac{\mu_0 I}{2\pi h}$ direction = $-\hat{y}$ from RHR

- The B-field from the y-wire: $B = \dfrac{\mu_0 I}{2\pi h}$ direction = $+\hat{x}$ from RHR

- The total B-field is therefore: $\boxed{\vec{B} = \dfrac{\mu_0 I}{2\pi h}\hat{x} - \dfrac{\mu_0 I}{2\pi h}\hat{y}}$



ex. 28.57 Suppose I swing a ball of mass m and charge Q=-e in a horizontal circle of radius r with a force F. What is the magnetic moment of the system? Which direction is it pointed?



- Recall that the magnetic moment is defined as: $\mu = IA$

- We already know the area A since this is a circle. Must find I:

$$I = \frac{dQ}{dt} = \frac{Q}{T} = \frac{Q}{2\pi r/v} = \frac{Qv}{2\pi r}$$

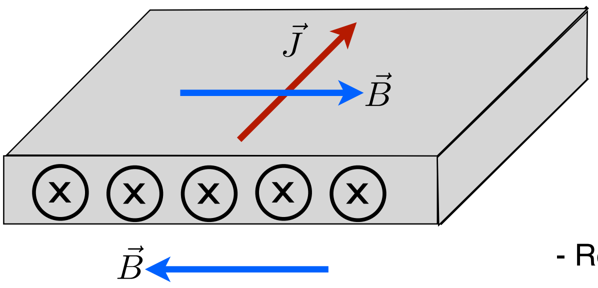
- We do not know the velocity directly, but we are given m and F:

$$F = \frac{mv^2}{r} \rightarrow v = \sqrt{\frac{Fr}{m}} \rightarrow I = \frac{Q}{2\pi}\sqrt{\frac{F}{mr}}$$

- Finally, substitute into the magnetic moment definition along with A

$$\boxed{\mu = \frac{Qr^2}{2}\sqrt{\frac{F}{mr}}}$$

- Q is moving CCW, but Q is negative, therefore current is moving CW -> magnetic moment is pointed down ex. 28.43

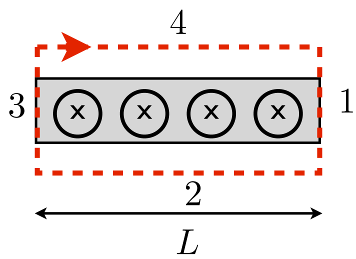


A large conducting sheet is in the xy-plane with a current density J (current/length) pointed in the y-direction. Using Ampere's Law, what is the B-field just above the sheet far away from any edges?

- Recall that Amperes law is: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

- Use RHR to find the direction of the B-field above and below the plane.
    - Magnitude of B-field on top and bottom must be the same by symmetry.

- Step #1: Pick your integration loop:



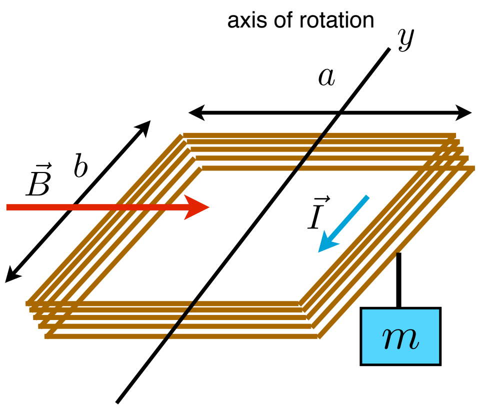
- Step #2: Pick your surface: Here flat surface
- Calculate LHS of Ampere's Law:

  B perpendicular to 1 & 3 -> only 2& 4 matter

- Calculate RHS of Ampere's Law:
    - Total enclosed charge is $I_{\text{enc}} = JL$

$$2BL = \mu_0 JL \rightarrow \boxed{B = \frac{\mu_0 J}{2}}$$

ex. 28.67



A rectangular wire of N-loops is horizontal with a B-field pointed in the x-direction. A mass m hangs from one side of the loop. What is the strength of the B-field needed to keep the loop horizontal?

- The current loop has a magnetic moment

$$\mu = NIA$$

and therefore a torque

$$\vec{\tau}_B = \vec{\mu} \times \vec{B} = -NIAB\hat{y} = -NIabB\hat{y}$$

- For the loop to remain horizontal, the torque must be balanced by the gravitational torque

$$\vec{\tau}_G = \vec{r} \times \vec{F} = rF\hat{y} = \frac{a}{2}mg\hat{y}$$

- Solving for the magnitude of the B-field: $NIabB = \dfrac{a}{2}mg \rightarrow B = \dfrac{mg}{2NIb}$