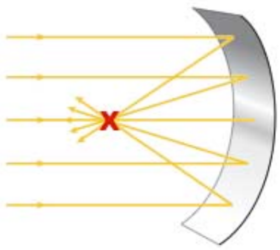
Spherical Mirrors:

- Not all mirrors are plane mirrors like we looked at last time.

- Many mirrors are also spherical.

- Spherical mirrors come in two different types depending on what side of the mirror reflects light.



Concave Mirror: Light reflects from the side facing the center of the spherical surface.

Convex Mirror: Light reflects from the outside surface.

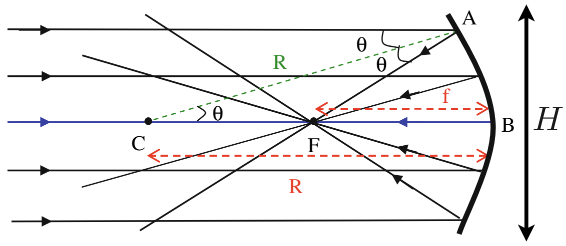
- Concave mirrors are used as makeup or cosmetic mirrors to enlarge images.

- Convex mirrors are used as security mirrors or side mirrors in cars where having a wider viewing range is important.

- Spherical mirrors work in the exact same way as flat mirrors do. The reflected light is follows the law of reflection.

- Here we assume that all light travels in a direction parallel to the normal vector for the mirrors center

- The radius of the mirror is called *R*, and defines the **center of curvature** *C*



Physical setup of light in a concave mirror.

- With all the light parallel to the normal vector, all of the light reflects off the mirror according to the law of reflection and will converge onto the **focal point** *F* of the mirror.

- This focal point is located a distance *f* away from the center of the mirror.

- If the height of the mirror *H*<*R* then using the law of reflection we have the following relation between the radius and focal point:

$$\boxed{f = \frac{R}{2}}$$

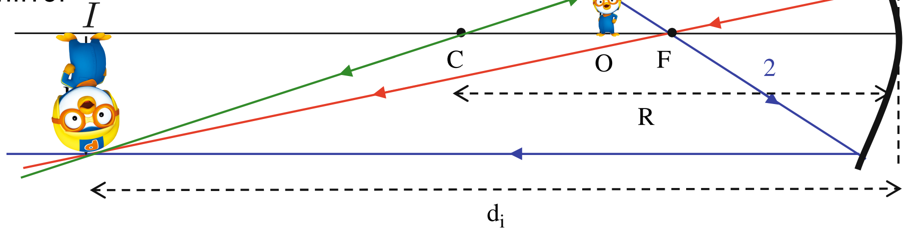
- As we will see, the focal length *f* is the key parameter for forming images in a curved mirror

Image Formation:

- An object placed some distance away from a concave mirror will produce an image in the mirror.

- In order to find the position and size of the resulting image, we will use a process called **raytracing** where we follow three special light rays.

- Here the object is at position *O* located a distance d0 from the mirror



#1: Draw a light ray from the top of the image and parallel to the normal vector at the center of the mirror. This ray will obviously reflect through the focal point.

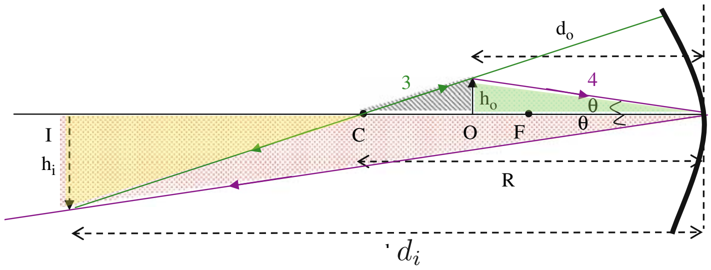
#2: Draw a ray from the top of the object through the focal point. The light ray will then reflect off the mirror and travel parallel to the left.

#3: A ray starting at the center of curvature and going through the top of the object will reflect backward through the center of curvature.

- The top of the image is located where these three rays meet.

The Mirror Equation:

- From the diagram on the right we can find a quantitative relationship between the object distance d_0, the image distance d_i, and the focal length f known as the mirror equation.



- Here we also call h_o the hight of the object and h_i the height of the image.

- We introduce another ray (#4) that goes from the top of the object, reflects off the center of the mirror with angle θ and passes through the top of the image.

- The #4 ray is the hypotenuse for both the red and green triangles giving the following relation

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$

- The #3 ray is also the hypotenuse for two similar triangles (black and yellow). From these triangles we have

$$\frac{h_i}{h_o} = \frac{d_i - R}{R - d_o}$$

- The ratio $m = h_i/h_o$ is called the **magnification**.

- The two equations can be combined by eliminating the magnification to yield

$$\frac{d_i}{d_o} = \frac{d_i - R}{R - d_o}$$

- Instead of the radius, we can inset the focal length using f=R/2.

$$d_i(2f - d_o) = d_o(d_i - 2f) \quad \Rightarrow \quad d_i f - d_i d_o + f d_o = 0$$

- Dividing all of the terms by $d_o d_i f$ we arrive at the **mirror equation** (“Lens Makers Equation”)

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

- The magnification can also be written as:

$$\boxed{m = -\frac{d_i}{d_o}}$$

- Here the minus sign is included to remember that the image is upside down

- Now we can determine where, and how big an image is for a concave mirror

Ex. A concave mirror has a radius of curvature of 25cm. A 2cm tall object is placed 20cm from the mirror along its axis. Find the location and size of the image.

Solution:

- Using do=20cm and f=R/2=12.5cm in the mirror equation we have

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{12.5} - \frac{1}{20} = 0.03 \text{ cm}^{-1} \Rightarrow \boxed{d_i = 33.3 \text{ cm}}$$

- The magnification is

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{33.3}{20} = -1.67$$

- The height of the image is therefore:

$$h_i = -1.67 h_o = -1.67(2) = \boxed{-3.33 \text{ cm}}$$

- Again, the minus sign here tells us that the image is upside down compared to the original object.

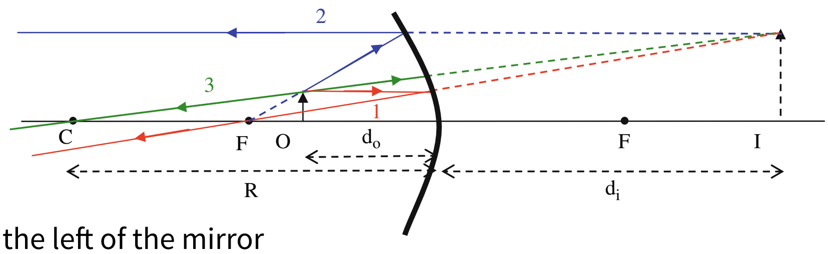
- The mirror equation can be rewritten to solve for the location of the image when we are given the object distance and focal length

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{f d_o} \Rightarrow d_i = \frac{f d_o}{d_o - f}$$

- Image that the focal length is fixed, but we vary the object distance d_o

- As long as $d_o > f$, then d_i is positive. But what happens when $d_o \leq f$ and the distance becomes negative of infinite?

- If $d_o \leq f$ then the object is placed inside the focal distance



- Performing the same raytracing as before, the 3 rays do not meet on the left of the mirror

- Instead, the three rays meet on the right side of the mirror forming a virtual image of the object.

- The image is not upside down and is magnified

Quantity	*Positive When*	*Negative When*
Focal length, f	Concave mirror	Convex mirror
Object distance, d_o	Real object (usual case)	*Virtual object (rarely)
Image distance, d_i	Real image (located in front of mirror)	Virtual image (located behind mirror)
Magnification, m	Erect	Inverted

Sign conventions for the mirror equation.

Ex. A 1cm tall object is located 10cm from a concave mirror with a radius of curvature of 40cm. find the location and size of the image and tell whether it is a real or virtual image.

Solution:

- Because f=R/2=20cm we can use the mirror equation to find the image distance:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{20} - \frac{1}{10} = -\frac{1}{20} \rightarrow \boxed{d_i = -20 \text{ cm}}$$

- We see that the image is 20cm behind the mirror and is therefore a virtual image.

- The magnification is:

$$m = -\frac{d_i}{d_o} = -\frac{-20}{10} = 2$$

- The image is therefore not upside down and has a height of 2cm

Ex. A mirror on the passenger side of a car is a convex mirror with radius of curvature of 150cm. If a car in the mirror is actually 20m away, describe the image in the mirror.

Solution:

- Since the mirror is convex, the focal length is negative: f=-R/2=-75cm.

- The mirror equation can be used to solve for the distance of the image.

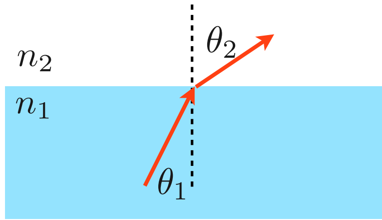
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = -\frac{1}{75} - \frac{1}{2000} = -0.0138 \text{ cm}^{-1} \rightarrow \boxed{d_i = -72.5 \text{ cm}}$$

- The image is a virtual image with a size determined by the magnification.

$$m = -\frac{d_i}{d_o} = -\frac{-72.5}{2000} = \boxed{0.04}$$

- Since the magnification is less than one, the image is reduced in size and will appear to be farther away than the object actually is.

Optical Fibers and Applications to Medicine:



- Consider the law of refraction for a light ray going from medium 1 to medium 2 where the index of refraction of 1 is larger than 2 $n_1 > n_2$ (for example glass to air)

- In this case, the light ray is refracted away form the normal vector because $n_1/n_2 > 1$

- We can see this by writing the law of refraction for $\sin\theta_2$:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

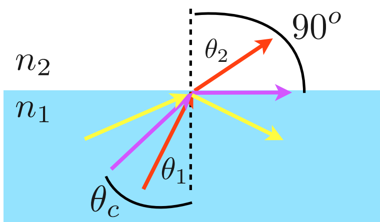
- Because the sine function has a maximum value of 1, this means that not all values of θ_1 will satisfy the law of refraction.

- The maximum incoming angle compatible with the law of refraction is called the **critical angle**

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}}$$

- What happens when the incoming angle is larger than the critical angle?

- Firstly, when the incident angle is equal to the critical angle, the light does not leave the first material but travels along the surface.



Refraction of light, above and below the critical angle

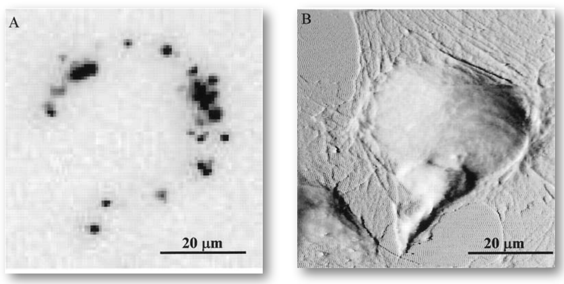
- At larger angles, there is no refraction and therefore all of the light is reflected back into the first material. This process is called **total internal reflection**.

- For the example of glass and air, $n_{\rm glass} = 1.5$, the critical angle is 42deg.

- Although none of the light escapes from the first material in total internal reflection, some of the lights E-field does leak outside.

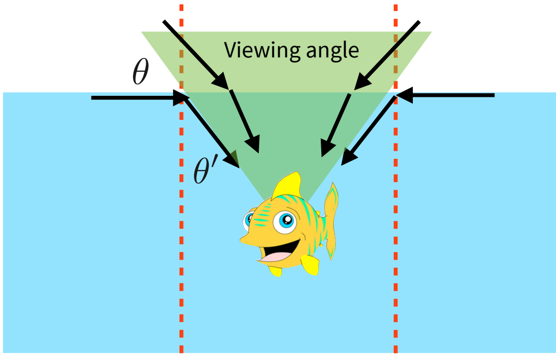
- This E-field is called the **evanescent field**.

- This weak E-field can be used to do useful things like fluorescence microscopy of things very near the interface between the two materials.



Imaging of a cell using the evanescent field generated by total internal reflection.

<u>Ex.</u> A fish sees the world above a smooth surface of water confined inside a circular disc above it. Find the angular width of this disc.



<u>Solution:</u>

- Light from above the surface of the water is refracted toward the normal vector as it enters the water.

- A ray of light just skimming the waters surface at $\theta \sim 90^o$ will bend toward the normal at the critical angle

$$\sin\theta' = \sin(90)\frac{1}{1.33} = 0.75 \qquad \theta' = 48.6^o$$

- Therefore, the fish sees the entire world above the water inside a circular disk that makes an angle of 48.6deg from the normal vector.

- Consider a solid cylinder of glass into which a beam of light, such as a laser, is aimed.

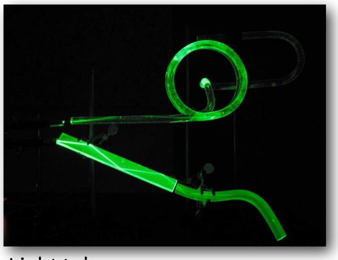
- As long as the light hits the walls of the cylinder at an angle <u>greater</u> than the critical angle, the light will continue to travel down the glass cylinder, even if it is bent.

- Such a device is called a **light tube**.

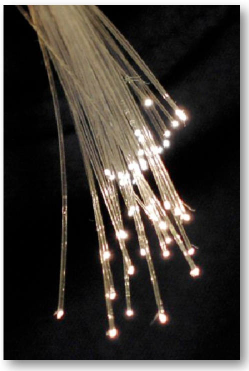
- Although these devices work, the light looses lots of energy as it travels. Therefore these devices are of limited use.

- Today, we use **optical fibers**, or "fiber optic cables" to transmit essentially all communications

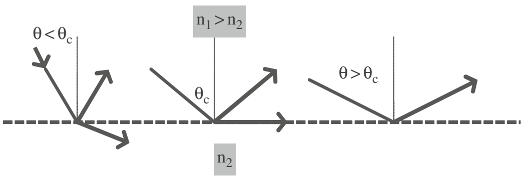
- Optical fibers are made of a small $\sim 10\ \mu\mathrm{m}$ wide piece of glass or plastic called the **core**, that is surrounded by a piece of *lower* index of refraction material called the **cladding**

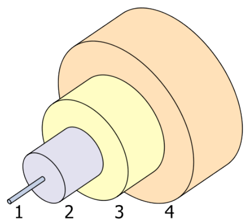


Light tube



Optical fibers





1=core, 2=cladding

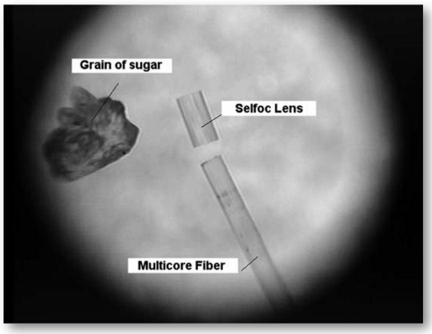
- Using one optical fiber along is not enough to capture an image

- However, if we combine many optical fibers together then we can get an image of what is on one end of the fiber from the other end.

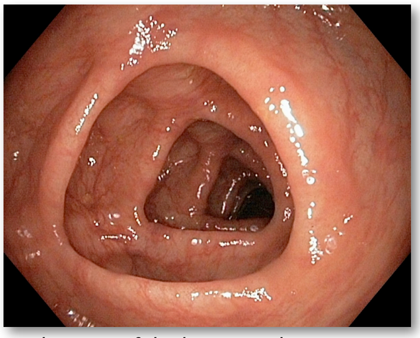
- Since the fibers are so small, we can combine many of them together and still have a very small diameter fiber.

- In medicine, these devices have made a large impact on how we take images inside of the body.

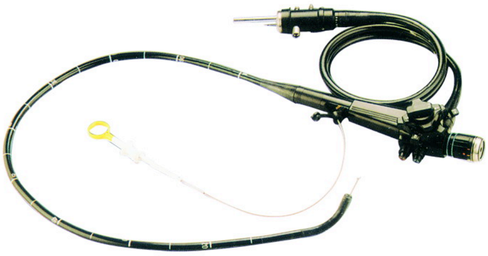
- Even surgery can be performed using these small fibers to see inside the body without creating a large incision.



Many optical fibers together are still smaller than a grain of sugar.



Endoscopy of the human colon.



Fiber optical endoscope