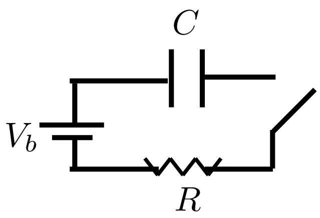
RC Circuits:

- Lets look at a circuit that has **both** a capacitor and a resistor in series with a voltage source:



- Suppose the circuit has a switch that is initially open.
- The capacitor has no net charge on its plates.
- Since the circuit is broken, charge cannot move: $I(0) = \frac{dq}{dt} = 0$
- Once the switch is closed, charge will build up on the capacitor until $q_{max} = CV$

- What is the charge on the capacitor as a function of time when the switch is closed?
  - We have only one loop, so Kirchoff #1 is trivial.
  - Using Kirchoff #2:

$$V_b - V_C - V_R = V_b - \frac{q(t)}{C} - R\frac{dq(t)}{dt} = 0$$

  - This can be turned into a 1st-order differential equation:

$$\frac{dq(t)}{dt} + \frac{q(t)}{RC} = \frac{V_b}{R}$$

- The solution to this equation can be guessed (see pg. 850 of your text), however I will show you how to solve this equation on this slide:

- We have the equation: $q'(t) + \frac{1}{RC}q(t) = V_b/R$
- Take the stuff multiplying q(t), and solve $e^{\int \frac{1}{RC}dt} = e^{\frac{t}{RC}}$
- Multiply both sides by this result:

$$e^{\frac{t}{RC}}q'(t) + e^{\frac{t}{RC}}\frac{1}{RC}q(t) = V_b/Re^{\frac{t}{RC}}$$

- The left-hand side now simplifies to: $\frac{d}{dt}\left[e^{\frac{t}{RC}}q(t)\right] = \frac{V_b}{R}e^{\frac{t}{RC}}$
- Integrating we get: $e^{\frac{t}{RC}}q(t) = CV_b e^{\frac{t}{RC}} + C_1$ where C1 is an integration constant
- At t=0 -> q=0 therefore $C_1 = -CV_b$ and:

$$q(t) = CV_b\left[1 - e^{-t/RC}\right] \qquad CV_b = q_{max} \quad \Rightarrow \quad \boxed{q(t) = q_{\max}\left[1 - e^{-t/RC}\right]}$$

- Either by guessing, or solving the equation for q(t), we get:

$$\boxed{q(t) = q_{\max}\left[1 - e^{-t/RC}\right]}$$

- When t=0 -> q=0 and when t >> RC -> $e^{-t/RC} \approx 0$ and q=qmax.

- It is typical to combine R and C into a single number with units of time

$RC = \tau$ **Time Constant** of the circuit

$$q(t) = q_{\max}\left[1 - e^{-t/\tau}\right]$$

- We can also easily solve for the current by differentiating q(t):

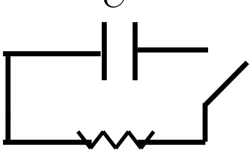
$$I(t) = \frac{q_{max}}{\tau}e^{-t/\tau} = \frac{q_{max}}{RC}e^{-t/RC}\boxed{= \frac{V_b}{R}e^{-t/RC}}$$

- For t >> RC, the current is nearly zero as the capacitor as built up all the charge it can store

- When $t = \tau$: $I = \frac{1}{e}\frac{V_b}{R}$

- Time constant tells you how long it takes for current to drop by 1/e.

- What if we start with a charged capacitor and no voltage source?



- At t=0 we close the switch as current starts to flow

- Want to know what q(t) and I(t) are

- Using Kirchoff voltage law: $-IR - V_C = 0$

- The equation of motion for charge is: $\frac{dq(t)}{dt} + \frac{q(t)}{RC} = 0$

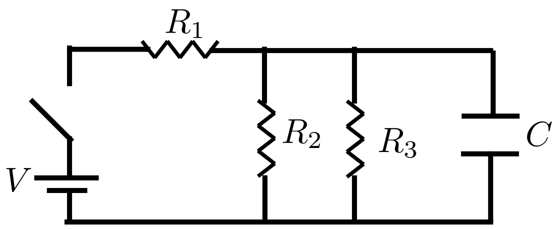
- Equation is similar to previous case, but will have no integration constant remaining.

- Try exponential solution for q: $q(t) = Ae^{-t/RC}$

- Arrive at $\boxed{q(t) = q_{max}e^{-t/RC}}$

- And differentiating: $\boxed{I(t) = -\frac{q_{max}}{RC}e^{-t/RC}}$

ex. RC Circuits:

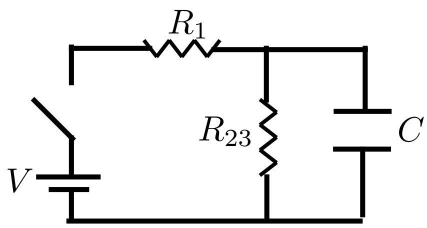


1) Determine potential across capacitor after switch has been closed for a long time.

2) Determine the energy stored in the capacitor after a long time.

3) After switch is closed, how much energy is dissipated in R3?

- Obviously R2 & R3 are in parallel, so the circuit can be simplified to:



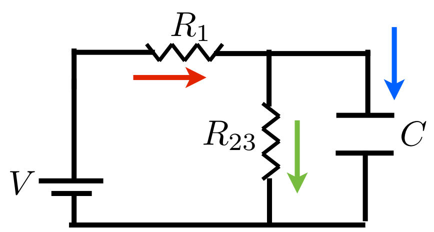
$$R_{23} = \frac{R_2 R_3}{R_2 + R_3}$$

- Since R23 & C are in parallel, the voltage across them is the same.

Can find Vc by calculating: $V_{23} = I_{23} R_{23}$

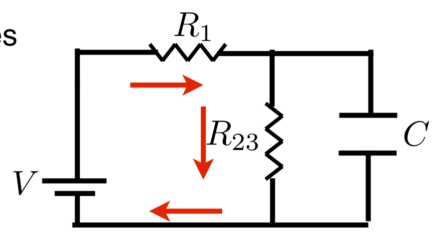
- Need to find the current through R23 after switch has been closed for a long time

short time after



capacitor becomes fully charged

long time later



- Once capacitor is fully charged no current can flow through it

$$I_{23} = \frac{V}{R_1 + R_{23}}$$

$$\boxed{V_C = V_{R_{23}} = I_{23} R_{23} = \frac{V R_{23}}{R_1 + R_{23}}}$$

-Now need to find the energy stored in the capacitor at late time:

$$\boxed{U = \frac{1}{2} C V_C^2 = \frac{1}{2} C \left(\frac{V R_{23}}{R_1 + R_{23}} \right)^2}$$

3) What is the energy dissipated by the resistor R3?

- Resistor is Ohmic so that the power (energy/time) is: $P(t) = \frac{V_{R_3}(t)^2}{R_3} = \frac{V_C(t)^2}{R_3}$

- Therefore by integrating the power we get energy: $E = \int_0^{t_f} \frac{V_C(t)^2}{R_3} dt$

- Since the capacitor is in parallel with R23:

$$V_C(t) = \frac{q(t)}{C} = \frac{CV_C}{C} e^{-t/R_{23}C} = V_C e^{-t/R_{23}C}$$

- Substituting into the power:

$$\boxed{E = \frac{V_C^2}{R_3} \int_0^{\infty} e^{-2t/R_{23}C} dt = \frac{R_{23}CV_C^2}{2R_3}}$$