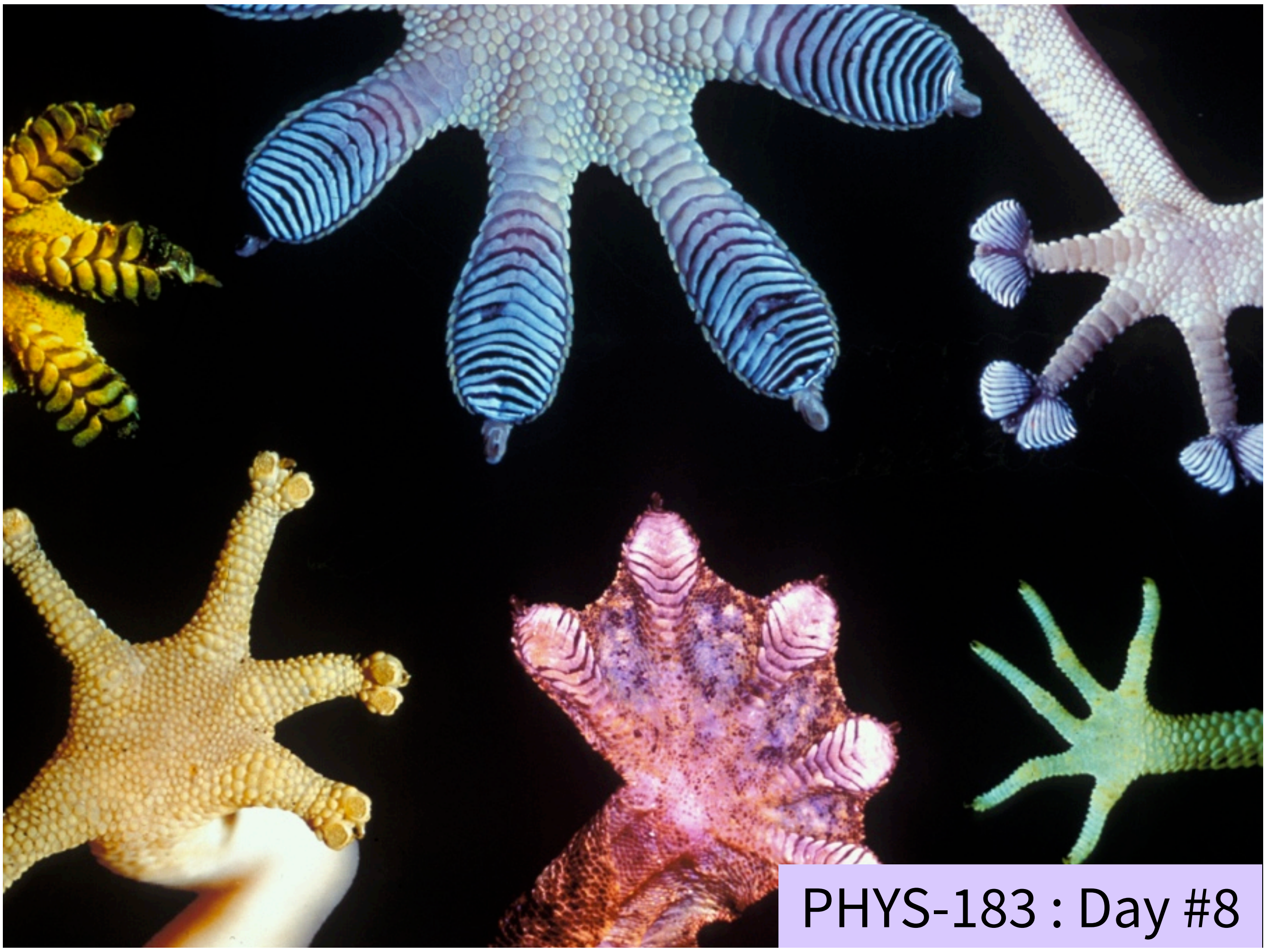
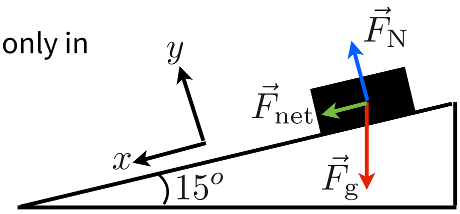


(3) Write down the equations of motion. Here we are interested only in the x-direction



$$F_x = F_{\mathrm{g}}\sin(15\,\mathrm{deg})$$

$$a_x = \frac{F_{\mathrm{g}}\sin(15\,\mathrm{deg})}{m} = g_{\mathrm{E}}\sin(15\,\mathrm{deg}) = 9.8*(0.26)\ \mathrm{m/s^2} = 2.54\ \mathrm{m/s^2}$$

- Since the initial velocity is zero, and we know how far the piano moves (x=5m) we can use

$$v^2(t) = 2a_x x = 2(2.54\ \mathrm{m/s^2})(5\ \mathrm{m}) \quad \Rightarrow \quad \boxed{v = 5.0\ \mathrm{m/s}}$$

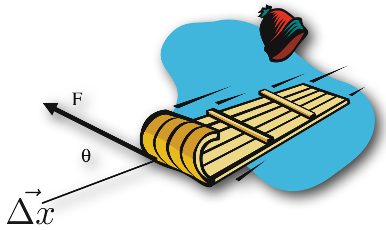
Work and Energy:

- Work and energy are scalar quantities, so we might think that we do not need to work about vectors.

- However, work is the product of a force and a displacement, both of which are vectors.

- In this section we generalize our definition of work to work in more than one-dimension.

- Suppose we pull a sled in the x-direction with a force that makes an angle θ with the x-axis.



- This force has two components, one in the y-direction, and one in the x-direction

- In this case, only the x-component of the force does any work.

- We need a more general definition of work that includes forces that are not in the same direction as the displacement:

$$W_F = F_x \Delta x \qquad \text{(constant force)}$$

or using the angle between the force and the displacement.

$$W_F = F_x \Delta x = F \cos\theta \Delta x$$

- If the force changes, then we have:

$$W_F = \sum \Delta W = \sum (F \cos\theta) \delta x \quad \text{(general definition)}$$

- The kinetic energy of an object also depends on the velocity which is a vector. But the equation is the same as long a we remember that

$$v^2 = v_x^2 + v_y^2 \quad \text{(2D)} \qquad v^2 = v_x^2 + v_y^2 + v_z^2 \quad \text{(3D)}$$

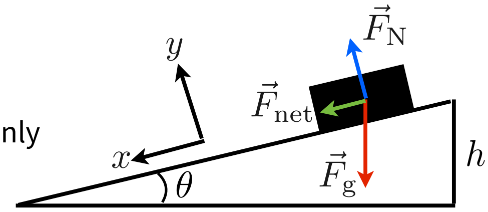
- The gravitational and elastic potential energies do not change. The forces that generate these potentials can only point in one direction at a time.

- The last thing we need to look at is power. The power is $P = \dfrac{\Delta W}{\Delta t}$

- So to get the correct power we must use the equation: $W_F = F_x \Delta x$

- Given these small changes, we can use work and energy to solve problems in more than one-dimension.

Ex. A piano of mass 100kg slides down a frictionless ramp that is 5m long and at an angle of 15 deg. If the piano starts from rest (v=0), use work and energy to find the velocity of the piano at the bottom.



(1) Use work-energy theorem:

- The work is generated by the force along the x-direction only

$$F_{\rm net} = mg\sin\theta$$

- The distance the force acts over is $d = h/\sin\theta$

- The total work is therefore: $W = mg\sin\theta\dfrac{h}{\sin\theta} = mgh$

- Since the piano starts at rest, the work is equal to the final kinetic energy (work-energy theorem)

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gh} = \boxed{5.0 \text{ m/s}}$$

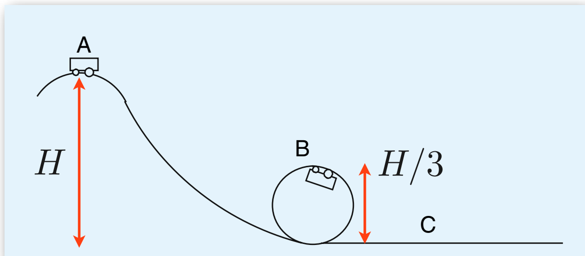
(2) Use energy conservation:

- Setting the ground as (y=0) the total initial energy is all potential $E = mgh$

- At the bottom, the energy is all kinetic $E = \frac{1}{2}mv^2$

- Get exact same answer!

Ex. In a roller coaster ride, the car of mass *m* starts from rest at point A and a height H. The loop has a height of H/3. Assuming no friction, (a) Find the speed of the roller coaster at point B, (b) the speed of the roller coaster at point C.



- Since the problem gives us initial heights and velocities, and is asking for speed. This is a perfect problem for solving using energy.

Part (a)

- The total energy is equal to the initial potential energy: $E = mgH$

- At point B, the total energy is a combination of kinetic and potential:

$$E = (1/2)mv_B^2 + mg(H/3)$$

- Setting these two equation equal, by energy conservation,and solving for v_B : $\boxed{v_B = \sqrt{\dfrac{4gH}{3}}}$

Part (b)

- At point C, all of the energy is now kinetic: $E = mgH = (1/2)mv_C^2$

$$\Rightarrow \quad \boxed{v_C = \sqrt{2gH}}$$

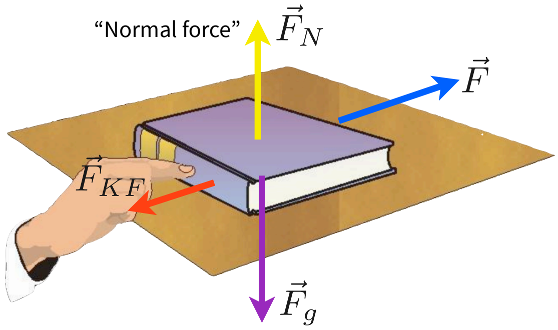
Contact Frictional Forces:

- In this class we have mostly ignored one of the two main forces in the world: Friction.

- We considered friction for an object in a liquid, but we did not discuss the friction between two solid objects.

- Friction is an essential force for most types of motion. Without friction we could not walk, cars could not move, machines would not work,...

- Imagine two objects sliding relative to each other.



All of these require friction

- The frictional force $\vec{F}_{KF}$ is always parallel to the surface of contact

- The frictional force always points in the direction opposite the velocity and is called the **Kinetic Friction** (KF)

- The kinetic friction is generated by atomic bonds being broken and formed at the surface of contact.

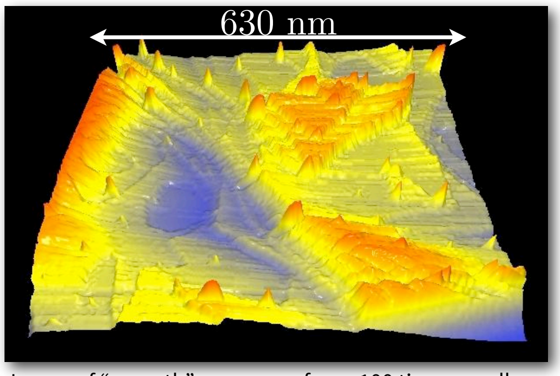


Image of "smooth" copper surface. 100 times smaller than width of human hair.

- These bonds are formed because the surface of every material is rough.

- The friction force depends on the two materials that are in contact.

- The friction force does <u>not</u> depend on the area of contact.

- The magnitude of the kinetic friction is proportional the magnitude of the Normal force:

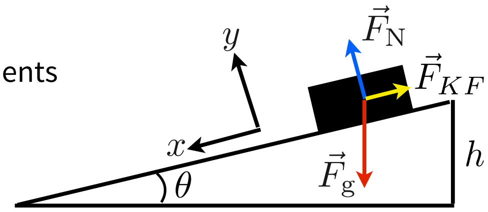
$$\boxed{F_{KF} = \mu_K F_N}$$

μ_K = **the coefficient of kinetic friction**

- This is <u>not</u> a vector equation! The normal force is perpendicular to the surface, but the friction force is parallel to the surface.

- The coefficient of kinetic friction depends on the two types of contact materials, but not the area.

Ex. A piano of mass 100kg slides down a ramp that is 5m long and at an angle of 15 deg with a coefficient of kinetic friction $\mu_K = 0.2$. If the piano starts from rest (v=0), find the acceleration and velocity at the bottom.



- We need to write Newton's 2nd law for the x and y components

$$F_g \sin\theta - F_{KF} = ma_x \qquad \text{(x-component)}$$

$$F_N - F_g \cos\theta = 0 \qquad \text{(y-component)}$$

- We also need the relation for the kinetic friction force: $F_{KF} = \mu_K F_N$

- For the normal force we find: $F_N = F_g \cos\theta = 100 \cdot 9.8 \cos(15\,\mathrm{deg}) = 950\ \mathrm{N}$

- The kinetic friction force magnitude is therefore: $F_{KR} = 0.2 \cdot 950\ \mathrm{N} = 190\ \mathrm{N}$

- Substituting the known values into the x-component equation we have:

$$a_x = \frac{mg\sin\theta - F_{KF}}{m} = \frac{[100 \cdot 9.8 \sin(15\,\mathrm{deg}) - 190]}{100} = \boxed{0.64\ \mathrm{m/s^2}}$$

- Knowing the acceleration, we can solve for the velocity after 5m:

$$v = \sqrt{2a_x x} = \sqrt{2 \cdot 0.64 \cdot 5} = \boxed{2.5\ \mathrm{m/s}}$$

- We could have also calculated the velocity at the bottom using work- energy.

- The total initial energy is all potential: $E = mgh$ $\qquad h = 5\sin(15\,\mathrm{deg}) = 1.29\ \mathrm{m}$

- At the bottom, the energy is all kinetic: $E = (1/2)mv^2$

- When there is friction, the initial energy is reduced because the <u>work is done by the force of friction that is always negative</u>; it points opposite the displacement.

$$W_{KF} = -F_{KF}x = -\mu_k F_N x = -0.2(950\ \mathrm{N})(5\ \mathrm{m}) = -950\ \mathrm{J}$$

- From the work-energy theorem we have:

$$W_{KF} = E_f - E_i \qquad \text{or} \qquad E_f = W_{KF} + E_i$$

Total initial energy Total final energy

Work done on object by friction

- In our example, this equation becomes: $-950 = \frac{1}{2}mv^2 - mgh$

- Solving for the velocity (v) we get the same answer as before (2.5m/s)

Static Friction:

- When two objects are in contact, but not moving, there are still atomic bonds between the objects

- When not moving, there is no net force parallel to the surface of contact.

- However, If we try to push the object, these bonds create a friction force that tries to keep the object from moving.

- The friction force that tries to keep an object from moving is called the **static friction force**

- The maximum static friction force depends only on the two materials that make contact and the normal force

$$F_{SF,\max} = \mu_s F_N$$ μ_s = **the coefficient of static friction**

- This is for the maximum static friction, typically the actual static friction is smaller:

$$\boxed{F_{SF} \leq \mu_s F_N}$$

- Again, this is not a vector equation, only for magnitudes.

- In almost all examples, the static friction force is stronger than the kinetic friction force

→ It is easier to keep a heavy object moving than it is to start its motion.

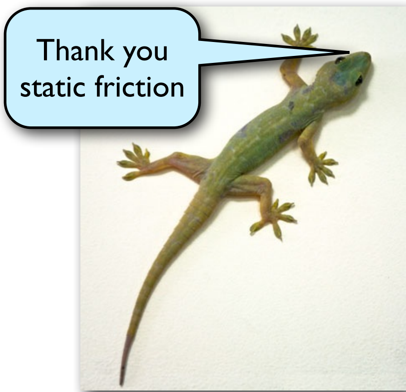
Below are some examples of kinetic and static friction coefficients:

Object and Surface	μ_s	μ_k
Steel on steel (dry)	0.7	0.6
Steel on ice	0.03	0.02
Metal on metal (lubricated)	0.15	0.07
Rubber on concrete (dry)	1.0	0.9
Human joints (lubricated with synovial fluid)	0.005	0.005

- If the force on an object is greater than the static friction, then the object will move. If not, the object will not move.

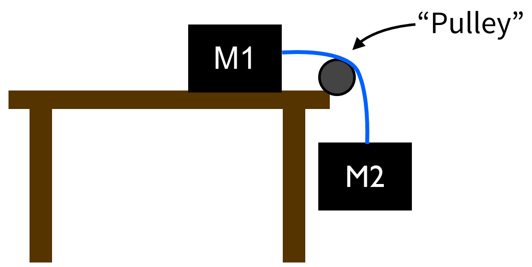
- If the object moves, use the kinetic friction to find the objects velocity, acceleration, position,...

When pushing a car, it is harder to start than it is to keep moving.

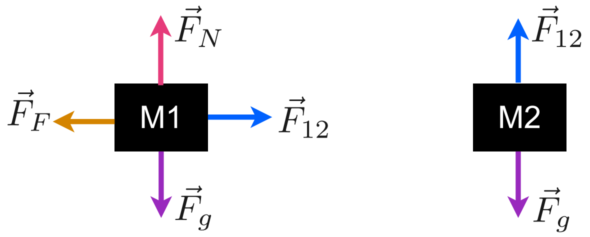


Friction holding gecko on the wall.

Ex. Two identical blocks of 20kg are attached by a string over a pulley at the edge of a table with one block on the table, and one block off the table. The coefficient of static friction is $\mu_s = 0.6$ and kinetic friction is $\mu_k = 0.4$. Do the blocks move, and if so, find their acceleration.



- Draw the forces on blocks 1 and 2:



- First we need to find out if the masses will move

  - The normal force on block #1 is found using Newtons 2nd law for y-direction

$$F_n - F_g = 0 \quad \Rightarrow \quad F_N = mg$$

  - So the maximum static friction is: $F_{SF,\max} = \mu_s mg = (0.6)(20)(9.8) = 120 \text{ N}$

  - This force is in the -x-direction

- In the +x-direction, the force on mass #1 is $F_{12} = mg = (20)(9.8) = 196 \text{ N}$

- This force is larger than the maximum static friction so **the masses will move**.

- Since the net force is in the +x-direction, block #1 will move to the right.

- The Newton's equations of motion for the blocks are (using the kinetic friction):

$$F_{12} - F_{KF} = m_1 a \qquad F_g - F_{12} = m_2 a$$

- We can get rid of F_{12} in these two equations by adding them together. Then we get

$$m_2 g - F_{KF} = (m_1 + m_2)a = 2ma$$

- Substituting in the kinetic friction equation:

$$mg - \mu_k mg = (1 - \mu_k)mg = 2ma$$

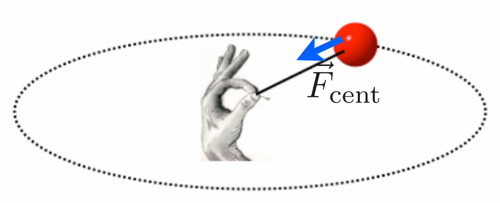
- Therefore the acceleration is: $a = (1 - \mu_k)g/2 = \boxed{2.94 \text{ m/s}^2}$

Circular Motion Dynamics:

- We have seen that an object traveling in uniform circular motion has an acceleration that is directed toward the center of the circle.

  - For example, swinging a ball on a string

  - A car turning on a road also has centripetal acceleration from the friction force.



- An object traveling in uniform circular motion still satisfies Newton's 2nd law, but with acceleration equal to centripetal acceleration

$$\vec{F}_{\rm net} = ma_{\rm cent} = \frac{mv^2}{r}$$

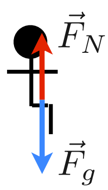
- The key to solving uniform circular motion problems is to first draw the force diagram, next solve for the net force, and finally plug the net force into the equation for $a_{\rm cent}$

Ex. A ferris wheel of radius 20m is rotating at 1.5 revolutions per minute. Find the force from the seat (normal force) on an 80kg man at the top and bottom of the wheel.

Ferris Wheel

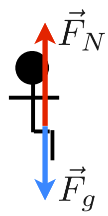
(1) Draw picture

(2) Draw force diagram on man



(Top)

Gravitational force must be bigger so that the net force points toward center of circle!



(Bottom)

Normal force must be bigger so that the net force points toward center of circle!

(3) Set the net force equal to the centripetal force

Top:

$$F_g - F_N = ma_{\rm cent} = mv^2/r \quad \Rightarrow \quad F_N = m(g - v^2/r)$$

$$v = \frac{1.5 \text{ rpm} * 2\pi\text{r}}{60 \text{ s/min}} = 3.1 \text{ m/s} \qquad F_N = 80(9.8 - 3.1^2/20) = \boxed{746 \text{ N}}$$

Bottom:

$$F_N - F_g = mv^2/r \quad \Rightarrow \quad F_N = m(g + v^2/r) = \boxed{822\ \mathrm{N}}$$

Artificial Forces in Circular Motion:

- Many people think that there is a net force that points away from the center of the uniform circular motion.

While turning in a car, you "feel" like something is pushing you outward

While washing your clothes, it looks like they are being pushed outward by some force

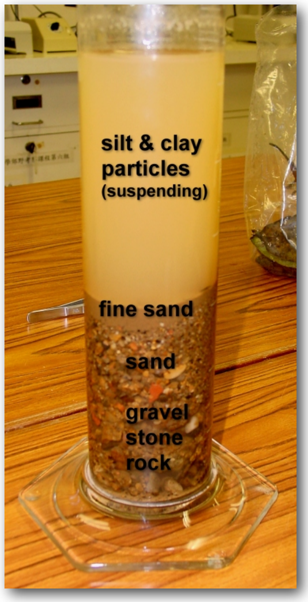
You are smarter than most people! There is NEVER a net force pointing away from the center of the circle!

- This is force is not real. Since the velocity is changing, we are in a non-inertial reference frame.

How a Centrifuge Works:

- We have seen that macroscopic objects in a fluid, where the force of gravity is larger than the buoyant force, will fall to the bottom of the liquid.
  - This is called **Sedimentation**

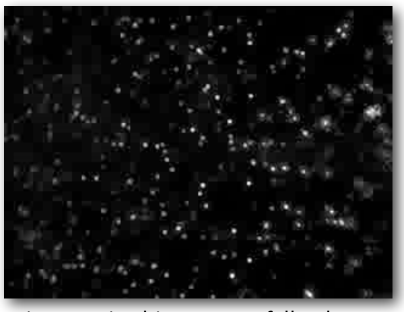
-However, microscopic objects never fall to the bottom because they are so small that the random motion of the fluid molecules is the largest force
  - Microscopic particle move in random directions in a fluid



-What if we want to get these microscopic objects to sediment so that we can collect them and study them?

- Generate a force in one direction that is much larger than the force due to the random fluid particle.

- This is exactly what a centrifuge machine in the laboratory does!



Microscopic objects never fall to bottom

- Suppose we have a test tube full of blood cells that is in uniform circular motion where the bottom of the tube is pointed away from circle

- These cells will have a small velocity that points toward the bottom of the tube v_r

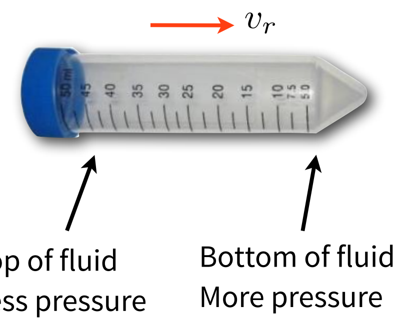
- There are two forces that create the net centripetal force
    - Friction force and the buoyant force

- Since our particles are small, the Reynolds number is small, and the friction force can be written as

Centrifuge test tubes point outward

$$F_f = f v_r \longleftarrow$$ Points in opposite direction from v_r

- We also have a buoyant force that points toward the center due to a difference in pressure between the top and the bottom of the test tube (we will see this later in class).



- Here Newton's 2nd law in the horizontal direction is

$$f v_r + F_B = m a_{\text{cent}}$$

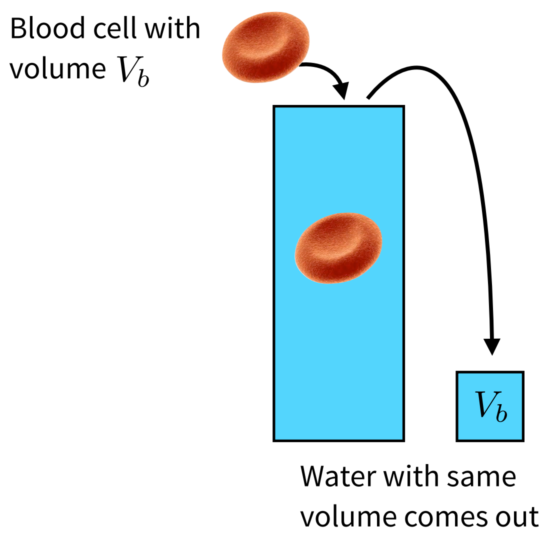
- The buoyant force typically depends on the weight of the fluid that is moved by the object $F_B = m_o a_{\text{cent}}$

-m_o is the mass of the water with the same volume as the particle

- Newton's 2nd law is now: $f v_r + m_o a_{\text{cent}} = m a_{\text{cent}}$

- We can solve for the ratio of velocity to acceleration called the **sedimentation coefficient**

$$s = \frac{v_r}{a_{\text{cent}}} = \frac{(m - m_o)}{f}$$ (Has units of seconds)



- This depends on the particles mass, the friction properties, and also the properties of the fluid the particle is in.

- The sedimentation coefficient is usually small so we define the Svedberg (S): $1S = 10^{-13}$ sec

Sample	*Sed. Coeff (S)*	*No. gs to Pellet*	*Time to Pellet*
Whole cells	10^6	100	10 min
Cell nuclei	10^5	700	10 min
Mitochondria	10^4	7000	10 min
Ribosomes	30, 50 S	100,000	2 h
Soluble proteins	1 – 5 (globular)	500,000	hours
	5 – 20 (elongated)		

Time it takes to get particle to form a layer