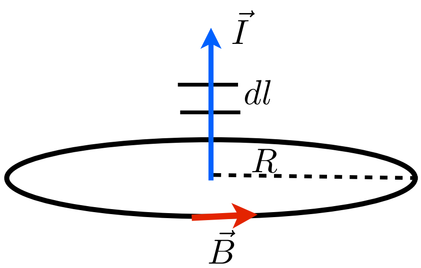
Magnetic Fields of Moving Charges

- Recall from last time that the direction of the magnetic field is given by the RHR:

current into the board   current out of the board

- Let us now look at the B-field from a loop of current:



- From experiment we know that: $B \propto \frac{I}{R}$

- Very similar result to E-field from line of charge:

$$E = \frac{\lambda}{2\pi\epsilon_0 R}$$

- For E-field, line of charge is like collection of monopoles each with $E_{\text{mono}} \propto 1/R^2$

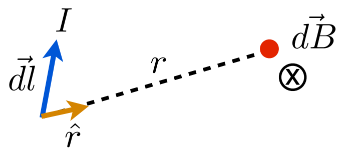
- Integration over all monopoles gives $E \propto 1/R$

- There are no monopoles for the B-field, but lets try to use this connection.

- Therefore, $B \propto 1/R$ suggests that if we chop up the wire into little pieces, then each piece should contribute a term $\propto 1/R^2$

- Integration over all $\vec{dl}$ pieces, like the E-field example, should give $B \propto 1/R^2$

- This thinking lead to the **Biot-Savart Law:**



to get direction correct

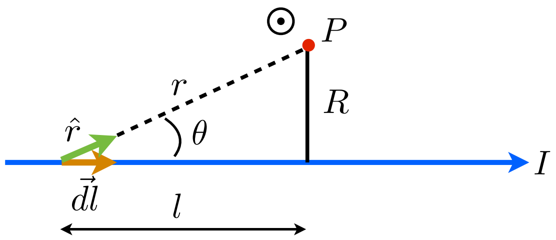
$$d\vec{B} = \frac{CI}{r^2}\left(\vec{dl} \times \hat{r}\right)$$

length = 1

- We can measure what the constant C is: $C = 10^{-7} \equiv \dfrac{\mu_0}{4\pi}$ Permeability of free-space

$$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi r^2}\left(\vec{dl} \times \hat{r}\right)}$$

- Lets apply Biot-Savart to line of current:



$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2}\left(\vec{dl} \times \hat{r}\right)$$

$$\vec{dl} \times \hat{r} = dl \sin\theta = dl\frac{R}{r}$$

$$r = \sqrt{l^2 + R^2}$$

$$dB = \frac{\mu_0 I R}{4\pi}\frac{dl}{(l^2 + R^2)^{3/2}}$$

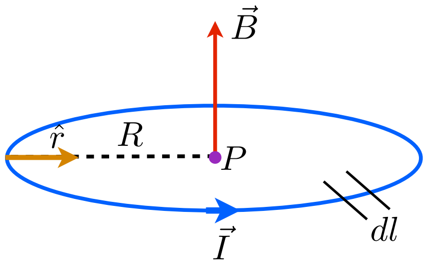
- Integration range is $l \in [-\infty, \infty]$ $$B = \frac{\mu_0 I R}{4\pi}\int_{\infty}^{\infty}\frac{dl}{(l^2 + R^2)^{3/2}}$$

- After integration $$B = \frac{\mu_0 I}{4\pi R}\left[\frac{l}{\sqrt{l^2 + R^2}}\Bigg|_{-\infty}^{\infty}\right]$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

- **Key Idea:** Fields from monopoles go as $\propto 1/R^2$

- Biot-Savart to for current loop:

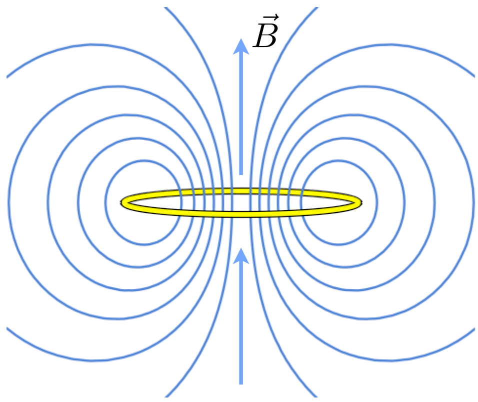


- r is always perpendicular to dl

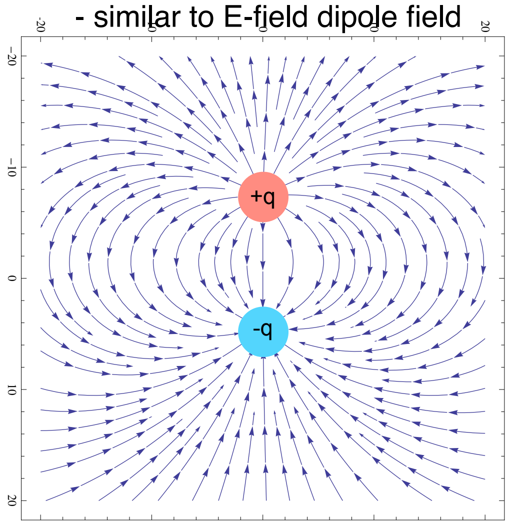
$$B = \int_{\text{loop}} d\vec{B} = \frac{\mu_0 I}{4\pi R^2} 2\pi R$$

$$B = \frac{\mu_0 I}{2R}$$

- lets put in the B-field lines from the loop:



- similar to E-field dipole field



- Recall Gauss' Law for E-field:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

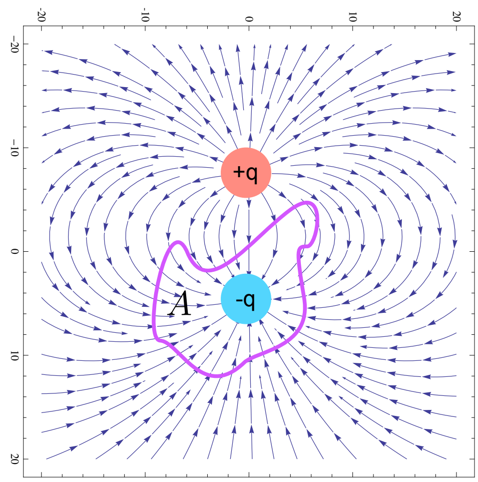
Maxwell Equation #1:

- This is non-zero only because there are electric monopoles.

- There are no magnetic monopoles (only dipoles)

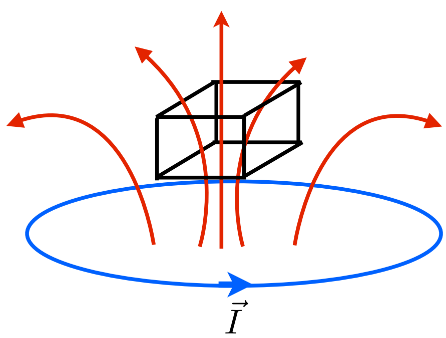
Maxwell Equation #2:

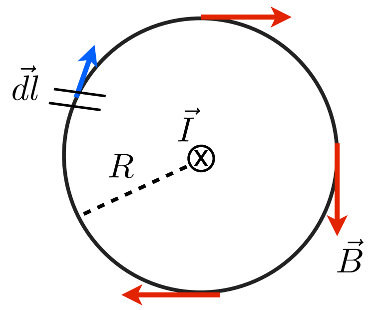
$$\boxed{\oint \vec{B} \cdot d\vec{A} = 0}$$



- There is no net magnetic flux through a surface

- The number of B-field lines going into A must be equal to number of B-field lines leaving A





- From Biot-Savart we have B-field:

$$B = \frac{\mu_0 I}{2\pi R}$$

- Now go around current wire in circle with radius R and chop into pieces $\vec{dl}$ pointed in same direction as B

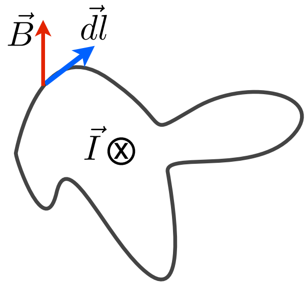
- Doing integral over closed circle, we get:

$$\oint \vec{B} \cdot \vec{dl} = B 2\pi R = \mu_0 I$$

NOT the same as $\vec{dl}$ that is small length wire with current

- Answer does not depend on distance R

- Ampere discovered that you do not need to walk in a circle; any closed path gives same answer.



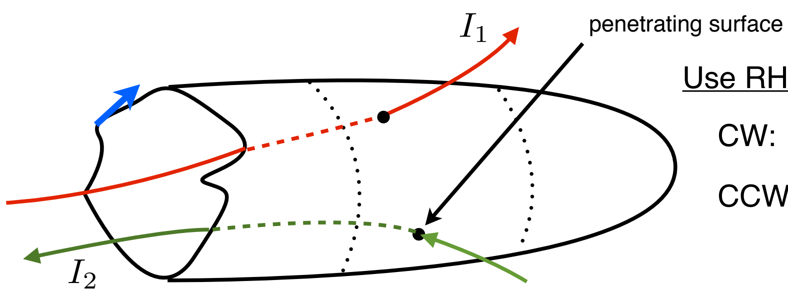
$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{\mathrm{enc}}}$$

Ampere's Law

- Can prove Ampere's Law with Biot-Savart

- Problem: How do we define enclosed current?

- Answer: Attach an **open** surface to the integration loop



Use RHR to determine sign of currents

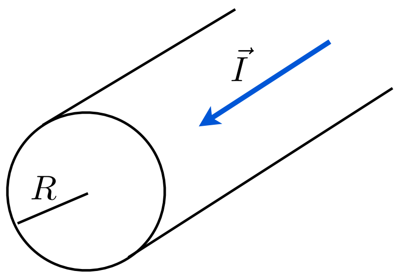
CW: $I_1 > 0, I_2 < 0$

CCW: $I_1 < 0, I_2 > 0$

- Enclosed current should be renamed "penetrating current": $\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{\mathrm{pen}}$

- Steps: You choose loop, then attach open surface. Surface defines sign of currents

Ex. Current carrying wire:

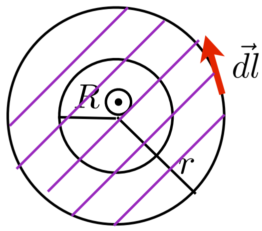


- Assume current is uniform in wire.

- What is the B-field inside and outside?

Outside: $r > R$

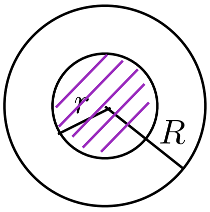
-Chose open surface to cover bigger circle



$$\oint \vec{B} \cdot \vec{dl} = B2\pi r = \mu_0 I \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

- Same answer as Biot-Savart, but much easier to get.

Inside: $r < R$



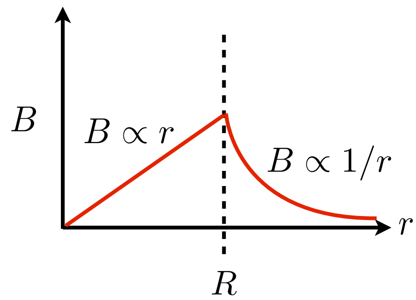
- Surface covers only part of wire

- Full Current does not go through surface

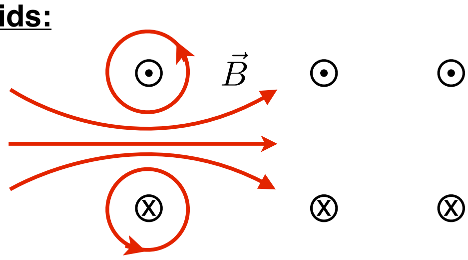
$$B2\pi r = \frac{\mu_0 r^2}{R^2} I \Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

- Lets plot the B-field as function of r:

- Of course the two equations are equal at r=R.



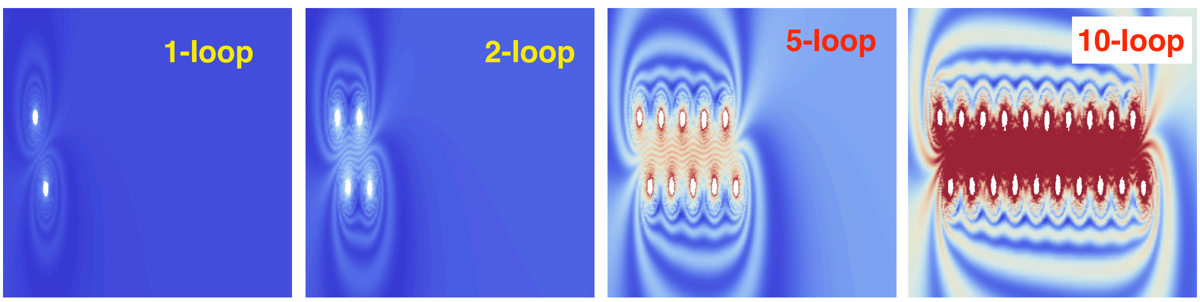
Solenoids:



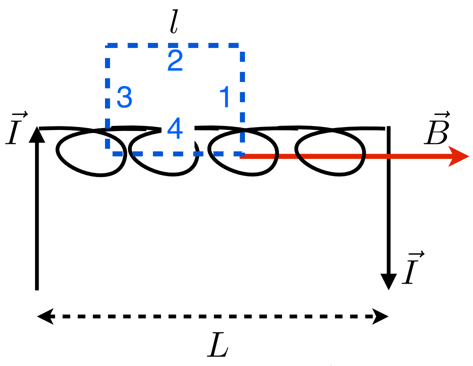
- Loops want to concentrate B-field at center

- Lots of loops -> almost uniform B-field inside of loops

- B-field is extremely weak outside of loops



- Use Ampere's Law to find B-field inside Solenoid:



- Wires turn clockwise
- Assume B-field outside is zero.
- Assume a total of N-loops.
- Must choose integration path (dotted blue path)

- No B-field outside so: $\int_2 \vec{B} \cdot \vec{dl} = 0$
- For paths 1 & 3: $\vec{B} \perp \vec{dl}$ -> $\vec{B} \cdot \vec{dl} = 0$
- Only path #4 contributes & B is constant inside
- Choose surface to find current I; just use flat surface over rectangle
- Not all loops go through rectangle: $Bl = \frac{l}{L} N \mu_0 I$

$\boxed{B = \frac{N}{L}\mu_0 I}$ Very good approximation if $L >> R_{\mathrm{loops}}$