PHYS-183 : Day #13

Ideal Fluids:

- In the next two weeks our goal will be to understand the physics of fluids

- Fluids are extremely important in biological systems. Essentially all life could not exists without water (the most important fluid)

- Moreover, the air we breath is a fluid, and our bodies obtain oxygen through the important process of diffusion

- A **fluid** is a gas or a liquid that, unlike a solid, moves to assume the shape of the container in which it is placed.

- The molecules in a fluid are randomly located, and the forces between the molecules in a liquid are much smaller than the forces in solid objects.

- If there are <u>no forces</u> between the molecules of a fluid, then the fluid is said to be **ideal**

- The most important ideal fluid is the **ideal gas**

- For gases, the average number of molecules per unit volume can easily change by a large amount.

ex. Air is easily compressed and can be used for many things

Compressed air for cleaning electronics.

- The average number of molecules per unit volume is used to define the density of the fluid

$$\rho = \frac{m}{V}$$ ⟵ Mass of molecules

- For gases, the <u>density is not a constant</u> so gases are said to be
compressible.

- In contrast, for liquids, the density is almost always a constant and liquids are said to be **incompressible**.

- To begin we will consider an ideal fluid, that is incompressible and with no friction.

Substance	*Density(10^3 kg/m^3)*
Water	0.998
Water, 4°C	1.000
Mercury	13.6
Sea water	1.025
Ice	0.917
Ethyl alcohol	0.791
Whole blood	1.06
Blood plasma	1.03
Bone	1.9
Air	0.0013
Water vapor, 100°C	0.006

[1]At 20°C and atmospheric pressure unless noted.

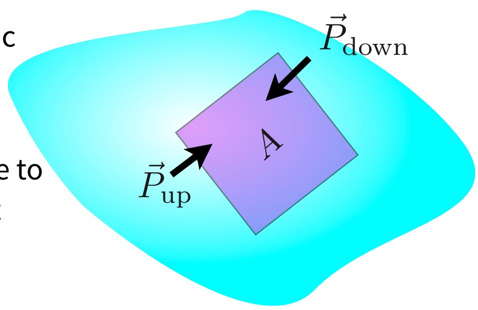
<u>Pressure:</u>

"static fluid"

- When a fluid is at rest, it is said to be in **hydrostatic equilibrium**, and there will be no net force on any part of the fluid, just like in solid objects

- Even though there is no <u>net</u> force on the fluid, we know from our earlier classes that there is always random motion due to the temperature of the fluid, we called this diffusion.

- Imagine we draw a surface with area A in a fluid in hydrostatic equilibrium



- If we look at the average momentum through the surface due to diffusion, we see that the momentum on one side is equal but opposite to the momentum on the other side.

$$\vec{P}_{\rm up,avg} = -\vec{P}_{\rm down,avg} \Rightarrow$$ No net force on *A*

- However, if we take just the force <u>on one side</u> and divide this by the total area we define this to be the **pressure** within the fluid.

$$P = \frac{F}{A}$$ Pressure has units of $\mathrm{N/m^2}$ which we call the Pascal (Pa)

- If there are no external forces, then the pressure is constant everywhere in the entire fluid

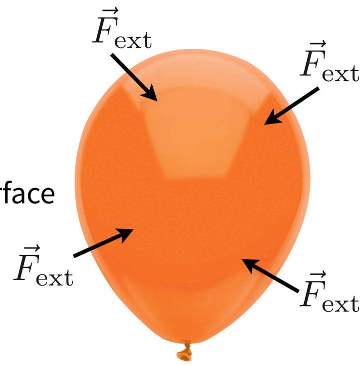
- If a fluid is put in a closed container, then there is an external force on the fluid, and therefore an external pressure is being applied.

- Although the pressure is on the surface, the external pressure on a fluid in a closed container increases the pressure everywhere in the fluid by the same amount; this is called **Pascal's Theorem**.

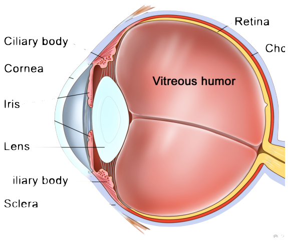
- The force from the container must be normal (perpendicular) to the surface if the fluid is at rest.

- If a fluid is put in a closed container, then there is an external force on the fluid, and therefore an external pressure is being applied.

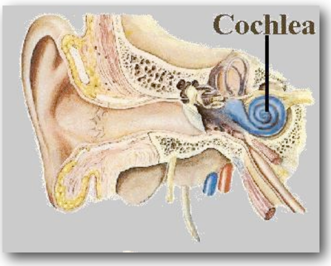
- Pressurized fluids occur in many parts of the human body



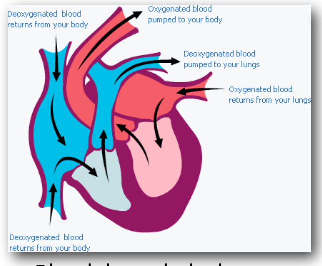
The pressure inside this balloon is the same everywhere (Pascal's theorem).



Vitreous Humor



Human Ear Cochlea

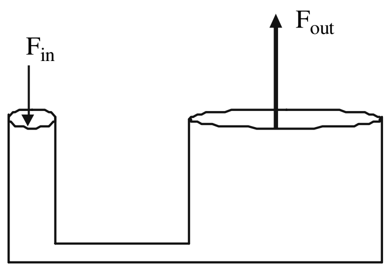


Blood through the heart

- Pascal's theorem can be used to amplify forces.

- Suppose I use a small force F_{in} acting over a small area A_{in} as shown

- The applied pressure is then $P = \dfrac{F_{\text{in}}}{A_{\text{in}}}$



Using Pascal's theorem to amplify forces

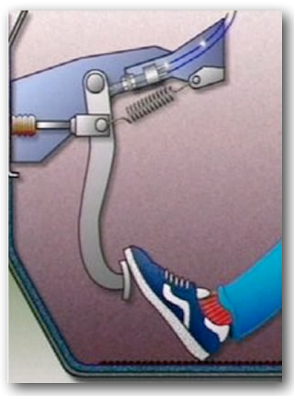
- Because the pressure must be essentially constant we have

$$P = \frac{F_{\text{in}}}{A_{\text{in}}} = \frac{F_{\text{out}}}{A_{\text{out}}}$$

- If A_{out} is larger than A_{in} then the initial force will get amplified

$$F_{\text{out}} = \frac{A_{\text{out}}}{A_{\text{in}}} F_{\text{in}}, \quad \frac{A_{\text{out}}}{A_{\text{in}}} > 1$$

- This is exactly how you stop a car by pressing on the brake pedal


The force from your foot get amplified to stop your car.

Ex. A cylindrical tube filled with blood is held vertically. The tube has radius of r=1cm and a height of h=10cm. Calculate the pressure at the bottom of the tube.

Solution:

- The pressure at the bottom of the tube is equal to the weight of the fluid divided by the area of the bottom.

- Using the density of blood from the table we have

$$P = \frac{mg}{A} = \frac{\rho_{\text{blood}}(\pi r^2 h)g}{\pi r^2} = \rho_{\text{blood}} g h$$

- We see that the pressure is independent of the radius of the container, this is true in general for a fluid in hydrostatic equilibrium

- Plugging in the numbers we find:

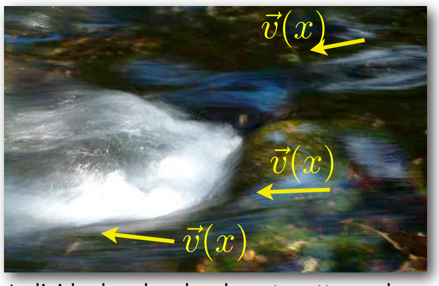
$$P = 1.06 \times 10^3 \cdot 9.8 \cdot 0.1 = \boxed{1040 \text{ Pa}}$$

Dynamics of Non-viscous Fluids:

- Now we begin our study of how fluids move.

- Although a fluid is made up of many many molecules, it is best to analyze it as a continuous medium.

- For an incompressible fluid, this means that we only care about the velocity of the fluid as a function of space and time.



Individual molecules do not matter, only the fluids velocity at each point

- In general, there are two-different types of fluid flow:

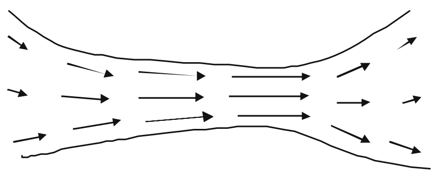
(I): **Steady-flow**, the flow does not depend on time.

(I): **Unsteady-flow**, the flow depends on time.

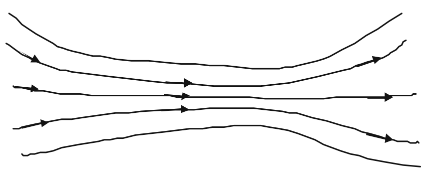
- In steady-flow, the velocity at each point in the fluid does not change in time.

- We can use a picture with vectors to represent steady-flow

-We can connect the velocity vectors together to get **streamlines**.



Velocity vectors for steady-flow.

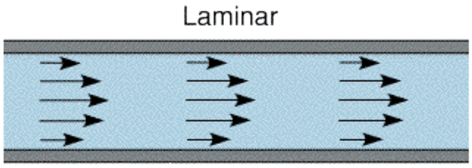


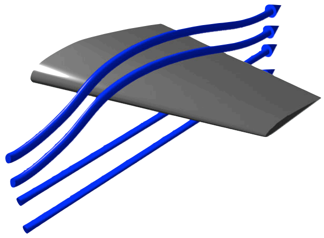
Streamlines for steady-flow.

Physics for Life Scientists (PHYS-183), Paul D. Nation

- In addition to steady and unsteady flows that tell us how a fluid depends on time, we have to types of fluid based on how they move in space

(1) **Laminar flow**: The fluid moves smoothly as if it were made out of a bunch of layers sliding over each other.

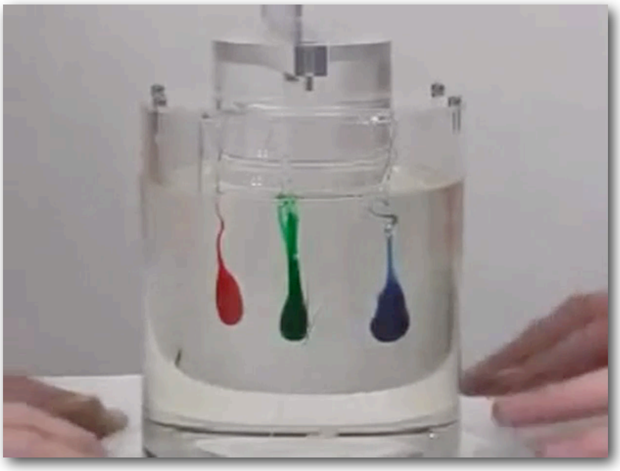




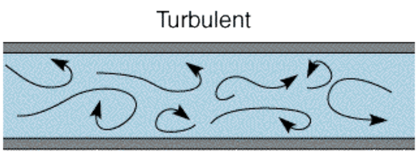
- The flow of air around a car, over over the wing of an airplane is an example of laminar flow.

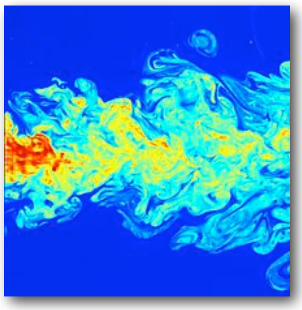
- Laminar flow also has a special property

- You can always go backward and get to the initial condition you started with



(2) **Turbulent flow**: The fluid moves in a chaotic fashion, very unsteady and very unpredictable.





- Turbulence is most common when larger objects (remember the Reynolds number?) move through fluids such as the air or water.

- Surprisingly, turbulent flow is very important for blood flow:

- The opening and closing of your heart valves requires turbulent flow because there are no muscles to generate a force

Human heart valve

- If you cut yourself, you stop bleeding faster if the cut is rough, because the blood will flow turbulently. Smooth cuts, such as paper cuts have laminar flow and take a long time to stop.

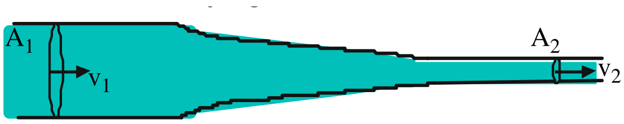
Conservation Laws in Fluids:

- We have already seen many times that using conservation laws makes solving physics problems quite easy.

- We can also apply conservation laws to the study of fluids with steady-flow

- For fluids we have the conservation of mass, and the conservation of energy.
    - Momentum is also conserved, but it is too difficult to discuss here.

- Suppose we have our fluid flowing in a tube that has a changing radius



- If the density of the fluid is ρ, then with area A1 and velocity v1, the mass of the fluid that passes through A1 in a time Δt is given by

$$\Delta m = \rho A_1 v_1 \Delta t$$

- Here $A_1 v_1 \Delta t$ is the volume of the fluid that will flow by in a time Δt

- We can also define $Q = A_1 v_1$ to be the volume flow rate, or volume per second.

- Because mass is conserved, the fluid is incompressible, and no fluid escapes from the tube, if we look at the fluid flow in the small part of the tube we must find the same amount of mass Δm flowing through it in the time Δt.

- For incompressible fluids, the density ρ is a constant and therefore:

$$A_1 v_1 = A_2 v_2$$

-The fluid flow rate $Q = Av$ is conserved.

Key Idea: The smaller the area of the tube is, the faster the velocity must be. This is a direct result of the conservation of mass.

- This relationship is between velocities and areas at two different locations.

- In order to calculate what the actual velocity of the fluid is, we will need to use conservation of energy.

Ex. Water exits a circular faucet (r_0) with a speed v_0. Find the expression for the diameter of the flowing water as it falls, assuming laminar flow.

Solution:

- The stream of water decreases in area as its velocity increases due to the conservation of mass.

- We can calculate the velocity after the water has fallen a distance *d* since this is a simple gravitational force problem. $v^2(t) = v^2(0) + 2gd$

- Therefore after a distance *d:* $v(t) = \sqrt{v^2(0) + 2gd}$

- Since the flow rate is conserved, the area will get smaller as the velocity increases

$$A(t) = \pi r^2 = A_0 \frac{v(0)}{v(t)} = \pi r_0^2 \frac{v(0)}{v(t)}$$

- Using r as the radius of the water after a distance d, then solving for 2r (diameter)

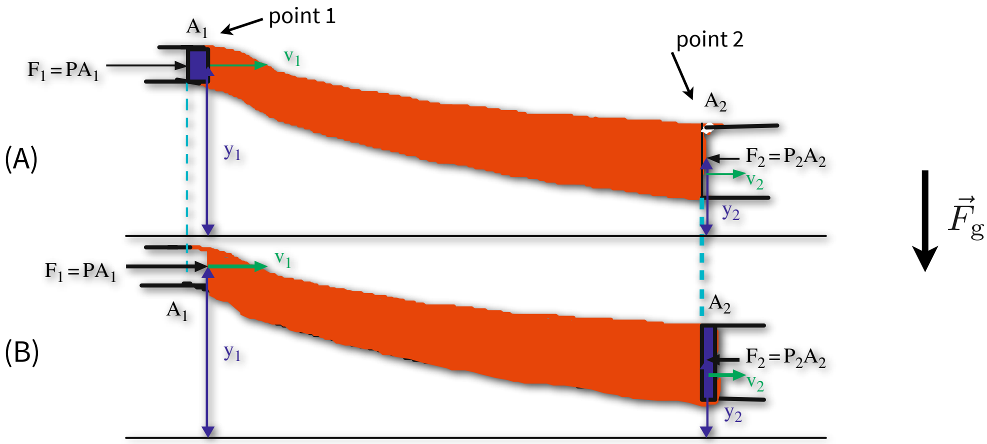
$$2r = 2r_0 \sqrt{\frac{v_0}{v}} = 2r_0 \sqrt{\frac{v_0}{\sqrt{1 + \frac{2gd}{v_0^2}}}} = 2r_0 \left(1 + \frac{2gd}{v_0^2}\right)^{1/4}$$

- The slower the initial water, the faster the area decreases, try it at home!

- We now want to look at the conservation of energy in a fluid.

- Lets consider a blood vessel with an ideal fluid flowing through it.

    - Blood is really not an ideal fluid and is very complicated, but for now we do not worry



- Here, the colored fluid between the two dotted lines in part (A) is our initial system

- Some time Δt later, we get the picture in part (B) and the blue section of the fluid has moved to the bottom of the blood vessel

- In going from (A) to (B), the CM of our fluid has moved, and has changed its velocity and height.

- We know that the fluid in our system has both kinetic and potential terms.

- We will show that the surrounding fluid does work on our system, so we will again use the Work-Energy Theorem

- Equal volumes of fluid at two different locations (1) and (2) will have different velocities because as we have shown $Q = A_1 v_1 = A_2 v_2 = \text{constant}$

- Therefore in going from point (1) to point (2), our fluid slows down and the systems CM loses kinetic energy

- To find the change in kinetic energy we only need to consider the fluid in the two blue regions

$$\Delta T = \frac{1}{2}\rho(v_2^2 - v_1^2)Q\Delta t \qquad \rho Q\Delta t = \text{mass of blue regions}$$

- According to the work-energy theorem, this change in kinetic energy must be equal to the net work done on the system

- We have two types of forces on our fluid that do work:

(I) Force of gravity (if there is a change in height) leads to a change in potential energy:

$$\Delta U = (\rho Q\Delta t)g(y_2 - y_1)$$

(II) There is work done by the pressure forces from the surrounding fluid

- One the left, the fluid exerts a positive force $F_1 = P_1 A_1$

- One the right, the fluid exerts a negative and smaller force $F_2 = P_2 A_2$

- The work due to force 1 is: $W_1 = F_1\Delta x_1 = P_1 A_1(v_1\Delta t) = P_1\Delta t(A_1 v_1) = P_1\Delta t Q$

- We get a similar equation for force 2, so net work is: $W_{\text{net}} = (P_1 - P_2)Q\Delta t$

- Combining kinetic energy, potential energy, and work terms we have

$$\left[\frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)\right] Q\Delta t = (P_1 - P_2)Q\Delta t$$

- Simplifying, we have an expression for the conservation of energy in an ideal fluid.

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

Bernoulli's Equation

- Since the points (1) and (2) are random, Bernoulli's equation says

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

for <u>all locations and for all times</u>.

- Again, blood is not an ideal liquid, and its flow is also turbulent

- However, we can understand several important diseases by using Bernoulli's equation

- Here we will assume that y=constant, so there is no potential energy term:

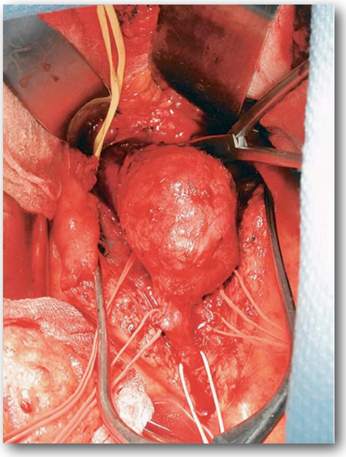
$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

- First, let us consider a blood vessel that has a weaken wall. This problem is called an **Aneurysm**

  - The wall of the blood vessel will get swollen ("bigger")

- If the cross-sectional area is bigger where the aneurysm is then the velocity of the blood in that region will be slower from conservation of mass.

- If aneurysm radius is N time bigger than the normal vessel, then the area will be N^2 times bigger



Aneurysm of the aorta

➔ - The velocity must be N^2 times <u>slower</u> in the aneurysm

- From Bernoulli's equation $P + \frac{1}{2}\rho v^2 = \text{constant}$

- If the velocity of the blood decreases by N^2 then the pressure must increase a lot to keep the right hand side constant

- Because the blood vessel is already weak, this increase in pressure can cause the vessel to break open, causing the person to bleed to death.

- Next we can consider a partially blocked artery due to the build up of plaque deposits, mainly made up of cholesterol.

- This disease is called **Arteriosclerosis**

- The analysis is similar to the previous example

- If the inside radius of the artery decreases by a factor of N, then the velocity of the blood will increase by N^2

A partially blocked artery (Atherosclerosis)

- Now, from Bernoulli's equation, the pressure in the region will drop by a lot.

- If the blocking deposit is big enough, the velocity can be so large that the outside pressure is much larger than the inside pressure and the artery is pushed closed

- If this happens to an artery in your heart, then this is called **Angina**, or a heart attack.

- If this happens to an artery in your brain, then this is called a stroke.