- We have shown that the maximum current occurs on resonance, but what about if we use a different frequency while keeping R, L, and C fixed?

$$I_{\max} = \frac{V_0}{Z} \qquad Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

<u>First Case:</u>

$$\omega \to 0, Z \to \infty \Rightarrow I_{\max} \to 0$$

- Due to the capacitor

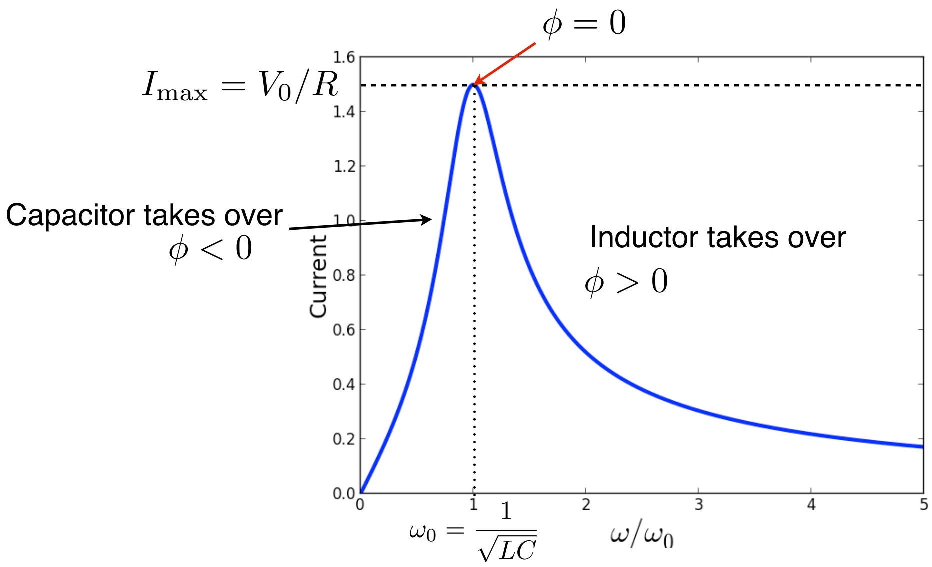
- As the frequency -> 0, the current is no longer AC but DC. DC-current builds up charge on the capacitor until no more current can flow.

<u>Second Case:</u>

$$\omega \to \infty, Z \to \infty \Rightarrow I_{\max} \to 0$$

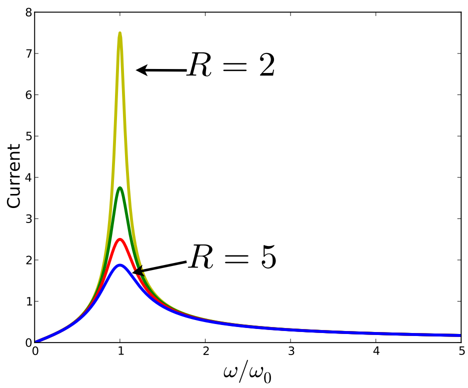
- Due to the inductor

- If frequency is very high then current changes extremely rapidly. Self-inductor tries really hard to fight rapid changes in current that generate rapid B-field changes



<u>Parameters</u>

$V = 7.5$ V
$R = 5\ \Omega$
$L = 8.2$ mH
$C = 100\ \mu$F

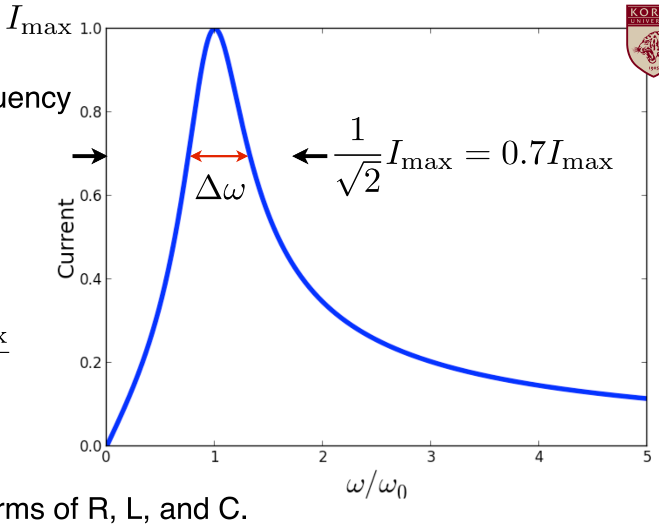


- What happens if R is decreased?

- The width of the peak decreases and the maximum current increases rapidly.

- If resistance is too low, can destroy your circuit if driving frequency is on resonance.

- We want some numerical way to describe the width of the peak at the resonance frequency



- This is called the **Quality Factor:**

$$Q = \frac{\omega_0}{\Delta\omega}$$

- $\Delta\omega$ is the width of the peak at $I = \frac{I_{\max}}{\sqrt{2}}$

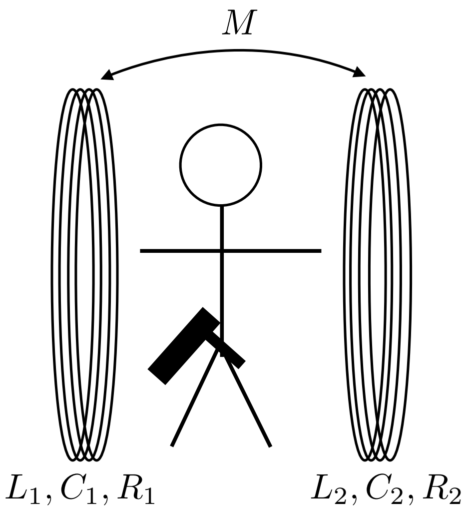
- The quality factor can also be written in terms of R, L, and C.

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{1}{\sqrt{LC}} \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- The quality factor tells us how strongly the system behaves on resonance, and how weakly it responses when not on resonance.

- Because Q~1/R, the Q-factor also tells us how much energy is lost during each period T.

Ex. Metal Detector:



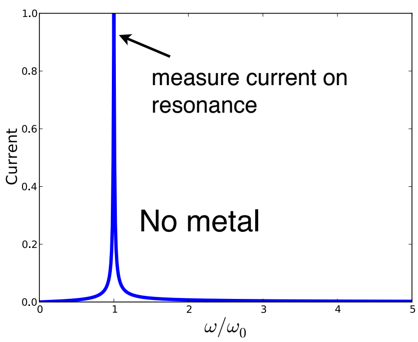
- Metal detector is made from two-coupled LRC Oscillators

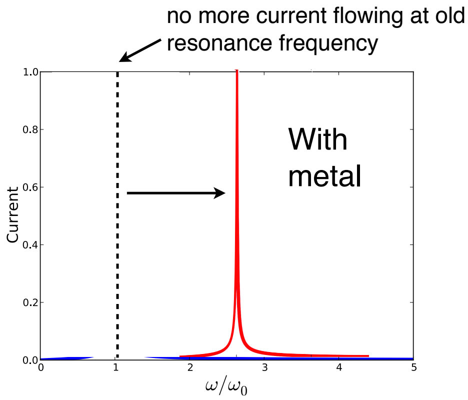
- Coupled via mutual inductance (haven't discussed this in detail)

- Two different resonance peaks (just going to show one)

- Oscillators have extremely large Q-factors (i.e. very sensitive to change in frequency)

- Eddy currents generated in metal, generated B-field that shifts resonance frequency





Power in AC-Circuits:

- Why did we define the Q-factor as the width at $I = \dfrac{I_{\max}}{\sqrt{2}}$ and not $I = \dfrac{I_{\max}}{2}$?

- In the real-world we are not interested in the current, but rather the power $P \propto I^2$

$$P = I^2 R = \left[I_{\max}\cos(\omega t - \phi)\right]^2 R = I_{\max}^2 R \cos^2(\omega t - \phi)$$

- The current oscillates in time, so lets calculate the average power using:

$$\langle I \rangle = \frac{1}{\sqrt{2}} I_{\max} \quad \text{Average over one-period} \rightarrow \quad \langle P \rangle = \frac{1}{2} I_{\max}^2 R$$

- It is also common to define the **RMS (Root-Mean-Squared) Current**: $I_{\text{rms}} = \dfrac{1}{\sqrt{2}} I_{\max}$

$\rightarrow$ $\boxed{\langle P \rangle = I_{\text{rms}}^2 R}$

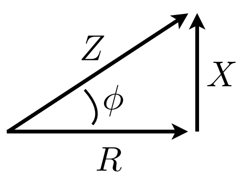
- In AC-circuits, when people talk about current, they always mean the RMS current.

- We can also define the **RMS Voltage**: $V_{\text{rms}} = V_{\max}/\sqrt{2}$

$\rightarrow$ $\boxed{I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}}$

- The average power can now be expressed using RMS values for both the current and the voltage:

$$\langle P \rangle = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}}{Z} I_{\text{rms}} R = \boxed{I_{\text{rms}} V_{\text{rms}} \frac{R}{Z}}$$



$\rightarrow$ $\dfrac{R}{Z} = \cos\phi$ $\rightarrow$ $\langle P \rangle = I_{\text{rms}} V_{\text{rms}} \cos\phi$