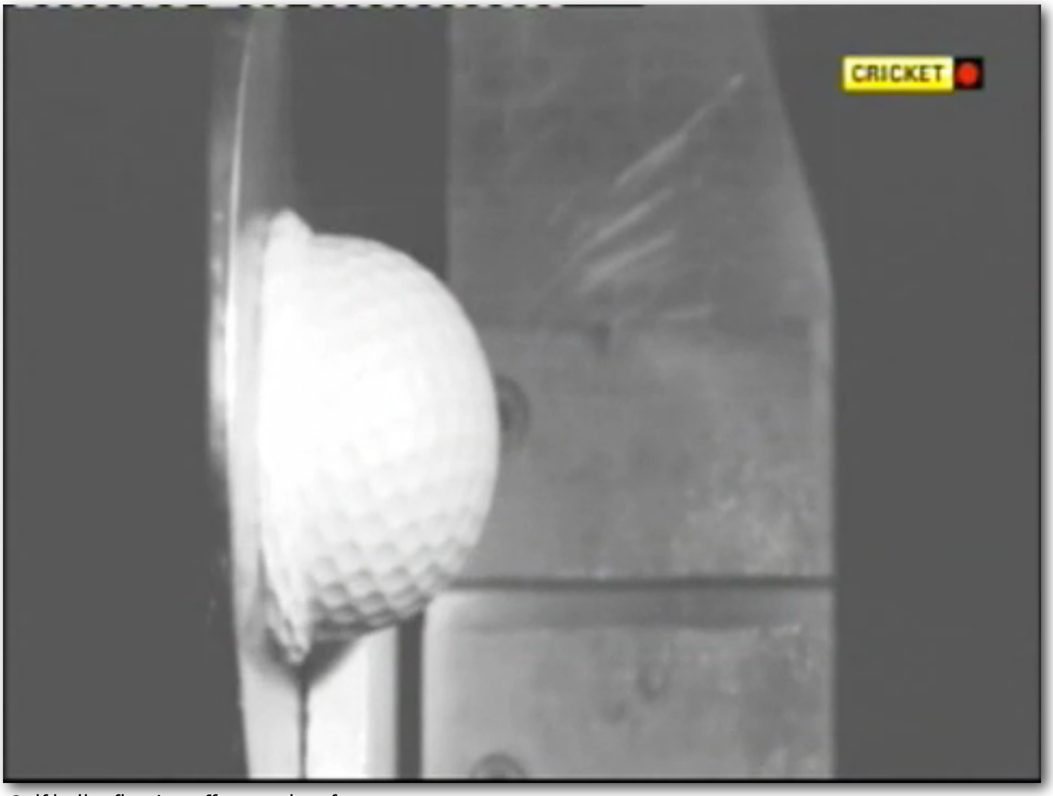
PHYS-183 : Day #9



Golf ball reflecting off a metal surface.

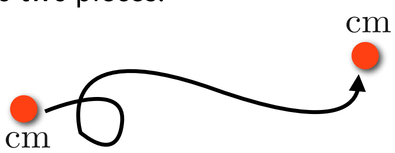
- This week, we will use our physics knowledge to begin to explore more realistic quantum systems.

- So far we have explained the center of mass motion using Newton's laws, but have not worried about the details of our object.

    - For example, things like the objects size and shape have not been important.

- For realistic objects we can divide the motion into two pieces:

    - Translational motion of the center of mass



    - Motion with respect to the center of mass

- This week we deal with translational motion, and next week rotational motion.

Momentum:

- So far we have focused on forces as the origin of motion according to the Newton's laws.

- We can alternatively describe motion based on the concept of momentum

- Many times, using momentum is easier because we may not know all of the forces on an object.

Def: For an object of mass *m* traveling at velocity $\vec{v}$, the **momentum** $\vec{p}$ is defined to be:

$$\boxed{\vec{p} = m\vec{v}}$$

- Momentum is a vector quantity that points in the same direction as the velocity.

- Using Newton's laws, an object with no net force, has a constant velocity, and constant momentum.

- If there is a net force, then the momentum will change.

- The bigger the net force, the bigger the change in momentum.

- Since momentum is proportional to mass, two object with the same velocity but different masses will have different momentum.

Ex: Suppose a 500kg car, and 2000kg truck are both traveling at 120km/hr:

$$p_{\text{car}} = (500\ \text{kg})(33\ \text{m/s}) = 16500\ \text{kg}\cdot\text{m/s}$$
$$p_{\text{truck}} = (2000\ \text{kg})(33\ \text{m/s}) = 66000\ \text{kg}\cdot\text{m/s}$$

- We have seen that the change in momentum is related to the force, now we must show this mathematically using Newton's 2nd law.

- Using $\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t}$, we can write Newton's 2nd law $\vec{F} = m\vec{a}$ as a change in momentum

$$\boxed{\vec{F}_{\text{net}} = \lim_{\Delta t\to 0}\frac{\Delta m\vec{v}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}}$$

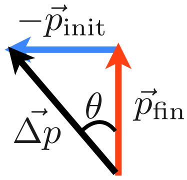
- This is the original way Newton wrote his second law of motion.

- This equation also works when the mass of the object changes as a function of time $m(t)$.

Mass changes as a rocket takes off

Ex. An E. Coli bacterium with mass $m = 6 \times 10^{-16}$ kg is initially swimming at a constant velocity of $8\ \mu\mathrm{m/s}$ to the east. One ms later it is found to be moving at $10\ \mu\mathrm{m/s}$ to the north. Find the change in the E. Coli's momentum and the average external force acting on the bacteria.

- Be careful, we can not do (10-8)=2 $\mu\mathrm{m/s}$ because this is not a 1D problem!



- We need to find the change in momentum $\vec{\Delta p} = \vec{p}_{\mathrm{final}} - \vec{p}_{\mathrm{initial}}$

- It is clear that the change in momentum is the hypotenuse of the triangle

$$\Delta p = m\sqrt{v_{\mathrm{init}}^2 + v_{\mathrm{fin}}^2} = (6 \times 10^{-16})\sqrt{(8 \times 10^{-6})^2 + (10 \times 10^{-6})^2}$$

$$\boxed{\Delta p = 7.7 \times 10^{-27}\ \mathrm{kg \cdot m/s}}$$

- The direction can be found using either the momentum or velocity vectors since they point in same direction

$$\theta = \tan^{-1}\left(\frac{8}{10}\right) = \boxed{39\ \mathrm{deg}}$$

- The average force can be found (without the limit):

$$\vec{F}_{\mathrm{net}} = \frac{\vec{\Delta p}}{\Delta t} = \boxed{7.7 \times 10^{-24}\ \mathrm{N}}$$

In same direction as $\vec{\Delta p}$

Ex. The fastest elevator in the world, in Yokohama Japan, has a maximum speed of 12.5m/s taking people from the bottom to the 70th floor in 40s. Find the maximum change in momentum for an 80kg person in the elevator. What is the net change in momentum for the entire trip?

Yokohama Japan.

- The maximum change in momentum would be during the initial acceleration or final deceleration.

- During this period, your momentum would increase from zero to $p = mv = (80)(12.5) = 1000\ \mathrm{kg \cdot m/s}$

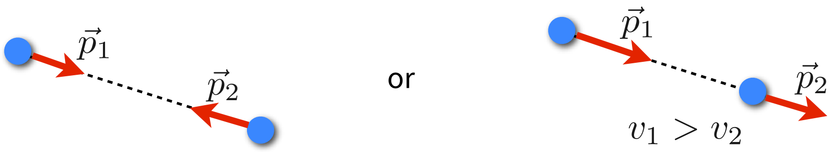
- Therefore, the total change in momentum is: $\boxed{\Delta p = 1000\ \mathrm{kg \cdot m/s}}$

- Since the elevator starts and stops with v=0, the net change in momentum for the entire trip is zero.

- Suppose we have two particles (1) and (2) that are initially very far apart and have momentums $\vec{p}_1$ and $\vec{p}_2$, respectively.

- At some point later in time, the particles collide with each other.

They must be moving along a line that connects the two particles.



- Here we will assume P1 and P2 point in same direction (it does not really matter).

- Using Newton's 3rd law (action-reaction force pairs) we have

$$\vec{F}_{1 \text{ on } 2} = \frac{\Delta \vec{p}_1}{\Delta t} \qquad \vec{F}_{2 \text{ on } 1} = \frac{\Delta \vec{p}_2}{\Delta t}$$

- We assume that the only these are the only forces on the objects.

- By the 3rd law, we know that these forces are equal in magnitude, and opposite in direction

The vector sum of $\vec{F}_{1 \text{ on } 2}$ and $\vec{F}_{2 \text{ on } 1}$ must be zero!

- Therefore, for all times:

$$\frac{\Delta \vec{p}_1}{\Delta t} + \frac{\Delta \vec{p}_2}{\Delta t} = 0 \quad \text{or} \quad \frac{\Delta(\vec{p}_1 + \vec{p}_2)}{\Delta t} = 0 \quad \text{or} \quad \Delta(\vec{p}_1 + \vec{p}_2) = 0$$

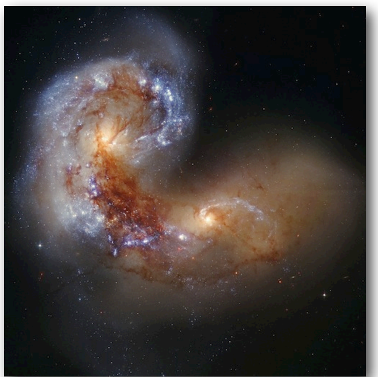
- In order for this to be true, **the net momentum of the two particles must be constant for all times**.

In this situation, **momentum is conserved**

- We now have two important conservation laws: Conservation of energy, and conservation of momentum.

- Like the conservation of energy, the conservation of momentum is always true.

- We do not need to know about the forces on the objects, only their momentum.

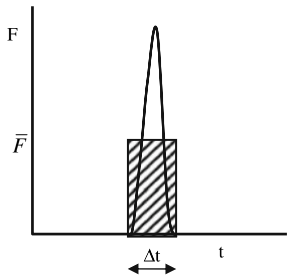


Every collision conserves momentum

- During the collision of the two objects, they will exert equal and opposite forces on each other for some amount of time Δt.

- The force of one particle on the other multiplied by the time Δt is called the impulse:

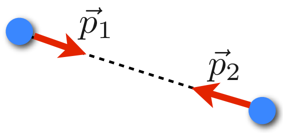
$$\boxed{\text{Impulse} = F\Delta t = \Delta p = p_{\text{final}} - p_{\text{initial}}}$$



Typical force vs. time graph for a collision. Impulse is the area under the curve (rectangle).

- The impulse is a single parameter that is able to predict the change in momentum for an object due to a collision.

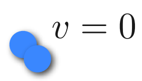
<u>Ex.</u> Suppose there are two identical particles with same mass *m* and velocity *v* traveling toward each other.

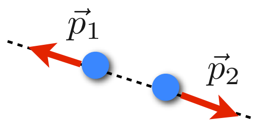


- In this case, the net momentum is zero: $\vec{p}_1 + \vec{p}_2 = \vec{p}_1 - \vec{p}_1 = 0$

- The net momentum must be zero before, during, and after the collision.

- In order for the momentum to be zero after the collision there are only two possible final states:





Particles stuck together   Particles go in opposite directions

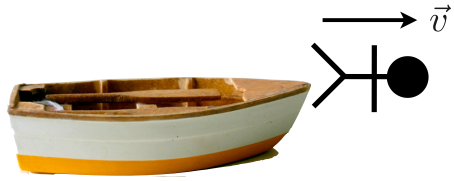
- Both of these conserve momentum but when the particles stick together, kinetic energy is not conserved.

  - In this case, the missing energy is usually turned into sound or heat.

- If two objects with unequal masses interact, things become more complicated, and what happens depends on whether kinetic energy is conserved or not.

- In these situations, conservation of momentum is extremely important for finding the answer.

Ex. A 60kg boy dives horizontally with a speed of 2m/s from a 100kg boat initially at rest. Ignoring the friction of the water, what is the boats final velocity?



- Since there are no external horizontal forces, the momentum is conserved.

- The initial momentum was zero, so the final momentum must be zero

- The momentum of the boy is $p_{\text{boy}} = (60)(2) = 120\ \text{kg}\cdot\text{m/s}$ (right)

- The final momentum of the boat must therefore be equal in magnitude, but opposite direction

$$p_{\text{boat}} = -120\ \text{kg}\cdot\text{m/s} \qquad \text{(left)}$$

- The boats velocity is equal to the momentum of the boat divided by the mass:

$$v_{\text{boat}} = \frac{p_{\text{boat}}}{m} = \frac{-120\ \text{kg}\cdot\text{m/s}}{100\ \text{kg}} = \boxed{-1.2\ \text{m/s}}$$

Ex. Two ice skaters, both traveling at a speed of 5m/s and heading straight toward each other, collide and grab each other. If their masses are 80kg and 50kg, find the velocity of the skaters together.

- Again, there are no net external forces, so we know momentum is conserved.

- The initial momentum is just the vector sum of the two individual skater momentum:

$$P_{\text{init}} = (80\ \text{kg})(5\ \text{m/s}) - (50\ \text{kg})(5\ \text{m/s}) = 150\ \text{kg}\cdot\text{m/s}$$

In same direction as 80kg skater

- When the skaters are together, their total mass is 130kg and the final momentum must equal the initial momentum

$$P_{\text{final}} = (130\ \text{kg})\text{v}_{\text{final}} = 150\ \text{kg}\cdot\text{m/s} = \text{P}_{\text{init}}$$

- Therefore, the final velocity of the skaters is:

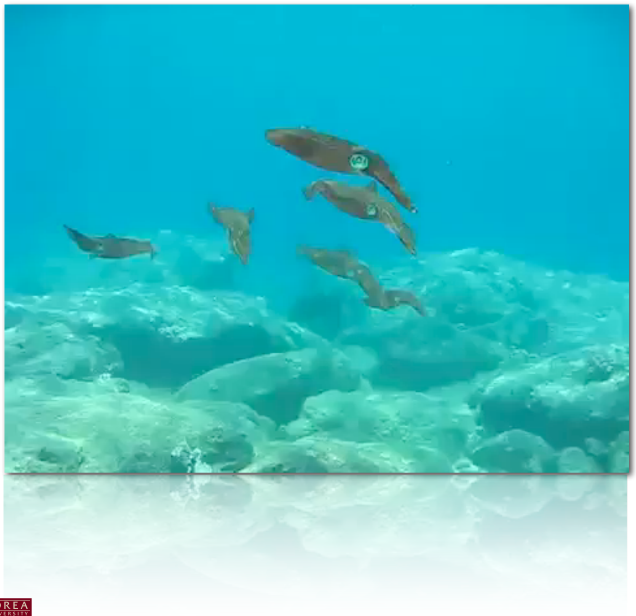
$$v_{\text{final}} = \frac{150\ \text{kg}\cdot\text{m/s}}{130\ \text{kg}} = \boxed{1.15\ \text{m/s}}$$

In same direction as 80kg skater

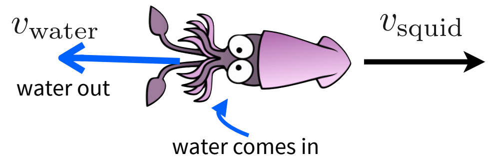
- When looking at the boy and the boat example, we ignored the friction force due to water pushing on the boat.

- For animals that swim (fish, jellyfish,whales) or fly (birds, bats) the reaction force due to the surrounding fluid supplies all the force necessary for motion.

- Let us look at how a squid moves through the water



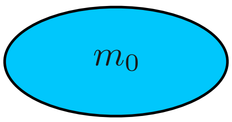
- The squids move by pushing water out of their body and then, due to the reaction force, their bodies go in the opposite direction of the water



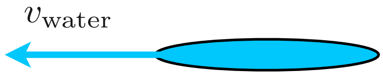
- Because water enters and leaves the squid, the mass of the squid is changing in time.

- This type of problem is easier to solve using momentum.

- We can model the squid a balloon that fills with water and then shrinks as water is released.



(Squid with water)



(Squid pushing water out)

- We will call the initial mass of water inside the squid m_0 , and assume that the water gets pushed out at a constant rate

$$\frac{\Delta m}{\Delta t} = \text{constant}$$

- Assuming all the water leaves with the same velocity v_{water} , the rate at which the momentum of the squid changes is

$$\frac{\Delta p}{\Delta t} = \frac{\Delta m}{\Delta t} v_{\text{water}}$$

- By Newton's 2nd law, this change in momentum must provide a net force on the squid.

$$F_{\text{net}} = \frac{\Delta p}{\Delta t} = \frac{\Delta m}{\Delta t} v_{\text{water}}$$

- For a typical squid modeled as a cylinder with radius 0.1m and length 0.3m, and empty mass 1kg, and pushes water with a velocity of 15m/s in one second, the net force can be calculated

$$V = \pi(0.1\ \mathrm{m})^2(0.3\ \mathrm{m}) = 0.01\ \mathrm{m}^3$$

$$m_0 = \rho_{\mathrm{H2O}} V = (1000\ \mathrm{kg/m^3})(0.01\ \mathrm{m}^3) = 10\ \mathrm{kg}$$

$$F_{\mathrm{net}} = \frac{\Delta m}{\Delta t} v_{\mathrm{water}} = \frac{10\ \mathrm{kg}}{1\ \mathrm{s}}(15\ \mathrm{m/s}) = 150\ \mathrm{N}$$

- The acceleration of the squid just after pushing out all the water can also be found:

$$a_{\mathrm{squid}} = \frac{F_{\mathrm{net}}}{m_{\mathrm{squid}}} = \frac{150\ \mathrm{N}}{1\ \mathrm{kg}} = 150\ \mathrm{m/s^2}$$