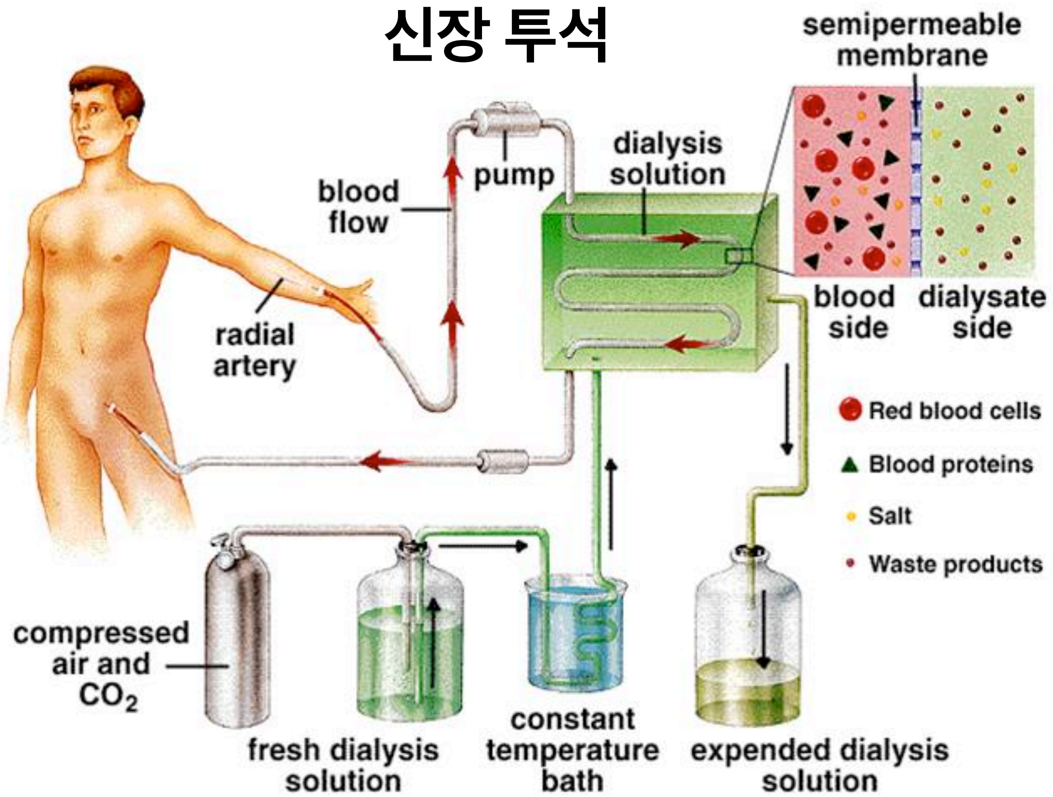
PHYS-183 : Day #4



Forces on Microscopic objects:

- **Microscopic** objects are objects that are so small that we need a microscope to see them

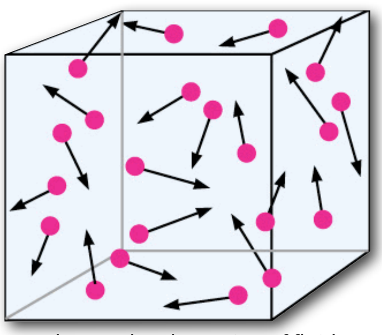
- Microscopic objects behave completely differently than macroscopic objects

- Every fluid (air, water, oil, alcohol) is made out of molecules

- All of these molecules are moving in random directions due to the fluids temperature

  Water molecules move around at $\sim 600\ \mathrm{m/s}$ at room temperature

- If you put an object in the fluid, then the fluid molecules will hit the object randomly with a force in a random direction



Random molecule motion of fluid

- For macroscopic objects, these random forces can be ignored because the object is too big.

- For microscopic objects, these random forces are the largest forces on the object!

  - These random forces are even stronger than the gravitational force

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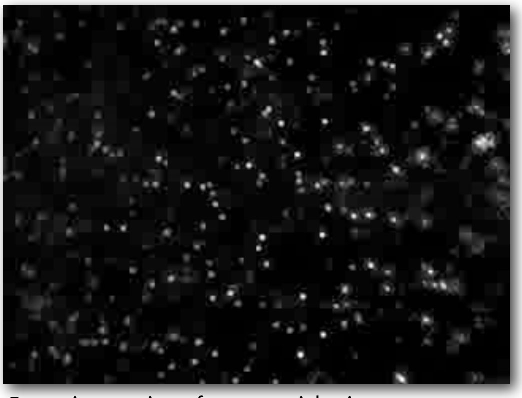
- In a fluid, microscopic objects move in random directions.

- This random motion is called **Brownian Motion**

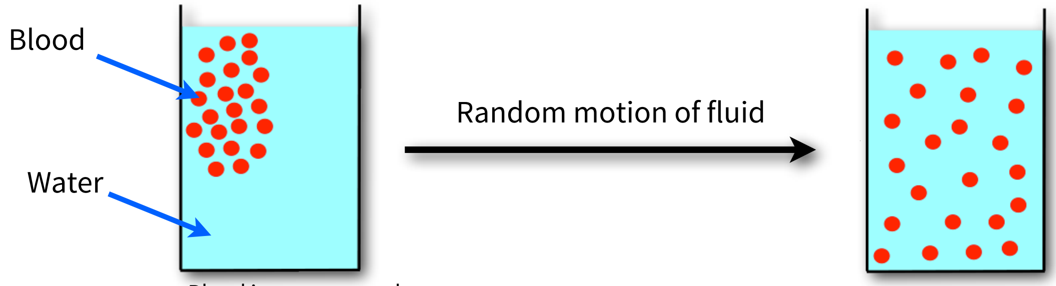
Discovered by a biologist:
Robert Brown (1827)



Brownian motion of nanoparticles in water

Diffusion:

- Although the motion of microscopic objects is random, the resulting motion is extremely important for biological molecules



Blood is concentrated

Blood is evenly distributed

- Suppose I have many objects (blood cells, sodium ions,...) in a small volume of fluid

We say that the objects are **concentrated** or have **high concentration**

- If I have a few objects in a large volume of fluid, then we call the objects **diluted** or have **low concentration**

Concentrated Bromine

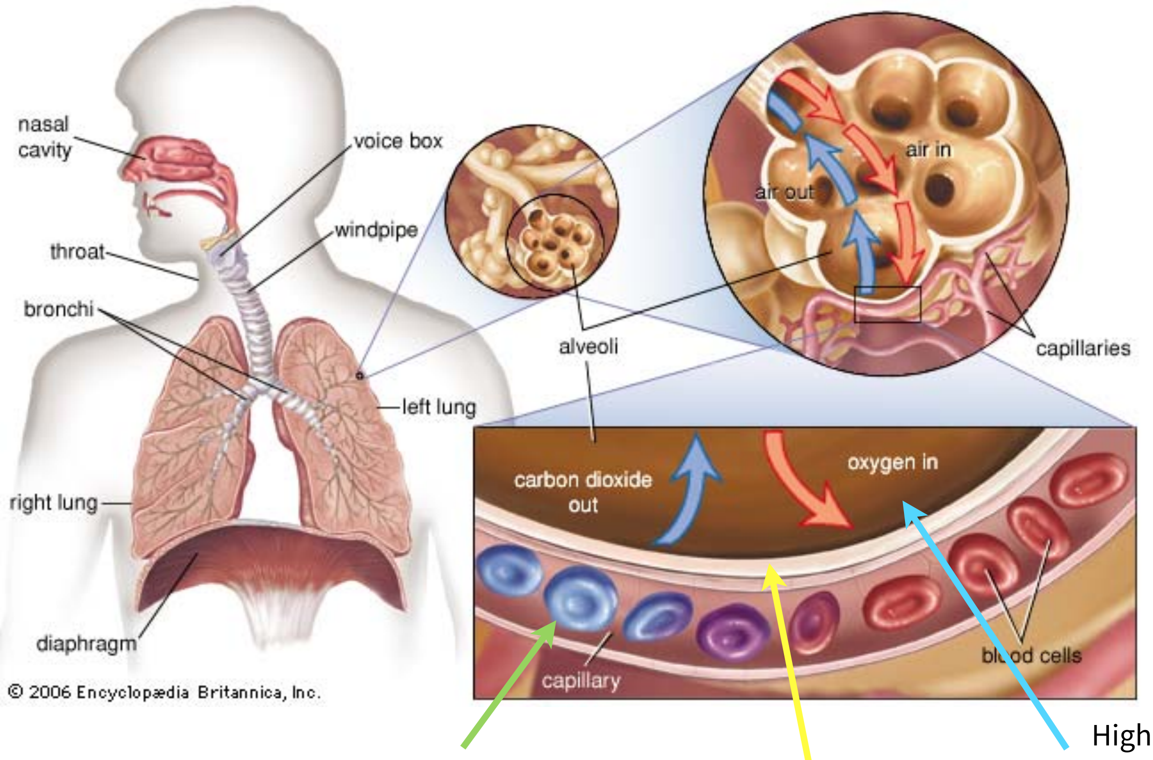
Diffusion of Bromine in air

- The random motion of fluid molecules takes concentrations of objects and makes them diluted

  - This process is called **diffusion**

- Diffusion is extremely important in biology!

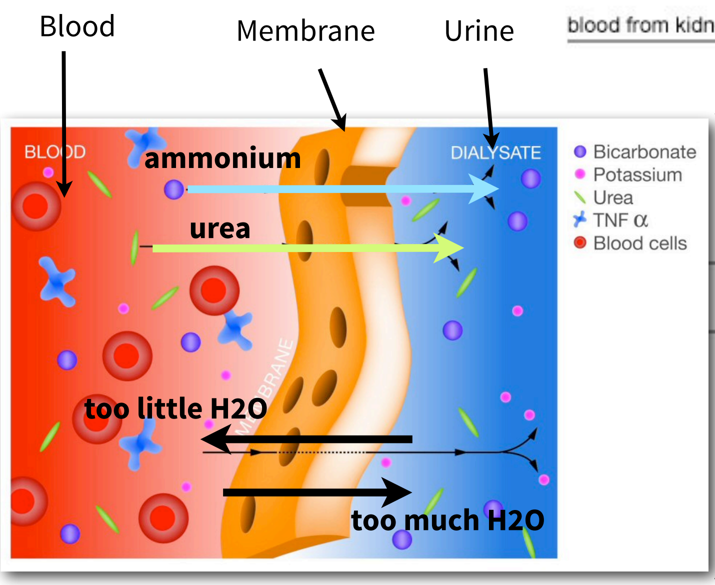
Ex. Diffusion of oxygen and carbon dioxide in lungs:

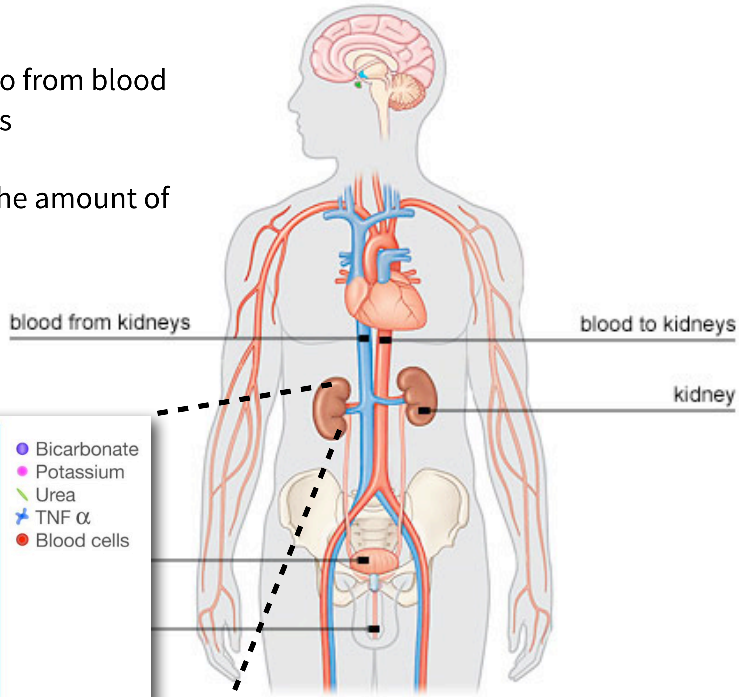


Ex. Diffusion waste products from kidneys:

- Waste products (urea, ammonium) to go from blood to your urine via diffusion in your kidneys

- Diffusion in the kidneys also balances the amount of water in your body





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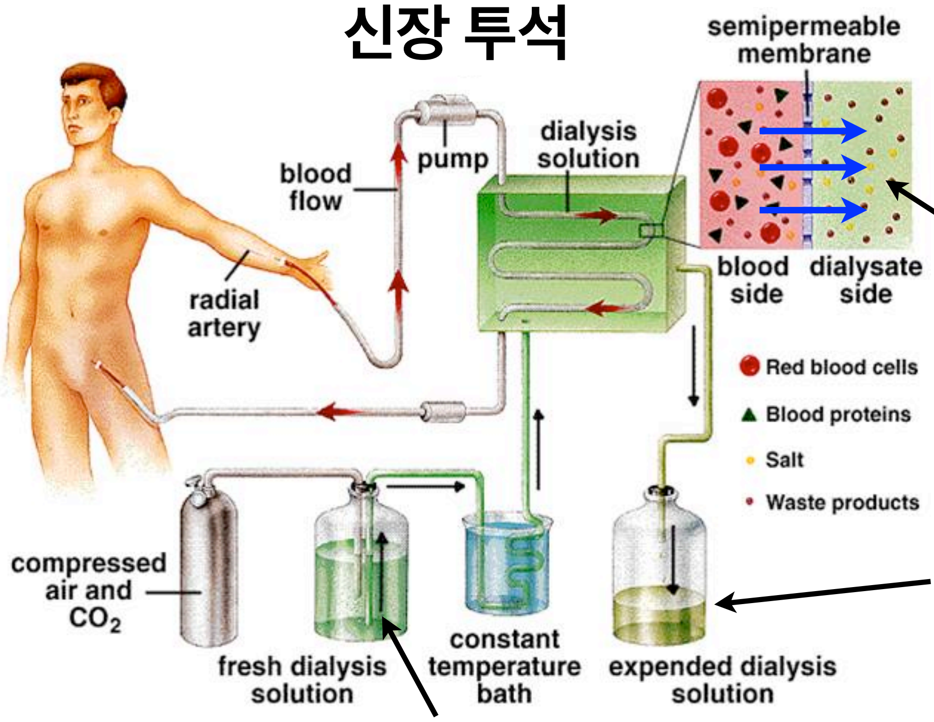
신장 투석



Always has low concentration of waste products

Old liquid with waste products from body

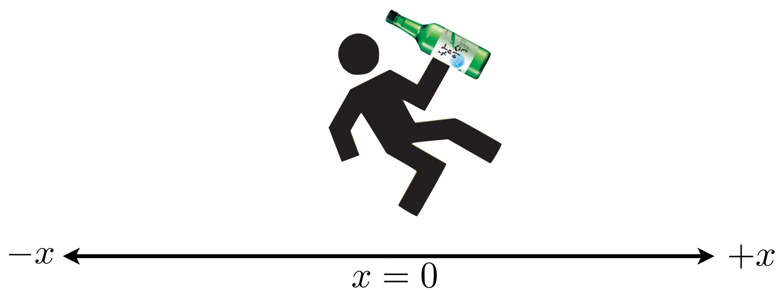
New liquid (no waste)

Understanding how diffusion works in kidneys save 2 millions lives every year!

- Since diffusion is caused by the random motion of atoms, how do we write this mathematically?

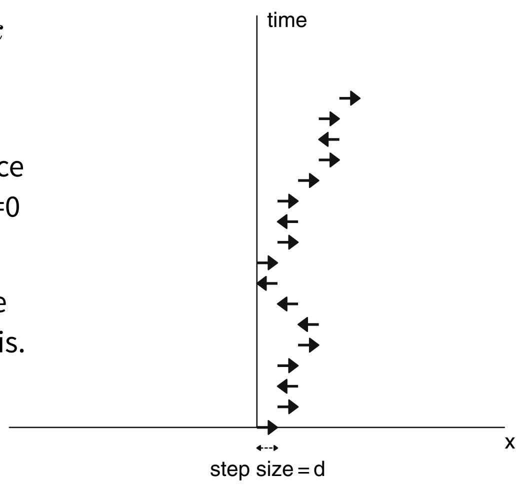
- The mathematical description is called a **random walk** or a **drunk walk:**

  - Suppose I have a drunk person who can only walk to the left or right



- If he is so drunk that he randomly walks either left or right for each step

- What does his motion look like?



- Since there is a 50% chance of going left, and 50% chance of going right, his average displacement will always be x=0

- However, as we see, over time the person starts to move away from x=0 so we need a new equation to describe this.

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- Instead of displacement, let us measure the average displacement squared: $\overline{(\Delta x)^2}$

  - Since this is the displacement squared, this number is always positive or zero.

  - Since the motion is random, we must always deal with averages.

  - The square root of this measures how far the average drunk man can walk away from x=0

- For the drunk walker, his averaged displacement squared is given by

$$\overline{(\Delta x)^2} = N d^2$$

number of steps he took how big his steps are

Ex. If N=10 and d=1m then $\overline{(\Delta x)^2} = 10\ \mathrm{m}^2$. To see how far the average person can walk, take square root $\Delta x_{\mathrm{avg}} = \sqrt{10}\ \mathrm{m} \approx 3\ \mathrm{m}$

- We also need an equation for other objects such as molecules, proteins,....

- Obviously things like molecules do not take steps, so we need to modify this expression

- For any object undergoing random motion in one-dimension the average squared displacement is

$$\boxed{\overline{(\Delta x)^2} = 2Dt}$$

time

D is called the **diffusion coefficient**

- The diffusion constant coefficient depends the size and shape of the object, the viscosity ("stickiness") of the fluid, temperature, ...

- **Key Idea**: Since $\overline{(\Delta x)^2}$ depends on time, the average displacement Δx_{avg} depends on the square root of time

$$\Delta x_{\mathrm{avg}} \propto \sqrt{t}$$

- compare to constant velocity in one-dimension: $\Delta x = vt \rightarrow x \propto t$

  If $\Delta x = 2$ after t=2 sec, then $\Delta x_{\mathrm{avg}} = 2$ would require about t=4 sec

- This is also true in two and three-dimensions

- Although objects in diffusion move quite fast in random directions, on average they do not move very fast away from where they started (x=0)

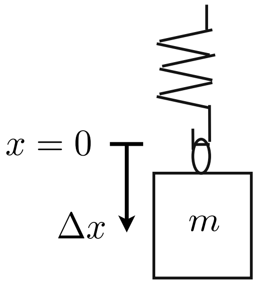
Hooke's Law and Oscillations.

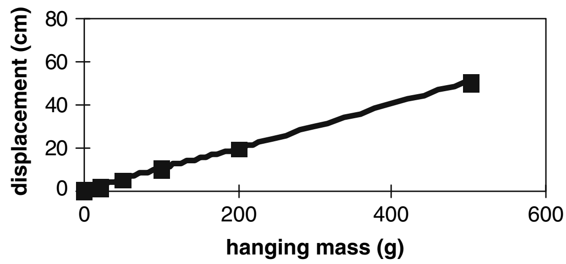
- In this section we are switching topics and are going to study springs and the motion of objects attached to springs

- Springs are some of the most important objects in all of physics!

    - About 50% of all of physics can be explained by using springs as a model.

- Suppose we hang a spring from the ceiling and measure the displacement of the spring for different masses





- For all of the masses tested we see that the displacement of the spring is linearly proportional to the mass on it

- Since the force of gravity is also proportional to the mass $F_g = mg_{\mathrm{E}}$, this tells us that the springs displacement is linearly proportional to the force on it

- If the spring and mass are not moving then we know by Newton's laws that the spring must also have a force equal in magnitude, but pointed in the opposite direction

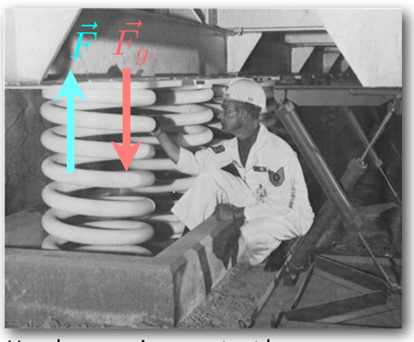
- If we let x be the displacement of the spring from equilibrium (x=0) then the force-displacement relationship can be written as

$$\boxed{F = -kx}$$ **(Hooke's Law)**

k is called the spring constant and tells us how hard we have push or pull on the spring to change its length

k has units of N/m

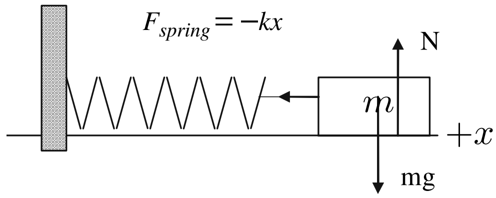
- The minus sign tells us that the force from the spring is always pointed in the opposite direction as the displacement x



Very large spring constant k

- Suppose we now take our mass and spring and place them sideways on the table

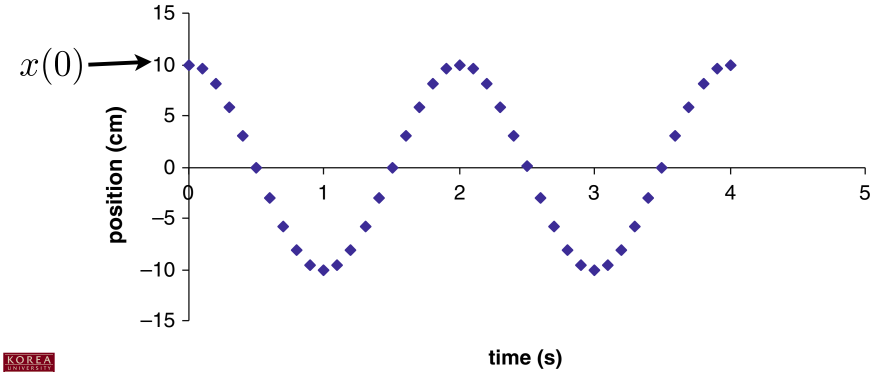
- Have three forces acting on mass, but two cancel



- Therefore, using Newton's 2nd law, we can find the springs acceleration

$$F_{\rm net} = -kx = ma$$

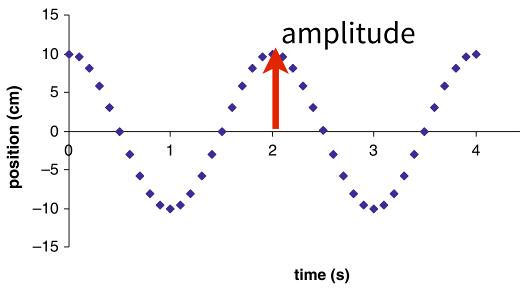
- If we stretch a spring with $k = 10\ \mathrm{N/m}$ from equilibrium by $x = 10\ \mathrm{cm}$ with a 1kg mass and then at time t=0 we let the mass go. What is the position as a function of time?



The mass undergoes **oscillatory motion**

- At t=0, x=10cm therefore the acceleration at t=0 is

$$a(0) = -\frac{k}{m}x = -\frac{10\ \mathrm{N/m}}{1\ \mathrm{kg}}0.1\ \mathrm{m} = -1.0\ \mathrm{m/s^2}$$



- The mass gains velocity in the -x-direction, but the acceleration gets reduced until x=0 where

$$a(0) = -\frac{k}{m}x = 0$$

- After x=0, the displacement becomes negative, so the acceleration becomes positive, and the mass begins to slow down

- At x=-10cm, t=1 sec, the velocity is once again zero, and the acceleration is

$$a(1) = -\frac{k}{m}x = -\frac{10\ \mathrm{N/m}}{1\ \mathrm{kg}}(-0.1\ \mathrm{m}) = +1.0\ \mathrm{m/s^2}$$

- The mass moves from +10cm to -10cm. This maximum displacement, 10cm, is called the **amplitude** of the motion

- With the mass initially displaced, the position of the mass as a function of time is given by

$$\boxed{x(t) = A\cos\left(\frac{2\pi t}{T}\right)}$$

$$A = \text{amplitude}$$
$$T = \text{period (or period of motion)}$$

- Amplitude are <u>always</u> positive

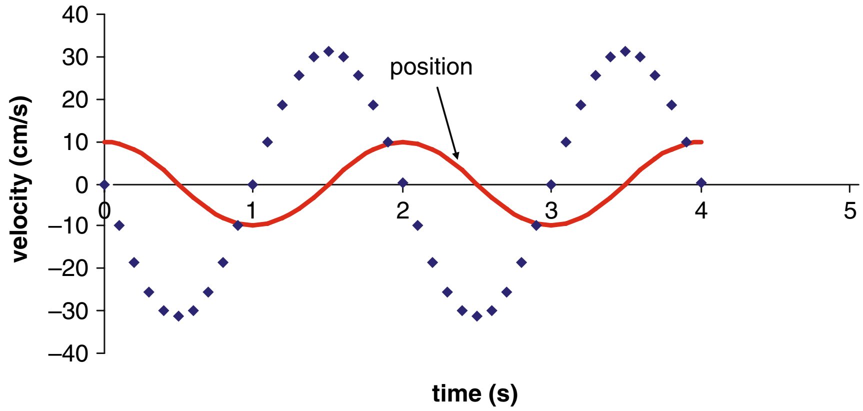
- The period tells us how long it takes for the mass to come back to its original location for the first time

- The equation is an example of **simple harmonic motion**

  - harmonic means "sine" or "cosine" functions
  - here simple means the motion goes forever

- If the mass started at x=0 at t=0, then we would need to use sin instead of cos

- To get the velocity of the mass, we need to find the slope of the positive curve at every point.



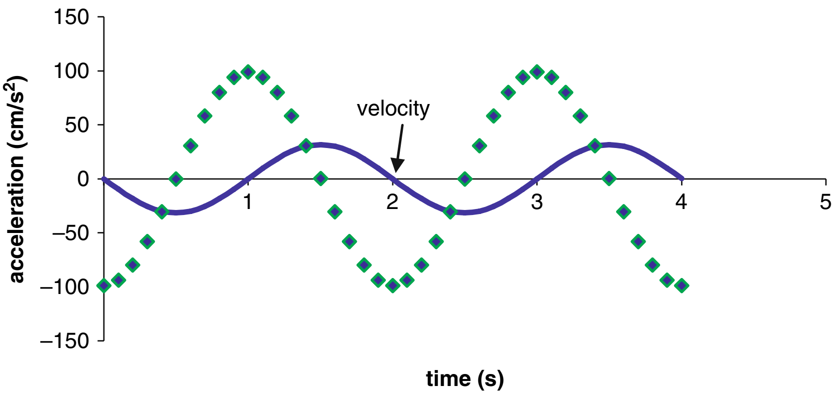
- The velocity is also a simple harmonic function : velocity starts out negative

$$v(t) = -v_{\max}\sin\left(\frac{2\pi t}{T}\right)$$

- The amplitude of the velocity $v_{\max}$ is given by:

$$v_{\max} = \left(\frac{2\pi}{T}\right)A$$

- We can do the same thing to get the acceleration (slope of velocity graph)



- Again, we get a simple harmonic function for the acceleration:

starts out negative

$$a(t) = -a_{\max}\cos\left(\frac{2\pi t}{T}\right)$$

with the amplitude:

$$a_{\max} = \left(\frac{2\pi}{T}\right)^2 A = v_{\max}\left(\frac{2\pi}{T}\right)$$

- Since both position and acceleration are cosine functions

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right) \qquad a(t) = -\left(\frac{2\pi}{T}\right)^2 A\cos\left(\frac{2\pi t}{T}\right)$$

this means that the acceleration and position are proportional to each other

$$a(t) = -\left(\frac{2\pi}{T}\right)^2 A\cos\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{T}\right)^2 x(t)$$

- But we already knew this since $F_{net} = -kx = ma$

- We can use these two equations to solve for the period of motion T

$$\frac{a}{x} = -\frac{k}{m} = -\left(\frac{2\pi}{T}\right)^2 \quad \Rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

- The bigger the mass, the longer the period.

- The stronger the spring, the shorter the period.

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- The period measures how long it takes for the spring to go from +A to -A and back one time.

- Another important quantity is the **frequency** (f) of the motion that measures how many times per second the mass goes back and forth

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- We can make things much simpler if we use the **angular frequency:**

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

- Since $(2\pi/T)$ is everywhere, all of our equations become very simple when using ω :

$$\boxed{\begin{array}{ll} x(t) = A\cos(\omega t) & \\ v(t) = -A\omega\sin(\omega t) & w = \sqrt{\frac{k}{m}} \\ a(t) = -\omega^2 x(t) & \end{array}}$$