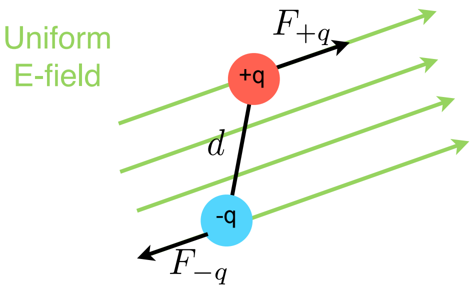
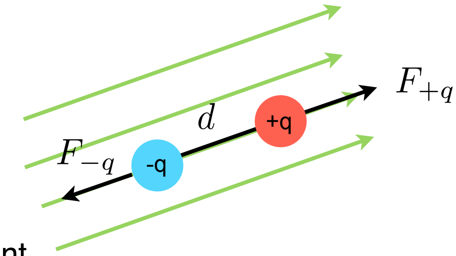
Electric Dipole in E-field:

- (+) charge will experience force in direction of E-field
- (-) charge will experience force opposite of E-field

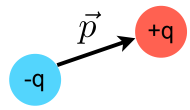


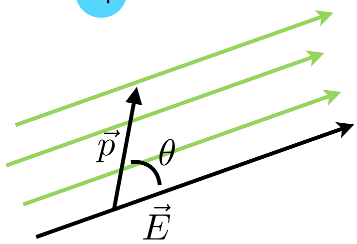
- E-field produces a torque on the dipole.
- Dipole rotates clockwise (in this example)
- Dipole wants to align with the E-field



- Torque reverses direction if dipole goes past alignment.
- recall that torque $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = |r||F| \sin\theta$
- also $\vec{F} = q\vec{E}$ so $\vec{\tau} = q\vec{r} \times \vec{E}$
- define "electric dipole moment": $\vec{p} = q\vec{d}$
- dipole moment always points from -q to +q
- torque on dipole: $\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = |p||E| \sin\theta$

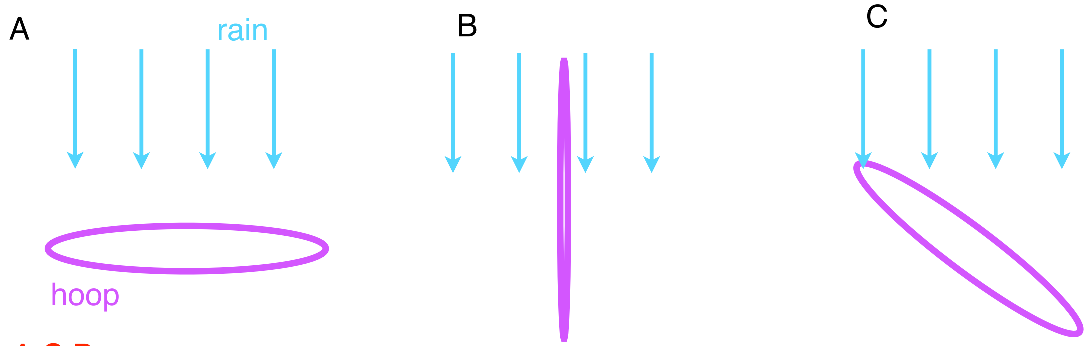
<u>dipole moment</u>





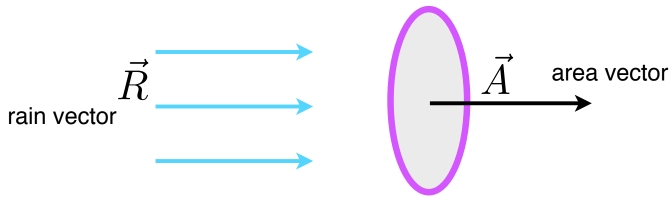
Electric Flux:

- Consider a hula-hoop on a rainy day. Rank (high to low) the following situations by how much rain goes through the hula-hoop.

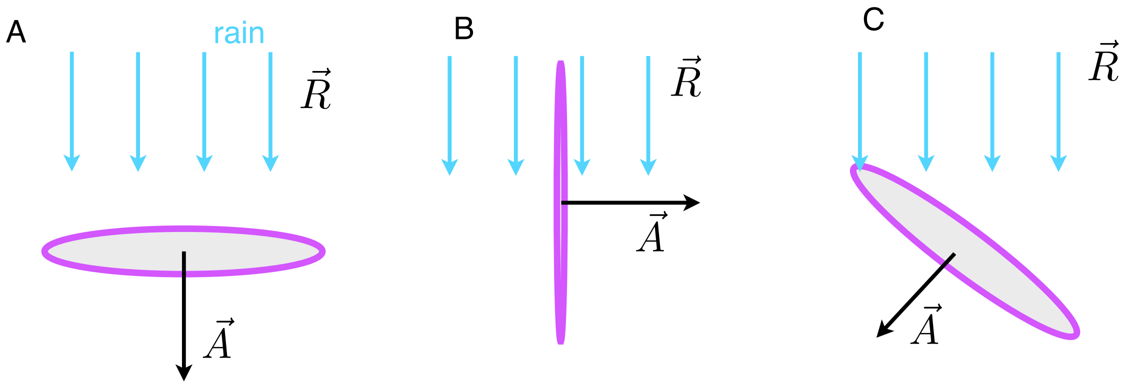


ANS: A,C,B

- The most rain enters the loop when it is held perpendicular to the direction that the rain falls.
- No rain goes through the loop when it is parallel to the rain fall.
- The loop forms a surface, to make this more mathematical we need to define the direction of the surface area formed by the loop:

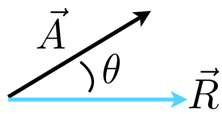


- Area vector is perpendicular to the surface



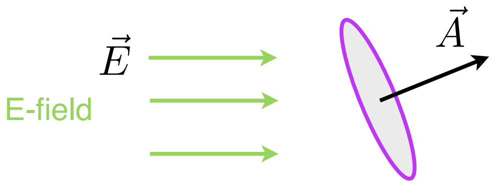
-Let R be the magnitude of rain and A be the magnitude of the loop area then the "rain flux" is:

"rain flux" $\phi = RA\cos\theta$



- Or in vector form: $\phi = \vec{R}\cdot\vec{A} = RA\cos\theta$

- Recall that the E-field can be thought of as a liquid -> define the **"electric flux"** as the amount of E-field (rain) flowing though a surface:

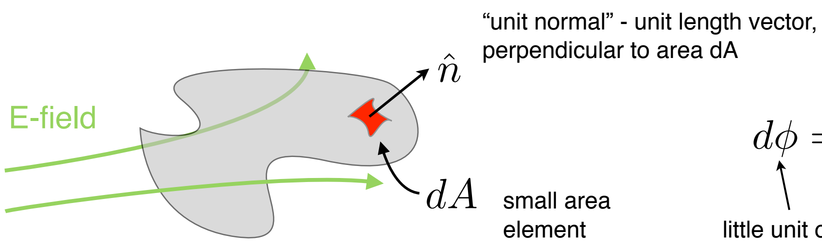


"electric flux"

$$\phi = \vec{E}\cdot\vec{A} = EA\cos\theta$$

- Not all surfaces are so simple, so the E-field can change direction though the surface

- Must break up surface area into tiny pieces:

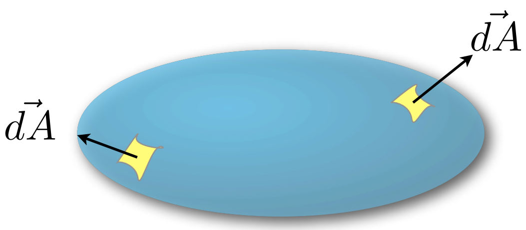


$$d\phi = \vec{E}\cdot\hat{n}dA = \vec{E}\cdot d\vec{A}$$

little unit of flux

always perpendicular

- So far only looked at open surfaces; can go through surface two ways. Now lets look at closed surfaces

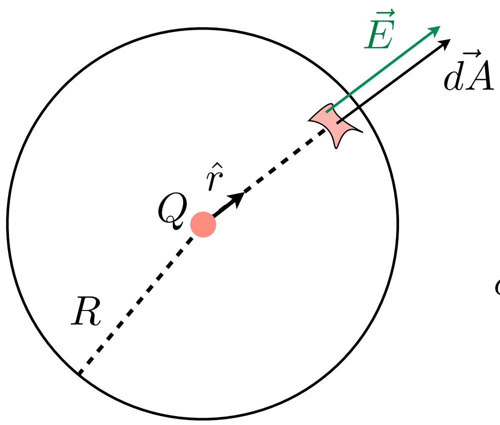


Area vector **always** points toward the outside

- Both E and dA can change, must do integral over all the closed surface:

$$\boxed{\phi = \oint_S \vec{E}\cdot d\vec{A}}$$

Ex. Point charge:



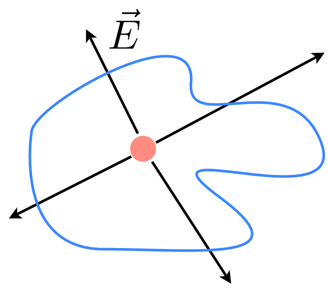
- Choose spherical surface
- dA is in radial direction
- since Q is point charge -> E-field is in radial direction as well
- |E| is the same everywhere on surface

$$\phi = \oint \vec{E} \cdot d\vec{A} = E \oint d\vec{A} = 4\pi R^2 E$$

since E is same everywhere on surface

- Since Q is point charge: $\vec{E} = \dfrac{Q}{4\pi\epsilon_0 R^2}\hat{r}$ $\Rightarrow$ $\boxed{\phi = \dfrac{Q}{\epsilon_0}}$

- ϕ does not depend on R
- All E-field lines must come out somewhere
- Think of E-field as water. Same amount of water comes out no matter what surface I pick.
- **Does not depend on surface shape!**



- Since we can use superposition, all E-fields add up vectorally, we can use this for any set of charges

- We have arrived at the first important law of electromagnetism: **Gauss' Law**

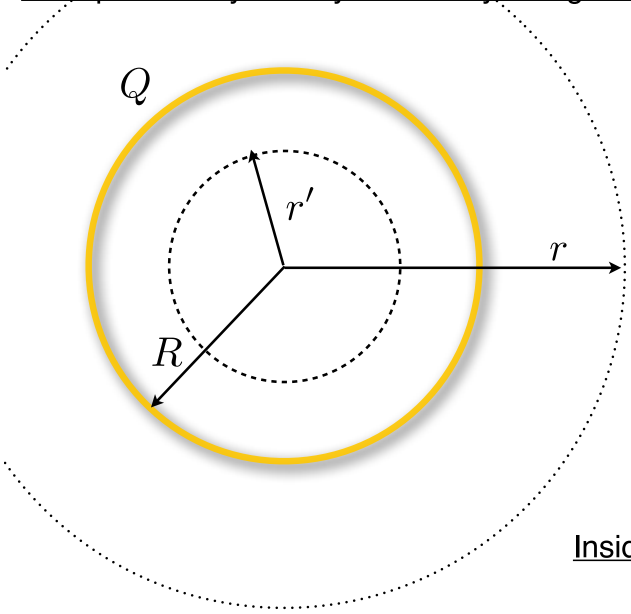
$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{\sum Q_{\text{enc}}}{\epsilon_0}}$$

- If $\phi = 0$ then no **net** charge inside surface.
- Gauss' Law always holds. But only useful if charges are arranged with some symmetry.
- **Gauss' Law is used to find E-field in cases with symmetry.**
- Three useful symmetries in this class: spherical, cylindrical, and planar.

Basic Idea of Symmetry:

If I can rotate or move the problem along a given direction (axis), and the problem does not change, then the E-field does not exist in that direction (along that axis).

Ex. Spherical symmetry: Uniformly charged shell



- Charge is uniformly distributed over shell
- Want to find E-field everywhere in space
- Must pick our Gauss surface for integral
    - Pick in smart way
    - Since problem is spherical -> spherical surface
- Now use symmetry:
    - E-field must be in radial direction
    - E-field is same everywhere on r' surface

Inside:

$$\boxed{\phi = 4\pi r'^2 E = \frac{Q_{\mathrm{enc}}}{\epsilon_0} = 0}$$

- E=0 everywhere inside shell!

- Coulomb's Law from all charges adds up to zero inside.

Outside:

$$\phi = 4\pi r^2 E = \frac{Q_{\mathrm{enc}}}{\epsilon_0} = \frac{Q}{\epsilon_0} \Longrightarrow \boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}}$$

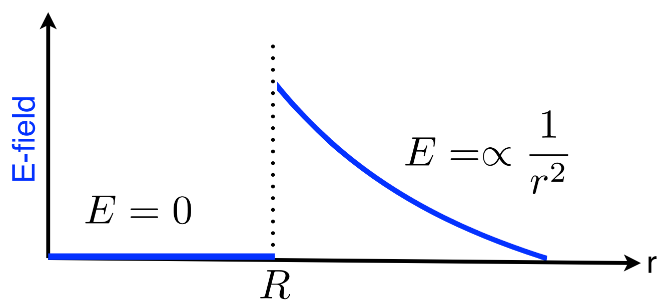
- This is the electric field from a point charge: we have proved the **shell theorem**.

- If charge is uniformly distributed on a shell, or concentrated at the center (point charge), it makes no difference to the E-field as long as you are outside.

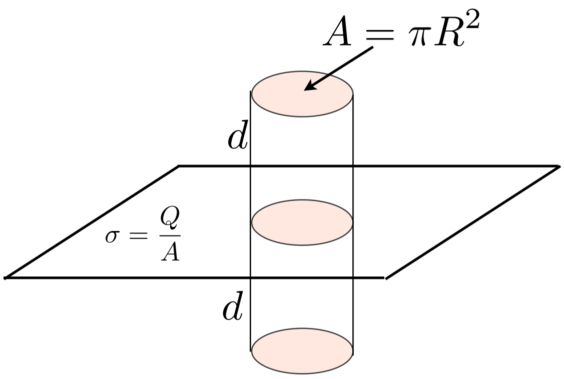
- Since E=F/q: $\vec{F} = \dfrac{q_1 Q}{4\pi\epsilon_0 r^2}\hat{r}$ Gauss' Law & Coulomb's Law are the same thing.

- Gauss' Law requires E-field goes as 1/r^2

E-field as function of radial distance



Ex. Planar Symmetry:



- Consider a plane of charge with charge density σ

- We will consider the plane to be infinite; it goes on forever in each direction.

- charge is again distributed uniformly

- What is the E-field everywhere in space?

- 1st, pick Gauss surface
    - Sphere is not good here
    - Choose cylinder for flat surface

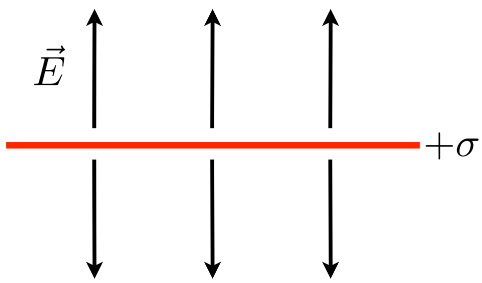
[note] Do not draw gauss surface until you get here

- Top & bottom same distance from plane
- Planar symmetry -> net E-field out of top & bottom only
- Calculate flux to find E-field:

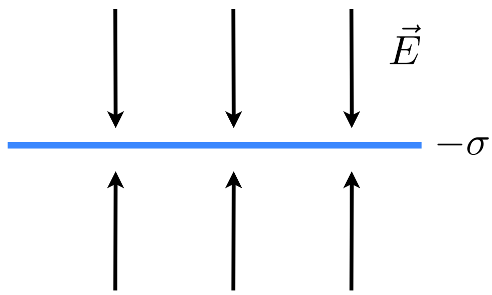
$$\phi = 2\pi R^2 E = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{\sigma \pi R^2}{\epsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z}}$$

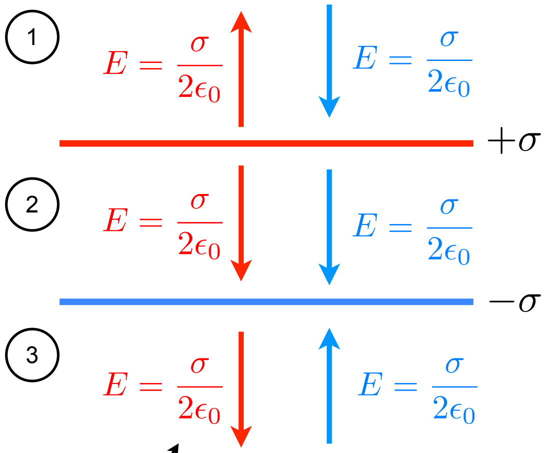
Does not depend on distance d!



$$|E| = \frac{\sigma}{2\epsilon_0}$$



Ex. Two Charged Planes:



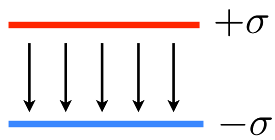
Remember, E does not depend on distance!

- What is the E-field everywhere?

- Use superposition; consider each plane individually, then add up.

1 $E = 0$ 3 $E = 0$

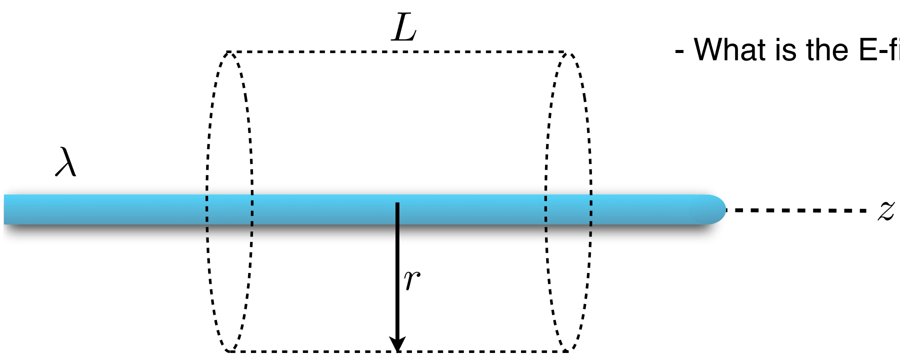
2 $E = \frac{\sigma}{\epsilon_0}$



Uniform E-field inside planes

- Planes assumed infinite. Uniform E-field obviously not true near edges

Ex. Cylindrical Symmetry:



- Consider an infinitely long rod with charge density λ

- What is the E-field everywhere outside?

- Choose gaussian surface:
    - Rod is cylindrical -> choose cylinder.
    - Now use symmetries
    - Can rotate about the z-axis and problem does not change -> E-field not in θ direction
    - Can move up and down z-axis and nothing changes -> E-field not in z-direction
    - E-field only in radial direction!

$$\phi = 2\pi r L E = \frac{Q_{\text{enc}}}{\epsilon_0} = \lambda L \quad \Rightarrow \quad \boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}}$$

Electric Fields & Gauss' Law Sample Problems