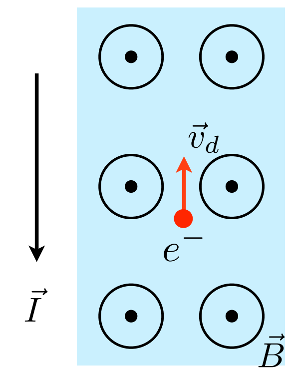
ex. Hall Effect

- We always think of the current I as being in the direction that (+)-charges move.

- But we also have mentioned several times that it is in fact the electrons that are the current carriers in conductors

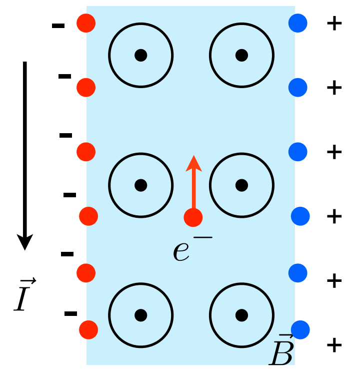
- Is there an experiment that can tell us that it is the electrons that move in a conductors. It can be seen in the **Hall Effect**



- Consider a conductor with a uniform B-field coming out of the page

- If the moving charges are positive, then using $\hat{F} = q\vec{v}_d \times \vec{B}$
    - Left side should have positive charge

But in experiment, negative charges build up on left side!



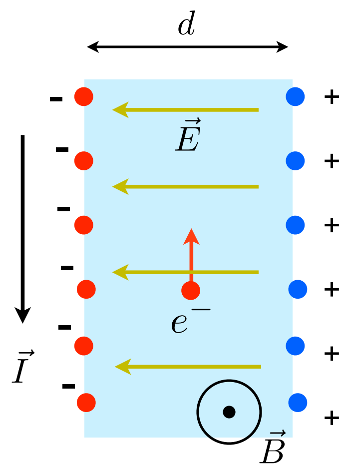
- Moving charges **must be** electrons moving in opposite direction.

- Since conductor, we have (+)-charges induced on right side

- Charges form two parallel plates of charge

- Charge builds up until repulsive force is larger than F_B

- We have seen this many times already:
    - Constant E-field: $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q}{\epsilon_0 A}$
    - Voltage across conductor simply $V = Ed$



- When $|F_E| = |F_B|$ force on electrons = 0

$$F_E = qE = F_B = qv_dB$$

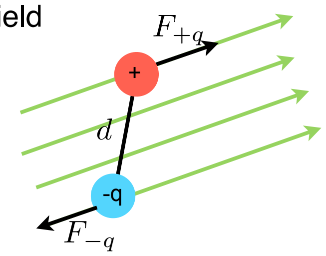
- Therefore: $E = v_dB$

- Drift velocity depends on the material $v_d \propto \tau$ mean-free path
    - Hall effect as tells you about the conductor material

- For $\vec{F}_E$ to cancel $\vec{F}_B$ you must have $\vec{E} \perp \vec{B}$

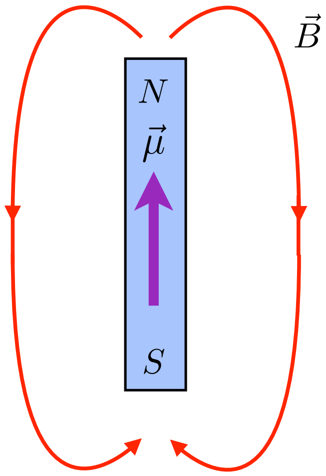
Magnetic dipoles:

- Recall that we had electric dipoles that rotate in an E-field



- Could define torque as $\tau_E = \vec{p} \times \vec{E}$

- Since all magnets are dipoles, we can define the magnetic dipole moment $\vec{\mu}$ that points from S- > N

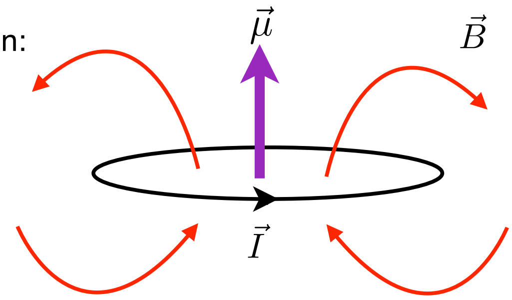


- If we can a loop of wire with current I and area A, then:

$$|\mu| = IA$$

- If we have N loops each with current I and area A:

$$|\mu| = NIA$$



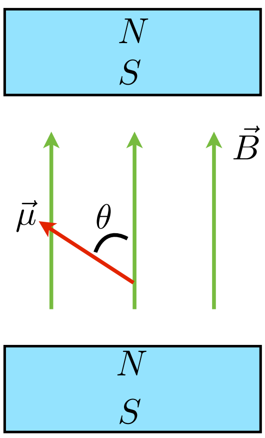
- The torque on a magnetic dipole is:

$$\tau_B = \vec{\mu} \times \vec{B} = |\mu||B| \sin\theta$$

- How much potential energy is stored in the dipole?

  - Recall that torque is angular equivalent of force

$$U = \int \vec{F} \cdot d\vec{r} \rightarrow \int \tau(\theta) d\theta$$



- For magnetic dipole:

$$U = \int \mu B \sin\theta d\theta = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$$

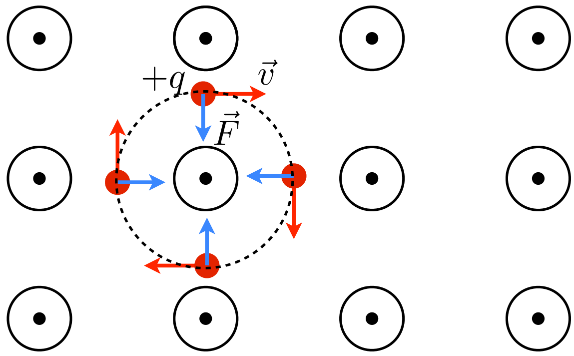
-> When mu and B is same direction, potential is minimized

-> When mu and B is opposite direction, potential is maximum

Recall that objects want to minimize their potential energy

Charged particle in constant B-field:

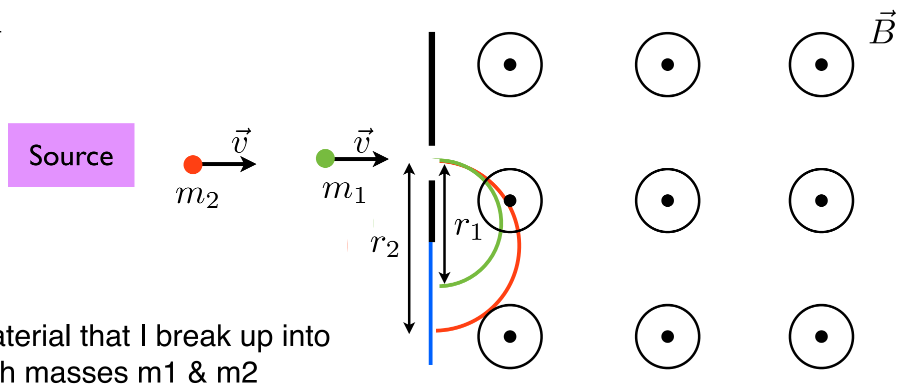
- Suppose I have a particle with charge +q, moving with velocity v
- At time t=0, I turn on a constant uniform B-field coming out of the page.



- What is the resulting motion?
  - Force is Lorentz force $\vec{F} = q\vec{v} \times \vec{B}$
  - Force is always perpendicular to the motion, gives rise to circular motion
  - The radius of the motion is proportional to the velocity and inversely to B & q

$$\frac{mv^2}{r} = qvB \rightarrow r = \frac{mv}{qB}$$

ex. Mass spectrometer:



- I have an unknown material that I break up into smaller components with masses m1 & m2

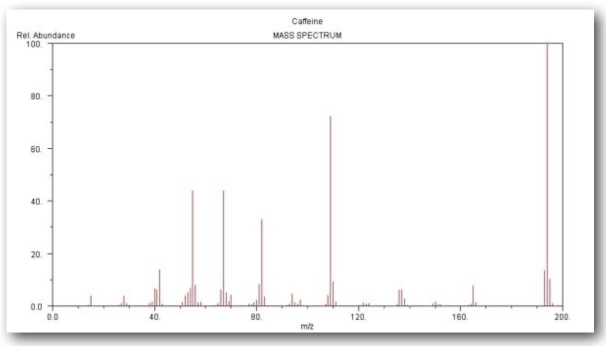
- Suppose that each piece is ionized with charge +1
- If each mass has velocity v & I know B then:

$$r_i = \frac{m_i v}{qB}$$

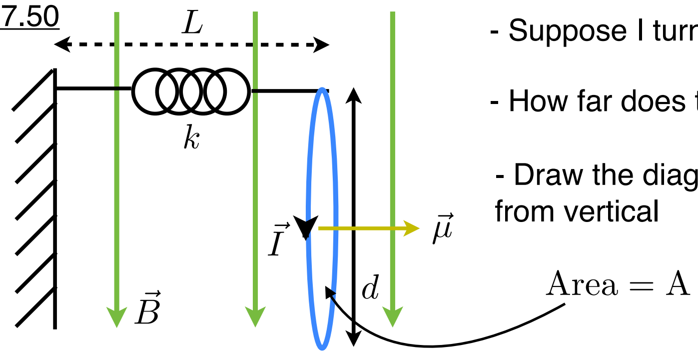
- If I don't know the charge either:

$$\frac{m_i}{q_i} = \frac{v r_i}{B}$$

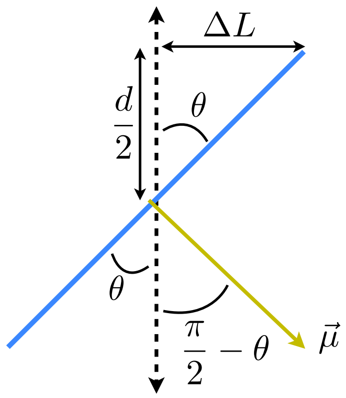
Mass spectrum for Caffeine



ex. 27.50



- Suppose I turn on the B-field at time t=0
- How far does the spring stretch?
- Draw the diagram when ring moves by angle θ from vertical



- What is force from spring?

$$|F_s| = k\Delta L \qquad \Delta L = \frac{d}{2}\sin\theta$$

- What is torque from spring?

$$\tau_s = \vec{r} \times \vec{F_s} = rF_s \sin\left(\frac{\pi}{2} - \theta\right) = \frac{d}{2}k\Delta L \cos\theta$$

- What is torque from B-field?

$$\tau_B = \vec{\mu} \times \vec{B} = \mu B \sin\left(\frac{\pi}{2} - \theta\right) = IAB\cos\theta$$

- Set torques equal to each other and solve for ΔL

$$\Delta L = \frac{2IAB}{dk}$$

- But we also know that $A = \pi\frac{d^2}{4}$

- Final answer: $\boxed{\Delta L = \frac{\pi d I B}{2k}}$