Energy in EM Waves:

- We have seen that plane waves of the form:

$$\vec{E} = E_0 \hat{x} \cos(kz - \omega t)$$
$$\vec{B} = B_0 \hat{y} \cos(kz - \omega t)$$

move in the z-direction at the speed of light $c = \omega/k$

- These waves also carry energy. Of course you have all experienced this standing outside on a sunny day.

- We first note that, over one period of the wave:

$$\int_0^{T=2\pi/\omega} \cos(kz - wt)dt = \int_0^{T=2\pi/\omega} \sin(kz - wt)dt = 0$$

- So on average the E-field and B-field are zero over one period.

- However, recall that the energy density for the E and B-fields are:

$$\tilde{U}_E = \frac{\epsilon_0}{2} E^2 \qquad \tilde{U}_B = \frac{1}{2\mu_0} B^2$$

- The energy densities are proportional to the square of the fields that are not zero!

$$\int_0^{T=2\pi/\omega} \cos^2(kz - wt)dt = \int_0^{T=2\pi/\omega} \sin^2(kz - wt)dt = \frac{\pi}{\omega}$$

- Furthermore, both E^2 and B^2 are also plane waves moving in the +z-direction

$$\vec{E}^2 = E_0^2 \hat{x} \cos^2(kz - \omega t)$$
$$\vec{B}^2 = B_0^2 \hat{y} \cos^2(kz - \omega t)$$

-> Energy is transported in the same direction as the wave travels.

- The rate, and direction, that energy is transported is defined using the **Poynting Vector**:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad [\mathrm{W/m^2}]$$

- Poynting vector gives the average power per unit area of energy

- Since $\vec{E} \perp \vec{B}$ for EM waves: $S = \frac{1}{\mu_0} EB = \frac{1}{c\mu_0} E^2$ since $B_0 = \frac{E_0}{c}$

- The average power per unit area is typically called the **Intensity**

$$I = S_{\text{avg}} = \frac{1}{\mu_0}\left[E_0^2 \cos^2(kz - \omega t)\right]_{\text{avg}}$$

- The average of $\cos^2(kz - \omega t)$ is 1/2. Therefore we can use $E_{\text{rms}} = \frac{1}{\sqrt{2}}E_0$

$$\boxed{I = \frac{1}{c\mu_0}E_{\text{rms}}^2}$$

- We can also ask how much energy is stored in the E-field vs the B-field?

$$\tilde{U}_E = \frac{\epsilon_0}{2}E^2 = \frac{\epsilon_0}{2}c^2B^2 = \frac{1}{2\mu_0}B^2 = \tilde{U}_B$$

- Therefore, the energy density of the E-field is the same as the density of the B-field.

Radiation Pressure:

- Because EM waves are radiated away from the source, the emitted waves are called **Radiation**.

- Since the waves carry energy, they also carry momentum from the relation: $E = pc$

- The change in momentum is thus related to the change in energy of the waves:

$$\Delta p = \frac{\Delta E}{c}$$

- If this change comes from the waves hitting a surface, then we can calculate the force

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta E}{c\Delta t} = \frac{IA\Delta t}{c\Delta t} = \frac{IA}{c}$$

- Since pressure is F/A we have:

$$P = \frac{I}{c}$$ Waves are absorbed

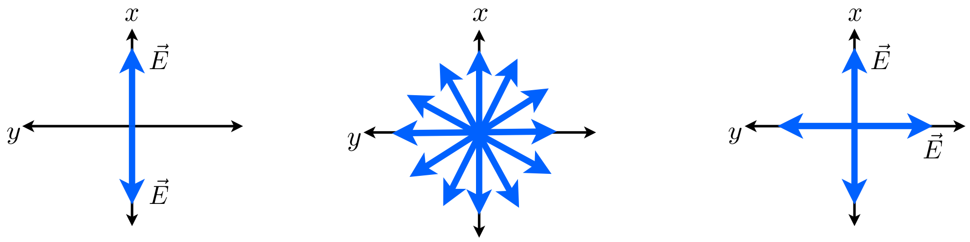
$$P = \frac{2I}{c}$$ Waves are reflected (2x momentum change)

Polarization:

- So far we have considered plane waves where the E-field is entirely in the x-direction and since $\vec{E} \perp \vec{B}$, the B-field is always in the y-direction

- If the fields always stay in the same orientation then the waves are called **Polarized**

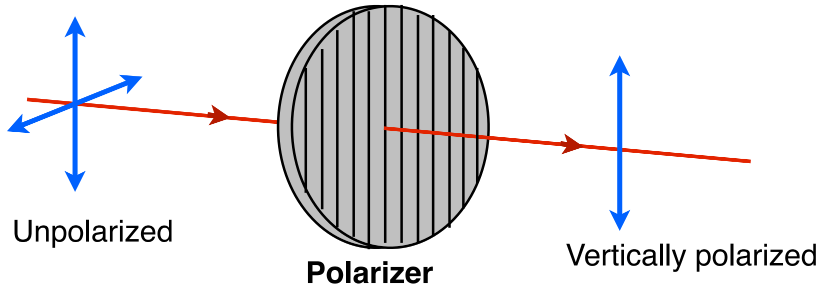
- If the each wave has a different field orientation, then the waves are **Unpolarized**



Polarized **Unpolarized** **Net unpolarized**

- The net polarization for unpolarized light has equal components in the x & y directions.

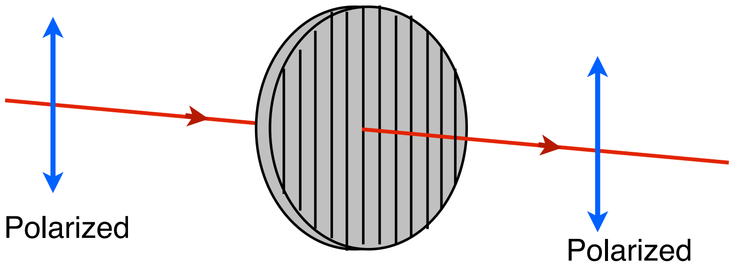
- We can always take unpolarized light and make it polarized by passing it through a polarizer



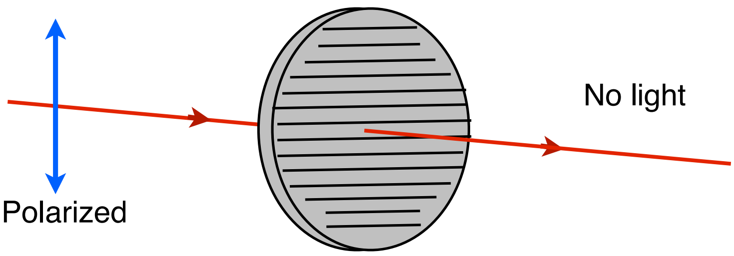
- After passing through the polarizer, the intensity of the transmitted light is half the original

$$I = \frac{1}{2} I_0$$

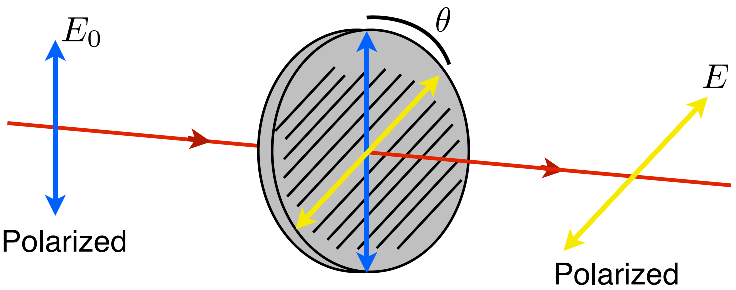
-If vertical polarized light goes through a vertical polarizer than 100% goes through



-If vertical polarized light goes through a horizontal polarizer than none goes through



-If light goes through polarizer with arbitrary angle



then magnitude of emitted E-field is: $E = E_0 \cos\theta$

- The intensity of the original light can be written in terms of the E-field as

$$I = \frac{1}{c\mu_0} E_{\rm rms}^2 = \frac{1}{2c\mu_0} E_0^2$$

- The intensity of the transmitted light can then be calculated as

$$\boxed{I = \frac{1}{c\mu_0} E_{\rm rms}^2 = \frac{1}{2c\mu_0} (E_0 \cos\theta)^2 = I_0 \cos^2\theta}$$