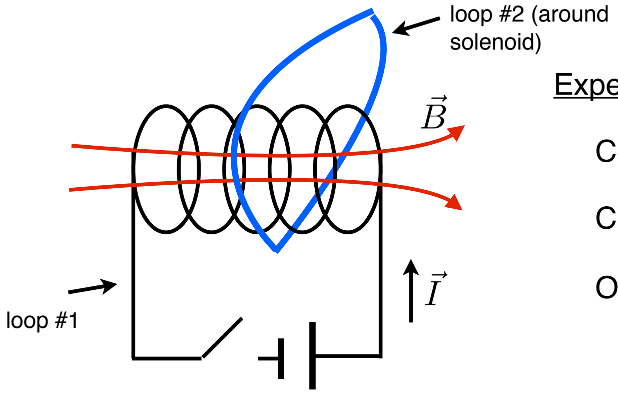
Induction

- Orstead (1821): Steady current produces a B-field

- Faraday (1831): Does a constant B-field generated a current?



<u>Experimental Results</u>

Constant current in #1 -> no current in #2

Close switch in #1 -> current in #2

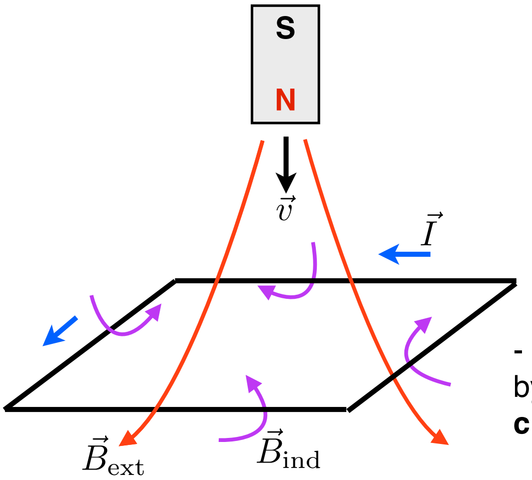
Open switch in #1 -> current in #2

- Changing current means changing B-field since: $B = (N/L)\mu_0 I$

- If current in #2, then force on electrons. So there is an E-field $\vec{F} = q\vec{E}$

From Faraday: Time-changing B-field produces a current in a wire around solenoid

Induction: E-field produced (induced) by time-changing B-fields through a loop of wire



- Bring bar magnet closer to loop -> number of B-field lines through loop is increasing

- Also induces E-field, and therefore current in loop

- Current tries to fight increase in B-field in loop $\vec{B}_{\text{ext}}$ by generating a B-field $\vec{B}_{\text{ind}}$ that **opposes the change** in B-field from the magnet.

- This effect is called **Lenz's Law**

- The <u>magnitude</u> of the external B-field <u>does not matter</u>, <u>only the rate of change</u>.

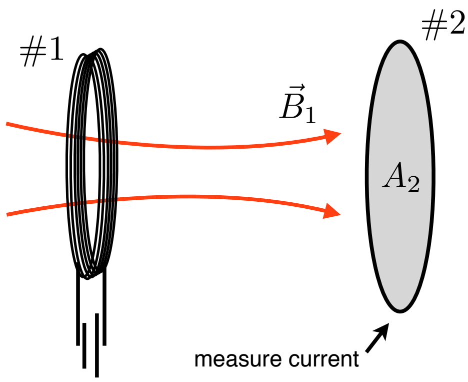
- Force is moving electrons around the wire -> work is being done

- This force comes from induced EMP that gives rise to current in loop

$$\mathcal{E}_{\text{ind}} = I_{\text{ind}} R_{\text{loop}}$$

resistance of wire loop (always there)

<u>Faraday's experiment:</u>



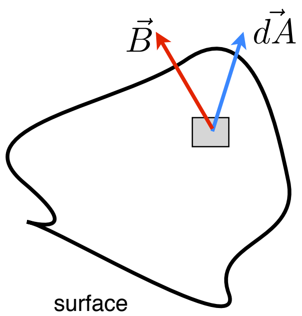
<u>Results:</u>

$$\mathcal{E}_2 \propto \frac{dB_1}{dt}$$

$$\mathcal{E}_2 \propto A_2$$

- EMF is proportional to the amount of changing B-field going through loop #2:

  -> EMF is from **change in flux** through the surface of loop #2

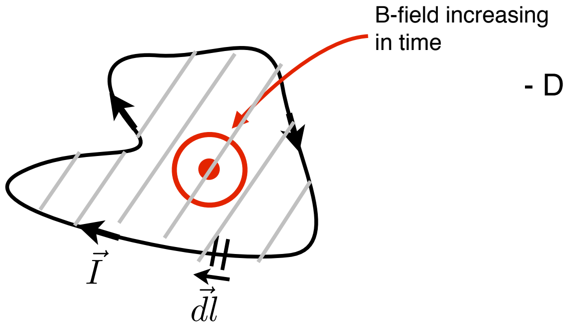


$$\phi_B = \int_{\text{open surface}} \vec{B} \cdot d\vec{A}$$ Magnetic Flux

Recall:

$$\phi = \int \vec{E} \cdot d\vec{A}$$ Electric Flux

- Suppose we have a conducting wire around a solenoid.
    - Since we are talking about flux, we need a surface



- dA and dl belong to the same surface. Always pick dl in same direction as current I.

- Direction of current is easy using Lenz's Law

because EMF opposes flux change

$$\mathcal{E}_{\text{ind}} = -\frac{d\phi_B}{dt}$$

$$= -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} \qquad *$$

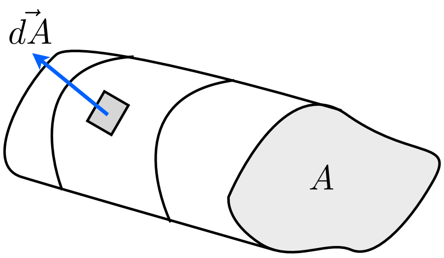
$$= \oint \vec{E}\cdot d\vec{l} \quad \text{by definition} \qquad *$$

- We get EMF and E-field without any battery! Only change in flux.

- Combining (*) we have arrived at **Faraday's Law:**

$$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}}$$

- We have to define an open-surface, so what is the direction of $d\vec{A}$?
    -Faraday does not tell you how to go around the loop.



<u>Use RHR:</u>

CW -> $d\vec{A}$ ⊗

CCW -> $d\vec{A}$ ⊙

<u>Steps for Faraday's Law:</u>

#1: Define loop

#2: Define direction around loop (always pick same direction as current from Lenz's Law)

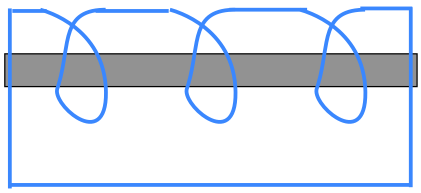
#3: Attach open-surface to loop.

-> can now calculate $\vec{B}\cdot d\vec{A}$ everywhere.

-If we know change in flux -> can calculate induced EMF

-Answer does not depend on loop area if all B-field is still enclosed: (i.e solenoid $B_{\text{ext}} = 0$)

- What happens if we wrap a wire around a solenoid 3 times?



- Must attach open surface to this loop
- B-field goes through surface three separate times.

  - EMF is three times larger since B-field "sees" 3x the loop area

- 1000 loops -> 1000x times more EMF

- Can get any EMF you want by adding more loops
- This is how a transformer changes voltages (i.e. 110 -> 220 V)

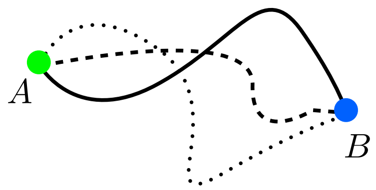
- Faraday's law for loops of wire is not very intuitive.
- Other circuits were easy to think about using Kirchoff's laws.

$$\sum_i V_i = 0 \quad \longleftrightarrow \quad \oint \vec{E} \cdot d\vec{l} = 0$$

NOT TRUE IF MAGNETIC FLUX IS CHANGING

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} \quad \text{(Faraday)}$$

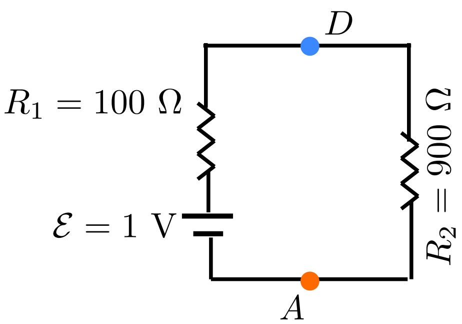
- Changing B-flux means that E-field inside the conducting wire is <u>non-conservative</u>
- Kirchoff's Laws are only valid for conservative E-fields.
- If E-field is conservative:



$$\int_A^B \vec{E} \cdot d\vec{l}$$ Is independent of the path

Non-conservative -> Path is important

<u>ex:</u>



$$\mathcal{E} = I\left(R_1 + R_2\right) \rightarrow I = 10^{-3} \text{ A}$$

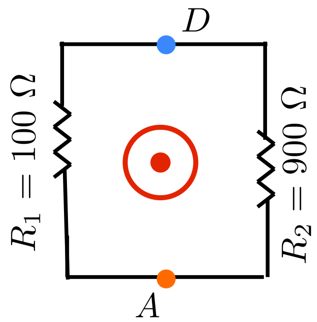
Can go from D->A in two ways:

$$V_D - V_A = IR_2 = +0.9 \text{ V}$$

$$V_D - V_A = \mathcal{E} - IR_1 = +0.9 \text{ V}$$

-Voltage drop is obviously the same (independent of path)

- Now replace the battery with a solenoid:



- Let $\mathcal{E}_{\text{ind}} = 1\ \text{V}$, same as battery.

- Find voltage drop along two directions:

$$V_D - V_A = IR_2 = +0.9\ \text{V} \qquad \text{(Same)}$$

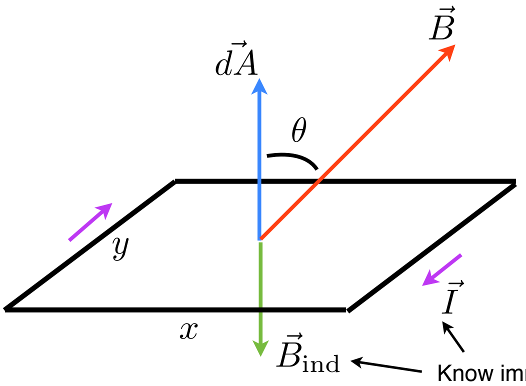
$$V_D - V_A = \cancel{\mathcal{E}} - IR_1 = -0.1\ \text{V}$$

Voltage drop depends on the path!

- Faraday's Law ALWAYS holds.
    - Kirchoff's voltage law is special case when: $\frac{d}{dt}\int \vec{B}\cdot d\vec{A} = 0$

$$V_D - V_A = IR_2 = +0.9\ \text{V}$$
$$V_A - V_D = IR_1 = +0.1\ \text{V}$$
$$V_D - V_D = IR_1 = +1.0\ \text{V} = \mathcal{E}_{\text{ind}}$$

We see that: $\oint \vec{E}\cdot d\vec{l} \neq 0$



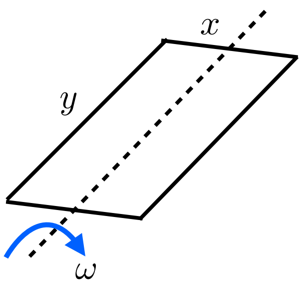
Know immediately from Lenz's Law

- Suppose ϕ_B is increasing

$$\phi_B = \int \vec{B}\cdot d\vec{A} = xyB\cos\theta$$

- Can change $|B|$
- Can change loop area A
- Can change θ

<u>Change θ :</u>



$$\omega = 2\pi/T$$

$$\theta = \theta_0 + \omega t$$

- Suppose $t = 0 \rightarrow \theta_0 = 0$

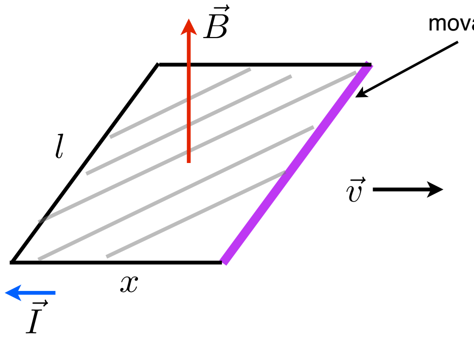
$$\phi_B = xyB\cos(\omega t)$$

double frequency, double EMF

$$\mathcal{E}_{\text{ind}}(t) = -\frac{d\phi_B}{dt} = xyB\omega\sin(\omega t)$$

$$I(t) = \mathcal{E}_{\text{ind}}/R_{\text{loop}}$$ "AC-Current"

Change area A :

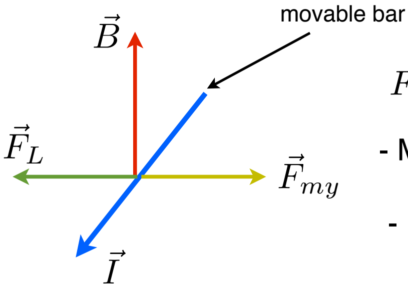


$$\phi_B = xlB$$

- Don't care about flux, just change in flux

$$\frac{d\phi_B}{dt} = lBv = |\mathcal{E}|$$

can always get direction from Lenz's law



$$F_L = ILB$$

- Must do work to overcome Lorentz force

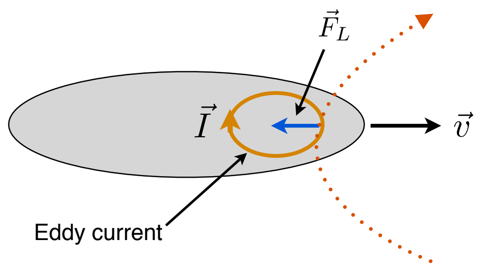
- Push/Pull doesn't matter, work is always positive

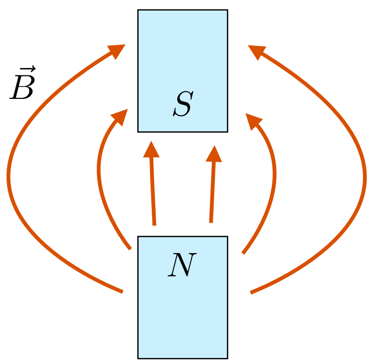
- What is power generated by my force?

$$P = \vec{F}_{my} \cdot \vec{v} = F_{my} v$$ always in same direction

$$P = IlBv = \mathcal{E}I \rightarrow \mathcal{E} = lBv$$ Same answer as Faraday via work as starting point

- Suppose I have a solid disc:





- As disc goes into B-field, area changes causing flux change

- Currents are generated in disc to oppose the change in B-flux: **Eddy Currents**

- Eddy currents always generate force that opposes the motion -> friction

- Energy comes from the objects kinetic energy and is dissipated as heat

- This process is called **Magnetic Braking**