PHYS-183 : Day #15

Viscosity of Simple Fluids:

- So far we have restricted our discussion to ideal fluids that do not have any frictional properties.

- However, real fluids are not ideal, they have friction, and do not conserve mechanical energy

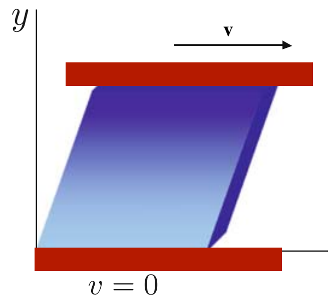
- The molecules in real fluids have attractive internal forces such that any relative motion of the molecules results in frictional forces.

- This is called **viscosity** and we will look at this effect for simple fluids.

- We can this of viscosity as the resistance of a fluid to flowing

Honey is a viscous fluid.

- To define the viscosity, think of a fluid with laminar flow moving between two plates



- The bottom plate is at rest, top plate has an external force F causing it to move at a constant velocity because of friction forces

- Because the fluid is viscous, it will stick to the to the surface of the plates forming a boundary layer

→ Velocity of fluid at bottom is zero
Velocity of fluid at top is *v*

- The viscosity of the fluid can be defined through the relationship between force / area and the change in the velocity as a function of height *y*

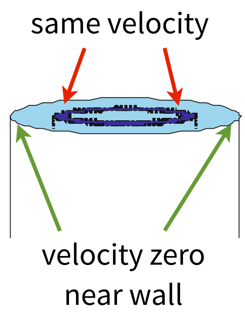
$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y} \qquad \eta = \text{viscosity of the fluid}$$

- Fluids that obey this relationship are called **Newtonian fluids**, otherwise called **non-Newtonian**.

Fluid	*Temperature*	*Viscosity (10^{-3} Pa-s)*
Water	0	1.8
	20	1.0
	37	0.7
Whole blood[a]	37	4.0
Blood plasma	37	1.5

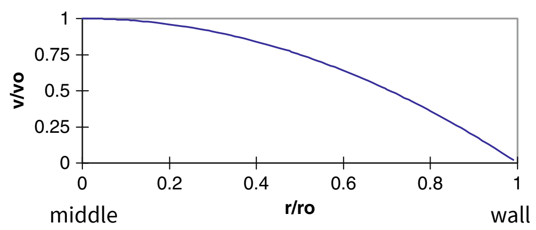
<u>Capillary Flow:</u>

- The capillary tube is a very important geometry for fluid flow in biology.



- When a liquid flows through a tube with no objects to block it, the flow at low velocities is laminar, and the liquid flows in layers of concentric cylinders

- The layer on the outside is at rest, and the fastest flow is in the middle of the tube



- The velocity across the tube is parabolic as a function of radius

- In order to calculate the flow rate, *Q=Av* , we must that the average across the cross-sectional area *A*

- For a tube, this average leads to the formula:

$$Q = \frac{\pi P r^4}{8\eta L} \qquad \begin{aligned} P/L &= \text{Pressure per length} \\ r &= \text{Radius of tube} \end{aligned}$$

- We can rewrite this equation to calculate the pressure difference ΔP across a tube length *L*

$$\Delta P = \left(\frac{8\eta L}{\pi r^4}\right) Q$$

Resistive to flow

- For a fixed ΔP the flow depends on the resistive term in the parenthesis

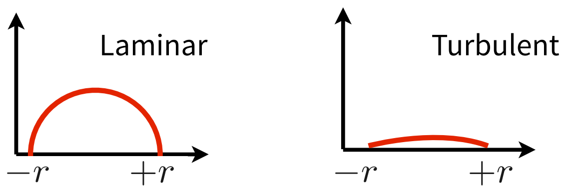
- The resistance is extremely sensitive to the radius of the tube

- The resistance increases rapidly as the radius of the tube gets smaller.

- For example, if your artery is partially clogged, then your blood pressure must be really high in order to keep the flow rate (rate of oxygen) the same.

- So far we have talked about the flow of viscous fluids at low velocities.

- If the velocity gets too large, then the fluid becomes turbulent and the velocity of the fluid <u>is not</u> a parabolic function of the tube radius.



- Turbulent fluids have friction forces that are <u>much bigger</u> than laminar flow.

<u>Fluids with Suspended Objects:</u>

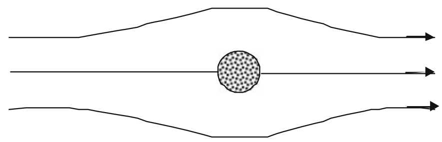
- What happens when a viscous flowing fluid encounters an object such as a biological macromolecule (protein, blood cell,...)

- The response of the fluid depends on the dimensionless Reynolds number:

$$\mathcal{R} = \frac{L\rho v}{\eta}$$

- Suppose the object is spherical with radius *r*

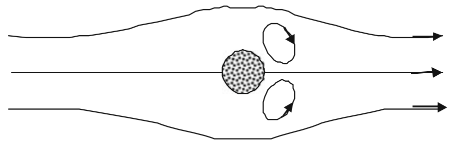
- If the Reynolds number of the object is on the order of 1 or less, then the fluid flow is laminar



Flow is laminar if Reynolds number is <=1

- In this case, the frictional force is proportional to the velocity of the fluid, viscosity, and size of the object.

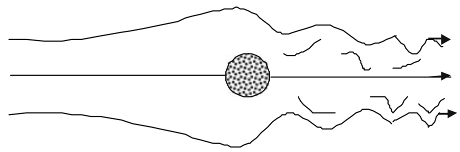
$$F_f = -6\pi\eta r v$$



Flow becomes unsteady with **vortices**.

- As the fluid velocity increases, the flow becomes turbulent and the friction force is proportional to the square of the fluid velocity.

- Careful design of an object can reduce the frictional force by a large amount (airplanes, cars, boats,...)



Flow in completely turbulent

- What is the viscosity of a fluid with a collection of suspended objects?

- In general this is an extremely hard problem. The turbulent flow from one object can affect another object, which can affect another object,...

- For a fluid with a collection of spherical objects in it, the viscosity was found by Einstein to be

$$\eta_s = \eta_0(1 + 2.5\Phi)$$

viscosity of bare fluid   fraction of total volume used by spheres

- The radius of the spheres is taken into account in the fractional volume Φ

<u>Blood and Other Complex Fluids:</u>

- Blood is an example of a fluid with many different kinds of suspended macromolecules.

- Blood is also an example of a non-Newtonian fluid.

- Blood has two primary components:

(I) Plasma: A newtonian liquid that is 92% water, 7%proteins, and other stuff.

(II) Red blood cells:

- Make up 50% of the volume of blood.

- Contain Hemoglobin (Iron-molecule) that carries oxygen through out your body

- The viscosity of blood depends on Φ , the fraction of volume taken up by the red blood cells.

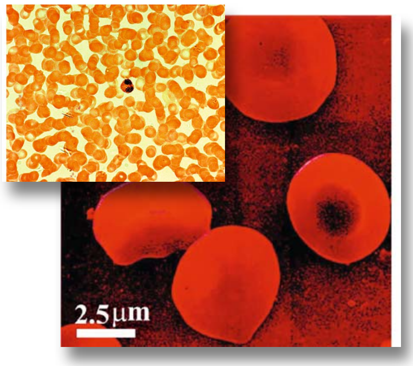
- Red blood cells are disk shaped, and thiner in the middle than the outside

- Red blood cells tend to stick together and form stacks.

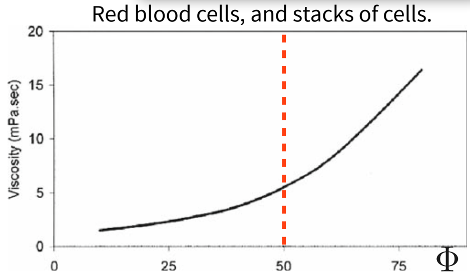
- If your blood cells were hard spheres, then your blood wouldn't flow.

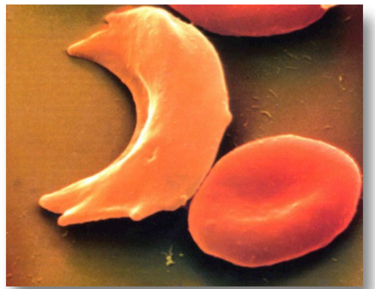
- Your blood flows because your blood cells have a special shape, and they can stretch and bend.

- Some diseases like Sickle cell, make blood cells hard and therefore blood does not flow well.



Red blood cells, and stacks of cells.



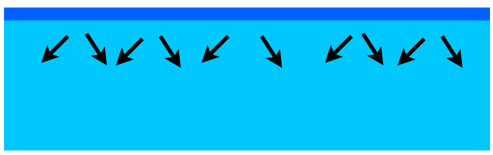


Sickle cell

Surface Tension:

- The surface of a fluid is a boundary that has many special properties.

- At the surface, there are attractive forces on only one side of the boundary



- This net force results in a slightly higher density at the surface of the liquid.

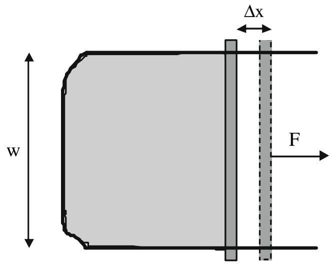
- Molecules that move into the surface layer have a higher energy, then those in the middle of the liquid, and therefore work must have been done on them

- The extra energy at the surface is called the surface energy per area γ.

- This value depends on the two fluids at the boundary

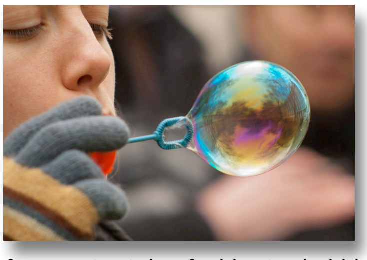
- For the boundary between air and water: $\gamma = 0.073\ \mathrm{J/m^2}$ (a large value)

- Suppose a layer of liquid in air is stretched across a device with a changeable surface area



- If we increase the surface area $(w\Delta x)$ using a force F, then then the work done must be equal to the change in the surface energy

$$F\Delta x = 2\gamma(w\Delta x)$$ (times 2 for top and bottom)



Surface tension is key for blowing bubbles

- Solving for the surface energy density: $\gamma = F/2w$

  - This is a force per length called the **Surface Tension**

- For some small insects, this surface tension is larger than the gravitational force and the insects can walk on the water

- Because a surface requires extra energy, if no external forces, fluids try to minimize their surface area and take a spherical shape

Water strider on the surface of a lake

Water with no external forces