Coulombs Law:

- We know everything there is to know about individual charges
- How do charges interact?
- Saw that charges exert forces on each other:

  Opposite charges attract
  Like charges repel

- Force between non-moving charges is called **Electrostatic Force.**
- Determined experimentally (like most of EM) by Coulomb (1785)

Coulomb's Law: Given two particles with charges q_1 and q_2, the resulting force is:

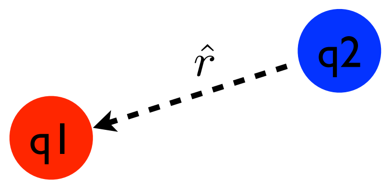
$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$

in radial direction

"electrostatic constant"

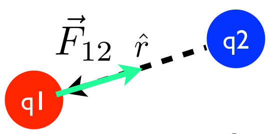
How to determine "radial direction"

Force of q2 <u>on</u> q1: $\vec{F}_{12}$



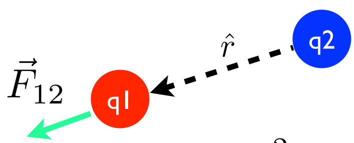
- Check for consistency:

$$q_1 = e \;\; ; \;\; q_2 = -e$$



$$\vec{F}_{12} = \frac{-ke^2}{r^2}\hat{r}$$

$$q_1 = e \;\; ; \;\; q_2 = e$$



$$\vec{F}_{12} = \frac{ke^2}{r^2}\hat{r}$$

<u>Conceptual Question:</u>

Suppose we are given the following formula for the electrostatic force:

$$\vec{F} = \tilde{k}\frac{q_1 q_2^3}{r^2}\hat{r}$$

Why is this not a valid force law? *Note: give a few minutes to discuss*

Ans: Force depends on which charge is labelled q2. Physics should not depend on how we label the charges

ex. q1=e, q2=2e

$$F = \tilde{k}\frac{8e^4}{r^2}$$

now q1=2e, q2=e

$$F = \tilde{k}\frac{e^4}{r^2}$$

Connection between EM and Gravity:

$$\vec{F}_{\mathrm{EM}} = k\frac{q_1 q_2}{r^2}\hat{r} \qquad \text{Coulomb's Law}$$

$$\vec{F}_{\mathrm{G}} = G\frac{m_1 m_2}{r^2}\hat{r} \qquad \text{Newton's Law}$$

-Both equations have same form:

- 1/r^2 spatial dependence
- interactions correspond to products of particle properties; charges for Coulomb, and masses for Newton's law.
- Both laws has a fundamental constant

- Amazing! Why should Gravity and EM follow same force law?
    - Hints at deeper connection
    - This connection used by Einstein in formulating relativity

Electrostatic constant

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

What is k?: $$k = \frac{1}{4\pi\epsilon_0} \qquad \epsilon_0 = 8.85 \times 10^{-12}\mathrm{C}^2/\mathrm{N}-\mathrm{m}^2$$

Permittivity of free space

$$k = 9 \times 10^9 \mathrm{C}^2/\mathrm{N}-\mathrm{m}^2$$

ϵ_0 is a material constant of empty space (like density of a solid or conductivity of a metal)

<u>- Ex. force of gravity vs. EM force:</u>

- Let us compare the force of gravity between an electron and a proton to the EM force.
- What is your guess? Which is stronger?

$$m_p = 1.67 \times 10^{-27}\mathrm{Kg}$$
$$m_e = 9.11 \times 10^{-31}\mathrm{Kg}$$
$$G = 6.67 \times 10^{-11}\mathrm{m}^3/\mathrm{Kg}-\mathrm{s}^2$$

$$\frac{|F_{EM}|}{|F_G|} = \frac{k}{G}\frac{q_1 q_2}{m_1 m_2} = \frac{k}{G}\frac{e^2}{m_p m_e}$$

$$\frac{|F_{EM}|}{|F_G|} \approx \frac{9 \cdot 10^9 * 2 \cdot 10^{-38}}{7 \cdot 10^{-11} \times 2 \cdot 10^{-27} \times 9 \cdot 10^{-31}} = \frac{1}{7}\frac{10^{-29}}{10^{-68}} \approx \boxed{10^{39}}$$

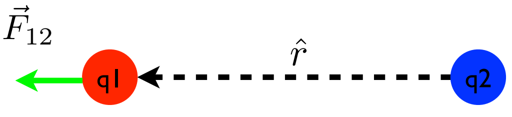
- EM force is huge compared to gravity!

- As mentioned, objects nearly neutral, and mass of Earth is huge -> not noticed so much.

Electrostatic Example Problems:

<u>Ex. force of between two charges</u>

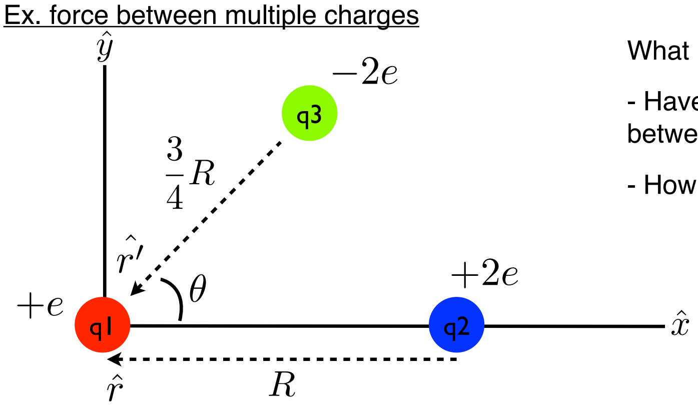
What is force on q1 from q2 when q1=e and q2=2e and separated by a distance of 4m?



$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r} = k\frac{2e^2}{4}\hat{r} = \frac{ke^2}{2}\hat{r}$$

in $\hat{r}$ direction since both charges same sign

<u>Ex. force between multiple charges</u>



What is the **net** force on q1?

- Have only considered force between two charges

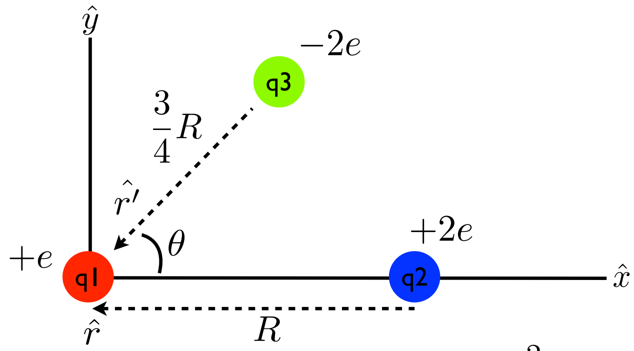
- How do we handle multiple particles?

- Because we know that $\vec{F}_{EM}$ has the same form as $\vec{F}_G$ we can borrow the concept of **superposition**:

The <u>**net**</u> force from a collection of charges on charge q1 is given by:

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^{N} F_{1i}$$

Superposition principle is not obvious. But it is confirmed by experiments.



$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{13}$$

- Find F12 (easy part first): $\vec{F}_{12} = k\frac{2e^2}{R^2}\hat{r} = -k\frac{2e^2}{R^2}\hat{x}$

- F13: $\vec{F}_{13} = -k\frac{2e^2}{\left(\frac{3}{4}R\right)^2}\hat{r}'$

r' is not along x or y directions; must decompose into x and y pieces

$$\vec{F}_{13} = k\frac{16(2e^2)}{9R^2}\cos\theta\hat{x} + k\frac{16(2e^2)}{9R^2}\sin\theta\hat{y}$$

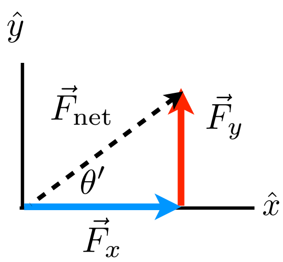
$$\boxed{\vec{F}_{1,\mathrm{net}} = k\frac{(2e^2)}{R^2}\left[\frac{16}{9}\cos\theta - 1\right]\hat{x} + k\frac{16(2e^2)}{9R^2}\sin\theta\hat{y}}$$

- We should check that this is right by <u>looking at limiting cases</u>:

$$\theta = 0 \Rightarrow \vec{F}_{1,\mathrm{net}} = k\frac{(2e^2)}{R^2}\left[\frac{16}{9} - 1\right]\hat{x}$$ Force is in X-direction OK

$$\theta = \pi/2 \Rightarrow \vec{F}_{1,\mathrm{net}} = -k\frac{(2e^2)}{R^2}\hat{x} + k\frac{16(2e^2)}{9R^2}\hat{y}$$

-What is the angle θ' of the resulting force?



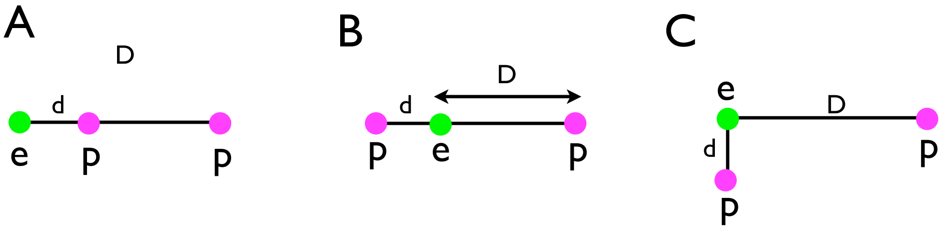
$$\tan\theta' = \frac{F_y}{F_x} \Rightarrow \theta' = \arctan\frac{F_y}{F_x}$$

$$\frac{F_y}{F_x} = \frac{16\sin\theta}{9\,(16/9\cos\theta - 1)}$$

$$\frac{F_y}{F_x} = \frac{\sin\theta}{\cos\theta - \frac{9}{16}}$$

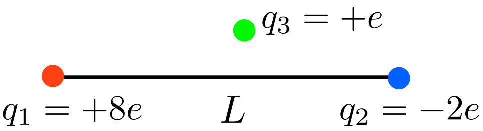
Conceptual Question:

Rank (high to low) the following situations in terms of the magnitude of the net force on the electron (e) from the protons (p).



ANS: A,C,B A & B are limiting cases of C! (or can us Pythagorean Theorem)

Ex. Equilibrium position of charges



- Where should I put q3 so it is in equilibrium?
- Is the equilibrium stable?

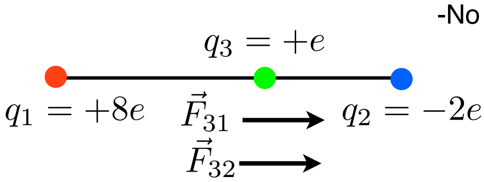
Recall: **Equilibrium** means the net force is zero

$$\Rightarrow \quad \vec{F}_{3,\mathrm{net}} = 0 \quad \Rightarrow \quad \vec{F}_{31} = -\vec{F}_{32}$$

Three situations:

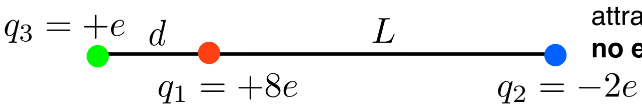
#1: Put q3 in between q1 & q2:

-No need to do any calculations, just think about it.



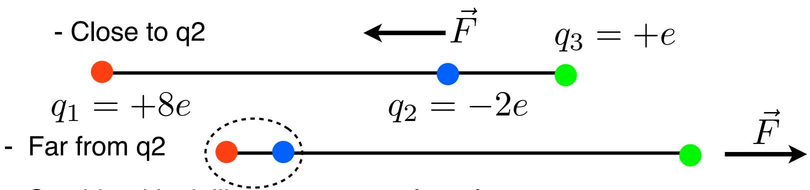
- Both forces in same direction; **no equilibrium**

#2: Put q3 to the **left** of q1 by distance d:



- Repulsive force from q1 always beats out attractive force from q2 since F~1/r^2; **no equilibrium**.

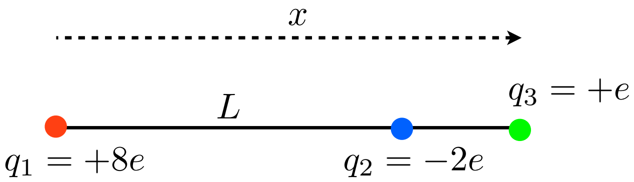
#3: Put q3 to the **right** of q2:



Combined look like +8e-2e=+6e from far away

Forces now in opposite directions; can **have equilibrium**

Calculate location of equilibrium point:



- Know forces are in opposite directions so only need magnitudes.

$$|F_{31}| = k\frac{8e^2}{x^2} = k\frac{2e^2}{(x-L)^2} = |F_{32}|$$

$$\frac{4}{x^2} = \frac{1}{(x-L)^2} \quad \Rightarrow \quad 2 = \frac{x}{x-L}$$

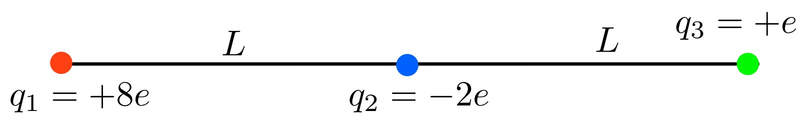
$$\boxed{x = 2L}$$

Is equilibrium point stable?:

- Recall that a equilibrium point is **stable** if any small displacement from equilibrium results in a force that is in the direction opposite that of the displacement

Two situations: move q3 slightly to the left or right.

Note: ask the class; give a few minutes to discuss

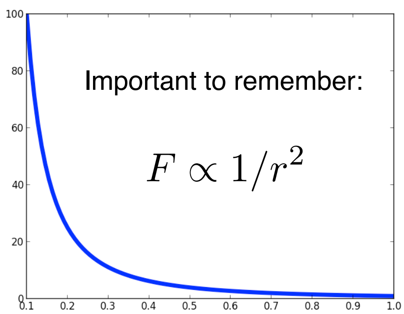


Case #1: deviation to the left:

- Both $\vec{F}_{32}$ and $\vec{F}_{31}$ increase, but

$\vec{F}_{32}$ grows faster than than $\vec{F}_{31}$ since q2 is closer to q3

net force directed **AWAY** from equilibrium



Case #2: deviation to the right:

- Both $\vec{F}_{32}$ and $\vec{F}_{31}$ decrease, but

$\vec{F}_{32}$ drops faster than $\vec{F}_{31}$;q3 pushed to the right net force directed **AWAY** from equilibrium