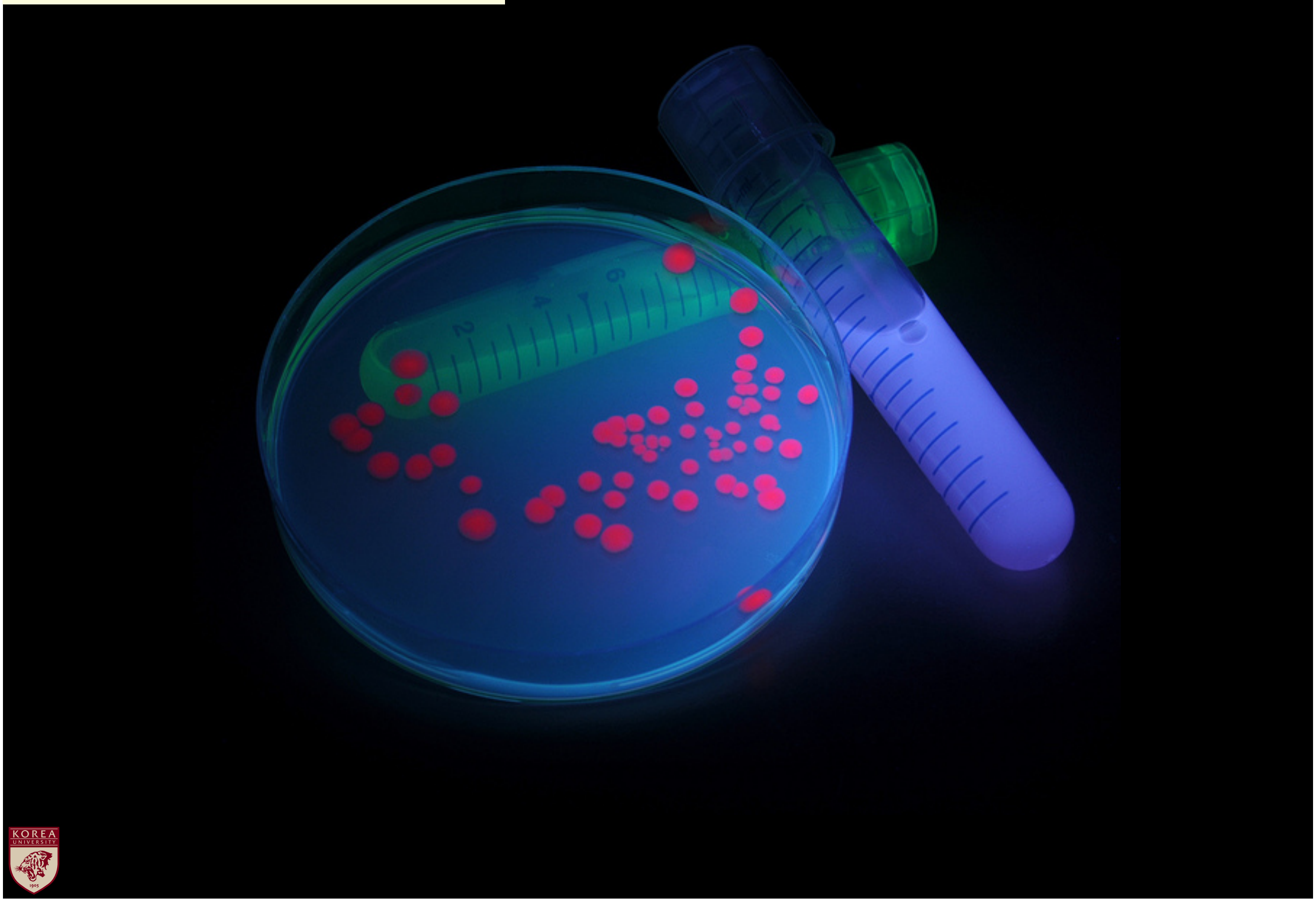
PHYS-183 : Day #27



Announcements

1) No class on Thursday (Holiday)

2) Next week, quiz on <u>both</u> Tuesday and Thursday

3) Final exam will be Wednesday June 19th from 19:00-21:00 in Room 633

- This week, we will look at electromagnetic (EM) waves generated from moving charges.

- Like all waves, EM waves transmit energy from one location to another.

- EM waves are perhaps the most important waves of all


Light/photosynthesis


Communications and electronic devices


Medical imaging and diagosis

- As their name suggests, EM waves are made from a combination of electric and magnetic fields.

- To understand EM waves, we must first take a brief look at magnetic fields and how they are created

Basics of the Magnetic Field:

- The magnetic field (B-field), like the E-field, is a vector field that has a magnitude and direction as every point in space.

$\odot$ Out of the board

$\otimes$ Into the board

- For the moment, lets assume that the magnetic field *B* is uniform in space.

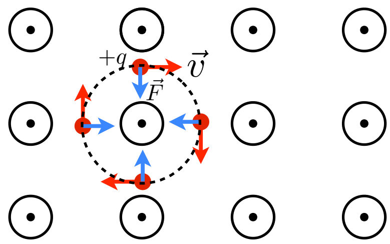
- We are interested in seeing what the force is on a charge *q* from the B-field.

- Experimentally we find that, if the charge is stationary with v=0, then there is no net force on it.

- Furthermore, if the charge is moving with a velocity that is directed along the magnetic field, then the particle also has no net force.

- However, if the charge is moving perpendicular to the magnetic field, then there is a net force on the charge with the magnitude

$$F_M = qvB$$



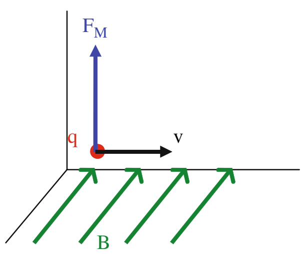
Motion of a charged particle in a constant B-field

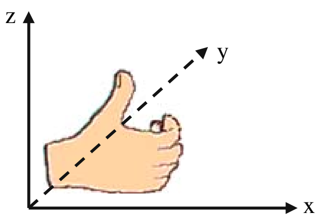
- The direction of the force is found to be perpendicular to both the velocity vector and B-field vector.

- Using vector mathematics, the direction and amplitude of the force is given by

$$\boxed{\vec{F}_M = q\vec{v} \times \vec{B} = qvB\sin\theta}$$

- The direction can be found using the so-called "right-hand rule"





1) Stick your top two fingers in the direction of the velocity

2) Stick your bottom two fingers in the direction of the B-field

3) The direction of your thumb is the same as the direction of the force

- As we have seen, in a uniform magnetic field, the motion of the charged particle is a circle.

- Like any circular motion, the centripetal acceleration is related to the force on the particle.

$$F_M = qvB = ma = \frac{mv^2}{r} \quad \Rightarrow \quad \frac{q}{m} = \frac{v}{rB}$$

- The magnetic field is measured in units of Tesla [T]

- Since the force is never in the same direction as the velocity, we have the very important result:

The magnetic field does no work on an object.

- The magnetic force and the displacement are never pointed in the same direction.

- Now that we know the basics of the B-field, our next task is to see how a B-field is generated.

- To generate an E-field, all we required was an individual particle with a non-zero charge.

- However, despite a lot of effort by physicist, we have never seen a single particle that has a "magnetic charge"

- The fact that these B-field charges do not exist tell us something important about the magnetic field.

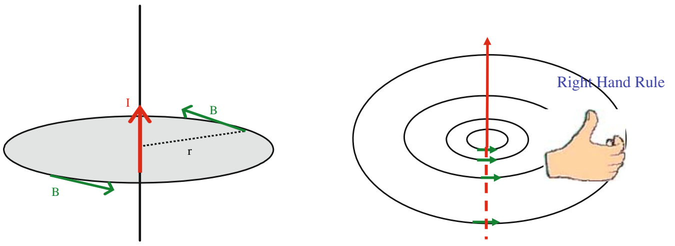
The magnetic field is not produced by charges. Instead, the magnetic field is produced by <u>moving</u> electrical charges.

- To see this, we can rewrite the magnetic force equation in terms of the electric current *I* flowing through a wire:

$$\boxed{\vec{F}_M = q\vec{v}\times\vec{B} = \frac{q}{t}\vec{l}\times\vec{B} = I\vec{l}\times\vec{B}}$$

- We see that the force on the wire is now perpendicular to the direction of the current and the B-field.

- For a long straight wire, the direction of the magnetic field is tangential to the direction of the current, making circles around the wire, with direction given by the right-hand rule.



1) Point your thumb in the direction of the current

2) The direction your fingers curl is the direction of the B-field

- In this case, the magnitude of the B-field as a function of the distance *r* away from the wire is

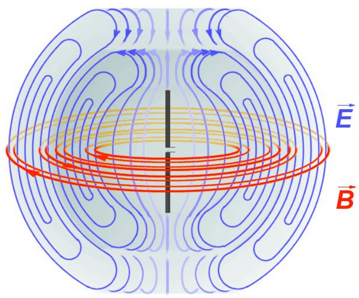
$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \qquad \mu_0 = 4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}$$

<u>Electromagnetic Waves:</u>

- Electromagnetic waves are made whenever charges accelerate.

- In particular, charges that oscillate back and forth as a function of time, or charges that undergo circular motion will emit some kind of EM waves (radiation).

- If the charges oscillate, then the direction of the E & B fields must also oscillate.



E & B fields generated by an antenna used for radio communication

- Recall that the equation for a wave moving in the +x direction with constant amplitude in the y & z directions is given by

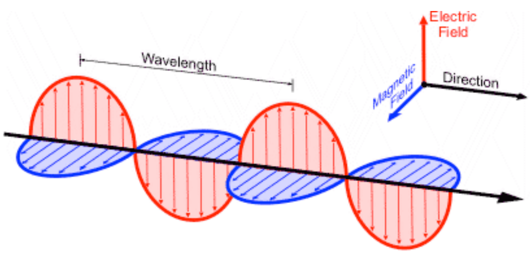
$$E(x,t) = E_{\max}\sin(kx-\omega t)$$

$$B(x,t) = B_{\max}\sin(kx-\omega t)$$

- We see that both E and B fields are transverse waves, and that whenever the E-field is large the B-field is large and when

- E and B are perpendicular to each other, oscillate in phase, and and travel at the speed of light *c*:

$$c = \frac{\omega}{k} = \lambda f$$



- Remember that for a wave equation we have the following relations:

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

where λ is the wavelength and $T = 1/f$ is the period.

- The magnitude of the electric and magnetic fields is related by the speed of light c: $\boxed{\frac{E_{\max}}{B_{\max}} = c}$

- The speed of light, in vacuum, is the same for all EM waves: $c = 3 \times 10^{8}\ \mathrm{m/s}$

- Since the purpose of a wave is to transmit energy, it is important to find out how much energy is stored in an EM wave.

- Recall that we have already found the amount of energy stored per unit volume for the electric field (from our capacitor example) to be

$$\frac{PE}{V} = \frac{1}{8\pi k}E^{2}$$

- A similar expression can be found for the B-field:

$$\frac{PE}{V} = \frac{1}{2\mu_0}B^{2}$$

- Therefore, the total energy in the EM wave, per unit volume is:

$$\frac{PE}{V} = \frac{1}{2}\left(\frac{1}{4\pi k}E^{2} + \frac{1}{\mu_0}B^{2}\right)$$

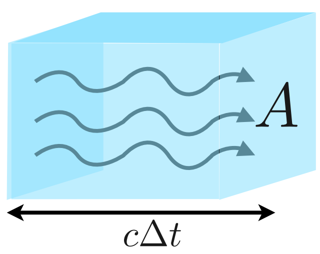
- We can simplify this equation by using $B = E/c$ and the relationship $\mu_0 c^{2} = 4\pi k$

$$\boxed{\frac{PE}{V} = \frac{1}{2}\left(\frac{1}{4\pi k}E^{2} + \frac{1}{4\pi k}E^{2}\right) = \frac{1}{4\pi k}E^{2}}$$

- The E-field and B-field both contain the same amount of energy/volume in an EM wave.

- Now that we have seen the an EM wave carries energy, let us consider a wave traveling in a time Δt

- In this time, the waves moves through a distance $c\Delta t$ and will sweep out a volume $V = Ac\Delta t$ where A is a cross-sectional area perpendicular to the wave velocity.



- The total energy transmitted in this time this therefore

$$\left(\frac{PE}{V}\right) Ac\Delta t = \frac{1}{4\pi k} E^2 Ac\Delta t$$

- The total energy per unit time per unit area is called the **Poynting Vector** $\vec{S}$ and points in the direction that the wave travels

$$S = c\left(\frac{PE}{V}\right) = \frac{c}{4\pi k} E^2$$

- The Poynting vector is measured in units of $\mathrm{J/s/m^2} = \mathrm{W/m^2}$

- The average value of the Poynting vector is called the **intensity** and is a quantity that can be directly measured (The Poynting vector itself is very difficult to measure)

$$\boxed{I = \frac{1}{2}\frac{c}{4\pi k} E_{\max}^2}$$

- Here, the factor of 1/2 comes from the fact that the average of $\sin^2(x)$ over one-period is 1/2.

__Ex.__ A EM wave traveling along the x-axis has an effective cross-sectional area of $1.5\ \mathrm{cm}^2$, a maximum E-field of 1500 N/C and a frequency of $f = 4 \times 10^{15}\ \mathrm{Hz}$. Find each of the following: __maximum B-field__, __energy density__, __Poynting vector__, __intensity__, and __total energy__ that hits a $0.5\ \mathrm{cm}^2$ in 10 seconds.

- The maximum B-field is related to the maximum E-field by the speed of light

$$B_{\max} = E_{\max}/c = \frac{1500\ \mathrm{N/C}}{3 \times 10^8\ \mathrm{m/s}} = \boxed{5 \times 10^{-6}\ \mathrm{T}}$$

- We can get the energy density directly from the E-field, or using B-field

$$\frac{PE}{V} = \frac{E_{\max}^2}{4\pi k} = \boxed{2 \times 10^{-5}\ \mathrm{J/m^2}}$$

- The Poynting vector is the energy density times the speed of light

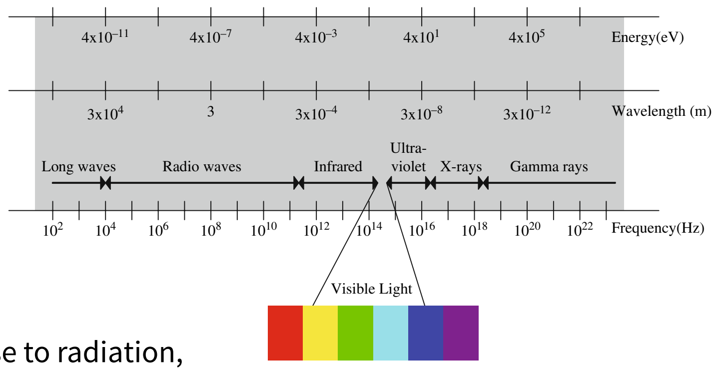
$$S_{\max} = \frac{PE}{V} c = 2 \times 10^{-5}(3 \times 10^8) = \boxed{6000\ \mathrm{W/m^2}}$$

- The intensity is 1/2 the Poynting vector: $I = (1/2)S_{\max} = \boxed{3000\ \mathrm{W/m^2}}$

- The total energy is the intensity times area times time:

$$E = 3000 \cdot (0.005)^2 \cdot 10 = \boxed{0.75\ \mathrm{J}}$$

Electromagnetic Spectrum:



- We have seen that accelerating charges produce EM waves

- Although the speed of these waves $c = \lambda f$ is fixed, the possible combinations of wavelength and frequency are quite large

- A large variety of different processes give rise to radiation, however all of them involve some kind of charge acceleration.

- The lowest energy, lowest frequency waves are generated by simple electrical currents that oscillate such as radio and television signals

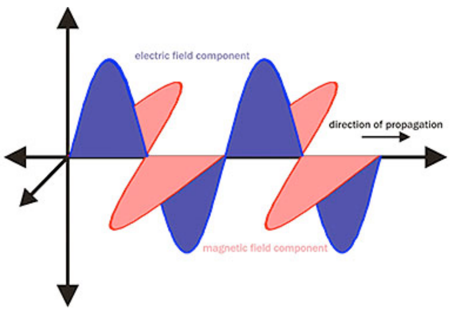
- Midrange energies are caused by electrons moving around the nucleus of an atom

- High energy EM waves come from the nucleus of an atom or from space.

- Although these waves share many of the same properties, the way in which they interact with material objects varies greatly.

The interaction of EM waves with materials depends on the frequency of the wave

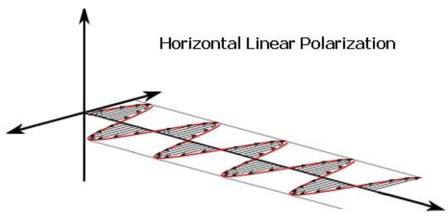
Polarization:



- We have seen that EM waves are transverse waves; both E and B are perpendicular to the direction of motion.

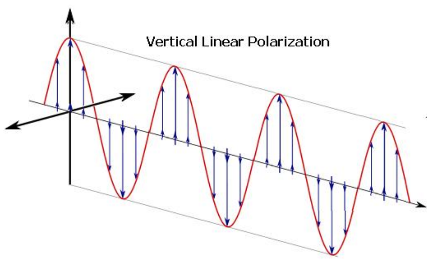
- The E and B fields are also perpendicular to each other.

- The direction of the E-field is an important property of an EM wave called its **Polarization**.



- If the E-field remains along the <u>same direction in space</u>, then the wave is called <u>linearly polarized</u>.

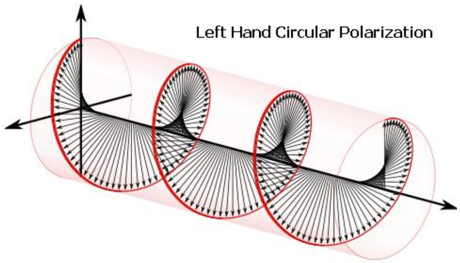
  - Sometimes these can be called horizontal or vertically polarized.

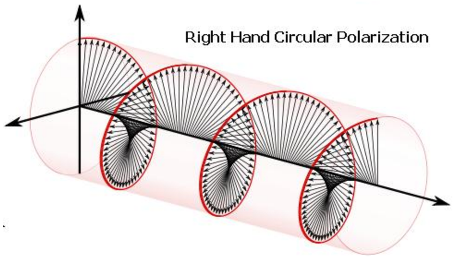


- In general, the E-field can point in any of the two directions perpendicular to the waves motion (i.e y & z), or any linear combination of these directions

- **Circularly polarized** light is light where the E-field rotates about the axis of motion as a function of time, and is a linear combination of the two perpendicular E-field components

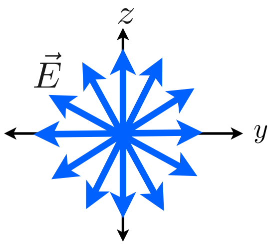
  - There is left and right circularly polarized light based on the direction in which the E-field rotates





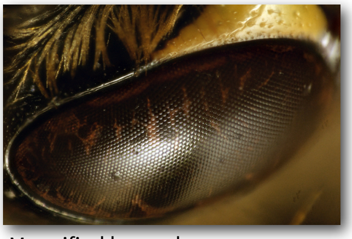
- Light can also have no polarization, in which case the light is called **unpolarized light**

- Unpolarized light as no definite direction for its E-field; the E-fields from all of the light sources has a random direction as a function of time.



Unpolarized light is generated by light where the E-field points in a random direction as a function of time

- Humans can not see the polarization of light. But many insects, such as bees, use this information for navigation.

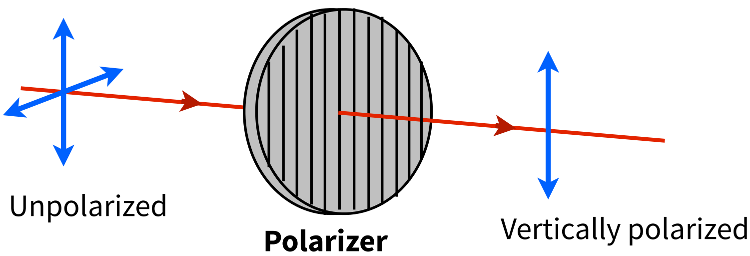


Magnified honey bee eye

- One can always create polarized light from unpolarized light by using a **polarizer**.

- Polarizers only let light that is polarized in the same direction as the polarizer through.

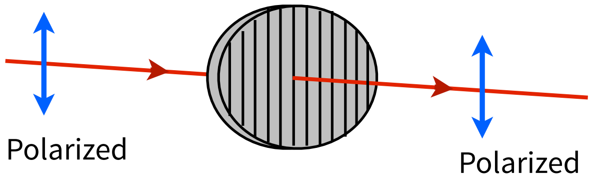
Thin film polarizers



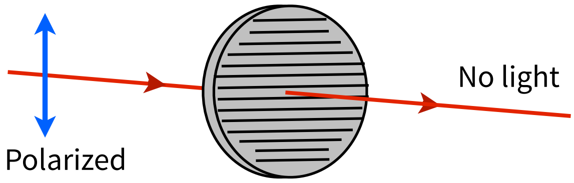
- The effect of the polarizer is best seen by seeing how the lights intensity is effected after passing through the polarizer.

- In the example above, the total intensity after passing through the polarizer is: $I = \frac{1}{2} I_0$

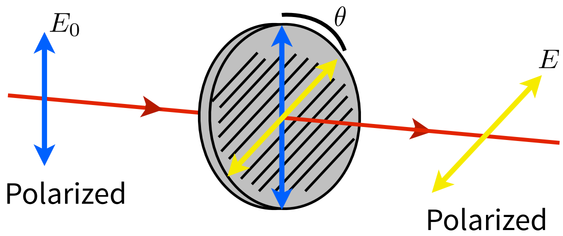
- If the input light is already polarized in the same direction as the polarizer, then 100% of light gets through



- Of course, if the light is polarized in a direction that is perpendicular to the polarizer then no light can pass through



- For polarized light in a polarizer with an arbitrary direction of polarization

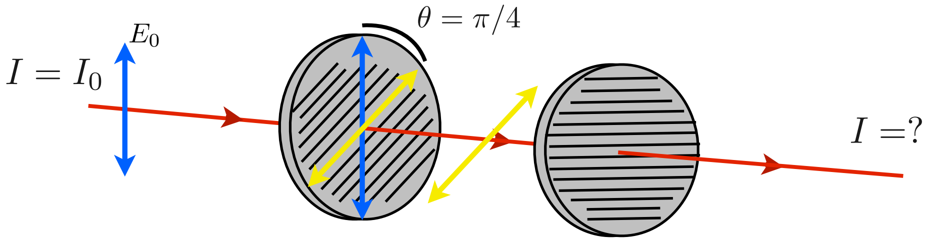


the E-field after passing through the polarizer is given by: $E = E_0 \cos\theta$

- Since the intensity is proportional to the E-field squared, the intensity as a function of angle is given by

$$\boxed{I = I_0 \cos^2\theta}$$

Ex. A vertically polarized 0.15W laser beam is incoming onto a polarizer with an angle of 45deg from the vertical. After this first polarizer, the beam passes through a second horizontal polarizer. What is the intensity of the transmitted beam? What would the intensity be if the first polarizer was removed or if the second polarizer was removed?



- The intensity after the first polarizer is: $I_1 = I_0 \cos^2(\pi/4)$

- The intensity after the second polarizer is:

$$I = I_1 \cos^2(\pi/4) = I_0 \cos^4(\pi/4) = 0.15\cos(\pi/4) = \boxed{0.038\ \mathrm{W/m^2}}$$

- If the first polarizer was removed, then the final intensity would be zero

- If the second polarizer was removed, then the final intensity would be

$$I_1 = I_0 \cos^2(\pi/4) = 0.15\cos^2(\pi/4) = \boxed{0.075\ \mathrm{W/m^2}}$$