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Course : EE656   Assignment-6

SVM Classifier

Case 1

$x^1 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $x^2 \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $x^3 \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$\hookrightarrow w = \begin{pmatrix} 0 \\ -1/2 \end{pmatrix}$

3 potential support vectors

Case 2

$x^1 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$x^2 \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$x^3 \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$x^4 \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$w = \begin{pmatrix} 3/5 \\ -4/5 \end{pmatrix}$

support vectors?

$$d(\omega, b, \alpha) = \sum_i \alpha_i$$
$$- \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j \, x^{iT} x^j$$
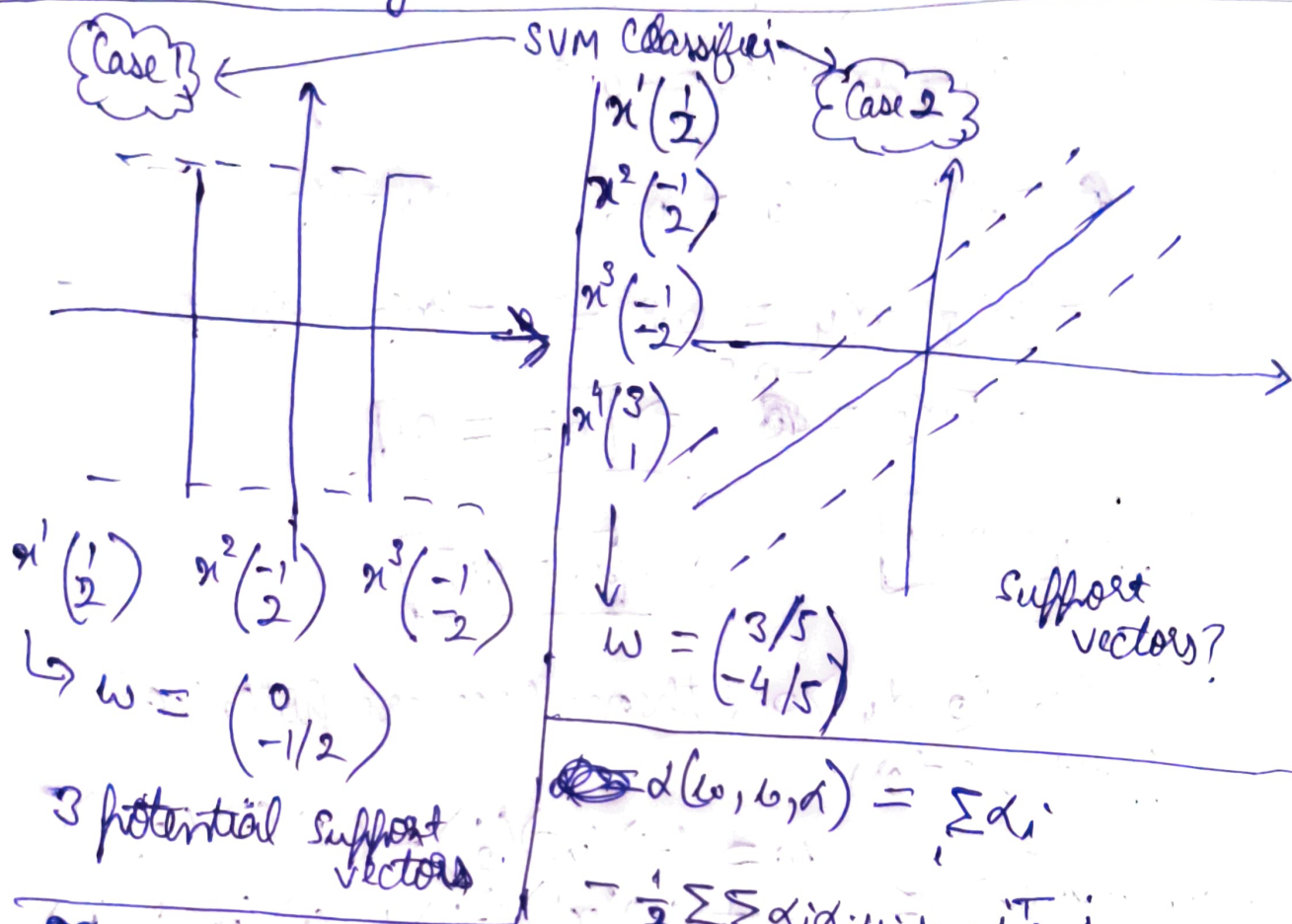
Label for points →

$w \cdot x^1 = (3/5)(1) - (2)(4/5) < 0 \Rightarrow \text{sign}(w x^1) = -1$

$w \cdot x^2 = -(3/5) - (2)(4/5) < 0 \Rightarrow \text{sign}(w \cdot x^2) = -1$

$w \cdot x^3 = (-3/5) + (2)(4/5) > 0 \Rightarrow \text{sign}(w x^3) = 1$

$w \cdot x^4 = (3)(3/5) - (1)(4/5) > 0 \Rightarrow \text{sign}(w \cdot x^4) = 1$

Bias-sum constraint

$\sum_i \alpha_i y_i = 0 \Rightarrow -\alpha_1 + \alpha_3 + \alpha_4 = 0 \qquad (\because \alpha_2 = 0)$

$(as \ 2.2 > 1)$

$-\alpha_1 - \alpha_3 + 3\alpha_4 = 0.6$

$-2\alpha_1 - 2\alpha_3 + \alpha_4 = -0.8$

$y_i (w \cdot x_i + b) > 1$

Now, $a_1 = \alpha_3 + \alpha_4$

$-(\alpha_3 + \alpha_4) - \alpha_3 + 3\alpha_4 = 0.6$

$-2(\alpha_3 + \alpha_4) - 2\alpha_3 + \alpha_4 = -0.8$

$\Rightarrow \quad -2\alpha_3 + 2\alpha_4 = 0.6 \Rightarrow \alpha_4 - \alpha_3 = 0.3$

$\Rightarrow \quad -4\alpha_3 - \alpha_4 = 0.8 \Rightarrow -4\alpha_3 - \alpha_3 - 0.3 = 0.8$

$\Rightarrow \quad -5\alpha_3 = 0.5^{\,-0.5} \Rightarrow \boxed{\alpha_3 = 0.1}$

$\therefore \boxed{\alpha_4 = 0.4}$

$\therefore \boxed{\alpha_1 = 0.5}$

$\therefore \alpha_1, \alpha_3, \alpha_4 > 0$, these three are potential support vectors

~~Hard margin width~~ $w = (-1)(0.5)\begin{pmatrix}1\\2\end{pmatrix} + (-1)(0)\begin{pmatrix}-\frac{1}{2}\end{pmatrix}$
$+ (1)(0.1)\begin{pmatrix}-\frac{1}{2}\\-2\end{pmatrix} + (1)(0.4)\begin{pmatrix}8\\1\end{pmatrix}$

$\Rightarrow w = \begin{pmatrix}3/5\\-4/5\end{pmatrix}$

Check for bias $\Rightarrow$ Using point x_1

$y_1(w x_1 + b) = 1 \Rightarrow (-1)(-1 + b) = 1$

$\Rightarrow 1 - b = 1 \Rightarrow \boxed{b = 0}$

Margin width $= \dfrac{2}{\|w\|^2} = \dfrac{2}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \boxed{2}$