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Distributed Delay Control Strategy for Leader-Following Cyber Secure Consensus in Riemann-Liouville Fractional-Order Delayed Multi-Agent Systems under Denial-of-Service Attacks

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Distributed Delay Control Strategy for Leader-Following Cyber Secure Consensus in Riemann-Liouville Fractional-Order Delayed Multi-Agent Systems under Denial-of-Service Attacks

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Abstract

Considered a Riemann-Liouville fractional-order delayed multi-agent systems (FDMASs), where the associated topology is a switching weighted digraph. By using the classical Lyapunov direct technique and algebraic graph theory. A new approach to leader-following cyber secure consensus analysis is taken in the present of denial-of-service attacks by means of distributed delay control strategy and some algebraic criteria are offered. The key advantage of our suggested strategy is that the matching Lyapunov function's first-order derivative may be obtained. Two illustrative cases are offered to highlight more logic of methodology.

Keywords: Fractional-order delayed multi-agent system; Consensus; Lyapunov direct method; Riemann–Liouville derivative

1 Introduction

Science and engineering have given system control [1], [2], [3] more attention recently. In particular, cooperative control of multi-agent systems has emerged as a contemporary study area of interest due to its widespread adoption in many industries, including smart grids [4] and urban traffic lights [5]. Several significant findings have been made on consensus control, including consensus problems of first-order [6], [7], [8], [9], [10], [11], second-order, and higher-order systems [12], [13], [14], [15], [16], and has drawn a lot of attention. Fractional calculus has received a lot of interest recently since it offers a great tool for describing the memory and inheritance characteristics of physical processes [17], [18], [19].

Designing consensus protocols is the key to achieving it. Different types of consensus protocols are used, such as the distributed control protocol [20], [21], [22], sampling event triggering control protocol [23],

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sliding mode control protocol [24], [25], observer protocol [26], adaptive pinning control protocol [27]. The main instrument to study consensus is stability theory, for Caputo delayed systems Chen et al. [28] were able to get a fresh understanding of finite-time stability . Using the traditional Lyapunov direct technique [29] discusses asymptotic stability of Riemann-Liouville nonlinear delayed systems. By creating a suitable Lyapunov function, Zhang et al. [30] hold Riemann-Liouville neutral systems in accordance with the method put forth by Liu et al[29].

The two primary techniques used in consensus analysis at this time are the time-domain method [31] and the frequency-domain method [32], [33], [34]. Time delays are an inevitable part of many real-world systems, which will negatively affect their performance and even make them unstable. Hence, taking time delays into account is crucial while performing system analysis and control. By expressing the answer to the error system, Zhu et al. [35] resolve the FDMAS consensus problem. To address FDMAS consensus issues In [36], [37], the fractional-order Razumikhin approach is employed. To examine consensus of FDMASs in Riemann-Liouville sense, there are just a few efficient techniques. To Caputo fractional-order systems majority of remaning solution yield particular observation.

The development of an attack-resistant, cyber secure control of MASs has included significant work. Currently, the most common attack categories consist of deception attacks replay attacks and DoS attacks [38]. DoS attacks are frequently utilized in real-world situations. Recently Dos attacks concerns in the MASs have also received a lot of attention , for example [39], in order to study MASs with connectivity-broken topologies or connectivity-maintained topologies, a number of attack models were developed. In many different fields applications for secure consensus control of MASs under Dos attacks exist . For example, by utilizing a fault-tolerant control strategy the problem of non-linear MASs being vulnerable to DoS attacks [40]. Secure cooperative event-triggered control for linear MASs under DoS attacks was investigated[41].

Our focus is on investigating of distributed delayed control for FDMASs with cyber secure consensus under DoS attacks with switching topologies. In the Riemann-Liouville sense we address the cyber consensus of FDMASs. With the use of algebraic graph theory and the classical Lyapunov direct method, a new approach to dealing with cyber secure consensus is taken, and a few algebraic criteria are offered. Large contribution of this article is the ability to take the first-order derivative of the corresponding Lyapunov technique, which offers efficient and practical way in a Riemann-Liouville sense to analyse the stability, cyber secure consensus and attractiveness of fractional-order delayed differential systems. In particular, approach suggested in this work is capable of effectively addressing the challenges given by discrete-time delays. Moreover, to verify linear matrix inequalities the conditions are specified. Finally numerical examples are offered to show effectiveness of suggested approach. Remainder of the paper is divided into the following sections: Several preliminaries are found in segment 2. Part 3 studies the leader-following cyber cyber secure consensus of FDMASs. The leaderless cyber secure consensus of FDMASs is examined in Segment 4. In segment 5, two instances are shown in further detail. This work is deduce in Segment 6.

2 Preliminaries

First, a number of foundational concepts and practical lemmas are introduced.

Assume that $G_\sigma = (V, F_\sigma)$ is a directed switching graph, where $F_\sigma = \{(v_p, v_q)$: v_q points to $v_p\}$ is the directed edge-set and vertex set V is the set $v_1, ...v_N$. The adjacency matrix of G_σ is defined by $B_\sigma = (b^\sigma_{pq})_{m\times m}$ matrix in which $b^\sigma_{pq} > 0$ if $(v_p, v_q) \in F$, else $b^\sigma_{(pq)} = 0$. $L_\sigma = (l^\sigma_{pq})_{m\times m}$ represent the laplacian matrix of G_σ, its entries are $l^\sigma_{pp} = \sum_{q=1,q\neq p}^{m} b^\sigma_{pq}$ and $l^\sigma_{pq} = -b^\sigma_{pq}(p \neq q)$. It is obvious that $\sum_{q=1,q\neq p}^{m} l^\sigma_{pq} = 0$.

Lemma 2.1. *[42] If and only if G_σ contains a root vertex that can get to each other vertex through directed edges, then all of L_σ's eigenvalues have positive real-parts, along the exception of a zero eigenvalue.*
For a function f(t)[43], the α-order Riemann-Liouville integral is defined by

$$_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-x)^{\alpha-1} dx f(x), \quad \alpha > 0. \tag{1}$$

v-order derivative in Riemann-Liouville sense is provided by

$$_{t_0}D_t^{v} f(t) = \frac{1}{\Gamma(m-v)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(x)}{(t-x)^{v+1-m}} dx, \quad m-1 \leq v < m, \tag{2}$$

Property 2.1. *Let $f(t)$ is a continuous function then*

$$_{t_0}D_t^{v}({}_{t_0}D_t^{-\alpha} f(t)) = {}_{t_0}D_t^{v-\alpha} f(t) \qquad \alpha > 0, \quad v > 0. \tag{3}$$

Lemma 2.2. *[29] If $y(t) \in R^N$ is a continuous differential function and $E > 0$ is an $N \times N$ matrix then.*

$$\frac{1}{2} {}_{t_0}D_t^{v}(y^T(t)Ey(t)) \quad \leq \quad (y^T(t)E {}_{t_0}D_t^{v} y(t), \qquad 0 < v < 1. \tag{4}$$

Lemma 2.3. *[44] If $B' \in R^{M\times M}$ and $A' \in R^{N\times N}$ have eigenvalues $\lambda'_1, \lambda'_2, ..., \lambda'_M$ and $\alpha'_1, \alpha'_2, ..., \alpha'_N$ respectively, then $\lambda'_p \alpha'_q$ are the eigenvalues of $B' \otimes A'$.*

3 Leader-following cyber seure consensus of FDMASs

Leader and follower syber secure consensus of FDMASs introduced in the following. To get syber secure consensus based on Lyapunov technique , two simple algebraic criteria are offered.

The state $y_i(t) \in R^N$ of the pth follower is specified by

$$_{t_0}D_t^{v} y_p(t) = Ay_p(t) + h_1(t, y_p(t)) + h_2(t, y_p(t - \tau_\sigma(t))) + w_p(t), \qquad p = 1, 2, ..., m. \tag{5}$$

Where the leader $y_0(t) \in R^N$ is satisfied and $A \in R^{N\times N}$ is a known matrix.

$$_{t_0}D_t^{v} y_p(t) = Ay_0(t) + h_1(t, y_0(t)) + h_2(t, y_0(t - \tau_\sigma(t))). \tag{6}$$

A switching signal $\sigma(t):[0,\infty)\rightarrow\Phi=1,2....,r(r\geq 1)$ is used to switch among different topologies. $\{H_1,...,H_r\},\{G_1,...,G_r\}$ are the information exchange matrix for consensus tracking and the set of directed switching graph G_σ Distributed delay control protocol under attack is designed as

$$w_p(t)=-K\sum_{v=1}^{m}b_{pq}^{\sigma}(y_p(t-\tau_\sigma(t))-y_q(t-\tau_\sigma(t)))+c_p^{\sigma}(y_0(t-\tau_\sigma(t))-y_p(t-\tau_\sigma(t))) \tag{7}$$

where $\tau_\sigma(t)$ is the discrete time delay $h_1(t,\ .\ )\in R^N$ and $h_2(t,\ .\ )\in R^N$ are given non linear functions. The entries of $B=(b_{pq}^{\sigma})_{m\times m}$ are provide by: $b_{pq}^{\sigma}>0$ if $y_q(t)$ points to $y_p(t)$, else $b_{pq}^{\sigma}=0$, $q=0,1,2,...,m$, $p=1,...,m$. Let $\bar{C}_\sigma\ =\ diag(c_{10},...c_{m0})$.

Definition 3.1. [7] If for any initial values, leader and follower global cyber secure consensus of FDMAS (5) -(6) under protocol (7) is attained.

$$\lim_{t\rightarrow\infty}||y_p(t)-y_0(t)||=0,\qquad p=1,...,m. \tag{8}$$

To arrive at our key results, we present some lemmas and assumptions.
(H1). Assume that h_p are Lipschitz continuous, which means that there are two scalars with $l_p>0$ such that for any $\phi,\psi\in R^N$,

$$||h_p(t,\phi)-h_p(t,\psi)||\leq l_p||\phi-\psi||,\qquad p=1,2. \tag{9}$$

(H2). In the appropriate digraph of system (5)–(7) to reach each other vertex assume that the directed edges can allow the leader $y_0(t)$.

Lemma 3.1. *[35] Let $W_\sigma=L_\sigma+\bar{C}_\sigma$. If and only if (H2) is true, all of the eigenvalues of W_σ have positive real-parts.*

Remark 3.2. *If (H2) is true, real parts all of eigenvalues of $L_\sigma\otimes\bar{C}_\sigma$ are positive according to Lemmas (2.3) and (3.1)*

Lemma 3.3. *[45] Let $\omega,\phi\in R^N$ and $K_1>0$, then inequality $2\omega^T\phi\leq K_1\omega^T\omega+\frac{1}{K_1}\phi^T\phi$ holds.*

Theorem 3.4. *Under $(H1)$ and $(H2)$, if there are two constant $K_p>0$, $p=1,2$ and a symmetric matrix $E>0$, FDMAS (5)-(6) with protocol (7) can realize the leader and follower global syber consensus such that*

$$I_N\otimes(A+A^T+2l_1I_n+k_1I_n+E)+K_2W_\sigma W_\sigma^T\otimes KK^T<0. \tag{10}$$

and

$$I_N\otimes\left(\frac{l_2^2}{K_1}I_n+\frac{1}{K_2}I_n-E\right)<0 \tag{11}$$

Proof. . Let $\delta_p(t) = y_p(t) - y_0(t)$, $p = 1, 2, ..., m$. One has from (5)-(7)

$$\begin{aligned}
{}_{t_0}D_t^v y_p(t) -_{t_0} D_t^v y_0(t) &= [Ay_p(t) + h_1(t, y_p(t)) + h_2(t, y_p(t - \tau_\sigma(t))) + w_p(t)] \\
&\quad -[Ay_0(t) + h_1(t, y_0(t)) + h_2(t, y_0(t - \tau_\sigma(t)))] \\
{}_{t_0}D_t^v y_p(t) -_{t_0} D_t^v y_0(t) &= A(y_p(t) - y_0(t)) + h_1(t, y_p(t)) - h_1(t, y_0(t)) + h_2(t, y_p(t - \tau_\sigma(t))) \\
&\quad -h_2(t, y_0(t - \tau_\sigma(t))) + w_p(t) \\
{}_{t_0}D_t^v \delta_p(t) &= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_0(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_0(t - \tau_\sigma(t))) \\
&\quad -K\sum_{q=1}^{m} b_{pq}^\sigma(y_p(t - \tau_\sigma(t)) - y_q(t - \tau_\sigma(t)) + c_p^\sigma(y_0(t - \tau_\sigma(t)) - y_p(t - \tau_\sigma(t))) \\
&= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_0(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_0(t - \tau_\sigma(t))) \\
&\quad +K\sum_{q=1}^{m} b_{pq}^\sigma(\delta_q(t - \tau_\sigma(t)) - K\sum_{q=1}^{m} b_{pq}^\sigma(\delta_p(t - \tau_\sigma(t)) \\
&\quad -c_p^\sigma(\delta_p(t - \tau_\sigma(t))) \qquad (12) \\
&= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_0(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_0(t - \tau_\sigma(t))) \\
&\quad -K\sum_{q=1}^{m} l_{pq}^\sigma(\delta_q(t - \tau_\sigma(t))) - c_p^\sigma(\delta_p(t - \tau_\sigma(t))).
\end{aligned}$$

Let $\delta(t) = (\delta_1^T(t), ..., \delta_m^T(t))^T$, $\qquad y(t) = (y_1^T(t), ..., y_m^T(t))T$,
$H_p(t, y) = (h_p^T(t, y_1), ..., h_p^T(t, y_m))^T$, $\qquad H_p(t, y_0) = (h_p^T(t, y_0), ..., h_p^T(t, y_0))^T$. p = 1,2, we have

$$\begin{aligned}
{}_{t_0}D_t^v \delta(t) &= (I_N \otimes A)\delta(t) + H_1(t, y(t)) - H_1(t, y_0(t) + H_2(t, y(t - \tau_\sigma(t))) - H_2(t, y_0(t - \tau_\sigma(t)) \\
&\quad -(L_\sigma \otimes K)\delta(t - \tau_\sigma(t)) - (C_\sigma \otimes I_N)\delta(t - \tau_\sigma(t)) \\
&= (I_N \otimes A)\delta(t) + H_1(t, y(t)) - H_1(t, y_0(t) + H_2(t, y(t - \tau_\sigma(t))) - H_2(t, y_0(t - \tau_\sigma(t)) \\
&\quad +[(L_\sigma + \bar{C}_\sigma \otimes KI_N]\delta(t - \tau_\sigma(t)) \\
&= (I_N \otimes A)\delta(t) + H_1(t, y(t)) - H_1(t, y_0(t) + H_2(t, y(t - \tau_\sigma(t))) \\
&\quad -H_2(t, y_0(t - \tau_\sigma(t)) - (H_\sigma \otimes KI_N)\delta(t - \tau_\sigma(t)). \qquad (13)
\end{aligned}$$

Pick Lyapunov function

$$V(t) =_{t_0} D_t^{v-1}(\delta^T(t)\delta(t)) + \int_{t-\tau_\sigma(t)}^{t} \delta^T(s)(I_N \otimes E)\delta(s)ds \qquad (14)$$

One can derive from Lemma (2.2) and Property (2.1)

$$
\begin{aligned}
V'(t) &= {}_{t_0}D_t^{v}(\delta^T(t)\delta(t)) + \delta^T(t)(I_N \otimes E)\delta(t) - \delta^T(t-\tau_\sigma(t))(I_N \otimes E)\delta(t-\tau_\sigma(t)). \\
&\leq 2\delta^T(t)_{t_0}D_t^{v}\delta(t) + \delta^T(t)(I_N \otimes E)\delta(t) - \delta^T(t-\tau_\sigma(t))(I_N \otimes E)\delta(t-\tau_\sigma(t)). \\
&= 2\delta^T(t)(I_N \otimes A)\delta(t) + 2\delta^T(t)[H_1(t,y(t)) - H_1(t,y_0(t))] + 2\delta^T(t)[H_2(t,y(t-\tau_\sigma(t))) \\
&\quad -H_2(t,y_0(t-\tau_\sigma(t)))] - 2\delta^T(t)(G_\sigma \otimes KI_N)\delta(t-\tau_\sigma(t)) \\
&\quad +\delta^T(t)(I_N \otimes E)\delta(t) - \delta^T(t-\tau_\sigma(t))(I_N \otimes E)\delta(t-\tau_\sigma(t)).
\end{aligned} \tag{15}
$$

We have from $(H1)$

$$
\delta^T(t)[H_1(t,y(t)) - H_1(t,y_0(t) \quad \leq \quad ||\delta(t)|| \ \ . \ \ ||\delta^T(t)[H_1(t,y(t)) - H_1(t,y_0(t)|| \leq l_1\delta^T(t)\delta(t). \tag{16}
$$

From lemma (3.3) is reported by

$$
\begin{aligned}
V'(t) &\leq 2\delta^T(t)(I_N \otimes A)\delta(t) + 2l_1\delta^T(t)\delta(t) + 2\delta^T(t)[H_2(t,y(t-\tau_\sigma(t))) \\
&\quad - H_2(t,y_0(t-\tau_\sigma(t)))] - 2\delta^T(t)(H_\sigma \otimes KI_N)\delta(t-\tau_\sigma(t)) \\
&\quad + \delta^T(t)(I_N \otimes E)\delta(t) - \delta^T(t-\tau_\sigma(t))(I_N \otimes E)\delta(t-\tau_\sigma(t)). \\
&\leq \delta^T(t)[I_N \otimes (A + A^T + 2l_1 I_n + E)]\delta(t) + K_1\delta^T(t)\delta(t) + \frac{1}{K_1}[H_2(t,y(t-\tau_\sigma(t)) \\
&\quad - H_2(t,y_0(t-\tau_\sigma(t))]^T[H_2(t,y(t-\tau_\sigma(t))) - H_2(t,y_0(t-\tau_\sigma(t)))] \\
&\quad - K_2\delta^T(t)(W_\sigma \otimes KI_N)(W_\sigma \otimes KI_N)^T\delta(t) - \frac{1}{K_2}\delta^T(t-\tau_\sigma(t))\delta(t-\tau_\sigma(t)) \\
&\quad -\delta^T(t-\tau_\sigma(t))(I_N \otimes +E)\delta(t-\tau_\sigma(t)) \\
V'(t) &\leq \delta^T(t)[I_N \otimes (A + A^T + 2l_1 I_N + K_1 I_N + E) + K_2 W_\sigma W_\sigma^T \otimes KK^T I_N]\delta(t) \\
&\quad + \frac{l_2^2}{K_1}\delta^T(t-\tau_\sigma(t))\delta(t-\tau_\sigma(t)) + \ \frac{1}{K_2}\delta^T(t-\tau_\sigma(t))\delta(t-\tau_\sigma(t)) \\
&\quad - \delta^T(t-\tau_\sigma(t))(I_N \otimes +E)\delta(t-\tau_\sigma(t)) \\
&= \delta^T(t)[I_N \otimes (A + A^T + 2l_1 I_n + K_1 I_n + E) + K_2 W_\sigma W_\sigma^T \otimes KK^T I_N]\delta(t) \\
&\quad + \ \delta^T(t-\tau_\sigma(t))\left[I_N \otimes \left(\tfrac{l_2^2}{K_1}I_n + \tfrac{1}{K_2}I_n - E\right)\right]\delta(t-\tau_\sigma(t)) \\
&= \Omega_r^T \begin{pmatrix} I_N \otimes (A + A^T + 2l_1 I_n + K_1 I_n + E) & 0 \\ +K_2 H_\sigma H_\sigma^T \otimes KK^T I_N & \\ 0 & I_N \otimes \left(\frac{l_2^2}{K_1}I_n + \frac{1}{K_2}I_n - E\right) \end{pmatrix} \Omega_r.
\end{aligned}
$$

where $\Omega_r = \delta^T(t), \delta^T(t-\tau_\sigma(t))^T$. Due to the inequality (10) and (11) the following matrix inequality hold

$$
\begin{pmatrix} I_N \otimes (A + A^T + 2l_1 I_n + K_1 I_n + E) & 0 \\ +K_2 W_\sigma W_\sigma^T \otimes KK^T I_N & \\ 0 & I_N \otimes \left(\frac{l_2^2}{K_1}I_n + \frac{1}{K_2}I_n - E\right) \end{pmatrix} < 0 \tag{17}
$$

. As a result, we have constant $\eta > 0$ such that $V'(t) \leq -\eta||\delta(t)||^2$.

However, it is evident that two real-value functions
$c_1(t) > 0$ and $c_2(t) > 0$ exist, with $||\delta(s)||^2_{t_0 \le s \le t} = c_1(t)||\delta(t)||^2$
and $||\delta(s)||^2_{t-\tau_\sigma(t) \le s \le t} = c_2(t)||\delta(t)||^2$.
It come after from (14) that

$$\begin{aligned}
V(t) &= \frac{1}{\Gamma(1-v)} \int_{t_0}^{t} (t-x)^{-v} \delta^T(x)\delta(x)dx + \int_{t-\tau_\sigma(t)}^{t} \delta^T(x)(I_N \otimes E)\delta(s)dx \\
&= \frac{c_1(t)||\delta(t)||^2}{\Gamma(1-v)} \int_{t_0}^{t} (t-x)^{-v} dx + \int_{t-\tau_\sigma(t)}^{t} \delta^T(x)(I_N \otimes E)\delta(s)dx \\
&\le \frac{c_1(t)||\delta(t)||^2}{\Gamma(1-v)} \int_{t_0}^{t} (t-x)^{-v} dx + \lambda_{max}(E) \int_{t-\tau_\sigma(t)}^{t} \delta^T(s)\delta(x)dx \\
&= \frac{c_1(t)||\delta(t)||^2}{\Gamma(1-v)} \int_{t_0}^{t} (t-x)^{-v} dx + \tau_\sigma(t)c_2(t)(E)\lambda_{max}(E)||\delta(t)||^2 \\
&= \frac{c_1(t)||\delta(t)||^2}{\Gamma(2-v)} (t-t_0)^{1-v} + \tau_\sigma(t)c_2(t)(E)\lambda_{max}(E)||\delta(t)||^2 \\
&= \left[\tfrac{c_1(t)(t-t_0)^{1-v}}{\Gamma(2-v)} + \tau_\sigma(t)\lambda_{max}(E)c_2(t)\right] ||\delta(t)||^2. \qquad (18)
\end{aligned}$$

Note that $V' \le -\eta||\delta(t)||^2$, one has from (18)

$$V' \le -\eta||\delta(t)||^2 \le -\eta s(t)V(t) \qquad (19)$$

where $s(t) = \frac{1}{\frac{c_1(t)(t-t_0)^{1-v}}{\Gamma(2-v)} + \tau_\sigma(t)\lambda_{max}(E)c_2(t)} > 0$
Integrating (19) from t_0 to t we gets

$$0 < V(t) \le V(t_0)e^{-\eta \int_{t_0}^{t} s(x)dx} \qquad (20)$$

As a result when $t \to +\infty$, $V(t) \to 0$, which leads $\lim_{t\to+\infty} ||\delta(t)|| = 0$,
namely, $\lim_{t\to+\infty} ||y_p(t) - y_0(t)|| = 0$. Consequently, the leader and follower global cyber secure consensus of fractional order multi-agent system (5)-(7) can acquired.
Because of Theorem (3.4), the following criterion is provided. □

Corollary 3.5. *Under (H1) and (H2), If two scalars $K_p > 0$, $p = 1,2$ and a symmetric matrix $E > 0$ exist, then the leader and follower global consensus of FDMAS $(5)-(6)$ under protocol (7) can be attained.*

$$\lambda_{max}\left[I_N \otimes (A + A^T + 2l_1 I_n + K_1 I_n + E) + K_2 W_\sigma W_\sigma^T \otimes KK^T\right] < 0 \qquad (21)$$

and

$$\lambda_{max}\left[\left(\frac{l_2^2}{K_1} + \frac{1}{K_2}\right) I_n - E\right] < 0 \qquad (22)$$

Proof. Relationships (10) and (11) are valid because of inequalities (21) and (22). Consequently, the proof is simple. □

4 Leaderless cyber secure consensus of FDMASs

This segment examine the provides two simple algebraic conditions and leaderless cyber secure consensus of FDMAS (5) .

Delayed control protocol under attack, is design as

$$w_p(t) = -K \sum_{q=1}^{m} b_{pq}^{\sigma}(y_p(t-\tau_\sigma(t)) - y_q(t-\tau_\sigma(t)), \qquad p = 1, 2, ..., m. \tag{23}$$

Definition 4.1. If any initial values exist, the FDMAS (5) under protocol (23) can reach global consensus.

$$\lim_{t\to\infty} ||y_q(t) - y_p(t)|| = 0, \qquad p, q = 1, 2, ..., m. \tag{24}$$

$(H3)$. Assume the matching digraph has vertices that can be reached by directed edges from one vertex to the other.

We simply need to swap out $(H2)$ for $(H3)$ and W_σ in (3.1) for $R_\sigma = (\tau_{\sigma(p-1,q-1)})_{(m-1)\times(m-1)}(t) = (l_{pq}^{\sigma} + a_{1q}^{\sigma})_{(m-1)\times(m-1)}$ to establish leaderless syber secure consensus, where $p, q = 2, ..., m$.

Theorem 4.1. *If there are two constants $K_p > 0$, $p = 1, 2$ and a symmetric matrix $E > 0$ under $(H1)$ and $(H3)$, the leaderless syber secure global consensus of FDMAS (5) and (23) is obtained such that*

$$I_{N-1} \otimes \Big[(A + A^T + 2l_1 I_n + K_1 I_n + E)\Big] + K_2 R_\sigma R_\sigma^T \otimes KK^T < 0 \tag{25}$$

and

$$I_{N-1} \otimes \left(\tfrac{l_2^2}{K_1} I_n + \tfrac{1}{K_2} I_n - E\right) < 0 \tag{26}$$

Proof. Consider $\delta_p(t) = y_p(t) - y_1(t)$, $p = 1, 2, ..., m$. One has from (5) and (23)

$$\begin{aligned}
{}_{t_0}D_t^v y_p(t) - {}_{t_0}D_t^v y_1(t) &= [Ay_p(t) + h_1(t, y_p(t)) + h_2(t, y_p(t - \tau_\sigma(t))) + w_p(t)] \\
&\quad - [Ay_1(t) + h_1(t, y_1(t)) + h_2(t, y_1(t - \tau_\sigma(t)))] \\
{}_{t_0}D_t^v y_p(t) - {}_{t_0}D_t^v y_1(t) &= A(y_p(t) - y_1(t)) + h_1(t, y_p(t)) - h_1(t, y_1(t)) + h_2(t, y_p(t - \tau_\sigma(t))) \\
&\quad - h_2(t, y_1(t - \tau_\sigma(t))) + w_p(t) \\
{}_{t_0}D_t^v \delta_p(t) &= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_1(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_1(t - \tau_\sigma(t))) \\
&\quad - K\sum_{q=1}^{m} b_{pq}^{\sigma}(y_p(t - \tau_\sigma(t)) - y_q(t - \tau_\sigma(t))) - K\sum_{q=1}^{m} b_{1q}^{\sigma}\delta_q(t - \tau_\sigma(t)) \\
&= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_1(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_1(t - \tau_\sigma(t))) \\
&\quad + K\sum_{q=1}^{m} b_{pq}^{\sigma}(\delta_q(t - \tau_\sigma(t)) - K\sum_{q=1}^{m} b_{pq}^{\sigma}(\delta_p(t - \tau_\sigma(t)) \\
&\quad - K\sum_{q=1}^{m} b_{1q}^{\sigma}\delta_q(t - \tau_\sigma(t)) \\
&= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_1(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_1(t - \tau_\sigma(t))) \\
&\quad - K\sum_{q=1}^{m} l_{pq}^{\sigma}(\delta_q(t - \tau_\sigma(t))) - K\sum_{q=1}^{m} b_{1q}^{\sigma}\delta_q(t - \tau_\sigma(t)). \\
&= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_1(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_1(t - \tau_\sigma(t))) \\
&\quad - K\sum_{q=2}^{m}(l_{pq} + b_{1q}^{\sigma})\delta_q(t - \tau_\sigma(t)) \\
&= A\delta_p(t) + h_1(t, y_p(t)) - h_1(t, y_1(t)) + h_2(t, y_p(t - \tau_\sigma(t))) - h_2(t, y_1(t - \tau_\sigma(t))) \\
&\quad - K\sum_{q=2}^{m} \tau_{\sigma(p-1,q-1)}(t)\delta_q(t - \tau_\sigma(t)) \qquad (27)
\end{aligned}$$

Let $\delta(t) = (\delta_2^T(t), ..., \delta_m^T(t))^T$, $\quad y(t) = (y_1^T(t), ..., y_m^T(t))T$,
$H_p(t, y) = (h_p^T(t, y_2), ..., h_p^T(t, y_N))^T$, $\quad H_p(t, y_1) = (h_p^T(t, y_1), ..., h_p^T(t, y_1))^T$. $\quad p$ = 1,2, we have

$$\begin{aligned}
&= (I_N \otimes A)\delta(t) + H_1(t, y(t)) - H_1(t, y_0(t) + H_2(t, y(t - \tau_\sigma(t))) \\
&\quad - H_2(t, y_0(t - \tau_\sigma(t)) - (R_\sigma \otimes KI_N)\delta(t - \tau_\sigma(t)). \qquad (28)
\end{aligned}$$

The remaining proof is exclude and is similar to Theorem (3.4)

Theorem (4.1) presents more simple criterion as follows. □

Corollary 4.2. *Leaderless global syber secure consensus of the FDMAS (5) and (23) have been reached under (H1) and (H3) if a symmetric matrix $E > 0$ and two scalars $K_p > 0$, $p = 1, 2$ exist such that*

$$\lambda_{max}\left[I_{N-1} \otimes (A + A^T + 2l_1 I_n + K_1 I_n + E) + K_2 R_\sigma R_\sigma^T \otimes KK^T\right] < 0 \qquad (29)$$

and

$$\lambda_{max}\left[\left(\frac{l_2^2}{K_1}+\frac{1}{K_2}\right)I_n-E\right]<0 \tag{30}$$

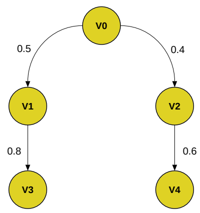
Remark 4.3. *In reality, the stability, syber secure consensus and attractiveness of Riemann-Liouville differential systems can be handled using both the classical Lyapunov direct methods and fractional Lyapunov direct methods. Here, the classical Lyapunov direct technique is used, allowing us to take the Lyapunov function's first derivative rather than its fractional derivative and avoiding complicated calculations. Most importantly, the strategy suggested in this work is capable of handling the challenge given by time delays effectively. When using the fractional Lyapunov direct technique, it is exceedingly challenging to build a Lyapunov function and evaluate its fractional-order derivative due to intricacy of fractional-order derivatives.*

Remark 4.4. *In [35–37], the Caputo sense secure consensus of FDMASs is taken into consideration.Consensus is reached in [35] found in the statement of solution and prperties of Mittag-Leffer functions, fractional Razumikhin technique is used in [36,37], we investigates the Lyapunov direct technique and FDMASs in Riemann-Liouville sense is used. The approaches used to analyze consensus vary because the definitions and features of the two derivatives differ. Due to the composition properties of operators, the application suggested in this study cannot be use in Caputo systems.*

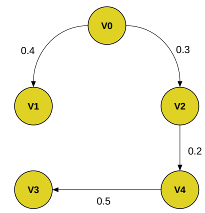
5 Numerical examples

We give two example to illustrate how well our findings work.

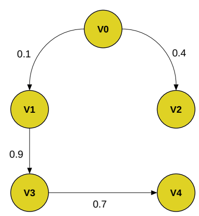
Example 5.1. Examine the FDMAS (5)-(7), the topology is depicted in Fig. 1.



(a) G_1



(b) G_2



(c) G_3

Figure 1: Topology switching among G_1, G_2 and G_3 caused by three different DoS attack.

Now, $h_1(t, y_p(t)) = \frac{1}{4} tanhy_p(t), h_2(t, y_p(t-\tau_\sigma(t))) = \frac{5}{11} siny_p(t-\tau_\sigma(t))$,
and

$$A \;=\; \begin{pmatrix} -1.4 & -0.2 \\ 0 & -1.6 \end{pmatrix}$$

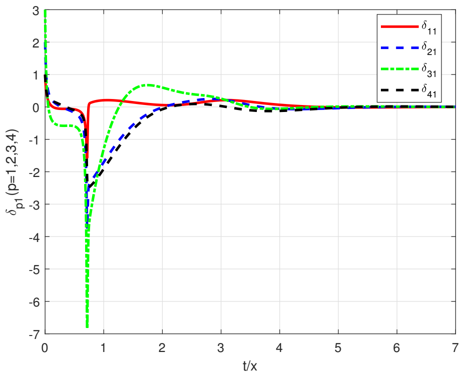
. According to $(H1)$ take $l_1 = 0.3, l_2 = 0.4$. From Fig. 1 we have

$$B_\sigma = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \end{pmatrix}$$

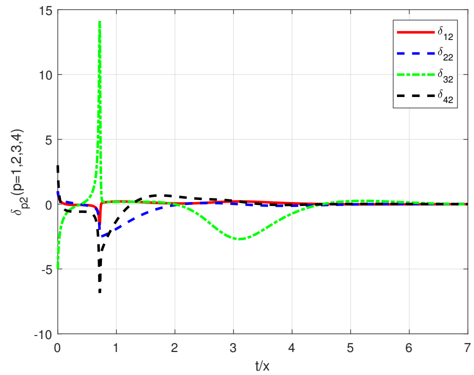
.

$$\bar{C_\sigma} = \begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$L_\sigma = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ -0.8 & 0 & 0.8 & 0 \\ 0 & -0.6 & 0 & 0.6 \end{pmatrix}$$



(a) α_{i1}



(b) α_{i2}

Figure 2: Resilient base CC errors $\alpha_i(t)$ of FOMAS (3) under communication topology with delayed control protocol (12) .

$$W_\sigma \;=\; L_\sigma \;+\; \bar{C_\sigma} \;=\; \begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ -0.8 & 0 & 0.8 & 0 \\ 0 & -0.6 & 0 & 0.6 \end{pmatrix}$$

$$Take\ K_1 \;=\; 0.6,\; K_2 \;=\; 1.3,\;\; then \;\; E \;\;=\;\; \begin{pmatrix} 1.4 & 0.2 \\ 0.2 & 1.4 \end{pmatrix}$$

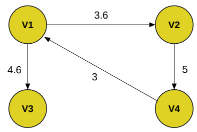
can be chosen to address the inequality (22 . A quick computation allows for the selection of

$$K \;\;=\begin{pmatrix} 0.3 & 0.4 \\ 0 & 0.2 \end{pmatrix}$$

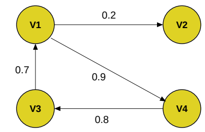
to address the inequality (21.

As a result, FDMAS (5-(6) under protocol (7) is obtained from leader and follower global cyber secure consensus , as stated by Corollary (3.5). Let $j = 1,\ \tau_\sigma(t) = 0.6$. Fig. 2 shows the syber secure consensus errror $\delta_p(t)$.

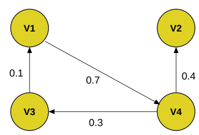
Example 5.2. Examine the FDMAS (5) and (23) have the topology is depicted in Fig. 3.



(a) G_1



(b) G_2



(c) G_3

Figure 3: Different switching topology under DoS attack.

We have, $h_1(t, y_p(t)) = \frac{5}{11} y_p(t), h_2(t, y_p(t - \tau_\sigma(t))) = \frac{2}{11} sin y_p(t - \tau_\sigma(t))$, and

$$A = \begin{pmatrix} -7 & -0.6 \\ 0 & -8 \end{pmatrix}$$

. According to $(H1)$ take $l_1 = 0.4, l_2 = 0.2$. From Fig. 3 we have

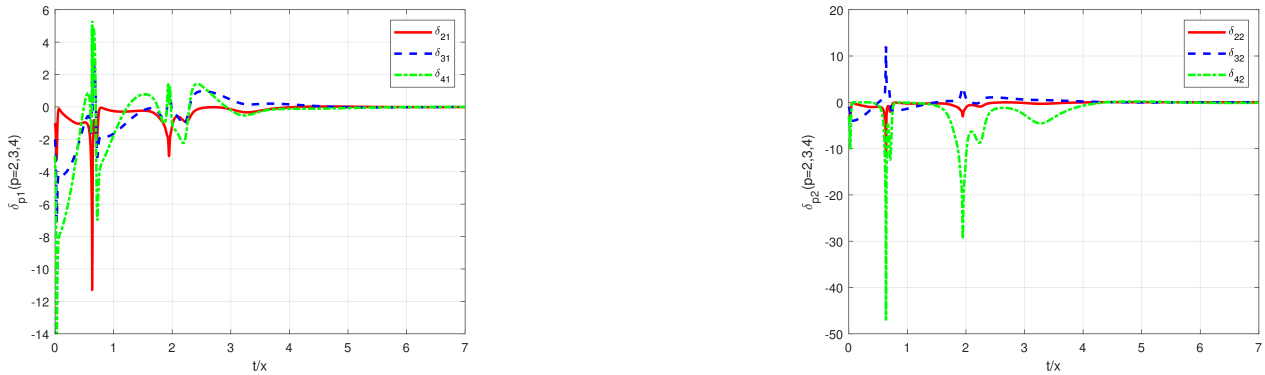
$$B_\sigma = \begin{pmatrix} 0 & 0 & 0 & 3 \\ 3.6 & 0 & 0 & 0 \\ 4.6 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \end{pmatrix}$$

.

$$L_\sigma = \begin{pmatrix} 3 & 0 & 0 & -3 \\ -3.6 & 3.6 & 0 & 0 \\ -4.6 & 0 & 4.6 & 0 \\ 0 & -5 & 0 & 5 \end{pmatrix}$$

$$R_\sigma = (l_{pq}^\sigma + b_{1q}^\sigma)_{(m-1)\times(m-1)} = \begin{pmatrix} 3.6 & 0 & 3 \\ 0 & 4.6 & 3 \\ -5 & 0 & 8 \end{pmatrix}, \quad p, q = 2, ..., m.$$

$$Take\ K_1 = 1.6,\ K_2 = 0.6,\ then\ \ E = \begin{pmatrix} 4 & 0.2 \\ 0.2 & 3.6 \end{pmatrix}$$



(a) α_{i1} (b) α_{i2}

Figure 4: Cyber secure consensus error $\delta_p(t)$ under switching topology.

can be chosen to satisfy the inequality (30).

A quick computation allows for the selection of

$$K = \begin{pmatrix} 0.4 & 0.2 \\ 0 & 0.3 \end{pmatrix}$$

to meet the inequality (30).

As a result, FDMAS (5)-(23) is obtained from leaderless global cyber secure consensus , as stated by Corollary (4.2). Let $q = 0.8$, $\tau_\sigma(t) = 0.7$. Fig. 4 shows the syber secure consensus errror $\delta_p(t)$.

6 Conclusion

The global cyber secure consensus of FDMASs in the Riemann-Liouville sense has been examined from both a leader-following and leaderless perspective. By using the classical Lyapunov direct approach many linear matrix inequalities are obtained . The method enables us to extract the Lyapunov function's integer-order derivative, which offers a quick and practical way in the Riemann-Liouville sense to talk about the cyber secure consensus, attractiveness and stability of fractional-order delayed differential systems . In particular, the suggested approach is capable of effectively addressing the challenge brought on by delays. In addition, FDMASs' coupling topology is a directed switching topology, which can be used to express a wide variety of real-world models. Future research interests include fractional-order multi-agent systems with switching topologies and the consensus of Riemann-Liouville fractional-order systems with distributed delay.

7 Declaration of Competing Interest

The authors state that they are clear of any financial conflicts of interest or close personal connections that might have seemed to have an effect on the research presented in this study.

Data availability statement:

No new data were created this study.

References

[1] X. Huo , L. Ma , X. Zhao , B. Niu , G. Zong , Observer-based adaptive fuzzy tracking control of MIMO switched nonlinear systems preceded by unknown backlash–like hysteresis, Inf. Sci. 490 (2019) 369–386.

[2] X. Zhao, X. Wang, L. Ma, G. Zong, Fuzzy-approximation-based asymptotic track- ing control for a class of uncertain switched nonlinear systems, IEEE Trans. Fuzzy Syst. (2019), doi: 10.1109/TFUZZ.2019.2912138.

[3] X.H. Chang, R. Huang, H. Wang, L. Liu, Robust design strategy of quantized feedback control, IEEE Trans. Circ. Syst. II Express Briefs (2019), doi: 10.1109/ TCSII.2019.2922311.

[4] F.D. Saraiva , W.M.S. Bernardes , E.N. Asada , A framework for classification of non-linear loads in smart grids using artificial neural networks and multi-a- gent systems, Neurocomputing 170 (2015) 328–338.

[5] M. Xu , K. An , L.H. Vu , Z. Ye , J. Feng , E. Chen , Optimizing multi-agent based urban traffic signal control system, J. Intell. Transp. Syst. 23 (4) (2019) 357–369.

[6] W. Zhu , Z.P. Jiang ,Event-based leader-following consensus of multi-agent systems with input time delay, IEEE Trans. Autom. Control 60 (5) (2015) 1362–1367 .

[7] W. Ni , D. Cheng , Leader-following consensus of multi-agent systems un- der fixed and switching topologies, Syst. Control Lett. 59 (2010) 209–217 .

[8] X. Wang , H. Su , Consensus of hybrid multi-agent systems by event-trig- gered/self-triggered strategy, Appl. Math. Comput. 359 (2019) 490–510 .

[9] J.Lu , X. Guo , T. Huang , Z. Wang , Consensus of signed networked multi-agent systems with nonlinear coupling and communication delays, Appl. Math. Com- put. 350 (2019) 153–162.

[10] J. Hu , Y. Wu , T. Li , B.K. Ghosh , Consensus control of general linear multiagent systems with antagonistic interactions and communication noises, IEEE Trans. Autom. Control 64 (5) (2019) 2122–2127.

[11] J. Hu , On robust consensus of multi-agent systems with communication delays, Kybernetika 45 (5) (2009) 768–784 .

[12] Y. Wu , Y. Zhao , J. Hu , Bipartite consensus control of high-order multiagent sys- tems with unknown disturbances, IEEE Trans. Syst. Man Cybern. Syst. 49 (10) (2019) 2189–2199.

[13] J. Qin , H. Gao , W.X. Zheng ,Second-order consensus for multi-agent systems with switching topology and communication delay, Syst. Control Lett. 60 (2011) 390–397.

[14] W. Ni , C. Xiong , J. Yang , Leader-following consensus of high-order multi-agent linear systems with bounded transmisstion channels, Int. J. Syst. Sci. 44 (9) (2013) 1711–1725.

[15] D. Shen , J.X. Xu , Distributed learning consensus for heterogenous high-order nonlinear multi-agent systems with output constrains, Automatica 97 (2018) 64–72 .

[16] J. Fu , G. Wen , W. Yu , T. Huang , X. Yu , Consensus of second-order multiagent systems with both velocity and input constraints, IEEE Trans. Ind. Electron. 66 (10) (2019) 7946–7955.

[17] L. Chen , R. Wu , Z. Chu , Y. He , L. Yin , Pinning synchronization of fraction- al-order delayed complex networks with non-delayed and delayed couplings, Int. J. Control 90 (6) (2017) 1245–1255 .

[18] Y. Zhou , B. Ahmad , F. Chen , A. Alsaedi , Oscillation for fractional partial differ- ential equations, Bull. Malays. Math. Sci. Soc. 42 (2) (2019) 449–465.

[19] S. Liu , W. Jiang , X. Li , X.F. Zhou , Lyapunov stability analysis of fractional non- linear systems, Appl. Math. Lett. 51 (2016) 13–19 .

[20] Z. Yu , H. Jiang , C. Hu , Leader-following consensus of fractional-order mul- ti-agent systems under fixed topology, Neurocomputing 149 (2015) 613–620 .

[21] P. Gong , Distributed consensus of non-linear fractional-order multi-agent sys- tems with directed topolpgies, IET Control Theory Appl. 10 (2016) 2515–2525 .

[22] F. Wang , Y. Yang , Leader-following exponential consensus of fractional or- der nonlinear multi-agents system with hybrid time-varying delay:a hetero- geneous impulsive method, Phys. A 482 (2017) 158–172.

[23] A. Wang , X. Liao , T. Dong , Fractional-order follower observer design for track- ing consensus in second-order leader multi-agent systems: periodic sam- pled-based event-triggered control, J. Frankl. Inst. 355 (11) (2018) 1628–4618

[24] J. Bai , G. Wen , A. Rahmani , Leaderless consensus for the fractional-order non- linear multi-agent systems under directed interaction topology, Int. J. Syst. Sci. 49 (5) (2018) 954–963.

[25] J. Bai , G. Wen , A. Rahmani , Y. Yu , Consensus for the fractional-order double-in- tegrator multi-agent systems based on the sliding mode estimator, IET Control Theory Appl. 12 (5) (2018) 621–628 .

[26] W. Zhu , W. Li , P. Zhou , C. Yang ,Consensus of fractional-order multi-agent systems with linear models via observer-type protocol, Neurocomputing 230 (2017) 60–65.

[27] Z. Yu , H. Jiang , C. Hu , J. Yu , Leader-following consensus of fractional-order multi-agent systems via adaptive pinning control, Int. J. Control 88 (9) (2015) 1746–1756.

[28] L. Chen , W. Pan , R. Wu , Y. He ,New result on finite-time stability of fraction- al-order nonlinear delayed systems, J Comput Nonlinear Dyn 10 (6) (2015) 064504.

[29] S. Liu , X. Wu , X.F. Zhou , W. Jiang , Asymptotical stability of Riemann–Liouville fractional nonlinear systems, Nonlinear Dyn. 86 (1) (2016) 65–71 .

[30] H. Zhang , R. Ye , J. Cao , A. Alsaedi , Delay-independent stability of Rieman- n–Liouville fractional neutral-type delayed neural networks, Neural Process. Lett. 47 (2) (2018) 427–442.

[31] D. Luo , J.R. Wang , D. Shen , Learning formation control for fraction- al-order multiagent systems, Math. Methods Appl. Sci. 41 (13) (2018) 5003–5014.

[32] X. Liu , Z. Zhang , X. Li , Consensus control of fractional-order systems based on delayed state fractional order derivative, Asian J. Control 19 (2017) 2199–2210.

[33] J. Liu , K. Qin , P. Li , W. Chen , Distributed consensus control for double-integrator fractional-order multi-agent systems with nonuniform time-delays, Neurocom- puting 321 (2018) 369–380 .

[34] J. Shen , J. Cao , Necessary and sufficient conditions for consensus of delayed fractional-order systems, Asian J. Control 14 (6) (2012) 1690–1697.

[35] W. Zhu , B. Chen , J. Yang , Consensus of fractional-order multi-agent systems with input time delay, Fract. Calc. Appl. Anal. 20 (2017) 52–70.

[36] R. Yang , S. Liu , Y.Y. Tan , Y.J. Zhang , W. Jiang , Consensus analysis of fraction- al-order nonlinear multi-agent systems with distributed and input delays, Neu- rocomputing 329 (2019) 46–52.

[37] S. Liu , R. Yang , X.F. Zhou , W. Jiang , X.Y. Li , X.W. Zhao , Stability analysis of frac- tional delayed equations and its applications on consensus of multi-agent sys- tems, Commun. Nonlinear Sci. Numer. Simul. 73 (2019) 351–362.

[38] Y. Yang, Y. Li, D. Yue, Y. Tian, and X. Ding, “Distributed secure consensus control with event-triggering for multiagent systems under DoS attacks,” IEEE Trans. Cybern., vol. 51, pp.2916–2928, 2021.

[39] D. Zhang, L. Liu, and G. Feng, “Consensus of heterogeneous linear multi agent systems subject to aperiodic sampled-data and DoS attack,” IEEE Trans. Cybern., vol. 49, pp. 1501–1511, 2019.

[40] L. Zhao, and G. Yang, "Adaptive fault-tolerant control for nonlinear multi-agent systems with DoS attacks," Inf. Sci., vol. 526, pp. 39–53, 2020.

[41] Z. Feng, and G. Hu, "Secure cooperative event-triggered control of linear multiagent systems under DoS attacks," IEEE T. Contr. Syst. T., vol. 28, pp. 741–752, 2020.

[42] W. Yu , G. Wen , G. Chen , J. Cao , Distributed Cooperative Control of Multi-agent Systems, Higher Education Press, Beijing, 2016.

[43] I. Podlubny , Fractional Differential Equations, Academic Press, New York, 1999.

[44] Z. Xu , K.Y. Zhang , Q. Lu , G.W. Leng , Concise Tutorial of Matrix Theory, Science Press, Beijing, 2001. (In Chinese)

[45] S. Liu , X.F. Zhou , X. Li , W. Jiang , Asymptotical stability of Riemann–Liouville fractional singular systems with multiple time-varying delays, Appl. Math. Lett. 65 (2017) 32–39.