Monte Carlo method applied to the reliability analysis referred to the Global Integrity Measure

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Summary. In continuation to [1], this paper gives a further step towards an improved reliability analysis of a dynamical system attractor, based on its dynamic integrity. Although the ideas are applicable to a broader class of problems within dynamical systems theory, they will be illustrated to discuss the load-carrying capacity of an imperfection-sensitive structure liable to buckle. The analysis uses the Monte Carlo method to assess the probability that the Global Integrity Measure is equal or larger than a prescribed value, considering the probability density functions of both the applied load and the imperfection parameter, provided the former is not larger than the buckling strength, Hence, a reliability measure can be made available to help engineers deciding whether a particular design is *reliable, given it is safe*, contributing to the improvement of current design practices.

Introduction: mathematical model

Effectively, this issue was initially addressed in [1], and a naïve reliability analysis was carried out for the load-carrying capacity assessment of the archetypal model liable to buckle that was formerly studied in [2] (see Fig. 1). It is a rigid bar hinged at the bottom and constrained laterally by an inclined linear elastic spring. The buckling strength (p_S) of the non-deterministic imperfect system under axial load (p) is sought in presence of an imperfection simulated by a small lateral load (q). Upper case parameters/variables are dimensional; lower case parameters/variables are dimensionless. The perfect model (q=0) exhibits asymmetric point of bifurcation. The safe basins of the imperfect model for different levels of the applied load are enclosed by homoclinic loops, as seen in [2].

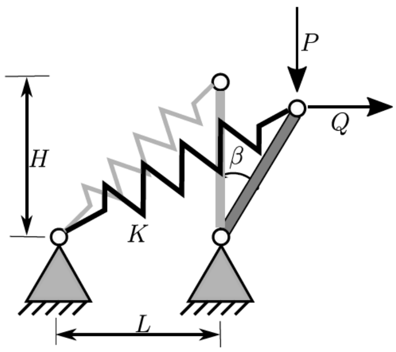

Fig. 1. Archetypal model, adapted from [2].

From [2], the area inside the homoclinic loops, herein considered as the one of interest in terms of reliability, is

$$A(p) = \frac{128}{15} 6^{5/8} \frac{q^{5/8}}{\alpha^{11/4}} (p - p_K)^{5/4} \tag{1}$$

where $p = \frac{P}{KL}$, $q = \frac{P}{KL}$ and $\alpha = \frac{2LH}{L^2+H^2}$. In Eq. 1, extracted from [2] after an asymptotic analysis, p_K is Koiter's load for the deterministic system, written as a function of the imperfection q, and $p_E = \alpha/2$ is Euler's load:

$$p_K = p_E - \alpha \frac{\sqrt{6}}{2} q^{1/2} \tag{2}$$

Monte Carlo method for assessing safety and reliability of the solution is set as follows. Firstly, we sample values for the load p and for the imperfection q. These two quantities are assumed to be independent and described by Gaussian distributions, according to Eq. 3.

$$f(p) = \frac{1}{\sqrt{2\pi}\,\sigma_p} e^{-\frac{(p-\bar{p})^2}{2\sigma_p{}^2}} \qquad\qquad f(q) = \frac{1}{\sqrt{2\pi}\,\sigma_q} e^{-\frac{(q-\bar{q})^2}{2\sigma_q{}^2}} \tag{3}$$

Next, for the sampled value of q, we compute the buckling strength p_S for the imperfect and stochastic system using Eq. 2. Notice that we define Koiter's load as the one for the imperfect and deterministic system, whereas the buckling strength refers to the imperfect and stochastic one. If $p \leq p_S$, the system is considered safe and the area within the corresponding homoclinic loop is obtained from Eq. 1. A first requirement for reliability of the solution is $p_S \geq p_{S,ref}$, where $p_{S,ref}$ is a reference value for the buckling strength. A second requirement for reliability is related to the Global Integrity Measure (GIM) [3], namely that it should be larger or equal to a reference value GIM_{ref}. In [2], GIM was directly associated to $A(p)$, after normalization of the area within the homoclinic loop with respect to that of the unloaded system.

$$GIM = \frac{A(p)}{A(0)} = \frac{A(p)}{1.720792} \tag{4}$$

Example

In the numerical analysis, $N_{Tests} = 50{,}000$ simulations of Monte Carlo method are carried out considering $\bar{p} = 0.1$, $\bar{q} = 0.063$, $\sigma_p = 0.02$ and $\sigma_q = 0.01$. Reference values for the reliability analysis are $p_{s,ref} = 0.155$ and $GIM_{ref} = 0.150$. The histograms of Fig. 2 give the relative frequency with respect to N_{Tests}.

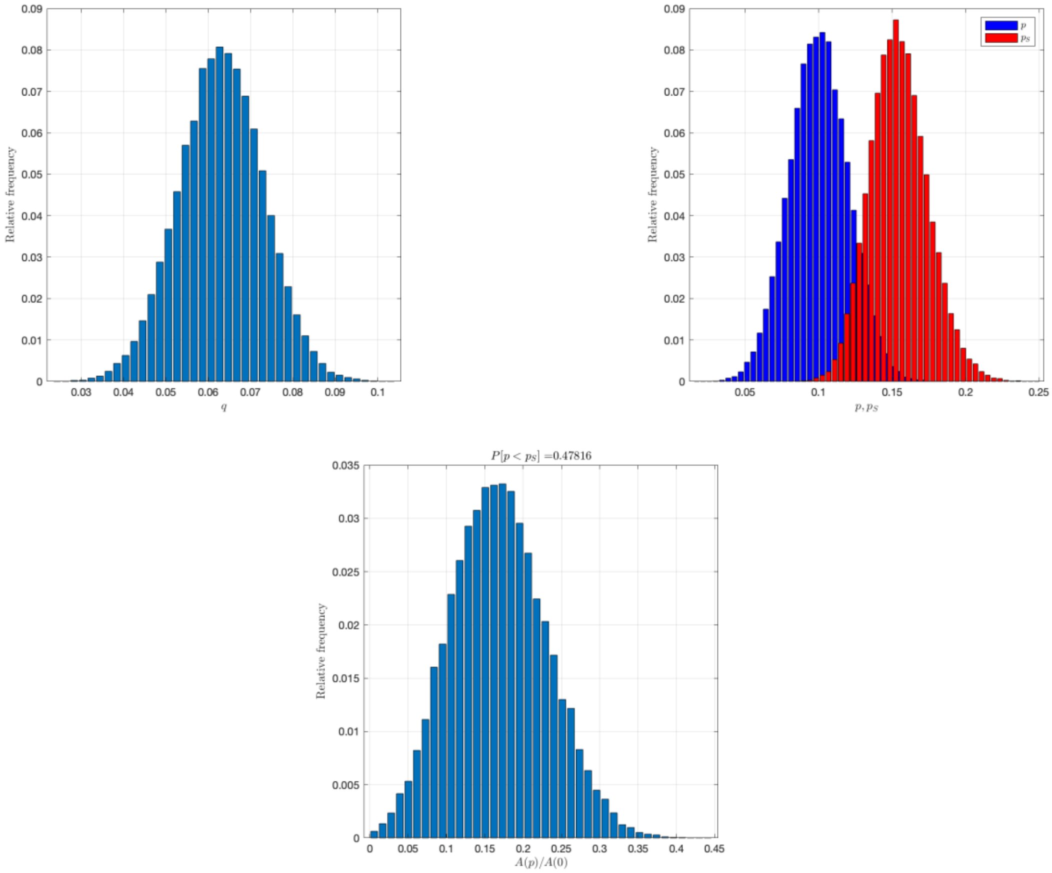

Fig. 2. Histograms obtained with Monte Carlo simulations.

The probability that GIM is at least the reference value $GIM_{ref} = 0.150$ among the safe solutions is found to be 29.3%.

Conclusions

The reliability analysis referred to the dynamic integrity has been pursued resorting to Monte Carlo method applied directly to the expression of the area of the attractor's safe basin, which is found to be dependent on the load parameter p and the imperfection parameter q of the studied system. Both p and q are assumed as stochastic variables, the latter being also used to determine the buckling strength p_s. Safety is imposed by $p \leq p_s$, whereas reliability is implied by imposing that the buckling strength is at least equal to a minimum reference value ($p_{s,ref} \leq p_s$) and GIM is at least a minimum reference value ($GIM_{ref} \leq GIM$). The result obtained in this example indicates that, with a probability of 29.3% of being reliable and safe, Thompson's load [2] could be chosen as $p_T = p_{s,ref} = 0.155$ (39% of Euler's load). It is a matter of engineering judgement to decide whether this is a good assumption for the system's load-carrying capacity or not. Should this probability be found low, the analysis would have to be repeated for a lower reference buckling strength.

References

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[3] Soliman, M.S.,Thompson, J.M.T. (1989) Integrity measures quantifying the erosion of smooth and fractal basins of attraction, *J. Sound Vib.* **135**: 453–475.