Piezoelectric energy harvesting from galloping of a prism mounted on a nonlinear support

Guilherme Rosa Franzini[*,†], Rodrigo Provasi[*,†], Guilherme Jorge Vernizzi[*], Carlos Eduardo Nigro Mazzilli[†] and Marcelo Amorim Savi[‡]

[*]*Offshore Mechanics Laboratory, Escola Politécnica, University of São Paulo, Brazil*

[†]*Department of Structural and Geotechnical Engineering, Escola Politécnica, University of São Paulo, Brazil*

[‡] *Centre of Nonlinear Mechanics, Federal University of Rio de Janeiro, Brazil*

Summary. The piezoelectric energy harvesting from the galloping of a rigid prism is numerically investigated. The prism is mounted on a bimorph piezoelectric cantilevered beam and on a nonlinear spring whereas the aerodynamic force is computed using the classical quasi-steady approach. The results for four configurations are presented in this extended abstract: two bistable nonlinear energy harvester, a monostable nonlinear energy harvester and a linear energy harvester. Among other aspects, the results show evidences of chaotic responses for the bistable configurations.

Introduction

Flow-induced vibration (FIV) phenomena consist of a potential source of energy for charging low-power equipment such as small sensors [1]. Among different FIV phenomena that can be employed for energy harvesting purposes, galloping is widely studied by the scientific community, as can be seen, for example, in [2].

Recently nonlinearities have been added to the energy harvesting systems as a form of improving their efficiency. In this scenario and depending on a set of parameters to be tuned, the nonlinear restoring force can lead to the presence of two or more stable equilibrium points (bistability or multistability, respectively). Reference [3] is an example of investigation of multistability as a way for improving energy harvesting from FIV.

This paper aims at contributing in the analysis of piezoelectric energy harvesting from the galloping phenomenon considering nonlinearities proportional to the prism displacement. Numerical simulations are carried out and bifurcation diagrams are obtained for some parameters associated with these nonlinearities.

Formulation and methodology

This contribution deals with the energy harvesting problem of a rigid prism under galloping, mounted at the tip of cantilevered piezoelectric beam and associated with a nonlinear spring. Figure 1(a) depicts a schematic representation of the investigated problem.

For conciseness of this extended abstract, details regarding the formulation are not shown but they arise from the formulation on the continuum for the piezoelectric beam, discretized using the Galerkin method. The dimensionless mathematical model is described by Eqs. 1 and 2, while the force coefficient due to galloping follows the quasi-steady approach [4].

$$\ddot{a} + 2\zeta\dot{a} + (1+\beta_1)a + \beta_3 a^3 - v = \frac{U_r^2}{2m^*}C_z = \frac{U_r^2}{2m^*}\sum_{k=1}^{N} C_k \left(\frac{\dot{a}}{U_r}\right)^k \tag{1}$$

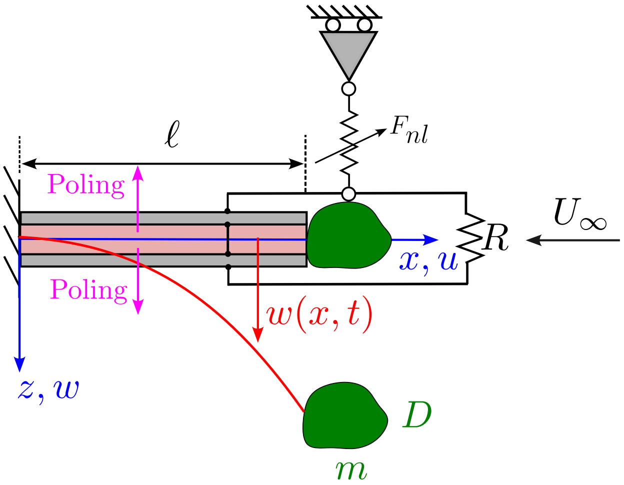
$$\dot{v} + 2\sigma_1\dot{a} + 2\sigma_2 v = 0 \tag{2}$$

In the above equations, ζ is the structural damping, β_1 and β_3 are terms related to the nonlinear spring, a is the prism displacement, v the electric tension, U_r the reduced velocity and m^* the mass ratio parameter. σ_1 and σ_2 are parameters associated with the electromechanical coupling the the electric resistance, respectively. N is the order of the polynomial function adopted in the quasi-steady hypothesis and C_k are the corresponding coefficients. The mathematical model is numerically integrated using DifferentialEquations.jl package written in Julia Language [5] and the bifurcation diagrams are associated with the intersection of the trajectories with the Poincaré section defined as $\Sigma = \{(a, v, \dot{a}) \in \mathbb{R}^3 : \dot{a} = 0, \ddot{a} < 0\}$, corresponding to local maxima of $a(t)$. For the first value of the bifurcation parameter, the initial condition employed in the numerical simulation corresponds to the equilibrium position and a small disturbance in the velocity $\dot{a}(0) = 0.1$.

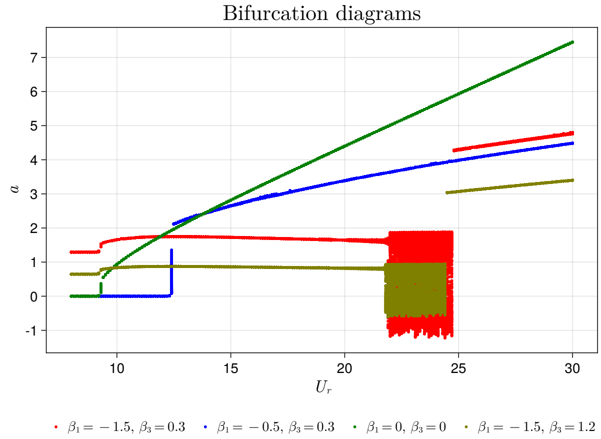
Results and Conclusions

We consider coefficients C_k from the experimental investigation with a square prism under galloping presented in [6]. Particular reference configurations for the piezoelectric beam and the electric resistance are taken for the sake of an example. The associated dimensionless quantities are: $m^* = 294$, $\sigma_1 = 0.162$, $\sigma_2 = 0.054$ and $\zeta = 0.01$. Reduced velocity U_r is considered as the bifurcation parameter. The configurations for the nonlinear spring are herein studied: $\beta_1 = -1.5$ and $\beta_3 = 0.3$ or $\beta_1 = 1.2$ (bistable nonlinear harvesters), $\beta_1 = -0.5$ and $\beta_3 = 0.3$ (monostable nonlinear harvester) and $\beta_1 = 0$ and $\beta_3 = 0$ (linear harvester).

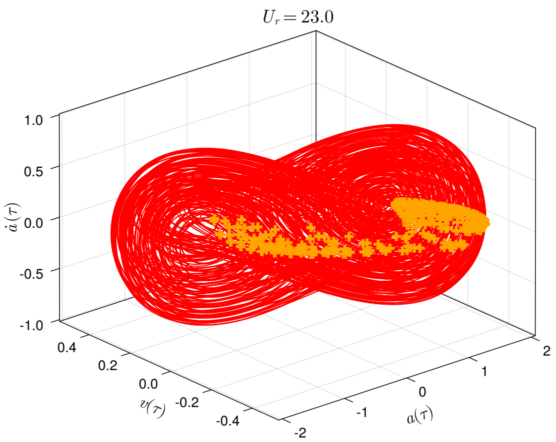
The bifurcation diagram shown in Fig. 1(b) reveals the presence of a possible chaotic response for the bistable systems in the interval $22 \leq U_r \leq 25$. After this value of reduced velocity, the maximum displacement of the bistable prism with $\beta_3 = 0.3$ is larger than the ones observed for the monostable nonlinear system.
Another two aspects should be emphasized in this extended abstract. The first one is the increase in the critical velocity associated with the Hopf bifurcation observed for the monostable nonlinear harvester. The second aspect is that, in opposition to commonly found in the literature, the energy harvesting performances of the bistable systems are the worst among the considered cases, as can be inferred from the 3D trajectories presented in Figs. 1(c) and 1(d). Comprehensive sensitivity studies focusing on the influence of both the parameters of the mathematical model and the initial conditions will be presented at the conference.



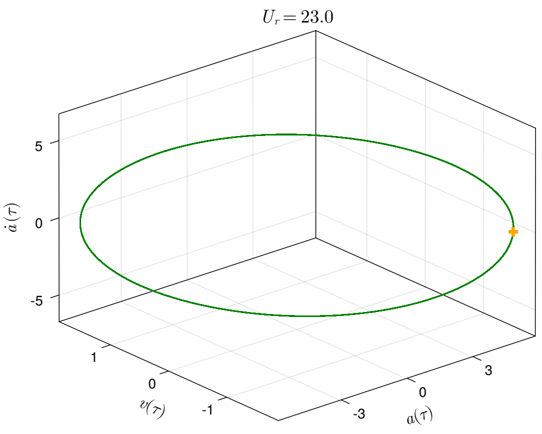
(a) Problem representation.



(b) Bifurcation diagram.



(c) 3D trajectory - $\beta_1 = -1.5, \beta_3 = 0.3$.



(d) 3D trajectory - $\beta_1 = 0, \beta_3 = 0$.

Figure 1: Problem representation and examples of results. The orange crosses represent the intersections with the Poincaré section.

Acknowledgments

The authors thank São Paulo Research Foundation (FAPESP) for the grants 2022/02959-9 and 2022/00770-0. First, fourth and fifth authors are also grateful to the National Council for Scientific and Technological Development (CNPq) for the grants 305945/2020-3 XX, YY).

References

[1] Wang J., Yurchenko D, Hu G. Zhao L, Tang L, Yang Y (2021) Perspectives in flow-induced vibration energy harvesting. *App. Phys. Letts.* **119(10)**.

[2] Hémon P, Amandolese X, Andrianne T (2017) Energy harvesting from galloping of prisms: A wind tunnel experiment. *J. Fluids and Struct.* **70**:390-402.

[3] Zhou Z., Qin W., Zhu P., Du W.(2021) Harvesting more energy from variable-speed wind by a multi-stable configuration with vortex-induced vibration and galloping. *Energy* **237**, 121551.

[4] Blevins R. D. (2001) Flow-induced Vibrations. Krieger Publishing Company, 2nd edition.

[5] Rackauckas C., Nie Q (2017) DifferentialEquations.jl-a performant and feature-rich ecosystem for solving differential equations in Julia. *J. Open Res Software*, **1, vol. 5**.

[6] Selwanis M.M., Franzini, G.R., Béguin, C., Gosselin, F.P. (2023) How a ball free to orbit in a circular track mitigates the galloping of a square prism. *Nonlinear Dyn.*, **11**:179-197.