

No. 1

a) $W = \{(x_1, x_2, x_1 + x_2) \mid x_1, x_2 \in \mathbb{R}\}$

zero vector

$$\begin{aligned} x_1 &= 0 & (0, 0, 0+0) \\ x_2 &= 0 & (0, 0, 0) \checkmark \end{aligned}$$

Closure under addition

$$\begin{aligned} \vec{u} &= (u_1, u_2, u_1 + u_2) \\ \vec{v} &= (v_1, v_2, v_1 + v_2) \end{aligned}$$

$$\vec{u} + \vec{v} = \left(\frac{u_1+v_1}{x_1}, \frac{u_2+v_2}{x_2}, \frac{u_1+u_2+v_1+v_2}{x_1+x_2} \right) \in \mathbb{R}^3$$

Closure under scalar multiplication

$$a\vec{u} = (au_1, au_2, au_1 + au_2) \in \mathbb{R}^3$$

a (any scalar number)

$\because a$ is a subspace of \mathbb{R}^3

$$0 = 0x_1, 0x_2, 0x_1 + 0x_2$$

$$\checkmark (0, 0, 0)$$

not closure under scalar multiplication

$$(0.0, 0.0) \in \mathbb{R}^2$$

$$(0.0, 0.0) \in \mathbb{R}^3$$

b) $W = \{x, x^2, x^3 \mid x \in \mathbb{R}\}$

zero vector

$$\begin{aligned} x &= 0 \\ (0, 0^2, 0^3) &= (0, 0, 0) \checkmark \end{aligned}$$

Closure under addition

$$\vec{a} = (a, a^2, a^3)$$

$$\vec{b} = (b, b^2, b^3)$$

$$\vec{a} + \vec{b} = \left(\frac{a+b}{x}, \frac{a^2+b^2}{x^2}, \frac{a^3+b^3}{x^3} \right)$$

$$a^2 + b^2 \neq (a+b)^2$$

Problem 2:

a) $W = \{(\alpha x, y) \mid x, y \in \mathbb{R}\}$

zero vector

$$\begin{aligned} x &= 0 \\ y &= 0 \quad (0, 0, 0) \checkmark \end{aligned}$$

Closure under addition

$$\vec{a} = (0, a_1, a_2)$$

$$\vec{b} = (0, b_1, b_2)$$

$$\vec{a} + \vec{b} = (0, a_1 + b_1, a_2 + b_2) \in \mathbb{R}^3$$

Closure under scalar multiplication

$$\alpha\vec{b} = (0, ab_1, ab_2) \in \mathbb{R}^3$$

b) $W = \{x, y, 5x+3y \mid x, y \in \mathbb{R}\}$

zero vector

$$\begin{aligned} x &= 0 & (0, 0, 5(0)+3(0)) \\ y &= 0 & (0, 0, 0) \checkmark \\ (0, 0, 0) &\checkmark \end{aligned}$$

Closure under addition

$$\vec{a} = (a_1, a_2, 5a_1 + 3a_2)$$

$$\vec{b} = (b_1, b_2, 5b_1 + 3b_2)$$

$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, 5a_1 + 3a_2 + 5b_1 + 3b_2) \in \mathbb{R}^3$$

→ closure under scalar multiplication

$$c\vec{a} = (ca_1, ca_2, ca_3) \in \mathbb{R}^3$$

$\boxed{a \text{ and } b \text{ of problem 2
are both subspaces
of } \mathbb{R}^3}$

problem 3

a) $W = \{(x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 0\} \subseteq \mathbb{R}^3$

zero vector

$$x_1 + x_2 + x_3 = 0$$

$$(0, 0, 0) \checkmark$$

closure under addition

$$\vec{a} = (a_1, a_2, a_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$= (0, 0, 0) \checkmark$$

closure under scalar multiplication

$$a\vec{b} = (a_1b_1 + a_2b_2 + a_3b_3)$$

$$= (0) \checkmark$$

$\boxed{\text{or is a subspace of } \mathbb{R}^3}$

b) $W = \{(x, y, z) \mid xy = 0\}$

zero vector

$$(0, 0, 0) \checkmark$$

closure under addition

$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} + \vec{b} = (b_1, a_2, a_3 + b_3)$$

$$(b_1)(a_2)(b_3 + a_3) \neq 0 \times$$