

no. 1

$$a) W = \{ (x_1, x_2, x_1 + x_2) \mid x_1, x_2 \in \mathbb{R} \}$$

zero vector

$$\begin{aligned} x_1 &= 0 & (0, 0, 0+0) \\ x_2 &= 0 & (0, 0, 0) \checkmark \end{aligned}$$

closure under addition

$$\begin{aligned} \vec{u} &= (u_1, u_2, u_1 + u_2) \\ \vec{v} &= (v_1, v_2, v_1 + v_2) \\ \vec{u} + \vec{v} &= (u_1 + v_1, u_2 + v_2, u_1 + u_2 + v_1 + v_2) \in \mathbb{R}^3 \end{aligned}$$

closure under scalar multiplication

$$a\vec{u} = (au_1, au_2, au_1 + au_2) \in \mathbb{R}^3 \checkmark$$

a (any scalar number)

$\therefore W$ is a subspace of \mathbb{R}^3

$$b) W = \{ (x, x^2, x^3) \mid x \in \mathbb{R} \}$$

zero vector

$$\begin{aligned} x &= 0 \\ (0, (0)^2, (0)^3) \\ (0, 0, 0) \checkmark \end{aligned}$$

closure under addition

$$\begin{aligned} \vec{a} &= (a, a^2, a^3) \\ \vec{b} &= (b, b^2, b^3) \\ \vec{a} + \vec{b} &= (a+b, a^2+b^2, a^3+b^3) \end{aligned}$$

$$a^2 + b^2 \neq (a+b)^2$$

Problem 2:

$$a) W = \{ (0, x, y) \mid x, y \in \mathbb{R} \}$$

zero vector

$$\begin{aligned} x &= 0 \\ y &= 0 \\ (0, 0, 0) \checkmark \end{aligned}$$

closure under addition

$$\begin{aligned} \vec{a} &= (0, a_1, a_2) \\ \vec{b} &= (0, b_1, b_2) \\ \vec{a} + \vec{b} &= (0, a_1 + b_1, a_2 + b_2) \in \mathbb{R}^3 \checkmark \end{aligned}$$

closure under scalar multiplication

$$a\vec{b} = (0, ab_1, ab_2) \in \mathbb{R}^3 \checkmark$$

$$b) W = \{ (x, y, 5x + 3y) \mid x, y \in \mathbb{R} \}$$

zero vector

$$\begin{aligned} x &= 0 \\ y &= 0 \\ (0, 0, 5(0) + 3(0)) \\ (0, 0, 0) \checkmark \end{aligned}$$

closure under addition

$$\begin{aligned} \vec{a} &= (a_1, b_1, 5a_1 + 3b_1) \\ \vec{b} &= (a_2, b_2, 5a_2 + 3b_2) \\ \vec{a} + \vec{b} &= (a_1 + a_2, b_1 + b_2, 5(a_1 + a_2) + 3(b_1 + b_2)) \in \mathbb{R}^3 \checkmark \end{aligned}$$

→ closure under scalar multiplication

$$\alpha \vec{c} = (\alpha a_1, \alpha b_1, \alpha(a_1 + 3b_1)) \in \mathbb{R}^3 \checkmark$$

a and b of problem 2
are both subspaces
of \mathbb{R}^3

Problem 3

$$a) W = \{ (x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 0 \}$$

zero vector

$$x_1 + x_2 + x_3 = 0$$

$$(0, 0, 0) \checkmark$$

closure under addition

$$\vec{a} = (a_1, a_2, a_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$= (0, 0, 0) \checkmark$$

closure under scalar multiplication

$$\alpha \vec{b} = (\alpha b_1, \alpha b_2, \alpha b_3)$$

$$= (0, 0, 0) \checkmark$$

W is a subspace of \mathbb{R}^3

$$b) W = \{ (x, y, z) \mid xyz = 0 \}$$

zero vector

$$(0, 0, 0) \checkmark$$

closure under addition

$$\vec{a} = (a_1, a_2, a_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$(a_1)(a_2)(b_3 + a_3) \neq 0 \checkmark$$