hw1

1.

```
fn RealRoot(a,b,c):
        if(a ==0)
                return -c/b
        if(b*b-4*a*c  >=0)
                return (-b+sqrt(b*b-4*a*c))/(2*a), (-b-sqrt(b*b-
4*a*c))/(2*a)
        else:
                return 'no real roots'
```

2.

a.

best case is choose first two can pair
best case: 2
worse case is first choose every shoes left side, and choose next will pair
worse case: 12

b.

in 1/9 case will remain 4 pair socks can wear,
in 8/9 case will remain 3 pair socks can wear
$E = \frac{1}{9} \cdot 4 + \frac{8}{9} \cdot 3$

3.

$f(n) = n^2 + 3n^3$

$3n^3 \leq n^2 + 3n^3$, $n \geq 0$
=>$\Omega(n^3)$
$n^2 + 3n^3 \leq 4n^3$, $n \geq 1$
=>$O(n^3)$

4.

1. $f_3 = 2 \geq (\frac{1+\sqrt{5}}{2})^{3-2}$ is true when n=3

2. let

$$f_n = f_{n-1} + f_{n-2} \geq (\frac{1+\sqrt{5}}{2})^{n-2}$$

is true

3. $(\frac{1+\sqrt{5}}{2}+1) = 2.61803398875 \geq (\frac{1+\sqrt{5}}{2})^2 = 2.61803398875$ is true

$$(\frac{1+\sqrt{5}}{2}+1) \geq (\frac{1+\sqrt{5}}{2})^n/(\frac{1+\sqrt{5}}{2})^{n-2} = (\frac{1+\sqrt{5}}{2})^2$$

$$(\frac{1+\sqrt{5}}{2})^{n-1} + (\frac{1+\sqrt{5}}{2})^{n-2} = (\frac{1+\sqrt{5}}{2})^{n-2} \cdot (\frac{1+\sqrt{5}}{2}+1) \geq (\frac{1+\sqrt{5}}{2})^n$$

$$f_{n+2} = f_{n+1} + f_n \geq (\frac{1+\sqrt{5}}{2})^{n-1} + (\frac{1+\sqrt{5}}{2})^{n-2} \geq (\frac{1+\sqrt{5}}{2})^n$$

$$f_{n+1} = f_n + f_{n-1} \geq (\frac{1+\sqrt{5}}{2})^{n-1}$$

so that, $f_{n+1} = f_n + f_{n-1} \geq (\frac{1+\sqrt{5}}{2})^{n-1}$ is true, Q.E.D.

5.

$$\begin{aligned} lg(n!) &= \Theta(nlg(n)) \\ lg(n!) &= lg(1 \cdot 2 \cdot 3 \cdot \ldots \cdot n) = lg(1) + lg(2) + \ldots + lg(n) \\ nlg(n) &= lg(n^n) = lg(n) + lg(n) + \ldots + lg(n) \end{aligned}$$

$$O(nlg(n)) : lg(1) + lg(2) + \ldots + lg(n) < lg(n^n) = lg(n) + lg(n) + \ldots + lg(n)$$

$$\begin{aligned} & lg(\frac{n}{2}) = lg(\frac{n}{2}) \\ & => lg(\frac{n}{2}+1) > lg(\frac{n}{2}) \\ \Omega(nlg(n)) : & lg(\frac{n}{2}) + lg(\frac{n}{2}+1) + \ldots + lg(n) > lg(\frac{n}{2}) + lg(\frac{n}{2}) + \ldots + lg(\frac{n}{2}) = \frac{n}{2} lg(\frac{n}{2}) \\ & = \frac{n}{2}(lg(n) - lg(2)) \end{aligned}$$

6.

(a)

$$c = 2, 2^{n+1} = 2 \cdot 2^n = O(2^n)$$

(b)

$$\text{c=}2^n \text{ can't be c. } 2^{2n} = 2^n \cdot 2^n \neq O(2^n)$$