LELEC2811 - E2
Data Acquisition

Exercise 1: Quantization and noise

The objectives of this exercise are (i) to derive the theoretical expression for the quantization noise floor, (ii) to discuss the hypotheses underlying this expression and (iii) to understand the impact of quantization on the power spectral density based on a practical study case.

1. From a mathematical point of view, the quantization noise, i.e. the difference between the actual and quantized signals, can be regarded as a random variable. Therefore, we ask you to compute the expected value and variance of the quantization noise. As a reminder,

$$\mathbb{E}(x) = \int_{-\infty}^{+\infty} x\, P(x) dx \quad \text{and}$$
$$\mathbb{V}\text{ar}(x) = \int_{-\infty}^{+\infty} \left[x - \mathbb{E}(x)\right]^2 P(x) dx,$$

   where x denotes a random variable, $P(x)$ its probability density function, $\mathbb{E}(x)$ its expected value and $\mathbb{V}\text{ar}(x)$ its variance.

   Hint: What assumption is done regarding the shape of the probability density function $P(x)$?

   Short answer: $\mathbb{E}(x) = 0$ and $\mathbb{V}\text{ar}(x) = \frac{\Delta^2}{12}$, where Δ stands for the least-significant bit (LSB).

2. Given that the quantization energy is the variance of the quantization noise, computed in the previous point, express the one-sided power spectral density of the quantization noise for a given sampling frequency f_s.

   Short answer: PSD = $\frac{\Delta^2}{6f_s}$.

3. Compute the LSB and quantization noise PSD in dB/Hz for a number of bits N = 2, 4, 8, assuming a full-scale voltage V_{FS} = 1V and a sampling frequency f_s = 100 kHz. A visualization is proposed through the Python code provided on Moodle for this exercise. In that code, the signal to be quantized is a sine wave at 100Hz with a peak-to-peak amplitude of V_{FS}.

4. Discuss the validity of the assumption done on the shape of the probability density function $P(x)$, based on the histogram of the quantization error.

Exercise 2: Oversampling

The objectives of this exercise are (i) to understand the impact of oversampling on the quantization noise PSD, (ii) to study an oversampled data acquisition system and (iii) to discover conventional digital implementations for the anti-aliasing filter.

1. Compute the one-sided PSD of the quantization noise for
   - A data acquisition system using an ADC operating at Nyquist's frequency, i.e. $f_s = f_{s,ny}$.
   - A data acquisition system using an ADC operating above Nyquist's frequency, i.e. $f_s = \text{OSR} \times f_{s,ny}$, where OSR denotes the oversampling factor.

   Both systems use an ideal anti-aliasing filter (AAF) with a cut-off frequency $f_c = \frac{f_{s,ny}}{2}$.

2. Let us now consider the oversampled data acquisition system presented in Fig. 1.
   - Describe the different operations performed by this system.
   - Draw the power spectral density at the output of the ADC and at the output of the sampling system.

3. The low-pass filter is usually implemented as a digital filter, more specifically a windowed-sinc finite-impulse response (FIR) filter.
   - What is the advantage of implementing the low-pass filter (LPF) in the digital domain, using a FIR filter, instead of the analog domain?
   - Two types of windowing are commonly used for the sinc: the Hamming and the Blackman window. Compare them from the point of view of roll-off and stop band attenuation, using the Python code provided on Moodle for this exercise.

     $$\text{Hamming:} \quad h[n] = K \times \frac{\sin\left(2\pi f_c (n - \frac{M}{2})\right)}{n - \frac{M}{2}} \times \left[0.54 - 0.46 \cos\left(\frac{2\pi n}{M}\right)\right]$$

     $$\text{Blackman:} \quad h[n] = K \times \frac{\sin\left(2\pi f_c (n - \frac{M}{2})\right)}{n - \frac{M}{2}} \times \left[0.42 - 0.5 \cos\left(\frac{2\pi n}{M}\right) + 0.08 \cos\left(\frac{4\pi n}{M}\right)\right]$$

     where $M + 1$ denotes the number of points to which the sinc is truncated, f_c the ratio between the filter cut-off frequency and the sampling frequency and K a scaling factor such that all filter coefficients sum to one. A good reference explaining windowed-sinc filter into more details is [1].

4. Discuss qualitatively the results obtained for N = 2, 4, 8 bits, for an oversampling factor of 64, based on the Python code provided on Moodle.
   - Does the theoretical quantization noise floor match the actual one?
   - How do you explain the peaks in the power spectral density of the quantization error?

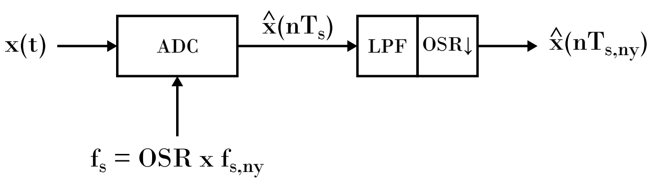

Figure 1: Oversampled data acquisition system.

References

[1] S. W. Smith, "The Scientist and Engineer's Guide to Digital Signal Processing", *California Technical Publishing*, 1998.