# LELEC2811 - E1
# Noise and Calibration

## Exercise 1: Magnetometer noise characterization

The objectives of this exercise are (i) to characterize the rms noise of a sensor in a given configuration and (ii) to represent the corresponding one-sided power spectral density in an appropriate way. This is done for two sensor configurations: low-power and high-performance.

In this exercise, we focus on the LIS2MDL magnetometer sensor of the X_NUCLEO_IKS01A3 sensor board. The two magnetometer sensor configurations to be tested are the following:

1. Low-power mode (`LP` = 1) without integrated low-pass filter (`LPF` = 0).
2. High-performance mode (`LP` = 0) with integrated low-pass filter (`LPF` = 1).

The piece of the main source code (`main.c`) defining the sensor configuration is shown in Fig. 1.

```c
#ifdef LIS2MDL_ENABLE
#define DATA_SIZE           12
#define FLOAT_CONVERSION    0
#define LIS2MDL_ODR         10.0    // Output data rate (10, 20, 50 or 100 Hz)
#define LIS2MDL_LP          1       // Power mode (0 for high-resolution mode, 1 for low-power mode)
#define LIS2MDL_LPF         0       // Bandwidth (0 for ODR/2, 1 for ODR/4)
#endif
```

Figure 1: LIS2MDL magnetometer configuration code.

In both cases, the output data rate of the sensor must be set to 10Hz and the sampling rate, fixed by the timer TIM2, to 10Hz. Those are the default settings of the code. Note that the output data rate is the rate at which data is sampled by the ADC internal to the magnetometer, while the sampling rate is the rate at which data is read from the magnetometer output register by the main function.

1. For the two sensor configurations described above, collect data from the three magnetometer axes.
2. Visualize the noise of one the magnetometer axes by plotting an histogram of the collected data. Then, compute the value of the rms noise and compare it to the specifications given in the sensor datasheet [1]. In which part of the datasheet do you expect this information to be located?
3. Represent the one-sided power spectral density (PSD) as a function of frequency. To do so, you should be able to answer the following questions:
   - What steps must be followed to go from the raw signal to its one-sided PSD?
   - What are the units of the x- and y-axis?
   - What is the most suitable choice of scale for both axes? Linear or logarithmic?

## Exercise 2: Altimeter noise characterization

The objectives of this exercise are (i) to visualize the pressure noise distribution and to understand how it translates into altitude noise, (ii) to evaluate the impact of environmental noise and device-to-device variations and (iii) to assess the altitude uncertainty related to these different sources.

In this exercise, we focus on the LPS22HH pressure sensor of the X_NUCLEO_IKS01A3 sensor board, which you have already used in the installation and hands-on session.

1. Collect pressure and altitude data for the following sensor configuration:
    - Output data rate (`ODR`): 100Hz;
    - Low-noise configuration: Enabled (1);
    - Device bandwidth: `ODR`/20 (3);
    - Sampling rate: 10Hz, fixed by the timer TIM2.

    The output data rate is the rate at which data is sampled by the ADC internal to the pressure sensor, while the sampling rate is the rate at which data is read from the pressure sensor output register by the main function.

```
#ifdef LPS22HH_ENABLE
#define DATA_SIZE            12
#define FLOAT_CONVERSION     1
#define LPS22HH_ODR          100.0 // Output data rate (one-shot, 1, 10, 25, 50, 75, 100, 200 Hz)
#define LPS22HH_LOW_NOISE_EN 1     // Low-noise (0 disabled, 1 enabled)
#define LPS22HH_LPF_CFG      3     // Device bandwidth (0 for ODR/2, 2 for ODR/9, 3 for ODR/20)
#endif
```

Figure 2: LPS22HH altimeter configuration code.

2. To compute the pressure noise, consider two different cases: (i) remove the mean of all collected data and (ii) remove the mean computed by the exponential moving average function (provided to you) using $\beta = 0.95$. In both cases, visualize the pressure noise by plotting an histogram of the collected data and describe its statistical distribution. Then, compute the pressure mean value and rms noise.

3. From a purely mathematical point of view, compute how the pressure rms noise translates into altitude noise, knowing that altitude is computed from pressure using the adiabatic height formula [2]

$$h(p) = 44330.77 \times \left(1 - \left(\frac{p}{p_0}\right)^{0.1902632}\right) \tag{1}$$

    where $p$ is the pressure in hPa, $p_0 = 1013.26$ hPa is the pressure at sea level and $h$ is the altitude in meters. More specifically, we ask you to:

    - Compute the altitude corresponding to $p = \mu_p - \sigma_p$, $p = \mu_p$ and $p = \mu_p + \sigma_p$ where $\mu_p$ and $\sigma_p$ are the pressure mean and rms noise computed previously from the experimental data, by removing the mean computed by the exponential moving average. Does your result match with the experimental altitude rms noise? What is the statistical distribution of the altitude noise?
    - Repeat the same computation by using $\mu_p = 1013.26$ hPa, corresponding to the pressure at sea level, and the same $\sigma_p$ as for the previous question. Is there a difference in altitude noise? If yes, how can you explain this difference?

4. Atmospheric pressure is strongly related to weather, resulting in an environmental noise on the altitude measurement. Pressure data for Louvain-la-Neuve from September 12, 2020 to September 26, 2020 was downloaded from [3] and is provided to you. Based on the min. and max. pressure value, compute the error (variation) in the perceived altitude due to environmental noise.

5. To assess device-to-device variations, data collected by the LELEC2811 groups in September 2019 is provided to you, together with a Python code to analyze it. What is the standard deviation of the pressure and altitude measurements provided by the groups?

6. Finally, assess the uncertainty related to each source (intrinsic noise, environmental noise and device-to-device variations). For the intrinsic noise and the device-to-device variations, you can compute the uncertainty as $\pm 3\sigma_h$, where $\sigma_h$ is the altitude rms noise or standard deviation.

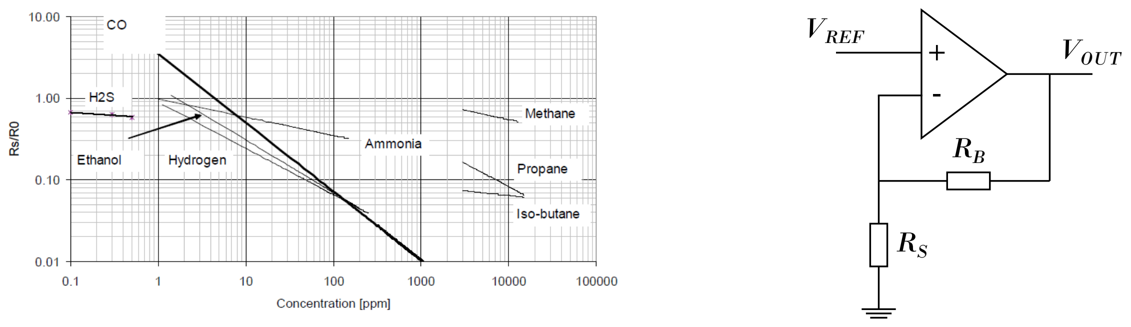
## Exercise 3: Transfer function and computation of stimulus

The objectives of this exercise are (i) to establish a mathematical expression for the transfer function based on the sensor model and the interface circuit equations, (ii) to calibrate the parameters of the sensor model based on measurements and (iii) to compute the stimulus by mathematical inversion of the transfer function.

In this exercise, we consider a CO gas sensing application based on the MICS-5524 metal-oxide (MOX) gas sensor from the company SGX Sensortech [4]. The electrical quantity through which the CO concentration is measured is the sensor resistance, whose value decreases exponentially when the CO concentration increases, as depicted in Fig. 3a. Therefore, the gas sensor resistance is given by

$$\frac{R_s(C)}{R_0} = a \times C^b \Leftrightarrow R_s(C) = \underbrace{R_0 \times a}_{R_0'} \times C^b \tag{2}$$

where $R_s(C)$ denotes the sensing resistance in Ohms, $R_0$ the sensing resistance in air in Ohms and $C$ the CO gas concentration in ppm. Furthermore, the interface circuit shown in Fig. 3b is used to measure the changes in gas concentration, by measuring the output voltage $V_{OUT}$. In this schematic, $R_S$ is the sensing resistor, $R_B = 50$ kΩ is the biasing resistor and $V_{REF} = 50$ mV is a fixed reference voltage.




(a) (b)

Figure 3: CO gas sensing application. (a) MICS-5524 gas sensor resistance with respect to gas concentration [4]. (b) Interface circuit schematic.

1. Determine the mathematical expression of the interface circuit output voltage $V_{OUT}$ as a function of the CO gas concentration $C$ and specify the assumptions you made to obtain this expression.
2. Compute the stimulus by mathematical inversion of the transfer function.

   *Short answer*: $C = \left(\frac{R_B}{R_0'} \frac{V_{REF}}{V_{OUT} - V_{REF}}\right)^{\frac{1}{b}}$.
3. Calibrate the parameters of the gas sensor model based on the two measurements of the interface circuit output voltage found below:

   $$C_1 = 100 \text{ ppm}, \quad V_{OUT1} = 0.787 \text{ V},$$
   $$C_2 = 500 \text{ ppm}, \quad V_{OUT2} = 2.085 \text{ V}.$$

   *Short answer*: $R_0' = 62$ kΩ/ppm$^b$ and $b = -0.631$.
4. Plot the interface circuit output voltage for gas concentrations ranging from 1 to 1000 ppm and discuss the sensitivity and linearity of this transfer function.

## Exercise 4: Input-referred noise

The objectives of this exercise are (i) to discuss the impact of crosstalk noise at the input or output of the circuit and (ii) to compute the input-referred thermal noise in an operational-amplifier-based gain circuit containing resistors.

Let us consider the non-inverting gain circuit depicted in Fig. 3b. The operational amplifier has a gain bandwidth product of 3 MHz and an input-referred voltage noise of 18 nV/$\sqrt{\text{Hz}}$ [5], while the resistors fixing the gain are $R_1$ = 1 M$\Omega$ and $R_2$ = 9 M$\Omega$, respectively.

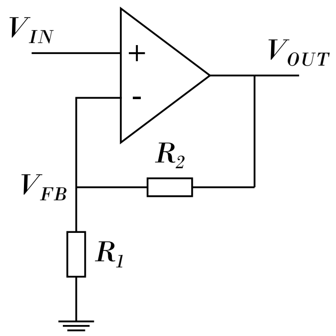



Figure 4: Schematic of the operational-amplifier-based non-inverting gain circuit.

Compute the transfer function $\frac{v_{out}}{v_{in}}(j\omega)$ of the circuit assuming that the operational amplifier is ideal and has a first-order gain response given by

$$A_v(j\omega) = A_{v0}\frac{1}{1+j\frac{\omega}{\omega_0}}$$

where $A_{v0}$ is the open-loop gain in V/V and GBW $= A_{v0}\dfrac{\omega_0}{2\pi}$ is the gain bandwidth product of the operational amplifier in Hz. Then, from this transfer function, identify the DC gain and the bandwidth of the circuit, denoted as $A$ and $f_c$, respectively.

*Short answer*: $A = \left(1+\frac{R_2}{R_1}\right)$ and $f_c = \frac{\text{GBW}}{\left(1+\frac{R_2}{R_1}\right)}$.

For the two situations herebelow, compute the input-referred rms noise. Then, explain how the impact of this noise source can be mitigated.

1. A peak-to-peak 2mV 50Hz crosstalk noise at the circuit output. Is there a difference if the crosstalk noise impacts the circuit input rather than its output?

   *Short answer*: $\overline{v_{in}} = 0.707$ mV (noise at the input) and $\overline{v_{in}} = 70.710\ \mu$V (noise at the output).

2. Thermal noise due to the resistors, when the circuit is operating at ambient temperature, i.e. $T$ = 298.15 K.

   - Compute the equivalent noise bandwidth (ENB) using the following formula

   $$\int_0^{+\infty} \left|\frac{v_{out}}{v_{in}}(f)\right|^2 df = A^2 \times \text{ENB}$$

     where $A$ is the DC gain of the circuit and ENB is the equivalent noise bandwidth, expressed as a multiple of the circuit bandwidth $f_c$.
     *Reminder*:

   $$\int \frac{1}{1+x^2}\,dx = \text{atan}(x) + C.$$

     *Short answer*: ENB $= f_c\frac{\pi}{2}$.

   - Compute the output-referred noise power spectral density (PSD) due to the thermal noise of resistors $R_1$ and $R_2$ and the input-referred voltage noise of the operational amplifier, in the bandwidth of the circuit. To do so, use the superposition principle through the formula

   $$\frac{\overline{v_{out}}^2}{\Delta f} = \sum_k \left|\frac{v_{out}}{v_k}\right|^2 \frac{\overline{v_k}^2}{\Delta f}$$

where $\frac{v_{out}}{v_k}$ denotes the transfer function between the output $v_{out}$ and noise source $v_k$ while all other sources are equal to zero and $\frac{\overline{v_k}^2}{\Delta f}$ denotes the PSD of noise source $v_k$. Which noise source is dominant in the output-referred noise PSD?
*Reminder*: The Boltzmann's constant value is $k$ = 1.381 $\times$ $10^{-23}$ J/K.

*Short answer*: $\frac{\overline{v_{out}}^2}{\Delta f} = 1.515$ pW/Hz.

- Compute the output-referred noise power $\overline{v_{out}}^2$ considering the appropriate bandwidth. Then compute the input-referred noise power $\overline{v_{in}}^2$ and the input-referred rms noise $\overline{v_{in}}$.

*Short answer*: $\overline{v_{out}^2} = 0.714\ \mu$W, $\overline{v_{in}^2} = 7.138$ nW and $\overline{v_{in}} = 84.485\ \mu$V.

## References

[1] STMicroelectronics, "Digital output magnetic sensor: ultra-low-power, high-performance 3-axis magnetometer", LIS2MDL datasheet, May 2017 [Revised Nov. 2018].

[2] G. Gerlich and R. D. Tscheuschner, "On The Barometric Formulas And Their Derivation From Hydrodynamics And Thermodynamics", *arXiv:1003.1508 [physics.ao-ph],* Mar. 2010.

[3] Meteoblue, "Weather in Louvain-la-Neuve". [Online]. Available: https://www.meteoblue.com/en/weather/week/louvain-la-neuve_belgium_2792073. [Accessed Sep. 26, 2020].

[4] SGX Sensortech, "MICS-5524 1084 rev. 8", MICS-5524 datasheet [Revised Jul. 2017].

[5] Texas Instruments, "TL08xx JFET-Input Operational Amplifiers", TL084 datasheet, 1977. [Revised Jan. 2014].