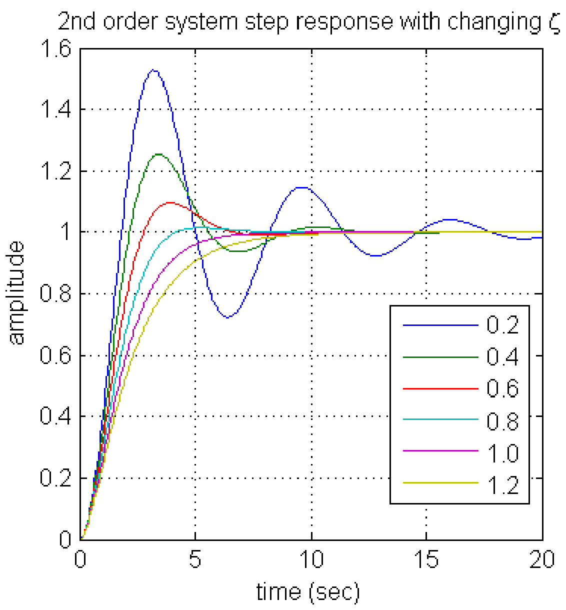
TEL AVIV UNIVERSITY אוניברסיטת תל אביב

Faculty of Engineering

<u>Lab Brief – Position Control</u>



<u>1. Lab Purpose.</u>

- Direct mathematical modeling of a dynamic system.
- Understanding the various components that form a controller.
- Design and simulation of a PD controller according to specification.
- Practical implementation of said controller.

2. System Description.

2.1. Mechanical System.

The module is called Quanser SRV-02, and is comprised of an outer aluminum shell, a DC motor, a gear transmission, an optical encoder, and dedicated connectors by which to read data and supply power to the motor.
Other sensors, such as a tachometer and a potentiometer are also present, but not used in this experiment.
To the output shaft, one can attach a variety of components, in our case, a black metal disc / rod.

2.2. Amplifier.

The Voltpaq-X1 amplifier get an analog signal from the data acquisition board, and supplies the necessary current in order to actuate the motor at that signal (voltage).

2.3. Data Acquisition Board.

The Quanser Q2-USB is connected at one end to the computer, and on the other end has:

- 2 Analog inputs (unused).
- 2 Analog outputs (1 connected to the amplifier).
- 2 Encoder inputs (1 connecter to the SRV-02).
- 8 Digital input/outputs (unused).

3. Theoretical Background.

3.1. Gear Basics.

Gears and transmissions are being used in many applications, which we will not cover here. The important thing there is to know, is that they can convert torque to angular velocity and vice versa, without adding or subtracting energy (aside heat lost to friction). Mechanical power in a rotary system is given by: $P = T \cdot \omega$, so as long as the product remains constant, it is the transmission that enables us to manifest this trade-off.

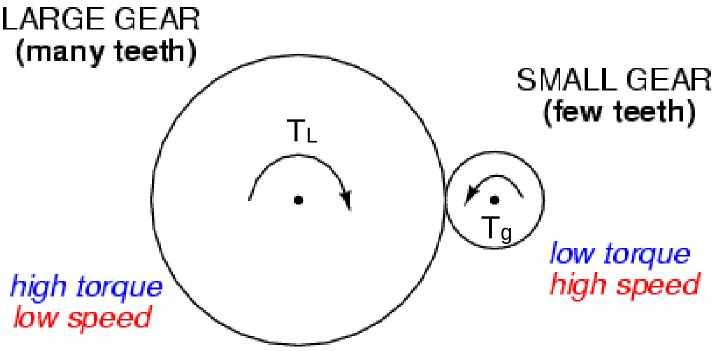


Figure 1: Scheme of a gear transmission

Given a gear ratio of K_g, assuming we enact a torque T_g over the small gear, it will be mirrored upon the large gear as: $T_L = T_g \cdot K_g$. To uphold the law of energy preservation, the angular velocity of the large gear will be $\omega_L = \dfrac{\omega_g}{K_g}$.

3.2. DC motor dynamics.

Figure 2 depicts an electrical scheme of a motor with a constant external magnetic field, generated by a permanent magnet.

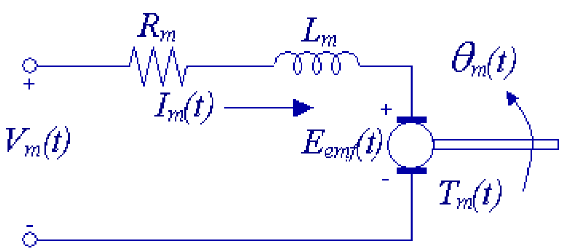


Figure 2: Scheme of the motor electrical parts

For ease of writing, we shall drop the notations off the parameters.
Applying Kirchhoff's law over the scheme will result in:

$$(1)\ V - R \cdot I - L \cdot \frac{dI}{dt} - E = 0$$

In general, mechanical systems react slowly and work in low frequencies (as opposed to electrical systems). Along with the fact that L has a typical magnitude of 10^{-4} [H], we could neglect it from the equation, even though $\dot{I}$ could <u>momentarily</u> reach very high values.
E is the back-emf created in the motor coils from the spinning of the shaft. It depends linearly on the shaft's angular velocity, and therefore we can rewrite equation (1):

$$(2)\quad I = \frac{V - K_e \cdot \dot{\theta}}{R}$$

That would be the complete electrical equation.
Now let's look on the mechanical part of the motor. Applying Newton's laws upon the torques around the shaft, will result in:

$$(3)\quad J_m \cdot \ddot{\theta}_m = T_m - T_g - B_m \dot{\theta}_m$$

While J_m is the shaft moment of inertia, T_m is the torque generated by the motor, T_g is the torque as seen from beyond the transmission, and B is the viscous friction coefficient of the shaft.
Looking at the other side of the transmission (the "big gear" where the load is connected to), the equation is:

$$(4)\quad J_L \cdot \ddot{\theta}_L = T_L - B_L \dot{\theta}_L$$

While J_L is the load moment of inertia, and B is the load viscous friction coefficient.
Converting T_L to T_g, θ_m to θ_L, and substituting equation (3) into equation (4) gives:

$$(5)\quad J_L \cdot \ddot{\theta}_L = K_g \cdot (T_m - J_m \cdot K_g \cdot \eta_g \cdot \ddot{\theta}_L - B_m \cdot K_g \cdot \eta_g \cdot \dot{\theta}_L) - B_L \dot{\theta}_L$$

While η_g, K_g are the gear's efficiency and the gear ratio, respectively.

We will now simplify and define:

$$\left(J_L \cdot \ddot{\theta}_L + J_m \cdot K_g^2 \cdot \eta_g \cdot \ddot{\theta}_L\right) = J_{eq} \cdot \ddot{\theta}_L$$
$$\left(B_L \cdot \dot{\theta}_L + B_m \cdot K_g^2 \cdot \eta_g \cdot \dot{\theta}_L\right) = B_{eq} \cdot \dot{\theta}_L$$

(notice how the inertia and friction are mirrored by a factor of K_g^2 (!), because both the torque has multiplied by K_g and the angular velocity divided by K_g).
and then rewrite:

$$(6)\quad J_{eq} \cdot \ddot{\theta}_L = K_g \cdot \eta_g \cdot T_m - B_{eq}\dot{\theta}_L$$

That would be the complete, post-gear mechanical equation.

3.3. Combining the Electrical and Mechanical Equations.
One connecting equation that is characteristic to the PMDC is the linear dependency between the torque and armature current: $T = I \cdot K_t$ while K_t is the motor torque constant, <u>which is equal to K_e</u> when it comes to a permanent magnetic field.
Using this relation, and remembering to convert the angular speed, we can combine equations (2) and (6) into one:

$$J_{eq} \cdot \ddot{\theta}_L + B_{eq}\dot{\theta}_L = K_g \cdot \eta_g \cdot \left(K_t \cdot \frac{V - K_e K_g \dot{\theta}_L}{R}\right) \Rightarrow collect \Rightarrow$$
$$(7)\quad \frac{RJ_{eq}}{K_t K_g \eta_g}\ddot{\theta}_L + \left(\frac{RB_{eq}}{K_t K_g \eta_g} + K_t K_g\right)\dot{\theta}_L = V$$

Applying the Laplace transform on this equation results in the transfer function:

$$\frac{\theta_{(s)}}{V_{(s)}} = \frac{1}{\left(\frac{RJ_{eq}}{K_t K_g \eta_g}\right)s^2 + \left(\frac{RB_{eq}}{K_t K_g \eta_g} + K_t K_g\right)s}$$

4. Pre-Lab Questions.

a) Derive a numerical presentation for the transfer function depicted above, using the table found in appendix 1.

b) A control loop is presented in the following manner:

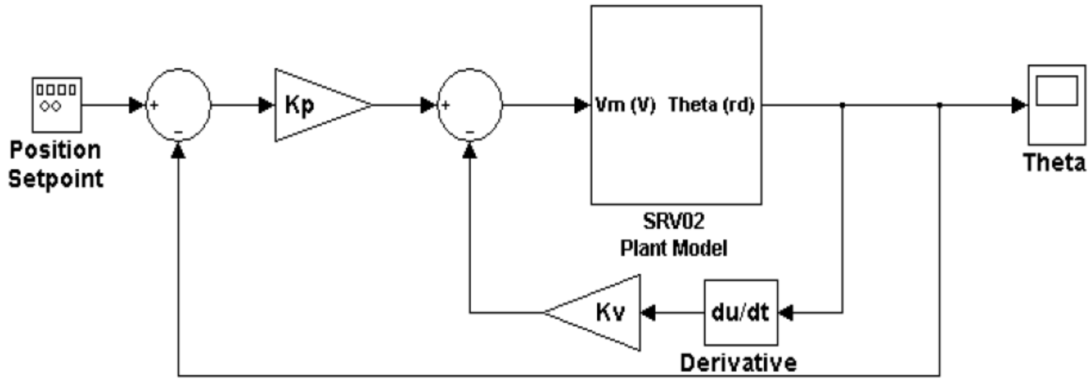


Figure 3: the PD controller Block diagram

Find the transfer function between the reference signal (Position Setpoint) and the output signal, θ_L , depending on the parameters K_p, K_v.

c) The characteristic form for second-order transfer functions is: $\dfrac{\omega_n^{\,2}}{s^2 + 2\cdot\zeta\cdot\omega_n + \omega_n^{\,2}}$.

if we increase/decrease K_p, K_v, how would that affect the parameters ζ, ω_n ?

d) Design a PD controller that would meet the following requirements:

- Overshoot: $OS \le 5\%$.
- Peak time: $t_p \le 0.15$ [sec].

Use the requirements to find a suitable pair of ζ / ω_n and then use them to calculate the appropriate K_p, K_v.
Note - the system must have SOME overshoot, meaning $\zeta < 1$.

e) Discuss about over-dampening, under-dampening, and critical dampening.

5. In-Lab Process.

5.1. Simulation.

First, open Matlab and make a copy of the folder "Position control", then change its name to your names.

Make sure that the system is connected correctly and that the black inertial disc is fastened to the system shaft.

Open the Matlab script: "setup_srv02_Exp02_pos.m", and run the code in order to initialize the control parameters. You may use the command line in order to enter the values found in the pre-lab questions as variables: kp, kv (case sensitive).

Now open the Simulink model: "s_srv02_pos.mdl".

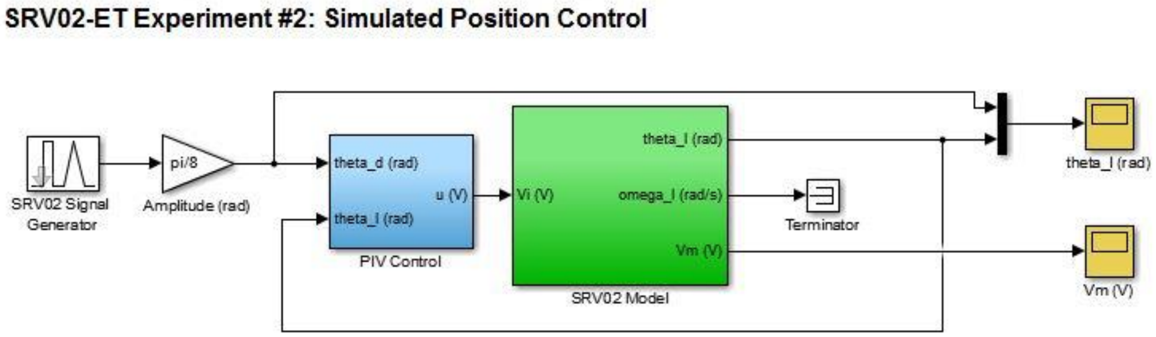


Figure 4: System simulation model

Note that instead of an ideal derivative, we have applied an actual HPF (High-pass filter), which approximates a derivative. This is because one cannot implement an effective derivation since it is an extremely noise-affected process, and, (as it has more zeros than poles) is not Strictly-Proper.

Click Run ("Play" icon) and open the position scope. You can plot the graph by running the script "Plot_position.m". <u>It should automatically plot consecutive graphs over 1 figure and change the curve colors, so there is no need to use hold on/off.</u>

Measure the peak-time and overshoot of the system output, and fill them in the following table:

K_p	K_v	t_p	os	ζ	ω_n

You are to calculate the resulting ζ and ω_n from the output.

* Read the following section (in the next page) all the way through *

While trying to meet the previously stated requirements, the table needs to have at least 10 measurements, in the following manner:

- One measurement with the kp, kv values found in the pre-lab.
- 4 measurements with a constant kp and varying kv.
- 4 measurements with a constant kv and varying kp.
- One measurement with values in which the response satisfies requirements.

During the process, you should save 3 figures:

- One containing all the constant-kp-varying-kv plots, alongside the results from the run using the original pre-lab values.
- One containing all the constant-kv-varying-kp plots, alongside the results from the run using the original pre-lab values.
- One containing the original alongside the corrected response.

Try to get a feeling from the table about the effects of kp and kv on the output.
If you managed to find a response that meets the requirements, move on. If not, keep changing the coefficients until you do.

5.2. Implementing the Controller.
After we managed to find a suitable controller, we may now proceed to testing it in the physical world (and its level of compatibility). In the Matlab command line, insert the coefficients from the previous section which gave the best system response.
Open the Simulink model: "q_srv02_pos.mdl".

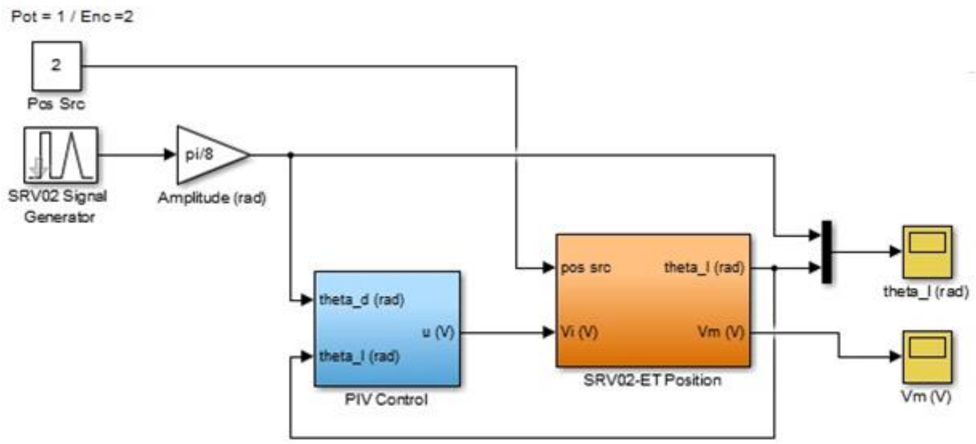
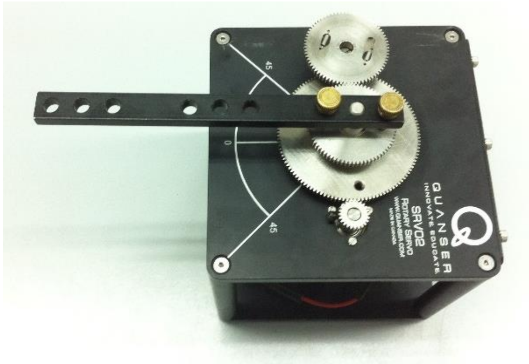


Figure 5: Real experiment Simulink model

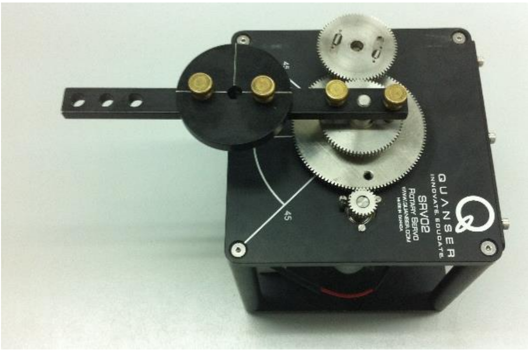
At the top menu, click Quarc → Build, and wait until Matlab finishes to compile.
After it does, click on "Connect to target", and then "Play".
The system will follow a square-wave input signal. Wait for a few cycles and then press stop (every 10 seconds the scope data elapses, try to stop near the end).

Now that you know how the mechanical system works, you will inspect how the system reacts to different types of load. We will test 4 configurations:

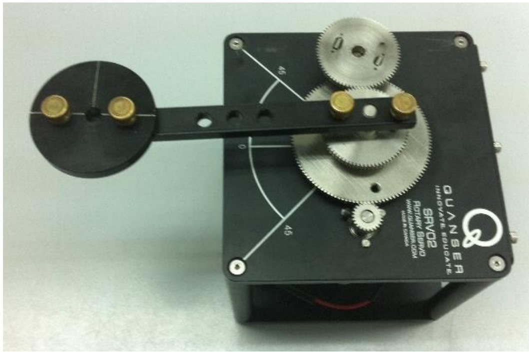
Disc Load



Bar without disc



Bar with a disc at half-distance



Bar with a disc at full-distance

It is important to make sure that the bolts are tightened thoroughly before activating the system!

For each configuration, you need to repeat the following sequence:

- Insert the best kp, kv values from the table.
- Run the system and check its response.
- Save a plot of the raw response.
- Change the values of kp, kv so that they meet the requirements.
- Save a plot of the response with the best values found, drawn on the same figure as the raw response.

Mark important points in every graph taken!
The idea of the process is the heart of classical control theory:

Design → Test → Tune → Test → Tune…

You might be needing to make multiple iterations in order to get it right.
Hint: in the tuning phase, use the table and graphs produced earlier in order to speed up the process instead of just guessing.

6. Post-Lab Questions.

A. Present the table made during section 5.1. Does the outcome match your statements in question (c) from the pre-lab?
B. Did every response have an overshoot? What are the causes for that?
C. Did your original kp, kv values meet the requirements designed by the controller? What might be the causes for that? Present the plots saved in section 5.1.
D. After finding the right values by simulation, did the real system respond in the same way the simulation did (t_p and OS, for disc-load only).
What might be the causes for that?
E. Present the final kp, kv values for each load configuration, as well as the plots from section 5.2.
F. Compare the results of the simulation (with the best values) to the results of the raw real system runs. For which configuration did you get the best match? Which one got the worse match?
G. What is the element that dictates the output difference between the 4 configurations? How does this element affect the system response? Explain.
H. Is it possible to analytically calculate the change in this element (theoretically)? Using which theorem?
I. Looking at the system's response, would you say that it is reasonable to assume we do not need an integral term in this particular controller?

Appendix 1: System Constants.

Symbol	Description	MATLAB Variable	Nominal Value SI Units
V_m	Armature circuit input voltage		
I_m	Armature circuit current		
R_m	Armature resistance	Rm	2.6
L_m	Armature inductance		
E_{emf}	Motor back-emf voltage		
θ_m	Motor shaft position		
ω_m	Motor shaft angular velocity		
θ_l	Load shaft position		
ω_l	Load shaft angular velocity		
T_m	Torque generated by the motor		
T_l	Torque applied at the load		
K_m	Back-emf constant	Km	0.00767
K_t	Motor-torque constant	Kt	0.00767
J_m	Motor moment of inertia	Jmotor	3.87 e-7
J_{eq}	Equivalent moment of inertia at the load	Jeq	2.0 e-3
B_{eq}	Equivalent viscous damping coefficient	Beq	4.0 e-3
K_g	SRV02 system gear ratio (motor->load)	Kg	70 (14x5)
η_g	Gearbox efficiency	Eff_G	0.9
η_m	Motor efficiency	Eff_M	0.69
ω_n	Undamped natural frequency	Wn	
ζ	Damping ratio	zeta	
K_p	Proportional gain	Kp	
K_v	Velocity gain	Kv	
T_p	Time to peak	Tp	